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***Abstract*—We propose an analytical approach to approximate the average two-terminal reliability ($ATTR$) for graphs where a fraction of the nodes is removed. The approximation is based on the generating function of the network's degree distribution under random node removals and stochastic degree-based node removals. Through validation on synthetic graphs, including Erdős Rényi random graphs and Barabási-Albert graphs, as well as four real-world networks from the Internet Topology Zoo, we observe that the analytical method effectively approximates the average two-terminal reliability under random node removals for synthetic graphs. In the case of real-world graphs under random and stochastic degree-based node removals or synthetic graphs under stochastic degree-based node removals, the analytical approximation yields reasonably accurate results when the fraction of removed nodes is small, specifically less than 10%, provided that the initial analytical approximation closely aligns with the real $ATTR$ values.**

I. Introduction

Designing robust and reliable systems is crucial in the real world, as high reliability ensures optimal functionality even in challenging situations such as failures or natural disasters. Various systems can be modeled as networked systems, including transportation systems [1], telecommunication systems [2], electrical systems [3], and more. Understanding the robustness of networks plays an important role in the design of reliable and resilient systems.

Network robustness assesses a system's capacity to withstand perturbations, such as link or node removals, within a specified time interval [4]. Connectivity metrics are commonly used to measure network performance in terms of robustness, as they reflect whether nodes can reach each other—a fundamental aspect of network functionality. The degradation of connectivity signifies the deterioration of functionality [5], [6]. The size of the largest connected component serves as a widely adopted connectivity metric [7], [8]. Additionally, average two-terminal reliability ($ATTR$) is a popular metric beyond those related to the largest connected component [9]–[11]. Specifically, in a graph, $ATTR$ is the probability that a random pair of nodes reside in the same connected component [10]. $ATTR$ can also be interpreted as the fraction of node pairs with a path between them [11]. If the graph is fully connected, $ATTR$ is one; otherwise, it can be calculated by dividing the total number of node pairs in each connected component by the total possible number of node pairs in the graph. A higher $ATTR$ value indicates a greater level of system robustness.

Currently, considerable effort is dedicated to exploring methods for predicting network robustness metrics under various attacks. Two primary directions in this research involve machine learning-based methods and analytical approaches. A substantial body of research employs machine learning-based approaches in the realm of robustness prediction. Noteworthy contributions include the work of Chen *et al.* [12]–[14]. They conducted experiments utilizing machine learning methods such as Learning Feature Representation-based Convolutional Neural Network (LFR-CNN) [12] and Spatial Pyramid Pooling Convolutional Neural Network (SPP-CNN) [13] to predict the connectivity robustness based on the largest connected component of different networks. Their research includes a comparative analysis of the performance of different methods and an in-depth investigation into the influence of the training data distribution [14]. On the analytical front, Newman *et al.* [15] have developed a method using the generating function of the degree distribution to calculate the size of the largest connected component.

While $ATTR$ stands as a popular connectivity metric for graphs, analytical approximation methods for $ATTR$ remain elusive. In this study, we first outline a method for approximating $ATTR$ when the size of the largest connected component of the network is known. Building on this, we present analytical approaches to approximate $ATTR$, leveraging results obtained from approximating the size of the largest connected component using the generating function of the degree distribution. Subsequently, we delve into the changes in the generating function of the degree distribution under two types of node removals: random node removals and stochastic degree-based node removals [16], [17]. Utilizing these insights, we demonstrate analytical approximation results for $ATTR$ under both types of node removals.

The paper is structured as follows: Section 2 presents the analytical approximation for $ATTR$; Section 3 introduces the graphs used in the study; Section 4 provides the simulation and analytical results; and the final section presents the conclusion and discussion.

II. Analytical approximation of $ATTR$

Consider a network $\mathcal{G}(N, L)$ consisting of a node set $\mathcal{N}$ with N nodes and a link set $\mathcal{L}$ with L links. If we denote P_{ij}, where $i \neq j$, as indicating whether there is a path between node $i \in \mathcal{N}$ and node $j \in \mathcal{N}$, $P_{ij} = 1$ represents the existence of a path between node i and node j, and $P_{ij} = 0$ represents the absence of a path between node i and node j. Based on the definition [11], $ATTR$ of a network $\mathcal{G}$ can be calculated by

$$ATTR = \frac{\sum_{i \neq j} P_{ij}}{\binom{N}{2}}, \tag{1}$$

or,

$$ATTR = \frac{\sum_{S_i} \binom{|S_i|}{2}}{\binom{N}{2}}, \tag{2}$$

where S_i is the i^{th} connected component in a network $\mathcal{G}$ and the number of nodes in the connected component S_i is denoted by $|S_i|$.

