Reliability of Intergreen Interval Based on Combined Dilemma and Option Zones

Easa, Said; Raju, Narayana; Arkatkar, Shriniwas

DOI
10.1061/AJRUA6.0001234

Publication date
2022

Document Version
Final published version

Published in
ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering

Reliability of Intergreen Interval Based on Combined Dilemma and Option Zones

Said M. Easa, M.ASCE[1]; Narayana Raju[2]; and Shriniwas S. Arkatkar[3]

Abstract: Currently, dilemma and option zones' failures are independently used to analyze the intergreen interval at signalized intersections. Therefore, the present research work was initiated to integrate these failures. First, the dilemma and option zones were modeled using the first-order second-moment method. Then, game theory was used to model the association between the dilemma and option failures. The failure probabilities of the dilemma and option zones were evaluated for various traffic conditions using Monte-Carlo simulation considering the Nash equilibrium. Next, the overall system probability was analyzed, based on the combined dilemma and option failures, given different intergreen intervals, speeds, and coefficients of variation. Finally, the study proposed a methodology for identifying the intergreen interval to limit system failure. This would aid practitioners in designing traffic lights at intersections and keeping proper intergreen intervals to limit the dilemma and option failures. **DOI: 10.1061/AJRUA6.0001234.**

Author keywords: Signalized intersections; Intergreen interval; Reliability; Dilemma zone; Option zone; Failure probability; Game theory.

Introduction

Intersections are one of the critical elements in urban transportation networks. The vehicular traffic over intersections must be handled for an efficient transportation network. At the same time, traffic throughput instability can accumulate and lead to network failure. In this direction, numerous studies have been carried out to address the intersections' problems. In this direction, initially, researchers focused on studies related to traffic signals and signal optimization to achieve maximum throughput over the intersections.

Further, due to controlled traffic movement and traffic signals at the intersections, the driver was found to experience a dilemma in the amber time. In this direction, Gazis et al. (1960) initially proposed the dilemma zone concept and explained the case as a zone within which a driver can neither make a safe stop nor clear the intersection before the traffic light turns red. On these lines, at high-speed signalized intersections, studies reported that the issue of the dilemma zone had been well recognized as a significant cause of read-end and right-angle crashes (Bar-Gera et al. 2013; Li 2010; Zhang et al. 2014). Parsonson (1992) uncovered another type of dilemma zone (Type-II dilemma) where vehicles at the onset of amber time can either clear the intersection or safely stop during the intergreen interval. On these lines, researchers focused on understanding the Type-II dilemma failure (Easa 1993; Hurwitz et al. 2012; Papaioannou 2007; Tarko et al. 2006). In the option zone case, if two cars are present and the leader vehicle decides to stop, the follower vehicle perceives enough time to cross the intersection and tends to cross the intersection. This phenomenon would increase the probability of rear-end collisions. Further, studies have shown that drivers in the option zone experience indecisiveness, resulting in rear-end collisions (Köll et al. 2004; Saito et al. 1990).

In connection with the dilemma zone, various studies have been reported, such as driving behavior biases (Lavrenz et al. 2014), road environment (Kim et al. 2016), rear-end collisions (Wu et al. 2013), the effect of countdown timer (Ma et al. 2010), and surrogate safety measures (Machiani and Abbas 2016). Along with that, researchers modeled the dilemma on various approaches, which included using a hazard function (Sharma et al. 2011), vehicle platooning (Gates and Noyce 2010), hidden Markov models (Li et al. 2016), drivers' perception using machine learning (Abbas et al. 2014), and logit Bayesian (Chen et al. 2018). Further, Peng et al. (2021) used the communication of autonomous vehicles to reduce the dilemma for human-driven vehicles at unsignalized intersections using deep reinforcement learning.

Further, it has been identified that as the failure probability of the dilemma zone (Type I) increases, the failure probability of the option zone (Type II) decreases and vice versa. However, most previous studies addressed only the dilemma or option zone. On the other hand, in actual traffic streams, both zones exist for vehicles and depend on the intergreen interval and vehicle speed. If the system is designed to reduce the failure probability of the dilemma zone, the failure probability of the option zone could increase. As a result, the dilemma and option failures still occur in signalized intersections, representing a significant gap in this research area.

This study aimed to address this research gap by simultaneously considering the dilemma and option failures. In line with previous studies (Easa 1993, 1994a, b; Easa et al. 1996), the first-order second-moment method (FOSM) quantified the failure modes. Further, game theory was used to integrate both failure modes. Games were played with the respective probabilities using individual failure probabilities to determine the systems failure probability for various intergreen intervals and approach speeds. Finally, design graphs were established to determine the required intergreen

[1]Professor, Dept. of Civil Engineering, Ryerson Univ., Toronto, ON, Canada M5B 2K3.

[2]Postdoctoral Researcher, Dept. of Transport and Planning, Faculty of Civil Engineering and Geosciences, Delft Univ. of Technology, Stevinweg 1, 2628 CN Delft, Netherlands (corresponding author). ORCID: https://orcid.org/0000-0002-3561-5676. Email: S.S.N.Raju@tudeflt.nl; s.narayanaraju.10@gmail.com

[3]Associate Professor, Dept. of Civil Engineering, Sardar Vallabhbhai National Institute of Technology, Surat, Gujarat 395007, India.

Note. This manuscript was submitted on October 9, 2021; approved on January 28, 2022; published online on March 21, 2022. Discussion period open until August 21, 2022; separate discussions must be submitted for individual papers. This paper is part of the ***ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering***, © ASCE, ISSN 2376-7642.

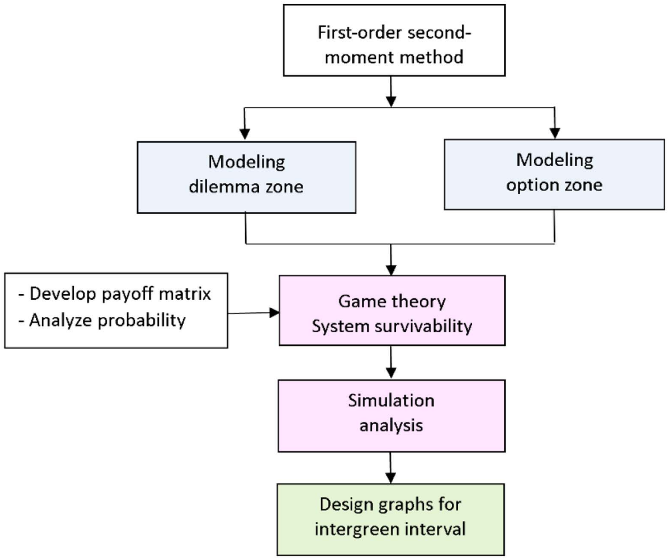

Fig. 1. Research methodology of the study.

interval for signalized intersections, given specified system survivability and traffic characteristics.

