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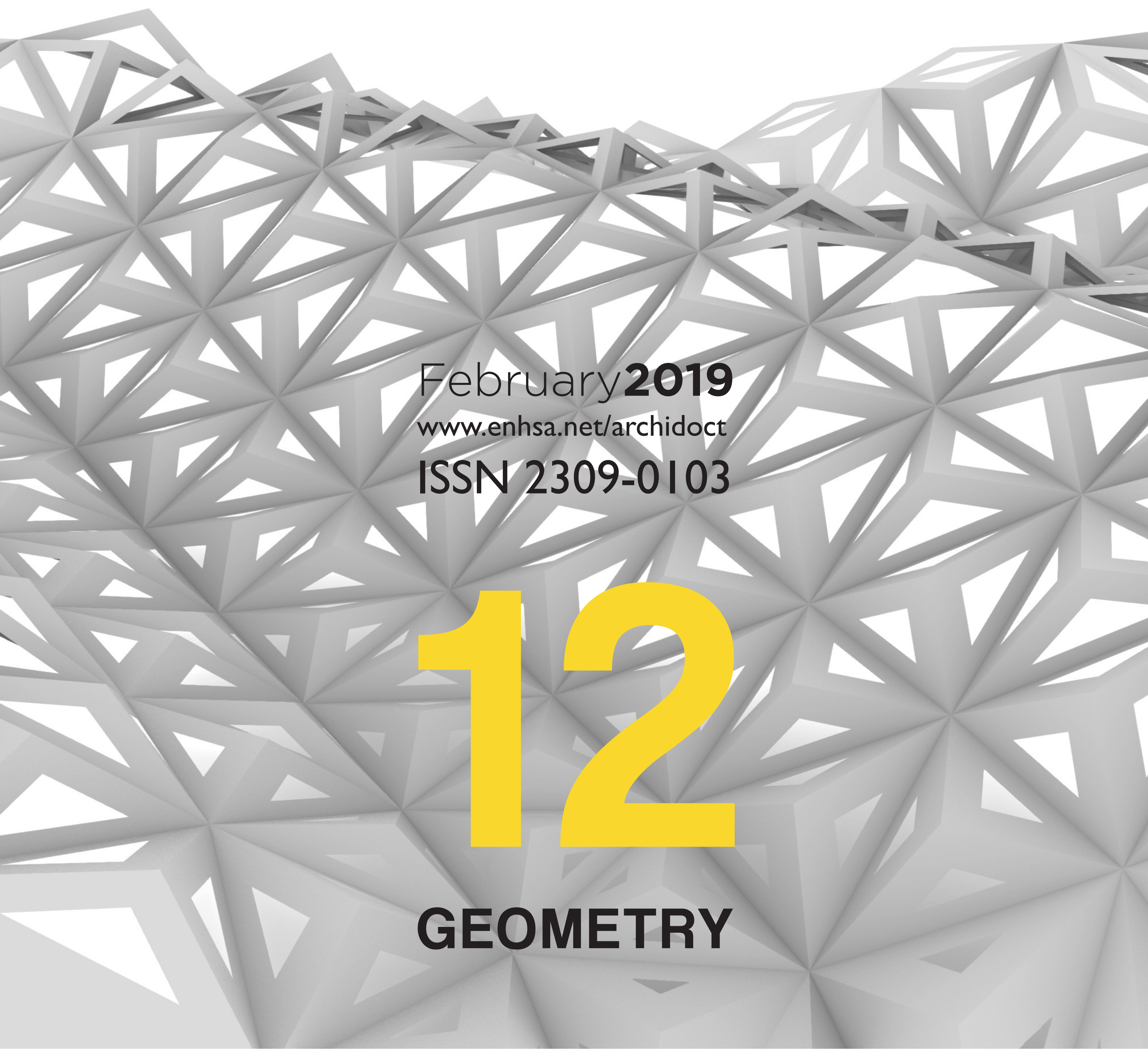

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# Programming Flat-to-Synclastic Reconfiguration

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## Abstract

*Advances in architectural geometry make free-form architecture explicitly definable and economically manufacturable. Enhancing the efficiency of fabrication, this research investigates strategies of translating free-form synclastic surfaces to flat pre-programmed reconfigurable mechanisms. The presented bi-stable mechanisms are produced by creating voids on flat materials. In such mechanisms, the generated blocks are outlined by the voids that are connected by the hinges. The position and the orientation of the hinges allow the blocks to rotate around each other, and then reconfigure from flat to synclastic. During the reconfiguration process, the blocks are temporarily deformed. As the elasticity brings the blocks back to the original dimensions, the materials reach the second stable states. Distribution of hinges on the flattened surface needs to be designed according to certain geometric constraints. This paper demonstrates the workflow of identifying the positions of the hinges. The developed methods are validated through prototypes such as a spherical surface and a free-form synclastic surface.*



## Keywords

bi-stable mechanism; auxetic mechanism; flat-to-curved reconfiguration; programmable material; discrete differential geometry

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## 1 Introduction

The emerging demand for bespoke double-curved surfaces challenges designers and manufacturers. Utilizing sheet materials to produce curved-surfaces is an economical solution, given that such materials are easy to be industrially produced and processed. Exploiting economic benefits of sheet materials, numerous researchers investigate how to decompose a free-form surface to planar components on architecture scale (Pottmann, 2013). Meanwhile, various advanced approaches have been proposed to transform sheet material to double-curved surface, and pilot prototypes have been produced on laboratory scales, such as paper origami (Tachi, 2013), reconfigurable prestressed composite (Aldinger, Margariti and Suzuki, 2018) or deployable auxetic shells (Konakovic-lukovic, Konakovic and Pauly, 2018). These approaches make materials reconfigurable, yet leave them vulnerable to bending stresses.

This paper proposes an approach that allows the reconfigured double-curved surface to resist bending stresses. The synclastic surfaces can be produced by introducing voids or slits on flat sheet materials. By either squeezing or expanding the materials, which closes the voids or opens the slits, then the sheet materials can be reconfigured into curved states (Figure 1). The voids creates gaps between the edges of blocks, while they remain interconnected via the tilted compliant hinges on the vertices. Additionally, these hinges allow the bending stresses to transmit across the blocks.

Geometrically, the major challenge lies in how to identify the set of hinges that can allow the blocks to be reconfigured from flat to curved state without residual strain or permanent deformation. Such hinges enable the material to stably maintain the desired shape instead of resuming its initial configuration. Figure 2 illustrates an overview of the methods, which are explained in the following sections.

