River Bed Response to Sea Level Rise

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by

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Preface

This report is the final component for the completion of the Master's program in hydraulic engineering at TU Delft. It has been a challenging process, exacerbated by the ever changing corona pandemic. I came to the Netherlands to learn from the experts in living with water and am excited to bring the knowledge and ethos I have learned with me to the US.

I have many people to thank for their support during this project. I would like to thank Astrid and Claudia for their patience and help during our meetings. Astrid's depth of knowledge often led to interesting questions and deeper exploration. Claudia's guidance and reassurance helped to keep me moving forward. Ralph and Martine both brought fresh perspectives and gave valuable feedback to make the report grounded and accessible.

I also want to thank my friends and family, in the Netherlands and the US, for all of their support during the many months of this project. Between lockdowns and quarantines, it was very solitary time, so I am grateful for all the calls, zooms, and outdoor meetups that kept us connected. Thank you to my classmates, Gijs, Madelief, Jan and Niels, for their comradery in the thesis process. Thank you to Marissa, Shelby, and Meghan, my American cohort here in Delft, for their friendship and encouragement. Thank you to my extended family for helping make the Netherlands really feel like home. Finally, this would not have been possible without the love and support of my parents and brothers, who have always been my greatest cheerleaders and continue to inspire me every day.

Mieke Scherpbier
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Abstract

Climate change is causing the global sea level to rise. Research and discussion of the effects of sea level rise are often focused along coastlines. However, the effects of higher water level and changing morphodynamics can reach far inland via rivers. This study uses a one-dimensional numerical model to analyze bed level response to sea level rise. The model simulates 100 years of steady sea level rise on a 1000 km long, fixed width channel.

Sea level rise creates a backwater curve which grows in the upstream and vertical directions. The transient response of the channel bed is an aggradation wave the grows in the upstream and vertical directions. We studied different cases which vary in sediment flux, flow discharge, grain size, and rate of sea level rise and found changes in the rate of growth of the aggradation in both directions. All runs start in an equilibrium state and run for 100 years.

This study finds a close relationship between the equilibrium slope and depth of the channel and the shape of the backwater curve. This relationship drives the aggradation patterns. For example, for the same amount of sea level rise, a flat, deep channel has a longer backwater curve with a smaller relative increase in depth than that of a steeper, shallower channel. The backwater curve drives the aggradation patters, such that the flat, deep channel then has a aggradation over a longer reach, and a smaller increase in bed level than the steeper shallower channel.

With the cases modeled in this study, three general trends in aggradation rates emerge: (1) an aggradation mound that grows quickly upstream, with slower increase in bed level as found in flatter, deeper channels; (2) a faster increase in bed level with slower upstream growth, found in cases with steeper slope and shallower depths; and (3) faster growth in both bed level and upstream direction caused by an increased rate of sea level rise.

Since natural channels are often complex with multiple sources of discharge inputs, a tributary case is also included. Starting from equilibrium state and applying sea level rise to the downstream boundary, the model shows aggradation waves in the regions downstream and upstream of the confluence, starting from the downstream boundary and the confluence. There is also degradation just downstream of the confluence. The scour hole grows at first, in depth and the downstream direction then reduces. The aggradation moving upstream intersects with the degradation moving downstream and fills in the scour hole. In some cases, we see the scour fills in within the 100 year time frame, resulting in a net increase in bed level. In a tributary system, the risk of scour is greatest for tributaries with high flow discharge or low sediment flux.

The transient response of the bed level to sea level rise is aggradation. As the water level continues to rise, the bed level is expected to do the same, but at a lower rate. In this study, the nominal rate of sea level rise is 10 mm/yr. The fastest rate of bed level rise in the model results is less than 6 mm/yr, creating an ever increasing water depth. This is beneficial for shipping and navigation in the channel, which would not require dredging of the aggradated material. However, the reduction in bankfull volume with rising water levels is dangerous for flood control.

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1
Introduction

1.1. Background

Rivers are essential to human life and human society. Most capital cities of countries are located along a river (Chan, Ao, Zakaria, Ab Ghani, & Rahaman, 2016), using them as sources of fresh water and shipping trade. A key example is the Netherlands, a delta region with two major rivers crossing through the country. The Rhine, the largest, enters along the eastern border and travels west to the North Sea. Over 90,000 inland vessels went through the Port of Rotterdam in 2020, carrying goods throughout the hinterland via rivers (*Facts and Figures*, 2020).

Climate change is altering rivers and putting this critical resource at risk. The warming climate is impacting the water cycle and weather patterns. In Europe, we already see drier summers and wetter winters (KNMI, 2015; Pachauri et al., 2014; Fox-Kemper et al., 2021). Warmer temperatures lead to less snow accumulation in the Alps, and the snow melts earlier in the spring. The combined effect yields lower river water levels in the summer and higher in the winter. The high water can cause flooding. Low water levels disrupt shipping, such as in the summers of 2018 and 2019 on the Rhine (Ellyatt, 2019).

Climate change is also causing an increase in sea level. The Intergovernmental Panel on Climate Change (IPCC) synthesizes climate change research and publishes a working report covering various impacts of a warming planet. The IPCC report outlines several climate scenarios for the future based on how many degrees the mean global temperature may rise by the year 2100. The most recent report, released in the summer of 2021, forecasts predictions for sea level rise based on the climate scenarios.

The IPCC quantifies a level of uncertainty for each scenario and indicates the likelihood for different climate impact effects. For example, the IPCC reports that it is “highly likely” that the mean temperature of the planet will increase by 2 to 3 degrees Celsius by 2100, including moderate changes to greenhouse gas emissions (Fox-Kemper et al., 2021). For sea level rise, there are a number of contributing climate driven effects, such as melting arctic ice sheets, thermal expansion of the ocean, and a decrease in storage of water on land (Pachauri et al., 2014; Fox-Kemper et al., 2021). To address the combination of the uncertainty in these factors, there is a range of confidence predicted for each scenario (Figure 1.1). The global mean sea level rise is expected to rise by 0.5 to 0.6 meters by 2100, possibly by as much as 0.8 meters within that scenario. In more extreme, less likely scenarios, the level is expected to exceed 1.0 meter by 2100. These scenarios were updated from the previous IPCC report in 2014, and scenarios that were once on the upper end of the expectations have become more likely, showing a worsening situation. Furthermore, the IPCC sounds the alarm that even if greenhouse gas emissions were to stop, the sea level will continue to rise.

While sea level rise has a great impact on coastlines, it can also have an effect further inland through rivers (Deltares, 2018). Higher sea levels make it more difficult for the river to drain into the sea. To better understand the impact of sea level on rivers, we have to consider the dynamics of the river. Rivers are complex systems, balancing water discharge and sediment transport with slope and grain size (Lane, 1955). Large scale

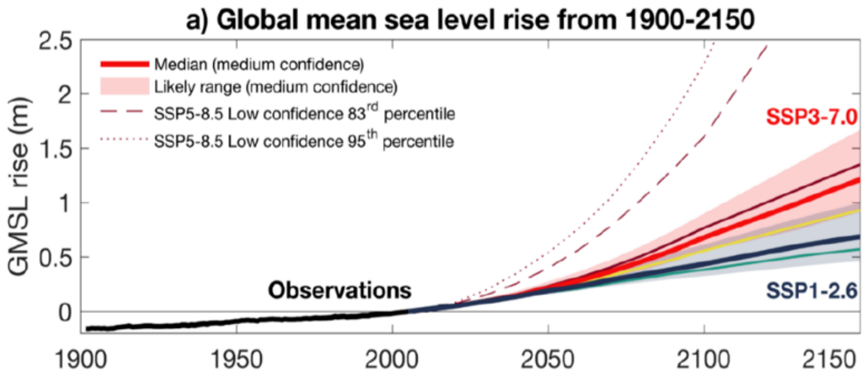


Figure 1.1: IPCC sea level rise predictions (Fox-Kemper et al., 2021). The rate is increasing over time, as is the uncertainty.

changes, such as sea level rise, can have large scale impacts on the river system. A channel adjusts towards an equilibrium state such that the gravitational acceleration of the water and sediment flowing downstream is abated by the frictional resistance from the bed. This state can be described by an equilibrium slope and depth and during which the flow is steady and uniform (Mackin, 1948).

Variations in flow discharge or downstream water level move the depth away from the original equilibrium depth, creating a backwater curve, a long smooth change in water level upstream of a disturbance. The backwater curve transitions the water level from current level to the level needed for the new equilibrium depth caused by the different conditions. Figure 1.2 shows a backwater curve caused by sea level rise. With the change in depth, comes a change in flow velocity. Flow velocity is directly related to sediment transport capacity. Faster moving water can transport more sediment. Spatial changes in velocity, either acceleration or deceleration, cause an increase or decrease in sediment transport, and therefore erosion or deposition of bed material, respectively. In the case of sea level rise, the increase in downstream water level creates a backwater curve that increases the water depth in the downstream direction. The resulting decrease in velocity leads to aggradation, increasing the bed level. The changing bed level is a transient response to the change in depth as the channel adjusts to reach a new equilibrium state.

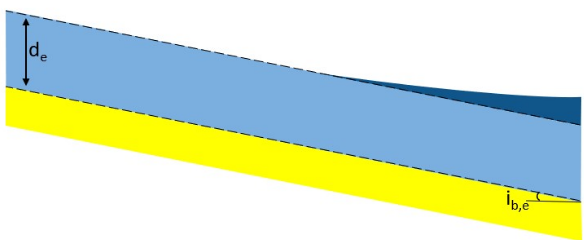


Figure 1.2: Channel with equilibrium slope and depth with backwater curve due to increase in downstream water level. The equilibrium state is defined by the depth, d_e, and slope, $i_{b,e}$.

Changes in river bed level can have large consequences. For example, a higher bed level reduces the bankfull discharge, making a channel more susceptible to flooding. Infrastructure along the river such as bridges, locks, and ports are designed for certain bed levels. A strong understanding of future changes in the river bed level is important for existing and future infrastructure projects along a river. Given the slow morphodynamic response rate of rivers, long term planning is necessary to determine the extent of any changes (de Vriend, 2015).

Previous research on both historic and future sea level rise suggests that rivers are vulnerable to large morphodynamic changes. Parker, Akamatsu, Muto, and Dietrich (2004) model sea level rise in the Holocene, caused by the melting from last glacial period. They show upstream growth of the sand bed region and the retreat of the delta over thousands of years. During this time, the sea level rose at a rate similar to that seen today. The scale of both the time and size of the impact on the river bed shows the extent to which rivers are vulnerable to sea level rise. While it may take a long time, the slow growth can add up to very large changes in bed level.

Rivers are most morphodynamically active due to bifurcations and confluence in the areas that are most vulnerable to sea level rise. In the Netherlands, the Rhine River bifurcates as it approaches the sea. Other rivers, such as the Delaware River in the United States, have tributaries that flow together and combine, increasing in size as they get closer to the sea. These physical mechanisms, bifurcation and confluence, create dynamic zones as flows separate or combine, respectively. At a bifurcation, changes in bed level can push more water to one branch or the other, effecting shipping channels and riverside infrastructure. At a confluence, the tributary increases the energy in the system by adding more water and sediment. For both, changes in one branch, such as rising sea level, impact the others, increasing the reach of the influence to a larger watershed.

1.2. Objective and Research Questions

The goal of this study is to examine the response of river bed level to sea level rise and the factors that influence the response, such as flow rate, sediment flux, grain size, and a confluence. The research questions that lead to achieving the objective are:

1. How does a river bed respond to sea level rise?
2. How does the response change due to changes in the flow discharge, sediment flux, grain size, or rate of sea level rise?
3. How does the response change with the presence of a confluence?

1.3. Methodology

The expected response of a river to sea level rise is hypothesized based on the physics of the river system in Chapter 2. The key components to the bed level response are the initial equilibrium state, a backwater curve created by rising sea level and the resulting aggradation trends. These conditions vary due to changes in water discharge, sediment flux, grain size, rate of sea level rise, and the addition of a tributary.

A one-dimensional model is used to simulate the river response to sea level rise. Simplifying to 1D flow reduces the computational time of the model which allows for large spatial scale (1000 km) and long time scale (100 years). A base case is set as a simple channel with constant water discharge and sediment flux. The downstream boundary is sea level, which rises at a constant rate. The simplifications and specifications, namely constant flow discharge, sediment flux, and grain size within each run, of the model are given in Chapter 3.

The first research question is answered by the results of the base case model, given in Chapter 4. The changes in the bed level due to sea level rise are compared to the expected response in Chapter 2.

To answer the second research question, the bed material grain size, water discharge, sediment flux, and rate of sea level rise are varied, expanding on the base case. Each parameter is varied independently, isolating their impact. The results are given in Chapter 5.

The final research question addresses how a tributary system, instead of a single channel, responds to sea level rise. Chapter 6 shows the results from a model with a confluence, including cases with varying properties of the tributary branch.

The limitations of the model and implications of this study on the Rhine and Delaware Rivers are discussed in Chapter 7.

2

Expected Bed Level Response to Sea Level Rise

The expected response of the bed level to sea level rise is discussed in this chapter. The physics of the river channel and a backwater curve are applied to the case of a simple channel with sea level rise. The bed level response is hypothesized for a base case and cases with varying water discharge, sediment flux, grain size, rate of sea level rise, as well as a tributary case. The expected response will be used in future chapters to analyze the results from the numerical model.