In addressing the research gaps from the literature, the present research work was carried in three stages, as shown in Fig. 1. Initially, an arbitrary symmetric intersection was assumed, with all arms having equal width W (m). In the first stage, using the FOSM method, both dilemma and option zones were modeled independently. In the second stage, after identifying the nature of the dilemma and option zones, both failure modes were combined using the game theory logic with the help of the Nash equilibrium. Finally, simulation was performed in the last stage to identify the optimum intergreen interval for the system survivability.

Dilemma and Option Zones

The existence of dilemma and option zones was determined using the relationship between a vehicle's minimum safe stopping distance X_s and maximum intergreen clearance distance X_c. As noted in Fig. 2, if $X_s > X_c$, the dilemma zone exists, and the drivers in this zone could neither stop nor pass. If $X_c > X_s$, the option zone

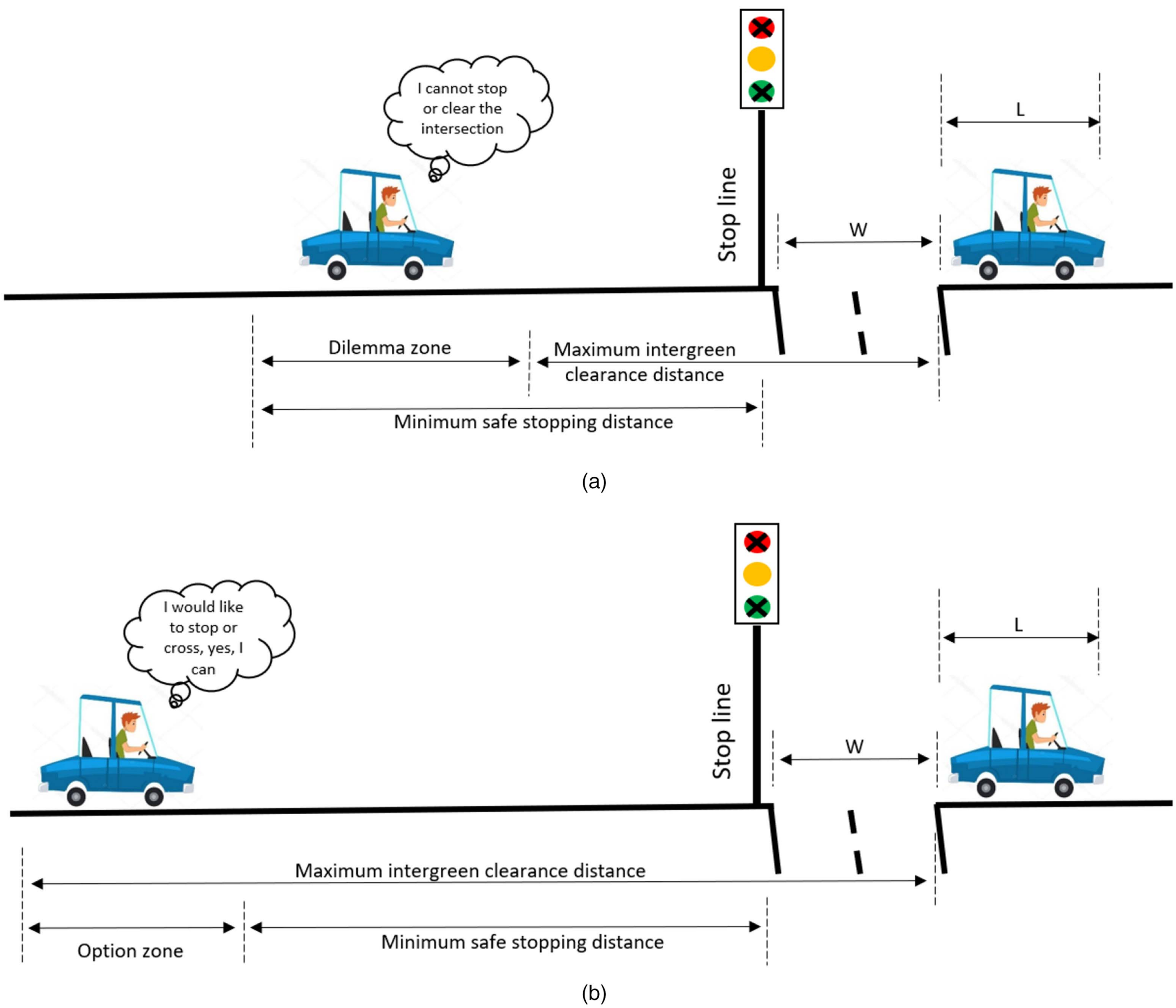

Fig. 2. Definition of dilemma and option zones: (a) dilemma zone; and (b) option zone.

exists, and the drivers in this zone have the choices of passing or stopping. If $X_c = X_s$, both types of zones are eliminated.

A driver approaching a signalized intersection will have to stop or clear the intersection if the green signal becomes amber. The minimum safe stopping distance is given by

$$X_s = v_0\tau + \frac{v_0^2}{2(d+Gg)} \tag{1}$$

where v_0 = approach speed (m/s); τ = driver reaction time to stop (s); d = deceleration rate (m/s^2); G = acceleration due to gravity (m/s^2); and g = approach grade (%). The maximum intergreen clearance distance X_c is the distance within which a vehicle can proceed to clear the intersection before the end of the intergreen interval. Therefore, during the intergreen interval, the vehicle travels a total distance equaling to the maximum intergreen clearance distance, the intersection width, and the length of the vehicle, mathematically given as follows:

$$X_c = Iv_0 - W - L + \frac{1}{2}(a+Gg)(I-\tau_1)^2 \tag{2}$$

where I = intergreen interval (s); W = intersection width (m); L = vehicle length (m); a = acceleration rate (m/s^2); and τ_1 = driver reaction time to clear the intersection (s). The intersection width W was measured along the actual vehicle path from the near-side stop line to the far-side edge of the conflicting traffic lane when there is no pedestrian traffic. When there is significant pedestrian traffic, or pedestrian signals protect the crosswalk, W is measured to the far side of the farthest conflicting pedestrian crosswalk. This research assumed that the driver will clear the intersection without acceleration, which is more conservative. Thus, Eq. (2) is simplified as follows:

$$X_c = Iv_0 - W - L \tag{3}$$

Because both the dilemma and option zones have adverse impacts on intersection safety, traffic authorities have been trying to reduce the size of these two types of zones at intersections. However, if the probability of occurrence of the dilemma zone decreases, the probability of occurrence of the option zone increases and vice versa. Therefore, in this study, the FOSM method and game theory were used to combine the dilemma and option zone failures and to determine the optimal intergreen interval.

Proposed Methodology

First-Order Second-Moment Method

One of the standard reliability methods is FSOM. The method simplifies the implied functional relationship using a truncated Taylor series expansion. The inputs and outputs of the method are expressed as means and variances. FOSM has been widely used because it involves straightforward mathematics (Haukaas and Gardoni 2011). A brief description of the FOSM method is first presented, followed by the proposed reliability analysis of the intergreen interval.