### *1.1 Outline*

In the following sections, relevant research and mechanisms are reviewed in section two. Section three introduces the mechanical principles and the geometrical features of the reconfigurable mechanism. With these fundamental insights in mind, section four explains how to employ the explored approaches to transform a synclastic surface into its flat configuration. For validation, pilot prototypes have been produced, and the production is reported in section five. Consequently, features of the current method and future works are summarized in section six.

## 2 Background

Since the 2000s, the demand for free-form architectures has gradually increased (Pottmann et al., 2015). The most effective way to build a large free-form surface is considered to be decomposing the curved surface into a series of flat panels (Pottmann et al., 2007). Although a considerable amount of unique components will be generated in the design process, the numerically controlled machinery can economically produce all the components from either 2D sheet materials (e.g., steel plates, float glass) or 1D profiles (e.g., steel tubes, extruded aluminum). However, the assembly of all the components is still a labor-intensive and challenging task for builders. Introducing bi-stable mechanism, the research aim is to develop a fabrication method of flat materials that can be reconfigured into the target curved states. With this objective, the research is built around the premises that such mechanisms can make the assembly process more efficient and less labour-intensive.

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In recent years, numerous ways have been proposed to deliver flat-to-curved reconfiguration for fast deployment. The reconfigurable systems may include embedded actuators or focus on the mechanisms to be actuated. In the first category, there are two distinct approaches. Some researchers control the reconfiguration through stacking materials with different expansion rates that create a composite system which respond to moisture or temperature changes (Tibbits, 2014; Reichert, Menges and Correa, 2015; van Manen, Janbaz and Zadpoor, 2017). During the ambient changes, the layered materials expand unevenly, cause curvatures on the composites. Meanwhile, some other researchers deposit stiff components on pre-tensioned membranes (Guseinov, Miguel and Bickel, 2017; Aldinger, Margariti and Suzuki, 2018). Once the pre-tensioning is removed, the contracting membranes actuate the composites to the curved configurations.

On the other hand, the research in the second category concentrates more on the mechanism to be actuated. The researchers investigate how to arrange the flexible joints to permit mechanisms to be reconfigured into desired shapes. In the research pursued by Konakovic et al. (2016), the sheet materials are homogeneously cut into triangle panels, and the connections between the triangles are considered as ball joints. Then, the material can be stretched and bent into various free-form shapes. In the cases of origami and kirigami explored by Tachi (2013) and Liu et al. (2018), all the components are connected by linear hinges, or the crease lines, laying in the plane of the sheet material. The sizes and shapes of the components are informed by the desired curved surface. The difference between these two approaches is that origami forbids the designer to cut the sheet materials while kirigami allows one to do so. In these approaches, the ball joints and the linear hinges make the flat materials pliable. This attribute makes the products, in their target configurations, incompatible with bending stresses.

In contrast to arrange linear hinges in the plane, Haghpanah et al. (2016) place the hinges vertically and produce another type of mechanism. The mechanism consists of multiple units that can be sequentially expended to other configurations and stably maintain the shapes. The features is termed bi-stability. Although both the initial shape and the reconfigured shape are confined in the plane, it suggests that the shape reconfigurable mechanism can also work on thick materials, which promise certain bending resistance.

A revised bi-stable mechanism is proposed in which the linear hinges are arranged in various orientation in the thickened sheet that allows one of the stable states to be on a plane and the other on a double-curved surface (Chiang, Mostafavi and Bier, 2018). However, the actuation of each bi-stable unit can happen independently or in sequence, which causes challenges of actuating mechanisms with in the multiple units. Figure 3 recapitulates the classification of the state of the art in reconfigurable mechanisms.

In this research, the approach of tilted hinges on thick materials is adopted. proposing a more applicable reconfibration method that avoids sequencial actuation, the approach is integrated into the bi-stable auxetic mechanism proposed by Rafsanjani and Pasini (2016). Auxetic mechanism (i.e., a mechanism shrinks in all directions when it is only compressed in one direction) can distribute actuating force to the whole system and activate all the reconfigurable units at once. Introducing tilted cutting in design and production process, the bi-stable auxetic mechanism can be compatible in transforming from flat to double-curved.

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**Figure 1.**

Two examples of the proposed programmable material. Both of them can be manufactured in the flat configuration (left column). By either compression (top row) or tension (bottom row), the material can be mechanically activated and then rests at the curved configuration (right column).

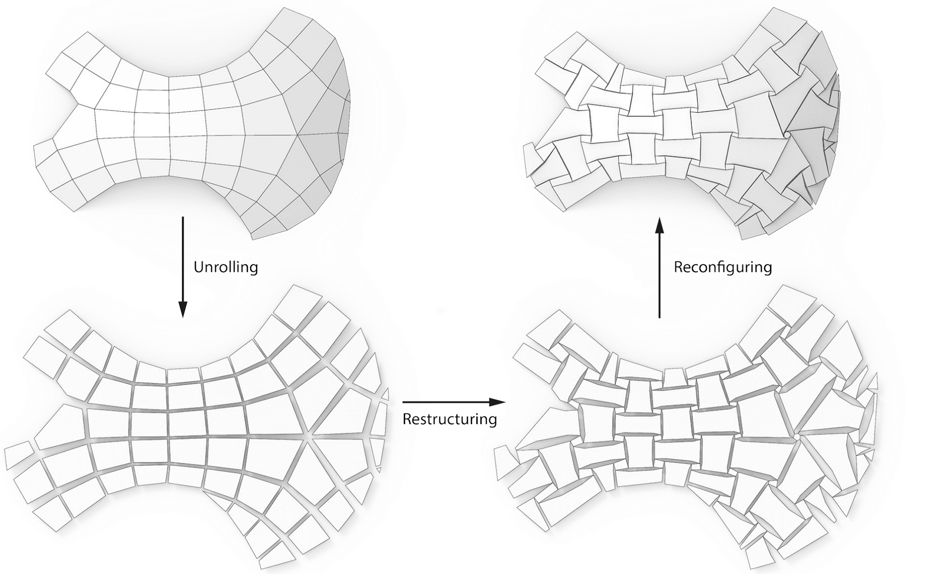


**Figure 2.**

The proposed workflow of the design process. a. The quadrilateral conical mesh. b. Unrolled panels. c. Restructured panels with connectors. d. The free-form synclastic surface formed by the reconfigurable mechanism.

