

Influence of Tidal Turbines on the Sediment Transport in the Eastern Scheldt

BSc Thesis

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This thesis has been written as being the final project of my Bachelor of Civil Engineering at the TU Delft. With this thesis I want to show what I have learnt during my Bachelor studies. But during this thesis, I have also learnt a lot of new skills, like writing a scientific report, that will be very useful for my further study.

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ABSTRACT

There is a growing interest in tidal energy, thanks to its constant availability in comparison to other renewable energy sources. Also in the Netherlands, tidal turbines have been installed. This has been done in the storm surge barrier that is positioned in the inlet of the Eastern Scheldt. These turbines have an impact on sedimentation.

Therefore, the objective of this study is to analyse the effect of tidal turbines on the sediment transport in the Eastern Scheldt. Within this research, there is a distinct focus on how the tidal situation plays a role in this. Literature research has been carried out to gain knowledge that is needed to fulfill this objective. From this literature research it could be concluded that the effect of tidal turbines on sediment transport is much greater when the tide is (horizontally) asymmetric. It means that the maximum flow velocity in ebb-direction does not equal the maximum flow velocity in flood-direction. This results in a net sediment transport in one direction. It can be concluded that the greater the tidal asymmetry, the greater the net sediment transport.

Now, it was time to qualify the aspects of the tide where the sediment transport depends on. This has been done by analytical elaborations of the relation between sediment transport and flow velocity. From this, it could be concluded that the sediment transport is depending on the wave height amplitude, discharge amplitude and the phase difference between the wave height and the discharge. The obtained relation was implemented into a Matlab model that computes the wave height and the discharge due to arbitrary tidal waves. The model has been extended to a network of channels that represents the Eastern Scheldt. With the extended model it was possible to simulate a net sediment transport, due to tidal asymmetry.

The storm surge barrier and the tidal turbines could be implemented in this model, represented by a higher resistance. This resulted in a lower wave height- and discharge amplitude and with that also the net sediment transport decreased. Since the net sediment transport is directed seawards, a decrease meant less erosion of the channels.

But from the literature research it was concluded that the construction of the storm surge barrier and of the tidal turbines resulted in more erosion of the channels. An explanation for this difference in results may be that in the model, only sediment transport due to tidal asymmetry is taken into account. Other forms of transport are not considered. Also, many assumptions have been made in the model and there is still room for improvement to represent the reality more accurate with this model, which could be subject to further research.

1. INTRODUCTION

1.1 Problem Definition

Recently, tidal turbines have been installed in the strom surge barrier of the Eastern Scheldt. Tidal energy is a very popular source of renewable energy, thanks to its constant availability. But tidal turbines cause changes in the currents as a result of an increased bed resistance, among other reasons. This, again, influences the sediment transport around the turbines. In other words, due to a change in the currents, the morphology of the hydraulic system will be influenced.

The Eastern Scheldt is a particular situation. The Eastern Scheldt already experienced many changes as a result of the construction of the storm surge barrier. For example, the storm surge barrier had a great impact on the sedimentation in the Eastern Scheldt and the estuary has not reached a new equilibrium state since then. This means that the Eastern Scheldt is still adjusting to the storm surge barrier. And now tidal turbines have recently been installed which have again an impact on the sediment transport and with that on the morphology as a whole. In combination with the already existing effects of the storm surge barrier, it is likely that the impact on the sedimentation is indeed significant now.

At the moment, there is still a great need for research into the consequences of tidal turbines on the morphology of a hydraulic system. There is not enough knowledge yet to be able to predict how the Eastern Scheldt will adapt to the new circumstances. There are various factors related to tidal turbines and to the original situation of the hydraulic system that affect the sediment transport in the Eastern Scheldt. One of these factors is for example the tidal situation of the estuary. According to a British study, it turns out that in regions of tidal asymmetry, the impact of energy extraction on the sedimentation is much greater, compared with energy extracted from regions of tidal symmetry [Neill et al., 2009].

1.2 Research Questions

Within this BSc-thesis there will be particular focus on the aspect of the effect of tidal turbines on the sedimentation in the Eastern Scheldt. The research question is therefore formulated as follows:

How does the tidal situation in the Eastern Scheldt affect the changes in sediment transport as a result of tidal turbines?

This research question can be answered by answering the following sub-questions:

- Question 1: What is the tidal situation in the Eastern Scheldt?

- Question 2: How did the construction of the storm surge barrier affect the tide in the Eastern Scheldt?

- **Question 3:** How did the construction of the storm surge barrier affect the sedimentation?

- **Question 4:** By what factors is the sedimentation influenced? And which of these factors are adjusted by tidal turbines?

- **Question 5:** Why do tidal turbines have more influence on the sedimentation at asymmetrical tide than at symmetrical tide?

1.3 Research Approach

To answer the above-mentioned questions, research will be done into existing knowledge about this subject. This will be done by means of literature research. Subjects to be discussed are general characteristics of the Eastern Scheldt, the impact of the storm surge barrier, tide in general, tidal (a)symmetry, sedimentation processes and tidal turbines. On the basis of this literature research, a concept for the modelling of this research question can be made. This is started by trying to understand the formulas and relations analytically. The goal is to create a model that describes the influence of the tidal turbines on the sedimentation processes in the Eastern Scheldt.

1.4 Thesis Outline

The structure of this thesis will be as follows:

Chapter 2 describes all the background information that is needed to answer the research questions, by means of literature research. This information consists of:

- The properties of the Eastern Scheldt
- Tide in general
- Sediment transport
- Tidal asymmetry
- Tidal turbines

Much time and effort will be spent for literature research. The reason for this is that a lot of knowledge is still to be gained to answer the research questions. With the gained knowledge, a concept can be made for the modelling. This is done in chapter 3. This chapter will deal with the analytical relations of formulas that can describe the sediment transport. Also, this chapter shows how these relations are affected by tidal turbines. When these formulas are understood analytically, the next step is to do the modelling in Matlab. This is explained in chapter 4 and 5. The knowledge and lessons learnt from both the theoretical and the modelling part are discussed in the discussion and conclusions & recommendations in chapter 6 and 7.

2. LITERATURE RESEARCH

2.1 Introduction

The objective of this chapter is to describe various elements that are important to know about in order to answer the research questions. Section 2.2 gives a short overview about the Eastern Scheldt and describes the impact that the storm surge barrier had. In section 2.3 it is explained what tide is and how it is generated. Then, in section 2.4 the sediment transport is analysed. It is explained what it is and how it can be described by formulas. With the information given in the sections 2.3 and 2.4, it is possible to go deeper into tidal asymmetry and how that influences the sediment transport. This is done in section 2.5. Section 2.6 describes tidal turbines and how they affect the sediment transport. The chapter can be closed with a conclusion in section 2.6 that combines the knowledge gained in this chapter.

2.2 The Eastern Scheldt and the Storm Surge Barrier

The Eastern scheldt is an estuary. An estuary is the transition between a river and a sea. It has characteristics of both a river and a sea; on one hand it has banks, sediment transport and flowing, fresh water in the upper part and. On the other hand it has tides and saline water. Thanks to this interaction between sea and river, an estuary is a very particular ecosystem with unique flora and fauna [Savenije, 2005]. And so is the Eastern Scheldt. The Eastern Scheldt is the biggest national park of the Netherlands [Rijkswaterstaat, n.d.]. The Eastern Scheldt is a tidal basin with a length of approximately 50 km and a surface area of 350 km² The total tidal prism passing through the inlet of the Eastern Scheldt before barrier

The total tidal prism passing through the inlet of the Eastern Scheldt before barrier construction was on average 1200 $Mm^3/tide$ [De Bok, 2001].

The storm surge barrier was built between 1983 and 1986, as a protection against high water. Also two more back-barrier dams were built with the aim of decreasing the tidal range by limiting the basin length and thereby increasing the decrease of the reflection and amplification of the tidal wave [Eelkema, 2013]. The combined effect was that the tidal prism decreased roughly by 25% from 1200 $Mm^3/tide$ to around 900 $Mm^3/tide$. And the same is valid for the maximum flow velocities, according to Eelkema (2013). Besides, the storm surge barrier reduced the effective cross-sectional area of the tidal inlet from ca. 80.000 m^2 to 17.900 m^2 [Vroon, 1994].

Moreover, the construction of the storm surge barrier resulted in decreasing average tidal currents inside and outside the basin. This means that flow velocities have decreased. Sediment transport is mainly done by tidal currents. So, the decrease in tidal currents has also led to a decrease in sediment transport into the flat areas of the Eastern Scheldt. On the other hand, the wind waves are the main reason for erosion of the flats in the Eastern Scheldt and these wind waves are not affected by the storm surge barrier. So as a result of these circumstances, the flats are being eroded more than that they are built up by sedimentation due to tidal currents [Eelkema, 2013]. It can be measured that since the construction of the storm surge barrier, the basin has received virtually no sediment from outside, according to Eelkema (2013). And most of the sediment that comes into the inlet of the storm surge barrier, is precipitated in the inlet channels and does not go further. This means that the storm surge barrier is acting as a blockage for sediment transport into the Eastern Scheldt. There is also a tide-driven transport of the eroded sediment from the flat areas, out of the Eastern Schelt through the storm surge barrier, as can be seen in figure 2.1. So, the Eastern Scheldt is basically losing more and more sediment.



Fig. 2.1.: Schematic overview of the main erosion and deposition areas on the ebb-tidal delta. Reprinted from "Ebb-tidal delta morphology in response to a storm surge barrier", by M. Eelkema, Z.B. Wang and A. Hibma, 2008, Delft, The Netherlands.

2.3 Tide

The existence of tide is the result of variations in time and space of the gravitational force exercized by the sun and the moon on the earth. The tide acts like a periodic function, consisting of different components, depending on the frequency of the earth's rotation. At most locations, the semi-diurnal tide M_2 , with a period of 12 hours and 25 minutes, is dominant. But there are more tidal components, with different frequencies. Linear combinations of two tidal components lead to subharmonic tides (e.g. spring – neap cycle). And non-linear interactions of tidal components lead to subharmonic, as well as superharmonic tides. These are the so-called overtides, e.g. M_4, M_6, M_8 etc., due to the interaction of M_2 with itself, but also many combined overtides, due to interactions of different components [Duijts, 2002]. Overtides can cause asymmetry of the tide and with that they are very important for sediment transport. But this will be further discussed in section 2.5.

The tide causes an up- and downward movement at the sea entrance of an estuary. This generates oscillations of the water mass inside the basin, at tidal frequency [Battjes & Labeur, 2017]. In an estuary, tide propagates as a wave with both a standing and a progressive character. For progressive waves, the phase lag between the velocity and the water elevation is zero. This means that both the discharge and the velocity are maximum when the water elevation is maximum. For standing waves the situation is different; there is a phase lag of $\pi/2$ between the water elevation and the velocity. Standing waves occur in semi-enclosed basins that are connected to the sea, like the Eastern Scheldt. Since the waves in the Eastern Scheldt are a combination of a progressive and a standing wave, the phase lag between velocity and elevation is between zero and of $\pi/2$ [Savenije, 2005].

2.4 Sediment Transport

The motion of particles depends among others on the properties of the transported material (grain size, fall velocity). Particles on the seabed will start moving when a so-called critical velocity is exceeded [Bosboom & Stive, 2015]. Two transport modes can be distinguished: The first is the bed load mode, which means that particles roll close to the seabed. The bed load mode dominates for low flows or large grains. The dimensionless transport number ϕ and the Shields number θ are used to describe the bed load mode. This will not be further discussed, since it is not going to be used in this thesis.

The second is suspended load mode, where grains are lifted up and are transported as a suspension until the velocity drops and they settle again. When the actual bed shear stress is larger than the critical bed shear stress, the particles will be lifted from the bed. If this lift is beyond a certain level, then the turbulent upward forces may be larger than the weight of the particles. In that case, the particles go into suspension, which means that they loose contact with the bottom for some time [Bosboom & Stive, 2015].