A. Approximation for $ATTR$ for a given network

To analytically approximate the $ATTR$ for a given network, we will first discuss a lower and upper bound for the $ATTR$. Consider a network $\mathcal{G}(\mathcal{N}, \mathcal{L})$, where the size of the largest connected component is denoted as M. The lower bound $ATTR_{min}$ of the $ATTR$ is derived under the assumption that the connected components, apart from the largest connected component, consist of isolated nodes. Hence, we obtain:

$$ATTR_{min} = \frac{\binom{M}{2}}{\binom{N}{2}}. \tag{3}$$

To establish the upper bound $ATTR_{max}$ of $ATTR$, we assume that the size H of the second largest connected component is as large as possible. Therefore, we define H as $\min\{M, N - M\}$. Next, we determine the integer division Q of $N - M$ over H and the remainder R. Q and R satisfy:

$$N - M = Q \times H + R. \tag{4}$$

Then, the upper bound $ATTR_{max}$ can be calculated by

$$ATTR_{max} = \frac{\binom{M}{2} + Q\binom{H}{2} + \frac{R(R-1)}{2}}{\binom{N}{2}}. \tag{5}$$

We will proceed by focusing on calculating the average size of connected components outside the largest connected component, denoted by A. If we denote the minimum average size as μ_{min}, we have $\mu_{min} = 1$ under the assumption that all other connected components are isolated nodes. For the maximum average size μ_{max} for the connected components outside the largest connected component, we have:

$$\mu_{max} = \begin{cases} \frac{N-M}{Q} & , R = 0; \\ \frac{N-M}{Q+1} & , R \neq 0. \end{cases} \tag{6}$$

According to [18], the average size of connected components outside the largest connected component A satisfies:

$$A = \frac{2}{2 - d_{av} u^2/(1 - S)}, \tag{7}$$

where d_{av} is the average degree of a network, S is the fraction of nodes in the largest connected component, which can be calculated by

$$S = 1 - G_0(u), \tag{8}$$

where u satisfies $u = G_1(u)$. Furthermore, $G_0(x)$ is the generating function of the degree distribution, and $G_1(x)$ is the generating function of the excess degree distribution in the network. Here $G_0(x)$ and $G_1(x)$ satisfy

$$G_0(x) = \sum_{k=0}^{\infty} p_k x^k, \tag{9}$$

$$G_1(x) = \frac{G_0'(x)}{G_0'(1)}, \tag{10}$$

where p_k denotes the probability that a node has degree k, $G_0'(x)$ is the derivative of $G_0(x)$ and $G_0'(1) = d_{av}$.

To calculate the expected size of the largest connected component M, we can use the fraction of nodes in the largest connected component S. This allows us to obtain the expected size of the largest connected component, denoted as $M = N \times S$. Note that the value of M may not be an integer.

With the average sizes of the connected components outside the largest connected component of the lower bound case and the upper bound case being derived and the average size of connected components outside the largest connected component being known, a parameter α can be derived such that the weighted average of μ_{min} and μ_{max} is equal to A:

$$A = \alpha \mu_{min} + (1 - \alpha)\mu_{max}. \tag{11}$$

Hence the parameter α satisfies:

$$\alpha = \frac{\mu_{max} - A}{\mu_{max} - \mu_{min}}. \tag{12}$$

The final estimate $ATTR^*$ for the $ATTR$ is derived by taking the weighted average of $ATTR_{min}$ and $ATTR_{max}$:

$$ATTR^* = \alpha ATTR_{min} + (1 - \alpha) ATTR_{max}, \tag{13}$$

B. Approximating $ATTR$ under random node removals

We have discussed how to approximate $ATTR$ using the generating function of the degree distribution of a network above. Similarly, if we want to approximate $ATTR$ under random node removals, we can use the same aforementioned framework. However, we need to know how to derive the generation function of the degree distribution after a fraction p of nodes is removed using the original degree distribution.