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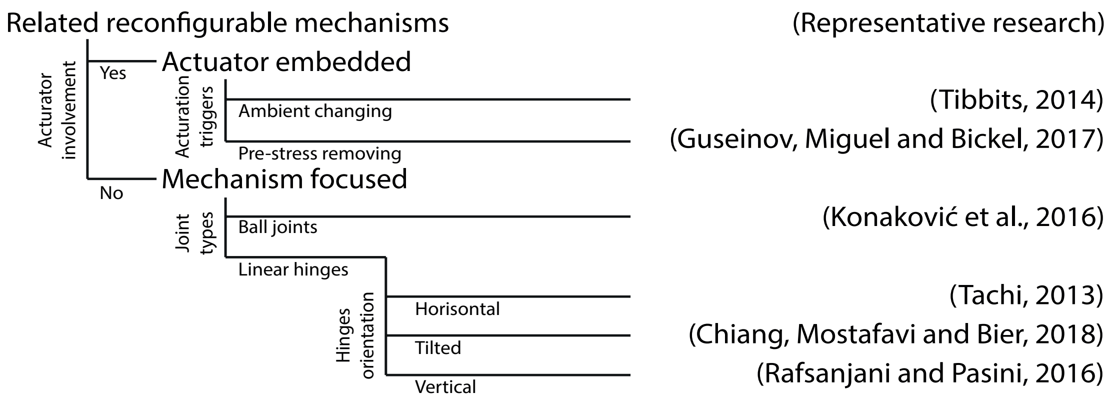


**Figure 3.**
The classification of related reconfigurable mechanisms

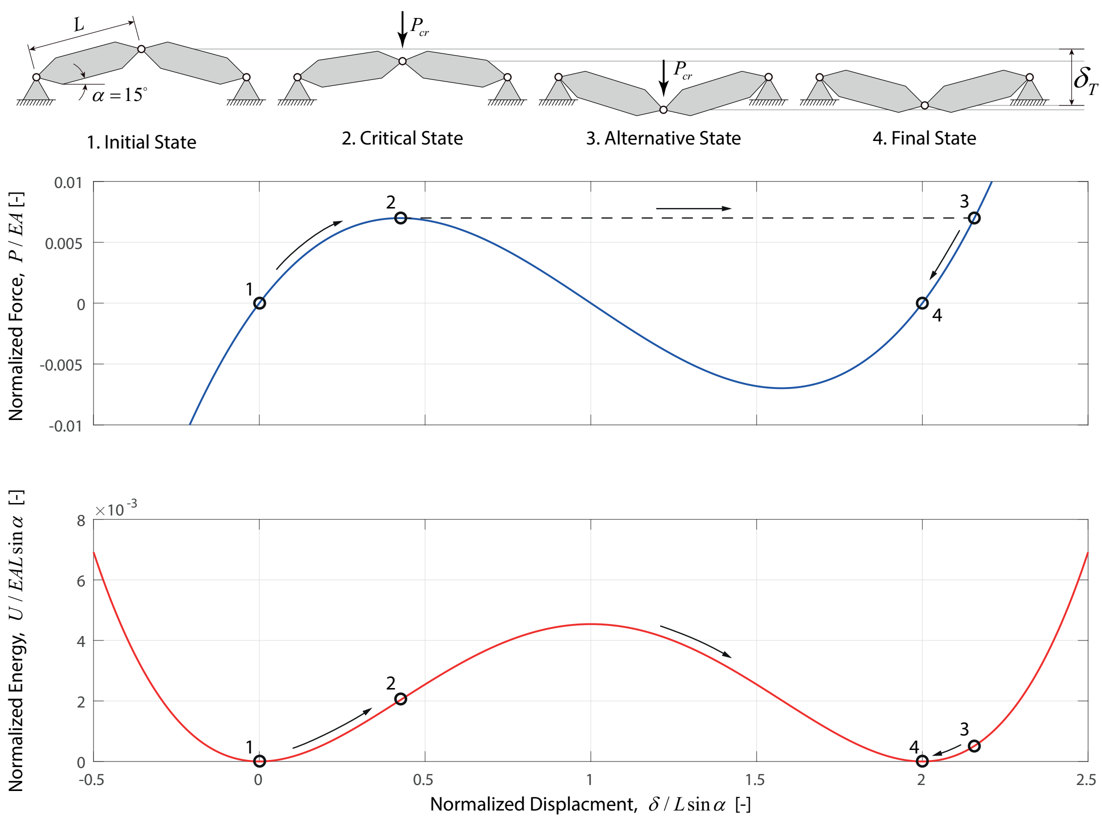


**Figure 4.**
The reconfiguration process of the idealized bi-stable mechanism with two elements.

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## 3 Principles of bi-stable reconfiguration

The bi-stable mechanism is also termed as snap-through buckling (Huang and Vahidi, 1971). The term refers to the features that when the mechanism is switched from one stable state to another, the mechanism snaps-though. This section reviews the mechanical features of snap-through buckling, and introduces the principles of designing a snap-through mechanism capable of spatial reconfiguration.

One of the simplest in-plane bi-stable units is illustrated in Figure 4, which consists of two linear structural members (with cross-sectional area *A* and material elastic modulus *E*) hinged to each other at one ends and pinned to the supports at the other ends. When an external force applies at the middle hinge, the two members are compressed and inclined. Until the external force exceeds the critical load, the reconfigurable unit will suddenly deviate from the critical state to the alternative state. After the external force is removed, the elasticity of the material brings the mechanisms back to the relaxed length, which leads the unit to the final state. The total displacement of the middle hinge during the reconfiguration follows

$$\delta\tau = 2L * \sin a \qquad (1)$$

which suggests that the total displacement $\delta\tau$ is proportional to the rotating arm $L$ and the sine of the tilting angle $a$.

Form the energy point of view, the stable states are corresponding to the local minimum points of the energy-displacement graph. When the hinges are ideally dissipating and storing no energy, the stable states are the mirror images of each other. In this paper, all the hinges are assumed to behave ideally, in order to design bi-stable mechanisms with geometric principals.

To make the mechanisms capable of conducting spatial reconfiguration, the hinges are designed to be not parallel to each other. As illustrated in the top row of Figure 5, given that three rigid blocks interconnected with ideal hinges and the four anchors are coplanar, the other stable configuration would be the mirror image against the plane defined by the four anchors. Between the two configurations, the centre blocks rotate around the dash-dotted line. To physically made a mechanism resemble the ideal case, the material must be thickened to approximate the stiff blocks while the compliant hinges must be notched to minimize the strain energy it might store.

To be noted that due to the hinges are not parallel to each other, the connecting blocks (i.e., side blocks) have different rotating arms at the top and bottom surfaces, which result in different displacements at the two surfaces. To be more precisely, the magnitude of the rotating arms are proportional to the distances from the rotation axis, so as the displacement. In the case shown in Figure 5, there are no residual stresses in the blocks. Only if the hinges are compliant hinges, some stresses will occur locally at the compliant hinges.

This section has described the temporary elastic deformation during the reconfiguration. After the bi-stable unit sets in the stable states, the deformation dissolves. Nevertheless, given that the hinges store negligible stain energy, the two stable states are simply mirror images of each other against the plane defined by the corner anchors. The next section introduces the method for applying this spatial reconfigurable unit to synclastic surfaces.

