MSc. Thesis

The effect of a multi-echelon model on the inventory investment for a supply chain of aircraft rotables

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by

I. Hofman

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Preface

This thesis is written to obtain the degree of Master of Science at the Delft University of Technology. The graduation project, of which this thesis is a part, is the concluding part of the Aerospace Engineering master degree at the Air Transport and Operations department of Delft University of Technology. The project could not have been conducted without the support of both the University and KLM. I would like to especially thank W. Verhagen for the ongoing support and mentoring from the University. Secondly, I would like to thank Thomas Knappers and Serge Wielemans for the opportunity and help over the course of the project. Without any of the three mentioned above, the results of this project could not have been achieved.

I. Hofman
Delft, June 2018

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1

Introduction

The Component Services group within KLM Engineering and Maintenance (KLM E&M) provides availability of aircraft spare parts. The end goal of this thesis is improving the operational performance of the component services group, while minimizing the financial cost. To do so, this thesis will first provide insight in the current supply chain structure of KLM E&M. Tailored to the supply chain structure, a new inventory optimization model will be developed. The purpose of the inventory optimization model is two folded. The model will increase insights in the current performance of the supply chain at KLM E&M, while simultaneously offering the possibility of determining the minimum required inventory size to fulfill customer demand.

The project is conducted in collaboration with Delft University of Technology (TU Delft). The thesis is concurrently used to obtain the degree of Master of Science at the faculty of Aerospace Engineering from the Delft University of Technology.

The remainder of this chapter will first elaborate on the specific case study at KLM E&M. Subsequently the research questions belonging to the project will be discussed. Finally the structure of the research project, and thesis will be discussed in chapter 1.3.

1.1. Problem description

KLM Engineering and Maintenance (KLM E&M) provides component availability for numerous airline customers. To ensure on-time performance, inventory is an indelible part of operations. Inventory models allow supply chain engineers to determine the required size of the component pool. The inventory system of KLM E&M consists of about 1500 different components distributed over 5 warehouses around the world. The current inventory optimization model used by KLM E&M is a multi-item single-echelon model. This model does not necessarily match with the actual operational conditions. A single-echelon model approaches the system as if all inventory is located at a single warehouse. However, the component pool of KLM E&M has multiple storage locations, each with a unique behavior and constraints.

The current supply chain of KLM E&M consists of one main warehouse in Amsterdam (AMS), and four remote warehouses located around the world. Figure 1.1 provides a graphical overview of the supply chain structure at KLM E&M. From Amsterdam items can be shipped to either, one of the remote locations, or a customer. Items are rarely shipped back from a remote location to Amsterdam, neither shipped between remote locations. Due to the geographical locations of the remote warehouses, do the shipping times between Amsterdam and remote location differ. Table 1.1 enlists the abbreviation and locations of the five warehouses of KLM, and lists the shipping time between Amsterdam and the remote warehouse.

The aircraft spare parts of this case study are classified in three layers. The first layer of an item is the part number. Each aircraft component will be assigned a part number by the manufacturer. The part number of an aircraft component is universal between all airlines, aircraft manufacturers, and aircraft maintenance providers. Some part numbers are fully interchangeable between each other. To keep track of interchangeable parts, KLM E&M assigns all part numbers to a code number. Fully interchangeable part numbers will get the

Table 1.1: Warehouses KLM E&M

Code	City	Country	Shipping time
AMS	Amsterdam	The Netherlands	N/A
RPA	Paris	France	3 days
RLO	London	United Kingdom	3 days
RKL	Kuala Lumpur	Malaysia	5 days
RMI	Miami	United States	5 days

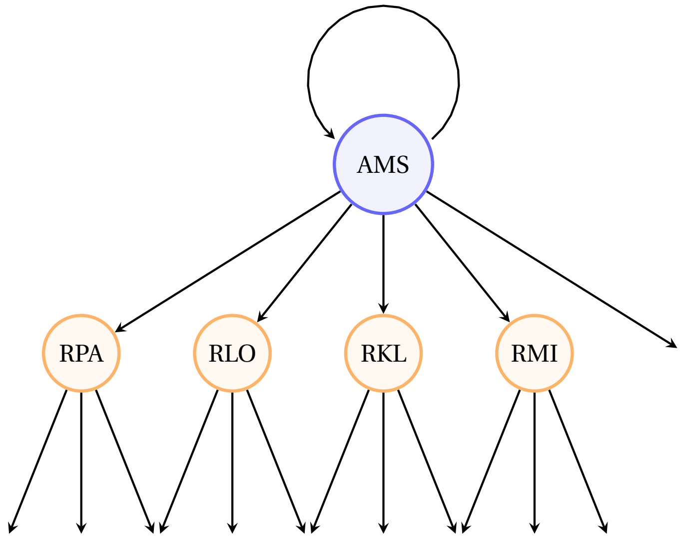


Figure 1.1: Supply chain structure at KLM E&M

same code number. During the lifetime of a component, it can get modified to a different code and part number. All code numbers which can be modified to each other, are grouped together by family numbers. Inventory optimization at KLM E&M will solely happen on family number level. However, it is good to keep in mind the under laying structure of the component pool. On the first of January 2018 the entire component pool consists of 1678 different family numbers.

The family number of the case study can be categorized in several groups. Each category of items is characterized by a different criticality, and therefore required delivery time. The four categories are, AOG (Aircraft on ground), critical, routine, and stock replenishment. Components with criticality AOG, and critical are bound to be delivered within the aforementioned shipping times between the warehouses. For these components it is necessary to have stock at the local warehouses. The components of the other two groups are allowed to have a longer lead time. It is therefore possible to ship the item on time from the main warehouse to each customer around the world.

By developing a more advanced optimization model, which takes into account the aforementioned aspect, and accounts for stochastic demand and supply behavior, it is expected that the size of the component pool could be reduced while maintaining or improving the component delivery time.

Many different inventory models have been developed in current literature. Tailoring a solution to the situation at KLM E&M is however challenging. Aircraft spare part items can generally be characterized as expensive low demand items. Such items do require different assumptions to be made during the development of the inventory model. The main goal of the project would therefore be to develop a multi location inventory model suitable for expensive and low demand items, as experienced by KLM E&M.

1.2. Research question

The research project will be conducted as a collaboration between KLM E&M and Delft University of Technology (TU Delft). Due to this collaboration the goal of the project will be two-folded. The goal related to KLM

E&M will be to develop a more advanced optimization model. The goal related to the TU Delft will be to fill some of the research gaps identified in the literature review and contributed to the knowledge of inventory optimization.

The main research question which needs to be answered for both stakeholders is the following: *What is the effect of a multi-echelon model on the inventory investment for a supply chain of aircraft rotables?* Before the research question can be answered the following three sub-questions have to be answered first.

1. What is the optimal allocation of inventory over the multi-echelon supply chain?
2. What is the gain on the service metric per inventory item if the stock level would be incremented?
3. What is the difference between the theoretically calculated, and the actual supply chain performance?

Next to the research question a couple of objectives are defined. The main objective for KLM E&M will be to develop a usable, understandable, and more advanced optimization model. Before achieving this objective, multiple sub-objectives have to be achieved. The sub-objectives related to KLM E&M are the following.

1. Determine the physical supply chain structure of KLM E&M
2. Determine which demand probability fits with the spare part demand experienced by KLM E&M
3. Select a optimization method suiting the inventory model and requirements of KLM E&M

The objectives related to the TU Delft are focused on filling the research gaps. Chapter 2 will provide an overview of the state of the art literature, and discuss the voids within the literature. The following sub-objectives related to the TU Delft have been determined.

1. Improve model predictions by creating a better fit between the model and reality
2. Increase the accuracy of the inventory optimization problem by improving the optimization method
3. Implement a more complex demand probability distribution in a multi-echelon inventory system

The research question will be answered using a mathematical inventory optimization model. With the inventory optimization model the optimal stock will be determined depending on the variables that will be described in Chapter 6.

1.3. Project and thesis structure

This thesis is structured somewhat chronological to the conducted project. The first step of the project is to determine the current state of the art of inventory optimization literature. The next chapter will first provide an overview of the current literature. The literature is, similar to the final structure of the model, divided in three parts. The first part will discuss model structure, the second part the demand probability, and the third part the solution techniques. With the state of the art in literature in mind, the novelty of this project is highlighted.

Chapter 3 describes the model developed over the course of this project. First the demand of components is analyzed. Secondly the structure and mathematics of the three inventory models used during the project is described. The chapter will end with the definition of different optimization parameters applicable to different objectives of the project.

The theoretical models of chapter 3 is implemented for the case study. The implementation is shown in chapter 4. The implementation is follow by the chapter containing the verification. The verification chapter makes a link between the mathematical model and the implementation, by showing the manual calculations for two components.

Chapter 6 will show the results obtained by the three inventory optimization models. Before the results are obtained, a small description of the scope is provided. To establish confidence in the obtained results, a sensitivity analysis is performed. The motivation and results of the sensitivity analysis are covered in chapter 7.

The report ends with a conclusion, providing a summary of the conducted research and an answer to the research question. The research on the topic is off course not finished yet. New opportunities will always arise to improve the fit between the model and reality. The second part of chapter 8 will describe some of the recommendations for further research.

2

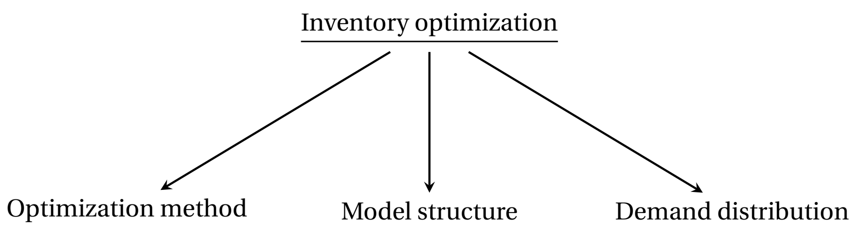
Theoretical context

In this chapter the reader will be presented with an overview of the literature relevant to the problem statement of chapter 1.1. In order to provide a clear overview of the current literature, the inventory optimization problem is divided into three parts. A graphical representation of the structure is provide in Figure 2.1.

The backbone of an inventory model is the way the demand for the inventory is modelled. The first part will discuss the probability distribution used to model the demand. The second part will go into detail on the actual model structure. In this part the effort of implementing the physical structure and constraints of the inventory model is covered. The third section contains an overview of the most important optimization parameters used in current literature. Finally, several solution techniques used to solve inventory optimization problems are discussed.

The chapter will end with a description of the novelty of the research conducted for this thesis.

Figure 2.1: Inventory optimization structure



2.1. Model structures

Current inventory models can be divided into two main categories. Namely the single-echelon and multi-echelon models. In a single-echelon model the system is represented as if all the inventory is located at the same location. The demand and resupply is all modeled to take place at one single location. The other main category is the multi-echelon model. Multi-echelon models optimize an inventory over several locations. One of the first implementations of the single-echelon model was presented by Feeney and Sherbrooke in 1966 [8].

A single-echelon model requires much less computational power compared to multi-echelon models. For this reason several authors have tried to approximate the results of a multi-echelon model by adapting the theory of a single-echelon model. An example of such an approach is presented by Muckstadt and Thomas in 1980 [13]. This paper shows that the approach works relatively well for high demand items. However, the larger the number of low demand items, the more important a multi-echelon approach will be. The KLM case described before involves low demand items, so a multi-echelon model will be desirable. The remainder of the literature discussed in this review will therefore mostly involve multi-echelon systems.

Multi-echelon models can be categorized on ordering policy. The main two categories are the batch ordering policy, and the lot for lot policy. The batch ordering category consist of the (Q,R) policy and the (S,s) policy [2]. The (Q,R) policy is described as the fixed replenishment point and fixed replenishment quantity inventory policy. When the on-hand inventory falls below the replenishment point R, an order will be placed of quantity Q. The (S,s) policy can be described as the minimum/maximum inventory policy [11]. When the on-hand inventory falls below the minimum s, the inventory will be replenished so that the on-hand inventory level is restored to level S. The (S,s) policy will have varying order sizes where the (Q,R) policy has fixed order sizes.

The lot for lot policy can be considered as a special case of the (Q,R) policy. The lot for lot policy, or (S,S-1) policy, places an order as soon as the inventory drops by 1. This can be seen as the (Q,R) policy with an order quantity Q of one. In general will the batch ordering policy be more economical for high demand items, and the lot for lot ordering policy more economical for low demand items. The low demand KLM case makes use of a lot for lot ordering policy. Main focus throughout this literature review will be on this policy, however some cases with an batch order policy are discussed as well.

2.1.1. The single-echelon structure

A single-echelon model approaches the system like all inventory is located at one location. All customers are served from this location. In case of repairable items, are all unserviceable items sent to the single location. The unserviceable items will be sent for repair and returned to the single location as well. A graphical overview of such an inventory system can be find in Figure 2.2 where the single location is identified by the green circle.

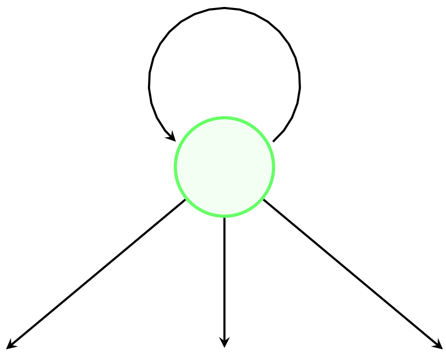


Figure 2.2: Single-echelon structure

Development of the single-echelon model started around 1966. As mentioned before, one of the first papers on single-echelon theory is written by Feeney and Sherbrooke in 1966 [8]. The authors prove that the single-echelon model can be used with any Poisson and compound Poisson demand process. The paper is written considering the aircraft spare parts industry. This industry is characterized by low-demand high value items. It is stated that the (S, S-1) reorder policy will suit these characteristics. Assuming that resupply is instantaneous will always result in an optimal inventory of zero. The authors of this paper relax this assumption, so with positive resupply times, the optimal inventory will usually be positive. The paper contains two measures of supply performance to which the model can be optimized. Namely, the number of back orders and the number of lost sales. The paper does not contain any implementation of both models, and does not provide any conclusions on which of those two models is preferable under which circumstances.

In 1968, Sherbrooke [19] continuous on the the work of Feeney and Sherbrooke and provides a clear description of the single-echelon back order model. To determine the number of back orders Sherbrooke calculates the probability that the number of items requested is larger than the number of items currently on stock. The amount of items currently on hand (OH) equals the total stock (s) minus the units due in (DI). Following this definition the stock can be defined as Equation 2.1.

$$s = OH + DI \tag{2.1}$$

The authors neglect shipping times of items from the customer to the warehouse. With this assumption the number of items due in equals the number of items in the repair cycle. The authors introduce the term *pipeline* (μ) to describe the amount of items in the repair cycle. With m as the average annual demand and T the average repair time in years, the pipeline is defined as the dimensionless value in Equation 2.2.

$$\mu = mT \tag{2.2}$$

To determine the number of back orders the authors look at the expected value of the probability distribution for the back orders. The expected value of a probability distribution is can be calculated with commonly known Equation 2.3.

$$E[x] = \sum_{x=1}^{\infty} xP(X = x) \tag{2.3}$$

Sherbrooke now defines the expected number of back orders according to Equation 2.4.

$$EBO(s) = \sum_{x=s+1}^{\infty} (x - s)P(\mu = x) \tag{2.4}$$

The inventory system is finally optimized using a marginal analysis. The marginal analysis is based on the marginal decrease in expected back orders divided by the item cost (Equation 2.5). This value is calculated for each stock level and item. Inventory is finally determined by repeatedly incrementing the stock of the item with the highest value of y by one.

$$y = \frac{EBO(s-1) - EBO(s)}{cost} \tag{2.5}$$

2.1.2. The multi-echelon structure

Multi-echelon models optimize inventory over multiple locations. A multi-echelon system is formed like a tree structure where each location can have one predecessor and multiple successor. The top level will always contain one single location. In case of repairable items all unserviceable items are sent to the top level where the items will be repaired. All serviceable items will be distributed from the top warehouse. A multi-echelon system can in theory contain an infinite number of levels. Figure 2.3 provides a graphical representation of a multi-echelon system with two levels. The central warehouse is represented with the green circle. The local warehouses are drawn as red circles.

The first occurrence of a multi-echelon model can be found in a paper written by Sherbrooke in 1968 [19]. In this paper the author developed the Multi-Echelon Technique for Recoverable Item Control (METRIC). The purpose of this model is again to optimize aircraft spare part inventory. The METRIC model is the basis for a large number of multi-echelon models developed later on. The model is build on the single echelon theory described in the previous section. The METRIC model optimizes the inventory level for every item at several bases. The objective of the model is to minimize the sum of the back orders across all bases. The authors assume a Poisson demand over all items. Next to this assumption the model is treated as the inventory is in a steady state. The number of aircraft and flying hours will remain the same over some period of time. The (S, S-1) inventory policy is applied for every item in every echelon. The model does not allow for lateral supply from another base. A base is solely resupplied from the depot in the echelon above.

Next to the written assumptions, four unwritten assumptions can be identified. The first assumption is regarding independent demand. The author would not be able to apply Palms theorem if failures of spare parts would be dependable of each other. The METRIC model is also unable to cope with irreparable items. It is assumed that every item can be repaired to its initial state. The third assumption relates to the back order queue. The METRIC model operates on a first come first serve basis. Finally, the METRIC model assumes items to be equally essential for operations.

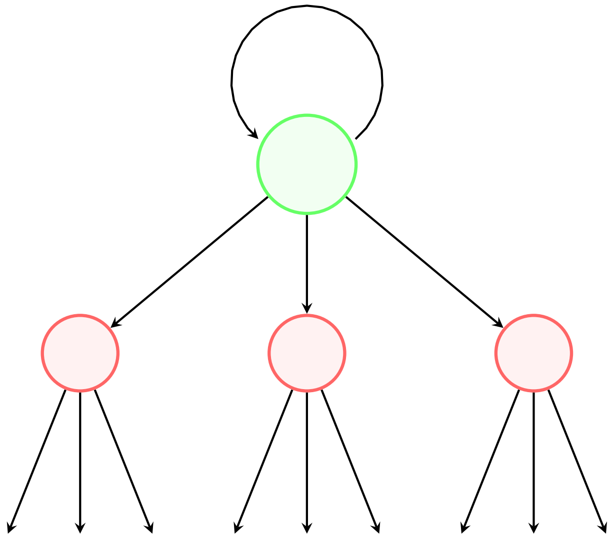


Figure 2.3: Multi-echelon structure

In the METRIC model can an unserviceable item be repaired at each echelon level. Sherbrooke defines the probability that an item is repaired at a certain base with the symbol r. In the METRIC model positive subscripts for j are used to refer to warehouses, with 0 indicating the parent warehouse. With these definitions the demand at a warehouse is defined as Equation 2.6.

$$m_0 = \sum_{j=1}^{J} (1 - r_j) \tag{2.6}$$

The pipeline for the parent warehouse $EBO(s_0|m_0T_0)$, is defined like the single echelon theory described in Equation 2.4. The average order and ship time form the the parent warehouse to the local warehouse is defined as O_j. With this, the pipeline for the local warehouse is determined by Equation 2.7.

$$\mu_j = m_j(r_jT_j + (1 - r_j)(O_j + \frac{EBO(s_0|m_0T_0)}{m_0})) \tag{2.7}$$

The calculated pipeline is subsequently substituted in Equation 2.4 to calculate the expected back orders. Finally the marginal analysis can be applied similar to the single echelon model with Equation 2.5. For a multi-echelon system the number of coefficients y will grow exponentially with the number of warehouses and items, which results in a much bigger optimization problem.

The METRIC model is used as basis for numerous other multi-echelon models. Each model tries to improve the performance of the model by relaxing some of the constraints. An example of such a model is presented by Muckstadt1973 [12], where the MOD-METRIC model is developed. The author relaxes the constraint of equal essentially, allowing for the modelling of assemblies and its components. The objective of the METRIC model is to minimize expected base back orders for all items. The objective of the MOD-METRIC model is to minimize the expected base back orders for the end item.

According to Sherbrooke [20] the MOD-METRIC model understates the delay in repair of a higher indenture item caused by back orders on the item's lower indenture components. The model also understates the delay in resupply of a base from a depot that has back orders.