The FOSM analysis approximates the first two moments (mean and variance) of a random variable, a nonlinear function of other random variables. Suppose that Y is a nonlinear function of several random variables, which is

$$Y = f(X_1, X_2, \ldots, X_n) \tag{4}$$

Then, the function Y can be expanded in a Taylor series about the means μ_{X_1} to μ_{X_n} (Berry et al. 2015). Considering the first order (linear) terms, then the expected value of Y, $E(Y)$, is given by

$$E(Y) = f(\mu_{X_1}, \mu_{X_2}, \ldots, \mu_{X_n}) + \sum_{i=1}^{n}(X_i - \mu_{X_i})\left(\frac{\partial f}{\partial X_i}\right) + \varepsilon \tag{5}$$

where the partial derivatives are evaluated at $\mu_{X_1}, \mu_{X_2}, \ldots, \mu_{X_n}$; and ε = higher-order terms. A simple expression of $E(Y)$ was obtained by considering the first term of Eq. (5). That is

$$E(Y) \cong f(\mu_{X_1}, \mu_{X_2}, \ldots, \mu_{X_n}) \tag{6}$$

The variance of Y, var(Y), is given by

$$\mathrm{var}(Y) \cong \sum_{i=1}^{n}\left(\frac{\partial f}{\partial X_i}\right)^2 \sigma_{X_i}^2 + \sum_{i\neq j}^{n}\sum_{i=1}^{n}\left(\frac{\partial f}{\partial X_i}\right)\left(\frac{\partial f}{\partial X_j}\right)\mathrm{cov}(X_i, X_j) \tag{7}$$

where the partial derivatives are evaluated at the mean values; $\sigma_{X_i}^2$ = variance of X_i; and cov(X_i, X_j) = covariance of X_i and X_j, which is given by

$$\mathrm{cov}(X_i, X_j) = \rho_{X_iX_j}\sigma_{X_i}\sigma_{X_j} \tag{8}$$

where $\rho_{X_iX_j}$ = coefficient of correlation between X_i and X_j.

The reliability method is based on Eqs. (6) and (7). A valuable measure of the dispersion of a random variable X_i that will be used later is the coefficient of variation, CV_{X_i}, which is defined

$$CV_{X_i} = \frac{\sigma_{X_i}}{\mu_{X_i}} \tag{9}$$

Eqs. (5)–(9) require no assumptions about the type of the probability distributions of the component random variables. To determine the system failure probability for the intergreen interval, the failure probabilities for the dilemma and option zones are formulated next.

Failure Mode 1: Dilemma Zone

For the dilemma zone, the safety margin is defined as the difference between the clearing distance and the stopping distance

$$Y_1 = X_c - X_s \tag{10}$$

where Y_I = safety margin of the dilemma zone. Eq. (10) is the limit state function. If $Y_1 > 0$, the state is considered as safe; if $Y_1 < 0$, the state is a failure state; if $Y_1 = 0$, the state is limit state. Substituting for X_c and X_s from Eqs. (3) and (1), then

$$Y_1 = Iv_o - W - L - v_o\tau - \frac{v_o^2}{2(d+Gg)} \tag{11}$$

According to Eqs. (6) and (7), the expected value and variance of Y_1, $E(Y_1)$, and var(Y_1), respectively, are given by

$$E(Y_1) = Iu_{v_o} - W - u_L - u_{v_o}u_\tau - \frac{u_{v_o}^2}{2(u_d+Gg)} \tag{12}$$

$$\begin{aligned}\mathrm{var}(Y_1) = {} & \left(I - u_\tau - \frac{u_{v_o}}{(u_d)}\right)^2\sigma_{v_o}^2 + \sigma_L^2 + (v_o)^2\sigma_\tau^2 + \left(\frac{u_{v_o}^2}{2(u_d)^2}\right)^2\sigma_d^2 \\ & + \left(I - u_\tau - \frac{u_{v_o}}{(u_d)}\right)(-v_o)\sigma_{v_o}\sigma_\tau\rho_{v_o,\tau} \\ & + \left(I - u_\tau - \frac{u_{v_o}}{(u_d)}\right)\left(\frac{u_{v_o}^2}{2(u_d)^2}\right)\sigma_{v_o}\sigma_d\rho_{v_o,d}\end{aligned} \tag{13}$$

ASCE-ASME J. Risk Uncertainty Eng. Syst., Part A: Civ. Eng., 2022, 8(2): 04022014

where $\rho_{vo,\tau}$ = correlation coefficient between vehicle speed and reaction times; and $\rho_{vo,d}$ = correlation of coefficient between vehicle speed and deceleration rate. The partial derivatives of Eq. (13) are given by

$$\left(\frac{\partial Y_1}{\partial v_o}\right) = I - u_\tau - \frac{u_{v_o}}{(u_d + Gg)} \tag{14}$$

$$\left(\frac{\partial Y_1}{\partial L}\right) = -1 \tag{15}$$

$$\left(\frac{\partial Y_1}{\partial \tau}\right) = -u_{v_o} \tag{16}$$

$$\left(\frac{\partial Y_1}{\partial d}\right) = \frac{u_{v_o}^2}{2(u_d + Gg)^2} \tag{17}$$

The reliability index is represented by the quotient of the expected value of the safety margin $E[Y_I]$ and its standard deviations. Mathematically

$$\beta_{Y_I} = \frac{[Y_I]}{\sqrt{(\mathrm{var}[Y_I]}} \tag{18}$$

A greater value of β_{Y_I} indicates that the failure probability is small. We assumed that Y_1 follows a normal distribution without loss of generality, as shown in Fig. 3(a). An estimate of the failure probability (dilemma zone), P_{Y_1}, is then given by

$$P_{Y_1} = 1 - \phi(\beta_{Y_1}) \tag{19}$$

where $\phi(\beta_{Y_1})$ = standard normal distribution. Eq. (19) makes no assumptions about the distributions of the component random variables. In many cases, especially when the nonlinear function has several random variables, the distribution of the safety margin tends to be close to normal even though the component random variables are not normal. However, for many skewed distributions, Monte Carlo simulation can achieve more accurate results.