In the random node removal process, each node has the same probability of being removed. According to [19], after a fraction p of nodes is randomly removed from the network, the updated generating function of the degree $\overline{G}_0(x)$ and excess degree distribution $\overline{G}_1(x)$ in the network become:

$$\overline{G}_0(x) = G_0(p + (1-p)x), \tag{14}$$

$$\overline{G}_1(x) = G_1(p + (1-p)x). \tag{15}$$

The average degree $\overline{d}_{av}$ after removing a fraction p of nodes is $\overline{G}_0(1) = (1-p)d_{av}$.

To obtain the size of the largest connected component $\overline{M}$ after removing a fraction p of nodes, we can employ Eq. (8) with the updated degree and excess degree distribution generating functions $\overline{G}_0(x)$ and $\overline{G}_1(x)$, to approximate the expected fraction of nodes in the largest connected component $\overline{S}$ as:

$$\overline{S} = 1 - \overline{G}_0(\overline{u}), \tag{16}$$

where $\overline{u}$ satisfies $\overline{u} = \overline{G}_1(\overline{u})$. Therefore, the size of the largest connected component $\overline{M} = N\overline{S}$. It should be mentioned that even though we remove nodes from a network, we still discuss $ATTR$, including all removed nodes, which means we take the removed nodes into account as isolated nodes in the original network. So, the lower bound $\overline{ATTRmin}$ after removing a fraction p of nodes becomes:

$$\overline{ATTR}_{min} = \frac{\binom{\overline{M}}{2}}{\binom{N}{2}}. \tag{17}$$

The upper bound $\overline{ATTR}_{max}$ can be calculated by employing Eq. (5)

$$\overline{ATTR}_{max} = \frac{\binom{\overline{M}}{2} + \overline{Q}\binom{\overline{H}}{2} + \frac{\overline{R}(\overline{R}-1)}{2}}{\binom{N}{2}}, \tag{18}$$

where $\overline{M}$, $\overline{Q}$, $\overline{H} = min\{\overline{M}, N - \overline{M}\}$ and $\overline{R}$ satisfy Eq. (4) as $N - \overline{M} = \overline{Q} \times \overline{H} + \overline{R}$.

Similarly, for the maximum average size $\overline{\mu}_{max}$ for the connected components outside the largest connected component after removing a fraction p of nodes, we have:

$$\overline{\mu}_{max} = \begin{cases} \frac{N-\overline{M}}{\overline{Q}} & ,\overline{R} = 0; \\ \frac{N-\overline{M}}{\overline{Q}+1} & ,\overline{R} \neq 0. \end{cases} \tag{19}$$

For the average size of connected components outside the largest connected component $\overline{A}$ after removing a fraction p of nodes, we have:

$$\overline{A} = \frac{2}{2 - \overline{d}_{av}\overline{u}^2/(1-\overline{S})}. \tag{20}$$

Then Eqs. (12) and (13) become the following:

$$\overline{\alpha} = \frac{\overline{\mu}_{max} - \overline{A}}{\overline{\mu}_{max} - \mu_{min}}, \tag{21}$$

$$\overline{ATTR}^* = \overline{\alpha}\overline{ATTR}_{min} + (1-\overline{\alpha})\overline{ATTR}_{max}. \tag{22}$$

The value of $\overline{ATTR}^*$ is an approximation of $ATTR$ after removing a fraction p of nodes.

C. Approximating $ATTR$ under stochastic degree-based node removals

In addition to random node removals, we also consider stochastic degree-based node removals in this study. For each node in the network, the probability q_i that node i will be removed is proportional to the power β of its degree k_i:

$$q_i = \frac{k_i^\beta}{\sum_j k_j^\beta}, \tag{23}$$

where β is a predefined parameter. When $\beta = 0$, stochastic degree-based node removals are equivalent to random node removals; when $\beta > 0$, nodes with higher degrees have a higher priority for removal. This study explores two cases: $\beta = 1$ and $\beta = 10$.

To approximate $ATTR$ under stochastic degree-based node removals, we adopt the approach presented in the paper [17] to map the fraction p of removed nodes under stochastic degree-based node removals to the fraction $\overline{p}$ of nodes under random node removals. Then, we can use the framework for approximating $ATTR$ under random node removals to approximate $ATTR$ under stochastic degree-based node removals.

When $\beta = 1$, according to [16], after removing a fraction p of nodes, the fraction $\overline{p}$ can be calculated by:

$$\overline{p} = 1 - \frac{fG'_\beta(f)}{d_{av}}, \tag{24}$$

where $f \equiv G_\beta^{-1}(1-p)$, $G_\beta(x) \equiv \sum_k p_k x^{k^\beta}$, d_{av} is the average degree of the initial network, and p_k is the probability that the degree of a node is k. If $\beta = 1$, $G_\beta(x) \equiv \sum_k p_k x^k$, which represents the generating function for the degree distribution. When $\beta > 1$, it is numerically challenging to determine the value of f.

