TU Delft

INVESTMENT OPTIMIZED AIRPORT INFRASTRUCTURE FOR BATTERY AND HYDROGEN CANISTER SWAPS

Simon van Oosterom

When once you have tasted flight,
you will forever walk the earth
with your eyes turned skyward.
For there you have been,
and there you will always
long to return.

LEONARDO DAVINCI

On cover
Artistic impression of the newly renovated B-terminal of New-York Laguardia Airport (NYC, 2015).

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INVESTMENT OPTIMIZED AIRPORT INFRASTRUCTURE FOR BATTERY AND HYDROGEN CANISTER SWAPS

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Nomenclature

Problems	
AC BS&CS	Aircraft Battery Swap and Charge Schedule problem
AC SA&BC	Aircraft Slot Allocation and Battery Charge problem

Sets	
J	Set of all arriving batteries
D	Set of all due dates/ departure times
B	Set of all battery types
T	Time interval in which all flights arrive and depart

Parameters	
r_j	Arrival time of battery $j \in J$
p_j	Required charging time of battery j
d_j	Departure time of the flight on which battery j arrived
n_b^n	Overnight stay batteries of type $b \in B$

Variables	
m	Number of charging stations
n_b^s	Spare batteries of type b
Arr_j	Allocated arrival time of battery j
Dep_j	Allocated departure time of battery j

Optimization methods	
(I)LP	(Integer) Linear Programming
MILP	Mixed-Integer Linear Programming
SPTF	Shortest Processing Time First algorithm
FIFO(X)	First-In-First-Out sorted by parameter X algorithm

Summary

In this thesis, we consider investment optimization for airport infrastructure which is required charge and refuel electric and hydrogen powered aircraft using battery and hydrogen canister swaps respectively.

In this report, we expand the model that was developed in [Justin et al., 2020]. In this situation, electric flights arrive at the airport with a near-depleted battery. Between landing and taking off for the next flight, the aircraft swaps its depleted batteries for ones which contain just enough energy to make it to the destination airport (plus safety buffer). Batteries are brought to a charging center where all have to be charged before the end of the day of operations, or used on a different flight. At each airport, the task at hand is to determine the most cost-effective infrastructure, consisting of (1) spare batteries, and (2) chargers. The optimization of hydrogen infrastructure is completely analogous, but batteries are replaced with canisters, and chargers with fuel points. We expand on this model by introducing the possibility of slot allocation, where flights that are not yet in possession of landing and take-off rights are assigned to them in such a way that requires the smallest extra infrastructure to be acquired.

We derive several (mixed) integer linear programming formulations to solve this problem and develop heuristics which are able to approximate the optimal solution using only open-source resources. These are expanded upon by the introduction of instances where more than one battery type is allowed, electricity pricing becomes dependent on the time-of-use, and storage of electricity at the airport is allowed such that the peak demand can be as low as possible. Finally, we incorporate a distinction between the long-, medium-, and short-term decisions which have to be made by the airport operator into the model analysis. This allows the user to determine the most cost-effective infrastructure combination which can meet a required level of service.

When testing these methods, we found that exact solutions can be found within reasonable time for cases with up to 200 batteries. Furthermore, an improved version of the algorithm used in [Justin et al., 2020] has shown to be capable of generating promising results while being applicable to larger instances. The models have been illustrated in a case study at the airports Schiphol (Amsterdam) and Zestienhoven (Rotterdam - The Hague), where they have proven to be able to solve all daily instances to optimality.

Further research may improve the quality of the model itself by integrating arrival delays of aircraft, and by relaxing the assumption that batteries have to be charged by the end of the day of operation. The scope of the model can be expanded by the introduction of transport equipment in the infrastructure, and using the topology of the airport. Finally, the sensitivity of the optimal solution to the required level of service can be studied further.

x

Preface

After long periods of finding the right focus of research, as well as a lot of hard work and late hours only to reach dead ends and throwing parts away, the puzzle finally fits together.

First and foremost, I would like to thank my graduation committee from NLR and TU Delft. Thank you Charlotte for your remarkable supervision of this thesis, from the first to the last day. Thank you for the energy which you brought to the intensive discussions which we had, which usually started out small but ended with discussing almost everything. Leo, thank you for the intensive reading of the various versions of my thesis and the large amounts of detailed feedback and suggestions. Your comments always provided excellent food for thought. Thank you Mihaela, for the suggestions which you provided while I was still trying to figure out where the focus of this work had to be.

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Finally, I would like to thank my family for their unwavering support. To my father Peter, my mother Angèle, and my brother Patrick. We got to spend a lot of time together last year, and while working from home was more often than not difficult, we helped each other through.

Yours sincerely,

Simon van Oosterom
May 21, 2021

Contents

CHAPTER 1

Introduction

Compare the awe of the readers of *around the world in 80 days* by Jules Verne with the possibilities which are at the disposal of everyone now, and it is hard to believe that the novel is only 147 years old. Over the course of the last century, civil aviation has become a vital part of modern life and the global industry, allowing people to reach almost every corner of the earth in less than 24 hours.

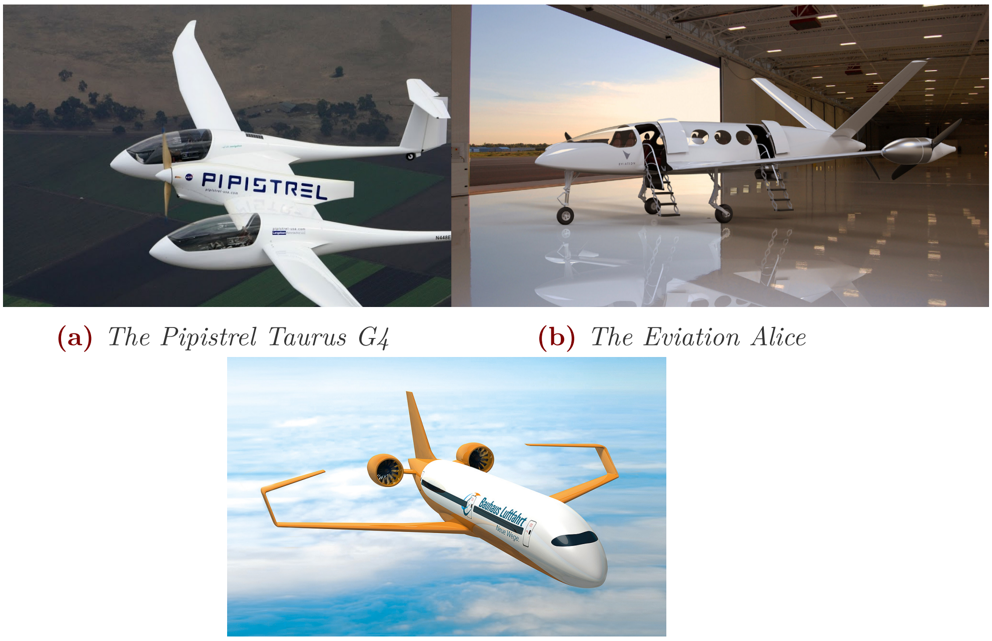
But despite these great achievements in the industry, aviation is now facing problems of a different kind. Estimates by the International Panel on Climate Change (IPCC), the European Commission, the National Aeronautics and Space Administration (NASA), the Boeing Company and Airbus SE report that the air travel and transport industry is responsible for up to 4% of the total global carbon-dioxide emissions [Lee et al., 2009].

In order to move against these issues which are faced by the industry, agencies are formulating plans and setting up programs which help to secure the future of global aviation. The European Commission has published their vision in *Flightpath 2050* in 2011 [Darecki et al., 2011] and recently in *Destination 2050* in 2021 [EC, 2021]. Most importantly, a severe decrease in the amount of greenhouse gasses is demanded: by the year 2050, inter-European flight travel should emit net zero emissions.

The need for alternative aircraft

These goals are highly unlikely to be achieved by the further development of conventional internal combustion engine (ICE) aircraft. In order to achieve these goals, alternative fuel aircraft have been gaining increasing attention in the research literature and by manufacturers. These alternative aircraft technologies include (hybrid-)electric and hydrogen propulsion systems. The former was long-thought to be impossible to achieve from a technical perspective, due to additional weight of the batteries which carry the electricity and the limits that this puts on the flight range, but every year these aircraft get closer to becoming a reality. Smaller recreational aircraft have already been realized in a full-electric version. Even though the question remains how to process used aircraft in a sustainable way, creating a circular aviation economy, these concepts are expected to be airworthy. Three examples are the realized Pipistrel Taurus Electro 4-seater, the in development Eviation Alice 12 seater, the concept of a commercially viable short-haul aircraft: the Bauhaus Luftfahrt Ce-Liner. These different aircraft can be seen in figure 1.1.

On the other hand, hydrogen powered aircraft have proven to work well over thirty years ago with the Russian Topolev TU-155, which had a range of over 2500 km [Tupolev, 2020]. This allowed it to perform almost any flight within the European continent without the need to refuel. The fact that this is a viable alternative to ICE aircraft was recently confirmed again when Airbus announced three new hydrogen aircraft which can carry up to 200 passengers and should have entered service by the year of 2035 [Duvelleroy, 2020]. These aircraft can be seen in figure 1.2. In these images, the disadvantage of the hydrogen aircraft can also be seen: hydrogen in liquefied form needs to be transported within a cylindrical container, similar to LPG. In a conventional aircraft, the only place to put such tanks is in the body, and as a consequence, about a third of the capacity is lost to placing the hydrogen tank. This can eventually be resolved by the use of a radically new blended-wing aircraft, seen in figure 1.2d, but is still far from realization.



(a) *The Pipistrel Taurus G4*

(b) *The Eviation Alice*

(c) *The Bauhaus Luftfahrt Ce-Liner concept*

Figure 1.1: *Examples of electric aircraft in different stages of development.*

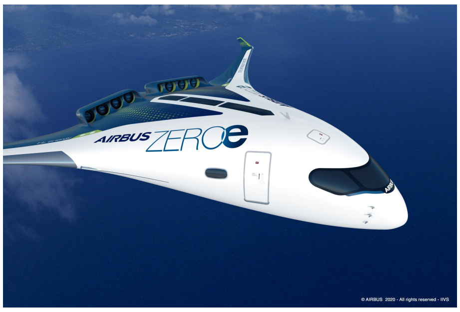
Refueling and recharging alternative aircraft

The question of how to refuel or recharge these new types of aircraft remains a major technical obstacle. For electric aircraft, this is caused by the relatively low specific energy density of the fuel. Even though batteries have seen a huge development in energy density, it is still fifty times less then that of kerosene. For hydrogen aircraft, this is caused by a combination of a low volumetric energy density, combined with the requirement to work under cryogenic conditions. Given these requirements and the fact

(a) *The Topolev TU-155 experimental hydrogen aircraft.*

(b) *The Airbus ZEROe turboprop concept*

(c) *The Airbus ZEROe turbofan concept*



(d) *The Airbus ZEROe blended concept*

Figure 1.2: *Examples of hydrogen aircraft in different stages of development.*

that for a commercial airline to be economically viable maximum refueling times are typically between 20 and 60 minutes, two recharging options remain.

The first one, which is more time consuming but requires less aircraft modifications, is to recharge or refuel the aircraft while the batteries or hydrogen canisters are on board. Fast recharging stations have seen considerable development, going from 50 kW to 250 kW now, while ones which can provide 1 MW are in development [Bigoni, 2018]. However, even if these chargers become available other problems remain. These are for example: maintaining battery lifetimes, and thermal management. Additionally, problems in the grid will emerge during the peak hours at the airport, when a large number of aircraft have to recharge simultaneously [Justin et al., 2020].

Hydrogen, is gravimetrically a more efficient energy carrier than both batteries and jet fuel, which it outperforms threefold [Marksel et al., 2019]. Refueling a hydrogen canister on-board can also be done in a process which bears more similarities to refueling kerosene. However, using liquid hydrogen also has a number of disadvantages which make the implementation of such a system very complex. The main disadvantages of hydrogen in aviation are that it must be cryogenically stored and transported to the aircraft, at approximately -250°C, that it is highly volatile, and that it still has a much lower volumetric energy density than kerosene.

Because of these disadvantages, swapping the used batteries and hydrogen tanks at the airport may become a good solution. In this concept of operations, the used energy carriers are transported to a central charging/fueling station at the airport where they are prepared for their next flight. This means that the recharging/fueling can be done with more freedom as to when this happens. The carriers can be charged/fueled at times which minimize the peak demand for electricity or hydrogen. Additionally, the procedure of swapping can be completed in a manner of minutes, which allows the airlines to operate their flights under normal turnaround times. Finally, the central charging/fueling station is a more controlled environment then the apron, and the hazardous batteries and hydrogen canisters can be stored in a more secure location. Of course, there are also disadvantages to this solution: standardization of the energy carriers is a prerequisite, the ownership becomes ambiguous, and the airworthiness of aircraft which allow the swap operation will have to be proven.

Nevertheless, aircraft which allow swaps are in development. The Pipistrel Taurus from figure 1.1a can swap its batteries. For hydrogen aircraft, the US based company Universal Hydrogen is looking into this possibility, aiming to retrofit a Bombardier Dash 8 with a hydrogen powertrain and the possibility of swapping the tank [Blain, 2020].

The existing airport infrastructure was not designed to accommodate these types of aircraft, and upgrades will be required. For hybrid-electric and full-electric aircraft, the current electricity grid will need to be enhanced in order to provide the power to recharge the batteries and aircraft, chargers will need to be bought which can recharge batteries when they are on and/or off-board. For hydrogen aircraft, infrastructure will have to be constructed in order to bring or generate the hydrogen respectively to and at the airport, and allow for operations to occur at cryogenic conditions.

Optimization of refueling and recharging infrastructure at airports

While a substantial amount of research has been focused on realizing the electric and hydrogen aircraft in the sense of producing airworthy vehicles, and making them commercially viable by lowering the flight costs per passenger kilometer, less research has been done as to how the airports should be redesigned in order to service these aircraft once they have been realized. To the authors knowledge, only three research programs have studied the problem of finding a recharge schedule and the required infrastructure for an electric fleet [Justin et al., 2020, Bigoni, 2018, Pereira, 2019], all of them with their own strengths and shortcomings. In addition, two reports have been published on how to accommodate hydrogen aircraft, but the sizing of this infrastructure was not calculated [EC, 2020, Bruce et al., 2020].

Research objective

The aim of this study is to produce an optimization model which computes the most cost effective infrastructure which is required to perform battery swaps for electric-, and tank swaps for hydrogen-, aircraft at airports. The main research question of this report is:

For which instance sizes is it possible to optimize the required infrastructure size to support the battery- and tank-swap of respectively electric and hydrogen aircraft, and what are suitable alternatives for beyond this limit?

In order to answer this question, we are going to perform the following steps: define the scope of our problem, perform a literature study to determine the research gaps, design, test, and upgrade the model, perform a case study, and evaluate the results.

Scope

The focus of this research will lie with the development of a mathematical optimization model which is able to solve instances to optimality which are as large as possible within as little time as possible. Background knowledge of the aviation industry is presented in the literature review, but this is solely used to develop a model which is relevant for the applications.
Within the scope of this study are the following aspects:

- **Maximum energy demand and spares**: The infrastructure which the core model should optimize consists of two categories: chargers/fuel points and spare batteries / hydrogen canisters. Both relate to the peak demand for electricity / hydrogen of the airport from the grid: the chargers relate directly to the peak power itself, while the spare batteries help spread the peak demand more evenly throughout the day.

- **Multiple battery/tank types**: the large differences in aircraft size may lead to the parallel use of more than one battery/hydrogen tank type at the airport. While the core of the model will only have a single battery type, the implementation will have multiple types.

- **Airport specific information**: Basic information about the layout of the airport is taken into account in the models. This includes the runway capacity, and the average taxi-time of aircraft and transport time of batteries and hydrogen canisters from the aircraft to the charging station.

Outside of the scope of this study are the following aspects:

- **Network problems**: We shall study the problem of infrastructure optimization within an airport without considering the decisions which have to be made at other airports. This is done by assuming either that the electric/hydrogen aircraft always arrive with minimal energy left, or that they depart with fully charged batteries/filled tanks.

- **Airport topology**: Detailed information about the layout of the airport is not used in this study. This includes the layout of runways, taxiways, and concourses. This is because of the belief that adding this information will only become relevant when society nears the implementation of the electric and hydrogen aircraft. Until that time, adding information with this level of detail will only generate marginal gains at a cost of adding a large level of complexity.

- **Ground support equipment**: The equipment which is responsible for transporting the batteries and hydrogen canisters is not considered in the optimization models. This choice is motivated by the same reason as mentioned for not including detailed airport layout information.

Structure of this Thesis

The rest of this report is organized as follows. Firstly, Chapters 2 and 3 make up the literature study: the former contains relevant information regarding electric and hydrogen aircraft, as well as airport operations, the latter contains the works which have been performed on this topic in the past.

Chapters 4, 5 and 6 present the contributions of this research to the scientific field are presented. In Chapter 4, the model and results are presented in a setting where the aircraft schedule is a predetermined part of the input. In Chapter 5, models and results are presented in the setting when the flight schedule is not predetermined. A case study where these models are applied is presented in Chapter 6. The discussion of the results and the conclusions of this thesis can be found in Chapter 7.

CHAPTER 2

Literature Study Industry: the Future of Aviation

In this chapter, the relevant background information on electric and (liquid-)hydrogen aircraft and on airport decision-making is provided. In the first section, aircraft sizing and range estimators are introduced, together with models on the charging and refueling process of electric and hydrogen aircraft.The second section is devoted to explaining the different stages of decision-making of airports, where special attention is given to the process of slot-allocation.

2.1 Electric and hydrogen aircraft technology

In this section, the models which describe the general properties of (future) aircraft together with the charging/refueling process of electric/hydrogen aircraft are discussed. This knowledge will be useful for generating realistic input values for the aircraft properties in the case study which shall be performed. This will be a brief introduction, but more background information on these new types of aircraft, together with a list of several concepts, can be found in Appendix A.

2.1.1 Estimating Aircraft Properties

In order to estimate the characteristics of the alternatively powered aircraft, we are going to use a model which calculates the Maximum Take-Off Weight (MTOW), and maximum fuel weight for an aircraft given a payload and range. Background information of the model can be found in [Raymer, 1992], [Pereira, 2019], and [Hepperle, 2012].

The MTOW is determined by solving the following equation, which depends on the empty operating weight EOW and the fuel weight W_f which are required to transport the given payload W_{pay} over a certain range R:

$$MTOW(W_{pay}, R) = EOW(MTOW) + W_{pay} + W_f(MTOW, R) \tag{2.1}$$

Historically, EOW is given by a near-linear function of MTOW:

$$EOW = A \cdot MTOW^{1+C}, \tag{2.2}$$

where (for a regional jet) $A = 0.882$ and $C = -0.048$. The fuel weight depends on the type of fuel which is used as well as the properties of the fuel. For regular (liquid or gaseous) fuels, it is given by the Breguet Formula:

$$W_f = \left(0.07\frac{E_k^* \cdot \eta_k}{E_f^* \cdot \eta_f} + 1 - e^{\frac{R \cdot g}{E_f^* \cdot \eta_f \cdot L/D}}\right) \cdot MTOW, \tag{2.3}$$

where g denotes the gravitational acceleration and L/D the lift-to-drag ratio[1]. E_f^* denotes the gravimetric energy density of the fuel and η_f the fuel to thrust efficiency, while E_k^* and η_k denote these values for kerosene: $E_k^* = 11$ kWh/kg and $\eta_k = 0.3$. For batteries, this function is modified to compensate for the fact that batteries do not loose weight as they are used:

$$W_f = \left(0.07E_k^*\eta_k + \frac{R \cdot g}{L/D}\right) \frac{MTOW}{\eta_f \cdot E_f^*} \tag{2.4}$$

In both cases, solving for MTOW needs to be done numerically with (for example) Picard iterations.

2.1.2 Refuelling and Recharging

Electric Aircraft

The battery recharging process has been studied in the literature extensively, and is a well known subject. Studies into charging larger batteries have also been performed due to the realization of electric vehicles. Under normal conditions, the recharging time is a linear function, given by equation 2.5:

$$t_{charge} = \frac{Q_{out} - Q_{in}}{P_{chg}}, \tag{2.5}$$

where t_{charge} is the recharge time, Q_{out} and Q_{in} are the states of charge at the beginning and end of the recharge, and P_{chg} is the charger power.

However, these charging times are in practice both very large, and underestimated by this formula. For up to around %80 of the capacity of the battery, the recharging time follows the linear relation from equation 2.5, but after this point, the battery starts to resist and the charging rate decreases. This relation is displayed in figure 2.1: the state-of-charge will approach the %100 limit, but this will never be reached [Domínguez-Navarro et al., 2019].

With the current state of technology, as well as in the foreseeable future, charging aircraft batteries can be a time-consuming process: with the current concepts (see Appendix A), a charging time of 4-5 hours is no exception. In order to be economically viable, airlines rely on turnaround times which are less then an hour for these types of aircraft, thus making the 5 hour wait an impossible option for a recharge, unless the aircraft spends the night at the airport. In order to deal with this, an alternative recharging option has been emerging in literature, taken from the automotive industry:

[1] A list of typical values for L/D ratio can be found here

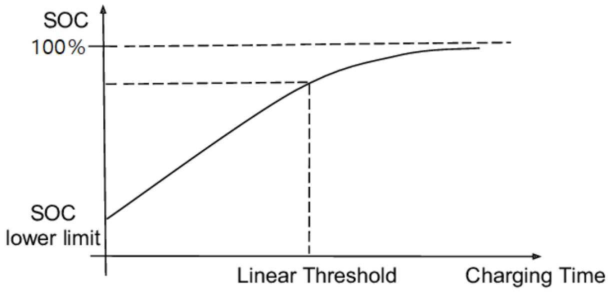


Figure 2.1: *State of Charge of a battery as a function of time [Pereira, 2019].*

swapping the batteries.

The process is simple: after arriving at the airport, ground handling removes the depleted batteries, which are taken to a battery recharging facility, and inserts new ones which arrived one a different flight and are either fully charged or sufficiently charged to complete the next flight. Performing this swap should take only a maximum of 15 minutes, comparable to refuelling an ICE aircraft [Plötner et al., 2013]. In addition to this, batteries can be recharged during off-peak hours, when electricity is less expensive, and at lower power, which is favourable for the batteries lifetime. This technique has already been implemented, the Tesla model S prototype could swap batteries in 90 seconds[2] and the Pipistrel Alpha Electro has batteries which can be removed by hand and taken, proving that it is conceptually possible.

There are also a number of downsides to this. Firstly, aircraft need to be designed and certified in such a way that swapping the batteries can be done safely and within a reasonable time. Since this operation is more invasive then charging the aircraft plug-in, this is bound to be a challenge. Secondly, in order to be economically viable, batteries need to be subject to a high level of standardization, airports need to have a supply of spare batteries and the size of this supply increases with the number of battery types which are used. Thirdly, since the batteries are no longer owned by neither the airlines nor the airports, regulations need to be in place in order to make sure that the batteries are used properly by all parties.

The charging rate of batteries is also determined by the so-called C-rates, which are measures to quantify the (dis)charging rate of a battery relative to its capacity. The relation between the maximum charging and discharging rate is given by:

$$\frac{P^C}{C^{Chr}} = \frac{P^D}{C^{Dch}}, \tag{2.6}$$

where P^C and P^D are the maximum charging and discharging rates, and C^{Chr} and C^{Dch} are the C-rates for charging and discharging respectively. P^D, given by the required power for taking-off, can be obtained from equation 2.4 by converting the battery weight to energy and taking $R = 0$ and dividing by the number of batteries which are on-board.

[2] This technology was demonstrated at a press event and can be seen over here

Hydrogen Aircraft

Hydrogen allows for a relatively fast refueling process at the airport and a number of studies have been conducted, standards have been made, and the technology is already implemented in hydrogen refueling stations for cars.

The safety requirements for refueling Hydrogen at airports, a large concern when it comes to this have been standardized in ISO/PASS 15594:2004. All operations need to be performed by authorized personnel in open-air, while staying clear of other combustible/flammable objects. Similar to refueling kerosene, other cabin operations can be performed simultaneously, no passengers are allowed on-board. Furthermore, two methods are available for refueling the aircraft. Firstly, for smaller airports, fueling can take place directly from the hydrogen transport truck, such that no stationary infrastructure is required at the airport. Secondly, for larger hubs, fueling can be done via a liquefied Hydrogen storage facility at designated gates which are equipped with a fuel point.

However, even when hydrogen is liquefied (when it is at its most compact form), and if it can be pumped into the aircraft at the same rate as kerosene, this will generally still lead to refueling times which are larger then the acceptable turnaround times. If we assume that hydrogen aircraft require the same amount of energy as kerosene to perform a similar flight, the lower volumetric density leads to refueling times which are three times as large. In addition to the fact that the refueling process is a highly specialized task because of the volatility of hydrogen and the low temperatures, using the conventional setup to refuel hydrogen aircraft is not possible in this environment. As an alternative to this, swapping the hydrogen tanks can be performed in a similar manner to swapping batteries. This also allows for storing and refueling hydrogen in a contained environment, away from the busy apron. Even though it is still early for this method, it is already being developed (see figure 2.2) by US-based company Universal Hydrogen [Blain, 2020]. The objective of this company is to have converted a 60-seater Bombardier Dash 8 by the year 2025.

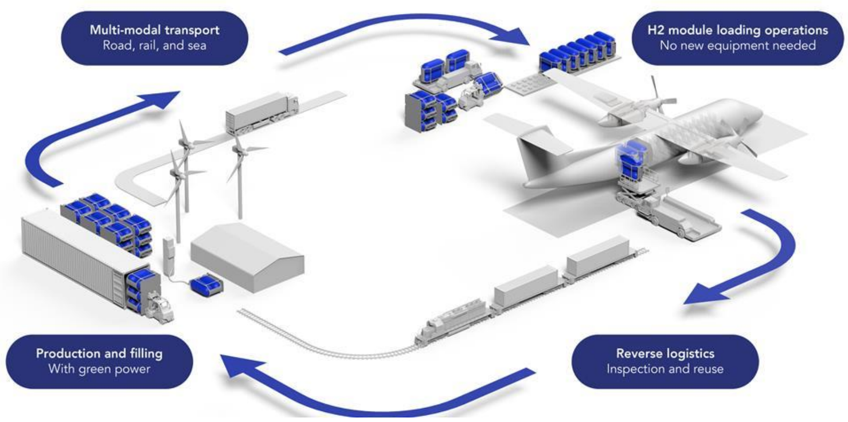


Figure 2.2: *Universal Hydrogen's concept of refuelling hydrogen aircraft by tank swaps [Blain, 2020].*

2.2 Airport Planning

In this Section, the two concepts in which this report aims to expand on the previously developed models for infrastructure optimization will be introduced. First, in Section 2.2.1, an overview is given of the long-term decision process of airports and the different phases of which this is constructed. In section 2.2.2, a part of this process is highlighted: the mechanism for allocating landing and take-off rights to flights. Understanding these two concepts will help with creating a model.

2.2.1 Airport development decision phases

The growth of the civil aviation industry combined with the limited available airport infrastructure and traffic management facilities have created large issues at the world's most used airports. In order to manage the demand for years into the future, airports need to think ahead in their planning, and a development overview needs to be created with only access to a number of forecasts to estimate the demand. In this section, the three phases which determine how the creation of this development plan is determined are discussed: the long-term strategical phase, the medium-long-term tactical phase, and the short-term operational phase. The distinction between these phases is frequently used throughout the aviation industry and can be applied to many decisions [Bazargan, 2010, De Neufville and Odoni, 2003, Belobaba et al., 2009]. Knowledge of this field will be used when the models which are created in this report are applied in case studies at airports.

These levels are separated by the time before the actions in this level affect the operated flights, and actions in the strategic, tactical, and operational level typically happen years, months, and days/hours before the flight respectively. Strategic level planning concerns itself with the long-term development of the airport, building new runways, concourses and attracting new carriers to expand the network are part of this phase. During the tactical phase, the objective is to match the resources obtained in the strategic phase to the demand as efficiently as possible. During this phase, airlines are assigned slots (see Section 2.2.2), and the flight timetable is created. Finally, during the operational phase, last minute decisions and alterations to this timetable are made. Delays need to be mitigated in this phase, and flights need to be assigned to the runways which are suitable for the weather conditions of this day. The phases are summarized in Table 2.1.

Table 2.1: *Decision phases*

Time before operations	Phase name	Phase description
Years	Strategic	Develop airport assets and infrastructure, as well as the flight network
Months	Tactical	Match the resources acquired in the strategic phase to the demand generated by the airlines
Days/Hours	Operational	Adjust the schedule developed in the tactical phase in order to mitigate delays as much as possible

Creating and operating the infrastructure for recharging electric aircraft can naturally be divided into the three phases: determine the size of the infrastructure is a strategic task, assigning what has been acquired to the requested flights is a tactical task, and handling possible delays and no-shows is a operational task. These three tasks shall be discussed in chronological order, because events which can occur during a later phase determine how an airport needs to plan ahead:

1. *Operational phase:* During the Operational phase, the flight timetable and recharging schedule are already known and are possibly partially executed, until an unpredictable event occurs. Breakdowns of chargers are out of the scope of this research, so the only common disruption which negatively influences the flight schedule is a delayed arrival: one or more batteries do not arrive at the airport on time and as a result of this, the planned charging schedule is no longer feasible. A new schedule needs to be made, and if there is no way that all other planned flights can arrive and depart as usual, we have to mitigate the effects as best as we can. Decisions from both the tactical and strategical phase need to be made such that this occurs as few times as possible: the schedule needs to be made in such a way that it is robust to reasonably expected disruptions.

2. *Tactical phase:* During the Tactical phase, the operators are given the infrastructure as determined during the strategic phase and are thus given the following task: to create a flight timetable which is as close to the requested time table as possible, and to create a charging schedule which is robust to delays. While creating the flight timetable, the coordinator needs to adhere to the IATA guidelines, which describe the priorities in which the requests are treated. This topic shall be discussed in detail in Section 2.2.2. If resources for charging electric- or refueling hydrogen-aircraft are tight, the two problems have to be solved simultaneously. Decisions from the tactical phase need to be made in a way such that the assigned flight timetable both reasonably fair and robust.

3. *Strategic phase:* Finally, during the Strategic phase, airport operations is provided with a forecast of the electric flights in the upcoming years and is tasked with creating an infrastructure which makes servicing this demand possible. This needs to be done with two objectives in mind: minimizing the acquiring and operating costs, and minimizing the risks associated with not knowing the flight schedule on forehand and the possibility of delays. The acquisition of the charging infrastructure and the spare batteries are very different: getting a new spare battery can be done relatively short before operations and is cheap, whereas bringing a new charger into service is associated with raising the peak power which is drawn from the grid, which implies that an expansion needs to be made to the electricity network. This is a process which is way more expansive and takes longer to execute. Thus on the one hand we need to make the infrastructure as small as possible, but on the other hand it needs to allow for changes in the flight schedule, both in the tactical and operational phase, without falling apart.

An overview of these three decision phases can be found in Figure 2.3. But before we focus on working with these different stages we need to build the basic element of this construction. In this report we shall focus on the strategical and tactical phase, but as we have indicated in this Section, an understanding of the operational phase is required to generate schedules which are robust enough to withstand the expected delays. This building block shall therefore be written from the perspective of the tactical phase, and combining a number of these units shall be used in the strategic phase.

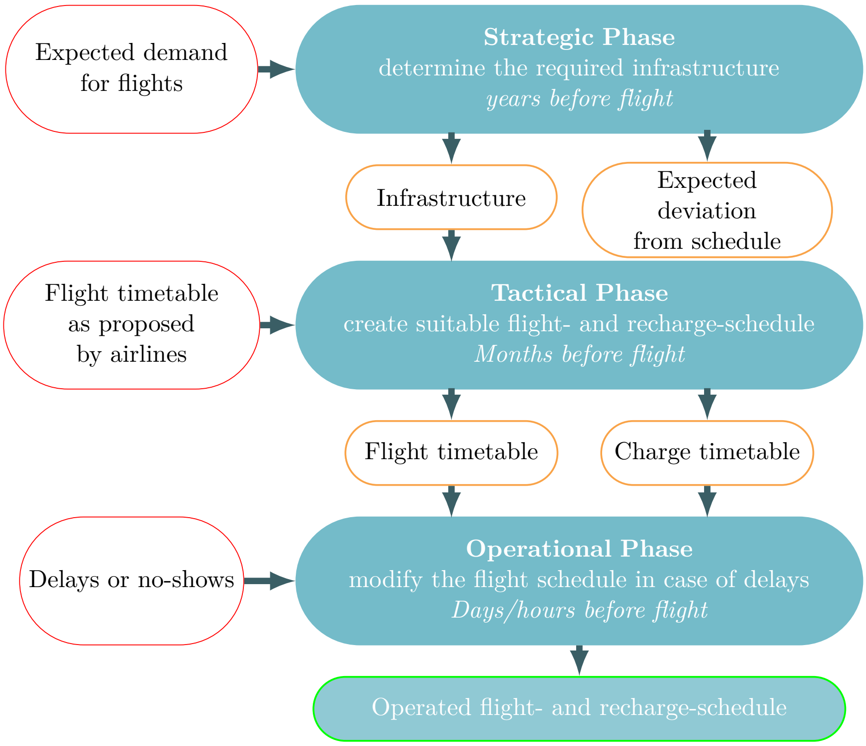


Figure 2.3: *Electric aircraft recharging with swaps and slot allocation model phases.*

2.2.2 Tactical airport demand management: Slot Allocation

The growth of the civil aviation industry combined with the limited available airport infrastructure and traffic management facilities have created large issues at the world's most used airports. The limited available capacity creates congestion, delays, and schedule unreliability [Ribeiro et al., 2018]. In order to mitigate these problems without upgrading airport infrastructure, a couple of demand management mechanisms have been introduced in the literature, or are already in place.

The foremost mechanism in use as of writing is the International Air Transport Association's (IATA) schedule coordination procedure [IATA, 2019]. This aims to allocate the supply in- to the demand for landing and take-off rights at given times at airports in a manner which is both transparent and fair to the users. It is currently applied in the 175 most congested airports around the world, which are so-called "schedule coordinated" [IATA, 2020].

The IATA schedule coordination procedure is repeated bi-annually (for a summer and winter season), and involves the allocation of *slots*: rights to land or take-off at an airport and to use the full range of it's infrastructure at a specific time and date. Five steps can be identified in the process, which can also be found in table 2.2:

1. *Declared capacity setting:* One year before the start of the season, each airport declares the number of slots which are available for each time. This may be given

in (any combination of) 5-minute, 15-minute, hourly, daily, weekly, monthly, or bi-annual period capacity.

2. *Slot requests:* Using the declared capacity, the airlines set out to produce a flight schedule. Once this is complete, they submit their requests for slots at the airport. If a flight takes place regularly, slots are requested in series, at most five months in advance of the season. If this is not the case, slots can be requested individually, which can be done up to one day before operations.

3. *Initial slot allocation:* Between the slot requests and at most four months before the start of the season, independent slot coordinators are given the task to create an initial slot allocation for the series of slots. This is done according to priorities and regulations set by IATA. The four primary request classes are, in order of priority: flights which have held the slots historically, small variations of these slots, new entrants, and others. In order to break ties between requests from the same primary priority class, secondary criteria are given by IATA. These are among others based on frequency, route, and aircraft and service type [IATA, 2019].

4. *Slot coordination conference:* After the initial slot allocation, IATA organizes a semi-annual conference, which is attended by all parties of interest. Slot coordinators and representatives from airports and airlines negotiate over slots in order to resolve scheduling conflicts.

5. *Slot Return:* If, for any reason, airlines decide not to use a slot, they are allowed to return it to the airport. For series of slots, this can be done up to two months before the start of the season. Up to one day before the flight, airlines may also request modifications to the slot coordinator, in order to resolve scheduling inefficiencies.

Table 2.2: *An overview of the IATA schedule coordination procedure for Schedule Coordinated airports, as prescribed in [IATA, 2019].*

	Timeframe	Action	Actor
Before start of the season	1 Year	Capacity Declaration	Slot Coordinator & Airport
	5 Months	Slot Requests	Airlines
	4 Months	Initial Slot Allocation	Slot Coordinator
	4 Months	IATA Slot Conference	Airlines & Slot Coordinator
	2 Months	Slot Return Deadline	Airlines
Before operations	1 Day	Changes and New Requests	Airlines

In a crowded market, this system provides a number of advantages. Firstly, delays are reduced significantly by preventing overloading of airports. Secondly, the utilization of an airport's capacity is improved by a more evenly spreading of flights across the day. Or, the other way to look at this, it protects airports from having to invest into building capacity for just a few hours of the day or a few days of the year. Finally, the system encourages "stability" and continuity of service by the provision of historic slot rights, and it is favored by large legacy carriers because of the orderly and transparent nature.

