Delft, University of Technology

Bachelor's thesis

Simulating the transition between two media in the Shive wave machine

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February 28, 2019

Abstract

In this research, a Shive wave machine is used to study (a) the velocity of waves throughout different media and (b) the transition of waves between two different media. The Shive wave machine used in this research consists of 32 parallel aluminum bars attached perpendicularly to three parallel central wires. When a perturbation is applied to one of the bars, a torsional wave is initiated in the Shive wave machine, which is mapped to a transverse wave at the extremities of the bars. The velocity of a wave in the Shive wave machine is theoretically determined by four variables: (i) The distance between the two outside wires and the central wire; (ii) the tension in the two outside wires; (iii) the distance between the bars; (iv) the moment of inertia of the bars. The transition of waves between two different media is theoretically determined by the wave velocities in the two media. The theory states that a part of the wave is reflected and a part is transmitted at the intersection between the two media. The results for (a) show that the measured velocity is higher than theoretically expected, which may be caused by an incorrect measurement of (ii). The results for (b) show that a change in (i) throughout the system does not comply with the theoretically expected ratios for reflection and transmission, whereas a change in (iv) does comply with the theoretically expected ratios.

Contents

Chapter 1

Introduction

Since Wilhelm Röntgen won the Nobel Prize in Physics for the discovery of X-rays in 1901 [1], there have been great developments in the field of medical imaging techniques. A part of these imaging techniques makes use of acoustic waves. These waves have the ability to penetrate the opaque media of the human body, just like X-rays, and thus give the possibility to make pictures and movies of the inside of the human body [2]. When acoustic waves are sent into a medium, the velocity of the waves depend among other parameters on the stiffness of the medium [3]. An example of an organ that continually stiffens and relaxes is the human heart, which is cyclically pumping, and therefore its medium changes both in a time- and in a space-depending way. The aim of several types of research such as [4] [5] is to investigate the velocity of waves throughout media varying over time and space, to optimize ultrasound imaging. Furthermore, when a wave reaches an intersection between two media, a part of it is reflected and a part of it is transmitted. This behavior is the focus of several researches such as [6] [7]. So, the velocity of waves in time- and space-varying media as well as the behavior of waves at intersections between different media should be included to optimize the imaging of a pumping heart using ultrasound waves.

The aim of this research is to find out whether the Shive wave machine can be useful for the researches about the velocity of waves throughout different media and about the behavior of waves between two media. To investigate this, I theoretically predict the behavior of the waves is in this report, after which I measure the behavior. Then, I compare the theoretical and practical behavior and conclude if the Shive wave machine is a useful tool or not. The Shive - or Bell - wave machine was founded by John Northrup Shive (February 22, 1913 – June 1, 1984), who created the machine as an educational apparatus used to illustrate wave motion [8]. The Shive wave machine consists of three parallel wires, to which a number of bars are attached perpendicularly. The bars are balanced at the wire in the middle. A torsional wave can be initiated in the machine by pushing the extremity of a bar up or down. The two outer wires will pass on the up- or down movement onto the next bar. The up- and down-movement of the extremities of the bars can be seen as a transverse mapping of the torsional wave, giving the image of a one-dimensional-wave on a string. This transverse form of the wave will be used for the measurements in this report. The wave velocity in the Shive wave machine is determined by several parameters, such as the tension in the outer wires or the distance between the three wires. By changing one of these variables throughout the machine, one can create two different media. In this research I will measure the reflection and transmission to test if they match the theoretically expected values.

This research is done as a Bachelor's thesis for the Bachelor Applied Physics at the Delft University of Technology. In chapter 2 of this report, the theoretical background of the project will be explained, which is followed by the experimental method in chapter 3. Afterward, the results will be presented and discussed in chapter 4 and concluded in chapter 5.

Chapter 2

Theory

The aim of this research is to investigate the waves traveling in the Shive wave machine and to explore whether this machine can be employed to observe wave behavior at the transition between two media. To be able to fully comprehend the scope of this study, it is necessary to give some theoretical background about the Shive machine, as well as about traveling waves.

2.1 Shive wave machine

As mentioned in the introduction, the Shive wave machine [8] was built by dr. J. Shive to help his students understand the traveling of waves. The system is schematically depicted in figure 2.1 and consists of a number ‘n’ of bars, which are equally distributed over and connected perpendicular to a central wire: The bars are placed parallel to the y-axis and are black in the figure, the central wire is placed parallel to the z-axis and is red. The bars are able to rotate around the central wire, so this wire is called the ‘pivot wire’ in the rest of this report. The rotation of the bars is in the XY-plane. Parallel to the pivot wire, there are two wires, which are connected to the bars and called the ‘tension wires’ in this report. These wires are blue in the figure. These wires are parallel to the pivot wire, and thus also span along the z-axis. When the system is at rest, the bars are balanced over the three wires, which means that each point of a single bar is at the same x-coordinate. In the rest of figure 2.1, the orange parts denote weight-carrying parts, ‘B’ is a cylinder around which the tension wires connect and ‘A’ is a tension meter. Furthermore, R is the distance from the end of a bar to the pivot wire and is half the length of a bar, r is the distance between a tension wire and the pivot wire, and Dz is the distance from one bar to another bar.

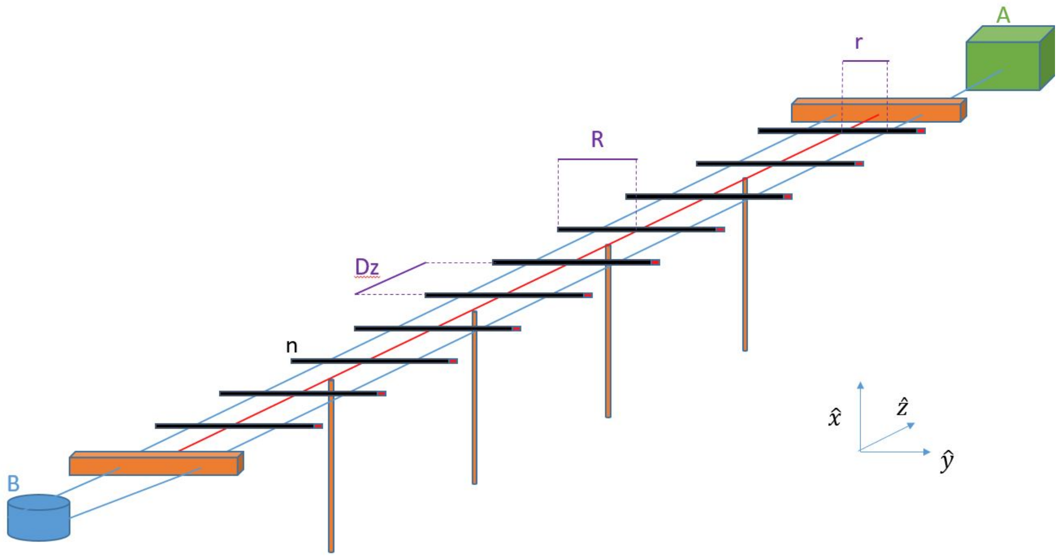

Figure 2.1: A schematic view of the Shive wave machine used in this experiment. The black lines are the (rotating) bars, the red line is the 'pivot wire' and the blue lines are the 'tension wires', which is actually one wire that rotates around cylinder 'B'. In this research, 32 bars were used. The structures attached to the lines are weight-carrying components. Furthermore, R is the distance from the end of a bar to the pivot wire, r is the distance from a tension wire to the pivot wire and Dz is the distance between two bars. The tension meter is denoted by 'A'.

When a perturbation is applied to one bar, i.e. the bar is pushed up or down in the (-)$\widehat{x}$-direction, the tension wires pass along the perturbation to the bars next to the perturbed bar. The initially perturbed bar will return to its position at rest, and the bars next to it will obtain the perturbed position. Then, the bars pass on the perturbation to their neighbors and so on. This results in a torsional wave. If one watches the projection of the displacement on the $\widehat{x}$-direction of the extremities of the bars, the torsional wave is mapped to a transverse wave. In this experiment, the transverse form of the wave is used for measurements, such as the velocity and amplitude of the wave.