2.1. Physics of backwater and bed level change

This study uses a simple channel with a fixed width. The equilibrium state of a river is described by flow depth and bed slope such that flow is uniform and steady – not accelerating or decelerating. The system is governed by the conservation of mass and momentum. To conserve mass, the amount of water and sediment entering at the upstream boundary is the same as is leaving at the downstream boundary. To conserve momentum, the downward gravitational force on the water and suspended sediment is balanced by the sheer force from the bed material for steady flow. For a channel with a fixed width, the main parameters that govern equilibrium state are flow discharge, sediment flux, grain size, and downstream water level.

Sea level rise increases the downstream water level. To study the river bed response to this change, sea level rise is applied to different cases, created by varying other parameters – flow discharge, sediment flux, and grain size. Each case begins in an equilibrium state, so that all changes are solely due to sea level rise. Each case has its own equilibrium state that alters depending on how the varying parameters effect the balance of the equilibrium system and change the equilibrium slope and depth. For example, a channel with greater flow discharge has a deeper water depth. To balance the added in gravitational force, the shear force increases with a flatter slope. The result is a channel with deeper water and flatter slope, or steeper slope and shallower depth. The equilibrium depth can be calculated by:

$$d_e^3 = \frac{c_f q^2}{g i_{be}} \tag{2.1}$$

where d_e is the equilibrium flow depth $[m]$,
c_f is the friction coefficient [-],
q is the water discharge $[m^2/s]$,
i_{be} is the equilibrium bed slope [-].

A channel in equilibrium is defined by the equilibrium slope and depth based on the boundary conditions. If a disturbance occurs, such as a change in boundary condition, channel width, or other structure, a backwater curve is formed. The backwater curve is a gradual change in depth in the upstream direction from the disturbance. It is the spatial adjustment in depth back to the equilibrium depth further upstream. The adjustment is nonlinear, such that the steepest slope is closest to the disturbance, decreasing with greater distance.

Here the disturbance is an increase in downstream boundary water level due to sea level rise. This study only includes mild slope rivers, resulting in M-1 type curves. The resulting backwater curve increases the water depth in the upstream direction (Figure 1.2). The greatest increase is at the downstream boundary, and reduces with distance upstream until the water level is such that the water depth returns to equilibrium depth. That distance is referred to as the adaption length, the length of the region effected by the backwater curve.

The adaption length is determined by two factors: the relative change in depth due to the disturbance and the equilibrium state of the channel. First, a large change in depth from equilibrium state creates a longer backwater curve than a small deviation. In this study, as the sea level rises, the depth is moving further away from equilibrium, resulting in a backwater curve that is growing longer and reaching further upstream. Second, the equilibrium state of the channel effects the adaption length. Equilibrium slope and depth are inversely related, i.e a channel with a steep slope has shallower depth, and a channel with deep water has a flatter slope. Flatter slope channels have longer adaption length than steeper slope channels.

These two factors have opposing effects. For the same increase in water level, a deeper channel has a smaller relative increase in depth, thus shorter adaption length. However, a deeper channel also has a flatter slope, and thereby a longer adaption length. In practice, the slope has greater influence on the adaption length than the relative change in depth. This can be seen in Bresse's approximation for the length of the backwater curve (de Vriend, Havinga, van Prooijen, Visser, & Wang, 2011):

$$L_{1/2} = 0.24\frac{d_e}{i_b}\left(\frac{d}{d_e}\right)^{4/3} \tag{2.2}$$

The equation calculates the half length, which is the distance at which the water depth is halfway between the level at the disturbance, d, and equilibrium level, d_e. The relative change in depth is shown as $\frac{d}{d_e}$ and the equilibrium state conditions as the ratio of depth to slope $\frac{d_e}{i_b}$. While the relative change in depth is to a higher power, recall from equation 2.1 that the slope is inversely related to d_e^3, such that that portion of the equation is related to d_e^4. Hereby the equilibrium state conditions have a greater influence on the adaption length than the relative change in depth. Figure 2.1 illustrates the relationship between equilibrium depth and slope of the channel and length of the backwater curve. For the same increase in downstream water level, the backwater curve extends farther upstream in case B, which has a flatter slope and deeper equilibrium depth The gradual increase in depth across the length of the back water curve leads to a gradual decrease in flow velocity. The slower moving water reduces sediment transport capacity and sediment deposits. Figure 2.2 shows the process of the transient state of the channel as it moves from one equilibrium state to a new one created by an increased in downstream water level. However, in the case of sea level rise, the new equilibrium state is constantly changing, and therefore never reached. The aggradation increases the bed level at a slower rate than the increase in water level, so the water depth continues to increase, and more aggradation occurs.

The change in bed level is driven by the change in depth from the backwater curve. The relationship between the spatial decrease in sediment flux and the temporal increase in bed level is given in the Exner equation (see Appendix A for more detail). With it, we can relate the spatial change in bed depth to the change in water level along the bed due to the backwater curve. To compare the influence of different parameters on the response of a river bed to sea level rise, the expected response can be derived by the effect the parameters have on the equilibrium state and thereby the backwater curve.

Sea level rise is a continual process, so the backwater curve will continue to grow, and so will the resulting aggradation. In this study, the aggradation mound is analyzed with three characteristics: growth in the vertical, z, direction, growth in the upstream, x, direction, and total volume deposited. The river response to sea level rise is analyzed for certain cases in the following sections.

2.2. Base Case

A base case is established as a simple channel with a constant rate of sea level rise. This will serve as the basis of comparison when we begin to vary parameters. Figure 2.3 shows the channel first in equilibrium. Then

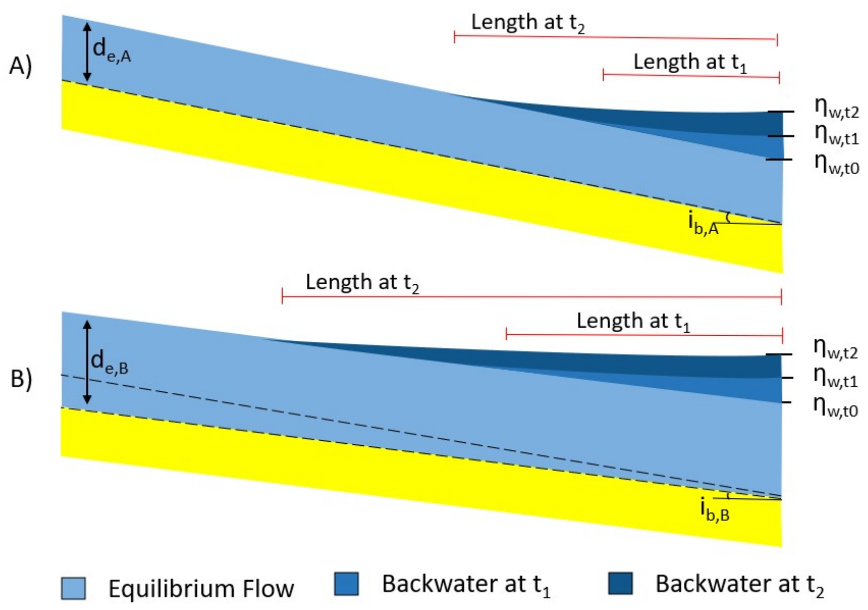


Figure 2.1: As the downstream water level increases over time, the backwater curve grows in depth and length. Case B has a flatter slope ($i_{b,B} < i_{b,A}$) and deeper equilibrium flow depth ($d_{e,B} > d_{e,A}$). The downstream water level, η_w, increases at the same rate in both cases. The adaption length grows faster in case B for the same increase in downstream water level.

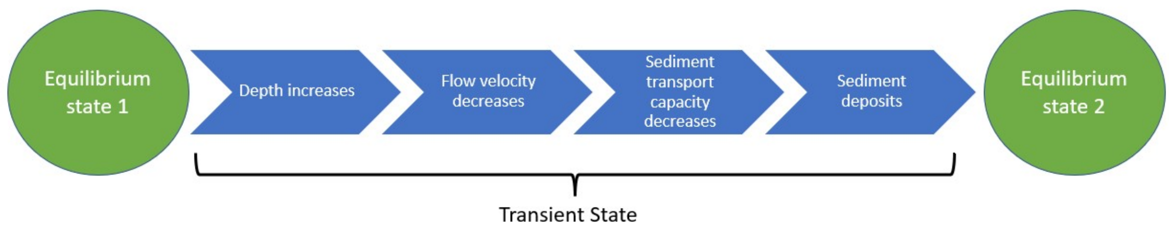


Figure 2.2: Process of transitioning from one equilibrium state to another due to increase in downstream water level from sea level rise.

the sea level begins to rise at the downstream end, creating a backwater curve and aggradation starts. As time goes on, the sea level continues to rise and the backwater curve grows further upstream. More sediment is deposited and the aggradation mound grows.

The aggradation mound is growing in both the vertical and upstream direction direction and migrating upstream. However, it is not possible to separate the changes in the aggradation mound due to the upstream movement of the mound or growth due to additional sediment deposition. The rate of growth of the aggradation mound is dependent on the influence of the backwater curve and the sediment supply. If sea level rise stops, the mound would continue to evolve until the river reaches a new equilibrium state.

Expanding from the base case, the water discharge, sediment flux, grain size, and rate of sea level rise are independently varied to increase the understanding of how these properties impact the river bed response. These trends are summarized in Table 2.1 at the end of the chapter.

2.3. Change in Water Discharge

For a system with a higher constant water discharge, the equilibrium state has a deeper water depth and a flatter slope to conserve momentum. The change in the equilibrium state also changes the backwater curve. Due to the different bed slope, the backwater curve is longer for the same rate of sea level rise. The relative

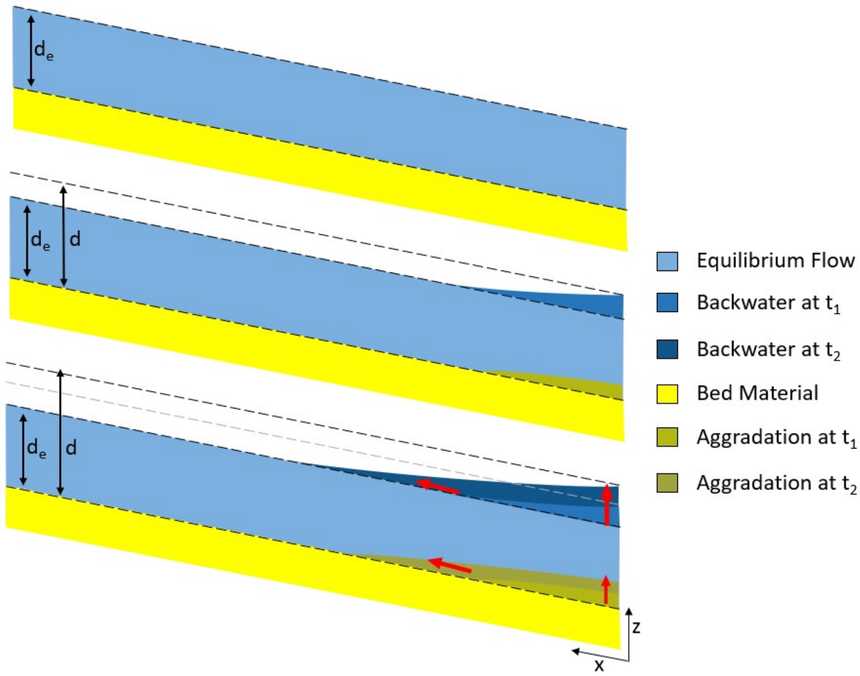
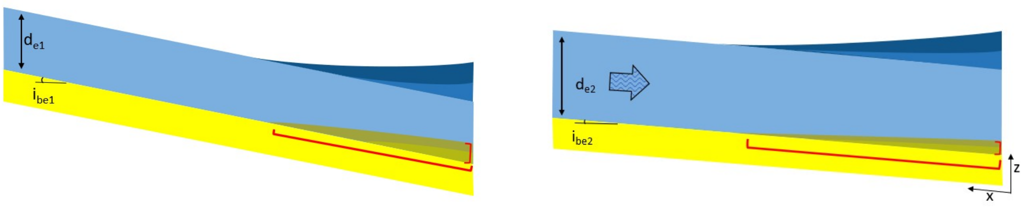


Figure 2.3: Simple channel starts in equilibrium state. The downstream water level increases with sea level rise at a constant rate, creating a backwater curve. Aggradation occurs as the depth increases from the initial equilibrium depth. The backwater curve grows as sea level rise continues; the bed level response is aggradation that grows as well.

change in depth due to sea level rise is smaller than the base case and the resulting spatial change in sediment mobility is smaller. We therefore expect that an increased water discharge results in longer and flatter aggradation mound that grows upstream faster, as illustrated in Figure 2.4.