Naturally, as the system is fifty years old as of writing, it also has its disadvantages. These are explained in two leading articles from The Economist [R., 2017a, R., 2017b]. First and foremost, the stability guaranteed by the historical rights inhibits competition in the market, and entering into an airport becomes hard. Secondly, slots represent economic value for which the airlines that hold them originally have not paid for. Even though buying slots from other airlines is strictly prohibited, the secondary slot market has exploded[3], creating a discrepancy between the regulated slots and the deregulated aviation industry.

Currently, slot allocation procedures are mainly handled with the help of specialized software like PDC Score, which helps the coordinators solve the problems heuristically. However, due to advances in computational power, optimization approaches have been developed in order to support this process, and have been subject to significant research. An overview of these methods can be found in the literature review from 2017 [Zografos et al., 2017]. Up to this date, the Priority Based Slot Allocation Model (PSAM) by [Ribeiro et al., 2018] provides the only model which accounts for IATA priorities and is able to solve the bi-annual slot allocation problem for a larger regional airport.

[3]For some airlines, the slots they hold are even more valuable then the aircraft they possess. Kenya airways, for example, managed to stay afloat after the economic crisis by selling its only London Heathrow slot pair to Oman Air.

CHAPTER 3

Literature Study Mathematics: Background and Previous Studies

The problem of managing a fleet of electric aircraft from a operational point of view has been studied in a number of research programs. While the core component of the research, making a schedule for when and how electric aircraft can recharge or swap batteries, remains the same, each of the solutions is different in the mathematical tools which are employed as well as the situations to which the models can be applied. In this chapter, the relevant mathematical background (in Section 3.1) and the previously created models themselves (in Section 3.2) are presented and discussed.

3.1 Mathematical Scheduling

Depending on the abstractions which are made, the problem of infrastructure sizing for aircraft fueling/charging can be described as one from scheduling theory. In this Chapter, the literature study is given into how these theories have been researched in the past and how they can be applied in this context. First, the concept of Generalized Due Dates and its applications are introduced in Subsection 3.1.1. Second, scheduling under variable electricity prices is treated in Subsection 3.1.2. A general overview of the concepts from scheduling theory is given in Appendix B.

3.1.1 Scheduling Under Generalized Due Dates

In this subsection, scheduling problems with generalized due dates, abbreviated as *gdd*, are discussed. In regular scheduling problems, jobs are specified by three properties: the release date r, the processing time p, and the due date d. Jobs have to be completed before their due date, or will be penalized. In *gdd* problems, the due dates d are given but are not coupled to the jobs: given is how many jobs should be completed at each moment, but it is of no importance which specific jobs are finished. Applications of problems with generalized due dates occur naturally when the products are identical after the tasks have been performed, and at each moment in time one knows how much completed tasks there have to be: no precedence or priority is specified by the customer.

Generalized Due Date problems have been introduced and formalized by Nicholas Hall in [Hall, 1986], where a number of problems have been considered. *gdd* problems which were proven to be polynomial-time solvable were: $1|gdd|\sum_j Tj$ and $1|gdd|\sum Uj$. Problems which were shown to be NP-complete were: $1|gdd|\sum_j w_jU_j$, $1|gdd, r_j|\sum_j T_j$, $P2|gdd|L_{max}$, and $P2|gdd|\sum_j U_j$. By extension, it follows that $Pm|gdd, r_j|L_{max}$ and $Pm|gdd, r_j|\sum_j U_j$ are also NP-complete.

Table 3.1: *Complexity of some scheduling problems with a generalized due date version (partially from [Hall et al., 1991]).*

Problem (notation for specific due dates)	Generalized due date version	Specific due date function
$1\|r_j\|L_{max}$	NP-complete [Hall et al., 1991]	NP-complete [Lenstra et al., 1977]
$1\|pmtn, r_j\|L_{max}$	Polynomially solvable [Hall et al., 1991]	Polynomially solvable [C.L. Liu and J.W. Layland, 1973]
$P\|pmtn\|L_{max}$	Polynomially solvable [Hall et al., 1991]	Polynomially solvable [Horn, 1974]
$P\|pmtn, r_j\|L_{max}$	Open problem	Polynomially solvable [Lawler and Labetoulle, 1978]

After the introduction of generalized due dates by Nicholas Hall in [Hall, 1986], a number of studies have extended the number of problems which have been shown to be polynomially solvable or are NP-hard. In [Sriskandarajah, 1990], Sriskandarajah et al. showed that $1|gdd|\sum_j w_jT_j$ and $1|prec, gdd|L_{max}$ are respectively weakly and strongly NP-hard. But the most significant theoretical research was reported by Hall et al. in [Hall et al., 1991], where a large number of *gdd* problems were considered.

In the next chapter, we shall see that generalized due dates can be used to model the recharging schedule for swapped batteries of electric aircraft. The model created in [Justin et al., 2020] can be formulated as a $Pm|gdd, pmtn, r_j|\gamma$ problem. The "regular" variant of this problem ($Pm|pmtn, r_j|\gamma$) is polynomial time solvable (see [Lawler and Labetoulle, 1978] or [Pinedo, 1995]), but if this is the case for the *gdd* variant is still an open question. The status of this, and related, problems are shown in table 3.1. Since this problem is a generalization of the $1|pmtn, r_j|\gamma$ or $P|gdd, pmtn|\gamma$ problem, it can be deduced from table 3.1 that it could be polynomial-time solvable. On the other hand, there are no generalizations of this problem which are also polynomial-time solvable, and therefor nothing about the complexity of this problem can be deduced from Table 3.1.

3.1.2 Scheduling Under Time-Of-Use Electricity Prices

In many industrialized countries, industries pay different electricity charges depending on the time during the day (referred to as peak-, mid-, and off-peak-load times). This is done to give the industry an incentive for not using the electricity network to a large extend during the times when there is a high electricity demand and there is a danger of an overload in the grid. In addition to this, the emergence of the smart-grid, which is proposed to counter the less predictable energy supply by renewable sources, will take this concept of Time-Of-Use (or TOU) electricity pricing to a higher level. Scheduling under TOU electricity prices the branch of scheduling theory which typically concerns itself with a two-objective optimization: how can the jobs be scheduled such that the already present objective function (e.g. the makespan) is optimized, while at the same time making sure that the prices which are paid for the electricity are minimized. These two objectives generally contradict each other, and the goal is to balance the two.

Scheduling under TOU electricity pricing is not a problem which is as well established as the scheduling theory from Appendix B. But driven by the pressure from the smart-grid and environment protection it has been studied by several research groups during the last fifteen years, for example in [Castro et al., 2009]. Due to the nature of the problem, the solution methods are very different when compared to the last Section: since they are known to be NP-hard, problems are usually only solvable with heuristics or via the MILP formulation. The heuristics which have been applied to solve these problems in a parallel machine environment can be found in Section B.3 on page 115.

3.2 Previous Studies in Scheduling Battery Swaps and Recharges

At the time of writing, three studies concerning the problem of scheduling aircraft energy carrier swaps are known to the author, and all of them are focused on electric aviation. All of these studies are relatively new, the oldest one was completed in 2018, which corresponds with the fact that the technology which will enable electric aircraft is also still in an early stage of development for aircraft which can offer commercially viable flights. The aerospace department of the Polytecnico di Milano was the first institution to publish about the subject. Models which were originally developed for electric vehicle recharge stations were converted to a model which could calculate the desired infrastructure size and recharge schedule for the batteries. The aerospace department at the TU Delft developed a model that worked from the perspective of an airline which flies round trips from a hub-airport and needs to perform a set of flights. It determines the assignment of aircraft to flights as well as the required infrastructure. Finally, the aerospace department at the Georgia Institute of Technology created a model for an airport in a network, where aircraft arrive according to a fixed schedule and need to swap their used batteries for a recharged one with which the next flight can be completed.

In the next subsections, each of the three studies is reviewed, and from these reviews the research gaps will emerge.

3.2.1 Bigoni et al.: Flight School Battery Charging (2018)

The aerospace department at the Polytecnico di Milano was the first group to study the the problem of assigning batteries to electric aircraft and generate a feasible recharging and battery swapping schedule [Bigoni, 2018]. Bigoni studied the situation which can generally be associated with a general aviation airfield, where aircraft take of once per day, perform a trip which takes a certain amount of time, and return to the same airport again with a (partially) depleted battery. At the airport, aircraft can be recharged either by replacing their batteries with new ones, or by recharging at a plug-in charging station (similar to the way that electric vehicles are charged currently). The problem was modelled using a Mixed Integer Linear Programming formulation, and was based on research done in the electric vehicle industry.

This formulation made the following additional assumptions: batteries can be charged preemptively on any of a number of machines, the time it takes to fully charge a battery is a linear function of the depth-of-charge, and batteries can be discharged in order to charge other batteries. Furthermore, there is a set of usable batteries I, indexed by i, and time is split up in constant sized intervals T. The following parameters and variables are used:

The program determines at which time to recharge each battery, and which battery should be swapped to perform the next flight of an aircraft. The goal is to minimize the amount of money which is paid for the electricity. This gives the linear programming formulation of equations 3.1 and 3.2.

The objective function gives the price paid for the electricity. The first constraint specifies which batteries are swapped to aircraft for each time, the second constraint limits the charge in each battery, and the third specifies that a battery can only be charged if it is not in the process of being swapped. The forth and fifth constraint

Table 3.2: *Parameters and variables from a simplefied version of the model by Federico Bigoni [Bigoni, 2018]. The two classes in the fourth column, B and C, denote that a variable can take on binary or continuous values respectively.*

Parameters		Variables		
notation	description	notation	class	description
N_t	demand for batteries at time t	$x_{i,t}$	B	indicator when a battery is used
BC^{max}	maximum battery capacity	$soc_{i,t}$	C	state-of-charge
P^{max}	maximum charger power	$bat^{chg}_{i,t}$	C	battery charge rate
soc^{in}	battery charge after use	β_t	B	auxiliary variables
λ_t	electricity price at time t	em_t	C	electricity used at time t
soc^{req}	state of charge required to be used			

determine how the state of charge evolves throughout the day: if a battery is not swapped, it is equal to the state of charge at the last time plus the amount which is added by charging, and if it is swapped, it is equal to the state of charge of a used battery. Finally, the sixth and seventh constraint specify respectively when a battery can be swapped, and how much electricity is used during one time step.

$$\min \quad \omega = \sum_{t \in T} \lambda_t \times em_t \tag{3.1}$$

$$\text{s.t.} \quad N_t = \sum_i x_{i,t} \qquad \forall t \in T \tag{3.2}$$

$$0 \leq soc_{i,t} \leq BC^{max} \qquad \forall i \in I, t \in T$$

$$|bat^{chg}_{i,t}| \leq P^{max}(1 - x_{i,t}) \qquad \forall i \in I, t \in T$$

$$soc_{i,t} = (soc_{i,t-1} + bat^{chg}_{i,t})(1 - x_{i,t}) + soc^{in} x_{i,t} \qquad \forall i \in I, t > 0$$

$$soc_{i,0} = soc_{i,|T|} \qquad \forall i \in I$$

$$x_{i,t} \leq 1 + (soc^{req} - soc_{i,t-1})/soc^{req} \qquad \forall i \in I, t \in T$$

$$em^{buy}_t = \sum_i bat^{chg}_{i,t} \qquad \forall t \in T$$

These equations describe the basics of the model, but in his thesis Bigoni describes a number of ways in which the model can be improved and extended to make it more realistic while not making it too hard to solve computationally. The first improvement which can be made accounts for the fact that the batteries are not always owned by a ground handling company, but possible also by the airline itself. Thus, the total number of batteries on the ground and in the air is fixed, but the distribution between the two can vary from time to time. In order to describe this, the variable $x_{i,t}$ is split up: $x^b_{i,t}$ and $x^o_{i,t}$ are introduced and depend on $x_{i,t}$. The former is equal to 1 when the battery is removed from the aircraft and transported to the charging station, while the latter is 1 when the battery is used in an aircraft:

$$x_{i,t} = 1 \text{ implies that:} \tag{3.3}$$

$$x^o_{i,t} = x^o_{i,t+1} = x^o_{i,t+2} = ...x^o_{i,t+t_m-1} = 1 \tag{3.4}$$

$$x^b_{i,t+t_m} = 1 \tag{3.5}$$

In the equations which describe the charging process of the battery, $x_{i,t}$ is replaced by $x^o_{i,t} + x^b_{i,t}$.

Assessing the model's strengths and shortcomings

Even though Bigoni's work was the first of its kind, it considered a large number of aspects to the problem. His research comprised: the technical properties of the currently available electric aircraft and how these technologies are expected to evolve in the near future, a large analysis of both the Capital and Operational Expenditures which airports would need to make to transition to a partial electric fleet, and a breakdown of the impact on the emission of the airport and the fleet of such a switch. These are some of the strengths and weaknesses of the model:

+ **It can handle both plug-in and battery-swap recharge methods.**

+ **It can deal with TOU-electricity pricing.** This allows the airport to buy electricity when it is cheap and use it when it is expensive.

- **It assumes conditions which (generally) only apply to specific airports.** The model assumes that aeroplanes return after a fixed time on the same day. This is acceptable for general aviation airfields or hubs of regional flights (such as Bresso and Athens), but is not realistic for a larger airport as a part of an airline network. Furthermore, the lay-out of the airport is not taken into account. Information on this may be used to determine how many chargers can be used at the same time without overloading the grid, or how many batteries can be safely stored at the airport at the same time.

- **It takes the departure schedule as an input.** In response to large changes in the infrastructure, a change within the schedule might be necessary to ensure that operations run as smooth and cost-effective as possible. Additionally, the flight schedule will be unknown to the airport operator when it needs to acquire the infrastructure.

- **It has the NP-hard time complexity, and is solved in discrete time.** Being in the NP-hard class can cause issues when solving the program for larger airports. Additionally, the use of discrete time intervals makes the problem formulation relatively inelegant. This manifests in a large number of variables and constraints, which makes the problem hard to solve for large instances.

After the completion of this thesis, the Aerospace Science and Technology department of the Polytecnico continued the research on this subject under the lead of Prof. Lorenzo Trainelli. The team worked on and published several papers [Salucci and Trainelli, 2019, Salucci et al., 2020], which are listed in the biography. The MAHEPA Project Consortium also published the Ground Infrastructure Investment Plan, which also includes details on the implementation of liquid hydrogen powered aeroplanes [Marksel et al., 2019].

3.2.2 Pereira: Electric Aircraft to Mission Assignment Problem (2019)

This study focusses on solving the EAMAP-problem: the Electric Aircraft to Mission Assignment Problem, and was worked on in the master thesis of Madalena Pereira [Pereira, 2019].

This study is quite different from the first one in terms of setting and mathematical tools. The model assumes a situation which is comparable to, but more general than, the situation in the work of Bigoni. This time, a fleet of identical electric aircraft is based at a large hub-airport and the airline needs to complete a number of round trips to smaller airports (of varying duration). Only the hub is large enough to accommodate the required infrastructure to recharge the aircraft, either via plug-in charging or battery swapping. The objective is to determine the most cost-effective way in which a number of these aircraft can perform these flights and recharge at the hub-airport. The situation is the one of Bigoni by allowing flights of different lengths, and by allowing aircraft to service any number of flights as long as the schedule remains feasible.

This problem has been heuristically solved by splitting it up into two phases. In the first phase, the flights are given as input, and the model assigns aircraft to these flights and determined how (if necessary) the aircraft should be recharged between jobs. In the second phase of the model, the output of the first phase is used as input, and the minimum amount of chargers and batteries is determined to successfully service this schedule. Both phases of the model are MILP formulations with one difference, they are respectively in continuous time and discrete time. This approach has a number of benefits but also generates some problems which did not exist in the previous section. In the remainder of this section, the method is explained as well as its strengths and limitations.

Phase 1: assigning aircraft to flights and recharging methods

As mentioned before, the first phase of the model takes a fleet of electric aircraft and a set of flights as input and outputs the assignment of aircrafts to flights and recharging methods. This is optimized with respect to the cost of operating the required number of aircraft and performing the battery recharges (either via swaps or via plug-in chargers). The variables and parameters which are used are given in table 3.3.

This problem is an interaction between two parts: the assignment of electric aircraft to flight, and the management of the batteries which are on board of these aircraft. The x, y, and z variables account for the former part, while the others are used for the latter. In between flights, batteries can either be charged by swapping, charged by plugging in, or not charged at all. The objective of the program is to minimize the cost of operations, associated with: using an aircraft, charging via swaps, or charging via plug-in. The problem can be found in equations 3.6 and 3.7: the first five constraints manage the flight assignment, while the last five manage the battery charging.

The first five constraints are self explanatory: the first one makes sure that each flight is executed once, while the others link the x, y, and z variables. The purpose of the other constraints is finding a suitable charging method. The reason why this has to be done with so many complicated constraints is that in order to determine how long a battery has to charge with the various methods, one needs to know the state with which the battery completed its last flight, as well as the charge required to begin the next one. Details of these constraints are beyond this review but are discussed in [Pereira, 2019].

$$
\begin{aligned}
\text{min} \quad & \omega_1 = C_{AC} \sum_k y_k + \sum_{i,j} C^{BC} w^{BC}_{i,j} + C^{BS} w^{BS}_{i,j} && (3.6)\\
\text{s.t.} \quad & \sum_k x_{i,k} = 1 && \forall i \in I\\
& x_{jk} \geq \sum_i z_{ijk} \qquad \forall j \in I, k \in K\\
& x_{ik} \geq \sum_j z_{ijk} && \forall i \in I, k \in K\\
& z_{ijk} \geq x_{ik} + x_{jk} - 1 - \sum_{l \in M, i<l<j} z_{ilk} && \forall i,j \in I, k \in K\\
& |I| y_k \geq \sum_i x_{ik} && \forall k \in K\\
& (1 - \sum_{k \in V} z_{ijk}) H \geq t^a_i + t^{LU} + w^{BS}_{ij} t^{BS} + w^{BC}_{ij} t^C_{ij} - t^d_j && \forall i,j \in I\\
& w^{BS}_{ij} + w^{BC}_{ij} \leq \sum_{k \in V} z_{ijk} && \forall i,j \in I\\
& q^S_j = (1 - \sum_{k \in V} \sum_{j>i \in M} z_{ijk}) Q && \forall j \in I\\
& + \sum_{j>i \in M} \left[(w^{BS}_{ij} Q + w^{BC}_{ij})(q^R_j + SF) + \left(\sum_k z_{ijk} - w^{BC}_{ij} - w^{BS}_{ij} \right) q^E_i \right]\\
& q^E_i = q^S_i - q^R_i && \forall i \in I\\
& q^C_{ij} = w^{BC}_{ij} (q^R_j + SF - q^E_i) && \forall i,j \in I
\end{aligned}
$$

An aspect of this research which also sets it apart from the previous one is the fact that the recharging time of batteries is not a linear function of the charge which needs to be transferred into the battery. This is something which a MILP normally cannot handle. In order to solve this problem, the charge time profile has been approximated with a piecewise linear function. These two pieces intersect at 90% of the battery capacity, and recharging beyond 90% is ten times slower than recharging before 90%. The inclusion of this aspect further enlarges the MILP formulation, with six additional constraints for each pair of flights. These can be found in [Pereira, 2019].

Phase 2: determining the recharging schedule

In the second stage of the model the flight assignment output from the first phase and the recharging methods are taken as input, and the recharging times as well as the required number of spare batteries are calculated. Similar to the research from [Bigoni, 2018], time is split up in evenly sized intervals, and for each battery, it is determined what to do in this interval.

The objective function and constraints of this phase can be found in the following

Table 3.3: *Parameters and variables from a simplefied version of the first stage the model by Madelena Pereira [Pereira, 2019]. The two classes in the fourth column, B and C, denote that a variable can take on binary or continuous values respectively.*

Parameters		Variables		
notation	description	notation	class	description
C_{AC}	cost of using an aircraft	$x_{i,k}$	B	indicator when flight i is serviced by AC k
C_{BC}	cost of charging a battery plug-in	$z_{i,j,k}$	B	AC k services flight j immediately after i
C^{BS}	cost of swapping a battery	y_k	B	AC k is used
H	cost of not fulfilling a mission	w_{ij}^{BS}	B	battery swap between flights i and j
Q	battery capacity	w_{ij}^{BC}	B	battery recharge between flights i and j
SF	battery charge safety buffer	t_{ij}^{C}	C	charge time between flights i and j
		q_i^S/q_i^E	C	battery capacity at the start/end of flight i
		q_{ij}^{C}	C	battery capacity required between flights i and j

equations.

$$\min \quad \omega_2 = b + \sum_k r_k \tag{3.7}$$

$$\text{s.t.} \quad 1 = \sum_k \sum_{t=e_i}^{l_i} u_{ik}^t \qquad \forall i \in I \tag{3.8}$$

$$o_k^t = \sum_{i:e_i \leq t \leq l_i + d_i} \sum_{t'=t-d_i, t' \geq e_i}^{t} u_{ik}^{t'} \qquad \forall t \in T, k \in K$$

$$\sum_i \sum_t u_{ik}^t \leq H r_k \qquad \forall k \in K$$

$$b \geq \sum_{t' \leq t} swap^{t'} - \sum_{i \in C_{ES}} \sum_k \sum_{t'':e_i \leq t'' \leq t-d_i} u_{ik}^{t''} \qquad \forall t \in T$$

The first equation makes sure that each job is executed, while the second and third make sure that a charger does not work on more then one job at the time. The final constraint counts the number of batteries the airline would be short if there were no spares, and concludes that the number of spares the the largest of these values.

Assessing the model's strengths and shortcomings

The problem which is described in this Section is a generalization of the problem which was discussed in the last Section: the flights which are performed are of different lengths and can be assigned to any aircraft within the fleet. However, the algorithm with which

Table 3.4: *Parameters and variables from the second stage of the model by Madelena Pereira [Pereira, 2019]. The two classes in the fourth column, B and I, denote that a variable can take on binary or integer values respectively.*

Parameters		Variables		
notation	description	notation	class	description
e_n, l_n	release and due time of battery n	o_k^t	B	charger k is working at time t
d_n	charging time of battery n	r_k	B	charger k is used
C^{BS}	cost of swapping a battery	$u_{i,k}^t$	B	charger k starts working on job i at time t
		b	I	number of spare batteries

this problem is solved is very different from the one which was presented in the last Section. Combined, the two give different strengths and limitations to the model:

+ **The model can handle both plug-in charging and battery-swap charging.** In this respect, the two studies mentioned in this and the last Section differ from the study in the next section.

+ **It can compute the total needs of a fleet.** The difference between this study and the one conducted in this report is that it does not solve the problem from the perspective of the airport but from the airline. Because of this, the most efficient recharging schedule can be calculated without the need to solve a network problem.

+/- **Splitting the optimization problem in two stages results in lower computation times, but does in general not guarantee that the output is optimal.** If the two were solved simultaneously the number of variables would explode because of the fact that the time dimension is attached to each variable. The use of the two stages comes at the cost that a choice which is made to give an optimal solution in the first stage forces the algorithm to choose for a non-optimal solution in the second stage. This can be justified if the costs associated with using a non-optimal solution in the first stage are larger in magnitude then the costs of making a non-optimal decision in the second stage.

+/- **The objective function of the second phase is simplistic, but can be generalized easily.** In order to determine the option which is as cost effective as possible, it would seem a better option to take the weighted sum of the number of batteries and chargers.

3.2.3 Justin et al. : Battery Swapping and Recharging Optimization for Commercial Aviation (2020)

Thus far, the final team of researchers that has published on the subject is the faculty of Aerospace Engineering of the Georgia Institute of Technology. Justin et al. published a paper on how to approach the battery swapping problem at airports with the help of scheduling theory instead of linear programming [Justin et al., 2020]. Even though this method places some restrictions on the input of the model it solves the problem in polynomial time, which is a significant improvement.

The setting which this model assumes is not a generalization of the setting of [Bigoni, 2018] but a completely new one. The model assumes that the airlines operate in a network of airports, where battery swapping and recharging facilities are present at each airport. Aircraft arrive and depart each airport according to a fixed schedule, with a battery state of charge which is exactly equal to the safety buffer which allows them to divert to other airports in case of an emergency. Upon arrival, the aircraft swaps the old battery for a new one, which has just enough charge to complete the next flight (+safety buffer). At the airport, batteries are recharged in charging stations such that they can be used in another flight. Finally, preemption is allowed during the recharging process of a battery.

All of these assumptions have been made such that the recharging problem at one airport becomes completely independent of the problem at another airport, and thus can be solved separately. At each airport, this problem can be formulated in terms of three-field terminology scheduling theory from Subsection 3.1.1 as a $Pm|pmtn, r_j, grd|L_{max}$[1]:

- During the time window, n flights arrive (and depart again). For each flight, a battery is swapped which needs to be recharged. In addition, some spare batteries may be present at the airport which also need to be recharged. These represent the *jobs* which have to be fulfilled. For each spare battery which is present at the airport, a battery which arrives on an aircraft does not need to be recharged on the same day, but can instead be used as a spare battery on the next.
- The processing time p_j of each job is equal to the amount of time it takes to recharge the battery for the next flight. Additionally, each job has a due date d_j.
- A list of release dates r_j are given. The release dates are either 0, for spare batteries, or the arrival times of aircraft which swap batteries at the airport.
- Each battery can be preemptively charged at any of m identical parallel chargers.
- The objective is to find a minimum maximal delay schedule, thus L_{max} has to be minimized.

In [Justin et al., 2020], batteries are assigned to aircraft on a First-In-First-Out (FIFO) basis, which means that it is known on forehand which battery is assigned to which flight(s), and the problem reduces to $Pm|pmtn, r_j|L_{max}$. However, as shall be explored in the following subsection, using the FIFO policy does not always lead to an optimal result. After this subsection, I shall present the method which is used to solve the $Pm|pmtn, r_j|L_{max}$ problem.

Limitations of solving the problem with FIFO

The FIFO policy does generally not give an optimal solution to the scheduling problem, and there are a number of counter examples which support this claim.

[1] *grd*, short for *generalized release dates*, are analogous to the *gdd* property. However, instead of the requirement to finish a number of jobs before a specific time, the number of jobs which can be worked on after a specific time are given. The problem here is identical to the *gdd* version under time reversal, thus we shall keep using *gdd* when possible.

Firstly, consider the following instance with two sets of jobs. The arriving jobs are characterized by $r_1 = 0$ and $r_2 = 1$, and the departing jobs are characterized by $(d_1, p_1) = (2, 3)$ and $(d_2, p_2) = (3, 6)$. These jobs can be scheduled FIFO or reversed, which give the Gantt charts which can be seen in figure 3.1.

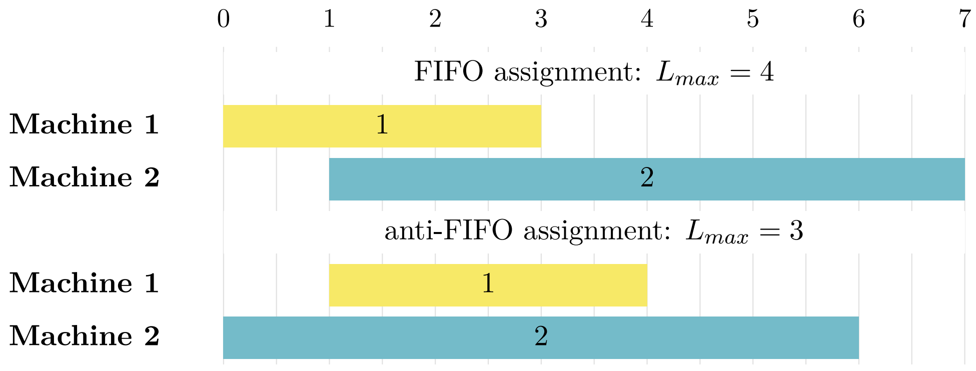


Figure 3.1: *Optimal schedule for the FIFO and anti-FIFO assignment, where the anti-FIFO assignment is able to generate a schedule with lower L_{max} value.*

One might think that FIFO is only guaranteed to be optimal in the case when there exists a schedule in which all aircraft are able to depart without delay, but this is also not true. Consider $r_1 = 0$, $r_1 = 1$, and $(d_1, p_1) = (2, 1)$ and $(d_2, p_2) = (6, 6)$. The FIFO and anti-FIFO optimal schedules give, assignments give the Gantt charts seen in figure 3.2.

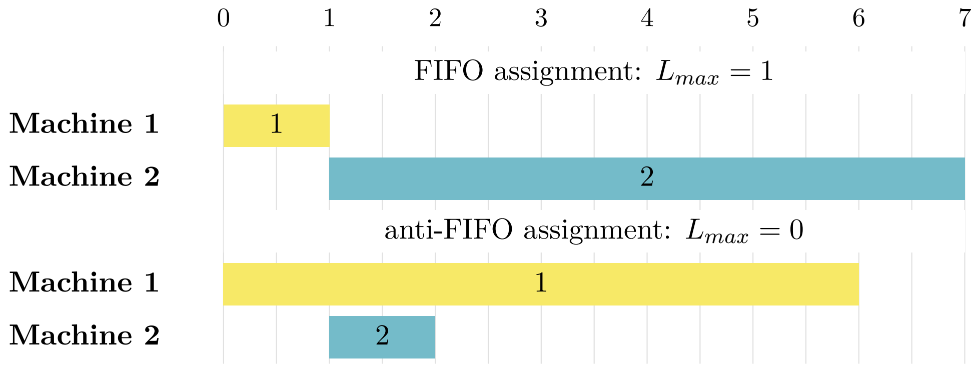


Figure 3.2: *Optimal schedule for the FIFO and anti-FIFO assignment, where the anti-FIFO assignment is able to generate a schedule with lower L_{max} value.*

In terms of scheduling theory, this problem can be described with generalized due dates. This can be seen by reversing the direction of the time in the problem: when this is done each job is described with a release date and a processing time, and there are a number of due dates which have to be satisfied. Transforming a solution between the two problems can also be done by reversing the time direction. Additionally, one needs to add the found value of L_{max} to the solution in order to make sure that no job is started before its release date. To the author's knowledge, finding an optimal solution to this problem in polynomial time is still an open problem.

Solving the Scheduling Problem with FIFO assignment

Justin et al. solved the scheduling problem in a number of phases. Firstly, the number of chargers and spare batteries is fixed and FIFO is applied, such that the problem reduces to $Pm|pmtn, r_j|L_{max}$. The maximum allowed delay for all flights is given by L^*, and it is determined if a schedule exists for this problem where $L_{max} \leq L^*$, which can be done in polynomial time. The book [Pinedo, 1995] presents an algorithm on how this can be done, and this is also used by Justin et al. We will not go into details on this algorithm here, but show how it is used to determine the size of the infrastructure. Two goals have been studied: minimum required peak power, and minimum infrastructure investment.

Firstly, the number of chargers and spare batteries are optimized such that the peak power is minimized. This is equivalent to minimizing the number of chargers at each airport. For this number of chargers, the minimum amount of spare batteries can be calculated, and this yields a feasible solution. These values can be found by iterating through possible combinations of spare batteries and chargers, starting with the theoretical maximum amount of spare batteries, the number of flights $|J|$, and the minimum number of chargers m_u:

$$m_u = \frac{E}{P\Delta T}, \tag{3.9}$$

where E is the total amount of electricity which is required for this operation, P is the maximum charging power, and ΔT is the time window (e.g. one day). If this combination yields a feasible charging schedule (using the algorithm from [Pinedo, 1995]), the optimal solution has been found. If not, a new charger is added and it is again determined if a feasible charging schedule exists. These steps are repeated until the smallest possible number of chargers has been found. This process can be seen in figure 3.3a.

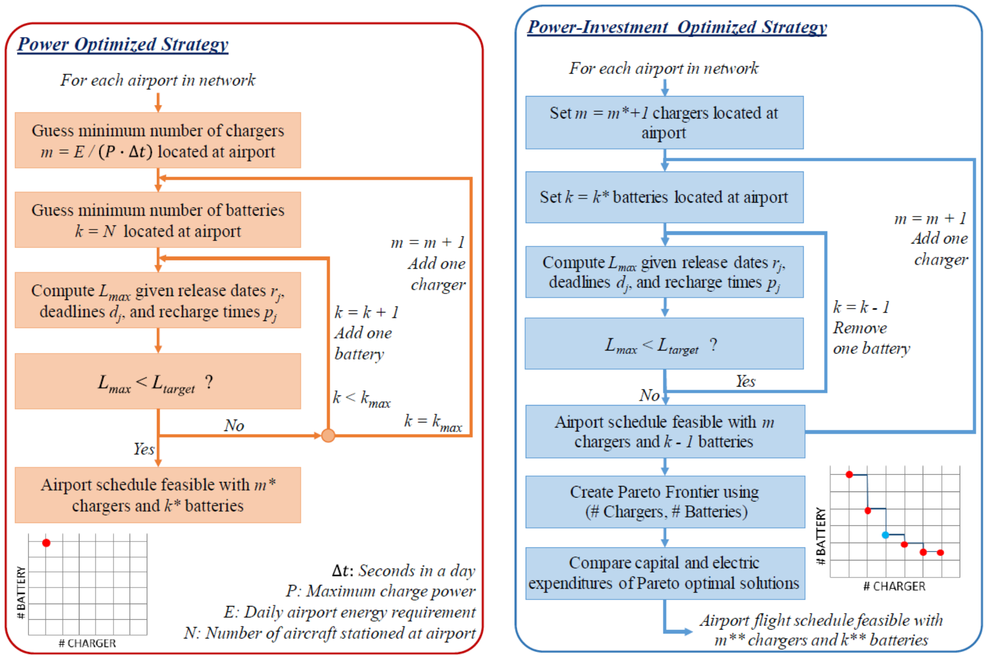
The second strategy aims at minimizing the operational costs of spare batteries and recharging stations. Fewer recharging stations imply a larger need for spare batteries and vice versa, there is a tradeoff between the two. The functions with respect to which we want to optimize this time is the weighted sum of the two numbers. In order to find the optimum, a similar procedure to the one of last paragraph is applied, but slightly modified.

The procedure to find the optimum is as follows: start from the position (number of spare batteries and chargers) which was optimal in the last paragraph, try to remove as many batteries as possible, add another charger, remove batteries again, and continue to iterate this procedure. This process terminates when the number of batteries has reached the lower bound (which has been defined in the last paragraph). This process has been summarized in figure 3.3b.

Assessing the model's strengths and shortcomings

This model used a completely different toolbox then the two others: the language and solution methods of scheduling theory. This has both advantages and disadvantages:

+ **The model optimizes the infrastructure needs for a network of airports**. Under the assumption that aircraft always arrive with a depleted battery, the infrastructure needs for every airport in a network can be optimized.



(a) *Flow-chart representation of the implementation of the process of finding the minimum amount of required chargers to operate an electric fleet.*

(b) *Flow-chart representation of the implementation of the process of finding the optimal weighted number of chargers and spare batteries.*

Figure 3.3: *Flow-chart representation for the two different strategies.*

+ **Using FIFO as an heuristic is a major improvement over charging the batteries on an as-needed basis.** The solution method has $\mathcal{O}(n^3)$ time complexity (independent of the number of chargers!) and can reduce the peak power or operation cost by over 60% for realistic instances, as shown in [Justin et al., 2020]. If FIFO is not used, the $Pm|grd, p_j, pmtn|L_{max}$ problem remains, which is still open.

- **However: theoretically, using FIFO to assign the batteries does not guaranty an optimal solution.** This has been shown with a counterexample.

- **Using the language of scheduling theory restrains the problem** There are a number of aspects which are either impossible or hard to describe with scheduling theory, or have never been combined before. Examples are TOU electricity pricing and cross-compatibility of different aircraft and batteries. Using a MILP formulation gives much more freedom to the user.

3.2.4 Research Gaps

All of these studies use a different approach and make different assumptions in order to solve the same problem. In this section, I'll explore the open questions which remain after going through all of these methods, and the research possibilities which they leave.

1. **The flight schedule has always been taken as input for the model.** The most flexible of the studies has been the one performed by Pereira, but even there the schedule of flights which have to be performed is still a part of the input of the model. However, during the stage when the airport needs to plan its infrastructure investment, the flight schedule which has to be serviced is unknown. Even if it were known, the model would need to consider every used schedule in order to determine the most demanding days.

2. **Flights are always assumed to be fixed.** However, in the acquired charging/fueling infrastructure at the airport is going to restrain the flight schedule. It may not be possible for flights to arrive and depart at the desired times, and a model which uses slot allocation could integrate this.

3. **Delays are not taken into account.** Operational delays may disrupt the charging schedule, and thus the optimal in

4. **How reasonable is the FIFO assignment in the heuristic of Justin et al.?** It has shown to outperform the as-needed heuristic, but how does it compare to others, such as the *Shortest Remaining Processing Time First*, and the assignment of the smallest release date to the smallest value of $d_j - p_j$? Additionally, is $Pm|grd, pmtn, r_j|L_{max}$ polynomially time solvable?

5. **Hydrogen-powered aircraft are not taken into consideration.** Hydrogen is a promising sustainable fuel source due to the ability of refuelling the aircraft in the conventional way, which is faster then recharging the batteries of an electric plane on-board. In addition to this, hydrogen powered planes are in a much later state of development than electric powered ones, and are expected to be able to carry more passengers over a larger distance.