As mentioned above, one of the things that are interesting for this research is the velocity of the wave throughout the system. The Shive wave machine has several parameters that determine the wave velocity. Skeldon et alia [9] make a derivation for the wave velocity throughout the Shive wave machine. The derivation starts with the situation as can be seen in 2.2, in which a small perturbation is applied to one bar. θ represents the angle of the 'tension wires' and ϕ represents the rotation of a bar compared to equilibrium.

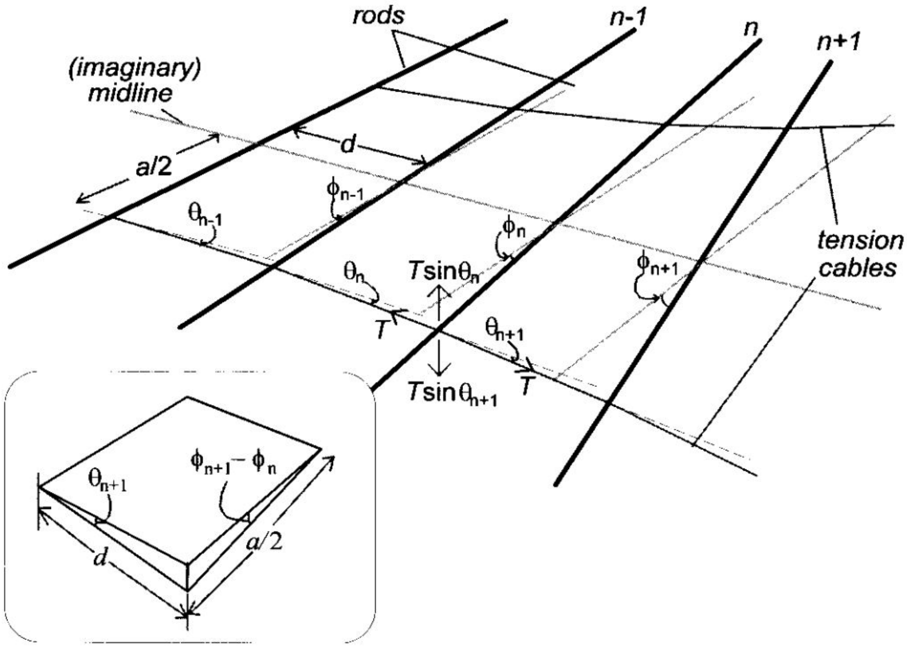

Figure 2.2: Detailed view of a small perturbation in the Shive wave machine. θ represents the angle of the 'tension wires', ϕ represents the rotation of the bar compared to equilibrium. The bars are numbered by n, n+1 etc. Skeldon labels the distance between the 'tension wire' and pivot wire $a/2$ and the distance between two bars d. Picture retrieved from [9].

The force F_n acting downward on bar n can then be described as

$$F_n = T\left(\sin\theta_{n+1} - \sin\theta_n\right), \tag{2.1}$$

in which T is the tension in the tension wire. We know that the torque is given by $\Gamma = F \times r$. For small angles θ_n, where the angle between F and r is approximately 0.5π, Γ_n is equal to:

$$\Gamma_n = Tr\left(\sin\theta_{n+1} - \sin\theta_n\right), \tag{2.2}$$

in which r is the position vector of the force, which is in picture 2.2 equal to $a/2$. The torque is also equal to $\Gamma = I\alpha$, which results in

$$\alpha I = \Gamma_n = Tr\left(\sin\theta_{n+1} - \sin\theta_n\right), \tag{2.3}$$

or,

$$\alpha = \frac{Tr}{I}\left(\sin\theta_{n+1} - \sin\theta_n\right). \tag{2.4}$$

If a complete bar is considered, the following equation for the angular acceleration is obtained:

$$\frac{\partial^2\phi_n}{\partial t^2} = \frac{2Tr}{I}(\sin\theta_{n+1} - \sin\theta_n). \tag{2.5}$$

In this equation, the force F of 2.1 is multiplied by two, because the bar experiences a force from the pivot wires on both ends. r is again the position vector and in this case the distance from the tension wire to the pivot wire.

To get the frequency of the torsional wave, we have to find an expression for θ expressed in ϕ. Figure 2.3 represents a detailed view of a perturbed bar and a bar at rest.

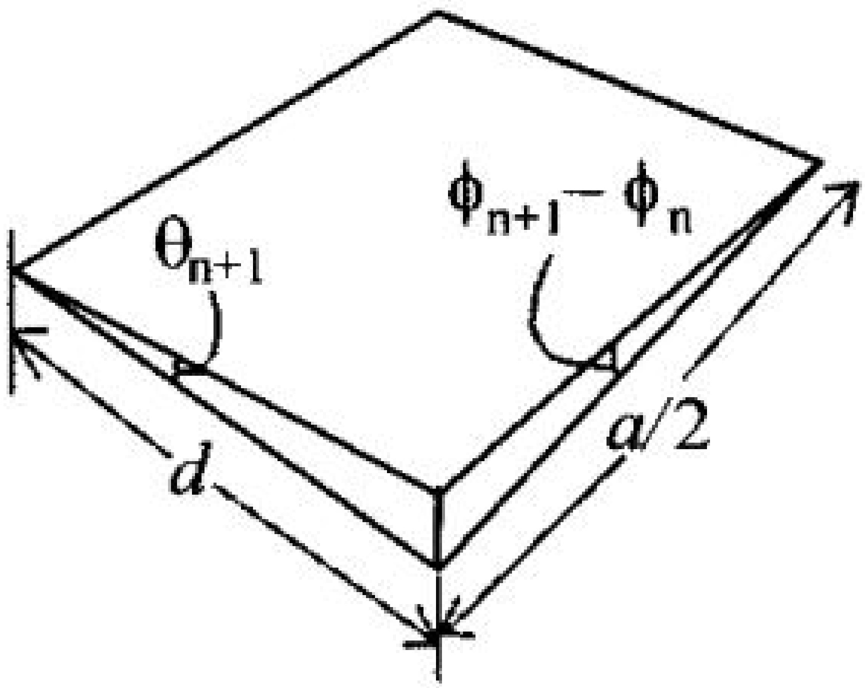

Figure 2.3: Detailed view of two bars in the Shive wave machine. θ represents the angle of the 'tension wires', ϕ represents the rotation of the bar compared to equilibrium. Picture retrieved from [9].

In the image, the left bar is put at rest so that the angle of the right bar is equal to the difference between both angles ϕ_n and ϕ_{n+1} of the two bars. If we use the small-angle approximation, which states that $\sin(\theta_{n+1}) \approx \theta_{n+1}$ and $\sin(\phi_{n+1} - \phi_n) \approx \phi_{n+1} - \phi_n$ for angles close to zero, we find

$$d\theta_{n+1} = a/2(\phi_{n+1} - \phi_n), \tag{2.6}$$

or, taking $d = Dz$ and $r = a/2$,

$$\theta_{n+1} = \frac{r}{Dz}(\phi_{n+1} - \phi_n). \tag{2.7}$$

Combining 2.5 and 2.7 and again using the small-angle approximation results in

$$\frac{\partial^2 \phi_n}{\partial t^2} = \frac{2Tr^2}{IDz}((\phi_{n+1} - \phi_n) - (\phi_n - \phi_{n-1})). \tag{2.8}$$

We can write this formula as

$$\frac{\partial^2 \phi_n}{\partial t^2} = \frac{2Tr^2}{IDz}((\phi_{n+1} - \phi_n) - (\phi_n - \phi_{n-1}))\frac{Dz^2}{Dz^2}, \tag{2.9}$$

or,

$$\frac{\partial^2 \phi_n}{\partial t^2} = \frac{2Tr^2 Dz}{I} \frac{\frac{\phi_{n+1}-\phi_n}{Dz} - \frac{\phi_n - \phi_{n-1})}{Dz}}{Dz}, \tag{2.10}$$

which can be rewritten as

$$\frac{\partial^2 \phi_n}{\partial t^2} = \frac{2Tr^2 Dz}{I} \frac{\partial^2 \phi_n}{\partial z^2} \tag{2.11}$$

if we assume that Dz is small enough and that the difference between ϕ_{n+1}, ϕ_n and ϕ_{n-1} is small enough. This is a wave euqation with velocity

$$v = \sqrt{\frac{2Tr^2 Dz}{I}} = r\sqrt{\frac{2TDz}{I}}. \tag{2.12}$$

In order to compute the uncertainty on our velocity measurements, we will use the 'calculus approach' [10]. This results in the following uncertainty for v:

$$u(v)^2 = (\frac{\partial v}{\partial r})^2(u(r))^2 + (\frac{\partial v}{\partial T})^2(u(T))^2 + (\frac{\partial v}{\partial Dz})^2(u(Dz))^2 + (\frac{\partial v}{\partial I})^2(u(I))^2. \tag{2.13}$$

In this equation, $u(v)$ denotes the uncertainty in v, $u(r)$ the uncertainty in r, $u(T)$ the uncertainty in T and $u(Dz)$ the uncertainty in Dz. 2.13 can be rewritten as:

$$u(v) = v\sqrt{(\frac{u(r)}{r})^2 + (\frac{u(T)}{2T})^2 + (\frac{u(Dz)}{2Dz})^2 + (\frac{u(I)}{2I})^2}. \tag{2.14}$$

The full derivation of the uncertainty in v is described in the appendix.