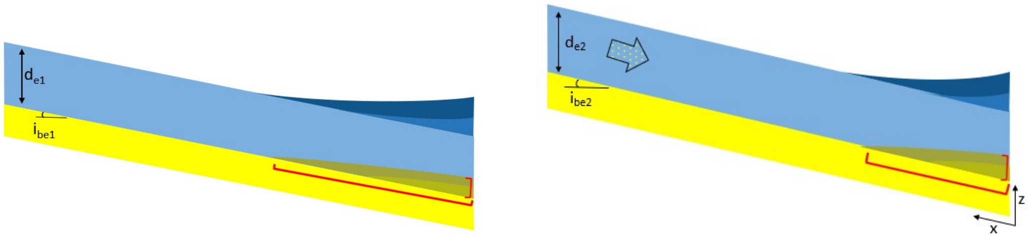


(a) Base Case (b) Increased Q_w

Figure 2.4: Increasing the constant water discharge creates an equilibrium state with deeper water depth ($d_{e,1} < d_{e,2}$) and flatter slope ($i_{b,1} > i_{b,2}$). Sea level rise creates a longer backwater curve, with a smaller relative increase in depth. The result is a longer, less tall aggradation mound.

2.4. Change in Sediment Flux

Increasing the constant sediment flux in a system leads to an increase in slope and decrease in water depth in the equilibrium state. The backwater curve is then shorter for the same rate of sea level rise, and the relative change in depth is higher. The resulting aggradation is therefore expected to be greater and over a shorter distance (Figure 2.5). The backwater curve grows more slowly upstream, so the upstream growth of the aggradation mound is also expected to be slower. Additionally, the sediment supply is higher which can further increase aggradation. In other words, if the river is carrying more sediment, it will also deposit more sediment, but due to the shorter backwater curve, it happens over a shorter distance. This leads to a taller aggradation mound and larger increase in bed level.



(a) Base Case (b) Increased Q_s

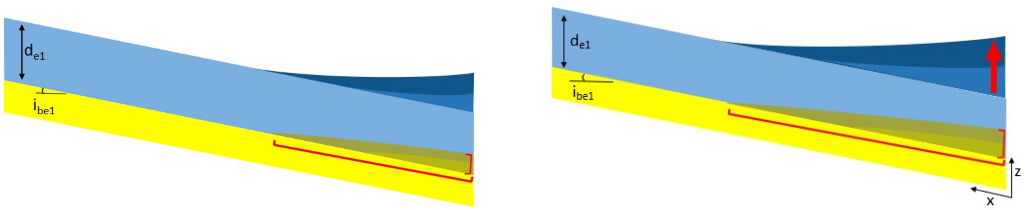
Figure 2.5: Increasing the constant sediment flux creates an equilibrium state that has a steeper slope ($i_{b,1} < i_{b,2}$) and shallower flow depth ($d_{e,1} > d_{e,2}$) . The effect on the backwater curve is a greater relative increase in depth and shorter length. The effect is a more compact aggradation mound that is taller and does not extend as far upstream.

2.5. Change in Grain Size

This study only considers cases with a uniform grain size, but examines the influence of grain size by varying it across cases. A system with an increased grain size has a steeper slope and shallower water depth. Sea level rise imposes a shorter backwater curve, and a greater increase in relative depth. This is similar to the effect of increasing the sediment flux (Figure 2.5). The upstream growth is expected to be slower, while the vertical growth will be faster. The total volume of aggradation is not expected to increase as much from a larger grain size than increased sediment flux because the sediment supply does not increase.

2.6. Change in Rate of Sea Level Rise

The rate of sea level rise determines the rate of growth of the backwater curve. In this study, the rate of sea level rise is constant, not accelerating, as it is in actuality (Fox-Kemper et al., 2021). An increase in constant rate of sea level rise causes the upstream extent of the curve to migrate upstream faster, and the relative depth increase with time will be larger. The bed level is expected to aggrade faster, both upstream and vertically, as shown in Figure 2.6.



(a) Base Case (b) Increased Rate of SLR

Figure 2.6: For the same initial state, increasing the constant rate of sea level rise leads to faster growth of aggradation in both depth and upstream reach

2.7. Simple Channel Summary

The bed level response in the simple channel is summarized in Table 2.1. Previous sections describe how changes in water discharge, sediment flux and grain size change the equilibrium state of the channel, and therefore the backwater curve. The aggradation patters reflect these differences.

2.8. Tributary System

We add a tributary to our model in an attempt to capture the higher degree of physical complexity present in natural river systems. The tributary increases the water discharge and sediment flux of the main channel downstream of the confluence. Figure 2.7 gives the layout of the tributary system. Branch 1 is the same as the base case. Branch 2 adds water and sediment to the main channel, resulting in Branch 3, which brings the

Case	Initial Slope	Initial Water Depth	Growth of aggradation in upstream direction	Increase in bed level
Increase in water discharge	−	+	+	−
Increase in sediment flux	+	−	−	+
Increase in grain size	+	−	−	+
Increase in rate of sea level rise	0	0	+	+

Table 2.1: Summary of expected response of simple channel cases. The relative changes in the growth of the aggradation with respect to the base case.

combined load to the sea. In the equilibrium state, Branch 3 has a new equilibrium depth and slope depending on the inputs of Branch 2. The same trends follow as with the simple channel: more water leads to flatter and deeper equilibrium state, where as more sediment leads to steeper and shallower conditions. Branch 3 is always deeper than Branch 1. However, the slope may be steeper or flatter depending on the inputs from the tributary branch. Figure 2.8 gives a profile of branches 1 and 3. There is a step at the confluence point to accommodate the spatial increase in flow depth.

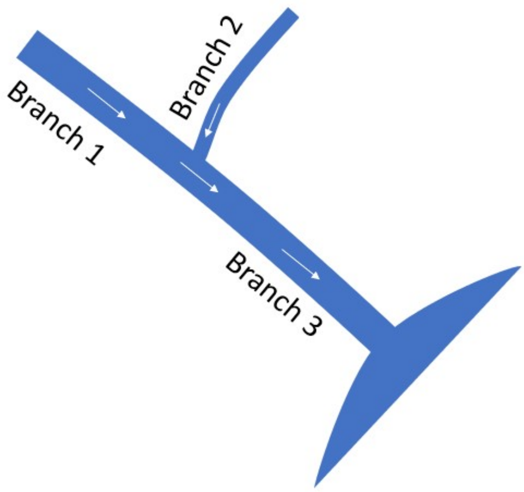


Figure 2.7: Branches of tributary system. Branches 1 and 3 make up the main channel. Branch 2 adds water and sediment to the system, increasing the flow rate and sediment flux of branch 3.

The river response resembles the single channel case at first, before the backwater curve reaches the confluence point. Then aggradation is expected to occur on the upstream branch as well. Since this branch is less deep, the relative increase of the flow depth due to sea level rise is larger. Sediment deposits more quickly for the same increase in water depth in the downstream branch. The sediment supply downstream is then reduced, slowing aggradation and causing some erosion just downstream of the confluence (Best, 1988). The degradation wave is expected to grow in depth and in the downstream direction.

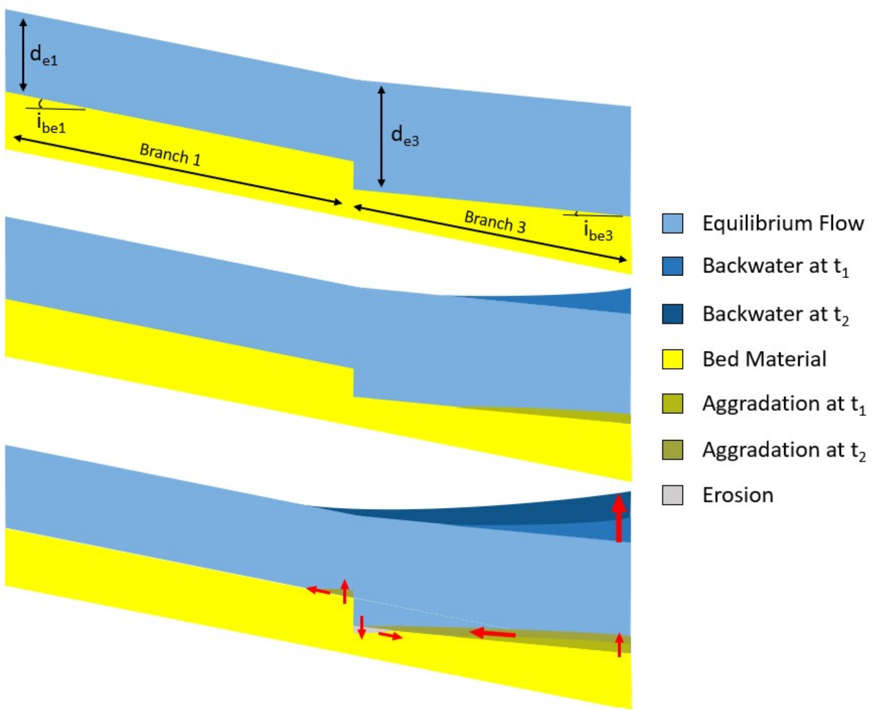


Figure 2.8: The main channel of a tributary system is shown as Branch 1 and Branch 3. The initial bed level response to sea level rise is similar to the response in the simple channel. As the backwater effects reach the confluence, aggradation begins in Branch 1 and degradation downstream of the confluence.

3
Model Plan

The response of a river bed to sea level rise is studied by modeling the hydraulic and morphological changes in a channel. To test the validity of the expected responses from the previous chapter, a model is created using Elv, a one dimensional research code.

3.1. The Model

Elv is a 1D model written in Matlab. The hydrodynamic and morphodynamic components are decoupled and solved with the Euler forward method in the upstream direction. The hydrodynamic calculations are driven by the backwater model and conservation of energy. For this study, the morphodynamic processes are modeled using the Exner equation for sediment mass conservation and the Ashida-Michue transport relation (Ashida & Michiue, 1972), which is applicable for the range of grain sizes used. See Appendix A for further explanation of these equations.

Given the long time scale and large spatial scale of this study, a 1D model is most feasible due to the shorter computational time. It still provides the detail of bed level changes needed to answer the research equations, especially in the simple channel cases. In the tributary cases, some nuance, such as the effects of the angle of the tributary, is lost by simplifying the mixing that occurs at the confluence point into 1D flow.

3.2. Parameters and Simplifications

The channel dimensions and parameters are displayed in Table 3.1. These values are loosely based on the Rhine and related studies, namely Nannenberg (2021) and Arkesteijn, Blom, and Labeur (2021). The channel is sufficiently wide to ignore the impact of wall friction. The length covers the area impacted over the simulation length, such that the upstream boundary stays in equilibrium during the simulation period[1]. The porosity and density of the bed material is constant and the values are applicable to the full range of bed grain size used.

Simplifications are made to isolate the impact of sea level rise on the bed level from other drivers of change in the bed level, such as changes in the hydrograph. In each scenario, a constant water discharge and sediment flux are applied. The flow is normal and steady. The sediment bed material is unisize. The model begins each run from an equilibrium state so the bed material grain size is the same as the sediment flux. Additionally, the sediment flux only contains bed material load; wash load is not included. At the downstream boundary, tidal influence, delta growth, or incoming sediment are not included in this study. Subsidence or uplift of the bed are also not included.

3.3. Boundary Conditions

The downstream boundary of this river model is the sea and the boundary condition is water level. Given the simplifications stated above, the initial water level is set at sea level. When applying sea level rise to the

[1] For one case, with high water discharge, the changes reached the upstream boundary, so the channel length was extended to 1500km to avoid interference of the boundary with the downstream effects

Parameter	Value
Channel Width	300 $[m]$
Channel Length	1000 $[km]$
Simulation Length	100 $[yr]$
porosity	0.4 [-]
density	2650 $[kg/m^3]$
c_f	0.008 [-]

Table 3.1: Model Parameters

model, the downstream water level increases at an interval of half a year, the length of each time step. The upstream boundary condition is set by the water discharge and sediment flux. Both are constant throughout a run (see Table 3.2). For the tributary cases, there are two upstream boundaries, each with a given water discharge and sediment flux (see Table 3.3).

3.4. Initial Conditions

All runs begin from an equilibrium state so any change in bed level is solely due to the response to sea level rise. The equilibrium slope is determined by preliminary runs without sea level rise to allow the slope to adjust to the constant boundary conditions.

For the simple channel, the initial state has normal flow conditions as determined by the boundary conditions. For the tributary case, the equilibrium state is set by the equilibrium slope and depth for each branch.

3.5. Base Case

A base case is set to use as a benchmark for comparisons and is loosely based on values for the Rhine River. The water discharge is 1000 m^3/s and sediment flux is 0.008 m^3/s for the base case. The sand bed is given by a characteristic grain size is 0.5 mm. These values are low for the Rhine, which averages about 2300 m^3/s but provide a starting point for the schematic analysis in this report (Frings et al., 2019).

The rate of sea level rise is 10 mm/yr. The current global average rate is 4 mm/yr and rising, depending on the increase in global temperature (Pachauri et al., 2014). The IPCC sets global warming scenarios based on global near surface air temperature. On the upper end of the likely scenarios, the rate of sea level rise likely reaches 10 mm/yr by 2080 (Fox-Kemper et al., 2021).

3.6. Run List

To explore the influence of different river characteristics on the river bed response to sea level rise, water discharge, sediment flux, bed grain size, and rate of sea level rise are varied in different runs. In each run, only one variable is changed, increased or decreased by a factor of 2 from the base case. The full list of scenarios for the simple channel is shown in Table 3.2.

