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Sparse Inversion for Solving the Coupled Marchenko Equations Including Free-surface Multiples

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Summary

We compare the coupled Marchenko equations without free-surface multiples to the coupled Marchenko equations including free-surface multiples. When using the conventional method of iterative substitution to solve these equations, a difference in convergence behaviour is observed, suggesting that there is a fundamental difference in the underlying dynamics. Both an intuitive explanation, based on an interferometric interpretation, as well as a mathematical explanation, confirm this difference, and suggest that iterative substitution might not be the most suitable method for solving the system of equations including free-surface multiples. Therefore, an alternative method is required. We propose a sparse inversion, aimed at solving an under-determined system of equations. Results show that the sparse inversion is indeed capable of correctly solving the coupled Marchenko equations including free-surface multiples, even when the iterative scheme fails. Using sparsity promotion and additional constraints, it is expected to perform better than iterative substitution when working with incomplete data or in the presence of noise. Also, simultaneous estimation of the source wavelet is a potential possibility.

Introduction

In recent years, much progress has been made in Marchenko redatuming. Starting with the work of Broggini et al. (2012), Wapenaar et al. (2014) extended the method to 2D and 3D, which was recently followed by the inclusion of free-surface multiples (Singh et al. (2015)). We will focus on the scheme including primaries, internal multiples and free-surface multiples, and study how its dynamics differ from the scheme without free-surface multiples. Iterative substitution is the conventional method for solving the coupled Marchenko equations (Slob et al. (2014)), but it might not be the most suitable method when including free-surface multiples due to a difference in dynamics. An intuitive and a mathematical explanation will be given. Based on these insights, we present an alternative method: sparse inversion.

Theory

The coupled Marchenko equations are displayed in equations 1 and 2. On the left side of the arrows are the equations without free-surface multiples, on the right side the equations that also include free-surface multiples. Both are written in a discretized notation, as introduced by Van der Neut et al. (2015). Here R_0 denotes the reflection response without free-surface multiples and R is the reflection response including free-surface multiples, which have been obtained by sources and receivers at the acquisition level. Both responses contain internal multiples. The star denotes complex conjugation. r represents the reflection coefficient of the free-surface, which we assumed to be equal to -1. Marchenko redatuming uses one-way focusing functions to focus both primary reflections and multiples at the desired depth level. The upgoing focusing function is represented by f_1^-. The downgoing focusing function has a direct part f_{1d}^+ and a coda f_{1m}^+. The direct part f_{1d}^+ is obtained by time-reversing the transmission response from the acquisition level to the focal level, modeled in a smooth velocity model. The estimate of this direct wavefield is needed to initiate the scheme and to determine the truncation times of the filter Θ. This filter is a time-symmetric windowing function that uses a difference in causality properties to separate the Green's functions from the focusing functions.

$$f_1^- = \Theta R_0 f_{1d}^+ + \Theta R_0 f_{1m}^+ \Rightarrow f_1^- = \Theta R f_{1d}^+ + \Theta R f_{1m}^+ - r\Theta R f_1^- \quad (1)$$

$$f_{1m}^+ = \Theta R_0^* f_1^- \Rightarrow f_{1m}^+ = \Theta R^* f_1^- - r\Theta R^* f_{1m}^+ \quad (2)$$

Conventionally, these equations are solved by iterative substitution (Slob et al. (2014)). The time-reversed direct wave f_{1d}^+ is used to find an initial estimate of f_1^-, using equation 1 and setting $f_{1m}^+ = 0$. Next, equation 2 is solved to obtain a first estimate of f_{1m}^+. The scheme continues by alternating between updating the upgoing focusing function f_1^- and the coda of the downgoing focusing function f_{1m}^+ according to these two equations. One iteration consists of updating both fields. When one wants to retrieve the one-way Green's functions instead of the one-way focusing functions, a filter $\Psi = 1 - \Theta$ can be applied. These Green's functions include primaries and all orders of multiples.