2.2 Transverse wave

We now know how fast the transverse wave is supposed to travel through a medium. The second part of this research is to investigate how the wave is transmitted and reflected when it travels from one medium to another. In chapter 4 of [11], when a one-dimensional wave on a string $\varphi(x,t)$, which moves along the x-axis from left to right, hits an interface between two media at $x = 0$, the wave can be split up into a left part $\varphi_L(x,t)$ and a right part $\varphi_R(x,t)$, defined by:

$$\varphi_L(x,t) = k_i(t - \frac{x}{v_1}) + k_r(t + \frac{x}{v_1}), \tag{2.15}$$

and,

$$\varphi_R(x,t) = k_t(t - \frac{x}{v_2}). \tag{2.16}$$

In these equations, k_i is the incident wave, k_r is the reflected wave and k_t is the transmitted wave. Furthermore, v_1 is the velocity in the 'first' medium (the left side), and v_2 is the velocity in the 'second' medium (the right side). Notice the '+'-sign in the reflected wave part; this part of the wave travels from right to left, in contrast to the other parts. Furthermore, considering a 1-D wave on a string, there are two boundary conditions:

1. The string is continuous, also at $x = 0$ at the intersection between the media. So, 2.15 and 2.16 must be equal at $x = 0$:

$$k_i(t) + k_r(t) = k_t(t). \tag{2.17}$$

2. The slope of the string is continuous. This is because if the slopes were not continuous, the tensions in the wires would result in a net force on the string. This would result in a displacement of the string, and eventually, the slope would be continuous so there would be no net force. It is helpful to think of a certain point in the wire as a massless atom as depicted in figure 2.4. There would be a net force acting on this atom if the slopes were not continuous and it would accelerate towards equilibrium:

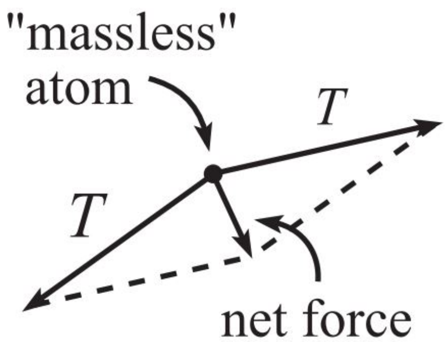

Figure 2.4: A wire with a discontinuity in its slope. Picture retrieved from [11].

Using formulas 2.15 and 2.16 and the condition that the slopes must be equal at the intersection at $x = 0$ gives:

$$\frac{\partial \varphi_L(x,t)}{\partial x}|_{x=0} = \frac{\partial \varphi_R(x,t)}{\partial x}|_{x=0}, \tag{2.18}$$

which results in:

$$v_2 k_i(t) - v_2 k_r(t) = v_1 k_t(t). \tag{2.19}$$

With 2.17 and 2.19, we get the following definitions for k_r and k_t, solely depending on k_i:

$$k_r(t + \frac{x}{v_1}) = \frac{v_2 - v_1}{v_2 + v_1} k_i(t - \frac{-x}{v_1}) = R k_i(t - \frac{-x}{v_1}), \tag{2.20}$$

and,

$$k_t(t - \frac{x}{v_2}) = \frac{2v_2}{v_2 + v_1} k_i(t - \frac{\frac{v_1}{v_2}x}{v_1}) = T k_i(t - \frac{\frac{v_1}{v_2}x}{v_1}). \tag{2.21}$$

In this research, we are interested in measuring the reflection R and the transmission T and compare them to their expected values. These expected values follow from 2.20 and 2.21, and are:

$$R = \frac{v_2 - v_1}{v_2 + v_1}, \tag{2.22}$$

and

$$T = \frac{2v_2}{v_2 + v_1}. \tag{2.23}$$

To be able to plot the data points of R and T versus one single variable, $f = \frac{v_2}{v_1}$ is introduced. The resulting formulas for R and T are:

$$R = \frac{f - 1}{f + 1}, \tag{2.24}$$

and

$$T = \frac{2f}{f + 1}. \tag{2.25}$$

It is important to stress that the expected velocity follows from 2.12, and the wave always travels from medium 1 to medium 2, in which respectively the velocity is v_1 and v_2.

The uncertainties in R, $u(R)$, and in T, $u(T)$ follow again out of the calculus approach and result in

$$u(R) = \frac{2}{(f + 1)^2} u(f), \tag{2.26}$$

and,

$$u(T) = \frac{2f}{f + 1} u(f). \tag{2.27}$$

In these equations, $u(f)$ is equal to

$$u(f) = f\sqrt{(\frac{u(v_2)}{v_2})^2 + (\frac{u(v_1)}{v_1})^2}. \tag{2.28}$$

In the appendix, the full derivation of the uncertainties is explained.

Chapter 3

Experimental method

3.1 Determination of the variables

In chapter 2, I explained that, according to the theory, the velocity of the transverse wave in the Shive wave machine is determined by four variables: a) the tension T of the tension wires, b) the distance r between the tension wires and the pivot wire, c) the distance Dz between two bars and d) the moment of inertia I of a rotating bar. In this section, I will describe how each of these four variables is measured.

(a) The tension T is measured with the 'Force Gauge PCE-FM 200' [12]. This machine measures the tension in Newton and is attached to one end of the tension wire of the system. The resolution of this machine is 0.1N. However, when there is a wave traveling through the wave system, the readings of the Force Gauge show fluctuations. The magnitude of these fluctuations depend on how tense the wire is; the tenser the wire, the bigger the fluctuations. In this report, we choose to take the magnitude of the fluctuations as uncertainty in the tension measurement so that $u(T)$ is in the range 0.1-1.00N.

(b) The distance r between the tension wires and the pivot wire can have five possible values. This is because the tension wire has to go through a hole in the bars, and each bar has five holes at each side of the pivot wire. The holes are placed so that r can be equal to 3, 6, 9, 12 or 15cm. The distance r is measured with a ruler with an uncertainty of 0.05cm, so $u(r) = 0.05$cm.

(c) The distance Dz between two consecutive bars can be determined by dividing the distance of the first bar to the last bar by the number of bars minus one. The distance is measured with a tape line and is equal to 341cm. The number of bars is 32. So, $Dz = \frac{341\text{cm}}{32-1} = 11$cm. The uncertainty in the tapeline is 0.5cm, so $u(Dz) = 0.5$cm.

(d) The moment of inertia I of a rotating bar is equal to

$$I = 1/12m(b^2 + l^2), \tag{3.1}$$

in which b and l are the dimensions of the bar perpendicular to the pivot wire, and m is the mass of the bar. The bars used in this research are made of aluminum and are 60cm x 1cm x 1cm. The mass of a single bar is the product of its volume, 60cm^3, and its density, 2702kg/m^3 [13], and is equal to $m = 0.16\text{kg}$. Filling in 3.1 results in $I = 0.0049\text{kgm}^2$. The uncertainty in the moment of inertia of a bar is equal to $u(I) = 0.0004\text{kgm}^2$. For the full derivation of the uncertainty in I, see A.