CHAPTER 4

Infrastructure Optimization with a Fixed Flight Schedule: the AC Battery Swap and Charge Schedule problem

In this Chapter, we shall look at the first model which is developed for this thesis: one which optimizes the charging and fueling process of electric and hydrogen aircraft respectively, under the assumption that the flight schedule is a known part of the input. In the first section we shall introduce the situation as well as the model assumptions. After this, we shall introduce linear programming formulations for this in Sections 4.2 and 4.3, and some heuristics shall be provided in Section 4.4. This chapter is concluded with performance results from all algorithms, which are presented in Section 4.5.

4.1 Model setting and mathematical description

In this chapter we shall study the situation which is depicted in figure 4.1. Hydrogen and/or electric aircraft from a set of flights J have to be serviced at an airport during a period of time $T = [0, T_{end}]$. Flight $j \in J$ arrives at time r_j and needs to depart again by time d_j. On board, these aircraft carry (partially) depleted batteries and hydrogen tanks which need to be swapped for full ones before the aircraft departs. In the mean time, the depleted battery/tank is transported to a central charging station somewhere at or around the airport, where it takes p_j time to recharge/refuel it such that it can be used to supply another flight. The airport needs to find a charging/fueling schedule which allows all flights to depart on time.

In order to create such a schedule, the airport can acquire two types of infrastructure. First are the chargers and fueling points, of which $m \in \mathbb{N}$ are bought, which resupply the depleted containers with energy until they are at full capacity again. Secondly, there are the spare batteries/hydrogen tanks, of which $n^s \in \mathbb{N}$ are bought, which act as a buffer for the airport during peak hours. These assets help the airport, but also come at a cost of $\omega^{charger}$ and ω^{spare} respectively. See the table below:

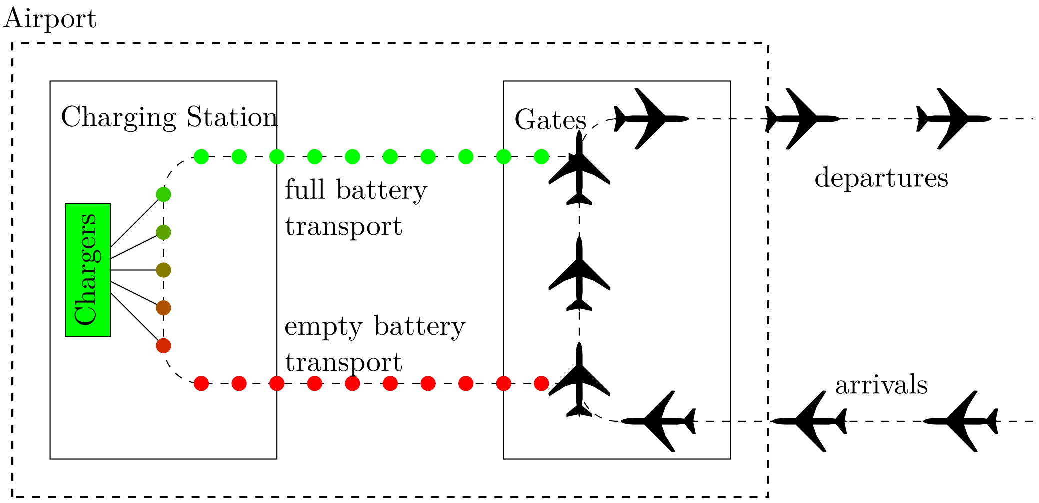


Figure 4.1: *Aircraft arrive and depart at the airport according to a fixed schedule. During their stay at the airport, the (partially) depleted batteries are removed from the aircraft and taken to the battery charging station for recharging. Aircraft are provided with (a) fully charged batteries (battery), which arrived on a different aircraft, and depart within the usual turnaround time.*

Asset	Number	Cost
Chargers	m	$m \times \omega^{charger}$
Spares	n^s	$n^s \times \omega^{spare}$

Our objective is to find the minimum cost infrastructure such that we can find a feasible charging/fueling schedule.

Remark. *Figure 4.1 shows only the service of (hybrid-)electric aircraft and not hydrogen aircraft. However, the treatment of hydrogen aircraft is completely analogous to that of electric aircraft in this model, where electricity is replaced by liquid hydrogen and batteries are replaced by cryogenic containers. Hereafter, the model is only discussed for electric aircraft (but also applies to hydrogen aircraft) unless explicitly stated otherwise.*

Now we can write down an exact formulation of the problem which we want to solve:

AC Battery Swap and Charge Schedule

Given: a set of flights J during a time interval T, where each is characterized by (r_j, d_j, p_j) and costs ω^{charge} and ω^{spare} associated with acquiring chargers and spare batteries.

Find: a minimum cost infrastructure such that a charging schedule for the batteries can be found where all flights can arrive and depart on time.

Assuming that:

1. Aircraft have to depart with a fully charged battery.
2. Spare batteries are fully charged at the start of T, and all batteries which have arrived at the airport need to be charged by the end of T.
3. There is an infinite storage buffer at the charging station.
4. Batteries can be placed or removed at a charger instantaneously.

Regarding the assumptions

Notice that first assumption implies that we can consider the problem of infrastructure sizing and recharge scheduling at each airport separately, instead of needing to consider an entire network of airport simultaneously. Alternatively, one can approach this problem with the assumption that the aircraft arrive with empty batteries and depart with just enough charge to complete the next flight, which again allows for the problem to be solved at each airport separately. These assumptions are theoretically identical, and solving the problem for one assumption means solving it for the other as well.

The second assumption contains two implications: we can repeat the charging schedule that we have created indefinitely, but as long as we return to the state at the start and end of T where all batteries which are present at the airport are fully charged. We need this assumption in order to ensure that the airport does not start of T with a full supply of spare batteries and ends T with non at all, thus causing problems for the period that follows T. This also implies that the best choices for T are an integer number of days.

Finally, the last assumption does not imply that batteries are moved to the charging station from the aircraft instantaneously. We can account for the time it takes to transport the batteries to the charging station by modifying the values of r_j and d_j.

Characterizing spare batteries and overnight stays

The final aspect of this model that we discuss is how we can characterize spare batteries. In our model, these can come in two forms: the regular spare batteries and the overnight stay batteries. The former type has already been discussed, these are the ones which the airport acquires to make sure that it can handle peak hour demand. We can use the same notation as we did with flights to denote a spare battery:

Definition 4.1. *A regular spare battery $j \in J$ is a battery which starts the period fully charged at the airport, and is thus available for each aircraft that arrives during this time. It is characterized by $r_j = p_j = 0$ and $d_j = T_{end}$. Acquiring this battery comes at a cost of ω^{spare}.*

where the deadline $d_j = T_{end}$ in this definition is included to reflect the fact that we need another battery to take the place of this spare battery at the start of the next interval.

The overnight stay batteries arrive at the airport on a flight near the end of T, but the aircraft itself departs near the start of the next period. This flight is characterized by $0 \leq d_j \ll r_j \leq T_{end}$. Since the second assumption of our problem required that all batteries need to be charged by the end of T, overnight stay batteries effectively turn into spare batteries for the next day. We can denote the number of these as n^n, and add these as "free" spare batteries:

Definition 4.2. *An overnight stay battery is a zero cost regular spare battery.*

4.2 The preemptive battery charging model

In this section, we are going to develop a mixed-integer linear programming model which can be used to solve the AC Battery Swap and Charge Schedule problem. Instead of using discrete time steps of equal length in order to calculate for each minute which battery should be charged by which charger, we are going to use variable sized time intervals. These time intervals are generated by the collection of release dates and due dates. This is illustrated in figure 4.2: all of these times are united in one large set, and for each pair of subsequent times, an interval is generated. In this way, no events occur in the interior of the intervals, only on the boundaries.

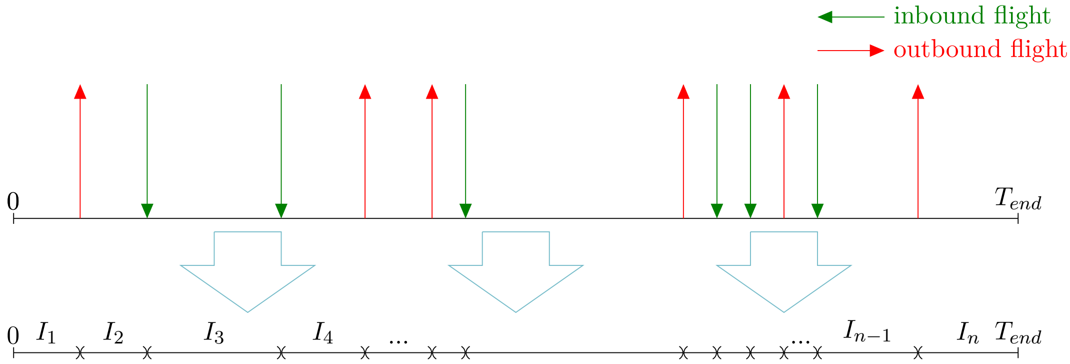


Figure 4.2: *The process of creating the set of intervals I from the arrival and departure times of the flights. No events occur in the interior of the intervals. The set of intervals I form a partition of the interval $[0, T_{end}]$*

The Mixed-Integer-Programming formulation

In order to write down the MILP formulation in a clean way, we introduce the following notation:

Sets

- J is the set of batteries which arrive at the airport during this period.
- D is the set of due dates during this period.
- I is the set of time intervals, indexed by i, such that $I_i = [s_i, e_i]$
- $J_i^I = \{j : s_i \geq r_j\}$ is the set of jobs which arrive before interval i starts.
- $I_j^J = \{i : s_i \geq r_j\}$ is the set of intervals which start after job j has arrived.
- $J_k^D = \{j : r_j \leq d_k\}$ is the set of jobs which arrive before due date k.
- $I_k^D = \{i : e_i \leq d_k\}$ is the set of intervals which are finished before due date k.

Parameters

- r_j is the release time of battery $j \in J$
- p_j is the processing time of battery $j \in J$
- d_k is the time of due date $k \in D$

- N_k is the cumulative demand for fully charged batteries up to and including due date d_k.

Variables

- $p_{ij} \in [0, \infty)$ is the time job j is charging during interval i.
- $y_{jk} \in \{0, 1\}$ is a binary value which is 1 if battery j is **not** charged yet before due date k.
- $m \in \mathbb{N}$ is the number of the to be acquired chargers
- $n^s \in \mathbb{N}$ is the number of the to be acquired spare batteries
- ω is the total cost of the infrastructure.

With this notation, the MILP model is given by:

$$\text{minimize } \omega = \omega^{spare} n^s + \omega^{charger} m \tag{4.1}$$

$$\text{subject to: } p_{ij} \leq |I_i| = e_i - s_i \qquad \forall i, j \in J_i^I \tag{4.2}$$

$$\sum_{j \in J_i^I} p_{ij} \leq m|I_i| \qquad \forall i \tag{4.3}$$

$$\sum_{i \in I_j^J} p_{ij} = p_j \qquad \forall j \tag{4.4}$$

$$y_{jk} \geq s \left[p_j - \sum_{i \in I_k} p_{ij} \right] \qquad \forall k, j \in J_k^D \tag{4.5}$$

$$\sum_{j \in J_k^K} (1 - y_{jk}) \geq N_k - n^n - n^s \qquad \forall d_k \in D \tag{4.6}$$

$$p_{ij} \geq 0, y_{jk} \in \{0, 1\}, n^s, m \in \mathbb{N} \qquad \forall i, j, k \tag{4.7}$$

The first constraint ensures that during each time interval, a battery can charge no longer than the length of this interval. In combination with the second constraint, this assures that no jobs are processed on two machines simultaneously. The third constraint ensures that each job is finished, and the fourth that the y variable can only be set to 0 once it is finished (s is a very small number). Finally, when there are $n^n + n^s$ batteries at the airport, the number of required batteries at each deadline is given by $N_k - n^n - n^s$, and constraint (4.6) makes sure that at least this number of batteries is recharged by this moment.

4.2.1 Example

We shall now take a look at a simple example, but which contains all the aspects of the problem. The flights are provided in Table 4.1, we consider a time period of one day (6 AM to 6 AM), and we have one flight which has an overnight stay (such that $n^n = 1$). The costs of acquiring and operating a charger are equal to that of a spare battery, such that $\omega^{charger} = \omega^{spare} = 1$.

The optimal objective value $\omega = 2$ can be reached with two different configurations: two chargers, or one charger and one spare battery, and the corresponding schedules are on display in Figure 4.3. These schedules are indeed optimal since first, we need

Table 4.1: *Input data for a simple example of the battery swap and recharging problem.*

Battery number j	release date r_j	charging time p_j	deadline d_j
1	8	3	10
2	9	1	11
3	24	5	6

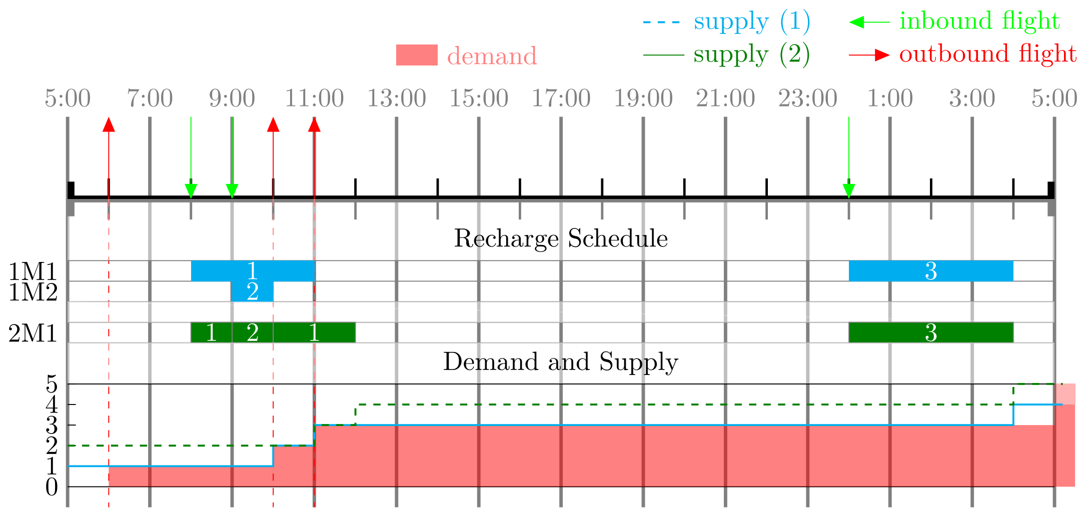


Figure 4.3: *Two optimal recharge schedules, given the data from table 4.1, for the battery swap and recharge problem with $\omega^{spare} = \omega^{charger} = 1$. The first solution, corresponding with machines 1M1, 1M2, and the continuous supply line, has $m = 2$ and $n^s = 0$. The second solution, corresponding with machine 2M1 and the green, dashed, line, has $m = 1$ and $n^s = 1$.*

at least one charger, and second, there does not exist a feasible schedule with only one charger and zero spare batteries.

At the top of the figure, we can see the times the batteries arrive at the depot from an inbound flight (given by green down-pointing arrows), and deadlines for a battery to depart with an outbound flight (red, up-pointing arrows). Below this, we can see two Gantt charts with the recharge schedules. Machines 1M1 and 1M2 correspond to the first solution, while machine 2M1 corresponds with the second. Finally, at the bottom of the figure, we have displayed the demand for and supply of fully charged batteries at each time during the day. The demand is given by N_k (from equation (4.6)) before 6:00, and by the total number of batteries which have been present at the airport at 6:00 (from equation (**??**). The supply of full batteries is given by the sum of the number of batteries which stay during the night(in this case, 1), the number of spare batteries, and the number of recharged batteries. The blue, continuous, line corresponds to the first solution, while the green, dashed, line corresponds to the second solution.

4.2.2 Improvements and expansions to the model

In this subsection, we shall discuss a number of options in which the model can be expanded in order to make it more realistic. These expansions will be used in the case study from Chapter 6.

Different types of batteries

It is not unthinkable that different types of aircraft are going to use different types of batteries. When these batteries cannot share the same chargers, the problem completely splits up in multiple sub-problems which can be solved separately. However, if the batteries share a set of chargers, the problems become intertwined and need to be solved collectively.

In order to denote the fact that batteries are from different groups, we use the following notation: let B be the set of battery (or hydrogen tank) types, where a single type is denoted by b. Let J_b^B denote the jobs which are of battery type b and D_b^B denote the deadlines which are of battery type b. From the last set, we deduce the parameters $N_{k,b}$, which denote the number of batteries which need to have been fully charged up to and including deadline d_k.

We assume that all batteries share the same set of chargers. In this case, we need to monitor the number of charged batteries for each type at all relevant deadlines. This can be done by replacing equation 4.6 by the following set of equations:

$$\sum_{j \in J_k \cap J_b^B} 1 - y_{jk} \geq N_{k,b} - n_b^n - n_b^s \qquad \forall d_k \in D, b \in B \tag{4.8}$$

where n_b^n and n_b^s are respectively the overnight staying and spare batteries of type b (analogous to the base case, the first one is a parameter, the second one a variable). Additionally, each type of battery comes with its own costs $\omega_{s,b}$, such that the objective function of equation (4.1) is modified to:

$$\omega = \sum_b \omega_b^{spare} n_b^s + \omega^{charger} m \tag{4.9}$$

If, additionally, the battery types have a different charging power level, this can be accounted for by modifying equation (4.3). Let s_b be the charging power for battery type b, and s be the maximum charging power of a charger. Equation (4.3) is now modified to:

$$\sum_{b \in B} s_b \sum_{j \in J_i^I \cap J_b^B} p_{ij} \leq s \cdot m \cdot |I_i| \qquad \forall i \in I \tag{4.10}$$

Variation in energy prices

Because energy companies prefer to spread the energy demand throughout the day as much as possible, they often employ a pricing construction. This consists of two components: firstly, the time of usage of the electricity determines the price which is paid for one kWh, and secondly, the consumers are also charged for the peak power which is drawn from the grid. This is detailed in Section 3.1.2 in the literature review.

We can integrate this in the model with only minor modifications. Firstly, assume that the electricity price is a piecewise constant function of time, and split the intervals in I until the electricity price is constant during each interval.

Parameters

- ω_i^{elec}: the electricity cost of using one charger during the entire length of interval i.

- ω_{peak}^{elec}: the cost associated with the electricity peak power.

Variables We only need one extra variable: P_{peak}, the peak electric power drawn from the grid.

The following terms are added to the objective function:

$$\sum_i \omega_i^{elec} \sum_j p_{ij} + \omega_{peak}^{elec} P_{peak} \tag{4.11}$$

And the following constraint, which defines the peak power:

$$P_{peak} \geq \sum_j p_{ij}/|I_i| \qquad \forall i \; inI \tag{4.12}$$

On-site energy production and storage

As an alternative to drawing (all) power from the grid, the airport can choose to produce and store energy on-site. These two infrastructures, production and storage, also come at a cost:

$$\omega^{produce} E_{max} + \omega^{store} S_{max} \tag{4.13}$$

where E_{max} and S_{max} denote respectively the maximum production and storage capacity of the installation. We optimize the cost of this infrastructure by determining the demand for both. First, define the total charging time at interval i, equivalent to the energy demand in that interval, as:

$$E_i^{dem} = \sum_{j \in J_i} p_{ij}, \tag{4.14}$$

For each interval, we need two additional variables which indicate how much electricity is produced and stored during this interval, denoted by E_i^{prod} and S_i. With these variables we can define the required constraints to determine the required size of the installation:

$$0 \leq S_i \leq S_{max} \qquad \forall i \qquad \text{storage sizing} \tag{4.15}$$

$$S_{i+1} = S_i + E_i^{prod} - E_i^{dem} \qquad \forall i = 0...i_{max} - 1 \qquad \text{calculating the next storage} \tag{4.16}$$

$$S_0 = S_{i_{max}} + E_{i_{max}}^{prod} - E_{i_{max}}^{dem} \qquad \text{periodicity} \tag{4.17}$$

$$E_i^{prod} \leq E_{max} \times |I_i| \qquad \forall i \qquad \text{peak power} \tag{4.18}$$

4.3 The non-preemptive battery charging model

In this section, we are going to solve the version of the AC BATTERY SWAP AND CHARGE SCHEDULE problem under the assumption that charging preemption is not allowed. There are reasons, especially for the process of refueling liquid hydrogen tanks, why a non-preemptive solution may be more desirable. Firstly, allowing for preemptions can over-complicate the logistical procedures at the charging station. Or, because of thermal constraints, the preemption of refueling a liquid hydrogen tank may not be possible in the first place.

AC Battery Swap and Non-Preemptive Charge Schedule

Given: a set of flights J during a time interval T, where each is characterized by (r_j, d_j, p_j) and costs ω^{charge} and ω^{spare} associated with acquiring chargers and spare batteries.
Solve: the AC Battery Swap and Charge Schedule problem.
Assuming: charging a battery cannot be preempted.

Remark. *A non-preemptive schedule might also be a optimal solution even when preemptions are allowed. An example of this can be found in Subsection 4.2.1, where the first solution does not contain any preemptions.*

Unfortunately, the number of required constraints and variables for the non-preemptive version of the problem is much larger than in the preemptive version. Because of the extra constraint, using the framework of last section would mean that we also need to account for which batteries are worked on at the start and end of each interval. In this section, we are going to present a solution that uses equally sized discrete time intervals to solve this problem.

In order to write the problem as an ILP, we shall use the same notation as in the last section. However, due to the slightly different problem structure, some new sets, parameters, and variables are used to describe this problem:
New sets

- T: the discretized set of all time instances, seperated by Δt. $T = \{0, \Delta t, 2\Delta t, ..., T_{end}\}$.
- T_j: the set of all time instances from T during which the battery from flight j can start charging: $T_j = T \cap [r_j, T_{end}]$.

New parameters

- t_k is the time of due date $k \in D$
- Dem_k cumulative demand for batteries at due date $k \in D$

New variables:

- $St_j^t \in \{0,1\}$ is equal to 1 if work on job j starts at time t. This variable exists if and only if $t \in T_j$.
- $Sup^t \in \mathbb{N}$ is the number of batteries which are charged before or at time t.

And we use the following objective function and constraints to model the problem:

$$\omega = \omega^{spare} n^s + \omega^{charger} m \tag{4.19}$$

$$\sum_{t \in T_j} St_j^t = 1 \qquad \forall j \tag{4.20}$$

$$\sum_j \sum_{t' \in (t-p_j, t]} S_j^{t'} \leq m \qquad \forall t \tag{4.21}$$

$$Sup^{t_k} = \sum_j \sum_{t' \leq t_k - p_j} St_j^{t'} + n^s + n^n \qquad \forall k \tag{4.22}$$

$$Sup^{t_k} \geq Dem_k \qquad \forall k \tag{4.23}$$

$$St_j^t \in \{0,1\}, \quad n^s, m \in \mathbb{N} \qquad \forall t, j$$

where equation (4.20) ensures that all batteries are charged by T_{end}, and equation (4.21) makes sure that no more batteries are charged at each time than the number of chargers. Equation (4.22) counts the cumulative number of usable batteries, given by the sum of all charged batteries, and the number of spares and overnight stays. Finally, equation (4.23) makes sure that the supply for charged batteries is always greater or equal to the demand.

The size of this problem does now not only depend on the number of charging jobs, but on the time partition T as well: the size is bound from above by $|J||T| + |J| + 2$ variables[1], and $2|J| + |T|$ constraints. This is the advantage of using this model: it grows linearly with the number of jobs, whereas in the last model adding new jobs also implied creating new intervals, and the problem grew quadratically.

Finally, all the expansions for the regular AC Battery Swap & Charge Schedule problem from Subsection 4.2.2 translate naturally to the non-preemptive version.

4.4 Heuristics

Solving this problem with the MILP models presented in the last section has exponential time complexity. In order to solve this problem faster, four heuristics are presented in this section: two of which follow a First-In-First-Out (FIFO) principle, as in [Justin et al., 2020], the other two use a Shortest-Processing-Time (SPT) principle.

FIFO heuristics

In [Justin et al., 2020], the conclusion is drawn that in order to generate the optimal schedule, the FIFO principle can be used: batteries are assigned to departing flights in order with which they arrived. This reduces the problem to an $P|r_j, pmtn|L_{max}$ scheduling problem (see [Pinedo, 1995] for details), which can be solved in polynomial time. In the case where the recharge times of the batteries are the same ($p_j = p$) this would be true, but it does not hold in general (see Section 3.2.3 for details and an example). Nevertheless, this method may work well as a heuristic approach, and we shall refer to this method as FIFO(R), as it sorts the flights by their arrival times r

In order to improve on this method, we introduce a new heuristic: FIFO(RP). In this method, batteries are assigned to departing flights in ascending order of the sum of their release dates plus processing times ($r + p$). The battery with smallest $r_j + p_j$ is assigned to the first departing flight, etcetera. This is still a heuristic, but since it accounts for the different charging times, it is expected to outperform the FIFO(R) heuristic.

[1] If we assume an average arrival time of a/T_{end}, the number of variables is roughly $(1 - a)|J||T| + |J| + 2$.

SPT heuristics

The SPT heuristics depend on a different principle: instead of assigning batteries to departing flights, which batteries are charged at each moment depends on their current state of charge.

The preemptive version of this heuristic is the shortest remaining processing time, or SRTP, rule. At each moment in time, the m batteries with the shortest remaining processing time are charged. This way, the algorithm tends to generate a large number of charged batteries on the short run, but procrastinates the more difficult tasks.

The non-preemptive version (and the only non-preemptive heuristic here) is the shortest available processing time, SAPT, rule. It is completely similar to the SRPT rule, but without allowing for preemptions.

An overview of all solution methods and heuristics for the AC Slot Allocation and Battery Charge Schedule problem are displayed in Figure 4.4.

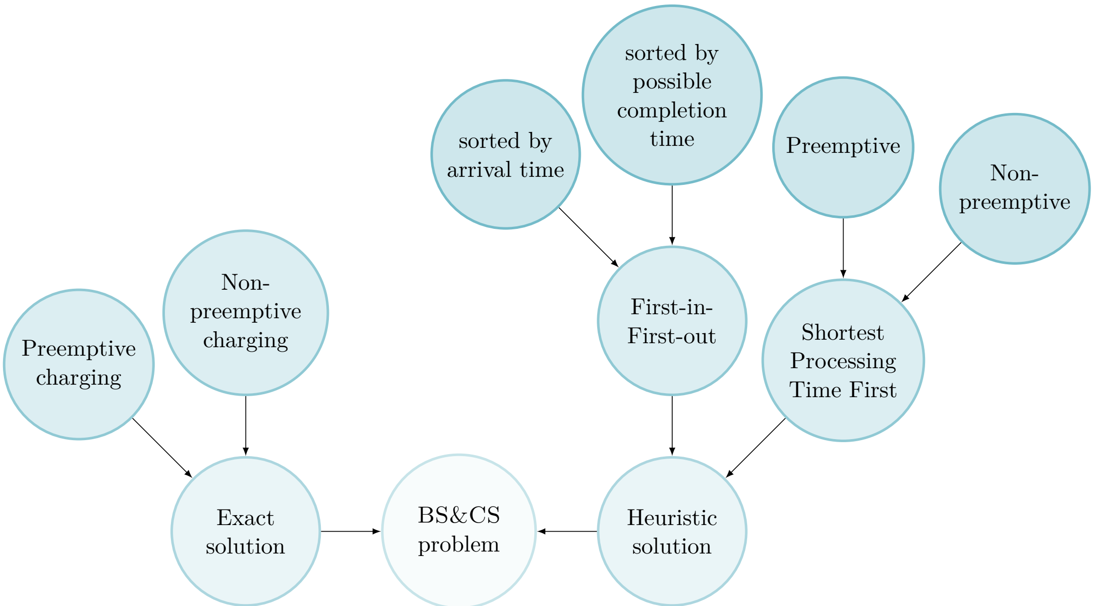


Figure 4.4: *Binary search algorithm to solve the AC Slot Allocation and Battery Charge Schedule problem with $\alpha = 1$, using the solutions to the AC BS&CS problem from last chapter. We are satisfied with ω if and only if the modified flight schedule is feasible for the BS&CS problem, and is infeasible for any value lower than ω.*

4.5 Experiments on an artificial flight schedule

In this section, we are going to use the models and heuristics which have been presented in this chapter to optimize the infrastructural investment and swapping schedule for artificially created flight schedules. The purpose of this is to analyse and compare the performance of the exact algorithms with each other (e.g. preemptive against non-preemptive), as well as measuring the performance of the various heuristics which have been introduced in Section 4.4.

We have created the datasets which have been used with the following parameters. A scheduling window of one day, $T = [0, 24]$ has been used. The arrival, processing and departure times of the batteries are identically uniformly distributed:

$$R_j \sim Unif[0, 18],$$
$$P_j \sim Unif[1, 3],$$
$$D_j \sim R_j + Unif[0.5, 1],$$

where hours are used as unit of time. Note that flights with overnight stays are not present in this dataset. Instances were either solved by self-programmed heuristics or with GUROBI, on a fourth generation Microsoft Surface. In the objective function, $\omega^{charger} = \omega^{spare} = 1$.

4.5.1 Scheduling battery recharges while allowing for preemptions (Section 4.2)

Firstly we are going to study the performance of the linear programming model from Section 4.2, which finds the optimal schedule when preemptions in the recharging process are allowed. The algorithm has been run on instances with up to sixty flights (batteries) each and was repeated 25 times for each number of flights. The time which it took to create the instance, as well as solve it to optimality, were stored. One can see the results in figure 4.5, where the times are presented as boxplots. Two are shown for each number of flights: the blue one, corresponding to the time to create the instance, and the red one, corresponding with the time to solve it. For example, one can see that the median time to solve an instance of fifty flights is somewhere at 0.2 seconds, and that the median time to create an instance of 10 flights is about 0.009 seconds. It was found that instances with up to 150 flights can be optimized within a reasonable time (5 minutes).

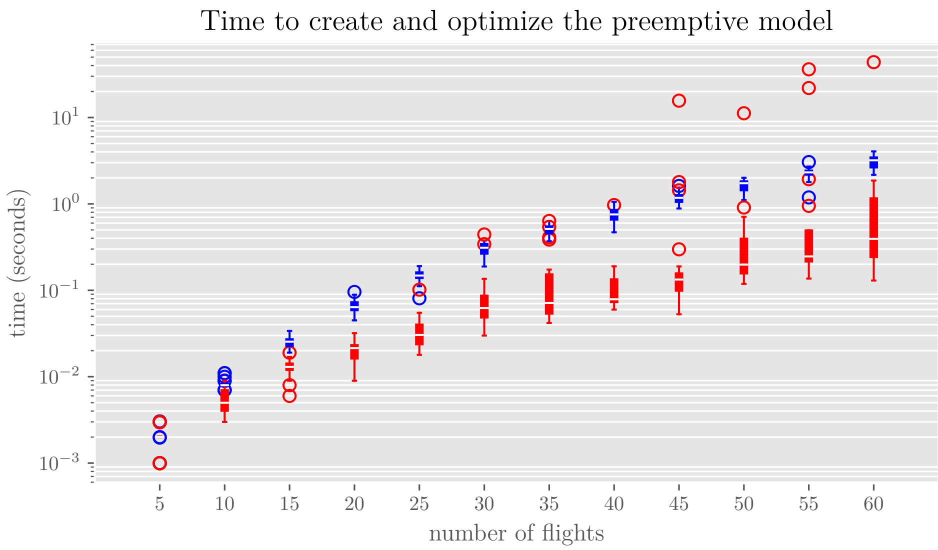


Figure 4.5: *Boxplots of the time it takes to create and solve equations 4.1 to ?? for instance sizes up to sixty flights. The blue and red boxplots respectively corresponds to the time it takes to create and solve the instances. The experiment was repeated 25 times for each number of flights with different samples.*

The performance of heuristics for preemptive schedules

Secondly, we can compare the optimal results generated by the LP model to the results generated by the heuristics. These are presented in figures 4.6, 4.8, for one instance per number of flights, and in 4.7, for multiple instances per number of flights.

In figure 4.6, instances are created with up to eighty flights, and are solved with the algorithm which finds the optimal solution and with the three heuristics. The values of the objective function which these algorithms found have been plotted, the blue dashed line indicates the optimal value. The green, orange, and purple points represent respectively the FIFO(R), FIFO(RP), and the SRPT heuristic. One can see that the FIFO(RP) heuristic generates the best results, it yields the optimal result in almost all of the instances, followed by the SRPT heuristic and the *Georgia* heuristic in the third place.

Figure 4.7 shows the time it takes to compute the results. Generally, calculating the optimal value takes longest, the FIFO heuristics are fastest, and the SRPT heuristic is somewhere between the two[2].

Figure 4.8 shows what happens to the objective function value if this experiment is repeated 30 times. For each number of flights, thirty instances are created and solved using the various heuristics. The difference between the value which they found and the optimal value is calculated, and the fraction of the times when this takes on specific values is plotted: green indicates that the optimal solution has been found, yellow indicates a gap of 1 to the optimal solution, orange a gap of 2 or 3, and red a gap of 4

[2] it should be the fastest in theory, that it is not might be due to the fact that I am an average programmer and can't beat a team of highly skilled GUROBI employees

or more. Again, the FIFO(RP) heuristic yields the best results, more then 30% of the cases are solved optimally even for instances with 75 flights.

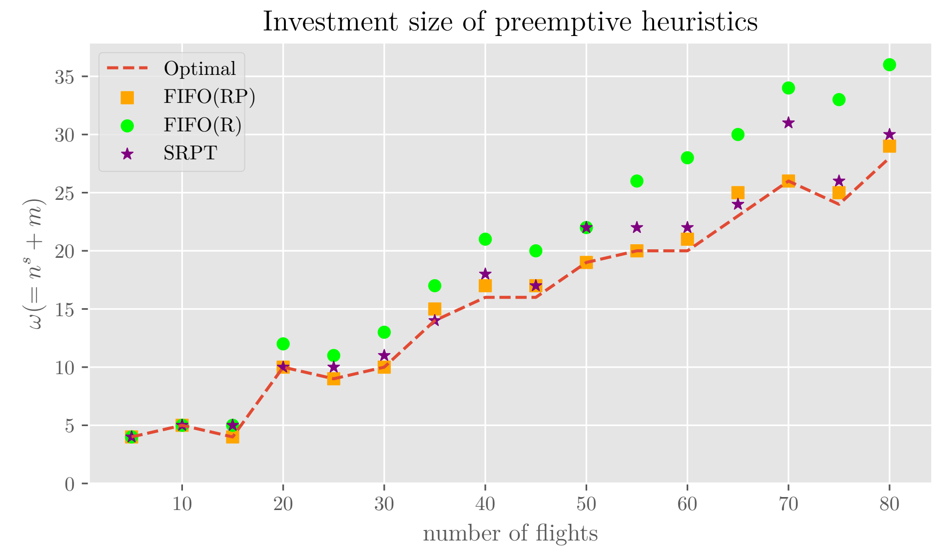


Figure 4.6: *Objective function value of instances with up to 80 flights for the optimal schedule and the ones obtained with the three heuristics. We have taken* $\omega^{spare} = \omega^{charger} = 1$.

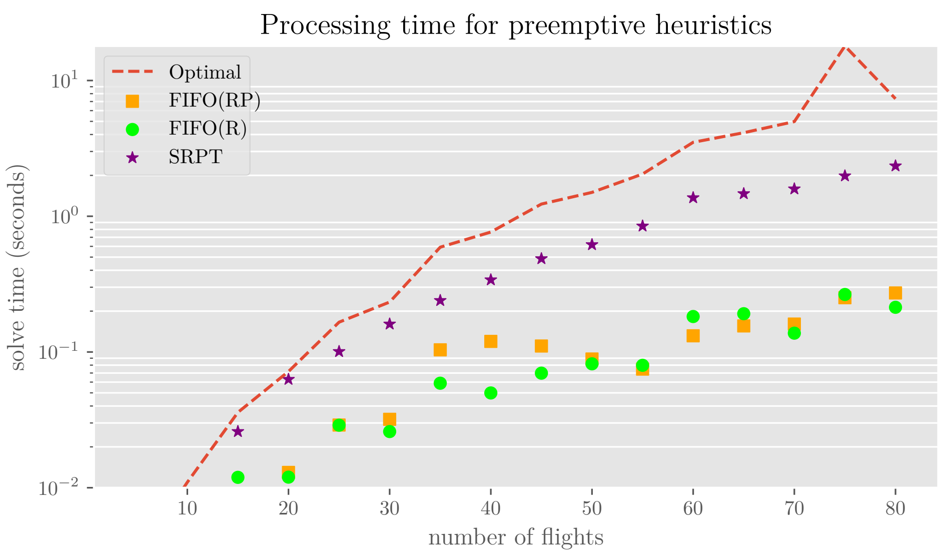


Figure 4.7: *Time it takes to solve instances with up to 80 flights to optimality and with the three heuristics. We have taken* $\omega^{spare} = \omega^{charger} = 1$.