The suspended sediment flux can be modelled as the product of the sediment concentration c and the horizontal velocity u of the water that is transporting the sediment:

$$q_s(z,t) = c(z,t)u(z,t)$$
 (2.1)

q is the sediment flux in $[m^3/s/m^2]$ c is the sediment concentration in $[m^3/m^3]$

The suspended sediment transport can be computed by integrating the suspended sediment flux $u \cdot c$ from the top of the bed load layer (z=a) to the water level:

$$S_{s}(t) = \int_{z=a}^{h} c(z,t)u(z,t)dz$$
 (2.2)

where $h = h_0 + \zeta$.

In coastal zones, as in our situation, waves play an important role in the water motion. This means that both the water velocity u and the sediment concentration c strongly vary as a function of time, on a scale comparable to the wave period [Bosboom & Stive, 2015].

Sediment transport can also be described as a total load transport. Then, the physics are represented in a less realistic way, but for practical application this makes it easier. An example of a formula that can be used to describe the total load is the formula of Engelund-Hansen (1967):

$$q_T = \frac{0.05 \cdot C_D^{3/2} \cdot \overline{U}^5}{(g(s-1))^2 \cdot d_{50}}$$
(2.3)

where C_D is the drag coefficient [-], s is the specific gravity [-] and d_{50} is the grain size [m].

This non-linear relationship leads to a net sediment transport in the direction of the faster flow velocity (ebb or flood), which will be discussed in the next section.

2.5 Relation Tidal Asymmetry and Sediment Transport

The tide shows a periodic behaviour with respect to water levels (vertical tide), flow velocities (horizontal tide) and discharges as well as associated sediment transport. For each of these aspects of the tidal motion it holds that one half of a period is not necessarily a mirror image of the other half. In such cases we speak of tidal asymmetry [Duijts, 2002]. The reason for this asymmetry is that a harmonic wave, propagating from deep into shallow water, cannot remain harmonic, because of decreasing water depth. This is called deformation of the wave and is caused by nonlinear effects. Nonlinear effects increase with increasing waveheight to depth ratio. There are two types of nonlinearities in the equations describing the flow: Terms that are nonlinear in the dependent variables (quadratic terms) and geometric nonlinearities, in particular the variation of the wet cross section with the water level [Battjes & Labeur, 2017].

We can distinguish vertical and horizontal asymmetry. Vertical asymmetry is related to ebb- or flood-dominant asymmetry. Flood-dominant asymmetry is the case when the duration of water level rise is shorter than the duration of water level fall.

Horizontal asymmetry is the most relevant case for this study. It is associated with a difference in maximum flow velocities for ebb and flood. The horizontal tide is called asymmetric if it causes a residual sediment transport. For example, if the maximum flood flow velocity is higher than the maximum ebb flow velocity, there is a residual sediment transport in the flood direction. So it can be concluded that, the greater the extent of horizontal asymmetry, the greater the net sediment transport in ebb or flood direction.

2.6 Sedimentation and Tidal Turbines

Unlike other forms of renewable energy, tidal energy is a consistent source of kinetic energy that is permanently available. The functioning is in principle very similar to windmills; under-water rotors use the kinetic energy of currents to drive generators, which in turn produce electricity. Since water is much denser than air, tidal turbines can be smaller than wind turbines. This makes it possible to place the turbines closer to each other, without decreasing the amount of electricity that they can produce (Marine Current Turbines, n.d.).

The presence of tidal turbines can have various impacts on the environment that have to be investigated. For example, the rotors can be dangerous for marine animals. Also, constructing tidal turbines in the inlet of an estuary can inhibit the incoming water flow, which can result in degradation of the water quality in an estuary.

But within this research, the focus is on the effect that the turbines have on the sediment transport that is generated by the current going through the turbines. The effect of tidal turbines can be seen as a drag force exerted on the current [Hill et al., 2014]. Due to energy extraction by tidal turbines, the flow velocity decreases. As seen in section 2.4, the sediment transport rate depends on the flow velocity. So, when the flow velocity decreases, also the sediment transport decreases. Beside that, an array of tidal turbines has the potential ability to divert peak flows away from their natural path and thus alter patterns of erosion and deposition of sediment within the channel where the tidal turbines are placed [Hill et al., 2014]. The effects of tidal turbines on sediment transport can be seen several kilometers from their location.

2.7 Conclusions

In section 2.2 it is described that since the presence of the storm surge barrier, there is almost no sediment transport going into the Eastern Scheldt. But there is still an outgoing flux of eroded sediment from the flats inside the Eastern Scheldt. It can be concluded from section 2.4 that sediment transport is strongly depending on flow velocity. In previous research [Hill et al., 2014] it has been found that tidal turbines influence the flow velocity and as a result of that, also the sediment transport. The

change in sediment transport is relatively stronger than the change in flow velocities, due to the non-linear relation between flow and sediment transport. As seen in section 2.5, horizontal tidal asymmetry has a great impact on the net sediment transport in the ebb or flood direction. The stronger the tidal asymmetry in the Eastern Scheldt, the more net sediment transport.

So, taking this all together, the tidal turbines in the Eastern Scheldt cause a decrease in flow velocity which will relatively stronger decrease the sediment transport, due to the nth power, seen in equation 3.1. Since there is already a great impact of the storm surge barrier on the sediment transport, the new changes as a result of the tidal turbines could be significant and they have to be studied.

In the next chapter research will be done on the factors that influence the sediment transport, thus the flow velocity and the variables that determine the flow velocity. In section 2.3 it is discussed that tide contains two main wave characters; a standing and a progressive wave. In the next chapter it will be investigated how the implementation of the equations for these waves influence the formulas for flow velocity and with that, the sediment transport. After that, it is possible to create a Matlab model that works with the derived formulas for the sediment transport.

3. CONCEPT FOR MODELLING

3.1 Introduction

After answering the research questions in a theoretical way, it is now possible to approach the problem in an analytical way. The goal is to formulate the sediment transport in such a way that it is possible to see what factors, concerning the tidal situation, can have an influence. Think of how the sediment transport for example depends on the discharge amplitude, or the phase angle between the discharge and the wave height of the tidal waves. Then, the aim is to investigate the impact of the tidal turbines on these factors and with that on the sediment transport.

As seen before, tidal waves have both a progressive and a standing character. So both the situations are analyzed analytically. After that a more general case is determined, using the formulas for arbitrary waves. These analytical derivations will be done in paragraph 3.2.

The end result of the derivations for arbitrary waves will then be used for the computations of sediment transport in Matlab. The concept and the approach of this model will be discussed in paragraph 3.3

3.2 Analytical Modelling of Sediment Transport

For simplification in the analytical modelling, it can be said that the sediment transport is proportional to the flow velocity to the nth power:

$$S \propto \overline{U}^n$$
 (3.1)

Where n is 3 or 5 [Wang, 2012]. The part containing the constants that also determine the total load, as discussed in section 2.4, are omitted for now, to make the elaborations more clear. These constants will be added again in the Matlab model.

The flow velocity is not constant, since we are dealing with oscillating depth and discharge, due to tide. Taking the time-average for one period, indicated by the bar, this can be further elaborated to:

$$\overline{S} = \overline{\left(\frac{q}{d}\right)^n} \tag{3.2}$$

In shallow water, like an estuary, we cannot neglect the contribution of the wave height ζ to the depth. To take this into account, we can write $d = d_0 + \zeta = d_0(1 + \frac{\zeta}{d_0})$. Substituting this into equation 3.2, gives:

$$\overline{\left(\frac{q}{d}\right)^n} = d_0^{-n} \cdot \frac{\overline{q^n}}{\left(1 + \frac{\overline{\zeta}}{d_0}\right)^n} \tag{3.3}$$

$$= d_0^{-n} \cdot \overline{q^n} \left(1 - n \cdot \overline{\frac{\zeta}{d_0}} \right) \tag{3.4}$$

$$= d_0^{-n} \cdot \overline{q^n} - n \cdot d_0^{-(n+1)} \cdot \overline{q^n \zeta}$$
(3.5)

Equation 3.5 will be the basis for further derivations for different waves. We will start by elaborating the situation for progressive waves. Then, the same will be done for standing waves. And in the end, a more general derivation will be given for arbitrary waves, which will be used for the modelling.

3.2.1 Progressive Waves

Since we are dealing with tidal waves, q (discharge per unit width) and ζ (wave height) are oscillating functions. If we are assuming to have a periodic progressive wave, we can write q^n and ζ as:

$$\overline{q^n} = \overline{\left\{\hat{q}cos(\omega t)\right\}^n} \tag{3.6}$$

$$\overline{\zeta} = \hat{\zeta} \cos(\omega t - \delta) \tag{3.7}$$

Where δ is the phase difference between the discharge and the wave height. Implementing these equations for progressive waves into equation 3.5, gives for the sediment transport:

$$S \propto -n \cdot d_0^{-(n+1)} \cdot \hat{q}^n \cdot \hat{\zeta} \cdot \cos(\delta) \cdot \frac{m-1}{m} \cdot \frac{m-3}{m-2} \cdots \frac{3}{4} \cdot \frac{1}{2}$$
(3.8)

with n is odd (mostly 3 or 5) and m = n+1 which is even. The result of 3.8 is a number that indicates the sediment transport due to progressive waves. The process of elaborations to get this result can be found in Appendix A.

3.2.2 Standing Waves

For periodic standing waves, we can write q and ζ as:

$$\zeta = 2\hat{\zeta}\cos(ks)\cos(\omega t) = \hat{\zeta}_{st}\cos(ks)\cos(\omega t)$$
(3.9)

$$q = 2c \cdot \hat{\zeta}sin(ks)sin(wt) = c \cdot \hat{\zeta}stsin(ks)sin(\omega t)$$
(3.10)

Substituting this into equation 3.5 and time-averaging it, gives zero. This means that the sediment transport due to a standing wave is zero. Elaborations to this result can be found in Appendix A.

3.2.3 Arbitrary Waves

For arbitrary waves, propagating in opposite directions, we can write q and ζ as:

$$\zeta = \hat{\zeta}^+ \cos(\omega t - ks + \alpha) + \hat{\zeta}^- \cos(\omega t + ks)$$
(3.11)

$$q = c \cdot \hat{\zeta}^+ \cos(\omega t - ks + \alpha + \delta) + c \cdot \hat{\zeta}^- \cos(\omega t - ks + \delta)$$
(3.12)

At the moment, variations in place are not taken into account, so s is taken as a constant. The derivation of a relation for arbitrary waves is more complicated then the ones for progressive and standing waves. But after some analytical elaborating steps, that can be found in Appendix A, the equation can be 'simplified' to:

$$\overline{S} \propto -n \cdot d_0^{-(n+1)} \cdot \overline{\left(\sqrt{C^2 + D^2}^n sin^n(\omega t + \phi) \cdot Acos(\omega t) + \sqrt{C^2 + D^2}^n sin^n(\omega t + \phi) \cdot Bsin(\omega t)\right)}$$
(3.13)

where

$$A = \hat{\zeta}^+ \cos(ks - \alpha) + \hat{\zeta}^- \cos(ks) \tag{3.14}$$

$$B = \hat{\zeta}^+ \sin(ks - \alpha) - \hat{\zeta}^- \sin(ks) \tag{3.15}$$

$$C = c \cdot \left(\hat{\zeta}^+ \cos(ks - \alpha - \delta) + \hat{\zeta}^- \cos(ks - \delta)\right)$$
(3.16)

$$D = c \cdot \left(\hat{\zeta}^+ \sin(ks - \alpha - \delta) - \hat{\zeta}^- \sin(ks - \delta)\right)$$
(3.17)

The relation described with equation 3.13 is too complicated to solve by hand. Also for the programming in matlab, it is not desirable to work with such a complicated description of the relation for sediment transport. Therefore, we will continue the modelling with a relation similar to the one derived for progressive waves, given by equation 3.8.

We will use the following relation for sediment transport by arbitrary waves:

$$S \propto -n \cdot d_0^{-(n+1)} \cdot \hat{q}^n \cdot \hat{\zeta} \cdot \cos(\alpha) \cdot \frac{m-1}{m} \cdot \frac{m-3}{m-2} \cdots \frac{3}{4} \cdot \frac{1}{2}$$
(3.18)

Where $\alpha = arg\{\tilde{q}_L\} - arg\{\tilde{\zeta}\}\)$, the phase lag between the specific discharge and the wave height.

