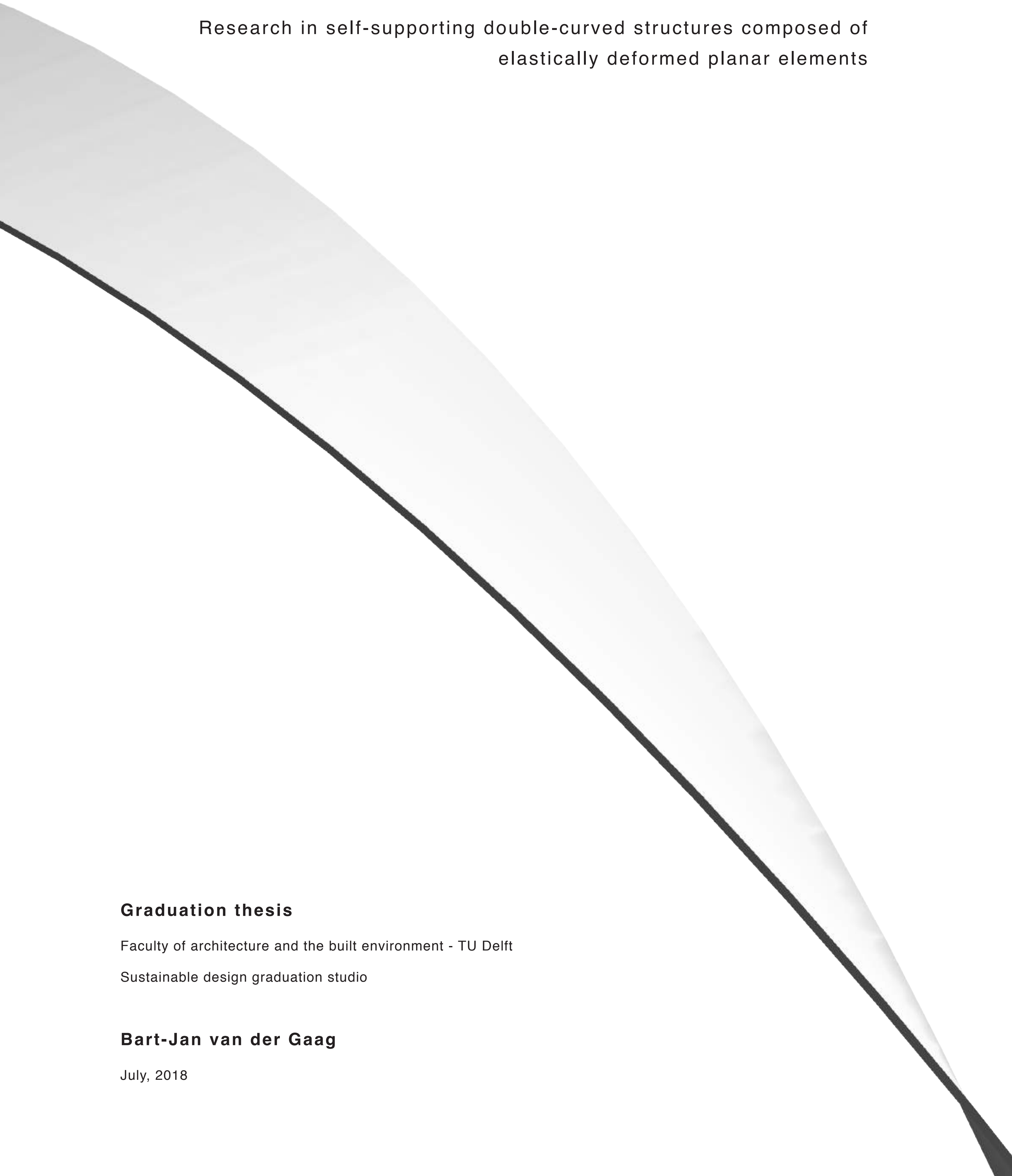
Exploiting active-bending for double curved structures

Research in self-supporting double-curved structures composed of elastically deformed planar elements



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Abstract

This thesis aims to exploit active bending as an approach to create complex curved geometries, which are structurally self-supporting systems. The simplicity of creating complex curved geometries from initially planar elements is the leading motivator for this research. The central research question is: ‘How can double curvature be exploited in a structural system composed of elastically deformed planar elements”? This research answers questions like what double curvature is and how it can be measured and controlled. What is the relationship between an elastically deformed double-curved geometry and the structural characteristics of active bending structures? The literature review is composed of part A: research into double-curvature, and part B: the structural behaviour of active bending structure.

For the question to what extent planar elements can deform, the measure of Gaussian curvature has been used. The Gaussian measure provides the relationship between bending and torsional curvatures. Bending moments relate to these curvatures. However, the Gaussian curvature is a purely geometrical measure without incorporation of material properties. The problem is that deformation of planar elements results in double curved geometries rather than, according to the Gaussian measure, in single curved geometries.

The inclusion of material properties such as Poisson’s ratio and bending rigidity in the Gaussian measure allows for prediction of curvature and the related bending moments. The basic geometry of a rectangular plate has been used to do physical tests, followed by computational test and FEA analysis. It has been proved that the stiffness of an elastically bend plate increases for a large Poisson’s ratio. This makes the Poisson’s ratio an essential parameter for active bending plate structures.

Further stiffness can be achieved through torsion. Torsional displacement of a clamped planar plate leads to increased tension, which leads to increased stiffness. The combination of bending and twisting a planar plate leads to a structurally stiff arch, which is both a structural element and an architectural design component.

Suitable materials for active bending plates structures are composites. They offer a relatively high Poisson’s ratio and a high strength to flexibility ratio.

For a planar plate of epoxy glass fibre composite of fifteen by one meter, only 20 mm thickness is required to make it structurally sufficient.

The feasibility and application of an active bending arch plate depend on the building method. The building method aims to keep the bending process and building sequence simple and modular. The erection process of the arch will be performed through the motion of the ends of the plate in a single direction over a modular gliding rail system, which allows for a simple assembly process.

The double curvature, the structural performance and the aligned building method prove the feasibility of active bending plate structures. Active bending does not have to be considered just a formation process, but a structural system itself. The exploitation of active bending is found in the nuance of double curvature as a result of the material dependent Poisson’s ratio. The simplicity of deforming planar elements, which is feasible with the developed building method results in an active bending arch system which can be used as a cantilever, bridge support, or cladding system.

Preface

This thesis was developed between November 2017 and July 2018 for the master track Building Technology at the faculty of architecture and the built environment - TU Delft.

To my first mentor Ir. A. Borgart, Andrew, I would like to express my gratefulness for helping me during my research process. “It doesn’t matter what you do as long as you like what you’re doing’’, is something he said to me at the very beginning of this research. Something with bending and double curvature was my response to that. His wide range of knowledge in the field of structural design and his ability to explain highly abstract theories with the use of very comprehensible and practical examples has helped me a lot during this research.

I would like to thank my second mentor Ir. K.B. Mulder, Koen, for helping me with design-related feasibility challenges. His guidance with the integration of material, architecture and functionality enabled me to look at this research from a different perspective. He helped me to develop my theoretical findings into something applicable, which resulted in a structural arch system as a product.

Lastly, I would like to thank Peter Eigenraam. Even though he wasn’t officially my mentor, he has been involved throughout the entire research and helped me with structural analyses and software related challenges.

To all three of you, a big thanks for helping me during my research. No question was too much and doors were always open. I enjoyed working together with the shared excitement for this research topic.

Delft, July 2018, Bart-Jan van der Gaag

Introduction

This research focusses on active-bending structures. “Active-bending structures are structural systems that include curved beam or shell elements which base their geometry on the elastic deformation from an initially straight or planar configuration” (Lienhard, 2014, p.13). The combination of the utilisation of the material properties, the simplicity of using planar elements, the interaction between force and form and the aim to find feasible building solutions for double curved objects are personal fascinations. The motivation to work with these fascinations is the lack of understanding of the translation from planar to double curved geometry. These days, computational-design allows architects and engineers to create arbitrary three-dimensional objects without requiring a broad knowledge of the geometric properties. These properties are highly relevant in the translation from computational design towards a buildable structure. A rational designed double curved object is better constructible compared to a non-rationalised double curved object. Non-rational geometry would first have to be rationalised for construction and therefore obtains a more significant deviation from what is initially designed. The computational geometric properties are of high importance for construction and buildability.

“In the inseparable relationship between form and force, the disciplines of architecture and structural engineering are equally addressed” (Lienhard,2014).

This research is not the answer to all the ongoing research and experiments in the creation of double curved objects, but rather a new perspective on active-bending as a structural approach with an emphasis on the geometric logic and simplicity of the used elements. The aim is to highlight the relationship between simple planar elements and complex double curved geometry, and what these two different geometries have in common. Understanding each of the geometric properties leads to an understanding of the geometric parameters which can be used for the translation of planar elements to double curved objects.

Table of contents

A. Introduction

A.1 Introduction 12
A.2 Problem statement 14
A.3 Research aim 15
A.4 Research questions 16
A.5 Research design 18
A.6 Approach and methodology 20

B. Research active bending

B.1 Active bending 24
B.2 Basic of structural behaviour active bending 30
B.3 Active bending and structural principles 40

C. Research Form and Curvature

C.1 Form and curvature 54
C.2 The geometry of triangular surfaces 62
C.3 Rationalised double curved surfaces 64
C.4 Continuity of curve 72

D. Implementation - design direction

D.1 Design methodology 78
D.2 A bottom up approach to obtain a double curved structure 80
D.3 Design parameters 81
D.4 Form creation process 84
D.5 Continuity and topology 90

E Two dimensional bending

E.1 Material properties in Gaussian curvature 102
E.2 Measurements 108
E.3 Double curvature through bending and torsion 112
E.4 Effect Poisson’s ratio on bending and torsional deformation geometry 114

F.design parameters

F.1 Design variables plate arch 124

F.2 Shape 126

F.3 Material selection based on geometric stiffening effect 138

G. Structural analyses

G.1 Weakness / loadconditions / test 134

G.2 Wind pressure load simulation 136

G.3 Form creation and structural behaviour 144

H. Building Methodology

H.1 Form creation process single arch 152

H.2 Plate 164

H.3 Support element 165

H.4 Coupling 172

H.5 Copy-paste - extenable structure 174

I Design application

I.1 Linear array 178

I.2 Squared array 182

J. Detailling and design freedom

J.1 Design freedom 188

J.2 Detailling 190

J.3 Future scenario's 194

K. Conclusions and future trends

K.1 Conclusion 210

K.2 Recommendations 214

Bibliography 216

Images and illustrations 220

A Introduction

Research layout

A.1 Introduction

Active-bending structural systems rely their simplicity on the creation of double curved objects from the elastic deformation of initially flat or planar materials. Many empiric examples of bending active structures known form vernacular architecture show a similar use of materials elastic deformation. In the past, such deformation processes have mainly been utilized for economic construction methods in the creation of double curved objects (Lienhard, 2014). The geometry was form-found through hanging- and simple analytical models. The lack of structural construction methods for double curved objects made active bending such a commonly used building technique. The computational abilities designers possess nowadays offer a wide range of tools to create double curved virtual geometry. Such virtual design tools have been the basis for explorations into form and design, leading to new surfaces, grid shells and membrane descriptions. Lienhard (2014) states that in the present build environment the creation of double curved objects is not the only motivation for active bending structure. Economic reasons such as transportation costs, assembly processes, performance and adaptability support the use of active-bending as well. The formation process of planar elements towards double curved elements requires enough elasticity in the material. This elasticity might contradict the structural performance of the structure. It is both the aim to obtain structural stiffness and double curved geometries, all based on the elastic deformation of initially planar elements.

Material, formation, form and force

Active bending as a structural system is composed of four components. These four components are the utilisation of the material properties, the simplicity of using planar elements and the interaction between force and form. This research aims to combine these four components to find feasible building solutions for double curved structural systems.

Active bending entirely relies the form and performance on the material behaviour. Common building structures base their performance on high materials stiffness without, or with a low flexible strain. In active-bending structures, the materials high breaking tension is a leading parameter for form and performance.

The form creation process is dependent on the material capacities as it determines how much an element can deform. The material selection needs to be adapted to the desired

Bending-active structure

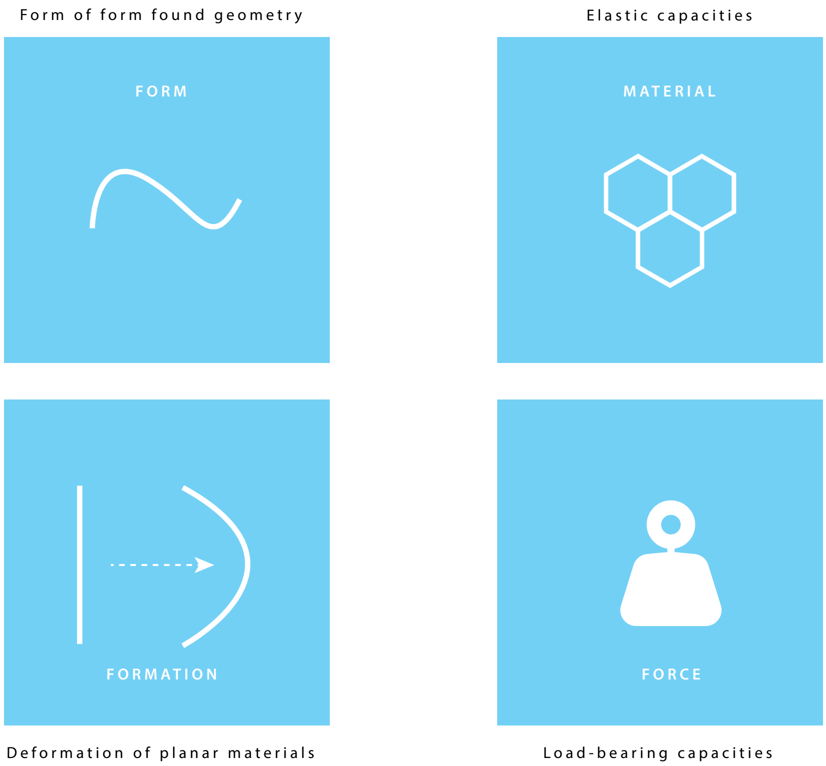


Figure [A1]. Bending active structural components

geometry and loadbearing capabilities of the design. The inseparable relation between the form and the structural capacities of the design both depend on the selected material. A form-finding process of planar geometry is the simulation of the deformation of the material. In active-bending structures, the form-found geometry is not necessarily structurally optimised. The form and structural performance of active-bending structures will always influence each other. It is up to the designers and engineers to obtain an equilibrium between the desires of form and structural performance.

A.2 Problem statement

These days arbitrary architecture is becoming more common. Arbitrary design may include high amounts of double curved geometry, leading to complex structural systems and unique construction methods. Such methods are expensive, labour intensive and structurally inefficient.

First, using active-bending as a form creation process to create a structural system requires knowledge about double curved geometry. The measure of double curvature, the magnitudes and the virtual description are therefore highly essential to know during the concept phase design of an active bending structure.

Secondly, the approach of how to use active-bending as a form creation process. There is a clear distinction between a bottom-up and a top-down design process where active-bending is used as structural system and form generation. The top-down design approach starts with a given geometry. Active-bending as structural approach can be used for the realisation of this given geometry, based on the motivation stated in the previous paragraph.

The parametric description and the double curvature of the input geometry do not necessarily match the requirements for active-bending structures. This means that the input geometry needs to be approximated, i.e. rationalised. Active-bending is the deformation of initial planar material into curved elements. Vice versa, the curved geometry has to be developable, which means it is planar in unrolled condition.

In the top-down approach, the initial design has to be rationalized to apply the active bending approach to the given structure. Consequently, this approximation or simplification of the input geometry might deviate from the desired geometry if it does not match active-bending requirements.

Bending active could be applied as a structural approach, but this is only after the approximation and rationalisation. Various examples show the ongoing interest in utilising an active-bending approach as a structural system, all aiming for the best approximation with the least deviation.

The bottom-up approach is a process where through element topology an entire geometry can be generated. In this case, active-bending is not a structural approach but rather a formation process leading to an entire stand-alone structural system. From the start of the design process, the building components can be aligned with the requirements for active-bending structures. There is no need for approximation and rationalisation in a later stage as all the elements will be rational and developable.

This research will focus on the bottom-up approach with the requirements for active bending as given input parameters. The obtained structure provides insight into the abilities of elastically deformed materials used to create double curved structures.

A.3 Research aim

This research aims to create a double curved structure which is entirely composed of actively-bent components. The bottom-up design approach allows to include the requirements of active-bending throughout the design process. Individual elements and the entire structure will match the active-bending requirements. It is not possible to approach this research entirely theoretical; the findings will be tested virtually through software simulations and practically through the prototype models. The objective is to create a self-supporting structure with double curvature in the overall design. The elastic deformation can be used to shape and to construct this structure. With active-bending as guiding-theme, this thesis will be built up from four parts.

A. Research into active-bending, providing a brief overview of the basics of active-bending.
B. A study in the measure of curvature and what kind of curvature descriptions can be distinguished.
Chapters A and B are the fundamentals of this research.

C. The implementation of the outcome gathered from chapter A and B.
The outcome helps to formulate the input for the first steps of the design approach. The geometry and the formation process will be the based on the knowledge gathered from chapter A and B. The geometry will be determined through experiments and software simulated form-finding. The geometry's' curvature and the structural performance will be analysed parallel to each other.
D. A prototype of the simulated element allows for additional analyses and validation.

A.4 Research questions

To formulate the central research question, a set of sub-questions will be formulated. These sub-questions relate to the four parts described in the previous paragraph. These questions help to systematically approach the central research question.

Research sub-questions

A.
What is the definition of active-bending?
What are the structural characteristics of active-bending?
What are the material requirements for active-bending structures?
What materials are best suitable for active-bending structures?
How does an active-bending structural system distinguish from other structural systems?

B.
What is line curvature?
What is surface curvature?
What sorts of curvature can be distinguished?
What kind of curved geometry can be distinguished?
How can double curvature be measured?
What are the curvature requirements for active-bending elements?
What are the most viable virtual descriptions of double curved surfaces?
How can these virtual descriptions be modified without losing the developability?

C.
What are the load bearing requirements for the structure?
What is the maximum possible curvature of the structure?
What external factors does the structure need to withstand?
What internal factors does the structure need to withstand?
What material is best to use considering internal and external factors?
What are the scale requirements for the structure?
What are size limitations for the single structural elements?
What is the difference between plate and beam bending?

D.

What is the loadbearing capacity of the structure?

Do the results from the prototype match the results from the software simulation?

What is a feasible construction approach for such structures?

What is the application for such structure, what time span does the structure has to withstand?

Based on the problem statement, the given research objectives and the sub-questions the central research question can be formulated.

Central research questions:

“How can double curvature be exploited in a structural system composed of elastically deformed planar elements??”

A.5 Research design

The sub-questions will be answered chronologically to answer the central research question. In general; this research can be determined as research by design or as an experimental design (Fellows, & Liu. 2015). This study contains two parts: literature research and empirical (experimental) research.

Literature research

The literature research functions as the framework of this thesis. The gathered information contains two parts: research in active-bending and research in form and curvature. Throughout the entire process, literature research will provide information used for evidence-based research validation.

Implementation

The form-creation process of the elastically deformed materials will be performed through software simulation and physical modelling. The results derived from these simulations will be analysed and reflected with the given design parameters. The primary input parameters for the simulations are the materials, the level of deformation and the scale and dimensions of the planar elements. These parameters will be further specified during the simulation process. The amount of simulation iterations is dependent on the outcome of the results related to the desired outcome.

Application

One selected active-bending element derived from the literature and simulation research will be built in a prototype/scale model. This prototype will give additional insight into the material properties, the construction method and the problems with cannot be foreseen in simulation analyses. This prototype will be one or several elements showing the formation process, structural behaviour and overall layout. The results will be compared with the initial data from the literature and simulation analyses. The aim is to compare the results from the as-built prototype with the theoretic results from literature and simulation. The reflection will be based on the possible deviations between theory and practice.

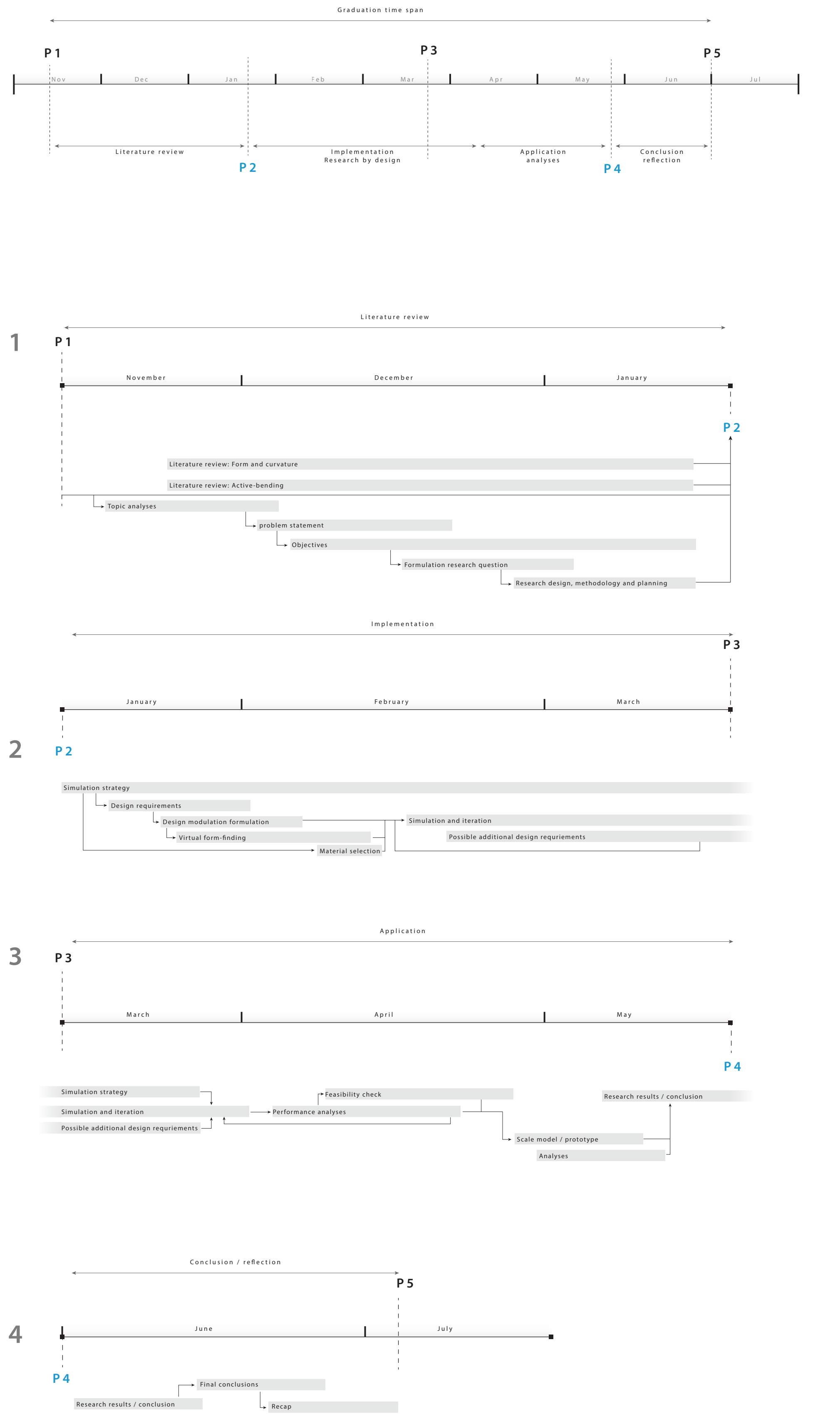


Figure [A2]. Graduation research time planning.

A.6 Approach and Methodology

This chapter describes the research approach, the research methodology and the data collection to obtain answers to the sub- and central research questions.

The results from the literature research, the software simulations and the practical experiments form the basis of the data collection.

Literature review

Literature research into active-bending as structural principle and formation process. This contains the mechanical behaviour, the viable materials, the various approaches to the use of active-bending and the position towards other structural principles. Secondly, literature research into curvature and form. A clear understanding of what curvature is, the various types of curvature and the level of complexity in the creation of double curved surfaces. Additionally, a clear understanding of computational surface descriptions and formations in their relation to rationalised and developable surfaces.

Both these research parts form the basic framework of the total research. The problem statement and research objectives, as well as the research questions, have been formulated during the literature review.

Simulation

Based on the outcome of the literature review, a set of rulings for active bending elements for will be formulated. These rulings are the basis of the first element design and topology. This simulation will be performed in the virtual environment through software modelling, regarding the geometry and disregarding the specific material properties. The used material properties will be a domain in which many materials could be applied. Ignoring the specific material properties at first allows for a geometric design process free of material restriction. This geometric design process is the exploration in shape of single elements and the topology possibilities.

In a later stage of the simulation process, the material properties are leading to the geometric behaviour of the deformed materials (Apperman, O., Christoph, G., Gengnagel, C., Lienhard, J., & Knippers, J. 2013). The form-finding of the structural elements and the topology of these elements towards an entire active-bending structure are not linear

"To build means to make architecture real, on the border of knowledge".

Frei Otto

processes. Based on the assumption that additional requirements will be formulated throughout the simulation process, several iterations have to be performed. The aim is to find viable elements and topology to meeting the given input requirements.

Parallel to and after the geometric simulation, the structural performance of the structure will be analysed. The aim is to strife towards both geometric and structural feasible design. However, the geometric layout and structural performance might contradict, as they both base their performance on different parameters. The structure has to be loadbearing and self-supporting but does not have to be structurally optimal. Based on the structural performance of the simulated geometry, several iterations will be required.

Application

The geometry derived from the virtual simulations will be further analysed in as-built prototype simulations. The prototype will be a deformed element or a fragment of the structure, based on the scale of single elements. First, the results from the prototype analyses will be compared with the results obtained from the virtual simulation. Secondly, the prototype provides additional insight into the deformation process. Furthermore, it includes information on the accuracy and the connection of individual elements, leading to an overall understanding of the entire structure.

Conclusion

A conclusion can be formulated based on the comparison of the virtual models and physical prototypes. These findings will be used to answer the sub- and central research questions.

B Research

Active bending

B.1 Active bending

This chapter gives an overview of the characteristics of active bending. The subchapters all briefly discuss each aspect to get a clear understanding of what active-bending is. The subchapters include information on how active-bending performs structurally and how it can be used as a design tool to create active-bending structures.

Definition active-bending

Active bending is the utilization of large elastic deformation to create curved structures from initially flat or linear elements.

(Chilton, Corne, Gosling, Mollaert, & Tejera 2017).

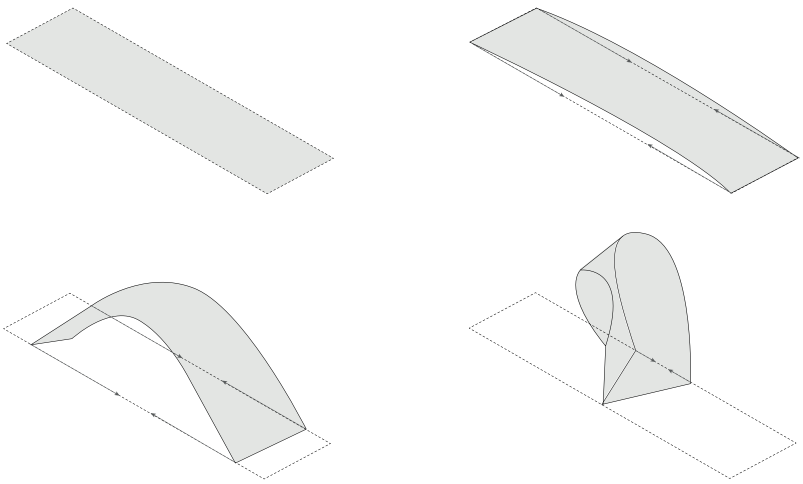


Figure [B1]: Active-bending principle (Schleicher at all 2015)

One-dimensional and two-dimensional

Bending-active structures distinguish two main characteristics. They relate to the geometrical dimensions of their fundamental components. Bending rods, for example, would be considered One-dimensional (1D) systems. Two dimensional systems (2D) use thin elements as basic building components (La Magna, , Schleicher, & Knippers, 2016). While the field for One-dimensional structures has been widely explored and frequently applied in the build-environment, the knowledge and applications of two-dimensional structures seem to be left apart (La Magna, 2016). These systems are considered more challenging to design. One of the main reasons is the limited formability of the plates as they only bend along the axis with the weakest moment of inertia. The limited formability makes it hard to bend these into complicated geometries. This thesis will focus on two-dimensional bending active structures.

Active bending approach development

Active bending as a design approach, to systematically elastically deform materials, has been going through many developments in building construction history (Apperman, O., Christoph, G., Gengnagel, C., Lienhard, J.,1 & Knippers, J. 2013). In the past, the primary motivation to use this approach was to create curved building components. The lack of alternative construction methods for curved building components made active bending a favourite construction technique. These days, the economic benefits, the advantages in transportation and the assembling-process, support the use of active-bending structures.

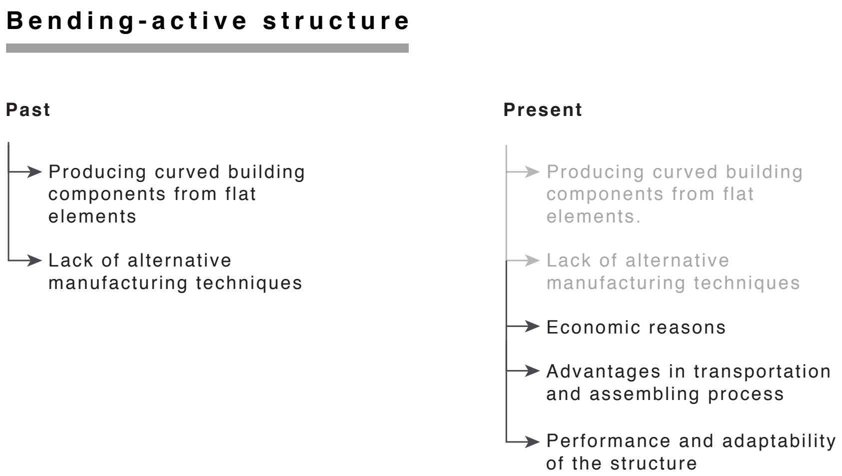


Figure [B2]. Past and present motivation use of active-bending.

Additionally, the adaptability and the performance of active bending structures are unique. Looking at the past, various examples of vernacular architecture made use of the elastic behaviour of the building materials. Bending-active structures were primarily found in areas where wood was rare, or in cultures where large timber pieces could not be developed, which led to the use of softwoods. During the industrial revolution, Iron became one of the most prominent building materials, which is one of the most used materials in construction in the 20th century. As Iron and reinforced concrete become more popular with architects and builders, a set of geometrical and structural typologies developed and functioned as the basis for a general architecture in the 19th and 20th century. It allowed to control structural complex building; however, it also limited the variety of geometric and structural designs. Only a few recognisable buildings have been built with the use of form-finding techniques based on materials and forces. At the time, such form found projects were incredibly time-consuming. The lack of advanced form-finding methods and structural analyses methods for bending-active structures is the reason why they are only limited to a few seminal projects.

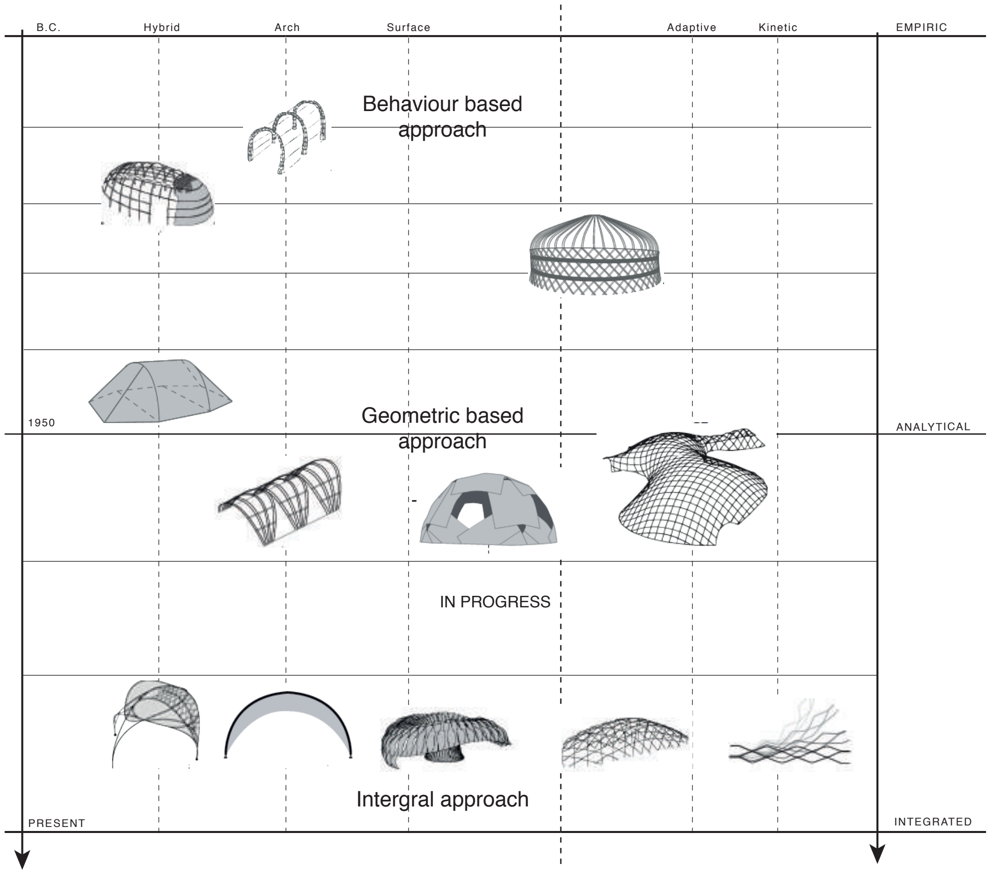


Figure [B3]. [Development of bending active structures] by Lienhard, J. 2014, *Bending active structures.*

The introduction of new lightweight building materials, such as Fiber Reinforced Polymers (FRP), offered new opportunities for bending active structures. FRP's are well known for their high strength to low bending stiffness ratio. New architectural constructions developed during the 1950s. Not only the introduction of new materials were the reason new building developments could arise. The development of analytical approaches for non-linear Finite Element Methods functioned as the basis of modern engineering mechanisms. This new analytical approach and the introduction of new building materials formed a new framework for form-finding and analyses of bending active structures.

To review and chronologically analyse these structures a framework will be presented, formulated by Lienhard (2014). He describes the development of bending active structures divided into three main approaches; Behavior-based approach, Geometry-based approach and integral approach. The scheme [figure B3] previews the approaches on a timeline divided per type of structures. In the next paragraph, the three approaches will be described supported by some example projects.

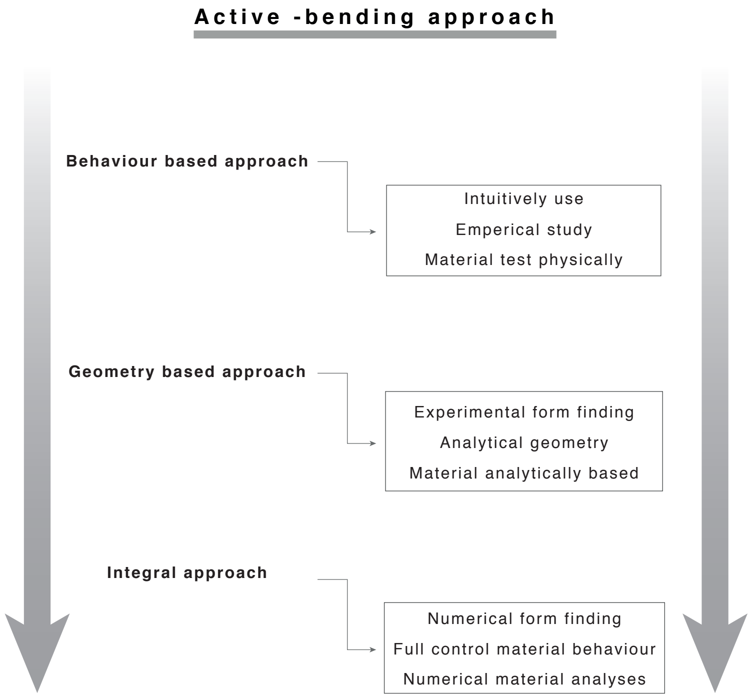


Figure [B4]. Various approaches active-bending

Behaviour-based approach

The behaviour based approached active-bending structures are well known from the vernacular architecture. With the elastic formation of local materials as a design driver, various examples of tension arch and shell systems can be found on every continent (Oliver, 1997) [figure B5].The empiric development of construction methods based on the elasticity of their building materials predominately resulted in similar structures (Lienhard 2014). Other applications are mobile structures or structures with a temporary use. The first tents were created through the bending of wood enclosed with the skin of local animals.
This concept has developed over the years and resulted in one of the first tent structures as we know them nowadays, used for camping. Thin rods can easily be bent by hand and put into an arch shape. A fabric membrane stabilises the rods and makes it an enclosed space. The production and development of such tents have been designed through physical tests on a 1:1 scale. This approach is a step towards a geometry-based approach where the design and the structural performance can be optimized through numerical analyses.

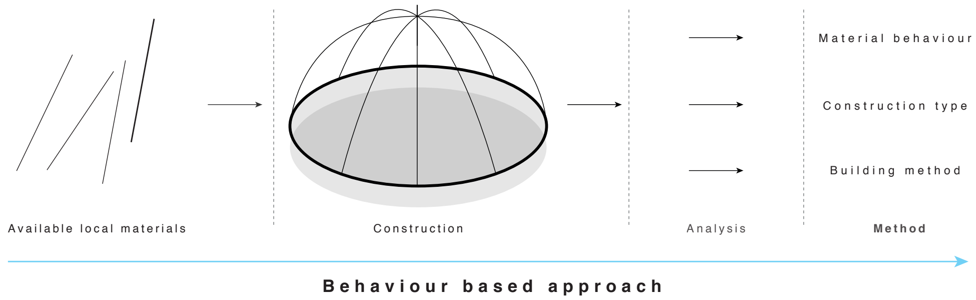


Figure [B5]. Behaviour based approach

Geometry based approach

Around the 1950s, simulation methods for lightweight structures started to develop. The main reason for this was the ongoing interest in the development of double curved shell and grid structures. However, the simulation techniques available at the time were not advanced in simulating sizeable elastic deformation, i.e. active-bending. An essential form-finding method was the hanging model. The geometry of double curved shells and grids were form found with the hanging model analyses [figure B6].

The desired shape of the shell given by the hanging model was the design intent, realised by pushing initially planar laths and plates into this desired shape. The first computational simulation methods at the time were not able to simulate deformations of the initially flat components. Therefore it wasn't possible to find the natural bending shape. It was not always possible to get to the desired geometry because the bent elements in the final erection process behaved differently from what was expected. Because of that, mainly grid structures have been built, covered by a flexible membrane. Only a few surface shells have been introduced by using plates instead of laths.

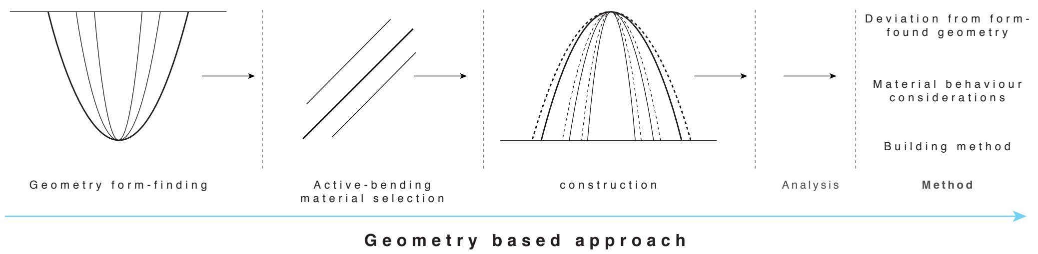


Figure [B6]. Geometry based approach

Integral approach

The simulation- and analyses software available nowadays offer a new approach to designing and analysing active-bending structures. Instead of starting with geometry and applying active-bending as a construction method, one can now develop structures that base their complex curved geometry from the erection process based on elastic deformation [figure B7]. Hybrid bending active structures are highly suitable for the creation of building components. These lightweight structures, also known as hybrid textile structures, has been a challenge in integral form-finding methods. The stability and stiffness of the hybrid system are much higher than the stiffness of the individual object, which allows for structures with a significantly small cross section. These hybrid systems stimulate ongoing research that explores possibilities for combining elastically bent elements with restrained membranes. The restrained membranes stabilise the bent beams and prevent further deformation and buckling. Researchers focus on the generation of developable grid structures. The final geometry of these grid structures is entirely based on the erection process of initially planar elements. Most of the executed projects are research projects and prototype construction to validate the feasibility of active bending hybrid structures as building components.

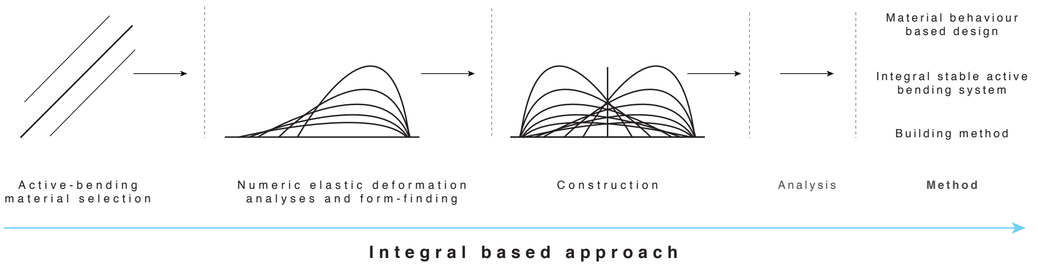


Figure [B7] Integral based approach


Figure [B8] [Fulani Dwelling dome house] Reprinted from maalidesign website, Reprinted fmor http://maalidesigns.blogspot.nl/2011/04/fulani-of-nigeria.html


Figure [B9] [Herzogenriedpark Mannheim] Reprinted from Wikipedia website, 2010, by Imanuel Giel, retrieved from https://commons.wikimedia.org/wiki/File:Herzogenriedpark_077.jpg


Figure [B10] [Interior view of research pavilion] Reprinted from Miesarch website, 2011 by Julien Lienhard, retrieved from http://miesarch.com/work/1053

B.2 Basics in structural behaviour Active bending

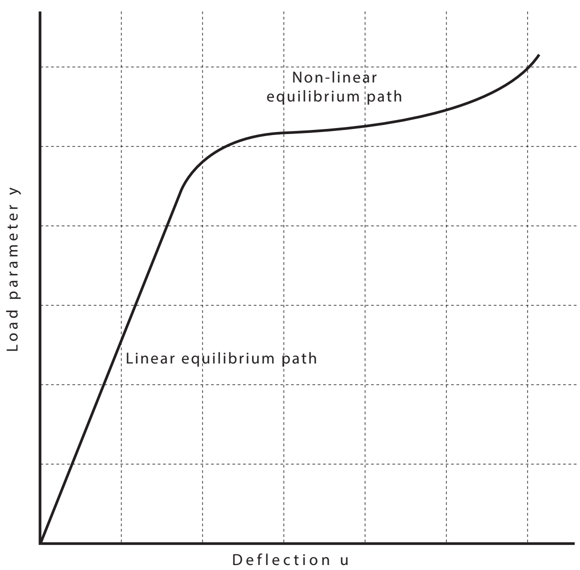
Properties active bending

Bending-active structures are known for their specific properties. This subchapter focusses on the structural behaviour of one-dimensional bending-active structures and the main factors that influence this behaviour. One- and two-dimensional descriptions have some similarities. The understanding of one-dimensional bending makes it easier to understand two-dimensional bending.

Non-linear structures

Bending-active structures do not always behave constant or linear but rather non-linear. Julien Lienhard (2014) describes linear systems as: ‘If the external forces of a linear system are multiplied by a factor n, the displacement and internal stresses also change with a factor n. If a system behaves differently, it is considered non-linear.

The equilibrium path is a representation of structural behaviour. The most common response diagram plots the equilibrium relation of external load and internal reaction forces or deflections [figure B11]. Load-deflection response diagrams representations are the key to the analysis of nonlinear structural behaviour. The non-linearity in a load-deflection curve directly simulates the non-linearity of structural behaviour. This is a helpful tool for later structural analyses of active-bending structures.



Load - deflection diagram

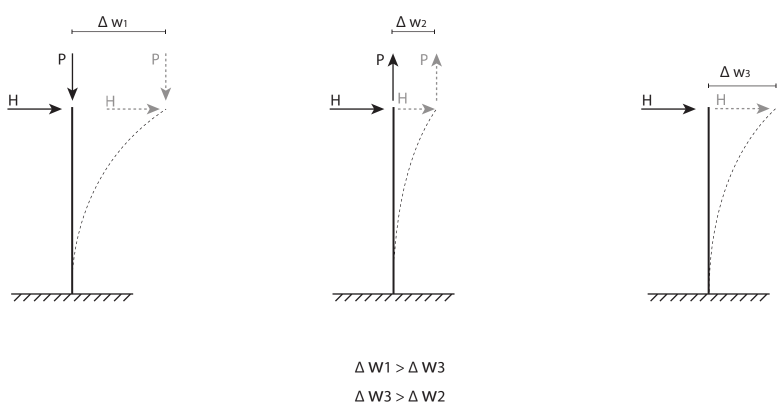
Figure [B11] [Load deflection diagram non-linear structures] by Lienhard, J. 2014, *Bending active structures.*

Relating non-linearity to active-bending, non-linear systems can be divided into systems' components: materials, boundary conditions and geometry.

1. Non-linear material behaviour: Non-linear equilibrium path (plastic deformation).
2. Nonlinear boundary conditions: Changing conditions of support after loads have been applied to a structure.
3. Non-linear geometry: a non-linear relationship between deflection and applied forces.

The materials plastic deformation will not be considered in active-bending structures because it is different for each material and structurally unpredictable. Changing support conditions may influence the structure, dependent on the design layout and the load conditions. Large deflections could lead to additional or fewer supports which is why it needs to be considered for active-bending. Geometric non-linearity must be considered for bending-active structures. The geometric non-linearity theory considers the static equilibrium on the deformed system. Thus, the reaction forces are composed of both linear and non-linear proportions.

The stresses in a structures' deformed state lead to the geometric stiffness. The geometric stiffness is depending on the load and the geometry of the structure. Tensile normal forces in a structure increase the stiffness and compressions normal forces decreases the structures' stiffness. A mast with a clamped support illustrates this [figure B12]. The moments and the deformation of the mast are dependent on the direction of load P. This effect is called the Delta-P effect, which is described as a bending moment from eccentrici-



P-delta effect of a mast with clamped support

Figure [B12]. [P-delta effect of mast with clamped support].by Lienhard, J. 2014, *Bending active structures.*

ties that appear in the equilibrium conditions of nonlinear analyses (Lienhard, 2014). As this stiffening effect is mainly visible in structures with a low elastic stiffness, it is primarily used for structures with a small bending stiffness such as cables and shells. The nonlinearity in the geometry of bending active structures may lead to this stiffening effect which works in favour of the structural system.

Pre-stressing

Prestressing or, pre-tensioning is used to increase the stiffness of a structure. It introduces small internal stresses that work opposite to stresses as results from external loads. Pre-stress increases the stiffness but does not increase the load-bearing capacity. Mainly for lightweight structures, i.e. active-bending structures, this potential of pre-stressing is used to stiffen an entire structure.

Here the difference can be made between linear elements, such as cables, and planar elements such as membranes. Linear elements are usually called pre-tensioned, whereas planar elements are pre-stressed.

Three main effects of both pre-tension and pre-stress are:
1. Increase beneficial material performance
2. Counteract external loads
3. Increase geometrical stiffness in flexible structures.

Among the number of prestressing types, residual stress is a form of pre-stress applicable in active-bending structures. The residual stresses are implemented before external loading. They remain in the structure after their original cause is removed. That includes pre-stress but also stress from elastic bending.

Elastica (beam theory)

The non-linear beam theory formed the basis for elastics in curvature. The most common model for beams in structural analyses is the Bernoulli Euler (BE) model [figure B14]. This model claims that plane sections remain plane and normal to the axis of the beam. This leads to a simplified differential equation of bending:

$$W''(x) = \frac{M(x)}{EI} \tag{1}$$

This leads to the formulation of curvature and deflection. The differential equation shows the reaction forces, deflection and curvature.

The equation below states that bending moment is always in proportion to the change in curvature based on the load quantity.

$$\frac{1}{r(x)} = \frac{w''(x)}{[1+(w'(x))^2]^{\frac{2}{3}}} = \frac{M(x)}{EI} \tag{2}$$

The solutions of this formula involve elliptic integrals which lead to the theory of elastic curves. Based on the understanding of structural mechanics, the elastic may be described as:

1. Post-buckling curve [figure B15]mare valid equilibrium states after passing a critical point on the load deflection curve)
2. Moment progression in a static system curve that generates a minimum of potential bending energy in the overall system.
3. Equilibrium of forces in a static system. (FEA)

The first two approaches include a lot of complex mathematical methods to develop closed-form solutions. These two will not be further discussed and used in the global understanding of elastica in this chapter. Julien Lienhard (2014) describes elastica as the understanding of a curve that generates minimal potential bending energy in a constrained system. This is the best description of the inseparable interdependency of mechanical behaviour and form present in all active bending structures. The local curvature characterises the elastica along the length of the arch. The curvature can be described by the local derivative of the tangent angle. Elastic bending of a beam produces potential energy in the system. The total amount of the potential bending energy of a beam can be described by

$$\widetilde{E}[K(x_0)] = \frac{1}{2}\int_l^0 EI \cdot K(x_0)^2 dx_0 \qquad (3)$$

$K(x_0)$	Local curvature	
$\widetilde{E}$	Total bending energy	$[J] = [\frac{kg \cdot m^2}{s^2}]$
E	Youngs modulus	$[N/mm^2]$
l	Length of deformed beam	$[m]$

In unconstrained conditions, a beam will be a straight line without any curvature. Therefore, the bending energy will be zero. If the beam is constrained, the bending energy tends to be the minimum possible (Levien 2009:17).

This formula formulation is a summary based on the mechanical fundamentals of active bending by Julien Lienhard (2014). For one-dimensional systems, there is a relationship between the local bending curvature and the bending moment which can be formulated as a differential segment. Figure B13 previews the relations of geometry in a deflected beam. The Thales theorem can be used to derive the relationship between the moment and the curvature:

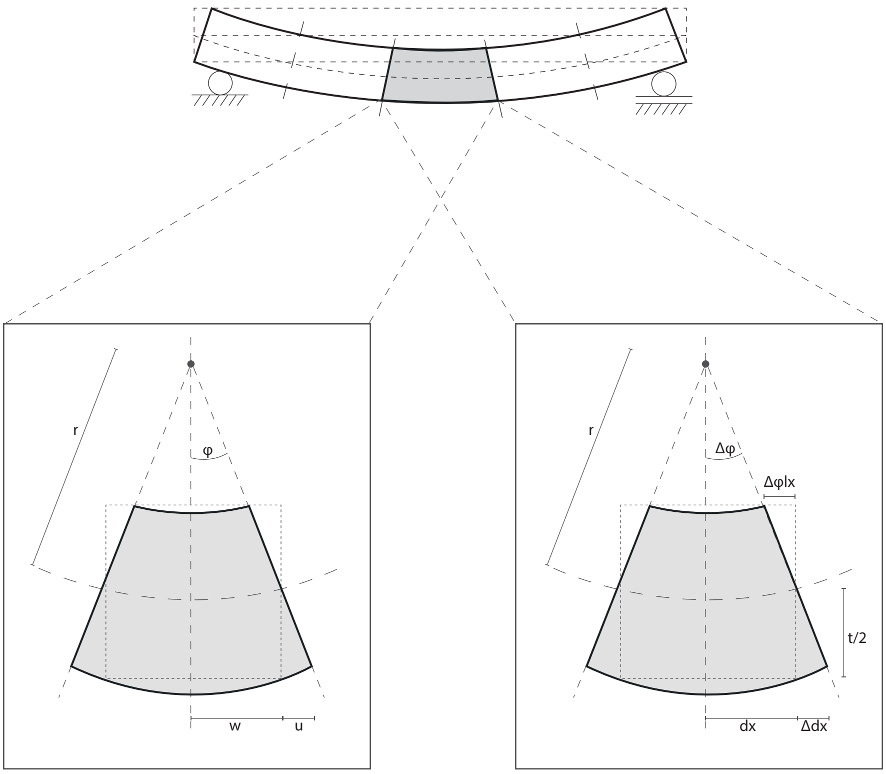


Figure [B13]. [Moment-curvature relation beam], by Lienhard, J. 2014, *Bending active structures*.

$$\frac{r(x_0)}{dx} = \frac{t}{\Delta dx} \tag{4}$$

From the relation between t and r and the introduction of the Hook's law the formula can be derived:

$$\frac{t}{r(x_0)} = \frac{\Delta dx}{dx} = \varepsilon = \frac{\sigma}{E} = \frac{M(x_0) \cdot t}{E \cdot I} \tag{5}$$

At the differential segment the bending stress to curvature relation can be derived:

$$\frac{1}{r(x_0)} = -\frac{M(x_0)}{E \cdot I} \tag{6}$$

$$\sigma(x_0) = \frac{E \cdot t}{2 \cdot r(x_0)} \tag{7}$$

This leads to the formula of the minimum bending radius for a given stress:

$$r_{min}(x_0) = \frac{E \cdot t}{2 \cdot \sigma_{rd}} \tag{8}$$

This relationship can also be derived from the strain of the outer fibre in a curved segment of a beam with planar sections. The arc length is proportional to the radius.

$$L = \frac{\sigma \cdot \pi \cdot r}{180} \tag{9}$$

The strain in this segment can be described by:

$$\varepsilon = \frac{\Delta L}{L} = \frac{\frac{t}{2}}{r} = \frac{t}{2 \cdot r} \tag{10}$$

The relation between bending curvature and bending moment is a highly valuable measure in active-bending structure. The above-presented relationship only relates to one-dimensional systems. For active-bending structure composed of two-dimensional plate elements, a similar relationship will be derived in chapter four.

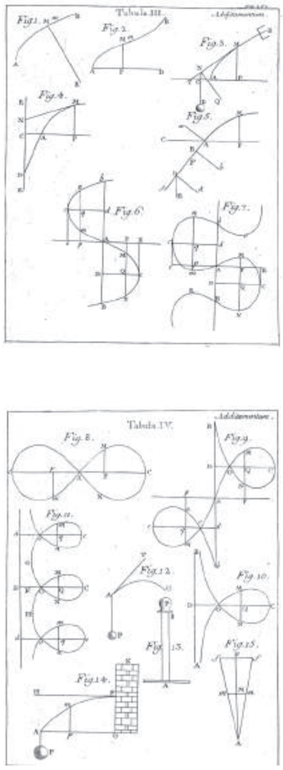
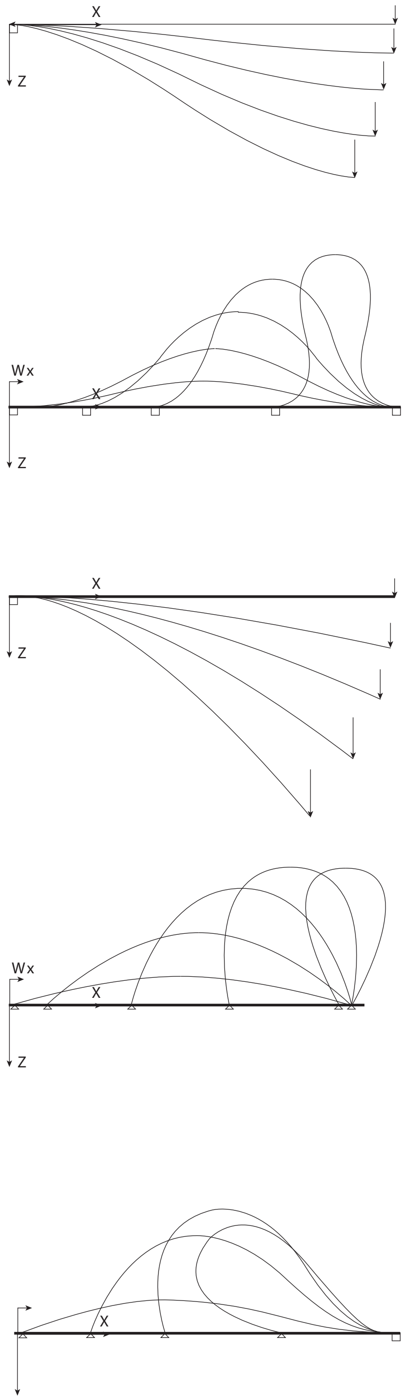


Figure [B14]. [Elastica figures by Euler] by Oldfather, Ellis, & Brown, 1933, *Leonard Euler Elastic curves*



Basic elastica curves based on the Euler cases of buckling

Figure [B15].[Basic elastica curves based on the Euler cases of buckling], by Oldfather et all, 1993, *Leonard Euler Elastic curves*

Materials active-bending structures

The geometric- and structural performance of bending active structures rely on a series of specific qualities of a material. The relationship between stiffness and maximum allowable stress is a relevant measure of how adequate material is for elastic deformation (La Magna, 2017.) The Ashby's Diagram from the Research of Julien Lienhard (2014) provides a comprehensive material classification. The diagram [figure B16] presents fields in the material property space and subfield per material class. The most suitable material for active bending structures falls in the dashed line area. The most common materials are wood and fibre reinforced polymers (Douthe, Baverel, & Caron. 2006). They share the characteristic of high flexural strength and medium to moderate stiffness.

Because active bending structures use the flexural capacities for form-finding, high material utilisation might be achieved before any load case, leading to a limited selection of materatiorials available. Dependent on the characteristics of an element, material, geometry and additional side factors, La Magna (2014) states that up to 60% utilisation of the material might be reached through form finding. The energy required for the erection process also depends on the flexural stiffness. More stiffness leads towards higher energy inputs to achieve the deformed state. Most of the active bending structures constructed so far have been erected by using manual labour. High embodied energy requirements would contradict with one of the main characteristics of active-bending. However, the problem arising from the use of low flexural stiffness is the overall stiffness of the structure. To cover the contradicting characteristics of the outcome structure, the global structural capacity is mainly generated by its geometric layout.

Wood, plywood and GFRP are the most suitable materials due to an appropriate ratio of flexural strength and stiffness. Based on previous research by La Magna (2017), the bending radii are usually between 5 to 30 m. Smaller curvatures with larger bending radii might require softer materials with a high flexural strength to keep the bending stresses after form-finding lower.

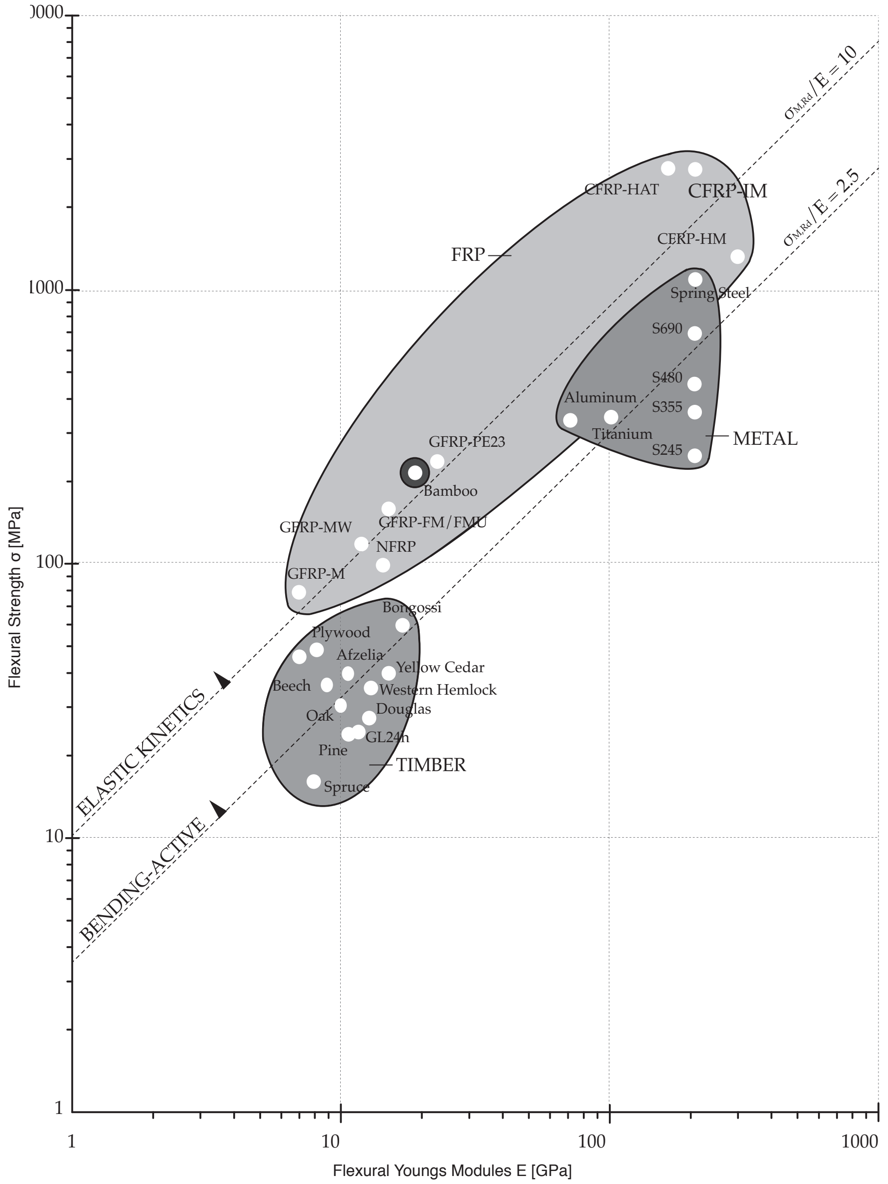


Figure [B16] [Ratio of strength σ [MPa] to stiffness E [GPa] of common building materials] by Lienahard, L. 2014, *Bending active structures*, p-37.