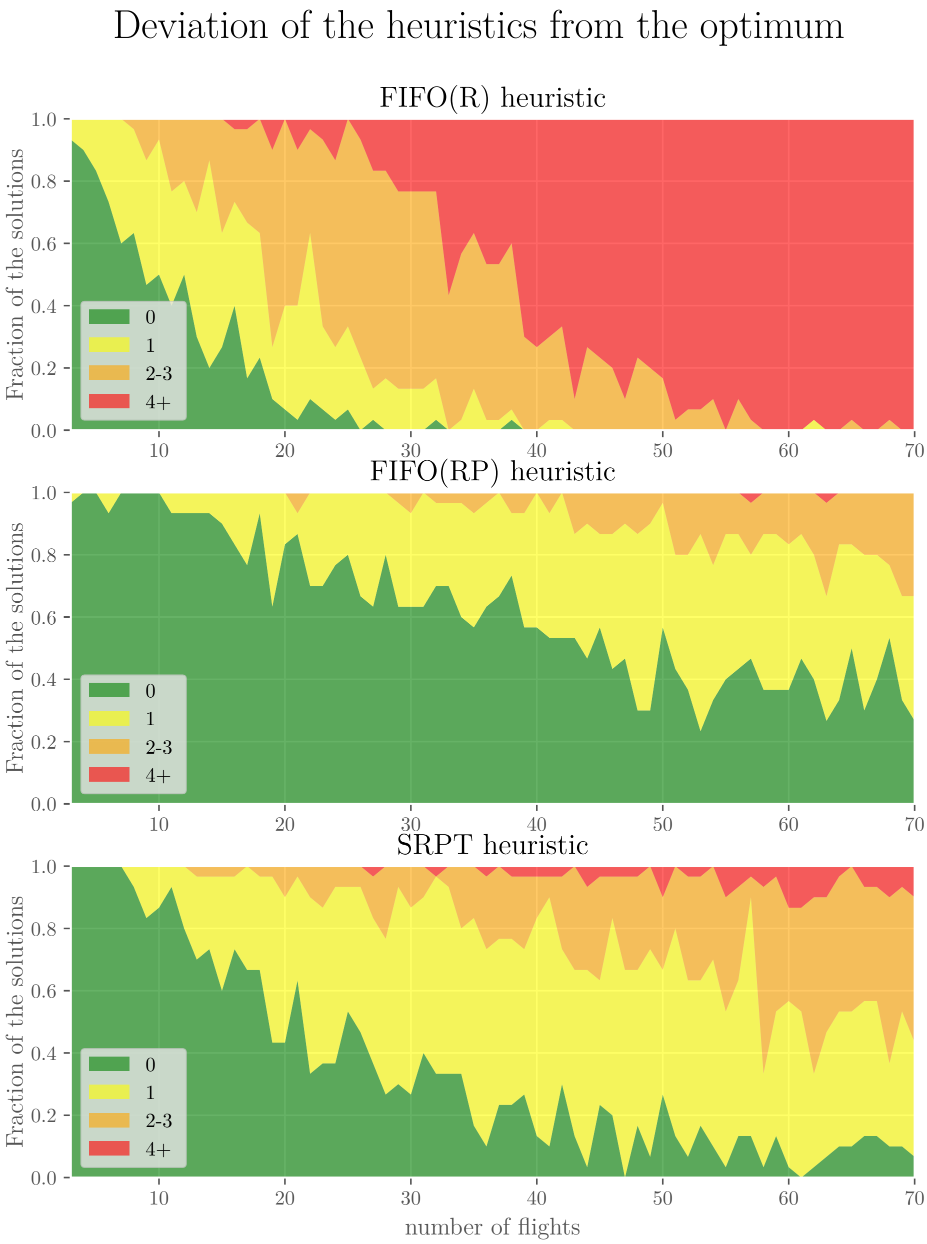


Figure 4.8: *Difference between the objective function value of the optimal schedule and the schedules obtained by the heuristics for instances with up to 75 flights. The experiment was repeated 30 times, and for each heuristic the fraction of the instances is displayed which are solved optimally, 1 from the optimum, 2 or 3 from the optimum, and 4 or more from the optimum. We have taken* $\omega_s = \omega_c = 1$*, and* $\omega_e = 0$.

4.5.2 Comparing preemptive to non-preemptive charging

Using the same dataset as before, we can analyse the trade-off which has to be made when choosing to recharge/refuel without preemptions. The results of this have been graphed in figures 4.9 and 4.10. In the first figure, four different algorithms were used to minimize the sum of the spare batteries and chargers: solved to optimality with and without preemptions, and approximated using the (preemptive) SRPT and (non-preemptive) SAPT algorithm. Since it takes much more time to solve the non-preemptive case to optimality, cases with only up to 60 flights were considered. The result is that using a non-preemptive schedule only in a single case (at 30 flights) requires a larger infrastructure investment, in all other cases the required infrastructure is identical to the preemptive solution. The SRPT and SAPT solutions remain most of the time within 1 or 2 extra chargers/batteries, with the SAPT algorithm giving only slightly worse solutions then the SRPT algorithm (at most one extra battery/charger).

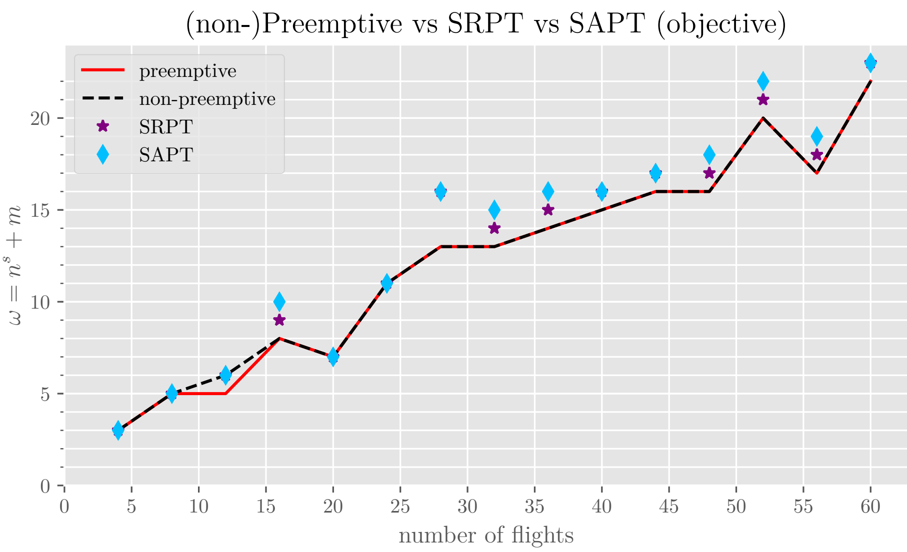


Figure 4.9: *Investment size of instances up to 60 flights when solved: to optimality with and without allowing for preemptions, and with the two SPT heuristics.*

Because the implementation for discrete time algorithm which finds the optimal solution when preemptions are not allowed is unable to handle instances of 100 batteries and up within a reasonable amount of time, we cannot compare the two for more flights per day. Regardless, we can still compare the two indirectly by using the fact that the SAPT algorithm finds a non-preemptive solutions and therefore provides an upper bound for the optimal non-preemptive solution. With the SAPT, SRPT and algorithm for finding the optimal preemptive solution, we can ramp up the number of flights (or batteries) to 170, and this results are given in figure 4.10. The SAPT generates comparable results to the SRPT algorithm. So for sure, we can say that the optimal non-preemptive result will stay within 5% of the preemptive solution. And probably, given that the non-preemptive solution followed the preemptive solution so well in figure 4.9, it performs better than that.

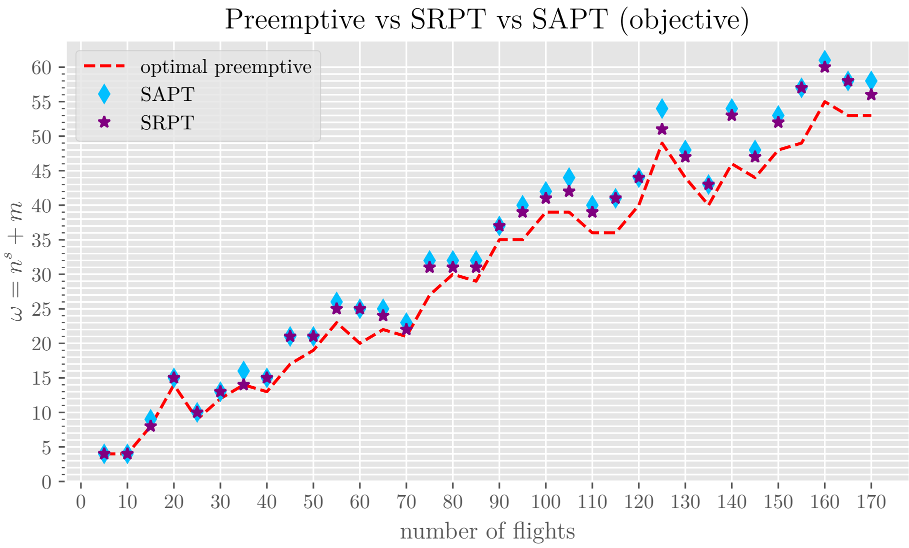


Figure 4.10: *Investment size (sum of spare batteries and chargers) of instances up to 170 flights when solved to optimality when allowing preemptions and with the SPT heuristics.*

CHAPTER 5

Infrastructure Optimization with a Flexible Flight Schedule: the AC Slot Allocation and Battery Charge Schedule Problem

In the last chapter, we considered flight schedules which are fixed in place. Although this simplified the problem and allowed us to write down two linear programs which are relatively straightforward, it also restricts the applications of this research. In this chapter, we shall investigate what happens if we do not restrict ourselves to fixed flight schedules. First, an updated problem description is given and its parameters are explained. Next, the linear programming formulations for this setting are introduced, together with two heuristics. Finally, we shall see how these algorithms perform when applied to an artificial dataset similar to the one from the last chapter.

5.1 Model setting and mathematical description

In this chapter, we shall study the situation which is depicted in figure 5.1, a generalization of figure 4.1 on page 34. The part of the research which has been discussed in last chapter can be found on the bottom half of the figure. Aircraft still arrive and depart at the airport according to a flight timetable, where they swap their depleted batteries (or liquid hydrogen canisters) for fully charged (filled) ones. The batteries which are disembarked from the aircraft are taken to a recharging facility, which is located somewhere at the airport, where they are charged. Once this is complete, the battery is assigned to a different aircraft.

However, instead of assuming that r_j and d_j represent the actual flight schedule, they are now the *desired* flight schedule. The actual flight schedule, which should deviate as less as possible from the desired one, is going to be determined via the process of slot coordination, as described in Subsection 2.2.2. For electric and hydrogen aircraft, there is an interaction between these two problems: a battery only becomes available for charging once the aircraft carrying it has arrived, and a departure can only be performed once a fully charged battery is available. By fixing the flight schedule in last

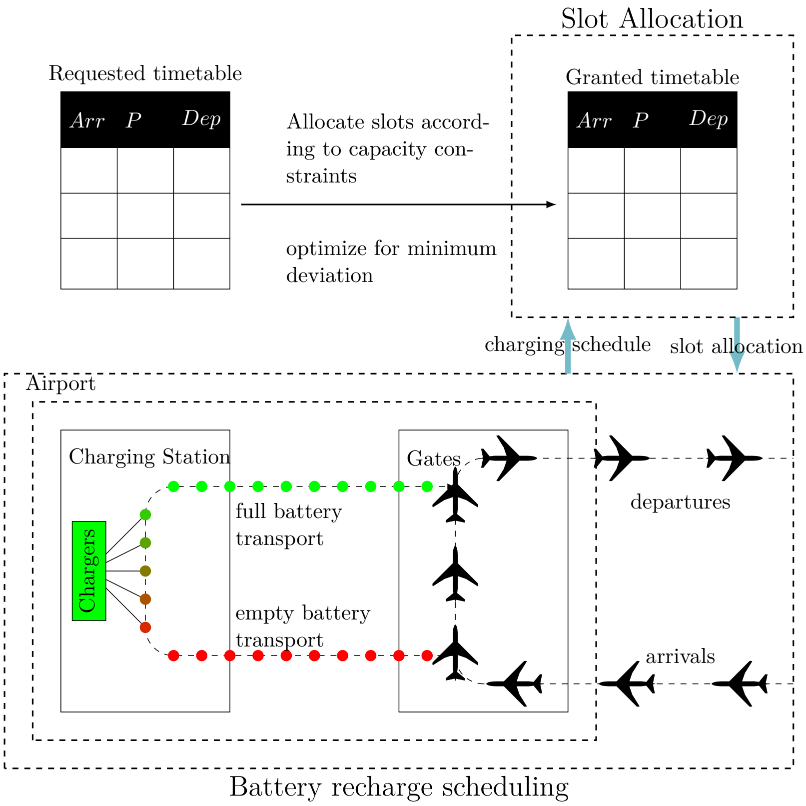


Figure 5.1: *Electric aircraft recharging with swaps and slot allocation model at the tactical level.*

chapter we eliminated one problem, but by including slot allocation, more resource efficient schedules are possible. Using the same notation as in last chapter, we formulate the following problem:

Slot Allocation and Battery Charge Schedule

Given: a desired flight schedule J during a time interval T, where each is characterized by (r_j, d_j, p_j) and infrastructure m and n^s.
Find: a feasible battery charging schedule and slot allocation which deviates as little as possible from the desired flight schedule.
Assuming that:

1. Aircraft have to depart with a fully charged battery.
2. Spare batteries are fully charged at the start of T, and all batteries which have arrived at the airport need to be charged by the end of T.
3. There is enough gate capacity to accommodate all flights, but runway capacity can be limited.
4. There is an infinite storage buffer at the charging station.
5. Batteries can be placed or removed at a charger instantaneously.

There is some ambiguity in the model as to how to define the deviation from a flight schedule to the desired one. In this report, we shall use a weighted sum of the average and the maximum time between the requested and granted slots, but many more possibilities for this can be explore.

All of the assumptions which were made for the AC Battery Swap & Charging

Schedule problem still hold, except for the assumption that the flight schedule is fixed.

5.2 The Preemptive Combined Slot Allocation and Battery Charging Model

In this section, we are going to work towards a model which can describe the interaction between the slot allocation and the battery charging model and is able to solve both problems simultaneously. We shall do this from the perspective of the tactical phase where we know the infrastructure that is available and the demand which has to be serviced. For the battery charging model, we shall use the groundwork which was laid in last chapter, and we will stick to the notation from that chapter as much as possible. But to see how the two aspects of the model are intertwined, lets first take a look at a simplified slot allocation model.

5.2.1 A slot-allocation model formulation

The airport in question is presented with a request for landing and departing slots by airlines. Ideally, all slots would be granted, but because of capacity constraints at the airport, that could imply that the airport becomes overloaded during certain moments. The slot coordinator has to give a proposal which satisfies the airlines as much as possible. We shall use the following notation:
Sets

- T is the set of times for each time slot. Elements from T are indexed by t. Elements of T are separated by distance Δt.
- J is the set of aircraft which would like to be served at the airport at some time during the time interval. Elements from J are indexed by j.
- $J' \subset J$ is the set of flights which do not require an overnight stay at the airport.

Parameters

- $ArrCst_j^t$ is the cost associated with assigning an arrival slot t to flight j.
- $DepCst_j^t$ is the cost associated with assigning a departure slot t to flight j.
- TAT_j^{min} and TAT_j^{max} are the minimum and maximum turnaround times accepted for flight j (which depends on the size of the aircraft etc.)
- Cap_{in}^t, Cap_{out}^t and Cap^t are the capacities for incoming, outgoing, and total flights during time slot t.
- $\alpha \in [0, 1]$ is the relative importance of the maximum deviation between the requested and granted slots when compared to the average deviation. When $\alpha = 1$ the goal is to minimize the maximum deviation, and when $\alpha = 0$ the goal is to minimize the average.

Variables

- $Arr_j^t \in \{0, 1\}$ is a binary variable equal to 1 if flight j is assigned to arrival slot t.

- $Dep_j^t \in \{0,1\}$ is a binary variable equal to 1 if flight j is assigned to departure slot t.
- $MaxCst$ is the cost associated with the requested slot with the largest deviation between the desired time and the allocated time.

With this notation, the ILP model is given by:

$$\begin{aligned}
&\text{minimize } \omega = \alpha MaxCst && (5.1)\\
&\qquad + \frac{1-\alpha}{|J|}\sum_{j\in J}\sum_{t\in T}\left(ArrCst_j^t \times Arr_j^t + DepCst_j^t \times Dep_j^t\right) &&\\
&\text{subject to: } MaxCst \geq \sum_t ArrCst_j^t \times Arr_j^t + DepCst_j^t \times Dep_j^t \qquad \forall j \in J && (5.2)\\
&TAT_j^{min} \leq \sum_t t \times (Dep_j^t - Arr_j^t) \leq TAT_j^{max} \qquad \forall j \in J' && (5.3)\\
&\sum_j Arr_j^t \leq Cap_{in}^t \qquad \forall t \in T && (5.4)\\
&\sum_j Dep_j^t \leq Cap_{out}^t \qquad \forall t \in T && (5.5)\\
&\sum_j Arr_j^t + Dep_j^t \leq Cap^t \qquad \forall t \in T && (5.6)\\
&Arr_j^t, Dep_j^t \in \{0,1\}
\end{aligned}$$

As can be seen, the objective function is a weighted sum of the maximal deviation (on the first line) and the average deviation (on the second line). There are only three types of constraints in this model: constraint (5.2) defines the maximum deviation, while (5.3) provides that the turnaround times are met, and constraints (5.4) through (5.6) makes sure that the capacities are not exceeded.

5.2.2 The Slot Allocation and Battery Charging Model Formulation

We now combine the Slot Allocation and the Battery Charging models into a single linear program which allows us to solve the Slot Allocation and Battery Charge Schedule problem. We start with the same sets, parameters and variables as in the slot allocation model, but we add the following notation from the battery charging model:

Parameters

- $m \in \mathbb{N}$ is the number of chargers
- $n^s \in \mathbb{N}$ is the number of spare batteries
- $n^n \in \mathbb{N}$ is the number of batteries which have an overnight stay at the airport.
- p_j is the charging time for the battery of flight j.

Variables

- $p_j^t \in \{0,1\}$ indicates when a battery is recharging
- $Ready_j^t \in \{0,1\}$ indicates when a battery is fully charged
- $Dem^t \in \mathbb{N}$ is the cumulative demand for batteries at time t
- $Sup^t \in \mathbb{N}$ is the cumulative supply of batteries at time t

And with this notation, we can provide the following ILP formulation:

$$\text{minimize } \omega = \alpha MaxCst + \frac{1-\alpha}{|J|}\sum_{j\in J}\sum_{t\in T} ArrDel_j^t \times Arr_j^t + DepDel_j^t \times Dep_j^t \tag{5.7}$$

subject to: Slot allocation problem:

$$MaxCst \geq ArrDel_j^t \times Arr_j^t + DepDel_j^t \times Dep_j^t \qquad \forall j \in J \tag{5.8}$$

$$TAT_j^{min} \leq \sum_t t \times (Dep_j^t - Arr_j^t) \leq TAT_j^{max} \qquad \forall j \in J' \tag{5.9}$$

$$\sum_j Arr_j^t \leq Cap_{in}^t \qquad \forall t \in T \tag{5.10}$$

$$\sum_j Dep_j^t \leq Cap_{out}^t \qquad \forall t \in T \tag{5.11}$$

$$\sum_j Arr_j^t + Dep_j^t \leq Cap^t \qquad \forall t \in T \tag{5.12}$$

Battery charging problem:

$$\sum_{t'\leq t} Arr_j^{t'} \leq p_j^t \qquad \forall j \in J, t \in T \tag{5.13}$$

$$\Delta t \sum_t p_j^t = p_j \qquad \forall j \in J \tag{5.14}$$

$$1 - Ready_j^t \leq p_j - \Delta t \sum_{t'<t} p_j^{t'} \qquad \forall j \in J, t \in T \tag{5.15}$$

$$\sum_j p_j^t \leq m \qquad \forall t \in T \tag{5.16}$$

$$Dem^t = \sum_{t'\leq t}\sum_j Dep_j^{t'} \qquad \forall t \in T \tag{5.17}$$

$$Sup^t = n^s + n^n + \sum_j Ready_j^t \qquad \forall t \in T \tag{5.18}$$

$$Sup^t \geq Dem^t \qquad \forall t \in T \tag{5.19}$$

This formulation is a direct combination of the slot allocation model and the battery charging model from the previous subsection and chapter respectively, although the battery charging model did not appear in the last chapter as such. Equations (5.7) through (5.12) have been explained in last subsection. Equation (5.13) makes sure that batteries can be recharged only after the flight which carries them has arrived at the airport, (5.14) makes sure that batteries are fully charged by the end of the day, (5.15) makes sure that a battery is ready to be used only once it is fully charged[1] and (5.16) makes sure that the recharging capacity is not exceeded. Finally, equations (5.17) through (5.19) make sure that the demand for batteries is matched by the supply.

[1]this is an assumption which has been explained in the previous chapter, see page 35

5.2.3 Improvements and Expansions to the Model

Most of the expansions and improvements to this model are very similar to the ones discussed in last chapter, and can be found in subsection 4.2.2 on page 38. There are however two improvements which were not applicable before and that we shall explain in this section: the introduction a maximum acceptable deviation from the requested schedule, and the presence of *grandfather rights*.

Firstly, in the Slot Allocation and Battery Charge Schedule problem we assumed that each flight has a minimum and maximum acceptable turnaround time, and the model penalizes flights which are assigned to slots which deviate far from their requested slots. We can make an additional assumption by introducing a maximum deviation of the assigned landing and take-off times from the requested times. We can use this to eliminate a large number of variables. If we assume a maximum deviation of 1 hour with $T_{end} = 24h$, we only need one twelfth of the Dep and Arr variables, and only half of the p and $Ready$ variables. In total, this reduced the number of variables to approximately 7/24 of the original number [2].

Secondly, we are able to modify this program such that it is also able to account for different priority classes of flights (see Subsection 2.2.2 on page 13). This can be done by iteratively solving the problem for different classes, starting with the highest priority and ending with the lowest. Suppose the flights from J are partitioned in different priority classes $J = J_1 \cup J_2 \cup ... \cup J_k$, where J_1 contains the highest priority flights (typically, these are the ones with grandfather rights). First, we solve the problem for only the flights which are from J_1, denote the optimal objective values of this subproblem as ω_1. After this, we are going to add the flights from J_2, but under the assumption that flights from J_1 are prioritized. We can do this by restricting the objective value for the flights from J_1 or fixing the flight schedule from all flights from this set altogether. We solve the problem again, which gives the optimal objective value ω_2. After this, we repeat the previous step: fix the flights from J_1 and J_2 and add the flights from J_3. We can repeat this until all sets have been considered.

5.2.4 Example

In order to see how this model works, let us return to the example from Subsection 4.2.1 on page 37, where three aircraft arrived and departed during a period of one day, one of them has an overnight stay. We have seen that if the problem has to be solved in such a way that there can be no deviation from the flight schedule, the smallest infrastructure required would be the size $m = 1$ chargers and $n^s = 1$ spares or $m = 2$ chargers and $n^s = 0$ spares.

However, suppose now that the airport only has access to one charger and zero spare batteries this time, which means that one or more flights have to be moved. Our primary objective is to minimize the maximum deviation of the flights, our secondary objective is to minimize the average deviation (such that α is almost equal to, but smaller then, 1). Conform with standard regulations, our slots are of fifteen-minute size.

[2] There are as much Arr and Dep variables as there are p and $Ready$ variables, so suppose there are 12 of each. Out of the 12 Arr and Dep variables, only 1 remains. Out of the 12 p and $Ready$ variables, only 6 remain. Thus in total, of the 24 original variables, only 7 remain.

Solving the problem with $MaxCst = 0$ is impossible, the next best option would be to solve it with $MaxCst = 15$ minutes. This also does not generate a feasible schedule, since battery 1 would still be charging by the time aircraft 2 has to depart. The first feasible solution is possible when $MaxCst = 30$ minutes. This solution is shown in Figure 5.2: by letting flight 1 arrive earlier and 2 depart later by half an hour, we can supply all flights with full batteries, even though we only have one charger and no spare batteries.

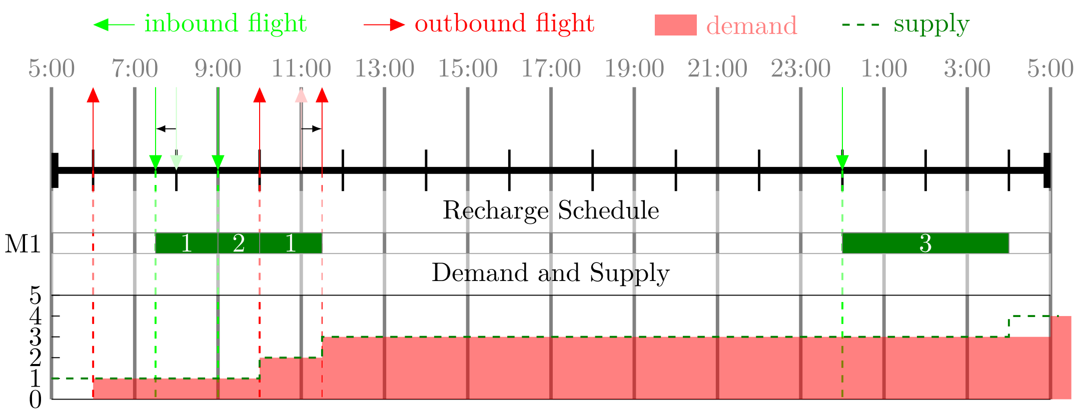


Figure 5.2: *An optimal slot allocation and recharge schedule given the data from example 4.2.1. Two slots have been relocated: the arrival from flight 1 occurs half an hour earlier, and the departure from flight 2 half an hour later.*

5.3 The Non-Preemptive Battery Charging Model

As in the previous chapter, we are also going to solve a non-preemptive processing version of our problem:

Slot Allocation and Non-Preemptive Battery Charge Schedule

Given: a set of flights J during a time interval T, where each is characterized by (r_j, d_j, p_j) and infrastructure m and n^s
Solve: the Slot Allocation and Battery Charge Schedule problem.
Assuming: charging a battery cannot be preempted.

We need to make two changes to the output variables of the problem. Firstly, we change the variable which indicates when a battery is charging to a variable which indicates when the charging process begins: $St_j^t \in \{0, 1\}$ is equal to 1 if battery j starts charging at time t and 0 otherwise. Secondly, there is no need for the variables which indicate when a battery is ready, this is now deducted directly from the St variables.

With everything else the same as in last section, the formulation for the discrete time charging problem looks like:

$$\text{minimize } \omega = \alpha MaxCst + \frac{1-\alpha}{|J|}\sum_{j\in J}\sum_{t\in T} ArrDel_j^t \times Arr_j^t + DepDel_j^t \times Dep_j^t \tag{5.20}$$

subject to: Slot allocation problem:

$$MaxCst \geq ArrCst_j^t \times Arr_j^t + DepCst_j^t \times Dep_j^t \qquad \forall j \in J, t \in T \tag{5.21}$$

$$TAT_j^{min} \leq \sum_t t \times (Dep_j^t - Arr_j^t) \leq TAT_j^{max} \qquad \forall j \in J' \tag{5.22}$$

$$\sum_j Arr_j^t \leq Cap_{in}^t \qquad \forall t \in T \tag{5.23}$$

$$\sum_j Dep_j^t \leq Cap_{out}^t \qquad \forall t \in T \tag{5.24}$$

$$\sum_j Arr_j^t + Dep_j^t \leq Cap^t \qquad \forall t \in T \tag{5.25}$$

Battery charging problem:

$$\sum_t t \times Arr_j^{t'} \leq \sum_t t \times St_j^t \qquad \forall j \in J \tag{5.26}$$

$$\sum_t St_j^t = 1 \qquad \forall j \in J \tag{5.27}$$

$$\sum_j \sum_{t' \in (t-p_j, t]} St_j^{t'} \leq m \qquad \forall t \in T \tag{5.28}$$

$$Dem^t = \sum_{t' \leq t} \sum_j Dep_j^{t'} \qquad \forall t \in T \tag{5.29}$$

$$Sup^t = n^s + n^n + \sum_j \sum_{t' \leq t - p_j} St_j^{t'} \qquad \forall t \in T \tag{5.30}$$

$$Sup^t \geq Dem^t \qquad \forall t \in T \tag{5.31}$$

Only minor modifications have been made to accommodate the change from the p_j^t to the St_j^t variables, these can be found in equations (5.26), (5.27), (5.28) and (5.30). Equation (5.26) makes sure that a battery can only start charging after the aircraft which brings it to the airport has arrived. Equation (5.27) ensures that a battery can only be charged once, and equation (5.28) makes sure that no more than m chargers are used at the same time. Finally, the last three equations manage the supply and demand for batteries, and make sure that the supply always exceeds the demand. Equation (5.15) has no counterpart in this formulation.

Notice one advantage of this model: the number of equations grows linearly with respect to both $|J|$ and $|T|$, and in total, the models size if given by $\mathcal{O}(|T||J|)$.

5.4 Heuristics

Solving the Slot Allocation and Battery Charging models under the assumption of both preemptive and non-preemptive charging as presented in the last two sections has exponential time complexity. In order to solve this problem efficiently for large instances while still generating reasonable results, we shall introduce two types of heuristics.

Greedy Bottleneck-resolving Heuristics

The first heuristic which we explore in this report is a greedy heuristic. This, as well as the next, work by alternating between modifying the flight schedule and solving the AC Battery Swap and Charging Schedule (AC BS&CS) problem given this (modified) flight timetable. This greedy algorithm looks in each iteration which adjustments to the flight schedule it can make to resolve scheduling conflicts with as little cost as possible.

The greedy algorithm is described in Algorithm 1, and consists of five basic steps. First, we start with a given deviation from the requested flight schedule (in the initialization, this is set to 0) and solve the AC BS&CS with the extra constraint that we have a fixed number of chargers. The output of this step consists of a minimum number of required spare batteries and a charging schedule. If the number of spare batteries is lower or equal to the amount which is available, we are done and if it is not, we proceed to the second step. In this step, we determine the demand for charged batteries (from the flight schedule) and the supply (from the charging schedule) and the points where the demand is equal to the supply. These times are the bottlenecks: we can only remove spare batteries further if there are no times at which the demand equals the supply. So, in the third step, we determine which flights should arrive earlier or depart later in order resolve the bottlenecks. For each bottleneck, these are the flights which depart at the start of the bottleneck, and the flight which battery is completing the charging process at the end of the bottleneck. Finally, we add deviation to these flights in a greedy approach: by adding deviation to the flights such that their final deviation is as small as possible. This step is regulated by two parameters: the maximum added deviation per flight, dt, and the maximum number of bottlenecks which are resolved per iteration, k. The steps are shown in figure 5.3. After these steps have been performed, the algorithm returns to the first step and determines a new charging schedule.

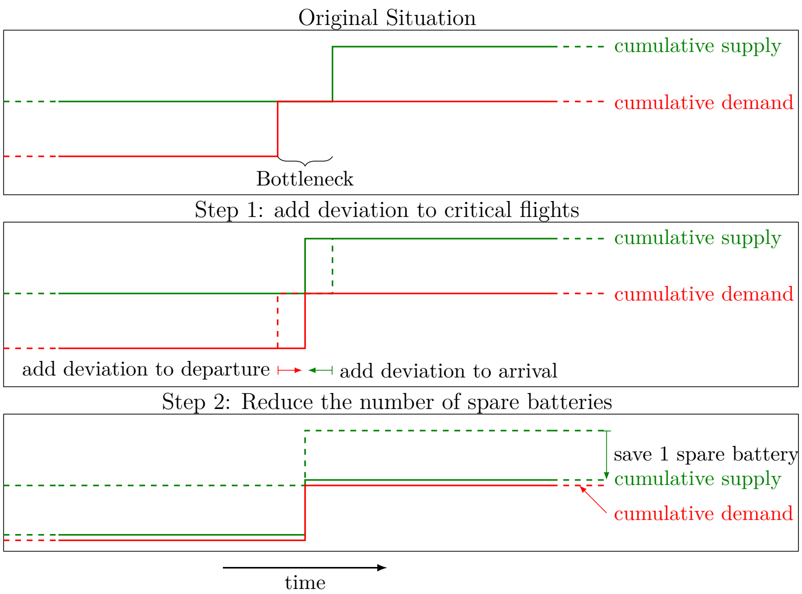


Figure 5.3: *Bottleneck resolving steps of the greedy algorithm used to solve the AC SA & BCS problem.*

Algorithm 1: Greedy algorithm for the AC SA & (NP) BCS problem.

Data: Requested flight schedule (Arr, Dep), processing times, infrastructure (m, n^s), and parameters of the heuristic k and dt
Result: Arrival and departure deviation for each flight $ArrDev$ and $DepDev$
Initialize $ArrDev = DepDev = 0$ for all flights, $n = |Arr|$;
while $n > n^s$ **do**
    Let $Arr^{out} = Arr - ArrDev$ and $Dep^{out} = Dep + DepDev$;
    Solve the AC BS&CS problem (heuristically), obtain charging schedule and n;
    Determine the cumulative demand and supply for batteries at each time;
    Determine the bottlenecks B: sets of connected times when the cumulative demand is equal to the supply;
    foreach $b \in B$ **do**
        Determine the critical departures: flights which depart at the start of the bottleneck;
        Determine the critical arrivals: flights of which the battery is finishing charging at the end of the bottleneck;
        Determine the critical arrival and departure with the smallest $ArrDev$ and $DepDev$;
        Determine the required deviation in which needs to be added to the flights in order to resolve the bottleneck;
        Store the total required deviation for both flights as A_b and D_b;
    end
    Determine the set of maximal k bottlenecks for which $\max\{A_b, D_b\}$ is smallest: B_s;
    foreach $b \in B_s$ **do**
        Add the maximum of A_b and dt to the $ArrDev$ of the most critical arrival flight;
        Add the maximum of D_b and dt to the $DepDev$ of the most critical departure flight;
    end
end

Binary Search Heuristics

Secondly, we can use a binary search algorithm in combination with any of the methods to solve the AC BS&CS problem. We do this by starting with the assumption that $\alpha = 1$, determining the optimal value of ω with a schedule where each slot grant deviates ω from the requested slot, and cleaning up the schedule as much as we can afterwards.

Let Arr_j and Dep_j be the desired arrival and departure times of the airlines. Notice that a solution to the AC SLOT ALLOCATION AND BATTERY CHARGE SCHEDULE problem exists with $MaxCst \leq \omega$, if and only if a solution to the AC BS&CS problem exists with $r_j = Arr_j - \omega$ and $d_j = Dep_j + \omega$. We can check if the latter is true by using the methods discussed in the previous chapter, both by solving the problem to optimal or by using a heuristic. By performing a binary search, we can thus determine the smallest value of ω for which the AC BS&CS problem can be solved with modified arrival and departure times.

However, now we have a flight schedule in which each granted slot deviates ω from the requested slot. If we take $\alpha = 1$ this is not a problem, but this is hardly a realistic objective. In order to improve on this schedule, which should almost always be possible, we use the following rule. We know after creating a flight and recharge schedule which battery is assigned to which departing flight, so let p_j^{min} and p_j^{max} denote the starting and ending time of the charging process for the batteries corresponding to flight j. The final granted arrival slot and departure slot are given by:

$$Arr_j^{out} = \min\{Arr_j, p_j^{min}\} \tag{5.32}$$
$$Dep_j^{out} = \max\{Dep_j, p_j^{max}\} \tag{5.33}$$

Since binary search algorithms have a logarithmic time complexity, using the FIFO and SPT heuristics generate polynomial time heuristics for this problem. Algorithm 2 summarizes the binary search heuristic.

Algorithm 2: Binary search algorithm for the AC SA & (NP) BCS problem.

Data: Requested flight schedule (Arr, Dep), processing times, infrastructure (m, n^s), and parameter of the heuristic ω_{max}.
Result: Allocated arrival and departure time for each flight Arr^{out} and Dep^{out}
Initialize $\omega = 0$;
while *Not satisfied with* ω **do**
  Let $Arr^{out} = Arr - \omega$ and $Dep^{out} = Dep + \omega$;
  Try to solve the AC BS&CS problem (heuristically), using infrastructure (m, n^s);
  if *A solution exists* **then**
    Update to a smaller ω using the binary search algorithm;
  else
    Update to a larger ω using the binary search algorithm;
  end
end
Let $\omega(OPT)$ be the largest value of ω for which the AC BS&SC problem could be solved;
Let Chr_j^{st} and Chr_j^{end} be the start and end charging time of the battery which arrives and departs on flight j respectively ;
foreach $j \in J$ **do**
  $Arr^{out} = \max\{Arr - \omega(OPT), Chr_j^{st}\}$;
  $Dep^{out} = \min\{Dep + \omega(OPT), Chr_j^{end}\}$;
end

An overview of all optimization models and heuristics for the AC Slot Allocation and Battery Charge Schedule problem can be found in figure 5.4.