To calculate the $ATTR$ approximation when $\beta = 10$, we assume that if a fraction p of nodes is removed, the nodes are removed starting from the one with the largest degree in descending order. While randomly removing a fraction p of nodes, a fraction p of links is also removed. Based on the total number of links removed under stochastic node removals with $\beta = 10$, we can acquire the corresponding mapping fraction $\overline{p}$ of removed nodes under random node removals as $\overline{p} = \frac{\sum_{k=k_{max}}^{k=\overline{k}} p_k N k}{N d_{av}} = \frac{\sum_{k=k_{max}}^{k=\overline{k}} p_k k}{d_{av}}$, where the largest degree value is denoted as k_{max}, the probability of removed nodes with degree k is denoted as p_k, and degree $\overline{k}$ satisfies $\sum_{k=k_{max}}^{k=\overline{k}} p_k = p$.

After obtaining the mapping fraction $\overline{p}$, we replace the fraction p with the mapping fraction $\overline{p}$ to perform the approximation under stochastic degree-based node removals. For Eqs. (14) and (15), we have:

$$\widetilde{G}_0(x) = G_0(\overline{p} + (1-\overline{p})x), \tag{25}$$

$$\widetilde{G}_1(x) = G_1(\overline{p} + (1-\overline{p})x). \tag{26}$$

Next, by employing Eqs. (16), (17), (18), (19), (20), (21), and (22), we can obtain the $ATTR$ approximation $\widetilde{ATTR}^*$ under stochastic degree-based node removals.

III. Data

To validate the analytical approaches, we use Erdős Rényi (ER) random graphs, Barabási-Albert (BA) graphs, and real-world networks. ER random graphs are generated with the following parameters: the number of nodes $N = 1000$ and link connection probability $p_{ER} = 0.008$. The average degree of ER random graphs is around eight.

Barabási-Albert (BA) graphs are created using the preferential attachment process. The initial graph is a star graph with four nodes. At each step, a new node with three links is added to the graph, and the new node has a higher probability of connecting with higher-degree nodes. The BA graph used in this study has a total of 500 nodes. The average degree of the BA graph is 5.96.

The real-world networks utilized are sourced from the Internet Topology Zoo [20], a collection of real-world communication networks. Table I displays the properties of these networks, including the number of nodes (N), the number of links (L), and the average degree (d_{av}).

Name	N	L	d_{av}
HinerniaGlobal	55	81	2.95
Interoute	110	146	2.65
Deltacom	113	161	2.85
Cogentco	197	243	2.47

TABLE I: Properties of four real-world communication networks. N represents the number of nodes, L exhibits the number of links in the network, and d_{av} is the average degree in the network.

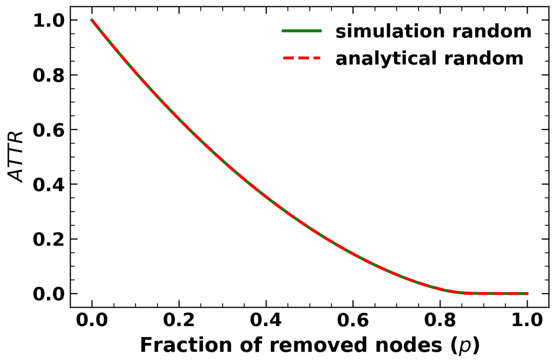
IV. Results

We conducted simulations on ER random graphs, BA graphs, and real-world graphs with 10,000 realizations to validate the analytical approaches. The average value of Average Two-Terminal Reliability ($ATTR$) was calculated over these realizations under random node removals. For ER random graphs, each realization involved generating a new graph with the same parameters, and we did not check whether the generated network was connected or not, followed by random node removals until all nodes were removed. For BA graphs, a single BA graph was generated in the case of BA graphs, and node removals were performed over 10,000 realizations. Similarly, random node removal processes were applied to real-world graphs over 10,000 realizations, with the average value obtained at each step. To calculate the analytical results under random removals, we obtained the generating function of the degree distribution for the BA graph and real-world graphs using their degree sequences. For ER random graphs, the degree distribution follows a Poisson distribution, expressed as $p(k) = \frac{(d_{av})^k}{k!} e^{-d_{av}}$, with the generating function as $G_0(x) = e^{d_{av}(x-1)}$. Here, the average degree d_{av} of ER random graphs is approximately 8.