Failure Mode 2: Wide Option Zone

As previously mentioned, the option zone exists if $X_c > X_s$. The width of this zone equals (X_c–X_s). If this width is large enough for two drivers to be present in the zone, this would be unsafe and considered a failure. The maximum safe width of this zone corresponds to a distance where the first driver is at the start of the zone (t near the stop line) and the second driver is at the other end of the zone. This width can be approximately formulated as the distance headway, between two vehicles, D. Thus, the safety margin for the option zone is defined as the difference between the distance headway and the option zone width, Y_2. Mathematically

$$Y_2 = D\text{–}(X_c - X_s) \tag{20}$$

Expressing the distance headway in terms of the time headway h and approach speed v_o, then

$$Y_2 = hv_o - Y_1 \tag{21}$$

The expected value of Y_2, $E(Y_2)$, is given by

$$E(Y_2) = \mu_h u_{v_o} - E(Y_1) \tag{22}$$

The variance of Y_2, $\mathrm{var}(Y_2)$, is given by

$$\begin{aligned}\mathrm{var}(Y_2) = {} & \left(\frac{\partial Y_2}{\partial v_o}\right)^2 \sigma_{v_o}^2 + \left(\frac{\partial Y_2}{\partial L}\right)^2 \sigma_L^2 + \left(\frac{\partial Y_2}{\partial \tau}\right)^2 \sigma_\tau^2 + \left(\frac{\partial Y_2}{\partial d}\right)^2 \sigma_d^2 \\ & + \left(\frac{\partial Y_2}{\partial h}\right)^2 \sigma_h^2 + \left(\frac{\partial Y_2}{\partial v_o}\right)\left(\frac{\partial Y_2}{\partial \tau}\right)\sigma_{v_o}\sigma_\tau\rho_{v_o,\tau} \\ & + \left(\frac{\partial Y_2}{\partial v_o}\right)\left(\frac{\partial Y_2}{\partial d}\right)\sigma_{v_o}\sigma_d\rho_{v_o,d} + \left(\frac{\partial Y_2}{\partial v_o}\right)\left(\frac{\partial Y_2}{\partial h}\right)\sigma_{v_o}\sigma_h\rho_{v_o,h}\end{aligned} \tag{23}$$

where the partial derivatives concerning L, τ, and d equal the right sides of Eqs. (15)–(17), respectively, multiplied by −1. The partial derivatives for v_o and h are given by

$$\left(\frac{\partial Y_2}{\partial v_o}\right) = u_h - \left(\frac{\partial Y_1}{\partial v_o}\right) \tag{24}$$

$$\left(\frac{\partial Y_2}{\partial h}\right) = u_{v_o} \tag{25}$$

Like Failure mode 1, the reliability index and failure probability for Failure mode 2 (option zone), β_{Y2}, and P_{Y_2}, respectively, are given by

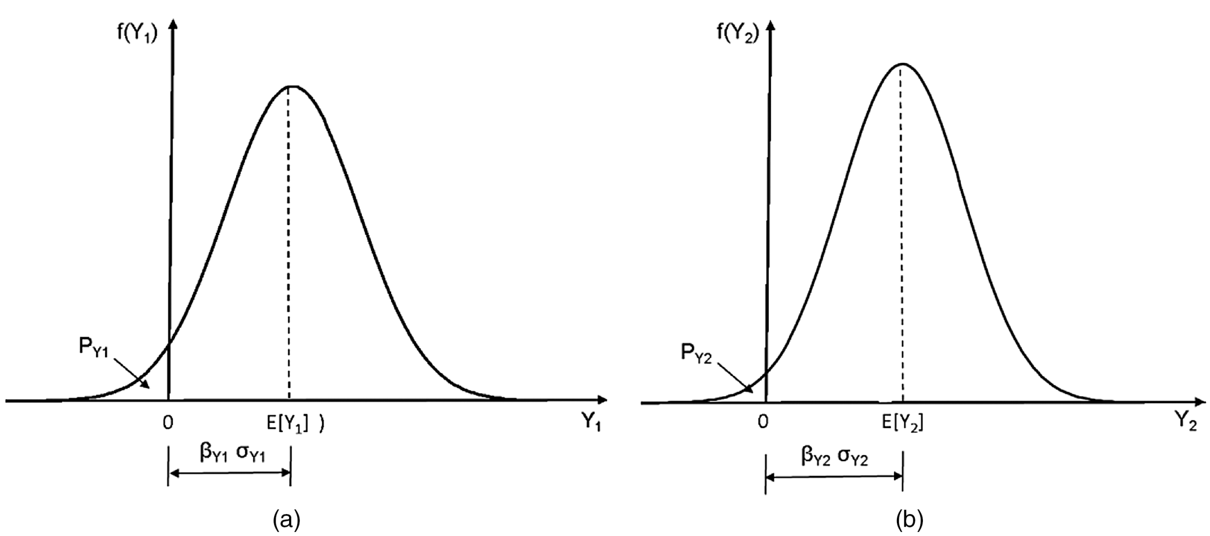

Fig. 3. Probability distribution of the safety margin for individual failure modes: (a) dilemma zone; and (b) option zone.

ASCE-ASME J. Risk Uncertainty Eng. Syst., Part A: Civ. Eng., 2022, 8(2): 04022014

$$\beta_{Y_2} = \frac{E(Y_2)}{\sqrt{(\mathrm{var}(Y_2)}} \tag{26}$$

$$P_{Y_2} = 1 - \phi(\beta_{Y_2}) \tag{27}$$

where Y_2 follows a normal distribution, as shown in Fig. 3(b).

Variation of Individual Failure Modes

The variations of the dilemma and option zones for different combinations of intergreen interval and mean approach speed are illustrated in Fig. 4. Using Monte Carlo simulation, the figure shows the distances X_C and X_S of Eqs. (1) and (3) and the safety margin Y_1 (dilemma) and Y_2 (option) of Eqs. (11) and (21). Negative values of Y_1 indicate a dilemma zone failure, whereas negative values of Y_2 indicate an option zone failure. The data used for simulation were $u_{vo} = 48$ km/h (30 mi/h), $u_t = 1.5$ s, $u_a = 3.1$ m/s^2 (10 ft/s^2), $u_L = 3.66$ m (12 ft), $h = 2$ s, and $W = 15.2$ m (50 ft). The coefficient of variation of u_{vo}, u_t, u_L, and u_h was 10%, and that of u_a was 15%.