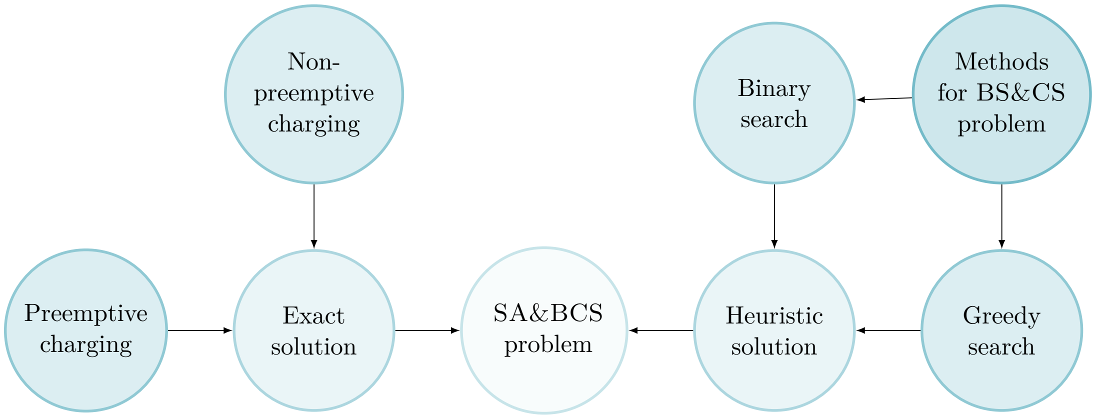


Figure 5.4: *Binary search algorithm to solve the AC SLOT ALLOCATION AND BATTERY CHARGE SCHEDULE problem with $\alpha = 1$, using the solutions to the AC BS&CS problem from last chapter. We are satisfied with ω if and only if the modified flight schedule is feasible for the BS&CS problem, and is infeasible for any value lower than ω.*

5.5 Experiments on an Artificial Flight Schedule

In this section, we are going to use the exact formulations and heuristics which we have been developing in this chapter in order to solve the Slot Allocation and Battery Charging problem for some artificially created flight schedules. The purpose of this is to analyze and compare the performance of the exact MILP formulations as well as the heuristics.

The artificial schedules are created in a similar manner as in the final section of last chapter. This time using a 15 minute window (comparable to the slot size at major airports), the arrival, departure and charging times are drawn independently from identical distributions:

$$\begin{aligned} Arr_j^i &\sim Unif[0, 18], \\ Dep_j^i &\sim Unif[0, 5, 1] + Arr_j^i, \\ P_j &\sim Unif[0.5, 5.5], \end{aligned}$$

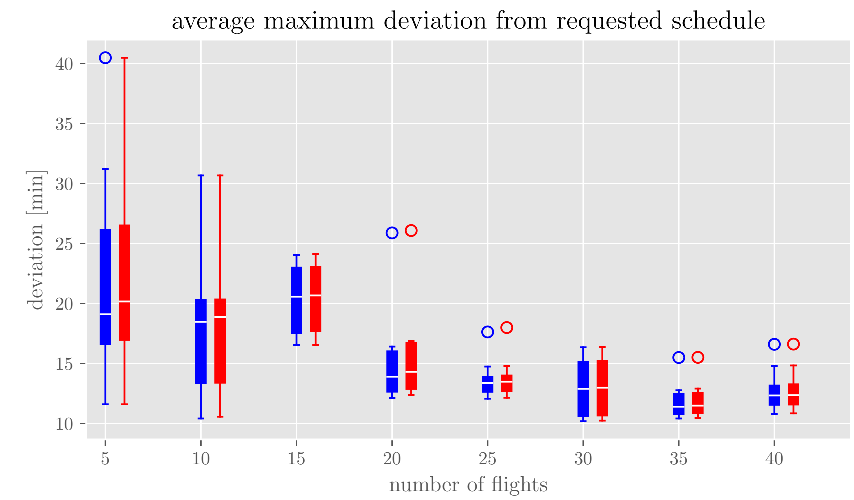
where the hours are used as unit of time. From these desired arrival and departure dates, the penalty functions $ArrCst$ and $DepCst$ are derived. The allocated flight times can be determined from the binary allocation variables:

$$ArrCst_j^t = |t - Arr_j^i| \qquad Arr_j^o = \sum_t t \times Arr_j^t,$$

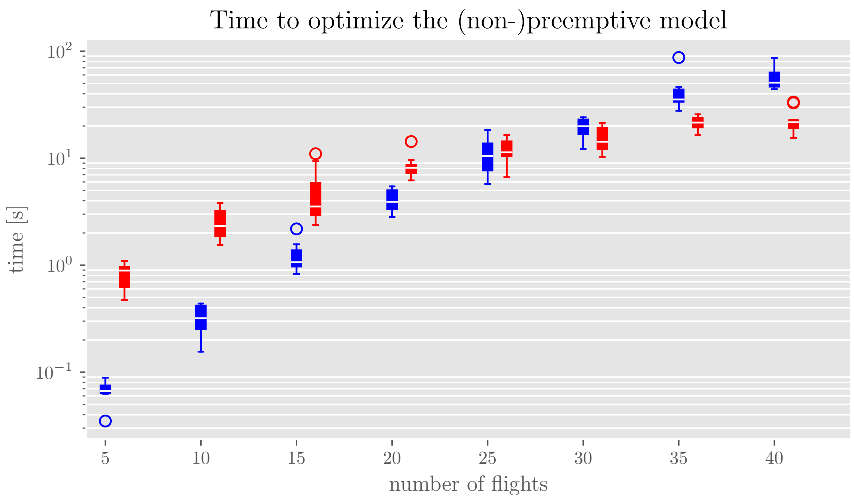
$$DepCst_j^t = |t - Dep_j^i| \qquad Dep_j^o = \sum_t t \times Dep_j^t.$$

5.5.1 Comparing Preemptive with Non-Preemptive Charging

Firstly, we are going to study the performances of the ILP models which provide an optimal solution to the Slot Allocation and Battery Charge Problem, which are presented in sections 5.2 and 5.3. On forehand, it has to be said that comparing these two methods is not as straightforward as it was when comparing the preemptive with the non-preemptive methods in the last chapter. This is because in the last chapter, for each flight schedule the output of the problem consisted of one number, the weighted sum of m and n^s, whereas now, the output is given as the deviation from the requested flight schedule as a function of m and n^s. In order to explain how we can still summarize the performances of the heuristics in one graph, we are going to start with a small solution and then scope out to the larger solutions. In short, the final results can be found in figures 5.5a and 5.5b, where respectively the quality of the optimal solution and the solve times for both methods have been graphed in boxplots for instances with up to 55 flights. In both graphs the principle holds: the lower the value, the higher the performance.



(a) *Quality of the flight schedule. Lower values indicate higher quality.*



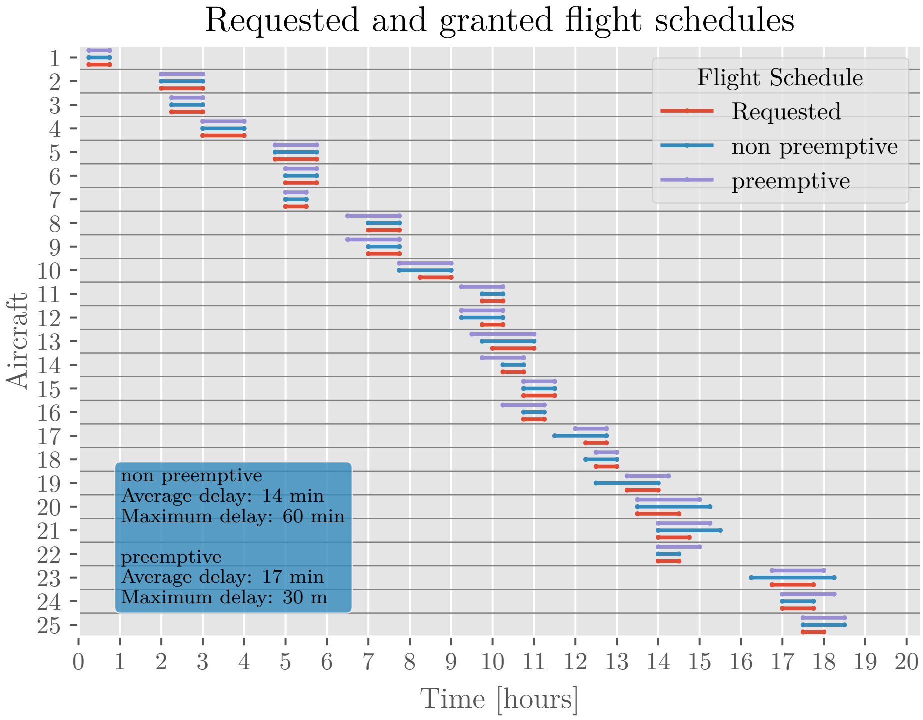
(b) *Time it takes to obtain the optimal solution.*

Figure 5.5: *Performance of the Slot-Allocation and Battery Charging problem ILP's from Sections 5.2 and 5.3, when applied to instances with up to 55 flights. Blue and red correspond to preemptive and non-preemptive solutions respectively. In Subfigure 5.5a one can see the quality of the flight timetables generated by the heuristics. In subfigure 5.5b, one can see the time it takes to solve the instances.*

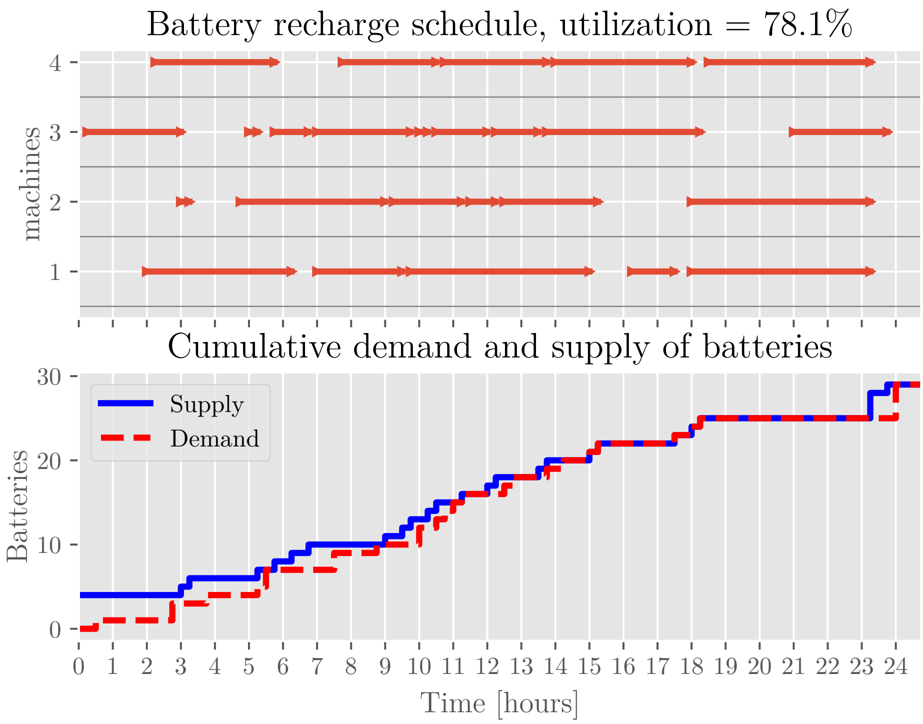
The rest of this section will be used to explain how Figures 5.5 and 5.5b have been constructed, building from the ground up. Let's first suppose we want to solve the most specific instance possible: we know the flights which want to visit the airport, and we know what infrastructure we have at out disposal, and suppose that $\alpha = 0.9$. We are presented with an instance of 25 flights, $m = 4$ and $n^s = 4$, and solve this to optimality with both preemptive and non-preemptive charging. the results are shown in figures 5.6a, where the flight timetable is shown, and 5.6b, where the recharging schedule for the batteries is shown. As can be seen, the granted flight schedules for preemptive and non-preemptive charging deviate with 30 and 60 minutes respectively at most from the requested flight schedule.

However, for a full analysis the quality of the methods we cannot just look at the performance when $m = 4$ and $n^s = 4$. In order to provide a complete picture of how the methods perform when applied to this flight schedule, we need to consider all pairs of m and n^s. So, for each of these pairs, both optimization algorithms have been performed on this flight schedule, and the objective values are graphed in figures 5.7a, for preemptive charging, and 5.7c, for non preemptive charging, as a heatmap. In this figure it can be seen that using preemptive charging especially improves the objective function between the area where $m = n^s = 4$ and $m = n^s = 6$. When we repeat the process which we used to create these two figures, we get an idea of how good the algorithms perform in general. This is done in figures 5.7b and 5.7d, where the values are averaged over 30 instances.

Finally we can make the step to figures 5.5 and 5.5b. The latter one is simple. For each amount of flights, we create a number of instances. For each instance, we determine the optimal solution for all pairs of m, n^s (as in the previous two figures) and store the time which that took. These times are graphed in figure 5.5b, and provide information on how fast the algorithms are working. Secondly, we summarize all information from the optimal solutions by taking the average value. Even though this removes a lot of information on how the methods perform, it gives a good general impression which is sufficient to compare one to another. These values are graphed in figures 5.5. Initially, the two methods generate comparable results, but after 25 flights the preemptive charging really starts to differentiate itself from the non-preemptive charging method, which is similar to last chapter. In contrast to that chapter, the preemptive charging now comes at the cost that it is significantly slower for large instances.

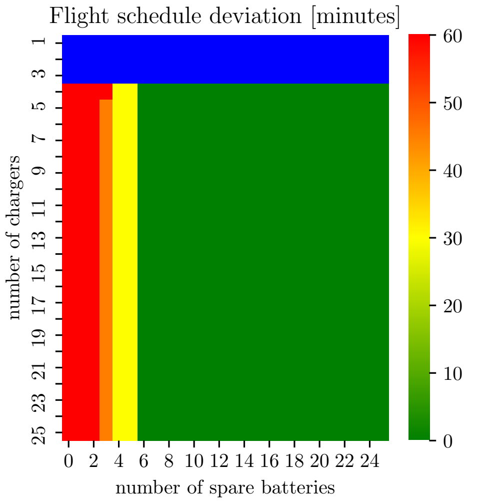


(a) *Requested and granted flight schedules.*

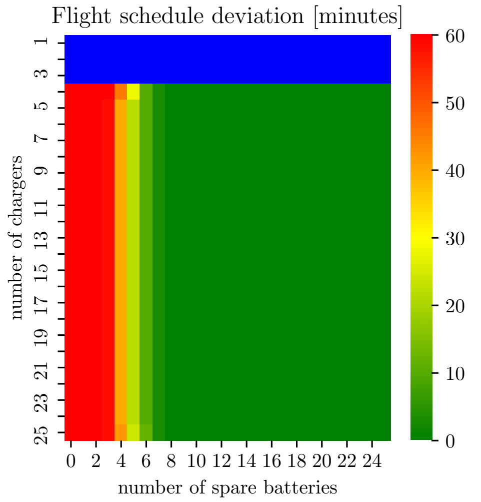


(b) *Battery charging schedule for non-preemptive charging.*

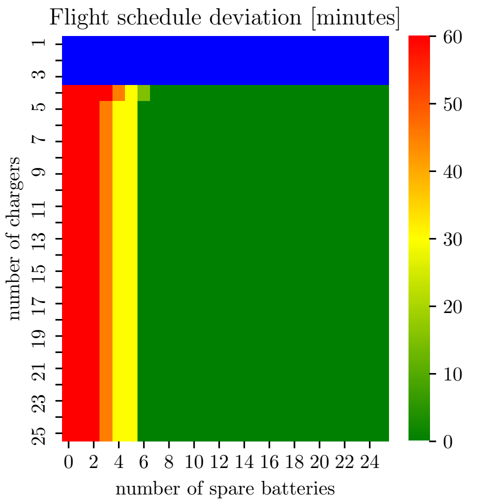
Figure 5.6: *Output of the Slot-Allocation and Battery Charging problem when applied to an instance with 25 flights in the case that preemption of charging is and is not allowed. In Subfigure 5.6a, one can see the requested flight schedule, and the granted flight schedules. In Subfigure 5.6b, one can see the charging process when charging non-preemptively: a Gannt chart with all the charging jobs in the top, and the supply and demand for batteries at the bottom.*



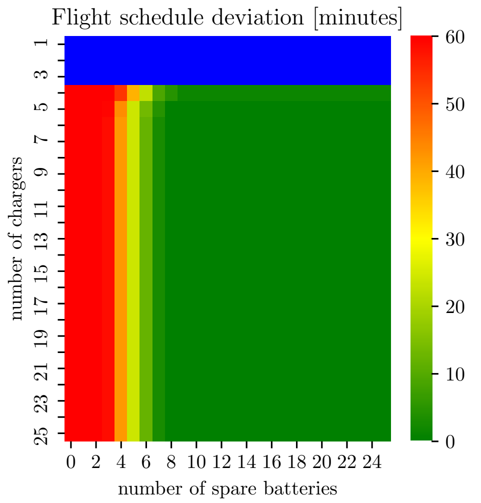
(a) *With preemption, single instance*



(b) *With preemption, multiple instances*



(c) *Without preemption, single instance*



(d) *Without preemption, multiple instances*

Figure 5.7: *Objective values of the optimal solution of the slot allocation and battery charging problem with 25 flights. The first row shows the results if preemption is allowed, while the second row shows the results it it is not. Secondly, In Subfigures 5.7a and 5.7c, the instance corresponding with Figure 5.6 is used. In Subfigures 5.7b and 5.7d, the average over multiple instances is taken.*

5.5.2 Comparing the Heuristics to the Exact Formulations

Now that we have seen how the non-preemptive and preemptive charging methods compare, and have observed that in order to obtain the optimal solutions both require a large amount of time, we are going to look at the heuristics. These are the four heuristics presented in Section 5.4: FIFO(R), FIFO(RP), SRPT and SAPT. The results of this are shown in figures 5.8, 5.9a and 5.9b.

In figure 5.8, one can see the performance of all heuristics, compared with the optimal solution with non-preemptive and preemptive charging, for the same instance with 25 flights as in the last subsection. The FIFO heuristics differ greatly in performance: FIFO(R) is by far the worst of all solutions, while FIFO(RP) comes pretty close to the optimal solution. The latter even generates a better solution then the optimal non-preemptive timetable. Somewhere between the three best solutions (optimal (non-)preemptive and FIFO(RP) and the worst one (FIFO(R)), we find the two SPT heuristics which provide comparable results. SRPT outperforms the SAPT heuristic, but not by a large margin.

The same pattern of performance can be found again if we look at figure 5.9a, where the quality of the solution found by all methods is graphed for instances with up to 120 flights. The FIFO(RP) heuristic performs on par with both optimal solutions, the FIFO(R) heuristic performs very poorly, and the SPT heuristics can be found somewhere in between the two. It should be noted that in this figure, as well as the next, the optimal preemptive solution has been calculated for instances up to 55 flights due to high computation times. This can be more clearly observed in figure 5.9b, where similar to figure 5.5b, the time it takes to perform the different methods has been displayed. One can see that solving the instances to optimality takes about 10 times as much time as solving them with the heuristics, all of which have a very similar running time.

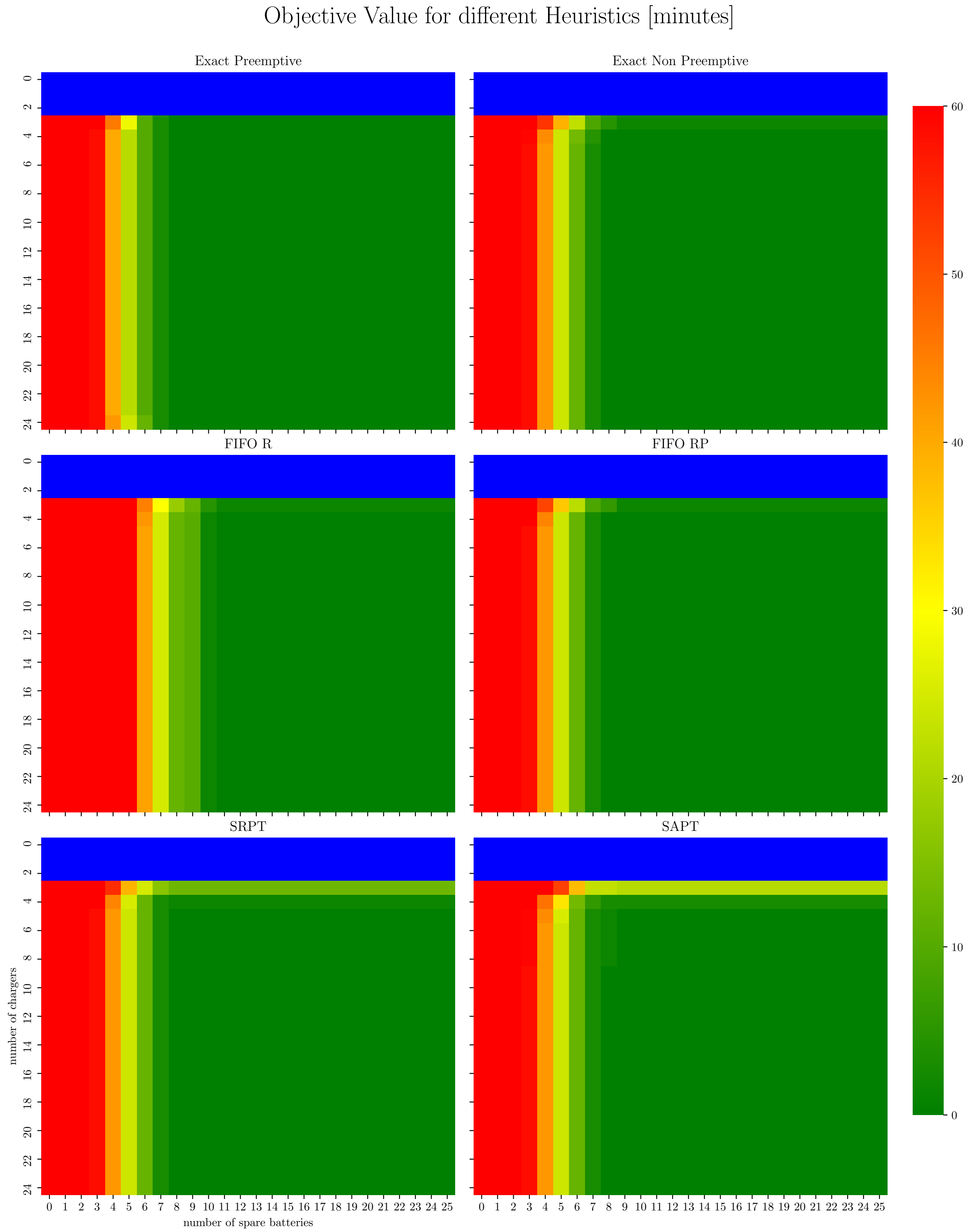
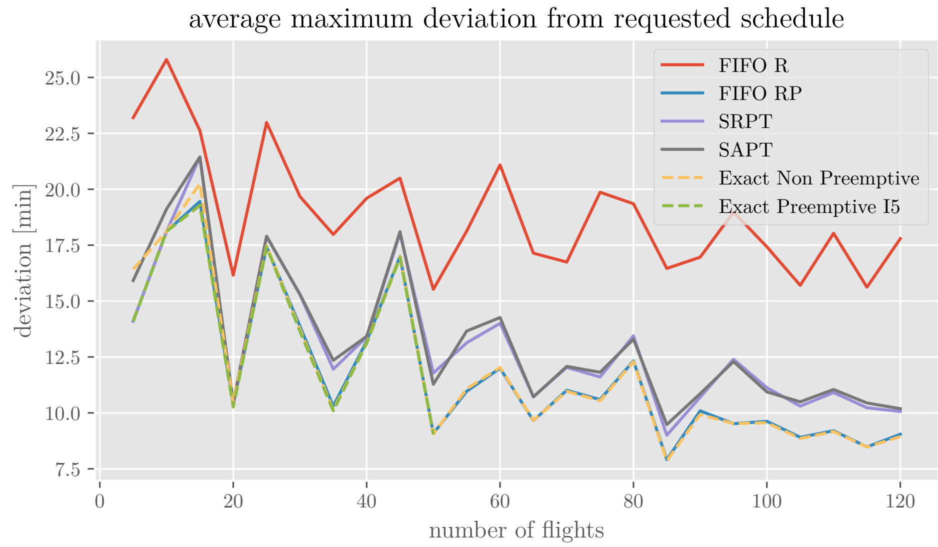
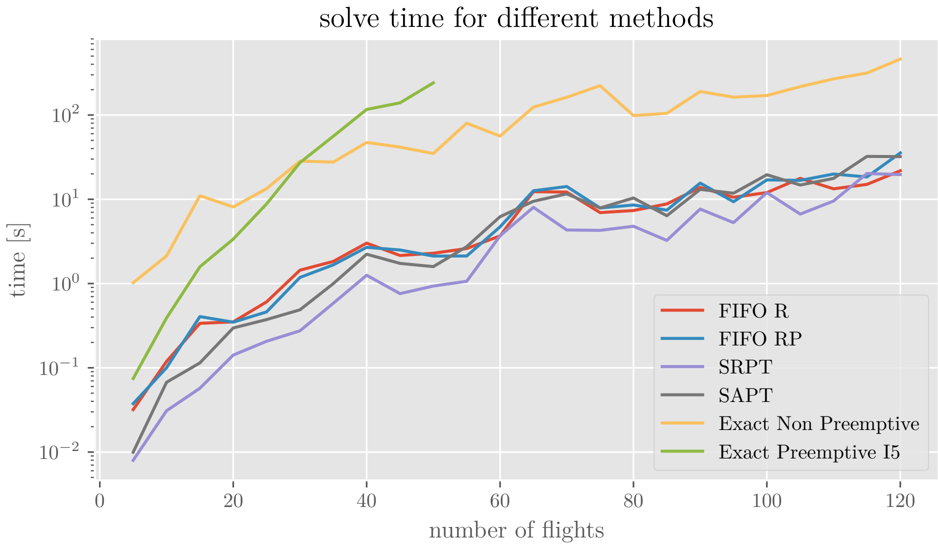


Figure 5.8: *Quality of the flight timetables generated for instances of 25 flights. These are generated by the preemptive and non-preemptive ILP's, which are able to solve the problem to optimality, and four different heuristics from Section 5.4.*



(a) *Flight timetable quality*



(b) *solve time*

Figure 5.9: *Performance of the different heuristics for the Slot-Allocation and Battery Charging problem, when applied to instances with up to 120 flights. In Subfigure 5.9a, similar to figure 5.5a one can see the quality of the flight timetables generated by the heuristics. In subfigure 5.9b, similar to subfigure 5.5b, one can see the time it takes to solve the instances. Notice that due to large computational times, the exact preemptive solution has been computed only for instances with up to 55 flights.*

CHAPTER 6

Case Study: Electrification and Hydrification at Amsterdam Schiphol and Rotterdam Zestienhoven

In this chapter we shall apply the knowledge that we have gathered in the literature review and obtained in Chapters 4 and 5 with a case study at two dutch airports: Amsterdam Airport Schiphol and Rotterdam-The Hague Airport Zestienhoven. Our objective is to determine the estimated required infrastructure to support the electric and hydrogen aviation operations at these two airports in 2030, 2040, and 2050. This includes the number of chargers and spare batteries for electric aircraft as well as peak power. For hydrogen, this includes the number of fuel points and spare tanks. The infrastructure is to be determined for different Levels of Service which the airports can provide, and should be sufficient to serve the airport for at least 95% of the days.

6.1 The airports

6.1.1 Background

In our case study, we are going to analyze two (very different) airports. Some technical specifications can be found in Table 6.1.

Amsterdam Airport Schiphol, hereafter referred to as *Schiphol*, is a major aviation hub, not only in The Netherlands, but also for Europe. It is the 9^{th} busiest airport in the world in number of movements (500,000 per year), the 12^{th} busiest in passengers (72 million per year) and the 18^{th} busiest in cargo tonnage (1.5 million tons per year). Schiphol serves as the main hub for the royal KLM, which operates just over half of the flights from and to the airport. Additionally, it is one of the most slot constrained airports in the world, limited in growth by its runway capacity and governmental regulations. Because of this, flights are near-exclusively commercial, and general aviation is heavily discouraged from landing at the airport by high landing fees.

Rotterdam The Hague Airport, hereafter *Zestienhoven*, on the other hand, is a regional airport. It is the third-busiest airport in The Netherlands, after Amsterdam and Eindhoven airport, with approximately 2 million passengers per annum. Besides commercial aviation, the airport extensively uses its single runway for general aviation, flight schools, flying clubs, and for business flights (demand for which is generated by the nearby port of Rotterdam).

Table 6.1: *Specifications for Amsterdam Schiphol and Rotterdam The Hague Zestienhoven.*

	Amsterdam Airport Schiphol	Rotterdam The Hague Airport Zestienhoven
Location	Haarlemmermeer, North Holland, The Netherlands	Rotterdam, Zuid-Holland, The Netherlands
IATA airport code	AMS	RTM
ICAO airport code	EHAM	EHRD
Number of runways	7	1
Passengers (2019)	71,706,999	2,133,976
Flights (2019)	514,625	52,439

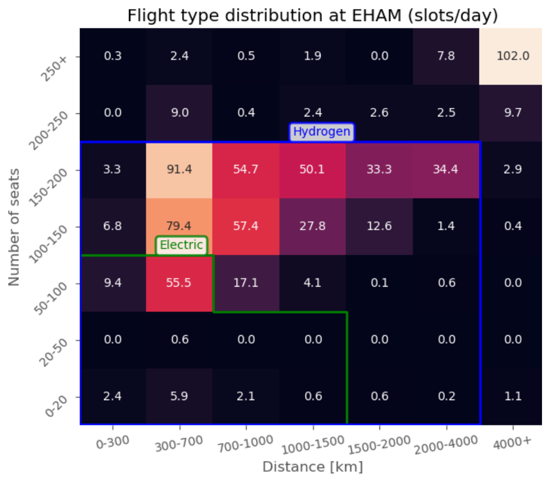
6.1.2 Historical flight data analysis

In order to simulate the flight schedules for electric and hydrogen aircraft in 2030, 2040, and 2050, we are going to use this subsection to analyze the current flight schedules at the airports. Specifically: the number of movements of a certain type per day, the distribution of arrivals and departures throughout the day, and the delay of the flights.

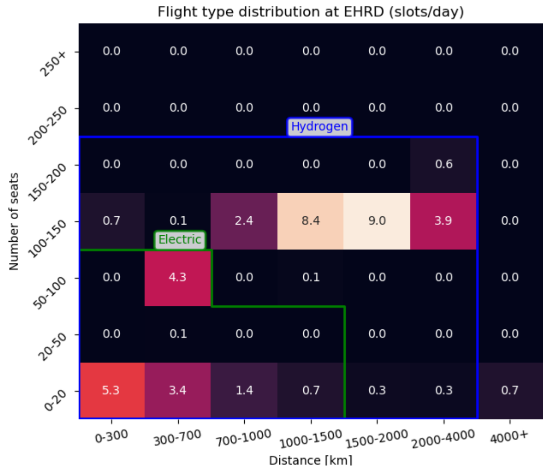
First, the number of flights per day is shown in Figures 6.1a for Schiphol, and 6.1b for Zestienhoven, together with a rough estimate of which flights could be captured by electric and hydrogen aircraft. For example, when we assume that the current schedule will remain in use, (hybrid-)electric aircraft could (in principle) service 65 arrival/departure pairs at Schiphol and 15 at Zestienhoven on average during the summer season. Additionally, hydrogen aircraft could capture 440 arrival/departure pairs at Schiphol and 26 at Zestienhoven. Together with a forecast on the fraction of flights from each class which are expected to be operated by electric/hydrogen aircraft, we are able to determine the number these flights throughout the day.

Second, once we have the number of flights for each class, we need to determine how to distribute them throughout the day. An uniform distribution might distribute the load on the network as best as possible, but does not capture the peak hours at Zestienhoven, or the bank-structure at Schiphol. Again, we use the flight data from 2019 which was used to create figure 6.1, but arrange the demand by arrival and departure time. The results can be found in Figures 6.3a for Schiphol and 6.3b for Zestienhoven. As can be seen, Schiphol has multiple peak banks spread throughout the day, alternating between arrival and departure banks. On the other hand, Zestienhoven has two peak moments: a departure rush in the morning and an arrival rush just before midnight.

Now that we know how to simulate the arrival and departure times, the range, and the seat capacity of each flight, we are going to complete our analysis of the airports



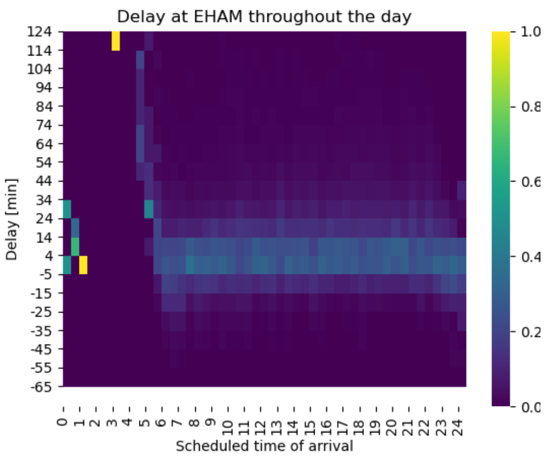
(a) *Amsterdam Schiphol*



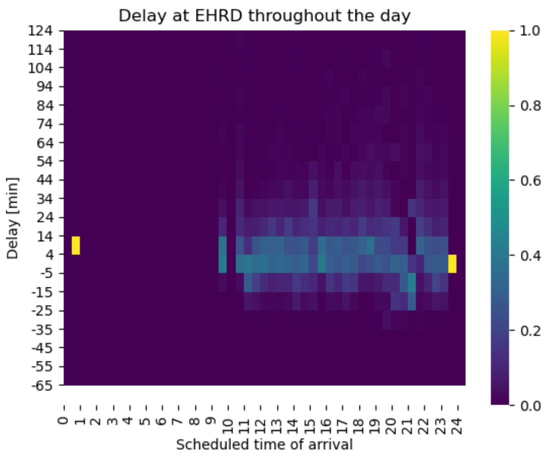
(b) *Rotterdam The Hague Zestienhoven*

Figure 6.1: *Breakdown of the average number of arrival/departure pairs per day for Schiphol and Zestienhoven in seat and distance classes. For Schiphol, the first week of the summer season of 2019 has been used, while for Zestienhoven, the entire summer season of 2019 has been used.*

by looking at the expected delay of the flights. This data will be used to determine the robustness of the recharge schedule and determine if the infrastructure is sufficiently large to respond to expectable delays. Using all available data from 2019, the delay of each flight has been stored, and the results are graphed in Figure 6.2a for Schiphol, and 6.2b for Zestienhoven. The graphs show for each half-hour of the day and for each 10 minute delay interval the fraction of flights which had a scheduled arrival time within the respective half-hour, and had a delay time of somewhere within the 10-minute interval. As can be seen, Schiphol has significantly higher delays then Zestienhoven, but is also more consistent in their delays throughout the day.