From these analytical derivations it can be concluded that it is possible to express the relation for sediment transport depending on the discharge- and waveheight amplitude and on the phase lag α between the discharge and the wave height as variables,

and depending on the depth and a value arising from n and m as constants. In the next section it will be explained how this expression is used in Matlab to create a model that computes the changes in sediment transport due to tidal waves.

3.3 The Matlab Model

The approach for this model is to simplify as much as possible to a situation that can be easily understood and checked. Then, add more and more factors to make the model more complex and to bring it closer to reality again. This was done by starting with a simple model of sections connected to each other in a straight line. The properties of these sections, like the dimensions and the resistance factor, are stored in vectors. For all these sections the discharge (in the starting node and end node of every section) and the waveheight (in every node) can be calculated within iterations. These results are stored in resp. matrices and vectors. The discharge and wave height are calculated with the formulas for arbitrary waves with bi-directional wave propagation, described in the book of J. Battjes and R.J. Labeur (2017). Then, the discharge- and waveheight amplitudes (absolute values of the discharge and wave height) are inserted in the expression for sediment transport as derived in the previous section (expression 3.18). Also the phase lag α has to be computed as described in section 3.2.3. As said before, the sediment transport that is now calculated, is not the real sediment transport. There is still missing a constant value of several factors of influence. To compute the real sediment transport, the formula of Engelund-Hansen (1967) is used:

$$q_T = \frac{0.05 \cdot C_D^{3/2} \cdot \overline{U}^5}{(g(s-1))^2 \cdot d_{50}}$$
(3.19)

where C_D is the drag coefficient [-], s is the specific gravity [-] and d_{50} is the grain size [m]. The values for these constants are assumed to be resp. 1.5, 2.65 and $4 \cdot 10^{-4}$ m. For \overline{U} , expression 3.18, for sediment transport due to arbitrary waves from the previous section can be implemented, since this equation is originally derived from U^5 .

Now, a matrix, containing the sediment transport in the beginning and in the end of every section, can be computed. Due to tidal asymmetry (resulting from nonlinear effects), these values will not be the same, which means that there is a net sediment transport into or out of the section. With this, it is possible to calculate how much sediment $[m^3]$ is deposited or eroded in a section in total during a certain time period. This gives a value for the volume change of a section during this time period. The depth and the length are considered as constant, so this volume change of a section is captured by the width. Both the storage and conveyance width of every section will therefore change and the next iteration of the whole computation will be done with new values for the widths. This means that with this approach it is possible to visualize changes in width of the channels, due to erosion or deposition of sediment. The Matlab code of these computations can be found in Appendix B.

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The modelling of this concept was started in Python, because knowledge about how to use Python was gained in the second year. But soon it was clear that it would be much easier to create the model in Matlab, because Matlab works better for computations with vectors and matrices. Also, there was an already existing Matlab model, made by R.J. Labeur, that computes the discharge and waveheight due to arbitrary tidal waves with bi-directional wave propagation and this model could be used as a base.

4. VALIDATION OF THE MODEL

Before continuing to implementing a more complex situation to the model, like the Eastern Scheldt, it is important to validate the model. For this model, validating is possible by looking at the shape that the sections as a whole take, after some time. From literature it is known that alluvial tidal rivers generally have a funnel shape, in which the cross section increases in seaward direction. The Western Scheldt is an example of this. The explanation for this phenomenon is that the discharge amplitude decreases in the propagation direction of the tidal wave, due to tidal damping. This influences the net sediment transport, which results in a narrowing of the cross section in the inland direction. In this way, the system tends to a morphological equilibrium characterized by a uniform distribution of the maximum tidal current velocities and surface elevation amplitudes along the river [Battjes & Labeur, 2017]. It is found that the storage width and conveyance area converge exponentially, at the same rate, while the depth is nearly constant.

The situation of a tidal river, like the Western Scheldt, can be simulated with our model. Since it is known what should happen to the sediment transport in this situation and with that to the storage width of the sections, this can be used as a validation of the model. It should be checked whether a state of (near-)equilibrium and a funnel shape will be reached after some time or not.

Simulating a situation like the Western Scheldt in our model can be done with sections in a line with the same dimensions for every section (in the original situation). The Matlab computations for this situation can be found in Appendix B.

As a result of these computations, the wave height- and discharge amplitude after 1000 iterations can be plotted, as seen in the figure below.



Fig. 4.1.: Plots for the water level amplitude and the discharge amplitude.

It can be seen that the discharge amplitude is decreasing in the propagation direction of the tidal wave, just as it is described in the theory before. Also, it can be seen that the surface elevation amplitudes are not uniform distributed along the river. This means that the system is not in a state of equilibrium. In reality, on the other hand, a state of (near-) equilibrium would be reached at some point. This means that the model is for some reason not able to exactly represent reality. A possible explanation for this is that the model describes only the net sediment transport due to tidal asymmetry. In reality, there are more phenomena that result in sediment transport, like wind waves and the flow of the alluvial river itself.

Secondly, it has to be checked if the mentioned funnel shape will be reached. This is checked by plotting the storage width, conveyance area and depth, as seen in the figure below.



Fig. 4.2.: Plots of the storage width, conveyance area and depth.

Fortunately, as can be seen in the graphs of figure 4.2, the funnel shape does indeed arise: The storage width and the conveyance area of the alluvial river converge exponentially, at the same rate. The depth is constant. From this, it can be concluded that the model is able to represent the changes in width due to sediment transport correctly. But, as could be concluded earlier, it has to be taken into account that the model does not represent all the processes that cause sediment transport and that this could be the reason why an equilibrium state is not reached. Keeping this in mind, it is now possible to continue to a more complex situation.

5. APPLICATION OF THE MODEL

5.1 Assumptions

Now, the model can be used to visualise the situation in the Eastern Scheldt. To achieve this, the Eastern Scheldt has to be divided into different sections along which the length, storage width and conveyance width can be taken as constant for that section. This is done by analysing the locations and dimensions of the flow channels and by simplifying these into a network of nodes and sections. The flat areas are included in the storage widths of the sections. A visualisation of this network can be seen in figure 5.1. Of course, this is a just an approximation and in reality the flow pattern can be different.



Fig. 5.1.: The Eastern Scheldt as a network of channels with nodes.

Now, the lengths and widths of the determined sections can be measured approximatley, by using satellite images (Google Maps). The values of the lengths and widths of every section are stored in vectors again. For simplification, the depth is assumed to be constant for the whole system, but in reality there are certainly variations in depth. In the begin situation, also the resistance factor is taken as constant over the whole system. In reality, there will be differences, for example due to changes in bed vegetation. Furtermore, the parameters for the Engelund-Hansen formula are assumed to be the same as assumed before. There has not been done any research to the exact value of for example the mean grain size in the Eastern Scheldt.

5.2 Eastern Scheldt in Original State

For the network visualised in the previous paragraph, the computations as described in paragraph 3.3 and in chapter 4] can be done, to get insight in the change in widths as a result of sediment transport. The Matlabscript can be found in Appendix B.2. The storm surge barrier and the tidal turbines are not taken into account yet. We are purely looking at the situation inside of the Eastern Scheldt. As a result, the wave height amplitude, the discharge amplitude and the phase difference between the discharge and the wave height can be plotted again, as seen in the figure below.



Fig. 5.2.: Wave height- and specific discharge amplitude and the phase lag for the Eastern Scheldt in original state after 1000 iterations.

The wave height amplitude plot does not show such a nice course as for the system with the channels connected in a line. Obviously, the reason for this is that we are dealing with a network now. The last two dots are the nodes 10 and 11 which represent a parallel channel to the three dots before that represent the nodes 7, 8 and 9. So, the last two dots should not be seen as following up the three dots before, but they follow up dot number 6. Also the discharge amplitude does not show a very nice curve. The main reason for this is that there are big differences in storage- and conveyance width of the sections. And there are channels that are dividing into more than one channel, which means that the discharge will be splitted. The pink line represents the discharge in the beginning of every section and the small dots represent the discharge in the end of every section.

The red dots in the phase lag plot represent the phase difference in the end of every section and the pink dots represent the phase difference in the beginning of every section. Why the phase lag plot looks like this, is not completely understood yet, but this will be further discussed in paragraph 5.5.

Now it may be clear that plotting the storage width like before, would not make the results very evident. Therefore, the network as seen in figure 5.1 is plotted in Matlab, with changing width of the sections. The result of this after 1000 iterations can be seen in figure 5.3. It is visible that the channels are becoming wider, due to erosion of sediment. Also, it can be seen that the sections closer to the sea undergo a greater change in width. This is comparable with the situation before, where we could observe the formation of a funnel shape.



Fig. 5.3.: Flow channel widths (Storage width B in m) of the Eastern Scheldt in the begin situation and after 1000 iterations.

5.3 Eastern Scheldt and the Storm Surge Barrier

Now, the storm surge barrier can be implemented. For this, a new small section $(L = 100, B = B_s = 1200 \text{ m})$ is added in front of the first section. In Appendix B.2 it can be seen how the situation changes by adding this section, without the influence of the storm surge barrier. It can be seen that the changes are relatively small, as long as the influence of the storm surge barrier is not implemented.

The storm surge barrier is in fact an extra resistance for the flow when it is entering the Eastern Scheldt. So, the influence of the storm surge barrier is represented by assigning an increased c_f -value to the new section. It is known that the contraction coefficient μ of the storm surge barrier has a value between 0.5 and 0.65. For this model, $\mu = 0.6$ is used. The following relations are used to determine the c_f -value of the storm surge barrier:

$$\xi_{kering} = \left(1 - \frac{1}{\mu}\right)^2 \tag{5.1}$$

$$c_f = \frac{1}{2} \xi_{kering} R/l \tag{5.2}$$

These relations are added in the model to define the new c_f -value for the first section. Also it is defined that there is no sediment transport possible in the new section (S=0 in section 1). This is done on the basis of the literature research, where could be concluded that the storm surge barrier is functioning as a blockage for sediment transport. Then, the same computations can be done as before. Again, the matlabscript can be found in Appendix B.2. As result, we can plot the wave height- and dischare amplitude again and also the phase lag.



Fig. 5.4.: Wave height- and specific discharge amplitude and the phase lag for the Eastern Scheldt + Storm Surge Barrier after 1000 iterations.

It can be seen that the wave height amplitude will be significantly lower as a result of the storm surge barrier, due to the higher resistance factor in the first section. The specific discharge in the new section is very high. A reason for this is that this section is placed in front of the other sections, before the flow is divided over more sections. In the other sections, the discharge amplitudes are a bit lower than they were in the original situation. But the changes are relatively small. And only a few values of the phase lags change significantly, due to the new circumstances, most of them do not change much.

Then, the change in widths of the sections after 1000 iterations can be plotted again. This can be seen in figure 5.5 below. It can be observed that the change in widths is smaller than it was in the original situation.



Fig. 5.5.: Flow channel widths (Storage width B in m) of the Eastern Scheldt + the Storm Surge Barrier in the begin situation and after 1000 iterations.

5.4 Eastern Scheldt, the Storm Surge Barrier and the Tidal Turbines

Now, the last step is to implement the tidal turbines. Due to energy extraction, also tidal turbines can be seen as an extra resistance to the flow. So, the impact of the tidal turbins can be defined in the same way as the storm surge barrier: The c_f -value will be increased. But then, the increase in c_f -value is smaller then for the storm surge barrier. It is estimated to be around 10% of the c_f -value for the storm surge barrier, since the impact of the tidal turbines is very small compared to the impact of the storm surge barrier. Then, the computations can be carried out in the same way as before. The Matlab Script can be found in Appendix B.2. The results for the wave height- and discharge amplitude and the phase lag between the discharge and the wave height can be seen below.



Fig. 5.6.: Wave height- and specific discharge amplitude and the phase lag for the Eastern Scheldt + Storm Surge Barrier + Tidal Turbines after 1000 iterations.