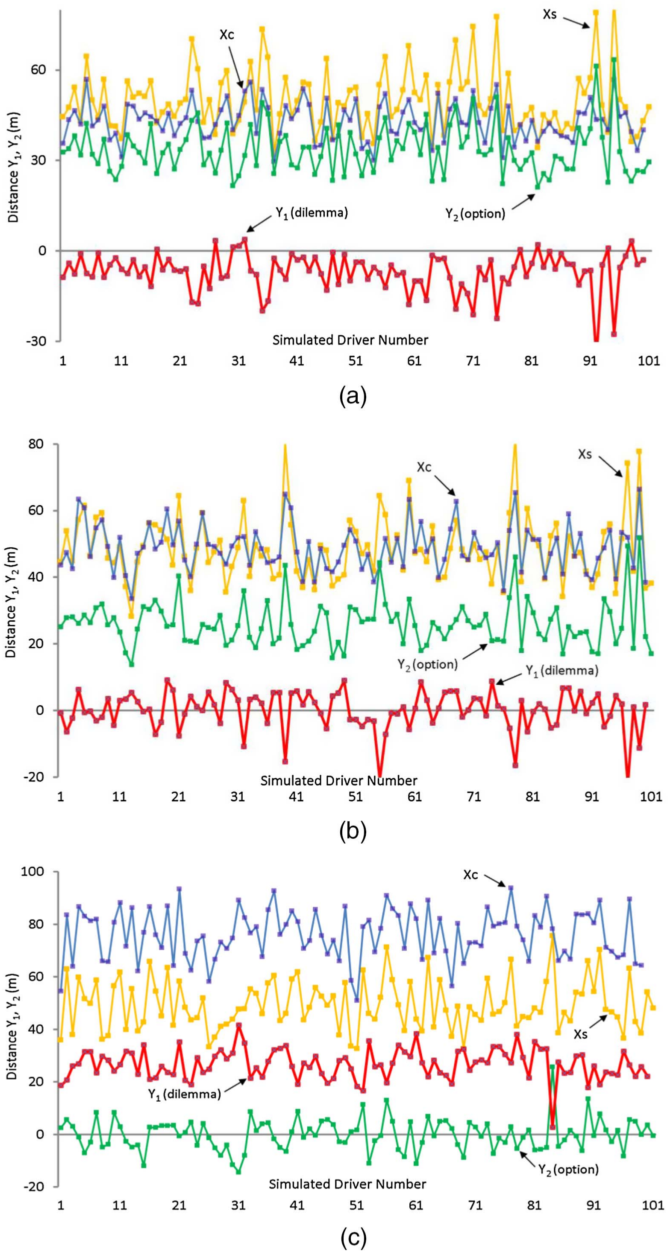

Fig. 4. Variation of the dilemma and option zones for different values of I ($u_{vo} = 48$ km/h): (a) $I = 4.5$ s; (b) $I = 5$ s; and (c) $I = 7$ s.

The results of Fig. 4(a) correspond to $I = 4.5$ s. As noted, Y_2 was entirely positive, indicating no option zone failure. In contrast, except for a few drivers, Y_1 was entirely negative, indicating that the option zone mostly exists. In Fig. 4(b), the intergreen interval was increased to 5 s. The increase in the intergreen interval has caused fewer drivers to experience the dilemma zone, and the option zone started to have smaller positive values. In Fig. 4(c), the intergreen interval was increased further to 8 s. As noted, the dilemma zone was eliminated entirely (Y_1 is positive), whereas the option zone became problematic because Y_2 was mostly negative (i.e., the distance headways are mostly less than the option zone width). It is evident from Fig. 4 that as the intergreen interval increased, the option zone failure increased, whereas that of the dilemma zone decreased.

The variation of the failure probability of the dilemma and option zones is conceptually shown in Fig. 5. For small I, $X_S > X_C$ and P_{Y1} (dilemma zone) would be high and P_{Y2} (option zone) would be low. As I increases, P_{Y2} increases and P_{Y2} decreases. The system failure probability, which is a function of P_{Y2} and P_{Y2}, would have a minimum point that corresponds to the optimal intergreen interval.

System Survivability: Game Theory

Furthermore, assessing the system survivability given the dilemma and option failures forms a complex problem by their nature. As previously shown, the dilemma and option failure probabilities vary in opposite directions. At the same time, both affect the system's functionality. Therefore, game theory was proposed to understand the trade-off between the failure modes and assess system survivability.

In general, a game theory is an architectural framework to assess the solutions based on the competing players (Breen 2017). One player's payoff depends on the other player's strategy, which is one of the fundamental notions in game theory. Even in the present case, the chance of the dilemma or option failure mode is conditional on the probability of the other failure mode. Further, in game theory, the players' strategies are assembled in the form of a payoff matrix (Matsumoto and Szidarovszky 2015). Additionally, the

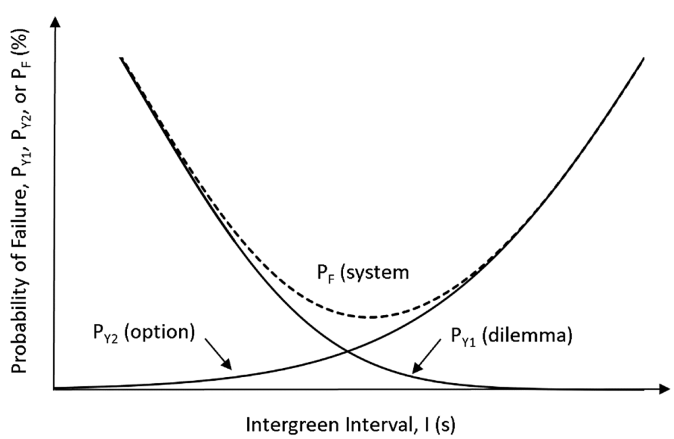

Fig. 5. Variation of failure probabilities of the dilemma zone, option zone, and system with I.

players can be divided into two forms as row and column players. In general, the row player applies the strategies over the rows of the payoff matrix. Similarly, the column player plays the game over the columns of the payoff matrix.

On these lines, initially the payoff matrix was developed. The system will be any of the three modes in the present system, such as dilemma failure, option failure, and no failure. In combination with the three modes, system failure was determined. For example, if the dilemma failure mode exists, there would not be an option failure or no failure mode. Similarly, there would not be a dilemma failure or no failure mode with the option failure mode. With the three modes, three strategies are defined as follows:

1. Reason 1: Dilemma (yes), Option (no), No failure (no). Result: Dilemma wins over the other two modes.
2. Reason 2: Option (yes), Dilemma (no), No failure (no). Result: Option wins over the other two modes.
3. Reason 3: No failure (yes):
   - No dilemma mode: there is a high chance that option failure can prevail, or
   - No option mode: there is a high chance that the dilemma failure can prevail.

To simplify, all three reasons are illustrated in Table 1. In line with the game theory, all reasons can be visualized through row operations. The dilemma win is represented by 1, the option win is represented by −1, and 0 represents no interaction.

Based on Table 1, the Payoff matrix A is given by

$$A = \begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{1} \\ \mathbf{-1} & \mathbf{0} & \mathbf{-1} \\ \mathbf{-1} & \mathbf{1} & \mathbf{0} \end{bmatrix}$$

where the matrix element A_{ij} = utility of the mode controlling row i when they play the ith row (dilemma) and the column player (option) plays the jth column. On these lines, the generally expected payoffs for the players (dilemma and option) are given by

$$E\,(\text{dilemma}) = P_{Y1}[P_{Y2}A_{11} + (1 - P_{Y2})A_{13}] + (1 - P_{Y1})[P_{Y2}A_{31} + (1 - P_{Y2})A_{33}] \quad (28)$$

$$E\,(\text{option}) = P_{Y2}[P_{Y1}A_{11} + (1 - P_{Y1})A_{13}] + (1 - P_{Y2})[P_{Y1}A_{31} + (1 - P_{Y1})A_{33}] \quad (29)$$

For example, scenario $A_{13} = 1$ indicates that the dilemma has occurred in the system, and there is no chance for option failure, resulting in dilemma victory. Further, a mathematical approach for the presented case can be represented in vector form with the number of strategies as its size. In the scenario $A_{13} = 1$, for the dilemma (row player), its strategy is (1, 0, 0) and for the option (column player), its strategy is (0, 0, 1). Given the probabilities of the dilemma and option zones, the strategies are (P_{Y1}, 0, $1 - P_{Y1}$) and (0, P_{Y2}, $1–P_{Y2}$).