Run	Water Discharge $[m^3/s]$	Sediment flux $[m^3/s]$	Grain Size [mm]	Rate of SLR [mm/yr]
Base	1000	0.008	0.5	10
1	500	0.008	0.5	10
2	2000	0.008	0.5	10
3	1000	0.004	0.5	10
4	1000	0.016	0.5	10
5	1000	0.008	0.25	10
6	1000	0.008	1.0	10
7	1000	0.008	0.5	5
8	1000	0.008	0.5	20

Table 3.2: Simple Channel Cases. Red cells indicate changes from the base case, either an increase or decrease by a factor of 2

The tributary case begins with the simple channel base case. A tributary branch is added, such that the upstream section of the original channel is the same, and the downstream section is the sum of the water discharge and sediment flux of the two. To start, the tributary carries 25% of the sediment and water in the main channel. The boundary conditions for the upstream end of the main channel remain the same across each case. The sediment flux and water discharge of the tributary branch are varied, as well as the location of the confluence point relative to the mouth of the river. Table 3.3 gives the values of the variables in each run.

Run	Water Discharge $[m^3/s]$	Sediment flux $[m^3/s]$	Length of Branch 3 [km]
1	250	0.002	100
2	125	0.002	100
3	500	0.002	100
4	250	0.001	100
5	250	0.004	100
6	250	0.002	50
7	250	0.002	200

Table 3.3: Tributary Cases: Water discharge and Sediment flux of the tributary branch. Distance from mouth is the location of the confluence point along the main channel. Red indicates changes from Run 1.

4 Model Results for Base Case of Simple Channel

To examine how a river bed responds to sea level rise, a simple channel base case is established in the Elv model described in Chapter 3. The simple channel starts in an equilibrium state with constant water discharge and sediment flux from the upstream boundary. The initial state is set by an equilibrium slope and water depth. To apply sea level rise, the downstream boundary water level is then increased at a constant rate for 100 years. The focus of this report is the bed level output data, which is given for each 50 m spatial step at half year intervals. This chapter analyzes the bed level changes of the base case model due to sea level rise.

4.1. Change in Bed Level

Figure 4.1 shows the change in bed level over time in the base case output from Elv. The sediment deposition starts at the downstream end and the aggradation mound grows both vertically and horizontally, further upstream. As discussed in Chapter 2, the behavior of the aggradation mound is due to the growth in the backwater curve and the changes in sediment supply due to the mound itself.

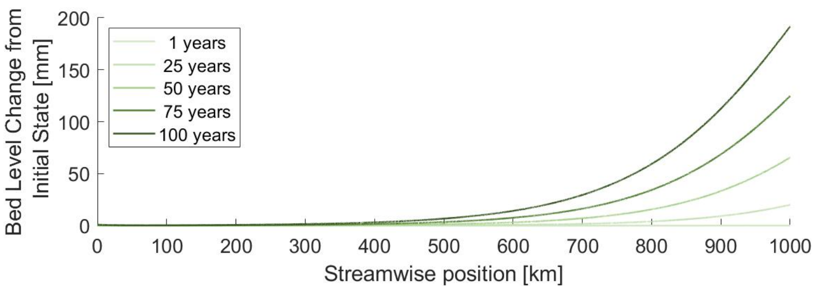


Figure 4.1: Output from base case model showing the bed level response to sea level rise over 100 years. The change in bed level from initial state increases in time and in the downstream direction.

To better characterize the bed level response to the rising sea level, the growth of the aggradation mound is categorized into three metrics: the rate of growth in the vertical direction, rate of growth in upstream direction, and total volume of deposited sediment.

4.2. Rate of Growth in Vertical Direction

Rate of growth in the vertical, z, direction measures how quickly the bed level is rising. The water level increases at a constant rate of 10 mm/yr at the downstream boundary. The bed level increases at a lower rate, reaching a maximum around 3 mm/yr during the 100 year time frame (Figure 4.2). The rate of the bed level rise is highest at the downstream boundary, where the rate of water level increase is the greatest. The backwater curve reduces the gradient in the upstream direction.

The bed level responds quickly at first, then slows over time, as seen by the spacing of the lines at the downstream boundary. There are two contributing factors to this. First, the same net increase in water level for each time period is a lower percentage increase in depth as time goes on. The bed level is rising at a slower rate than the water level, so the depth is increasing over time. The increase in water level then has a lower relative change in depth, and lower response to sediment mobility. Second, as the water level increases, the backwater curve grows in the upstream direction and has greater reach. The relative increase in water level, and subsequent relative decrease in sediment mobility, is greater upstream and more aggradation occurs. The sediment supply reaching the downstream portions of the channel is then reduced, causing a decrease in aggradation.

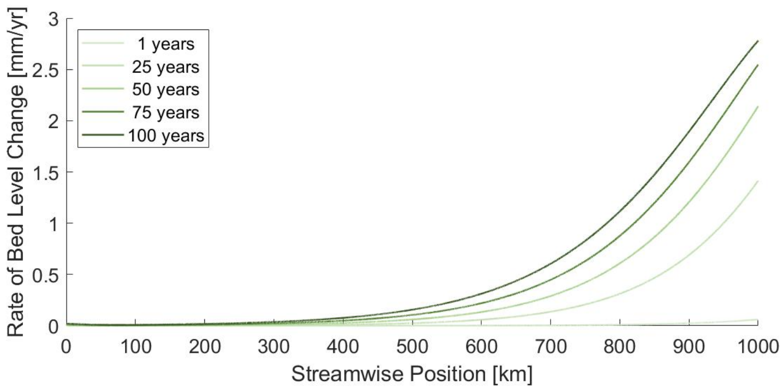


Figure 4.2: Rate of change of bed level increases over time and in the downstream direction.

4.3. Rate of Growth in Upstream Direction

The rate of growth in the upstream, x, direction characterizes how quickly the effect of sea level rise moves upstream, further inland. This rate is calculated by setting a height for the front of the aggradation wave, such as 5 mm, and tracking the time at which the bed level difference has become 5 mm from the initial level, as shown in Figure 4.3. The rate is derived by dividing the length of the step by the time it took for the front end height to reach the next grid point. The results are given in Figure 4.4 for various heights of the front end of the wave. The grid has a cell length of 50 meters and the model records data every half year. Therefore, both the time and location of the upstream migration is rounded, causing the staircase behavior in the graph.

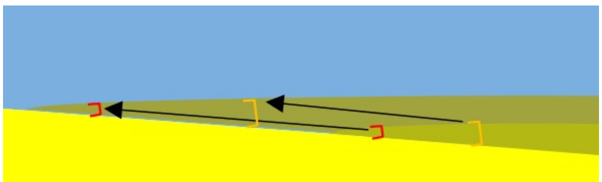


Figure 4.3: Measuring upstream migration rate: A chosen bed level difference of the front end of the aggradation mound is recorded as it moves upstream

The bed level response can again be attributed to the same two factors - relative increase in depth and sediment supply. In a backwater curve, the slope of the water level increases in the downstream direction. Therefore, the change in depth that drives sediment deposition is greater downstream leading to faster aggradation. In the upstream direction, the relative increase in depth is smaller, so aggradation slows. Additionally, sediment supply is spread throughout a larger reach, so the net increase in bed level is reduced.

4.4. Total Volume of Deposited Sediment

The third metric to characterize the changes in the bed level is total volume. This model is one dimensional, so volume is given in square meters, as the area under the curve in Figure 4.1. The volume increases exponentially at first, when the system is first out of equilibrium. The relative increase in water depth due to sea level rise is greatest at this time so aggradation increase quickly. As time goes on, the total volume deposited

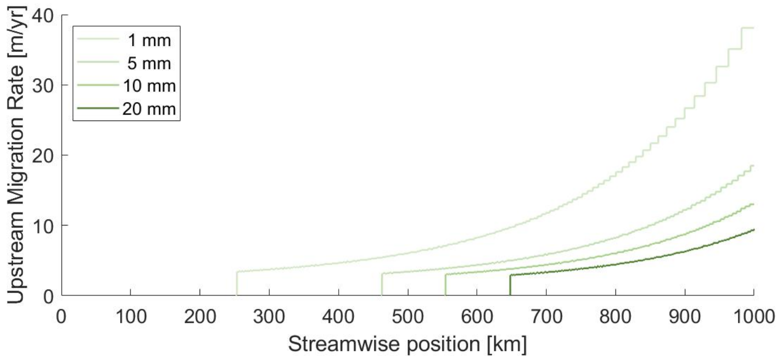


Figure 4.4: Upstream velocity of the front edge of the aggradation mound at various heights

continues to increase but at a more linear rate as shown in Figure 4.5. Despite the increase in bed level due to the aggradation, the water depth continues to increase as the sea level rises. As such, the sediment mobility continues to decrease in the downstream direction, causing more deposition of sediment. Since the sediment flux is constant at the upstream boundary, there is a maximum sediment supply to the channel, so there is expected to be a maximum rate of volume deposited. The limit is determined by the sediment supply, and was not reached during the 100 year time frame of this model.

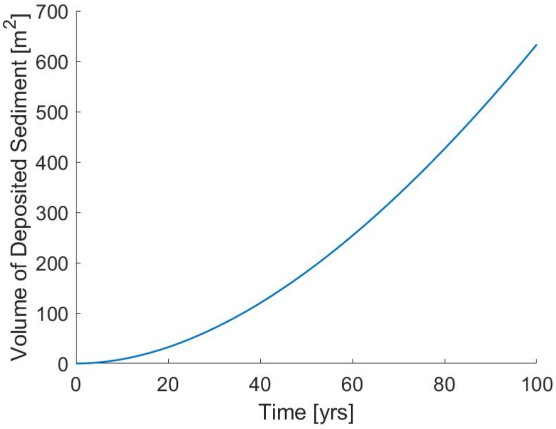


Figure 4.5: Volume of deposited sediment increases over time

5

Results from Simple Channel Model Runs

The simple channel base case in the previous chapter is the basis for the scenarios discussed in this chapter. Each scenario changes one parameter, water discharge rate, sediment flux, grain size, or rate of sea level rise, to asses how each parameter influences the changes of the bed level. The output from the 1D models is analyzed and compared with the expected response from Chapter 2 and with the other case to assess the relative influence of the parameter on bed level response to sea level rise. For each case, the value of the parameter is either increased or decreased by a factor of 2. See Table 3.2 for more details of the inputs.

5.1. Water Discharge

Varying the water discharge in the system varies the equilibrium state. A higher water discharge leads to a flatter slope and a deeper equilibrium flow depth. The sediment flux is the same as in the base case, so the equilibrium flow velocity is the same. The slope and depth adjust to create an equilibrium system with steady flow conditions. The flow velocity determines the sediment transport capacity. In the equilibrium state, the bed level is not changing, there is no erosion or deposition. The transport capacity must then match the sediment flux, thereby linking the flow velocity to the sediment flux. Fast moving water carries more sediment. This relationship plays a larger role in the following section, when sediment flux is varied.

As explained in Chapter 2, the expected response of the channel bed with a higher water discharge, and subsequent flatter slope, is a longer aggradation mound with a smaller change in bed level. The system has a slower response to the rising sea level.

First, the rate of growth of the aggradation mound in the vertical direction at the downstream boundary is shown in Figure 5.1. With the values used in this study, the initial depth of each scenario approximately doubles as the flow discharge doubles. The aggradation rate more than doubles for each decrease in water discharge. The relative increase in water depth due to sea level rise on the shallower water leads to a larger change in flow velocity and sediment transport capacity. This change is also occurring over a shorter length because the increased initial slope shortens the backwater curve. The result is a larger change over a shorter period, which causes faster aggradation. Recall that the applied rate of sea level rise is 10 mm/yr. The fastest bed level increase observed is only about 60% of the rate of sea level rise. The bed level is expected to continue to rise as long as sediment supply is available.

The rate of upstream migration of the aggradation wave for each case is shown in Figure 5.2. This plot tracks an increase in bed level of 5 mm from the initial state as it moves upstream. As expected, the case with the flattest slope – higher Q_w – moves fastest upstream.

The total volume deposited over the 100 years is given in Figure 5.3. The low water discharge case experiences the greatest relative change in depth from the sea level rise, and therefore has the greatest response. The difference in the response is not directly proportional to the discharge rate. Increasing the water discharge has a stronger relative decrease in the deposited volume. Despite the decrease in total volume, the effects are greater in the upstream reaches.

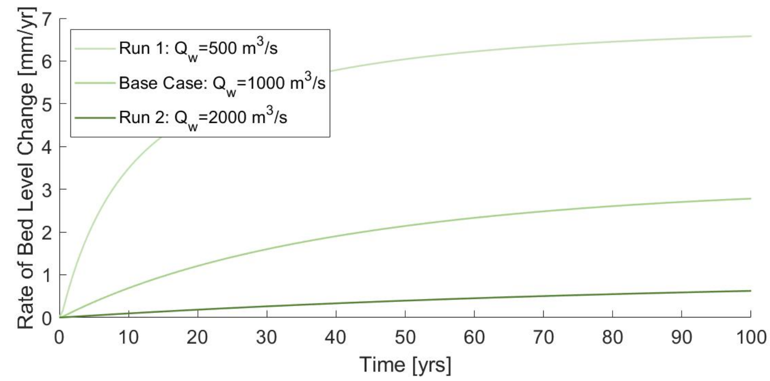


Figure 5.1: Bed level rate of change for cases with varying water discharges. The bed level increases the fastest in the case with low water discharge

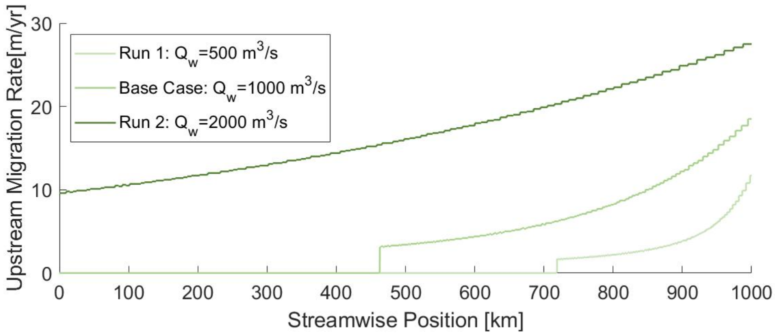


Figure 5.2: Upstream velocity of front edge of the aggradation wave, identified by tracking the movement of a 5mm increase bed level. Aggradation moves upstream faster in a high water discharge channel

5.2. Sediment Flux

The analysis of the rate of bed level increase and upstream migration of the aggradation wave is repeated for changes in sediment flux. Increasing sediment flux leads to a steeper equilibrium slope and subsequent shallower equilibrium depth in the initial state. The result is a shorter backwater curve and a greater relative increase in depth due to sea level rise. The expected response is an aggradation mound that increase the bed level more quickly, but has a lower upstream migration.