B.3 Active bending and structural principles

Structural princples as a framework

The categorisation of basic structural principles and systems in a structural framework is a helpful tool for understanding structural behaviour. It allows developing analytical calculation techniques that help to analyse structures by abstract static systems (Lienhard, 2014). Finite Element Modeling software and computational workflows allow us to add a new degree of complexity and efficiency in today's structural design. A designer or structural engineer does not need to think and act in structural systems anymore as the computational capabilities break loose from this framework. The result is a more free and open design environment which could potentially lead towards new structural systems. However, this raises the question what the structural criteria are for developing an efficient structure. How can we use the modern computational design techniques and align these to functional, structural systems, to create smart structures?

Bending-active structures are not the complete answer to this question. However, it could be an essential element in the development of designing and constructing such new structural systems, based on the physical behaviour of elastic bending. It is not possible to assign active-bending towards a restricted typology, as the dynamic appearance and diversity is the primary quality. In addition to that, active-bending structures are known for their load bearing behaviour which is based on nonlinearities, which can both be used for stabilising and destabilising the structure. Lienhard (2014) states that for these reasons, a bending activation is an approach rather than one type of structural system.

The book 'Structure systems' by Heino Engel (1997) provides a comprehensive overview of the described structural system. A brief overview of the structural systems will be provided to show how active bending as a structural approach might evolve within these types and sub-types. Lienhard (2014) describes a structural action as the process of receiving, transferring and transmitting a load. Residual bending stress plays an essential role in the load transfer of bending active structures, which is why they stand out from the structural systems described by Engel (1997).

Structural types	Appearance
Section active	Profiles
Vector active	Trusses
Surface active	Shell
Hybrid active	Profile & membrane
Form active	Arch
Height active	Wall

Figure [B17]. Structural systems with appearance.

Section-active

Structures systems of solid, rigid linear elements, in which the redirection of the forces is effected through mobilization of section forces. They are primarily subjected to bending; inner compression tension and shear stress. Examples of section-active members are beams, frames and plates.

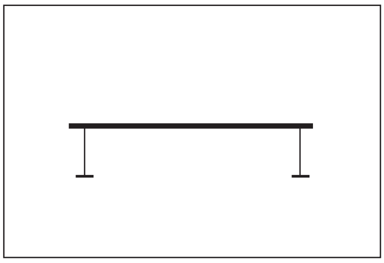


Figure [B18]. Section active principle. Engels 1997

Vector-active

Solid straight systems of solid straight-line elements in which the direction of forces is effected through vector partition. Splitting the overall multi-direction forces into individual vectors. The members, in for example a truss, are subjected to compression in one part and compression in the other part. The stabilisation in such structures is mainly achieved through triangulation assembled with point connection.

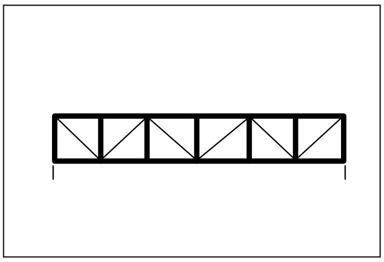


Figure [B19]. Vector active principle. Engels 1997

Form-active

Form-active structures are systems with flexible, non-rigid members, in which the particular design layout follows the direction of forces. The main characteristic of these structures is the stabilisation form design. The primary components are mainly subjected to a single kind of normal stresses which is either tension or compression. They are considered structures in single stress condition. Because external loads such as snow and wind vary, the overall shape of such a structure has to be able to change and deform as well. Typical structural features are thrust line, catenary and circle shapes.

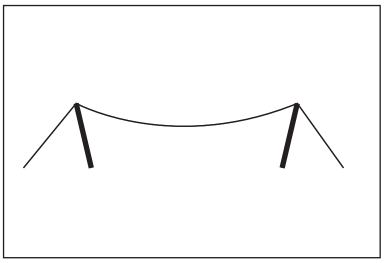


Figure [B20]. Form active principle. Engels 1997

Surface-active

Surface active structures systems are either flexible or compression-, tension-, shear-resistant surfaces. The forces are redirected by the surfaces resistance and the matching surface design. The systems members are mainly subjected to membrane stresses which are acting parallel to the surface, without significant bending components. They are systems in membrane stress condition, considered space enclosing structures.

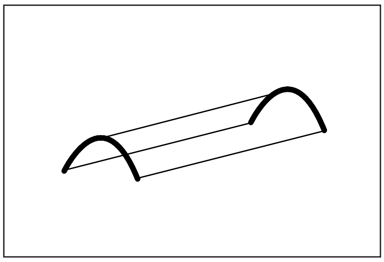


Figure [B21]. Surface active principle. Engels 1997

Height-active

Height-active structures are systems with rigid solid elements in mainly vertical extension. The redirection of forces, which are height loads (story loads and wind loads), is effected through typical ‘height-resistant elements’. The system members are the load transmitters and the stabilisers, both dealing with complex and diverse changing forces.

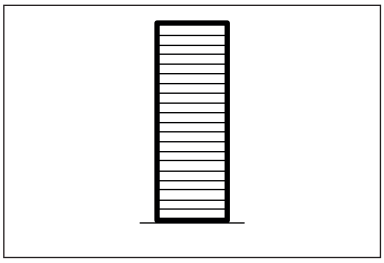


Figure [B22]. Height active principle. Engels 1997

Hybrid-active

In general, hybrid structure systems result from the combination of two structural systems of varying internal load transmission into a coupled system (Lienhard, J., Bergmann, C., Magna, R. L., & Runberger, J. 2017). If the individual structural systems are similar regarding their structural capacity, their coupling to a hybrid system may be in favour of the two, creating a new combined structural system. When additional rigidity through opposite system deflection is enabled, we can speak of a hybrid structural system.

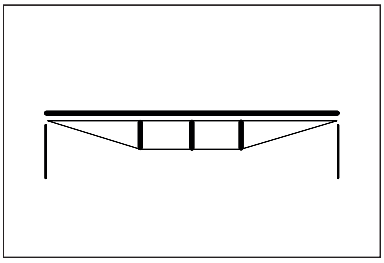


Figure [B23]. Hybrid active principle. Engels 1997

Bending-active

Curved beam and surface structures that base their geometry on the elastic deformation from initially straight or planar elements are considered bending active structures. (Knippers 2011). Active- bending is therefore not considered a structural principle, but rather an approach to creating specific geometries through the systemized elastic deformation process. The force flow through the structure which the determines the structures for and efficiency cannot be generalised as with the structural systems mentioned above. The main characteristic of bending-active structures is not the load bearing capacity or the geometric layout, but it is the formation process where bending is used to get to the final geometry.

The load transfer through an active bending structure includes bending and normal forces. These match with the load bearing of section-active structures. Because bending-active structures can be formed similar to the shape of shells and arches, the load bearing behaviour also acts similar to that of surface- and form-active structures. Although active bending is considered a formation process rather than a structural system, Lienhard (2014) describes bending active structures as:

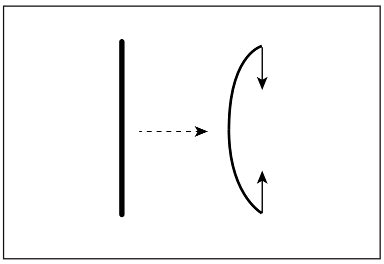


Figure [B24]. Bending active principle.

Bending active structures are structural systems that include curved beam or shell elements which base their geometry on the elastic deformation from initially straight or planar configuration. A bending-active formation is a form of pre-stressing the elements. This is generated through large nodal displacement which results in constant exposure to bending stress. The bending stress of the elements should never reach the elastic limit state of the used materials.

From a structural perspective bending active is considered a constrained statically indetermined structure with residual bending stress (Lienhard 2014). It is obvious that the reason to use active-bending structures lies in the simplicity of creating double curved complex geometries from initially flat elements.

Active bending as structural approach

The matrix of the structural system families presents a systematic overview how they physically appear, i.e., what kind of components are mainly used in such structures, including the primary load transfer and occurring forces. As mentioned before, active-bending can both be seen as a structural system family as well as an approach to create structures with complex double curved geometry (La Magna, Schleider, 2016). For that reason, both the structural system and the approach ave been placed in the matrix on the next page.

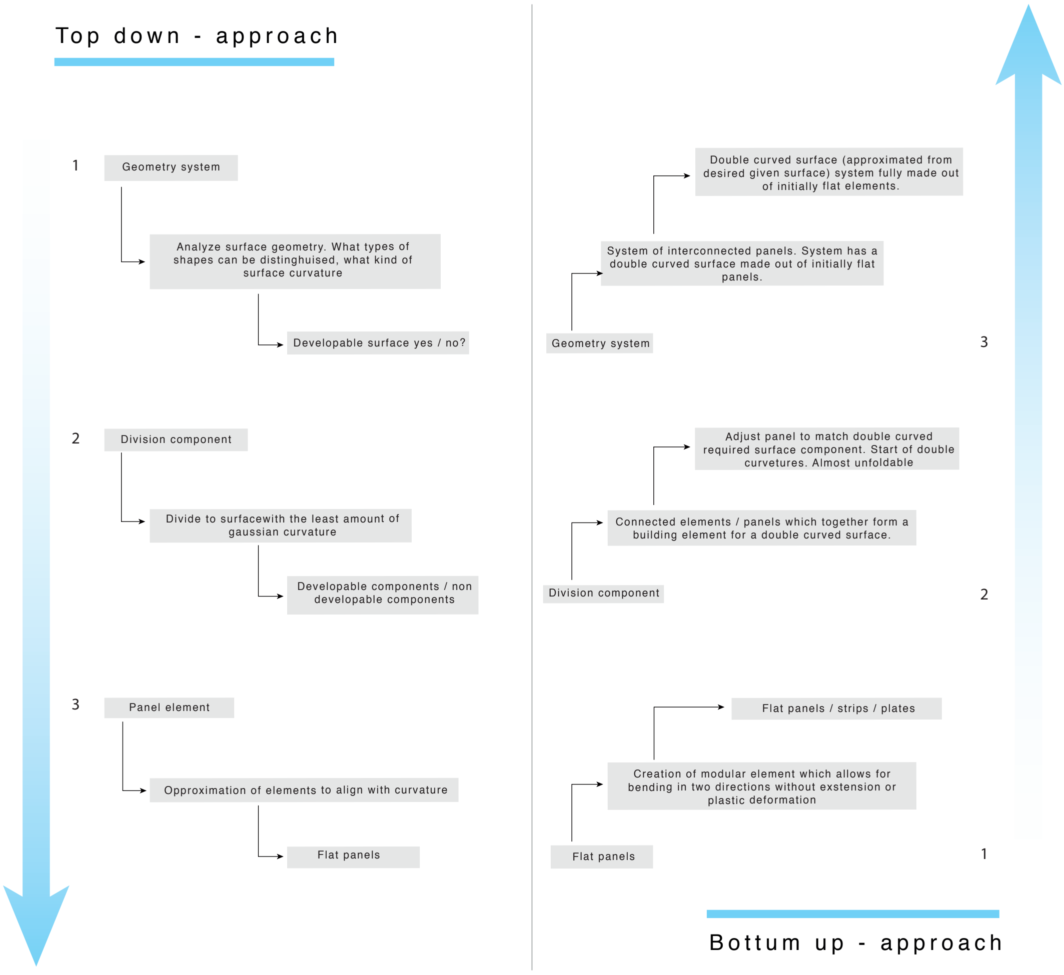


Figure [B25]. Bottom up and top down design approach

Structural system matrix

Because the active-bent shapes are similar to the shapes of arches, vaults and shells and because the transfer of loads through an actively bent shape act similar to that form surface- and form-active driven structures, active bending can be applied as an approach on those three structural systems: section-, surface- and form-active structures.

Vector-active structures are build up from straight elements which are either in compression or tension with force vectors only in the form of axial forces. Height-active structures are composed of rigid elements in a vertical direction, which transfers the combined loads and forces from vector-, surface-, and form-active structures. For these reasons, active-bending as a design approach is not applicable on vector- and height active structures.

Hybrid structures, however, are composed of the combination of different structural

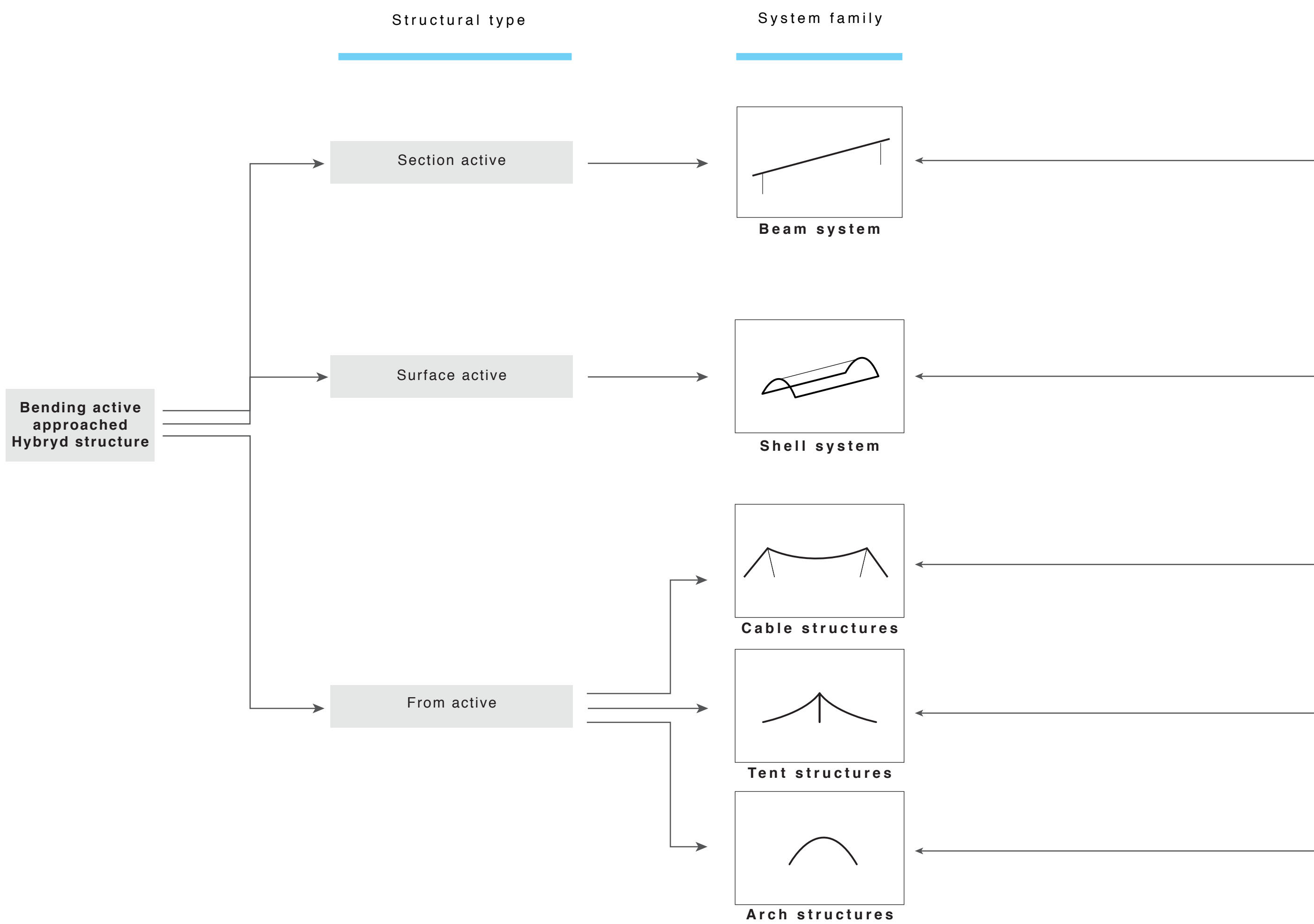
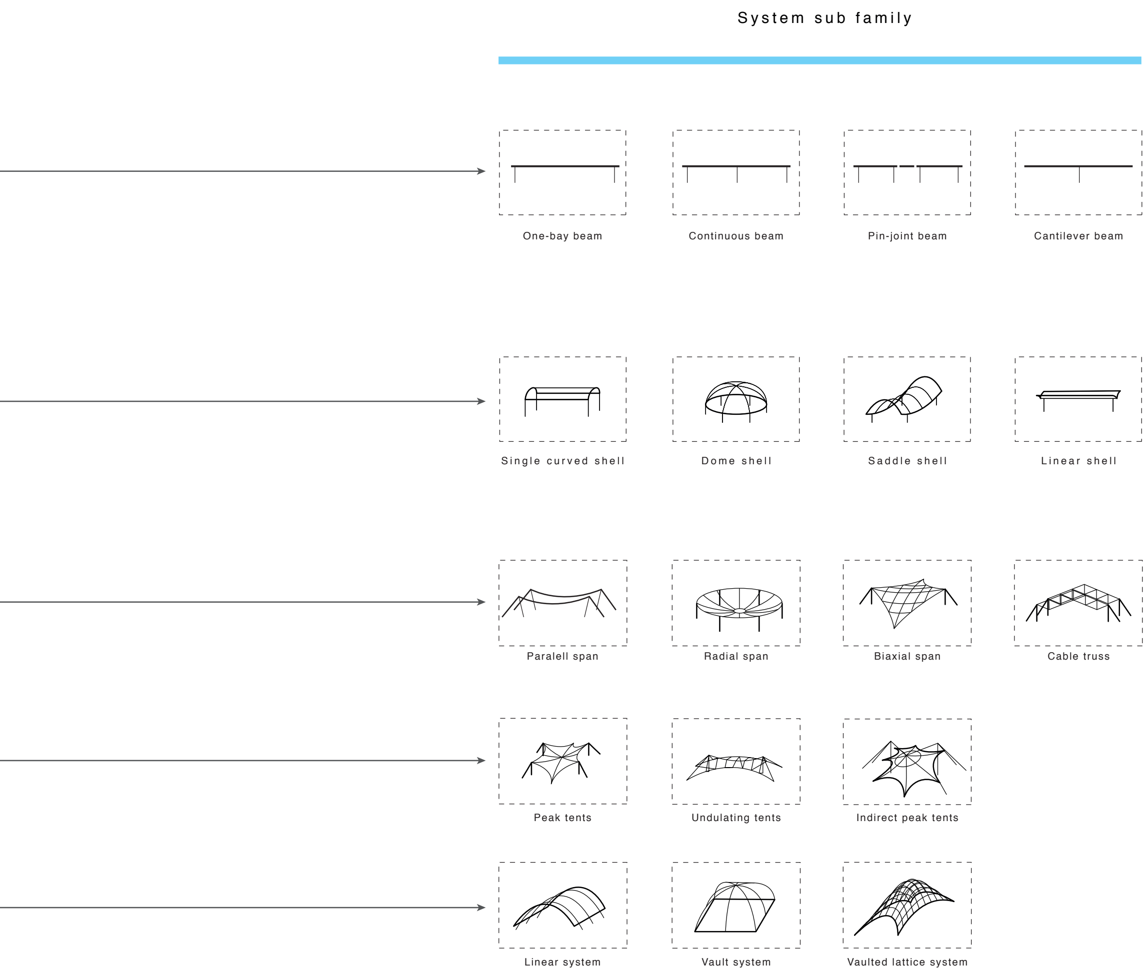


Figure [B26]. Bending active approached stucture composed from existing structural types with their system family.

systems such as section-, form- and surface-active structures. Bending-active structures are very likely to be build up from the combination of these structural systems. The aim is to get familiar with individual structural families and how active bending could be applied as an approach to create such structures. The combination of structural families, i.e., hybrid structures, will not be discussed in this research.

The load-bearing capacities and the design appearance of active bending structures fit within the parent families: section-, form- and surface-active structural systems. A more in-depth matrix including these three-structural systems with their sub-systems provides a better understanding of all the varying configurations to be distinguished. This matrix can be a helpful tool in a later design phase, to refer to existing structural systems and related geometric layout of the structure.



Structural types	Appearance	Forces
Section active	Profiles	Bending, normal force
Vector active	Trusses	Tension, compression
Surface active	Shell	Membrane stress
Hybrid active	Profile, membrane	Superposition and force coupling
Form active	Arch	Tension, compression
Height active	Wall	Complex changing stress condition
Bending active	**profile, membrane, plates**	**residual stress**

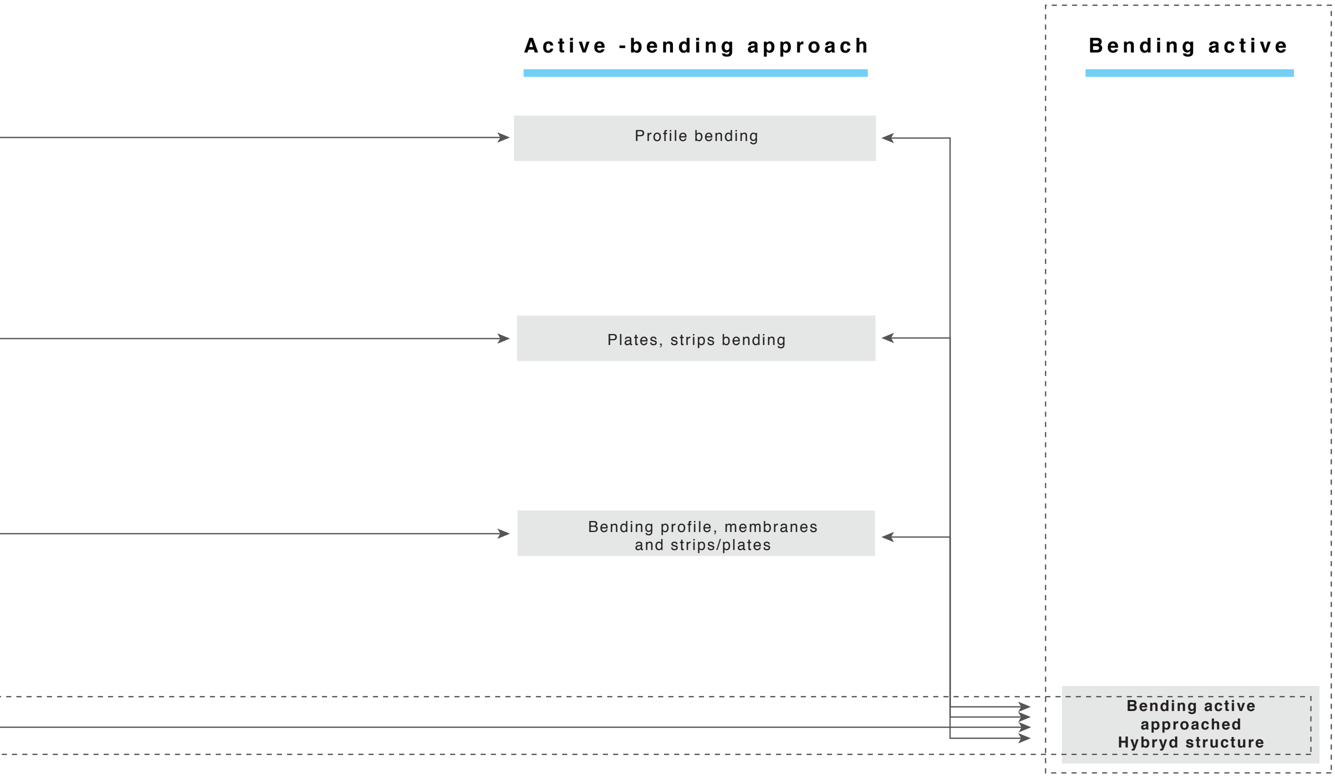
Figure [B27]. Active bending approach emerging from existing structural systems

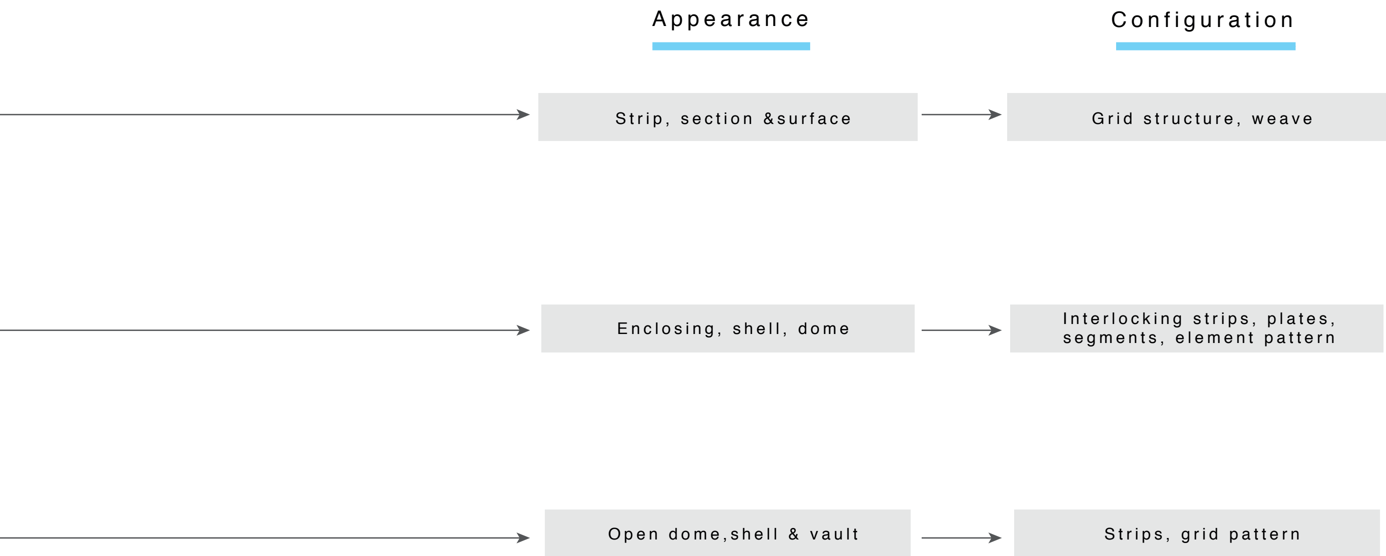
Bending active approached Hybryd structure

Structural type	S Element	M Component	L System
Section active	Rod profile, section	Arch	Grid
Surface active	Strip, plate	Surface	Shell
From active	Rod profile, section	Arch	Vault

Figure [XXXX]

Figure [B28]. Various scale types and appearnces within bending active structural system.


Active -bending approach
Bending active
Profile bending
Plates, strips bending
Bending profile, membranes and strips/plates
Bending active approached Hybryd structure


Appearance
Configuration
Strip, section &surface
Grid structure, weave
Enclosing, shell, dome
Interlocking strips, plates, segments, element pattern
Open dome,shell & vault
Strips, grid pattern

Traditional - integral design approaches

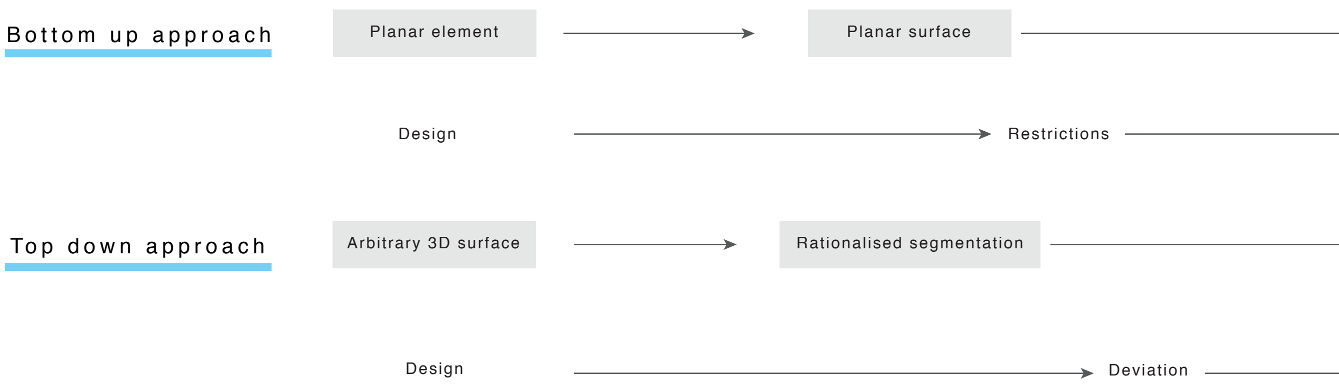


Figure [B29]. Traditional design approach

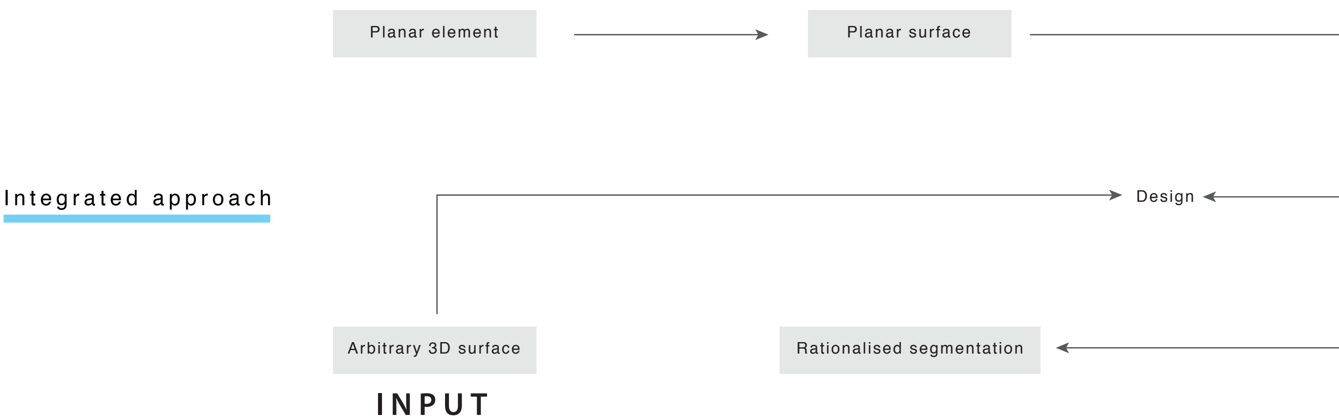
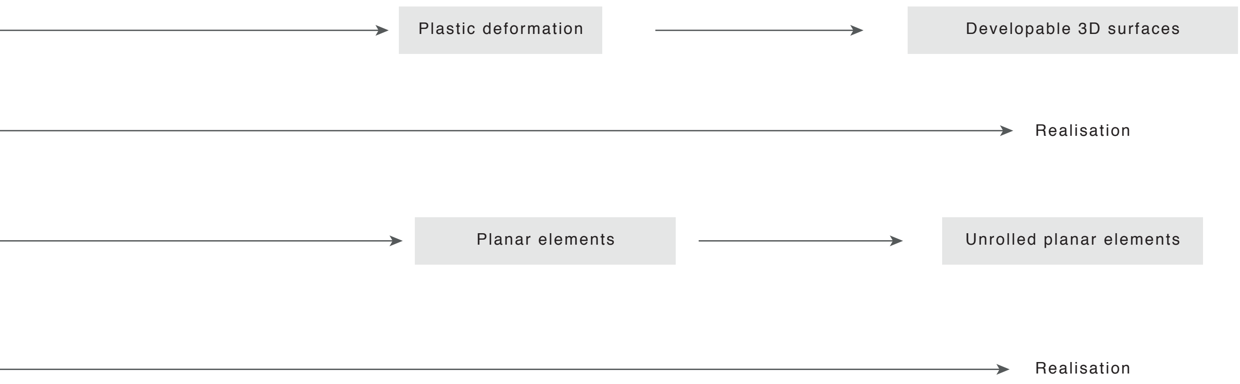
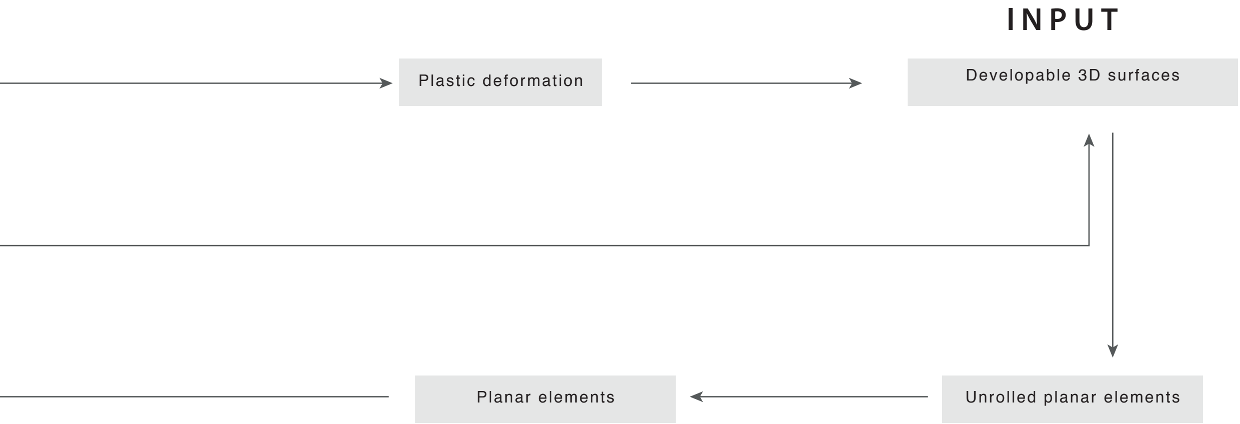


Figure [B30]. Integral design approach


Plastic deformation
Developable 3D surfaces
Realisation
Planar elements
Unrolled planar elements
Realisation


INPUT
Plastic deformation
Developable 3D surfaces
Planar elements
Unrolled planar elements

C Research

Form and Curvature

C.1 Form and curvature

Active bending for double curvature

The motivation of the research in the application of active bending in the building construction lies in the simplicity of creating double curved geometry from initially planar elements. The vernacular architecture shows some construction methods where the elastic deformation of local materials has been used. The introduction of advanced computational software and analyses methods available nowadays, make it possible to create more complex and unique geometry. This software also possesses the ability to simulate materials elastic deformation and erection processes. This results in new types of geometric shell- and grid-structures and membrane hybrid structures. They all vary in construction type, but all share the same approach to the creation of these structures: two-directional curved geometry from initially flat building components. The previous chapter focused on the structural behaviour, the different methods and the structural principles known in today's build environment.

This chapter emphasises on the designed geometry and the double curvature within an arbitrary shape. The creation of these double curved surfaces requires a comprehensive understanding of the types of curvature occurring. Elastic deformation of specific building materials allows for the production of double curvature; however, there are certain extents in the creation of this double curvature, developable with active-bending. Although some geometry may seem visually complicated, most of them can be derived from rationalised elements, reducing the complexity of the fabrication of the elements and the assembly of the final structure. Construction principles may benefit from the rationalisation of initial design intent. The first part of the chapter explains the various types of curvature in both one and two directions, evolving in several types of surfaces that can be distinguished. The last section focuses on the general characteristics of surfaces and the rationalised surfaces that can be derived from complex geometry. The theory of Beranek (1979) and Calladine (1983) will be used as a guideline to explain and describe curavture.

What is surface curvature

To describe and categorize surface curvature it has to be clear what planar curvature is. Line (or planar) curvature is the amount by which a curve deviates from being straight, i.e. a line. Surface curvature is the amount by which a geometric object deviates from being a flat plane, i.e. planar. The description of the curvature in a surface is based on the theory of the curvature of a plane curve. The tangents at two points on a plane curve define the angle of embrace of a segment of this curve. The curvature of this segment can be described by the following formula. The theory of Gauss is used to explain this curvature. This formula is based on an arc with a uniform curvature (a segment of a circle).

$$\text{Curvature} = \frac{\text{Angle of embrace}}{\text{Length of arc}} \quad (1) \qquad \text{Curvature} = \frac{\Delta\alpha}{\Delta s} \quad (2)$$

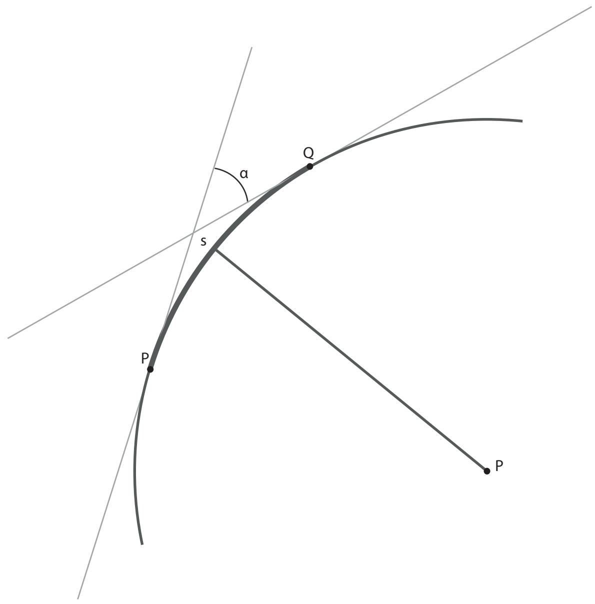


Figure [C1].[The segment of a plane curve substends ‘angle of embrace’]. By Calladine, 1983, Reprinted from Gaussian curvature and shells structures

A differential formula describes a more accurate description of the curvature in a given point on a surface. If point one moves on the curve towards point two and then merge, the difference quotient changes to as differential quotient, which is curvature k in point P [figure C2].

$$\frac{d\alpha}{ds} = \lim_{\Delta s \to 0} \frac{\Delta\alpha}{\Delta s} \quad (3)$$

$$k = \frac{y''}{1+y'^2} \cdot \frac{1}{\sqrt{1+y'^2}} = \frac{y''}{(1+y'^2)^{\frac{3}{2}}} \quad (4)$$

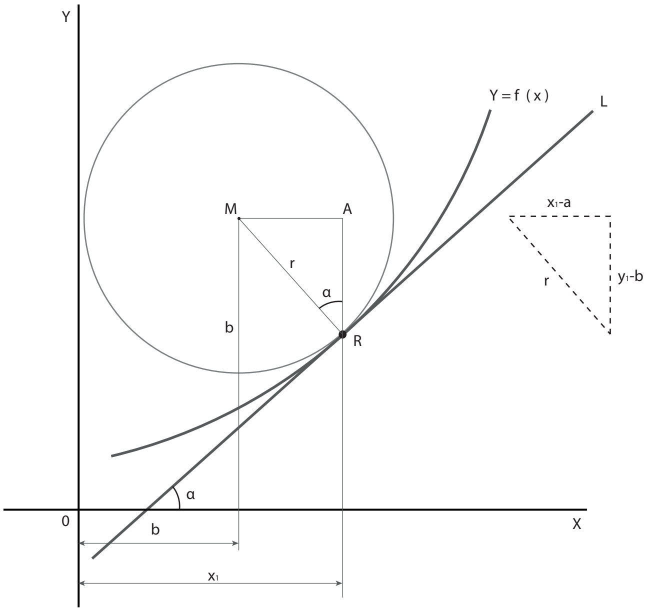


Figure [C2]. Measure of single curvature

$$r = \frac{\left(1 + y'^2\right)^{\frac{3}{2}}}{y''} \qquad (5)$$

Three essential approaches and definitions can be derived from the formula of the osculating circle radius:

1. In the extreme values of curvature is $y' = 0$ which gives $r = \frac{1}{y''}$. r Is negative for the maximum value and positive for the minimum value.

2. In a concavity $y'' = 0$ which makes r = ∞ . The osculation circle disconnects from the tangent curve. The curvature on each side of the concavity is of opposite value.

If y' is small relative to 1, r is $r \approx \frac{1}{y''}$. For example, the tangent of a curvature with a small angle relative to the x-axis in case of a minor load on a horizontal beam.

For a broader definition of the curvature of a smooth curve in a specific point, one can use the limit of 1 as the arc length of the segments tends to go zero.

If 1 is taken as the definition of curvature, it is easy to show through elementary geometry that:

$$\text{Curvature} = \frac{1}{\text{Radius of curvature}} \tag{6}$$

Where the radius of the curvature is the radius of the circle that is tangent to the curve at the point considered. Gauss introduced the idea of an auxiliary circle, with a unit radius, onto which points on a curve can be mapped to a rule of parallel normal [figure C3]. Point Q and D on the curve map to q and d on the auxiliary circle. The angle of embrace of the segment QD is equal to the arch length dq of the circular image on the auxiliary circle. For that reason, the curve can be defined by the appropriate limit of:

$$curvature = \frac{\text{Arc length pq of circular image}}{\text{Arc length PQ of curve}} \tag{7}$$

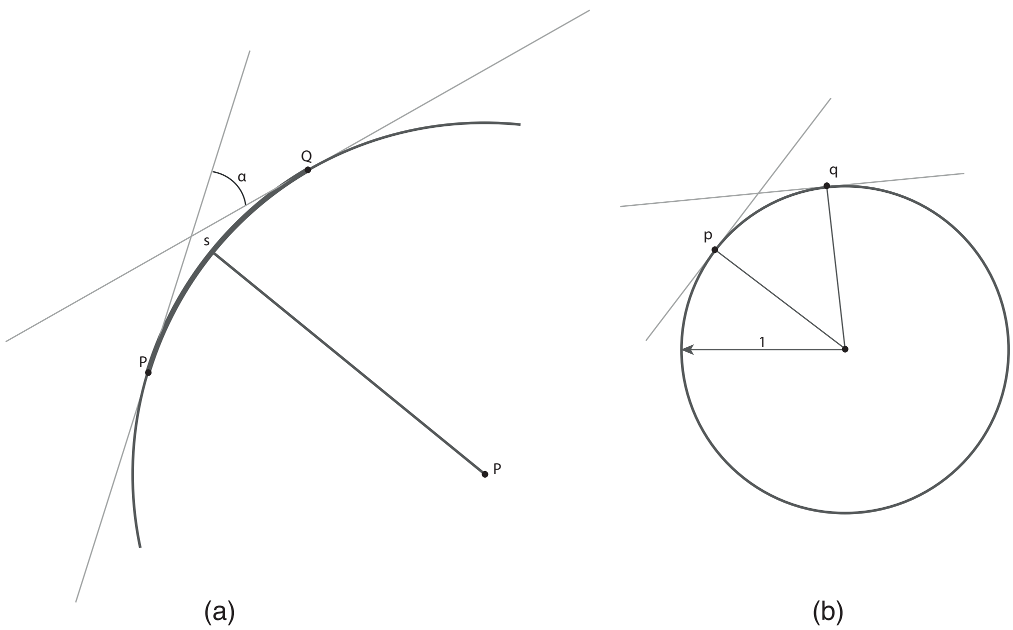


Figure [C3]. [(a) the segment PQ of a plane curve subtends an angle of embrace alfa. (b). point qp on the curve map to point qp on the auxiliary circle of unir radius, by a rule of paralell normals]. By Calladine, 1983, Reprinted from Gaussian curvature and shells structures

Gaussian curvature

Similar theories applied to general surfaces in a three-dimensional space show similar results. Points on a surface can be projected along the parallel normal to points on an auxiliary sphere with a unit radius. The curve through these points is a closed boundary curve defining a region of a surface which has a spherical appearance in the form of a closed curve on the auxiliary sphere. The theory of Gauss defines the area of a curve on the auxiliary sphere as the 'entire curvature' of the region of the original surface, by direct analogy with the plane curve and its circular image (calladine, 1983)

The surface area of the spherical image is what he called the 'solid angle of embrace' of the region of the surface. With the theory of the planar curvature and the curvature stated above, Gauss created a definition for the measurement of the curvature of a surface:

$$\text{Gaussian curvature} = \frac{\text{Area of spherical image}}{\text{Area of region of surface}} \tag{8}$$

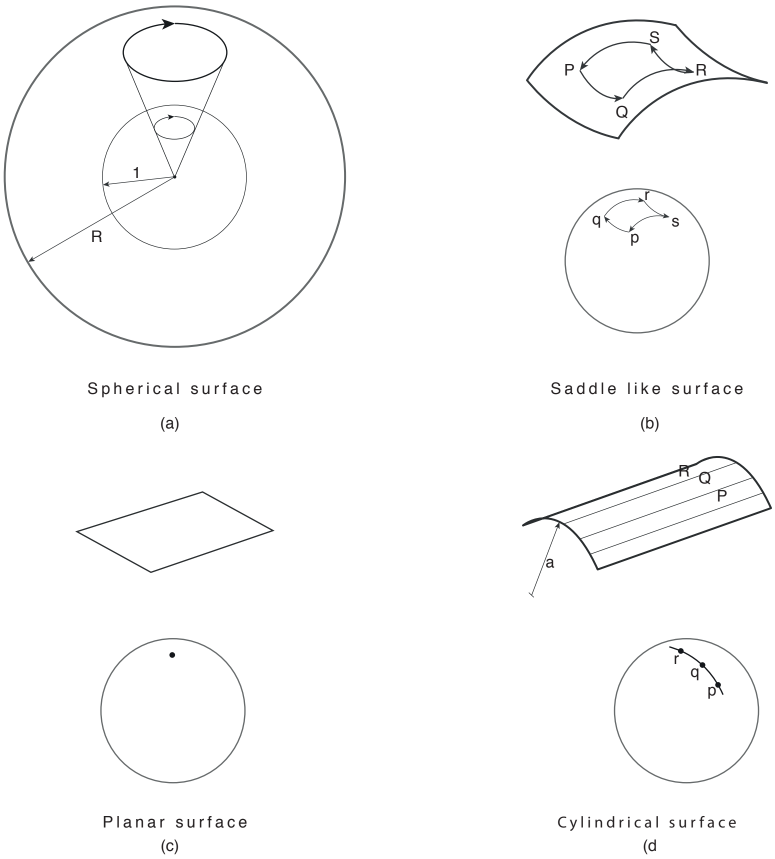


Figure [C4] [Examples of the projection of curved surfaces onto the unit sphere: (a) spherical surface, (b) saddle like surface, (c) planar surface, (d) cilindrical surface]. By Calladine, 1983, Reprinted from Gaussian curvature and shells structures

The formula for a planar curve (formula 1) leads straight to formula 2, which is an alternative definition. The same account in this three-dimensional situation where Gauss showed that on a smooth surface in a certain point the Gaussian curvature is equivalent to:

$$\text{Gaussian curvature} = R^{1^{-1}} \cdot R^{2^{-1}} \tag{9}$$

Principle curvature

R 1 and R2 are the minimum and maximum radii of curvature of the surface in one specific point. These two radii are the principal radii. R1-1 and R2-1 are the principal curvatures. Principle curvatures can be described as the minimum and maximum curvature of sections of the surface cut by a rotating plane around the n axis at a specific point [figure C5].

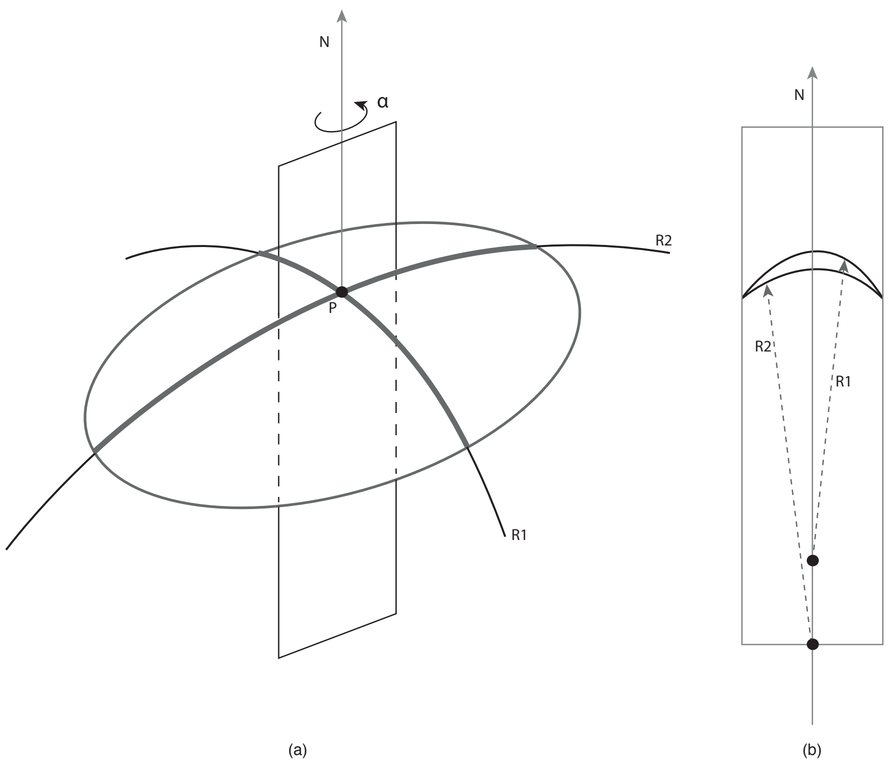


Figure [C5].[Definition of principal radii, r1, r2 at a typical point P on a smooth surface]. By Calladine, 1983,
Reprinted from Gaussian curvature and shells structures

Mean curvature

Mean curvature is the extrinsic measure of curvature from the differential geometry, that measures the curvature of a surface in space. As described and illustrated in the previous paragraph, a point on a surface has a maximum, and minimum curvature called the principal curvatures. The mean curvature is the average of all curvatures over an angle α in this specific point. The mean curvature is equivalent to the average of the principal curvature:

$$\text{Mean curvature} = H \tag{10}$$

$$H = \frac{P_1 + P_2}{2} \tag{11}$$

Because the Gaussian curvature is the product of the principal curvatures, the mean curvature can also be described as the average Gaussian curvature.

$$H = p^2 - 2Hp + K = 0. \tag{12}$$

The mean curvature is used in the expression of the geometrical phenomena: minimal surface. If the mean curvature is equivalent to zero, a surface has the least area.

$$H = 0 \tag{13}$$

The various types of Gaussian curvature

The easiest way to prove that formula (4) and formula (5) have equal results is through the example of a sphere with radius R. The linear dimensions of an arbitrary curve on the surface are R times those of the exactly similar image on the auxiliary sphere, so the ratio of areas is R2. This leads to:

$$\text{Gaussian curvature} = R^{-2} \tag{14}$$

In this example, both the radii are positive. The resulting Gaussian curvature is therefor also positive. A certain point on the surface and the according image point on the auxiliary sphere move in the same sense round their respective closed perimeters. In the case of

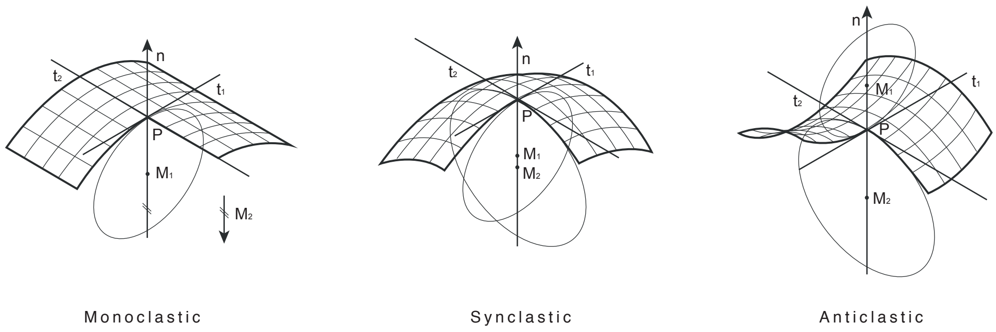


Figure [C6]. Three Gaussian curvature measures. Monoclastic, Anticlastic and Synclastic

one negative radius and one positive radius, the Gaussian curvature becomes negative. The principle curvatures have their centers of curvature on opposite site of the surface. This corresponds to the fact that the point on the surface and the spherical image move in opposite senses round their respective closed curves. The shape has a saddle like geometry. In case of planar and cylinder formed surfaces, the Gaussian curvature is zero. Every point on the plane maps to a single point on the auxiliary sphere. Any closed trajectory on the plane maps to a point. Because there is no enclosed area, the Gaussian curvature is zero. With a cylindrical surface, every point maps to a point on a single great circle of the sphere, normal to the axis of the cylinder. Because no area can be enclosed on the auxiliary sphere, the Gaussian curvature is zero.

C.2 The geometry of triangular surfaces

The construction of a double curved synclastic surface can be approximated by replacing the smooth surface by a network of smaller triangular planar surfaces. This approximation is called the triangulation version of a smooth surface. Some features will be lost in this approximation. A planar curve is approximation is similar to that of a surface. The smooth curve may be approximated by some straight-line segments, set off at a small angle α from each other (a). If all the segment lengths are equal to l, and the angles are all equal, then the points lie on a circle. The approximated curvature of that circle is :

$$\text{curvature} = \frac{\alpha}{l} \tag{15}$$

With α in radians.

It is simplest to use segments of constant length l. At each kink α is chose so that:

$$\alpha = curvature \cdot l \tag{16}$$

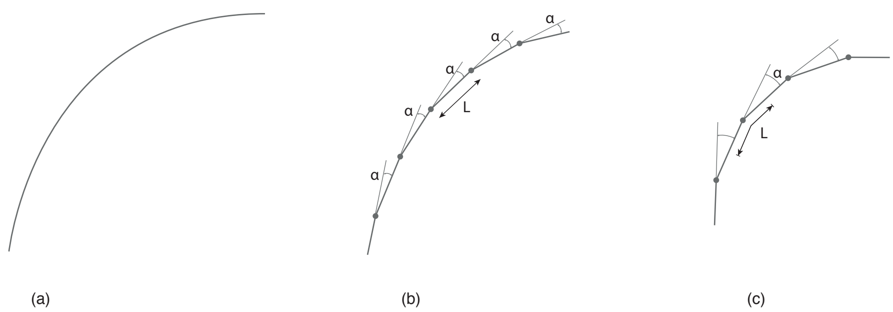


Figure [C7] [Planar curve can be represented by a series of straight lines (b) which are only uniforn in a circle, but generally non-uniform (c), vertexes in (c) are associated with the sum of half-lengths of the segments], By Calladine, 1983, Reprinted from Gaussian curvature and shells structures

In the case of smooth curve set out by straight curves with a varying segment length, L now represents the sum of the two half-segments which meet at the angle α.
The corresponding rule for the construction of a surface with a given Gaussian curvature can be described by:

$$\text{Gaussian curvature} = \frac{\text{angular defect at a vert}}{\text{area associated with the v}} \tag{17}$$

The angular defect is 360 degrees, or 2π – a sum of interior angles of faces meeting at the vertex. The angle of embrace of a vertex is always equal to the angular defect of the vertex.

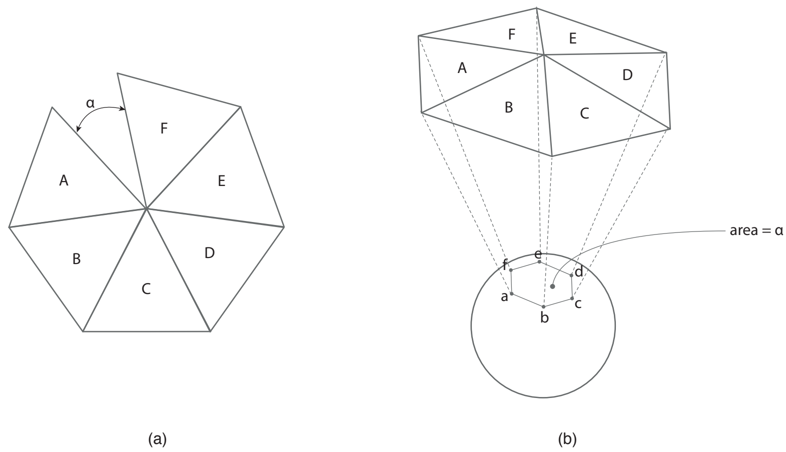


Figure [C8]. [The segment of a plane curve substends 'angle of embrace'] (a) when cut and flattened. (b) Solid angle of embrace = (enclosed area on unit sphere). By Calladine, 1983, Reprinted from Gaussian curvature and shells structures

C.3 Rationalised double curved surfaces

The construction of complex geometric surfaces can be highly influenced by the amount of curvature, the level of continuity and the description of their offset surfaces (Straat, 2011). In general, three distinct manufacturing process are distinguished.

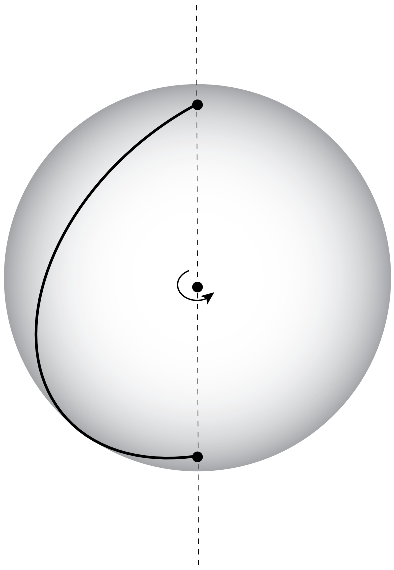
1. Construction processes based on the removal of material
   CNC milling
2. Material deformation
   Bending, **extending**
3. Mould casting
   Cast concrete

These processes have been used in building construction for a long time. The modelling software available nowadays renders a new approach for these methods with more detailed complexity. When a surface serves a load-bearing function, it is considered both the external shape and the internal design volume. An important step is a translation from an initial desired design towards a feasible construction method. The constructability of a surface depends on the ability to rationalise the surface. These rationalised surface 'classes' all have their qualities and disadvantages for construction. To understand and distinguish the varying surface classes, the focus will be on three main surface construction types: Surfaces of revolution, ruled surfaces and translational surfaces.

surfaces of revolution

The best example to explain a surface of revolution is a spherical image. The 360 degrees rotation of half a circle around an axis generates this simple surface of a sphere. The profile curve can have any form of open or closed planar curvature. The final shape of the surface depends on the location and the direction of the generator and the shape and location of the profile curve. This small amount of input parameters describe a surface of revolution. The main advantage is the fact that offsets of surfaces of revolution are easy to describe than other types of surfaces, as they all base their geometry from the same generator axis. This is positive for architectural design as the initial design often changes due to approximation based on leading geometric load capacities. A negative quality is the relative difficult constructability of such shapes. Additionally, final surface shapes based on curved profiles

Figure [C9] [Shaping on potter’s wheel] Reprinted from Flickr website, by Ethan Schoonhoven, 2004, retrieved from https://www.flickr.com/photos/ejas/272228452



360 degrees revolving curve around generative axis

Figure [C10]. Curve rotation around central axis, surface of revolution

are non-developable. The complexity of constructing such shapes is highly reduced when straight line profiles are used to define the shape of a surface.

ruled surfaces

Ruled surfaces base their shape on a sweep motion of a straight line along a curve. The surfaces contain straight line generators, which makes them ideal for architectural application. Pottman (2007) distinguishes two methods to create a rules surface.

-The connection of corresponding points on two generating curves.

-Moving of straight lines on a directive curve.

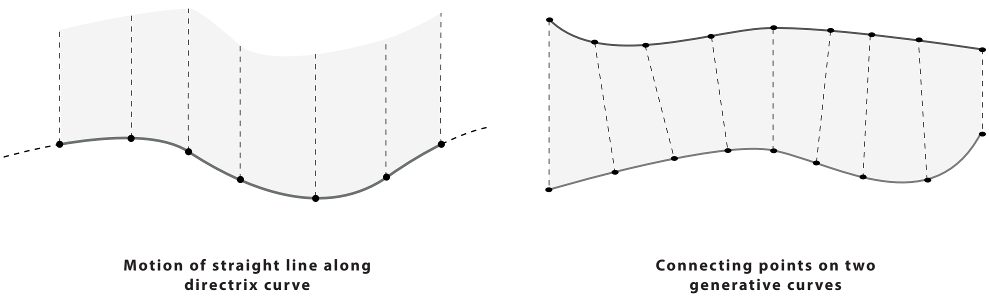


Figure [C11] (a) straight lines over directrix curve. (b) two point connection generative curves

The sweeping motion of a straight line determines different subclasses of ruled surfaces. Each of these subclasses has their advantages but also come with limitations related to the architectural design freedom. Two subclasses are:

-Extruded surfaces
-Developable surfaces

Extrudes surfaces

An extrude surface originates its shape from the translation of a generative curve in the direction of a straight line. Extruded surfaces are subclasses of translational surfaces (see next chapter) if the path the generative curve follows is a straight line. The motion along a straight-line results in two types of extrusions. A parallel extrusion generates a cylinder surface, and a central extrusion creates a cone surface. These two types are both monoclastic and are the basis of developable surfaces.

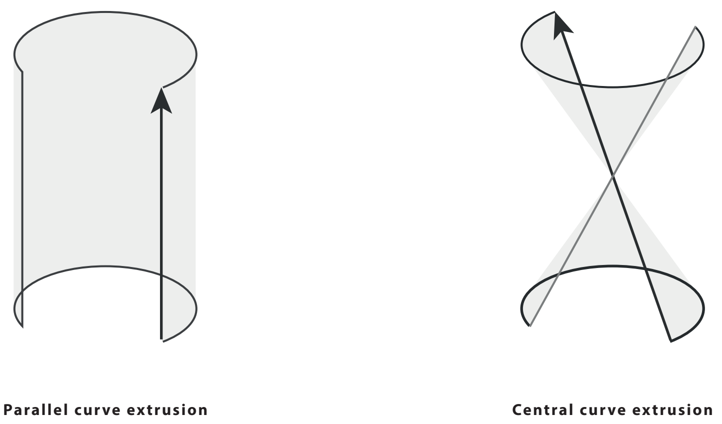


Figure [C12] (a) cylindrical shape, (b) conical shape. Two extruded surface generations

Developable surfaces

Developable surfaces have the advantage of being created from a planar surface material, without plastically deformation. Huffman(1976) states that a developable surface offers a complexity that is precisely the midway between that of a completely random surface and that of a plane surface. In other words, developable surfaces provide a more fertile potential compared to planar surfaces and more tractable analytical than entirely random surfaces. The critical characteristic of a developable surface is that all points on a given isoline embedded in the surface have the same tangent plane. The neighbourhood of a point on a paper surface can be defined by a single-parameter family of tangent planes. Additional to the two-described cone and cylinder surfaces, tangent surfaces of space curves are considered developable surfaces (Straat, 2011).

In a ruled surface which is non-developable, different points on the same generator curve have different tangent planes; they rotate around the generator (Huffman 1976). These are called non-torsional generators. Developable surfaces only contain rulings with just one tangent plane touching the surface along the complete line. These are called torsal generators.