The system failure depends on how both the dilemma and option players respond to each other. Based on this, the Nash equilibrium between the failure modes was analyzed in the present case. From a game theory point of view, if the dilemma (Player 1) and the option (Player 2) failures are in the Nash equilibrium, then Player 1 decides considering Player 2 unchanged decision. Similarly, Player 2 decides considering Player 1 decision. This process is repeated until the players' decisions remain unchanged. Further, this can be extremely helpful in analyzing system failure probability, given the dilemma and option failures.

Table 1. Payoff matrix of the dilemma and option failures

Condition	Dilemma	Option	No failure
Dilemma	0	1	1
Option	−1	0	−1
No failure	−1	1	0

Typically, the Nash equilibrium is considered as a profile of strategies from each player. At equilibrium conditions, all players do not have any incentive to change their strategies. In the present case, let $(P_{Y1})_1, (P_{Y1})_2, \ldots, (P_{Y1})_n$ be the strategies for a dilemma zone player, where $(P_{Y1})_i \in P_{Y1}$ (P_{Y1} is the player's strategy set). Also, let $(P_{Y2})_1, (P_{Y2})_2, \ldots, (P_{Y2})_n$ be the strategies for an option zone player, where $(P_{Y2,})_i \in P_{Y2,}$ ($P_{Y2,}$ is the player's strategy set). Then, the Nash equilibrium is given by

$$\max_{(P_{Y1,})_i \in P_{Y1}, (P_{Y2,})_i \in P_{Y2}} U(P_{Y1,} P_{Y2}) \quad (30)$$

where U($P_{Y1,}P_{Y2}$) = combined utility of P_{Y1} and P_{Y2}.

Simulation Analysis

Monte Carlo simulation was performed for different conditions to understand system survivability, given the intergreen intervals over the speeds of the approaching vehicles. In the present case, the intergreen interval was varied from 3 to 10 s, with an interval of 0.25 s. The speed was varied in the range of 16 to 128 km/h (10 to 80 mi/h) with an interval of 16 km/h (10 mi/h). The probabilities P_{Y1} and P_{Y2} were computed individually for four different coefficients of variations (COVs). Finally, the entire game theory was conceptualized in the programming language Python to compute the system probability. Using the probabilities P_{Y1} and P_{Y2}, with the help of the Nash game, the system failure probability (P_F) was computed as shown in Fig. 6.

Thus, the probability of system survivability (P_S) is given by

$$P_S = 1 - P_F \quad (31)$$

Further, P_S for various scenarios is presented in Fig. 7. The analysis shows that the system survivability has been varied over speed and intergreen interval. As noted, low speeds, say 16 km/h (10 mi/h), require more time for the vehicles to cross, resulting in

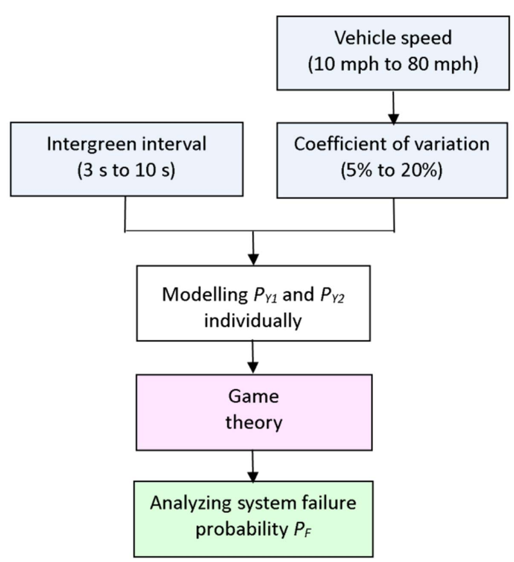

Fig. 6. Simulation process for analyzing system failure probability.