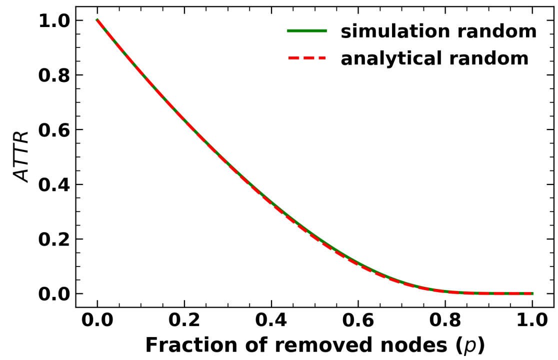
We present the analytical results of different graphs under random node removals along with the simulation results in Fig.1. Notably, we observe a close fit between the analytical and simulation results for synthetic graphs. However, for real-world graphs, a discernible gap exists between the analytical and simulation outcomes under random node removals. This discrepancy is expected, considering that the analytical results, based on generating functions, represent averages across all random graphs with the same degree sequence, whereas a real-world graph is just one instance within this set. Interestingly, we find that the analytical curves align well with the simulation curves under random node removals for synthetic graphs. Furthermore, for real-world graphs, when the analytical approximation closely matches the actual value of the $ATTR$ before nodes are removed, the approximation results are reasonably accurate, particularly when the fraction of removed nodes is small, such as below 10%.

We also conducted simulations and obtained analytical results for stochastic degree-based node removals with $\beta = 1$ and $\beta = 10$ for synthetic and real-world graphs. The average simulation values and corresponding approximation values of $ATTR$ during node removals based on the stochastic degree are illustrated as green solid curves and red dashed curves in Figs.2-3. Notably, we observed an increase in the differences between analytical and simulation results with a higher value of β for synthetic graphs. However, for synthetic graphs, the approximation results remain acceptable when the fraction of removed nodes is relatively small. In contrast, for real-world graphs, the approximation results improve with a higher value of β, and the overall performance is comparable to the results under random node removals for real-world graphs.

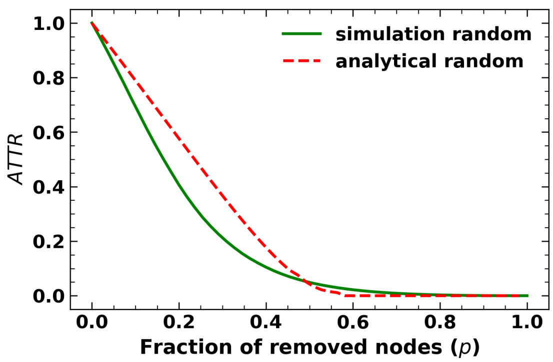
The interesting part is that for random node removals and stochastic degree-based node removals with $\beta = 1$, the analytical approximations of $ATTR$ values are equal to or larger compared to the $ATTR$ values obtained from the simulation results. However, for node removals with $\beta = 10$, the analytical approximation of $ATTR$ values is equal to or smaller than the $ATTR$ values obtained from the simulation results for most networks. This is because we assume that nodes are removed starting from the node with largest degree



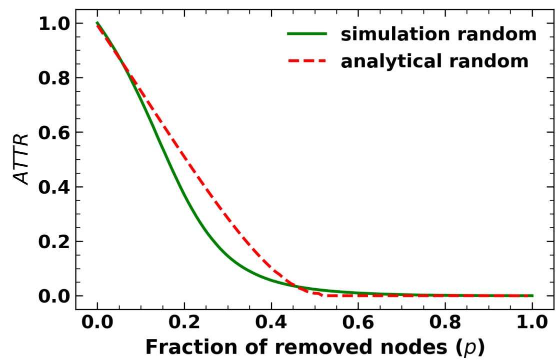
(a) ER(1000, 0.008)



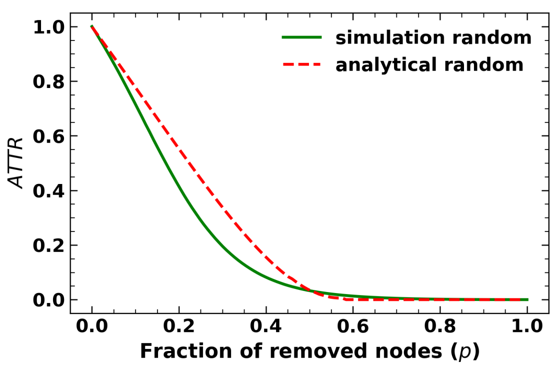
(b) BA(500, 3)