Figure 5.4 shows the channel bed aggradation rate at the downstream end. Locations further upstream have the same trend, but dampened with distance from the downstream boundary. The backwater curve created by sea level rise reduces the net increase in water level in the upstream direction, so the bed level response is reduced as well.

The sediment aggradation increases in cases with higher sediment fluxes. These results can be compared with those from changing the water discharge rate in Figure 5.1. Runs 1 and 4 have the same ratio of sediment flux to water discharge, as do Runs 2 and 3 (see values in Table 3.2). Using these similarities, we can compare the relative impact of changes in sediment flux and water discharge. For cases with the same ratio of water discharge and sediment flux, the aggradation rate is higher for the case with a smaller water discharge. This suggests that water discharge has a larger influence on the increase in bed level than the sediment supply. Changes in the water discharge have a greater influence on the equilibrium slope than the changes in sediment flux. The slope increases due to a factor 2 increase in sediment flux was less than the slope increase due to a factor 2 decrease in water discharge. The subsequent change in the backwater curve alters the aggradation rates. A low water discharge channel has a steeper slope, which shortens the backwater curve. Therefore the aggradation occurs over a shorter spatial length, allowing for greater increase in bed level.

Similarly, in comparing the volume of deposited sediments in Figures 5.3 and 5.5, we also see that the sediment flux has a greater influence on the total volume of deposited sediment. Cases with a higher sediment

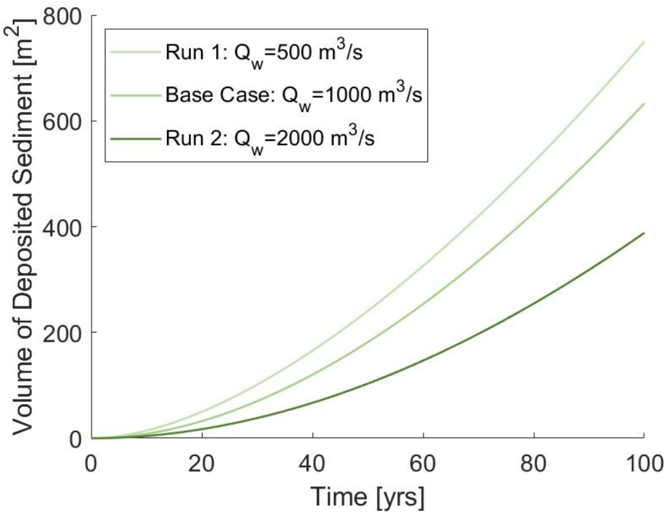


Figure 5.3: Volume of deposited sediment for cases with varying water discharge. Low discharge leads to higher volume of sediment

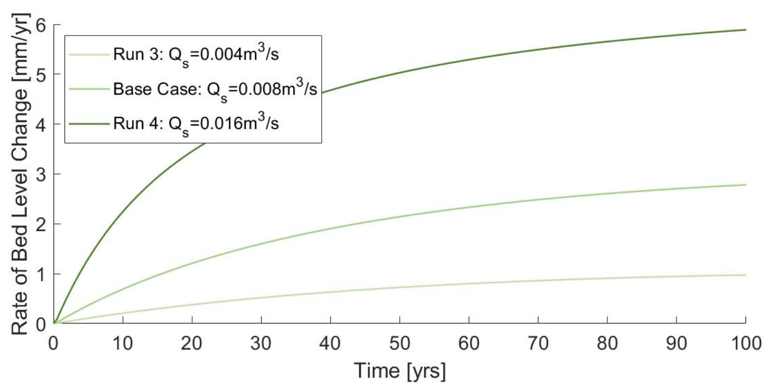


Figure 5.4: Rate of bed level change in cases with varying sediment flux. Aggradation is faster in a channel with higher sediment flux

flux to water discharge ratio deposited more volume of sediment. In cases with the same ratio, the case with higher value of sediment flux had greater volume of deposited sediment. Sediment supply is higher, so there is more sediment available for deposition.

Growth of the aggradation mound in the upstream direction is shown in Figure 5.6. Systems with a steeper slope (higher sediment flux) have have slower growth of the aggradation mound in the upstream direction. These cases have less variability in slope than the cases for different water discharge (Figure 5.2). This translates to a smaller change in upstream growth rate. There is a close relationship between the slope and the upstream migration of the aggradation wave due to the corresponding length of the backwater curve.

In summary, increasing the sediment flux leads to an initial state with a steeper slope and shallower depth. The backwater curve caused by sea level rise creates a more compact aggradation mound, one that is taller and does not extend as far upstream. The steeper slope also slows down the upstream migration rate of the mound. Sediment flux has less of an impact on the rate of bed level change than water discharge, but more influence on the total volume of deposited sediment.

5.3. Grain size

In this section, the grain size is varied for each case. Changes in grain size have a similar effect to the equilibrium slope as changes in the sediment flux. Larger grain size channels have a steeper slope and shallower flow depth. The initial states of the cases are very similar to that of the sediment flux cases as shown in Table 5.1. Runs 3 and 5 differ in slope by about 10% and in depth by about 4%. Runs 4 and 6 differ by 4 and 2%

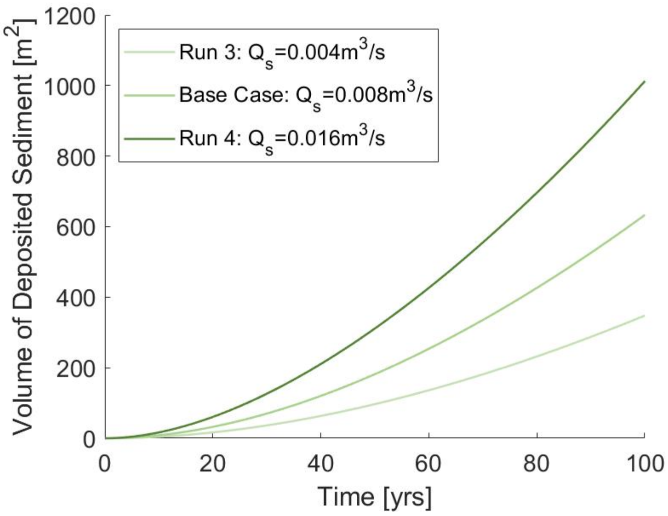


Figure 5.5: Volume of deposited sediment for simple channel cases with varying sediment fluxes

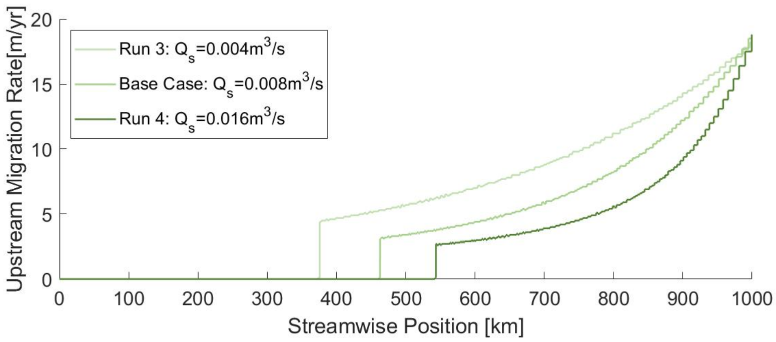


Figure 5.6: Upstream velocity of the front edge of aggradation mound in simple channels with varying sediment fluxes.

for slope and depth, respectively. This allows for a comparison between the impact of sediment supply and equilibrium state properties. The sediment flux is constant across all cases, and the backwater curve grows in the same way as in the previous section.

Run	Grain Size [mm]	Sediment flux [m^3/s]	Equilibrium Slope [-]	Equilibrium Depth [m]
Base	0.5	0.008	$2.28 * 10^{-5}$	7.35
3	0.5	0.004	$1.50 * 10^{-5}$	8.46
4	0.5	0.016	$3.67 * 10^{-5}$	6.27
5	0.25	0.008	$1.67 * 10^{-5}$	8.15
6	1.0	0.008	$3.51 * 10^{-5}$	6.37

Table 5.1: Equilibrium slope and depth for the runs with varying grain size and sediment flux. Note the similarities between runs 5 and 3 and runs 4 and 6

The response of the bed level with increasing grain sizes is expected to be similar to that of cases with increasing sediment flux with the exception of deposition volume. The sediment supply is the same across the cases with varying grain size, so while the volume is expected to increase in cases with steeper slopes (larger grain size), the extent is smaller than the result of an increase in sediment flux. Small grain size sediment is more mobile, and therefore less likely to settle. Large grain size requires greater sediment transport capacity, so smaller changes in flow velocity lead to greater deposition.

To easily compare the effect of grain size and sediment flux, Figures 5.7, 5.8, and 5.9 show both the results of

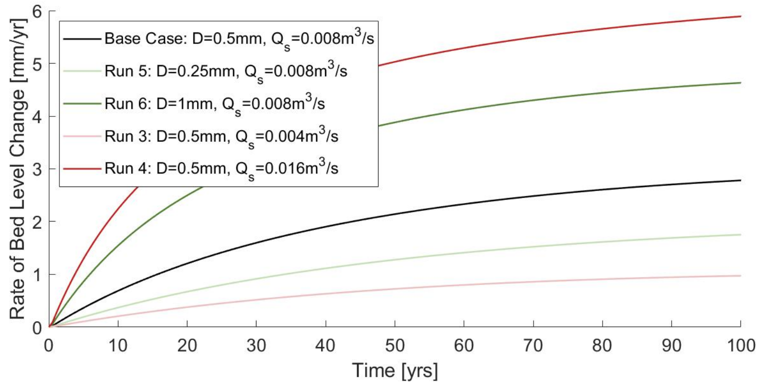


Figure 5.7: Rate of bed level change for varying grain size (green) and sediment flux (red). The base case is shown in black.

the grain size (in red) and sediment flux cases (in green). The base case is shown in black. Looking first at the effect of grain size, the green lines, the growth of the aggradation mound follows the expected trends: a system with larger grain size will have faster aggradation in the vertical direction, slower aggradation in the upstream direction, and more total volume deposited (Section 2.5). By comparing these results with those of the sediment flux scenarios (Section 5.2), the influence of the initial state can be partially distinguished from the influence of the increasing sediment flux.

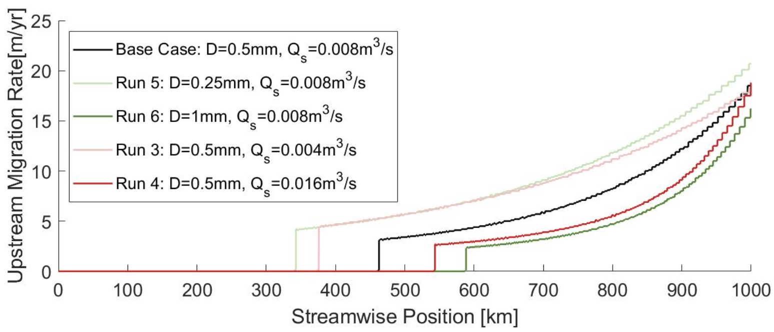


Figure 5.8: Upstream velocity of aggradation wave for cases with varying grain size (green) and sediment flux (red). The base case is shown in black.

Figure 5.7 shows that the aggradation mound grows faster with a larger sediment flux than a larger grain size. The amount of sediment in the system determines in how much sediment is available for deposition. Comparing the total volume of the deposited sediment supports this as well (Figure 5.9). However, that effect does not hold true for growth in the upstream direction. In Figure 5.8, there is a closer relationship between the cases with similar equilibrium states. This suggests that the streamwise growth of the aggradation mound is more heavily influenced by the slope. The slope heavily determines the length of the backwater curve, so it follows that the upstream migration rate is closely related to the upstream growth of the backwater curve as sea level continues to rise.

To summarize, grain size has more influence on the upstream migration rate of the aggradation mound than sediment flux, but less influence on the rate of growth of the bed level or the total volume deposited. These effects extend past the influence that grain size and sediment flux have on the equilibrium slope and depth.