Fig. 5.7.: Flow channel widths (Storage width B in m) of the Eastern Scheldt + the Storm Surge Barrier + Tidal Turbines in the begin situation and after 1000 iterations.

It can be observed that the situation does not change that much in comparison with the situation of the storm surge barrier only. This is logical, since the impact of the tidal turbines is small with respect to the impact of the storm surge barrier. But if more tidal turbines will be built in the storm surge barrier, the c_f -value will increase of course. This will result in a greater change of the situation. It will cause a further decrease of the wave height and discharge amplitudes and with that a smaller change in widths with respect to the original situation . To complete the results, also the new flow channel widths can be plotted again, as seen in figure 5.7 on the previous page.

5.5 Analysis of the Results

A table with the exact values for the wave height- and discharge amplitudes for the original situation, the situation with the new section, with the implementation of the storm surge barrier in the new section and with the tidal turbines can be found in Appendix B.2. From the results of the three different situations discussed in this chapter, it can be concluded that for an increasing resistance coefficient c_f , the wave height amplitude is decreasing. Also the discharge amplitude is decreasing, but to a lesser extent. These results are logical, since an increased resistance factor causes a higher resistance parameter χ , which in turn increases the dimensionless resistance factor σ . By some more steps through the complex part of the calculation this results in a decrease of the wave height- and discharge amplitude. The sediment transport is depending on both the wave height amplitude and the discharge amplitude. When they are both decreasing, also the sediment transport is decreasing. Moreover, the decrease is to a higher extent, due to the 5th power of the flow velocity. This decrease in sediment transport is visible in the fact that the widening of the sections with respect to the begin situation is smaller for the situation with storm surge barrier and turbines than for the situation in original state.

Another aspect that should be analysed is the phase lag. In section 5.2 it could be observed that the values for the end of section 6 (between nodes 4 and 5) and for the sections 11 and 13 (the end sections) are differing significantly from the other values. During the iterations in the Matlab model it was visible that the end values of the sections 11 and 13 were oscillating up and down from negative to positive. The reason that these two values are varying and the others are not is that in these points Q=0. So α is only depending on the wave height, which causes these oscillations. Why the end value of section 6 is higher than the others is still unclear at the moment. However, it is also notable that the change in width is relatively small for this section, in comparison to other sections. In the begin situation this section had a very small conveyance width and a relatively large storage width. The same description is valid for the plots of the phase angle in the sections 5.3 and 5.4. But now all the dots moved one position to the right, because of the new section in the beginning. Also it can be observed that the first value (pink) is higher when the storm surge barrier (& turbines) are implemented than in the original situation, while the other values stay more or less the same.

6. DISCUSSION

The main objective of this thesis was to find out how the tidal situation of the Eastern Scheldt affects the changes in sediment transport due to tidal turbines. This goal is achieved as follows: A model has been created to simulate the net sediment transport due to tidal asymmetry, depending on the wave height and the discharge of tidal waves, which are properties of the tidal situation. The result of this model is that the net sediment transport due to tidal asymmetry is decreasing when tidal turbines are implemented. Since this net sediment transport is directed outward the Eastern Scheldt, this means a decrease in erosion of the flow channels. In our model, this results in a decrease of the widening process of the flow channels, compared to the situation if there would be no storm surge barrier and tidal turbines. This is a remarkable result, because it would mean that tidal turbines and the storm surge barrier have a rather positive effect on net sediment transport due to tidal asymmetry: The erosion process of the flat areas in the Eastern Scheldt would decrease.

But this is not in agreement with previous research about the Eastern Scheldt and the Storm Surge Barrier. As mentioned before, other research says that since the construction of the storm surge barrier, there is almost no sediment transport from the sea into the Eastern Scheldt. Which means that the flats are eroding more and more, due to wind waves [Eelkema, 2013]. There are several explanations why the result of the model is not in accordance with previous research.

The first is that in this model, only sediment transport due to tidal asymmetry is taken into account. So, for example erosion of sediment due to wind waves is not considered. From the literature research, in paragraph 2.2, it could be concluded that there was in incoming flux of sediment transport in the original situation of the Eastern Scheldt and that the storm surge barrier was blocking this flux. But in this model, there is no incoming net sediment transport visible. Only an outgoing flux (also mentioned in paragraph 2.2) is simulated in the model. A possible explanation for this could be that the incoming flux is not the result of tidal asymmetry.

Secondly, there have been made several assumptions which are not entirely correct in reality: It is assumed that the depth is constant over the entire system, which is not the case in reality. Also it is assumed that the net sediment transport only results in changes of the width, but in reality also the depth and the length of the sections may undergo some changes. Also, it is assumed that the sections keep their initial locations. This would mean that flow patterns do not change. In reality, the flow channels change their position a bit after time, due to changes in the tidal situation and also due to new implementations like the storm surge barrier. New flow patterns also change the net sediment transport patterns.

And the storm surge barrier is modelled as a small new section with a higher resistance factor. This is a very simplified representation and in reality the storm surge barrier probably causes more effects than a higher resistance only. The same is true for the tidal turbines. Beside a higher resistance, turbines also cause turbulences of the water, for example. And turbulences also influence sediment transport. Furthermore, the model is only taking into account the sediment transport that is developing inside every section due to the tidal situation of the Eastern Scheldt. As mentioned before, it is not taking into account the amount of sediment that is coming in from the sea through the inlet due to other forms of sediment transport than tidal asymmetry.

Moreover, there are 'jumps' in the width at the nodes. The model calculates the change in volume due to sediment transport for each section. And with this value, the width is adjusted to a constant value for the whole section. In the next section, this value for the width is different. So at the nodes there are jumps. In reality, the width is not constant over the length of a section of course. It will vary and therefore the sections would have smooth transitions in the nodes.

Also, the phase lag should be considered again. Now, the changes in phase lag as a result of the storm surge barrier and the tidal turbines are small. And there are a few values that stand out. But at the moment, not enough knowledge is gained to give an explanation of this result. This could be the subject of further research.

Another aspect that should be considered again in other research is that the model is not turning to a state of (near-) equilibrium. In reality this would be the case, after some time. A reason for this is that an equilibrium would be reached for a situation that includes all the occuring sediment transports, which is not the case for this model.

Besides, it can be concluded from the model that the impact of the tidal turbines is very small compared to the impact of the storm surge barrier. This is based on the assumption that the influence of the turbines is approximately 10% of the impact of the storm surge barrier. This result agrees with the expectations from the theory.

Thanks to these results, we now know to what extent the tidal situation contributes to a change in net sediment transport due to tidal turbines and the storm surge barrier. But we should keep in mind, that there are still a lot of uncertainties and assumptions within this research. And a lot of research still needs to be done to give better insight in the effect of tidal turbines on sediment transport. The analysis carried out here can be used to draw conclusions and to define the lines for future research.

7. CONCLUSIONS AND RECOMMENDATIONS

Based on the research questions of this BSc thesis about the influence of the tidal situation on the change of the sediment transport due to tidal turbines, we may conclude that:

- The tidal situation of the Eastern Scheldt is that of an estuary which means that it has both a standing and a progressive character. For a more general formulation, the situation can be described with the formulas for arbitrary waves. This results in a wave height amplitude that is increasing in landward direction and a discharge amplitude that is decreasing in landward direction.
- The construction of the storm surge barrier resulted, among others, in a higher resistance in the inlet. This caused a decrease in both the wave height- and discharge amplitude.
- Due to a decreased wave height- and discharge amplitude, as a result of the construction of the storm surge barrier, also the net sediment transport caused by tidal asymmetry, decreased inside of the Eastern Scheldt. But within this thesis, nothing can be concluded about the total net sediment transport, since only the sediment transport due to tidal asymmetry is investigated.
- The net sediment transport can be described by a derivation containing the following variables that are influenced by tidal turbines: The specific discharge, the wave height amplitude and the phase difference between the specific discharge and the wave height. Other factors in the formulation of the net sediment transport are the depth, grain size, drag coefficient and specific gravity.
- It can be confirmed that the sediment transport due to tidal asymmetry is indeed influenced by tidal turbines. Due to a higher resistance, the wave height and discharge decrease. This results in a decreased flow velocity and with that in a decrease of the net sediment transport due to tidal asymmetry. Since the net sediment transport due to tidal asymmetry is directed outwards of the Eastern Scheldt, this leads to a decrease of the erosion of the channels.
- The effect of the tidal turbines on the sedimentation is small compared to the effect of the storm surge barrier.

But this research also gives rise to a lot of questions about this topic that can not be answered yet. Research should be done on the total sediment transport due to different phenomena. For example, the effect of windwaves should be added to the model. Also, other aspects of the model can be refined. For example, a variation in the depth could be implemented. Or, the model could be extended to a situation where the sediment volume change not only adjusts the width, but also for example the depth. And more knowledge should be gained about the phase lag between the discharge and the wave height, to give a better validation and explanation of the results obtained for the phase lag. Furthermore, it should be investigated why the model is not able to reach a state of equilibrium for the situations described in this thesis. More factors and mechanisms may be added to reach this. Moreover, a better analysis of how to implement the storm surge barrier and the tidal turbines into the model should be done. This should result in a more realistic simulation of the effects on sedimentation.

All in all it can be said that much has been learnt about sediment transport and the tidal situation in the Eastern Scheldt during this thesis. But there is definitely still a lot of research to be done before we can say that we completely understand the effect of tidal turbines on sediment transport. LIST OF REFERENCES

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APPENDICES

A. ANALYTICAL DERIVATIONS OF CHAPTER 3

A.1 Progressive Waves

The equations for progressive waves can be written as:

$$\overline{q^n} = \overline{\left\{\hat{q}cos(\omega t)\right\}^n} \tag{A.1}$$

$$\overline{\zeta} = \overline{\hat{\zeta}cos(\omega t - \delta)} \tag{A.2}$$

Where δ is the phase difference between the specific discharge and the wave height. Which gives:

$$d_0^{-n} \cdot \overline{\hat{q}^n \cos^n(\omega t)} - n \cdot d_0^{-(n+1)} \cdot \overline{\hat{q}^n \cos^n(\omega t) \cdot \hat{\zeta} \cos(\omega t - \delta)}$$
(A.3)

$$= d_0^{-n} \cdot \overline{\hat{q}^n \cos^n(\omega t)} - n \cdot d_0^{-(n+1)} \cdot \hat{q}^n \cos^n(\omega t) \cdot \hat{\zeta} \cdot \left(\cos(\omega t)\cos(\delta) + \sin(\omega t)\sin(\delta)\right)$$
(A.4)

Time-averaging can be achieved by using the following rule:

$$\bar{f} = \frac{1}{T} \int_0^T f(t) dt \tag{A.5}$$

For n is odd (3 or 5), it can be concluded from plotting the function (as seen in figure A.1) that:

$$\overline{\hat{q}^n \cos^n(\omega t)} = \frac{1}{T} \int_0^T \hat{q}^n \cos^n(\omega t) dt = 0$$
(A.6)



Fig. A.1.: Plot of $cos^n(\omega t)$ for one period.

So this results in:

$$-n \cdot d_0^{-(n+1)} \cdot \overline{\hat{q}^n \cos^{n+1}(\omega t) \cdot \hat{\zeta} \cdot \cos(\delta)} - n \cdot d_0^{-(n+1)} \cdot \overline{\hat{q}^n \cos^n(\omega t) \cdot \hat{\zeta} \cdot \sin(\omega t) \sin(\delta)}$$
(A.7)

But also here, we can eliminate the cosine to the nth power, if n is odd as in this case. Which gives:

$$-n \cdot d_0^{-(n+1)} \cdot \overline{\hat{q}^n \cos^{n+1}(\omega t) \cdot \hat{\zeta} \cdot \cos(\delta)}$$
(A.8)

Saying that $n \cdot d_0^{-(n+1)} \cdot \hat{q}^n \cdot \hat{\zeta} \cdot \cos(\delta) = \alpha$ and n+1 = m, which is an even number, gives:

$$-\frac{1}{T}\int_0^T \alpha \cos^m(\omega t)dt \tag{A.9}$$

where $T = \frac{2\pi}{\omega}$.