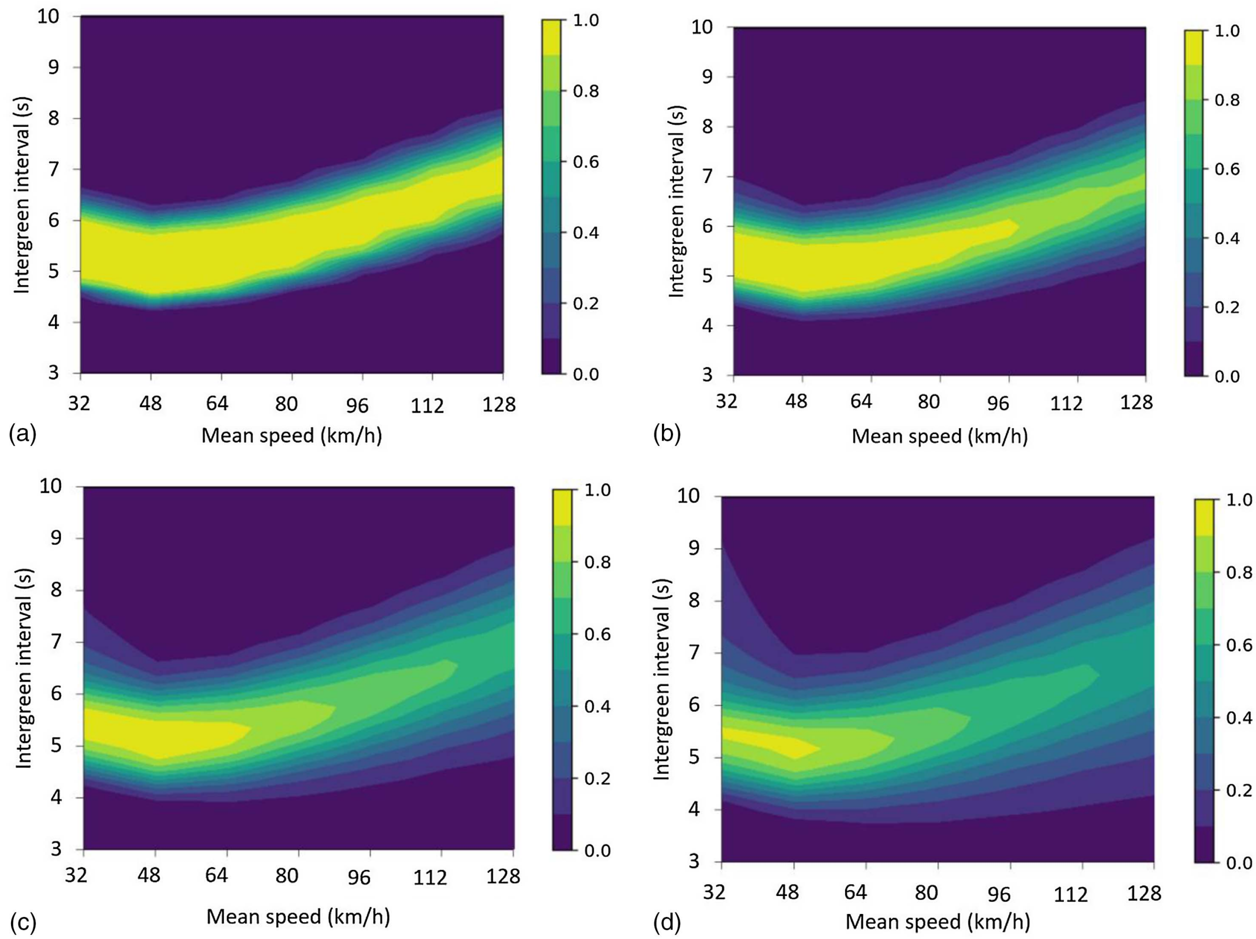

Fig. 7. Probability of system survivability for different conditions: (a) COV = 5%; (b) COV = 10%; (c) COV = 15%; and (d) COV = 20%. Legend on the right side of the plots shows the system survivability scale.

option failure at fewer intergreen intervals. For the present case, around 6.5 to 8.5 s, the intergreen interval was required for system survival. On the other hand, from speeds of 32 km/h (20 mi/h) and greater, the vehicles can overcome the dilemma and option failures, for intergreen intervals between 5 and 6 s. Further, with an increase in speeds, the survival intergreen interval increased.

Similarly, with an increase in the coefficient of variation, the probability of system survival has been diminished. For example, at COV = 5%, the system will survive at even 128 km/h (80 mi/h), whereas with an increase in the coefficient of variation, the probability is reduced. Given this at the same 128 km/h (80 mi/h), the system hardly survived at a higher coefficient of variation. This clearly explains that by a decrease in the discipline among the vehicles, system failure has increased.

Design Graphs

The proposed method can be used to develop design tools for determining the intergreen interval, given a speed's mean and coefficient of variation and the desired failure probability. An example of a design graph for COV = 10% is shown in Fig. 8. As noted, the intergreen interval varied with the change in the speed and the system survival probability. For example, for a mean approach speed of 48 km/h (30 mi/h), if the intergreen interval is 8 s, the probability is zero, indicating that a complete system failure will occur. For the same scenario, if the intergreen interval is 5.5 s, the probability of system survival equals, indicating that the system will function without failure. Consider another scenario with an intergreen interval of 5 s for a mean approach speed of 80 km/h (50 mi/h) (Fig. 8). The corresponding $P_S = 0.8$ ($P_F = 0.2$). By reducing the mean speed to 72 km/h (45 mi/h), P_S will increase to 0.95 ($P_F = 0.05$). Thus, the necessary traffic regulatory measures can be taken to reduce the system failure probability.

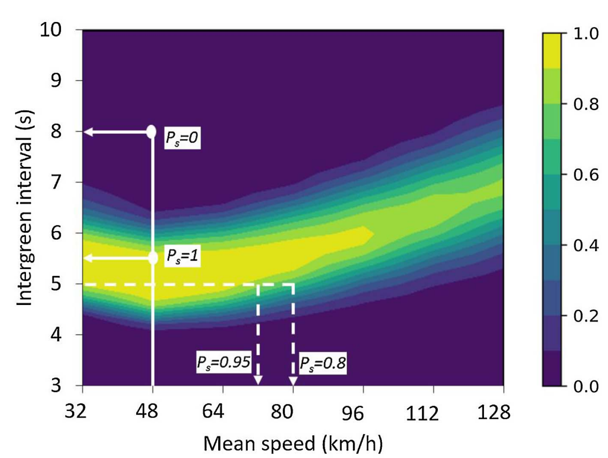

Fig. 8. Mapping the intergreen intervals for system survivability (COV = 10%).

Traditionally, practitioners and enforcement officers evaluate the intergreen intervals, primarily considering the dilemma failure and in some instants the option failure. However, in this study, both dilemma and option failures are combined. This proposed method would help practitioners identify suitable intergreen intervals for signalized intersections based on prevailing conditions.

Concluding Remarks

Transport agencies have been concerned about the crash risk caused by the dilemma and option zones at intersection approaches. However, previous studies have considered only the individual effects of these zones. This paper has presented a method that combines the safety effects of the dilemma and option zones to determine the optimal intergreen interval. Based on this study the following comments are offered:

- This research has proposed a FOSM reliability method integrated with game theory to determine the optimal intergreen interval that minimizes the system failure probability associated with combined dilemma and option zones. The results showed that when vehicles approach the intersections at higher speeds, the dilemma zone will dominate the failure probability. However, the option zone will dominate the failure probability for high-volume intersections with low limits.
- A game theory between the dilemma and option failure modes was conceptualized using the Nash equilibrium. The failure modes were combined to evaluate the probability of system survivability, which was computed for various intergreen intervals, average speeds, and coefficients of variation. Design graphs were developed to help practitioners determine the optimal intergreen interval based on intersection characteristics.
- For signalized intersections with fixed timings, the developed method can be used to determine the optimal intergreen interval that eliminates or minimizes the system failure probability. The method could also be used in a dynamic manner where the intergreen interval of each cycle is varied within specified limits to minimize system failure probability. This would require collecting real-time data related to the intersection approach speeds for each cycle to determine the optimal intergreen interval, which is then fed to the signal controller. Such data may be collected through an intelligent highway infrastructure. For human-driven vehicles (HDV) mixed with connected and autonomous vehicles (CAV), the arrival times and speeds of the HDV vehicles could be estimated using CAV's trajectories.

Data Availability Statement

Some or all data, models, or codes that support the findings of this study are available from the corresponding author upon reasonable request:

- Generated data from simulation runs; and
- Python codes.

Acknowledgments

The authors are grateful to two anonymous reviewers for their thorough and most helpful comments. This research is financially supported by the Natural Sciences and Engineering Research Council (NSERC) of Canada. In addition, the lead author is grateful for valuable discussions with Dr. Xiaobo Qu and Dr. Tarek Sayed.

References

Abbas, M., S. G. Machiani, P. M. Garvey, A. Farkas, and R. Lord-Attivor. 2014. "Modelling the dynamics of driver's dilemma zone perception using machine learning methods for safer intersection control." *J. Transp. Eng.* 142 (11): 1–10. https://doi.org/10.1061/(ASCE)TE.1943-5436.0000885.

Bar-Gera, H., T. Oron-Gilad, and O. Musicant. 2013. "In-vehicle stopping decision advisory system for drivers approaching a traffic signal." *Transp. Res. Rec.* 2365 (1): 22–30. https://doi.org/10.3141/2365-04.