5.4. Sea Level Rise

The influence of the rate of sea level rise on the river bed response is also analyzed. The initial conditions in all three runs are the same. The imposed rate of sea level rise varies per case, but is a constant rate throughout each run. The expected bed level response, discussed in Section 2.6, is larger for higher rates of sea level rise. The backwater curve is longer, increasing the upstream migration of the aggradation, and the water depth increases faster as the water level increases faster. We have already seen that the bed response is slower than

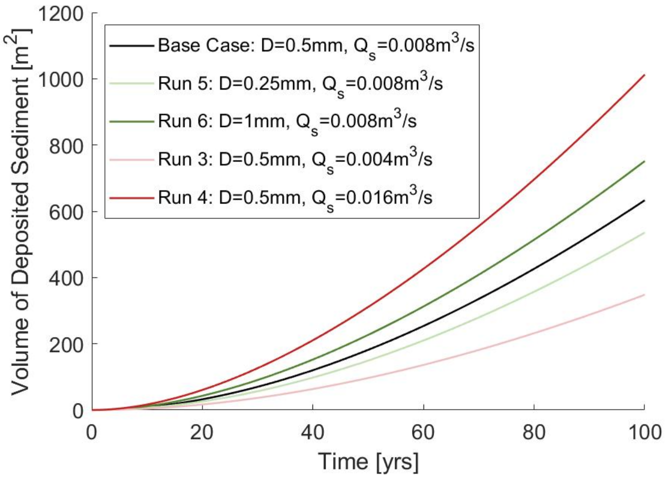


Figure 5.9: D: Volume of deposited sediment for cases with varying grain size (green) and sediment flux (red). The base case is shown in black.

the water level increase, so depth increases faster, leading to an increased change in flow velocity, and therefore sediment transport capacity, all leading to faster aggradation. In Figures 5.10, 5.11, and 5.12, we see that the bed level response is as expected: a higher rate of sea level rise leads to a greater response in the bed level.

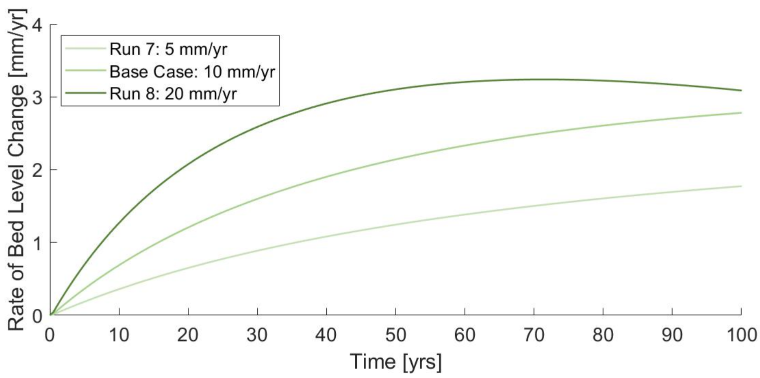


Figure 5.10: Bed level rate of change in cases with varying rates of sea level rise. A faster rate of sea level rise causes a faster bed level response

5.5. Summary of Simple Channel Results

The general trends of the aggradation patterns in the simple channel model is given in Table 5.2. The results match with the expectations in Table 2.1.

For each tested parameter, water discharge, sediment flux, grain size and rate of sea level rise, the parameter was increased or decreased by a factor of 2. We can therefore roughly determine the relative weight of each parameter by comparing with change in bed level response of each case with respect to the base case. Bed level change has the slowest response to an increase in rate of sea level rise. The rate at which the downstream water level increases has the least influence on the rate of bed level change, but greatest influence on the volume of deposited sediment. The backwater curve grows more quickly in both the vertical and horizontal directions for a higher rate of sea level rise. This results in a greater drop in velocity, and therefore sediment transport capacity, causing more deposition. The increasing length of the backwater curve means the deposition occurs over a longer stretch of the channel.

In the cases that vary other parameters, the aggradation mound responds faster in either upstream migration

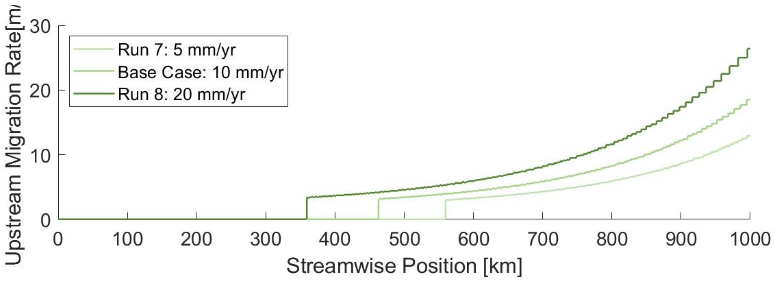


Figure 5.11: Upstream velocity of aggradation mound in channel for varying rates of sea level rise. The aggradation moves upstream faster for higher rates of sea level rise

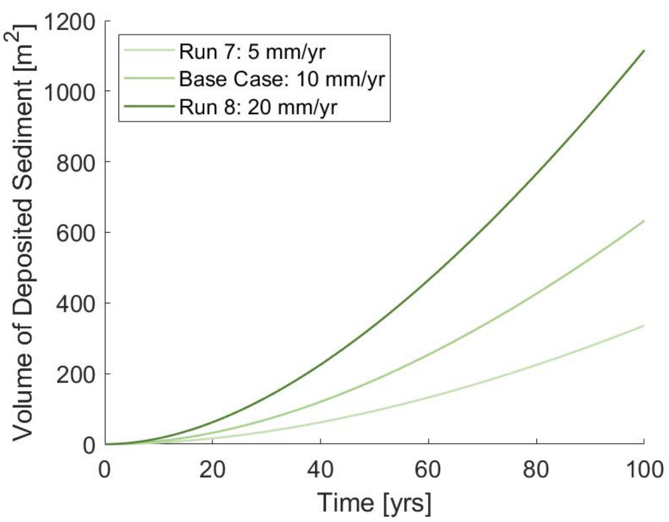


Figure 5.12: Volume of deposited sediment for cases that vary in rate of sea level rise. High rate of sea level rise increased the volume deposited

or bed level rise due to an increase in the parameter. For example, an increase in water discharge increases the upstream migration rate, but decreases the rate of bed level change. In the case of sea level rise, the aggradation mound responds by increasing in both upstream migration and bed level change. The mound grows more quickly in all metrics. So even though the influence of the rate of sea level rise may be smaller than the influence of other parameters, the rising rate of sea level rise amplifies the aggradation that occurs.

Case	Growth of aggradation in x direction	Growth of aggradation in z direction	Volume of deposited sediment
Increase in water discharge	+	−	−
Increase in sediment flux	−	+	+
Increase in grain size	−	+	+
Increase in rate of sea level rise	+	+	+

Table 5.2: Summary of results of simple channel cases. The relative changes in the growth of the aggradation with respect to the base case.

6

Results from Tributary Model Runs

A tributary river is added to the base case, increasing the flow downstream of the confluence point creating a more complex system. The system can be divided into three different branches, the main channel upstream of the confluence point, labelled Branch 1, the incoming tributary, Branch 2, and the channel downstream of the confluence point, Branch 3 (see Figure 2.7). The focus of this chapter is the main channel, made up of branches 1 and 3. In the 1D model of the tributary system, the discharge rate and sediment flux of the tributary and the distance between the confluence and the river mouth are independently varied in each run. The results from the model are discussed in this chapter.

6.1. Base Case with Tributary

As with the simple channel scenarios, each run begins from an equilibrium state. Figure 6.1 shows the equilibrium slopes of the three branches and the water level. The step at the confluence point is to accommodate the increase in depth in Branch 3. The slope and depth of Branch 3 varies depending on the inputs from Branch 2.

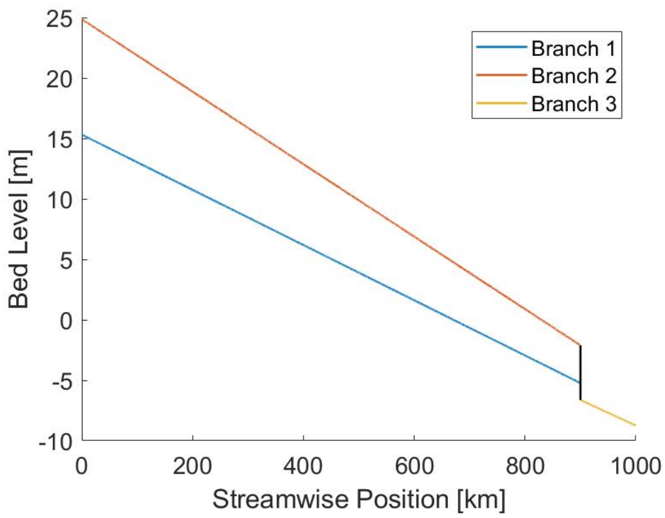


Figure 6.1: Layout of bed level of tributary model with equilibrium slopes of each branch. The confluence creates a step in bed level.

When sea level rise is applied to the downstream boundary of the system, the bed level beings to rise, like in the simple channel system. As discussed in the previous chapter, river channels respond differently to the backwater curve depending on the different properties of the channel. With a tributary system, there are three different channels, each with different properties and subsequent different responses. They are all connected, which further influences the response. The downstream water level influences the upstream branches water level and the sediment supply from the upstream branches influences the downstream branches.

Figure 6.2 shows the change in bed level of branches 1 and 3 over time. Looking first at the upstream section, Branch 1, the response looks very similar to that of the simple channels previously discussed. The downstream end of Branch 3 looks like the downstream end of the simple channel cases. The notable difference is the area near the confluence where degradation occurs. There is a steep decrease in bed level, including erosion of bed material. The scour hole grows at first, then decreases in depth over time.

In this run, branches 1 and 3 have the same ratio of water discharge to sediment flux. As discussed in Section 5.2, the channel with higher sediment flux will deposit volume at a faster rate than a channel with the same ratio, but lower sediment flux. In this case, that means that Branch 1 will respond faster to the changes in depth than Branch 3. This can also be viewed through the lens of relative change in depth. At the confluence point, both branches have approximately the same water level, and thus net increase in depth. Since Branch 3 is deeper, the relative change in depth is smaller, causing it to have a smaller response. As such, the faster rate of aggradation in Branch 1 influences Branch 3 in two ways. First, aggradation upstream reduces the sediment supply that reaches Branch 3. Erosion occurs to compensate for the negative sediment budget. Second, the upstream aggradation further decreases the sediment mobility in Branch 1. In equilibrium, there is a spatial increase in mobility at the confluence point. The increase is larger if the upstream mobility is decreased. The larger change in mobility also contributes to the degradation.

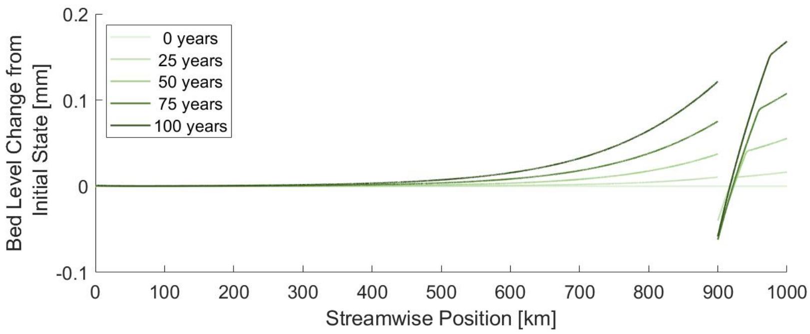


Figure 6.2: Change in bed level of main channel of tributary model. Branch 1 (upstream of the confluence at 900 km) has an aggradation pattern similar the the simple channels in the previous chapter. Branch 3 (downstream of the confluence) shows erosion in the upstream section and aggradation in the downstream section

While the scour hole grows both deeper and further downstream, the aggradation mound in Branch 3 is also growing taller and further upstream. The meeting point the aggradation and degradation waves creates the angle in the bed level change seen in Figure 6.2 near 950 km. The growth of the aggradation mound is dominant and starts to fill in the scour hole. The forces driving the erosion decrease as the water level increases. The relative difference in the two branches is reduced as the water depths increase. If the model was extended for a longer time frame, the scour hole would likely be filled in and the bed level would be expected to increase past its initial level.

To characterize the response of a tributary system to sea level rise, the response is categorized into the aggradation, scour, and total change in volume over time.

6.2. Water Discharge

The water discharge of the tributary branch, Branch 2, is varied to model how the properties of the tributary influence the main channel's response to sea level rise. The results from the model are compared by the total volume of aggradation in branches 1 and 3 and the volume of eroded sediment in Branch 3. There is no erosion in the upstream branch.

Based on the results of the simple channel model, we expect a decrease in deposited sediment volume in Branch 3 for system with a high water discharge tributary. This will increase the water level at the confluence, increasing the water depth for Branch 1, promoting more aggradation. The results of the model are given in Figures 6.3 and 6.4. Increasing the water discharge of the tributary leads to more aggradation upstream of the confluence, and less aggradation downstream. The scour occurs just downstream of the step, in the upper extent of Branch 3. Increasing the water discharge rate in the tributary leads to more scour.