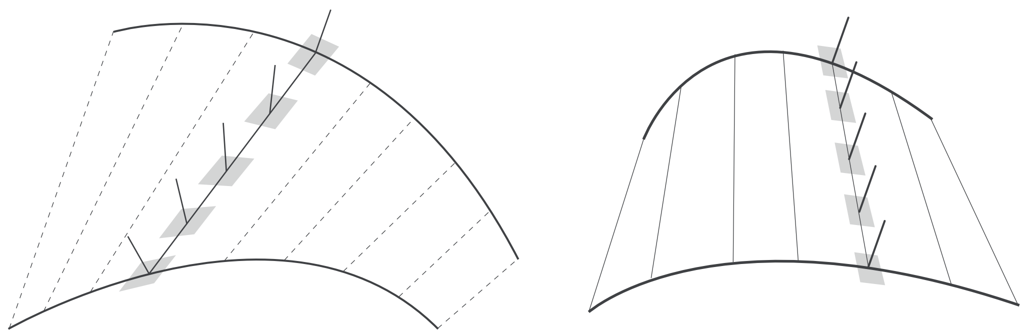


Figure [C13] Normal axis of generator curve. (a) non developable, (b) developable.

To design developable surfaces, it is required to generate the input curves based on specific rules. Rulings reflect the character of the curves they span. If the boundary curve is fair, the locus of rulings defining a developable surface between them will also be fair an continuous. The problem is only reduced to finding a fair representation of the boundary curves (Nolan,1970).

Developable surfaces have a Gaussian curvature value zero. Vice versa, surfaces with a Gaussian curvature value zero contain generating lines in at least one direction. They can also be mapped isometrically. The planar isometric image of a developable surface is the planar unfolding of the surface.

Translational surfaces

The third rationalising of a surface is the translational surface. Complex surfaces often get rationalised into subdivided individual planar components. This meshing, or subdivision, splits the surface into tiles. This phenome is called tessellation, or a tessellated surface. The initial designed form or surface has to be constructed which requires a matching strategy to translate the shape into buildable components. The size of these components, or mesh faces, are based on the building strategy and the overall layout. Commonly a surface gets divided into three, four or six-sided polygons: triangles, squares or hexagons. They are symmetric, have equal side lengths and same angles (Blackwell, 1984). A lot of today's building costs of such shapes goes to the curvature of the mesh faces and the connections in between the faces (Blackwell, 1984). The benefit of a triangle mesh is that it is always planar. The easiest way to approximate a complex surface is through triangular meshing. It is evident that for both construction and economic reasons, triangular meshes are mostly used in construction of complex surfaces. However, twice as many triangles are needed to present the same shape as a quad mesh face. A planar quad mesh would be the most economical approach. To be able to approximate complex surfaces with planar quad meshes on complex surfaces, specific design regulations have to be followed. Translational surfaces are surfaces designed with these regulations.

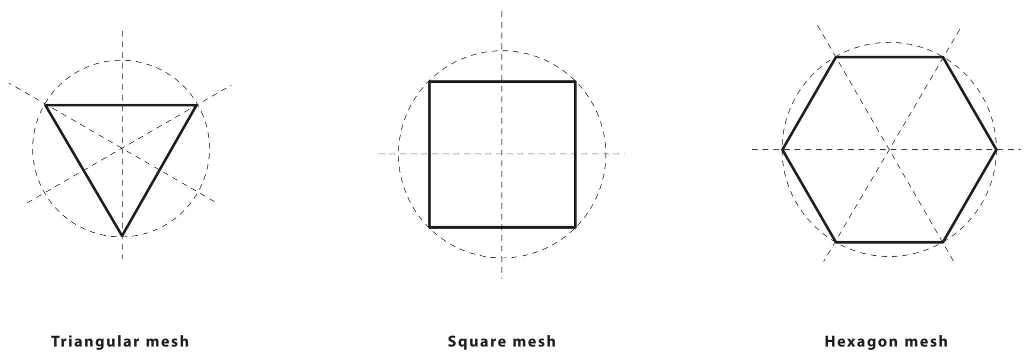


Figure [C14] Rationalised surfaces. (a) triangular, (b) squared, (c) hexagonal.

Translational surfaces are very similar to extrude surfaces, discussed in the previous paragraphs. The translation of a generative curve, also called the generatrix, along a trajectory curve, also called the directrix, is the generalisation of extruded surfaces. If both the generatrix and the directrix are arbitrary shaped curves, the surface may have a high overall curvature. A translation surface always has two sets of section curves which are precisely similar to the profile curves. This method doesn't result in any arbitrary surface shape to be created. Still, a large number of varieties of surface forms are possible.

The method for the creation of translational surfaces is based on the mathematical description of two parallel vectors in space, always defining a planar surface. If the vectors or non-parallel but remain their position in the same plane, the surface will be planar as well.

If only the vectors of one direction are parallel, a different type of translational surface is generated. This is called a scale-trans surface. When the section curves in the lateral direction are scaled, the vectors in the other direction are not parallel anymore. The scale of the section curves causes each lateral vector to become longer or shorter by the same scaling factor, without change of direction.

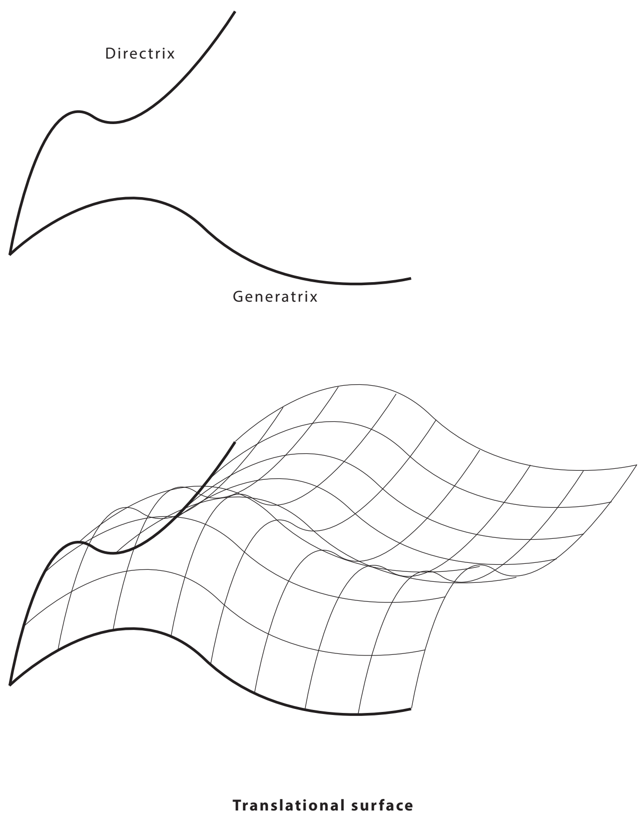


Figure [C15]. Surface based on generatrix and directrix.

Translational surfaces have frequently been used in construction with the core motivation of creating surface meshes in which all four joints are on one plane so that they can be covered with planar panels.

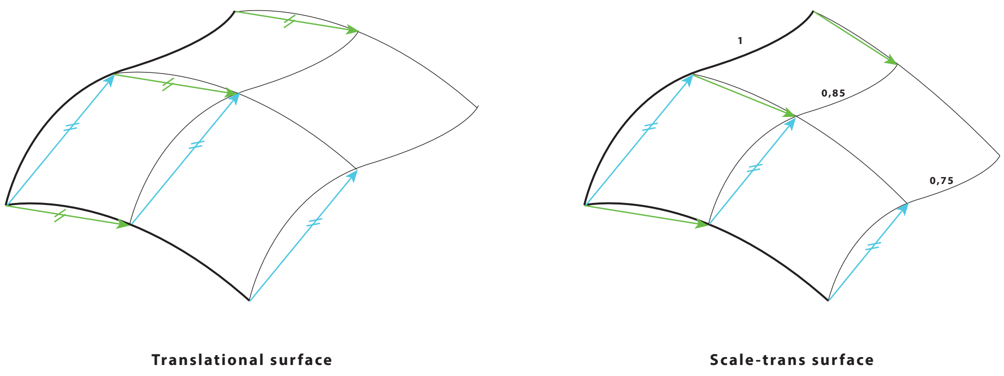


Figure [C16](a) translation surface, paralell vectors. (b) scaled translational surface, non-paralell vectors.

From arbitrary surfaces towards rationalised surfaces

Today's software and simulation techniques offer a wide range of modelling tools to create arbitrary shapes and surfaces. However, in architectural design, double curved surfaces are not directly based on a parametric surface description. Therefore, it is hard to derive their local properties such as normal direction and curvature. Various modelling methods of generating developable surfaces, either via approximation or direct digital modelling (Rose, Sheffer, Wither, Cani, & Thibert. 2007). Some of these methods will be discussed below.

Developable approximation

With a nondevelopable surface as given input, a large number of methods aim at approximating it with one or more developable surface patches. These are based on, for example, the combination of conical surfaces, the generation of strips or the deformation of double curved surfaces to approximate developable surfaces (Gonzalez-Quintial, Barrallo, & Artiz-Elkarte. 2015).

The approximation approach is highly restricted in the amount of Gaussian curvature of the given input surface. In most cases, the final result of the approximation is mathematically not developable. If the surfaces need to be realised from planar elements, this might cause problems in the manufacturing setup. In such setups, the distortion caused by the use of unfolded approximated developable surfaces can be significant (Straat, 2011). The approximated developable would have to deviate more from its original surface.

Direct modelling

Most of the modelling methods for developable surfaces require a clear specified ruling direction for the surface. Most of the developable surfaces are based on the input of polyline directrices. The output is a developable surface strip where the interior edges approximate a ruling by connecting the two polylines.

A more advanced direct model technique is the deformation of an original planar strip into the desired shape using bending and physical deformation.

Characteristics of developable surfaces

Developable surfaces embed specific geometric characteristics. The possibilities for active bending to employ these characteristics are related to the restrictive boundary conditions of developable surfaces. This chapter presents an overview of the developable surface properties and focusses on the elementary differential geometry concerning a parametric description of developable surfaces.

Ruled surfaces are the fundamentals of surface design and are very important for the design of rationalised surfaces. A ruled surface is the definition of a surface between two given space curves and a set of straight line segments. Ideally, these two lines would have the same degree and are parameterised over the same interval. If all points on the curves are joint by straight line segments, a ruled surface connecting the curves is obtained. Because through every point on a rule surface runs a curve that has normal curvature of zero, the principal curvatures are not the same. This is why the Gaussian curvature is negative.

Ruled surfaces follow the concept of interpolation. Every curve over the surface with a constant u value is a straight-line segment. The tangent plane at a point on a ruling varies as the point moves over the ruling. A group of different tangent planes define the double curvature of that surface. Ruled surfaces mostly appear to be straight in one direction which is why they are easily confused with developable surfaces (Straat, 2011).

As discussed in the previous paragraph, developable surfaces have a zero Gaussian curvature. However, practical developability is obstructed by self-intersections of a surface (Straat, 2011). If the definition of a tangent surface, tangents of the input curve in both directions are taken, the surface is self-intersecting at this curve. This is called the curves of regression. If generators do not intersect anywhere except for the corve of regression or a cone's apex, then the surface is developable.

Three developable surfaces can be distinguished; cylinders, cones and tangential developables. Cylinders and cones are ruled surfaces where the generators are respectively parallel to a fixed direction passing through a single point. This makes them relatively restricted in their shape variation which is why they often tend to be difficult to use. The main advantage is that they can be easily subdivided into flat faces; rectangle faces for cylinders and triangle faces for cones.

A tangent surface is a surface which spans between two ruling which are the tangent lines of a space curve. This space curve is called the edge/curve of regression. The entire tangent surface is determined by means of its edge of regression (Glaeser, 1972).

C.4 Continuity of curves

The continuity of a composite curve (a curve combined with two or more single curves) depends on the joint conditions and the geometrical relationship of the control points adjacent to the joints. When segments are joint at a common point, their first (n) derivatives should be equal at that point. This is called the nth order of continuity. If two curves are joined at their endpoints, the continuity is n = 0. If the tangent vectors at the joint of both curves have the same direction, n = 1. If at the joining point the second derivative is parallel to the tangent vector, n = 2.

For n = 0, 1 or 2, different characteristics of G continuous curves can be distinguished.

G0 Positional continuity: The end positions of two cures are coincidental. They may meet at an angle leading to sharp corners and edges. Visually this will cause broken highlights.

G1 Tangential continuity: The end vectors of the curves required to be parallel. Considered sufficiently smooth.

G2 Curvature continuity: The end vectors of the curves required to be of the same length, direction and rate of length change. Considered perfectly smooth.

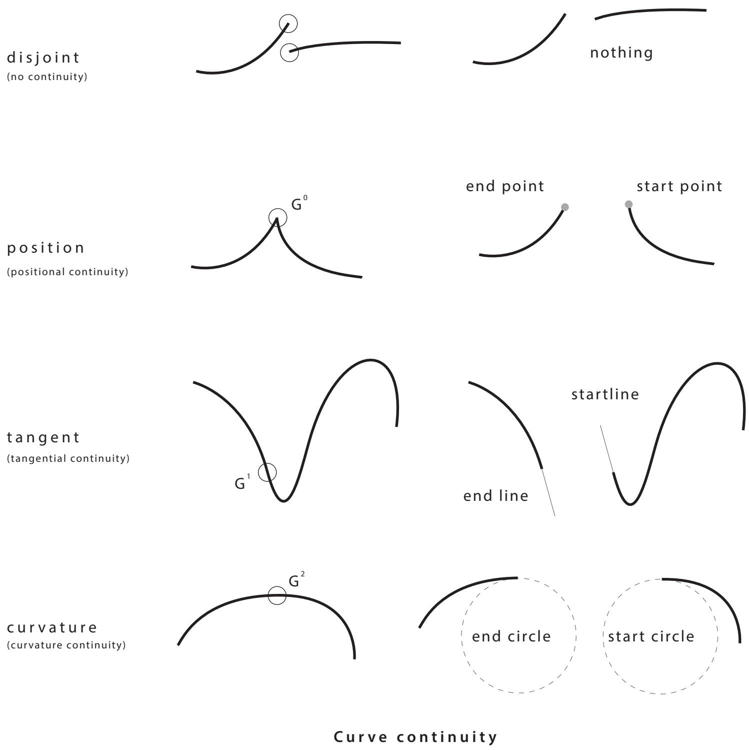


Figure [C17] Levels of continuity curves for curves

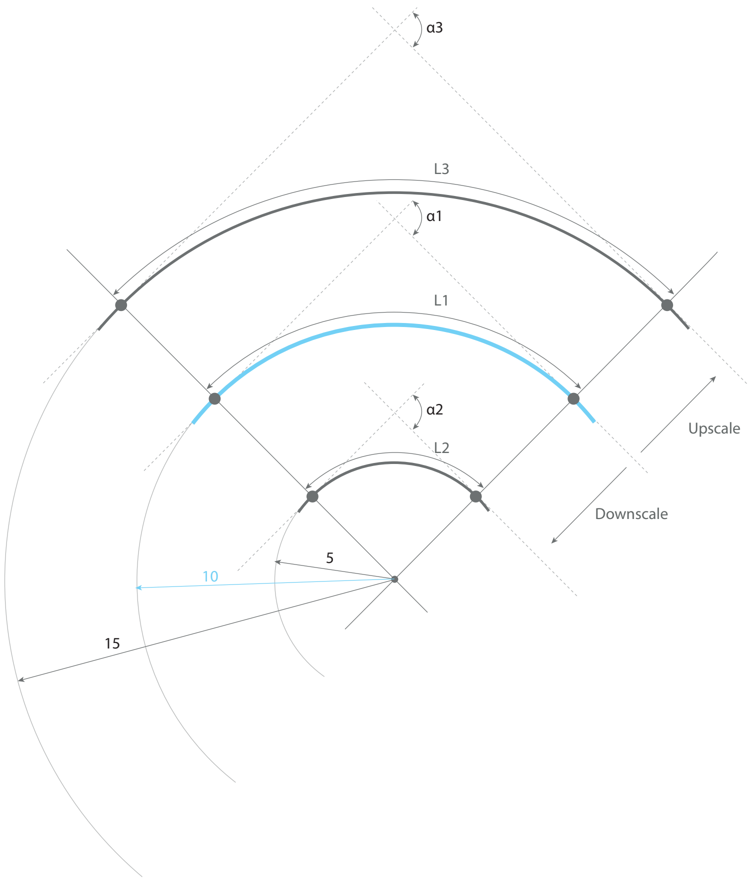
Surface continuity

Form conversion is the process of creating developable surfaces from arbitrary double curved surfaces. The level of continuity of the initially double curved surface must usually be sacrificed to be able to build a double curved surface. However, by adjusting the layout and topology of the initial surface in the design phase, the form conversion approach guarantees the bendability of the elements and in this way the ability for two-dimensional elements to span multiple directions (La Magna, & Knippers. 2017).

It is therefore highly relevant to understand the adjustment of the layout and topology of any arbitrary double curved surface. The elaborated modelling techniques and double curved surface description provide valuable knowledge with can be used in any design phase of arbitrary geometry.

Scale and offset double curved surfaces

The offset of a planar surface is a one to one copy of the original surface, moved along the axis perpendicular to the original surface. The offset of a single- and double-curved surface is different from the offset of a planar surface. The radii of a curved surface and the surface area changes, leading to a change in curvature. In figure C18, the blue curve is scaled proportionally, both up and down. The initial radius of 10 moves to a radius of 5 and 15. The length of the arches changes from 5π to 2,5π and 7,5π. The only constant parameter is the angle of embrace, which is the angle at which the two tangent curves meet.



r1	5
r2	10
r2	15
L1	2,5 π
L2	5 π
L3	7,5 π
α1	0,25 π
α2	0,25 π
α3	0,25 π
curvature 1	1/10
curvature 2	1/20
curvature 3	1/30

Figure [C18] Linear scaling curved curve

The increased arc length of a scaled curve is linearly proportional to the increased curvature radius. The increased surface area is quadratic proportional to the increased curvature radius. The offset of a double curved surface can be determined using the increased radius and the given constant angle of embrace of the principal radii.

D Implementation

Design direction

D.1 Design methodology

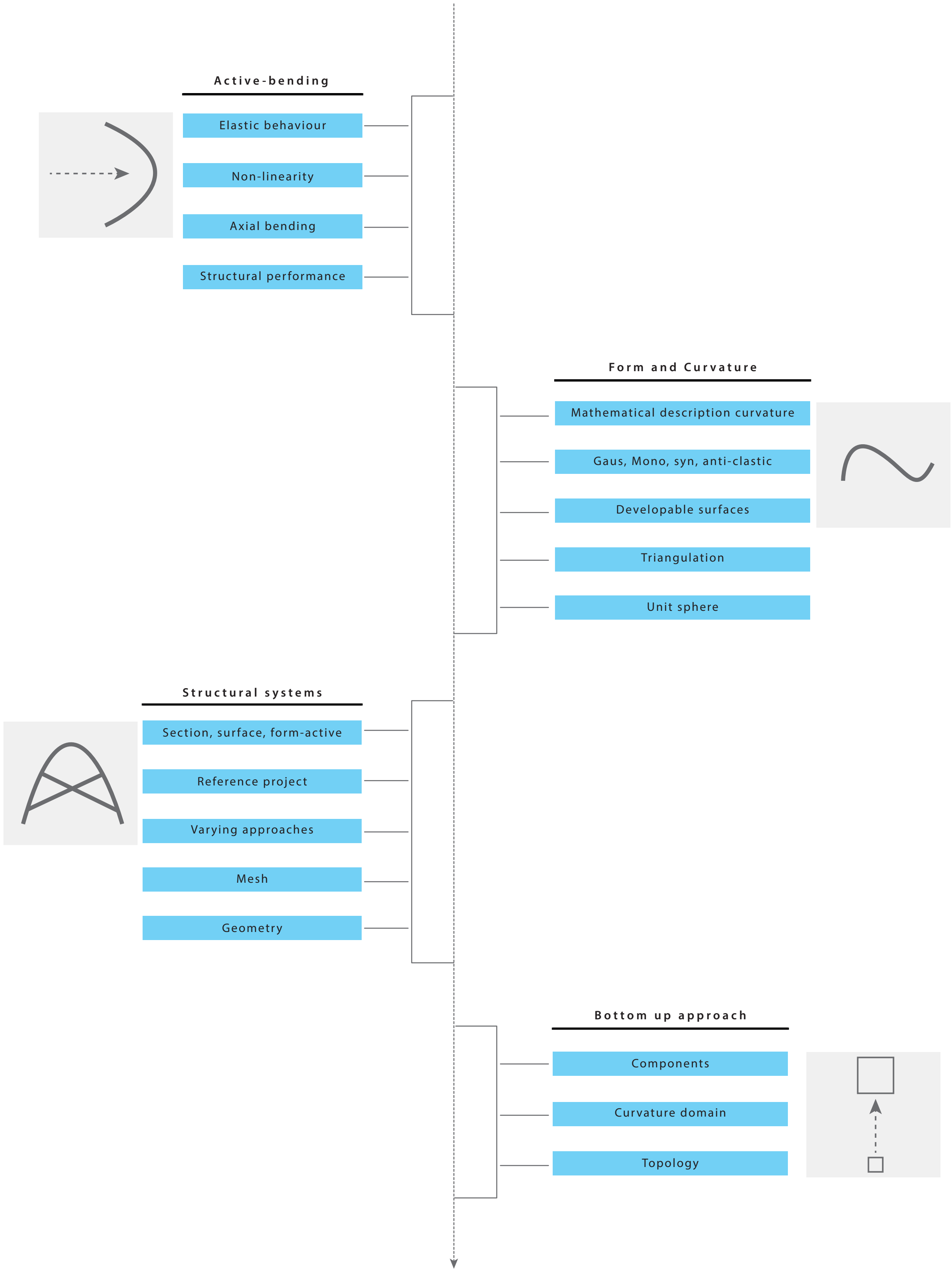
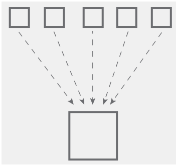


Figure [D1]Design methodology.

Implementation

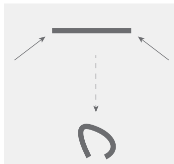
Design parameters



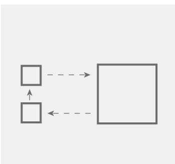
- Geometric element layout
- Continuity
- Maximum curvature
- Material
- Thickness
- Plate bending

Form creation

- Continuity
- Bending
- Torsion
- Global curvature
- Relation structural systems
- Element resistance
- Constraints
- Poisson’s ratio



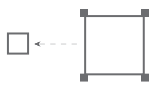
Analyses



- Simmulation
- Structural performance
- continuity in curvature
- Typologies

Concept Application

- Prototype
- Detail design
- Analyses iteration



D.2 A bottom-up approach to obtain a double curved structure

Geometrical and mechanical limits of plate bending lead to the understanding that significant deformation of plates can only occur under axial bending. This means that the only configuration achievable with bending, are the combinations of developable strips (Magna la, 2017). The question here is how planar elements can be employed to induce curvature in a structure. Geometrically speaking, with the use of planar or single curved developable elements, some characteristics of the base surface must be sacrificed. Instead of an overall continuous curvature, the structure will be an assembly of local smooth curved elements with zero Gaussian curvature. The individual elements will either be flat of a section of a cylinder or cone. Only the global structure will perceive double curvature.

Dependent on the geometrical quality, the findings of La Magna (2017) will be used to distinguish three different approaches.

1. Shells made of planar plates. This is called segmented shells. Curvature happens in the vertices (la Magna et al. 2016). [figure D2]
2. Edge coupling of developable strips. The curvature is concentrated along the edges of the adjacent strips (Bruetting, Körner, Sonntag, & Knippers. 2016). [figure D3]
3. Overlapping of developable strips. The voids completely take the curvature (Schleicher, Rastetter, La Magna, Schönbrunner, Haberbosch, & Knippers. 2015). [figure D4]

Figure [D2] [Forstpavilion] Reprinted from Itke website, by Roland Halbe, retrieved from https://www.itke.uni-stuttgart.de/de/archives/portfolio-type/landesgartenschau

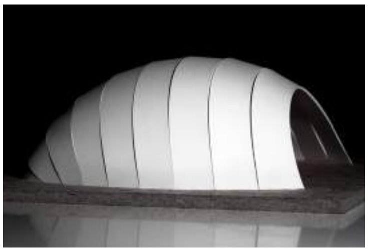


Figure [D3] [Bending active segmented shell] Reprinted from Itke website, retrieved from https://www.itke.uni-stuttgart.de/de/archives/portfolio-type/bending-active-segmented-shells

Figure [D4] [Panikkar] Reprinted from coda-office website, 2014, retrieved from http://coda-office.com/cat/work/Panikkar

The first approach uses non-deformed planar elements. The curvature of the overall shape moves to the edges, leaving the faces straight. The second approach is similar to the first approach in the edge coupling principle of the elements (Schleicher, S., La Magna, R., & Zabel, J. 2017). The difference is the use of developable strips, which are elastically deformed in one direction. Both these approaches lead to a segmented overall double curved shape. The third approach connects the individual elements through overlapping. The overlap couplings are rather planar, moving the curvature to the voids between the elements. The overlap of the developable elements allows for a higher level of continuity of the overall geometry. This third approach will be used for the simulation process.

An essential question that rises based on the bottom-up approach is what the requirements are for the geometry of the components of the structure. The geometry and the topology of the individual elements determine the overall structural layout and performance.

D.3 Design parameters

The ability to create form, design and construct with active-bending elements depends on a number design parameters. These parameters can be categorized into two main groups: topology and material [figure D5].

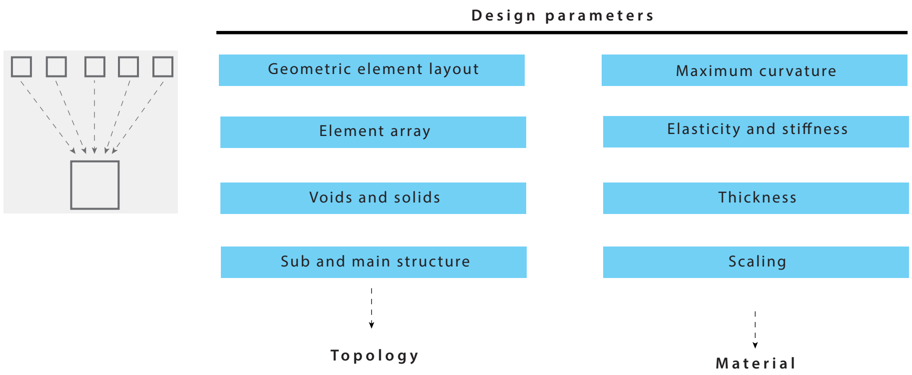


Figure [D5]Design parameters devided by topology and material.

Topology

Individual structural elements are the starting point for this designs approach, leading to the question how global double curvature can be obtained in a structure. The geometric layout of individual components determines the overall topology and design. For example, a quad shaped element topology looks and performs differently from a triangularly shaped element topology. Changing the elements geometric layout simultaneously changes the structural, bending and structural performance. Based on this effect, the first design parameter is the geometric layout of the individual building components. Although the geometry has to be similar for all components, the aim of this research is not to design with one unique element. The geometry is identical, but sizes and angles at which edged meet may vary per element along the overall structure.

The most important effect of the topology of individual elements is whether the structure is continuous or not. By arranging the elements in a particular array, a grid, or pattern, will be obtained. Continuity in this research is defined as: regardless of the number of elements in an arrangement, a pattern will always be continuous without obstructions such as open spaces, or arbitrary geometric conversions. The importance of continuity is the force-flow throughout the structure and the overall coherence in a structure. Additionally, the ability to be able to apply this design method to a broader range of structures rather than for one structure only.

The pattern or topology of individual elements defines where a structure has material functioning as force transfer, and where it has voids to capture double curvature. Thus, the geometry and the topology determine the ratio between open and close surface in a structure. This rate may influence the level of applicability when the structure has a space enclosing function.

Material

One of the parameters regarding the material of the structure is the maximum bending radius. This radius, i.e. the curvature, depends on the ratio of flexural strength and flexural stiffness (La Magna, 2017). Torsional rigidity plays an important role as well, as torsion might be used to obtain negative Gaussian curvature. The minimum radius defines the maximum bending formation possible to which a material can deform. This determines the amount of curvature in a geometric element. With given element dimensions and a minimum radius, the amount of curvature over the area of an element can be calculated by the following formula:

$$\text{Curvature} = \frac{\text{Angle of embrace}}{\text{Length of arc}} \qquad \text{Curvature} = \frac{\Delta\alpha}{\Delta s}$$

Axial bending will occur over the section with the lowest moment of inertia. This needs to be considered during the deformation process of the geometric elements as well as the deformation of the global structure. If the moment of inertia changes over the span of an individual component, the amount of curvature will not be linear relative to the length of the element. Additionally, the amount of embodied energy required for the deformation process depends on the material properties and the width and thickness of the element at the bending axis.

Bending plates requires a relatively low moment of inertia compared to non-flexible structures. Non-flexible structures are build to have the least bending, i.e., least deformation possible. The formation process of active-bending structures contradicts the eventual load transfer through the structure. The formation process requires highly flexible members, and the load transmission requires rigid structural members. Structural rigidity partly depends on the thickness of the erected surface, as it has to withstand shear stresses and bending moments.

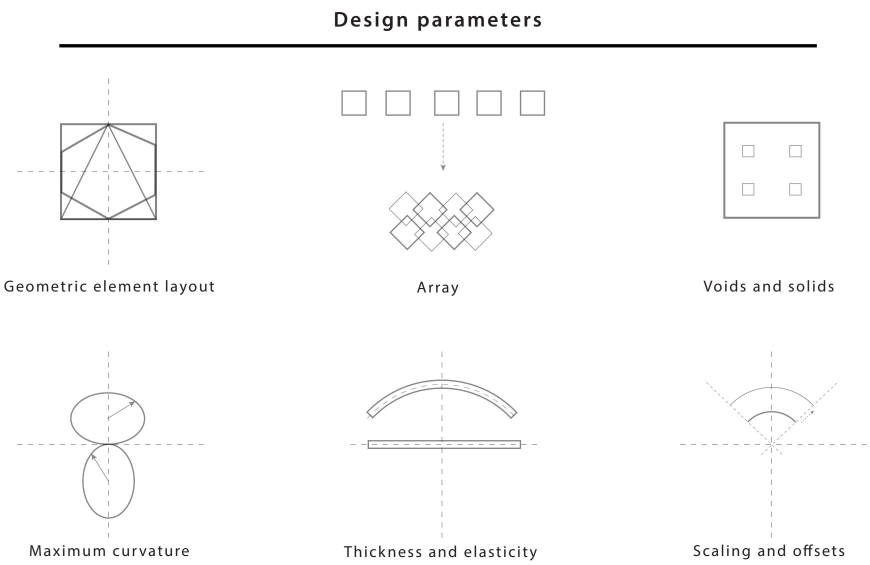


Figure [D6] Design parameters geometric topology

D.4 Form creation process

The design parameters are the global ingredients for an active-bending structure. They describe the primary variables for the structures' appearance and the structural performance. These fundamental parameters are divided into sub-parameters relating to a specific design process, for example, the form finding process. The derived parameters for the form finding process are categorised into three groups: similarity, topology and material.

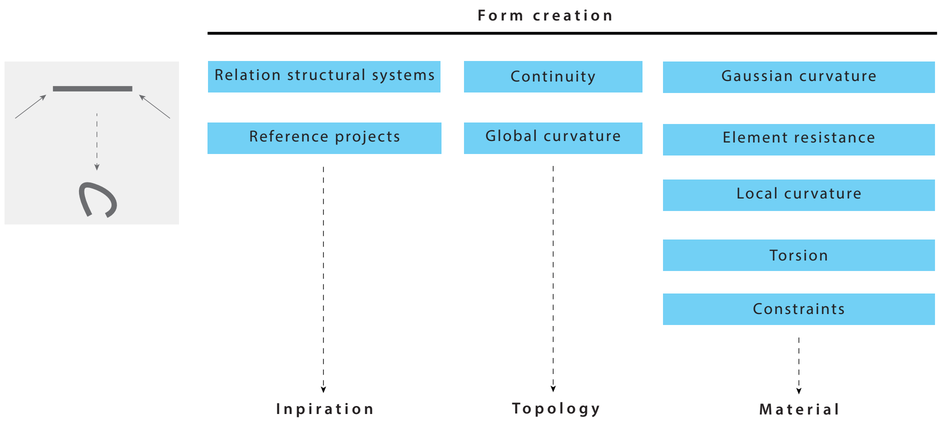


Figure [D7] Form creation process, similarity topology and material

Similarity structural systems

The first chapter of this research lists a range of existing structural systems described by Heino Engel (1997). Some of these structural systems have similar design principles and structural behaviour compared to active-bending structures. These similar structural systems are section-, surface- and form-active structures. The load transfer through an active-bending structure includes bending and normal forces. The deformation in an active bending structure is intended for formation, contradicting with section-active structures, where large deformations tend to be avoided. Even though the amount of deformation and shape is different, the load transfer shows similarities with the load bearing behaviour of section-active structures. Because active-bending structures can be formed to a similar shape of shells and arches, the load bearing behaviour also acts similar to that of surface and form-active structures. Observing the similarities between these three structural systems allows for inspiration which can be used as input for the design and the structural performances of an active-bending structure.

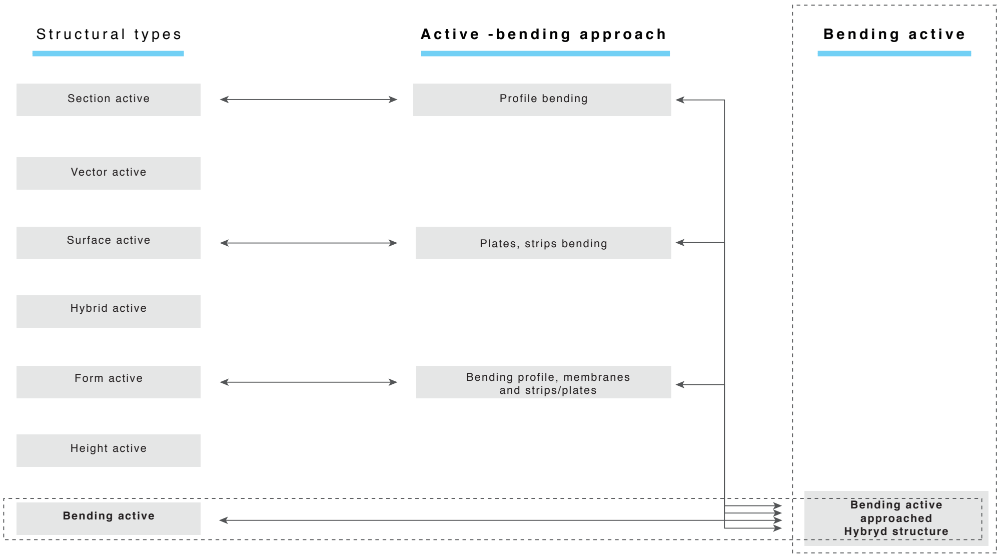


Figure [D8] Similarities between structural system and active-bending design approach

Inspiration

This research aims to create and design a double curved active-bending structure. The similarities between existing structural systems and reference projects are to be found in structural performance, erection process, types of curvature and final shape. The aim of observing reference projects is to provide an input for the active bending structure and to gain knowledge from previously built projects. In particular, the observation of double curvature and the distinction between the various types of double curvature could potentially provide valuable insight into the creation and performance of double curved structures.

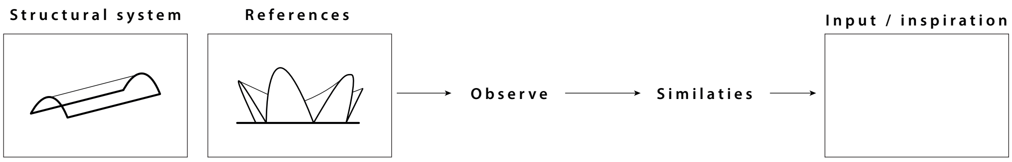


Figure [D9] Similarity workflow

Reference projects

l’oceanogràfic, Valencia, Spain.

The l’oceanogràfic is a concrete shell structure. The polar array of eight shells form the primary structural enclosure of the building. The shape of the shells are anticlastic and can be classified in the surface-active structural system. The creation of the shells has been performed with the use of casts and moulds. Planar wooden strips have been slightly bent in position to form the anticlastic curvature. The wooden mould is a ruled surface, created by generative curves around a directrix curve. Because the tangent lines of the generative curves are non-parallel, the obtained surface becomes double curved.

Figure [D10] [The restaurant of L’Oceanografic in Valencia, Spain as viewed from across the water.] Reprinted from Wikipedia website, by David Illif, 2007, retrieved from https://nl.wikipedia.org/wiki/L%27Oceanogr%C3%A0fic License: CC-BY-SA 3.0

O2 Arena, London, United Kingdom.

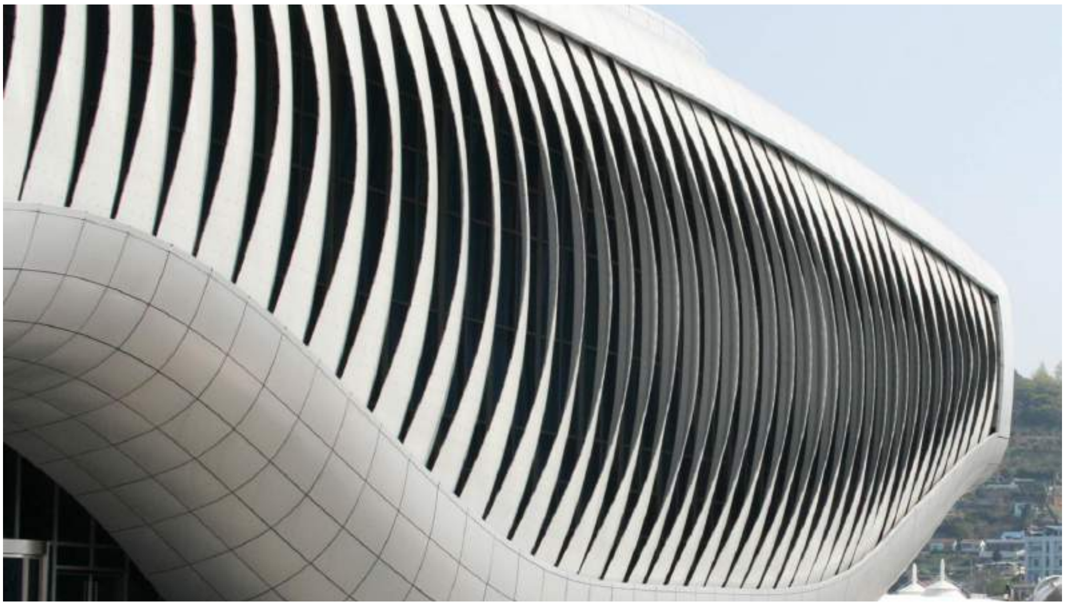
The O2 arena in London is a membrane tensile structure. The global shape is a segment of a sphere, which is a synclastic surface. The membrane is suspended from twelve pillars with the use of tension cables. Horizontally, tension cables make the synclastic surface segmented, as the entirety of the structures is composed of several membrane strips. This structure is both surface-and hybrid active. The axial tensile stress stiffens the fabric. The tensile cables hold the membrane in place.

Figure [D11] [The O2 Arena London] Reprinted from Stubhub website, 2017, retrieved from https://www.stubhub.co.uk/magazine/the-o2-london

Heydar Aliyev Center, Baku, Azerbaijan.

The smooth surface of the Heydar Aliyev Center skin mainly fulfils cladding purposes, as it is held in place by an entire standalone steel structure. Nevertheless, all kind of curvatures can be distinguished. The global surface contains syn, anti and mono-clastic curvatures. The steel structure is clad with small composite tiles. In between the panels is a small seem, leaving the curvature in the edges. Only at places where the curvature is significantly small, the panels have single curvature themselves. The entirety of the buildings cladding surfaces s a segmented double curved surface.

Figure [D12] [Heydar Aliyev Center, Zaha Hadid Architects] Reprinted from Archdaily website, 2017, by Iwan Baan, retrieved from https://www.archdaily.com/448774/heydar-aliyev-center-zaha-hadid-architects,



Multihalle, Mannheim, Germany.

The Multihalle is a large grid-shell structure composed of thin slats. The overall shape of the structure has both syn-clastic and anticlastic curvature. The form of the structure is obtained through bending of thin slats and restraining them where they intersect. Even though the shape itself is a shell, the structural system relates more to that of active-bending. Through the elastic deformation of planar strips is double curvature obtained. The slats themselves are single-curved, they bend in one direction. The voids in between the slats take all the double curvature. For space enclosing purposes a membrane has been pulled over the slats.

Figure [D13] [Herzogenriedpark Mannheim] Reprinted from Wikipedia website, 2010, by Imanuel Giel, retrieved fromhttps://commons.wikimedia.org/wiki/File:Herzogenriedpark_077.jpg

Denver union station, Denver, United States.

The Denver Union station is a large cantilever covering the platforms underneath. The structure is composed of several steel arches with varying dimensions. The steel arches can be classified in the form-active structural systems. The forces will be transferred through the arch to its support. A fabric membrane clads the arches, leading to double curved membrane surfaces. They appear to be synclastic curved surfaces, but a closer look reveals the segmentation of the fabric. The arches form the basic structure, and the membrane is the space enclosure.

Figure [D14] [Denver union station] Reprinted from structurflex website, 2018, retrieved from https://www.structurflex.com/projects/denver-union-station

Expo 2012 pavilion, Seoul, South-Korea.

The Expo 2012 pavilion is a unique project regarding behaviour and performance. The facade is composed of kinetic louvres with can open and close. The fibre reinforced polymer louvres are single curved in their closed position. In open configuration, the bottom and the top of one side of the louvres move towards each other. The louvres' shape changes from monoclastic curved to anticlastic curved elements. Besides the fact that they do not have a structural load bearing capacity, they will be exposed to significant variable load cases such as wind. The anticlastic surface erecting process is through active bending, leaving the louvres with surface-active structural capacities.

Figure [D15] [One ocean, Thematic pavilion 2012] Reprinted from archdaily website, 2018, by Soma retrieved fromhttps://www.archdaily.com/236979/one-ocean-thematic-pavilion-expo-2012-soma

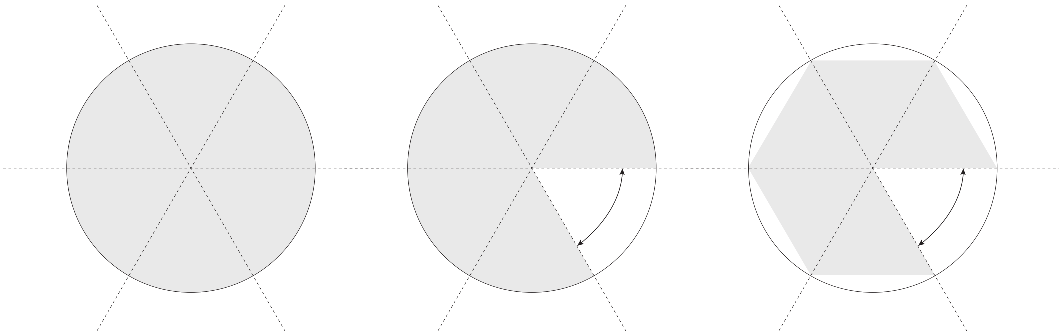
D.5 Continuity and topology

The geometric layout and topology principle can be explored through paper modelling. Although paper modelling does not take any applicable material capacities into account, it allows for quick and feasible hand-on geometric topology.

The triangulation approach described in the previous chapter is an approximation of a double curve smooth sphere. The faces are planar, and the curvature happens in the vertices. Overlap coupling of these planar faces distributes the curvature from the vertices equally over the faces. This requires a void as the faces cannot bend in two directions, i.e. they are developable.

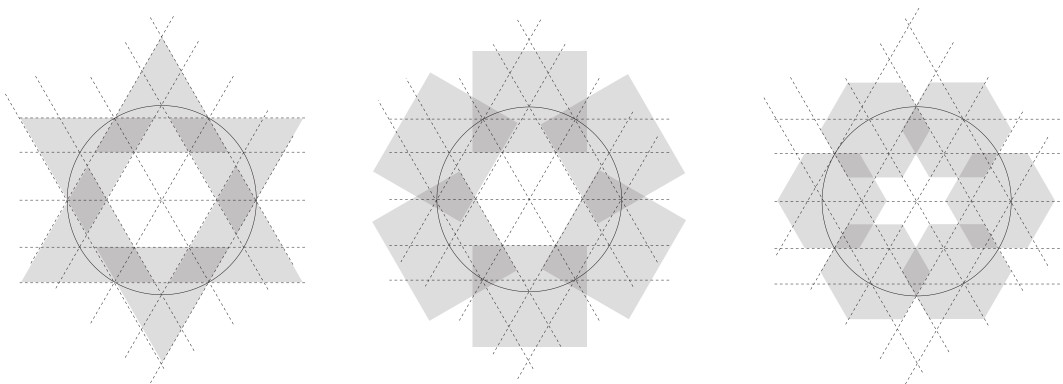
A section of a sphere is used to simulate this theory. A planar circle can be divided into some faces. The triangulation of a segment of a sphere approximates the double curved surface through a polar array of triangular faces around the midpoint of the sphere (Wester, 1984). The angle of embrace is the same as the angular defect in unrolled condition. With the angular defect, the angle of the faces can be calculated. Thus, with the given Gaussian curvature, the area of the curved surface to construct and the number of faces, the angle of the triangles that meet at a vertex can be calculated (Calladine, 1983).

A planar array of triangles, hexagons and quads, all with equal edge lengths, represent a planar circle. The planar circle becomes three dimensional by the reduction of the number of faces from six to five. The angle at which the faces meet as well as the angular defect is equal to the angle of the six faces in the planar circle. The result of the reduction of one face is the Gaussian curvature. The angles at which the faces meet changes with a different Gaussian curvature as given input. The aim is to obtain a repetitive topology in which the varying Gaussian curvature can be accumulated.



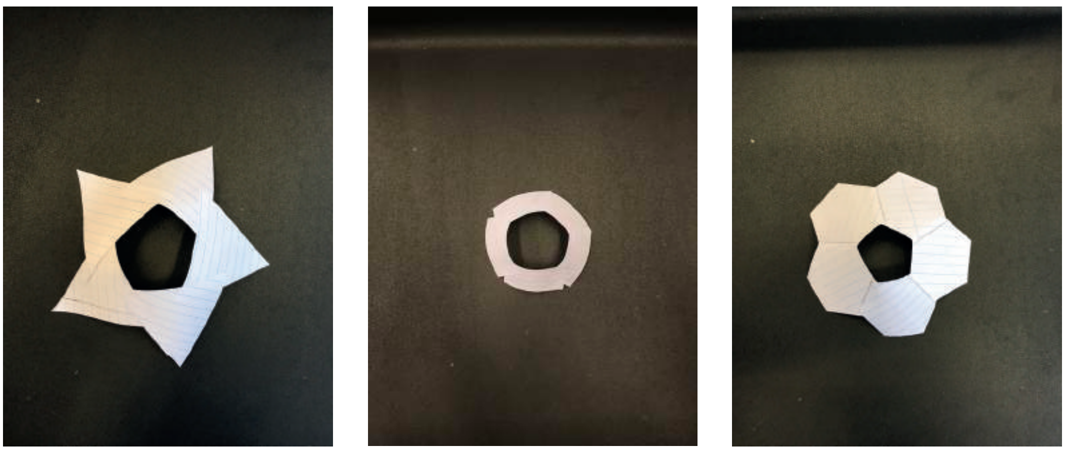
(a) (b) (c)

Figure [D16]: triangulation of spherical section. (a) planar circle composed of six triangular shapes. (b) angle of defect similar to triangles vertex angles. (c) triangulated flattened representation of spherical image.



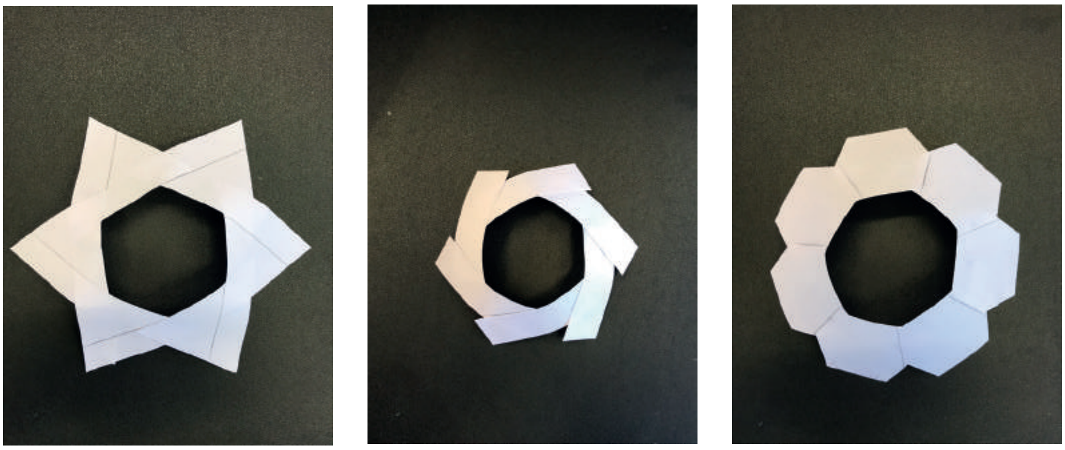
(a) (b) (c)

Figure [D17]: planar circle based on the tesselation of six triangles, expressed with alternative geometry. (a) triangles, (b) rectangular strips, (c) hexagonal element.



(a) (b) (c)

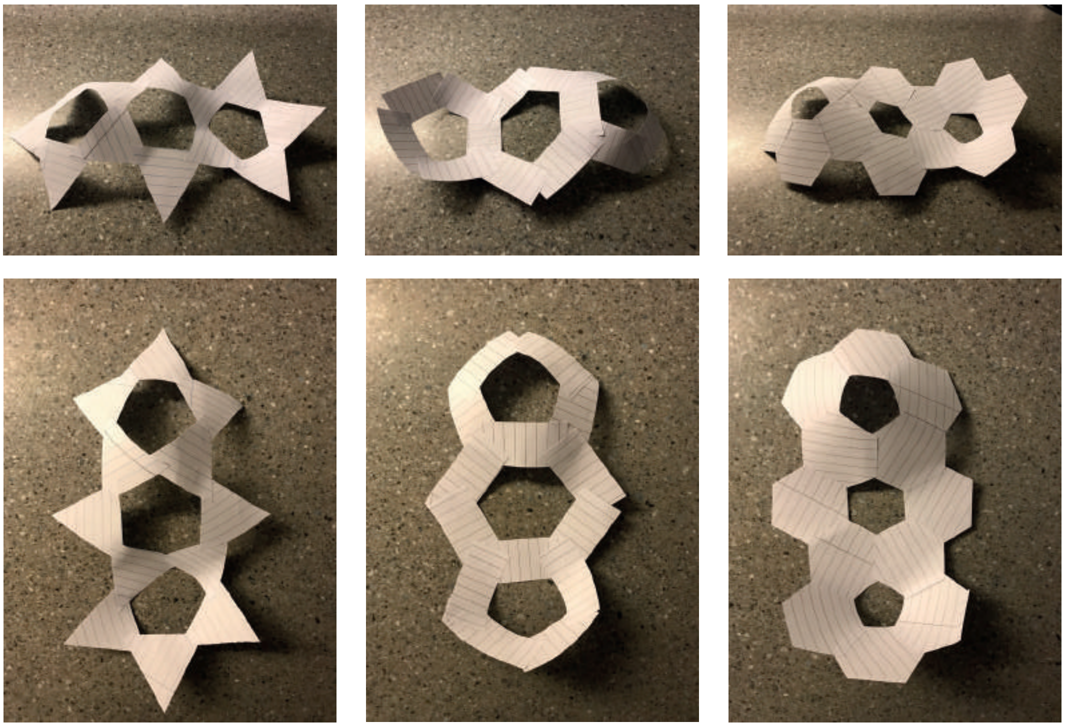
Figure [D18]: Reduction of one ellement from planar configuration resulting in three dimensional object. Individual elements have identical edge lengths. (a) five triangles, (b) 5 quads, (c), 5 hexagonals.



(a) (b) (c)

Figure [D19]: (a). three dimensional description composed of six and 7 elements. Gaussian curvature as given input. Resutling in varying angles. (a) triangle tesselation, (b) quad tesselation, (c) hexagon tesselation.

The paper models in [figure D19] present the triangulation theory constructed with the approach of overlap coupling of developable surfaces. These models do not have any Gaussian curvature as they can all be projected on a conical shape. No curvature has to be taken by the voids. A double curved surface is a coupling of two or more developable surfaces. The surface connecting the developable surfaces consist of double curved geometry, making the overall surface double curved [figure D20].



(a) (b) (c)

Figure [D20]: Double curved surface through overlap connection of developable surfaces. Curvature is take by the voids. Triangle (a), quad (b), and hexagonal (c) tesselation. All tesselation share the capacity of hexagonal voids in the center and pentagonal voids in the curved end surfaces.

Just with two developable surfaces, one single-curved and one planar, several free-form configurations can be obtained. These free-form structures are not designed proposals, but rather models which show the various shapes that can be erected by using just two elements. A planar component consists of six triangles with all equal corner angles. Because the angles at which the triangles meet is always similar, the array of six triangles form a flat surface.

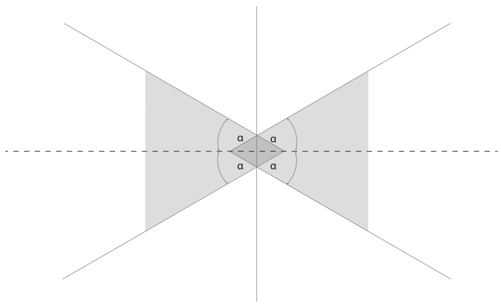


Figure [D21] Continuity in surface if angles at which triangles meet is similar.

A polar array of five triangles leads to an angle of defect of 60 degrees, which can be considered a conical single curved shape. The combination of these two components leads to an overall double curved structure. The connection between the planar components and the conical single curved components are the transition from planar to curved. The double curvature is obtained when the transition components have to connect negative and positive curvature.

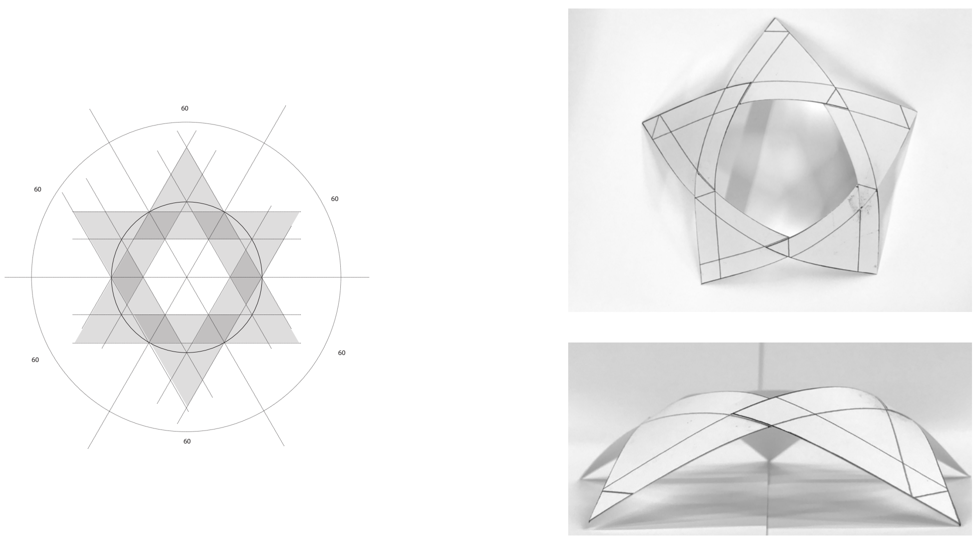


Figure [D22] (a) planar surface composed from six triangles. (b) double curved surface composed from five triangles.

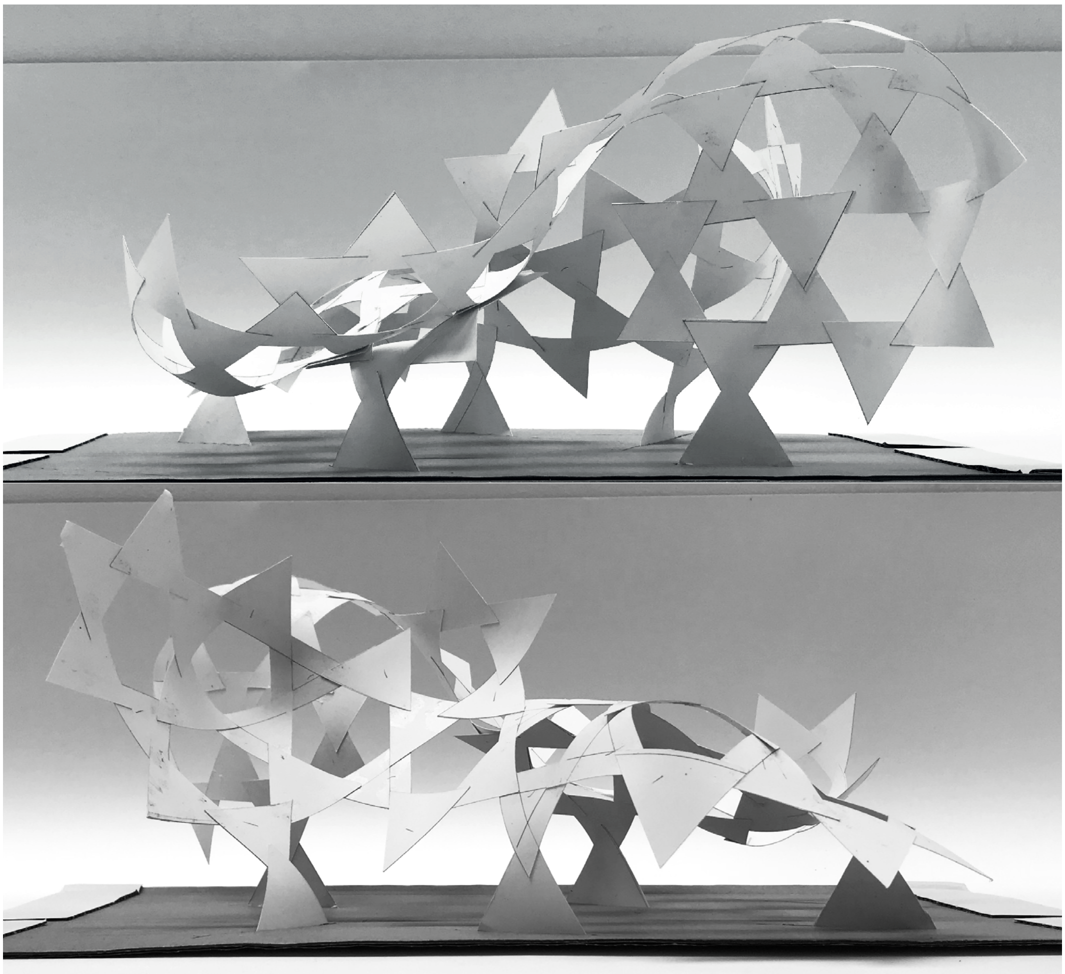


Figure [D23] Form study using triangulation method. Composed of planar and double curved elements.

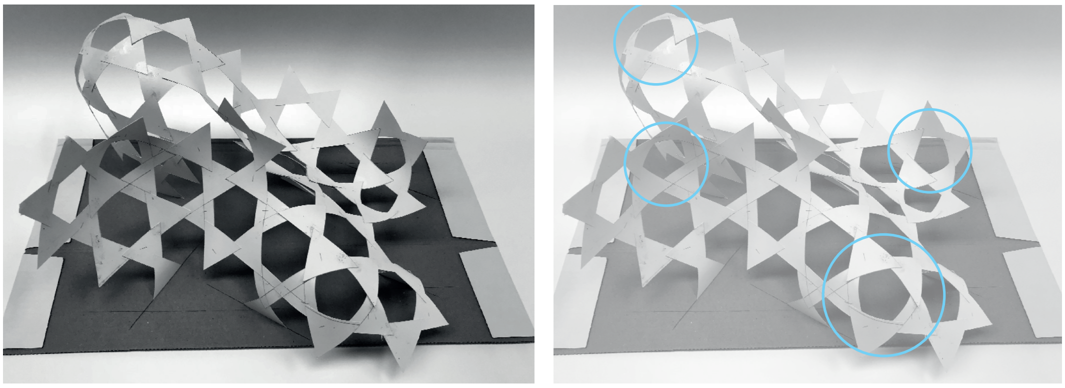


Figure [D24] Blue cricles alocate the double curved components composed of five planar triangles.

A chain of connected components allows creating a global curved structure. Two different elements, one planar with six triangles, one curved with five triangles, accumulate the global double curvature. However, the use of only two different elements makes it hard to control the amount of curvature as there is only one fixed curved element. This paper model is a first approach to creating and managing double curvature in a structure. Actual building materials do not allow such bending over a short length, which will lead to less curved elements. The aim of analysing this first model is to add components with varying curvature to be able to manage the amount of double curvature in the overall structure.

In the structure shown above the aim was to vary the amount of curvature over the span of the structure. By changing the angle at which (in this case triangles) the elements meet, the amount of curvature also changes over the span. A subtle curve has been obtained by introducing a small angular defect in an array of six triangles. One side of the structure is planar, and only curved in one directs. The other side of the structure is double curved [figure D25, D26].