Ideally, the iterative scheme converges within a few iterations. Figure 1 shows the convergence of the Green's function for 3 different models, comparing the solutions of the Marchenko equations without free-surface multiples to the Marchenko equations including free-surface multiples. The error in the convergence plots has been calculated by taking the l_2-norm of the absolute difference between the modeled Green's function and the Green's function at every iteration in the scheme. Clearly, the scheme without free-surface multiples converges in a fast and straightforward manner. However, a completely different behaviour is observed for the scheme without free-surface multiples: the Green's function is not always correctly retrieved. For the first model, convergence takes place within a few iterations, but it occurs less fast and direct compared to the scheme without free-surface multiples. A behaviour of converging and diverging is observed for the second model, and complete divergence is found for the third model. This suggests that iterative substitution is perhaps not the most straightforward method for solving the coupled Marchenko equations including free-surface multiples.

The underlying dynamic by which the equations with free-surface multiples are iteratively solved is different from the dynamics when iteratively solving the equations without free-surface multiples (Staring et al. (2016)). When no free-surface multiples are present, the dynamics are straightforward. Internal multiples can generate artefacts in the focusing functions during the first iteration. A form of multiple

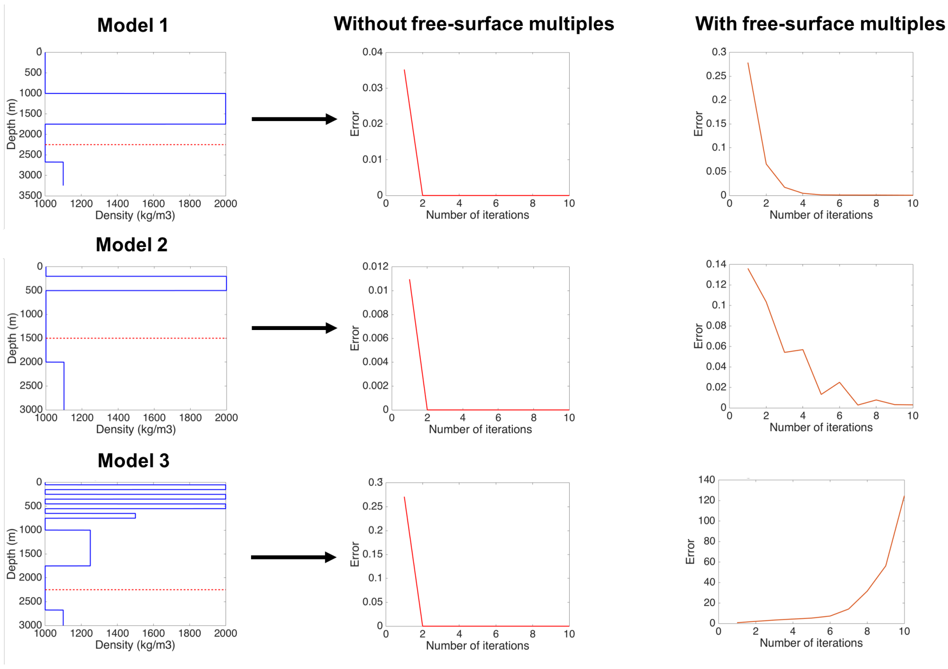

Figure 1 *Three models and their convergence, for the scheme without free-surface multiples and the scheme including free-surface multiples. The redatuming level is indicated by the dotted red line.*

prediction is used to immediately generate counter-events for these artefacts in the next iteration. The following iterations contain only amplitude updates of these counterevents, until the removal of artefacts is complete. No new artefacts are being created after the first iteration. This can be seen when looking at the convergence plots in the middle of figure 1.

In contrast, a completely different mechanism is found when studying the scheme including free-surface multiples. Free-surface multiples can also create artefacts in the focusing functions during the first iteration, but their removal is not straightforward. Although all necessary counter-events are being generated during the second iteration, they are being sabotaged by reproductions of the artefacts themselves, and new artefacts are being added. These new artefacts will not receive counter-events in next iterations, but will only be removed once the original artefacts are being eliminated. This is delicate, since the original artefacts are boosting and reproducing themselves while the scheme tries to remove them. Depending on the play of these dynamics, the scheme can produce convergence or divergence behaviour as illustrated by the three plots on the right in figure 1.