Using the following trigonometric integral law for m is even (Wikipedia):

$$\int_{0}^{\frac{1}{2}\pi} \{\cos(x)\}^{m} dx = \frac{m-1}{m} \cdot \frac{m-3}{m-2} \cdots \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}$$
(A.10)

For one period, it is possible to simply multiply by four.

$$-\frac{\alpha\cdot\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} \{\cos(\omega t)\}^m dt = -\frac{\alpha\cdot\omega}{2\pi}\cdot\frac{m-1}{m}\cdot\frac{m-3}{m-2}\cdot\cdot\cdot\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{2\pi}{\omega}$$
(A.11)

$$= -\alpha \cdot \frac{m-1}{m} \cdot \frac{m-3}{m-2} \cdots \frac{3}{4} \cdot \frac{1}{2}$$
(A.12)

Implementing alpha again, this gives for the sediment transport:

$$S \propto -n \cdot d_0^{-(n+1)} \cdot \hat{q}^n \cdot \hat{\zeta} \cdot \cos(\delta) \cdot \frac{m-1}{m} \cdot \frac{m-3}{m-2} \cdots \frac{3}{4} \cdot \frac{1}{2}$$
(A.13)

with n is odd and m = n+1 which is even. The result of A.13 is a number that indicates the sediment transport due to progressive waves.

A.2 Standing Waves

For periodic standing waves, we can write q and ζ as:

$$\zeta = 2\hat{\zeta}\cos(ks)\cos(\omega t) = \hat{\zeta}_{st}\cos(ks)\cos(\omega t) \tag{A.14}$$

$$q = 2c\hat{\zeta}\sin(ks)\sin(wt) = c\hat{\zeta}_{st}\sin(ks)\sin(\omega t)$$
(A.15)

Substituting this in equation 3.5 gives:

$$d_0^{-n} \cdot \overline{(2c\hat{\zeta})^n sin^n(ks)sin^n(wt)} - n \cdot d_0^{-(n+1)} \cdot \overline{(2c\hat{\zeta})^n sin^n(ks)sin^n(wt)} \cdot 2\hat{\zeta}cos(ks)cos(\omega t)$$
(A.16)

This is zero, due to the time-averaging of $sin^n(\omega t)$, which means that the sediment transport due to a standing wave is zero.

A.3 Arbitrary Waves

For arbitrary waves, propagating in opposite directions, we can write Q and ζ as:

$$\zeta = \hat{\zeta}^+ \cos(\omega t - ks + \alpha) + \hat{\zeta}^- \cos(\omega t + ks)$$
(A.17)

$$q = c\hat{\zeta}^+ \cos(\omega t - ks + \alpha + \delta) + c\hat{\zeta}^- \cos(\omega t - ks + \delta)$$
(A.18)

At the moment, variations in place are not taken into account, so s is taken as a constant. Then, these can be written as:

$$\zeta = \hat{\zeta}^{+} \cdot \left(\cos(\omega t)\cos(ks - \alpha) + \sin(\omega t)\sin(ks - \alpha) \right) + \hat{\zeta}^{-} \cdot \left(\cos(\omega t)\cos(ks) - \sin(\omega t)\sin(ks) \right)$$
(A.19)

This can be further derived to:

$$\left(\hat{\zeta}^{+}\cos(ks-\alpha) + \hat{\zeta}^{-}\cos(ks)\right) \cdot \cos(\omega t) + \left(\hat{\zeta}^{+}\sin(ks-\alpha) - \hat{\zeta}^{-}\sin(ks)\right) \cdot \sin(\omega t) \quad (A.20)$$

Saying that $A = (\hat{\zeta}^+ \cos(ks - \alpha) + \hat{\zeta}^- \cos(ks))$ and $B = (\hat{\zeta}^+ \sin(ks - \alpha) - \hat{\zeta}^- \sin(ks))$ gives:

$$\zeta = A\cos(\omega t) + B\sin(\omega t) \tag{A.21}$$

The same can be done for q:

$$q = c\hat{\zeta}^{+} \cdot \left(\cos(\omega t)\cos(ks - \alpha - \delta) + \sin(\omega t)\sin(ks - \alpha - \delta)\right) + c\hat{\zeta}^{-} \cdot \left(\cos(\omega t)\cos(ks - \delta) - \sin(\omega t)\sin(ks - \delta)\right)$$
(A.22)

$$q = c \cdot \left(\hat{\zeta}^+ \cos(ks - \alpha - \delta) + \hat{\zeta}^- \cos(ks - \delta)\right) \cdot \cos(\omega t) + c \cdot \left(\hat{\zeta}^+ \sin(ks - \alpha - \delta) - \hat{\zeta}^- \sin(ks - \delta)\right) \cdot \sin(\omega t)$$
(A.23)

Saying that $C = c \cdot (\hat{\zeta}^+ \cos(ks - \alpha - \delta) + \hat{\zeta}^- \cos(ks - \delta))$ and $D = c \cdot (\hat{\zeta}^+ \sin(ks - \alpha - \delta) - \hat{\zeta}^- \sin(ks - \delta))$ gives:

$$q = C\cos(\omega t) + D\sin(\omega t) \tag{A.24}$$

$$=\sqrt{C^2 + D^2}sin(\omega t + \phi) \tag{A.25}$$

Where $\phi = atan2(B, A)$. Now, we want to substitute these equations into 3.5, which gives:

$$\overline{S} \propto d_0^{-n} \cdot \overline{\left\{\sqrt{C^2 + D^2}sin(\omega t + \phi)\right\}^n} - n \cdot d_0^{-(n+1)} \cdot \overline{\left\{\sqrt{C^2 + D^2}sin(\omega t + \phi)\right\}^n} \cdot \left(Acos(\omega t) + Bsin(\omega t)\right)$$
(A.26)

Time-averaging shows that $\{sin(\omega t + \phi)\}^n = 0$. But the other terms are not zero. So, the equation can only be simplified to:

$$\overline{S} \propto -n \cdot d_0^{-(n+1)} \cdot \overline{\left(\sqrt{C^2 + D^2}^n sin^n(\omega t + \phi) \cdot Acos(\omega t) + \sqrt{C^2 + D^2}^n sin^n(\omega t + \phi) \cdot Bsin(\omega t)\right)}$$
(A.27)

where

$$A = \hat{\zeta}^+ \cos(ks - \alpha) + \hat{\zeta}^- \cos(ks) \tag{A.28}$$

$$B = \hat{\zeta}^+ \sin(ks - \alpha) - \hat{\zeta}^- \sin(ks) \tag{A.29}$$

$$C = c \cdot \left(\hat{\zeta}^+ \cos(ks - \alpha - \delta) + \hat{\zeta}^- \cos(ks - \delta)\right) \tag{A.30}$$

$$D = c \cdot \left(\hat{\zeta}^+ \sin(ks - \alpha - \delta) - \hat{\zeta}^- \sin(ks - \delta)\right) \tag{A.31}$$

It is also possible to simplify the relation for sediment transport in the case of arbitrary waves to: m = 1 - m - 2 - 2 - 1

$$S \propto -n \cdot d_0^{-(n+1)} \cdot \hat{q}^n \cdot \hat{\zeta} \cdot \cos(\alpha) \cdot \frac{m-1}{m} \cdot \frac{m-3}{m-2} \cdots \frac{3}{4} \cdot \frac{1}{2}$$
(A.32)

Where $\alpha = arg\{\tilde{q}_L\} - arg\{\tilde{\zeta}\}.$

B. MATLAB SCRIPTS AND PLOTS

B.1 Basic model: Sections connected as a line

```
clear all
im = complex(0,1);
% INVOERGEGEVENS
% NETWERK
nx=10;
                          % aantal vakken
np=11;
                          % aantal knooppunten
vak=[1 2;
                          % knoopnummers per vak
    2 3;
     3 4;
     4 5;
     5 6;
     67;
     7 8;
     8 9;
     9 10;
     10 11];
% VAKCONSTANTEN
L=repmat(15000,nx,1); % vaklengte [m]
Bs=repmat(600,size(L)); % stroomvoerende breedte [m]
                            % bergende breedte
B=2*Bs;
                                                      [m]
D=repmat(15, size(B)); % stroomvoerende diepte [m]
cf=repmat(.005, size(B)); % weerstandsfactor [-]
% RANDVOORWAARDEN
omega=1.4e-4;
                            % frequentie
                                                      [rad/s]
hrand=[1];
                            % knoopnummers randpunten
                            % compl ampl randpunt
hzee=1.25;
                                                      [m]
% zwaarteversnelling (m/s2)
q=9.81;
S EINDE INVOER
% probleemvariabelen
h=repmat(0,np,1);
                         % oppervlakte-uitwijking per knoop
Q=repmat(0,nx,2);
                         % debieten per vak (positief naar knoop toe)
```

```
% start iteratie
close all
niter = 1000;
Bplot=repmat(0,nx,niter);
for iter=1:niter
  iter;
   % aanvullende vakconstanten
   As=D.*Bs;
                              % stroomvoerend oppervlak
  R=As./(Bs+2*D);
  R=As./(Bs+2*D); % hyraulische straal
chi=8*cf./(3*pi*As.*R); % weerstandsparameter
   k0=omega./sqrt(g*As./B); % golfgetal (wrijvingsloos
     % matrix opstellen
     X=repmat(0,np,np);
     % loop over alle vakken
     for j=1:nx
        k=vak(j,:);
        Qe=abs(Q(j,:));
        Qrep=(sum(Qe)*sum(Qe.*Qe)/4)^.333;
        sigma=chi(j)*Qrep/omega;
        p=im*k0(j)*sqrt(1-im*sigma);
        a=coth(p*L(j));b=sinh(p*L(j));
        A=im*omega*B(j)*[a -1/b;-1/b a]/p;
        X(k,k) = X(k,k) + A;
     end
     % randvoorwaarden
     h=repmat(0,np,1);
     for j=1:length(hrand)
         k=hrand(j);
         X(k,:)=0;
         X(k, k) = 1;
         h(k) = hzee(j);
```

```
end
```

% stelsel oplossen
h=X\h;

```
% debiet berekenen
 for j=1:nx
    k=vak(j,:);
    Qe = abs(Q(j,:));
    Qrep=(sum(Qe)*sum(Qe.*Qe)/4)^.333;
    sigma=chi(j)*Qrep/omega;
    p=im*k0(j)*sqrt(1-im*sigma);
    a=coth(p*L(j)); b=sinh(p*L(j));
    A=im*omega*B(j)*[a -1/b;-1/b a]/p;
    Q(j,:) = (A*h(k))';
 end
 % sedimentttransport berekenen (op constante factor na)
 S = repmat(0, nx, 2);
 Sk = repmat(0, np, 1);
 n = 5.;
 m=n+1;
 s1 = 2.65; %specific gravity
 Cd = 1.5; %drag coefficient
 d50 = 0.0004; %mm grain size
 F=((0.05*(Cd).^(3/2))/((g*(s1-1)).^(2)*d50));
 for j=1:nx
    k = vak(j,:);
    U = abs(Q(j,:))/As(j);
    alfa = angle(Q(j,:)) - angle(h(k)');
     S(j,:) = -(U.^5).*abs(h(k)'/D(j)).*cos(alfa)*Bs(j)*n*((m-1)/m)*((m-3)/(m-2))* <
((m-5)/(m-4)) *F;
     Sk(k) = Sk(k) + [1,-1]' \cdot S(j,:)';
  end
  % geul aanpassen
  if (iter>50)
    beta = 4.e4;
    dV = -(sum(S'))';
    dV = (diff(sk'))';
    dB = beta*dV./(D.*L);
     aa = B./Bs;
    Bs = Bs + dB;
    B = aa.*Bs;
  end
  Bplot(:,iter)=Bs;
end
```

```
%pause
% plotten
hold off
subplot(1,2,1)
plot(abs(h), 'ro')
set(gca, 'fontsize',15)
xlabel('node','FontSize',18)
ylabel('amplitude wave height [m]', 'FontSize',18)
subplot(1,2,2)
plot(linspace(1,nx,nx),abs(Q(:,1)),'-m')
hold on
plot(linspace(2,nx+1,nx),abs(Q(:,2)),'.g')
set(gca, 'fontsize',15)
xlabel('node','FontSize',18)
ylabel('discharge Q [m^3/s]', 'FontSize',18)
hold on
drawnow
figure
%subplot(1,3,1)
%plot(Bplot(:,iter),'ro')
%xlabel('section')
%ylabel('Conveyance Width, Bs [m]')
subplot(1,3,1)
plot(B, 'ro')
set(gca, 'fontsize',15)
xlabel('section','FontSize',18)
ylabel('Storage Width B [m]', 'FontSize',18)
subplot(1,3,2)
plot(As, 'ro')
set(qca, 'fontsize',15)
xlabel('section','FontSize',18)
ylabel('Conveyance Area As [m^2]', 'FontSize', 18)
subplot(1,3,3)
plot(D, 'bo')
```

```
set(gca,'fontsize',15)
xlabel('section','FontSize',18)
ylabel('Depth D [m]','FontSize',18)
```