3.2 Validating the wave velocity theory

To validate the theory of the wave velocity in the Shive wave machine, the first part of this research consists of measuring wave velocities v for different tensions T. The other parameters are constant, and equal to $Dz = 11 \pm 0.5\text{cm}$, $r = 6.0 \pm 0.05\text{cm}$ and $I = 0.0049 \pm 0.0004\text{kgm}^2$. I calculate the velocity of a wave by starting a wave in the Shive wave machine and by measuring the time and distance it takes the maximum to travel from one point to another. This process is repeated five times at a set tension. Out of each set of five velocities, the mean velocity is calculated with an error. The measured velocity is then equal to

$$v_{meas} = \frac{\Delta z}{\Delta \bar{t}}, \tag{3.2}$$

in which Δz is the distance travelled by the wave and $\Delta \bar{t}$ is the mean time. The uncertainty in v_{meas} is equal to

$$u(v_{meas}) = v_{meas}\sqrt{(\frac{u(\Delta z)}{\Delta z})^2 + (\frac{u(\Delta \bar{t})}{\Delta \bar{t}})^2}. \tag{3.3}$$

I present the measured results of v in chapter 4 together with the theoretically predicted results of v, which follow from 2.12. I will also present the uncertainties in these results.

3.3 Simulating two media in the Shive wave machine

In wave physics, the velocity of a wave is determined by the medium it travels through. The four mentioned variables can be seen as medium-defining. In this experiment, I want to simulate the transition of a transverse wave from medium one to medium two, so, I need to set different variables for medium one and two. To fully see the influence of a single variable on the transition and reflection of a wave, I chose to only alter one of the four variables throughout the system. For example, if I set r_1 for the first part and r_2 for the second part, the velocities, defined by 2.12, are respectively $v_1 = r_1\sqrt{\frac{2TDz}{I}}$ and $v_2 = r_2\sqrt{\frac{2TDz}{I}}$. The expected values for the reflection R and the transmission T can then be calculated via 2.22 and 2.23.

Of the four variables, it is not possible to set two different tensions T_1 and T_2, because the tension wires in the system consist of a single wire which goes across all the bars, and tension, therefore, is equally distributed across its whole length. Furthermore, it is very time-consuming to change the distance Dz in the system, because the bars are screwed onto the pivot wire. So, in this project, I chose to simulate the two media by changing r or by changing I. I stress again that these two parameters were not changed at the same time, so either r was constant throughout the system and I was changing over the intersection, or I was constant and r was changing.

In this part of the research I will first apply the theory of chapter 2 to calculate a theoretical prediction for R and T with uncertainties, called R_{cal} and T_{cal}, followed by an explanation of the practical measurements of R and T, called R_{meas} and T_{meas}.

3.3.1 Distance of tension wire r_i

r is the distance from the tension wire to the pivot wire. In figure 3.1, a schematic example with r_1 and r_2 is given in the YZ-plane. We see that in this example r_1 is larger than r_2, but this can also be vice versa.

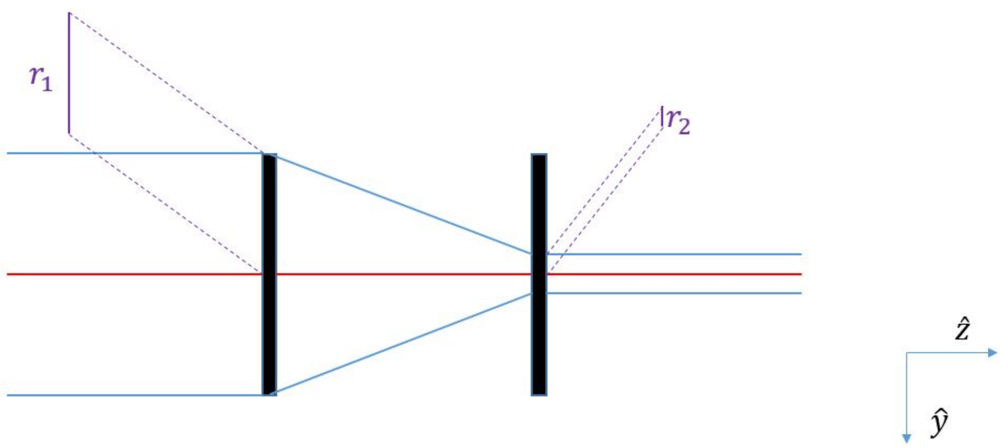

Figure 3.1: A schematic overview of system 2.1 with two different r's, r_1 and r_2, in the YZ-plane.

When one sets two different r's for the system, r_1 and r_2, the expected reflection and transmission values will be

$$R_{cal} = \frac{v_2 - v_1}{v_2 + v_1} = \frac{r_2 - r_1}{r_2 + r_1}, \tag{3.4}$$

and

$$T_{cal} = \frac{2v_2}{v_2 + v_1} = \frac{2r_2}{r_2 + r_1}. \tag{3.5}$$

With $f = \frac{v_2}{v_1} = \frac{r_2}{r_1}$, the uncertainty relations of 2.26, 2.27 and 2.28 can be simplified to:

$$u(f) = f\sqrt{(\frac{u(r_2)}{r_2})^2 + (\frac{u(r_1)}{r_1})^2}, \tag{3.6}$$

$$u(R_{cal}) = \frac{2}{(f+1)^2} u(f), \tag{3.7}$$

$$u(T_{cal}) = \frac{2f}{f+1} u(f). \tag{3.8}$$

In the results, the measurements of f, R_{cal} and T_{cal} and their uncertainties will be given.

3.3.2 Moment of inertia I_i

The moment of inertia I of a set of N_{mass} point masses is equal to

$$I = \sum_{i}^{N_{mass}} m_i r_i^2, \tag{3.9}$$

in which m_i denotes all independent weights i, and in which r_i denotes the distance from each weight i to the axis of rotation of the system. Equation 3.1 gives the specific moment of inertia for a bar. In order to change the moment of inertia of the system, one has to add or subtract weight, or he has to distribute the weight otherwise. In this study, I chose to go with the first option and to add weight to the system in the form of small weights, which are assumed to be point masses. The total moment of inertia of the system will then be equal to

$$I_{tot} = \sum_{i}^{N_{mass}} m_i r_i^2 + 1/12m(b^2 + l^2). \tag{3.10}$$

The first part of this equation represents the added point masses, the second part represents the aluminum bar. The added masses in this experiment have a weight of $m_i = 0.080 \pm 0.001$kg. Furthermore, the distance r_i in this equation is the distance of the center of the masses to the pivot axis and can vary in the range of $0 - 30$cm (which is half the length of the bars). A schematic figure of how the point masses are added to the system is given in 3.2:

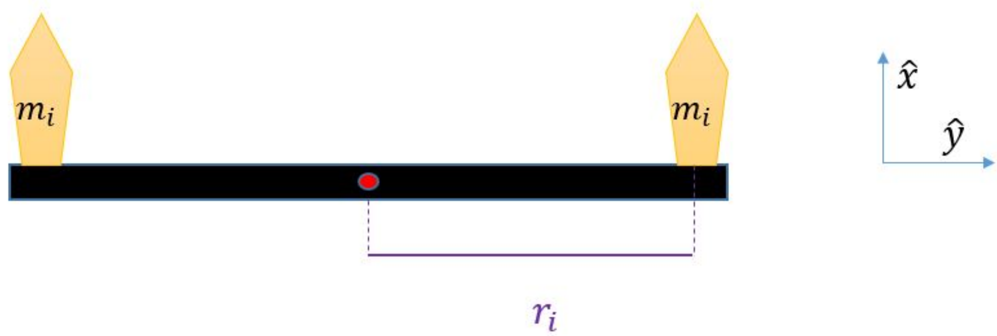

Figure 3.2: A schematic overview in the XY-plane of system 2.1 with added point masses.