It is also tried to derive exact solutions for the METRIC problem. To accomplish this authors had to make more restrictive assumptions. An example of such an approach can be found in Simon [22]. In this paper, the resupply times are assumed to be constant instead off arbitrary and the demand is assumed to be Poisson instead of compound Poisson. An exact solution will result in a more optimal solution, but will require substantial computation time. This, combined with the more restrictive assumptions, motivated researchers to come up with a different way of improving the METRIC model. An example can be found a paper written by Graves in 1985 [9], where the VARI-METRIC model is developed. Graves improves the performance of the METRIC model by relaxing the assumption that the shape of the demand distribution will remain constant. Modelling the demand with a negative binomial distribution does accomplish this relaxation. Graves does assume a deterministic shipment time between the different echelons. Sherbrooke [20] shows that the VARI-METRIC model allows for arbitrary shipment times as well. It is shown that the VARI-METRIC results in a much better solution compared to the METRIC model without sacrificing largely on computational time. Graves shows that in 11,5% of the cases, the METRIC stock levels differ by at least one unit from the optimal results. Contrarily, the VARI-METRIC model only deviates in 0,9% of the cases from the exact result.

There are many examples where the multi-echelon model has been adjusted to serve specific characteristics of real life supply chains. In the paper of Shtub and Simon [21], the multi-echelon model is utilized to determine the reorder point of a supply chain in order to maximize the fill rate of the local warehouses. The model is applied to the consumable high value spare parts domain. An interesting extension of the model is the implementation of non identical local warehouses. The warehouses are given weights to determine the priorities in supply form the depot.

A different direction within multi-echelon modelling is the implementation of lateral transshipment. Possible savings within inventory optimization could be achieved by relaxing the assumption that a warehouse can only receive resupply from a warehouse in a echelon above. Resupply between warehouses in the same echelon is also called lateral transshipment. Paterson et al. provided an overview of papers utilizing this concept[17]. Current literature taking into account lateral transshipment, is limited on a couple of areas [17]. There is little known about determining when it is best to perform the redistribution. Lateral transshipment theory is also solely applied to systems with a small number of locations. Increasing the number of locations, exponentially increases the dimensions of the problem. Resulting in very long computational times. For the same reason, most of the papers discussed by Paterson et al. [17] involve single item systems.

2.2. Demand probability distributions

The backbone of each inventory optimization model is the demand probability. This chapter will provide an overview of the frequently used probability distributions, and most interesting probability distributions.

2.2.1. Poisson distribution

The first and still most widely used demand probability is the Poisson distribution. Current literature covers two types of the Poisson distribution to model demand, namely the basic Poisson distribution and the compound Poisson distribution. Both probabilities will be discussed in more detail below.

Basic Poisson distribution

The Poisson distribution is a discrete probability distribution that gives the probability of a given number of events occurring in a fixed interval of time. The probability of the distribution is based on the average number of events occurring in the time interval. Equation 2.8 provides the probability mass function of the Poisson distribution where λ denotes the average number of occurrences of events.

$$P(X = k) = e^{-\lambda} \frac{\lambda^k}{k!} \tag{2.8}$$

An early implementations of the Poisson distribution can be found in [19]. Sherbrooke shows that the if the time between demands has an exponential distribution, the probability distribution for the demand follows a Poisson. The author finally utilizes the Poisson distribution to determine the probability of a stock-out for a given amount of stock. The exact implementation is described before in section 2.1.

The Poisson distribution itself is characterized as the "memoryless" distribution. The time of the previous demand has no influence on the time of the next demand. Many authors, e.g. Axsäter [1], Hopp et al. [10], Nozick and Turnquist [16], Axsäter [3], Caglar et al. [6], do make the assumption of independent demand and therefore use the Poisson distribution.

A useful characteristic of the Poisson distribution lays in the queuing theorem of Palm. This theorem states that if demand at each local warehouse is Poisson, the demand at the central warehouse can also follows a Poisson distribution.

The basic Poisson distribution gives the probability of a certain number of events occurring in a fixed time interval. This makes the basic Poisson distribution ideal for inventory systems with fixed ordering size, like the (S,S-1) replenishment policy. The basic Poisson distribution only provides the probability of the occurrences of an event. It is not able to differentiate in different types of events.

Compound Poisson distribution

The compound Poisson distribution is a extension of the basic Poisson distribution. The compound Poisson distribution provides the probability distribution of the sum of a number of independent identically distributed events, where the events itself are Poisson-distributed. This probability distribution allows for the implementation of non identical events. In contrary to the basic Poisson distribution, which is only capable of providing information on the probability of the occurrences of events, provides the compound Poisson process information on the event itself as well. The ability of representing different events make a compound Poisson distribution useful for inventory systems where demand occurs in clusters of varying size and inventory systems with varying order sizes. The (S,s) ordering policy described in Section 2.1 is an example of ordering policy with varying order sizes.

Feeney and Sherbrooke [8] has shown that Palms theory does apply for each Compound Poisson distribution, which allow the compound Poisson distribution to be used for multi-echelon systems as well. Examples of papers utilizing the compound Poisson distribution are Sherbrooke [19], Simon [22], and Muckstadt [12].

2.2.2. Negative binomial distribution

The negative binomial distribution is discrete probability distribution. The distribution describes the number of successes in a sequence of independent and identically distributed Bernoulli trials before a number of failures occurs. A Bernoulli trial is a random experiment with exactly two possible outcomes, namely success and failure. The probability of success and failure is the same every time the experiment is conducted. Equation 2.9 provides the probability mass function of the negative binomial distribution. The negative binomial distribution depends on two parameters. The first parameter is the number of failures until the experiment is stopped which is denoted as r. The second parameter is the success probability for each experiment, denoted as p.

$$P(X = k) = \binom{k+r-1}{k} p^k (1-p)^r \tag{2.9}$$

When using the Poisson distribution to model demand, the mean and variance of the distribution are assumed to be the same. This assumption can be relaxed by modelling the demand with the negative binomial distribution. Similar to the Poisson process, Sherbrooke [18] shows that if the time between demands is characterized by the logarithmic Poisson process, the probability distribution for the demand follows a negative binomial distribution. The complexity of determining the parameters of negative binomial distribution limits the usage of this distribution. Graves [9] first tried to utilize the distribution to improve the METRIC model by Sherbrooke, as discussed in chapter 2.1.

2.2.3. Weibull distribution

The demand process for some items is not random, but results from wear out [18]. Such behaviour can be modelled using a the Weibull distribution. Most inventory models discussed in this literature review do require a discrete demand probability as demand is a discrete phenomenon. The Gamma and Weibull distribution are both in origin a continuous probability distribution. In order to use such a distribution the probability distribution should be made discrete. For the Weibull distribution this is done by Nakagawa and Osaki [14]. The discrete form of the Weibull distribution according to Nakagawa and Osaki can be found in

Equation 2.10. The shape parameter of the distribution is denoted with β. The q is the success rate of each event, which is per definition equal to 1 minus the failure rate $(1-p)$.

$$P(X=k) = q^{k^\beta} - q^{(k+1)^\beta} \quad (2.10)$$

Unfortunately no implementation of the Weibull distributions for a supply chain model can be found in current literature.

2.2.4. Choosing a demand distribution

Effort has been made to construct a method to determine the most suitable demand distribution for a certain demand type. Syntetos et al. [24] compared the Poisson distribution, negative binomial distribution, compound Poisson distribution, normal distribution, and the gamma distribution. The author analyzes three different component pools varying between 3000 to 5000 stock keeping units.

Syntetos et al. shows that the negative binomial distribution and the compound Poisson distribution provide the most frequent fit. The gamma and normal distribution do not perform well. Syntetos et al. claims that the misfit may be caused by the continuous nature of this distribution. During the research the fit of the distributions is tested on discrete observations. The Poisson distribution provides an reasonable fit. However, it is interesting to mention that the fit increases for slow moving items.

2.3. Optimization parameter

In order to be able to optimize a system, it is necessary to determine which parameter of the system should be minimized or maximized. Brooks et al. [4] lists four main optimization parameters for aircraft spare parts optimization, which are commonly used. Although the paper is written in 1969, current literature on inventory optimization is almost always utilizing one of those four service metrics. The four optimization parameters described by Brooks et al. are, fill rate, back orders, operational rate, and average aircraft on ground.

Fill rate is the portion of demand met with on hand stock over a certain time period. This service metric does not allow for any time based service agreements. The back order service metric is similar to the fill rate. However, this metric will also take into account the duration of the shortage. The operational rate is the probability that, at any given point in time, there will be no due-out from base supply. The average aircraft on ground service matrix is the number of aircraft grounded for lack of spare parts at any given point in time. This last service metric is rather specific. It is designed to fit the application of the paper by Brooks et al..

More recently a fifth service metric has been employed for a supply chain optimization. Caggiano et al. [5] emphasizes the inability of implementing time based service level constraints. According to Caggiano et al. are many supply chain contracts based on a service level which depends on time. To be able to include these time based contract statements, the time bases fill rate service metric is developed.

The usage of service metrics in current literature is not evenly distributed across the five metrics mentioned above. The literature discussed in this paper mainly utilizes the fill rate, back order, and time based fill rate service metric.

2.4. Solution technique

The current literature covers a wide range of solution techniques for inventory optimization models. This chapter discusses a selection of the most used, and most relative solution techniques. The solution techniques can be divided into three groups. Namely, exact solutions, linear programming, and simulation based optimization. The three different groups are highlighted in the remaining of this chapter.

2.4.1. Exact solution

Exact solutions inherently provide an optimal solution to a problem. Due to this characteristic, exact solutions are a popular way of solving mathematical problems. Inventory optimization problems usually involve a large capital investment, which means that even a small percentage reduction in inventories may correspond to savings on the order of hundreds of thousands of euros [25]. Therefore, being able to find the exact solution instead of an approximation of an inventory optimization problem is desirable. Deriving an exact solutions may be challenging and complicated for complex systems, like inventory optimization systems.

Solving an average real life inventory process exactly is nearly impossible. In order to be able to derive an exact solution for such problem, extensive assumptions do have to be made. In papar written by Axsäter [1], a two echelon stock policy for consumable items is determined utilizing an exact solution. The authors develop a cost function representing the system. This is different compared to the papers by Sherbrooke [19] and Simon [22], where the objective is to determine the steady state stock distribution. The outcome of such a system can thereafter be used to calculate the cost.

The cost function derived by Axsäter takes into account the delays experienced by the customer and the unit's storage time at each of the facilities. It is assumed that the holding cost and stock out cost incurred by the customers delay are linear with the time. The ordering costs are disregarded. Both the depot and the individual warehouses follow the (S,S-1) replenishment policy. If an order from a warehouse cannot be full filled, the items are back ordered on a first come first serve basis. With these assumptions the author states that the computational efforts are similar to the METRIC model. The author does not provide an implementation of the developed theory. The author recommends to compare the method with the METRIC model and the VARI-METRIC model to evaluated the computational performance and accuracy of the model. However, no implementation of the theory can be found in current literature.

A more recent example of an exact solution procedure applied to inventory optimization can be found in a paper by Topan et al. [25]. The proposed approach utilizes the branch-and-price algorithm. The (S,S-1) policy for the depot used by Axsäter is relaxed to the more general (Q,R) policy. On the other hand, the shipping time between depot and warehouses is more restricted. Topan et al. do assume the shipping times to be deterministic.

The developed model is applied to a range of problems. The authors consider problems with 5-30 items and 2-4 warehouses. It is shown that the computational time increases drastically when the number of items increases. For the maximum case, with 20 items and 4 warehouses, the computational time on an average desktop is more than 5 hours. The results of the paper confirm that an exact solution can be used as long as the number of items and especially the number of warehouses are limited. Unfortunately the results have not been compared to an approximate solutions of for example the METRIC model.

The theory of the two papers above is extended by Stenius et al. in 2016 [23]. The previous mentioned papers both model the demand with the Poisson distribution. This assumption is relaxed by Stenius et al., where the demand is represented by a compound Poisson process. The theory developed in the paper is applied to a single item, two echelon supply chain concerning the distribution of sheet metal products. Is is shown that deriving the exact solution is computationally challenging. Therefore, the authors do not recommend to apply the theory to an larger inventory system. The authors do believe the theory presented in the paper will form a good foundation for future research on accurate heuristics.

2.4.2. Linear programming

Most of the used solution techniques for inventory optimization problems belong in the linear programming category. Within linear programming there are two frequently applied methods, namely the Greedy algorithm and the Lagrangian relaxation.

Greedy algorithm

A greedy algorithm solves an optimization problem by making the local optimal choice at every step. The solution obtained by a greedy algorithm will rarely be the global optimal solution. However, a greedy algorithm generally yields a local optimal solution close to the global optimal solution in a reasonable time. This characteristic of the greedy algorithm makes it a regular choice or researchers within the inventory optimization domain.

An early implementation of a greedy algorithm can be found in the single echelon model of Feeney and Sherbrooke [8] discussed before. The proposed Greedy algorithm calculates the back order reduction at each step. By dividing the back order reduction with the investment cost, the marginal improvement can be calculated. The marginal analysis approximates the maximum back order reduction with the lowest investment costs.

The METRIC model presented by Sherbrooke [19] is optimized in a similar way as the single echelon model mentioned before. Sherbrooke proposes a marginal back order analysis of the system as well. Unfortunately, this optimization function is not necessarily convex. This may result in sub optimal distributions of the inventory. However, due to the relative simplicity and good computational performance the Greedy algorithm is still applied. Many of the improved models based on METRIC model use a similar Greedy algorithm. Examples of these papers are, Muckstadt [12], Graves [9], and Sherbrooke [20].

A different implementation of the Greedy algorithm is presented by Caggiano et al.[5]. The FastIncrement procedure developed in this paper is an improved version of the approach presented in based on a Greedy algorithm. The procedure employs a marginal analysis technique that greedily increments the stock levels at all bases until all service-level constraints are satisfied.

Lagrangian relaxation

A different linear approach is the Lagrangian relaxation. Hopp et al. [10] introduced the Lagrangian relaxation method to improve the performance of the Greedy algorithm. Instead of minimizing the back orders, Hopp et al. minimized the Lagrangian multipliers. The advantage of this approach lays in the insensitivity for small changes of the Lagrangian multipliers. For small changes in the system it is not required to resolve the entire model. According to the author the results of the proposed Lagrangian method are reasonably close to the back order optimization. However no comparison has been provided.

The method introduced by Hopp et al. divides the problem in two parts. First the optimal stock of the central warehouse constrained to the service level is calculated. After the central warehouse stock is determined the optimal local warehouse stock is calculated. The local warehouse stock is constrained to the average total delay.

Caglar et al. [6] continue on the research of Hopp et al.. The method proposed by Caglar et al. uses a combination between a Greedy algorithm and the Lagrangian relaxation as well. The objective of the model is to minimize the system wide inventory holding cost, constrained to an average response time to the customers. Also, Caglar et al. considers repairable items, where Hopp et al. only considers consumable items. The computational experiments presented reveal that the developed heuristic works much better on large-sized problems compared to the heuristic developed by Hopp et al..

2.4.3. Simulation based optimization

The most recent development in supply chain optimization is a form of simulation based optimization. A simulation based optimization allows for wider, and more complex implementation of the supply chain. Due to significant interactions between planning and scheduling for the different echelons, it is necessary to consider the simultaneous optimization and scheduling decisions in order to determine the global optimal solution [15]. A simulation based optimization can capture the behaviour of all the entities involved, their interactions, and the uncertainties associated with these systems.

An example of a simulation based optimization is presented by Nikolopoulou and Ierapetritou [15] where the authors implement a hybrid approach. The author combines the optimization, and the simulation modelling approach by applying a Mixed Integer Linear Programming formulation in the context of an agent based simulation. The model presented minimizes the inventory, back order, transportation, and production costs. A limitation of the proposed system lays in the inability to accommodate for any stochastic characteristics. For example, the model does not include a stochastic demand, or lead-time. With this limitation the authors are able to iterate to an optimal solution in a reasonable time. The model is applied to a small 3 echelon, 2 item supply chain. The supply chain network contains two suppliers that provide raw materials to 3 production sites, which server 3 markets. The computation time will increase drastically when the system would be implemented for a larger scale supply chain.

A different simulation based optimization framework is presented by Chu et al. [7]. The objective of this paper is to minimize the inventory cost while maintaining a service level quantified by the fill rates. The inventory system is modeled by an agent-based system, which returns the performance functions. A limitation of an agent-based simulation method is that the output does not represent the uncertainty of the input parameters. To overcome this limitation, Chu et al. developed a computational algorithm to estimate the expectations of these uncertainties. The developed method is applied to two single item case studies. The first case study is two echelon system with four facilities while the second was a three echelon system with seven facilities. The local optimal solutions were found in 270 and 751 seconds respectively. Chu et al. did not compare the obtained results with other methods. However, it can be seen that the developed method will quickly become computational demanding for larger multi-item systems.

2.5. Novelty of the project

The academic novelty of the project will lay in combining the several elements described in this chapter. First the demand probability with the best fit will be determined. In the second step the physical supply chain structure is captured in a mathematical model. The third, and most novice step, will be to implement a new service metric on the developed supply chain model. The new service metric will optimize the system with regards to the achieved service level. Optimizing the model to service level will result in a better fit with the current operations of KLM E&M, and will contribute to the state of the art literature.

The new service metric, combined with the categorization of components as discussed in chapter 1.1 is believed to be novice. Extensive research is conducted on inventory optimization models, as described in this chapter. The characteristics of aircraft spare part supply chains do however require a slightly different approach. This research will contribute to the knowledge of such supply chains, by entirely focusing on the operations of KLM E&M.

3

Inventory optimization model

The theoretical content of chapter 2 is used to develop a single-echelon, METRIC, and VARI-METRIC model. This chapter will discuss the complete derivation and mathematical foundation of these models. The structure of the chapter is according the structure of a typical inventory optimization problem shown in figure 2.1. First the demand of the supply chain items is discussed. Secondly the theoretical models are provided, and finally the solution technique is determined.

3.1. Demand probability distribution

Before the inventory pool of KLM E&M can be optimized, a probability distribution modelling the demand is determined. As discussed in chapter 2, different discreet probability distributions can be utilized. This section will provide a concise analysis of the demand for components as seen at KLM E&M.

The demand analysis is performed on the demand data from two consecutive years (2 January 2016 - 1 January 2018). During this time span a total of 9707 components are requested. On average this will result in about 13,2 component requests per day. The component pool of KLM E&M consists of 1678 different family numbers as mentioned in chapter 1.

The first step of the analysis will provide a general insight in the demand of the different components. The requests are distributed over the different family numbers. Similar to many processes in nature, are the number of requests divide over the different family numbers according to the Pareto distribution. Figure 3.1 shows the theoretical Pareto distribution, combined with the data of the actual components. It can be seen that the distribution of demand over the family numbers generally follows the Pareto distribution. During the further analysis of the demand it is important to keep this characteristic in mind. Majority of the items is only requested rarely.

Figure 3.2 shows in blue the distribution of the number of requests received by KLM E&M each day. The input of the inventory optimization model should as best as possible represent this distribution for each individual component. Two different probability distribution from chapter 2.2, namely the Poisson distribution and the negative binomial distribution, are fitted to the actual data.

Generally looking at the date, it can be seen that the negative binomial distribution results in a better fit compared to the Poisson distribution. Statistical tests do unfortunately not confirm the similarity between the distribution. To quantify the match between the theoretical distributions, the chi-squared test is performed. According to this test the negative binomial distribution is more likely to be a match with the data compared to the Poisson distribution. However, it is not possible to qualify the data as the negative binomial distribution with any statistical confidence. Table 3.1 shows the results of the chi-squared test, tested for both distribution.