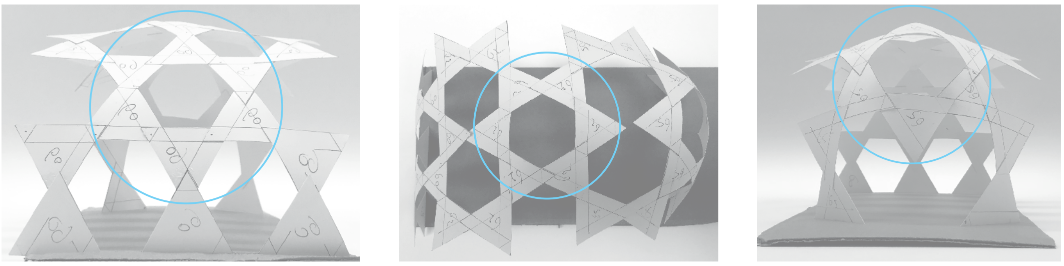
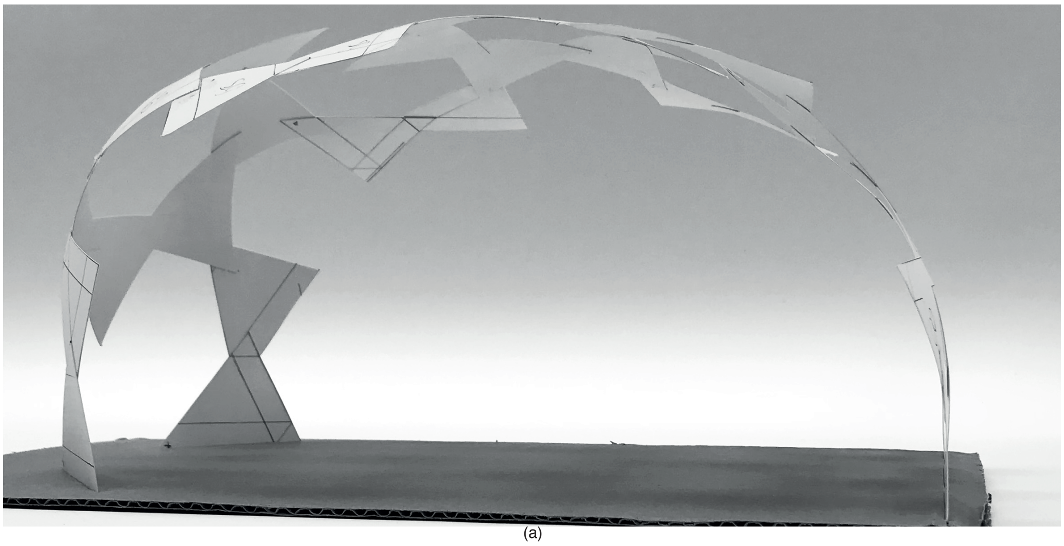


Figure [D25] (a) Planar arangement six triangles. (b) slightly curved arrangement six planar triangles. (c) double curved arrangement six triangles

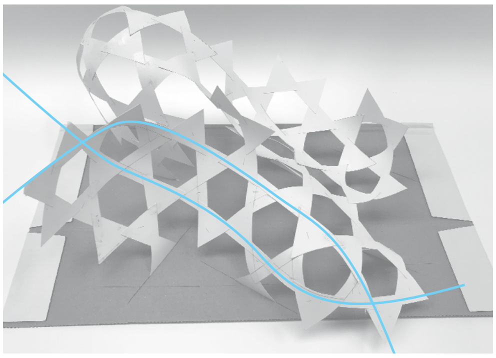


(a)

(b)

Figure [D26] Increasement of double curvature over the span of the arch by alternating angle at which triangles meet. (a) front view, (b) isometric view.

Continuous and non-continuity



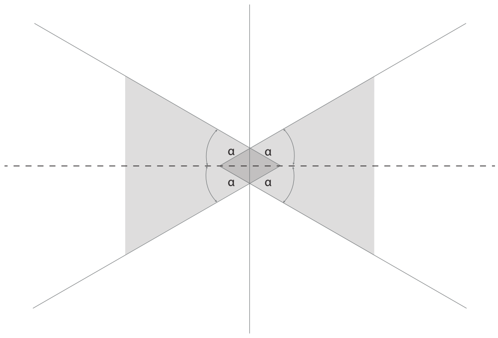
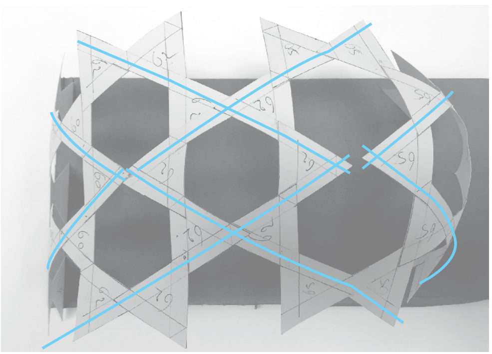


Figure [D27] Continuity in double curved structure. (a) contious lines in triangulated shape. (b) similar angle at which triangles meet.

Because the connected triangles all meet at the same angle, the structure can be considered continues. [Figure D27] shows the continuous lines going through the structure. The lines are straight in planar elements and diverge when passing curve elements. The continuity in the structure is essential for the performance and the design freedom. In a continuous structure, the number of geometric elements is free to choose because the topology will always be a constant pattern. The continuity in the structure depends on the angles at which the geometric components meet and the dimensions of each element.



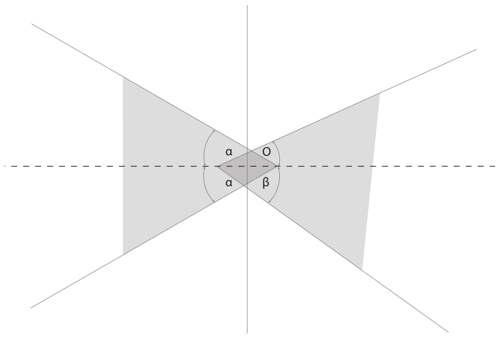


Figure [D28] Discontinuity in double curved structure. (a) Discontinuous lines in triangulated shape. (b) varying angle at which triangles meet.

The structure is composed of three elements with a varying curve [figure D28,a]. An increasing angular defect obtains this variation in curvature. Because the curvature varies over the span, the angle at which the geometric components meet also varies. For continuous structures, it is essential that the angle at which the elements meet is similar. [figure D28,a] shows the discontinuity in the structure as a result of varying angles at which the triangles meet.

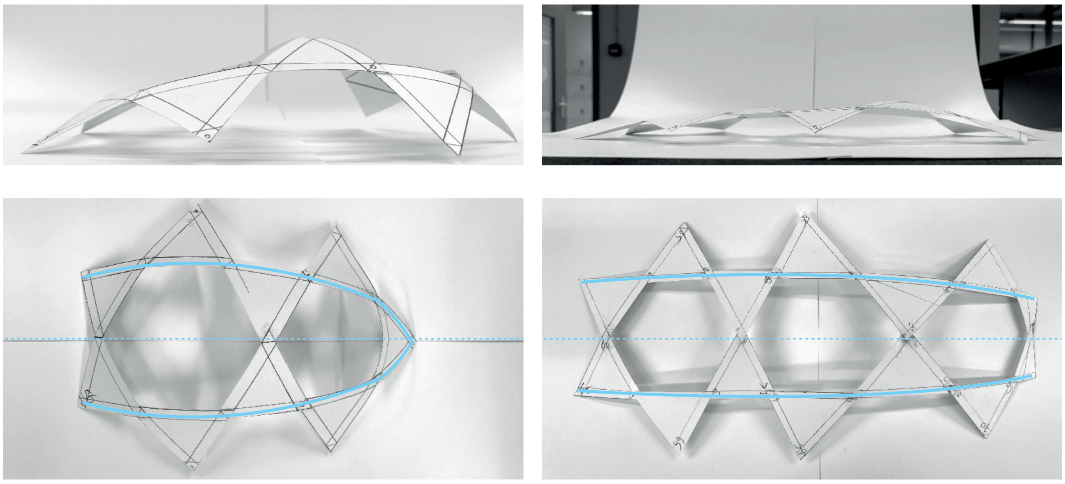


Figure [D29] Varying curvature measures based on triangulation method. Angles at which triangles meet is equal. Edge length per triangle alternates.

The paper model studies, presented in [figure D29], present the attempt at obtaining continuity in the structure with control of the amount of curvature in the geometric components. The different curvature depends on the angular defect in an array of six triangles. For example, an array of six triangles has an angular defect of five degrees. The triangles all meet at the same angle. This results in a curved component (A component is considered an array of six triangles with an angular defect) if the triangles have equal corner angles. If they do not have equal corner angles, they are either planar, positive curved or negative curved. If they are curved, the edges of the individual components do not have equal lengths.
The triangles adjacent to the array of six triangles have to adapt to the corner angles to remain the continuity in the structure.

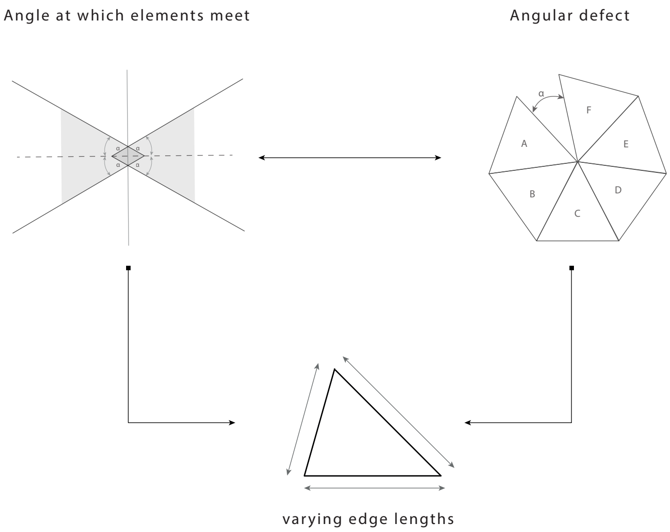
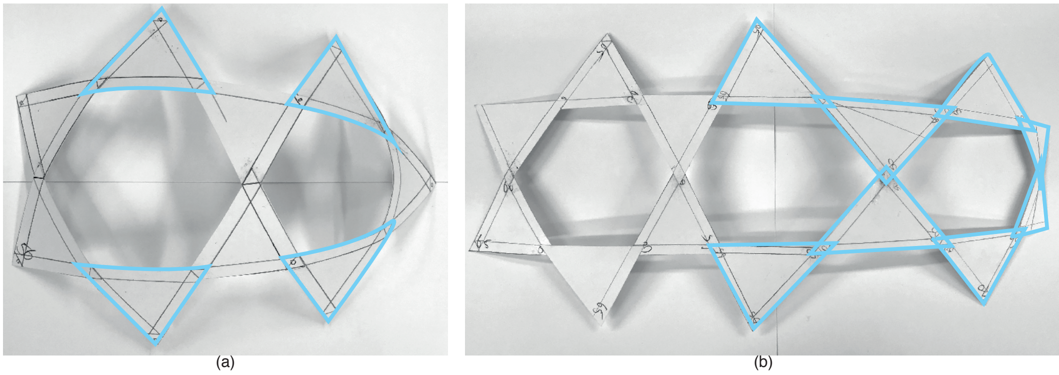


Figure [D30] Three parameters in triangulation method. 1, angle at which triangles meet. 2, Edge length. 3, Angular defect.



(a) (b)

Figure [D31] Continuous structure with varying triangle sizes.

An analysis of the paper models distinguishes two observations. First, continuity and controlling the amount of curvature at the same time is possible. Secondly, as a result of the varying angular defects used to control the amount of curvature, the edge lengths change. Consequently, the shape of the triangles changes. [Figure D31,a] shows a large angular defect leading to diverging continuity lines but remaining triangles with almost similar edge lengths. [Figure D31,b] shows a slight curvature (small angular defect), leading to nearly straight continuity lines. The continuity is possible because the triangle edges adapt to the angular defect. This results in triangles varying in shape. The continuity and the amount of curvature depend on three essential parameters. The angle at which the elements meet, the angular defect and the edge lengths of the geometric elements. The angles at which the element meet determines the continuity and the angular defect determines the amount of curvature. The edge length is a result of these two parameters.

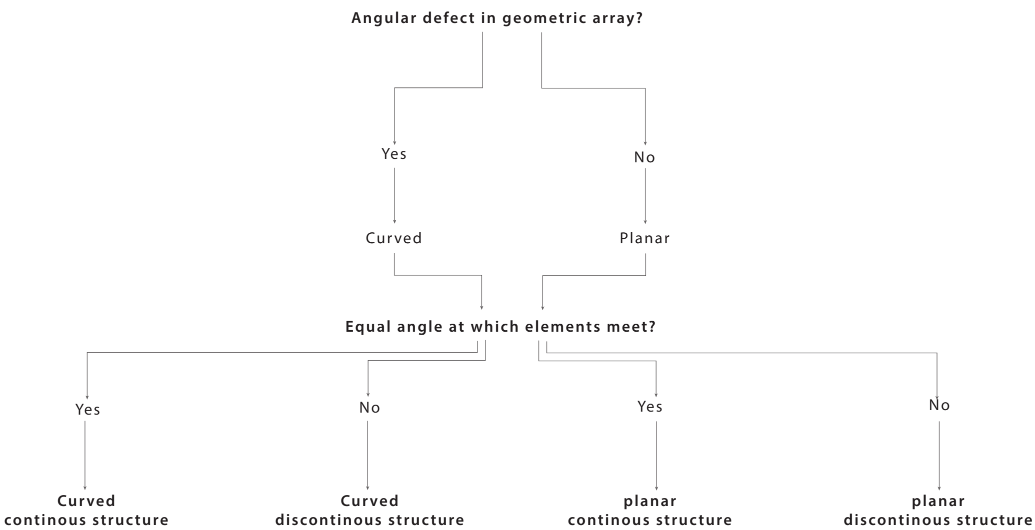


Figure [D32] Continuity in a triangulated structure.

Triangulation elaboration

The analyses method through paper models is a first attempt to get familiar with bending, continuity and double curvature. The triangulation method by Calladine (1983) is a useful tool to quickly approximate and control curvature. The different angular defects and the number of triangles in an array are easy to change based on the desired curvature. Here, the triangles have been used to get familiar with the manipulation of curved elements. The triangular geometry is not the final geometry used in this research. The same triangulation method can be applied to, for example, rectangular or hexagonal geometries. Further design development and design parameters determine which geometry suits best for an active-bending structure. The final geometric components can be arbitrary shaped, as long as the angle at which they meet is similar.

[figure D33] Illustrates the different geometries applied to the triangulation method. A highly important discovery is that regardless of the applied geometry, triangular patterns are always visible in the topology. In other words, every double curved structure generated with the triangulation method can be made of rectangular strips which are continuous throughout the entire structure. This means that these planar strips must have some measure of double curvature. This will be further elaborated in the next paragraph, where the measure of double curvature will be aligned with the bending of planar elements.

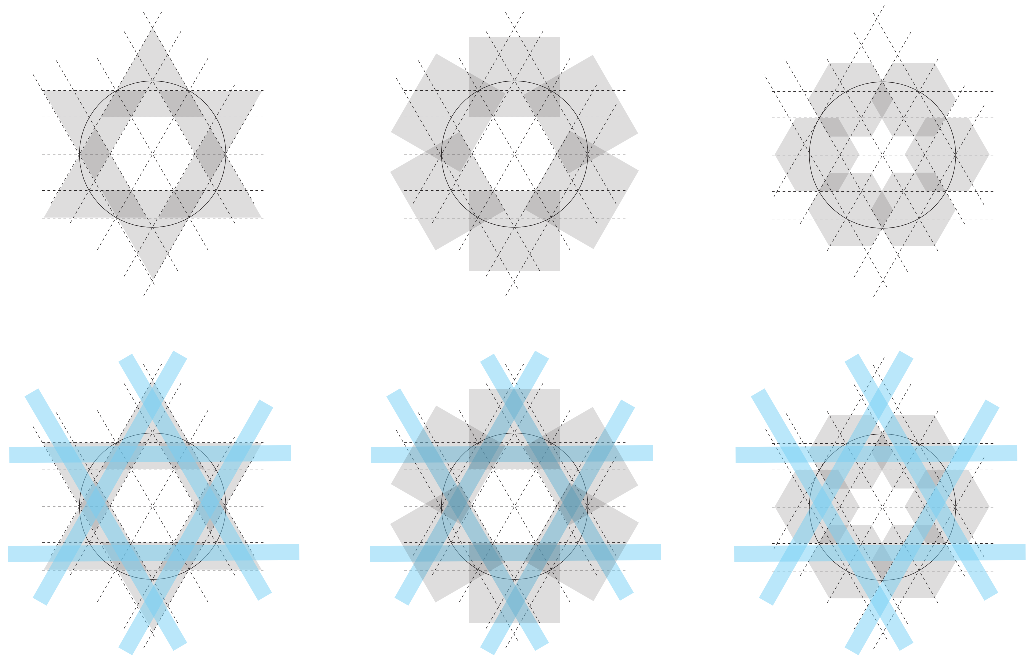


Figure [D33] Continuous curve in topology of varying geometric elements. (a) Trianglular, (b) squared, (c) Hexagonal.

E Two-directional bending

Bending curvature-moment relationship

E.1 Material properties in Gaussian curvature

The theory of Beranek (1979) has been used to describe a double curvaed surface. [figure E1] Presents a double curved surface in space which can be described by:

$$z = ax^2 + bxy + cy^2 + ... \quad (1)$$

The second derivative describes:

$$\frac{\partial^2 z}{\partial x^2} = 2a \qquad k_{xx} = 2a \quad (2)$$

$$\frac{\partial^2 z}{\partial x \partial y} = b \qquad k_{xy} = b \quad (3)$$

$$\frac{\partial^2 z}{\partial y^2} = 2c \qquad k_{yy} = 2c \quad (4)$$

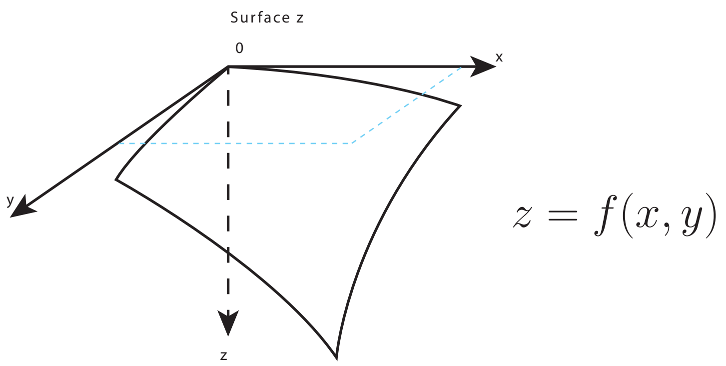


Figure [E1] [Three dimensional surface description], by Beranek, 1979, reprinted from K3 dictaat ruimtelijke constructies deel 2.

With both the bending and the torsion the principal curvatures can be calculated with the Mohr's circle. The product of the two principal curvatures is the Gaussian curvature (explained in chapter two).

The formula for the principal curvatures is:

$$k_{1,2} = \frac{k_{xx} + k_{yy}}{2} \pm \sqrt{\left(\frac{k_{xx} - k_{yy}^2}{2}\right) + k_{xy}^2} \quad (5)$$

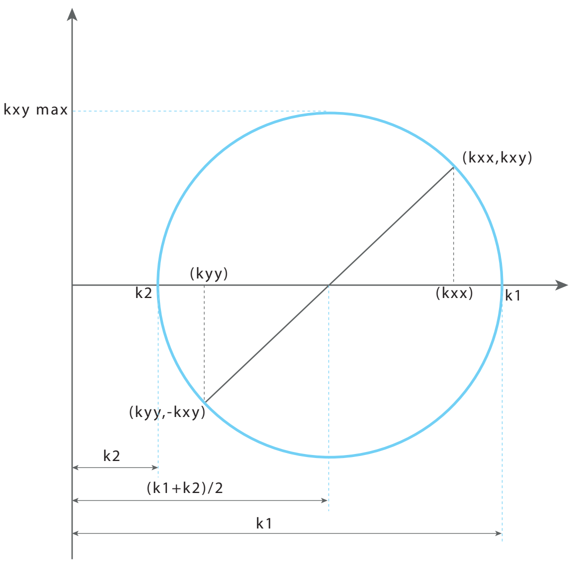


Figure [E2] Mohrs circle, curvature relation between bending and torsion.

the sum of the principal curvature is:

$$k_1 \cdot k_2 = k_{xx} \cdot k_{yy} - k_{xy}^2 \tag{6}$$

Deformation of curved surfaces

The same surface in space can be descirbed with:

$$z = f(x, y) \tag{7}$$

After deformation, every point on the surface in both x-and y-direction has shifted over a distance ux and uy as a result of deformation w in the z-direction:

$$g = w(x, y) \tag{8}$$

The z coordinates after deformation is:

$$z' = z + w \tag{9}$$

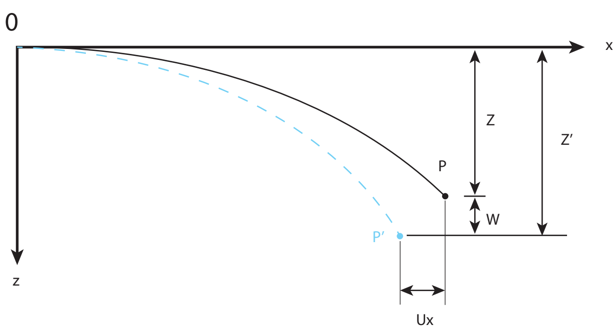


Figure [E3] [Curve deformation with distance 'w' as displacement]. by Beranek, 1979, reprinted from K3 dictaat ruimtelijke constructies deel 2.

Before deformation, the Gaussian curvature value is:

$$k_1 \cdot k_2 = \frac{\partial^2 z}{\partial x^2} \cdot \frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial x \partial y}^2 \tag{10}$$

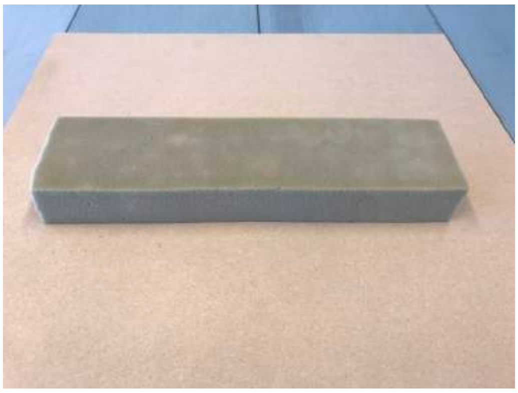
After deformation, the Gaussian curvature value is:

$$k_1' \cdot k_2' = \left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 w}{\partial x^2}\right) \cdot \left(\frac{\partial^2 z}{\partial y^2} + \frac{\partial^2 w}{\partial y^2}\right) - \left(\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial y}\right)^2 \tag{11}$$

If deformation only occurs as a result of bending:

$$k_1 \cdot k_2 - k_1' \cdot k_2' = 0 \tag{12}$$

The theory presented above is an approach to geometrically describe the curvature relation between a surfaces' initial shape and the deformed shape of this same surface. The relation between kxx, kyy and kxy can be calculated with the Mohrs' circle. However, the product of the principal curvatures does not take any material properties into account. The Poisson' ratio is the measure of the Poisson effect, the phenomenon in which material tends to expand in directions perpendicular to the direction of compression. Conversely, if the material is stretched rather than compressed, it usually tends to contract in the directions transverse to the direction of stretching. If only the geometric formula of the Gaussian curvature were used, the material would not undergo any contraction in the transition from flat to deformed. [figures E4, E5] Show, in an exaggerated proportions, this is not correct when actual material is used. The Gaussian curvature in the deformed stage is not equal to the Gaussian curvature in the flat state.



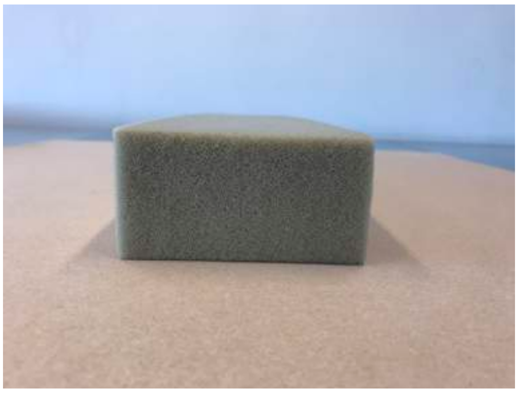


Figure [E4] Planar surface. k1*k2

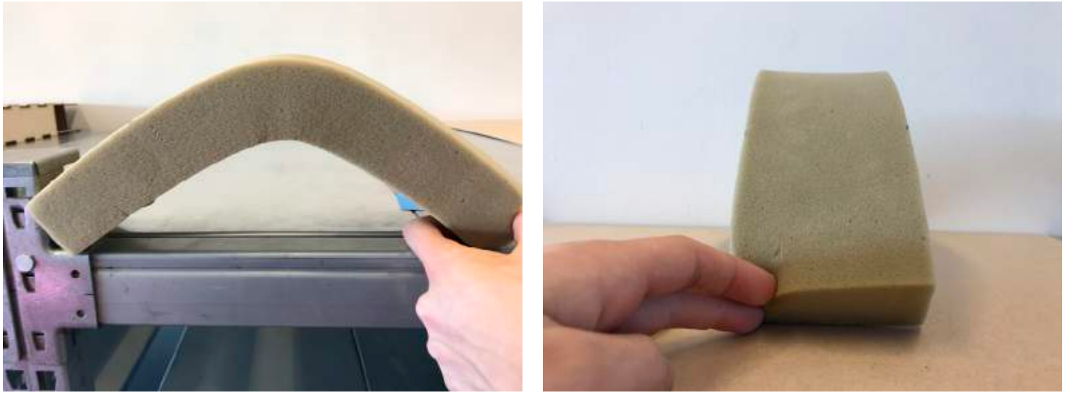


Figure [E5] deformed planar surface. k1'*k2'

The aim is to be able to measure and simulate curvature with the inclusion of material properties. The contraction and the materials bending rigidity have to be included to control and predict the material behaviour during bending. The stresses in a plate can be described with the Youngs modulus, the Poisson's ratio and the strain with the following formulas (Wierzbicki, 2006).

$$\sigma_{xx} = \frac{E}{1-v^2}\left(\varepsilon_{xx} + \nu\varepsilon_{yy}\right) \quad (13)$$

$$\sigma_{yy} = \frac{E}{1-v^2}\left(\varepsilon_{yy} + \nu\varepsilon_{xx}\right) \quad (14)$$

$$\sigma_{xy} = \frac{E}{1-v}\varepsilon_{xy} \quad (15)$$

The bending rigidity can be described with:

$$D = \frac{Eh^3}{12(1-v^{2)})} \quad (16)$$

Leading to bending moments in all directions:

$$M_{11} = D(\kappa_{11} + \nu\kappa_{22}) \quad (17)$$

$$M_{22} = D(\kappa_{22} + \nu\kappa_{11}) \quad (18)$$

$$M_{12} = D(1-\nu)\ \kappa_{12} \quad (19)$$

By substituting these bending moments in the formula of the Gaussian curvature, the Poisson's ratio can be dissolved. This new formula allows to predict and calculate the ratio between the different bending moments in a deformed plate.

$$z = f(x, y) \tag{20}$$

$$g = w(x, y) \tag{21}$$

$$k_1 \cdot k_2 = k_{xx} \cdot k_{yy} - k_{xy}^2 \tag{22}$$

$$k_1 \cdot k_2 - k_1' \cdot k_2' = 0 \tag{23}$$

$$ii = \left(\frac{\partial^2 z}{\partial x^2}\right)\left(\frac{\partial^2 z}{\partial y^2}\right) - \left(\frac{\partial^2 z}{\partial x \partial y}\right)^2 \tag{24}$$

$$hh = \left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 z}{\partial y^2} + \frac{\partial^2 w}{\partial y^2}\right) - \left(\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial y}\right)^2 \tag{25}$$

If the initial Gaussian curvature is zero, the result of ii-hh should be equal tot the formula of the initial surface. z = o leads to ii.

$$hh = -\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right) + \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 \tag{26}$$

Now, the aim is to check if this formula also applies to the formula with the derived bending moments substituted in the Gaussian curvature equation:

$$k_1(M) \cdot k_2(M) - k_1'(M) \cdot k_2'(M) = 0 \tag{27}$$

$$ii = \left(\frac{\partial^2 z}{\partial x^2} + \nu\left(\frac{\partial^2 z}{\partial y^2}\right)\right)\left(\frac{\partial^2 z}{\partial y^2} + \nu\left(\frac{\partial^2 z}{\partial x^2}\right)\right) + (1 - \nu) \cdot \left(\frac{\partial^2 z}{\partial x \partial y}\right)^2 \tag{28}$$

$$ii = \left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 w}{\partial x^2} + \nu\left(\frac{\partial^2 z}{\partial y^2} + \frac{\partial^2 w}{\partial y^2}\right)\right)\left(\frac{\partial^2 z}{\partial y^2} + \frac{\partial^2 w}{\partial y^2} + \nu\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 w}{\partial x^2}\right)\right) - \left((1-\nu)\left(\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial y}\right)^2\right) \tag{29}$$

Now, the aim is to check if this formula also applies to the formula with the derived bending moments substituted in the Gaussian curvature equation:

$$pp = \left(\frac{\partial^2 z}{\partial x^2} + \nu\frac{\partial^2 z}{\partial y^2}\right)\left(\frac{\partial^2 z}{\partial y^2} + \nu\frac{\partial^2 z}{\partial x^2}\right) - \left((1-\nu)\frac{\partial^2 z}{\partial x \partial y}\right)^2 - \left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 w}{\partial x^2} + \nu\left(\frac{\partial^2 z}{\partial y^2} + \frac{\partial^2 w}{\partial y^2}\right)\right)\left(\frac{\partial^2 z}{\partial y^2} + \frac{\partial^2 w}{\partial y^2} + \nu\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 w}{\partial x^2}\right)\right) - (1-\nu)\left(\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial y}\right)^2 \tag{30}$$

Again, with the initial surface of zero Gaussian Curvature, and a Poisson's ratio of zero. The resulting formula should equal to the formula of the initial surface.

$$\nu = 0 \tag{31}$$

$$z = f(x, y) = 0 \tag{32}$$

$$pp := ii - ff \tag{33}$$

$$pp = -\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right) + \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 \tag{34}$$

The resultating formula is similar to the formula of the initial surface. With the inclusion of the Poisson's ratio, the relation between the bending moments can be calculated.

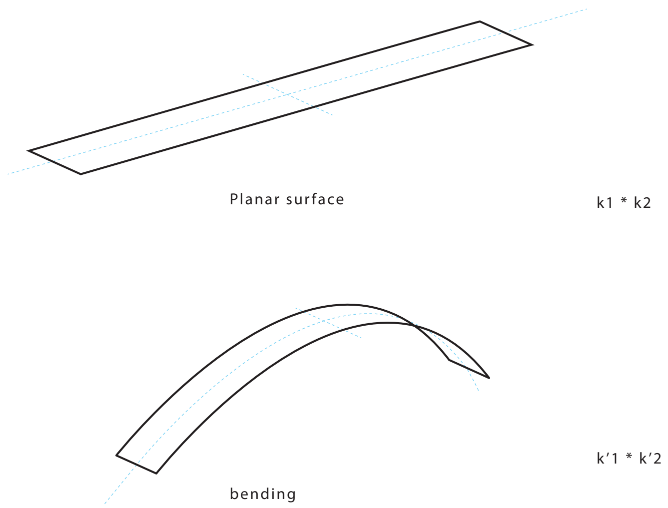
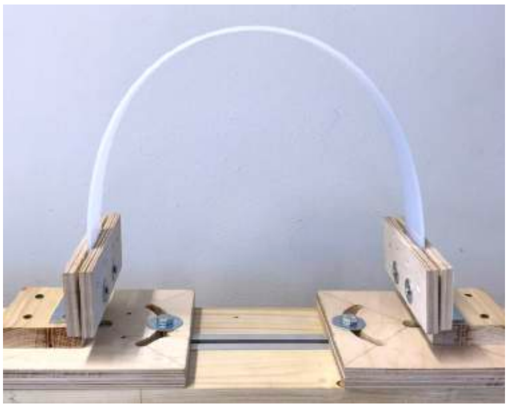


Figure [E6] Bending deformation plate.

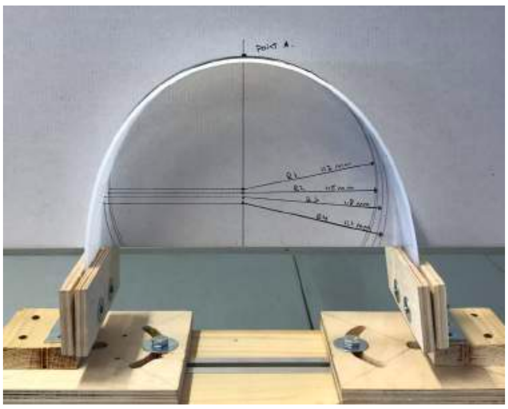
E.2 Measurements

A simulation with actual material allows for validation of this theoretical approach. Deforming a flat strip into one strip with bending and one strip with torsion provides for measuring the individual curvatures. The strip with only bending will be measured on their principle radii. The strip with torsion will be measured on the torsional radius. The important considerations that have to be taken into account are the accurate dimensions of the strips, the pureness of the material and the applied forces to deform the strip, which can be bending moments only. In the comparison of the measured values with the calculated values, small deviations must be accepted due to possible inaccurate measurements.

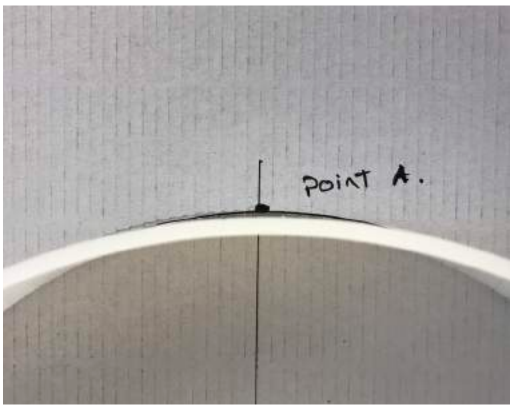


(a)

(b)



(c)



(d)

Figure [E7] (a) Bending plate front view, (b) bending plate side view, (c) four principle curvature measures, (d) point A on surface as measure reference.

The Poisson's ratio influences the deformation of a bent strip. In this measurement, an Acrylic cast sheet will be used because of its relatively high Poisson's ratio. A 3 mm thick and 48 mm wide strip is placed in a bending rig [figure E7 a,b], allowing to control the principle radius 1. The principle radius 2 is the result of the amount of curvature in direction 1. [figures E7 c,d] show the tangent circles in point A on the bent strip. To make sure the approximation of the tangent circle is accurate, four circles with varying radii have been used.

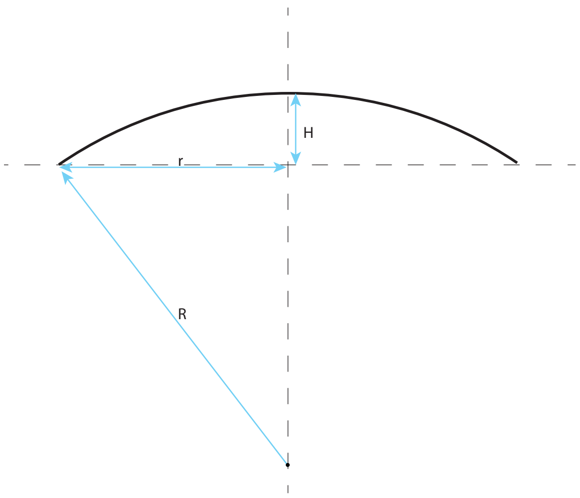


Figure [E8]] Sphere segment, radius calculation

[figure E8] Shows a segment of a circle. H is the height between the border of the strip and the centre of the strip, r is half the width of the strip and R2 is principle radius 2. R2 will be calculated leading to an H value using formula (35). This is the value to measure in order to validate the calculated value. The average for H is 0.9 mm [table E10].

$$H = R^2 - \sqrt{R^2 - r^2} \tag{35}$$

$$My = D(\frac{1}{R_1} + \frac{\nu}{R_2}) = 0 \tag{36}$$

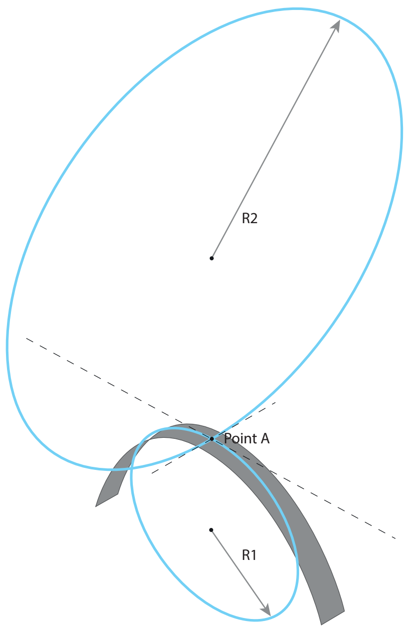
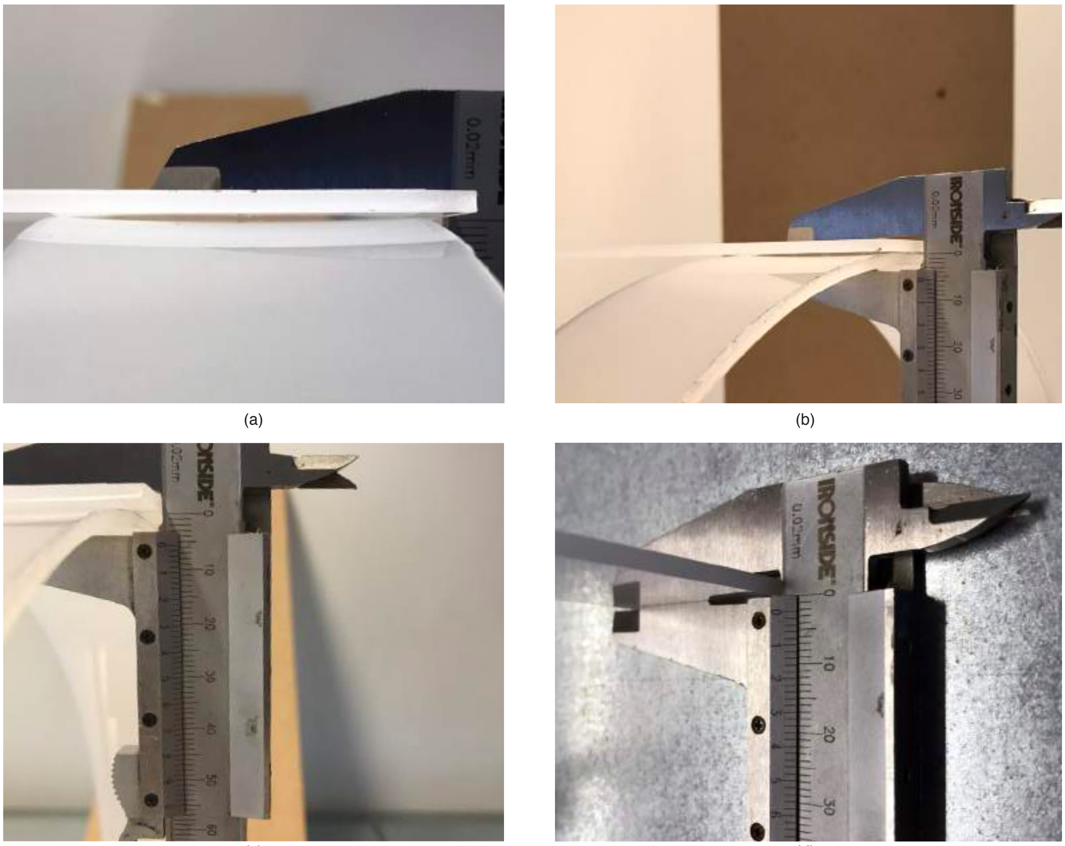


Figure [E9] Input principle radius 1, Output principle radius 2.

Measure parameters	Material properties	Variable parameter
Bending rigidity		3588,48
Youngs modules	2700	
Poissons ratio	0,365	
Width plate		48,00
Thickness plate		2,40

Aproximation	Measured R1 (mm)	Calculated R2(mm)	Calculated H (mm)
1	112,00	306,85	0,94
2	115,00	315,07	0,91
3	118,00	323,29	0,89
4	121,00	331,51	0,87
Avarage			**0,90**

Table [E10] Material properties and measure results



(a) (b) (c) (d)

Figure [E11] (a)Anticlastic deformation, (b) measure of anticlastic deformation, (c) measure of two sheets including void, (d) measure of single planar sheet.

The measured value is the thickness of the bent strip with a flat strip on top minus the thickness of two flat strips [figure A11]. The measured value is between 0.8 mm and 0.9 mm. These values are very close to the calculated values. It has to be noted that more accurate measurement techniques allow for more precise measure values.

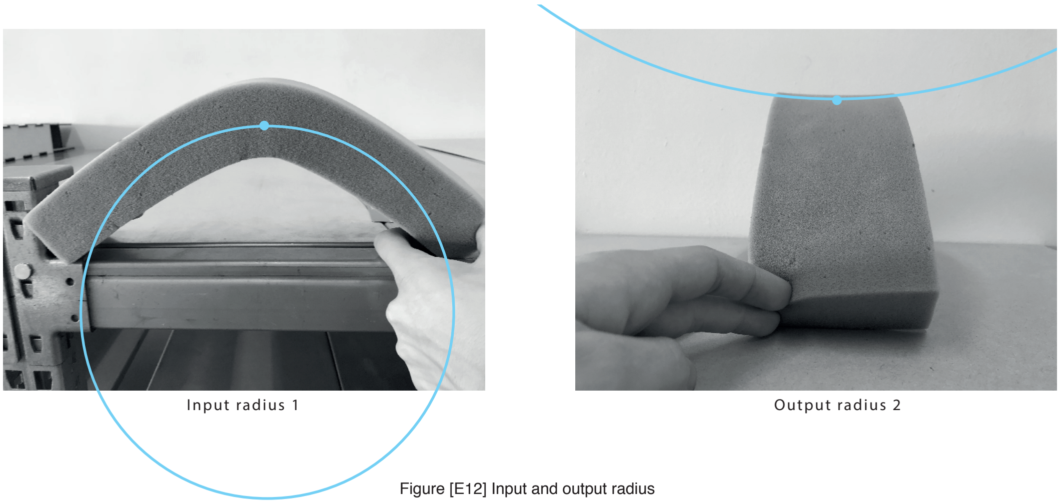


Figure [E12] Input and output radius

Deforming a flat strip into a twisted strip without any bending leads to torsional radii only. The torsional radii are always equal in both directions. The Poisson's ratio influences the magnitude of the torsional moment based on the torsional radii but does not influence the torsional radii themselves. Formula (17,18,19) show the influence of the Poisson's ratio on the bending- and torsional moments. In a situation with fixed pure torsion, the torsional moments are lower for materials with a high Poisson's ratio compared to materials with a low Poisson's ratio. Vice versa, higher Poisson's ratio lead to higher bending moments compared to a lower Poisson's ratio. For bending only, the Poisson's ratio determines a relation between the principal radii.

$$M_{11} = D(\kappa_{11} + \nu\kappa_{22}) \tag{17}$$

$$M_{22} = D(\kappa_{22} + \nu\kappa_{11}) \tag{18}$$

$$M_{12} = D(1 - \nu)\ \kappa_{12} \tag{19}$$

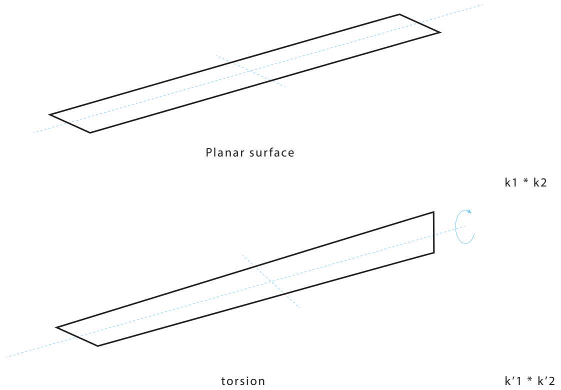


Figure [E13] Torsional deformation of planar surface

E.3 Double curvature through bending and torsion

Continuous linear strips represent the continuity in the triangular paper models. If the strip has a cut or a significant change of direction, the structure is non-continuous. A set of differences can be distinguished by observing the strips in a single curved and double curved structure. In a double curved structure, the continuous strips have bending, torsion or a combination of those two. Continuous strips in single curved structures only bent in one direction, without torsion. The difference between bending and bending in combination with torsion makes a significant change in structural performance. This chapter focusses on the differences in structural performance of single curved elements and double curved elements. Additional factors that influence the structural performance are the constraints, the dimension of the elements and the applied material. A rig [figureE14] allows for testing single elements. This rig enables experimented with bending and torsion through adjustable constraints.

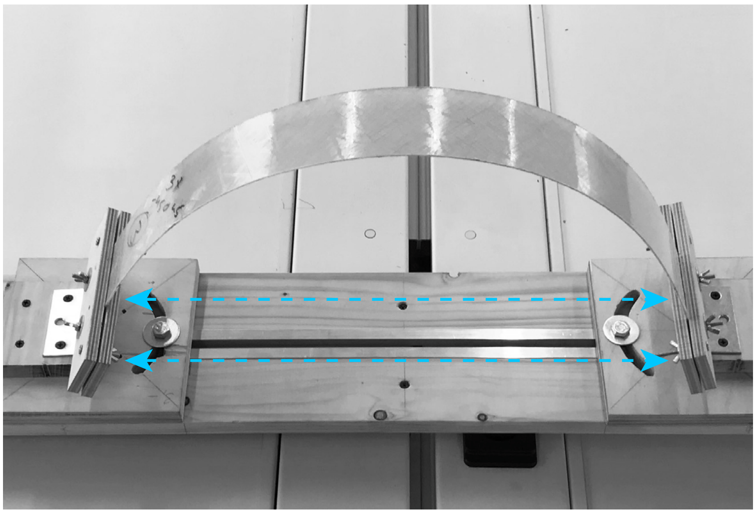


Figure [E14] Bending deformation of planar surface.

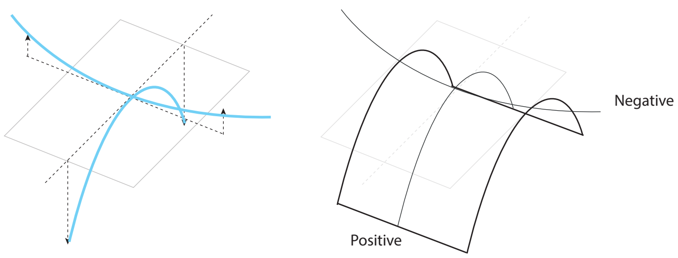


Figure [E15] Form conversion from planar surface to anticlastic surface through bending

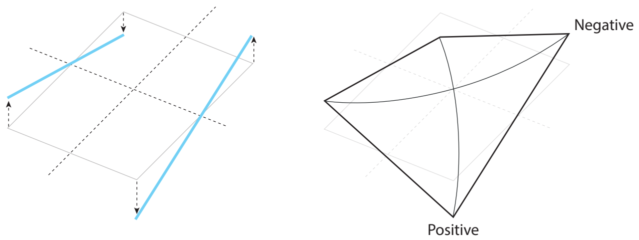


Figure [E16] Form conversion from planar surface to anticlastic surface through torsion

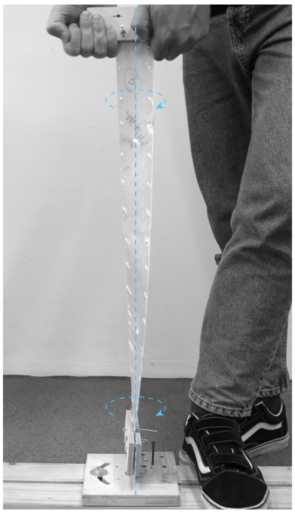


Figure [E17] Torsional deformation of planar surface.

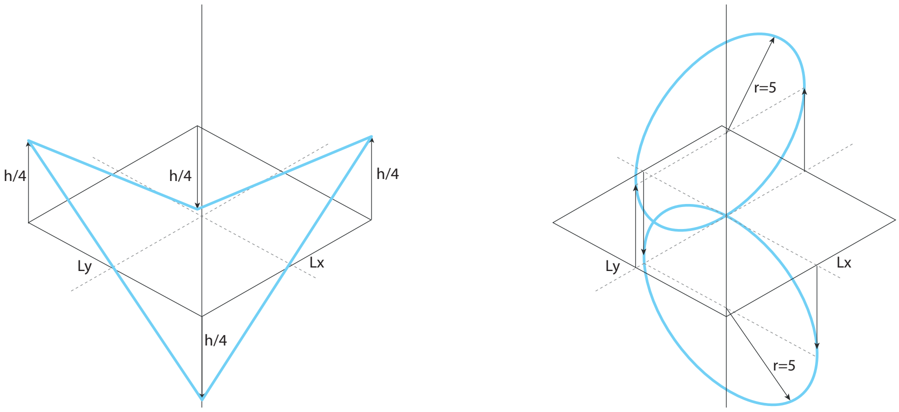


Figure [E18] Measure of anticlastic curvature, bending and torsion.

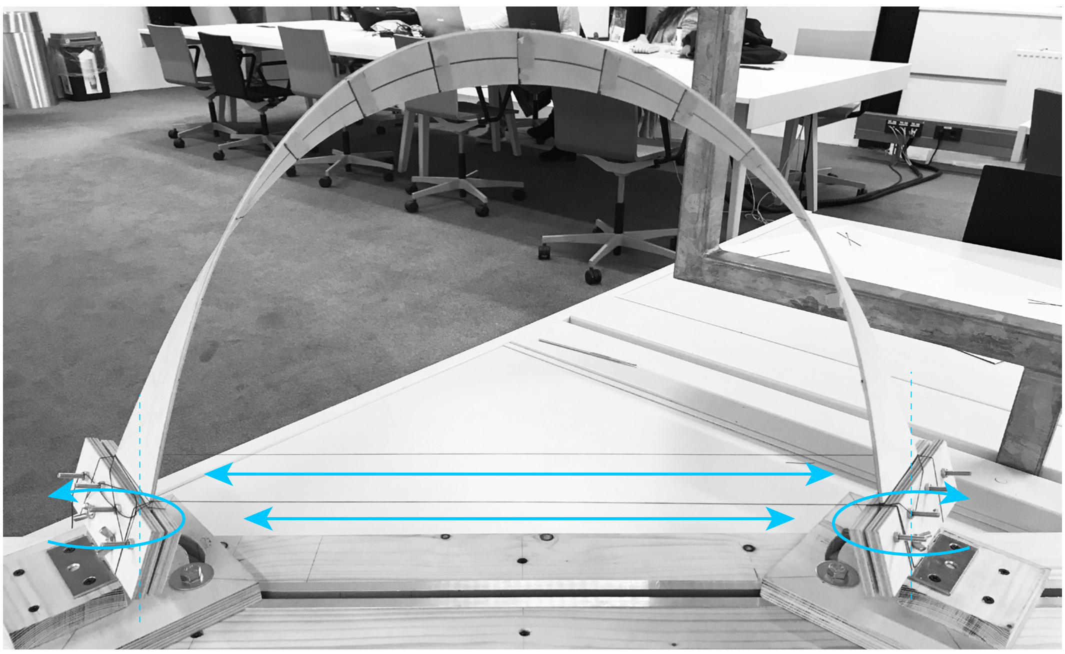


Figure [E19] Bending and torsion combined

E.4 Effect Poisson's ratio on bending and torsional deformation geometry

With the formulas given in the previous chapters, it is easy to control and manipulate the geometrical behaviour of a plate in the deformation phase. The different deformation behaviour of plates based on the Poissons's ratio shows the influence on the geometry. Two individual plates will be compared. Two plates will have an extreme Poisson's ratio with a value of 0.0 and 0.5. These are the most extreme values just for research's purposes. These factors are very unlikely to find in any applicable material. The structural analyses software Karamba 3D has been used to simulate these two different behaviours. Two plates will both equal width, length, thickness, Young's modulus and bending rigidity will be compared.

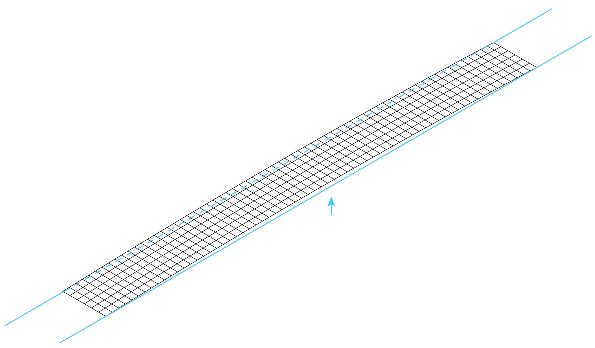
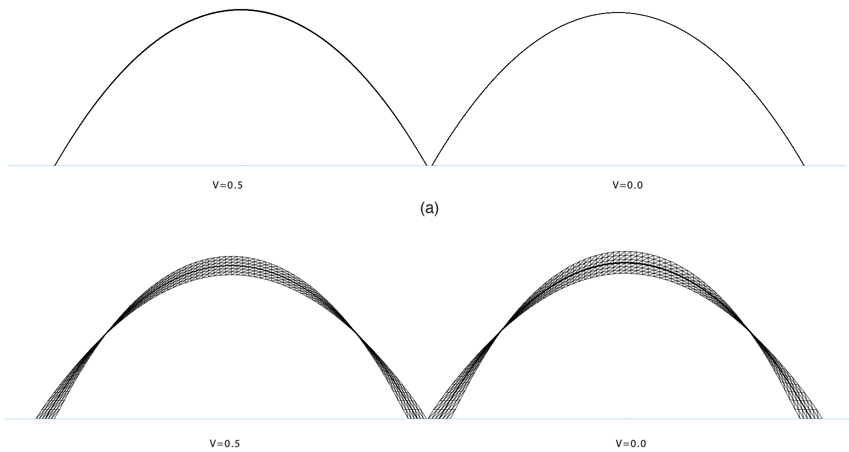


Figure [E20] Planar mesh subject to bending and torsion. Slightly curved for form finding pruposes.

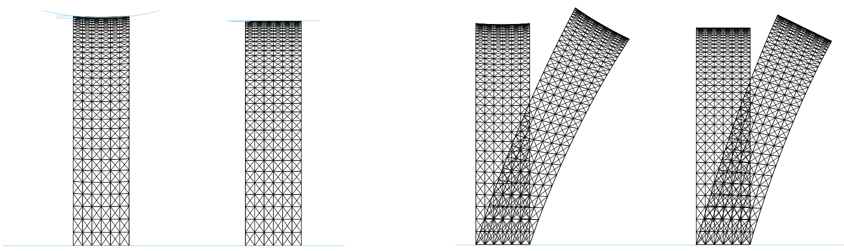
The deformation of these plates has been executed in two steps. The first step is bending only. The second step is torsion, using the deformed strip from bending as initial shape. These two steps will show the influence of the Poisson's ratio on bending and torsion. The formulas for bending moments in a plate show that a high Poisson's ratio leads to bigger bending moments and smaller torsional moments. This influences the deformation as smaller bending moments lead to less resistance.

The blue and red lines are the principal stress lines. The line pattern for the strip with a 0.0 nu value shows a logical arrangement. The strip does not deform in the direction perpendicular to the bending direction. The strip with a nu value 0.5 has a particular line pattern, leaving the most principle stress lines toward the edges as the bending moments get higher at the edge. As a result of a high nu value, there is a bending deformation perpendicular to the bending direction [figure E22].



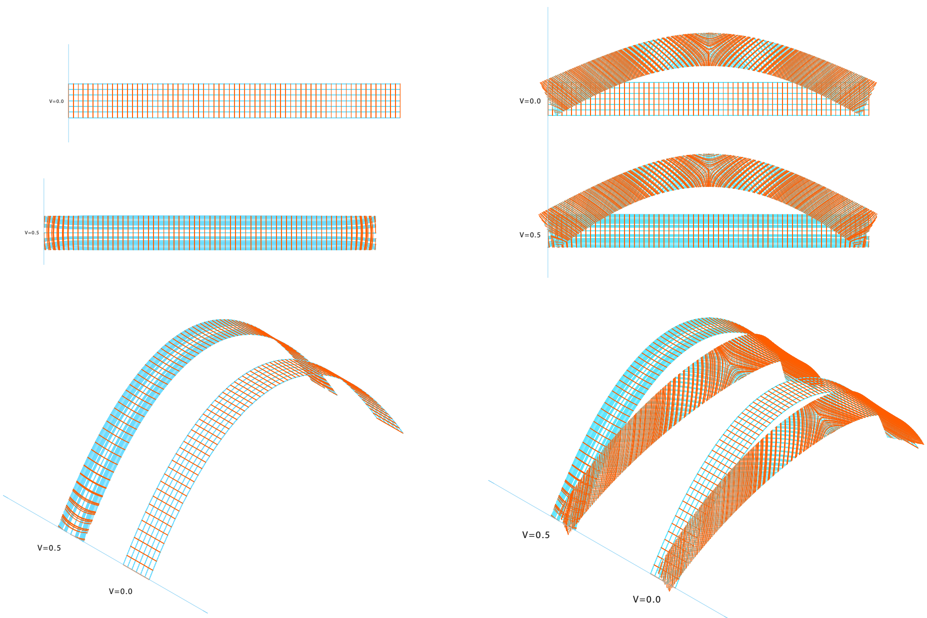
(b)

Figure [E21] Deformation plots side. (a) bending Poisson's ratio 0,5 and 0,0. (b) bending and torsion Poisson's ratio 0,5 and 0,0.



(a) (b)

Figure [E22] Deformation plots. (a) anticlastic an monoclastic through bending (b) Varying cantilevers due to bending and torsion.



(a) (b)

Figure [E23] Princple stress plots. (a) Principle stress direction bending, (b) principle stress direction bending and torsion

The front views [figure E21 a] show the increased functional thickness of the plate as a result of a high nu value. The thickness rises to the top of the arch where the principle radius is the smallest. For bending only, the bending moments increase with a high nu value. For torsion, however, the torsional moments become lower with a high nu value. [figure E21 b] shows both the bent arches with a prescribed angular displacement at the supports, both 45 degrees in opposite directions. [figure E22 b] Shows the difference in sideways motion of both arcs. Because the torsional moments are less for high nu values, the resistance to torsion is less, leading to a more deformed arch. There is no absolute measure on wheater a more sideways deformed geometry is structurally better, as this depends on the load case. The ability to control the amount of sideways motion as a result of the torsional moment can be an architectural advantage, as it allows the designer to influence the geometry.

Loads on twisted and bent arches

More importantly is the way the deformed geometry behaves under load conditions. The plates with both deformations, bending and torsion, have been exposed to a point load in the middle of the arch. This point load provides a general understanding of the load transfer throughout the arches as well as the structural improvement through prescribed displacements. [figure E24] Shows the two arches which are bent in a front view. It is evident that the arch with a high nu value deforms less because the thickness at the top of the arch has increased. Also, the bending moments are bigger, leading to more resistance to deformation. [figure E25] shows the arches with the torsional displacement. Even though the sideways deformation is larger for arches with a high nu value, the deformation as a result of the point load is less. This is because a point load results in bending moments rather than torsional moments. An applied torsional load would lead to more deformation in arches with a low nu value. Arches with a high nu value lead to stronger geometries after bending, being the main structural optimiser. Torsion can be used as structural stiffener leaving the bent arch with more stability in both horizontal directions.