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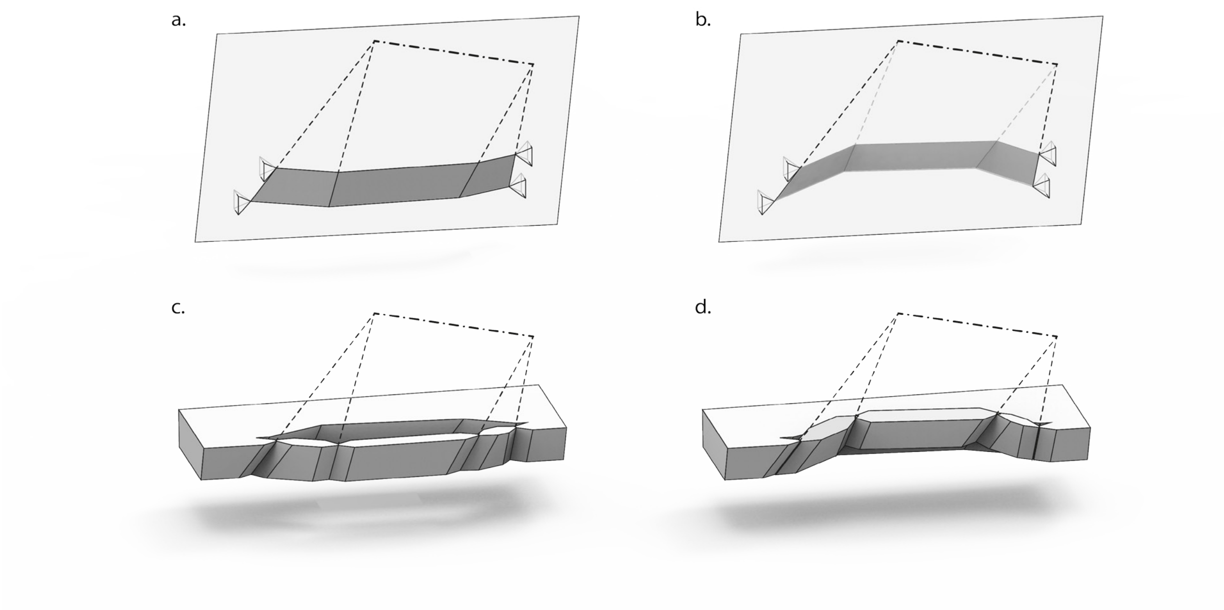


**Figure 5.**
The applied spatial bi-stable unit. a. and b. are the two stable states of a conceptual bi-stable unit. They are mirror images of each other against the mirror defined by the four anchors. c. and d. are the two stable states of the corresponding physically manufacturable bi-stable unit. The dashed lines are the extension of the hinges. They intersect at points defining the dash-dotted line which serve as the virtual rotation axis of the center piece.

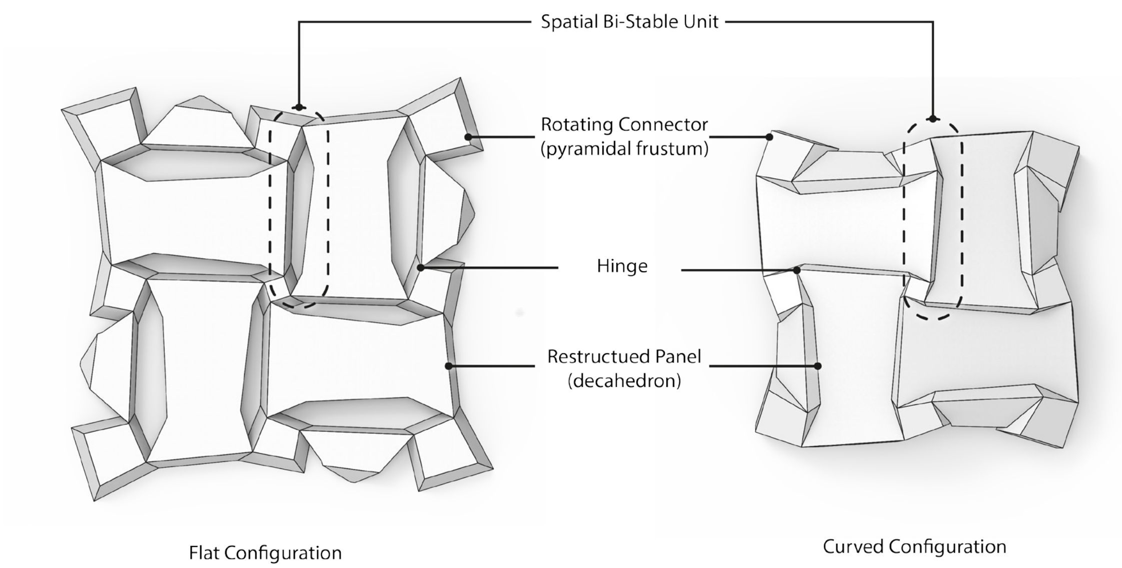


**Figure 6.**
The reconfigurable material consists of two types of blocks: rotating connectors and restructured panels. The blocks are interconnecting each other with hinges.

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## 4 Geometrical design processes

### *4.1 Basic elements in the proposed mechanism*

As demonstrated by Rafsanjani and Pasini (2016), the bi-stable auxetic mechanism can be achieved by arranging rotating quadrilaterals around concave octagons for in-plane reconfiguration. To create a flat-to-curve mechanism, this paper revises two approaches of the previous research. The homogeneously repeated pattern and the perpendicular cutting are replaced by a heterogeneously graded pattern and tilted cutting. In this way, the thick sheet material can be reconfigured from flat to doubly curved.

Figure 6 illustrates how to build up the proposed mechanism with the spatial bi-stable units. Basically, every spatial bi-stable unit defines a hexagonal void in the sheet material, and the void can be closed in the other configuration. Between the hexagonal voids, there are the rotating connectors and the restructured panels, which are discussed in section 4.3 in more detail. Additionally, the rotating connectors were prisms in the case presented by Rafsanjani and Pasini (2016). Here, the connectors are rendered as pyramidal frustums. The different lengths at the top and the base cause different displacements as suggested by equation (1). In the case depicted in Figure 6, the bottom surfaces with larger rotating arms introduce larger displacement during the reconfiguration and deliver the desired curvature without troubling bending stresses, which are commonly observed in other formative manufacturing processes, e.g. cold forging.