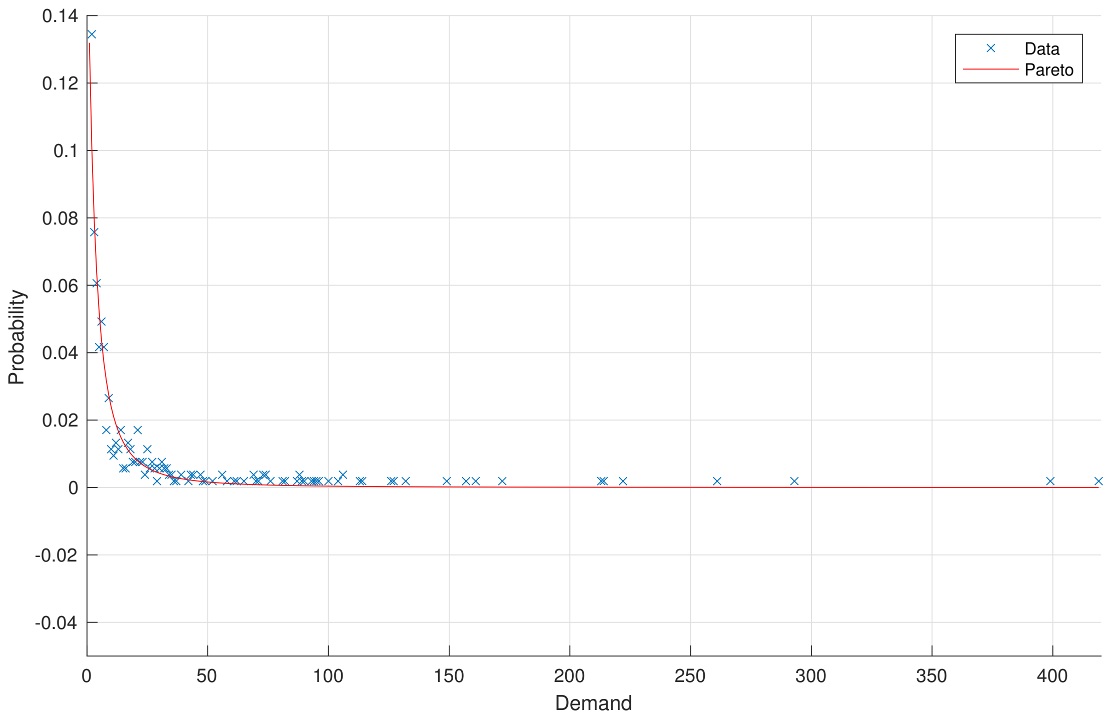


Figure 3.1: Total demand per family number

Table 3.1: Details chi-squared test demand probability

	Poisson	NBD
h	1	1
p	0	$5.4 * 10^{-8}$

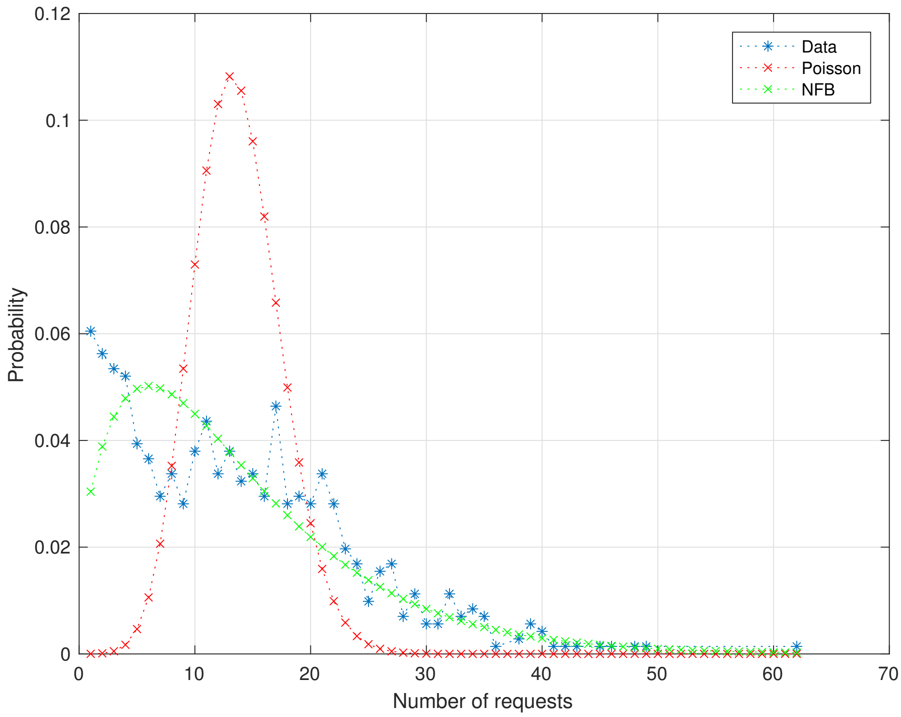


Figure 3.2: Total demand per day

3.2. single-echelon

The purpose of this section is to describe the mathematical single-echelon model applicable to the supply chain of KLM E&M. The theoretical single-echelon model of Sherbrooke [19] is used to develop the model described in this section. The physical structure is identical to the structure described in depicted in Figure 2.2.

3.2.1. Mathematical single-echelon model

The demand (m) for an aircraft rotable from the inventory pool is equal to the expected number of removals of this item. A common reliability parameter of a component in the aviation industry is the mean time between removals ($MTBR$). In case the $MTBR$ is expressed in hours, can the expected removals be determined using the amount of flying hours with this component. The total flying hours of the entire fleet is captured in a parameter called fleet hours (FH). For redundancy purposes, aircraft often contain several times the same component. The quantity of identical components in an aircraft is given by the quantity per aircraft parameter (QPA). With these three variables the expected number of removals can be defined according to Equation 3.1.

$$m = \frac{FH * QPA}{MTBR} \tag{3.1}$$

The model neglects handling and shipping times. This means that the pipeline of a component equals the number of components in the repair loop. The number of components in the repair loop depends on the number of components put in the repair loop, times the time a component remains in the repair loop. The repair loop does not experience queuing of items. This assumptions can be seen as if the repair capacity is infinite. With this assumption, the time a component remains in the repair loop is constant. The constant representing the repair time is given by the turn around time (TAT) of the component. The pipeline for each component can therefore be calculated according to Equation 3.2, where the TAT is given in days.

$$pipeline = m * \frac{TAT}{365} \tag{3.2}$$

The expected number of items on back order can be derived form the definition in Equation 2.4, where the probability distribution depends on the pipeline. Equation 3.3 shows the expected back orders for the single-echelon model.

$$EBO(s) = \sum_{x=s+1}^{\infty} (x-s)P(x = pipeline) \tag{3.3}$$

For computational purposes, can the definition of the expected back order be rewritten to the recursive form according to Equation 3.5. If the stock is zero, all orders will result in a back order. Therefore will $EBO(0)$ per definition be equal to the pipeline.

$$EBO(s) = \sum_{x=s}^{\infty}(x-(s-1)P(x) - \sum_{x=s}^{\infty} P(x) \tag{3.4}$$

$$= EBO(s-1) - 1 - \sum_{x=0}^{s-1} P(x) \tag{3.5}$$

The model is finally optimized using a marginal analysis. The marginal analysis parameter called $secret$, is defined as the interval of the optimization parameter (X_{opt}) divided by the cost of this improvement. The definition of the optimization parameter can be found in Equation 3.6. The optimization parameter itself is discussed later on in chapter 3.5.

$$secret = \frac{X_{opt}(s) - X_{opt}(s+1)}{price} \tag{3.6}$$

3.3. multi-echelon: METRIC

Similar to the single-echelon model, can the multi-echelon METRIC model be applied to the aviation spare parts industry as well. This chapter will apply the theory of the METRIC model described in chapter 2 to the general case at KLM E&M.

3.3.1. multi-echelon model structure

The structure of the model described in this chapter is slightly different from the model structure described and depicted in Figure 2.3. The model in this chapter will include the capability of local repair. In the multi-echelon model defined in chapter 2, are unserviceable components always returned to the main warehouse. This type of operations is not necessarily the most efficient method for a supply chain. It might be beneficial to repair unserviceable components locally at the remote warehouse. Such an operations is called local repair. To incorporate local repair in the model, the model structure is modified according to Figure 3.3. The dashed lines represent the flow of components repaired locally.

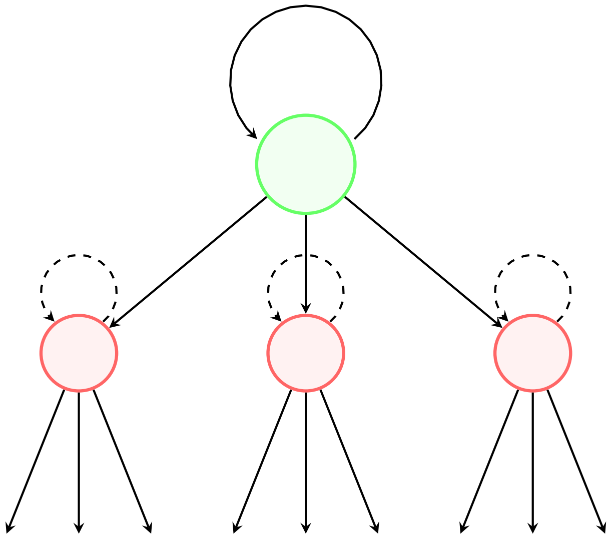


Figure 3.3: Multi-echelon structure with local repair

3.3.2. Mathematical multi-echelon METRIC model

The definition of demand for components at the local warehouses is identical to the demand for the single-echelon model (Equation 3.1). The model structure does not allow for any demand directly from the main warehouse. The demand at the main warehouse is therefore equal to the sum of the components shipped from the remote warehouses to the main warehouse. The number of components shipped to the main warehouse depends on the fraction of items locally repaired. This fraction is defined by r_j, where j identifies each local warehouse. The definition of the demand at the main warehouse can be found in Equation 3.7.

$$m_{main} = \sum_{j=1}^{J} (1 - r_j) m_j \tag{3.7}$$

The definition of the pipeline for the main warehouse in Equation 3.8, is comparable to the pipeline definition of the single-echelon model.

$$pipeline_{main} = m_{main} * TAT_{main} \tag{3.8}$$

The pipeline definition for a remote warehouse is less trivial. The model structure reveals that a component in the pipeline for a remote warehouse can be located at two positions. The component can reside in the local pipeline, or the pipeline between the main warehouse and the local warehouse. The definition of the pipeline for a remote warehouse (Equation 3.9) is therefore separable in two different pipelines. The local pipeline, as depicted in Equation 3.10, is derived similar to the pipeline of the main warehouse. The parent pipeline itself can be divided into two parts as well. One part represents the orders which are on stock at the main warehouse, and therefore can be shipped to the remote warehouse instantly. The second term represents the orders which are back ordered at the main warehouse. Combining these two terms results in Equation 3.11.

$$pipeline_j = localPipeline_j + parentPipeline_j \tag{3.9}$$

$$localPipeline_j = m_j * r_j * TAT_j \tag{3.10}$$

$$parentPipeline_j = m_j * (1 - r_j) * \left(shippingTime_j + \frac{EBO_{main}}{m_{main}} \right) \tag{3.11}$$

With the definition of the pipeline, the expected back orders at the remote warehouse can be determined. The definition of the expected back orders (Equation 3.12) can be rewritten to the form of Equation 3.14. It is worth mentioning that for a stock level of 0, the expected back orders will per definition be equal to the expected removals. Evidently, if there is no stock, each order will result in a back order. Therefore it can be stated that $EBO(0) = E[X]$, where $E[X]$ stands for the expected demand at each point in time.

$$EBO(s) = \sum_{x=s+1}^{\infty} (x-s)P(x = pipeline) \tag{3.12}$$

$$= pipeline - \sum_{s=0}^{s=stock} P(X \geq s-1) \tag{3.13}$$

$$= pipeline - \sum_{s=0}^{s=stock} 1 - P(X < s-1) \tag{3.14}$$

The expected back orders experienced by the customers of the entire supply chain is the sum of the expected back orders at the remote warehouses (Equation 3.15). Note that the expected back orders of the main warehouse are not included in this sum. The structure of the model does not connect any customers to the main warehouse, therefore the back orders at the main warehouse should be excluded from the system back orders summation. The back orders of the main warehouse do influence the system back orders through the pipeline of the remote warehouse (Equation 3.11).

$$EBO_{system}(s) = \sum_{j=1}^{J} EBO_j(s) \tag{3.15}$$

The data structure containing the optimization parameter (Equation 3.16) for the multi-echelon model differs from the single echelon model. The data structure contains the optimization parameter for each component at each location. Equation 3.17 shows the construction of the optimization parameter vector.

$$\underline{\Delta X_{opt}(s)} = X_{opt}(s) - X_{opt}(s_j + 1) \tag{3.16}$$

$$\underline{\Delta X_{opt}(s)} = \begin{cases} X_{opt}(s) - X_{opt}(s_{main} + 1), & \text{for } main \\ X_{opt}(s) - X_{opt}(s_j + 1), & \text{for } \forall \text{ j} \end{cases} \tag{3.17}$$

The marginal analysis variable *secret* is changed from a single value, to a vector as well. The length of the vector is the same as the amount of warehouses in the supply chain structure. Equation 3.18 shows the multi-echelon form of the secret parameter.

$$\underline{Secret(s)} = \frac{\underline{\Delta X_{opt}(s)}}{price} \tag{3.18}$$

3.4. multi-echelon: VARI-METRIC

The VARI-METRIC model is an extension of the METRIC model. The previous section showed that the expected back orders of the METRIC model are solely based on the expected value of the pipeline. The VARI-METRIC model relaxes this assumption. By determining the variance of the pipeline, the probability distribution in the expected back order equation can be chosen more freely. The remainder of this chapter will show the derivation of the variance of the pipeline. Chapter 4 will discuss the implementation of the variance in the probability distribution of the expected back orders.

3.4.1. Mathematical multi-echelon VARI-METRIC model

For the METRIC model the expected number of components between the main and local warehouse is continuously described as the pipeline. A different, more mathematical notation, is $E[X_j]$ where j denotes each local warehouse. This section will use the mathematical notation to make the derivation more recognizable.

The variance of the pipeline will be derived from the general definition of the pipeline. The general definition of the variance can be found in Equation 3.19.

$$Var[X] = E[X^2] - E[X]^2 \tag{3.19}$$

Equation 3.19 can be rewritten in the following two forms. The proof of Equation 3.20 and 3.21 can be found in Appendix A.1 and A.2 respectively.

$$Var[X] = E\Big[(X - E[X])^2\Big] \tag{3.20}$$

$$Var[X] = E\Big[Var[X|Y]\Big] + Var\Big[E[X|Y]\Big] \tag{3.21}$$

Applying Equation 3.21 to the pipeline from the main warehouse to the remote warehouse results in Equation 3.22.

$$Var[X_j] = E\Big[Var[X_j|X_{main}]\Big] + Var\Big[E[X_j|X_{main}]\Big] \tag{3.22}$$

To determine the first term of the variance, the expectation is rewritten. The variance differentiates between the case where the number of items on order is more than the items on stock at the main warehouse ($x_{main} > s_{main}$), and less or equal to the items on stock at the main warehouse ($x_{main} \leq s_{main}$).

If the number of items on order at the main warehouse is less or equal to the number of items on stock. The average number of items in the pipeline from the main warehouse to the remote warehouse is the average number of items on its way to the remote warehouse (Equation 3.23).

$$E\big[X_j|x_{main}\big] = m_j * shippingTime_j \qquad x_{main} \leq s_{main} \tag{3.23}$$

When the number of items on order is more than the items on stock at the main warehouse ($x_{main} > s_{main}$), the number of back orders will be $x_{main} - s_{main}$. Thus, the expected number of items in the pipeline to the remote will be the average items on its way to the warehouse, plus the fraction of back orders at the main warehouse designated for the remote (Equation 3.24).

$$E\big[X_j|x_{main}\big] = m_j * shippingTime_j + \frac{m_j}{m_{main}}(x_{main} - s_{main}) \qquad x_{main} > s_{main} \tag{3.24}$$

With the seperate equation for the expectation, the first term of the variance can be derived. Equation 3.22 shows the derivation of the first term of the variance.

$$Var\Big[X_j|x_{main}\Big] = \begin{cases} m_j * shippingTime_j, & x_{main} \leq s_{main} \\ m_j * shippingTime_j + \frac{m_j}{m_{main}}\Big(1 - \frac{m_j}{m_{main}}\Big)\Big(x_{main} - s_{main}\Big), & x_{main} > s_{main} \end{cases} \tag{3.25}$$

Determining the expectation of the variance results in equation 3.26.

$$E\Big[Var[X_j|X_{main}]\Big] = m_j * shippingTime_j + \frac{m_j}{m_{main}}\Big(1 - \frac{m_j}{m_{main}}\Big)EBO_{main} \tag{3.26}$$

The second term of Equation 3.22 is derived as follows:

$$\begin{aligned}
Var\Big[E[X_j|X_{main}]\Big] &= Var\Big[parentPipeline_{main}\Big] && (3.27)\\
&= Var\Big[m_j * shippingTime_j + \frac{m_j}{m_{main}}EBO_{main}\Big] && (3.28)\\
&= Var\Big[m_j * shippingTime_j\Big] + Var\Big[\frac{m_j}{m_{main}}EBO_{main}\Big] && (3.29)\\
&= 0 + \left(\frac{m_j}{m_{main}}\right)^2 Var\Big[EBO_{main}\Big] && (3.30)\\
&= \left(\frac{m_j}{m_{main}}\right)^2 VBO_{main} && (3.31)
\end{aligned}$$

Substituting Equation 3.26 and Equation 3.31 into Equation 3.22 gives the following variation for the pipeline at the remote locations.

$$Var[X_j] = m_j * shippingTime_j + \frac{m_j}{m_{main}}\left(1 - \frac{m_j}{m_{main}}\right)EBO_{main} + \left(\frac{m_j}{m_{main}}\right)^2 VBO_{main} \tag{3.32}$$

To determine the variance of the pipeline it is required to determine the variance of the back orders at the main warehouse. The variance of the back orders can be derived using the alternative form of the variance in equation 3.20. From this definition the recursive form of the variance of back-orders can be determined (Equation 3.33-3.34). Similar to the expected back-orders, $VBO(0)$ will per definition be equal to the variance of the demand.

$$\begin{aligned}
VBO(s) &= E[BO^2(s)] - (EBO[s])^2 && (3.33)\\
&= VBO(s-1) - EBO(s) - EBO(s-1) - (EBO(s))^2 + (EBO(s-1))^2 && (3.34)
\end{aligned}$$

3.5. Optimization parameter

Before the system can be optimized, a solution technique has to be determined. This section will first elaborate on selecting the solution technique. After this, the section continues with the definitions of the different optimization parameters.

3.5.1. Selecting the solution technique

Chapter 2.4 described a wide variety of solution techniques which can be used to optimize a inventory optimization model. This section will commence shortly on the process behind choosing the optimization parameter used during the project.

The introduction stated that the project is a collaboration between the TU Delft and KLM E&M. The selection of the solution technique is therefore based on the requirements of both stakeholders. Finding an exact solution for the inventory optimization problem can be ruled out. The high complexity of the multi-echelon problem does not allow for an exact solution, without making critical simplifications. Simplifying the model to a point where a exact solution is possible, does limit the usability of the model for KLM E&M and the academic novelty for the TU Delft.

The end users of the to be developed inventory optimization model are the specialists at KLM E&M. These specialists are required to perform analyses on the inventory levels, multiple times a day. The final inventory optimization model is therefore required to limit the computational load.

Considering computation time, the choice between solution techniques is quickly narrowed down to a linear programming approach. Simulation based approaches do provide better insight in the behaviour of the supply chain, but are computational to demanding.

The Lagrangian relaxation algorithm does entail benefits regarding the optimization procedure. The complexity of such a algorithm does unfortunately not fit in the scope of the project. This project will therefore uses a greedy algorithm as solution technique of the model. A secondary advantage of a greedy algorithm lays in the comparison with the current inventory optimization tool at KLM E&M. The current optimization program used by KLM E&M does utilize the greedy algorithm as well. Developing a new model with this same algorithm enables the specialists at KLM E&M to easily compare the two tools to each other.

The objective of KLM E&M is to increase operational profitability. To achieve the maximum operational profitability, investment costs should be minimized while still achieving the desired service level target. A marginal analysis lends itself eminently for this purpose. The marginal analysis makes the optimal decision during each iteration for that moment in time. The decision is based on the gain of service metric relative to the cost of this gain. The model developed over the course of this project will therefore use a greedy algorithm based on a marginal analysis. The next section will describe in more detail what the specific definition of marginal analysis parameter for this project.

3.5.2. Expected back order as optimization parameter

As mentioned in the definition of chapter 3.2, a back order occurs if an inventory item is not on hand when an order is placed. If the assumption that every item is critical for operation is made, consequently each back order will result in downtime of the end product. To minimize the downtime of the end product, the event of a back order should therefore be limited.

The expected back order based optimization parameter can be defined by substituting the expected back orders into equation 3.6. The final definition of the secret can be found in equation 3.35.

$$secret_{EBO} = \frac{EBO(s) - EBO(s+1)}{price} \tag{3.35}$$

3.5.3. Back order cost as optimization parameter

For most supply chain items it is possible to borrow an item externally if necessary. Especially the aviation spare part industry is well know for this type of operation. The cost of such a borrow may differ between the items in the supply chain. So, instead of solely optimizing the number of back orders, it may be interesting to minimize the cost of back orders as well.