The velocity throughout the wave machine can then be calculated with formulas 3.10 and 2.12, and with formulas 2.22 and 2.23, R and T can be calculated:

$$R_{cal} = \frac{v_2 - v_1}{v_2 + v_1} = \frac{\frac{1}{\sqrt{I_2}} - \frac{1}{\sqrt{I_1}}}{\frac{1}{\sqrt{I_2}} + \frac{1}{\sqrt{I_1}}} = \frac{\sqrt{I_1} - \sqrt{I_2}}{\sqrt{I_1} + \sqrt{I_2}}, \tag{3.11}$$

and

$$T_{cal} = \frac{2v_2}{v_2 + v_1} = \frac{\frac{2}{\sqrt{I_2}}}{\frac{1}{\sqrt{I_2}} + \frac{1}{\sqrt{I_1}}} = \frac{2\sqrt{I_1}}{\sqrt{I_1} + \sqrt{I_2}}. \tag{3.12}$$

With $f = \frac{v_2}{v_1} = \frac{\sqrt{I_1}}{\sqrt{I_2}}$, the uncertainty relations of 2.26, 2.27 and 2.28 can be simplified to:

$$u(f) = f\sqrt{(\frac{u(I_2)}{2I_2})^2 + (\frac{u(I_1)}{2I_1})^2}, \tag{3.13}$$

$$u(R_{cal}) = \frac{2}{(f+1)^2}u(f), \tag{3.14}$$

$$u(T_{cal}) = \frac{2f}{f+1}u(f). \tag{3.15}$$

The uncertainty in I_1 and in I_2, or in I_{tot}, $u(I_{tot})$, is equal to

$$u(I_{tot}) = \sqrt{(u(I_{bar}))^2 + N_{mass}(u(I_{mass}))^2}, \tag{3.16}$$

in which $u(I_{bar}) = 0.0004\text{kgm}^2$, the uncertainty of the moment of inertia of the bars, and the uncertainty in the moment of inertia of a single mass is

$$u(I_{mass}) = I_{mass}\sqrt{(\frac{u(m_i)}{m_i})^2 + (\frac{u(r_i)}{r_i})^2}, \tag{3.17}$$

in which I_{mass} follows from 3.9 and m_i and r_i are respectively the mass and distance to the pivot wire of an applied mass. The maximum values for m_i and r_i are 0.080 ± 0.001kg and 0.30 ± 0.01m. The factor N_{mass} is the number of applied masses and is in this research equal to 0, 2, 4 or 6. In the results, the measurements of f, R_{cal} and T_{cal} and their uncertainties will be given.

3.3.3 Imaging and data processing

If the results of the theoretical values match that of the measured values, it is possible to conclude that the Shive wave machine is a reliable simulation tool for the study of waves across multiple media, which may be relevant in the field of acoustic imaging.

To measure the reflection and transmission, the Shive wave machine is placed in a darkened room. The endpoints of the bars of one side of the system ($+\hat{y}$) are painted with fluorescent paint. To light up these fluorescent points, UV lamps are placed around the system. One can shoot videos of a traveling wave, which would result in videos with frames like the one displayed in figure 3.3. In this research, I made use of the camera 'Nikon D5300' [14], which shoots videos at 50 frames per second.

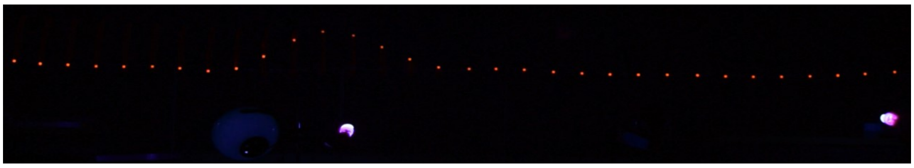

Figure 3.3: Example of a frame shot by the video camera used in this experiment. The camera shoots 50 frames per second.

I used the program ImageJ [15] to turn the red dots (as the ones shown in figure 3.3) into coordinates. This program is able to do the following processes:

1. Set scale. You can select two points in a frame and set the distance between them. ImageJ then calculates the number of pixels per unit length and uses this scale for all of your frames.

2. Split color channels. ImageJ splits the channels into a red, green and blue channel. For the fluorescence shown in 3.3, the red channel is the most useful channel.

3. Threshold. ImageJ sets a threshold so that if a pixel meets this threshold, it is assigned the value '1', and if it does not meet the threshold, it is assigned the value '0'. The example image from 3.3 will then look like the binary figure 3.4:

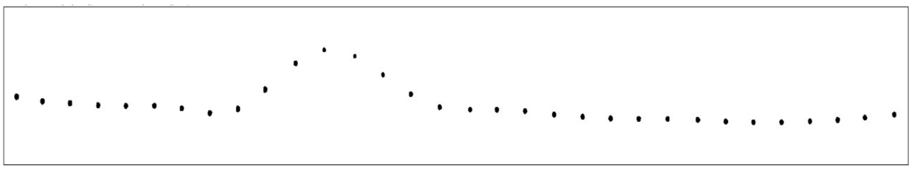

Figure 3.4: The same example shot from 3.3 is turned into a binary image.

4. Measure particles. ImageJ presents a data sheet with the x- and y-coordinates of the particles of the binary image. In 3.5, the example shot is used to shown how ImageJ finds the coordinates:

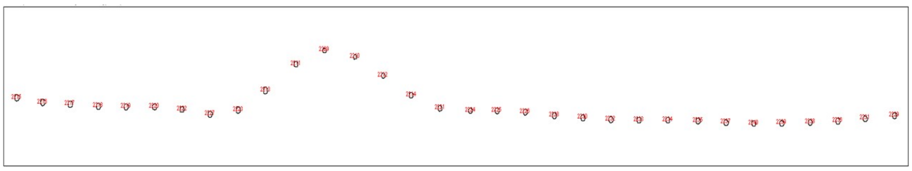

Figure 3.5: ImageJ finds the coordinates of the binary image.

The coordinates can be stored in a data-sheet and imported into Matlab for data processing.

3.3.4 Measuring amplitude

In figure 3.5 it can be seen that the dots at rest do not always have the same height. There may be a number of explanations, such as that the weight carrying pivot wire, which should be completely straight, is bent by the weight of the bars, or that the video camera is tilted during the filming. One can correct for the height of the weights, by subtracting the height of each data point by its initial height in rest.

To obtain the reflection R_{meas}, I chose to measure the maximum amplitude of a bar before the intersection of medium 1 to medium 2 and to measure the maximum amplitude of the same bar when the reflected wave passes back through it. Note that the amplitude of the reflected wave can be both positive and negative, according to the theoretical prediction in formula 2.22. The reflection coefficient is then measured by

$$R_{meas} = \frac{A_{bar,r}}{A_{bar,i}}, \tag{3.18}$$

in which $A_{bar,r}$ is the amplitude of the reflected wave and in which $A_{bar,i}$ is the amplitude of the initial wave for the same bar. To obtain the transmission T_{meas}, I chose to measure the maximum amplitude of a bar before the intersection of medium 1 to medium 2, and to measure the maximum amplitude of a bar when the transmitted wave passes through it. Note that the amplitude of the transmitted wave can only be positive, according to the theoretical prediction in formula 2.23. The transmission coefficient is then measured by

$$T_{meas} = \frac{A_{bar,t}}{A_{bar,i}}, \tag{3.19}$$

in which $A_{bar,t}$ is the amplitude of the transmitted wave.

The uncertainty in R_{meas} and T_{meas} are equal to

$$u(R_{meas}) = R_{meas}\sqrt{(\frac{u(A_{bar,r})}{A_{bar,r}})^2 + (\frac{u(A_{bar,i})}{A_{bar,i}})^2}, \tag{3.20}$$

and,

$$u(T_{meas}) = T_{meas}\sqrt{(\frac{u(A_{bar,t})}{A_{bar,t}})^2 + (\frac{u(A_{bar,i})}{A_{bar,i}})^2}. \tag{3.21}$$

The uncertainties $u(A_{bar,i})$, $u(A_{bar,r})$ and $u(A_{bar,t})$ follow from the uncertainty in position of the binary dots and are equal to 0.50cm.

Chapter 4

Results and Discussion

4.1 Validating the wave velocity theory

The first experiment in this research was to measure wave velocities in the Shive wave machine against different tensions. The measured times and lengths from which the velocity is calculated are shown in table B.1. The variables during these measurements were equal to $Dz = 11 \pm 0.5$cm, $r = 6.0 \pm 0.05$cm and $I = 0.0049 \pm 0.0004$kgm^2. In table 4.1 the velocities v_{cal} calculated with equation 2.12 and the measured velocities v_{meas} are shown together with their uncertainties. The results and uncertainties of the measured results of v_{meas} are also plotted in figure 4.1, together with the curve of formula 2.12:

Table 4.1: The results of the measurements of validating the Shive wave machine.