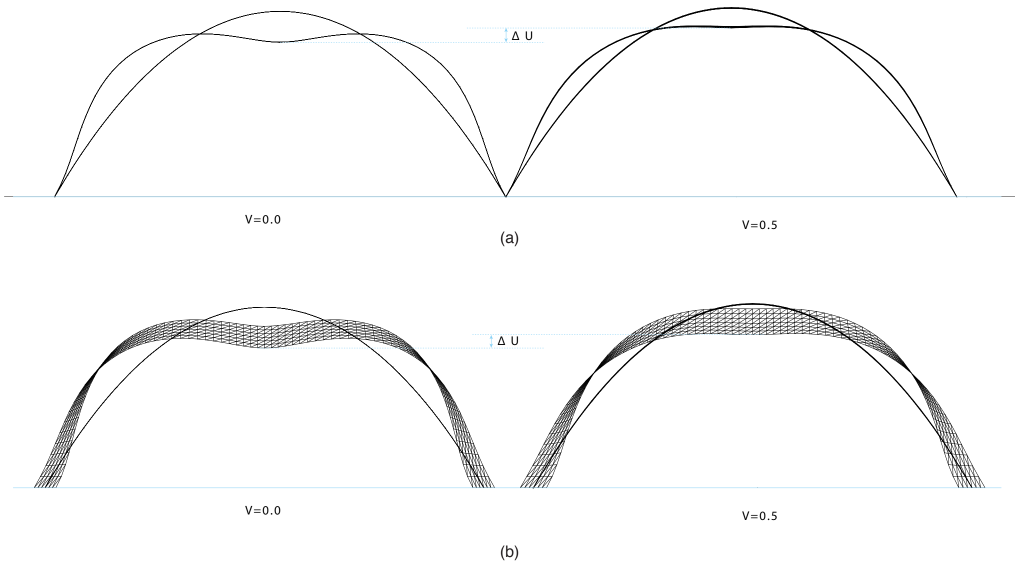


Figure [E24] Front view deflection. (a) deflection arch bendiing only, (b) deflection arch bending and torsion

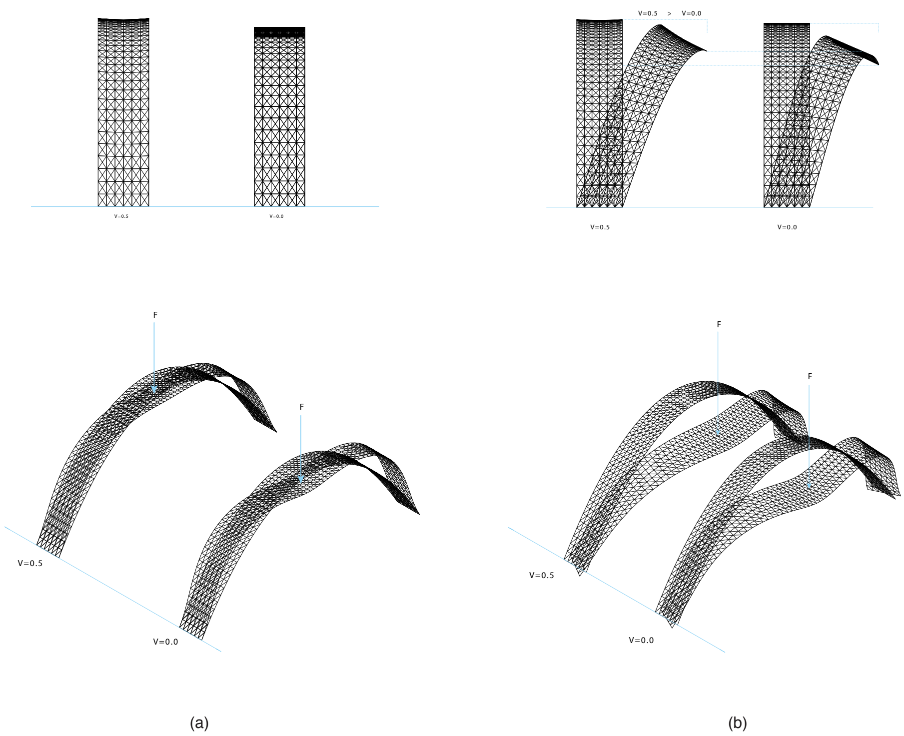
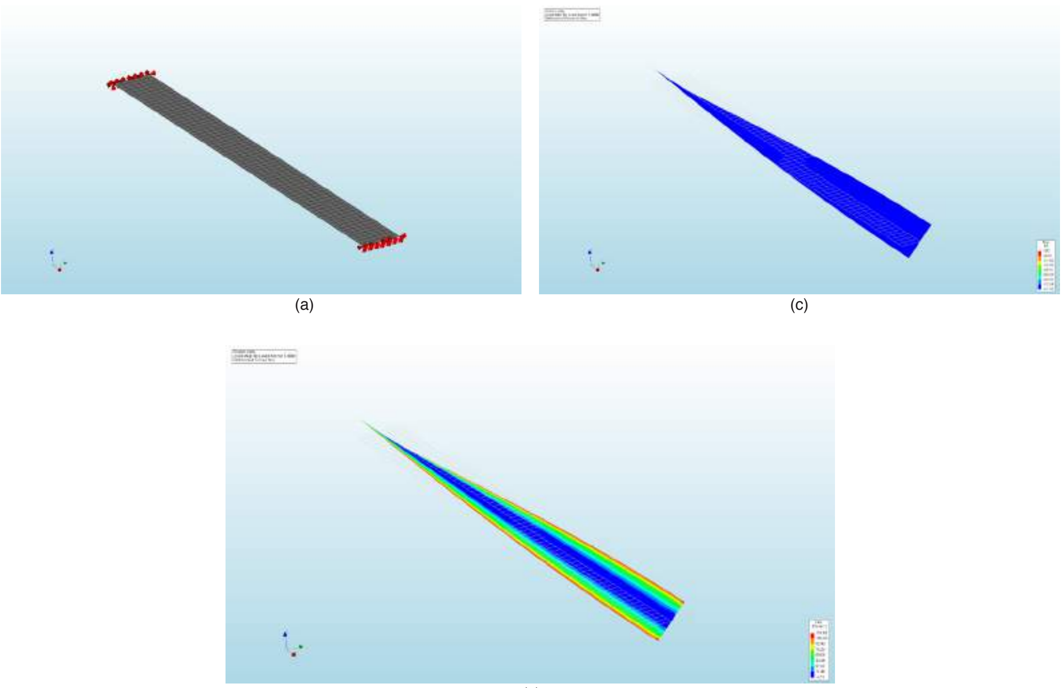


Figure [E25] side view defelction. (a) deflection arch bendiing only, (b) deflection arch bending and torsion

Stiffening effect torsion

A bent plate can develop additional stiffness from torsional prescribed displacement in the supports. For the understanding of this stiffening effect, a flat sheet will be suspect to torsion (figure xxx). Rotation of both supports around the middle axis of the plate with a 45 and -45 degrees angle leads to elongation of the outer fibers of the plate. Fibre elongation results in tensile stresses. These tensile stresses add up to the geometric stiffness of the plate and become dominant in a structure where prescribed torsional displacement has a significant magnitude (Lienhard,2014). [figure E26] shows the stress plot of a flat strip subject to torsion. It shows the tensile bands on the borders of the plate without compression in the centre of the plate.



(a) (c)

(c)

Figure [E26] Structural analysis torsional deformation. (a) planar plate, (b) torsional deformation, (c) stress magnitudes

The stress-stiffening effect can be applied on the elastically bent arch. Further deforming the elastic arch through torsional displacement increases the stiffness. Looking at the stress plot of a bent and twisted arch, it shows that the tensile stress in the outer fibres is still dominant, due to torsion. The plate resists the tension of the outer fibres, leading to compression in the centre of the plate. The magnitude of tensile stress at the edges is bigger than the compression stresses in the centre, which leads to increased geometrical stiffness of the plate.

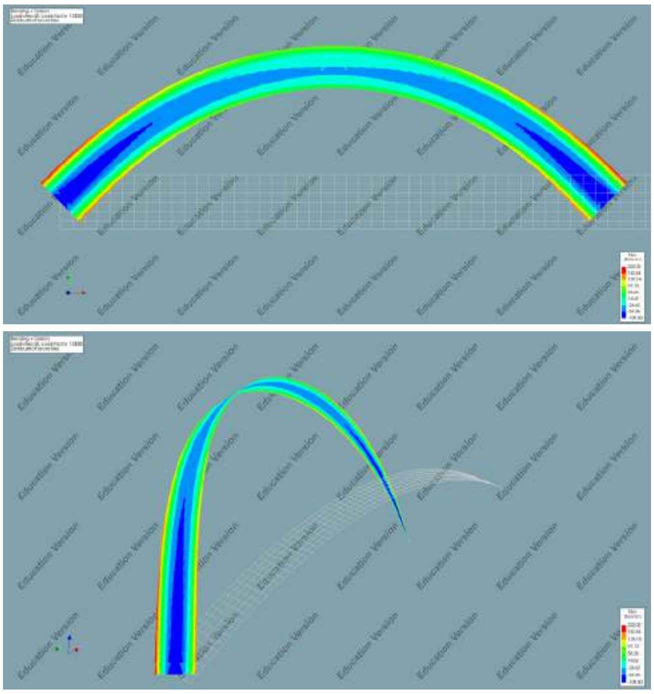
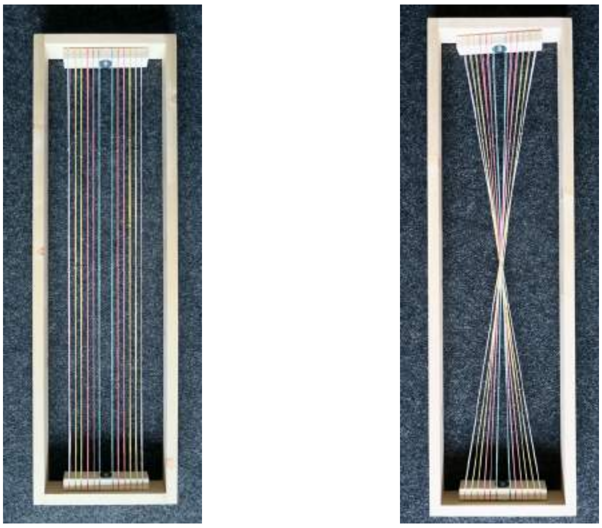


Figure [E27] Structural analysis bending and torsional deformation. Magnitude of stresses

The initially flat plate undergoes two distinct deformations steps. The first step is bending deformation. Bending increases the stiffness of the plate due to material behaviour, especially for materials with high Poisson's ratio. The flat strip deforms into an anticlastic surface which increases the geometric stiffness. The second step is torsional displacement of the clamped supports, around the middle axis of the plate. Torsion increases the tension in the outer fibres of the plate. These two steps together both determine the highly increased stiffness of the shape as well as the architectural geometry which can be used and manipulated as both structural and architectural elements [figure E27].

Torsion model elastic bands

A physical model allows for further elaboration on the structural stiffening effect of torsion. A planar strip can be considered an array of fibres in one direction. In the model shown in [figure E28], the fibres are represented by the elastic bands. Torsional displacement at both ends of the plate, or at both ends of the elastic bands, results in elongation of the fibres. The outer fibres, white elastic bands in the physical model, undergo the biggest elongation. The middle fibres, the green elastic bands, the least elongation. The elastic bands tend to take the shortest path resulting in compression which in this model results in an intersection of all the elastic bands.



(a) (b)

Figure [E28] Structural analysis torsional deformation. (a) no torsion, (b) torsion

(a) (b)

Figure [E29] Structural analysis torsional deformation. (a) no torsion, (b) torsion

The significant difference in structural performance between a plate and a model with elastic bands is that the plate resists the compression of the fibres. If the plate resists the

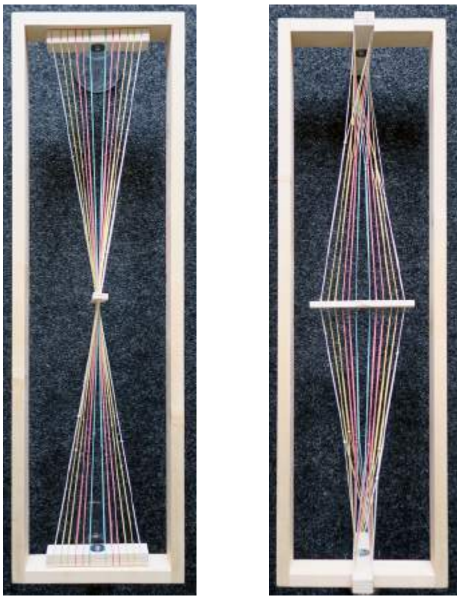


Figure [E30] Structural analysis torsional deformation with plate spacer

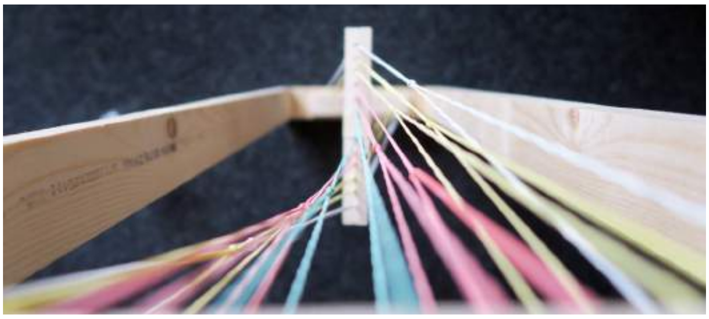


Figure [E31] Structural analysis torsional deformation with plate spacer, top view

compression as a result of torsion, the fibre elongates more, as they can not follow the shortest path. This results in more considerable tensile stresses in the outer fibres. The ends of the plate are clamped which holds them in place. The deformation as a result of torsion is restrained, disallowing surface curvature at the top of the plate and making the fibres elongate. This effect increases the tensile stress in the plate. In [figure E30], the model includes a spacer in the middle of the elastic bands. This spacer represents a section of a plate where warping of the section is restrained (which only occurs at both ends of the plates). The increased tensile stresses in the elastic bands result in increased stiffness of the entire strip. As long as the tensile stresses are more significant than the compression stresses, the plate has an increased stiffness.

F Design variables

Parameters active bending plate arch

F.1 Design variables plate arch

With bending and torsion both as design driver and structural stiffener, some individual design parameters can be distinguished. First of all, the ratio between the width of the arch and the height to the top of the arch, measured from the ground. This ratio determines the force-flow through the structure. There is no absolute optimum value for this ratio because it is highly dependant on the load cases. On average for active-bending structures, a ratio of 1:3 may be used. During the form creation of a flat plate into a curved arch, the supports are free to move. After the formation process, the supports have to be fixed in all directions. Free rotation around the bending axis allows for low bending moments in the supports but may lead to insufficient stability of the overall structure. Completely fixed supports without rotation adds up to the stability of the structure. However, the plate requires additional strength at the support to prevent it from failing as a result of local peak bending moments.

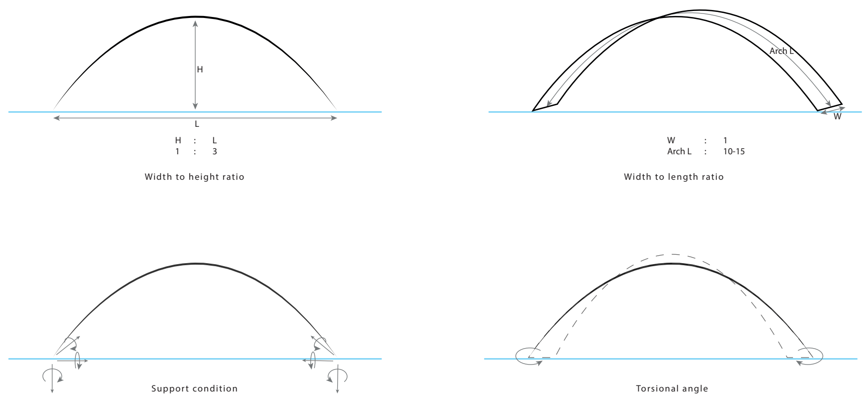


Figure [F1] Design variables arch structure

The previous chapter showed the stiffening effect of a high Poisson's ratio as a result of bending a plate. The wider the plate, the higher the deformation perpendicular to the bending direction. For active-bending structure, the width of the plate has certain limits. First, the wider a plate, the harder it is to deform the plate. Secondly, the deformation perpendicular to the bending direction gets disrupted at the support. Clamping the plate at the supports prevents the plate from deforming as a result of bending. As a rule of thumb, the plate starts to deform at one width distance from the support [figure F2]. A minimal width may lead to insufficient structural capacities. A ratio 1:15 between width and length is a sufficient measure to both be able to form the plate and use the stiffening effect as a result of bending (Baratta, 1981).

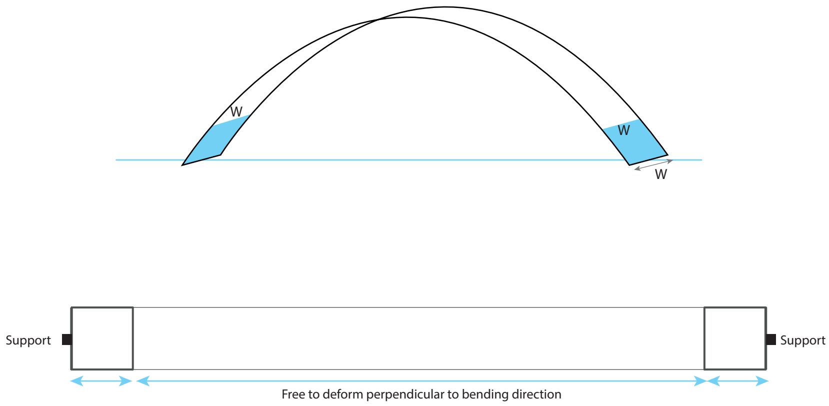


Figure [F2] Distorted deformation due to clamped plate supports.

Additional to the stiffening effect of bending, torsion adds up to the stiffness of an arch as well. Both the supports rotate around the vertical axis in the centre of the strip. The stiffening effect as a result of torsion depends on the amount of twist. [figure F3] Shows three distinct twist angles. Of both the supports are aligned, they are only stable in one direction. If both the supports get a twist angle of 45 degrees and - 45 degrees, they end up in a position perpendicular to each other. The total amount of twist in the plate is 90 degrees. The orientation of the supports adds up to the overall stiffness of the arch.

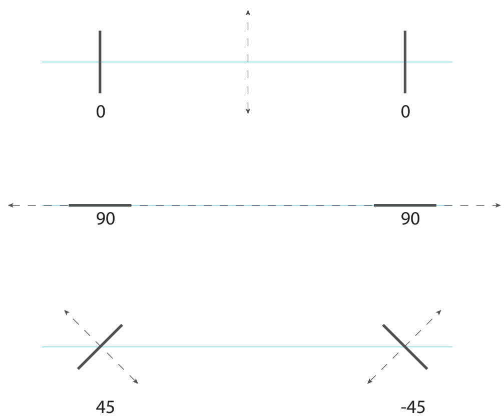


Figure [F3] Torsional angles of respectively 0, 90 and 45 degrees.

F.2 Shape

As a result of rotation around the vertical axis in both supports, the arch deforms sideways. One edge of the plate stretches, resulting in a lower but wider curve. One edge of the plate compresses, resulting in a higher but smaller curve. Because the length of the plate's edges does not shrink or stretch (disregarding creep) and because of the distance between the edged remains equal, the plate deforms out of its original vertical plane. The top of the bent and twisted arch now cantilevers sideways rather than being vertically aligned with its supports. A vertical load has to be transferred sideways to the supports [figure F4,F5]. This load transfer may seem to contradict with the structural logic that the arch is stiffer when it bents out of plane. However, through twisting the supports, additional stiffness is obtained, leading to a more rigid and stronger arch.

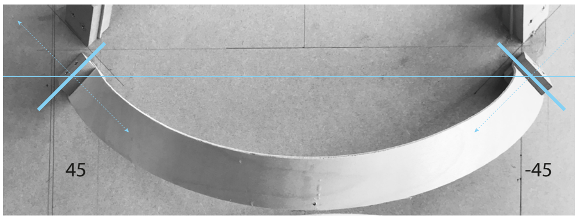


Figure [F4] 45 and -45 degrees twist at supports

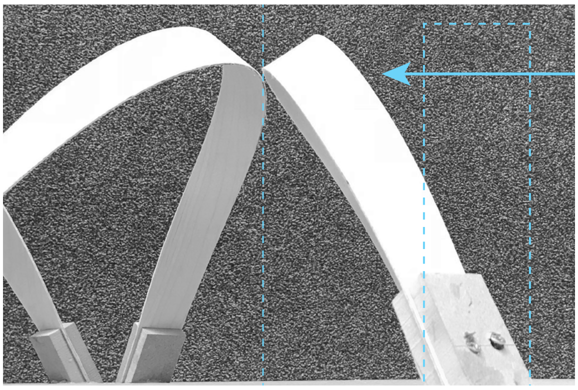


Figure [F5] Cantilever deformation arch as a result of torsional displacement

Even though the arch which is both bent and twisted is stiffer than the arch with only bending, the out of plane motion is a weak element in the arch. Strategic placement of multiple arches benefits the overall stiffness, supporting the individual arch at the structurally most vulnerable location. Most simply and logically, the arrangement of two arches next to each other, with their lower tops connected, allows for horizontal support preventing from significant horizontal displacement due to loads [figure F6, F7]. Two interconnected arches create a stable structure with supports in 4 directions, all pointing towards the top of the structure. The simplistic deformation and topology of two plate strips now form the base of a system which entirely relies its structural performance on the elastic behaviour of the material.

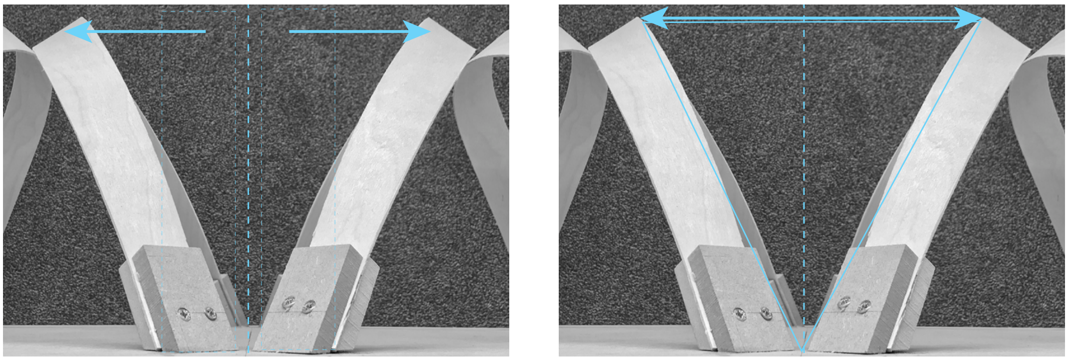


Figure [F6] Tension cable connection between two opposite arches

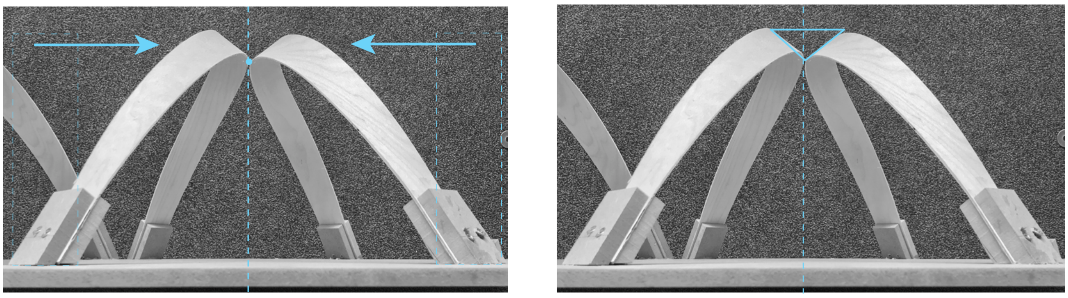


Figure [F7] Fixation connectino between two arches leaning against each other

F.3 Material selection based on Geometric stiffening effect

As mentioned in chapter B, suitable materials for active-bending structures depend on the ratio between maximum tensile strength and stiffness. One of the defining aspects of active-bending structures is the ability to deform planar sheets easily. Considering both the formation process and the final load bearing capacities, active-bending structures require contradicting properties. The formation process requires high flexibility, large deformation and the ability to deform without high embodied energy. The outcome structure needs different properties, high stiffness, low deformation and high load-bearing capacities. This contradiction can be mediated with the right material selection and the outcome geometry. Besides the proper ratio between the strength and flexibility of the material, it also requires a high Poisson's ratio, as it increases the geometric stiffness during the elastic formation process. Fibre reinforced polymers and woods have proved to be suitable materials for active-bending structures. Both timber and GFRP's are anisotropic materials. Additionally, according to La Maga, ''Wood and GFRPs are known to be susceptible to long-lasting creeping effects, especially in combination with high strain and deformations which is typically the case of bending-active structures.''(La Magna, 2017, p 20).

Fibre reinforced polymers offer a higher Poisson's ratio and are easier to adapt to the structural desires. Pure bending has principle stress directions parallel and perpendicular to the direction of the element; 0 and 90 degrees. Pure torsion has principle stress directions at 45 and -45 degrees relative to the direction of pure bending. Using GFRP's allows for manual manipulation of the fibre direction. The deformation of bending and torsion depends

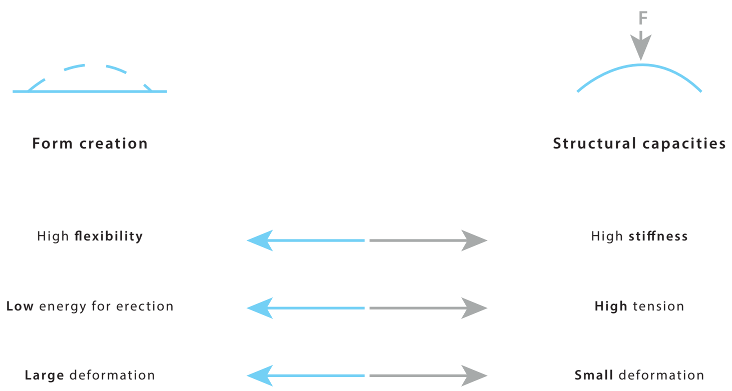
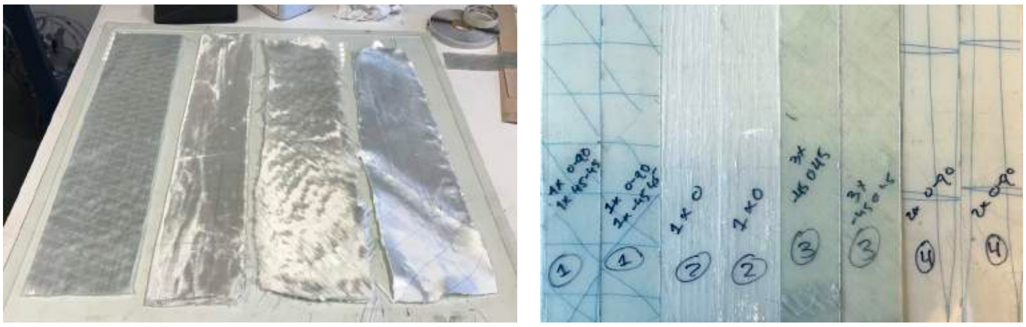


Figure [F8] Contradiction between form funding purposes and structural capacities

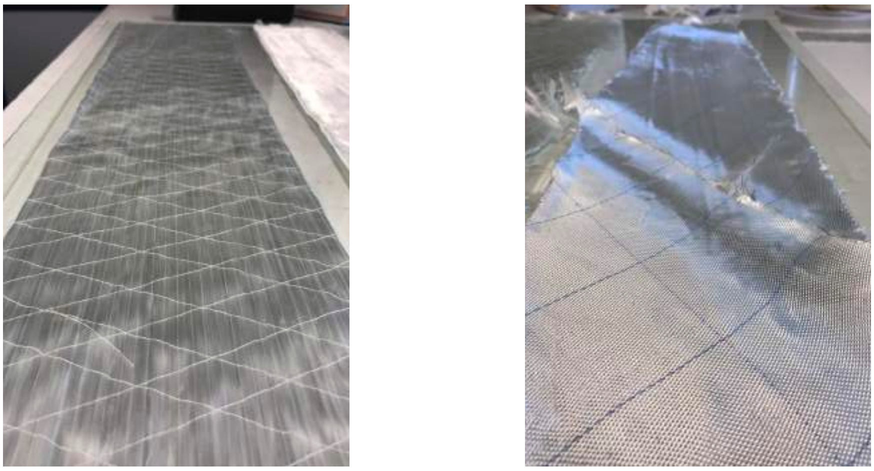
on the amount of resistance in the principal stress directions. Manual placement of the number of fibre layers and the direction of fibres determines the deformation, purely based on the amount of resistance. Working with GFRP's in active-bending structures enables full control of how much and how a strip deforms based on the local resistance from the amount and direction of fibres.

Considering the need for the isotropic behaviour of the material, fibres in all directions are required. A quasi-isotropic laminate is an FRP with fibres in 0-90 and 45-45 direction and shows isotropic Young's modulus and Poisson's ratio behaviour (Van Otterloo, 2003). Unidirectional fibre lay-up allows for high strength in a single direction. For bending only this would be highly suitable, for bending and torsion, strength in all directions is required. Additionally, The Poisson's ratio is a structural stiffener in this structure. Composites with a unidirectional lay-up cannot make use of the beneficial effect of the Poisson's ratio, which is deformation as a result of bending in a perpendicular direction.



(a) (b)

Figure [F9] Four varying Glass fibre layup. (a) naked fibres, (b) composite strips



(a) (b)

Figure [F10] Naked glass fibres layup. (a) lineair direction, (b) cross direction

The first experiment with composites directly shows the influence of the number of fibres and fibre direction. [figure F9] Shows four different fibres directions, placed on a glass sheet, ready for the vacuum infusion production process. The first two sheets only have

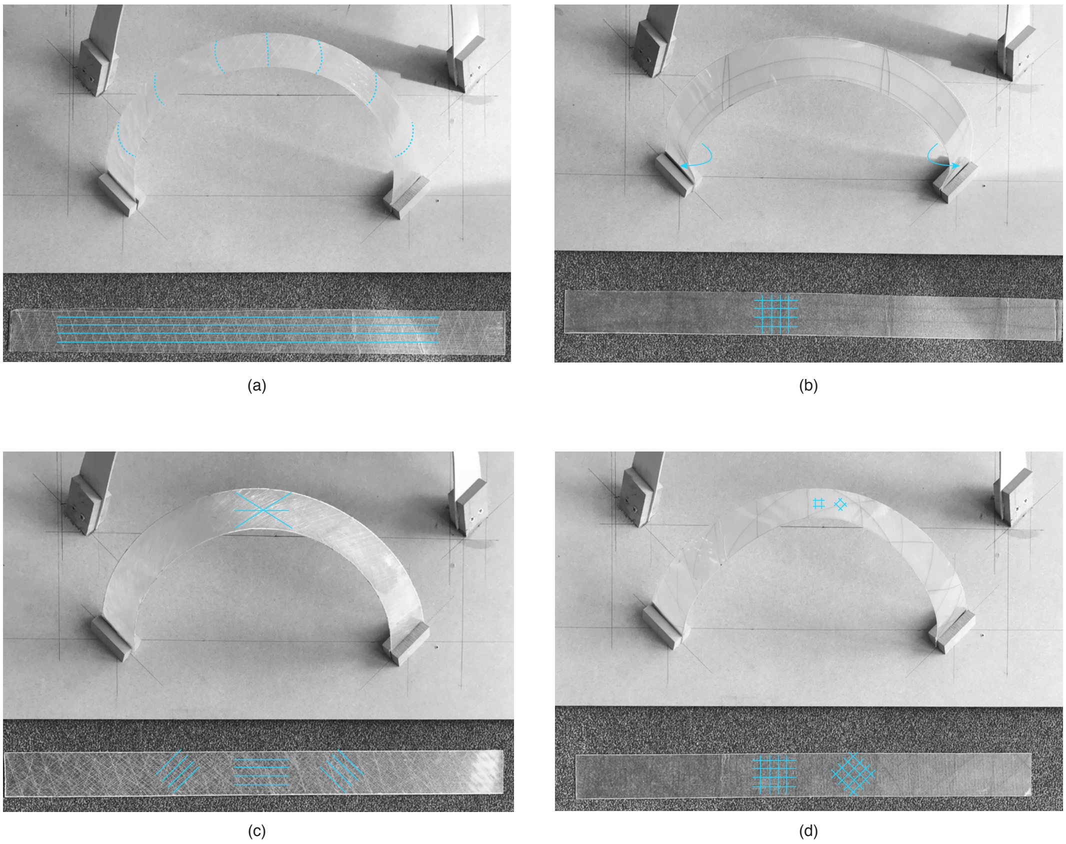


Figure [F11] Behaviour bending and torsional deformation varying fibre directions (a) lineair direction 0 degrees, (b) cross lineair direction 0 and 90 degrees, (c) Lineair and cross direction 0, 45 and -45 degrees, (d) Cross directions 0, 90, 45, -45 degrees direction.

fibres in the bending direction; the second two sheets have fibres in both bending and torsional direction. All the strips have an equal thickness of 0.5 mm. Even though this is too thin for structural purposes, it allows for relative comparison of the influence of the fibre directions. All the strips will be suspect to bending and torsion of the same magnitude.

The first strip has only fibres in the direction of bending. These fibres do not give any resistance perpendicular to the bending direction. This strip is weak, and it tends to curve along the weak axis quickly, leading to unstable and insufficient load bearing capacities. The second strip has fibres in both the perpendicular and aligned to the bending direction. Compared to the first strip it remains relatively flat. However, where the strip undergoes torsional displacement, it results in an outwards deflection as a result of weak resistance to torsional bending moments. Strip three and four show similar deformations. They possess of fibres in both the bending and torsional directions, 0-90 45 and -45 degrees. A clean twisted arch is the result of resistance in both the bending and torsional direction [figure F11].

This proves the direct influence of the fibre directions to the deformation of a strip suspect to bending and torsion. With a given twist angle of 45 degrees, the amount of outwards deflection can be manipulated with the number of fibres in a particular direction. More fibres lead to more resistance which leads to less deformation. This design freedom must always be within the bounds of a sufficient load bearing capacity.

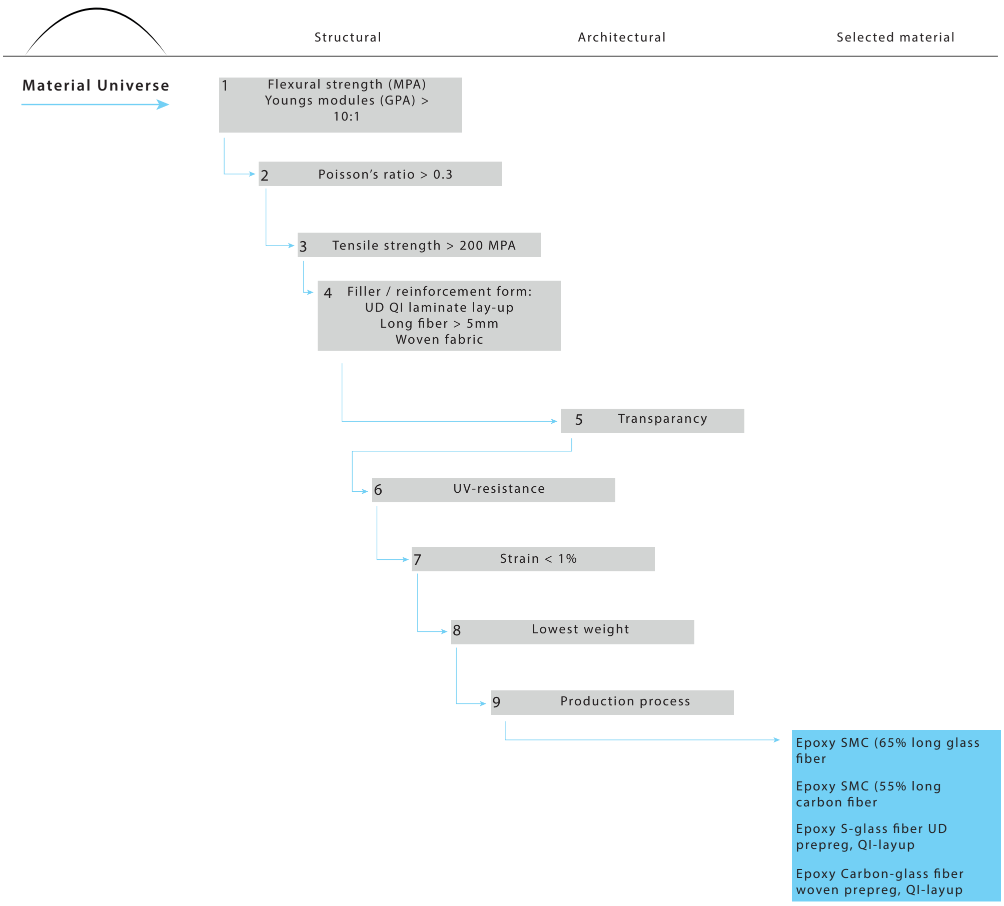


Figure [F12] Material selection diagram

The applied structural material for the arch has been selected based on some parameters. Some of these parameters are based on structural performance, and some are based esthetic values. The structural parameters are the Poisson's ratio, the ratio between flexural strength and stiffness and the quasi-isotropic laminate lay-up. The minimum Poisson's ratio is 0.3. The flexural strength to stiffness ratio is at least 10:1. The Lay-up angles of the fibres are at least 90-0 45 and -45. Additional parameters such as UV resistance, low strain, weight and production processes have been taken into account.

The transparency of the composite is a rather aesthetic parameter. Lightweight, thin active bending structures expose a certain lightness. The contradiction of transparency in the structural arches adds up to the lightness of the entire structural system.

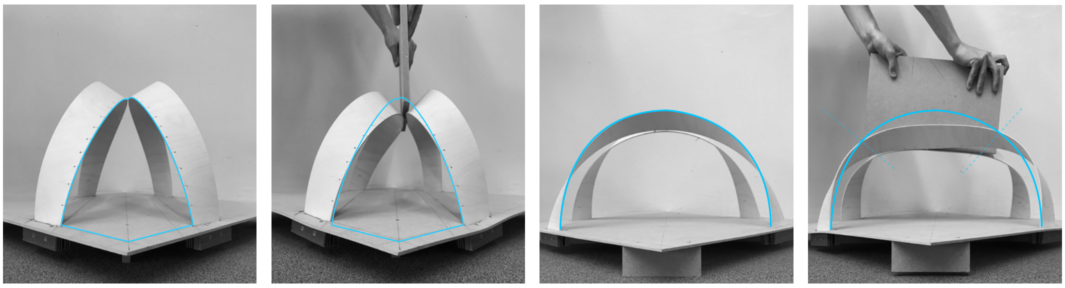
G Structural Analyses

Structural performance active bending arch

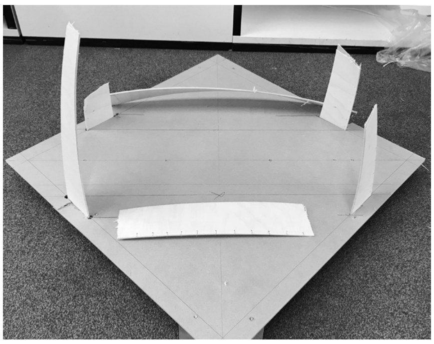
G.1 Weakness / load conditions / test

The research aim is to create a structural system out of initially flat plates, deformed through elastic deformation. Through bending and torsion the plate develops a significant stiffness. Even though the plate with its increased stiffness is suitable for a structural arch, there are still weak spots. Finding and understanding these weak spots will be done through empirical analyses. The results gathered from this analysis will be used for further studies through software simulation.

As a result of torsion, the arch deforms out of plane towards one side. Therefore, the minimal amount of plates in this active-bending structure is two. They are positioned in the opposite direction to stabilise each other. The first load analysis is a distributed load on two connected arches. The second analysis is a point load on four connected arches.



(a) (b)



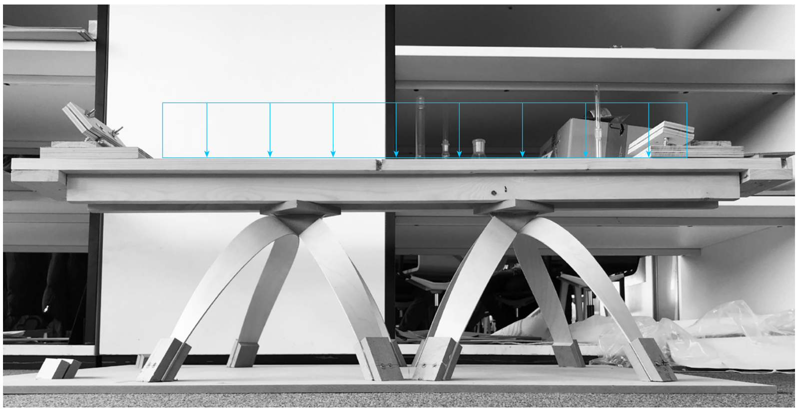
(c)

Figure [G1] Physical model simulation coupled arches. (a) side views deformation, (b) front view deformation, (c) strucutres failiure, breaking points.

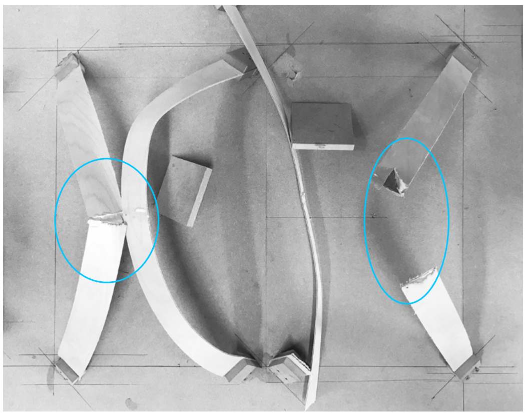
[figure G1 a,b] show the initial shape of the two arches. One side view and one front view. The front view shows the deformation going into two directions. The sides of the arch move outwards as a result of pressure forces. The top of the arch deflects vertically. Both these deflections rotate around an axis perpendicular to the plate, indicated with a dashed line [figure G1, c]. Increasing the load till failure of the structure shows the breaking points of the plate, precisely at these two axes. A vertical point load results in plate failure at the position where the plate deformation direction switches.

The second empiric analysis is a point load on four connected arches [figure G2]. A triangular connection where two arches meet prevents the arches from moving away from each other. Interesting to see how only the outer arches fail. This failure is most likely the result of instability in the structure, direction all the load to the outer arches. Sideways sway leads to significant loads in the outer arches and minimal load in the middle arches. In a structure like this, the outer arches would have to be stronger to withstand extra loads due to sway in the entire structure.

These analyses are a first approach to find the weaker spots in an active bending structure. The first study with two arches showed the weakest position in the plate under vertical load conditions. The second study showed the effect of a load on an array of arches, leading to failure in the outer arches.



(a)



(b)

Figure [G2] Physical model simulation four coupled arches. (a) point load at top. (b) failure outer arches

G.2 Wind pressure load simulation

scale model wind simulation

In classical structural engineering, three scales can be distinguished: the physical structural model, a reduced scale structure and a large-scale structure. ''In today's engineering practice analysis is mostly based on Finite-Element-Modelling (FEM) in which dimensions are considered by the relation between geometrical and mechanical input variables. Structural analysis is therefore always done on a virtual 1:1 model'' Lienhard, & Knippers (2013, p-137). Scale models are highly relevant in the development of active bending structures. They provide valuable insight into stability behaviour. However, the dead load plays an essential role in the stability, which is usually different in scaled models. These models will be evaluated on their axial forces only, i.e. what load case leads to tension and what to compression. Increase of tension leads to increased stiffness, compression leads to instability in the structure resulting in buckling.

Active-bending structures are known as flexible structures. Structural analyses for active-bending structures are different from general rigid structures such as concrete floor plates or steel beams. The form creation process for an active-bending arch has been done through non-linear prescribed displacement analyses. Large deformations are the result of multiple load step iterations of the specified displacement of the supports. This analysis results in shapes which undergo large deformations relative to the initial shape.

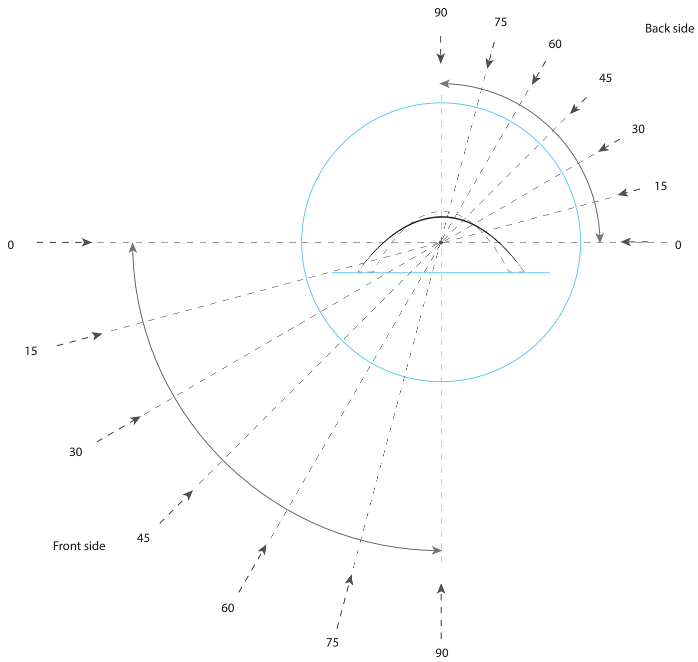


Figure [G3] Wind simulation horizontal windload angles

Because the structure is flexible, variables load cases such as rain, wind and possible snow may cause large deformations as well. The previous analyses of a vertical load case on top of the two arches results in failure of the strip at the position of the weakest axis relative to the load direction. A wind test allows for quick analyses of the weak points in a twisted and bent arch, relative to the varying loadcases. The use of a fan and a scaled model of a twisted and bent arch allows for quick and effective analyses of any load direction. Not only does the wind pressure from the fan simulate the wind load in an actual situation, but it also functions as a simulation for a vertical load case such as rain or snow. Even though structurally insufficient, setting the stiffness, i.e. the Youngs Modules to 10-20% of the actual material, deformation becomes more visible. Large deflection with there correspon-ding load directions provides inside in the performance of the arch.

The first analysis is a wind load to the side of the arch. From this position, the arch will rotate in six steps over 90 degrees, 15 degrees per step, until it faces the front of the arch. The same analyses will be performed starting at the same side, rotating 90 degrees to the back of the arch. These 12 steps cover all the possible horizontal load cases [figure G3]

The second analysis is a vertical load case. Again, starting at the side of the arch, rotating over 90 degrees in 6 steps until the wind load is vertical. Because the arch is symmetric over the z-axis, six steps are sufficient for all vertical load conditions [figure G4].

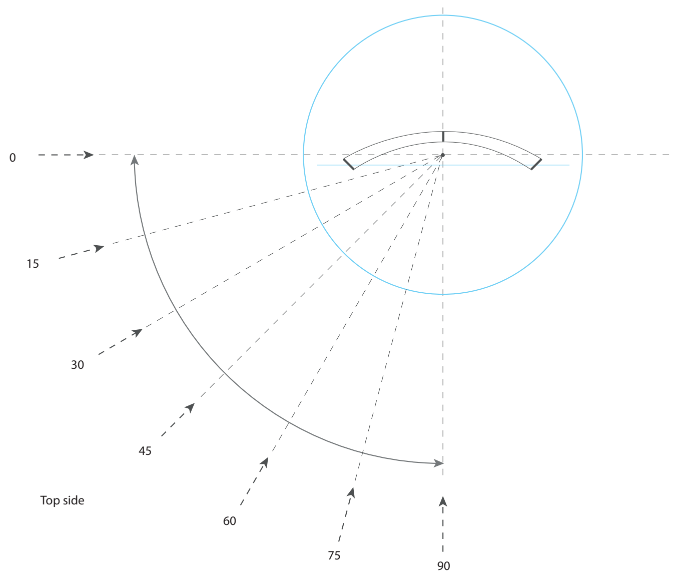


Figure [G4] Wind simulation vertical windload angles

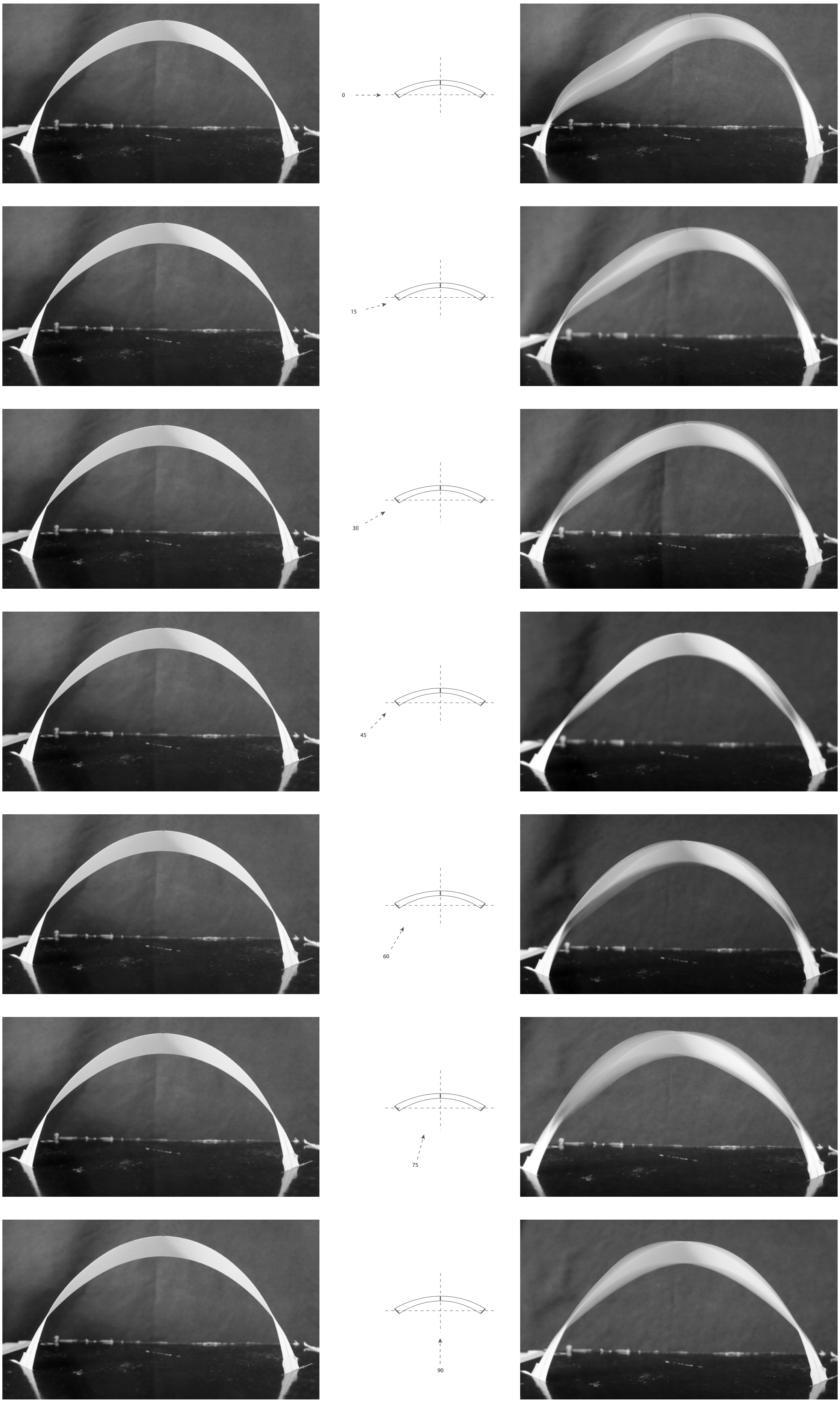


Figure [G5] Wind simmulation front side through seven wind pressure angles.

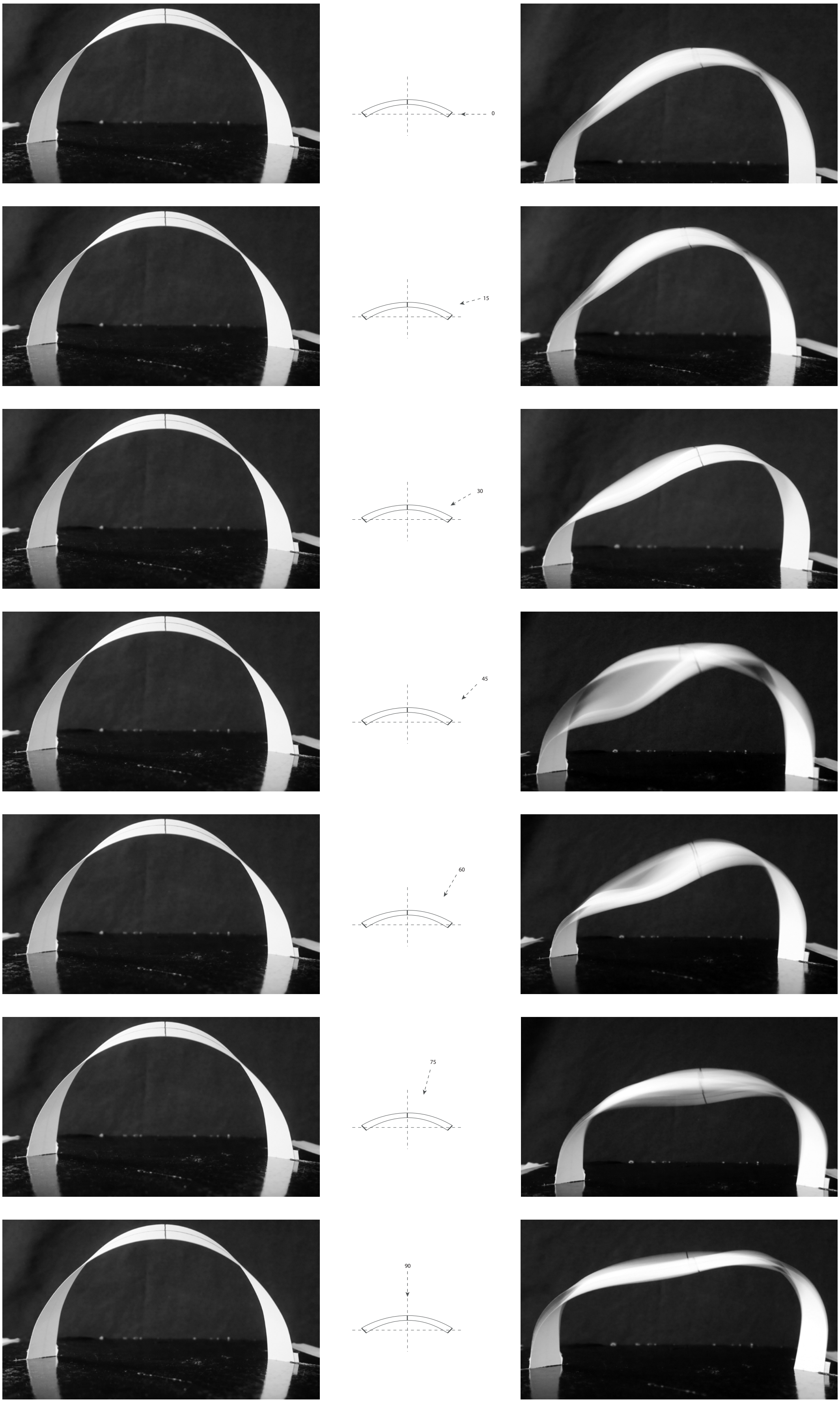


Figure [G6] Wind simmulation back side through seven wind pressure angles.

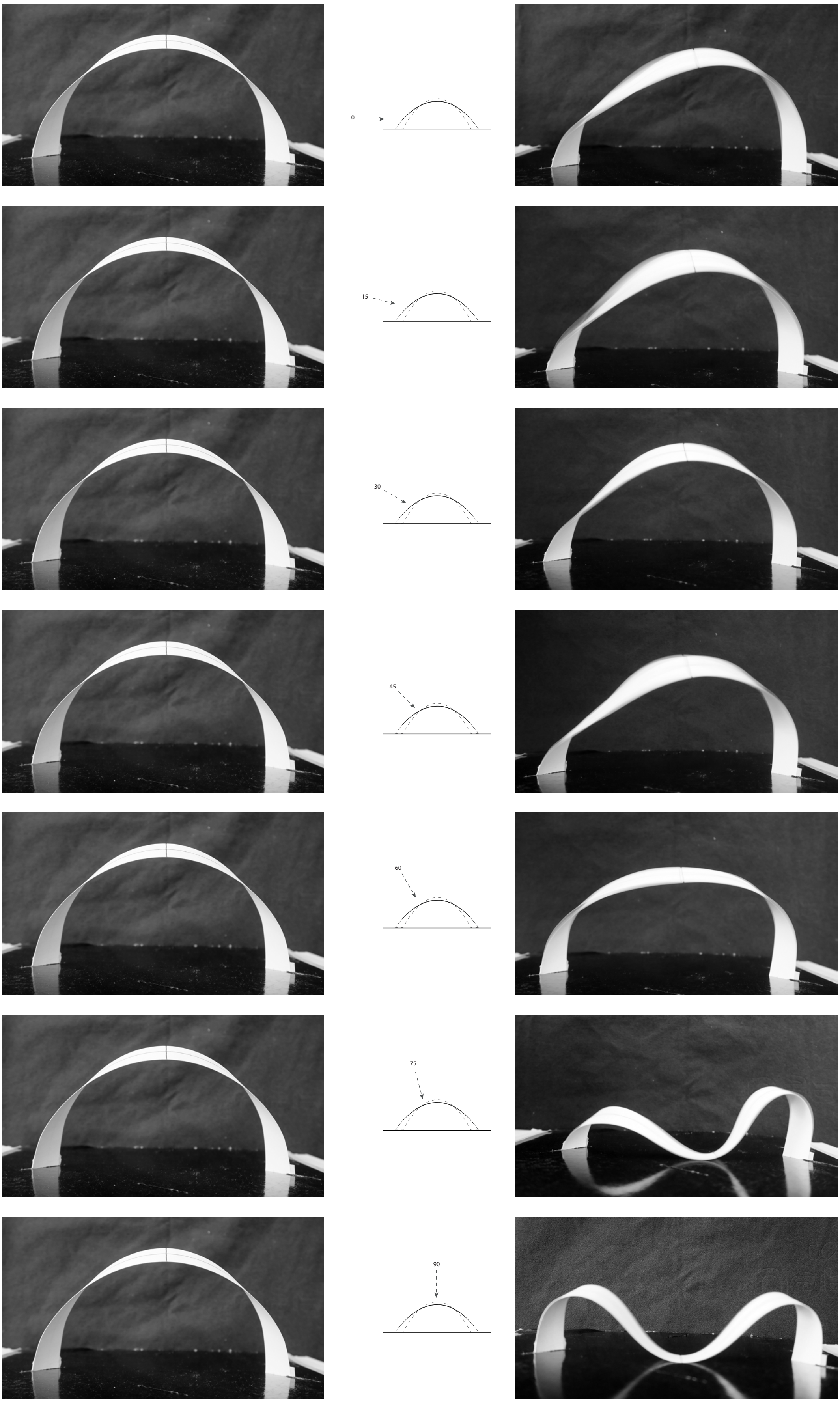


Figure [G7] Wind simmulation top through seven wind pressure angles.

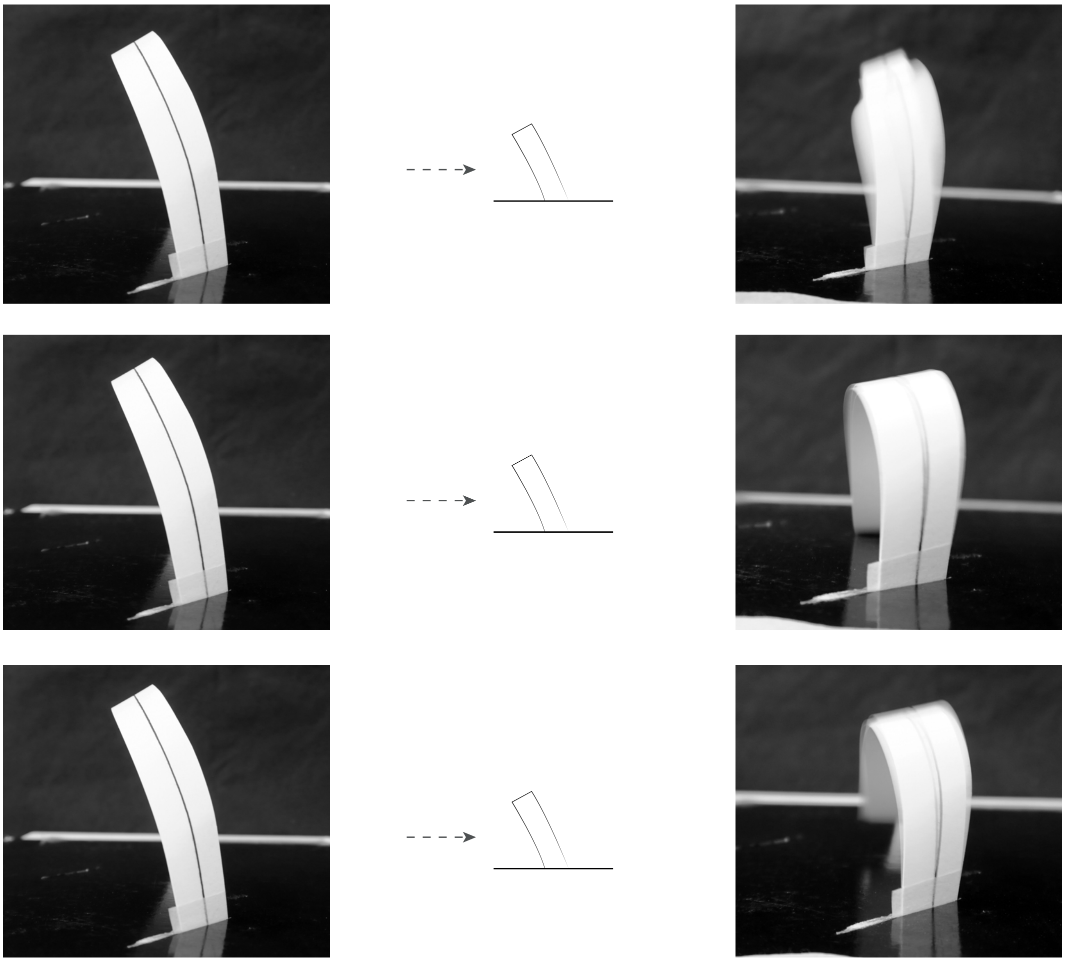


Figure [G8] Wind simmulationback side, right view

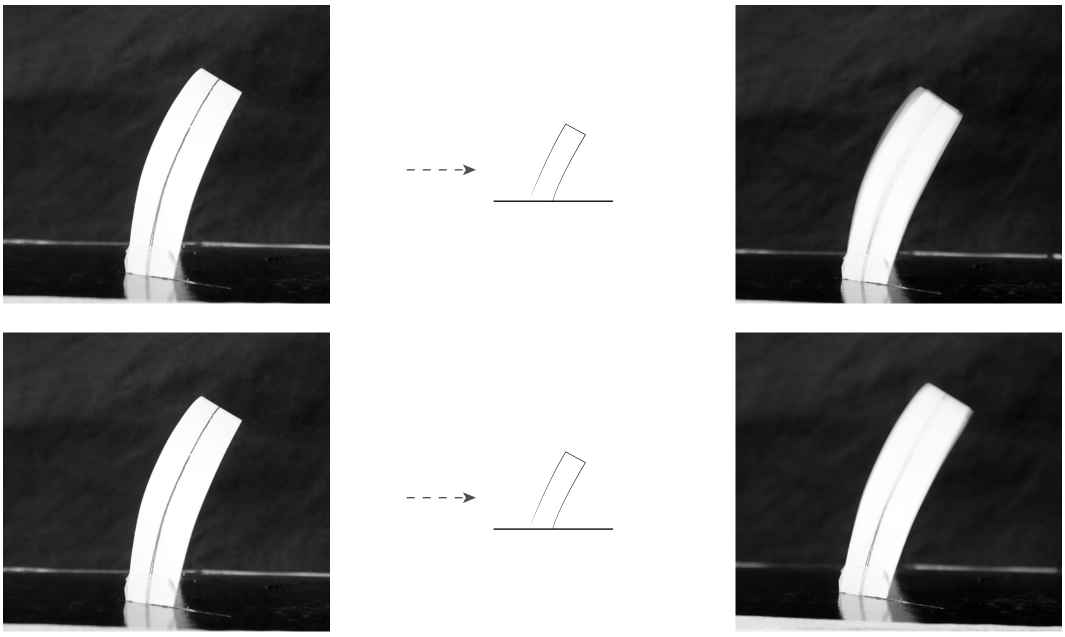


Figure [G9] Wind simmulation front side, right view

Results wind test

The three wind-simulations provide valuable insight into the deformation of a bent and twisted strip as a result of wind pressure. The wind-simulations aim is to find weak spots in the arch structure. These weak spots are the input for the topology of arches. For every load condition, the left column shows the initial shape, and the right column shows the deformed shape.

