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Improved Direction Finding Accuracy for A Limited Number of Antenna Elements with Harmonic Characteristic Analysis

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Abstract — **A direction-finding approach for arrays with a limited number of antenna elements has been investigated. A method based on the harmonic analysis of the received signal has been proposed to solve it. The angle estimation accuracy has been improved by angle searching and peak detection. The proposed method is theoretically described and numerical simulations are provided to verify its effectiveness. Compared with classical direction-finding methods with limited antenna elements, significant improvements have been demonstrated.**

Keywords — **Harmonic Characteristic analysis, limited antenna elements, direction on arrival (DOA), array signal processing.**

I. INTRODUCTION

Most radar-based direction-finding algorithms use an array with a physical or virtual aperture of several elements and implement either phase-comparison or amplitude-comparison of the received signals to determine the direction of arrival information. In both approaches, the direction of arrival (DoA) information is derived primarily from the characteristics of the received signals at such antennas/array [1]. These algorithms include digital beamforming (DBF) [2], Minimum Variance Distortionless Response (MVDR) [3], subspace-based methods, such as multiple signal classification (MUSIC) [4], [5], and estimation of signal parameters via rational invariance techniques (ESPRIT) [6], among others.

These algorithms have degraded performances with fewer antenna elements, because the angle resolution decreases significantly. However, in some applications, the number of antenna elements has to be limited because of power consumption and size constraints, resulting in a trade-off between elements used for azimuth rather than elevation estimation. Such scenarios may include radar on unmanned aerial vehicles (UAVs), or uniform linear arrays (ULA) in automotive radars with few elements for elevation estimation. To the best of our knowledge, designing an effective direction finding algorithm for these applications is still an outstanding challenge, without an established solution in the literature.

Potential solutions could be adding more virtual array elements using the platform motion information, or developing a novel direction-finding framework to extract more information from the available limited elements. Using the motion information to form a synthetic virtual aperture on automotive MIMO radar for higher angular resolution is an established approach [7], [8], [9]. However, they require accurate radar position information, enough movement in the antenna directions, and will be generally valid only for side-looking radar. These are not always realizable in the erratic dynamic scenarios of radars on UAVs. Furthermore, even in case of automotive radar, the vehicle's dynamic will not allow enough movement in the vertical direction for the elevation estimation.

Time-modulated arrays (TMAs) were first proposed as a means of producing low side-lobe antenna patterns by using simple on-off switching of the array elements [10]. In 2010, an experimental result of direction finding by using only a two-element TMA was reported [1]. In [11] [12] the fundamental and the first harmonic components were then used to estimate the DoA, and they also extended the framework to different configurations and waveform types. However, these direction-finding approaches based on only 2 antennas all suffer from high error, even with some research improving the DoA estimation accuracy [13].

In this paper, inspired by the TMA signal processing, we propose the use of harmonic analysis to deduce the mathematical expression between incident angle (i.e., the DoA angle) and the harmonic coefficients. Based on this, we can easily calculate the fundamental components via the discrete Fourier transform (DFT) to perform angle searching. Then, with simple peak finding on the resulting fundamental component curve, we can quickly implement a simple direction-finding algorithm with low computational cost, which is specifically suitable for platforms with limited power and size of the radar.

The rest of the article is organized as follows. In Section II, the signal model and the problem formulation is given. The proposed approach is demonstrated in Section III. The simulation results and their analysis are provided in Section IV. Finally, conclusions are drawn in Section V.

II. THE SIGNAL MODEL AND PROBLEM FORMULATION

A. *The Signal Model*

Frequency modulated continuous wave (FMCW) radar with one transmitter and two receiver antennas is considered here. Without losing generality, the omnidirectional antenna pattern is considered for the transmitter and receiver elements, and the targets are located in the far-field of the array [14].

With a distance between the two receivers equal to d, the range differences between the point target and the two receiver antenna pairs will be equal to $dsin\theta$, where θ is the incident angle of the target, and $d = \frac{\lambda}{2}$ to avoid ambiguity in the spatial domain. λ is the wavelength of the signal.