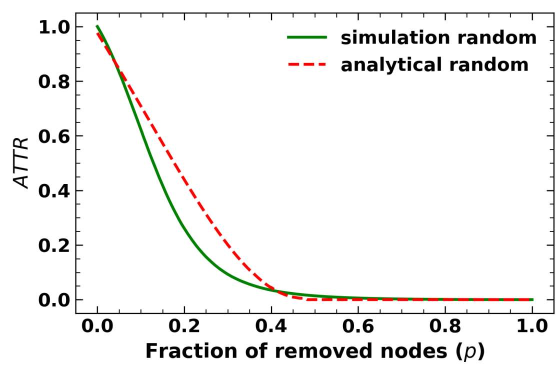
(c) HinerniaGlobal



(d) Interoute



(e) Deltacom



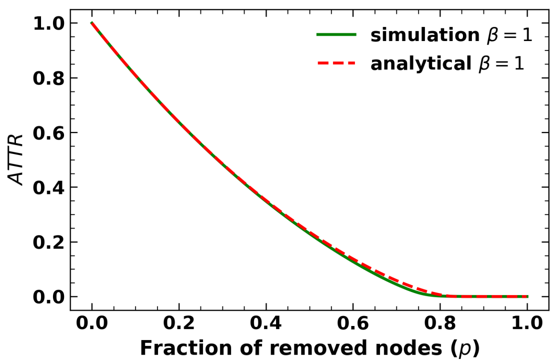
(f) Cogentco

Fig. 1: The results for the $ATTR$ by simulations and the analytical approximation under random node removals for synthetic and real-world graphs. The x-axis denotes the fraction of nodes removed, and the y-axis denotes the $ATTR$ value. The green solid curves represent the simulated results, and the dashed red curves represent the corresponding analytical approximations.

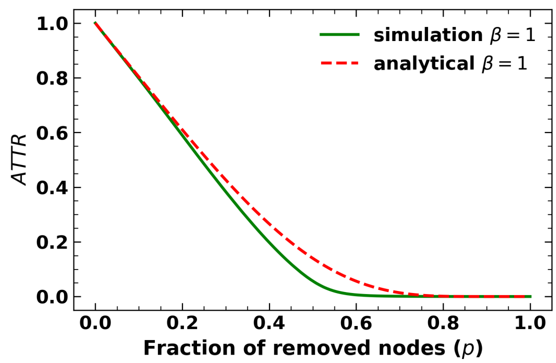
in descending order while performing the approximation. However, when removing nodes based on degree with $\beta = 10$, it is not always the case that the node with the largest degree is removed first.

V. Conclusion and Discussion

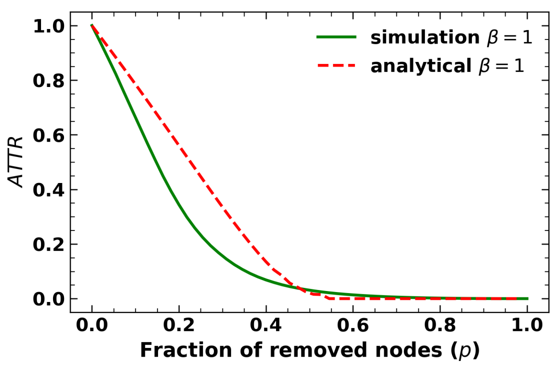
As Average Two-Terminal Reliability ($ATTR$) can be used to measure the connectivity of a network, developing analytical approximation approaches under different types of node removals can assist decision-makers in efficiently comparing



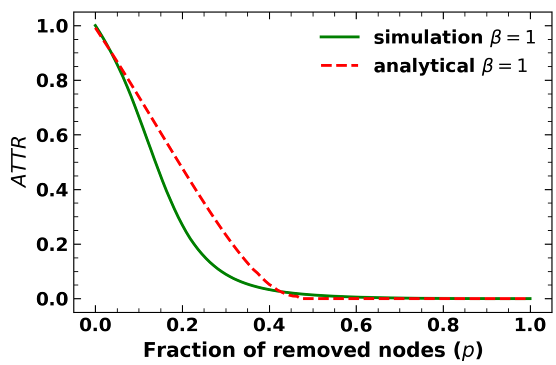
(a) ER(1000, 0.008)