To do so the expected borrow cost need to be defined. The expected borrow cost is simply defined by multiplying the expected number of back orders by the price of a back order. Equation 3.36 depicts the mathematical definition of the expected borrow cost, where the borrow cost is defined by the variable $cost$.

$$EBO_{cost}(s) = EBO(s) * cost \tag{3.36}$$

To minimize the total cost of the back orders, the secret defined in equation 3.37 can be used. The optimization parameter multiplies the cost of a back order, by the reduction in back orders if the stock would be increased. The profit gain by reducing the expected borrow cost is then divided by the price of the item, similar to the expected back order secret of equation 3.35.

$$secret_{cost} = \frac{EBO_{cost}(s) - EBO_{cost}(s+1)}{price} \tag{3.37}$$

3.5.4. Service level as optimization parameter

The objective of most supply chain optimization problems is to maximize the operational availability. The general definition for operational availability of maintenance supply chain is if it is not down for either maintenance or supply [18]. The operational availability can be calculated according to equation 3.38. Where $MTBM$ is the mean time between maintenance, and MDT the mean downtime due to spares, maintenance, and delays.

$$A_{operational} = \frac{MTBM}{MTBM + MDT} * 100\% \tag{3.38}$$

For this project the downtime due to maintenance is not relevant. Therefore it is useful to derive two separate expressions for the maintenance availability and supply availability. Sherbrooke [18] states that the maintenance and supply availability can be determined by equation 3.39 and equation 3.40 respectively.

$$A_{maintenance} = \frac{MTBM}{MTBM + MCTM + MPMT} * 100\% \tag{3.39}$$

$$A_{supply} = \frac{MTBM}{MTBM + MSD} * 100\% \tag{3.40}$$

In these equations $MCTM$ is the corrective maintenance time, $MPMT$ the preventive maintenance time, and MSD the mean supply delay. The operational availability, as defined in equation 3.38, can be approximated by multiplying the maintenance and supply availability. This approximation will result in an underestimation of the operational availability. However, for high availability rates will the error be minimal [18].

For the single echelon model described in Chapter 3.2 the supply availability can be rewritten to the mathematical form of equation 3.41. Where i denotes any item in the supply chain.

$$A_{Availability} = \prod_{i=1}^{I} \left(1 - \frac{EBO(s_i)}{FH * QPA_i}\right)^{QPA_i} * 100\% \tag{3.41}$$

The multi-echelon model of section 3.3 and 3.4 required an extra step to determine the availability. In a multi-echelon model the supply availability is determine by the combined availability at the remote warehouses. Equation 3.42 shows the mathematical representation of the availability parameter for a multi-echelon system. In this equation, j denotes any remote warehouse in the system. The availability at the main warehouse does not effect the system availability due to the assumption that all customer requests will take place at the remote warehouses.

$$A_{Availability} = \frac{\sum_{j=1}^{J} A_j * FH_j}{\sum_{j=1}^{J} FH_j} * 100\% \tag{3.42}$$

The optimal availability for the supply chain of KLM E&M differs from the availability rates described above. KLM E&M is not necessarily concerned with the mean supply delay. The contractual agreements of the KLM E&M customers are service level based. The service level is determined by the fraction of components delivered within the contractually agreed time span. To facilitate an optimization based on the achieved service level a new optimization parameter needs to be defined. The new optimization parameter is defined as the fraction of on time deliveries ($D_{achieved}$) compared to the total deliveries (D_{total}). The mathematical representation of the optimization parameter can be found in equation 3.43.

$$A_{servicelevel} = \frac{D_{achieved}}{D_{total}} * 100\% \tag{3.43}$$

The same availability parameter can be used in the marginal analysis as well. Equation 3.44 depicts the marginal analysis parameter based on the on time removals.

$$secret_{OTR}(s) = \frac{D_{achieved}(s+1) - D_{achieved}(s)}{price} \tag{3.44}$$

4

Implementation

4.1. Single-echelon model

The implementation of the single-echelon model depends heavily on the probability distribution forming the basis of the model. The equation defining the expected number of back orders (Equation 3.5) has a variable P, representing a discrete probability distribution. Chapter 3.1 elaborated on the fit between the supply chain of KLM E&M and different probability distributions. It is shown that the Poisson distribution results in the best fit, with the available resources and data sets. Assuming a Poisson distribution, will change the expected back order equation of chapter 3.2 to Equation 4.1. Instead of taking the sum the Poisson distribution, the cumulative distribution can be used.

$$\begin{aligned} EBO(s) &= EBO(s-1) - 1 - \sum_{x=0}^{s-1} Poisson(s, pipeline) \\ &= EBO(s-1) - 1 - Poisson.CDF(s-1, pipeline) \end{aligned} \tag{4.1}$$

The final single-echelon model implemented, consists of two steps. First the model is initialized, after which the model gets optimized. This section will provide details on both steps of the implemented model.

Initialization

During the initialization, the starting state of the model is determined. The starting state of the model consists of two parts, the initial service level and the optimization parameter of each component. The initial service level of the system is determined according to Equation 3.43. The optimization parameter is component specific, and is therefore initialized for each component individually. The optimization parameter is based on Equation 3.6. The specific *secret* value will depend on the chosen optimization parameter as explained in 3.5. For clarification, the complete initialization procedure is provided as pseudo code in Algorithm 1.

Algorithm 1 Initialization single-echelon

```
procedure INITIALIZE
    onTimeRemovals ← 0
    for all components do
        ΔserviceLevel ← serviceLevel(stock+1) − serviceLevel(stock)
        secretList[component] ← ΔserviceLevel

        onTimeRemovals ← onTimeRemovals + serviceLevel(stock) ∗ expectedRemovals
    systemServiceLevel ← onTimeRemovals / totalRemovals
```

Optimization

The optimization procedure of the single-echelon contains a while loop repeating itself until the optimization target is reached. Line 3 of Algorithm 2 selects the component with the highest value for the optimization parameter, as it is most beneficial to increase the stock of this component. To increase the stock of a component a couple of variables do require modification. Line 6 of the algorithm calls a different procedure to modify the variables required to increase the stock by one.

The INCREASESTOCK procedure increments the stock level of the component by one. After the new stock level is set, a new service level delta is determined. Finally, the new service level delta is saved to the data structure containing the reference between component and service level delta.

Increasing the stock of a component will naturally impact the service level of the system. The initialization procedure reveals that the system service level is determined based on the on time removals. To limit computational load, the on time removals variable is modified instead of recalculated. Line 4 of Algorithm 2 deducts the on time removals caused by the selected $maxComponent$. After the method to increment the stock is finished, a the new amount of on time removals is added to the total on time removals again (Line 8).

Algorithm 2 Optimization single-echelon

```
1:  procedure OPTIMIZE
2:      while finalServiceLevel < systemServiveLevel do
3:          maxComponent ← max(secretList)
4:          onTimeRemovals ← onTimeRemovals − serviceLevel(stock) ∗ expectedRemovals
5:
6:          goto INCREASESTOCK(maxComponent)
7:
8:          onTimeRemovals ← onTimeRemovals + serviceLevel(stock) ∗ expectedRemovals
9:          systemServiceLevel ← onTimeRemovals/ totalRemovals
10:
11: procedure INCREASESTOCK
12:     stock ← stock + 1
13:     ΔserviceLevel ← serviceLevel(stock+1) − serviceLevel(stock)
14:     secretList[component] ← ΔserviceLevel
```

4.2. Multi-echelon model

The implementation of the METRIC and VARI-METRIC model is identical. The difference between the model lays in the definition of the secret value. The implementation described in this section can therefore be applied to the METRIC and VARI-METRIC model.

The fist step of the implementation lays in the model structure. In the problem description of chapter 1 the supply chain structure of KLM E&M is described. This structure shows a direct link between the main warehouse and a customer. A multi-echelon model does not facilitate such a connection. During the development of the multi-echelon model it is assumed customer demand will only occur at the remote warehouses. To cope with this restriction a fifth virtual local warehouse is introduced. The new local warehouse, called SPL, is located in Amsterdam as well. Customer requesting components from the main warehouse will be server via this virtual warehouse. Figure 4.1 shows the implemented supply chain structure for the case study.

Similar to the single echelon model, does the multi-echelon model depend on a discrete probability distribution. Equation 3.14 contains the same variable P. Substituting the Poisson distribution will finally result in Equation 4.2 for the expected number of back orders.

$$P(X < s-1) = Poisson.CDF(s, pipeline) \tag{4.2}$$

The VARI-METRIC model utilizes the negative binomial distribution as well. Equation 4.3 shows the implementation of the negative binomial distribution for the variable P.

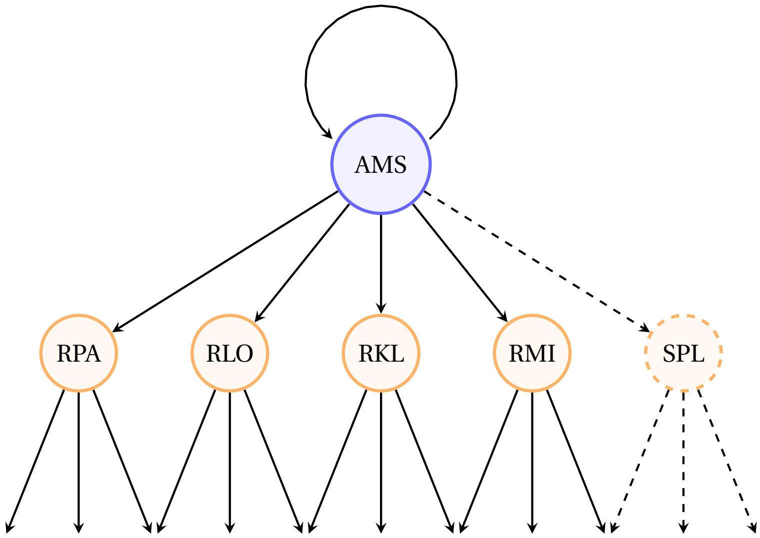


Figure 4.1: multi-echelon representation of the KLM E&M supply chain

$$P(X < s - 1) = NBD.CDF(s, pipeline, Var[X_j]) \tag{4.3}$$

The implementation of the multi-echelon model is closely related to the implementation of the single echelon model. It consists of two phases, an initialization phase, and an optimization phase. Due to the complexity of the model structure, the implementation increases in complexity too. Both phases will be discussed in more detail below.

Initialization

During the initialization phase, each component, at each location is initialized. At line 4, a different optimization parameter for an increase in stock at each location is determined.

Line 8 of Algorithm 3 shows the calculation of on time removals. The final on time removals to the customers are achieved at the local warehouses only, as the customer requests do only take place at the local warehouses.

Algorithm 3 Initialization multi-echelon

1: **procedure** INITIALIZE
2:     **for all** $components$ **do**
3:         **for all** $locations$ **do**
4:             Δ*serviceLevel* ← *serviceLevel(stock+1)* − *serviceLevel(stock)*
5:             *secretList*[$component, location$] ← Δ*serviceLevel*
6:
7:             **if** $location \neq main$ **then**
8:                 *onTimeRemovals* ← *onTimeRemovals* + *serviceLevel(stock)* ∗ *expectedRemovals*

Optimization

The optimization of the multi-echelon model is depicted in Algorithm 4. The optimization procedure is repeated until the final service level goal is reached. During each optimization step, the stock of component with the highest $secret$ value is increased by one. After each increment in stock, the service level of the system will increase as well. Due to the multi-echelon structure, the increase of stock of a component will effect the service level of the entire system. For the single echelon mode, it was sufficient to determine the increase in on time removals for the modified component. However, for the multi-echelon model the on time removals for the entire system have to be redetermined.

Algorithm 4 Optimization multi-echelon

```
procedure OPTIMIZE
    while finalServiceLevel < systemServiveLevel do
        maxComponent ← max(secretList)

        goto INCREASESTOCK(maxComponent)

        for all components do
            for all locations do
                if location ≠ main then
                    onTimeRemovals ← onTimeRemovals + serviceLevel(stock) ∗ expectedRemovals

        systemServiceLevel ← onTimeRemovals/ totalRemovals

procedure INCREASESTOCK
    stock ← stock + 1
    ΔserviceLevel ← serviceLevel(stock+1) − serviceLevel(stock)
    secretList[component, location] ← ΔserviceLevel
```

4.3. Optimization parameter

The expected back order, and expected back order cost optimization parameter can be calculated directly from the equations of the model. The service level optimization parameter does however require an extra step during implementation. Equation 3.43 depicts the mathematical definition of the optimization parameter. Determining the total demand of an item is fairly straight forward, by calculating the sum of the demand at each location. With the assumption of Poisson demand for each component, the $D_{achieved}$ can be determined according to Equation 4.4. The parameters of the cumulative distribution function ($Poisson_{CDF}$) are the stock level (s) and the expected number of units in the pipeline ($pipeline$). The definition of the pipeline varies between the single- and multi-echelon models. For the single-echelon model the pipeline is defined according to Equation 3.2. The definition of the pipeline for the multi-echelon models can be found in Equation 3.9.

$$D_{achieved} = Poisson_{CDF}(s-1, pipeline) * D_{total} \tag{4.4}$$

5

Verification

To ensure validity of the developed inventory optimization model, a verification of different steps of the optimization procedure is performed. The purpose of this section is threefold. The section lists the different verification steps with results, it motivates the chosen verification steps, and finally will discuss the limitation of the verification steps.

5.1. Motivation for verification procedure

The verification procedure of this chapter lays a relation between the mathematical model of chapter 3 and the implementation of chapter 4. The purpose of the verification procedure is to increase insight in the calculations performed during the optimization of the model. With this chapter, the reproducibility of the conducted research is meant to increase as well.

The mathematics of the single and multi-echelon model are closely related. To a large extend are the equations of the multi-echelon model a complex version of the single echelon model. Therefore, for the sake of simplicity does this chapter only perform the verification of the equations once. Each verification step highlights the single and multi-echelon equations belonging to the particular step.

The verification is performed in a bottom up procedure. The final optimization parameter of the system are the on time removals (OTR). In order to determine the on time removals, several calculation steps have to be performed. The verification procedure on the next chapter will therefore start on the bottom, and step wise proceed to the final on time removals of the system.

5.2. Verification of the inventory optimization model

In order to perform a verification of the inventory optimization model, a couple of input parameters are defined. Table 5.1 lists the parameters belonging to a typical fast moving and slow moving component of the KLM E&M case described in chapter 1. The fast moving component selected as example is a crew oxygen cylinder. The slow moving component selected is a radar antenna placed in the nose of the aircraft.

For the multi-echelon model it is necessary to determine the number of flight hours for customers related to a remote warehouse. KLM E&M does currently not have these numbers. Therefore an estimation has been made. It is assumed that a customer request is always fulfilled from the warehouse closest to the hub airport of the customer. For example, all component requests from airlines located in Asia will be shipped from the warehouse in Kuala Lumpur. Using this assumption, the number of aircraft served by each warehouse can be determined. The final flight hours are calculated by multiplying the number of aircraft by the average flight hours of an aircraft per year. For KLM E&M, the average flight hours per year is set to 3190. The estimated number of flight hours for the two components are provided in Table 5.2.

Next to the flight hours, the multi-echelon model depends on the shipping time between the warehouses. The supply chain of KLM E&M is a global operation. The shipping time between the warehouses therefore

Table 5.1: Verification components parameters

	Fast mover	**Slow mover**
Family number	221	62
Name	Oxygen cylinder	Radar antenna
QPA	1	1
$MTBR$ (hours)	2172	515568
Price	$ 4.319	$ 23.735
Flight hours	535.421	520.905
TAT (days)	30	34
Exp removals (year)	246,6	1,12
AIP	24,86	0,10

Table 5.2: Components flight hours and shipping times

	Fast mover	**Slow mover**	**Shipping time**
RPA	73.297	51.580	3 days
RLO	117.007	0	3 days
RKL	150.095	397.337	5 days
RMI	0	0	5 days
SPL	195.022	71.988	0 days
Total	535.421	520.905	-

varies per location. For the European warehouses a shipping time of 3 days is the norm. The shipping time between Amsterdam and a non European warehouses is 5 days. An overview of the shipping times can be found in Table 5.2.

The first step of the model is to determine the expected removals (m) per year. Equation 3.1 and 3.7 are used to calculate the expected removals for the single- and multi-echelon model. As explained in Chapter 1, does KLM E&M at this moment not perform repair actions at the local warehouses. The local repair parameter (r_j) is therefore set to 0 for all remote warehouses. Equation 5.1 provides the demand calculation for one local warehouse. The demand at RPA for the fast moving component is used as an example. The calculation show that the demand for the fast moving component at RPA is expected to be 33.75 per year

$$m_{RPA} = \frac{FH * QPA}{MTBR} = \frac{73.297 * 1}{2172} = 33.75 \text{ per year} \tag{5.1}$$

The demand calculation for the main warehouse of the multi-echelon model differs from the equation described above. Equation 5.2 shows the demand calculation of the main warehouse for the multi-echelon model. Again, the fast mover component is used as an example. Calculating the demand for the single echelon model, or different components is considered trivial. The demand at the main warehouse for the fast moving component is 302.55 per year.

$$\begin{aligned} m_{main} &= \sum_{j=1}^{J} (1 - r_j) m_j \\ &= (1-0) * m_{RPA} + (1-0) * m_{RLO} + (1-0) * m_{RKL} + (1-0) * m_{RMI} + (1-0) * m_{SPL} \\ &= 33.75 + 53.87 + 69.10 + 0 + 89.79 \\ &= 246.51 \end{aligned} \tag{5.2}$$

The next step is to determine the number of components in the pipeline. The pipeline calculation for the single echelon model (Equation 3.2) is equal to the pipeline of the main warehouse in a multi-echelon model (Equation 3.8). Equation 5.3 shows that the expected number of components in the pipeline of the main warehouse for the fast moving component is 20.26 at any given point in time.

$$pipeline_{main} = m_{main} * TAT_{main} = 246.51 * \frac{30}{365} = 20.26 \tag{5.3}$$

The pipeline of the remote warehouses for the multi-echelon model depends on the expected back orders (EBO) of the main warehouse, as can be seen Equation 3.11. The definition of expected back orders at the main warehouse is given in Equation 3.12. Combined with the discussed implementation of the probability distribution in Chapter 4, are the expected back orders determined according to Equation 5.4. The verification of the expected number of back orders for the main warehouse is performed for two iterations, the case where stock is 0 and stock is 1. The parameters of the fast moving component are substituted in Equation 5.4. For the case where stock is 0, the number of expected back orders is 20.26. Chapter 3.3 already stated the fact that, in case of zero stock, the expected back orders always equals the pipeline. For the case where stock is 1, the recursive form of the back orders from Equation 4.1 is used. Due to the high demand and low stock, decreases the number of expected back orders by approximately one. As the stock increases the decrease in expected back orders for each step will convert to zero. Chapter 7 will further elaborate on this behaviour.

$$\begin{aligned}
EBO_{main}(stock=0) &= pipeline_{main} - \sum_{s=0}^{s=0} 1 - P(X < s-1) \\
&= 20.26 \\
EBO_{main}(stock=1) &= EBO(stock-1) - \Big(1 - \sum_{s=0}^{s=stock} P(X < s-1)\Big) \\
&= EBO(0) - \big(1 - Poisson_{CDF}(0, pipeline_{main})\big) \\
&= 20.26 - (1 - 1.58 * 10^{-11}) \\
&= 19.26
\end{aligned} \tag{5.4}$$

The pipeline of the remote warehouse consists of two terms. The first term, the local pipeline, represents components in the local repair loop. The second term, the parent pipeline, represents components shipped from the main warehouse. The assumption of no local repair, results in a local pipeline of 0. Equation B.5 shows the calculation of the local pipeline. .

$$localPipeline_{RPA} = m_{RPA} * r_{RPA} * TAT_{RPA} = 33.75 * 0 * TAT_{RPA} = 0 \tag{5.5}$$

As mentioned before, the parent pipeline depends on the expected number of back orders. Equation B.6 shows the parent pipeline for the case of 1 stock at the main warehouse. The shipping time between the main warehouse and the Paris warehouse is 3 days.

$$\begin{aligned}
parentPipeline_{RPA} &= m_j * (1 - r_j) * \left(shippingTime_j + \frac{EBO_{main}}{m_{main}}\right) \\
&= m_{RPA} * (1 - r_{RPA}) * \left(shippingTime_{RPA} + \frac{EBO_{main}}{m_{main}}\right) \\
&= 33.75 * (1 - 0) * \left(\frac{3}{365} + \frac{19.26}{246.51}\right) \\
&= 2.91
\end{aligned} \tag{5.6}$$

The final pipeline of the remote location with 1 stock at the main warehouse can be found in Equation 5.7. Due to the assumption of no local repair, the final pipeline will be equal to the parent pipeline.

$$pipeline_{RPA} = localPipeline_{RPA} + parentPipeline_{RPA} = 0 + 2.91 = 2.91 \tag{5.7}$$

With the pipeline of the local warehouse, the expected back orders at the local warehouse can be determined. The calculation of the expected back orders at the local warehouse is different for the METRIC and VARI-METRIC model, due to the different implementation of the probability distribution P. The procedure for the METRIC model is identical to the expected back order calculation of the main warehouse. The verification of the local warehouse expected back orders for the METRIC model can be found in Equation 5.8.

$$\begin{aligned} EBO_{RPA}(stock=0) &= pipeline_{RPA} \\ &= 2.91 \\ EBO_{RPA}(stock=1) &= EBO(0) - \left(1 - Poisson_{CDF}(0, pipeline_{RPA})\right) \\ &= 2.91 - (1 - 0.05) \\ &= 1.96 \end{aligned} \tag{5.8}$$

The expected back orders for a local warehouse of the VARI-METRIC model is defined differently. Chapter 4 discussed the probability required for the VARI-METRIC model. Next to the pipeline, this model requires the variance of the pipeline as well. Before determining the variance of the pipeline, it is necessary to calculate the variance of the back orders at the main warehouse (VBO) first. The variance of the back order can be obtained by substituting the corresponding values in equation 3.34. Equation 5.9 contains the variance for the case of 0 and 1 stock at the main warehouse.

$$\begin{aligned} VBO_{main}(stock=0) &= pipeline_{main} \\ &= 20.26 \\ VBO_{main}(stock=1) &= VBO_{main}(0) - EBO_{main}(1) - EBO_{main}(0) - \left(EBO_{main}(1)\right)^2 + \left(EBO_{main}(0)\right)^2 \\ &= 20.26 - 19.26 - 20.26 - 19.26^2 + 20.26^2 \\ &= 20.26 \end{aligned} \tag{5.9}$$

The variance of the local pipeline can be determined according to equation 3.22. The variance of the pipeline of RPA, with a stock level at the main warehouse of 1, can be found in equation 5.10.

$$\begin{aligned} Var\left[X_{RPA}\right] &= m_{RPA} * shippingTime_{RPA} + \frac{m_{RPA}}{m_{main}}\left(1 - \frac{m_j}{m_{main}}\right)EBO_{main} + \left(\frac{m_{RPA}}{m_{main}}\right)^2 VBO_{main} \\ &= 33.75 * \frac{3}{365} + \frac{33.75}{246.51}\left(1 - \frac{33.75}{246.51}\right) * 19.26 + \left(\frac{33.75}{246.51}\right)^2 * 20.26 \\ &= 2.93 \end{aligned} \tag{5.10}$$

With the variance of the pipeline it is possible to determine the expected back orders according to the VARI-METRIC model. Equation 5.11 shows the calculation of the expected back orders at RPA with a stock level at the main warehouse of 0 and 1.

$$\begin{aligned} EBO_{RPA}(stock=0) &= pipeline_{RPA} \\ &= 2.91 \\ EBO_{RPA}(stock=1) &= EBO(0) - \left(1 - NBD_{CDF}(0, pipeline_{RPA}, Var\left[X_{RPA}\right])\right) \\ &= 2.91 - (1 - 0.051) \\ &= 1.96 \end{aligned} \tag{5.11}$$

With the expected back orders of the local warehouse, the final optimization parameter can be determined. Depending on the desired optimization parameter the final calculations of the model differ. The solution for the three different optimization parameters will be shown in detail below.