B.2 Specialised model: Eastern Scheldt

B.2.1 Eastern Scheldt in Original State

```
clear all
im = complex(0,1);
% INVOERGEGEVENS
% NETWERK
nx=13;
                          % aantal vakken
np=11;
                           % aantal knooppunten
vak=[1 2; 1 3; 1 5; 3 4; 2 5; 4 5; 2 6; 5 6; 6 7; 7 8; 8 9; 6 10; 10 11 ];
% VAKCONSTANTEN
L=[7000; 11000; 11000; 4000; 10000; 5000; 11000; 4000; 7000; 10000; 9000; 6000; ¥
10000 ]; % vaklengte[m]
Bs= [2000; 700; 1200; 500; 1400; 500; 2000; 1100; 1600; 1600; 1600; 1800; 1000 ] ; #
% stroomvoerende breedte [m]
B= [2800; 2000; 3300; 1400; 3200; 2200; 2400; 2800; 3500; 4400; 6000; 4000; 2000] ; "
% bergende breedte [m]
D=repmat(15,size(B)); % stroomvoerende diepte [m]
cf=repmat(.005,size(B)); % weerstandsfactor
                                                   [-1
% RANDVOORWAARDEN
omega=1.4e-4;
                        % frequentie
                                                   [rad/s]
hrand=[1];
                          % knoopnummers randpunten
                           % compl ampl randpunt [m]
hzee=1.25;
% zwaarteversnelling (m/s2)
q=9.81;
% EINDE INVOER
% probleemvariabelen
                     % oppervlakte-uitwijking per knoop
% debieten per vak (positief naar knoop toe)
h=repmat(0,np,1);
Q=repmat(0,nx,2);
% start iteratie
close all
niter = 1000;
Bplot=repmat(0,nx,niter);
Bplot1=repmat(0,nx,niter);
for iter=1:niter
   iter;
    % AANVULLENDE VAKCONSTANTEN
   As=D.*Bs; % stroomvoerend oppervlak
                      % hyraulische straal
   R=As./(Bs+2*D);
   chi=8*cf./(3*pi*As.*R); % weerstandsparameter
   k0=omega./sqrt(g*As./B); % golfgetal (wrijvingsloos)
   % matrix opstellen
   X=repmat(0,np,np);
   % loop over alle vakken
```

```
37
```

```
for j=1:nx
     k=vak(j,:);
     Qe=abs(Q(j,:));
     Qrep=(sum(Qe)*sum(Qe.*Qe)/4)^.333;
     sigma=chi(j)*Qrep/omega;
     p=im*k0(j)*sqrt(l-im*sigma);
     a=coth(p*L(j)); b=sinh(p*L(j));
     A=im*omega*B(j)*[a -1/b;-1/b a]/p;
     X(k,k) = X(k,k) + A;
  end
  % randvoorwaarden
  h=repmat(0,np,1);
  for j=1:length(hrand)
      ml=hrand(j);
      X(ml,:)=0; X(ml,ml)=1;
      h(ml)=hzee(j);
  end
  % stelsel oplossen
  h=X\h;
  % debiet berekenen
  for j=1:nx
     k=vak(j,:);
     Qe=abs(Q(j,:));
     Qrep=(sum(Qe)*sum(Qe.*Qe)/4)^.333;
     sigma=chi(j)*Qrep/omega;
     p=im*k0(j)*sqrt(l-im*sigma);
     a=coth(p*L(j)); b=sinh(p*L(j));
     A=im*omega*B(j)*[a -1/b;-1/b a]/p;
     Q(j,:) = (A*h(k))';
  end
  % sedimentttransport berekenen (op constante factor na)
  S = repmat(0, nx, 2);
  Sk = repmat(0, np, 1);
  alfa = repmat(0,nx,2);
  n = 5.;
  m=n+1;
  s1 = 2.65; %specific gravity
  Cd = 1.5; %drag coefficient
  d50 = 0.0004; %mm grain size
  F = ((0.05*(Cd).^{(3/2)}) / ((g*(s1-1)).^{(2)}*d50));
  for j=1:nx
     k = vak(j,:);
     U = abs(Q(j,:))/As(j);
     alfa(j,:) = angle(Q(j,:)) - angle(h(k)');
     S(j,:) = -(U.^5).*abs(h(k)'/D(j)).*cos(alfa(j,:))*Bs(j)*n*((m-1)/m)*((m-3)/u)
(m-2))*((m-5)/(m-4))*F;
     Sk(k) = Sk(k) + [1,-1]'.*S(j,:)';
  end
```

```
V= repmat(0,nx,1);
   for j=1:nx
       V(j)=Bs(j)*D(j)*L(j);
   end
 dV=repmat(0,nx,1);
  for j=1:np
      sumV=0;
      k1=[];
      for jl=1:nx
          k = vak(j1,:);
          if j==k(1) || j==k(2)
              sumV=sumV+V(j1);
              k1 = [k1; j1];
          end
      end
      sumV;
      k1;
      for j2=kl
          dV1=(Sk(j)/sumV)*V(j2);
          dV(j2)=dV(j2)+dV1;
      end
   end
   % geul aanpassen
   if (iter>50)
      beta = 4.e4;
     %dV = -(sum(S'))';
    % S1 = [1,-1].*S;
     % dV = diff(S1,1,2);
      dB = -beta*dV./(D.*L);
      aa = B./Bs;
      Bs = Bs + dB;
      B = aa.*Bs;
   end
   Bplot(:,iter)=Bs;
  Bplotl(:,iter)=B;
end
%pause
   % plotten
  hold off
  subplot(1,3,1)
  plot(abs(h), 'ro')
   set(gca,'fontsize',15)
  xlabel('node','FontSize',18)
  ylabel('amplitude wave height [m]', 'FontSize', 18)
   subplot(1,3,2)
  plot(linspace(l,nx,nx),abs(Q(:,1)),'-m')
  hold on
  plot(linspace(2,nx,nx),abs(Q(:,2)),'.g')
   set(gca,'fontsize',15)
   xlabel('section')
```

```
ylabel('Discharge Q [m^3/s]')
   %hold off
   $subplot(1,1,1)
   %plot(linspace(1,nx,nx),abs(Q(:,1))./Bs,'-m')
   %hold on
   $plot(linspace(2,nx,nx),abs(Q(:,2))./Bs,'.b')
   %set(gca,'fontsize',15)
   %xlabel('section','FontSize',18)
   %ylabel('Specific Discharge g [m^2/s]', 'FontSize',18)
   subplot(1,3,3)
   plot(alfa(:,1)*180/pi, '*m')
   hold on
  plot(alfa(:,2)*180/pi,'*r')
   set(gca, 'fontsize',15)
  xlabel('section','FontSize',18)
  ylabel('Phase lag alpha [degrees]', 'FontSize', 18)
figure
s = vak(:,1)';
t = vak(:,2)';
x= 10e3*[0 3 9 13 11 15 18 26 33 23 30];
y=10e3*[0 -3 5 3 0 -2 -9 -12 -16 -3 3];
weights1 = (Bplot1(:,1))';
Gl= graph(s,t,weightsl);
LWidths1 = G1.Edges.Weight/1000.;
subplot(1,2,1)
plot(G1, 'XData', x, 'YData', y, 'EdgeLabel', G1.Edges.Weight, 'LineWidth', LWidths1)
set(gca,'fontsize',15)
xlabel('B [m] begin situation', 'FontSize', 18)
weights = (Bplot1(:,iter));%[100 90 20 80 90 90 30 20 100 40 60 30 60];
G = graph(s,t,weights);
LWidths = G.Edges.Weight/1000.;
subplot(1,2,2)
plot(G,'XData',x,'YData',y,'EdgeLabel',G.Edges.Weight, 'LineWidth',LWidths)
set(gca,'fontsize',15)
xlabel('B [m] end situation', 'FontSize', 18)
```

B.2.2 Eastern Scheldt with a new section in front

With the new section, the numbering of the sections and nodes is changed as seen ain B.1. Also for this situation the plots for the wave height- and discharge amplitude, the phase lag and the changes in widths are shown, in order to compare to the original situation.



Fig. B.1.: The Eastern Scheldt with the new section as a network of channels with nodes.



Fig. B.2.: Wave height- and specific discharge amplitude and phase lag between these two for the Eastern Scheldt + new section.



Fig. B.3.: Flow channel widths (Storage width B in m) of the Eastern Scheldt + new section in the begin situation and after 1000 iterations.