T (N)	v_{cal} (m/s)	v_{meas} (m/s)
9.40 ± 0.25	1.23 ± 0.12	1.32 ± 0.04
11.40 ± 0.20	1.36 ± 0.13	1.49 ± 0.02
13.75 ± 0.20	1.50 ± 0.14	1.65 ± 0.02
16.00 ± 0.40	1.61 ± 0.15	1.75 ± 0.03
18.00 ± 0.20	1.71 ± 0.16	1.87 ± 0.06
21.00 ± 0.90	1.85 ± 0.17	1.96 ± 0.05
22.00 ± 0.50	1.89 ± 0.18	2.06 ± 0.04
24.60 ± 0.40	2.00 ± 0.19	2.13 ± 0.02
27.60 ± 1.00	2.12 ± 0.20	2.25 ± 0.02

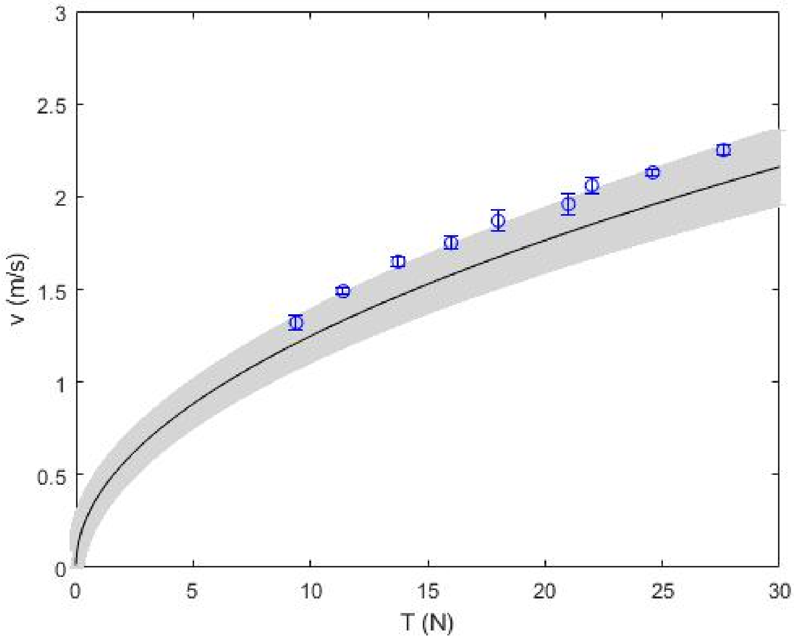

Figure 4.1: The blue points represent the measured velocities and their uncertainties. The black line represents the curve of formula 2.12 and the gray area represents the uncertainty in the formula. The uncertainty bars vary in amplitude as can be seen in tables B.1 and 4.1.

4.2 Simulating two media in the Shive wave machine

4.2.1 Distance of tension wire r_i

In this research, r can be equal to 3, 6, 9, 12 and 15cm. Furthermore, the theory states that v depends linearly on r via formula 2.12. I have chosen to limit the value of r to 3, 6 and 9 cm to keep the velocity as low as possible. The combinations used for r_1 and r_2 were:

1. $r_1 = 9$cm, $r_2 = 3$cm
2. $r_1 = 3$cm, $r_2 = 9$cm
3. $r_1 = 6$cm, $r_2 = 3$cm
4. $r_1 = 3$cm, $r_2 = 6$cm

I chose to include two binary (z,x)-frames of a video of a traveling wave for each combination. In the frames are 32 black dots visible, which represent the bars, and in the bottom right corner is a black legend, which represents a length of 20cm. In the first frame, the initial wave is visible; its direction is denoted by a red arrow. In the second frame, the reflected and transmitted waves are visible; their directions are denoted by a green and a blue arrow. The intersection between the two media lays between the 16th and 17th bar. I also included a

(t,x)-plot of the 20th bar and the 13th bar of the system. The 20th bar and the 13th bar are denoted by red and blue circles respectively. The amplitudes of these bars are used in formulas 3.18 and 3.19 to calculate R and T.

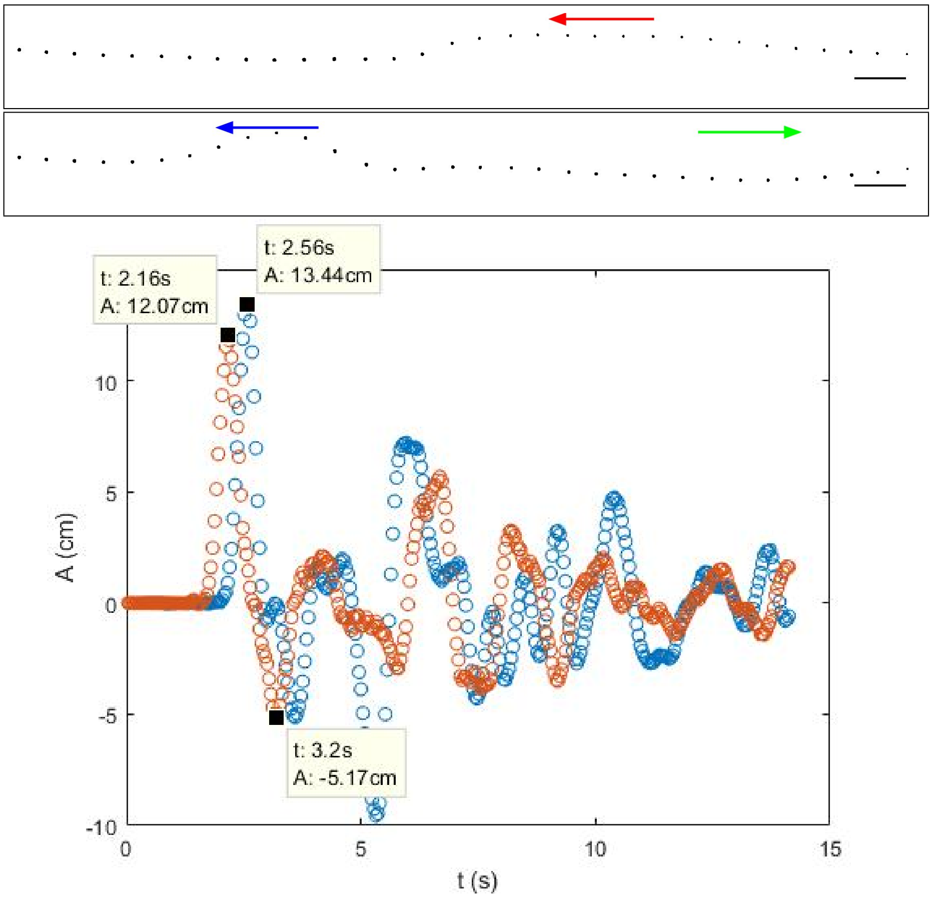

Figure 4.2: The binary frames (first two images) and the plot of the amplitudes (third image) of a wave through system 1. In the first two images, the red, green and blue arrow respectively denote the direction of the initial, reflected and transmitted wave, and the black bar represents a distance of 20cm. In the third image, the red and blue circles respectively denote bars of the right and left side of the system.