To arrange the hexagonal voids, the designers have to consider the voids as an interrelated system rather than multiple independent units. Given the fact that each hinge affiliates to two hexagonal voids, the two units have to agree on the position and the orientation of the shared hinges. Furthermore, the rotation angle of the hinge is also shared by the two voids. However, there is an outward method to design the interrelated voids, for an arbitrary conical synclastic mesh; in other words, no recursive iteration is required. In the following sub-sections, a workflow is proposed to locate such solutions of the synclastic surfaces. The process starts with a conical mesh, which guarantees that the solution exists.

### *4.2 Unrolling an arbitrary synclastic conical mesh*

This research designs the bi-stable auxetic mechanisms from conical meshes following the recommendation from Chiang, Mostafavi and Bier (2018). In conical mesh, each node has an axis intersected by all the bisector planes of the dihedral angles between surrounding facets. For detail concerning the definition and the features of conical mesh, readers are referred to the paper presented by Liu et al. (2006).

Here, a method for unrolling a synclastic conical mesh is proposed and demonstrated of flattening a mesh to make all the normal vectors of the facets point up. The targeted normal vector can be expressed as $(1, 0, 1)$ in Cartesian coordinates or as $(1, 0, \varphi)$ in spherical coordinates, where the vector has a unit length, a zero polar angle, and an azimuth angle $\varphi$ that can be any real number. Let $\vec{n}_{P,i}^{\ c}$ be the normal vectors of panels in the curved mesh (where $P$ stands for panels and $c$ for the curved mesh), which can be expressed as $(1, \theta_i, \varphi_i)$ in spherical coordinates. Here, a "Neutral Surface" is proposed, which is defined that all the normal vectors of the mesh panels on the neutral surface have half as much polar angles as their corresponding panels have. As a result, the corresponding normal vectors $\vec{n}_{N,i}^{\ c}$ (where $N$ refers to the neutral surface) equal to $(1,\theta_i /2, \varphi_i)$. Regarding the position of the neutral surface, the distance between it and the mesh can be arbitrarily decided. Once the distance is set, the vertices of the neutral surface can be determined by

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the extensions of the nodes' axes.

To unroll the conical mesh, the neutral surface would be turned concave-side convex, or be turned inside out (or be mirrored against the horizontal plane). Therefore, the normal vector of the flipped neutral surface $\vec{n}_{N,i}^{\ f}$ (where $f$ stands for both flipped and flattened) becomes $(1,-\theta_i/2, \varphi_i)$ . Before and after the unrolling, the facets of the neutral surface are turned $-\theta_i$ in total. Let every mesh panel of the curved conical mesh be turned as the corresponding facet on the neutral surface does, which means that the inclination of the mesh panel will also be turned $-\theta_i$. Then the normal vectors of the unrolled panels will be $\vec{n}_{P,i}^{\ f} = (1, 0, \varphi_i)$. During the flipping of the neutral surface, the rotation angles make the normal vectors point upright as desired. The unrolling process is illustrated in Figure 7. Unrolling a synclastic conical mesh via such a neutral surface guarantees a smooth journey to locate the legitimate hinges. The application of the unrolling method is also demonstrated in Figures 12 & 13.

The term neutral surface echoes the neutral plane in the conventional bending theory. In the bending, all the lengths on the neutral plane are preserved, which means that there is no compression or tension. While the material in one side of the neutral plane get either compressed or tensioned. A similar feature can be observed in the reconfiguration process, as shown in Figure 10. The material above or below the neutral surface reconfigures to the curved state by either stretching or contracting.

*4.3 Connecting panels with rotating pyramidal frustum*

The primary task of this section is to locate the legitimate hinges, which dictate how the blocks rotate around each other. Ideally, the hinges should bring all scattered nodes in the flat configuration back to the same position in the curved configuration. Figure 8 shows a set of functional hinges (dashed lines) that merges the panels and closes the gaps.

There are only three independent degrees of freedom for a node to design the hinges, considering the hinges and the rotating have to coordinate with each other. In general, there are five degrees of freedom in a rotation in 3D space. Two of them are the position of the rotation axis, two of them are the orientation of the axis, and the other one is the rotation angle. For a unrolled node (as shown in Figure 8), the rotation axes should pass through the node on the neutral plane which fixes two of the degrees of freedom for every panel. If one of the panel is assigned with the three undetermined, all the other panels have to rotate dependently to the first panels. Therefore, there are only three independent degrees of freedom; two of them can be regarded as the orientation of the merged axis (black dotted line in Figure 8), and the other one is the magnitude of the rotation angle.

Once the hinges are determined, the intersection points of the hinges and the mesh panels define a polygon. For a four-edge node, the polygon is quadrilateral. When the sheet material has a certain thickness, the quadrilateral turns out to be a frustum of a quadrilateral pyramid. The gaps between the mesh panels have to be restructured accordingly as shown in Figure 8.

As discussed, for each node, there are three independent degrees of freedom to define the rotating connectors. But the nodes on the same edge of the mesh still have to agree on the inclination of the gap in the flat configuration. In other words, for a mesh with $n$ nodes and $m$ edges, there are $3n-m$ degrees of freedom to be determined for all the rotating connectors. One way to omit the iteration is to determine the rotating connectors node by node. A node that is determined later has to align itself to the previously determined nodes. Therefore, only the first node has three degrees

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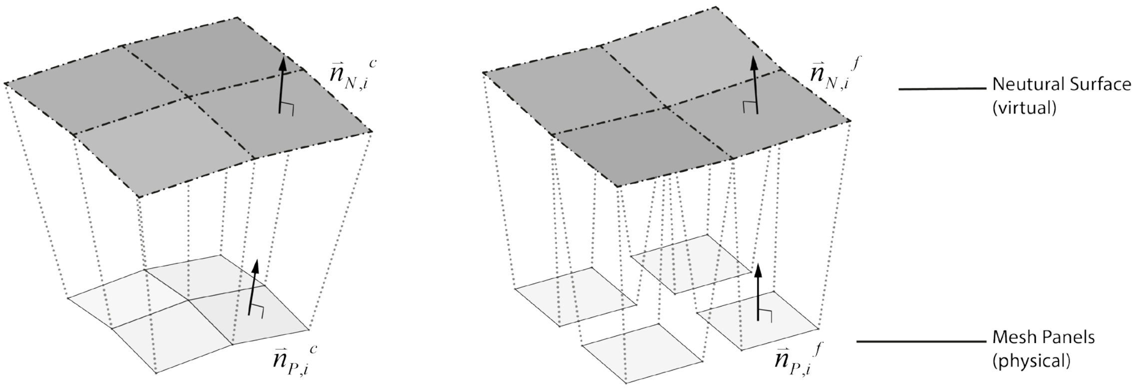


**Figure 7.**
The axes (dash-dotted lines), which the mesh panels rotate around, form the "Neutral Surface." The inclination of the neutral surface is half of the corresponding mesh panel. During the reconfiguration, the neutral surface is turned inside out, which suggest that the inclination angles are opposite to the original. Then the corresponding mesh panels will follow the rotation of the neutral surface and turned to be zero inclination in the end.