The optimization parameter based on the expected back orders, as described in Equation 3.35, can be determined directly from the values discussed before. The implementation of the expected back order optimization parameter can be found in Equation 5.12.

$$secret_{EBO}(stock = 0) = \frac{EBO(0) - EBO(1)}{price} = \frac{2.91 - 1.96}{4319} = 2.20 * 10^{-4} \tag{5.12}$$

The optimization parameter based on the increase in the theoretical on time removals is defined before in Equation 3.44. Before this optimization parameter can be determined, the on time removals need to be determined. Equation 5.13 shows the on time removals based on Equation 4.4 for the multi-echelon method. The implementation for the single echelon model is similar to the implementation shown below.

$$\begin{aligned} D_{achieved}(stock = 0) &= 0 * D_{total} \\ &= 0 \\ D_{achieved}(stock = 1) &= Poisson_{CDF}(0, pipeline_{RPA}) * D_{total} \\ &= 0.05 * 33.75 \\ &= 1.69 \end{aligned} \tag{5.13}$$

The marginal analysis based on the on time removals can be found in Equation 5.14.

$$secret_{OTR}(stock = 0) = \frac{D_{achieved}(1) - D_{achieved}(0)}{price} = \frac{1.69 - 0}{4319} = 3.91 * 10^{-4} \tag{5.14}$$

The final service level at the remote location is obtained according to equation 5.15. If the stock levels at the main and remote warehouse are both 1, the service level will be 5%.

$$serviceLevel(stock = 1) = \frac{D_{achieved_{RPA}}}{m_{RPA}} * 100\% = \frac{1.69}{33.75} * 100\% = 5\% \tag{5.15}$$

The procedure can be repeated for the slow moving component. After the performed verification of the fast moving component, are the calculation for the slow moving component trivial. For the sake of fullness are the calculations included in appendix B.

5.3. Limitations of model verification

The model verification of the previous sector is limited to a single item. Performing a verification for a multi item system is to computational demanding to be done by hand. The verification of the multi item model will be covered during the sensitivity analysis in chapter 7.

The optimization based on service level is not included in the verification procedure. The unexpected complex characteristic of this optimization parameter, could not be translated in a usable model over the course of this project. The obtained results concerning the service level optimization are discussed in more detail in chapter 6.

6

Results

The main goal of the project is to determine the required amount of stock to achieve a set service level target. Over the course of this thesis, three different models have been suggested to determine the required stock. Namely, the single echelon model, the METRIC model, and the VARI-METRIC model.

This chapter will discuss the ideal amount and distribution of aircraft components according to the three different models. To do so, the parameters of the different components need to be defined. The fist section will therefore discuss the scope of the specific case study at KLM E&M. The second section will discuss the obtained results. A requirement of the model is a reasonable computational load. The last section of this chapter will provide insight in the computation time of the different models.

6.1. Scope of the case study

Chapter 3.1 already stated the fact that the entire pool of components at KLM E&M consists of 1678 different family numbers. The results obtained in this chapter are based on this entire pool of components. The parameters of the supply chain components of the case study vary greatly. Chapter 3.1 discussed the variety and distribution of the demand for the different components. This analysis revealed large differences in demand for the components. Next to the demand, other parameters do vary between the components as well. To give insight in the parameters of the different components, this section will show and discuss the limits of the different parameters.

Table 6.1 provides an overview of the 5 main defining parameters of the aircraft components. Additionally the expected number of removals are included as well. The expected number of removals can be calculated using the TAT, $MTBR$, and QPA according to equation 3.1. The final expected removals do however provide an insightful look on the characteristics of the component pool.

Table 6.1: Limits of the parameters within the scope

Parameter	**Minimum**	**Maximum**	**Mean**	**Median**
$MTBR$ (hours)	949	99.999.999	38.622.004	540.000
QPA	1	22	1,71	1
TAT (days)	23	58	38,99	38
Price	$ 114	$ 965.759	$ 38.440	$ 12.567
Flight hours	0	1.595.000	302.601	178.640
Exp removals (year)	0	348,1	6,65	0,01

The widest range in the parameters can be found in the $MTBR$. The component with the shortest lifetime will on average be removed after 949 flight hours. On the other hand, there are components which practically never fail. The model can not cope with infinite numbers. Components which barely fail are therefore given a $MTBR$ of 99.999.999 hours. The mean $MTBR$ is disproportionately influenced by the components with the maximum $MTBR$. The median of 540.000 hours does therefore provide a better insight in the distribution of the $MTBR$.

Most components on an aircraft are unique. Occasionally, it does happen a component is present multiple times. This can happen due to, for example, redundancy purposes. The mean and median of the QPA do confirm this. The highest QPA of the component pool of this case study belongs to a light assembly. This component does occur 22 times in a Boeing 737.

The TAT of the components within the supply chain of the case study varies between 23 and 58 days. The majority of the components have a TAT between 30 and 40 days, as is indicated by the mean and median as well. Chapter 7 will provide some insight and motivation for the goal of KLM E&M to reduce the TAT.

As mentioned in the introduction, can aircraft components be considered as expensive items. The prices in table 6.1 do confirm this. The most expensive component, the airborne weather radar, is almost 1 million dollars. Although this component is an extreme example, the average price lays still above 300.000 dollars. The price of a component plays an important factor in the determination of the optimization parameter.

The expected removals provide a good insight on the characteristics of the component pool. Some components with a very large MTRB and low amount of flight hours are not expected to be removed at all. Table 6.1 confirms this by showing that the minimum expected number of removals is 0. Components with a small $MTBR$ and/or a large number of flight hours are expected to be removed often. The most removed item of the case study is the oxygen cylinder, with 348 expected removals per year. The demand analysis of chapter 3.1 already revealed the fact that the majority of items do only get removed rarely. The median of the expected removals does again confirm this statistic.

6.2. Results of the case study

This section contains the results obtained by the single echelon, METRIC, and VARI-METRIC model for the scope described in the previous section. Each optimization is iterated until an overall service level of 95% is reached. First the results of the three models optimized to a minimum number of expected back orders is discussed. Secondly, the results of the models for the expected back order cost optimization are discussed. At the end of the section a short overview of the required computational time is provided.

6.2.1. Expected back order optimization

Figure 6.1 shows the investment required to achieve a theoretical 95% service level across all locations and components, according to the three different models. The investment is broken down into the different locations for the multi-echelon models.

The total investment is the lowest for the single echelon model. This characteristic may suggest that the single echelon is preferable. However, as discussed in chapter 2.1, does the single echelon model underestimate the losses induced by the shipping time between the different locations. The multi-echelon models do take into account the shipping times and will therefore be more expensive. The total investment of the METRIC and VARI-METRIC model are similar. The investment for the VARI-METRIC model is slightly less compared to the METRIC model. Compared to the total investment, this difference can however be neglected.

The difference between the METRIC and VARI-METRIC model do reveal them self in the number of items in the warehouses. The different approach on the probability distribution for the pipeline to the remote warehouses, does result in an increase in bought items for the VARI-METRIC model. The VARI-METRIC model prefers to increase the stock of cheaper items which get removed more often.

It is interesting to note the relatively large number of items placed at the RKL warehouse. The fraction of demand for this warehouse (±19%) is slightly larger compared to the demand at RPA (±16%). However, the difference in investment and number of items is higher. The main driver for this difference lays be in the different shipping time between the main warehouse and the remote warehouse.

Besides looking at the results of the total system. It is interesting to look at the results of a single component as well. The output of the model contains an overview of the performance of each individual component in the pool. To provide insight in the output of a specific component, the results of the fast mover of chapter 5

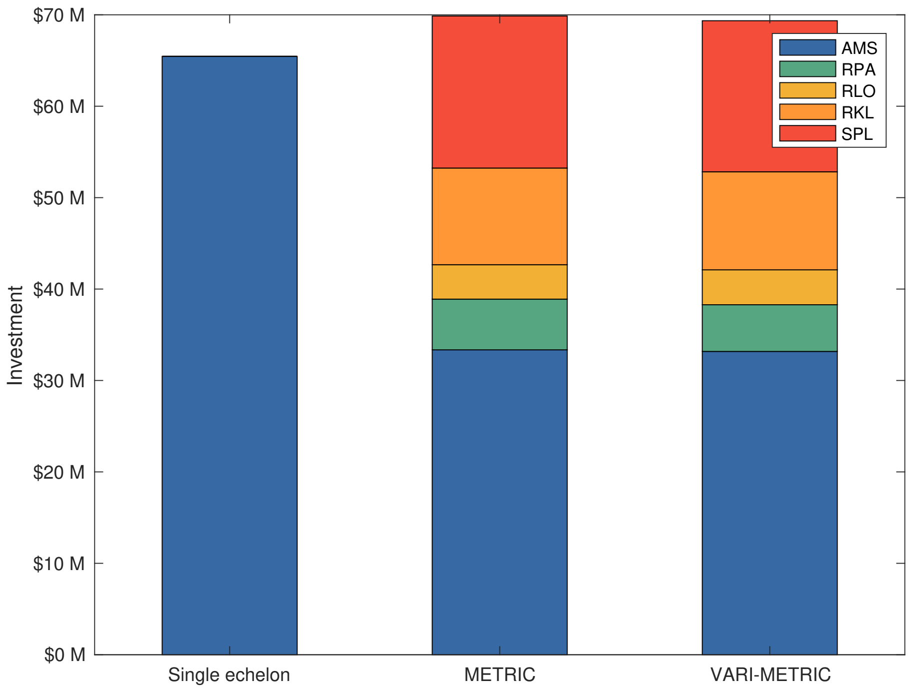


Figure 6.1: Expected back order optimization investment

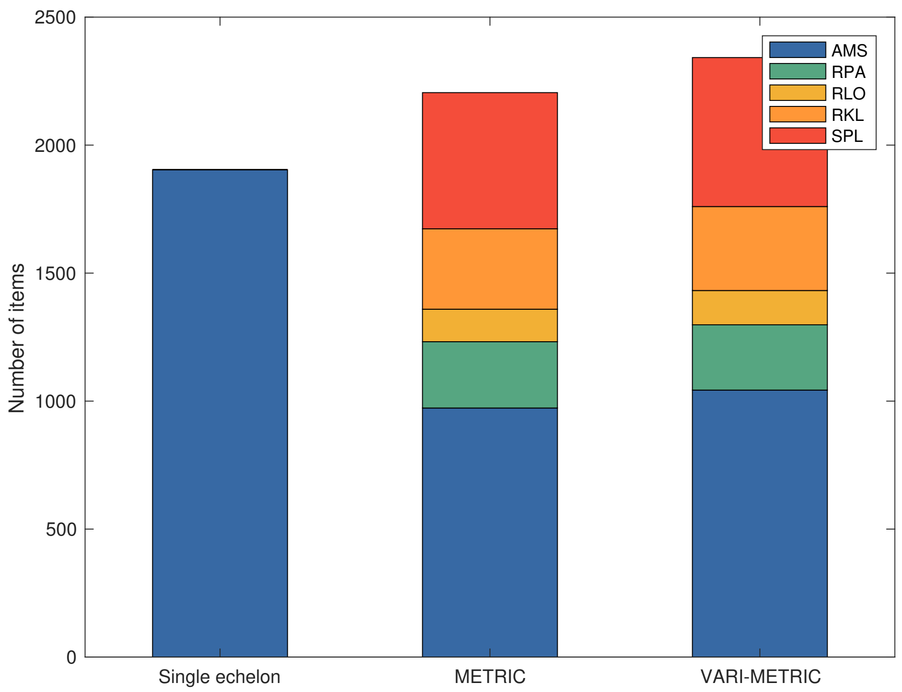


Figure 6.2: Expected back order optimization number of items

are graphically represented in figure 6.3. The figure contains the following information. Within the circle, the optimal stock at each location is provided. The average number of components for the fast moving component in the repair loop of the main warehouse, known as the pipeline, is about 20. Finally, the theoretical service level to the customers at the individual locations is provided. The weighted average service level for the entire fast moving component is 99.96%.

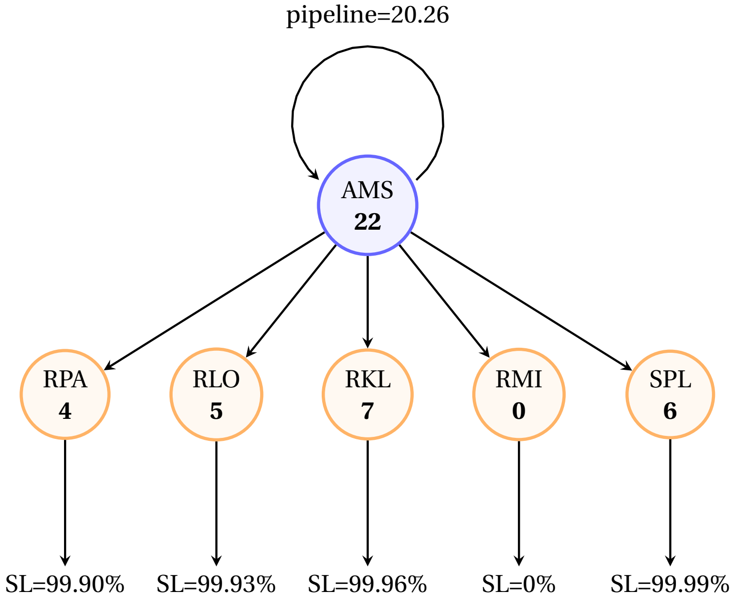


Figure 6.3: Expected back order optimization detailed result family 221

Similar to the fast moving component, the results of the slow moving component are provided. Figure 6.4 provides a graphical overview of the results. The expected number of components in the pipeline is 0.094. The model does only forward deploy stock to RKL. This warehouse is able to provide a service level of 98.62%. The other warehouses wil therefore per definition experience 0% service level. The weighted average of the service level is 75.23%. The results do correspond to the values of the verification procedure of the slow moving component in appendix B.

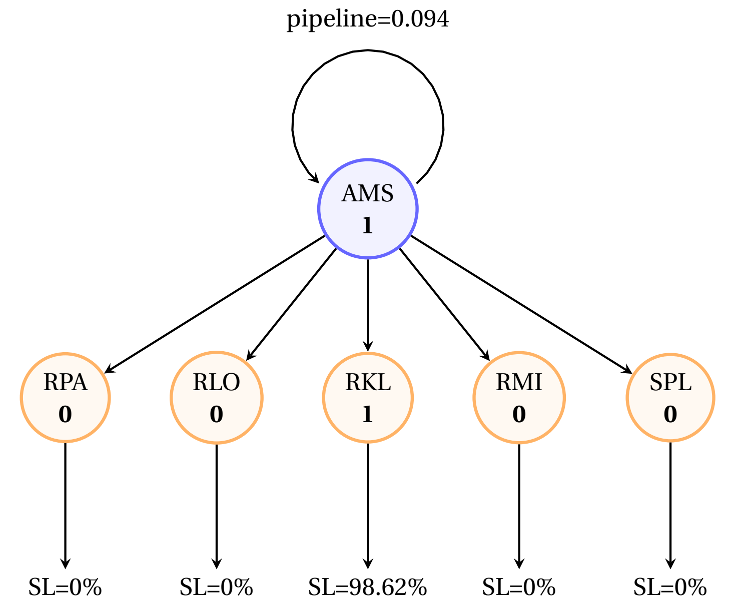


Figure 6.4: Expected back order optimization detailed result family 62

The service level of the fast moving component is above the 95% service level target of the optimization. The

Figure 6.5: Weighted service level versus expected number of removals

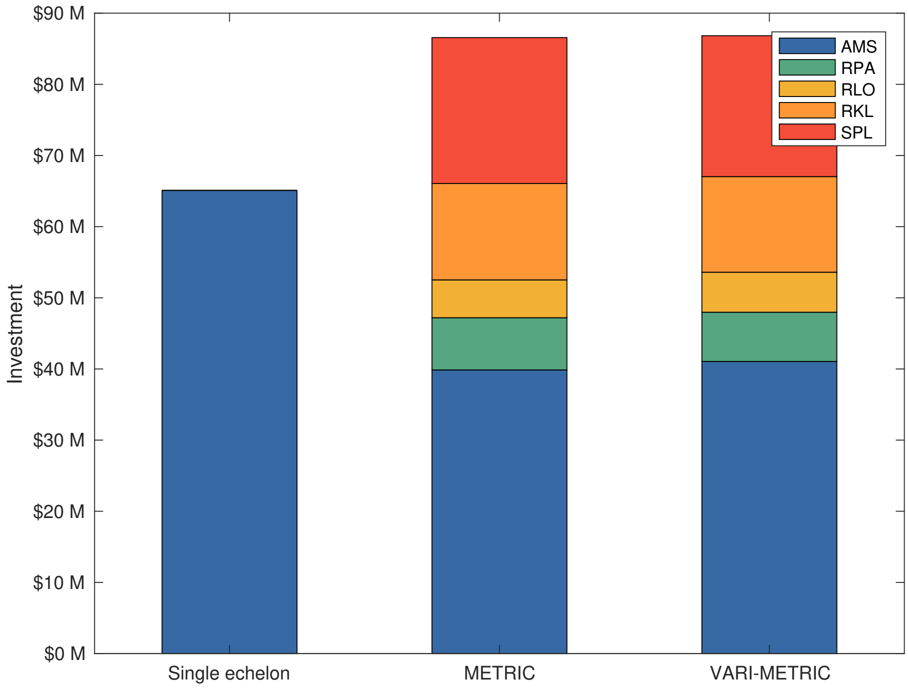


Figure 6.6: Expected back order cost optimization investment

high amount of expected removals, combined with the relatively low price makes this component preferable for the greedy algorithm. In general it can be stated that components with these characteristics achieve a high service level. Figure 6.5 provides a graph containing the average service level of each individual component. The figure contains a couple of outliers, marked in red. The high price of these components reduces the preferability of the components drastically. Although the relatively high amount of expected removals, is the average weighted service level below the 95% target.