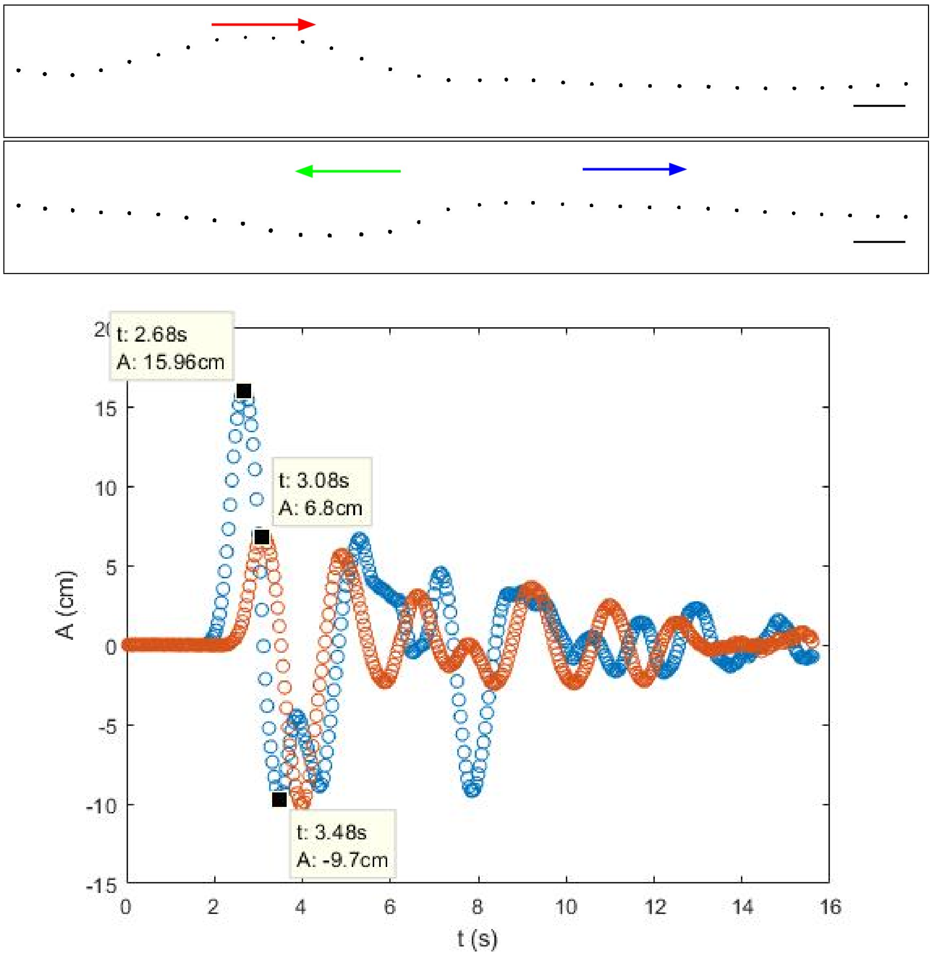

Figure 4.3: The binary frames (first two images) and the plot of the amplitudes (third image) of a wave through system 2. In the first two images, the red, green and blue arrow respectively denote the direction of the initial, reflected and transmitted wave, and the black bar represents a distance of 20cm. In the third image, the red and blue circles respectively denote bars of the right and left side of the system.

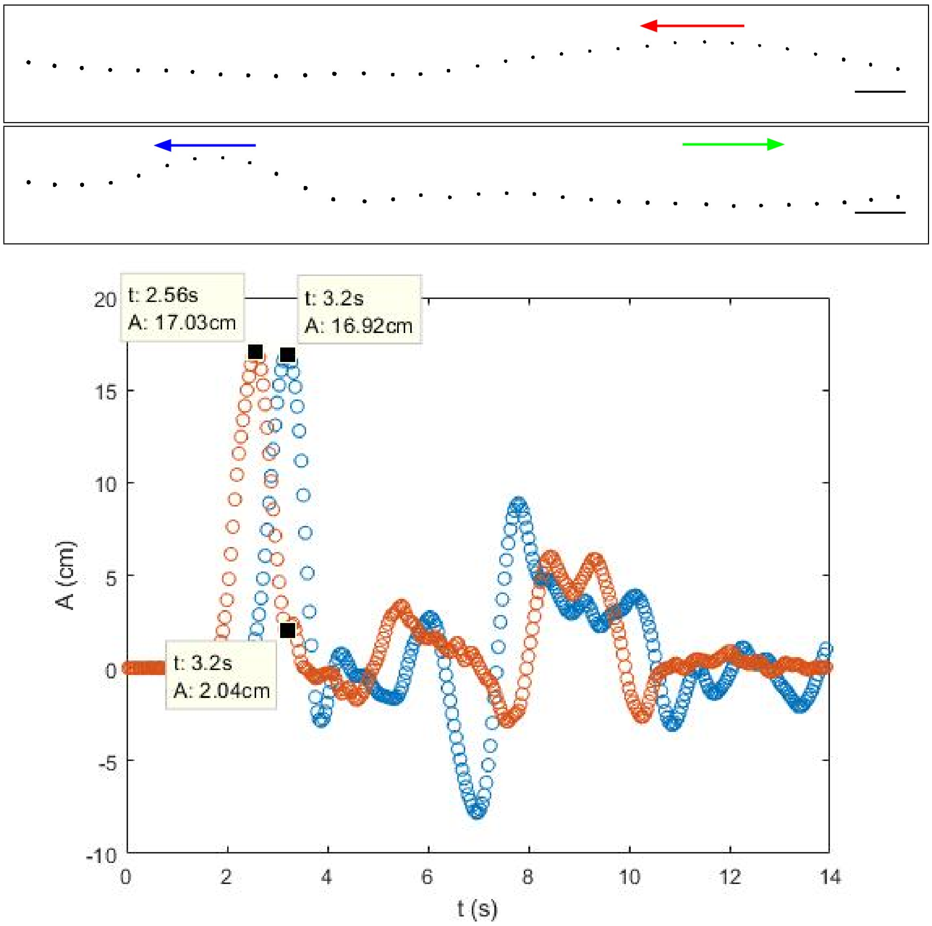

Figure 4.4: The binary frames (first two images) and the plot of the amplitudes (third image) of a wave through system 3. In the first two images, the red, green and blue arrow respectively denote the direction of the initial, reflected and transmitted wave, and the black bar represents a distance of 20cm. In the third image, the red and blue circles respectively denote bars of the right and left side of the system.

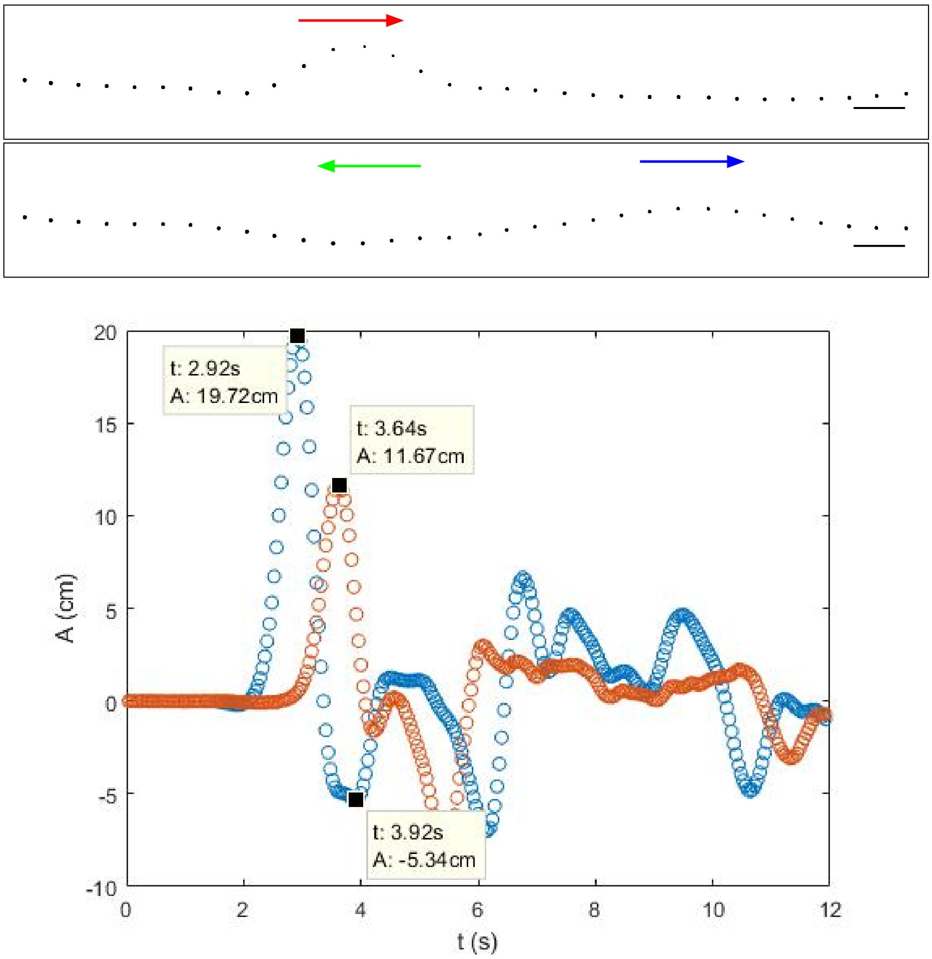

Figure 4.5: The binary frames (first two images) and the plot of the amplitudes (third image) of a wave through system 4. In the first two images, the red, green and blue arrow respectively denote the direction of the initial, reflected and transmitted wave, and the black bar represents a distance of 20cm. In the third image, the red and blue circles respectively denote bars of the right and left side of the system.