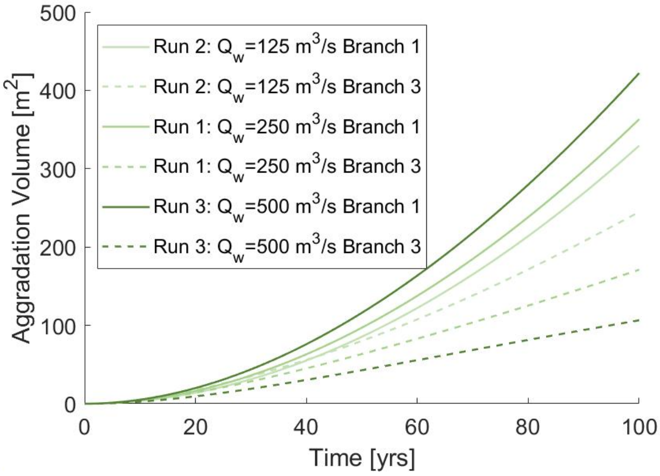


Figure 6.3: Volume of deposited sediment in tributary cases with varying water discharge of Branch 2. Increasing the water discharge of the tributary increases the aggradation upstream of the confluence (Branch 1) and decreases the aggradation downstream of the confluence (Branch 3)

Higher water discharge in the tributary creates a larger difference in discharge, and therefore water depth, between branches 1 and 3. Branch 1 is already more sensitive to changes in depth, because it is shallower; this is further amplified as the difference between the branches initial states increases due to changes in the inputs in Branch 2. Additionally, the slope of Branch 3 changes in the initial state due to changes in the water discharge of Branch 2. As in the the simple channel, a higher water discharge leads to a flatter slope, which leads to a backwater curve that extends further upstream faster. This means the increase in water level reaches up the upstream branch sooner, which further increases the aggradation volume in Branch 1 relative to cases with a lower water discharge and subsequent steeper slope in Branch 3.

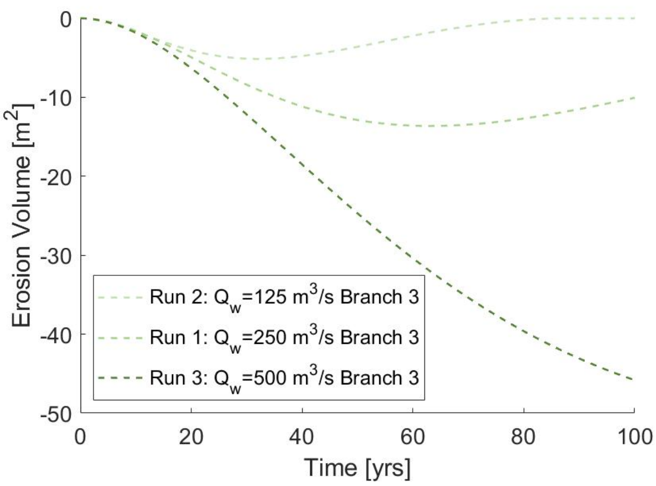


Figure 6.4: Volume of eroded sediment in Branch 3 over time. Higher water discharge in the tributary increases the erosion downstream of the confluence.

The volume of eroded sediment in Branch 3 increases as the water discharge of the tributary branch increases. As previously discussed, the scour is due to two factors, the reduced sediment supply from the upstream branch and the increase change in mobility across the confluence region. Both of these factors are amplified by increasing the flow discharge in Branch 2. First, there is more aggradation in Branch 1, which limits the sediment supply to Branch 3. Erosion then occurs to increase the supply to the downstream reaches of Branch 3. Secondly, the flow velocity, and therefore sediment mobility, has a greater change across the confluence.

In the initial state, the flow velocity of branches 1 and 3 are the same across the three cases. Increases in the water discharge of Branch 2 increase the aggradation in the upstream branch, resulting in a greater decreases in flow velocity for the water entering the confluence. In the initial state, there is an increase in flow velocity and subsequent sediment transport in this region. The step is greater due to the decrease in the upstream velocity, therefore contributing to additional scour.

Over time, the size of the scour hole decreases as the aggradation mound in Branch 3 migrates upstream and the flow velocity decreases due to the increasing depth from the growing backwater curve. We see that for the low water discharge case, run 2, erosion begins, creating scour, then decreases. The scour hole is fully filled by approximately year 80. The base case also shows a reduction in scour volume.

6.3. Sediment Flux

The sediment flux of the tributary branch is varied in this section. As in the simple channel, the results of increasing the sediment flux show an opposite trend to increasing the water discharge. An increase in sediment flux in the tributary leads to less aggradation in the upstream branch, more in the downstream branch, and less scour near the confluence. The increase in sediment added by the tributary counteracts the reduction in sediment supply by the faster aggradation in Branch 1, resulting in less scour.

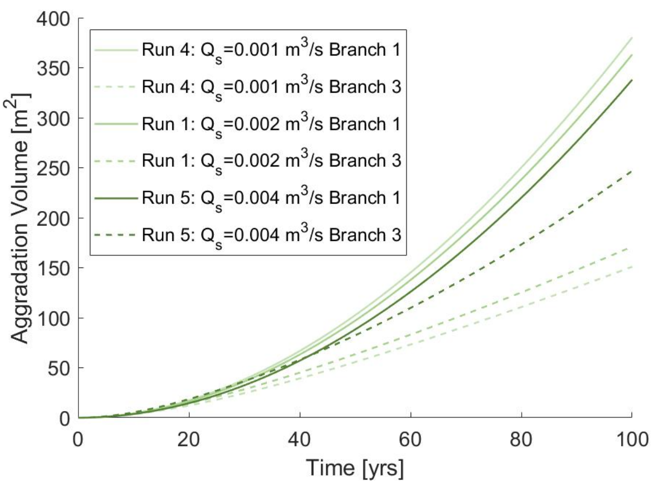


Figure 6.5: Volume of deposited sediment over time for cases with varying sediment fluxes in tributary. Higher sediment flux in the tributary leads to greater volume of aggradation downstream of the confluence, and reduces the aggradation upstream.

In Figure 6.6, the scour in the the case with high sediment flux is gone after about 90 years. In the section 5.2, the aggradation mound in the channel with higher sediment flux moved less quickly upstream. In this case, we see that the mound has moved upstream enough to fill in the scour and create a net increase in bed level. Of the three cases, the upstream growth rate is the slowest, but the scour volume is the smallest, so it requires less volume to fill in. The scour is caused by an increase in sediment mobility across the confluence. In the case of a sediment rich tributary, the change in flow velocity between branches 1 and 3 is greater in the equilibrium state than that of a system with a sediment poor tributary. As such, when aggradation begins on Branch 1 and the change in mobility increases, the relative increase is smaller in this system, leading to less scour.

In runs 2 and 5, the slope of the downstream branch, Branch 3, is steeper than that of the upstream branch. These cases are characterized by a high ratio of sediment flux to water discharge. They have more aggradation in Branch 3, and less in Branch 1. In the simple channel, steeper slope cases have more compact backwater curves that do not reach as far upstream. For the tributary system, this means that the upstream branch has a smaller backwater curve, resulting in a greater impact downstream, and less of an impact upstream. Additionally, runs 2 and 5 have less scour and more total aggradation. This is more to due with the greater sediment supply than the resulting slope.

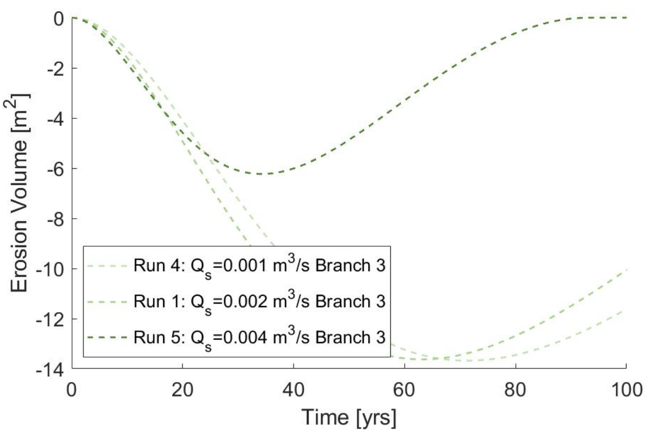


Figure 6.6: Volume of eroded sediment downstream of the confluence in cases with varying sediment fluxes in the tributary branch. Higher sediment flux reduces the amount of scour

6.4. Location of Confluence

The length of Branch 3 is the distance between the confluence and the mouth of the river. It is varied to examine how the location of the confluence influences the river bed response. The closer the confluence was to the mouth, the greater the response in the upstream branch. The downstream branch sees less aggradation, but that is influenced by the fact that it is shorter.

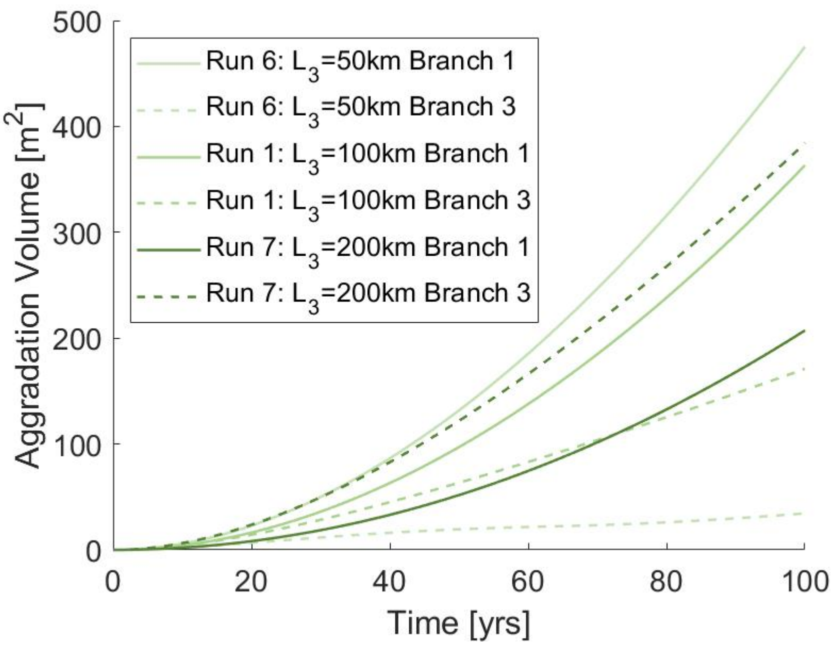


Figure 6.7: Aggradation volume for cases with varying lengths of Branch 3. A shorter Branch 3 moves the confluence closer to the sea, resulting in an increase in aggradation in Branch 1

Instead, looking at Figure 6.8, the system with the confluence closest to the mouth also has the largest response. While the case with the longest Branch 3 has a slower start to the scour, as the effects from the sea travel upstream, the maximum depth is smaller and earlier than the other cases. The increase in water depth at 200 km from the sea is less than that at 50 km, so the change in mobility at the confluence is smaller, resulting in less scour.

In all the cases in this chapter, the upstream branch always has a greater aggradation volume than the downstream branch except for the case with the confluence 200 km from the sea. In this case the downstream

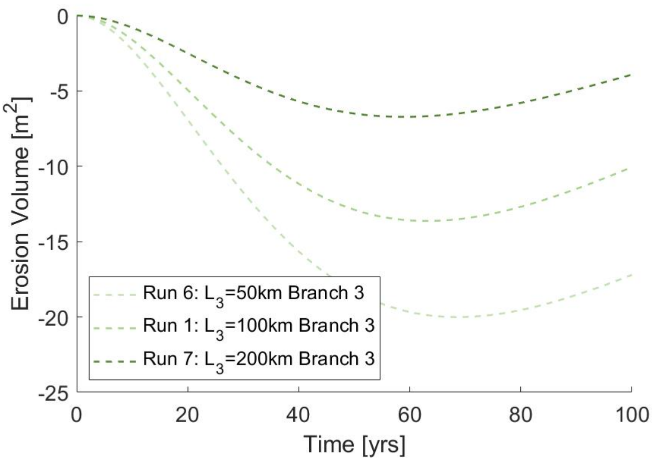


Figure 6.8: Volume of eroded sediment in Branch 3 for varying lengths of the branch.

branch had a greater volume of deposited sediment. Furthermore, the system has the largest volume of total deposited sediment. While it may seem that the further a confluence is from the sea, the less response it will have to changes in the sea, the opposite is shown to be true. The upstream branch is less affected, but the downstream branch has a larger response.

7 Discussion

The limitations of the study due to the simplifications in the model and implications of this study on rivers of the world are discussed in this chapter. This study isolates the effect of sea level rise on a river bed, which removes the noise of other influences, but also removes some of the connections between those influences. The effects of these choices are divided by simplifications made of the boundary conditions and in the channel.

7.1. Boundary Condition Simplifications

The boundary conditions in the model are simplified to steady flow from the upstream end and negating inputs from the downstream end other than the changing downstream boundary.

The upstream boundary condition does not account for variable flow. Climate change is changing precipitation patterns such that the winters are wetter, but warmer, reducing the snow pack, and the summers are drier (KNMI, 2015; Fox-Kemper et al., 2021). The extreme high and low water periods are more common and a cause of concern. Nannenberg (2021) modelled the river bed response to these changes and found degradation wave at the upstream and downstream end of the simulation region. The degradation at the upstream reaches increase the sediment supply to the downstream regions, which would increase the aggradation trends seen in this report. However it is unclear which trend would be more prominent in the downstream reaches - aggradation due to sea level rise or degradation due to greater variation in flow.

In a tributary system, changes in the hydrograph are included in the models by Verhaar (Verhaar, Biron, Ferguson, & Hoey, 2008, 2010). These studies use one-dimensional models to predict morphological changes in a gravel beds in the St Lawrence River water shed. The model uses a decreased water level at the downstream boundary of the channel as a result of reduced flow in the main channel. The lower water level has a reduced sediment transport capacity, which starves the downstream regions of sediment supply. The river bed response is erosion to increase the sediment supply. This is the opposite effect of the increase in sea level rise in the tributary models in this study.