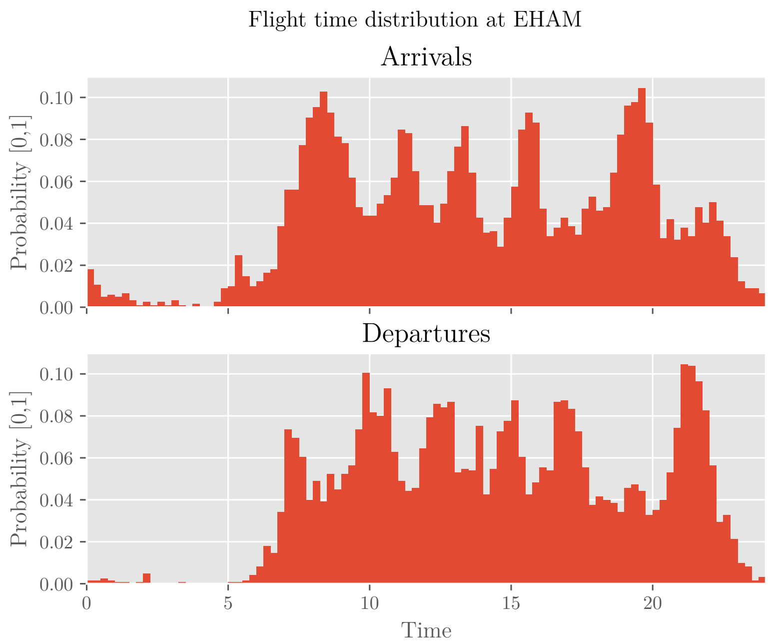


(a) *Amsterdam Schiphol*

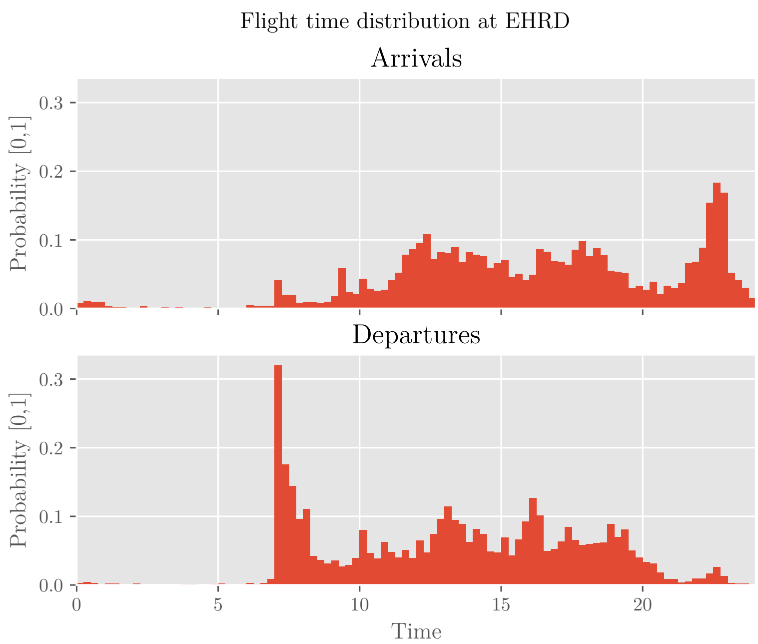


(b) *Rotterdam The Hague Zestienhoven*

Figure 6.2: *Distribution of the delay of arrival flights at Schiphol and Zestienhoven, separated by the scheduled time of arrival.*



(a) *Amsterdam Schiphol*



(b) *Rotterdam The Hague Zestienhoven*

Figure 6.3: *Historical probability density function of arrival and departure flights from Schiphol and Zestienhoven throughout the day. The same data which has been used to create figure 6.1 has been used.*

6.1.3 Demand

At the moment, not much is known about what fraction of the flights which are operated now are going to be executed by electric or hydrogen aircraft. Therefore, we are going to make do with the following data for 2030, 2040, and 2050, tabulated in 6.2. It is assumed that between electric and hydrogen aircraft, all flights below 20 passengers will be electric, half of the flights between 20 and 100 passengers will be electric and the other half hydrogen, and above 100 passengers all flights will be hydrogen.

Table 6.2: *Forecast demand fraction of flights captured by electric and hydrogen aircraft for different classes of capacity and range. The values are given in the format [2030]/[2040]/[2050].*

	distance [km]	0-1000	1000-1500	1500-3000	3000-6000	6000+
		Electric				
Passengers	0-20	18/63/88	18/63/88	5/38/63	-/18/38	-/5.0/18
	20-100	8.8/31/44	2.5/19/31	-/8.8/19	-/2.5/8.8	-/-/2.5
	100+	-	-	-	-	-
		Hydrogen				
	0-20	-	-	-	-	-
	20-100	8.8/31/44	2.5/19/31	-/8.8/19	-/2.5/8.8	-/-/2.5
	100-150	2.5/38/63	-/18/38	-/5.0/18	-/-/2.5	-/-/-
	150-210	-/18/38	-/5.0/18	-/-/2.5	-/-/-	-/-/-
	210-300	-/5.0/18	-/-/2.5	-/-/-	-/-/-	-/-/-

6.2 The Aircraft Fleet

6.2.1 Aircraft sizing

With the models for electric and hydrogen aircraft from Section 2.1, a number of surrogate aircraft for concepts from Appendix A were created. These will make up the fleet of aircraft which is going to be used in this case study. An overview of the modeled aircraft can be found in table 6.3. We have used the values $E_f^* = 33$ kWh/kg for hydrogen, and $E_f^* = 1$ kWh/kg for batteries. The former is, obviously, fixed by God, while the latter can change due to technological advancements.

Table 6.3: *Overview of the modeled aircraft. The columns give: the aircraft name, the fuel type, the number of passengers, the range, the fuel efficiency, the L/D ratio and the total weight of the fuel. The fuel type is given by either E[lectric] or H[ydrogen].*

Make	Model	Fuel	Pax	R [km]	η_f	L/D	W_f [kg]	EIS
EVIATION	Alice	E	12	1000	0.93	25.0	5072	2022
EMBRAER	E175e	E	96	700	0.93	16.5	18064	2035
Universal Hydrogen	Dash-8	H	50	930	0.70	16.5	200	2025
AIRBUS	ZEROe turboprop	H	100	2000	0.70	18.0	532	2035
AIRBUS	ZEROe turbofan	H	200	4000	0.33	20.0	1292	2035

With these aircraft, The total number of jobs for each airport and year is given in table 6.4. As can be seen in this table, the average demand at Zestienhoven in 2030 is only one electric aircraft per day. Because of this low demand, we shall discard it in this study.

Table 6.4: *The number of to be charged batteries and hydrogen canisters for Schiphol and Zestienhoven for one day, assuming a single battery/canister per aircraft. The number of jobs will grow linearly with the number of days and the number of batteries/canisters per aircraft.*

	Schiphol		Zestienhoven	
	Electric	Hydrogen	Electric	Hydrogen
2030	6	10	1	0
2040	25	114	5	3
2050	35	205	9	10

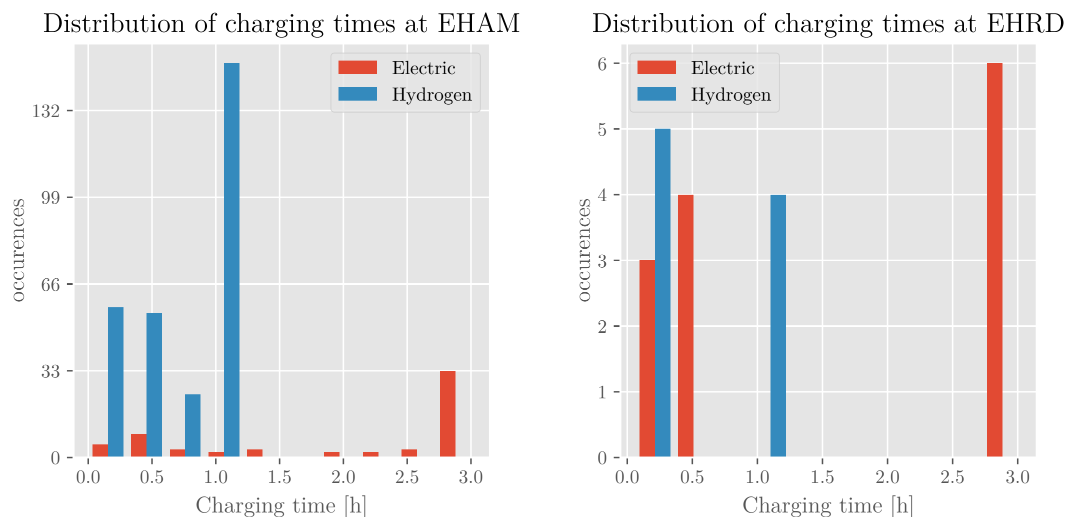
6.2.2 Refueling and recharging

The recharging/refueling rate of the batteries/hydrogen tanks is limited by both the properties of the aircraft and the infrastructure which is available at the airport. Limits due to the former are described in Section 2.1, while limits due to the latter are provided in table 6.5. The projection of the charging speed of batteries is based on [Marksel et al., 2019], while the hydrogen fuel speed is based on the assumption that this process can be implemented with the same flow rate as kerosene [EC, 2020].

Table 6.5: *Fueling/Charging speed*

Technology	Year	capacity per point
Electric	2030	1 MW
	2040	2 MW
	2050	2 MW
Hydrogen	2030-50	500 l/min

Given these values, the following distributions for charging time for Schiphol and Zestienhoven can be deduced:



(a) *Amsterdam Schiphol* **(b)** *Rotterdam The Hague Zestienhoven*

Figure 6.4: *Example of the distribution of the battery charging and hydrogen tank filling times at Schiphol and Zestienhoven in 2040, assuming 3 tanks per using the data presented in Sections 6.1 and 6.2.*

6.2.3 INFRASTRUCTURE COST

Finally, we need an indication of the cost which is associated with the infrastructure. In order to do this, we assume that the cost of a battery or hydrogen container grows linearly with it's capacity, and that the same goes for the battery charging stations and hydrogen fuel points. The parameters which have been used can be found in table 6.6. We shall explain how these were determined in this subsection.

Battery cost forecast have been well researched. Every 10 years, the price of a battery with a given capacity drops by roughly 50%. In 2020, we reached the point where batteries cost €100 per kWh. Battery lifetime is dependent on the number of charge/discharge cycles. We have assumed a maximum of 1000 cycles at a rate of 2 cycles per day, giving a 500 day life-span.

The cost of a cryogenic hydrogen tank is not as well known. We use the assumption

Table 6.6: *Daily cost for each type of infrastructure*

Year	Battery [€/kWh]	Charger [€/MW]	Hydrogen canister [€/kg]	Hydrogen fuel point [€/(kg/hr)]
2030	0.11	560	0.080	2.80
2040	0.056	650	0.093	3.25
2050	0.028	750	0.108	3.75

from [EC, 2020], which sets the price of a hydrogen tank at 250 €/kg. Assuming a lifetime cycle of 10 years, this gives 6.9 cents per kg per day. This is increased by inflation at an average rate of 1.5% per year.

The price of a charger has been deduced from transport cost of electricity, which makes up for 25% of the total cost, or approximately 2 cents per kWh. The transportation cost of electricity per day which is thus associated with a charger of 1 MW is thus given by (24 hr/day·1 MW =)$24 \cdot 10^3$ kWh/day times 0.02 €/kWh = 480 €/day. After this, inflation is added

The price of hydrogen fuel points has been determined in a similar way, using a transport cost of €0.10 per kg.

6.3 Infrastructure sizing

In this section, we shall use the knowledge of the demand for electric and hydrogen flights at Schiphol and Zestienhoven from Section 6.1 and combine this with the aircraft properties provided in Section 6.2 to determine the required recharging/refueling infrastructure. First, we assume that the generated flight schedules are fixed, and use the AC Battery Swap and Charging Schedule problem formulation to determine the optimal size of the required infrastructure for each instance. This shall be compared with the infrastructure which is required to accommodate plug-in recharges/refueling. Second, we shall assume that a fraction of the flights does not have a fixed but requested arrival and departure time, and shall use a modified version of the AC Slot Allocation and Battery Charge Schedule problem to perform a strategic analysis to determine which infrastructure combination is most cost effective for each desired level of service.

The case of Schiphol in 2040 shall be discussed it in full detail in Subsection 6.3.1. The final result of all cases is presented in Subsection 6.3.2.

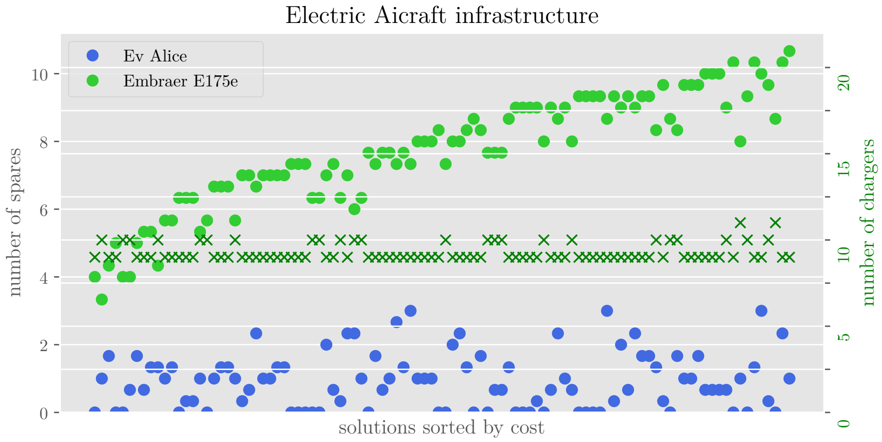
6.3.1 Schiphol in 2040

In this subsection, we shall study Schiphol in the year 2040, which is the first year when all aircraft from table 6.3 are in service. During the indicative first week of April, the Eviation Alice services 5, the Embraer E175e 20, the Airbus ZEROe turboprop 27, and the ZEROe turbofan 87 flights per day. At this time, there is no demand for flights which can be serviced by the Bombardier Dash-8 retrofitted by Universal Hydrogen. With this demand for different flights, 150 flight schedules are created and used as input, and each of these instances has been solved without the assistance of heuristics.

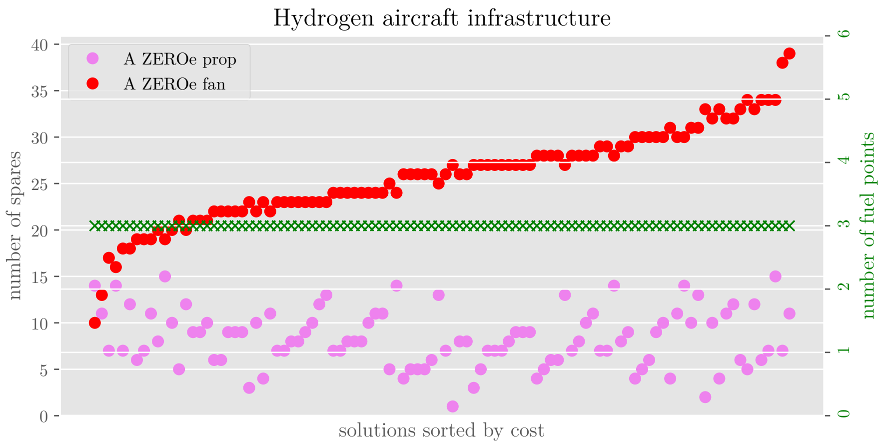
The AC Battery swap and charge schedule problem

First, we shall use the flight schedule as input for the AC Battery Swap and Charge Schedule problem. The infrastructure of all optimal solutions is shown in Figure 6.5, sorted by the investment size. These solutions are analyzed in this subsubsection, where we shall compare the differences between charging via the *swap* and *plug-in* strategy, and between drawing the required energy from the grid on a as-needed base and using a storage facility at the airport as a buffer to lower the peak demand.

The first set of results in Figure 6.6 provide a detailed breakdown of the average demand for electricity and hydrogen throughout the day. In both figures, the power demand for plug-in charging is also shown. Several features can be observed. First, the plug-in strategy results in a very peaky demand relative to the *swap* strategy, which correspond to the multiple arrival banks observed in figure 6.3a. This indicates that the charging stations and hydrogen fuel points are relatively expensive compared to the spare batteries/hydrogen canisters. Indeed, the cost of a spare battery pack for the Eviation Alice and the Embraer E175e are €280 and €1150, whereas the cost of one charging station is €840. Similarly, the cost of a spare hydrogen canister set for the ZEROe turboprop and turbofan are €36 and €237 , whereas a fuel station costs just over €5000 (all of these prices are on a daily basis). Second, between approximately 4:00 in the night and 7:00 in the morning, the demand for electricity and hydrogen almost vanishes. This is a result of the assumption that each battery has to be charged before the end of the day of operations (at approximately 6:30 in the morning).



(a) *Electricity*

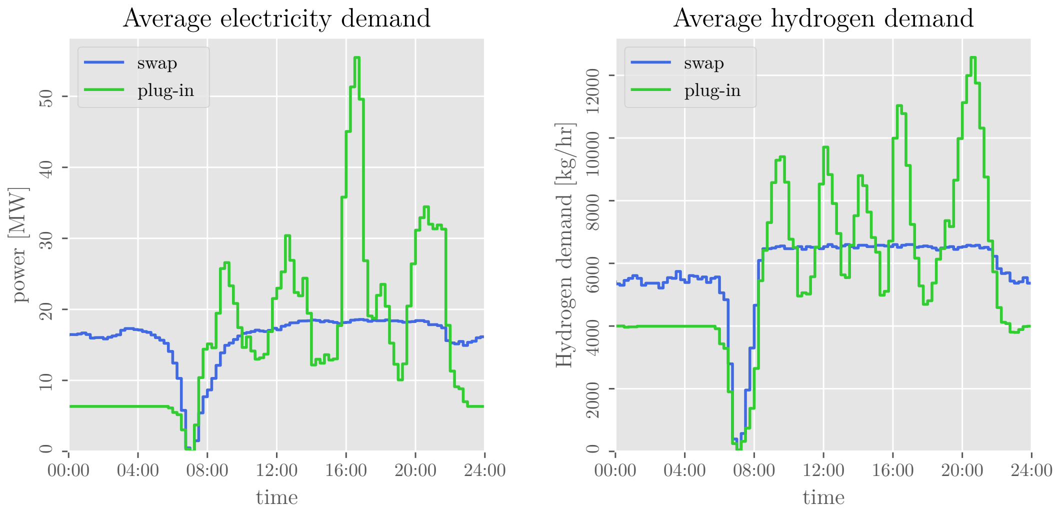


(b) *Hydrogen*

Figure 6.5: *Optimal infrastructure size for all of the 150 generated flight schedules for electric (6.5a) and hydrogen (6.5b) aircraft at Schiphol in 2040 for a reference summer season week. The number of batteries are counted as battery packs per aircraft, and the solutions are sorted by increasing investment size. Spare batteries are shown as circles, while the chargers/fuel points are shown as green crosses (×).*

The large difference in peak demand for energy between the *swap* and *plug-in* strategies can be seen in figure 6.7. In accordance with Figure 6.6, the required peak power for the *swap* strategy is highly concentrated around the minimum acceptable value. If the assumption that batteries need to be fully charged by the end of the day of operations can be improved, the required peak power could be brought down further. Salient is the large difference in the demand between the *swap* and the *plug-in* strategy, as well as the spread of demand within *plug-in* itself.

Alternatively, the peak load on the grid can be spread more evenly throughout the



(a) *Electricity* **(b)** *Hydrogen*

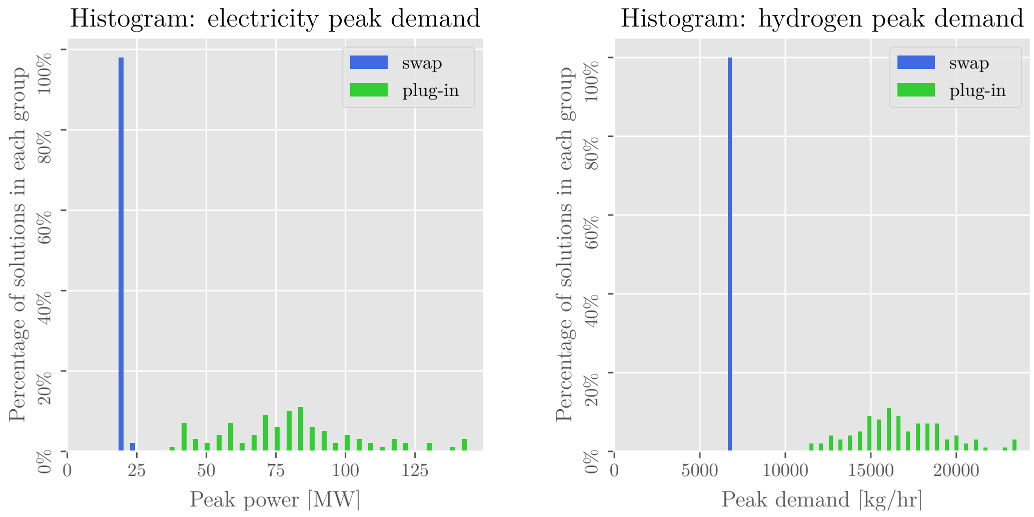
Figure 6.6: *Average demand for electricity (6.6a) and hydrogen (6.6b) for the optimal infrastructure combinations at Schiphol in 2040 for a reference summer season week throughout the day. Charging is done via either the swap (corresponding to the solutions in Figure 6.5) or plug-in strategy, and no deviations are allowed from the requested flight schedule.*

day by the introduction of a storage facility for electricity or hydrogen at Schiphol. Assuming a constant supply of electricity and hydrogen from the grid to the airport, the size of these storage facilities has been determined. This has been done with the following conversion: let E_i be the demand for energy during each time instance $t_i \in T$, such that the total demand for energy is given by $E = \sum_i E_i$. In order to minimize the tension on the grid, the airport demand is given by a constant $E/|T|$. Using this, the cumulative energy demand at time t_i is given by $\text{CSup}_i = i \cdot E/|T|$, and the cumulative demand by $\text{CDem}_i = \sum_{i' \leq i} E_i$. Notice that at the last time instance, the cumulative supply is equal to the demand. The required storage facility size S at the airport to handle this demand is then given by:

$$S = \max_i \{\text{CSup}_i - \text{CDem}_i\} - \min_i \{\text{CSup}_i - \text{CDem}_i\}, \tag{6.1}$$

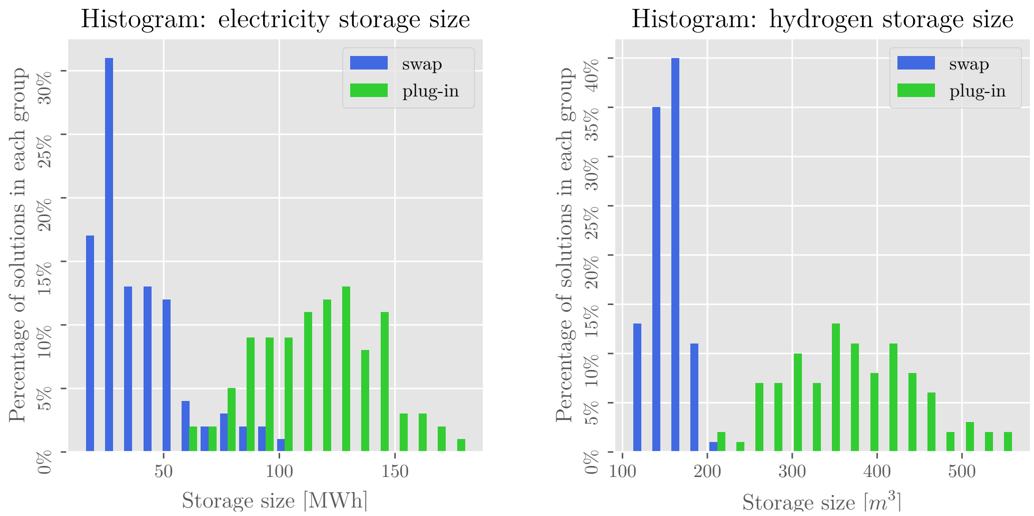
where this is the smallest possible size which ensures that there is enough energy to supply the demand to the aircraft. The distribution of S is graphed in Figure 6.8. The *swap* strategy still requires a smaller infrastructure size, but the difference between the two is less extreme when compared to the previous result.

Third, similar to Figure 6.7, the spread in the required number of batteries for each type of aircraft is graphed in Figure 6.9. These graphs show a histogram with the required amount of spare battery packs which are sufficient to power one aircraft of the corresponding type.



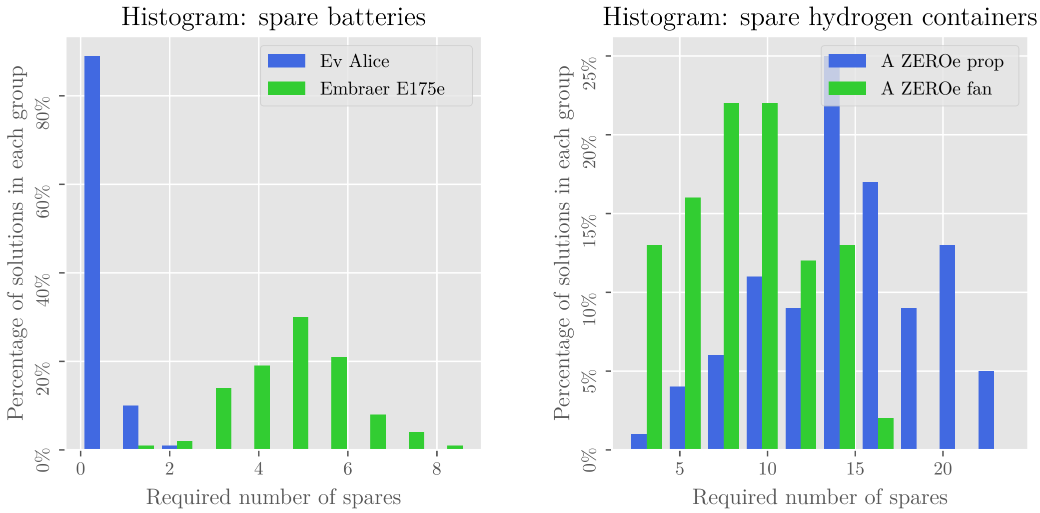
(a) *Electricity* **(b)** *Hydrogen*

Figure 6.7: *Distribution of the required peak supply of electricity (6.7a) and hydrogen (6.7b) at Schiphol in 2040 for a reference summer season day. Charging is done via either the swap (corresponding to the solutions in Figure 6.5) or plug-in strategy, and no deviations are allowed from the requested flight schedule.*



(a) *Electricity* **(b)** *Hydrogen*

Figure 6.8: *Distribution of the required storage size for electricity (6.8a) and hydrogen (6.8b) at Schiphol in 2040 for a reference summer season day. We assume a constant supply of electricity/hydrogen from the grid to the airport. Charging is done via either the swap (corresponding to the solutions in Figure 6.5) or plug-in strategy, and no deviations are allowed from the requested flight schedule.*

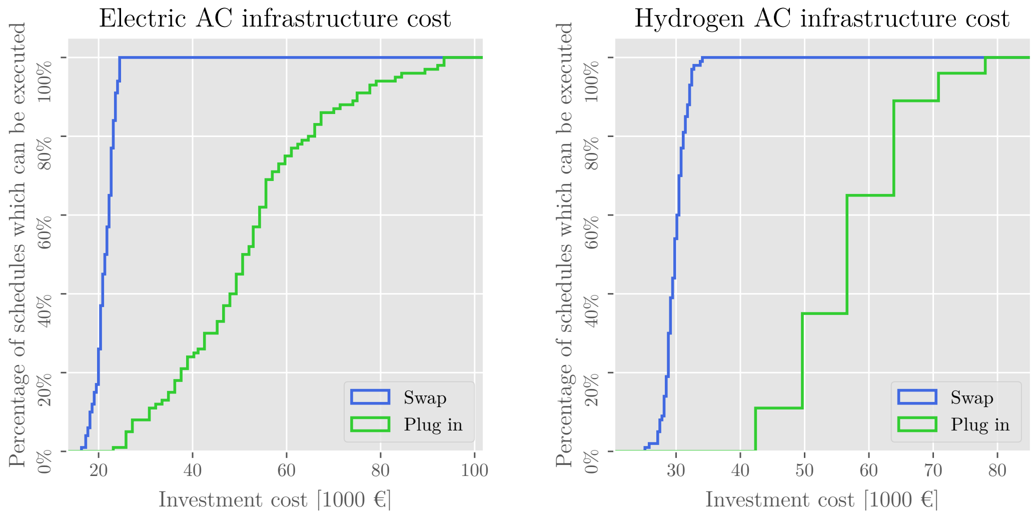


(a) *Electricity* **(b)** *Hydrogen*

Figure 6.9: *Histogram of the required amount of spare battery packs (6.9a) and spare hydrogen canister sets (6.9b) for the optimal infrastructure combinations from Figure 6.5 at Schiphol in 2040 for a reference summer season day.*

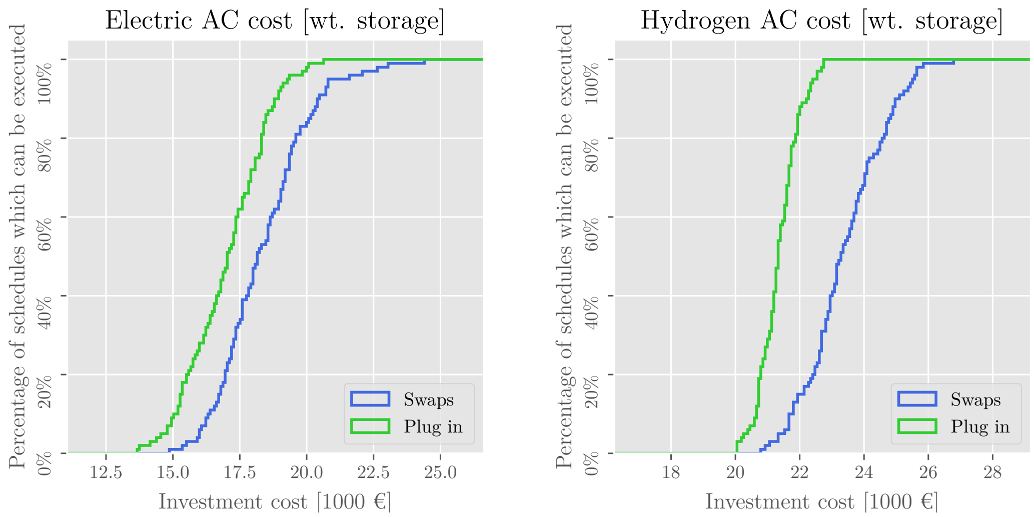
Finally, the cumulative distribution of the required investment of the optimal solutions is graphed in Figures 6.10 (without an energy storage facility) and 6.11 (with an energy storage facility). One can observe that without a energy storage facility, the peak demand for energy with the *plug-in* strategy is so large that the costs dwarf the cost of using the *swap* strategy. On the other hand, when using an on-site energy storage, the investment sizes become very similar.

There is, however, a very important difference between the results for the *swap* and *plug-in* strategies, which is related to how the curves can be interpreted. Since there is only one piece of infrastructure on which the investment is used for the *plug-in* strategy, this investment distribution is directly related to the percentage of the days for which the infrastructure bought with this investment is sufficient: if a hydrogen storage tank is acquired with an operating cost of 22k€, one can deduce from Figure 6.11b that it will be sufficiently large to serve %90 of the days. On the other hand, a swap strategy infrastructure consists (in this example) of three components (the chargers/storage facilities, and two types of spares) and in each optimal solution the ratio between the number of components is different (as shown in Figure 6.5). Therefore, Figures 6.10 and 6.11 do not provide the investment size, given a percentage of the flight schedules which have to be executed, thus the costs will be higher then the ones which are on display in these Figures. In the next subsubsection, we shall perform a strategic analysis to determine the actual investment cost.



(a) *Electricity* **(b)** *Hydrogen*

Figure 6.10: *Required investment in electric (6.10a) and hydrogen (6.10b) infrastructure at Schiphol in 2040 for a reference summer season day. Charging is done via either the swap or plug-in strategy.*



(a) *Electricity* **(b)** *Hydrogen*

Figure 6.11: *Same as figure 6.10, but with a storage facility for hydrogen and electricity.*

AC Slot-Allocation and Battery Charge Schedule problem

Now we shall perform a strategic analysis of the required infrastructure size, using the AC Slot-Allocation and Battery Charge Schedule problem. We use the following parameters for this problem:

Number of flights without slots	20 %
Minimum acceptable solve	95 %

Thus, 80% of the flights already has hold of a slot and have to arrive and depart exactly on the given times. The other 20% of the flights are requesting a slot, and the coordinator can deviate from the request, but at a cost of lowering the level of service which the airport provides. The other aspect of the level of service is given by the minimum percentage of the days on which a solution cannot be provided without requesting at least one flight which already has a slot to deviate from it. This value has been set to 95%.

Since the introduction of multiple batteries increases the number of possible infrastructure combinations far beyond what can be handled using the regular strategic analysis algorithm (even with the enhancements detailed in Appendix C), only a subset of all interesting infrastructure combinations shall be studied. This set is deduced from figure 6.5, and is listed below:

	Electric		Hydrogen	
Chargers		9 - 11		3
Spares	Alice	2 - 8	ZEROe turboprop	4 - 14
	E175e	22 - 36	ZEROe turbofan	18 - 38

Each infrastructure combination has been tested on 150 new instances and the maximum deviation of the slots has been stored. Infrastructure combinations which failed to provide a schedule without moving a fixed flight less on less then 95% of the instances were discarded, and the maximum deviation of the 95% percentile is used as the performance for each infrastructure combination. For levels of service ranging from 0 to 90 minutes, the most cost effective solution which could provide the desired level was determined. The results are graphed in Figure 6.12 and in Tables 6.7 and **??**.

Figure 6.12 shows the required operational investment to provide a given level of service on at least 95% of the days. As can be seen, the required investment ranges from 22 to 24 k€ for electric aircraft, and 27 to 30 k€ for hydrogen aircraft.

Table 6.7 provides an analysis of the performance for the optimal infrastructures for levels of service of 0, 30, 60, and 90 minutes. The table lists, from left to right, the level of service, the number of chargers, the number of spares divided by the number of spares per aircraft, the investment size, the percentage of the days on which the level of service was attained, the average value of the maximum deviation from the requested slots, and the robustness to delays. This robustness is measured as the average percentage of flights which need to be delayed at the airport because there are not enough spare batteries/ hydrogen canisters to ensure that the scheduled departure time is met. The data from Figure 6.2 has been used as input for the delays. How these values are determined can be found in Appendix D. The corresponding distribution of

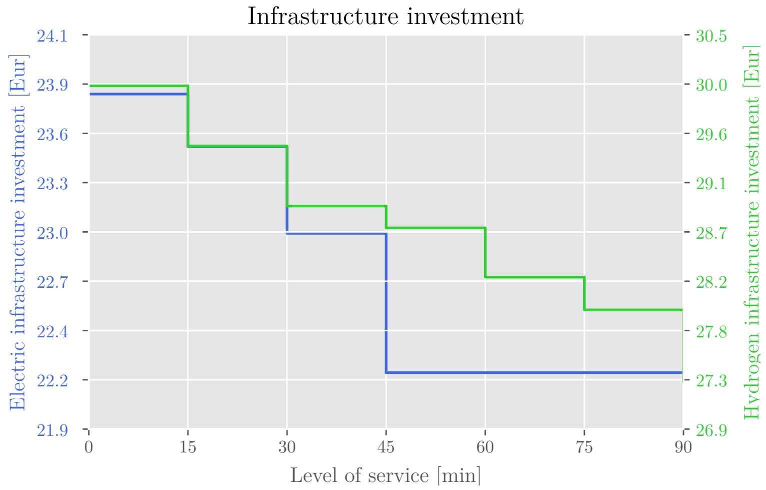
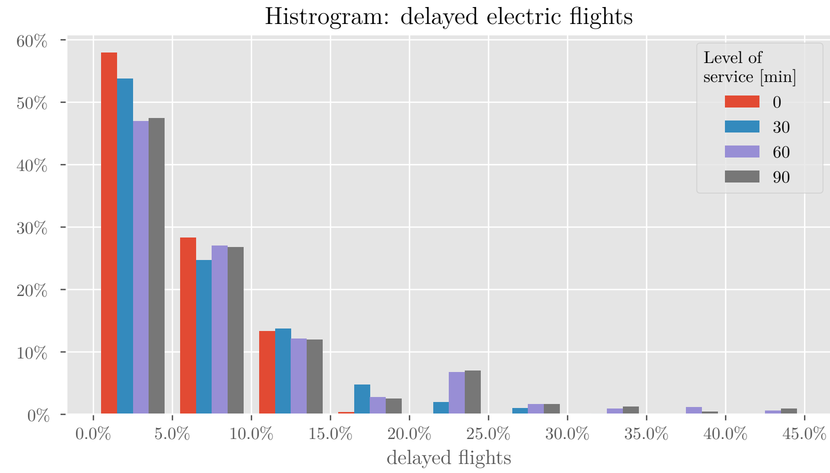


Figure 6.12: *Required investment in electric and hydrogen aircraft infrastructure at Schiphol in 2040 for a reference summer season day for different levels of service. The level of service is given by the 95 percentile of the maximum deviation of the slot allocation from the requested slots. The investment is given in k€.*

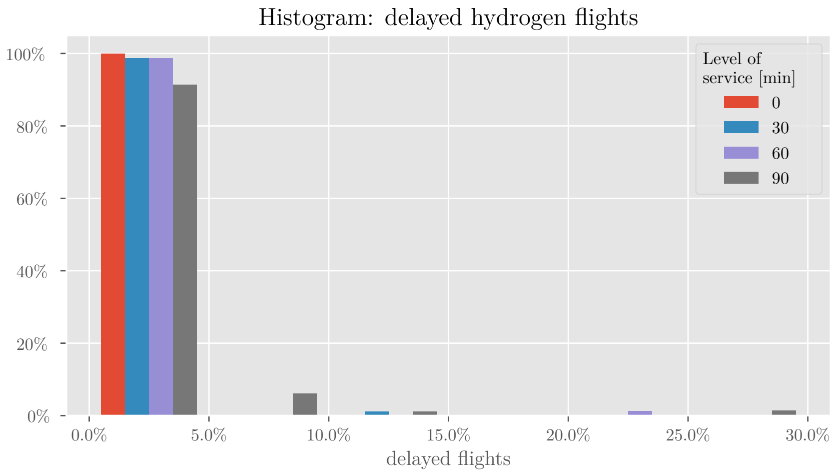
the percentage of delayed flights can be found in Figure 6.13. These results, together with the ones from the other five cases shall be discussed in the next subsection.