Below, the Matlabscript for the situation with a new section can be found:

```
clear all
im = complex(0,1);
% INVOERGEGEVENS
% NETWERK
nx=14;
                          % aantal vakken
np=12;
                          % aantal knooppunten
vak=[1 2; 2 3; 2 4; 2 6; 4 5; 3 6; 5 6; 3 7; 6 7; 7 8; 8 9; 9 10; 7 11; 11 12 ];
% VAKCONSTANTEN
L=[100; 7000; 11000; 11000; 4000; 10000; 5000; 11000; 4000; 7000; 10000; 9000; 4
6000; 10000 ]; % vaklengte[m]
Bs= [1200; 2000; 700; 1200; 500; 1400; 500; 2000; 1100; 1600; 1600; 1600; 1800; ∠
1000 ]; % stroomvoerende breedte [m]
B= [1200; 2800; 2000; 3300; 1400; 3200; 2200; 2400; 2800; 3500; 4400; 6000; 4000; ε
2000]; % bergende breedte [m]
D=repmat(15, size(B)); % stroomvoerende diepte [m]
cf=repmat(.005,size(B)); % weerstandsfactor
                                                   [-]
% RANDVOORWAARDEN
omega=1.4e-4;
                         % frequentie
                                                  [rad/s]
hrand=[1];
                          % knoopnummers randpunten
                          % compl ampl randpunt [m]
hzee=1.25;
% zwaarteversnelling (m/s2)
g=9.81;
% EINDE INVOER
% probleemvariabelen
h=repmat(0,np,1);% oppervlakte-uitwijking per knoopO=zeros(nx,2);% debieten per vak (positief naar knoop toe)
% start iteratie
close all
niter =1000;
Bplot=repmat(0,nx,niter);
Bplotl=repmat(0,nx,niter);
for iter=1:niter
   iter;
    % AANVULLENDE VAKCONSTANTEN
   As=D.*Bs; % stroomvoerend oppervlak
                            % hyraulische straal
   R=As./(Bs+2*D);
   k0=omega./sqrt(g*As./B); % golfgetal (wrijvingsloos)
    %cf=[0.5*(1-(1/0.6)).^(2)*R(1)/L(1); .005; .005; .005; .005; .005; .005; .005; .005; .005;
.005; .005; .005; .005; .005; .005;]; % weerstandsfactor [-]
   chi=8*cf./(3*pi*As.*R); % weerstandsparameter
   % matrix opstellen
   X=repmat(0,np,np);
```

```
% loop over alle vakken
for j=1:nx
   k=vak(j,:);
   Qe=abs(Q(j,:));
   Qrep=(sum(Qe)*sum(Qe.*Qe)/4)^.333;
   sigma=chi(j)*Qrep/omega;
   p=im*k0(j)*sqrt(l-im*sigma);
   a=coth(p*L(j));b=sinh(p*L(j));
   A=im*omega*B(j)*[a -1/b;-1/b a]/p;
  X(k,k) = X(k,k) + A;
end
% randvoorwaarden
h=repmat(0,np,1);
for j=1:length(hrand)
    ml=hrand(j);
    X(ml,:)=0; X(ml,ml)=1;
    h(m1) = hzee(j);
end
% stelsel oplossen
h=X\h;
% debiet berekenen
for j=1:nx
   k=vak(j,:);
   Qe=abs(Q(j,:));
   Qrep=(sum(Qe)*sum(Qe.*Qe)/4)^.333;
  sigma=chi(j)*Qrep/omega;
  p=im*k0(j)*sqrt(l-im*sigma);
  a=coth(p*L(j)); b=sinh(p*L(j));
  A=im*omega*B(j)*[a -1/b;-1/b a]/p;
   Q(j,:) = (A*h(k))';
end
% sedimentttransport berekenen (op constante factor na)
S = repmat(0, nx, 2);
Sk = repmat(0, np, 1);
alfa = repmat(0, nx, 2);
n = 5.;
m=n+1;
s1 = 2.65; %specific gravity
Cd = 1.5; %drag coefficient
d50 = 0.0004; %mm grain size
F=((0.05*(Cd).^(3/2))/((g*(s1-1)).^(2)*d50));
for j=1:nx
   k = vak(j,:);
   U = abs(Q(j,:))/As(j);
   alfa(j,:) = angle(Q(j,:)) - angle(h(k)');
   if j==1
       S(j,:)=0;
   else
```

```
S(j,:) = -(U.^5).*abs(h(k)'/D(j)).*cos(alfa(j,:))*Bs(j)*n*((m-1)/m)*((m-3)/ w
(m-2))*((m-5)/(m-4))*F;
        Sk(k) = Sk(k) + [1,-1]'.*S(j,:)';
      end
  end
  V = repmat(0, nx, 1);
  alfa;
   for j=1:nx
      V(j)=Bs(j)*D(j)*L(j);
  end
  dV=repmat(0,nx,1);
  for j=2:np
     sumV=0;
     kl=[];
      for j1=2:nx
          k = vak(j1,:);
          if j==k(1) || j==k(2)
              sumV=sumV+V(j1);
              kl = [kl; j1];
          end
      end
      sumV;
     k1;
      for j2=kl
          dVl = (Sk(j) / sumV) *V(j2);
         dV(j2)=dV(j2)+dV1;
     end
  end
   % geul aanpassen
   if (iter>50)
     beta = 4.e4;
     %dV = -(sum(S'))';
    % S1 = [1,-1].*S;
    % dV = diff(S1,1,2);
     dB = -beta*dV./(D.*L);
     aa = B./Bs;
     Bs = Bs + dB;
     B = aa.*Bs;
  end
  Bplot(:,iter)=Bs;
  Bplotl(:,iter)=B;
end
%pause
  % plotten
  hold off
```

```
noid off
subplot(1,3,1)
plot(abs(h),'ro')
set(gca,'fontsize',15)
xlabel('node','FontSize',18)
```

```
ylabel('amplitude wave height [m]', 'FontSize', 18)
   subplot(1,3,2)
   plot(linspace(1,nx,nx),abs(Q(:,1)),'-m')
   hold on
   plot(linspace(2,nx,nx),abs(Q(:,2)),'.g')
   set(gca,'fontsize',15)
   xlabel('section')
   ylabel('Discharge Q [m^3/s]')
   %hold off
   $subplot(1,1,1)
   $plot(linspace(1,nx,nx),abs(Q(:,1))./Bs,'-m')
   %hold on
   $plot(linspace(2,nx,nx),abs(Q(:,2))./Bs,'.b')
   %set(gca,'fontsize',15)
   %xlabel('section','FontSize',18)
   %ylabel('Specific Discharge q [m^2/s]', 'FontSize',18)
  subplot(1,3,3)
  plot(alfa(:,1)*180/pi,'*m')
  hold on
  plot(alfa(:,2)*180/pi,'*r')
  set(gca,'fontsize',15)
  xlabel('section','FontSize',18)
  ylabel('Phase lag alpha [degrees]', 'FontSize',18)
   %drawnow
   $subplot(1,4,4)
   %plot(Bplot(:,iter),'ro')
   %hold on
   %plot(-0.5*Bplot(:,iter),'ro')
   %xlabel('vak')
   %ylabel('Bs')
   %hold on
  %drawnow
figure
s = vak(:, 1)';
t = vak(:,2)';
x= 10e3*[-0.1 0 3 9 13 11 15 18 26 33 23 30];
y=10e3*[0 0 -3 5 3 0 -2 -9 -12 -16 -3 3];
weights1 = (Bplot1(:,1))';
Gl= graph(s,t,weightsl);
LWidthsl = G1.Edges.Weight/1000.;
subplot(1,2,1)
plot(G1, 'XData', x, 'YData', y, 'EdgeLabel', G1.Edges.Weight, 'LineWidth', LWidths1)
set(gca,'fontsize',15)
xlabel('B [m] begin situation', 'FontSize', 18)
weights = (Bplot1(:,iter))';%[100 90 20 80 90 90 30 20 100 40 60 30 60];
G = graph(s,t,weights);
LWidths = G.Edges.Weight/1000.;
subplot(1,2,2)
plot(G, 'XData', x, 'YData', y, 'EdgeLabel', G.Edges.Weight, 'LineWidth', LWidths)
set(gca,'fontsize',15)
xlabel('B [m] end situation')
```

B.2.3 Eastern Scheldt with Storm Surge Barrier

```
clear all
im = complex(0,1);
% INVOERGEGEVENS
% NETWERK
                          % aantal vakken
nx=14:
                           % aantal knooppunten
np=12;
vak=[1 2; 2 3; 2 4; 2 6; 4 5; 3 6; 5 6; 3 7; 6 7; 7 8; 8 9; 9 10; 7 11; 11 12 ];
% VAKCONSTANTEN
L=[100; 7000; 11000; 11000; 4000; 10000; 5000; 11000; 4000; 7000; 10000; 9000; 
u
6000; 10000 ]; % vaklengte[m]
Bs= [1200; 2000; 700; 1200; 500; 1400; 500; 2000; 1100; 1600; 1600; 1600; 1800; ∠
1000 ]; % stroomvoerende breedte [m]
B= [1200; 2800; 2000; 3300; 1400; 3200; 2200; 2400; 2800; 3500; 4400; 6000; 4000; μ
2000]; % bergende breedte
                                 [m]
D=repmat(15, size(B)); % stroomvoerende diepte [m]
%cf=repmat(.005,size(B)); % weerstandsfactor
                                                   [-]
% RANDVOORWAARDEN
omega=1.4e-4;
                         % frequentie
                                                  [rad/s]
hrand=[1];
                           % knoopnummers randpunten
hzee=1.25;
                           % compl ampl randpunt [m]
% zwaarteversnelling (m/s2)
g=9.81;
% EINDE INVOER
% probleemvariabelen
h=repmat(0,np,1); % oppervlakte-uitwijking per knoop
Q=zeros(nx,2); % debieten per vak (positief naar knoop toe)
% start iteratie
close all
niter = 1000;
Bplot=repmat(0,nx,niter);
Bplotl=repmat(0,nx,niter);
for iter=1:niter
   iter;
    & AANVULLENDE VAKCONSTANTEN
               % stroomvoerend oppervlak
2*D).
% humeulische strool
    As=D.*Bs;
    R=As./(Bs+2*D);
                             % hyraulische straal
    k0=omega./sqrt(g*As./B); % golfgetal (wrijvingsloos)
    cf=[0.5*(1-(1/0.6)).^(2)*R(1)/L(1); .005; .005; .005; .005; .005; .005; .005; .005; .v
005; .005; .005; .005; .005; .005;]; % weerstandsfactor
                                                               [-]
   chi=8*cf./(3*pi*As.*R); % weerstandsparameter
   % matrix opstellen
  X=repmat(0,np,np);
```

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```
% loop over alle vakken
for j=1:nx
   k=vak(j,:);
   Qe=abs(Q(j,:));
   Qrep=(sum(Qe)*sum(Qe.*Qe)/4)^.333;
   sigma=chi(j)*Qrep/omega;
  p=im*k0(j)*sqrt(l-im*sigma);
  a=coth(p*L(j)); b=sinh(p*L(j));
   A=im*omega*B(j)*[a -1/b;-1/b a]/p;
   X(k,k) = X(k,k) + A;
end
% randvoorwaarden
h=repmat(0,np,1);
for j=1:length(hrand)
    ml=hrand(j);
    X(ml,:)=0; X(ml,ml)=1;
    h(ml) = hzee(j);
end
% stelsel oplossen
h=X\h;
% debiet berekenen
for j=1:nx
   k=vak(j,:);
   Qe=abs(Q(j,:));
   Qrep=(sum(Qe)*sum(Qe.*Qe)/4)^.333;
  sigma=chi(j)*Qrep/omega;
  p=im*k0(j)*sqrt(l-im*sigma);
  a=coth(p*L(j)); b=sinh(p*L(j));
  A=im*omega*B(j)*[a -1/b;-1/b a]/p;
  Q(j,:) = (A*h(k))';
end
% sedimentttransport berekenen (op constante factor na)
S = repmat(0, nx, 2);
Sk = repmat(0, np, 1);
alfa = repmat(0, nx, 2);
n = 5.;
m=n+1;
s1 = 2.65; %specific gravity
Cd = 1.5; %drag coefficient
d50 = 0.0004; %mm grain size
F=((0.05*(Cd).^(3/2))/((g*(s1-1)).^(2)*d50));
for j=1:nx
   k = vak(j,:);
   U = abs(Q(j,:))/As(j);
   alfa(j,:) = angle(Q(j,:)) - angle(h(k)');
   if j==1
       S(j,:)=0;
   else
```

```
S(j,:) = -(U.^5).*abs(h(k)'/D(j)).*cos(alfa(j,:))*Bs(j)*n*((m-1)/m)*((m-3)/ ~
(m-2))*((m-5)/(m-4))*F;
        Sk(k) = Sk(k) + [1,-1]'.*S(j,:)';
      end
   end
   V= repmat(0,nx,1);
   for j=1:nx
       V(j)=Bs(j)*D(j)*L(j);
   end
  dV=repmat(0,nx,1);
  for j=2:np
     sumV=0;
      kl=[];
      for j1=2:nx
         k = vak(j1,:);
          if j==k(1) || j==k(2)
             sumV=sumV+V(j1);
              k1 = [k1; j1];
          end
      end
      sumV;
      kl;
      for j2=k1
         dV1 = (Sk(j) / sumV) *V(j2);
          dV(j2) = dV(j2) + dV1;
      end
   end
    % geul aanpassen
   if (iter>50)
     beta = 4.e4;
     %dV = -(sum(S'))';
   % S1 = [1,-1].*S;
    % dV = diff(S1,1,2);
     dB = -beta*dV./(D.*L);
      aa = B./Bs;
      Bs = Bs + dB;
      B = aa.*Bs;
   end
   Bplot(:,iter)=Bs;
   Bplotl(:,iter)=B;
end
%pause
   % plotten
  hold off
   subplot(1,3,1)
  plot(abs(h), 'ro')
   set(gca,'fontsize',15)
```

xlabel('node','FontSize',18)

ylabel('amplitude wave height [m]', 'FontSize', 18)