The received signal is correlated with the conjugate copy of the transmitted signal to get the de-chirped signal at the two antenna elements. This can be written as in the following equations, where for simplicity it is still indicated as a complex amplitude:

$$z(l, t') = \sum_{i}^{N} \alpha_i e^{j2\pi(f_0 \frac{ld}{c} sin\theta - \mu\gamma(i)t')} \quad (1)$$

where N is the total number of point targets, α is the constant complex amplitude associated to a target, f_0 denotes the starting frequency, μ denotes the frequency modulation rate equal to the ratio between bandwidth B and chirp duration T_c, and c is the speed of light. $t' \in [0, T_c]$ is the index in the fast-time domain, $l \in [0, 1]$ is the index of the receiver antenna. The initial round trip delay of the i-th target is

$$\gamma(i) = \frac{2D(i)}{c} \ll T_c \quad (2)$$

where D is the range between the transmitter antenna, the target, and the receiver antenna.

B. The Problem Formulation

Inspired by the TMA signal processing, the proposed technique allows conventional array amplitude weighting patterns to be synthesised in a time-average approach by digitally switching the array elements on for a period that corresponds to the array element's relative amplitude weight. For this, we reshape one chirp signal received (1) as:

$$\mathbf{Z}_i = [z(0, t'); z(1, t')] \quad (3)$$

As more chirps, for example L in total, are received and concatenated in the time domain, then $\mathbf{Z}_i$ can be extended into the vector $\mathbf{Z}$:

$$\mathbf{Z} = [\underbrace{z_1, .., z_i, .., z_l}_{L}] \quad (4)$$

The new formed vector can be expressed as:

$$z(t) = \alpha U(t) e^{-j2\pi\mu\gamma t} \quad (5)$$

where $U(t)$ is a periodical function, which is expressed as:

$$U(t) = \begin{cases} 1 & t \in [(n-1)T_c, nT_c] \\ e^{j2\pi f_0 \frac{d}{c} sin\theta} & t \in [nT_c, (n+1)T_c] \end{cases} \quad (6)$$

Because of its periodical nature, it can be represented by its Fourier series as:

$$U(t) = \sum_{k=-\infty}^{\infty} a_k e^{-j2\pi k F_p t} \quad (7)$$

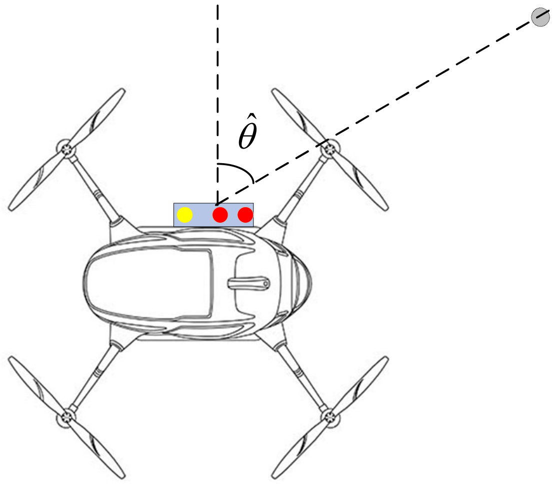

Fig. 1. The assumed scenario for side-mounted UAV radar; the transmitter is marked in yellow, while the receivers are marked in red.

where F_p is the modulation frequency and is equal to $\frac{1}{T_c}$, and a_k is the Fourier coefficient of the k-th harmonic component. This is calculated by:

$$a_k = \begin{cases} \frac{1 + e^{j2\pi f_0 \frac{d}{c} sin\theta}}{2} & k = 0 \\ \frac{j(e^{-jk\pi} - 1)}{2k\pi}(1 - e^{j2\pi f_0 \frac{d}{c} sin\theta}) & k \neq 0 \end{cases} \quad (8)$$

The fundamental component a_0 has a direct relationship with the incident angle θ. The amplitude of a_0 will reach its peak value when $sin\theta = 0$

$$|a_0| = \sqrt{\frac{1 + cos(2\pi f_0 \frac{d}{c} sin\theta)}{2}} \quad (9)$$

Suppose we perform an angle searching operation for each angle value, only when the term $sin\theta$ of the second element's received signal $z(1, t')$ is compensated. In that case, the fundamental components will reach its peak. This can provide the DoA estimation.

III. The Proposed Method

As discussed in Section II, the fundamental component a_0 can be used to estimate the DoA angle. As depicted in Fig.1, let us assume a scenario where there is a point-like target located in the field of view of a drone-mounted radar.

Generally, the point-like target corresponds to one range bin after the Fast Fourier Transform (FFT) on the range domain. Once the target is detected, the range of this target can be measured. By using harmonic analysis on target's range frequency, the incident angle can be extracted from the harmonic coefficient. The steps of the proposed approach are described as follows.

Step 1: Calculate an estimation of the target range.