Table 6.7: *Properties of the optimal infrastructure combinations to support electric and hydrogen aviation for different levels of service [LoS] at Schiphol in 2040. The infrastructure size, given by the number of chargers and spare batteries, as well as the investment size are shown. In the last two columns the result of the numerical simulation is shown. The column Sufficient denotes the percentage of the days on which the infrastructure is sufficiently large, and Max dev. gives the value of the maximum deviation averaged over all instances. Finally, the column Delays denotes the percentage of flights which is delayed due to insufficient charging infrastructure.*

LoS [min]	chargers	spares		Investment [k€]	Sufficient [%]	Max dev [min]	Delays [%]
Electric		Alice	E175e				
0	10	1.3	8.6	23.8	100.0	0.00	3.61
30	10	1.3	8	23.0	96.2	2.10	4.71
60	10	1.3	7.3	22.2	96.2	6.54	7.17
90	10	1.3	7.3	22.2	97.5	6.54	7.19
Hydrogen		prop	fan				
0	3	6	26	29.99	97.5	0.36	0.09
30	3	10	22	28.90	97.5	3.36	0.17
60	3	10	20	28.26	97.5	9.96	0.30
90	3	4	18	27.32	95.0	25.44	1.51



(a) *Electricity*



(b) *Hydrogen*

Figure 6.13: *Histogram of the percentage of delayed electric (6.13a) and hydrogen (6.13b) flights caused by an insufficient infrastructure size, for each of the combinations listed in Table 6.7.*

6.3.2 Final results

In a method which is completely analogous to the one discussed in the last subsection, the optimal infrastructure sizes for the other five cases have been determined for each level of service ranging from 0 to 90 minutes. These results can be found in tables 6.8 through 6.9, which are organized in the same way as table 6.7. The investment in infrastructure for electric aircraft range from 4.5k€ (Zestienhoven 2040, regardless of the level of service) to 29k€ per day (Schiphol 2050, highest level of service). The investment in hydrogen infrastructure ranges from 7.5k€ to 65.3 k€ per day (again for Zestienhoven in 2040, Schiphol in 2050).

Several features can be observed in the tables:

- **Economics of scale:** First, note that the infrastructure price for low demand are relatively large compared to the cases in which the demand is large, which is a consequence of the fact that the relatively expensive charging/fuel points have a low utilization for those cases.

- **Level of service genuinly affects the investment size:** Second, in the two cases with large demand for electric/hydrogen aircraft, lowering the promised level of service causes a significant decrease in the required infrastructure investment. For electric/hydrogen aircraft in Schiphol 2040/2050, the drop is respectably given by 7, 10, 12, and 8% for lowering the level of service from 0 to 90 minutes.
- **The solutions overperform on each level of service except 0:** Finally, when one regards the average value of the maximum deviation and observes that this is often significantly smaller than the indicated level of service[1], it is possible to lower the expenses even further while still generally being able to offer the promised level of service on the vast majority of the days. This is, naturally, a consideration which will have to be made at the airports.

Table 6.8: *Same as Table 6.7, but for Schiphol in 2030 and 2050*

Schiphol 2030

LoS [min]	chargers	spares		Investment [k€]	Sufficient [%]	Max dev [min]	Delays [%]
Electric		Alice	E175e				
0	8	1.0	3.3	12.77	93.0	1.08	5.24
30	8	1.0	3.3	12.77	94.0	1.08	5.22
60	9	1.0	3	12.55	93.0	2.70	8.96
90	8	1.0	3	11.99	95.0	3.00	8.80
Hydrogen		prop	fan				
0	1	1	1	6.45	96.0	0.0	14.80
30	1	1	1	6.45	96.0	0.0	14.86
60	1	1	1	6.45	96.0	0.0	14.89
90	1	1	1	6.45	96.0	0.0	14.86

Schiphol 2050

LoS [min]	chargers	spares		Investment [k€]	Sufficient [%]	Max dev [min]	Delays [%]
Electric		Alice	E175e				
0	14	1.7	13	28.99	99.0	0.60	0.08
30	13	1.7	11.7	26.70	96.0	3.06	0.11
60	13	1.7	11.7	26.70	98.0	3.06	0.10
90	13	1.7	10.3	25.90	96.0	14.82	0.16
Hydrogen		prop	fan				
0	6	8	41	65.29	98.0	0.48	0.00
30	6	11	37	63.97	98.0	3.12	0.00
60	6	8	33	62.30	98.0	11.40	0.00
90	6	5	29	60.64	97.0	26.46	0.01

[1] which is given by the 90 percentile value of the maximum deviation

Table 6.9: *Same as Table 6.7, but for Rotterdam in 2040 and 2050*

Zestienhoven 2040

LoS [min]	chargers	spares		Investment [k€]	Sufficient [%]	Max dev [min]	Delays [%]
Electric		Alice	E175e				
0	2	1.0	1.3	4.49	96.0	1.08	0.0
30	2	1.0	1.3	4.49	97.0	1.08	0.0
60	2	1.0	1.3	4.49	97.0	1.08	0.0
90	2	1.0	1.3	4.49	98.0	1.08	0.0
Hydrogen		prop	fan				
0	1	0	1	7.43	99.0	0.0	21.77
30	1	0	1	7.43	99.0	0.0	21.77
60	1	0	1	7.43	99.0	0.0	21.77
90	1	0	1	7.43	99.0	0.0	21.77

Zestienhoven 2050

LoS [min]	chargers	spares		Investment [k€]	Sufficient [%]	Max dev [min]	Delays [%]
Electric		Alice	E175e				
0	7	1.3	2.3	13.69	99.0	0.12	4.35
30	7	1.0	2.3	13.59	99.0	0.60	6.19
60	7	1.0	2.3	13.59	99.0	0.60	6.19
90	7	1.0	2.3	13.59	99.0	0.60	6.20
Hydrogen		prop	fan				
0	1	1	2	8.95	100.0	0.0	27.84
30	1	1	2	8.95	100.0	0.0	27.82
60	1	1	2	8.95	100.0	0.0	27.86
90	1	1	2	8.95	100.0	0.0	27.84

CHAPTER 7

Discussion and conclusion

The aim of this research was to develop an extensive optimization approach to the problem of determining the required infrastructure for electric and hydrogen aircraft on a medium- and long-range time scale. In the first chapter, we performed a literature review on the relevant aviation aspects, studying both future aircraft and airport operations. In Chapter 3, we used this knowledge to assess the existing optimization models for electric aircraft operations and defined the research gaps. In Chapters 4 and 5, we developed an optimization framework and model that partially filled these gaps, created several heuristics, and tested its performance. Finally, in Chapter 6, the models were applied in a case study at Amsterdam airport Schiphol and Rotterdam - The Hague airport Zestienhoven. We shall start this chapter by presenting our main conclusions, and discuss each component in further detail in the following sections.

Main conclusions and answer to the research question

This study is, within the scope of the assumptions made in the introduction, concerned with the following research question:

For which instance sizes is it possible to optimize the required infrastructure size to support the battery- and tank-swap of respectively electric and hydrogen aircraft, and what are suitable alternatives for beyond this limit?

We have seen that the answer to this question depends on the used charging policy (preemptive, non-preemptive), the acceptable deviation from the requested flight schedule (with and without slot-allocation), and the number of used battery types. Under the strongest conditions (with only a single battery type), a model has been developed which is able to optimize instances with up to around 200 batteries when each aircraft arrives and departs at the requested times, and 50 batteries with slot allocation is allowed. Increasing the number of different battery types has been shown to increase the instance sizes which can be optimized. It has been shown that the processing time can be reduced by a factor ten when using one of the heuristics, which also allows much larger instances to be handled. Finally, larger instances will be solvable with either updates to the linear programming solvers, or with the use of corporate computers.

Three types of recommendations can be made for future study. First, the model can be improved within the scope of this research by taking operational delays into account when creating recharge schedules, by introducing day interweaving, and by developing multi-battery-type heuristics. The scope of the research can be expended by introducing ground support equipment as a part of the infrastructure, by determining the optimal charging station location, and by solving the infrastructure optimization as a network problem. Finally, the model is ready to be applied to many more applications beyond the case study. One can use this model to research the asymptotic infrastructure cost per flight, the sensitivity to more levels of service, and the importance of battery/hydrogen canister standardization.

Tactical infrastructure optimization (Chapters 4 and 5)

In Chapters 4 and 5, we have created an optimization model for the infrastructure requirements in the tactical planning phase (Section 2.2.1). In total, four different model variants have been created, summarized in the following table:

model	version	
	preemptive	non-preemptive
AC BATTERY SWAP & CHARGE SCHEDULE	page 36	page 40
AC SLOT ALLOCATION & CHARGE SCHEDULE	page 53	page 57

In the AC BATTERY SWAP & CHARGE SCHEDULE problem and the AC SLOT ALLOCATION & CHARGE SCHEDULE problem, the infrastructure requirements are determined for a fixed and flexible flight schedule respectively. The latter model is a generalization of the former. The preemptive and the non-preemptive version of the models can be applied to both hydrogen and electric aircraft operations, though we have observed in the literature study that the non-preemptive version may be needed for liquid hydrogen fueling operations and that the preemptive version may be favorable for electric charging because of smaller investment requirements.

In addition to the basic model versions, we have developed several extensions which are used to generalize the model and make it more applicable in practical situations. These are the inclusion of multiple battery types, the possibility of on-site energy generation and storage, and a time-dependent cost of energy. The first two of these have been implemented in the case study.

Several heuristics have been developed with two purposes: being able to solve larger problem instances within reasonable time, and being able to do so with open source software. For the AC BATTERY SWAP & CHARGE SCHEDULE problem, we created various versions of the SPT algorithms and improved the FIFO algorithm proposed in [Justin et al., 2020]. The runtime of the SPT and FIFO algorithm to assess the feasibility of one infrastructure set is given by $\mathcal{O}(|J|)$ and $\mathcal{O}(|J|^4)$ respectively, where $|J|$ is the number of batteries which have to be processed. In order to find the optimal infrastructure combination, the running time is multiplied with $\mathcal{O}(|J|)$. For the AC SLOT

Allocation & Charge Schedule problem, we created the binary search and bottleneck solving local search heuristic, both of which work in tandem with the algorithms created for the AC Battery Swap & Charge Schedule problem. The running time of each of these heuristics is given by the running time of the used AC Battery Swap & Charge Schedule method multiplied with $\mathcal{O}(\ln T)$ and $\mathcal{O}(|J| \ln T)$ respectively, where T is the maximum accepted deviation from the requested flight schedule.

The performance of the models and heuristics for the base problems have been extensively studied in Sections 4.5 and 5.5. Implementations of the (M)ILP formulations for all of the problem combinations were able to solve instances with up to 200 batteries, except for the preemptive version of the AC Slot Allocation & Charge Schedule problem, which was limited to around 100 batteries. The computation time of the algorithms can be drastically reduced by the use of the various heuristics.

For the AC Battery Swap & Charge Schedule problem, the FIFO(RP) heuristic outperforms the others by a significant margin. Most notably, it outperforms the FIFO(R) algorithm from [Justin et al., 2020] on the test data. Finally, in this report the FIFO algorithms were solved using the LP formulation of the max-flow problem, replacing a combinatorial optimization approach for the Max Flow problem like the Ford-Fulkerson or Edmonds-Karp algorithms.

For the AC Slot Allocation & Charge Schedule problem, the binary search algorithm combined with the FIFO(RP) policy has generated the results which most closely approximate the optimal solutions. The main disadvantage of this method is that it is restricted to minimizing the deviation of the maximum deviated slot. The method is in itself unable to provide a solution for which the flights are as close to their requested slots as possible. The alternative, the bottleneck resolving greedy algorithm, was designed to resolve this issue, but performed very poor in practice. Developing a more advanced version of the binary search heuristic is therefore an interesting topic for future research.

Strategic infrastructure optimization (Chapter 5)

The discussed models were created in order to determine the most cost-effective infrastructure combination for a given flight schedule. However, this flight schedule is only known to the airport a couple of months before the day of operations, too late for the acquisition of extra charging/fueling stations or the construction of a larger storage. In order to determine the *real* optimal infrastructure, a strategic analysis has to be performed. This is an important differentiation between this study and the previously conducted ones from Section 3.2. Our approach to strategic scheduling is presented in Sections 2.2.1 and 5.5 for a single battery type problem, and in Section 6.3 for the multi-battery problems encountered in the case study.

The problem faced with the strategic analysis, that a large number of infrastructure combinations have to be evaluated, has been mostly eliminated with the efficient searching algorithm proposed in Appendix C. This allows instances of up to 200 batteries for the AC Battery Swap & Charge Schedule problem, and for up to 50 and 120 for respectively the preemptive and non-preemptive versions of the AC Slot Allocation & Charge Schedule problem to be evaluated in a strategic analysis.

The heuristics which have been used in the tactical analysis exhibit a similar performance in the strategic analysis, but it must be said that the metric which was used to measure the performance of the heuristics (which can be found in figure 5.9a) is not a qualitatively high measure for their performance. The reason for this is the fact that the metric fails to recognize that the performance at possibly interesting infrastructure combinations is more relevant than the performance at other combinations. A better metric could consider the investment size which needs to be made to guarantee a certain level of service with each heuristic, and compare this to the investment size when using the optimal solutions.

Case study (Chapter 6)

The solution methods developed for the AC Battery Swap & Charge Schedule and AC Slot Allocation & Charge Schedule problem have been applied in a tactical and strategic analysis in a case study at Amsterdam Schiphol and Rotterdam - The Hague Zestienhoven airports. A comparison has been made of the investment size between using battery swaps (which are assumed in both models) and conventional charging/fueling, and between using and not using an on-site storage facility.

In order to perform the study, the following assumptions have been made. In brackets, the relative strength of the assumptions is given.

- **Aircraft model (Medium)**: The models from Section 6.2 have been used to determine the energy demand for different flights. This model is a crude approximation of reality, given that it is based on empirical data for kerosene aircraft. Nevertheless, is is able to produce values for the Eviation Alica and the Universal Hydrogen version of the Bombardier Dash-8 which are relatively close to the official issued figures.

- **Aircraft demand (Weak)**: The demand for electric and hydrogen flights has been estimated using the matrix in Table 6.2. However, there is, as of writing, no accurate prediction for the demand of these flights. This value is therefor the weakest assumption made in the case study.

- **Infrastructure cost (Medium)**: The cost of a charger/fuel point does only include the transportation cost of the energy, and the cost of on-site storage only includes the cost of the battery size/fuel tank. The ground-support equipment and labor cost are not included, and the cost of a hydrogen tank is assumed to grow linearly with its volume. The infrastructure cost is thus expected to be underestimated in this case.

- **Level of service (Strong)**: Finally, the metric by which airports are going to measure the performance of charging/fueling infrastructure is uncertain. The percentage of days on which the infrastructure does not cause delays was used in this report, but there are more criteria which can be used as a metric for the level of service. Some examples are: the average value of the maximum deviation, the number of days on which a feasible solution is produced, or a shift in the percentile at which the maximum deviation is measured (which has been set at 95% in this study). Especially the latter can cause a large difference in the infrastructure size.

Because of the different types of aircraft which are used, each type having its own battery/hydrogen tank, we were able to optimize instances with input sizes way beyond which were achieved for the base model. This is caused by the very little interaction that the different battery/tank types have, only encountering each other in the constraint which determines the peak demand. It is expected that the instance size of the problems which can be solved on a tactical level will grow linearly with the number of battery types (in a regime with large demand and a relative small number of different types), since the model size decreases linearly with the battery types and increases linearly with the number of batteries.

Solving the strategic phase with an increasing number of battery types is more difficult, because the number of possible infrastructure combinations is given by the product of the total number of batteries, and the product of the number of batteries from each type. In our case study this issue has been resolved in a semi-automated way, by selecting a subset of all infrastructure combinations and discarding those that appear implausible to be optimal, reducing the number of combinations to under a hundred.

The required investment size per outbound passenger is summarized in Table 7.1 for two different levels of service. The models show a decrease in the investment cost per passenger with increasing demand, but for hydrogen also a high dependency on the utilization of the available refueling station. More cost-effective solutions may be achieved by reducing the size of the fuel station, even though this may lead to an increase in the required number of spare batteries. This study shows a tendency of the cost per passenger to converge towards €10 for electric- and €1.50 for hydrogen aviation, but the dependency of this cost as a function of the demand will have to be verified in future work.

Table 7.1: *Optimal required investment in infrastructure per outbound passenger using battery/tank swaps, as displayed in Tables 6.7 through ??. All displayed values are in €, and inflation has been taken into account.*

Energy	LoS	Schiphol			Zestienhoven	
	[min]	2030	2040	2050	2040	2050
E	0	25.00	11.55	10.00	33.10	36.83
	30	25.00	11.16	8.93	33.10	36.56
H	0	4.60	1.49	1.75	24.77	4.26
	30	4.60	1.34	1.61	24.77	4.26

Finally, we have shown in the case study that the cost of the infrastructure when using a conventional (plug-in) fueling/charging method is heavily dependent on the decision whether or not on-site storage of electricity and hydrogen is going to be implemented. Without this storage, the costs due to the large peak demand far exceed the costs with plug-in charging, but with storage, they are on a very similar order of magnitude. When a storage facility is used, the answer to which method is most cost-effective will depend on the cost of the ground support equipment, responsible for transporting the energy from the storage to the aircraft. Obtaining these costs and determining its implications could be studied in future work, but was not the focus of this research.

Recommendations for further research

A few points for future work are already mentioned in the discussion sections above. This section will provide an overview of the steps on which future research could focus. The recommendations are categorized in three classes: improvements of the model within the existing scope, expansion of the scope, and improvements in the applications. Within each class, recommendations are sorted by added value to the field.

Model improvement recommendations

First, there are a couple of manners in which the models from Chapters 4 and 5 can be improved further within the scope of this research:

- **Delays**: In this study, the effects of operational delays on the supply and demand for assets has only been studied after the creation of the swap- and charge-schedules. No rigorous method has been developed for optimizing a charge schedule's robustness to delays. Future research could focus on how to use, e.g. stochastic programming and Monte Carlo sampling, to optimize the models while taking this operational performance into account.

- **Day interweaving**: A flaw of the model, which can be noticed best when looking at the average energy consumption figures on page 81, is that the demand for energy is nearly constant throughout the day except for the couple of hours just before the start of the day. This causes higher average demand levels throughout the rest of the day and increases the required storage facility size, and is a result of the assumption that jobs have to be processed by the end of the day of operations. By easing this assumption and interweaving the charging process of neighboring days, more cost-effective infrastructure solutions may be found.

- **Multiple battery heuristics**: As we have seen in the case study, the optimization models which were developed have been capable of solving all generated instances to optimality within a reasonable time. However, problems for which an aircraft carries more batteries/hydrogen tanks, or for which the time limit will be longer than a single day, may not be solved to optimality within this timeframe. The FIFO(RP) heuristic, which was found to be recommendable (see Sections 4.5 and 5.5), is for a strategic analysis still limited to a single battery type. With a couple of minor modifications, and the use of the same method to solve multi-battery type strategic analysis from Section 6.3, we expect that this heuristic can be expanded to cope with multiple types.

Scope recommendations

Secondly, in order to move the model closer to real-world applications, the following subjects can be added to the scope:

- **Ground Support Equipment**: A number of infrastructure components has not been taken into account during our analysis, notably: spare battery housing, connectors of energy from the grid to the batteries, labor, and ground support equipment (GSE) for battery transportation. Each one of these is a vital part of the infrastructure, but the last is missed most as its cost cannot be added to an existing component[1], but needs to be a component by itself. Each GSE unit will be tasked with transporting batteries to several flights, which has to be done such that the GSE fleet is as small as possible.

- **Topology**: With the inclusion of transportation of the batteries/hydrogen canisters across the airport, the topology of the airport starts to pour in. The question of where to place the charging station(s) at the airport such that the expected transportation cost become minimal becomes relevant. Before the models which are able to optimize this can generate relevant results, other studies will have to be conducted which are able to determine the positions where the charging stations can be placed at an airport. This study will at least involve combined knowledge of airport safety and GIS/geomatics.

- **Network problems**: When airports and airlines are able to fully collaborate, and spread the demand at each airport, the required infrastructure size will be much smaller. Researching this scenario would be interesting to evaluate the possible gains, although this seems far away from reality (at least for now). First, optimizing the flight schedule for the required infrastructure can cause problems in other aspects of operations like transfer connections. Second, airlines live in a competitive world, and this level of attunement requires a high level of collaboration. Last, the coordination of all flights over a network of airports will induce so many constraints that the problem sizes may be too large to handle.

Application recommendations

Finally, the implementation of the developed models for case studies and other real world applications can be improved. A number of ways in which this can be done are:

- **Infrastructure dependency per flight**: Since the estimations for the demand of electric/hydrogen flights are still in an early stage, and the infrastructure size heavily depends on this, it may be more relevant to determine the required infrastructure size per flight (or passenger) as a function of the number of flights which uses these facilities.

- **The importance of battery standardization**: In the conducted case study, aircraft types of different sizes used different battery types. However, the developed models can also be used to determine the effects of the introduction of multiply battery types for the same aircraft classes, simulating non-compatibility between different manufacturers. Computing the increased infrastructure investment can give insight in the importance of battery standardization.

- **Sensitivity to operational parameters**: A second interesting application of the models would be to study the sensitivity of the investment size (per flight / passenger / PK[2]) to more of the input to more input operational parameters. In the case study, the percentage of fixed flights was set to 80, and the percentile at which the maximum delay was measured was set at 95, but these values can differ between airports.

[1] Spare battery housing cost can be included in the spare battery cost, the connectors in the charger cost, and labor within both.

[2] passenger kilometer

- **Storage size optimization**: In our case study, the model objective was set to the combined cost of the spare batteries and chargers, and the resulting solutions were converted to include an on-site energy storage which was as small as possible (results at Figures 6.8 on page 82 and 6.11 on page 84). This has been done to determine the possible gains of using on-site storage. But these gains can possibly be even larger when the storage cost is also included in the cost function before the model is optimized.

Bibliography

[Bazargan, 2010] Bazargan, M. (2010). *Airline Operations and Scheduling*. Ashgate Publishing Company, Surrey, 2 edition.

[Belobaba et al., 2009] Belobaba, P., Odoni, A. R., and Barnhart, C. (2009). *The Global Airline Industry*. John Wiley & Sons, Ltd, Hoboken, 1 edition.

[Bigoni, 2018] Bigoni, F. (2018). Impact of hybrid-electric aircraft operations. Master's thesis, Polytecnico di Milano, Milan.

[Blain, 2020] Blain, L. (2020). Universal Hydrogen and Magnix building world's largest hydrogen plane. https://newatlas.com/aircraft/universal-hydrogen-magnix-largest-hydrogen-plane/.

[Brelje and Martins, 2019] Brelje, B. J. and Martins, J. R. R. A. (2019). Electric, hybrid, and turboelectric fixed-wing aircraft: A review of concepts, models, and design approaches. *Progress in Aerospace Sciences*, 104:1 – 19.

[Bruce et al., 2020] Bruce, S., Temminghoff, M., Hayward, J., Palfreyman, D., Munnings, C., Burke, N., and Creasey, S. (2020). Opportunities for hydrogen in aviation. Technical report, CSIRO.

[Castro et al., 2009] Castro, P. M., Harjunkoski, I., and Grossmann, I. E. (2009). New Continuous-Time Scheduling Formulation for Continuous Plants under Variable Electricity Cost. *Industrial & Engineering Chemistry Research*, 48(14):6701–6714. Number: 14 Publisher: American Chemical Society.

[Che et al., 2017] Che, A., Zhang, S., and Wu, X. (2017). Energy-conscious unrelated parallel machine scheduling under time-of-use electricity tariffs. *Journal of Cleaner Production*, 156:688 – 697.

[C.L. Liu and J.W. Layland, 1973] C.L. Liu and J.W. Layland (1973). Scheduling algorithms for multiprogramming in a hard-real-time environment. *Journal of the Association for Computing Machinery*, pages 46–61.

[Darecki et al., 2011] Darecki, M., Edekstebbe, C., Enders, T., Fernandez, E., Hartman, P., Herteman, J.-P., Ky, P., Schotman, G., and Smith, C. (2011). Flightpath 2050. Flightpath 2050: Europe's Vision for Aviation, European Union.

[De Neufville and Odoni, 2003] De Neufville, R. and Odoni, A. R. (2003). *Airport systems : planning design, and management*. McGraw-Hill, New York, 15 edition.

[Ding et al., 2016] Ding, J., Song, S., Zhang, R., Chiong, R., and Wu, C. (2016). Parallel Machine Scheduling Under Time-of-Use Electricity Prices: New Models and Optimization Approaches. *IEEE Transactions on Automation Science and Engineering*, 13(2):1138–1154. Number: 2.

[Domínguez-Navarro et al., 2019] Domínguez-Navarro, J. A., Dufo-López, R., Yusta-Loyo, J. M., Artal-Sevil, J. S., and Bernal-Agustín, J. L. (2019). Design of an electric vehicle fast-charging station with integration of renewable energy and storage systems. *International Journal of Electrical Power & Energy Systems*, 105:46 – 58.

[Duvelleroy, 2020] Duvelleroy, M. (2020). Airbus Reveals New Zero Emission Concept Aircraft. https://www.airbus.com/newsroom/press-releases/en/2020/09/airbus-reveals-new-zeroemission-concept-aircraft.html.

[EC, 2020] EC (2020). Hydrogen-powered Aviation. https://www.fch.europa.eu/sites/default/files/FCH%20Docs/20200720_Hydrogen%20Powered%20Aviation%20report_FINAL%20web.pdf.

[EC, 2021] EC (2021). Destination 2050 - a route to net zero european aviation. https://www.destination2050.eu.

[Fang and Lin, 2013] Fang, K.-T. and Lin, B. M. T. (2013). Parallel-machine scheduling to minimize tardiness penalty and power cost. *Computers & Industrial Engineering*, 64(1):224 – 234. Number: 1.

[Hall, 1986] Hall, N. G. (1986). Scheduling Problems With Generalized Due Dates. *IIE Transactions*, 18(2):220–222. Number: 2 Publisher: Taylor & Francis _eprint: https://doi.org/10.1080/07408178608975351.

[Hall et al., 1991] Hall, N. G., Sethi, S. P., and Sriskandarajah, C. (1991). On the complexity of generalized due date scheduling problems. *European Journal of Operational Research*, 51(1):100 – 109. Number: 1.

[Hepperle, 2012] Hepperle, M. (2012). Electric Flight - Potential and Limitations. https://elib.dlr.de/78726/1/MP-AVT-209-09.pdf.

[Horn, 1974] Horn, W. (1974). Some simple scheduling algorithms. *Naval Research Logistics (NRL)*, 21:177–185.

[IATA, 2019] IATA (2019). Worldwide Slot Guidelines (10th Edition). https://www.iata.org/contentassets/4ede2aabfcc14a55919e468054d714fe/wsg-edition-10-english-version.pdf.

[IATA, 2020] IATA (2020). Worldwide Slot Guidelines - Annex 12.7 - Contact List for Level 2/3 Airports. https://www.iata.org/contentassets/4ede2aabfcc14a55919e468054d714fe/wasg-annex-12.7.xlsx.

[J. Cheng et al., 2018] J. Cheng, F. Chu, and M. Zhou (2018). An Improved Model for Parallel Machine Scheduling Under Time-of-Use Electricity Price. *IEEE Transactions on Automation Science and Engineering*, 15(2):896–899. Number: 2.

[Justin et al., 2020] Justin, C. Y., Payan, A. P., Briceno, S. I., German, B. J., and Mavris, D. N. (2020). Power optimized battery swap and recharge strategies for electric aircraft operations. *Transportation Research Part C: Emerging Technologies*, 115:102605.

[J.Y. Moon et al., 2013] J.Y. Moon, K. Shin, and J. Parl (2013). Optimization of production scheduling with time-dependent and machine-dependent electricity cost for industrial energy efficiency. *International Journal of Advancements in Manufacturing Technology*, 68:532–535.

[Kurniawan et al., 2020] Kurniawan, B., Chandramitasari, W., Gozali, A. A., Weng, W., and Fujimura, S. (2020). Triple-chromosome genetic algorithm for unrelated parallel machine scheduling under time-of-use tariffs. *IEEJ Transactions on Electrical and Electronic Engineering*, 15(2):208–217. Number: 2 _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1002/tee.23047.

[Kurniawan et al., 2017] Kurniawan, B., Gozali, A. A., Weng, W., and Fujimura, S. (2017). A genetic algorithm for unrelated parallel machine scheduling minimizing makespan cost and electricity cost under time-of-use (TOU) tariffs with job delay mechanism. In *2017 IEEE International Conference on Industrial Engineering and Engineering Management (IEEM)*, pages 583–587. ISSN: 2157-362X.

[Lawler and Labetoulle, 1978] Lawler, E. L. and Labetoulle, J. (1978). On Preemptive Scheduling of Unrelated Parallel Processors by Linear Programming. *J. ACM*, 25(4):612–619. Number: 4 Place: New York, NY, USA Publisher: Association for Computing Machinery.

[Lee et al., 2009] Lee, D. S., Fahey, D. W., Forster, P. M., Newton, P. J., Wit, R. C. N., Lim, L. L., Owen, B., and Sausen, R. (2009). Aviation and global climate change in the 21st century. *Atmospheric Environment*, 43(22):3520 – 3537. Number: 22.

[Lenstra et al., 1977] Lenstra, J. K., Kan, A. H. G. R., and Brucker, P. (1977). Complexity of Machine Scheduling Problems. In Hammer, P. L., Johnson, E. L., Korte, B. H., and Nemhauser, G. L., editors, *Studies in Integer Programming*, volume 1 of *Annals of Discrete Mathematics*, pages 343 – 362. Elsevier. ISSN: 0167-5060.

[Liu et al., 2018] Liu, C.-H., Nanthapodej, R., and Hsu, S.-Y. (2018). Scheduling two interfering job sets on parallel machines under peak power constraint. *Production Engineering*, 12(5):611–619. Number: 5.

[Marksel et al., 2019] Marksel, M., Brdnik, A. P., and Kamnik, R. (2019). Ground infrastructure investment plan. Technical Report D10.1, MAHEPA Consortium. Issue: D10.1.

[NYC, 2015] NYC (2015). Governor Cuomo Unveils Vision For Transformative Redesign of LaGuardia Airport. https://www.governor.ny.gov/news/governor-cuomo-unveils-vision-transformative-redesign-laguardia-airport.

[Pereira, 2019] Pereira, M. (2019). Short-range route scheduling for electric aircraft. Master's thesis, Delft University of Technology, Delft.

[Pinedo, 1995] Pinedo, M. (1995). *Scheduling: Theory, Algorithms, and Systems*. Industrial and systems engineering. Prentice Hall.

[Plötner et al., 2013] Plötner, K., Schmidt, M., and Baranowski, M. (2013). Operating Cost Estimation for Electric-Powered Transport Aircraft.

[R., 2017a] R., C. (2017a). Runway robbery: The case for reforming airport-slot allocation. *The Economist*.

[R., 2017b] R., C. (2017b). Winning the slottery: The system for allocating airport slots is broken. *The Economist*.

[Raymer, 1992] Raymer, D. (1992). *Aircraft design: a conceptual approach*. Americal Institute of Aeronautics and Astronautics, Washington D.C.

[Ribeiro et al., 2018] Ribeiro, N. A., Jacquillat, A., Antunes, A. P., Odoni, A. R., and Pita, J. P. (2018). An optimization approach for airport slot allocation under IATA guidelines. *Transportation Research Part B: Methodological*, 112:132–156.

[Saberi-Aliabad et al., 2020] Saberi-Aliabad, H., Reisi-Nafchi, M., and Moslehi, G. (2020). Energy-efficient scheduling in an unrelated parallel-machine environment under time-of-use electricity tariffs. *Journal of Cleaner Production*, 249:119393.

[Salucci and Trainelli, 2019] Salucci, F. and Trainelli, L. (2019). An Optimization Model for Airport Infrastructures in Support to Electric Aircraft.

[Salucci et al., 2020] Salucci, F., Trainelli, L., and Riboldi, C. (2020). Optimal Rechaging Infrastructure Sizing and Operations for a Regional Airport. Bordeaux, France.

[Sriskandarajah, 1990] Sriskandarajah, C. (1990). A note on the generalized due dates scheduling problems. *Naval Research Logistics (NRL)*, 37(4):587–597. Number: 4 _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1002/1520-6750%28199008%2937%3A4%3C587%3A%3AAID-NAV3220370411%3E3.0.CO%3B2-O.

[Troeltsch, Florian et al., 2020] Troeltsch, Florian, Hornung, Mirko, Kaiser, Jochen, Scholz, Anna, and Engelmann, Marc (2020). Hydrogen Powered Long Haul Aircraft with Minimized Climate Impact.

[Tupolev, 2020] Tupolev (2020). History of tupolev. http://www.tupolev.ru/en/about/.

[Yang and Ogden, 2007] Yang, C. and Ogden, J. (2007). Determining the lowest-cost hydrogen delivery mode. *International Journal of Hydrogen Energy*, 32(2):268 – 286. Number: 2.

[Zeng et al., 2018] Zeng, Y., Che, A., and Wu, X. (2018). Bi-objective scheduling on uniform parallel machines considering electricity cost. *Engineering Optimization*, 50(1):19–36. Number: 1 Publisher: Taylor & Francis.

[Zografos et al., 2017] Zografos, K. G., Madas, M. A., and Androutsopoulos, K. N. (2017). Increasing airport capacity utilisation through optimum slot scheduling: review of current developments and identification of future needs. *Journal of Scheduling*, 20(1):3–24.

Appendices

APPENDIX A

Background on Electric and Hydrogen Aviation

In this Chapter, we shall study the state of technology of the electric and hydrogen aircraft. We shall use this knowledge in later chapters to develop the refuelling and recharging schedule model. In the first section, the different avionic types of electric and hydrogen aircraft are presented. After this, we shall discuss the energy consumption and range of the aircraft which are in development today, the possible recharging methods, and the possibilities for producing and transporting the required energy. The Chapter is concluded with a Section where the research possibilities and open questions are presented.

A.1 Avionics of Electric and Hydrogen Aircraft

A.1.1 Electric Aircraft

In contrast to the traditional ICE aircraft, whose hydraulic and pneumatic features are boosted by a jet engine which is fuelled by kerosene, (Hybrid-)Electric Aircraft, or (H)EA in short, all of these systems are replaced by their electric counterparts. With the current technology and expected technological advances, these aircraft are limited by the low gravimetric energy density of their batteries, which impairs their potential range and capacity [Marksel et al., 2019]. On the other hand, they benefit from the high efficiency of electric motors and the relative simplicity of the propulsion system, which lowers the maintenance requirements and enhances the reliability. These aircraft come in a number of different configurations, which shall be explained in this section.

Firstly, there is the pure Electric Aircraft (EA), where the energy which is delivered by the battery is used to power a single or multiple electric motor(s) which rotates the propellers. This configuration can be seen in figure **??**. If the energy which is used to power this aircraft is produced in a sustainable way, the advantage of this aircraft is that there in no greenhouse gas emission involved during the flights. This comes at the cost that when compared to comparable hybrid-electric and conventional aircraft, this type is the most limited in amount of passengers it can carry and/or the distances it can service.

Smaller aircraft of this type have already been introduced as stated in the introduction of this literature review: the Pipistrel Alpha Electro is a full-electric aircraft with the capability to transport two passengers for a flight of roughly one hour. Research has been done into a concept of a short-haul full electric aircraft, the Bauhaus Ce-Liner, which could transport 189 passengers for over a distance of 1700 kilometres [Brelje and Martins, 2019]. However, this relies on the assumption that the battery storage efficiency increases eight- to tenfold.

The other two configurations are hybrids between a conventional ICE aircraft and a electric aircraft, and come in two architectures. The parallel ICE-Electric Aircraft (pH-EA) architecture powers the propeller of the aircraft via either a battery and a electro-motor, or via a kerosene engine powertrain. The serial ICE-Electric Aircraft (sH-EA) architecture always powers the propeller via an electro-motor, but uses the combustion engine to generate electricity and recharge the battery ??. The former architecture can be used to reduce emissions around airports, while the latter can be used to reduce emissions throughout the flight.