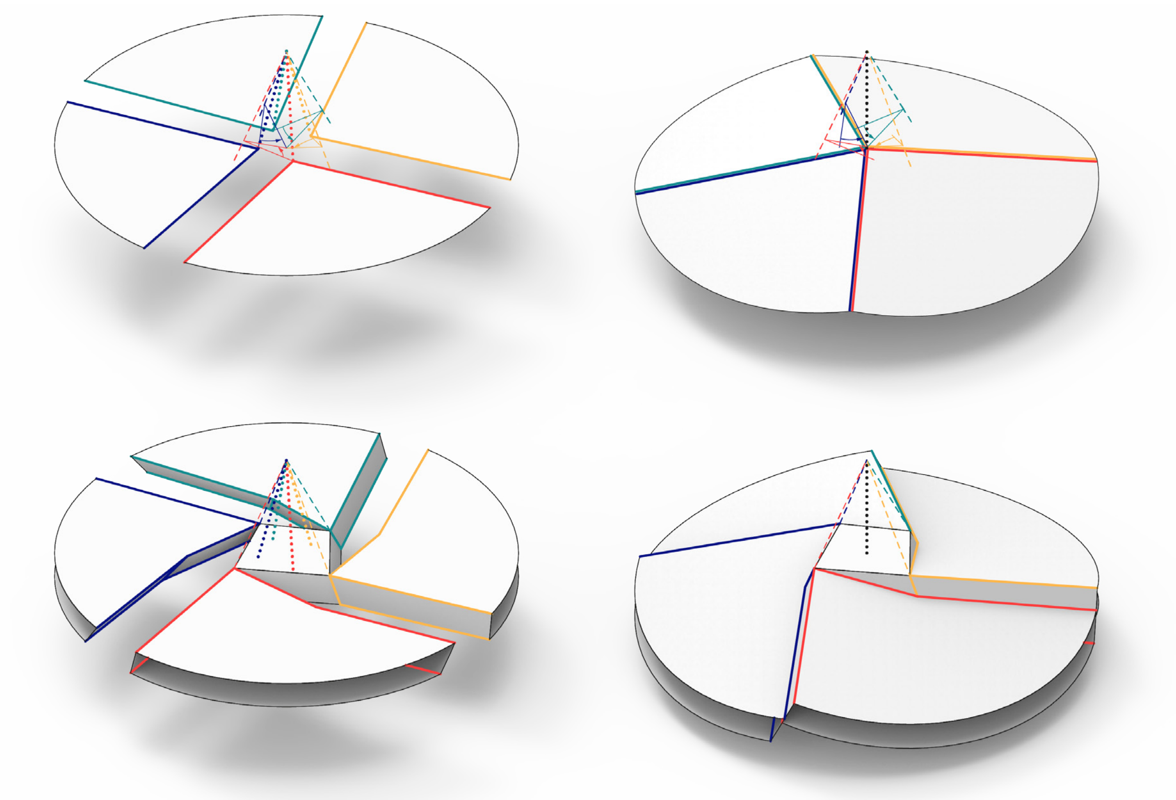

**Figure 8.**
Close up on panels rotated around the hinges (dashed lines). Each panel has its hinge (in the same color), while all the panels merge at the same axis (the center black dotted line).

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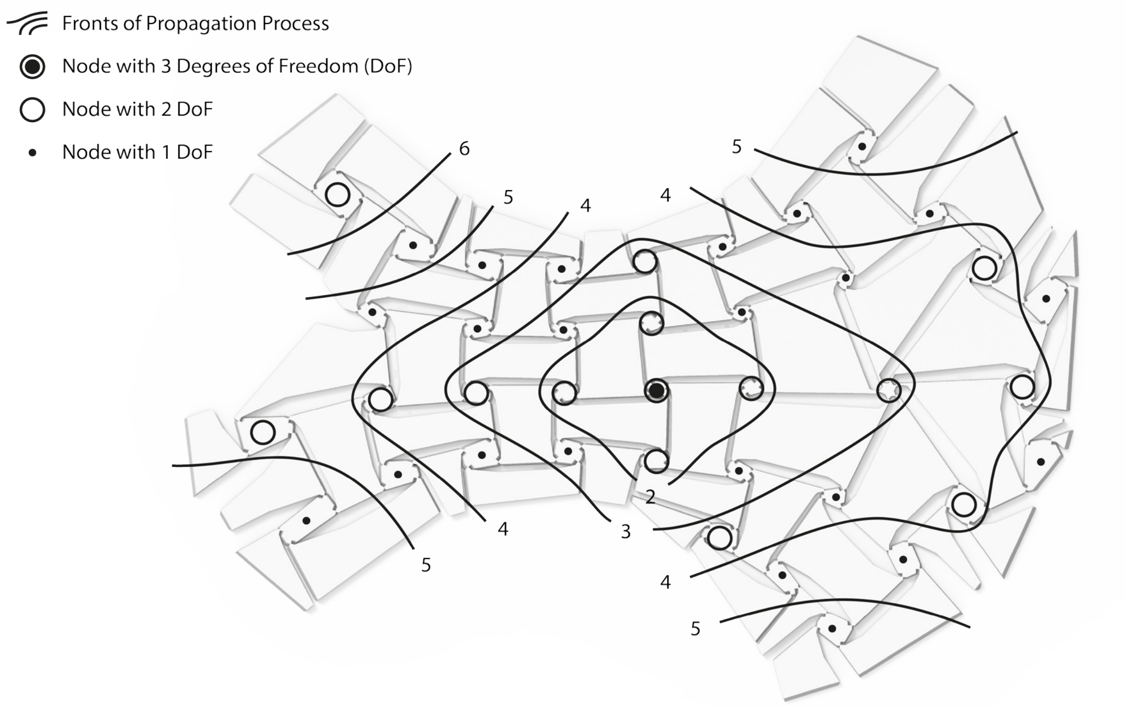