(b) BA(500, 3)



(c) HinerniaGlobal



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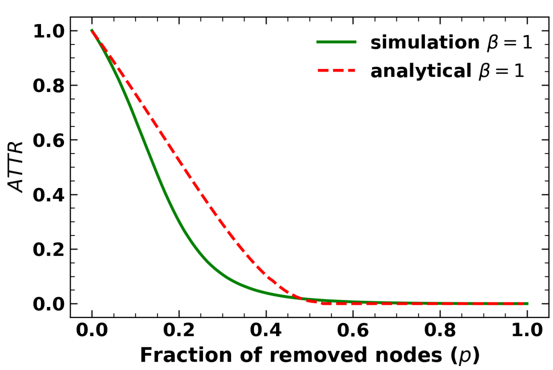
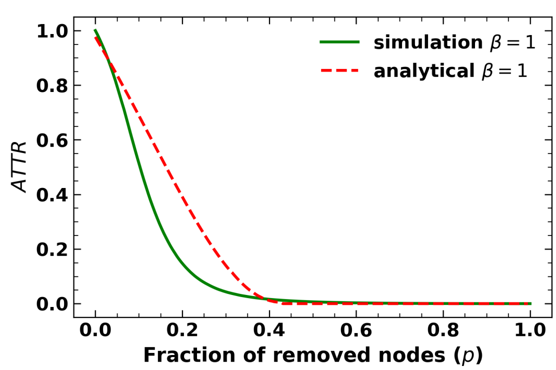