The first wind-simulation [figure G5] is a wind pressure starting from the side of the arch, rotating through 6 steps until the wind pressure faces the front of the arch. The first two steps show a significant deformation at the place where the face of the arch is perpendicular to the wind pressure, i.e. where the arch has the weakest moment of inertia, relative to the wind pressure. The 3rd, 4rth and 5th step have a wind pressure almost perpendicular to the support. The reason why the deflection is less compared to the first two steps is that the supports are clamped. The 6th and 7th step show the least deformation. The wind pressure has the same direction as the cantilever direction of the arch. A load condition in the same direction as the cantilever adds up to the tension in the arch. Due to the relatively low youngs modules, the model resonated with the wind pressure but does not deflect significantly. The arch is very resistant to frontal wind loads.

The second wind simulation [figure G6] is equal to the first but instead rotating towards the back of the arch. The first two steps show similar results to that of the first simulation. Step three four and five show significantly more substantial deformations. The wind load is in opposite direction of the cantilevered arch, meaning that wind load results in pressure in opposite direction of the tension from bending and torsion. The arch deflects over the weakest section. Step six and seven show a somewhat symmetrical deflection as the pressure is facing the back of the arch perpendicular. The wind pressure pushes the top of the arch down, leading to curvature densification at both sides of the arch.

The third wind test [figure G7] simulates a load in a vertical direction applied on the arch. The first four steps show similar results from the previous two tests, deflections at the position of the weakest moment of inertia. The fifth step is a wind pressure almost straight on top of the arch, leading to unequal curvature division over the full length of the arch. Besides large deflection downwards, the increased curvature at both sides of the arch leads to locally increassed stresses. The last two steps show similar results with drastic vertical deflections. Even though these deflections are unrealistic, they clearly show the weaker areas in the arch as a result of a distributed vertical load.

[figures G8, G9] Show the results of the first two simulations from the side, with clear differences in resistance to an equal wind load. Based on these wind analyses, two conclusions can be formulated. First, a bent and twisted arch is highly resistant against horizontal loads facing the front of the arch. Wind pressure in this directions results in increased tension in the arch. In the opposite direction, the arch is not resistant against horizontal loads. Wind pressure from the back leads to unequal curvature and tension division over the length of the arch. Locally densified curvature might result in material failure.

Vertical load leads to similar deflections. The sides of the arch tend to spread, which results in a large deflection of the middle of the arch and locally increased stresses.

Many effective ways can be used to make an active bending plate arch stronger. An increased thickness of the plate will increase the strength. A double-layered structure with a load transferring core in the middle of the section will lead to better shear stress resistance. Multiple plates interconnecting at the weaker spots increases the stability of the plate, preventing it from large deformation. Spanning steel cables between the plate to prevent it from deflection, making the structure hybrid with actively bent elements and tension cables (Van Mele, De Laet, Veenendaal, Mollaert, & Block. 2013).

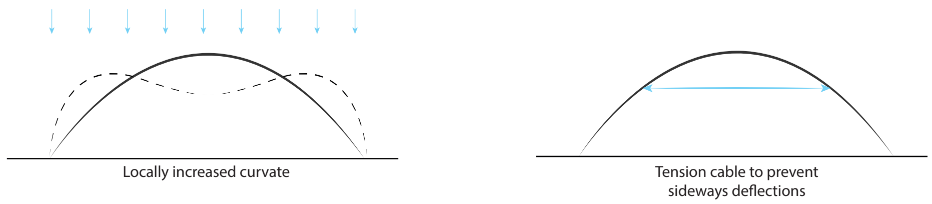


Figure [G10] Cable arch strengthning hybrid structure

Even though all these options add up the strength of the plate, they all go against the initial concept of the active-bending plate structure. The simplicity and the elegance of obtaining structural rigidity with just single planar elements is the objective of this research. Hierarchically, an individual plate should always be dominant. The aim for this research is to use topology of several arches to support each other at weaker spots.

G.3 Form creation and structural behaviour

The chapter on the structural behaviour of a bent and twisted arch investigates the form finding process, stiffness and the deflection behaviour as a result of several load cases. The structural analyses are divided into two parts. The first part is the form creation process, where bending and torsional prescribed displacement is individually analysed. The second part is the analysis of the erected arch subject to several load cases, such as wind rain and snow. The software Diana has been used for the form creation process and the FEM analyses.

Phased non-linear structural analyses

The plate behaves non-linear in both the form creation process and the load cases application. Large deformations are the result of prescribed deformations in both the supports. Each prescribed deformation per direction has been performed through 40 iteration load steps. Each load case will be performed in 20 iteration load steps, as the deformations will be less compared to the form creation deformation. Each load case analyses will have two phases. Phase one is the form creation process, where the supports at both ends of the arch are fixed in translation and free in rotation. The second phase is the application of the load case. This phase will have the same supports, with an added support in the y-direction at the top of the arch. This supports simulates the coupling of two arches against each other.

Form creation

The initial shape of the arch prior to the form creation process is a plate with a minor linear curvature [figure G11]. In a real situation, this will be the result of the dead load after lifting the plate in its position. Both the supports on each side are fixed in the x,y and z-direction, free in rotation. The first step is the bending deformation. One support will have a prescribed displacement over the x-axis towards the other support, leading to a pre-stressed bent arch [figure G12]. After the bending deformation, both the supports will be subject to a rotational displacement of 45 and -45 degrees [figure G13]. The end position of both support is perpendicular to each other.

The arch will first be analysed on a distributed load on the arch itself, simulating wind pressure and suction. Secondly, four point-loads which simulate the fixation of the steel cables for the membrane suspension.

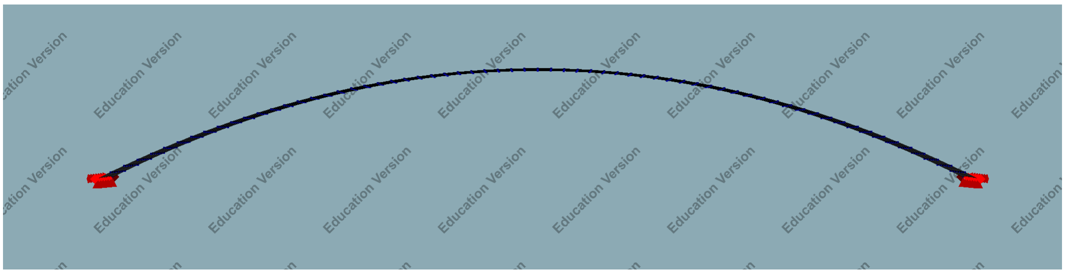
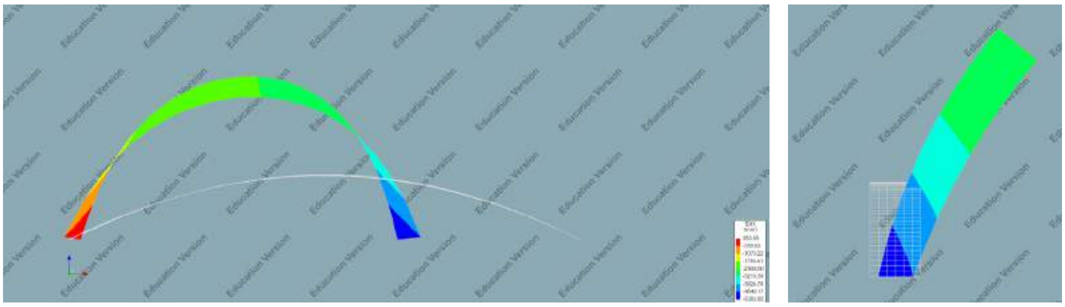


Figure [G11] Planar plate with slight curvature prior to deformation process

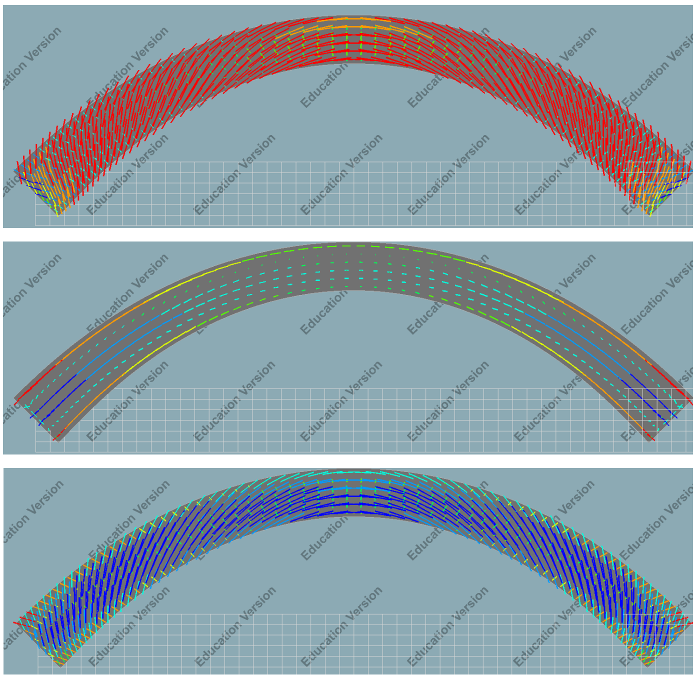
(a) (b)

Figure [G12] Pure bending deformation, (a) front view, (b) side view



(a) (b)

Figure [G13] Bending and torsional deformation, (a) front view, (b) side view



(a)

(b)

(c)

Figure [G14] Principle stress lines form creation process, (a) layer 1, (b) layer 2. (c) layer 3

Stress behaviour form creation process

The material selection filter described in chapter F resulted in four suitable composites. Two epoxy resin glass-fibre composites and two epoxy resin carbon-fibre composites. The material properties are shown in [table G15]. Two composites are composed of long fibres, with makes them resistant to tension in all directions. Two composites are unidirectional quasi-isotropic with a layup in 0,90,45 and -45 degrees.

The criteria for the selected material are the magnitude of internal stresses, deformation, flexibility and reaction forces.

Material	Tensile strength [MPA]	Poisson's ratio	Shear strength [MPA]	Youngs Modulus [GPA]	Flexural strength [MPA]	σ[MPA]/ E[GPA]
Epoxy SMC (65% long glass fiber)	267,00	0,33	153,00	22,40	476,00	21,25
Epoxy SMC (55% long carbon fiber)	278,00	0,31	198,00	60,60	607,00	10,01
Epoxy S-glass fiber(UD prepreg, QI-layup)	504,00	0,31	-	21,00	121,00	5,76
Epoxy Carbon fiber (woven prepreg, QI-layup)	649,00	0,34	-	48,20	649,00	13,46

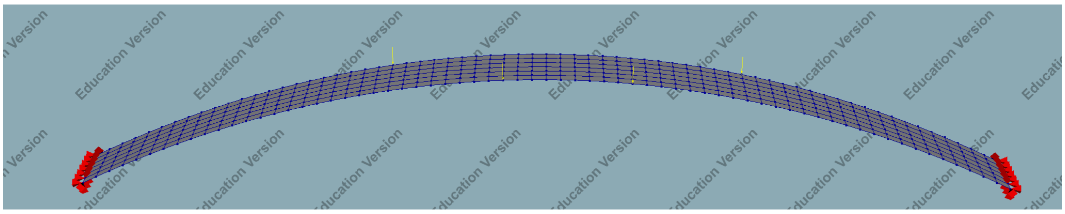
Table [G15] Structural material properties

Form creation process Bending + Torsion	σxx [MPa]	σyy [MPa]	σzz [MPa]	σxy[MPa]	σyz [MPa]	σzx[MPa]
Epoxy SMC (65% long glass fiber)	66,13	57,44	39,11	35,77	48,34	47,22
Epoxy SMC (55% long carbon fiber)	175,23	154,16	105,24	96,73	132,91	128,77
Epoxy S-glass fiber(UD prepreg, QI-layup)	60,98	53,51	36,49	33,52	45,96	44,55
Epoxy Carbon-glass fiber (woven prepreg, QI-layup)	127,44	110,15	74,92	68,37	91,51	90,05

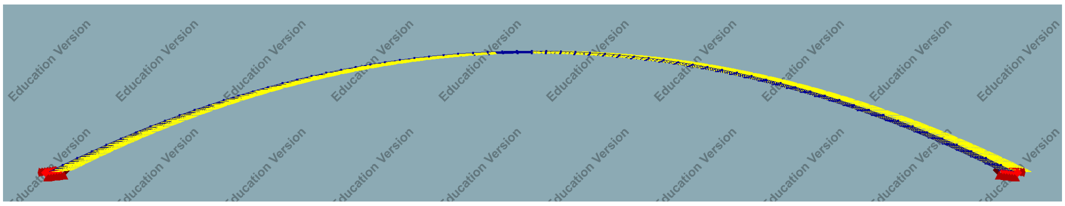
Table [G16] Internal stresses form creation process

The flexibility of the material is measured by the relative ratio of flexural strength divided by the flexural stiffness σ [MPa] / stiffness E [GPa]. The epoxy glass fibre is the most flexible with a ratio of 21,25. The relative stiffest ratio is epoxy glass fibre with a ratio of 5,76. I

The internal stresses in the plate after the form creation process clearly show high internal stresses as a result of high Young's modulus. The plate's with a lower youngs modules requires less embodied energy for the form creation process, which results in smaller reaction forces. The most substantial tensile stresses occur in the bending direction; the most substantial shear stresses occur in the yz-plane.



(a)



(b)

Table [G17] Loadcases arch, (a) pointload membrane fixture, (b), windpressure and suction

Load application

The first load application is the points load from the suspension cables attached to the plate. The bending stresses in the x-direction increase for all four materials. The shear stresses in the xy-plane increase and becomes dominant for the carbon fibre arches, while for the glass fibre arches the yz-shear stresses remain dominant.

The wind pressure and suction, both in the x-direction, lead the minor increase of the bending stresses. The shear stress in the yz-plane increases largely, almost 50% extra for the glass fibres and twenty percent for the carbon fibres.

Pointload membrane fixture	σxx [MPa]	σyy [MPa]	σzz [MPa]	σxy[MPa]	σyz [MPa]	σzx[MPa]
Epoxy SMC (65% long glass fiber)	82,85	69,93	41,60	46,11	42,66	31,67
Epoxy SMC (55% long carbon fiber)	179,93	160,77	99,84	106,43	120,50	113,05
Epoxy S-glass fiber(UD prepreg, QI-layup) (structure failed)	78,76	66,59	40,85	43,48	40,89	30,82
Epoxy Carbon-glass fiber (woven prepreg, QI-layup)	133,63	117,29	71,42	78,20	79,15	74,16

Table [G18] Internal stresses form creation process + membrane point loads

Windload pressure + suction	σxx [MPa]	σyy [MPa]	σzz [MPa]	σxy[MPa]	σyz [MPa]	σzx[MPa]
Epoxy SMC (65% long glass fiber)	70,26	73,24	71,51	42,45	79,41	37,20
Epoxy SMC (55% long carbon fiber)	176,90	158,42	134,98	101,97	157,77	105,70
Epoxy S-glass fiber(UD prepreg, QI-layup)	65,01	69,31	67,77	40,17	76,47	35,72
Epoxy Carbon-glass fiber (woven prepreg, QI-layup)	129,38	115,97	103,84	73,72	116,93	68,33

Table [G19] Internal stresses form creation process + wind pressure

Deformation x and z direction	Vertical deformation [mm]	Horizontal deformation [mm]
Epoxy SMC (65% long glass fiber)	-103	-115
Epoxy SMC (55% long carbon fiber)	-18	-77,1
Epoxy S-glass fiber(UD prepreg, QI-layup) (failure)	-1526	-1225
Epoxy Carbon-glass fiber (woven prepreg, QI-layup)	-67	-107

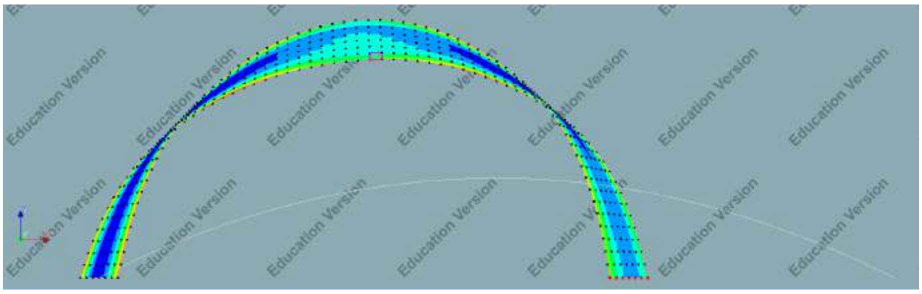
Table [G20] Vertical and horizontal deflections all laodcases

Both load cases combined lead to vertical and horizontal deformations shown in [table G18, G19]. The Epoxy glass fibre plate does not resist both forces cases as a result of low flexural strength. The Epoxy SMC glass fibre shows the most deformation, roughly 100 mm in both vertical and horizontal direction. The epoxy SMC carbon fibre shows the least deflection.

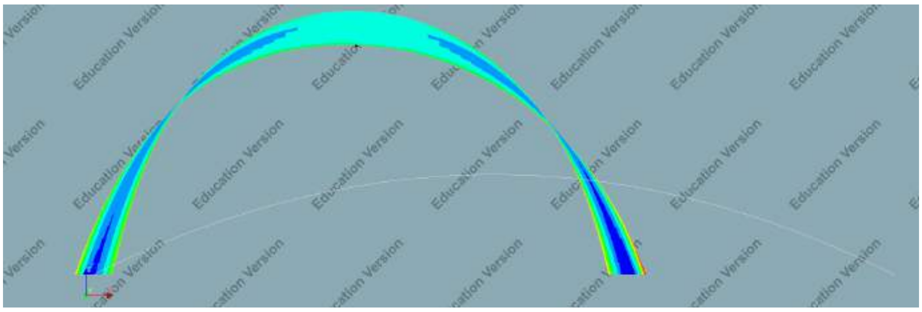
The aim is to have minimum reaction forces in both supports. The reaction forces as a result of the formation process highly depends on the stiffness of the material. The reaction forces shown in [table G21] are not adequate measures for actual reaction forces, but rather a measure of the forces in the support nodes. The reaction forces will be distributed over the full width of the support, leading to smaller values. The nodal reaction forces can be used to relatively compare the magnitude of the forces. The epoxy SMC carbon fibre proved to be the most suitable material regarding the stiffness and the according deflection of the arch. However, the large stiffness requires a significantly large force to erect the arch. Compared to the reaction forces of the Epoxy SMC glass fibre, the reaction forces of the Epoxy SMC Carbon fibre are ten times larger.Based on the flexibility, the internal stresses within the limit state, the deformation and the reaction forces, Epoxy SMC (65% long glass fibre) is the most suitable material for this arch structure.

Reaction forces xyz [N]	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6	Node 7
Epoxy SMC (65% long glass fiber)	21872	16832	-18893	-17803	-7210	5043	9520
Epoxy SMC (55% long carbon fiber)	145711	85650	-176710	-209559	-119160	231852	85726
Epoxy S-glass fiber(UD prepreg, QI-layup) (failure)	16957	13961	-14071	-12067	-4011	2951	9099
Epoxy Carbon-glass fiber (woven prepreg, QI-layup)	109299	65461	-135241	-158818	-90995	180643	69626

Table [G21] Nodal reacion forces all loadcases for relative comparison



(a)



(b)

(c)

(d)

Figure[G22]. Deflection diagrams all load cases, (a) Epoxy SMC (65% long glass fiber), (b) Epoxy SMC (55% long carbon fiber), (c) Epoxy S-glass fiber(UD prepreg, QI-layup) (failure), (d) Epoxy Carbon-glass fiber (woven prepreg, QI-layup)

H Building methodology

Application and feasibility

H.1 Form creation process single arch

The construction method for this active-bending plate structure is based on some fundamental boundary conditions. The development of this structure undergoes four design stages until its completion. Each construction stage will have its boundary conditions, which will be further explained per stage later in this chapter. The four design stages are (1) the creation of the plate, (2) the form conversion of the plate, (3) the coupling of two plates adjacent to each other and (4) the space enclosure attached to the plates.

Planar strip development

The strips will be composed of Glass fibre reinforced polymers. The production process is relatively easy as the entire plate is planar. The creation of planar elements allows for high production speed and repetitive operations.

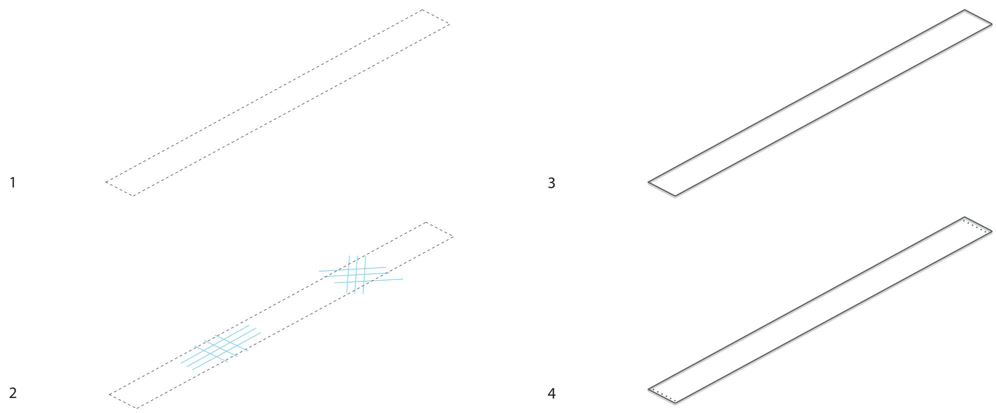


Figure [H1] Planar strip development. (1) bottom resin layer, (2) multidirectional fiber layup, (3) top resin layer, (4) Hole drill preperation for support attachment.

Form conversion

The form conversion of the plate will be performed through several construction steps. At first, the plate will be placed in its supports on both sides. The supports are clamped over the full width of the plate. The first step is the bending of the plate. Bending the plate can be achieved by a prescribed displacement of the supports. If the supports are fixed in y- and z-direction and only allow movement over the x-direction, only horizontal loads are required for the first form conversion. After application of horizontal loads, the supports will be closer to each other, and the plate will have the desired bending curvature. The erected arch is still in the same x-plane as in the starting position.

The final form conversion step is torsion of both supports over a 45 degrees angle. Without further movement of the entire support over the x-axis, each side of the support will be moved over the x-axis in the opposite direction. The x-coordinates of the centre of the support stays at its position and only rotates around the z-axis. Adjusting the amount of rotation per individual support allows for full control of the deformed shape of the arch. After adapting both supports to the desired shape of the plate arch, the rotation around the y-axis will be fixed. To improve the stiffness of the arch, the end condition of both supports are fixed, for both rotation and movement, in all directions. Figure xx shows the four construction steps towards a bent and twisted arch.

Boundary conditions for form conversion.
-The starting position of the plate before form conversion is slightly curved
-Only free movement over the x-axis
-Individual rotation adjustment per support
-Fixed supports after bending and twisting the plate

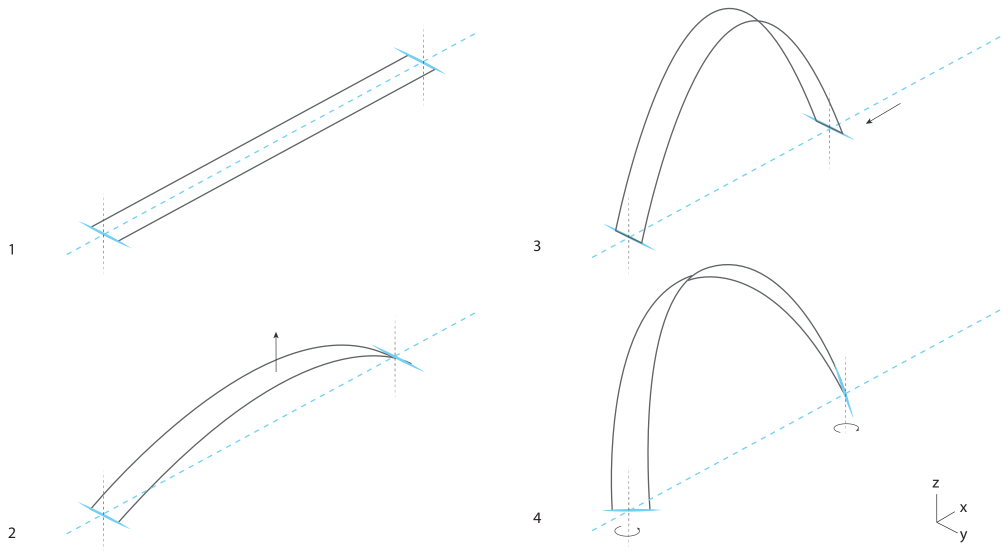


Figure [H2] Form conversion steps. (1) planar strip, (2) initial shape prior to bending, (3) bending deformation, (4) torsional deformation

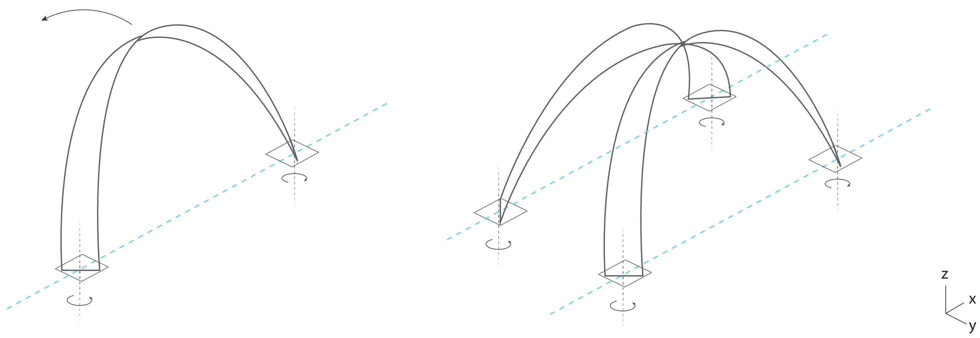


Figure [H3] Coupling arches.

Coupling

The wind simulations and the structural analyses show the strength and weaknesses of the arch. The topology of several arches will be used to obtain a secure and stable structural system. The most straightforward topology is two arches with their backsides against, i.e. the weaker sides facing each other. The connection between the two arches at the top functions both as horizontal support and stabiliser. Connecting two arches results in a double arch structure with four legs in four different directions.

As a result of wind pressure or vertical load, the arch deflects and deforms. The connection at the top of the arches functions as a support to withstand large deformations. The connection between the two arches is a small element, which has to be resistant to considerable pressure and tension forces. All the loads going from one arch to the other will be transferred through this connection element, leading to large local peak stresses in the arch. High local peak stresses might result in local failure of the arch. To prevent large peak stresses, it is highly essential to divide the force over the arch as much as possible.

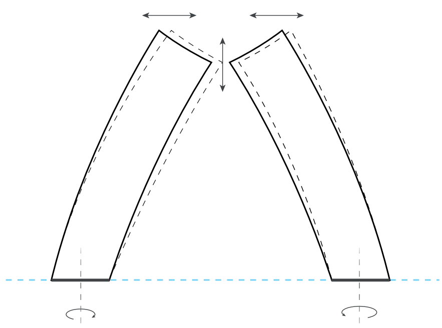


Figure [H4] Torsional adjustments for arch alignment.

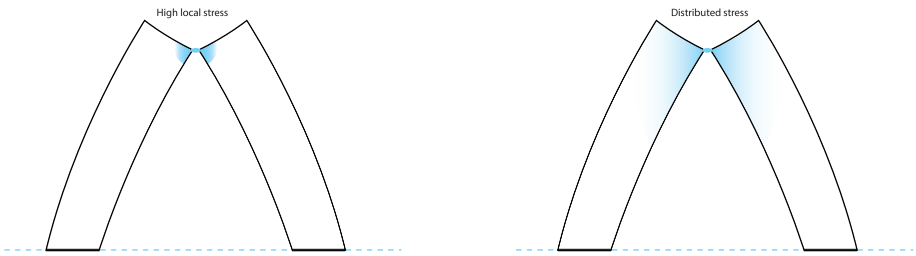


Figure [H5] Peak stresses at coupling location two arches.

Space enclosure

The shape of the arch clearly defines a space and distinguishes what is inside and what is outside the structure. The design freedom of arrangement and topology of several arches allow for a large variety of such enclosed spaces. Even though the surface area of the arches already covers a significant amount of space, an additional enclosure is required to define inside and outside. Within the active-bending structural systems, the use of membranes and tensile-fabric is a conventional material to create complex double curved surfaces. “Combining elastically bent elements with a tension structure creates a hybrid bending-active construction” (Laet, Slabbinck, Mele, Block, & Mollaert, 2013). In this research, a coated tensile fabric will be used as additional space enclosure.

Generally, there are three enclosure principles to distinguish. The fabric can be stretched over the arch, under the arch or in-between the arch [figure H6]. Pulling the fabric over the arch allows for complete space enclosure with the arches visible from inside. The membrane allows for restrainment of the arch (Alpermann, Lafuente, & Gengnagel, 2012). The structure appears as a smooth cylinder from outside leaving all the structural components inside. Suspending the fabric under the arch leaves the inside space completely smooth, with visible structure from outside. The third option is strategic placement of multiple tensile membranes as such that both the arches and the fabric work as space enclosure. This option leaves the structural arches dominant in the structure and allows for minimal use of tensile fabric.

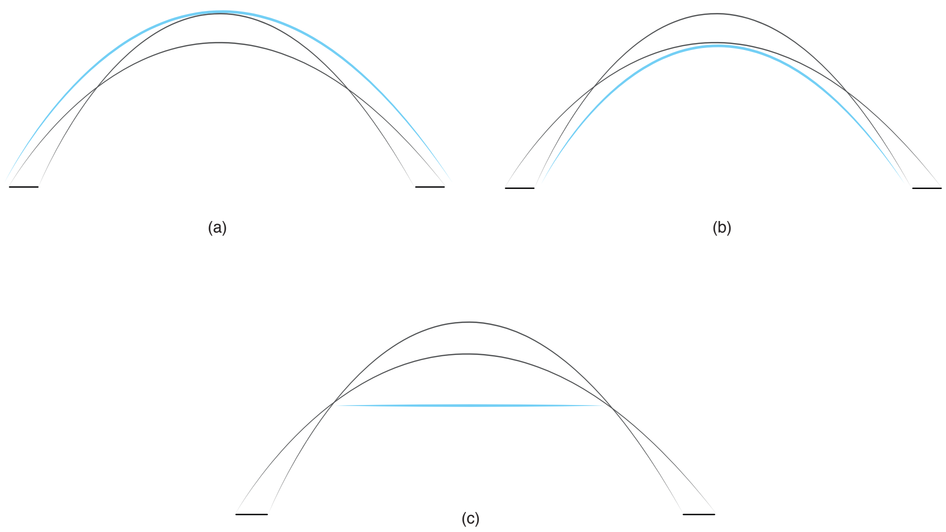


Figure [H6] Space enclosing membrane placement. (a) over the arch, (b) under the arch, (c) suspended between arch

Just four general fabric shapes can be used to enclose an entire grid of arches [figure H7]. First, the space between two arches facing the backsides. Second, the space between two arches facing the front sides. Third, the space between two arches adjacent next to each other. And Fourth, The space between the four arch legs adjacent to each other. Within a rational and repetitive array of arches, strategic placement of these four components adds up to the diversity of the enclosed space.

The most critical parameter in the design of these four tensile shapes is the flow of water. Even though the fabrics do not always connect, water should flow towards outside the arch system. Looking closer at the water flow on the arch themselves leads to strategic placement of fabric membranes.

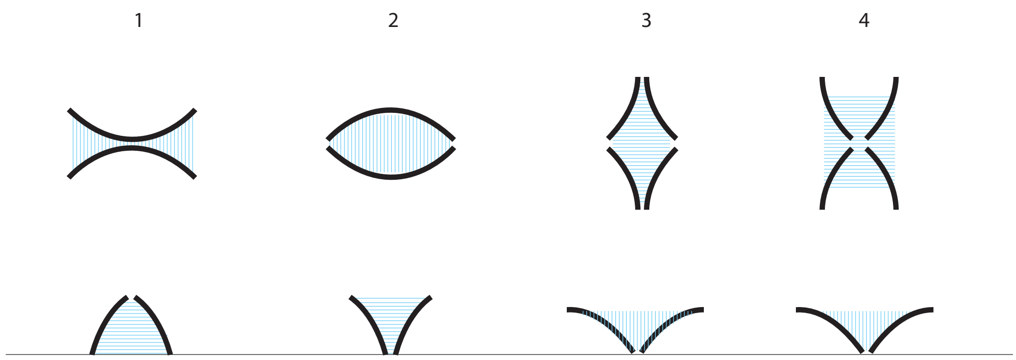


Figure [H7] Four generic space enclosure membrane shapes

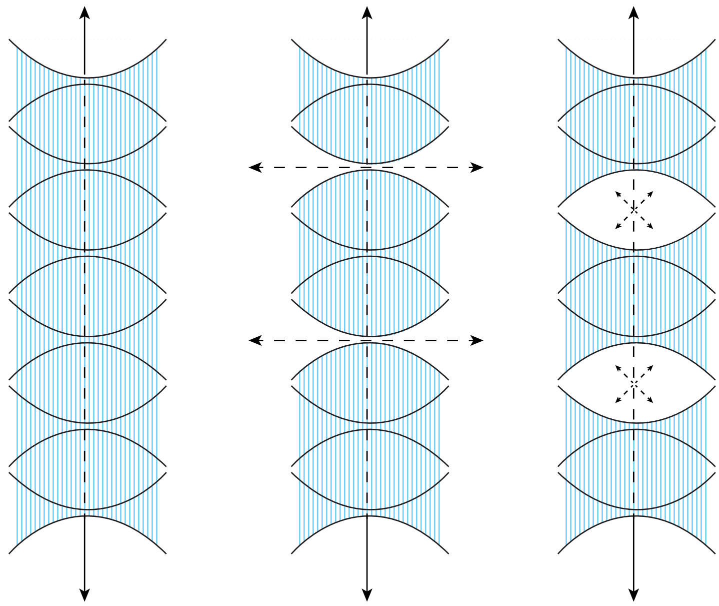


Figure [H8] Lineair membrane enclosure shape one and two.

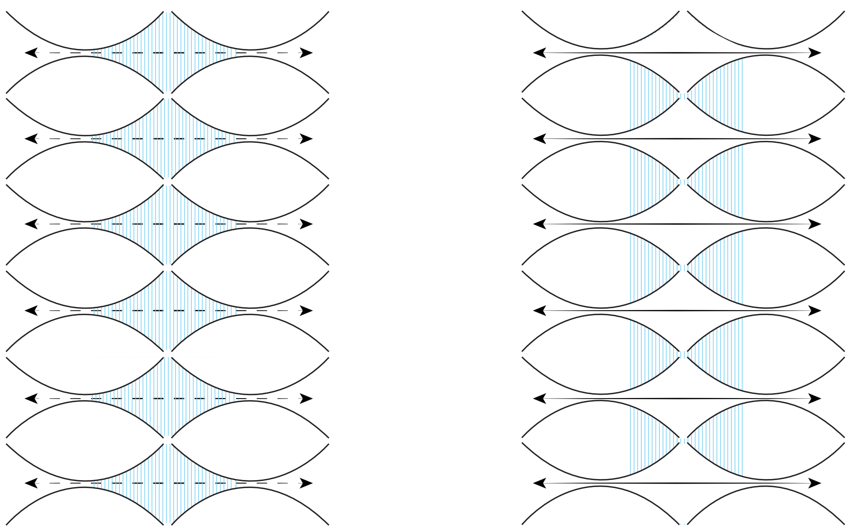


Figure [H9] Lineair membrane enclosure shape three and four

The normal direction of the face of two arches is always in the opposite direction, mirrored over the central axis. After plotting these vectors of both arches, it becomes clear that on both sides of the arch there are two places where the normal faces are parallel [figure H11]. These parallel faces also have the weakest moment of inertia relative to a vertical load condition, meaning that the arch is likely to warp at these places.

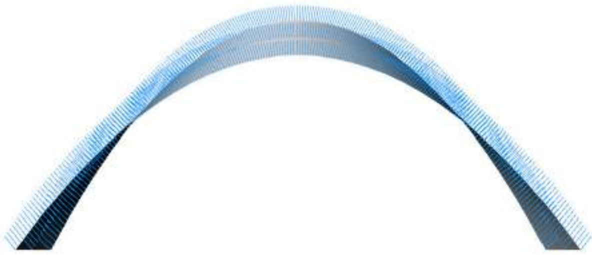


Figure [H10] Normal vector direction two arches

Figure [H11] perpendicular normal vector two arches

These parallel faces are very useful for strategic placement of a membrane space enclosure from one arch to the other. By suspending membrane under multiple arches, the legs of the arches ‘penetrate’ the membrane. Aiming for a minimum amount of membrane fixtures to the arch and standard straight membrane shapes, the best place to penetrate the membrane is at the place where the arch has parallel faces. For both the membrane inside and outside the arch, the same mounts can be used without large gaps between the membranes.

Membrane fixture

Suspension of the membranes under the arch requires mounts and steel cables. The steel cables are the edges of the membranes, allowing for tensioning the membrane. The fixture of the steel cables to the arch requires a connection mount in the arch which leads to weakening of the plate locally as well as local peak stresses as a result of applied force. It would only be logical to integrate the steel suspension cables with the structural performance of the arch. The structural capacity of the arch will be enhanced if tension cables are applied, obtaining a cable restained system as descirbed by Alpermann, & Gengnagel (2012). The addition of steel cable will aesthetically weaken the concept of the plate structure, which is why the membrane will cover the steel cables. Only plate arches and membranes will be visual.

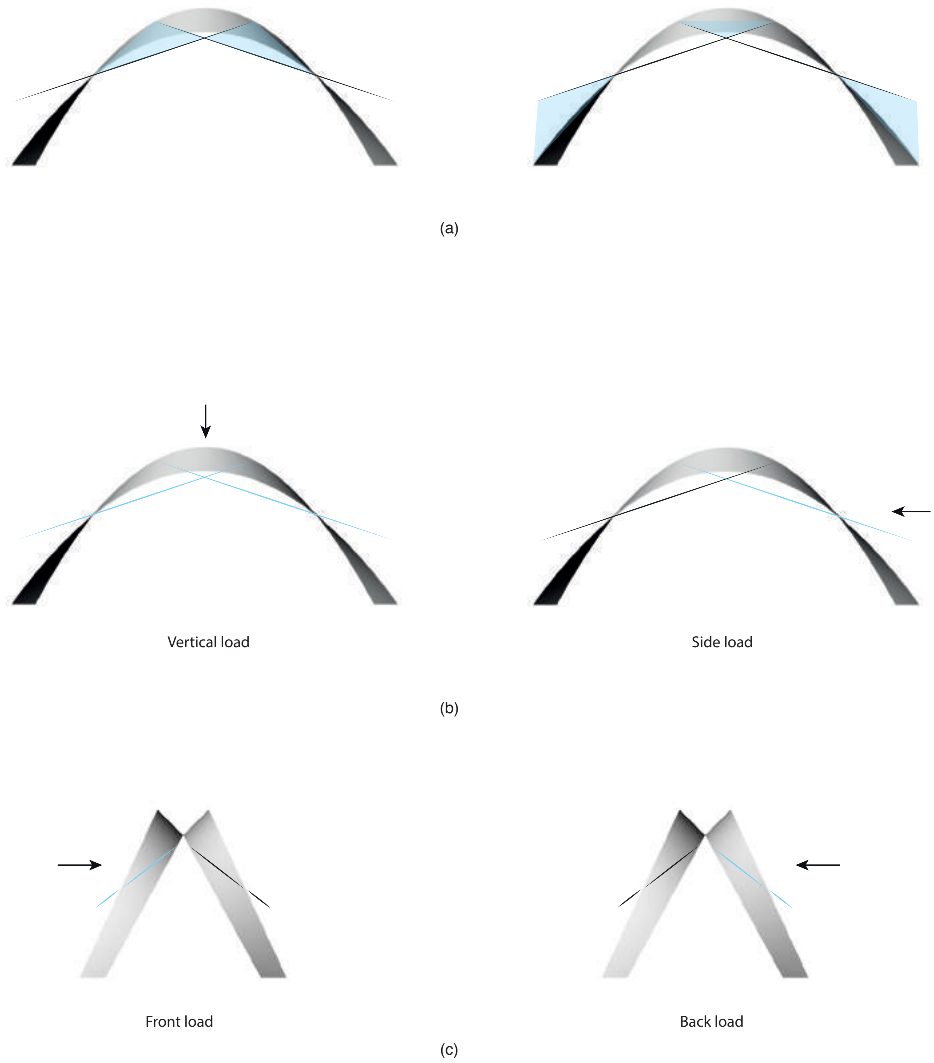
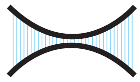


Figure [H12] Tension cable arch. (a) Triangle connection arch, (b) Cable activation side and top load, (c) cable activation back and front load.



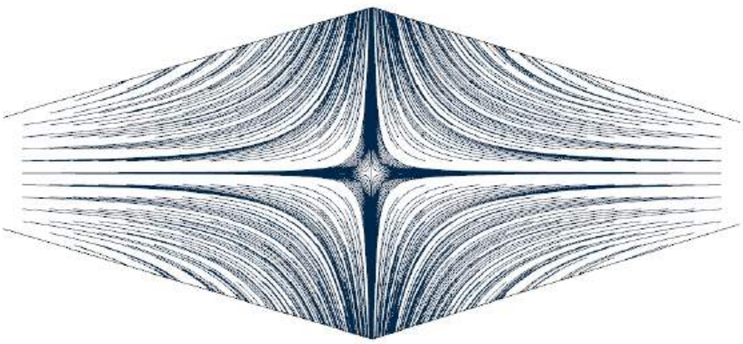
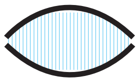


Figure [H13] Water flow mebrane shape one.

The first membrane shape covers the four cables in a coupled arch structures. It is composed of three membranes. The first two are planar membranes which will be suspended by the first section of the cables. The opening in the middle of these two planar membranes enhances the lightness of the structure. An anticlastic membrane surface, also known as deep surface (Ahlquist, Lienhard, Knippers, & Menges. 2013) covers this opening, suspended by the four shorter cables at the top. The shape of the membrane directs the water to the sides. From inside the arch, these three membranes have a dynamic dialogue of varying forms and water flow. From outside the arch, the front view shows the lightness of the membrane almost being invisible suspended in between the arches.

Figure [H14] Arches with membrane space enclosure shape one.



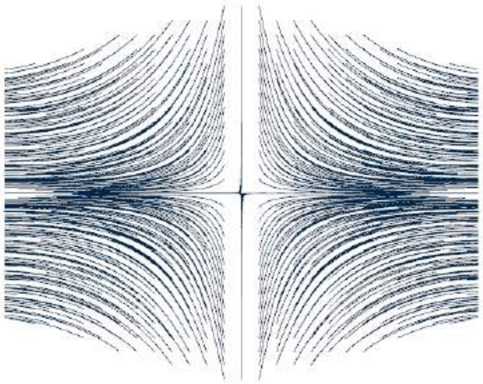


Figure [H15] Water flow mebrane shape two.

The second membrane shape covers the space between two arches in opposite position of each other. This membrane uses the same cables as the first planar membranes use. Because the cables have opposite direction next to each other, the surface between the cables is anticlastic. The water from the first membrane shape flows directly to the second membrane. From here the water will be guided to the edge of the membrane, which is parallel to the face of the arches. From here the water will flow outside the arches.

Figure [H16] Arches with membrane space enclosure shape two.

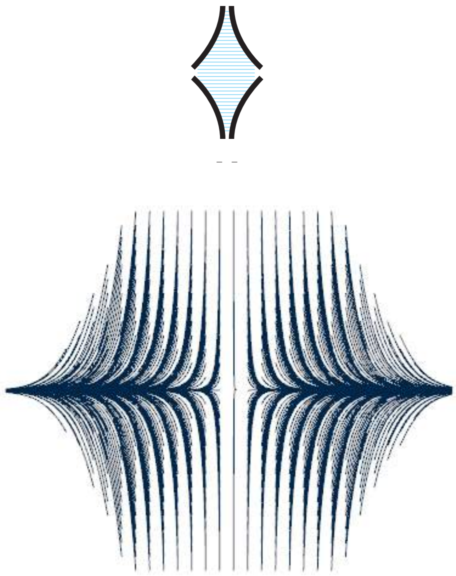


Figure [H17] Water flow mebrane shape three

The thirds membrane covers the space under four arches next to each other. The cables inside the arch can be extended outwards where they can be connected to the base of the arches. Again, the shape of the membrane is an anticlastic surface which allows the water to flow towards the collection points outside the arch structure. The cable connection to the base of the arch can be adjusted in height to allow full control of the shape of the membrane. Lowering the cable connection to the base will ‘close’ the arch. A high connection allows walking underneath the membrane, connecting the space under four connected arches.

Figure [H18] Arches with membrane space enclosure shape three.

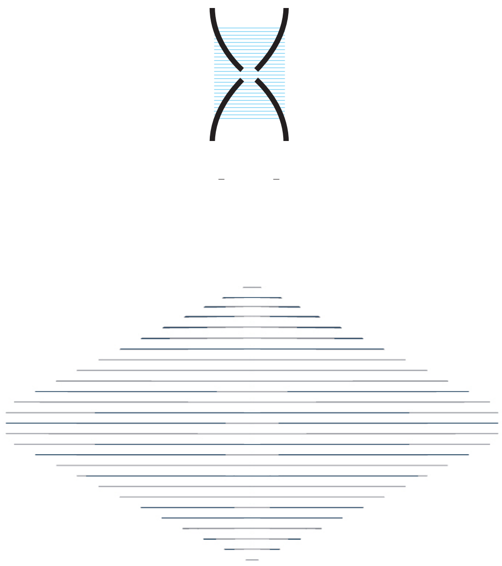


Figure [H19] Water flow mebrane shape four.

The fourth membrane shape is a planar shape and finishes the closure of all the arches. Even though it covers an area where people will not be able to walk, it is essential to flow the water to the water collection points above the base. The water from the second membrane flows over the fourth membrane towards the arch bases. The height adjustment for the thirds membrane will also be applicable to the fourth membrane. Vertical height alternation leads to different shapes for the membranes outside the arches.

Figure [H20] Arches with membrane space enclosure shape four.

H.2 Plate

The fibre-reinforced polymer plate is composed of many fibre layers and epoxy resin. The direction of the fibres alternate per layer to obtain quasi-isotropic material qualities. These qualities are required because of the different stresses and deformation directions the plate will be subject to. To be able to undergo these deformations, the plate needs two supports at both ends. The connection of the plate to the supports requires a local connecter at each side of the arch.

The plate is first clamped between two layers which are significantly stronger than the plate itself. The end of the clamp has a circular hollow void which allows for the placement of a steel tube. This steel tube is a sleave, fixed to the clamped layers. The tube is a sleave for the steel axis bar. This bar enables free rotation in the direction perpendicular to the plate, which is required for the bending process. The end of the connection will be wrapped with a circular layer to hold every component in place. Fifteen bolts will hold these layers together when they are subject to significant forces.

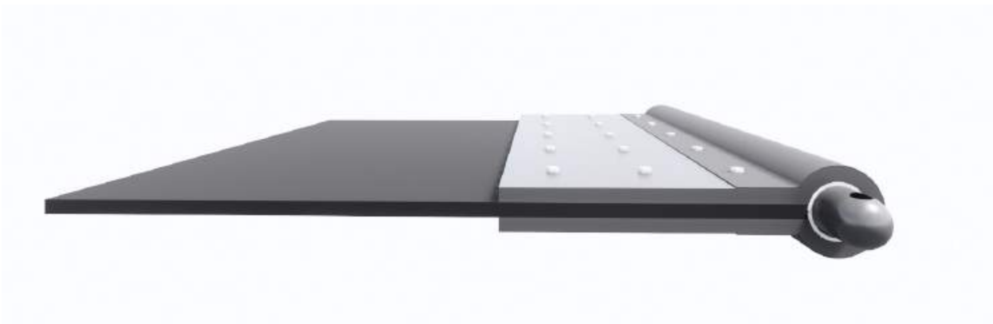


Figure [H21] Clamped plate connector.

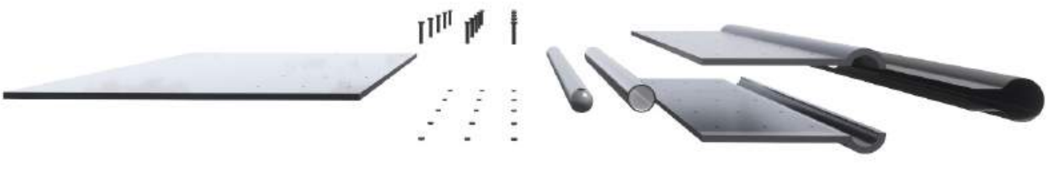
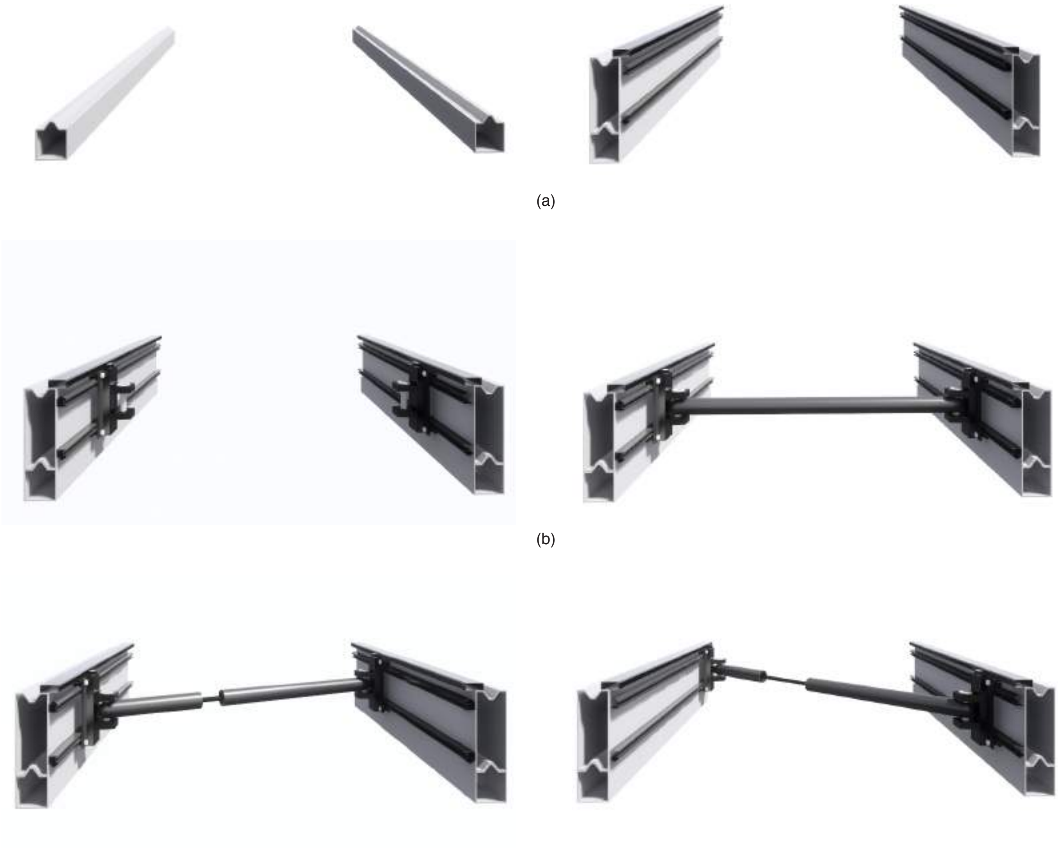


Figure [H22] Exploded view clamped plate connector.

H.3 Support element



(a)

(b)

(c)

Figure [H23] Arch support principle building sequence. (a), ground frame with gliding rails, (b) Sliding mount elements, (c) rotational freedom with extandable steel axis.

The plate with the connection elements attached will be placed in a support element, i.e. the base for the arch. This support will be used for the torsional deformation as well as the fixation to the ground. The aim is to have motion in one direction only, parallel to the initial shape of the plate.

[figure H23] Shows the composition sequence of the support element. First, a frame will be attached to the ground. This frame allows for alignment and horizontal adjustments. Two extrude profiles will be connected to the frame, functioning as the main gliders for the plate. These profiles have glider mounts attached to it, which will perform as the connection between the steel axis bar to the glider profiles. The steel axis bar is extendible. Linear motion of the mounts over the gliders extents the axis bar, obtaining freedom in the rotation angle of 45 to -45 degrees.

(a) (b)

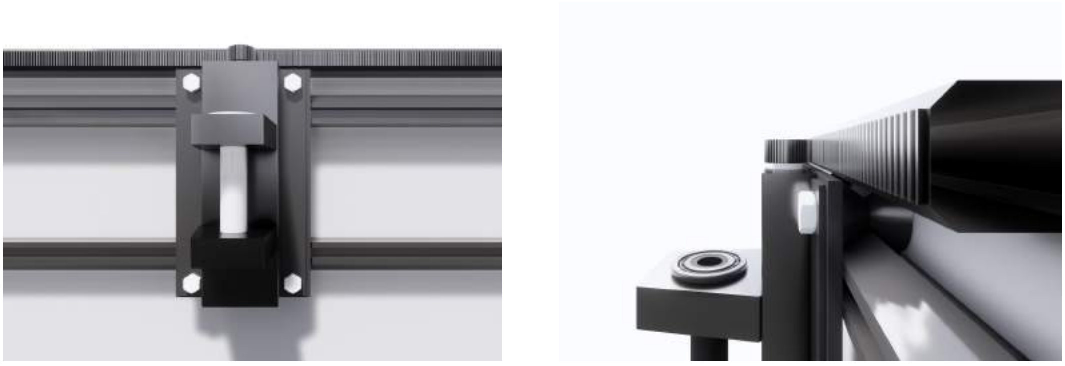
Figure [H24] Arch support free rotational motion,(a) position after bending, (b) position after torsion.

[figure H24] shows the plate attached to the support. The steel axis bar in the clamped support of the plate allows for free bending motion of -90 to 90 degrees. The support itself with the linear glider system provides for a rotation angle. The simplicity of movement in only one direction and the obtained rotational freedom over two axes makes this support highly suitable for mobile arch structures.

(a) (b)

Figure [H25] Extendable central axis, (a) open view straight position, (b), open view rotate position

The steel axis in the clamped support rotates free and can extend. Through rotational deformation the middle angle elongates. The elongation in the axis happens inside the clamped support. [figure H25] shows the different variations of the axis, in both short and long circumstances. To prevent the axis from popping out, both sides of the support enclose minimum 300 mm of the circular axis. The plates plate cannot move freely over the axis. Rotation makes the plate deform towards on side. The plate stays at the opposite position of the cantilever during the rotational deformation.

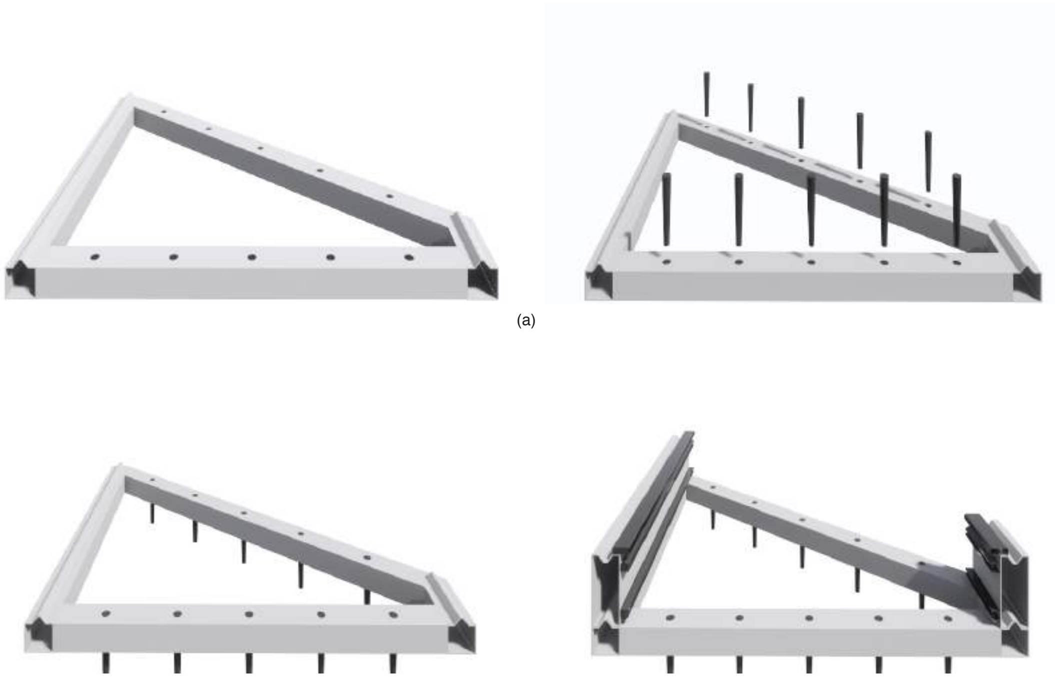


(a) (b)

Figure [H26] Gliding mounts system. (a) interlocking gliding system steel wheels, (b) gear wheel with gear rail for lineair motion

The top of the mount consists of a small gearwheel. This gearwheel is the ‘engine’ of the gliding system. The gearwheel rotates against a gear glider which is attached at the top of the main profile. Through manual or automatic rotation, the mount moves over the slider in the x-direction. The form creation process of the arch requires a significant force. The small gearwheel allows for minimal movement with manual or small automatic force. Motion through rotation of a small gearwheel provides for high precision form creation. After the desired form is obtained, the mount has to be held in its position. Two fixation elements, which make use of the same small glider profiles, are attached to each side of the mount. Fixation through bolds leaves them in place, interlocking the mounting element in all directions. The arch is now fixed in its support.

Figure [H27] Gilding mount locking system.



(a)

(b)

Figure [H28]Triangle shaped arch support, (a) ground fixation, (b) rail attachement.

With the given support principle and gliding mount system, the question raises how to create the desired form of the arches, with minimal support elements. In chronologic sequence, the first step is to bent the arch. Secondly, after bending, a torsional displacement will lead to the final shape. The aim is to leave the size of the support elements minimal. Application of a rotational angle requires only motion of one gliding mount, while the other gliding mount can remain in its position. The shape of the frame to which the mounts are connected can, therefore, be triangular. [figure H28] shows the first sequence of attaching the triangular structure to the ground. The two gliders, one small glider and one more extended glider, will be placed on top of this frame. This frame is the final support of the arch.

(a) (b)

Figure [H29] Rail extension for bending deformation sequence one, (a) modular extending elements into sleave, (b) modular one meter long rail elements.

(a)

(b)

Figure [H30] Rail extension for bending deformation sequence two, (a) extension of rail for bending purposes, (b) seamless rail extension

The second sequence is the attachment of two gliders to the triangular support. These gliders allow for the bending deformation process. Modular sized gliding rails of one meter long can be attached through connection elements [figure H30]. The desired length can easily be adjusted through the addition of multiple modular gliding rails. The modular rail elements are raised from the ground by small mount elements. These elements hold the rails in place during the formation process.

The third sequence starts after obtaining the desired length of the rails. The mounts with the plate attached will be placed on the rail gliding system. The modularity of the rail elements and the gear glider leads to a simple entirety in the rail system. As the mounts glide over the rails towards the final support element, the plate will bend into an arch shape. The gliding rails can be removed after the bending process, leaving the triangular rail shape as final support.

(a)

(b)

Figure [H31] Rail extension for bending deformation sequence three, (a) placement arch, (b) bending deformation over rails

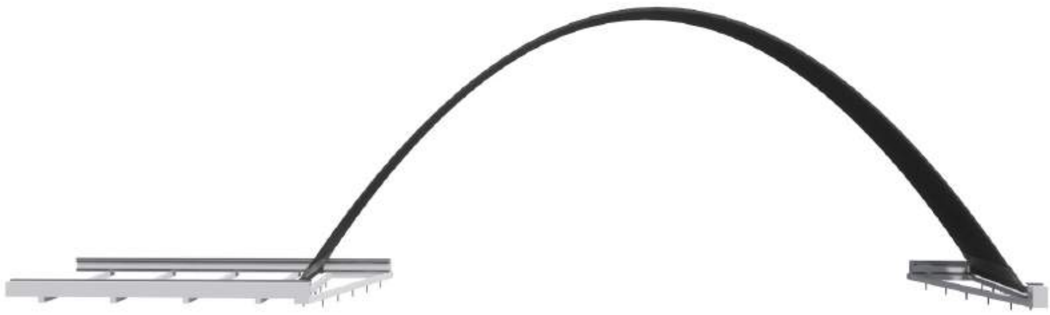
(a) (b)

Figure [H32] Bending procces, (a) plate reaches supporte element through sliding over rail, (b) disassambly rail.