**Figure 9.**
The propagation map of the free-form conical mesh.

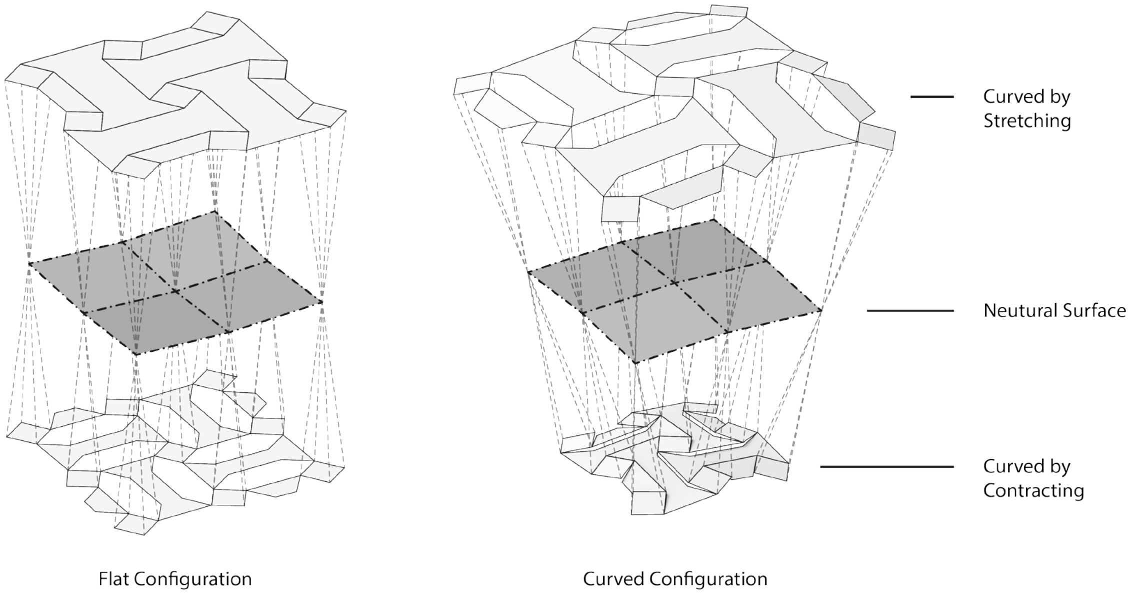


**Figure 10.**
The focal points of the hinges locate on the neutral surface. The neutral surfaced is mirrored during the reconfiguration, which means that all the length on the neutral surface remains constant before and after the reconfiguration. On the other hand, above and below the natural surface are expanding the zone and contracting zone respectively.

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of freedom. Other nodes may have two or only one degree of freedom, depending on how the determination propagate. Figure 9 shows an example of how the process propagates throughout all the nodes of a free-form mesh. So far, this section has presented the methods to unroll a synclastic conical mesh, to locate the legitimate hinges, and to restructure the mesh panels. Figure 10 visually recapitulates the relationship between the neutral surface and the legitimate hinges. It is noteworthy that if hinges are extended to the other side of the neutral surface, a solution of curved by stretching can also be identified.

## 5 Prototyping with 5-axis CNC waterjet

The previous sections have introduced the mechanical property of the proposed spatial bi-stable unit and the method of applying such units to form an auxetic mechanism. In this section, the discussion is focusing on how to transform the geometrical solution to physically producible cutting tool path.

To provide the general idea, Figure 11 shows a glimpse of the process, modifying a geometrical solution to a producible solution, which leads to the waterjet cutting path. To make the design physically producible, the hinges must have physical widths. On the contrary, the hinges are regarded as mathematical lines with zero widths, in the previous geometrical analysis. To make the hinges have a physical width, the process explained by Figure 5c & 5d is adopted, thickening the bi-stable units and notching the compliant hinges. The 200mm-by-200mm prototype shown in Figures 11 & 12 is made from 4mm thick polypropylene sheet with 5-axis CNC waterjet machine, and the width of the compliant hinges is set to be 0.8 mm after a few trial and error tests. Polypropylene is a flexible and resistant to fatigue. These material properties make the prototype repeatedly reconfigurable.

The proposed workflow has been validated on a spherical surface. The design and analysis methods have also been applied to a free-form surface (Figure 13). The production with a 5-axis waterjet is under the arrangement.

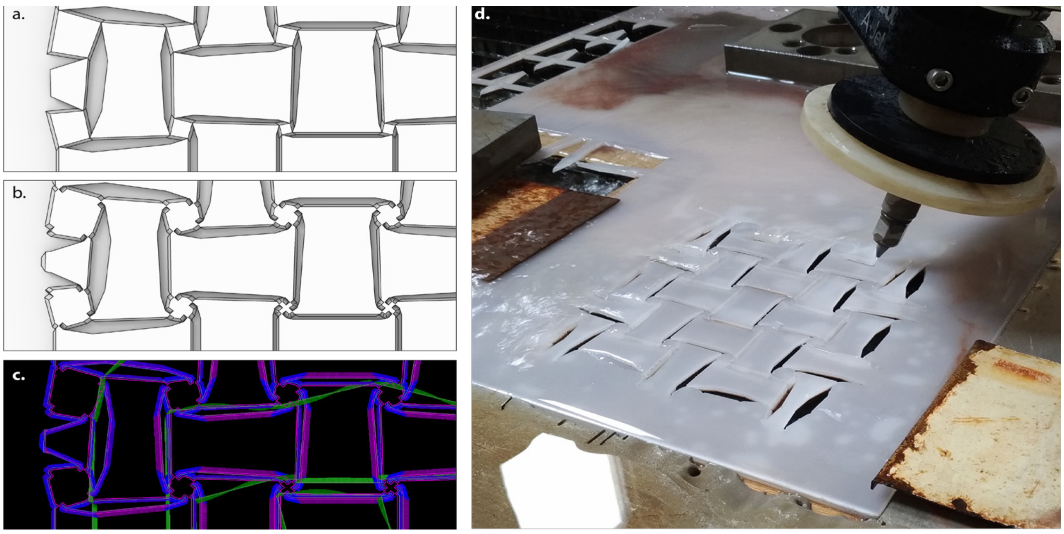


**Figure 11.**

The production process and the result of a 25-panel spherical surface. a. The close up of the geometrical solution. b. The producible solution. c. The tool path for waterjet cutting. d. The production with a 5-axis CNC waterjet machine.