Berry, D. W., A. M. Childs, R. Cleve, R. Kothari, and R. D. Somma. 2015. "Simulating Hamiltonian dynamics with a truncated Taylor series." *Phys. Rev. Lett.* 114 (9): 090502. https://doi.org/10.1103/PhysRevLett.114.090502.

Breen, R. 2017. "Game theory." In *The Oxford handbook of analytical sociology.* Oxford, UK: Oxford University Press.

Chen, C., Y. Chen, J. Ma, G. Zhang, and C. M. Walton. 2018. "Driver behavior formulation in intersection dilemma zones with phone use distraction via a logit-Bayesian network hybrid approach." *J. Intell. Transp. Syst.* 22 (4): 311–324. https://doi.org/10.1080/15472450.2017.1350921.

Easa, S. M. 1993. "Reliability-based design of intergreen interval at traffic signals." *J. Transp. Eng.* 119 (2): 255–271. https://doi.org/10.1061/(ASCE)0733-947X(1993)119:2(255).

Easa, S. M. 1994a. "Reliability analysis of open drainage channels under multiple failure modes." *J. Irrig. Drain. Eng.* 120 (6): 1007–1024. https://doi.org/10.1061/(ASCE)0733-9437(1994)120:6(1007).

Easa, S. M. 1994b. "Reliability-based design of sight distance at railroad grade crossings." *Transp. Res. Part A Policy Pract.* 28 (1): 1–15. https://doi.org/10.1016/0965-8564(94)90038-8.

Easa, S. M., A. Shalaby, and A. A. E. Halim. 1996. "Reliability-based model for predicting pavement thermal cracking." *J. Transp. Eng.* 122 (5): 374–380. https://doi.org/10.1061/(ASCE)0733-947X(1996)122:5(374).

Gates, T. J., and D. A. Noyce. 2010. "Dilemma zone driver behavior as a function of vehicle type, time of day, and platooning." *Transp. Res. Rec.* 2149 (1): 84–93. https://doi.org/10.3141/2149-10.

Gazis, D., R. Herman, and A. Maradudin. 1960. "The problem of the amber signal light in traffic flow." *Oper. Res.* 8 (1): 112–132. https://doi.org/10.1287/opre.8.1.112.

Haukaas, T., and P. Gardoni. 2011. "Model uncertainty in finite-element analysis: Bayesian finite elements." *J. Eng. Mech.* 137 (8): 519–526. https://doi.org/10.1061/(ASCE)EM.1943-7889.0000253.

Hurwitz, D. S., H. Wang, M. A. Knodler, D. Ni, and D. Moore. 2012. "Fuzzy sets to describe driver behavior in the dilemma zone of high-speed signalized intersections." *Transp. Res. Part F Traffic Psychol. Behav.* 15 (2): 132–143. https://doi.org/10.1016/j.trf.2011.11.003.

Kim, S., Y. J. Son, Y. C. Chiu, M. A. B. Jeffers, and C. D. Yang. 2016. "Impact of road environment on drivers' behaviors in dilemma zone: Application of agent-based simulation." *Accid. Anal. Prev.* 96 (Nov): 329–340. https://doi.org/10.1016/j.aap.2015.08.019.

Köll, H., M. Bader, and K. W. Axhausen. 2004. "Driver behaviour during flashing green before amber: A comparative study." *Accid. Anal. Prev.* 36 (2): 273–280. https://doi.org/10.1016/S0001-4575(03)00005-8.

Lavrenz, S. M., V. Dimitra Pyrialakou, and K. Gkritza. 2014. "Modeling driver behavior in dilemma zones: A discrete/continuous formulation with selectivity bias corrections." *Anal. Methods Accid. Res.* 3–4 (Oct): 44–55. https://doi.org/10.1016/j.amar.2014.10.002.

Li, J., Q. He, H. Zhou, Y. Guan, and W. Dai. 2016. "Modeling driver behavior near intersections in hidden Markov model." *Int. J. Environ. Res. Public Health* 13 (12): 1265. https://doi.org/10.3390/ijerph13121265.

Li, Z. 2010. "Highway work zone safety audits for safety improvements." *Eng. Constr. Archit. Manage.* https://doi.org/10.1108/09699981011074592.

Ma, W., Y. Liu, and X. Yang. 2010. "Investigating the impacts of green signal countdown devices: Empirical approach and case study in China." *J. Transp. Eng.* 136 (11): 1049–1055. https://doi.org/10.1061/(ASCE)TE.1943-5436.0000181.

Machiani, S. G., and M. Abbas. 2016. "Safety surrogate histograms (SSH): A novel real-time safety assessment of dilemma zone related conflicts at signalized intersections." *Accid. Anal. Prev.* 96 (Nov): 361–370. https://doi.org/10.1016/j.aap.2015.04.024.

Matsumoto, A., and F. Szidarovszky. 2015. *Game theory and its applications*. Tokyo: Springer.

Papaioannou, P. 2007. "Driver behaviour, dilemma zone and safety effects at urban signalised intersections in Greece." *Accid. Anal. Prev.* 39 (1): 147–158. https://doi.org/10.1016/j.aap.2006.06.014.

Parsonson, P. S. 1992. "Signal timing improvement practices." In *NCHRP synthesis of highway practice*. Washington, DC: USDOT Federal Highway Administration.

Peng, B., M. F. Keskin, B. Kulcsár, and H. Wymeersch. 2021. "Connected autonomous vehicles for improving mixed traffic efficiency in unsignalized intersections with deep reinforcement learning." *Comm. Transp. Res.* 1 (Dec): 100017. https://doi.org/10.1016/j.commtr.2021.100017.

Saito, T., N. Ooyama, and K. Sigeta. 1990. "Dilemma and option zones, the problem and countermeasures: Characteristics of zones, and a new strategy of signal control for minimizing zones." In *Proc., IEEE Conf. Publication*. New York: IEEE.

Sharma, A., D. Bullock, and S. Peeta. 2011. "Estimating dilemma zone hazard function at high speed isolated intersection." *Transp. Res. Part C Emerging Technol.* 19 (3): 400–412. https://doi.org/10.1016/j.trc.2010.05.002.

Tarko, A., W. Li, and L. Laracuente. 2006. "Probabilistic approach to controlling dilemma occurrence at signalized intersections." *Transp. Res. Rec.* 1973 (1): 55–63. https://doi.org/10.1177/0361198106197300107.

Wu, M., W. Ma, and L. Li. 2013. "Characterize dilemma zone and minimize its effect at coordinated signalized intersections." *Procedia—Soc. Behav. Sci.* 96 (Nov): 766–778. https://doi.org/10.1016/j.sbspro.2013.08.087.

Zhang, Y., C. Fu, and L. Hu. 2014. "Yellow light dilemma zone researches: A review." *J. Traffic Transp. Eng.* 1 (5): 338–352. https://doi.org/10.1016/S2095-7564(15)30280-4.

ASCE-ASME J. Risk Uncertainty Eng. Syst., Part A: Civ. Eng., 2022, 8(2): 04022014