The range estimation of the target can be determined by the position of the point corresponding to the target after FFT. Through (2), we can obtain the frequency $f = \mu\hat{\gamma}$ of the far-field sinusoidal wave received by the array and associated to the target.

Step 2: Phase compensation on the 2nd receiver signal by angle searching. The compensation term is given as:

$$Z_e(t) = e^{j2\pi f_0 \frac{d}{c} sin\theta_c} \tag{10}$$

Note that during the compensation, the fundamental component as in (9) will vary. For each possible angle value θ_c, we can obtain the value of the fundamental component and form a curve.

Step 3: Construct the proposed received signal formulation.

As in the discussion in Section II, a new signal formulation can be constructed to further explore its phase information via harmonic components analysis. Here we obtain the vector **Z** of multiple chirps from different elements as in (4).

Step 4: Peak searching to obtain the DoA result.

In the DoA angle searching operation, for each vector used we can calculate its fundamental component via Fourier analysis, and form a curve with the fundamental component values as a function of the different DoA angle values. The peak will be reached only when the searching angle is equal to the target's angle. Hence, this can be estimated via simple peak search. The algorithm is summarized in the pseudocode of Algorithm 1.

Algorithm 1 Proposed direction finding

Get the range estimation $\hat{r}$

for θ_c in $[-90°, 90°]$ do

$z_{sea}(1, t') = |z(1, t')| \times z_e$

Reshape the data as in (4) to get **Z**

$a_0(\theta_c) = \frac{1}{N} \sum_{n=1}^{N} \mathbf{z}(n) e^{-j\frac{2pi\mu\hat{\gamma}}{Fs}(n-1)}$

endfor

Use the fundamental component curve to get the peak value, which is the target DoA $\hat{\theta}$ from the considered $\hat{\gamma}$ range bins.

IV. Results and Discussion

The proposed simulation is based on a radar with limited antenna elements, only 1 transmitter and 2 receivers. The center frequency is set as 77 GHz, the chirp slop is 62.5 MHz/us, the bandwidth is 1 MHz, and the sampling rate is 8 Msps. All the simulations are performed in MATLAB.

To test the effectiveness of the idea, 36 chirps are collected for the harmonic analysis. The antenna with one transmitter and two receivers on the radar was located at the coordinate center, with a target placed at an incident angle of 10° and range bin of 5m to meet the Fraunhofer distance [14]. The SNR (signal-to-noise ratio) after de-chirping is set at 20 dB.

In Fig.2, the harmonic analysis of the received signal is shown. The fundamental component's position relates to the position of the target, while the position of the maximum fundamental component provides information on the direction of arrival. After performing the angle searching operation, we can obtain the curve of the fundamental component with respect to the angle, shown with red line in Fig. 3. From (9), only after matching the value of the searching angle to the true target angle, the component will reach its peak, as all the harmonic components will disappear and the energy will be added together into the fundamental component. In this case, the ground-truth angle is $10.1°$ in Fig. 3, and we obtain an estimation of $10.1°$, which is very close to the ground truth. And the radiation pattern of FFT is given in blue line too.

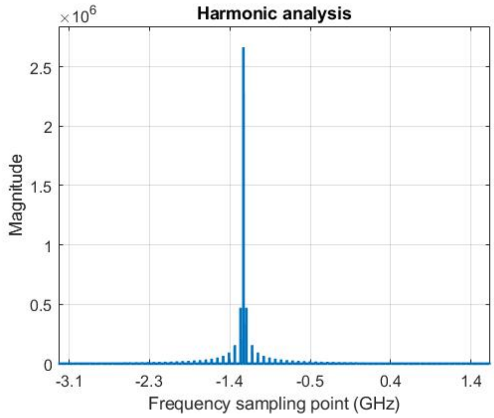

Fig. 2. The harmonic analysis on the received radar signal

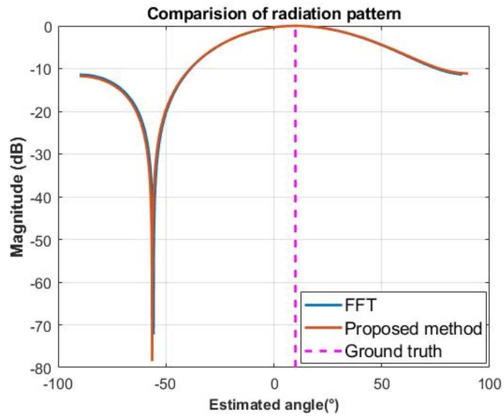

Fig. 3. The comparison of radiation patter between FFT and proposed method.