```
subplot(1,3,2)
   plot(linspace(l,nx,nx),abs(Q(:,1)),'-m')
  hold on
   plot(linspace(2,nx,nx),abs(Q(:,2)),'.g')
   set(gca,'fontsize',15)
   xlabel('section')
   ylabel('Discharge Q [m^3/s]')
   %hold off
   %subplot(1,1,1)
   %plot(linspace(l,nx,nx),abs(Q(:,1))./Bs,'-m')
   %hold on
   $plot(linspace(2,nx,nx),abs(Q(:,2))./Bs,'.b')
   %set(gca,'fontsize',15)
   %xlabel('section','FontSize',18)
   %ylabel('Specific Discharge q [m^2/s]', 'FontSize',18)
   subplot(1,3,3)
  plot(alfa(:,1)*180/pi,'*m')
   hold on
  plot(alfa(:,2)*180/pi,'*r')
   set(gca,'fontsize',15)
   xlabel('section','FontSize',18)
   ylabel('Phase lag alpha [degrees]', 'FontSize', 18)
   %drawnow
   $subplot(1,4,4)
   %plot(Bplot(:,iter),'ro')
   %hold on
   %plot(-0.5*Bplot(:,iter),'ro')
   %xlabel('vak')
   %ylabel('Bs')
   %hold on
   %drawnow
figure
s = vak(:,1)';
t = vak(:,2)';
x= 10e3*[-0.1 0 3 9 13 11 15 18 26 33 23 30];
v=10e3*[0 0 -3 5 3 0 -2 -9 -12 -16 -3 3];
weights1 = (Bplot1(:,1))';
Gl= graph(s,t,weightsl);
LWidths1 = Gl.Edges.Weight/1000.;
subplot(1,2,1)
plot(G1, 'XData', x, 'YData', y, 'EdgeLabel', G1.Edges.Weight, 'LineWidth', LWidths1)
set(gca,'fontsize',15)
xlabel('B [m] begin situation', 'FontSize', 18)
weights = (Bplot1(:,iter))';%[100 90 20 80 90 90 30 20 100 40 60 30 60];
G = graph(s, t, weights);
LWidths = G.Edges.Weight/1000.;
subplot(1,2,2)
plot(G, 'XData', x, 'YData', y, 'EdgeLabel', G.Edges.Weight, 'LineWidth', LWidths)
set(gca,'fontsize',15)
xlabel('B [m] end situation')
```

B.2.4 Eastern Scheldt with Storm Surge Barrier and Tidal Turbines

```
clear all
im = complex(0,1);
% INVOERGEGEVENS
% NETWERK
nx=14;
                          % aantal vakken
np=12;
                           % aantal knooppunten
vak=[1 2; 2 3; 2 4; 2 6; 4 5; 3 6; 5 6; 3 7; 6 7; 7 8; 8 9; 9 10; 7 11; 11 12 ];
% VAKCONSTANTEN
L=[100; 7000; 11000; 11000; 4000; 10000; 5000; 11000; 4000; 7000; 10000; 9000; ư
6000; 10000 ]; % vaklengte[m]
Bs= [1200; 2000; 700; 1200; 500; 1400; 500; 2000; 1100; 1600; 1600; 1600; 1800; ∠
1000 ]; % stroomvoerende breedte [m]
B= [1200; 2800; 2000; 3300; 1400; 3200; 2200; 2400; 2800; 3500; 4400; 6000; 4000; μ
2000]; % bergende breedte
                                [m]
D=repmat(15, size(B)); % stroomvoerende diepte [m]
%cf=repmat(.005,size(B)); % weerstandsfactor
                                                   [-1
% RANDVOORWAARDEN
omega=1.4e-4;
                          % frequentie
                                                  [rad/s]
hrand=[1];
                          % knoopnummers randpunten
hzee=1.25;
                           % compl ampl randpunt [m]
% zwaarteversnelling (m/s2)
q=9.81;
% EINDE INVOER
% probleemvariabelen
h=repmat(0,np,1); % oppervlakte-uitwijking per knoop
Q=zeros(nx,2); % debieten per vak (positief naar knoop toe)
% start iteratie
close all
niter = 1000;
Bplot=repmat(0,nx,niter);
Bplotl=repmat(0,nx,niter);
for iter=1:niter
    iter;
    % AANVULLENDE VAKCONSTANTEN
    As=D.*Bs;
                             % stroomvoerend oppervlak
    R=As./(Bs+2*D);
                             % hyraulische straal
   k0=omega./sqrt(g*As./B); % golfgetal (wrijvingsloos)
    cf=[0.5*(1-(1/0.6)).^(2)*R(1)/L(1)+0.1*0.5*(1-(1/0.6)).^(2)*R(1)/L(1); .005; . v
005; .005; .005; .005; .005; .005; .005; .005; .005; .005; .005; .005;]; %*
weerstandsfactor [-]
   chi=8*cf./(3*pi*As.*R); % weerstandsparameter
   % matrix opstellen
```

```
X=repmat(0,np,np);
% loop over alle vakken
for j=1:nx
   k=vak(j,:);
   Qe=abs(Q(j,:));
   Qrep=(sum(Qe)*sum(Qe.*Qe)/4)^.333;
   sigma=chi(j)*Qrep/omega;
   p=im*k0(j)*sqrt(l-im*sigma);
   a=coth(p*L(j));b=sinh(p*L(j));
   A=im*omega*B(j)*[a -1/b;-1/b a]/p;
   X(k,k) = X(k,k) + A;
end
% randvoorwaarden
h=repmat(0,np,1);
for j=1:length(hrand)
   ml=hrand(j);
   X(ml,:)=0; X(ml,ml)=1;
   h(ml) = hzee(j);
end
% stelsel oplossen
h=X\h;
% debiet berekenen
for j=1:nx
   k=vak(j,:);
   Qe=abs(Q(j,:));
   Qrep=(sum(Qe)*sum(Qe.*Qe)/4)^.333;
   sigma=chi(j)*Qrep/omega;
  p=im*k0(j)*sqrt(l-im*sigma);
   a=coth(p*L(j));b=sinh(p*L(j));
   A=im*omega*B(j)*[a -1/b;-1/b a]/p;
   Q(j,:) = (A*h(k))';
end
% sedimentttransport berekenen (op constante factor na)
S = repmat(0, nx, 2);
Sk = repmat(0, np, 1);
alfa = repmat(0,nx,2);
n = 5.;
m=n+1;
s1 = 2.65; %specific gravity
Cd = 1.5; %drag coefficient
d50 = 0.0004; %mm grain size
F=((0.05*(Cd).^{(3/2)})/((g*(s1-1)).^{(2)*d50}));
for j=1:nx
   k = vak(j,:);
   U = abs(Q(j,:))/As(j);
   alfa(j,:) = angle(Q(j,:)) - angle(h(k)');
   if j==1
       S(j,:)=0;
```

```
else
       S(j,:) = -(U.^5).*abs(h(k)'/D(j)).*cos(alfa(j,:))*Bs(j)*n*((m-1)/m)*((m-3)/ u
(m-2))*((m-5)/(m-4))*F;
       Sk(k) = Sk(k) + [1,-1]'.*S(j,:)';
      end
   end
  alfa;
  V= repmat(0,nx,1);
   for j=1:nx
       V(j)=Bs(j)*D(j)*L(j);
   end
  dV=repmat(0,nx,1);
  for j=2:np
     sumV=0;
      kl=[];
      for j1=2:nx
          k = vak(j1,:);
          if j==k(1) || j==k(2)
             sumV=sumV+V(j1);
              kl = [kl; j1];
          end
      end
      sumV;
      k1;
      for j2=kl
         dV1 = (Sk(j) / sumV) *V(j2);
         dV(j2)=dV(j2)+dV1;
      end
   end
   % geul aanpassen
   if (iter>50)
     beta = 4.e4;
     %dV = -(sum(S'))';
    % S1 = [1,-1].*S;
    % dV = diff(S1,1,2);
     dB = -beta*dV./(D.*L);
     aa = B./Bs;
     Bs = Bs + dB;
     B = aa.*Bs;
   end
  Bplot(:,iter)=Bs;
  Bplotl(:,iter)=B;
end
```

```
$pause
    % plotten
    hold off
    subplot(1,3,1)
    plot(abs(h),'ro')
    set(gca,'fontsize',15)
    xlabel('node','FontSize',18)
```

```
vlabel('amplitude wave height [m]', 'FontSize', 18)
  subplot(1,3,2)
  plot(linspace(l,nx,nx),abs(Q(:,1)),'-m')
  hold on
  plot(linspace(2, nx, nx), abs(Q(:, 2)), '.g')
  set(gca,'fontsize',15)
  xlabel('section')
  ylabel('debiet Q [m^3/s]')
   %hold off
   %subplot(1,1,1)
   %plot(linspace(1,nx,nx),abs(Q(:,1))./Bs,'-m')
   %hold on
   $plot(linspace(2,nx,nx),abs(Q(:,2))./Bs,'.b')
   %set(gca,'fontsize',15)
   %xlabel('section','FontSize',18)
   %ylabel('Specific Discharge g [m^2/s]', 'FontSize',18)
  subplot(1,3,3)
  plot(alfa(:,1)*180/pi,'*m')
  hold on
  plot(alfa(:,2)*180/pi,'*r')
  set(gca,'fontsize',15)
  xlabel('section','FontSize',18)
   ylabel('Phase lag alpha [degrees]', 'FontSize', 18)
   %drawnow
   %subplot(1,4,4)
   %plot(Bplot(:,iter),'ro')
   %hold on
   %plot(-0.5*Bplot(:,iter),'ro')
   %xlabel('vak')
   %ylabel('Bs')
   %hold on
   %drawnow
figure
s = vak(:,1)';
t = vak(:,2)';
x= 10e3*[-0.1 0 3 9 13 11 15 18 26 33 23 30];
y=10e3*[0 0 -3 5 3 0 -2 -9 -12 -16 -3 3];
weights1 = (Bplot1(:,1))';
Gl= graph(s,t,weightsl);
LWidths1 = G1.Edges.Weight/1000.;
subplot(1,2,1)
plot(G1, 'XData', x, 'YData', y, 'EdgeLabel', G1.Edges.Weight, 'LineWidth', LWidths1)
set(gca,'fontsize',15)
xlabel('B [m] begin situation', 'FontSize', 18)
weights = (Bplot1(:,iter))'; % [100 90 20 80 90 90 30 20 100 40 60 30 60];
G = graph(s,t,weights);
LWidths = G.Edges.Weight/1000.;
subplot(1,2,2)
plot(G, 'XData', x, 'YData', y, 'EdgeLabel', G.Edges.Weight, 'LineWidth', LWidths)
xlabel('B [m] end situation')
```

B.2.5 Results

Two tables are made: one with the wave height amplitudes in every node and one with the discharge amplitudes in the two nodes of every section. This is done for all the situations that are simulated in Matlab. The numbering as indicated in figure B.1 is used.

Node	13 sections	14 sections	14 sections+S	14 sections + S + T		
1	-	1.2500	1.2500	1.2500		
2	1.2500	1.2393	1.0861	1.0722		
3	1.2967	1.2859	1.1317	1.1176		
4	1.3007	1.2898	1.1338	1.1196		
5	1.3203	1.3093	1.1518	1.1375		
6	1.3290	1.3180	1.1610	1.1467		
7	1.3635	1.3523	1.1936	1.1791		
8	1.4143	1.4031	1.2435	1.2290		
9	1.4861	1.4747	1.3113	1.2964		
10	1.5166	1.5050	1.3382	1.3230		
11	1.3786	1.3674	1.2075	1.1929		
12	1.3971	1.3858	1.2237	1.2089		

Table B.1: Wave height amplitudes [m] for the different modelled situations.

Section	Section 13 sections		14 sections		14 sections+S		14 sections+S+T	
1-2			125250	125220	95506	95488	92987	92969
2-3	62031	49862	61000	49110	47551	39148	46403	38288
2-4	17349	5387	17040	5330	12988	4542	12646	4472
2-6	48219	22779	47270	22430	35012	17754	33981	17347
4-5	5387	3970	5330	3930	4542	3405	4472	3359
3-6	15630	4413	15380	4350	12071	3499	11787	3427
5-6	3970	966	3930	960	3405	978	3359	982
3-7	34232	23960	33730	23640	27079	19347	26503	18975
6-7	27260	22378	26870	22080	21677	18069	21226	17720
7-8	35143	24333	34650	24060	28168	20328	27610	20001
8-9	24333	11446	24060	11360	20328	10078	20001	9962
9-10	11446	0	11360	0	10078	0	9962	0
7-11	11209	3895	11080	3860	9263	3411	9100	3370
11-12	3895	0	3860	0	3411	0	3370	0

Table B.2: Discharge Amplitudes $[m^3/s]$ for the different modelled situations.