Since this is only an intuitive explanation found by studying the different updates of the one-way wavefields, a more exact and mathematical explanation is required to confirm these findings. We start by rewriting equations 1 and 2 including free-surface multiples in matrix form:

$$\underbrace{\begin{pmatrix} -r\Theta R & \Theta R \\ \Theta R^* & -r\Theta R^* \end{pmatrix}\begin{pmatrix} 0 \\ f_{1d}^+ \end{pmatrix}}_{\text{b}} = \underbrace{\left[\begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} - \begin{pmatrix} -r\Theta R & \Theta R \\ \Theta R^* & -r\Theta R^* \end{pmatrix}\right]}_{\text{A=I-M}}\underbrace{\begin{pmatrix} f_1^- \\ f_{1m}^+ \end{pmatrix}}_{\text{x}}. \tag{3}$$

This is a Fredholm equation of the second kind, which can be expanded as a Neumann series:

$$\begin{pmatrix} f_1^- \\ f_{1m}^+ \end{pmatrix} = \sum_{k=1}^{\infty} \underbrace{\begin{pmatrix} -r\Theta R & \Theta R \\ \Theta R^* & -r\Theta R^* \end{pmatrix}^k}_{\text{M}} \begin{pmatrix} 0 \\ f_{1d}^+ \end{pmatrix}. \tag{4}$$

Thus, our iterative solution can be interpreted as a Neumann series, where k indicates the iteration

Table 1 *l_2 norms of the matrix M*

	Model 1	Model 2	Model 3
Without free-surface multiples	0.58	0.82	0.98
With free-surface multiples	1.25	3.74	4.55

number. This series is guaranteed to converge if $\left\| \begin{pmatrix} -r\Theta R & \Theta R \\ \Theta R^* & -r\Theta R^* \end{pmatrix}^k \begin{pmatrix} 0 \\ f_{1d}^+ \end{pmatrix} \right\|_2 \to 0$ as $k \to \infty$ (Fokkema and van den Berg (2013)), where the subscript 2 indicates the l_2-norm. In order for this to hold, $\|M\|_2 < 1$ has to be satisfied, such that $\|MP\|_2 \leq \|P\|_2$ for any wavefield P. Note that when the norm is above 1, convergence is no longer guaranteed, but may still occur. The value of this norm can only be found by numerical testing. Table 1 displays the norms of matrix M of the three models displayed in figure 1. Without exception, the norms for the scheme without free-surface multiples are below 1 and the iterative scheme is guaranteed to converge. However, norms are above 1 when including free-surface multiples, meaning that convergence is no longer guaranteed. This confirms the intuitive explanation and the observations in figure 1. Therefore, we can conclude that the iterative scheme might not be the most suitable method for solving the coupled Marchenko equations including free-surface multiples.

Sparse inversion: an alternative approach

In order to find a more suitable method, we need to specifically look for something that is capable of tackling the convergence issue. We propose a sparse inversion for solving the linear system of equations $b = Ax$, as given in equation 3. Here the vector $x = A^{-1}b$, containing the coda of the one-way focusing functions, is the unknown that we wish to solve for. The Neumann lemma states that if $\|M\|_2 < 1$ then the matrix $A = I - M$ is invertible. In order to avoid running into the same problem as encountered using the iterative approach, the sparse inversion aims to solve an under-determined system of equations by minimizing a norm, thereby making the matrix A invertible. In addition, it allows us to enforce sparsity on the focusing functions, which should be free of artefacts. We use the SPGl_1 solver (Van den Berg and Friedlander (2008)), that seems to be particularly suitable for this type of problem. It attempts to minimize $\|x\|_1$, where the subscript 1 indicates the l_1-norm. In addition, solving the coupled Marchenko equations by inversion gives us the possibility to add additional constraints, for example in the form of smoothing and damping. When data is incomplete or contaminated with noise, an inversion with these extra constraints might still lead to an acceptable result. Also, simultaneous inversion for the source wavelet, that is usually not accurately known, might be possible.