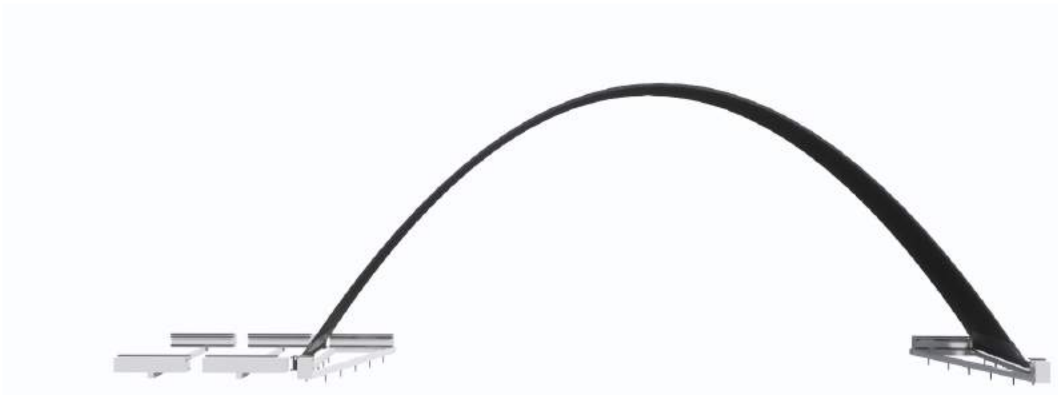
Figure [H33] Rotational motion over gliding rails

Figure [H34] Exploded view gliding mount

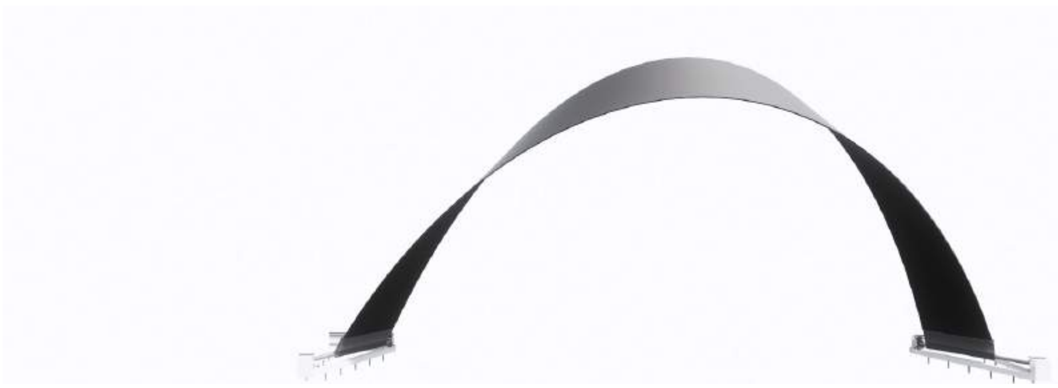
(a)



(a)



(a)



(a)

Figure [H35] Bending and torsional deformation sequence, (a) start position, (b) bending deformation, (c) torsional deformation, (d) removing modular rail system.

H.4 Coupling

The last and the fourth sequence is the rotational deformation to both ends of the plate. One mount will glide over the long rails till a 45 degrees angle is obtained. After the bending and torsion process, the mounts can be fixed in its position. The support base is triangular shaped with one longer rail for the rotation motion and one short rail to which the mount is attached. The coupling of two arches at the top will be done through a connection element. This element will be clamped to the arches at the top over a length of 400 mm. Smaller connection elements will lead to sizeable local peak stresses in the plate. A connection over the edge of the plate distributes the loads equally over an edge segment.

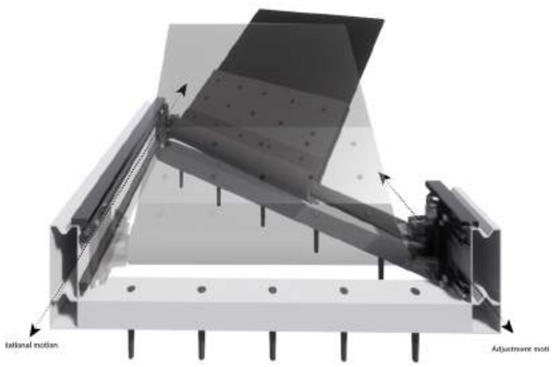


Figure [H36] Adjustment freedom within support frame four alignment of two arches.

Figure [H37] Edge coupling element at top of arches.

Even though the supports can be placed with high accuracy, unforeseen deviations may not be rare. The top of two arches may not align horizontally or may differ in height. The ability to adjust the arch after the form creation process is highly essential for the feasibility of the structure. Therefore, the ability to move the mounts within the final support element is necessary. The small rail and the more extended rail are not only suitable for the rotational form creation but also for final adjustments of the overall arch. Minor motion in the mounts leads to a correction in the shape of the arch This leaves the ability to align the top of two arches perfectly.

Figure [H38] Adjustment position arch within support system

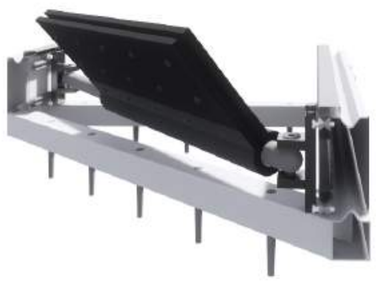
Figure [H39] Edge coupling element

(a)

(b)

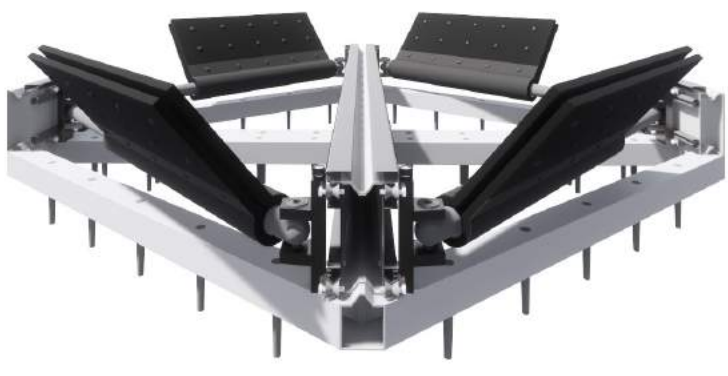
Figure [H40] Coupled arches. (a) fornt view, (b) side view.

H.5 Copy-paste - extenable structure



(a)

(b)



(c)

Figure [H41] Support expanding sequence, (a) single support, (b) coupled support using the same center rail, (c) four supports combined form one support.

The adaptive and mobile qualities of this structure require an easy formation process with repetitive and modular components. The base of the structure, the arches supports, are crucial for the extendability of the structure, i.e. they have to adapt to the size of the required structure. The support has to be modular to remain a non-complex structure regardless of the number of arches required.

The simplest form of this structure is a single arch with two supports on each side [figure H35, b]. Coupling one arch to another arch requires double the amount of support elements. The triangular support frame is able to work together. Adjacent supports can be coupled in a way that a minimum space is required for two supports next to each other. Additionally, the adjacent gliding mounts make use of the same gliding rail. In a sideways coupling, the long rail can be used on both sides [figure H41, b]. The gliding mounts of both the arches glide over the saim rail. In a coupling in a linear direction, the rails can simply be extended [figure H41, c]. Two rails become one rail, making a single support work together as one shared base support builds from four single supports.

Not only does this reduce the complexity of coupling multiple arches, the double use of the rails also allows for additional strength as forces counteract each other. [figure H42] shows the coupled support with four arches attached.

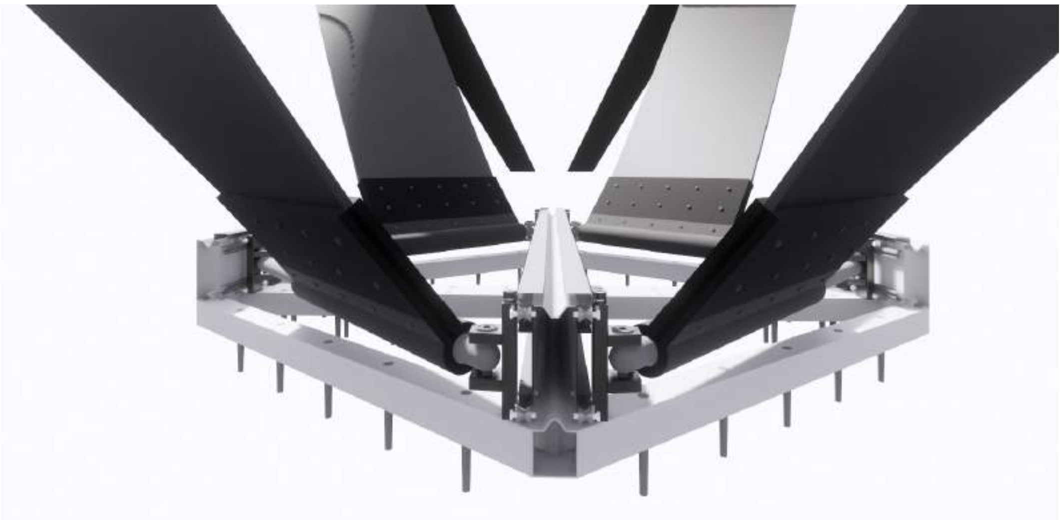


Figure [H42] Coupled support with four arches connected with multipurpose middle rail

I Design application

Active bending arch canopy

I.1 Linear array

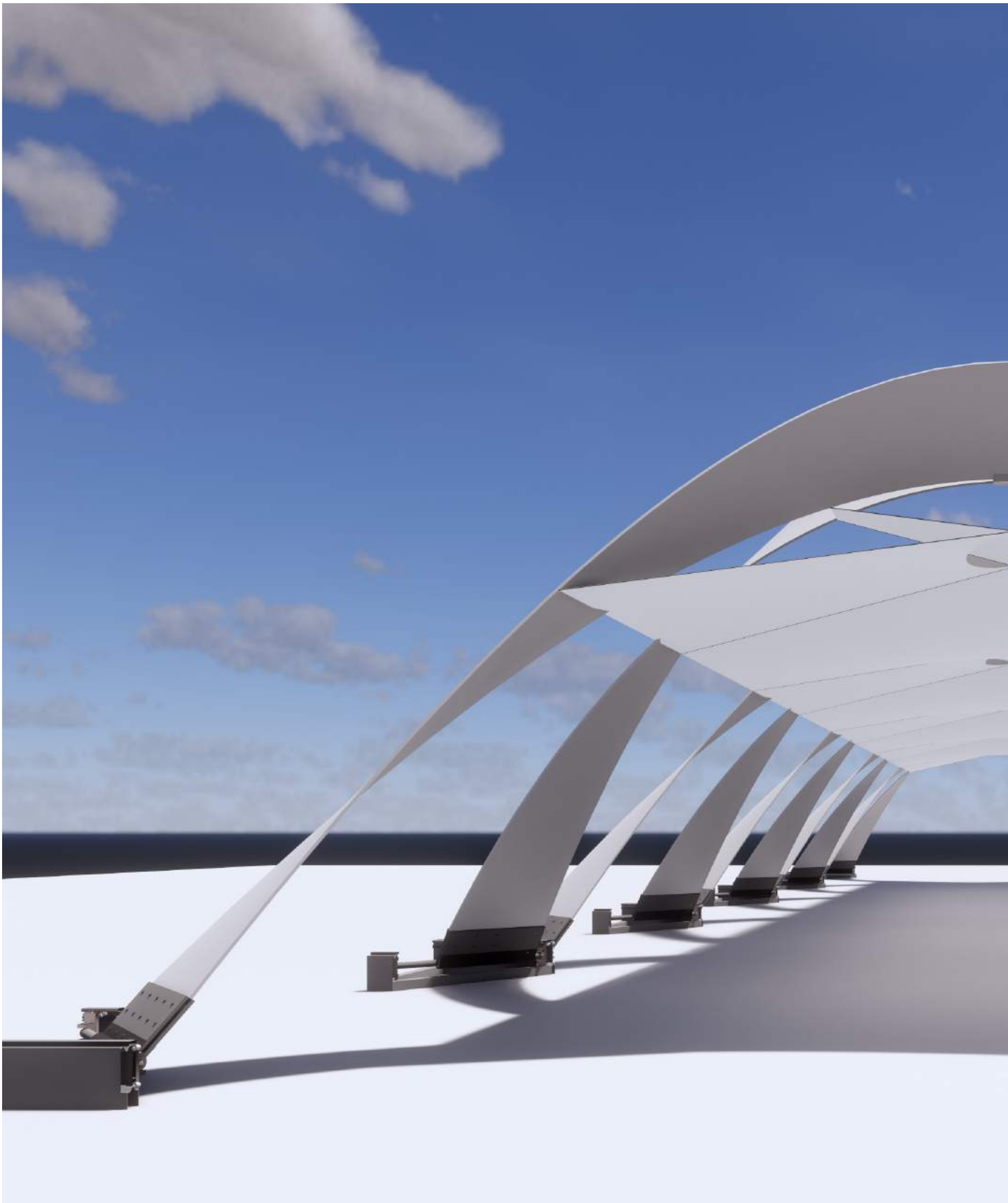
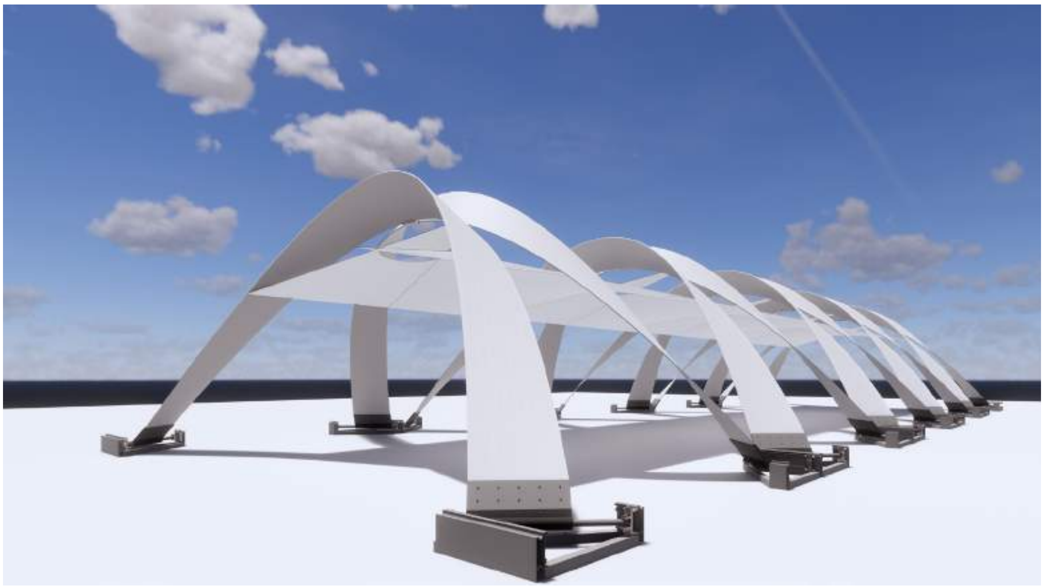
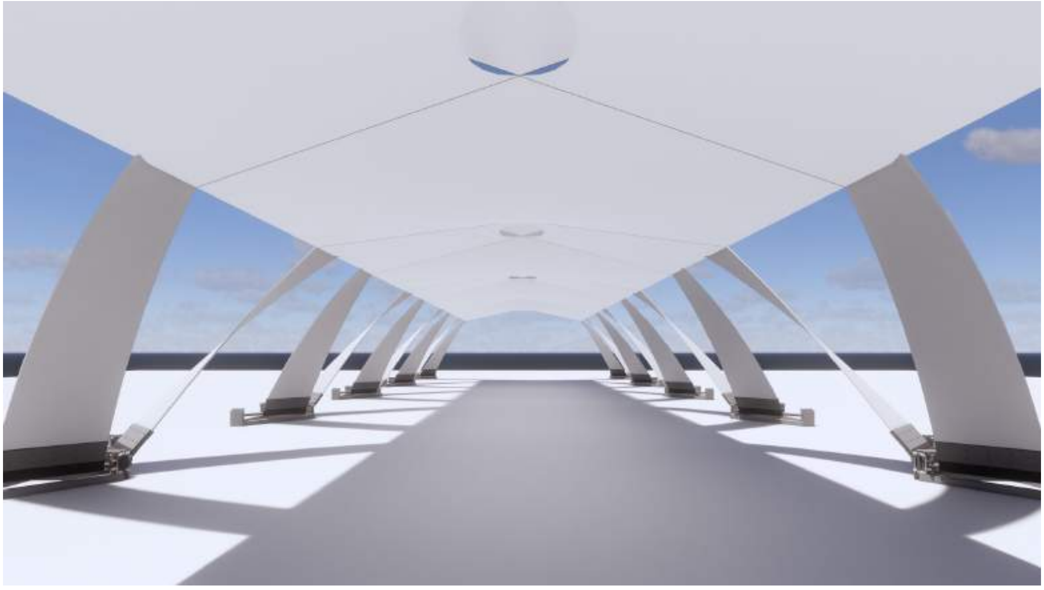


Figure [I1]Design intent linear array, front.

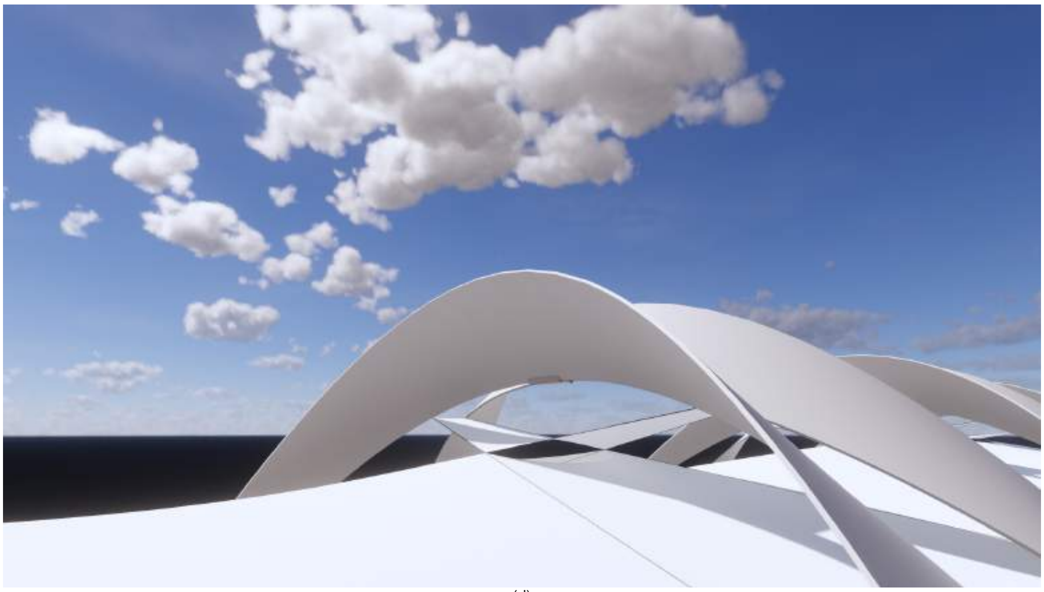


(a)

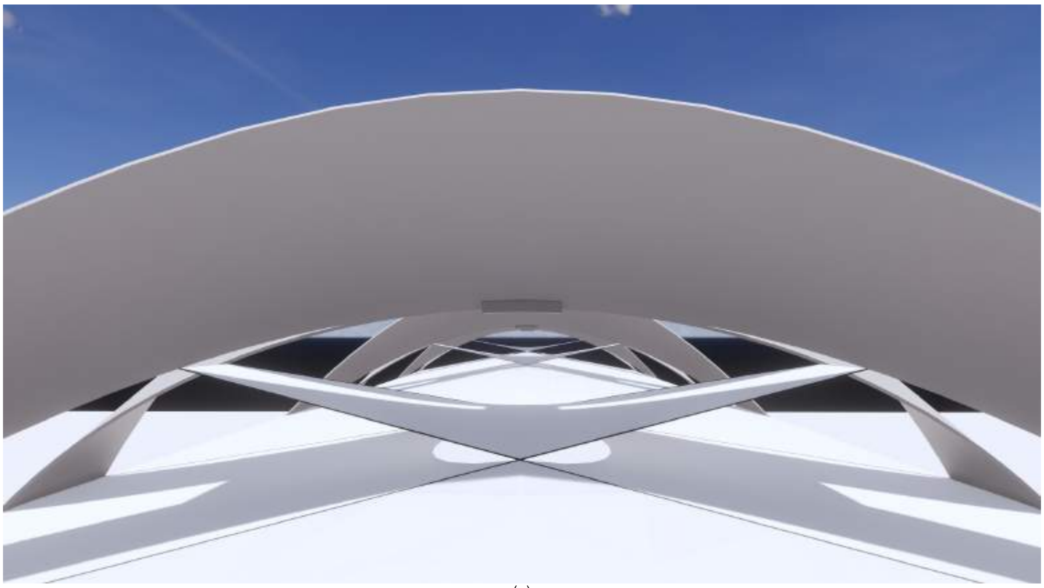
(b)



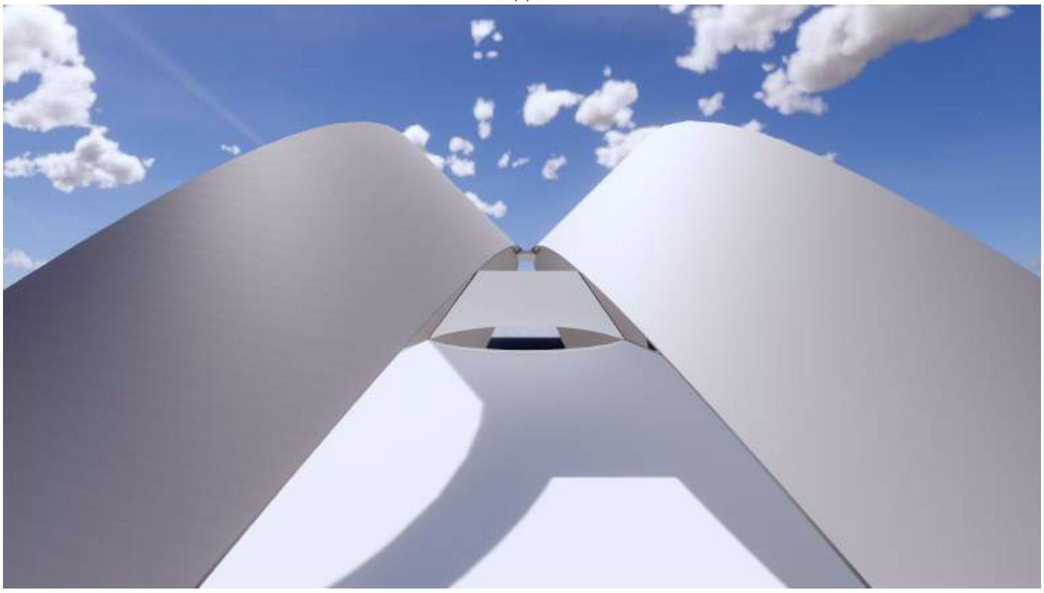
(c)



(d)



(e)



(e)

Figure [I2]Design intent linear array. (a) isometric view, (b) side view, (c) inside view, (d) membrane suspension side view, (e) mebrane suspension vront view.

I.2 Squared array

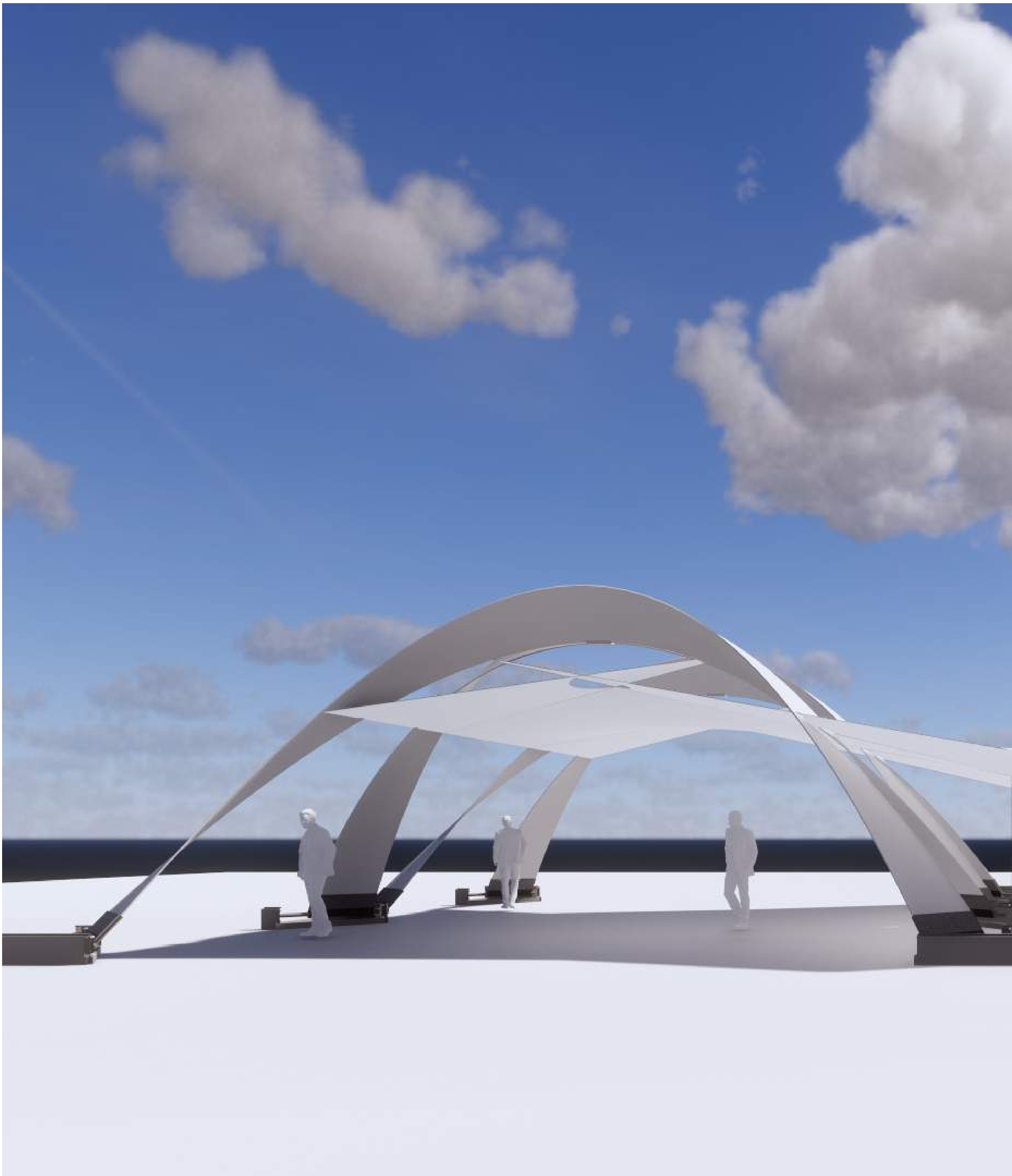
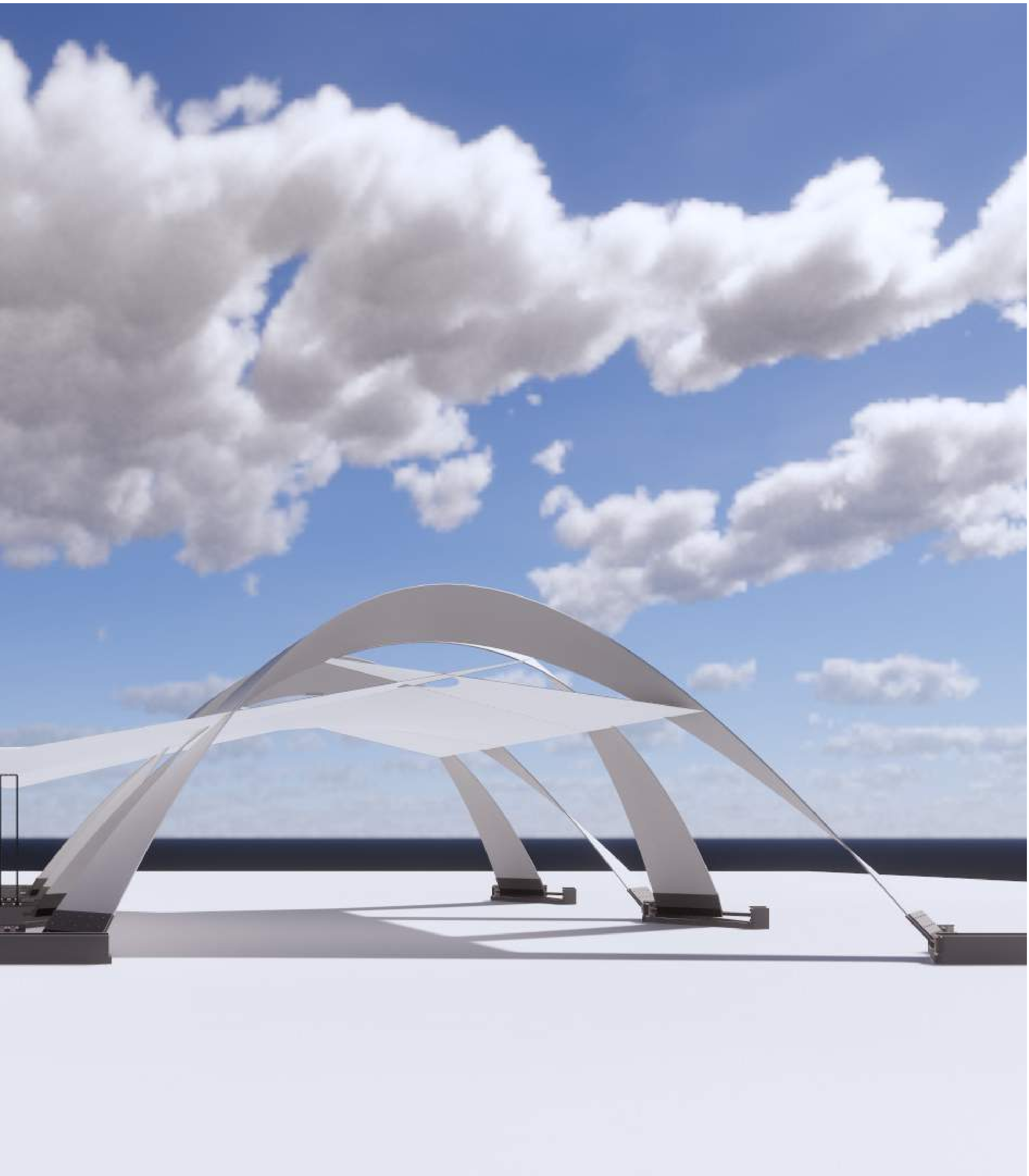
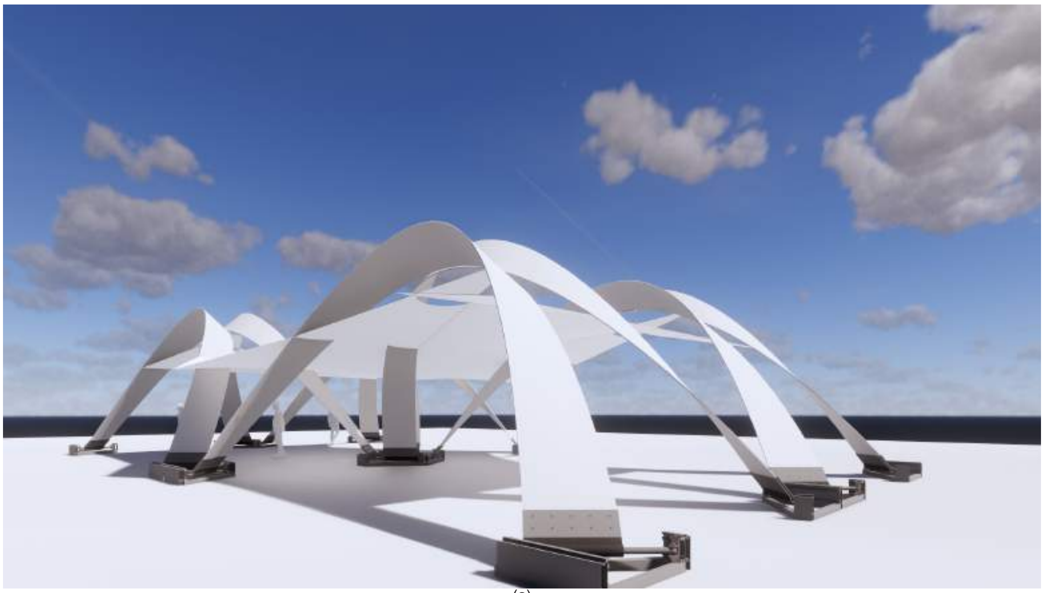
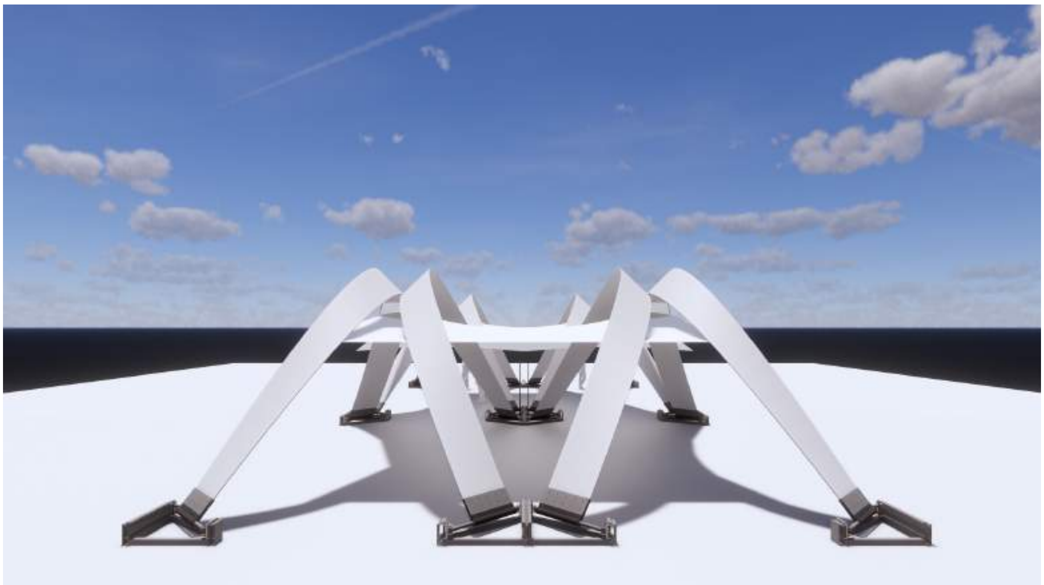
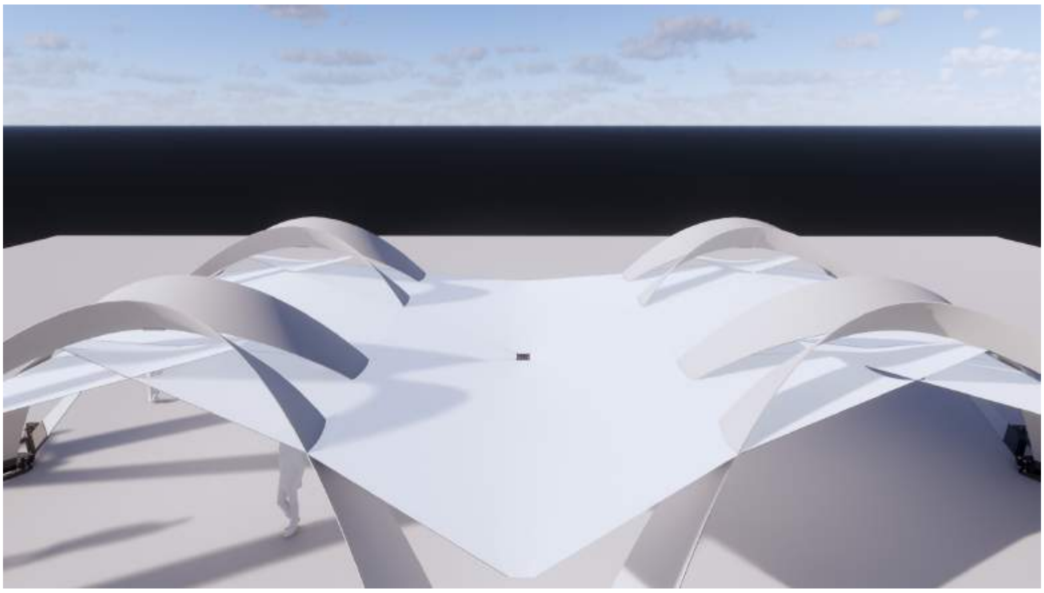


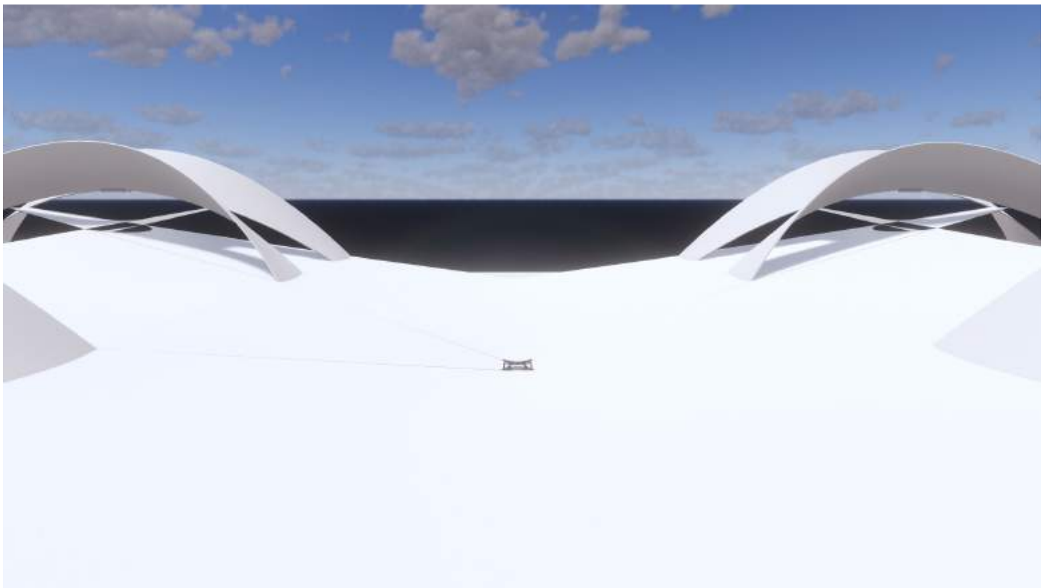
Figure [I3]Design intent linear array, front.



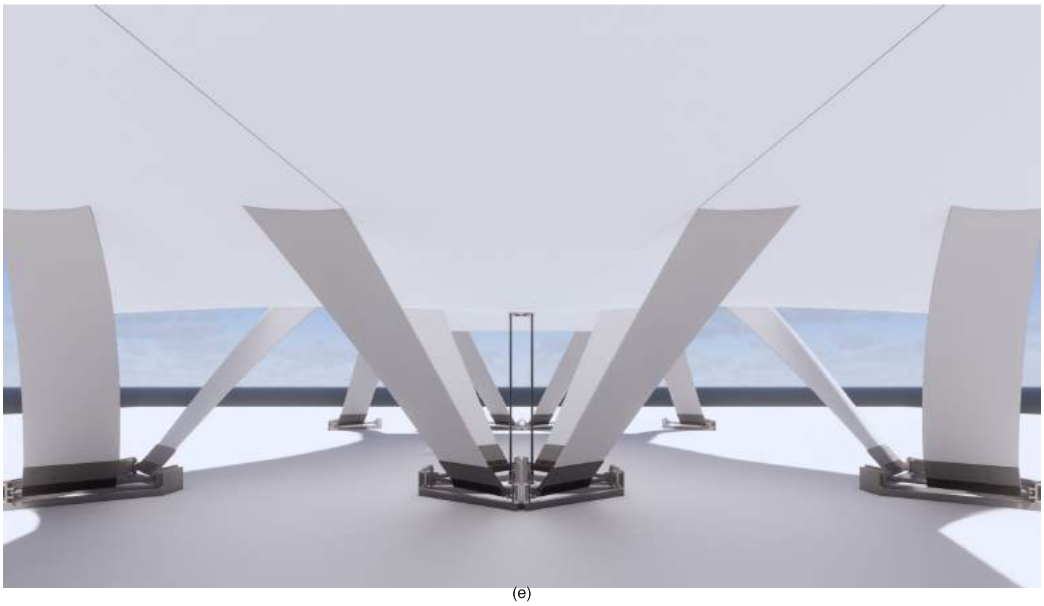

(a)


(b)

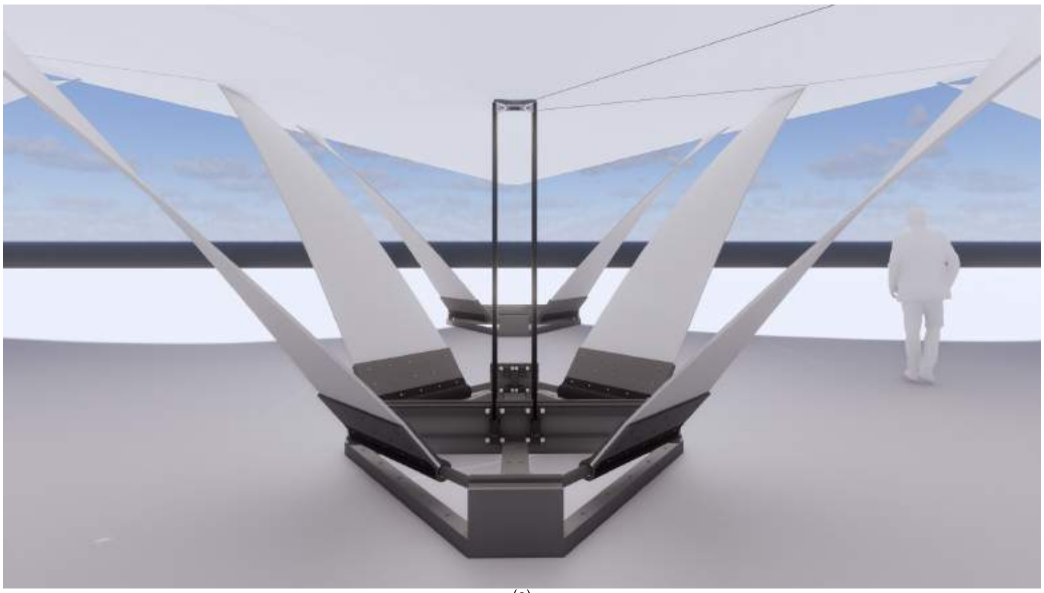

(c)



(d)



(e)



(e)

Figure [I4] Design sideways linear array. (a) isometric view, (b) side view, (c) birds eye view, (d) inside structure view, (e) inside structure support view.

J. Detailing, design freedom and future scenarios

J.1 Design freedom

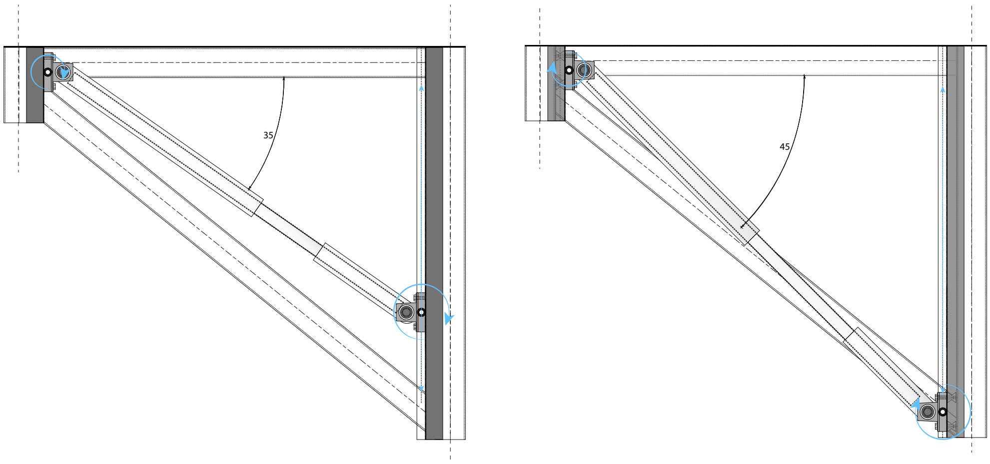


Figure [J1]Design freedom plate positions minimum and maximum, (a) 35 degrees, (b) 45 degrees.

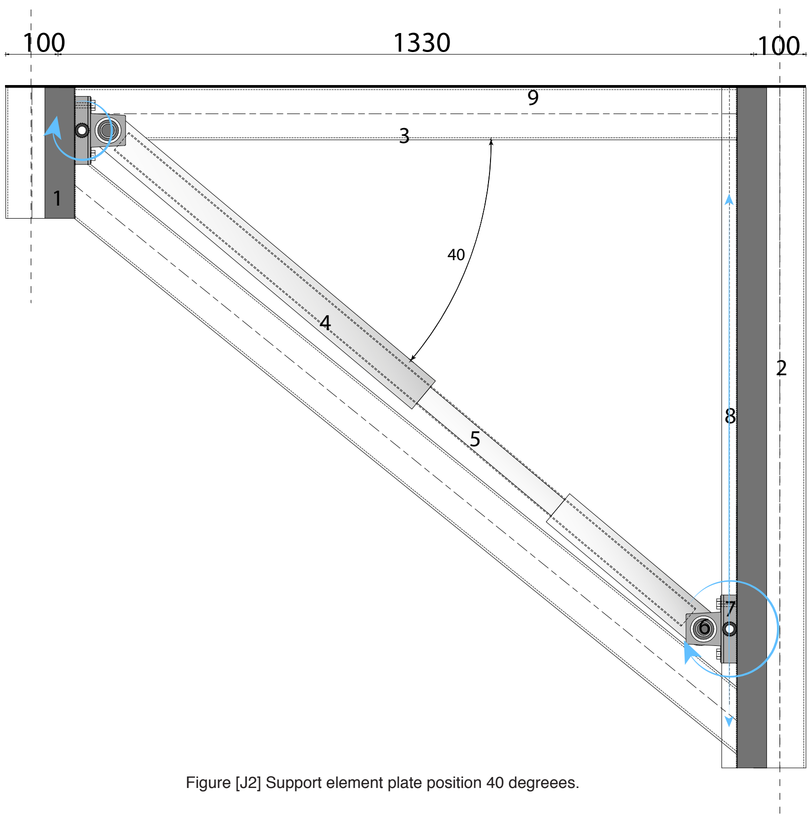


Figure [J2] Support element plate position 40 degreees.

short gliding rail **1**
Long gliding rail **2**
Base frame mount support **3**
Rotational axis plate **4**
Extendable rotational axis **5**
Rotational bearing **6**
Gear wheel **7**
Gear gliding rail **8**
Finish plate reaction forces **9**

The final geometry and the structural performance of a planar plate depend on how much bending and torsional deformation the plate is subject to. Regarding torsion, the plate performs optimally after 45 degrees torsional displacement at both support. The normal vector direction of the plate at both ends are perpendicular to each other. The less torsional movement would mean the less pre-stress in the plate, as well as a non-perpendicular position of the two supports. Even though this would not be structurally optimal, it allows for design freedom as the geometry of the plate varies for different rotational angles. With a domain of ten degrees, the rotational angle bounds are set to 35 to 45 degrees. Within this torsional range, the geometry varies while retaining its structural capacities.

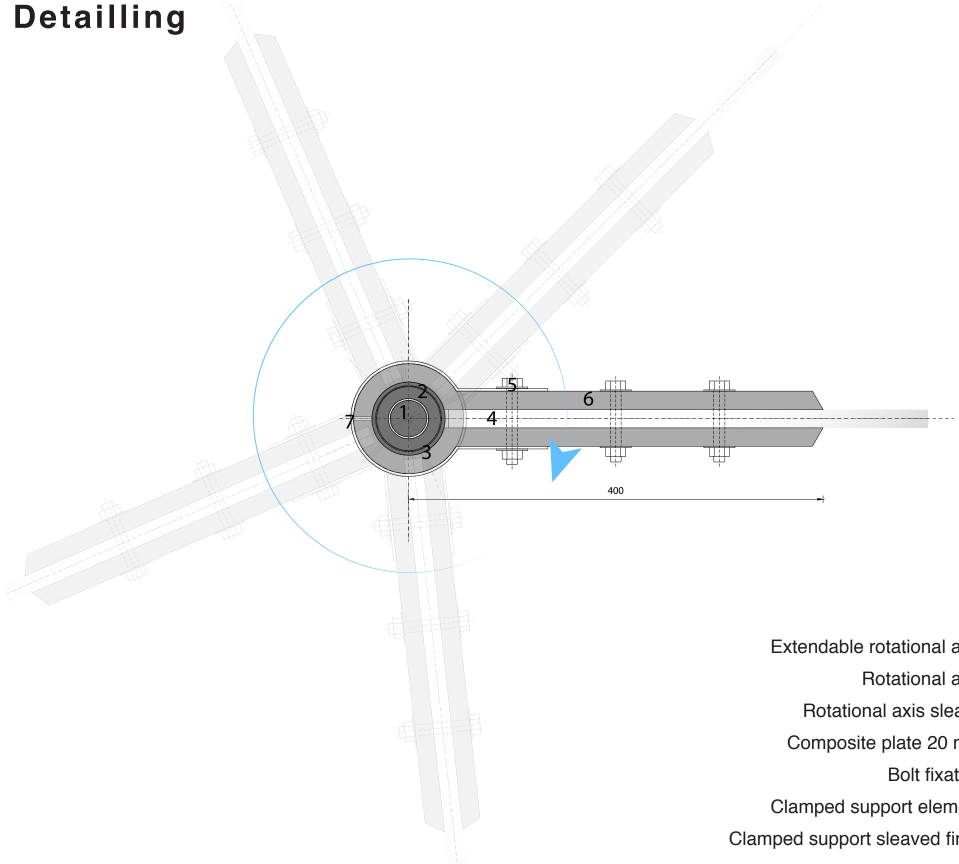
(a) (b) (c)

Figure [J3] Deesign freedom plate positions, (a) 45 degrees position, (b) 40 degrees position, (c) 35 degrees position.

(a) (b) (c)

Figure [J4] Plate shape design freedom positions, (a) 45 degrees position, (b) 40 degrees position, (c) 35 degrees position.

J.2 Detailling



Extendable rotational axis **1**
Rotational axis **2**
Rotational axis sleave **3**
Composite plate 20 mm **4**
Bolt fixation **5**
Clamped support element **6**
Clamped support sleaved finish **7**

Figure [J5] 360 degrees rotational freedom plate element

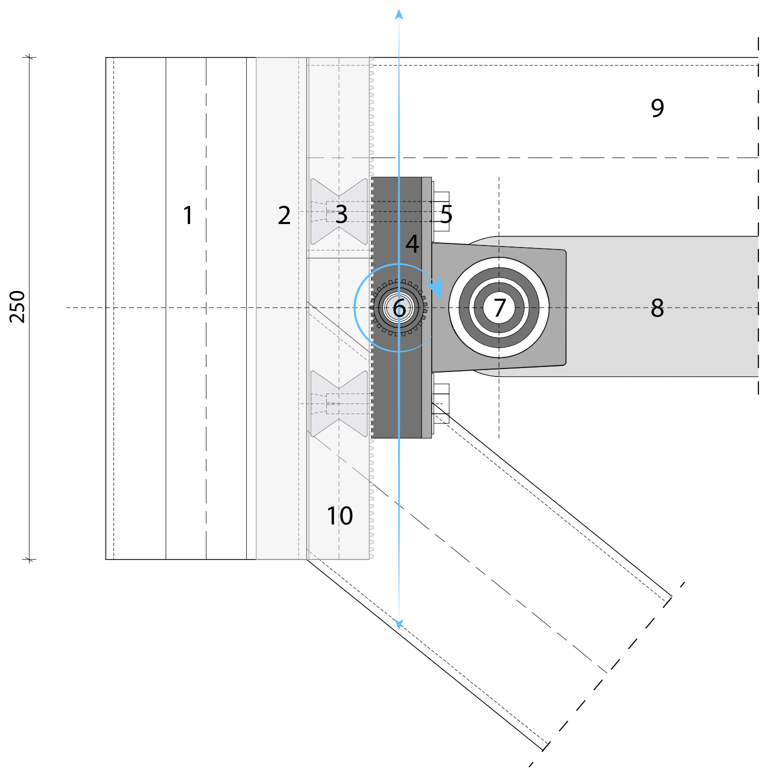


Figure [J6] Gliding mount

Short gliding rail **1**
Gear gliding rail **2**
Wheels linear motion support **3**
Mount plate **4**
Bearing bolt gliding wheels **5**
Gear wheel **6**
Rotational bearing plate **7**
Rotational axis plate **8**
Base frame mount support **9**

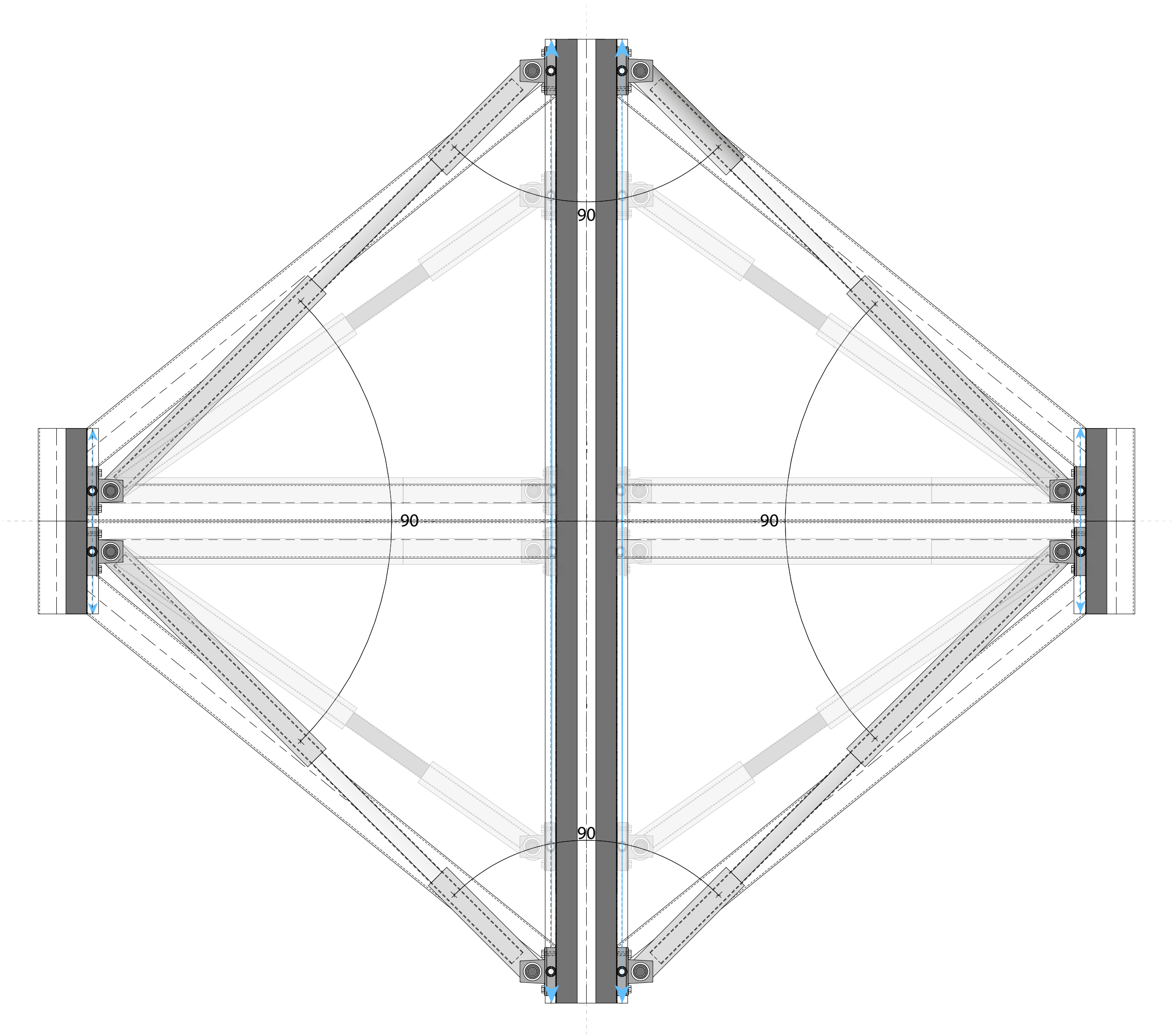


Figure [J7] Combined support elements

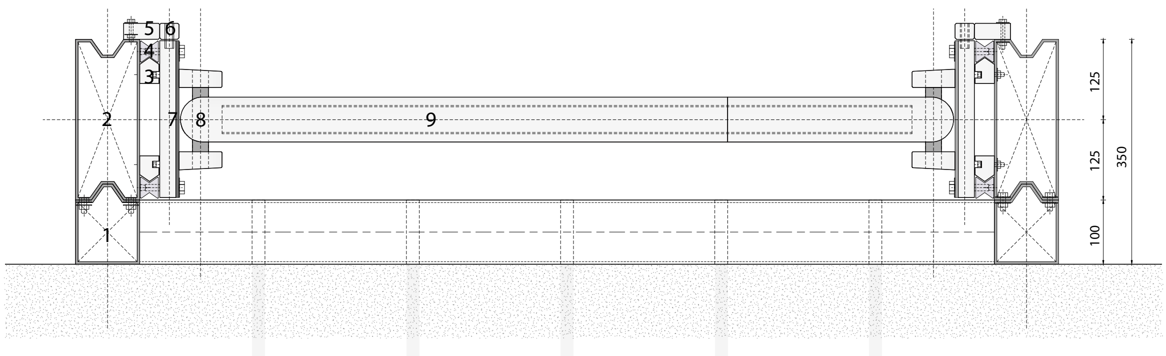


Figure [J8] Single support element section

1 Base frame mount support
2. Profile gliding rail
3. Rail wheel mount
4. Wheel linear motion support
5. Gear gliding rail
6 Gear wheel
7 Mount plate
8 Rotational bearing plate
9 Rotational axis plate

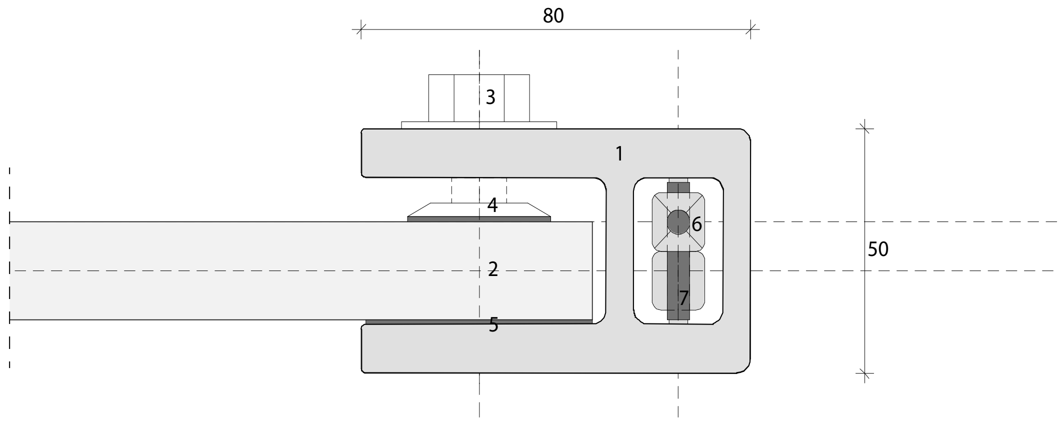


Figure [J9] Clamb to plate cable connector, back view

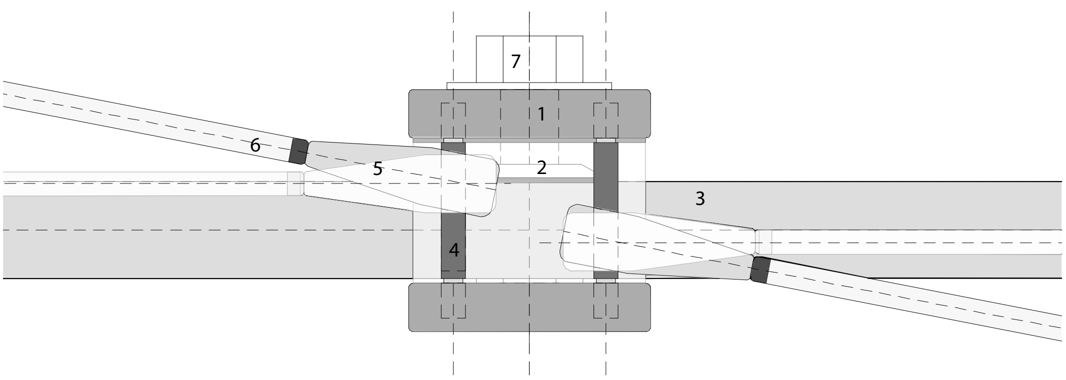


Figure [J10] Clamb to plate cable connector, side view

Figure [J9]

1 clamb element
2 composite plate
3 Fixation bolt
4 distributed soft pressure on plate
5 Soft membrane plate protection
6 Steel cable mount
7 Bearing axis steel cable

Figure [J10]

Clamb element **1**
distributed soft pressure on plate **2**
composite plate **3**
Bearing axis steel cable **4**
Steel cable mount **5**
steel cable **6**
Fixation bolt **7**