Tests with different range bins and different angles values are also performed in a Monte Carlo fashion. These are compared with the TMA direction finding algorithm for two elements in [11], [12]. The Monte Carlo results under SNR 20 dB are shown in Fig. 4, where the mean error decreased from $0.601°$ to $0.1°$. The results on lower SNR of 5dB are also shown in Fig. 5. As expected the results worsen, but a good improvement can still be seen from the conventional TMA approach, where the mean error drops from $0.693°$ to $0.365°$.

A more traditional DOA estimation, for example, beamscan, is also tested here. The proposed method can reach comparable error levels but only with one multiplication, whereas the beamscan needs the same computation complexity as the 3-dimensional DFT process. So the computation decreases from $O(K * M * N * log(K * M * N))$ to the $O(M + N + K)$, where K, M, N is the size of the radar data in range-Doppler and angle domain, respectively.

Further simulations also test the case of 2 targets at different range bins and different angle locations of 0° and

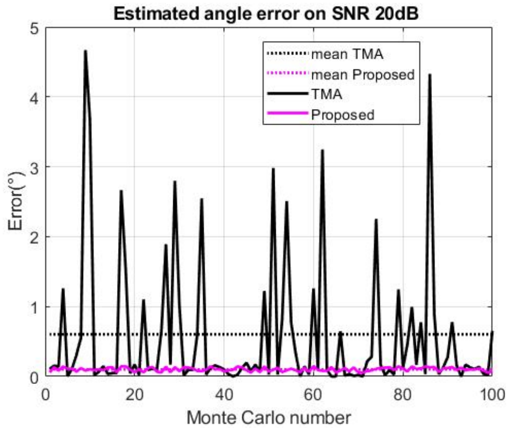

Fig. 4. The Monte Carlo test results with high SNR (20dB)

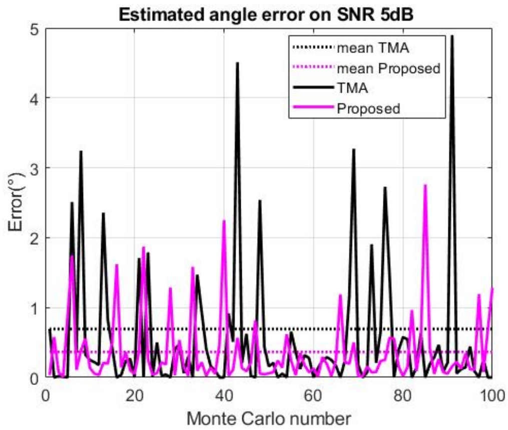

Fig. 5. The Monte Carlo test results with low SNR (5dB)

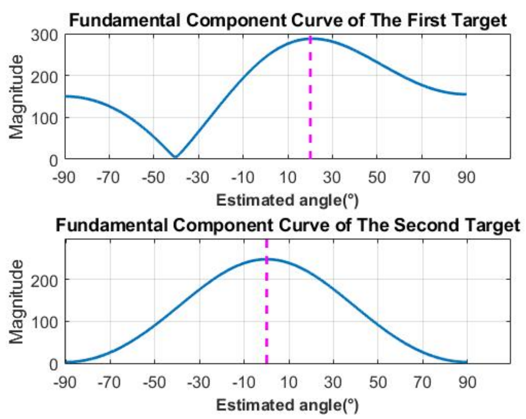

Fig. 6. The fundamental component curves of 2 different targets

20° . The proposed method can provide the correct direction estimation separately, as shown in Fig. (6). It's worthy mentioning that the algorithm doesn't have the resolution ability, as it alleviates the computation times of FFT, from 3D to 1D, but still it can provide more accurate estimated angle compared with any other methods for a limited number of antennas.[12]

V. Conclusion

In this paper, a novel DOA inspired by the TMA signal processing has been proposed for an array with a limited number of elements. Based on harmonic analysis, mathematical expressions for the algorithms have been given. We can easily calculate the fundamental components via the discrete Fourier transform (DFT) by performing angle searching. Via peak finding on the fundamental component curve, we can implement direction-finding with low complexity. The simulation results show that the angle estimated algorithm is more accurate than the TMA direction finding algorithm under different SNRs. Future work will focus on validating the method on more realistic simulations, such as experimental data. Also, this approach can be implemented on current TMA processing systems.

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