6.2.2. Expected back order cost optimization

This section provides the results of the single echelon, METRIC, and VARI-METRIC model optimized to the cost of an expected back order. Determining the cost of an expected back order, or in the case of KLM E&M the borrow cost, is not trivial. For the case study of this thesis does the borrow cost vary significantly per event. The cost of a borrow among other things depend on the current market behaviour, duration, and availability per vendor. Analyzing and forecasting the cost of a borrow is outside the scope of this study. As a rule of thumb it can be said that the borrow cost is about 50% of the price of the component. The results in this section are obtained applying this rule of thumb. The results are therefore merely useful for the analysis of the behaviour of the model. Further research will be required to fit this model to the supply chain at KLM E&M.

Figure 6.6 contains the investment required to achieve the 95% service level target for the three different models. The results are inline with the results of the expected back order models. The investment of the single echelon model is less compared to the multi-echelon models. The investment of the METRIC and VARI-METRIC models are similar, with a slight advantage of the VARI-METRIC model. The investment of all three models is consistently higher compared to the expected back order models. The borrow cost decreases the effect of the prize on the optimization parameter. Therefore, more expensive items are held on hand.

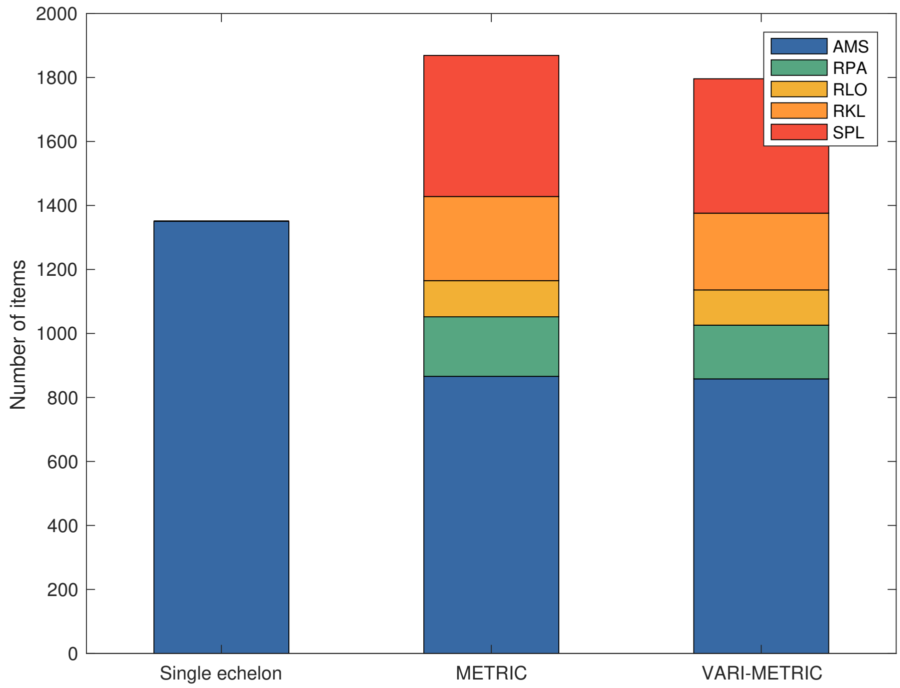


Figure 6.7: Expected back order cost optimization number of items

The increase in preferability of more expensive items does result in a lower total number of components. Figure 6.7 shows the number of items required to achieve the target service level by the three different models. Interestingly does the VARI-METRIC model this time invest in more expensive items compared to the METRIC model. An explanation for this behaviour can be given with the help of figure 6.8. This figure shows the final weighted service level of each component, versus the demand of the component. In general, if the demand for a component is high, the service level of this component will be high as well. The expected back order model does however contain a few outliers. Three outliers are highlighted with a red circle in figure 6.8. These three outliers are relatively expensive items, with a prices of $437.442, $524.137, and $350.037. The model optimized to the expected back orders does not invest to heavily in these items, as it is expensive to do so. Contrarily, the expected back order cost model does invest in these items. The cost of a back order is high as well, so the negative effect of these items is reduced.

6.2.3. Required computational time

A requirement opposed by KLM E&M for the optimization model is the computational load of the model. The specialists at KLM E&M do regularly need to perform inventory optimization calculations. A model relying on extensive computational performance is therefore not sufficient. This section will discuss the required computational time for the inventory optimization model developed for this project.

The results of this section are obtained by running the optimization model on an average end user laptop. An overview of the specific configuration of the laptop can be found in Appendix E.

The greedy algorithm increments the inventory by one unit during each optimization step. Existing supply chains usually have an initial amount of stock. Including the initial stock in the input of the optimization model, reduces the computational time. Hence, the model requires less iterations to achieve the desired service level target. The results of the this entire chapter, including the computational times of this section assume that there is no initial stock. The comparison of computational time can therefore be considered as a worst case scenario for the case study.

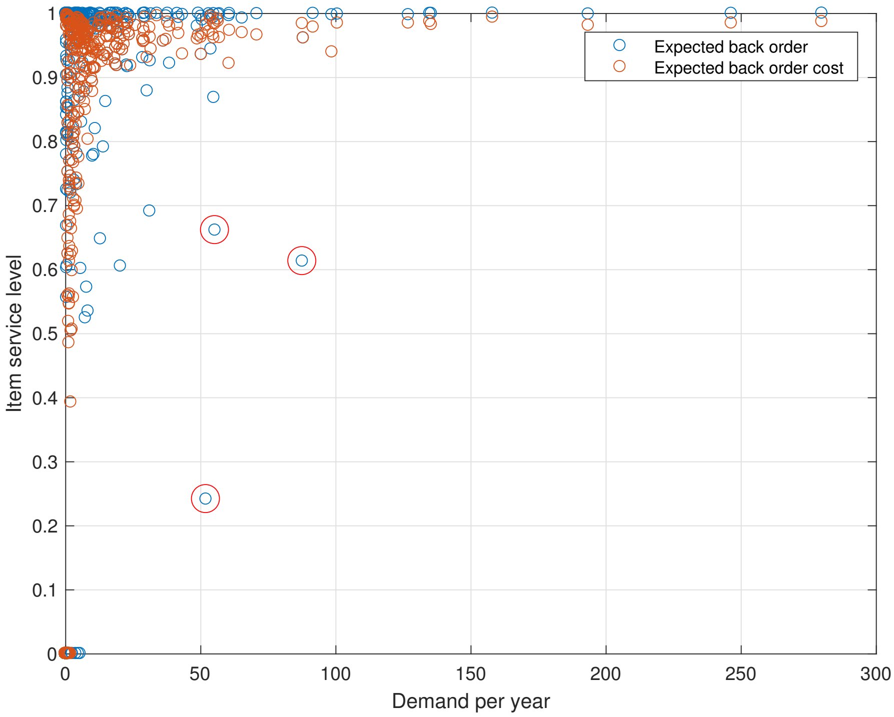


Figure 6.8: Weighted service level versus expected number of removals for the VARI-METRIC model

Table 6.2: Computation times METRIC and VARI-METRIC model

Method	Model	Time (s)	Increase
Expected back order	METRIC	532	7,0%
	VARI-METRIC	569	
Expected back order cost	METRIC	480	-5,8%
	VARI-METRIC	452	

Table 6.2 lists the computation time of the two multi-echelon models. The single echelon model is included in the final multi-echelon models. Each time a multi-echelon model is optimized, the single-echelon model will be optimized as well. It can be seen that the computation time of the models lays between 7,5 and 9,5 minutes. As mentioned in chapter 2.1 does the implementation of the VARI-METRIC model come with a increase in computation time. This behaviour does indeed appear if the model is optimized according to the expected back order method. For the expected back order cost optimization it is more complicated to observe this behaviour. Due to the increase in item count, and therefore iteration steps, is the computation time of the METRIC model slightly higher. Although significant, are these computation times acceptable for the usage case at KLM E&M.

7

Sensitivity analysis

The validation of the developed models is done through the technique of a sensitivity analysis. The complexity of the iterative METRIC and VARI-METRIC model does not allow it to manually check each step of the calculation. To ensure the correct performance of the model, a sensitivity analysis is performed. This chapter discusses the sensitivity analysis of the METRIC and VARI-METRIC model. The sensitivity analysis is divided in two sections. The first section discusses the sensitivity analysis performed for the METRIC and VARI-METRIC model, using the expected back order optimization parameter. The second section will discuss the sensitivity analysis for the METRIC and VARI-METRIC model optimized by the expected back order cost parameter.

7.1. Expected back order optimization

The sensitivity analysis is performed on two parameters of the aircraft components, namely the $MTBR$ and the TAT. The $MTBR$ does directly effect the demand for the component. Decreasing the $MTBR$, will increase the demand for the component.

To solely test the effect of the $MTBR$ on the model, the scope of the sensitivity analysis is reduced to only one item. The results shown in this section are obtained with the parameters of the fast moving component described in chapter 5. Each run is optimized until an overall service level of 95% is reached. Figure 7.1 and figure 7.2 show the results for the METRIC and VARI-METRIC model respectively. Each stacked bar in the figures represent a single run of the model. The $MTBR$ of the input is varied for each of the seven runs. The actual $MTRB$ of the component is 2172 hours. During the analysis the $MTBR$ is varied between 500 hours and 3000 hours with steps of 500 hours.

The METRIC and VARI-METRIC model behave in a very similar way. As expected, does the number of items increase when the $MTBR$ decreases. If the average time a component stays on an aircraft is shorter, more stock will be required to still achieve the 95% service level. The final investment required to achieve the desired service level, increases as well. The scope of the sensitivity analysis only consist of one component, with one price. The final investment of the model is therefore trivial and not provided in a figure.

Similar to the $MTBR$, the TAT can be varied. If the TAT decreases, a component will reside shorter in the repair loop of the supply chain. A lower TAT is therefore expected to reduce the required number of components. The sensitivity analysis for the TAT is performed on the same scope as the $MTBR$ analysis described before. Only this time, the TAT is varied between 15 and 35 days with steps of 5 days. Figure 7.3 and figure 7.4 show the results for the METRIC and VARI-METRIC model respectively. As expected does the number of items decrease when the TAT is reduced.

Besides analyzing the behaviour of the model, does the above performed analysis fulfill a second goal. To increase the operational profitability at KLM E&M, it is necessary to achieve the contracted 95% service level with the least amount of investment as possible. As proven in the analysis of this chapter, can a reduction

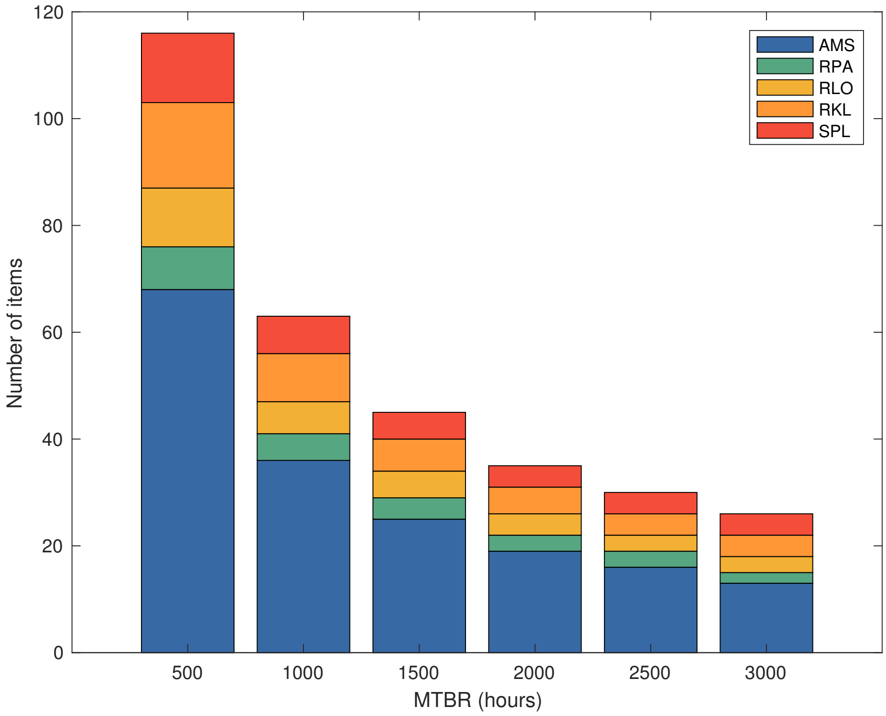


Figure 7.1: Total stock METRIC model for fast moving component with varying MTBR

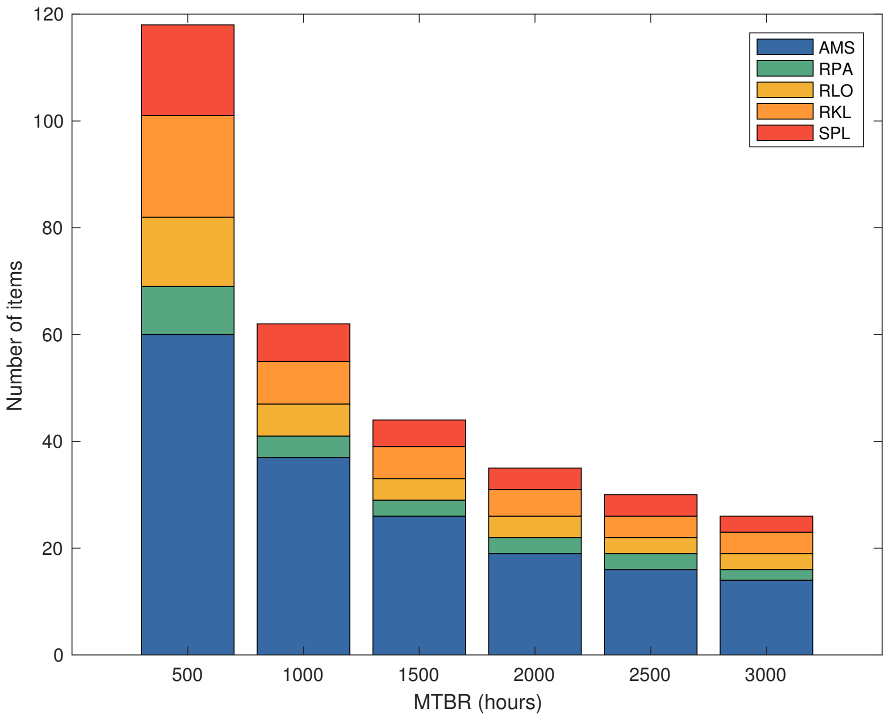


Figure 7.2: Total stock VARI-METRIC model for fast moving component with varying MTBR

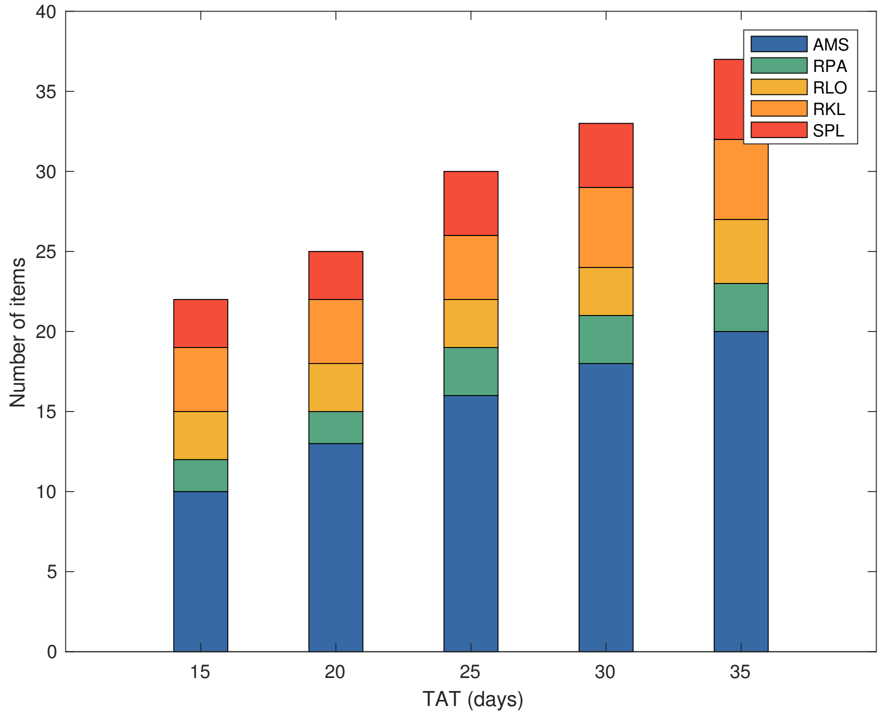


Figure 7.3: Total stock METRIC model for the fast moving component with varying TAT

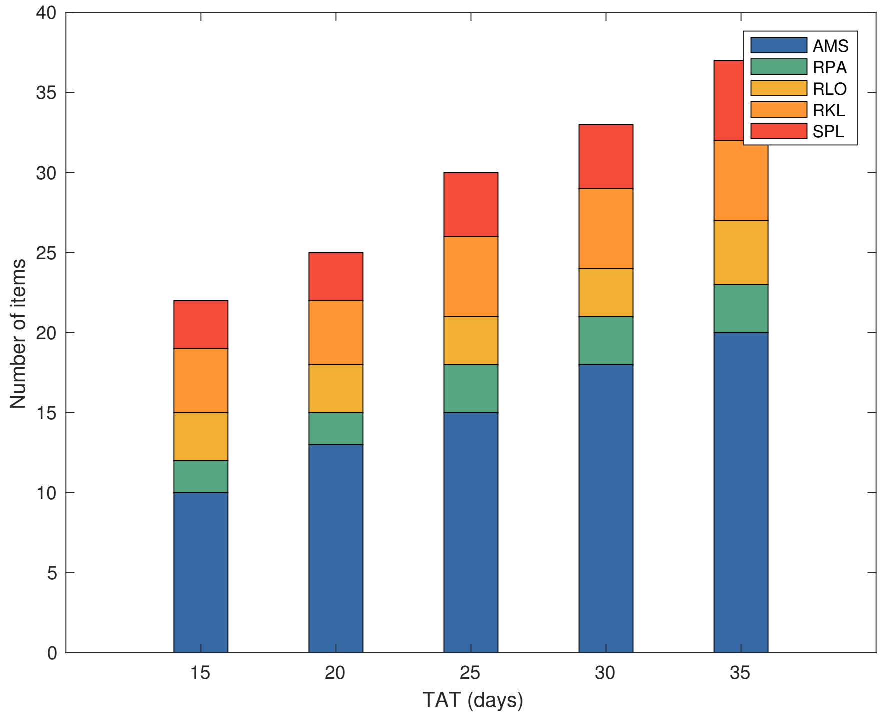


Figure 7.4: Total stock VARI-METRIC model for the fast moving component with varying TAT

in required number items be achieved at two ways. KLM E&M can either increase the $MTBR$ of the components, or decrease the TAT.

It is difficult to increase the operating time of a component before it fails. Some components do get a fixed $MTBR$ from the manufacturer. After this fixed amount of flight hours an airline is obligated to replace the component. In case the component does not contain such a restriction the reliability of a component could be increased. This can be achieved by performing a modification campaign on the component. Such campaigns do however require investment, so significant improvement in the $MTBR$ has to be obtained to make such a campaign profitable. The analysis of this chapter enables insight in the potential decrease in investment if the $MTBR$ would increase.

Decreasing the TAT is a second way to increase operational profitability. The repair loop of a component at KLM E&M consists of a couple of sections, varying per component. In general the repair loop does at least contain the sections shipping, logistic handling, and repair process. By reducing the time required for any of the sections the overall TAT can be reduced. At the time of writing KLM E&M is conducting a business redesign process to, among others, reduce the TAT. The objective of KLM E&M is to reduce the total TAT to 14 days. The analysis of this chapter indicates that such a reduction can significantly reduce the required inventory investment.

7.2. Expected back order cost optimization

This section describes the sensitivity analysis of the METRIC and VARI-METRIC model optimized by the cost of an expected back order. The first step of the sensitivity analysis is identical to the analysis performed in the previous section. Varying the TAT and $MTBR$ does result in the same characteristics as if the model is optimized by the expected back orders. The results of this analysis can be found in appendix D.

However, the sensitivity analysis for the cost optimization can be taken a step further. Changing the cost of a back order will influence the preferability of the component during the optimization. If the cost of a back order is increased, the component will get less preferable. For the case at KLM E&M the cost of a back order is defined by the borrow cost. If a component is back ordered, KLM E&M will borrow the component from a competitor to fulfill the customer demand.

Figure 7.5 and figure 7.6 show the effect of varying the borrow cost of a component. Increasing the borrow cost does result in a decrease in bought items. The results are obtained with a scope specifically designed for the sensitivity analysis, containing variations of the fast moving component. The system is optimized until an overall service level of 95% is reached. Changing the borrow cost for a run with one component does therefore not result in any change in output. The scope of this analysis does therefore consists of 5 copies of the fast moving component, with a different borrow cost for each component. The borrow cost for this analysis is varied between 20% and 100% of the price of the component, with steps of 20%.