In order to satisfy the causality requirements that form the base of the Marchenko scheme, a restriction matrix containing the filter Θ on its diagonal should be applied to the matrix x to ensure that we obtain a solution inside the domain of the focusing functions:

$$\underbrace{\begin{pmatrix} -r\Theta R & \Theta R \\ \Theta R^* & -r\Theta R^* \end{pmatrix} \begin{pmatrix} 0 \\ f_{1d}^+ \end{pmatrix}}_{\text{b}} = \underbrace{\left[\begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} - \begin{pmatrix} -r\Theta R & \Theta R \\ \Theta R^* & -r\Theta R^* \end{pmatrix} \right]}_{\text{A}} \underbrace{\begin{pmatrix} \Theta & 0 \\ 0 & \Theta \end{pmatrix} \begin{pmatrix} f_1^- \\ f_{1m}^+ \end{pmatrix}}_{\text{x}}. \quad (5)$$

Results

In this section, we compare iterative substitution to sparse inversion, by looking at the retrieved downgoing Green's functions. Data including free-surface multiples is used, such that we can see whether the sparse inversion is capable of overcoming the convergence issue from which the iterative scheme suffers. Figure 2 shows the results, where the red line represents the correct downgoing Green's function that was obtained by modeling and the blue line represents the results obtained by solving the coupled Marchenko equations including free-surface multiples. The top windows show the results of the iterative substitution and the bottom windows contain the results of the sparse inversion using the SPGl_1 solver.

To test whether the sparse inversion is indeed capable of solving the coupled Marchenko equations, we start by looking at model 1 for which the iterative scheme still finds the correct solution. The sparse inversion retrieves the correct downgoing Green's function, proving that it is indeed capable of solving the coupled Marchenko equations. Next, we perform the sparse inversion for models 2 and 3, where iterative substitution does not find the correct result. The sparse inversion does find the correct solution.

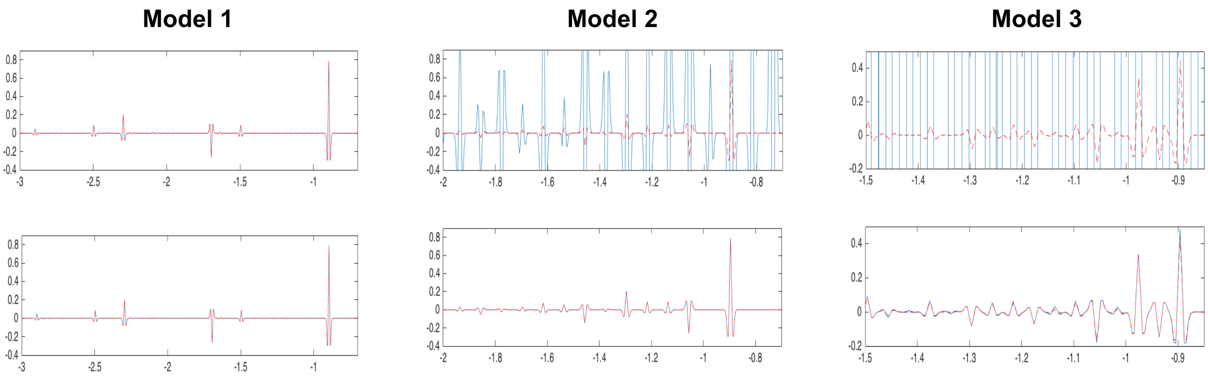

Figure 2 *Comparison of two methods: the iterative substitution in the upper row of figures, the sparse inversion with SPGl$_1$ in the lower row of figures. The red line indicates the modeled Green's functions and the blue line represents the retrieved Green's functions.*

Conclusions

While iterative substitution provides a straightforward and natural way to solve the coupled Marchenko equations without free-surface multiples, it does not for the coupled Marchenko equations including free-surface multiples. Convergence of the iterative scheme is not guaranteed, often resulting in incorrect Green's function retrieval. The interferometric interpretation of both schemes has shown that this difference can be attributed to the underlying dynamics. We suggest a sparse inversion as an alternative method. While this method is computationally more expensive, we have demonstrated that it is capable of correctly solving the coupled Marchenko equations including free-surface multiples where the iterative scheme fails. In addition, sparse inversion allows for more flexibility and is more robust. Using sparsity promotion and additional constraints, it might be able to deal better with noise and incomplete data. Simultaneous estimation of the source wavelet also belongs to the potential possibilities.

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