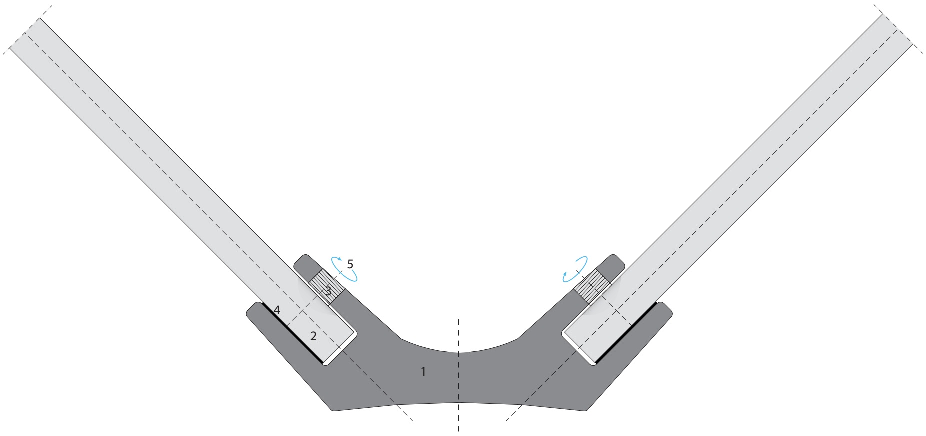


Figure [J11] Connection elements two plates

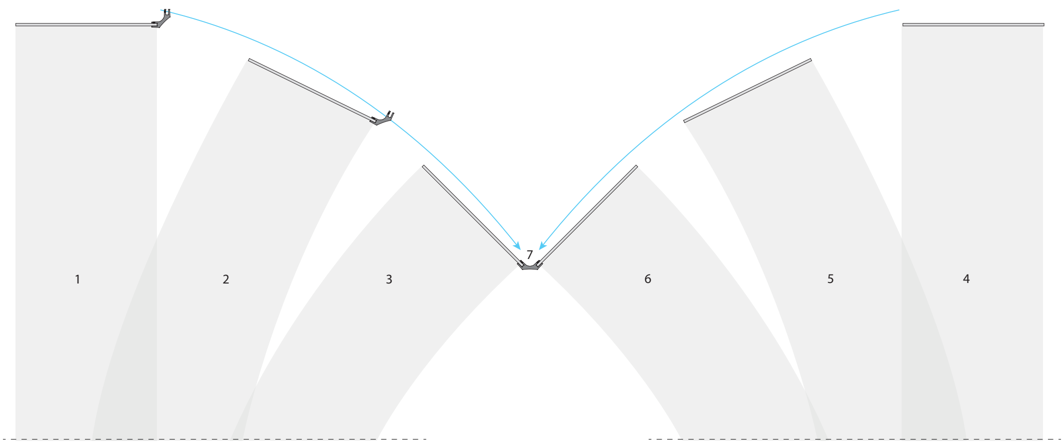


Figure [J12] Bending and torsion sequence into connection element

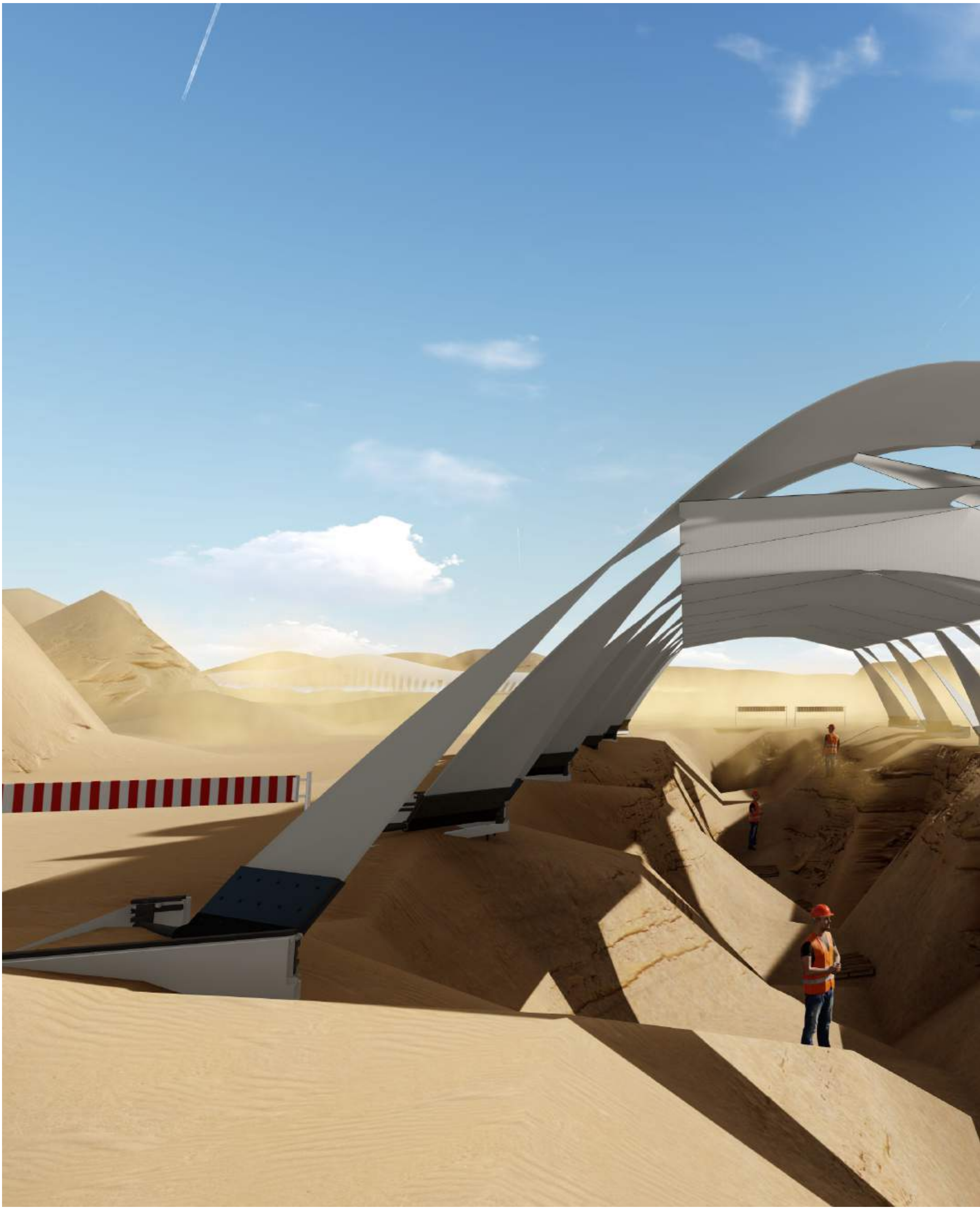
Figure [J11]

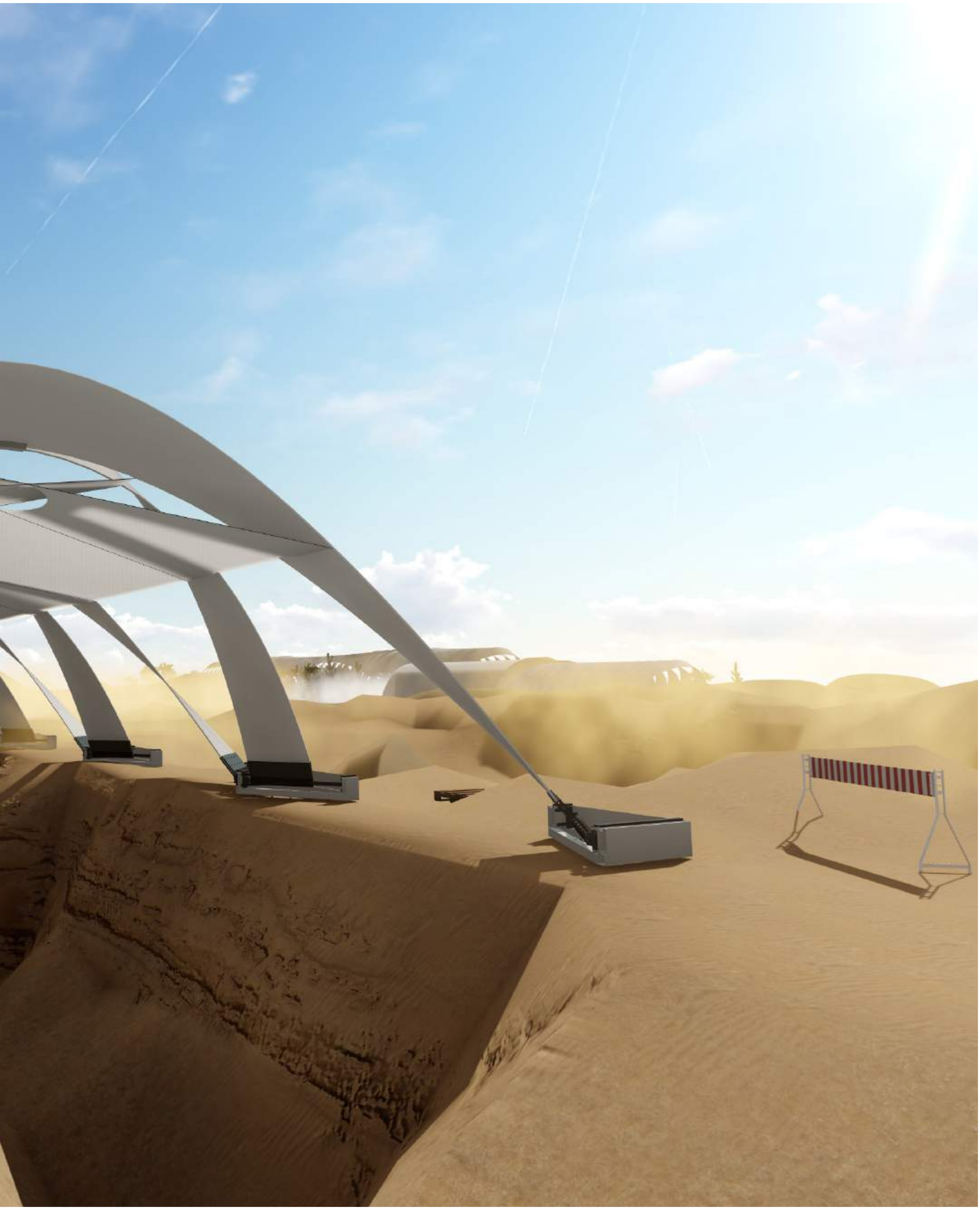
1 Connection element
2 composite plate
3 Fixation bolt in element
4 Soft membrane plate protection
5 Fixation to plate, temporary hold

Figure [J12]

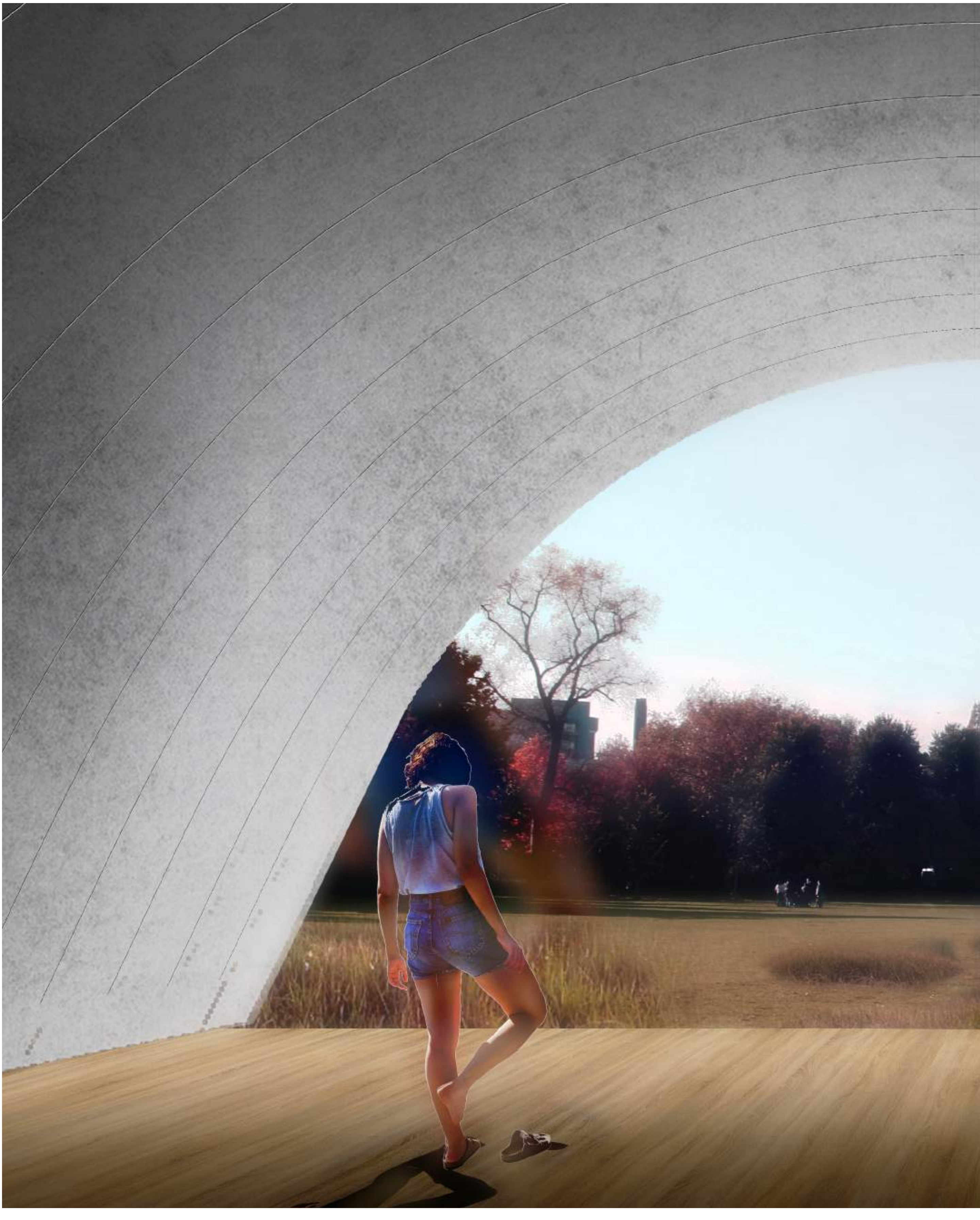
Bending only plate 1 attached connection element **1**
20 degrees torsional displacement **2**
45 degrees torsional displacement **3**
Bending only plate 2 attached connection element **4**
20 degrees torsional displacement **5**
45 degrees torsional displacement **6**
Final connection - dry connection - plate connector **7**

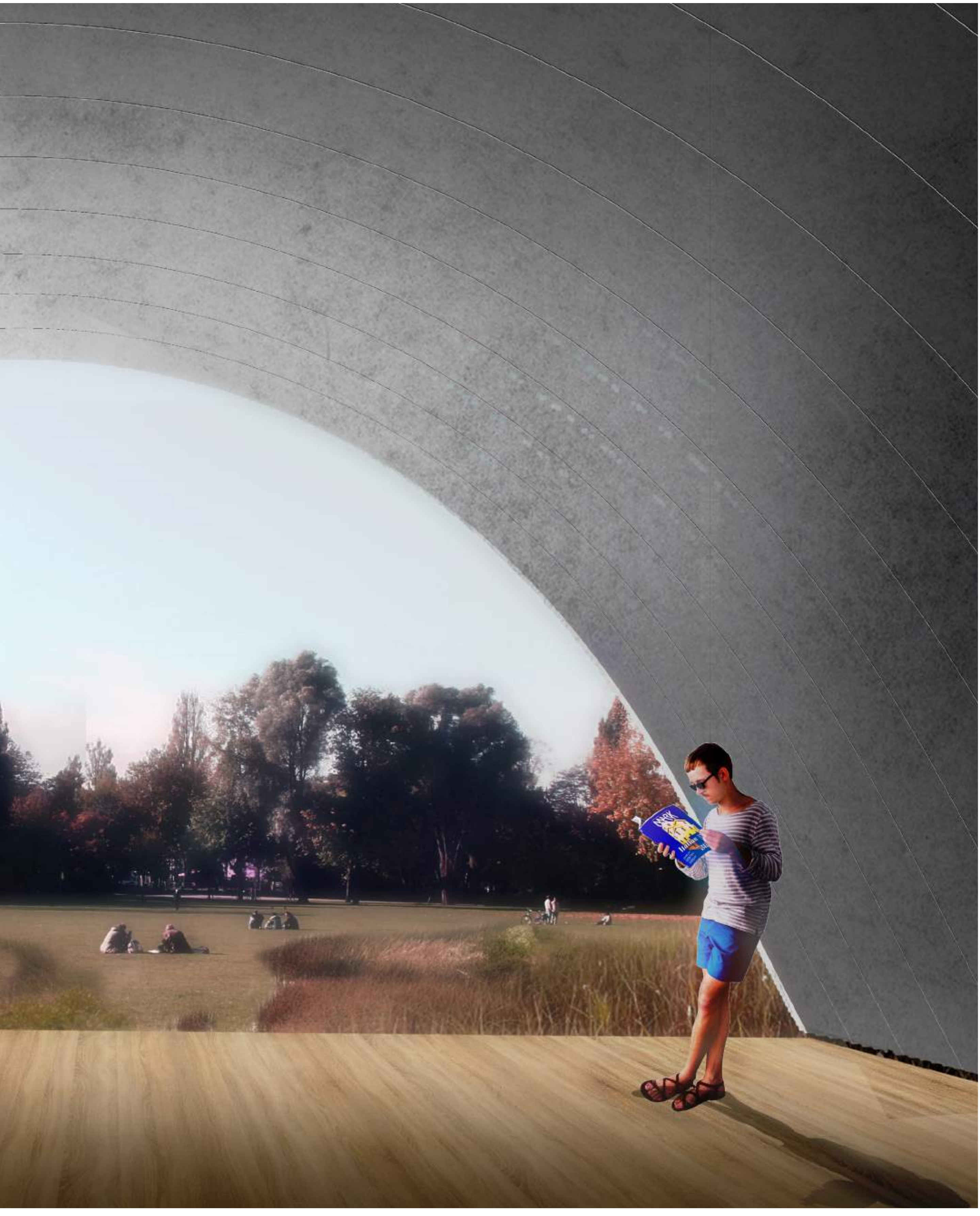
J.3 Future scenario's
[J.13] Structures for remote & fragile building sites



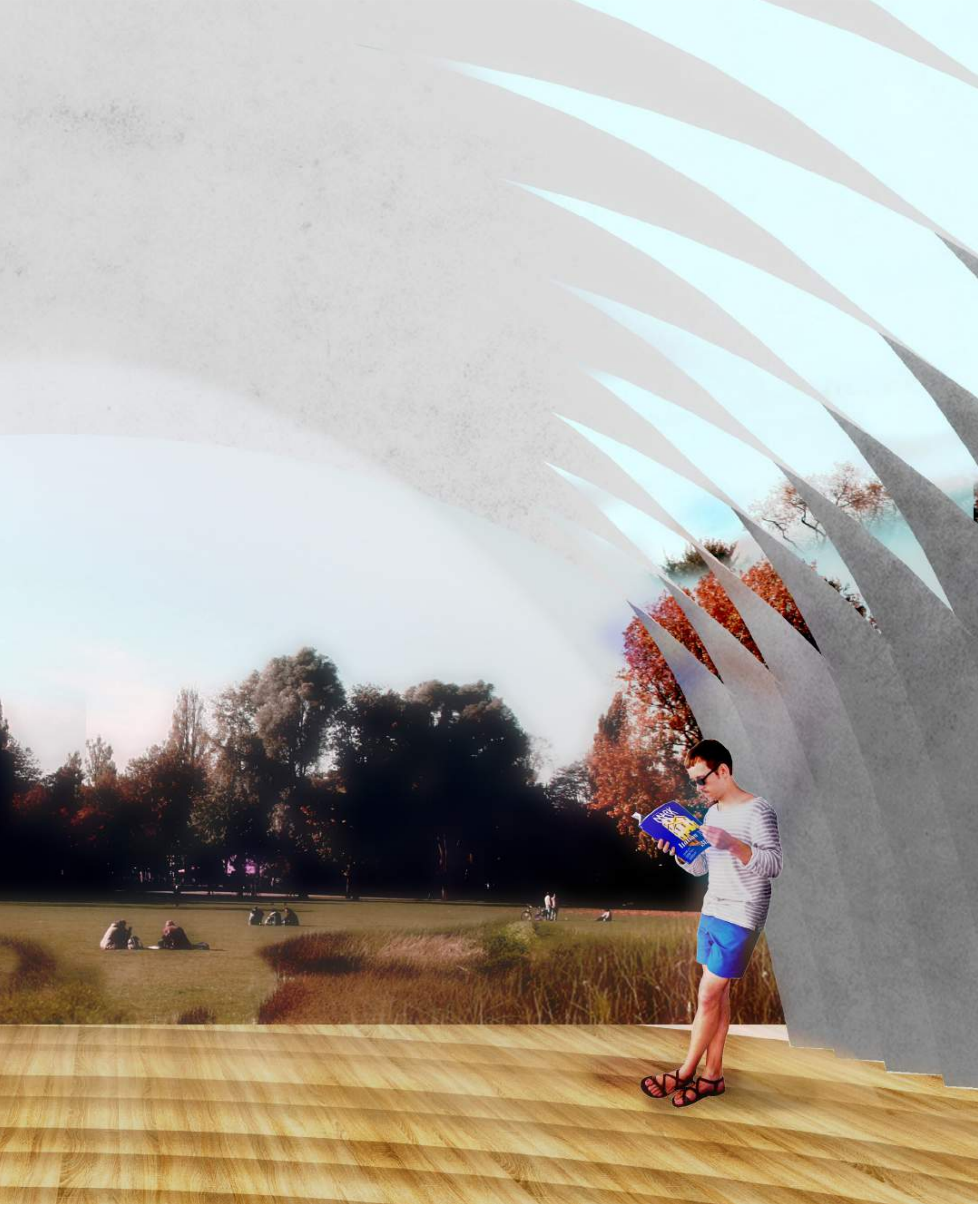


[J.14] Kinetic architecture pavilion - closed

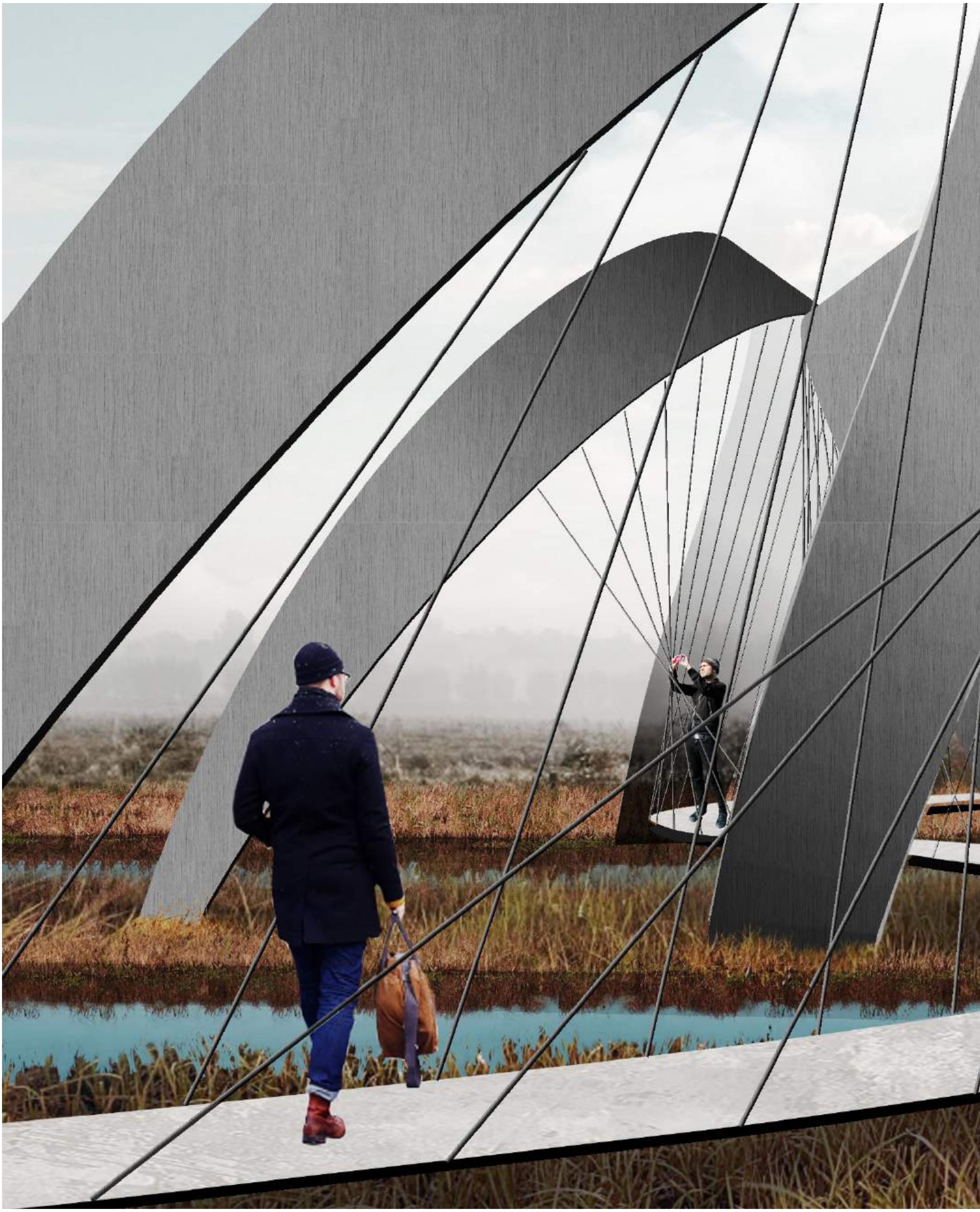


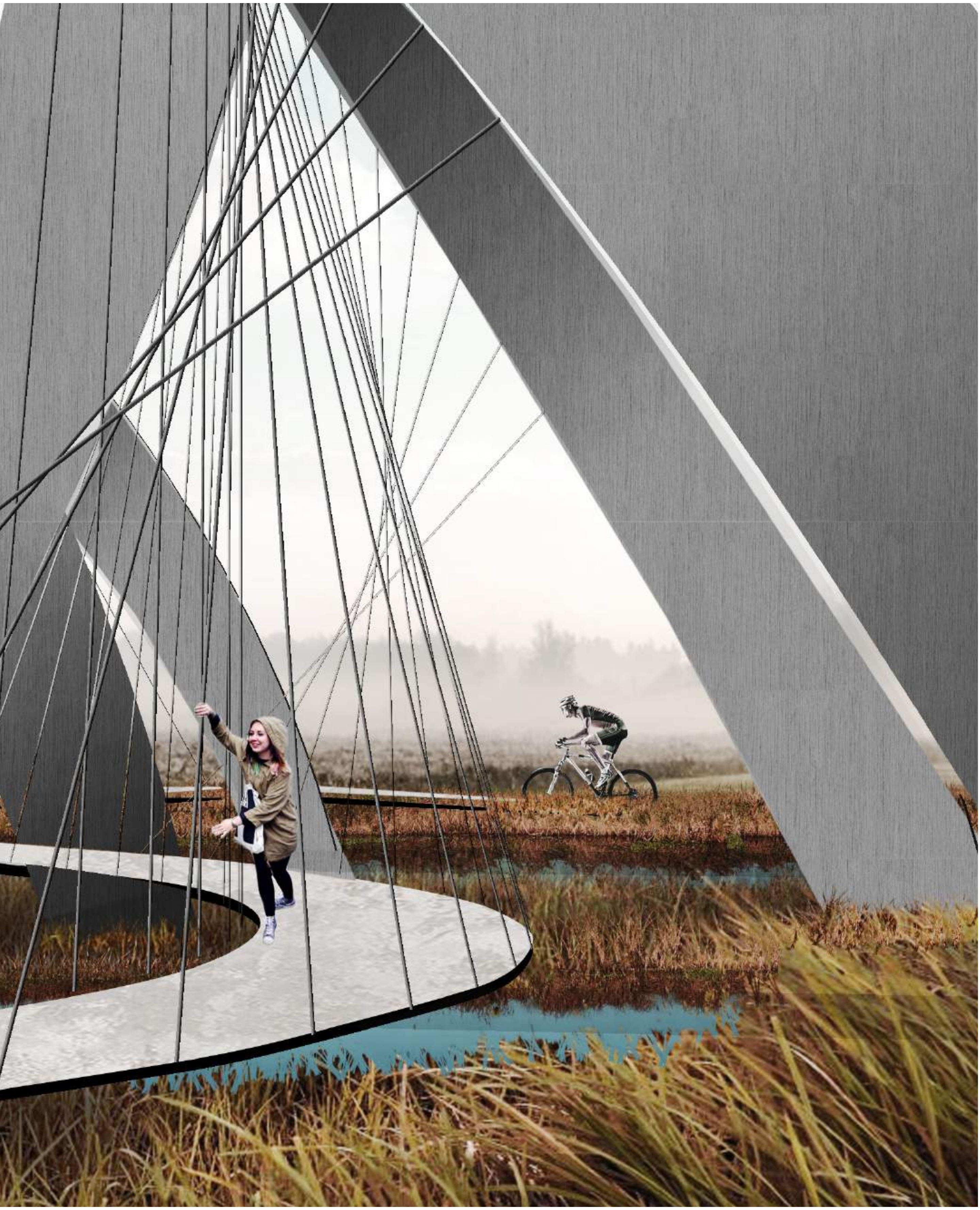


[J.15] Kinetic architecture pavilion - Open



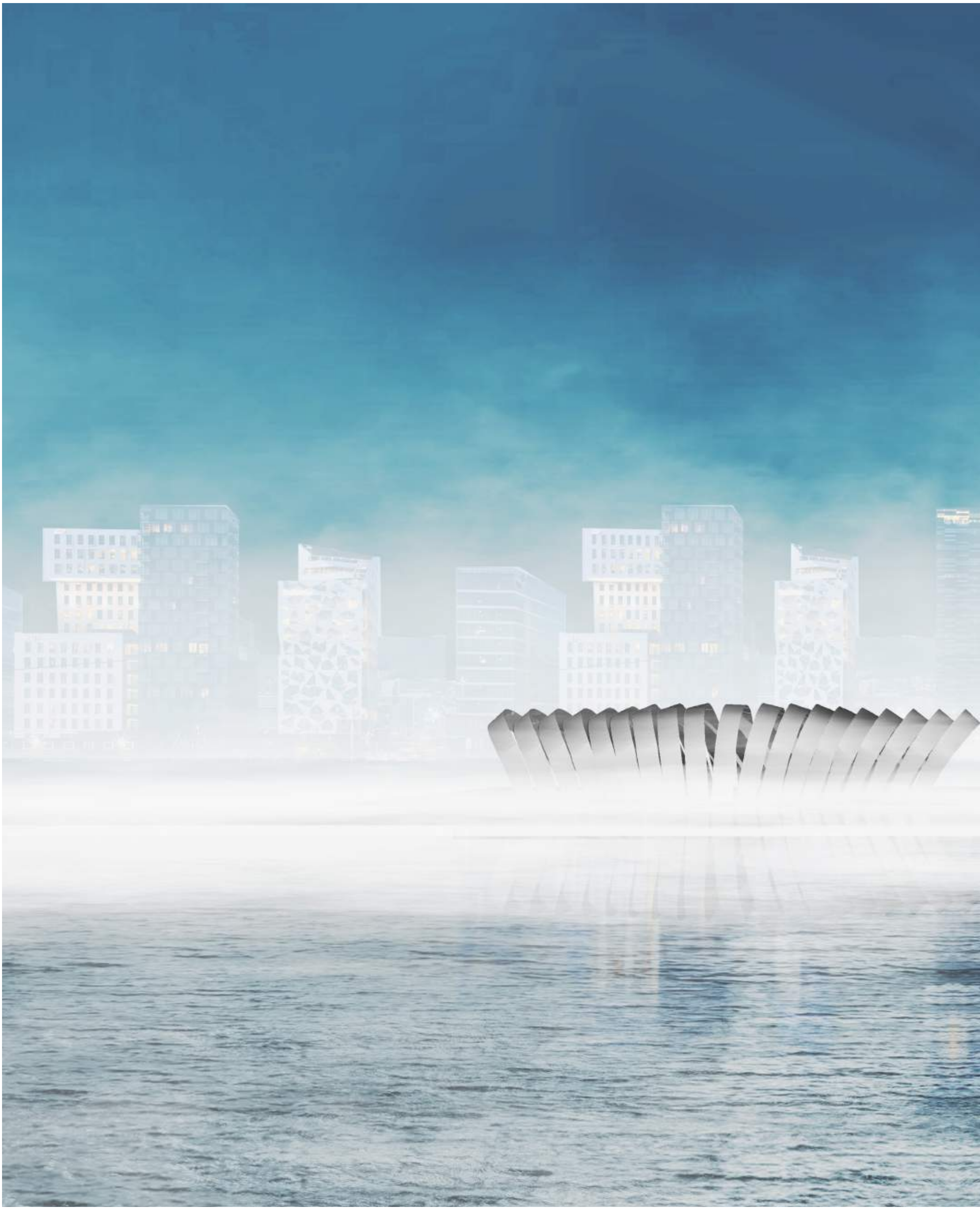
[J.16] Arch bridge structure





[J.17] Arch bridge design

[J.18] Upscale and randomisation



[J.19] Entire active-bending building envelope enclosure

K Conclusions and future trends

K.1 Conclusion

Active bending

Active-bending is the elastic deformation of initially flat or linear elements and is considered as a non-linear deformation process. There is a non-linear relationship between deformation and applied forces. The geometric behaviour and structural performance of active-bending structures rely on the relationship between the stiffness and the maximum allowable stress in a material. Typical materials for active-bending structures are woods and FRP's. The form creation process of active-bending structures takes up to 60% utilisation of the materials.

The behaviour and performance for active bending structures can be considered one-dimensional, for linear elements, or two-dimensional, for plate elements. Linear active-bending relies on the youngs modulus, the moment of inertia and the length of the component. The moment-curvature relation allows for the calculation of the maximum allowable curvature in the bending direction. For plate structures, the maximum permissible moment can be calculated with the curvature in x-, y- or xy-direction, the bending rigidity and the Poisson's ratio.

Form and curvature

The advantage of active-bending lies in the simplicity of creating double curved geometry from initially planar elements. Single curvature, i.e. one-dimensional curvature is the amount by which a curve deviates from being straight. The measure of double curvature in this research is the Gaussian curvature, which is the product of the principal curvatures in a specific point. With the use of the Mohr circle, the relation between the principle radii and the torsional radii can be derived. The Gaussian curvature in a particular point on a double curved surface remains equal after deformation of the surface. If the principle radius in one direction increases, the principle radius in the other direction decreases.

Three geometric measures can be distinguished, mono-clastic, syn-clastic and anti-clastic. Geometrically, mono-clastic surfaces are developable, which means they can be unrolled to planar surfaces. Syn-and anticlastic surfaces are non-developable. Non-developable surfaces need to be rationalised to be able to create them from planar surfaces. The triangulation method has been used in this research to rationalise double-curved geometries. A polar array of planar triangles with an angular defect simulate a double curved surface.

Implementation

Many varying geometric shapes can be used for the triangulation method, such as quads or hexagons. The angle at which the geometric elements meet determines the continuity in a triangulation topology. The curvature of an approximated double curved surfaces can be adjusted through the change in angle at which single components meet. Any arbitrary surface with any curvature can be rationalised with the use of the triangulation method, without losing continuity in the topology.

Regardless of the geometric shape used for the triangulation method, in a continuous topology, continuous rectangular strips can be derived. In other words, all double curved surfaces created with the triangulation method can be created from linear quad shapes strips. These strips are subject to bending and torsion. The application of bending- and torsional displacement to a rectangular strip increases the stiffness of the strip. These two stiffening effects are based on two different phenomena's.

Theoretical framework bending and torsion

Geometrically, bending deformation of a planar strip into a single curved shape leads to a monoclastic surface, i.e. a developable surface. However, the application of the same bending deformation to a planar strip composed of actual materials leads to an anticlastic surface as the results of the Poisson's ratio.

The formula for the Gaussian curvature allows for the calculation of the relationship between the principle bending radius and the torsional radius. This formula does not take any material properties into account. The bending and torsional radii with their related bending and torsional moments cannot be calculated with the formula of Gaussian curvature for surfaces with actual material. The inclusion of material properties in the Gaussian curvature formula allows to calculate and predict the bending and torsional curvatures and moments. With only one principle radius, the bending radius, and the material properties, the bending and torsional moment relation can be derived.

One of the key structural phenomena of an active-bending structure is the increased tension, which leads to increased stiffness. Torsional displacement of a planar strip leads to high tensile stresses in the outer fibres and compression in the centre of the strip. If the magnitude of tensile stresses is more significant than the compression stresses, as a result of torsional displacement, the strip increased in geometric stiffness.

The Poisson's ratio is an essential property for both bending and torsion. For bending, a high Poisson's ratio results in an anticlastic surface with larger bending moments, compared to a low Poisson's ratio. This results in a geometrically stiffer surface. For torsion, a high Poisson's ratio results in smaller torsional moments compared to the torsional moments with a low Poisson's ratio. The Poisson's ratio does not affect the geometric behaviour of a surface subject to torsional displacement.

Application of bending and torsional displacement to a planar strip leads to a structurally and geometrically stiff arch, where the Poisson's ratio has a significant influence on the structural performance.

Design application

Fibre reinforced polymers are materials with a high Poisson's ratio, relatively large tensile strength and flexural stiffness. These properties make it a suitable material for active bending plate structures. Generally, FRP's are anti isotropic. Because the form creation process leads to stresses in both bending and torsional direction, a quasi-isotropic fibre arrangement is required. Glass and Carbon fibres with an Epoxy resin are the two composite components used in this research. For the Carbon fibre composite, the dimensions of the plate are 1000 by 15000 by 15 millimetres. The thickness to length ratio is 1:1000, which is an extremely thin measure for such an arch structure.

As a result of bending and torsional displacement, the arch cantilevers towards one side. Coupling to arches in the opposite direction results in a structural component which is stiff in four directions.

Aesthetically, two coupled arches define a space underneath. The topology of two coupled arches allows for design freedom in both linear and polar arrays. The space enclosure has been accomplished through membrane suspension under the arches. The design of the membranes is entirely based on drainage, leaving water outside the arch structure.

Building methodology

The arch structure is considered a mobile structure, which requires an easy assembly and disassembly process. Modular and repetitive components increase the feasibility of the structures application. In chronologic sequence, the arch will first undergo bending deformation by sliding the supports over a rail until it reaches its final position. When attached to the final support, the torsional deformation results in the final shape of the arch. The membrane suspends from the arches by steel cables. The steel cables are mounted to the arch and are positioned to increase the strength of the arch.

Final statement

The research question is “How can double curvature be exploited in a structure composed of elastically deformed planar elements??”. The first approach to answer this question was to create double curved structures through the elastic deformation of planar elements and looking for the extent till which this was possible and structurally feasible. In other words, active-bending was used as an approach to creating double curvature.

During this exploration process, the nuance of double curvature as a result of bending and torsion has been the design driver throughout the entire research. Pure bending and torsion results in complex double curved surfaces as a result of the material behaviour. Here, the entirety of the structure’s performance and aesthetics is based on the elastic deformation process, which makes active-bending a new, unique structural system itself.

K.2 Recommendations

Poisson's ratio bound

The Poisson's ratio is a leading parameter for active-bending plate structures. The theoretical domain of the Poisson' ratio bounds from -1,0 to 0,5, with practical values between 0,0 to 0,5. A high Poisson's ratio increases the geometric stiffness of a plate suspect to bending deformation. Anisotropic materials might offer possibilities to go beyond the incompressible limit of the Poisson's ratio of 0,5. With an orthotropic definition of the material, the Poisson's ratio is not bound anymore.

Design freedom

This research only discussed the elastic deformation of rectangular planar plates. Over the length of the plate, the section does not change. For example, an oval-shaped plate leads to a different deformation over the span of the plate. Within the bounds of structural feasibility, this might offer possibilities for increased design freedom. In addition to alternating between the amount of bending and torsional deformation, the shape of the original plate becomes a design driver as well.

Clamped supports

The form creation process for the arches requires free rotation in all directions. The structural analyses in this research have been performed with fixed supports, free in rotation. A clamped support adds up to the structural performance of an arch. With clamped supports, the arch is better resistant to load cases such as wind and rain. Further design development of the support frames may include clamping possibilities to fixate the plate entirely after form creation.

Wind resonate own frequency.

Active bending structures are flexible structures. In the structural analyses of the plate subject to wind load, the own frequencies have not been taken into account. It is very likely for flexible structures to resonate with the wind, leading to significant, undesirable deformation and possible disruption.

Creep

The effects of creep influence the structural performance of active bending structures drastically, as it relies on the tension in the fibres and the friction between the resin and the fibres. The ability to be kinetic and adaptive is a unique quality of this active-bending structure. However, to remain feasible, it relies on the long-term ability to be exposed to cyclic sizeable elastic deformation. In the non-erected phase, when the plates are non-stressed in a planar condition, the effect of resetting the creep works advantageously. However, this can only be done if no permanent creep deformation occurs.

For now, resetting the plates is essential for active bending plate structures, which means they only perform optimally for a specified period. Permanent active bending structures require materials with minimum to no creep effects.

Double layer

Even though the structural thickness of the plate is only twenty millimetres, with a length of fifteen meters and a width of one meter, the plate becomes relatively heavy. Additionally, the costs of a composite plate are significantly substantial with such dimensions. Commonly used methods to decrease the weight of composites is to make sandwich panels, two thin composite outer layers and a lightweight middle layer. The intermediate layer would require the same Poisson's ratio to deform simultaneously with the outer layer. The friction between the outer layers and the intermediate layer is an essential parameter. Metamaterials might offer possibilities for the interlayer. The curvature behaviour of metamaterials is manually adaptable, making it possible to align with the required curvature properties of the outer layers.

Double curved architecture

The canopy structures prove the feasibility of active bending plate structures. The future application of such structures might emerge in large-scale canopy structures with permanent use. Further development of plate bending allows for broader applicability. The kinetics and adaptability allow for application in dynamic facade elements or adjustable sunshade canopies. The building method enables building at places which do not allow for large construction sites. This active bending plate structure has a low impact on its surroundings which make them feasible for application in small or protected areas. Future development might replace the large structural backing systems required for double curved facades nowadays. The double curved active bending plate structure is both the structure and the double curved surface cladding of a building.

Bibliography

Ahlquist, S., Lienhard, J., Knippers, J., & Menges, A. (2013, February). Exploring material reciprocities for textile-hybrid systems as spatial structures. In Prototyping Architecture: The Conference Papers, edited by Michael Stacey (pp. 187-210).

Ahlquist, S., Lienhard, J., Knippers, J., & Menges, A. (2013). Exploring material reciprocities for textile-hybrid systems as spatial structures. In Prototyping Architecture: The Conference Papers, edited by Michael Stacey (pp. 187-210).

Alpermann, H., & Gengnagel, C. (2012). Shaping actively-bent elements by restraining systems. In Conference proceedings IASS-APCS symposium: from spatial structures to space structures.

Alpermann, H., & Gengnagel, C. (2013). Restraining actively-bent structures by membranes. In International Conference on Textile Composites and Inflatable Structures.

Alpermann, H., Lafuente Hernández, E., & Gengnagel, C. (2012). Case-studies of arched structures using actively-bent elements. In Proceedings of the IASS-APCS Symposium: From Spatial Structures to Space Structures.

Apperman, O., Christoph, G., Gengnagel, C., Lienhard, J.,1 & Knippers, J. (2013). Active Bending, A Review on Structures Where Bending Is Used as a Self-Formation Process. International Journal of Space Structures 28 (3–4): 187–196. doi:10.1260/0266-3511.28.3-4.187.

Baratta, F. I. (1981). When is a Beam a Plate?. Journal of the American Ceramic Society, 64(5).

Beranek, W.J. (1979), K3 dictaat ruimtelijke constructies, deel 2. Technische Hogeschool Delft, Delft

Blackwell, W. (1984). Geometry in architecture. New York et al: Wiley.

Bruetting, J., Körner, A., Sonntag, D., & Knippers, J. (2016). Bending-Active Segmented Shells. Doctoral dissertation, Master's thesis, University of Stuttgart.

Calladine, C.R. (1983). Theory of shell structures, Gaussian curvature and Shell structures. Department of Engineering, University of Cambridge.

De Laet, L., Slabbinck, E., Van Mele, T., Block, P., & Mollaert, M. (2013). Case study: A modular, self-tensioned, bending-active canopy. Journal of the International Association for Shell and Spatial structures.

Douthe, C., Baverel, O., & Caron, J. F. (2006). Form-finding of a grid shell in composite materials. Journal of the International Association for Shell and Spatial structures, 47(1), 53-62.

Drew, P. (1976). Frei Otto form und konstruktion. Hatje.

Engel, H. (1997). Structure Systems. Ostfildern-Ruit, Germany: Verlag Gerd Hartje.

Fellows, R. F., & Liu, A. M. (2015). Research Methods for Construction. Manhatten: John
Glaeser, L. (1972). The work of Frei Otto. The museum of Modern Art, New York.

Gonzalez-Quintial, F., Barrallo, J., & Artiz-Elkarte, A. (2015). Freeform surfaces adaptation using developable strips and planar quadrilateral facets. Journal of Facade Design and Engineering, 3(1), 59-70.

La Magna, R. (2017). Bending-active plates: strategies for the induction of curvature through the means of elastic bending of plate-based structures. Stuttgart: Institut für Tragkonstruktionen und Konstruktives Entwerfen, Universität Stuttgart.

La Magna, R., & Knippers, J. (2017). On the behaviour of bending-active plate structures. the International Association for Shell and Spatial Structures (IASS).

La Magna, R., Schleicher, S., & Knippers, J. (2016). Bending-Active Plates: Form and Structure. Advances in Architectural Geometry 2016. vdf Hochschulverlag AG. doi:10.3218/3778-4.

La Magna, R., Schleider, S., (2016). Bending-active plates: Form-finding and form-conversion. Modelling Behaviour: Design Modelling Symposium (pp. 53-64).

Li, J. M. (2017). Timber shell structures: form-finding and structural analysis of actively bent grid shells and segmental plate shells. Stuttgart: Institut für Tragkonstruktionen und Konstruktives Entwerfen, Universität Stutgart.

Lienhard, J., & Knippers, J. (2013). Considerations on the scaling of bending-active structures. International Journal of Space Structures, 28(3-4), 137-148.

LIENHARD, J., BERGMANN, C., MAGNA, R. L., & RUNBERGER, J. (2017). A Collaborative Model for the Design and Engineering of a Textile Hybrid Structure.

Lienhard, J., La Magna, R., & Knippers, J. (2014). Form-finding bending-active structures with temporary ultra-elastic contraction elements. Mob Rapidly Assem Struct IV, 136, 107.

Maurin, B., Motro, R., Raducanu, V., & Pauli, N. (2008). Soft 'tensegrity like'panel: Conceptual design and Form-Finding. Journal of the International Association for Shell and Spatial Structures, 49(158), 77-87.

Nicholas, P., Hernández, E. L., & Gengnagel, C. (2013, April). The Faraday Pavilion: activating bending in the design and analysis of an elastic gridshell. In Proceedings of the Symposium on Simulation for Architecture & Urban Design (p. 21). Society for Computer Simulation International.

Nolan, T.J. (1970). Computer-aided design of developable hull surfaces. Delft: The society of naval architects and marine engineers.

Oldfather, W. A., Ellis, C. A., & Brown, D. M. (1933). Leonhard Euler's elastic curves. Isis, 20(1), 72-160.

Oliver, P. (1997). Encyclopedia of vernacular architecture of the world. Cambridge University Press.

Otto, F. (1978). Multihalle Mannheim Band IL 13. Mitteilungen des Instituts für leichte Flächentragwerke (IL) Universität Stuttgart.

Pottmann, H. (2007). Architectural geometry (Vol. 10). Bentley Institute Press.

Rose, K., Sheffer, A., Wither, J., Cani, M. P., & Thibert, B. (2007). Developable surfaces from arbitrary sketched boundaries. In SGP'07-5th Eurographics Symposium on Geometry Processing (pp. 163-172). Eurographics Association.

Sapidis, N. S. (Ed.). (1994). Designing fair curves and surfaces: shape quality in geometric modeling and computer-aided design. Society for Industrial and Applied Mathematics.

Schleicher, S., & La Magna, R. (2015). Bending-active plates: form-finding and form-conversion. In Modelling Behaviour: Design Modelling Symposium (pp. 53-64).

Schleicher, S., La Magna, R., & Zabel, J. (2017). Bending-active Sandwich Shells: Studio One Research Pavilion 2017.

Schleicher, S., Rastetter, A., La Magna, R., Schönbrunner, A., Haberbosch, N., & Knippers, J. (2015). Form-Finding and Design Potentials of Bending-Active Plate Structures. In Modelling Behaviour (pp. 53-63). Springer International Publishing.

Schleicher, S., Rastetter, A., La Magna, R., Schönbrunner, A., Haberbosch, N., & Knippers, J. (2015). Form-finding and design potentials of bending-active plate structures. In Modelling Behaviour (pp. 53-63). Springer, Cham.

Seggern, von, D.H. (1994). Practical handbook of Curve Design and Generation. Reno, University of Nevada: CRC Press.

Sheil, R., Menges, A., Glynn, R., & Skavara, M. (2017). Fabricate (pp. 1-304). UCL Press.

Steele, C. R., & Balch, C. D. (2009). Introduction to the Theory of Plates. Dept. of Mechanical Engineering, Stanford University.

Tenpierik, M. (2015). BT research methodology, an introduction. Delft: TU Delft.

Van de Straat, R. J. (2011). Parametric modelling of architectural developables., Delft, Delft University of technology.

Van Mele, T., De Laet, L., Veenendaal, D., Mollaert, M., & Block, P. (2013). Shaping tension structures with actively bent linear elements. International Journal of Space Structures, 28(3-4), 127-136.

Van Otterloo, D. L., & Dayal, V. (2003). How isotropic are quasi-isotropic laminates. Composites Part A: Applied Science and Manufacturing, 34(1), 93-103.

Wester, T. (1984). Structural order in space: the plate-lattice dualism. Plate Laboratory, Royal Academy of Arts, School of Architecture.

Wierzbicki, T. (2006). 2.081 J/16.230 J Plates and Shells, Spring 2006.
Wiley & Sons.

Images and illustrations

A. Introduction - Research layout

[A1] Bending active structural components - by author
[A2] Graduation research time planning.

B. Research - Active bending

[B1] Active-bending principle (Schleicher at all 2015)
[B2] Past and present motivation use of active-bending - by author
[B3] Development of bending active structures] by Lienhard, J. 2014, Bending active structures.
[B4] Various approaches active-bending - by author
[B5] Behaviour based approach - by author
[B6] Geometry based approach - by author
[B7] Integral based approach - by author
[B8] Fulani Dwelling dome house]Reprinted from maalidesign website, Reprinted fmor http://maalidesigns.blogspot.nl/2011/04/fulani-of-nigeria.html
[B9] Herzogenriedpark Mannheim] Reprinted from Wikipedia website, 2010, byImanuel Giel, retrieved from https://commons.wikimedia.org/wiki/File:Herzogenriedpark_077.jpg
[B10] [Interior view of research pavilion] Reprinted from Miesarch website, 2011 by Julien Lienhard, retrieved from http://mies-arch.com/work/1053
[B11] Load deflection diagram non-linear structures] by Lienhard, J. 2014, Bending active structures.
[B12] [P-delta effect of mast with clamped support].by Lienhard, J. 2014, Bending active structures.
[B13] [Moment-curvature relation beam], by Lienhard, J. 2014, Bending active structures.
[B14] [Elastica figures by Euler] by Oldfather, Ellis, & Brown, 1933, Leonard Euler Elastic curves
[B15] [Basic elastica curves based on the Euler cases of buckling], by Oldfather et all, 1993, Leonard Euler Elastic curves
[B16] [Ratio of strength σ [MPa] to stiffness E [GPa] of common building materials] by Lienahard, L. 2014, Bending active structures, p-37.
[B17] Structural systems with appearance. - by author
[B18] Section active principle. Engels, 1997
[B19] Vector active principle. Engels, 1997
[B20] Form active principle. Engels, 1997
[B21] Surface active principle. Engels, 1997
[B22] Height active principle. Engels, 1997
[B23] Hybrid active principle. Engels, 1997
[B24] Bending active principle - by author
[B25] Bottom up and top down design approach - by author
[B26] Bending active approached stucture composed from existing structural types with their system family - by author
[B27] Active bending approach emerging from existing structural systems - by author
[B28] Various scale types and appearnces within bending active structural system - by author
[B29] Raditional design approach - by author
[B30] Integral design approach - by author

C. Research - From and curvature11

[C1] The segment of a plane curve substends ‘angle of embrace’]. By Calladine, 1983, Reprinted from Gaussian curvature and shellsstructures
[C2] Measure of single curvature - by author
[C3] The segment PQ of a plane curve subtends an angle of embrace alfa - by author
[C4] [Examples of the projection of curved surfaces onto the unit sphere - by author
[C5] [Definition of principal radii, r1, r2 at a typical point P on a smooth surface] - by author
[C6] Three Gaussian curvature measures. Monoclastic, Anticlastic and Synclastic - by author
[C7] Planar curve can be represented by a series of straight lines - by author
[C8][The segment of a plane curve substends ‘angle of embrace’]. By Calladine, 1983, Reprinted from Gaussian curvature and shells structures
[C9] [Shaping on potter’s wheel] Reprinted from Flickr website,by Ethan Schoonhoven, 2004, retrieved from https://www.flickr.com/photos/ejas/272228452

[C10] Curve rotation around central axis, surface of revolution - by author
[C11] straight lines over directrix curve - by author
[C12] Two extruded surface generations - by author
[C13] Normal axis of generator curve - by author
[C14] Rationalised surfaces - by author
[C15] Surface based on generatrix and directrix - by author
[C16] Translation surface, paralell vectors. Scaled translational surface, non-paralell vectors - by author
[C17] Levels of continuity curves for curves - by author
[C18] Linear scaling curved curve - by author

D. Implementation - Design direction

[D1] Design methodology - by author
[D2] [Forstpavilion] Reprinted from Itke website, by Roland Halbe, retrieved from https://www.itke.uni-stuttgart.de/de/archives/portfolio-type/landesgartenschau
[D3] [Bending active segmented shell] Reprinted from Itke website, retrieved from https://www.itke.uni-stuttgart.de/de/archives/portfolio-type/bending-active-segmented-shells
[D4] [Panikkar] Reprinted from coda-office website, 2014, retrieved from http://coda-office.com/cat/work/Panikkar
[D5] Design parameters devided by topology and material - by author
[D6] Design parameters geometric topology - by author
[D7] Form creation process, similarity topology and material - by author
[D8] Similarities between structural system and active-bending design approach - by author
[D9] Similarities workflow diagram - by author
[D10] [The restaurant of L'Oceanografic in Valencia, Spain as viewed from across the water.] Reprinted from Wikipedia website, by DavidIllif, 2007, retrieved from https://nl.wikipedia.org/wiki/L%27Oceanogr%C3%A0fic License: CC-BY-SA 3.0
[D11] [The O2 Arena London] Reprinted from Stubhub website, 2017, retrieved from https://www.stubhub.co.uk/magazine/the-o2-london
[D12] [Heydar Aliyev Center, Zaha Hadid Architects] Reprinted from Archdaily website, 2017, by Iwan Baan, retrieved from https://www.archdaily.com/448774/heydar-aliyev-center-zaha-hadid-architects,
[D13] [Herzogenriedpark Mannheim] Reprinted from Wikipedia website, 2010, by Imanuel Giel, retrieved fromhttps://commons.wikimedia.org/wiki/File:Herzogenriedpark_077.jpg
[D14] [Denver union station] Reprinted from structurflex website, 2018, retrieved from https://www.structurflex.com/projects/denver-union-station
[D15] [One ocean, Thematic pavilion 2012] Reprinted from archdaily website, 2018, by Soma retrieved fromhttps://www.archdaily.com/236979/one-ocean-thematic-pavilion-expo-2012-soma
[D16] Triangulation of spherical section - by author
[D17] Planar circle based on the tesselation of six triangles, expressed with alternative geometry - by author
[D18] Reduction of one ellement from planar configuration resulting in three dimensional object - by author
[D19] Three dimensional description composed of six and 7 elements - by author
[D20] Double curved surface through overlap connection of developable surfaces - by author
[D21] Continuity in surface if angles at which triangles meet is similar - by author
[D22] A planar surface composed of six triangles - by author
[D23] Form study using triangulation method. composed from planar and double curved elements. planar surface composed from six triangles - by author
[D24] Blue cricles alocate the double curved components composed from five planar triangles - by author
[D25] Double curved arrangement six triangles - by author
[D26] Increasement of double curvature over the span of the arch by alternating angle at which triangles meet - by author
[D27] Continuity in double curved structure - by author
[D28] Discontinuity in double curved structure - by author
[D29] Varying curvature measures based on triangulation method - by author
[D30] Three parameters in triangulation method - by author
[D31] Continuous structure with varying triangle sizes - by author
[D32] Continuity in a triangulated structure - by author
[D33] Continuous curve in topology of varying geometric elements.

E. Two-dimensional bending - Bending curvature

[E1] [Three dimensional surface description], by Beranek, 1979, reprinted from K3 dictaat ruimtelijke constructies deel 2
[E2] Mohrs circle, curvature relation between bending and torsion - by author
[E3] Curve deformation with distance ‘w’ as displacement]. by Beranek, 1979, reprinted from K3 dictaat ruimtelijke constructies deel 2.
[E4] Planar surface. k1*k2 - by author
[E5] deformed planar surface. k1’*k2’ - by author
[E6] Bending deformation plate - by author
[E7] Bending plate front view - by author
[E8] Sphere segment, radius calculation - by author
[E9] Input principle radius 1, Output principle radius 2 - by author
[E10] Material properties and measure results - by author
[E11] Anticlastic deformation measure - by author
[E12] Input and output radius - by author
[E13] Torsional deformation of planar surface - by author
[E14] Bending deformation of planar surface - by author
[E15] Form conversion from planar surface to anticlastic surface through bending - by author
[E16] Form conversion from planar surface to anticlastic surface through torsion - by author
[E17] Torsional deformation of planar surface - by author
[E18] Measure of anticlastic curvature, bending and torsion - by author
[E19] Bending and torsion combined - by author
[E20] Planar mesh subject to bending and torsion. Slightly curved for form finding pruposes - by author
[E21] Deformation plots side - by author
[E22] Deformation plots - by author
[E23] Princple stress plots - by author
[E24] Front view deflection - by author
[E25] side view defelction - by author
[E26] Structural analysis torsional deformation - by author
[E27] Structural analysis bending and torsional deformation. Magnitude of stresses - by author
[E28] Structural analysis torsional deformation - by author
[E29] Structural analysis torsional deformation one - by author
[E30] Structural analysis torsional deformation with plate spacer - by author
[E31] Structural analysis torsional deformation with plate spacer, top view - by author

F. Design variables - Paramteres active bending plate arch

[F1] Design variables arch structure - by author
[F2] Distorted deformation due to clamped plate supports - by author
[F3] Torsional angles of respectively 0, 90 and 45 degrees - by author
[F4] 45 and -45 degrees twist at supports - by author
[F5] Cantilever deformation arch as a result of torsional displacement - by author
[F6] Tension cable connection between two opposite arches - by author
[F7] Fixation connection between two arches leaning against each other - by author
[F8] Contradiction between form funding purposes and structural capacities - by author
[F9] Four varying Glass fibre layup - by author
[F10] Naked glass fibres layup - by author
[F11] Behaviour bending and torsional deformation varying fibre directions - by author
[F12] Material selection diagram - by author

G. Structural analyses - structural performance bent and twisted arch

[G1] Physical model simulation coupled arches - by author
[G2] Physical model simulation four coupled arches - by author
[G3] Wind simulation horizontal windload angles - by author
[G4] Wind simulation vertical windload angles - by author
[G5] Wind simmulation front side through seven wind pressure angles - by author
[G6] Wind simmulation back side through seven wind pressure angles - by author
[G7] Wind simmulation top through seven wind pressure angles - by author

[G8] Wind simmulation back side, right view - by author
[G9] Wind simmulation front side, right view - by author
[G10] Cable arch strengthning hybrid structure - by author
[G11] Planar plate with slight curvature prior to deformation process - by author
[G12] Pure bending deformation - by author
[G13] Bending and torsional deformation - by author
[G14] Principle stress lines form creation process - by author
[G15] Structural material properties - by author
[G16] Internal stresses form creation process - by author
[G17] Loadcases arch - by author
[G18] Internal stresses form creation process + membrane pointload - by author
[G19] Internal stresses form creation process + wind pressure - by author
[G20] Vertical and horizontal deflections all laodcases - by author
[G21] Nodal reacion forces all loadcases for relative comparison - by author
[G22] Deflection diagrams all load cases - by author

H. Building methodology - application and feasability

[H1] Planar strip development - by author
[H2] Form conversion steps - by author
[H3] Coupling arches - by author
[H4] Torsional adjustments for arch alignment - by author
[H5] Peak stresses at coupling location two arches - by author
[H6] Space enclosing membrane placement - by author
[H7] Four generic space enclosure membrane shapes - by author
[H8] Lineair membrane enclosure shape one and two - by author
[H9] Lineair membrane enclosure shape three and four - by author
[H10] Normal vector direction two arches - by author
[H11] perpendicular normal vector two arches - by author
[H12] Tension cable arch - by author
[H13] Water flow mebrane shape one - by author
[H14] Arches with membrane space enclosure shape one - by author
[H15] Water flow mebrane shape two - by author
[H16] Arches with membrane space enclosure shape two - by author
[H17] Water flow mebrane shape three - by author
[H18] Arches with membrane space enclosure shape three - by author
[H19] Water flow mebrane shape four - by author
[H20] Arches with membrane space enclosure shape four - by author
[H21] Clamped plate connector - by author
[H22] Exploded view clamped plate connector - by author
[H23] Arch support principle building sequence - by author
[H24] Arch support free rotational motion - by author
[H25] Arch support with rotational motion freedom sequence - by author
[H26] Gliding mounts system - by author
[H27] Gilding mount locking system - by author
[H28] Triangle shaped arch support - by author
[H29] Rail extension for bending deformation sequence one - by author
[H30] Rail extension for bending deformation sequence two - by author
[H31] Rail extension for bending deformation sequence three - by author
[H32] Bending procces - by author
[H33] Rotational motion over gliding rails - by author
[H34] Exploded view gliding mount - by author
[H35] Bending and torsional deformation sequence - by author
[H36] Adjustment freedom within support frame four alignment of two arches - by author
[H37] Edge coupling element at top of arches - by author
[H38] Adjustment position arch within support system - by author
[H39] Edge coupling element - by author
[H40] Coupled arches - by author

[H41] Support expanding sequence - by author
[H42] Coupled support with four arches connected with multipurpose middle rail - by author

I. Design application

[I1] Design intent linear array, front - by author
[I2] Design intent linear array - by author
[I3] Design intent linear array, front - by author
[I4] Design intent linear array - by author

J. Detailing, design freedom and future scenarios

[J1]Design freedom plate positions minimum and maximum - by author
[J2] Support element plate position 40 degreees - by author
[J3] Deesign freedom plate positions - by author
[J4] Plate shape design freedom positions - by author
[J5] 360 degrees rotational freedom plate element - by author
[J6] Gliding mount - by author
[J7] Combined support elements - by author
[J8] Single support element section - by author
[J9] Clamb to plate - cable connector - back view - by author
[J10] Clamb to plate - cable connector - side view - by author
[J11] Connection elements two plates
[J12] Bending and torsion sequence into connection element
[J13] Structures for remote & fragile building sites
[J14] Kinetic architecture pavilion - closed
[J15] Kinetic architecture pavilion - open
[J16] Arch bridge structure
[J17] Arch bridge design
[J18] Upscale and randomisation
[J19] Entire active-bending building envelope enclosure