(e) Deltacom

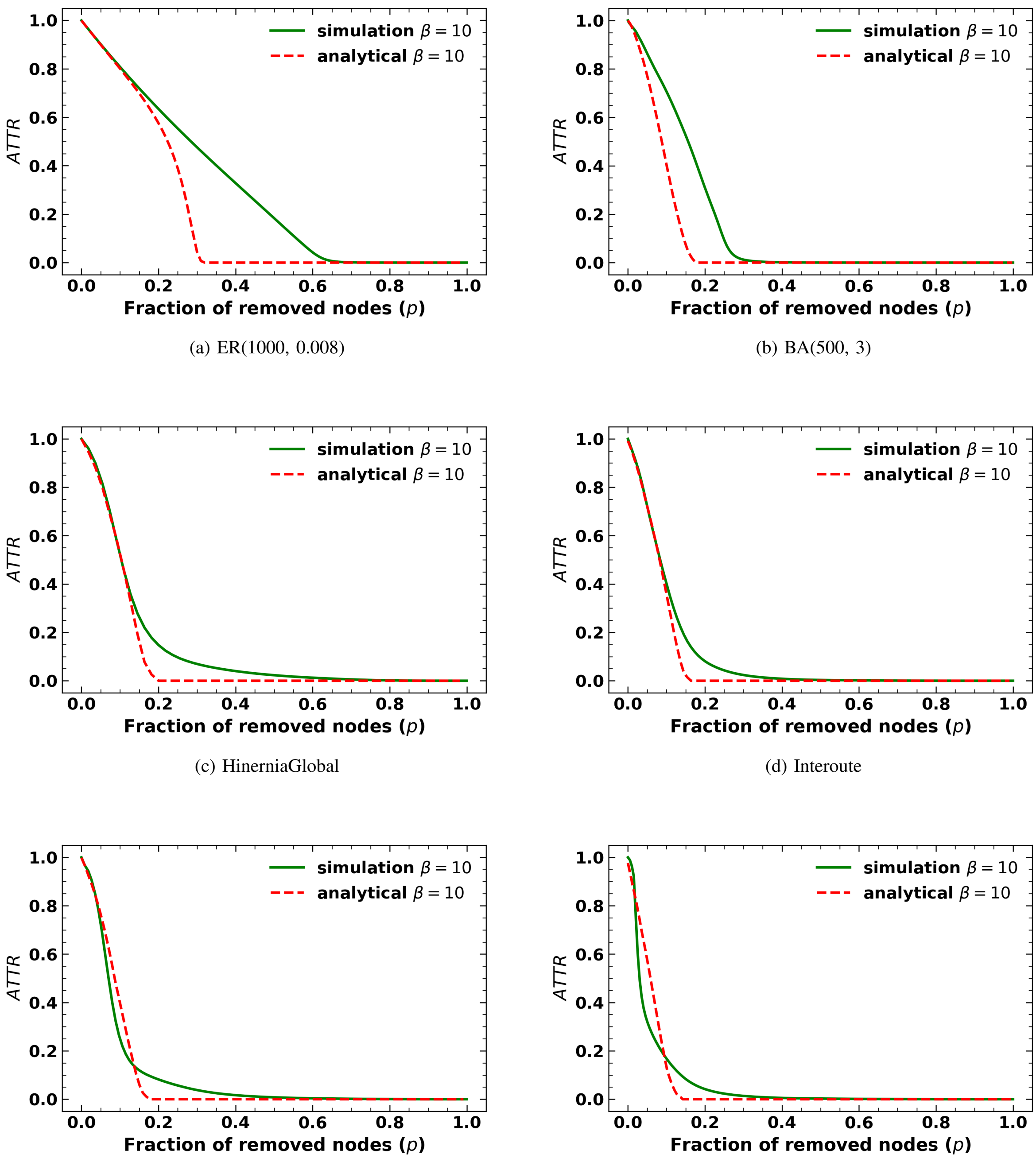


(f) Cogentco

Fig. 2: The results for the $ATTR$ by simulations and the analytical approximation under stochastic degree-based node removals with $\beta = 1$ for synthetic and real-world graphs. The x-axis denotes the fraction of nodes removed, and the y-axis denotes the $ATTR$ value. The green solid curves represent the simulated results, and the dashed red curves represent the corresponding analytical approximations.

network performance among various network topologies under node attacks. This approach can save time and costs compared to running simulations. For example, we can utilize the $ATTR$ approximation approaches under node attacks to design a robust communication network.

Specifically, we have introduced analytical approximations for $ATTR$ utilizing the generating function of degree distributions. Additionally, we presented analytical approaches to approximate $ATTR$ under both random node removals and stochastic degree-based node removals. Validation of the



(a) ER(1000, 0.008)

(b) BA(500, 3)

(c) HinerniaGlobal

(d) Interoute

(e) Deltacom

(f) Cogentco

Fig. 3: The results for the $ATTR$ by simulations and the analytical approximation under stochastic degree-based node removals with $\beta = 10$ for synthetic and real-world graphs. The x-axis denotes the fraction of nodes removed, and the y-axis denotes the $ATTR$ value. The green solid curves represent the simulated results, and the dashed red curves represent the corresponding analytical approximations.

analytical approximations was conducted using two types of synthetic graphs and four real-world graphs. Upon comparing simulation and analytical results, we observed that the approximations of $ATTR$ for random node removals and stochastic degree-based node removals align well with simulation outcomes for ER random graphs, particularly in the case of random node removals. However, the analytical results for BA graphs exhibit strong performance only under random node removals and reasonable accuracy only when the fraction of removed nodes is small, around up to around 10%. For real-world graphs, even in the random node removal scenario, the performance of analytical approximations is less satisfactory. This discrepancy is $ATTR$ contributed to the fact that the results from generating functions represent averages across all graphs with the same degree sequence. Nevertheless, if the approximation results for real-world graphs closely match the initial real $ATTR$ values, the approximations perform reasonably well, particularly when the fraction of removed nodes is small.

As the approximation method is based on the generating function of the degree distribution, networks with the same degree distribution will yield the same approximation results. Additionally, the method does not always perform well for sparsely connected networks. If the network is connected and very sparse, the $ATTR$ approximation using the generating function of the degree distribution may incorrectly infer that the network is disconnected, implying an approximated value for $ATTR$ smaller than one, before the network is attacked.

For synthetic networks, the analytical approximations perform better under random node removals and stochastic degree-based node removals with $\beta = 1$ than the approximation under stochastic degree-based node removals with $\beta = 10$. However, for real-world networks, it seems that the analytical approximations perform worse under random node removals and stochastic degree-based node removals with $\beta = 1$ than the approximation under stochastic degree-based node removals with $\beta = 10$. The reason may be a result of the network density, as the average degree in real-world networks is smaller than the average degree in synthetic networks. This aspect should be investigated in the future.

The approximations under stochastic degree-based node removals are not always accurate. Therefore, further investigation is warranted to refine the analytical approximation approaches, particularly focusing on the limitations observed in stochastic degree-based node removals for both synthetic and real-world networks.

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