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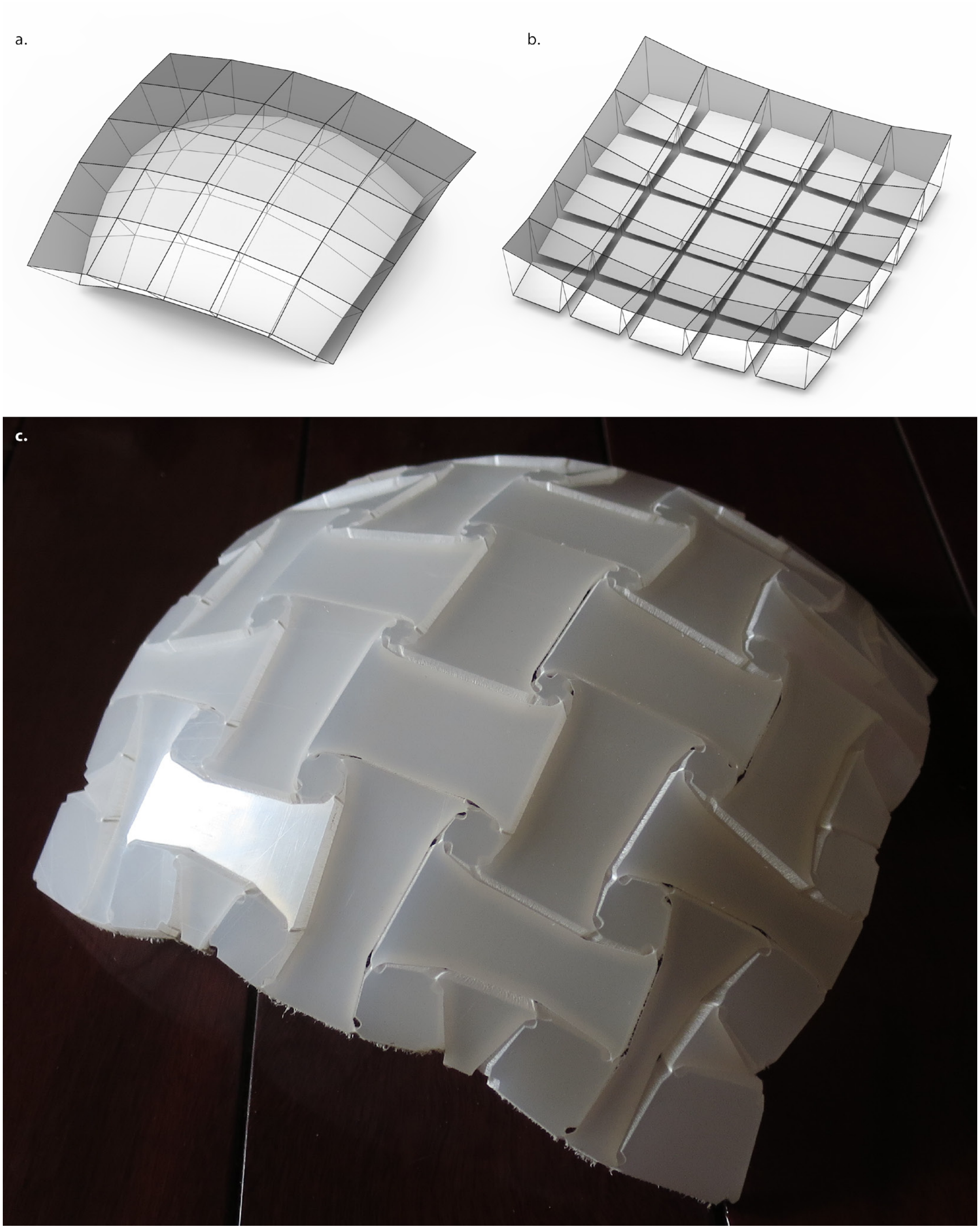


**Figure 12.**
The process of designing the spherical surface. a-b. Unroll the mesh panels with the Neutral Surface. c. Physically produced results.
The video of the flat-to-curved reconfiguration process can be accessed via https://youtu.be/KvPXyMupNOA.

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## 6 Conclusion and future works

This paper has presented a method to translate free-form synclastic conical meshes to cutting patterns on sheet materials. Then, the cutting patterns enable the sheet materials to be transformed into the desired shapes when the mechanisms are mechanically activated. The activation can also be pre-designed as either stretching or contracting. The design processes of the cutting patterns consist of three steps. In the first step, unrolling the synclastic conical mesh with the proposed neutral plane automatically introduces the gaps with appropriate widths. In the second step, the proposed frustum connectors can automatically distribute different reconfiguring displacements at top and bottom surfaces. With these two steps, there is neither bending stress nor residual strain in the blocks. In the third step, the geometrical solution is revised into the producible solution for 5-axis waterjet cutting machine. Due to the hinges are compliant hinges in this production method, there will be local strains at those hinges.

To investigate the capacity of the proposed mechanisms, some future works have to be continued. So far, neither the applicability on larger scales nor the dynamic behavior of the mechanism during the reconfiguration has been explored yet. Additionally, the proposed neutral plane are not compatible with anticlastic surfaces. Methods to unroll an anticlastic surface are important topics to increase the applicability. After these topics are addressed, broader applications may be achieved. I believe a pavilion-like shell structure can be erected with this mechanism in the coming years.

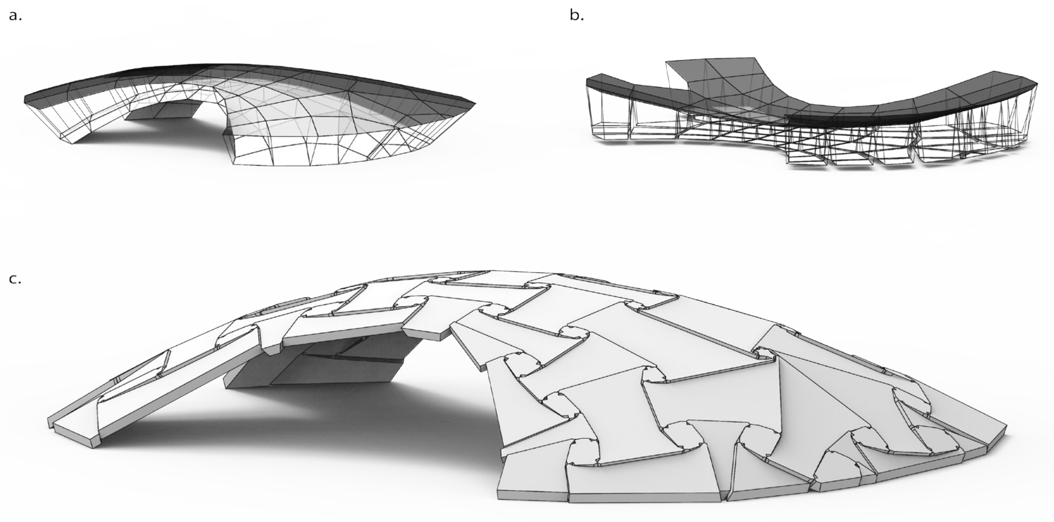


**Figure 13.**
The case study of a free-form surface. a-b. Unroll the mesh panels with the Neutral Surface. To be noted that, there is a six-edge node which is the umbilical point of the curved surface. c. Rendering of the expected result (to be updated with the physically produced prototype, waterjet cutting is under arrangement at the moment of manuscript submission).

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