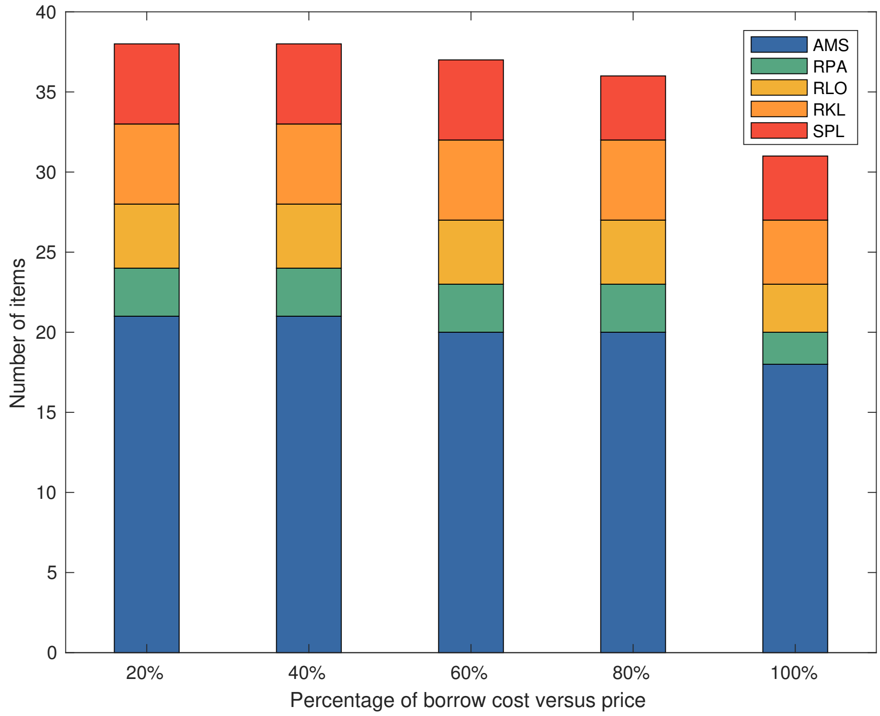


Figure 7.5: Total stock METRIC model for the fast moving component with varying borrow cost

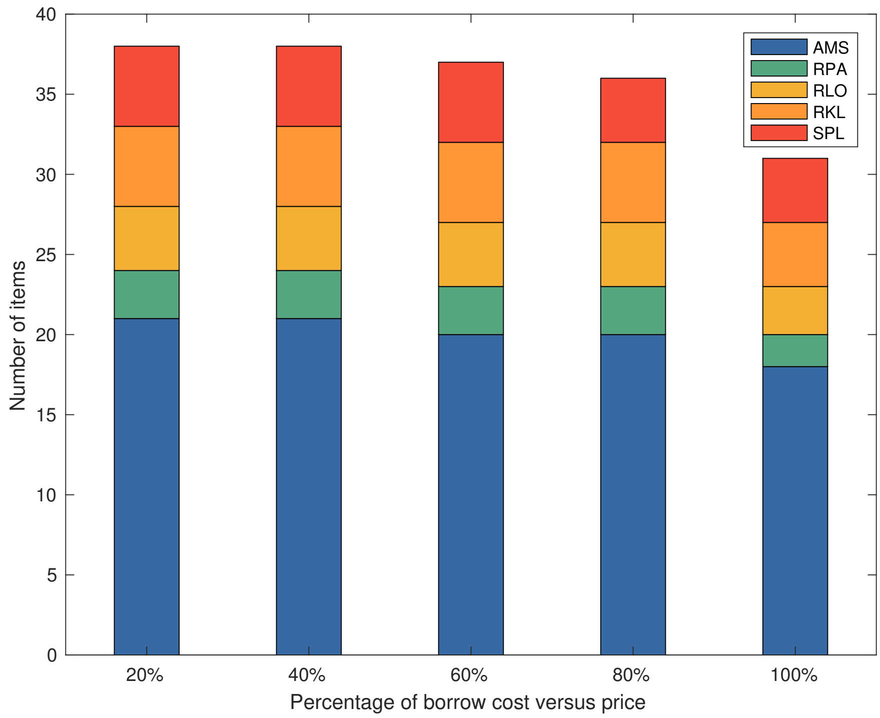


Figure 7.6: Total stock VARI-METRIC model for the fast moving component with varying borrow cost

8

Conclusion

8.1. Conclusion

The research project started with the objective to determine the effect of a multi-echelon model on the investment of a supply chain of aircraft rotables. The project is conducted with the supply chain of KLM E&M in mind. The final inventory model developed is tailored to the operations of KLM E&M, and all results are obtained with data of this same supply chain.

To answer the research question, a systematic approach is applied. First the structure and requirements of an inventory optimization model are discussed. A inventory optimization model can be divided into three main sections. Chapters 1 showed that the three sections can be identified as the demand probability, the model structure, and the solution technique. The three sections combined, will eventually determine the behaviour and computational requirements of the model.

Starting point of the research is the current state of the art in literature. Chapter 2 provides an overview of current literature on the three different sections. Different probability distributions are applied to model the demand for an inventory optimization model. The theoretical context chapter discussed the Poisson, Negative Binomial, and Weibull distributions as options to model the demand. Analysis of the demand made clear that with the current available data, the Poisson distribution is the best way to model demand. The Negative Binomial distribution tends to better represent the demand. However, more extensive research is required to statistically proof the fit between data of low demand items, and this distribution.

Within current literature, two theories on inventory optimization can be distinguished. The simplest approach to the problem is a single-echelon model. Such a model assumes all demand is taking place at the same warehouse. The second theory, namely the multi-echelon model, relaxes this assumption by introducing local warehouses. Many variations of this model are developed, each aiming to reduce the impact of an modelling assumption.

The solution technique determines mainly the computational load of the model. The theoretical context chapter discusses the possibility of an exact solution, linear programming, and a simulation based optimization. The selected optimization algorithm for this project belongs to the linear programming category. The inventory model is optimized utilizing the greedy algorithm. This technique comes with a low computational demand, while still being a viable solution technique in current literature.

An inventory model does require an optimization metric to be optimize to. Current literature covers a couple of metrics, each suiting different types of operations. The most novice step of this project lays in the implementation of a new optimization metric, suitable for a supply chain containing low demand high value items. This thesis proposed three optimization metric, namely the expected back orders, the expected back order cost, and the service level. The expected back order cost and service level metric are two new metrics, representing the novelty of the project.

Chapter 6 showed and discussed the effect on the investment of the different service metrics. The results of the expected back order metric, and expected back order cost metric are compared to each other. According to the expectations, does the investment increase if the model is optimized to investment cost. The motivation of the cost optimization lays in the reduction of total operational cost of the supply chain. The complex operational conditions make the validation of this behaviour unfortunately challenging, and therefore forced to be left out of the scope of this project.

Next to the expected back order, and expected back order cost optimization, a service level optimization metric is developed. The complex behaviour of this optimization metric does not permit the development of a usable optimization metric jet. The early results are however promising. As soon as the initialization problem is solved, it is expected that the service level optimization will outperform the other two optimization metrics.

The research performed over the course of the project allows two of the three sub-questions to be answered. The first sub question, what is the optimal allocation of inventory, is answered in chapter 6. According to the two models, the majority of stock needs to be located at the main warehouse. The remaining stock needs to be distributed over the local warehouses. Where the distribution depends on mainly on, the demand, and turn around time.

The answer on the second sub-question, what is the gain on the service metric per item, depends per service metric and stock level. The inventory model is optimized according to this research question. The marginal analysis divides the increase in service metric by the cost of this increase. Naturally, there is no definitive answer on this sub-question. The different possible service metrics are discussed in chapter 4.

The third sub-question, what is the difference between the theoretical calculated and actual supply chain performance, is left unanswered. The complexity of the operation of the case study, makes it difficult to validate the obtained results. The model does behaves as expected, as shown in chapter 7. However, a definite validation can not be given due to the many unknown parameters influencing the supply chain.

With the answers to the three sub-questions, the research question can be answered. The investment required for a supply chain of aircraft rotables increases for a multi-echelon model compared to the single-echelon model. The increase varies with the choice in optimization model and optimization metric. The increase in complexity of the model and required investment is necessary to reduce the underestimation of losses in the supply chain due to the global network. The increase in investment is caused by thee main factors. The multi-echelon model allocates stock to remote warehouses. This allocation provides a better representation of the reality, but does require more items to fulfill the optimization target. The multi-echelon model takes into account the shipping times between the different locations as well. The single echelon model essentially assumes instant delivery. Taking into account the shipping times will increase the investment cost, but again provide a better fit with reality. The VARI-METRIC does not only look at the expected back order between the local- and main warehouse, but also takes into account the variance. Taking into account the variance will require more items to achieve the optimization target, and requires therefore a higher investment.

8.2. Recommendations

There are definitely possibilities to improve the results obtained by the inventory optimization model. Improving the inventory optimization model will achieve two goals. The model will provide a better insight in the behaviour of a supply chain, enabling a better analysis of operational decisions. Secondly, the optimization algorithm is more reliable, and can reduce the required investment to achieve the desired optimization target. This section will discuss some recommendation for further research to improve the fit between the model and the actual supply chain.

To be able to implement a demand probability capable of producing a better fit to the failure data, more extensive research is required. Fitting a probability distribution to low demand items is challenging. Low demand items will inevitably result in a limited amount of data to determine the fit of the probability distribution. To justify the usage of for example the Negative Binomial distribution, further research can look at the fit between many slow moving items. The approximately 1000 slow moving items at KLM do not produce enough data to determine this fit. It is hard to say how many items are exactly required to determine this fit, as this varies by the behaviour of the items as well.

Major possibilities of improvements lay in the extension of the model structure. The model developed during this project utilizes a virtual warehouse to accommodate for customer request at the main warehouse. Locating stock at the virtual warehouse places it out of reach for the other local warehouses. Eliminating the need of a virtual warehouse will reduce the required investment to achieve the desired target.

The current model does not allow for any shipment between local warehouses. If an item is placed at a local warehouse, it will stay there until a customer request is placed from that specific warehouse. In practice it may occur that, an item at a certain moment may be more desirable at a different location. The shipment of items between warehouses in the same echelon is called lateral transshipment in literature. Introducing the possibility of lateral transshipment in the model, can increase the achieved service level with the same amount of stock. Current literature covers extensive research on lateral transshipment. Applying the theory to a multi-echelon, multi-item model is to this date not performed before. Models containing lateral transshipment are computational demanding and therefore difficult to implement for such supply chains.

This paper already elaborated on optimizing the model to the achieved service level. Especially parties providing item availability can profit from a service level optimization. In order to successfully apply the developed service level optimization metric to a supply chain model, a way to cope with the initial characteristic of the metric needs to be developed. It is believed, that the work conducted during this project can provide a good starting point for further research on this topic.

All inventory optimization models use a constant to model the TAT of the items. It might be interesting to analyze the effect of variation of the TAT on the required investment to achieve the optimization target. To analyze this effect, the TAT can be represented by a probability distribution. Research on the fit between the demand and a probability distribution will be useful for this purpose as well. Many of the same challenges encountered during the demand analysis will apply the this problem as well.

The greedy algorithm combined with a marginal analysis provides opportunity for further research as well. Implementing an epsilon greedy algorithm increases the change of escaping from a local optimum during optimization. Changing the solution technique will almost always increase the computational load of the model. Off all alternatives provided in this thesis, the epsilon greedy algorithm is considered the most evident next choice. The effect on computational load can be limited, while improving the estimation of the true global optimum.

A

Mathematical proof

A.1. Proof of first variance form

The correctness of Equation A.1 can be proven by a lemma assuming the equation. The proof of the lemma can be found below.

$$Var[X] = E\left[(X - E[X])^2\right] \tag{A.1}$$
$$= E\left[X^2 - 2 * X * E[X] + E[X]^2\right] \tag{A.2}$$
$$= E[X^2] - E\left[2 * X * E[X]\right] + E\left[E[X]^2\right] \tag{A.3}$$
$$= E[X^2] - E[2]E[X]E[X] + E[X]^2 \tag{A.4}$$
$$= E[X^2] - 2E[X]^2 + E[X]^2 \tag{A.5}$$
$$= E[X^2] - E[X]^2 \tag{A.6}$$

A.2. Proof of second variance form

Equation A.7 is also know as the law of total variance.

$$Var[X] = E\left[Var[X|Y]\right] + Var\left[E[X|Y]\right] \tag{A.7}$$

The proof of the above form is given below.

$$\left.\begin{array}{l} Var[X] = E[X^2] - E[X]^2 \\ E[X] = E\left[E[X|Y]\right] \end{array}\right\} Var[X] = E\left[E[X^2|Y]\right] - E\left[E[X|Y]\right]^2 \tag{A.8}$$

$$Var[X] = E\left[Var[X|Y] + E[X|Y]^2\right] - E\left[E[X|Y]\right]^2 \tag{A.9}$$
$$= E\left[Var[X|Y]\right] + E\left[E[X|Y]^2\right] - E\left[E[X|Y]\right]^2 \tag{A.10}$$
$$= E\left[Var[X|Y]\right] + Var\left[E[X|Y]\right] \tag{A.11}$$

A.3. Recursive formulas probability distributions

A.3.1. Poisson distribution

The Poisson distribution is solely dependable of the mean (μ). The definition of the recursive form for the Poisson distribution can be found in Equation A.13.

$$P(0,\mu) = e^{-\mu} \tag{A.12}$$

$$P(i+1,\mu) = \frac{\mu}{i+1} P(i,\mu) \qquad \forall i \in \mathbb{Z}^{\geq} \tag{A.13}$$

A.3.2. Negative binomial distribution

The Negative binomial distribution is dependable of two parameters (p, r). These two parameters can be defined in terms of the mean (μ) and variance (σ). The definitions of p and r can be find in Equation A.14 and A.15 respectively.

$$p = \frac{\mu}{\sigma^2} \tag{A.14}$$

$$r = \frac{\mu^2}{\sigma^2 - \mu} \tag{A.15}$$

The recursive form of the Negative binomial distribution is provided in Equation A.17.

$$P(0,p,r) = p^r \tag{A.16}$$

$$P(i+1,p,r) = \frac{i+r}{i+1}(1-p)P(i,p,r) \qquad \forall i \in \mathbb{Z}^{\geq} \tag{A.17}$$

B

Verification slow moving item

This appendix contains the verification of the slow moving component defined in chapter 5.

The expected removals per year can be defined for each location individually. The calculation is performed for the RKL warehouse, as example.

$$m_{RKL} = \frac{FH * QPA}{MTBR} = \frac{397.337 * 1}{515568} = 0,77 \text{ per year} \tag{B.1}$$

Equation B.2 shows the demand calculation of the main warehouse for the multi-echelon model.

$$\begin{aligned} m_{main} &= \sum_{j=1}^{J} (1 - r_j) m_j \\ &= (1-0) * m_{RPA} + (1-0) * m_{RLO} + (1-0) * m_{RKL} + (1-0) * m_{RMI} + (1-0) * m_{SPL} \\ &= 0,10 + 0 + 0,77 + 0 + 0,14 \\ &= 1,01 \text{ per year} \end{aligned} \tag{B.2}$$

The expected number of components in the pipeline is given by Equation B.3.

$$pipeline_{main} = m_{main} * TAT_{main} = 1,01 * \frac{34}{365} = 0,094 \tag{B.3}$$

The expected back orders at the main warehouse can be found in Equation B.4.

$$\begin{aligned} EBO_{main}(stock = 0) &= pipeline_{main} - \sum_{s=0}^{s=0} 1 - P(X < s - 1) \\ &= 0,094 \\ EBO_{main}(stock = 1) &= EBO(stock - 1) - \big(1 - \sum_{s=0}^{s=stock} P(X < s - 1)\big) \\ &= EBO(0) - \big(1 - Poisson_{CDF}(0, pipeline_{main})\big) \\ &= 0,094 - (1 - 0,91) \\ &= 0,004 \end{aligned} \tag{B.4}$$

Equation B.5 shows the calculation of the the local pipeline for the remote warehouse.

$$localPipeline_{RKL} = m_{RKL} * r_{RKL} * TAT_{RKL} = 0,77 * 0 * TAT_{RKL} = 0 \tag{B.5}$$

Equation B.6 shows the parent pipeline for the case of 1 stock at the main warehouse. The shipping time between the main warehouse and the Kuala Lumpur warehouse is 5 days.

$$\begin{aligned} parentPipeline_{RKL} &= m_j * (1 - r_j) * \left(shippingTime_j + \frac{EBO_{main}}{m_{main}}\right) \\ &= m_{RKL} * (1 - r_{RKL}) * \left(shippingTime_{RKL} + \frac{EBO_{main}}{m_{main}}\right) \\ &= 0,77 * (1 - 0) * \left(\frac{5}{365} + \frac{0,004}{1,01}\right) \\ &= 0,014 \end{aligned} \tag{B.6}$$

The final pipeline of the remote location with 1 stock at the main warehouse can be found in Equation B.7. Due to the assumption of no local repair, the final pipeline will be equal to the parent pipeline.

$$pipeline_{RKL} = localPipeline_{RKL} + parentPipeline_{RKL} = 0 + 0,014 = 0,014 \tag{B.7}$$

The verification of the local warehouse expected back orders for the METRIC model can be found in Equation B.8.

$$\begin{aligned} EBO_{RKL}(stock = 0) &= pipeline_{RKL} \\ &= 0,014 \\ EBO_{RKL}(stock = 1) &= EBO(0) - \left(1 - Poisson_{CDF}(0, pipeline_{RKL})\right) \\ &= 0,014 - (1 - 0,99) \\ &= 0,004 \end{aligned} \tag{B.8}$$

Equation B.9 contains the variance for the case of 0 and 1 stock at the main warehouse.

$$\begin{aligned} VBO_{main}(stock = 0) &= pipeline_{main} \\ &= 0,094 \\ VBO_{main}(stock = 1) &= VBO_{main}(0) - EBO_{main}(1) - EBO_{main}(0) - \left(EBO_{main}(1)\right)^2 + \left(EBO_{main}(0)\right)^2 \\ &= 0,094 - 0,004 - 0,094 - 0,004^2 + 0,094^2 \\ &= 0,005 \end{aligned} \tag{B.9}$$

The variance of the pipeline of RKL, with a stock level at the main warehouse of 1, can be found in equation B.10.

$$\begin{aligned} Var\left[X_{RKL}\right] &= m_{RKL} * shippingTime_{RKL} + \frac{m_{RKL}}{m_{main}}\left(1 - \frac{m_j}{m_{main}}\right)EBO_{main} + \left(\frac{m_{RKL}}{m_{main}}\right)^2 VBO_{main} \\ &= 0,77 * \frac{5}{365} + \frac{0,77}{1,01}\left(1 - \frac{0,77}{1,01}\right) * 0,004 + \left(\frac{0,77}{1,01}\right)^2 * 0,005 \\ &= 0,014 \end{aligned} \tag{B.10}$$

With the variance of the pipeline it is possible to determine the expected back orders of according the the VARI-METRIC model. Equation B.11 shows the calculation of the expected back orders at RKL according with a stock level at the main warehouse of 1.

$$
\begin{aligned}
EBO_{RKL}(stock=0) &= pipeline_{RKL} \\
&= 0,014 \\
EBO_{RKL}(stock=1) &= EBO(0) - \big(1 - NBD_{CDF}(0, pipeline_{RKL}, Var[X_{RKL}])\big) \\
&= 0,014 - (1 - 0,99) \\
&= 0,004
\end{aligned} \tag{B.11}
$$

The implementation of the expected back order optimization parameter can be found in Equation B.12.

$$
secret_{EBO}(stock=0) = \frac{EBO(0) - EBO(1)}{price} = \frac{0,014 - 0,004}{23.735} = 4,21 * 10^{-7} \tag{B.12}
$$

Equation B.13 shows the on time removals for the multi-echelon method. The implementation for the single echelon model is similar to the implementation shown below.

$$
\begin{aligned}
D_{achieved}(stock=0) &= 0 * D_{total} \\
&= 0 \\
D_{achieved}(stock=1) &= Poisson_{CDF}(0, pipeline_{RKL}) * D_{total} \\
&= 0,99 * 0,77 \\
&= 0,76
\end{aligned} \tag{B.13}
$$

The marginal analysis based on the on time removals can be found in Equation B.14.

$$
secret_{OTR}(stock=0) = \frac{D_{achieved}(1) - D_{achieved}(0)}{price} = \frac{0,76 - 0}{23.735} = 3,2 * 10^{-5} \tag{B.14}
$$

The final service level at the remote location is obtained according to equation B.15.

$$
serviceLevel(stock=1) = \frac{D_{achieved_{RKL}}}{m_{RKL}} * 100\% = \frac{0,76}{0,77} * 100\% = 99\% \tag{B.15}
$$

C

Service level optimization

This appendix will show and discuss the results obtained by the VARI-METRIC model optimized using the service level parameter.