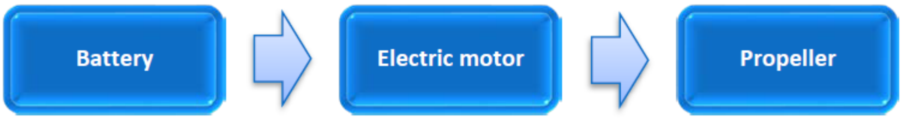


Figure A.1: *Electric Aircraft powertrain scheme [Marksel et al., 2019].*

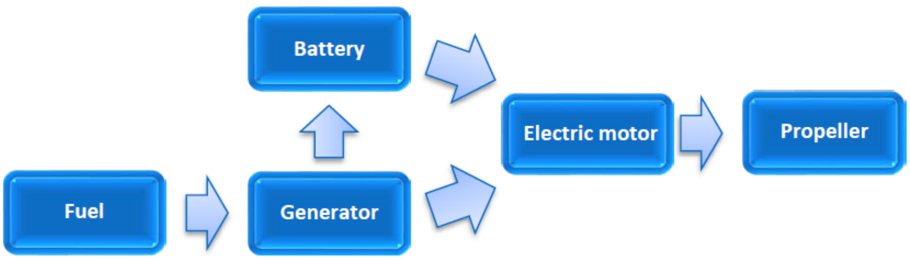


Figure A.2: *Serial Hybrid ICE/Electric Aircraft powertrain scheme [Marksel et al., 2019].*

A.1.2 Hydrogen Aircraft

A second option for sustainable flight is the use of planes which are powered by Hydrogen, HA aircraft. Like a performer in a balancing act, these aircraft try to find the middle ground between the ones powered by an Internal Combustion Engine (ICE aircraft, ICE-A), and the ones powered by electricity from batteries (EA). Despite the fact that Hydrogen is the most common element in the known universe, composing over three quarters of all visible mass, it is relatively rare on earth. But, much alike the electricity which powers EA, it can be produced as an renewable resource. However, due to the much higher energy density per mass, hydrogen planes are much lighter. In addition, the fact that it can be converted to a liquid form allows it to be fuelled on-board in a matter of minutes, much like LPG/Autogas cars, making these aircraft the aviation industries counterpart to hydrogen cars.

Hydrogen aircraft also have their shortcomings. Generation of hydrogen from electricity has a energy efficiency between 60 and 80 percent, and is combined with a lower efficiency of the power unit of the aircraft. And not unlike LPG compared to petrol, it also has its disadvantages when compared to kerosene [Marksel et al., 2019]. It is highly volatile and has a lower energy to volume ratio, it has to be transported in cryogenic tanks which have to be placed inside the fuselage, since it cannot be contained in the wings. As a result of this, the body of an HA is to be longer and/or wider then a similar ICE aircraft [Marksel et al., 2019].

Roughly speaking, and similar to EA, HA also comes in two different forms. The first variant of Hydrogen Aircraft are the most comparable to the LPG cars. A jet-engine is used to propel the aircraft, but instead of using kerosene, one burns Hydrogen. The disadvantage of this type of aircraft is that it emits water in vaporized form, which is actually the strongest greenhouse gas, but the advantage is that it only requires way smaller modifications with respect to conventional aircraft. Due to this, large versions of this type have been around for a long time: in 1989, Russian aircraft manufacturer JSC Tupolev built the TU-155 (as shown in the introduction), a narrow-body aircraft with a capacity of 120 passengers and a range of over 3000 km [Tupolev, 2020]. This shows that the technology of HA is much closer to implementation then the EA counterpart.

The second type of HA uses a fuel-cell in the propulsion system. A fuel-cell is a system which produces electricity from Hydrogen: Hydrogen molecules flow through a membrane where they are split into protons and electrons, this produces a direct current electricity, and electrons rejoin the protons together with Oxygen at the other side of the membrane to create water, which is the only residual product. The current which is generated can either be stored in batteries to support the aircraft systems or deliver large Wattages, or can be fed directly to an electric motor which turns a propeller. The lay-out of the power train can be seen in figure ??. Hydrogen aircraft of this type are still mostly smaller and experimental: the HY4, created by the German Aerospace Centre and based on the Pipistrel Taurus G4, was the first aircraft of this type in 2016 and can carry 4 passengers. Larger aircraft, such as the Airbus ZEROe turboprop and blended wing [Duvelleroy, 2020], and the Bauhaus Luftfahrt Hy-ShAir are in development and can hit the market around 2035-2040 [Troeltsch, Florian et al., 2020].

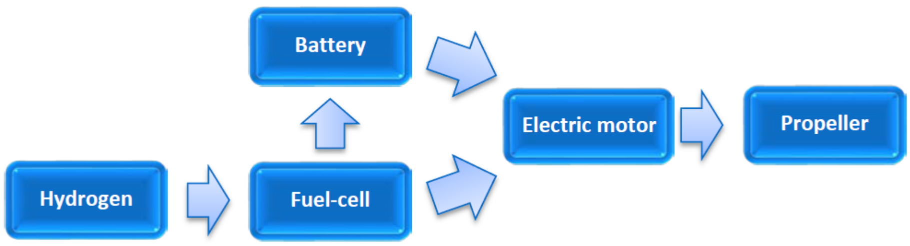


Figure A.3: *Fuel-cell based Hydrogen aircraft powertrain.*

A.2 Future aircraft Concepts

Table A.1: *Hello there*

Electric Aircraft Model	first flight	EIS	seats	capacity (kWh)	range (km)
Lange Antares 20E	2003	-	1	10	700 (gliding)
Pipistrel Taurus G4	2011	-	4	5	300 (gliding)
Pipistrel Alpha Electro	2015	-	2	21	120
Eviation Alice	-	2022	11	920	1000
Bauhaus Luftfahrt Ce-Liner	-	2035	189	50.000	1667
Airbus VoltAir	-	2035	33	?	?
IMOTHEP	-	2040	19	1210	500
NOVAIR	-	2040	150	90.000	2200
Hydrogen Aircraft Model				capacity (m^3)	
Tupolev TU-155	1988	-	?	163	2600
HY-4	2011	-	4	0.25	1500
Airbus ZEROe Turboprop	-	2035	100	?	2000
Airbus ZEROe Turbofan	-	2035	200	?	4000
Airbus Airbus ZEROe blendwing	-	2035	200	?	4000
Bauhaus Luftfahrt Hy-ShAir	-	2045	400	371	11.800

A.3 Energy Production and Transportation

A.3.1 Electric Aircraft

Finally, all of this power required to recharge this aircraft has to come from somewhere. It can either be drawn from the grid, produced at the airport itself, and/or be drawn from a buffer battery at the airport. Additionally, the fact that electricity production will be moving away from fossil to renewable sources and the implications that this has do require attention.

If the power is drawn from the grid on an as-needed basis, the electricity grid will deal with high peak powers and needs to be up to the demand. A large electricity consumption at the airport can cause different types of problems for the state of the grid, as has already been shown to be the case for charging stations for electric vehicles: it can cause fluctuations in the supply power quality, voltage instability, and increased loss of energy. Most countries employ a pricing system which creates incentives for the user to manage the use in a favourable way: there is a distinction in price for consumption during peak- and off-peak-hours, an additional charge for the maximum power demand.

The use of renewable sources for electricity, such as solar panels and wind turbines, will add a new dimension to this problem. Due to the fact that the power generated by these sources is less predictable then it is now, network operators will look for new ways to manage the energy demand, which has to be equal to the supply at all times. A way of handling with these problems is by employing a smart grid: in which the network communicates with the users and the suppliers in order to synchronize one to the other.

Airports can anticipate these challenges in a number of ways [Justin et al., 2020]. Firstly, the peak capacity of the electricity grid can be raised by constructing new power lines. This step may be necessary independent of what other measures are taken, but when taken alone it is hardly elegant. In order to relieve the grid from the large demand, the swapping schedule can be optimized in such a way that the grid can cope. Additionally, the airport can produce some of the electricity by itself, and can create storage facilities which can be charged up when the demand is low and the electricity is cheaper, and can be discharged to power aircraft batteries when the demand is high.

A.3.2 Hydrogen Aircraft

So far, we have treated hydrogen in such a way that we could've replaced the word with LPG and everything (except the part of the fuel cells) would hold up mostly. Even the experimental aircraft Tupolev TU-155 switched midway during testing from Hydrogen to LPG. What actually distinguishes Hydrogen from (liquid) natural gas is that it can be synthesized in a sustainable way. In this section, the production, transport, and storage of Hydrogen to and at the airport is discussed.

There are a number of ways in which one can synthesize hydrogen on an industrial scale. These methods are categorized in three levels of sustainability: grey, blue, and green production. Grey production is characterized by the fact that it consumes non-renewable resources in the process and greenhouse-gasses are released as a by-product. The blue production is similar to grey production, but instead of releasing the greenhouse gasses into open air, it is stored underground. Finally, during green hydrogen production, only renewable resources are used, and no greenhouse gasses are released. While the first category is now the most common (steam reforming, where Hydrogen is produced from methane accounts for 68% of the world total and is currently the cheapest method), the latter category is suitable for industrial production [Marksel et al., 2019]. Electrolysis, the most popular green method, where hydrogen is produced by separating the oxygen atom in water, accounts for 5% of the production. The cost of this depends on the method which is used, but ranges from just over €1 up to €8 /kg [?].

If the Hydrogen is not produced at the airport, there are a three of ways it can be transported towards it: in gaseous form with a tube truck or with a pipeline, or in liquid form with a cryogenic truck. As is to be expected, deliveries by truck are cheaper and more suitable if the production facility is not very far from the airport and the demand is not very large. Delivery by pipeline on the other hand is the more efficient method for airports which have a large demand. Additionally, when the hydrogen is delivered at the airport in gaseous form, the airport needs to have the infrastructure to convert it to liquid form. A study from the University of California looked into the most cost-effective way of transporting Hydrogen [Yang and Ogden, 2007]. A short summery of their findings is displayed in figure ??, which aligns with the given explanation.

Finally, when the Hydrogen is delivered at the airport, it can either be transferred to the aircraft directly, or be stored. Most large airports now have such a facility for kerosene, and it is to be expected that the same will hold for Hydrogen. In gaseous form, it needs to be stored in spherical or cylinders at 300 bar, depending on the configuration. In liquid form, thesis high pressure levels are not required, but the tanks are typically more expensive nonetheless because of the extensive isolation which needs to be used. Such a storage unit can be seen in figure ??.

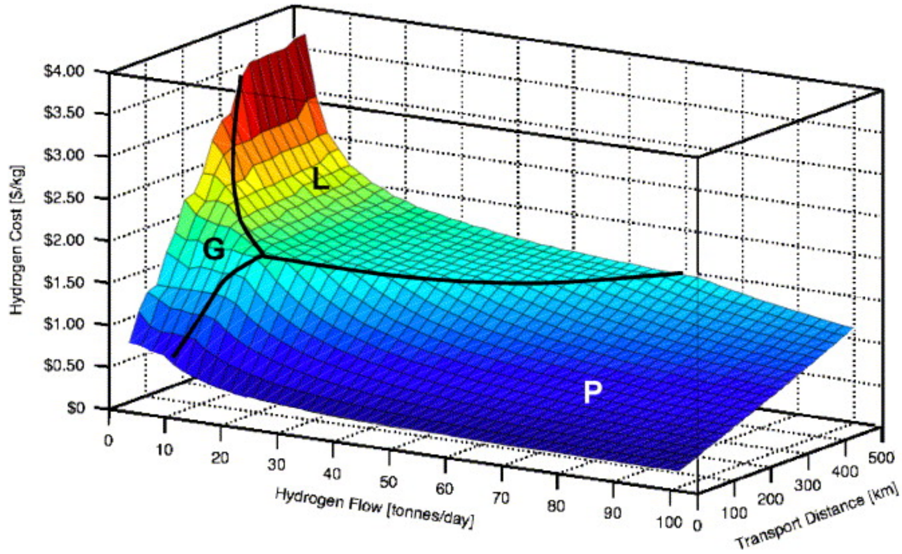


Figure A.4: *Most cost effective delivery method of Hydrogen as a function of the demand and distance to the production plant, taken from [Yang and Ogden, 2007].*

Figure A.5: *Large Liquid Hydrogen storage tank.*

A.4 Research gaps

For larger (hybrid-)Electric Aircraft, a number of open questions remain. The answer to these questions depends on the course of the industry and regulations by the air traffic institutions and governments. In this section, the most important questions which have arisen from this chapter have been enumerated. The answer to these questions depends on the course of the industry and regulations by the authorities. The most probable answers to these questions are used as assumptions and input for the model which we are going to develop, but different courses may lead to other approaches.

1. For what types of aircraft are battery swaps going to be a relevant recharging method?
2. What level of mobility and automation are the swapping and recharge stations going to have?
    - Plugging the charger into an aircraft or a battery should not be a very

complex task to perform and will likely be performed manually. However the question of whether or not these chargers are going to be bound to a single gate is still open.

- How battery swaps for larger aircraft are going to be performed is still open. If the task is relatively easy to perform, it can be done manually or semi-automated with a piece of support equipment that can drive around the airport and can (un)load and transport the batteries. On the other hand, if the task becomes to complex, an immobile battery swapping station analogous to the ones proposed by Tesla Inc. [1] has to be used.

3. How well are the types of batteries and the swapping systems going to be standardized by the aircraft manufacturers?

4. Finally, how is the electricity grid going to be able to cope with the huge demand of larger airports?

Even though the development of larger Hydrogen Aircraft is in a much further stage then larger Electric Aircraft, there are still some open questions.

1. Are airports prepared and allowed to produce their own hydrogen on-site?

2. Similarly, are airports willing to store hydrogen on-site? Hydrogen storage containers possibly present a large security risk and airports may want to place it somewhere away from the airport.

[1] Over here

APPENDIX B

Framework and Notation of Scheduling Theory

Scheduling is a decision-making process that is used on a regular basis in many manufacturing and services industries. It deals with the allocation of resources to tasks over given time periods and its goal is to optimize one or more objectives [Pinedo, 1995]. These resources and tasks can take many forms in applications. At airports, resources can be runways and gates, but also maintenance crew, flight crew, baggage handling vehicles or refuelling vehicles, the list can go on and on. The tasks to which these resources can be allocated to are for example: incoming and outgoing flights, turnarounds or maintenance jobs. Each task can take a certain amount of time to be completed, can be available only at a specific time window, or can have a high or low priority level. Additionally, the objective which determines how resources should be allocated optimally can take on many forms, depending on the application. Examples are: minimizing the completion time of the last task, minimizing the number of late jobs, or minimizing the maximum delay.

In this literature review, only a small part of this theory can be discussed. I shall discuss the framework in which deterministic scheduling theory operates, as well as the used notation, in Subsection B.1. Secondly, the most used method to report and visualize output of scheduling problems is used in Subsection B.2: Gantt Charts.

B.1 Notation and Framework of Scheduling Theory

In the framework which we use, tasks and resources are referred to as jobs and machines respectively. Of each type, the numbers are considered to be finite. Jobs need to be assigned to machines at specific times, and each machine can be occupied with at most one job at a time. The set of jobs is denoted with J, with $\#J = (|J| =)n$, and each job is labelled with subscript j. Similarly, the set of machines is denoted as I, with $\#I = (|I| =)m$, and each machine is labelled with subscript i. Each job has a processing time, which is denoted with p_{ij} (or p_j if it is independent of the machines) [Pinedo, 1995].

A scheduling problem is described with a triplet of characteristics: $\alpha|\beta|\gamma$, where each of the fields contains information about one aspect of the problem.

Firstly, the field α contains information about the machine environment of the problem. Some of the possible (and relevant) values of this field are:

- **Single Machine** (1): Only one machine is available.
- **Identical Parallel Machines:** (Pm): m machines with the exact same characteristics are available. Each job may be processed on any of the machines.
- **Different Speed Parallel Machines:** (Qm): m machines with different speeds v_i are available. Each job can be processed on any of the machines, but the processing times of the jobs are give by $p_{ij} = p_j / v_i$.
- **Flow Shop** (Fm): There are m machines available, and each job has to be processed on each machine according to a specific order (first machine 1, then 2, then 3, etc. until it is processed on machine m).
- **Flexible Flow Shop** (FFc): This is a combination of Pm and Fm: there are c stages, each with a number machines, and each job needs to processed on every stage in a specific order (first 1, then 2, etc.).

Secondly, the field β specifies the constraints and possibilities for the processing. The field may contain zero, one, or multiple entries. Some of these are:

- **Release dates** (r_j): When release dates are present, working on job j cannot start before r_j.
- **Due dates** (d_j): When due dates are present, work on job j should be finished before d_j. If it is not, this will translate in a penalty.
- **Preemption** (*pmtn*): If preemption is allowed, work on jobs may be interrupted to be resumed (possibly on a different machine) later.
- **Precedence** (*prec*): Precedence constraints specify if a certain operation has to be completed before work is being started on a different operation.
- **Blocking** (*block*): In the flow show environment, there may be a limited amount of jobs which are allowed in the queue between two stages.

Finally, the field γ defines which objective function needs to be optimized. By convention, the objective function always needs to be minimized. A number of choices for this field are:

- **Completion time related:** The completion time of a job j, denoted with C_j, gives the time at which work on the job has been finished. If multiple operations on a job have to be performed (in the Fm and FFc environment), it is the completion time of the job at the last stage.
- **Lateness related:** The lateness of a job is given by the difference between the completion time and the due date:

$$L_j := C_j - d_j \tag{B.1}$$

- **Tardiness related:** The tardiness of a job is given by the maximum of the lateness and 0:

$$T_j := max\{L_j, 0\} \tag{B.2}$$

  Objectives related to this can be used if it is irrelevant how much before its due date a job is delivered.

- **Unit Tardiness related:** The unit tardiness is given by a unit step function:

$$U_j := \begin{cases} 1 & \text{if } C_j > d_j \\ 0 & \text{else} \end{cases} \tag{B.3}$$

  Objectives related to this can be used if it is only relevant if a job is completed on time or not.

Objectives based on these concepts can be the maximum, the sum, or the weighted sum. The maximum of these are denoted as: C_{max}, L_{max}, and T_{max} respectively. Additionally, weights may be assigned to specific jobs. For example, if the maximum of all completion times, also known as the makespan, needs to be minimized, the objective function is given by $C_{max} := \min \max_j C_j$. If one wants to minimized the weighted sum of the tardiness, the objective is given by: $\sum_j w_j T_j$

B.2 Gantt Charts

One of the most common ways to present the solution to a scheduling problem is a Gantt chart, named after its inventor Henry Gantt. It displays when which machines are working on which jobs. Machines are arranged vertically, while the time axis is displayed horizontally.

We can illustrate how a Gantt chart looks like with an instance from the example of the last Subsection. Suppose we have four jobs with:

j	1	2	3	4
p_j	2	2	3	4

There are a lot of options in which these jobs can be scheduled, three of these are presented in a Gantt chart in figure B.1. In the first two solutions, jobs 1 and 3 are scheduled at machine 1 and jobs 2 and 4 at machine 2. Both of these solutions have $\max_j C_j = 6$. These would be the optimal solutions if preemption was not allowed, but because it is, the schedule can be improved upon, and the optimal schedule is given as solution 3. In this case, job 4 is processed on both machines 1 and 2.

B.3 Application: TOU Electricity price scheduling for identical parallel machines

For the parallel machine environment, Moon et al. [J.Y. Moon et al., 2013] were the first to investigate the problem under TOU electricity pricing. They considered an objective function which was composed of a weighted sum of the makespan and the electricity consumption, and used a genetic algorithm as an heuristic to find good

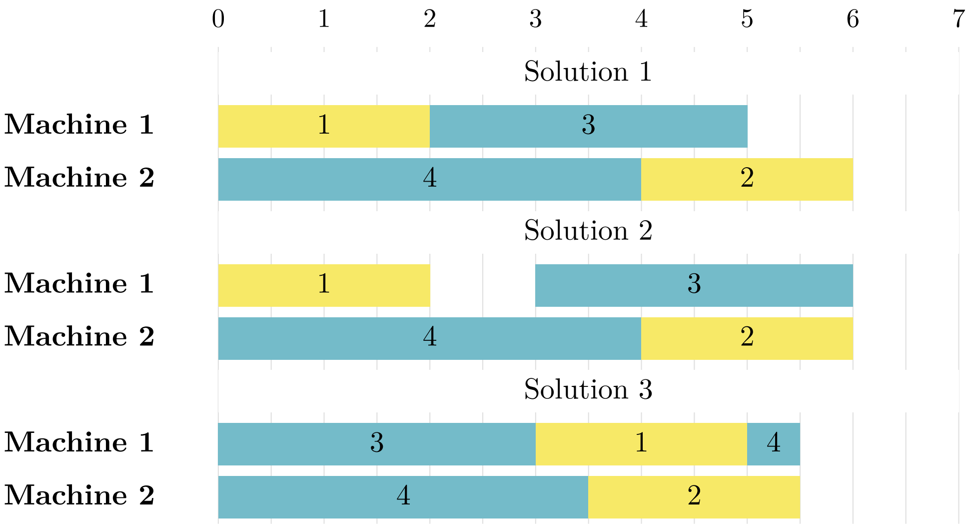


Figure B.1: *Three possible solutions to the scheduling problem given above. Both the first and second solution give* $\max_j C_j = 6$ *while the lower solution gives* $\max_j C_j = 5.5$.

solutions to the problem. For this genetic algorithm, each solution to the problem was encoded as the sequence of the jobs and the possible lengths of the intervals between two processes. With this, they were able to solve a problems of up to 65 jobs on 20 machines. The use of genetic algorithms in this setting was further explored by Kurniawan et al. in [Kurniawan et al., 2017, Kurniawan et al., 2020]. Ding et al. [Ding et al., 2016] continued working on this objective, but instead formulated it as an MILP problem and used a column generation heuristic to solve it, and were able to solve problems for up to 200 jobs on 20 machines. Che et al. [Che et al., 2017] further improved this model by splitting the search heuristic up into two stages, where the first one allows preemption in order to solve the problem efficiently. This heuristic improved the search times somewhere between ten- and hundredfold. Simultaneously, Cheng et al. [J. Cheng et al., 2018] simplified the model, leading to a reduction in the number of variables and constraints, which also reduced the computation time.

In addition to makespan optimization, a number of different objective functions have also been investigated in the literature. Fang et al. [Fang and Lin, 2013] used a Particle Swarm Optimization algorithm in order to minimize the sum of the sum of weighted tardiness and the electricity consumption, where instances with fifty jobs on five machines were solved. Similar research was performed in [Zeng et al., 2018], but with an insertion algorithm. Liu et al. [Liu et al., 2018] optimized the schedule for the minimum number of required machines, using a genetic algorithm.

Table B.1: *Scheduling problems with TOU electricity pricing for the parallel machine environment with different objective functions and solution heuristics.*

Machine environment	Objective function	Solution heuristic
Parallel machines (Pm)	C_{max}	Genetic algorithm [J.Y. Moon et al., 2013, Kurniawan et al., 2017, Kurniawan et al., 2020], Column generation [Ding et al., 2016, Che et al., 2017, J. Cheng et al., 2018], Fit-and-relax [Saberi-Aliabad et al., 2020]
	$\sum w_j T_j$	Particle Swarm Optimization [Fang and Lin, 2013], Insertion algorithm [Zeng et al., 2018]
	m	Generic algorithm [Liu et al., 2018]

APPENDIX C

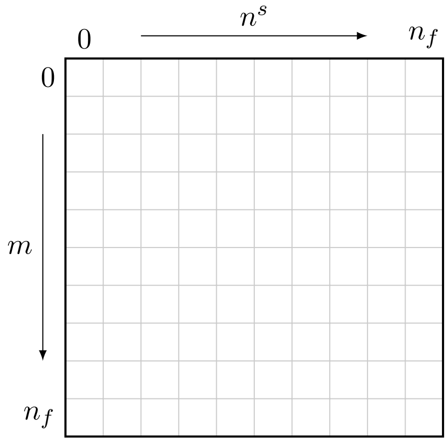
Solving the Slot Allocation and Battery Charge Problem on a Strategic Level

In this appendix, we are going to discuss a method to solve the AC Slot Allocation and Battery Charge problem at a point in time when strategic decisions have to be made. The airport is in a master-planning phase and needs to decide the number of chargers and spare batteries, m and n^s, which it is going to acquire.

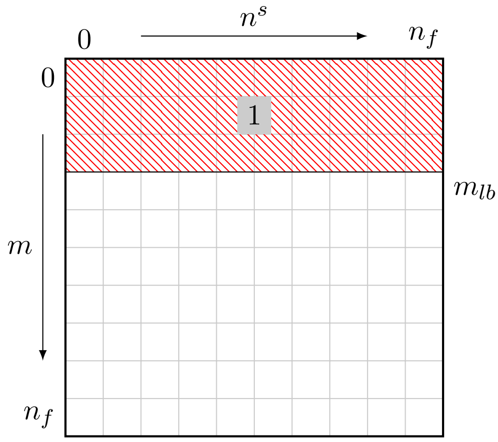
In order to do this, all possible combinations of m and n^s need to be evaluated, but doing so is very time consuming and can be omitted with a pattern. This has been used in the code of this report.

Let $\omega : \mathbb{N} \times \mathbb{N} \to \mathbb{R}$ denote the optimal value of this problem for a given pair of m and n^s.

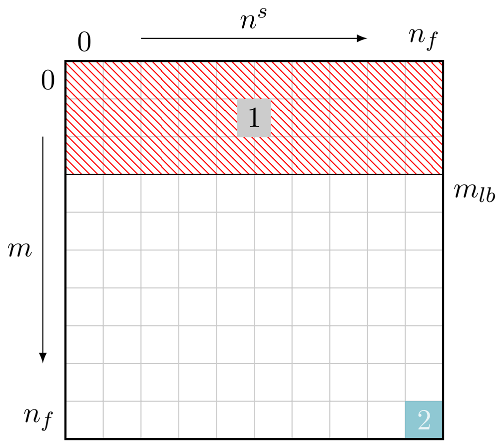
We start the program with an empty matrix, which is eventually going to contain the optimal values of ω for each combination of m and n^s. We know a natural upper bound for both of these parameters: the number of flights in the input, n_f. Secondly, we know (and are going to use) that $\omega \geq 0$ for all solutions.



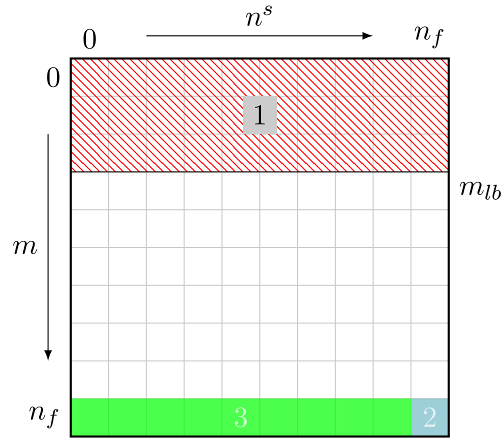
Our first step is to eliminate the values of m for which we on forehand know are never feasible. This is the case if the amount of work which arrives, given by $\sum_j p_j$, is larger then the amount which can be processed if the machines are on at all times, given by $m \times |T|$. The first feasible value of m is denoted by m_{lb}, and is given by $\left\lceil \sum_j p_j / |T| \right\rceil$.



The second step is to solve the problem with the largest possible infrastructure: $m = n^s = n_f$. Regularly, these input values will yield $\omega = 0$ (although this cannot be guaranteed).



In the third step, we calculate ω for all other values of n^s, with $m = n_f$, starting from $n^s = 0$ and working our way up. In this step, as well as the next two, we are going to use the fact that ω is a non-increasing function of both m and n^s, and this is what is going to reduce the computation time. Because, once we have found a value of n^s for which $\omega(m, n^s) = \omega(m, n_f)$, we know that $\omega(m, n^s + 1) = \omega(m, n^s + 2) = ... = \omega(m, n_f - 1) = \omega(m, n_f)$, and we don't need to use our solution methods for each of these values of n^s.



In the fourth and fifth step, we iterate over the values of m, starting from m_{lb} and working our way up to $n_f - 1$. For each value of m, we start by computing $\omega(m, n_f)$. Similar to the last step: if $\omega(m - 1, n_f) = \omega(n_f, n_f)$, we know that $\omega(m, n_f) = \omega(n_f, n_f)$ and we can omit solving the problem directly. After we have determined $\omega(m, n_f)$, we are going to repeat step 3 for this value of m, and determine $\omega(m, n^s)$ for $0 \leq n^s \leq n_f - 1$. Additionally, we can use the knowledge from step 4: of $\omega(m - 1, n^s) = \omega(n_f, n^s)$, then $\omega(m, n^s) = \omega(n_f, n^s)$

.Steps 4 and 5 are shown in figure C.1

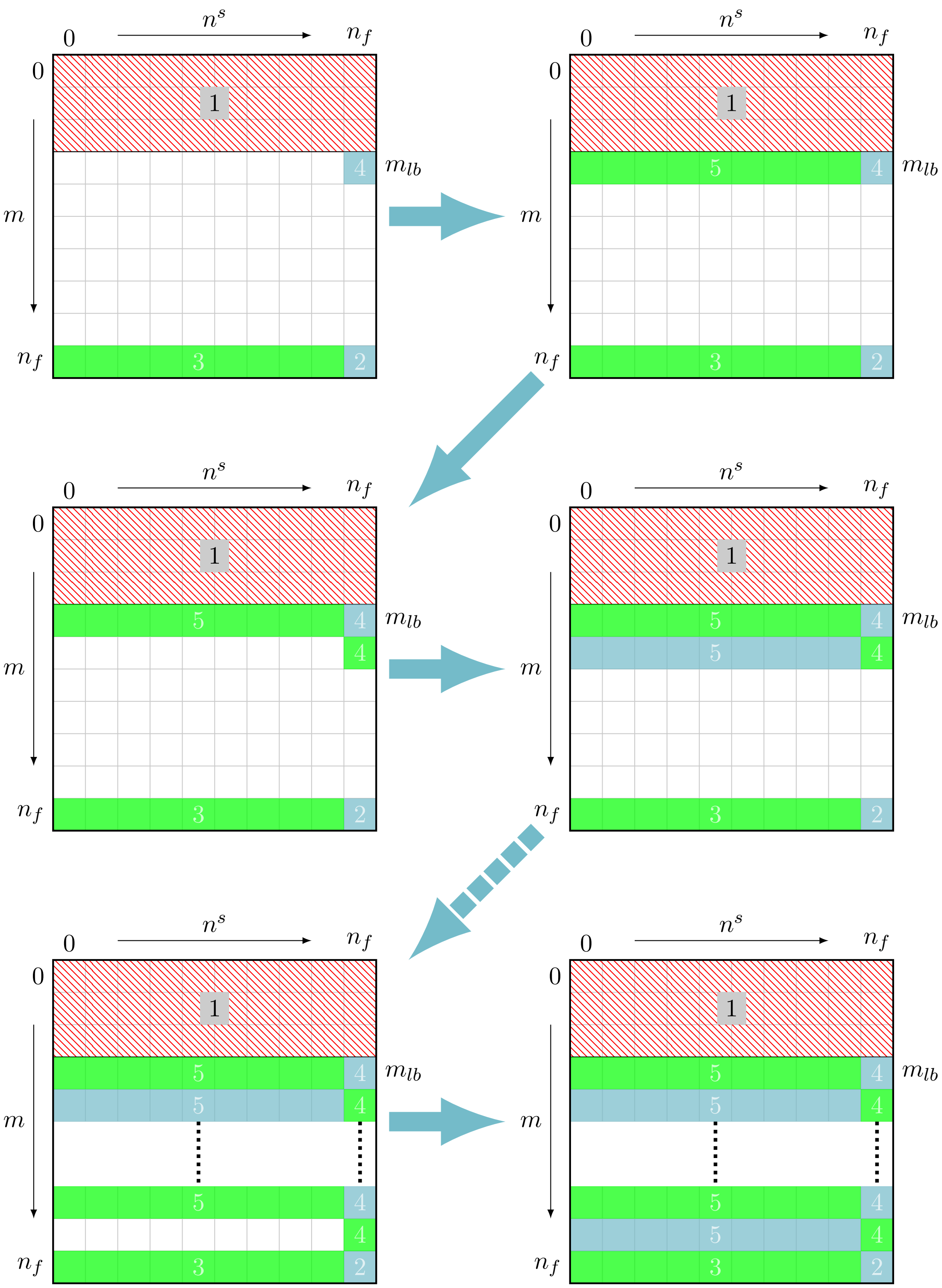


Figure C.1: *Final two steps*

APPENDIX D

Determining the robustness of a flight- and charge schedule

In this appendix, we are going to evaluate how to determine the required infrastructure for a flight timetable which is sufficiently robust to expectable delays.

In the last chapter, we determined how to create a flight and recharge schedule given a certain infrastructure. From this, we can determine at which times each flight arrives, departs, and its battery starts recharging. At each moment in time, the shortage of batteries is known, and when all flights arrive and depart on time, it is always smaller then the number of spare batteries.

However, if flights arrive later then planned, batteries can arrive late at the charging station, and a shortage of batteries may emerge later. This can cause other flights to be delayed. Therefore, we want to find the number of spare batteries for which the probability of such an event falls below a certain threshold.

Suppose the following: let J be the set of all flights, and let sta_j and std_j denote the scheduled time of arrival and departure of flight j, and let DEL_j be stochastic variable which denotes the delay of flight j[1]. The actual time of arrival and departure are denoted by ATA_j and ATD_j. Each flight needs a turnaround time of at least tat_{min} and the battery needs to spend p_j time at the charger. The scheduled and actual charging completion time of battery j are denoted by scc_j and ACC_j. Finally, scs_j denotes the scheduled charging start time of flight j. The following equations relate the variables:

$$ATA_j = sta_j + DEL_j \tag{D.1}$$

$$ATD_j = \begin{cases} std_j & \text{if aircraft } j \text{ stays overnight} \\ std_j & \text{if } ATA_j + tat_{min} \leq std_j \\ ATA_j + tat_{min} & \text{else} \end{cases} \tag{D.2}$$

$$ACC_j = \begin{cases} scc_j & \text{if } ATA_j + p_j \leq scs_j \\ ATA_j + p_j & \text{if } ATA_j + p_j > scs_j \end{cases} \tag{D.3}$$

[1]In this appendix, we use lower-case letters to denote deterministic variables, and upper-case letters for stochastic variables.

D.1 The number of required spare batteries

Let $S : T \to \mathbb{N}$ be a stochastic variable which denotes the number of required spare batteries. Our first objective is to provide an answer to the question:

What is the probability that S is larger then the number of spare batteries at the airport at some time during T?

When this situation occurs, the delay of one(or more) flight(s) will cause other delays due to the limited charging infrastructure.

We are going to determine S by looking at the effects of each flight individually. For each battery, the shortage function for flight j: $B_j : T \to \{0, 1\}$ is given by:

$$S_j = \mathbb{1}_{ATD_j \leq t} - \mathbb{1}_{ACC_j \leq t} \qquad \Rightarrow \tag{D.4}$$

$$S_j(t) = \text{Ber}(p = \mathbb{P}[Del_j \in T_j^t]), \tag{D.5}$$

where the set $T_j^t \subset T$ denotes all times by which flight j can be delayed such that is a shortage of batteries at time t. The total shortage of batteries is given by the sum of the shortage functions over all flights, minus the number of batteries which have stayed at the airport during the night:

$$S = \sum_j S_j - nn, \tag{D.6}$$

where nn denotes the number of batteries which stay at the airport during the night. For each time instance, we that $S(t)$ is given by a Poisson binomial distribution :

$$S(t) = \sum_j \text{Ber}(p = \mathbb{P}[Del_j \in T_j^t]) \tag{D.7}$$

Thus, we can determine the distribution of the shortage for each $t \in T$. If these were independently distributed, our knowledge of $S(t)$ would be enough to determine the distribution of S. They are however, obviously, in general not independent of one another: the knowledge of the number of spare batteries at some moment in time influences the distribution of S in the near-future.

For this reason, it is not straight-forward to compute the distribution of S. Fortunately, given a realization of $(DEL_j)_{j \in J}$, S can be computed with very little effort: with 1 vector addition to compute ATA, one vector addition and one point-wise comparison to compute each of ATD and ACC. From the latter two, we can determine the number of required spare batteries from Equations D.4 and D.6. First, start with $s_{max} = s = nn$ and $t = 0$, and increase t until the end of T is reached. Whenever an element of ATD or ACC is equal to t, s is respectively decreased or increased by 1, and s_{max} is updated as the maximum of the previous s_{max} and s. In order to do this efficiently, we only need to sort ADT and ACC.

D.2 The number of delayed flights

Secondly, we are interested in determining the following:

What is the probability that n or more departures are delayed because there are not sufficient fully charged spare batteries?

Given a realization of DEL, the number of delayed aircraft can be determined in a similar way as the number of required spare batteries. Note that when fully charged batteries are allocated to departing aircraft in a manner that minimizes the maximum caused delay, this is done on a first-in-first-out basis: the first fully charged battery is assigned to the first departing flight et cetera. The supply and demand times for fully charged batteries are given by:

$$Sup = (\underbrace{0, 0, ..., 0}_{nn \text{ times}}, \text{sort}(ACC))$$
$$Dem = (\text{sort}(ATD), \underbrace{0, 0, ..., 0}_{nn\text{times}})$$

The number of delayed flights is then given by:

$$N_{del} = \sum_j \mathbb{1}_{Dem_j < Sup_j} \tag{D.8}$$

APPENDIX E

Code

All of the code that has been used in this report is available on request at:
`S.J.M.vanOosterom@student.tudelft.nl`