The combination of changes in the hydrograph and sea level rise are combined by (Gomez, Cui, Kettner, Peacock, & Syvitski, 2009), in a model of the Waipaoa River in New Zealand. The increase in sea level resulted in a rising bed level in the downstream segment of the river. The drier conditions reduced the flow velocity, such that sediment transport was reduced. This may cause a reduction in the volume of sediment deposited in response to the sea level rise. Case studies supported by measurements are necessary to better predict the balance between aggradation and degradation.

The downstream boundary is more difficult to expand with the model used in this study. The mouth of a river is a dynamic region with tides, mixing of salt and fresh water, and sediment entering the system, none of which is included in this study. The Elv research code is not appropriate to model these components. It is possible to include tidal effects on the water level, but the short term variations are not expected to have significant influence on the long term changes of the bed level. The density difference of salt water creates mixing and effects the buoyancy of sediment (Wright, 1977). A more robust, 2D or 3D model is needed to further study these changes. Case studies are also useful in that they can determine the dominant forces at the mouth of the river. A tidal dominant river mouth responds differently to sea level rise than the river dominant system in this study.

7.2. Channel Simplifications

The river channel in this study is simplified to a fixed width with a constant slope in the equilibrium state. By using unisize bed material, the slope is constant for the whole reach of the channel. This study uses a channel length of 1000 km. On the Rhine, 1000 river kilometers from the sea is in the Alps, where the slope is on the order of 10^{-3} an order of magnitude steeper than in the Dutch delta region, where the slope is on the order of 10^{-4} or less (Frings et al., 2019). Using a wider range of grain size creates a varying slope and a transition region from coarser to finer grain in the streamwise direction. The result is a backwater curve that does not reach as far upstream, and has a larger depth in the upstream end. Aggradation would then be expected to be faster than what the model in this study shows in the downstream section. Additionally, the sand-gravel transition is expected to move further upstream (Blom, Chavarrías, Ferguson, & Viparelli, 2017). This will further support the upstream growth of the adaption length of the backwater curve and subsequent aggradation.

While the tributary model increases complexity of the study to a larger system, a bifurcating model is also useful. We were not able to reliably include a bifurcation in the model for this study, nevertheless, some insight can still be gained. Bifurcations can be seen as the inverse of a tributary - an extraction rather than addition. In a tributary, there is an increase in sediment mobility at the confluence point, which is exacerbated by effects of sea level rise. The faster aggradation of the upstream branch slows the flow velocity, creating a larger difference in velocity upstream and downstream of the confluence. Hypothesising the same effect for a bifurcation, the downstream branch will have faster aggradation which will the decrease in sediment mobility. The rising bed level will effect how the flow is distributed between two branches (Wang, De Vries, Fokkink, & Langerak, 1995). This could pose an increased risk to the Rhine and other bifurcating rivers that rely on a stable bifurcation to create sufficiently deep water for shipping.

While it is possible to create a 1D bifurcation model with nodal point relationships, a 2D model is more robust. For both a bifurcation and confluence, 2D or 3D modeling allows for more insight to the local changes at the nodal point. The angle of flow and mixing in a confluence will influence where in the horizontal cross section the erosion will occur (Best, 1988). Similarly, for a bifurcation system, rising bed level in the downstream branches adjust the division in water and sediment, which in turn influences the bed level response.

7.3. Applications to the Rhine and Delaware Rivers

In the model used in this study, sea level rise caused an increase in the river bed level that reached on the order of 100's of kilometers inland, with a height on the order of 10 centimeters over a 100 year time span. The scale of the aggradation is enough to have an impact on infrastructure in and near the river. The extent of the increase varies strongly based on the properties of the river.

For infrastructure in the river - such as bridge abutments – degradation is more dangerous than aggradation. The scour seen downstream of the confluence can be a vulnerability for rivers such as the Delaware, which has over 200 tributaries and runs through in highly developed areas (Schmidt, 2019). The results in this study show scour that grows, reaches a maximum depth, then becomes more shallow as it moves downstream. The extent of the scour depends on the difference between the three different branches of the confluence. With so many tributaries along the Delaware, it is useful to know which confluences pose the largest risk of erosion: tributaries with high water discharge or low sediment flux.

In some cases, the degradation is minimal and resolves in a short period of time. In others, like the case with a high water discharge tributary, have much more scour that continues to grow for nearly 100 years. Since sea level rise is already well underway, it is useful to know what stage of scour a confluence is in. Sufficient time scale data is needed to determine if the scour is just beginning, or has already reached a maximum and is decreasing.

Aggradation can be of great concern for flood control. The Room for the River project in the Netherlands focused on increasing the safety in the case of river floods. Rising bed level will decrease the bankfull flow, increasing flood risk. However, the Rhine currently has a negative sediment budget, and nourishment projects are underway (Cox et al., 2021). Degrading bed level pose other threats, namely to infrastructure along the rivers. Following the results of this study, it is possible that sea level rise may reduce the need for nourishment over time.

8

Conclusion and Recommendations

The response to the research questions are given in this chapter, as well as recommendations for expansion of the model used in this study.

8.1. Conclusion

1. How does a river bed respond to sea level rise?

   In an fixed width alluvial river, the rise in sea level, the downstream water level, leads to aggradation and a rising bed level. The bed level increase begins at the downstream end and grows both taller and further upstream. The extent of the aggradation is on the order of 100 km inland and a height on the order of 10 cm over 100 years in this study. The rate of bed level rise is slower than that of sea level rise, so the channel is always in a transient state.

2. How does the response change due to changes in the flow discharge, sediment flux, grain size, or rate of sea level rise?

   Flow discharge and sediment flux have opposing influences on the aggradation behavior. An increase in flow discharge or decrease in sediment flux leads to an decrease in the height of the aggradation mound and increase in the length. The reverse is also true. For the same ratio of flow discharge and sediment flux (Q_w/Q_s), flow discharge has more influence on the growth rate of the aggradation mound where as the sediment flux has more influence on the total volume deposited. Grain size has more influence on the growth in the upstream direction than sediment flux, but less so for growth in the vertical direction or total volume. The rate of sea level rise has the largest change on the volume of the deposited sediment, increasing the response of the bed level.

   The main determinant in bed level change is the equilibrium state of the channel. The slope and depth determine the length of the backwater curve, which in turn determines the spatial change in depth along the channel, driving bed level change.

3. How does the response change with the presence of a tributary?

   The branches upstream of the confluence behave similarly to a simple channel: the downstream water level of the reach rises and aggradation occurs. The same behavior occurs on the downstream segment, except for just downstream of the confluence. The relative decrease in sediment mobility is larger for the upstream branch, so the sediment supply is decreased to the downstream branch. This leads to scour. The pit grows at first, then decreases as it moves downstream and the aggradation mound moves upstream. In some cases, the scour is completely filled in and a net increase in bed level occurs.
   Similar to the simple channel, the sediment flux and water discharge of the tributary branch have opposing effects on the main channel. High water discharge or low sediment flux tributary increased the volume of deposited sediment on the upstream reach, decreased the volume deposited on the downstream reach and increased the amount of scour. The reverse is also true.

All changes in the system are derived by changes in the depth due to sea level rise. The scour is due to the difference in relative change in mobility between the upstream and downstream segments. Systems with a smaller change in depth in the equilibrium state have a smaller difference in the relative change in depth, and therefore also sediment mobility during sea level rise conditions, resulting in less scour. To summarize, a tributary with a smaller step, has less aggradation in the upstream reach, and less scour, whereas a larger step has more upstream aggradation and more scour.
The distance that the confluence is from the sea also influences the amount of scour. If the confluence is further from the sea, the rate of increase of water level is lower, so there is less scour.

8.2. Recommendations

The following recommendations are made as possible avenue to extend the research conducted in this study:

- Expansion of the theoretical model: Including variable flow to the upstream boundary conditions will allow for combining the effects of climate change through sea level rise and the hydrograph. Additionally, expanding the model to include multi-size bed material results in non-uniform slope in the equilibrium state. This can result in a more complex relation between slope and the sea level rise induced backwater curve.

- Validating the model with a case study: This study uses generic values loosely based on the Rhine River. There is already a long history of sea level rise, so some of the trends shown in this study may be present. However, dredging projects and fluctuations in flow may make it difficult to find an applicable river.

- More complex modeling of confluence: the region around the confluence in the tributary model is a highly dynamic region. A 2D, or even 3D, model can better simulate the mixing and the scour. It also allows for other factors to be studied, such as the confluence angle or a bifurcation

A

Additional Explanation of Relevant Equations

The Exner equation and Ashida-Michiue transport relation are used in the calculations within the Elv model. Further explanation of these equations is given here.

A.1. Exner Equation

The bed level response to sea level rise is described in Chapter 2. Bed level change is quantified by the Exner equation, which relates a spatial change in sediment flux to a temporal change in bed level.

$$\frac{\delta z_b}{\delta t} = -(\frac{1}{1-p})\frac{\delta Q_s}{\delta x} \tag{A.1}$$

Where, p = porosity
z_b = bed level
Q_s = sediment flux

The backwater curve created by sea level rise increases the water depth spatially in the downstream direction. The flow velocity also decreases in the same way, which decreases the sediment flux. Reduction in sediment flux causes aggradation. The bed level then increases as aggradation continues.

A.2. Sediment Transport Equation

In this study, the sediment flux is described using the Ashida Michiue (1972) transport relation, given as:

$$q_s* = a(\theta - \theta_c)(\sqrt{\theta} - \sqrt{\theta_c}) \tag{A.2}$$

Where, q_s is the dimensionless sediment flux [-]
θ is the Shields parameter
a = 17, a coefficient
θ_c = 0.05, critical Shields parameter

The sediment flux is related to the Shields parameter, which accounts for grain size and porosity. A larger grain size is less mobile and requires a higher flow velocity to remain suspended.

B

Additional Tributary Figures

Additional figures from Chapter 6 are given below. From the tributary model, the total change in volume for the main channel was compared for each variable, flow discharge and sediment flux of the tributary and location of the confluence, given in Figures B.1, B.2, and B.3. As with the simple channel cases, the deposited volume increases for cases with lower flow discharge or higher sediment flux.

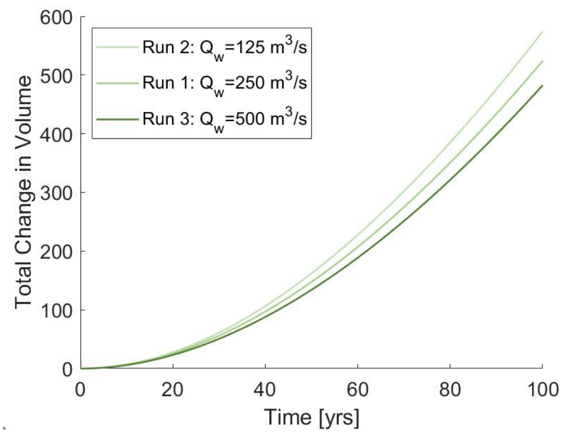


Figure B.1: Total change in volume of bed material in tributary model with cases varying in water discharge of Branch 2. Aggradation continues to increase over time, at a faster rate in cases with lower water discharge in the tributary branch

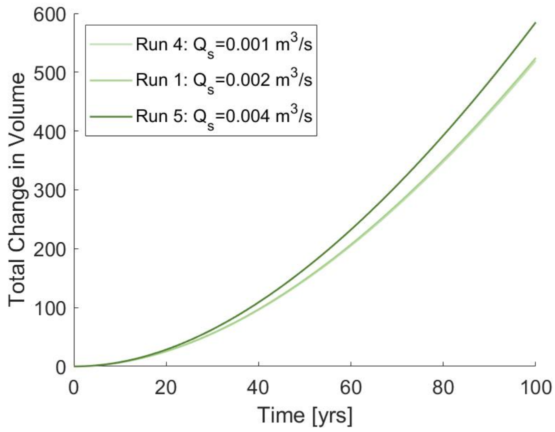


Figure B.2: Total change in volume of bed material in tributary model with cases varying in sediment flux of Branch 2. Aggradation continues to increase over time, at a faster rate in cases with higher sediment flux in the tributary branch. There is little difference between the sediment flux, and the change in volume, of Runs 1 and 4.

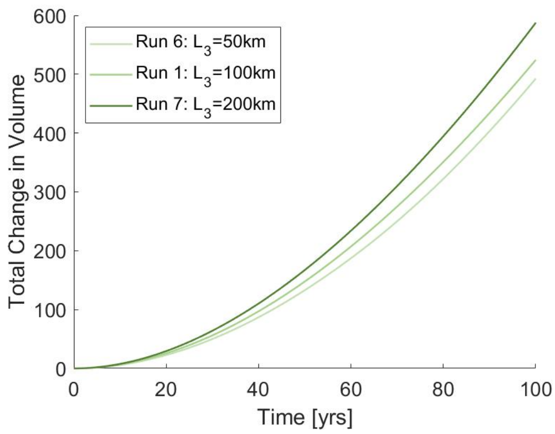


Figure B.3: Total change in volume of bed material in tributary model with cases varying in distance of the confluence from the sea. Shorter Branch 3 leads to greater volume of deposited sediment in the main channel over time.

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