Figure C.1 shows the investment required to achieve the a 95% service level. As the final requirement of the system is a certain service level, optimizing the system to service level should result in an investment less compared to the expected back order, and expected back order cost models. Clearly this is not the case. The final investment for the service level optimization is much higher compared to both other models.

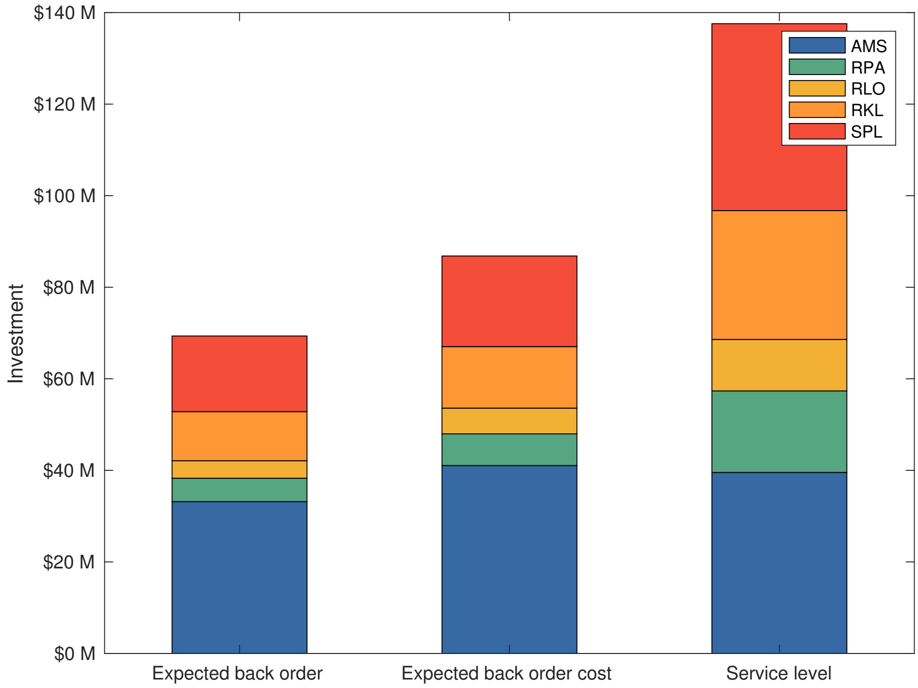


Figure C.1: VARI-METRIC investment required for varying optimization parameters

An explanation for this behaviour can be found in figure C.2 and figure C.3. These figures show the behaviour of the optimization parameter for varying MTBR and TAT respectively. The figures show the increase in service level for a component if the stock of this component would be increased by one. It can be seen that for each MTBR, the increase in service level at very low stock levels is low as well. When the stock increases, the

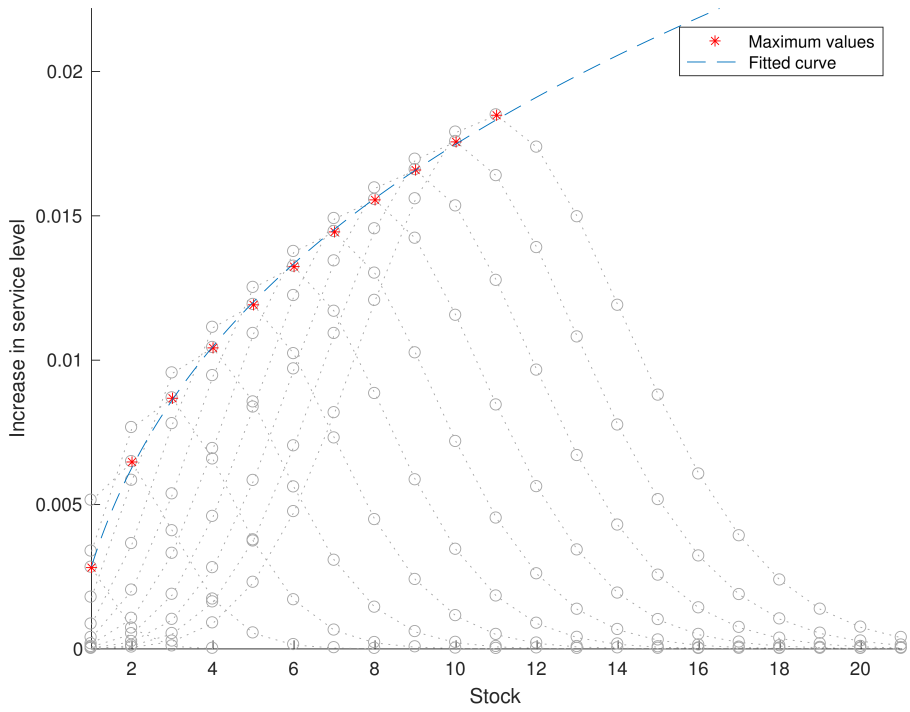


Figure C.2: Service level analysis variable removals

delta in service level increases as well. After reaching a maximum, the service level delta decreases again. This, Poisson like, curve makes it very challenging to optimize the system. At low stock levels, it is difficult to know the future maximum increase in service level. Therefore, it may not be beneficial for the optimization algorithm to invest in a model at low stock levels, but will be beneficial at higher stock levels.

To be able to optimize the system on service level, the relation between service level delta and stock delta needs to be determined. The blue line in figure C.2 shows the potential of determining this relations ship. The large number of variables make it however very tricky to design a rule uniform for each item in the supply chain. Figure C.3 does contain a similar fitted curve for varying TAT. This curve does show a complete different shape compared to the curve belonging to the demand.

Despite the complex characteristic of the service level parameter. Effort has been made to verify the correct behaviour of the model. To verify the behaviour of the model, the effect of the service level characteristic should be avoided. This is done by imposing a minimum full rate for each supply chain item. By setting a minimum fill rate for each component, the optimization parameter of each component is started after reaching the maximum delta in service level.

To test the behaviour of the model, the minimum fill rate of each component is set to 10%. Figure C.4 show the final investment of the three models with the minimum fill rate constraint. It can be seen that, in this case, the investment of the service level optimization is lower compared to the other two models. This is according to the expectations mentioned above.

The total investment of the three models with minimum fill rate constraint are much higher compared to the normal models. Defining a minimum fill rate forces the model to keep at least 1 stock at each single location. Placing 1 stock at locations with an expected demand of one per ten years, is off course not ideal. The solution of setting a minimum fill rate is therefore not useful for the case study of this project.

Concluding, it can be said that optimizing the system to service level is potentially beneficial for the supply chain of the case study. The complex characteristic of the service level parameter does it however make challenging to perform this optimization. To be able to use the service level optimization, future research will have to define a way of coping with the complex characteristic.

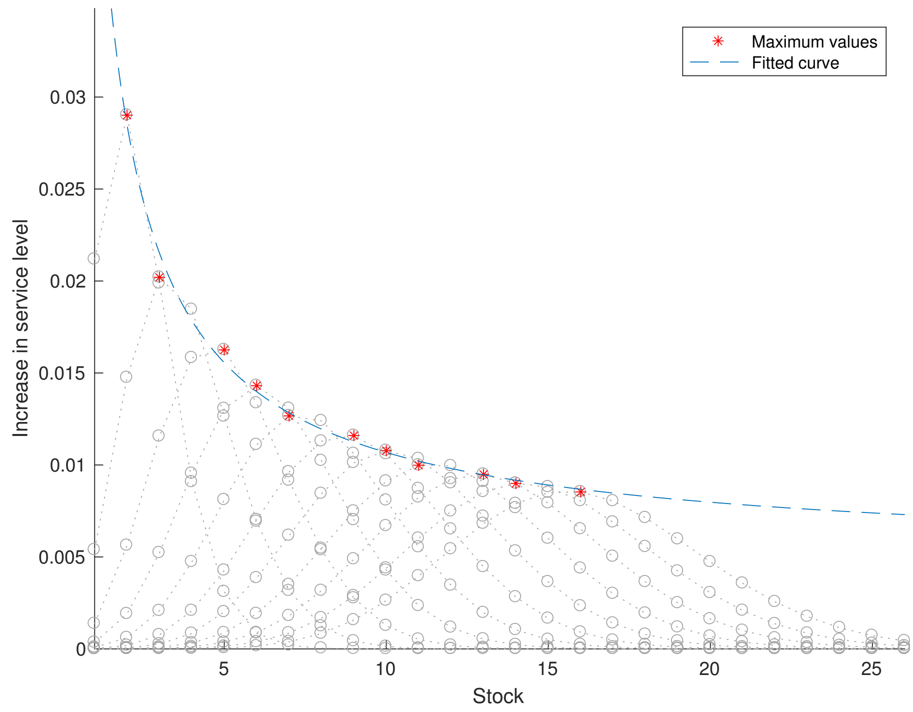


Figure C.3: Service level analysis variable TAT

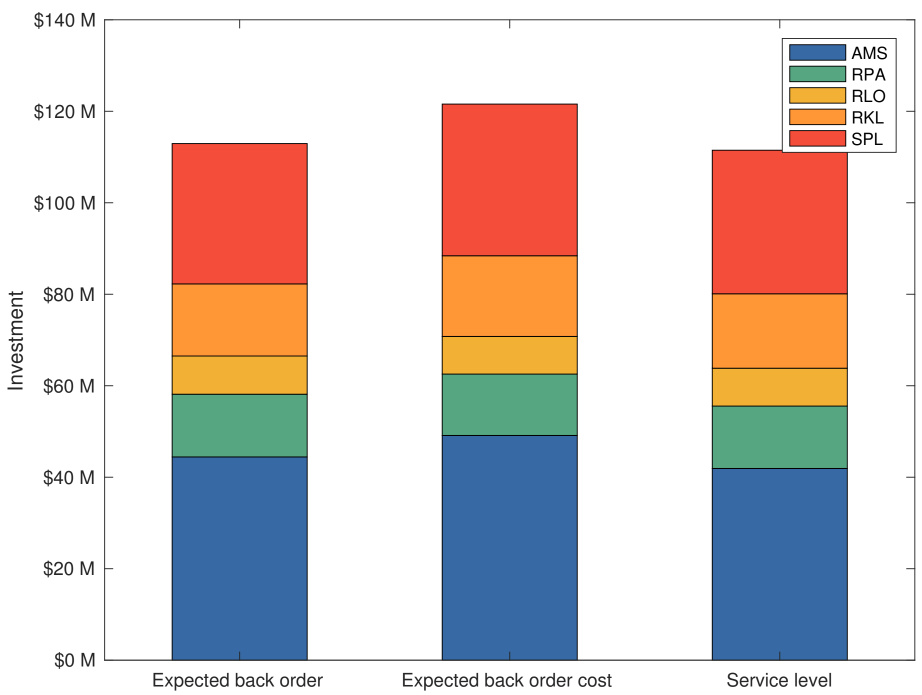


Figure C.4: VARI-METRIC investment required for varying optimization parameters with a minimum fill rate of 0.1

D

Sensitivity analysis expected back order cost optimization

This appendix contains the graphs belonging to the sensitivity analysis of the expected back order cost optimization model. The sensitivity analysis is performed for the METRIC and VARI-METRIC model.

Figure D.1 shows the output of the METRIC model for varying MTBR. It can be seen that the number of items decreases if the MTBR increases.

Figure D.2 shows the output of the VARI-METRIC model for varying MTBR. The trend showing in this graph is again similar to trend described above.

The second step of the sensitivity analysis is varying the TAT. Figure D.3 and D.4 show the output of the METRIC and VARI-METRIC model respectively. The stock level increases when the TAT increases as well. Due to the longer repair times, it will be necessary to keep more stock to fulfill customer demands.

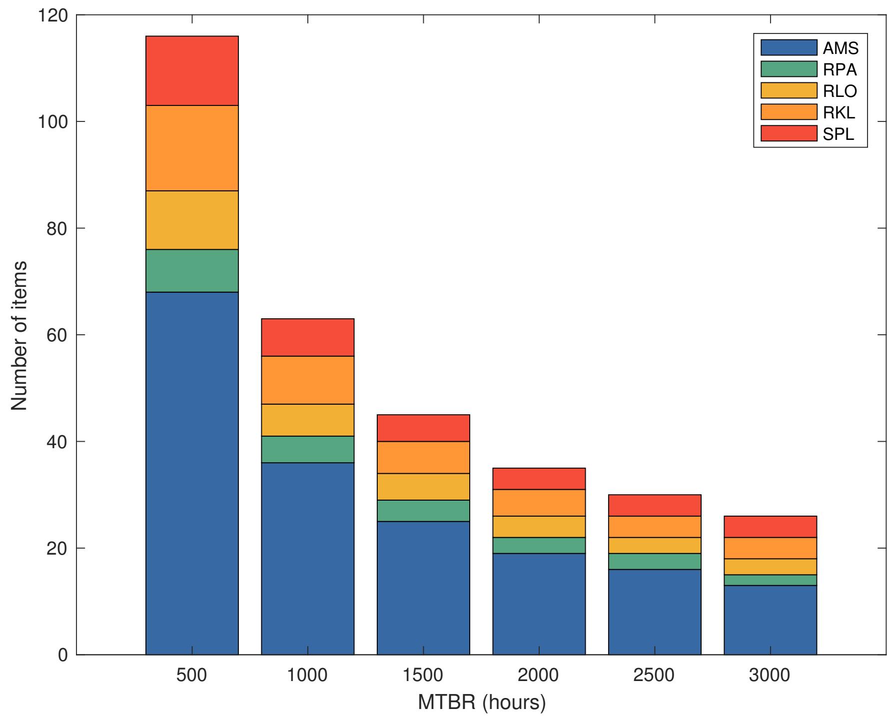


Figure D.1: Total stock METRIC model for fast moving component with varying MTBR

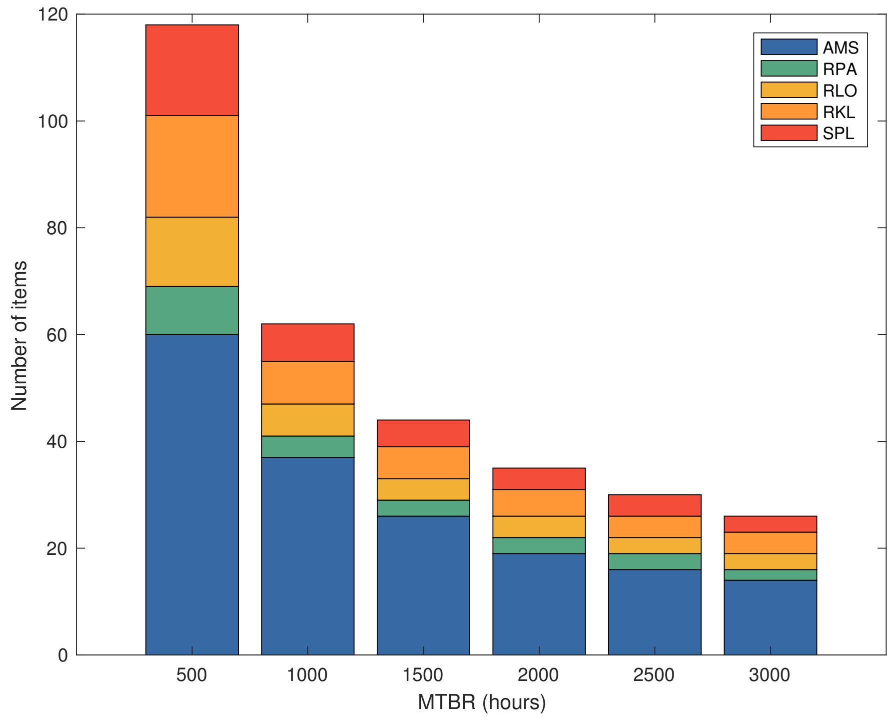


Figure D.2: Total stock VARI-METRIC model for fast moving component with varying MTBR

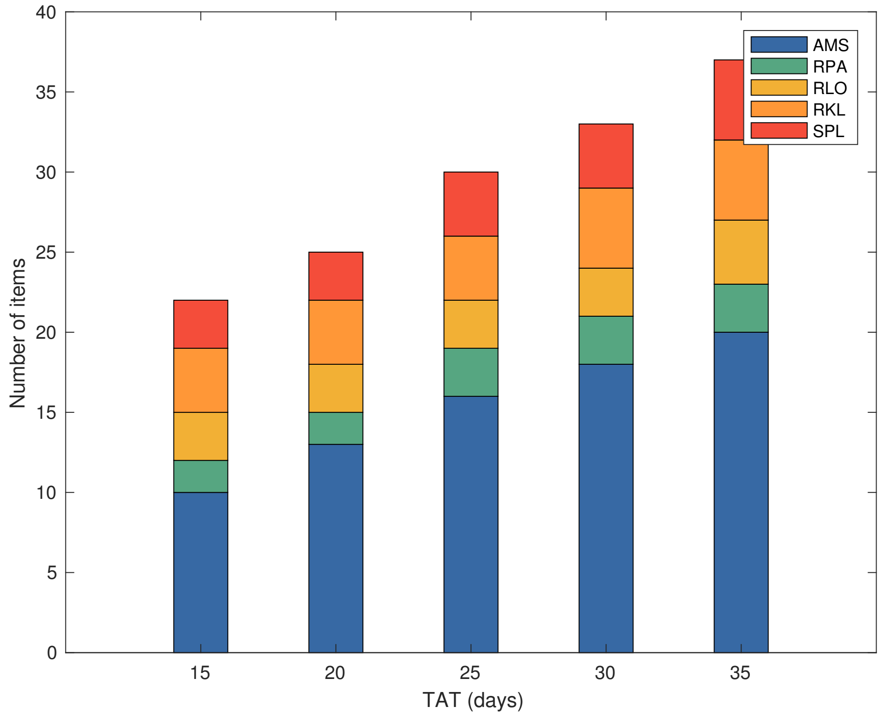


Figure D.3: Total stock METRIC model for the fast moving component with varying TAT

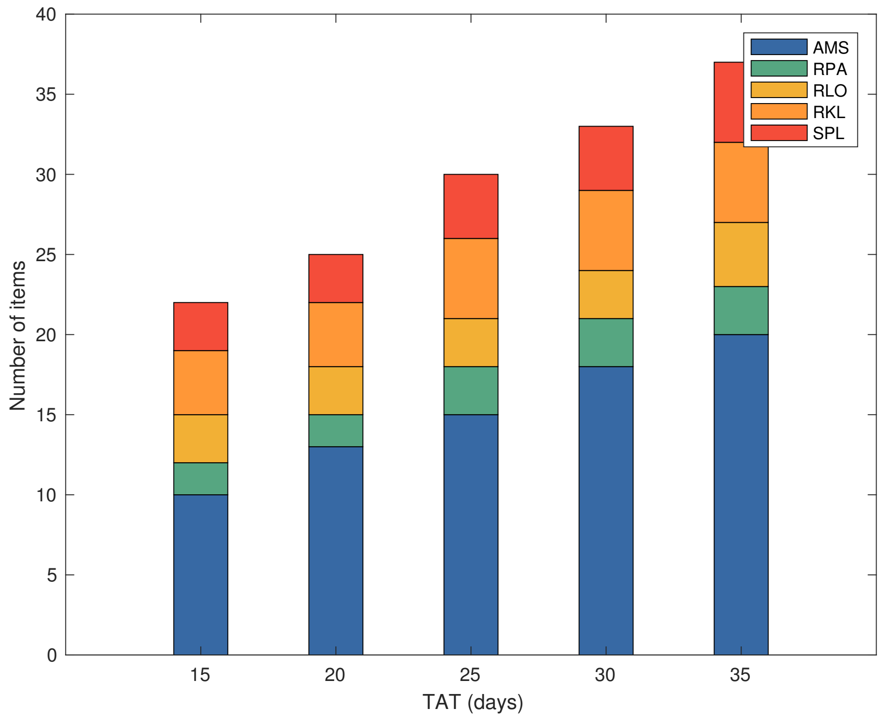


Figure D.4: Total stock VARI-METRIC model for the fast moving component with varying TAT

E

Hardware configuration

This appendix shows the configuration details of the computer used for all calculations performed over the course of the project. To achieve similar performance it is recommended to use at least a machine with comparable specifications.

Table E.1: Hardware configuration

Manufacturer	Lenovo
Product	ThinkPad L570
Processor	Intel Core i5 2.4GHz
RAM	8GB
Operating system	Windows 7 Professional
System type	64-bit

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