Out of the Matlab-plots, the reflection R and transmission T can be calculated via formulas 3.18 and 3.19. The results are given in B. The measured and expected values for R and T are shown in table 4.2:

Table 4.2: The results of the measurements for R and T by changing r throughout the Shive wave machine. The theoretically expected values of R and T are also given.

f(-)	R_{cal}(-)	T_{cal}(-)	R_{meas}(-)	T_{meas}(-)
3.00 ± 0.05	0.50 ± 0.01	1.50 ± 0.08	-0.43 ± 0.05	1.11 ± 0.06
0.33 ± 0.01	-0.50 ± 0.01	0.50 ± 0.01	-0.61 ± 0.04	0.43 ± 0.03
2.00 ± 0.04	0.33 ± 0.01	1.33 ± 0.05	0.12 ± 0.12	0.99 ± 0.04
0.50 ± 0.01	-0.33 ± 0.01	0.67 ± 0.01	-0.27 ± 0.03	0.59 ± 0.03

4.2.2 Moment of inertia I_i

In 3.3.2 is mentioned that the extra masses applied to the bars are equal to $m_i = 0.080 \pm 0.001$kg. The increased moment of inertia I_{tot} can be calculated via formula 3.10. The combinations used for I_1 and I_2 were:

1. $I_1 = 0.0049$kgm^2, $I_2 = 0.0183$kgm^2
2. $I_1 = 0.0183$kgm^2, $I_2 = 0.0049$kgm^2
3. $I_1 = 0.0049$kgm^2, $I_2 = 0.0300$kgm^2
4. $I_1 = 0.0300$kgm^2, $I_2 = 0.0049$kgm^2
5. $I_1 = 0.0049$kgm^2, $I_2 = 0.0336$kgm^2
6. $I_1 = 0.0049$kgm^2, $I_2 = 0.0219$kgm^2

Again, I included the binary frames of the combinations. The intersection between the two media now lays between the 24th and 25th bar. I included a (t,x)-plot of the 28th bar and the 21st bar of the system. The 28th bar and the 21st bar are denoted by red and blue circles respectively. The amplitudes of these bars is used in formulas 3.18 and 3.19 to calculate R and T.

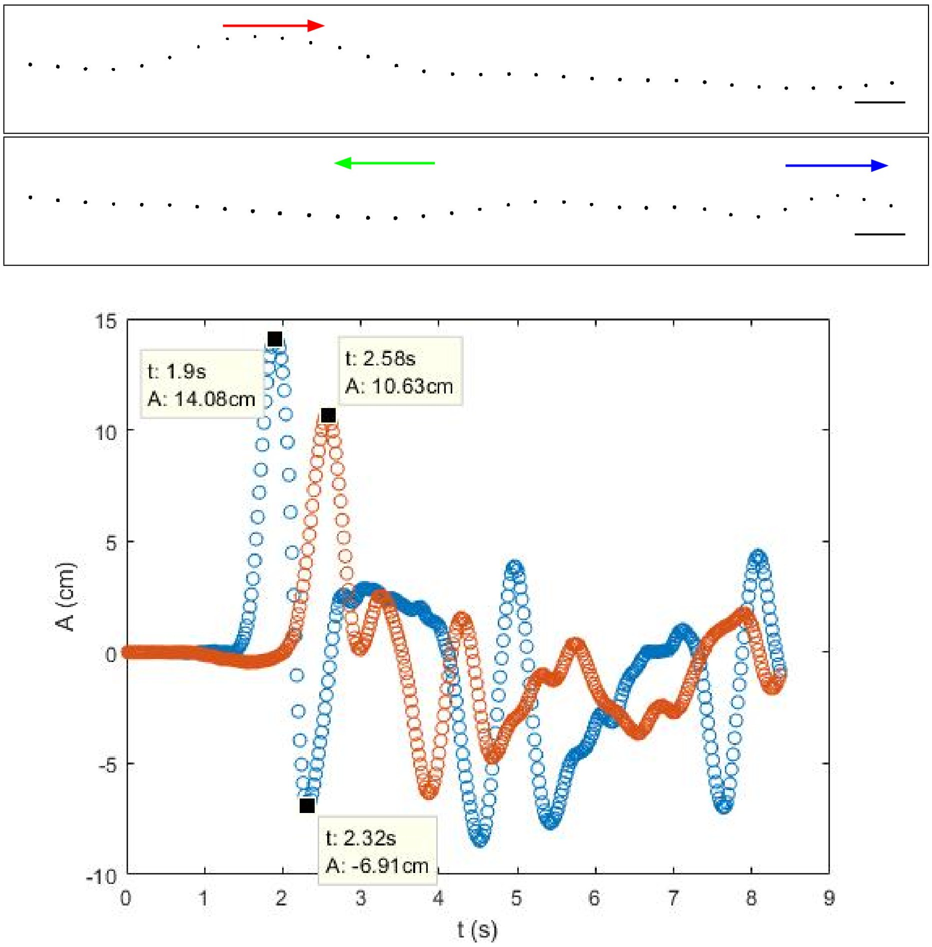

Figure 4.6: The binary frames (first two images) and the plot of the amplitudes (third image) of a wave through system 1. In the first two images, the red, green and blue arrow respectively denote the direction of the initial, reflected and transmitted wave, and the black bar represents a distance of 20cm. In the third image, the red and blue circles respectively denote bars of the right and left side of the system.

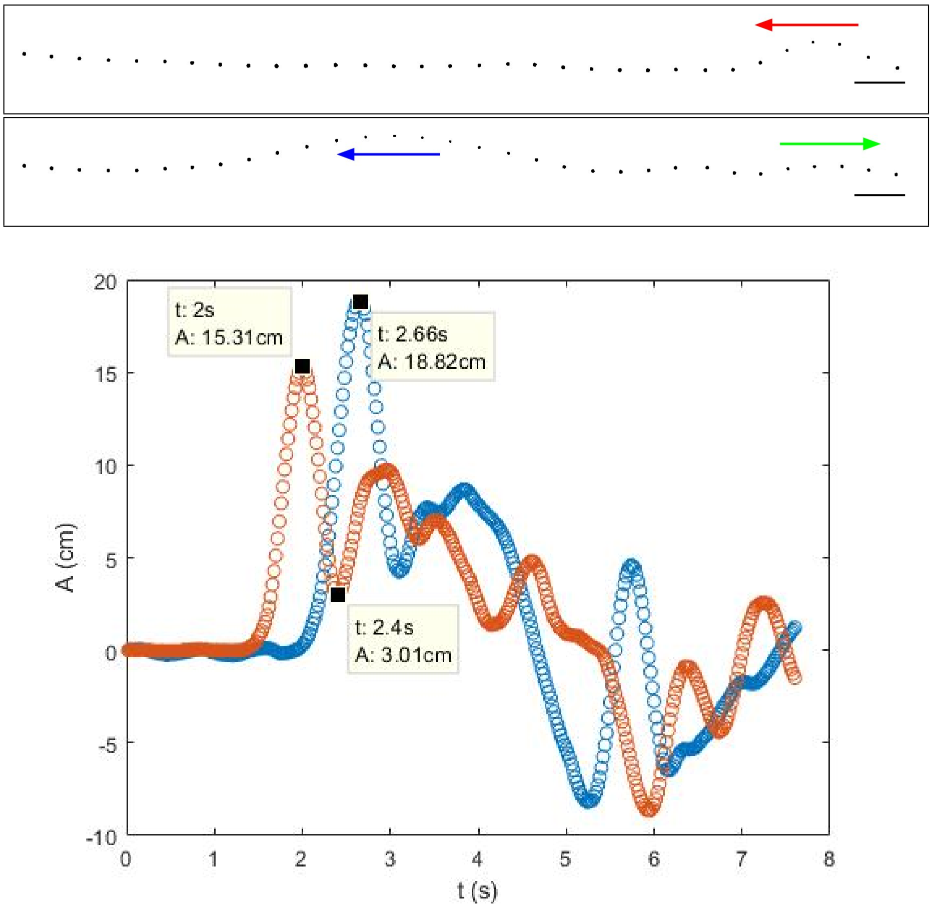

Figure 4.7: The binary frames (first two images) and the plot of the amplitudes (third image) of a wave through system 2. In the first two images, the red, green and blue arrow respectively denote the direction of the initial, reflected and transmitted wave, and the black bar represents a distance of 20cm. In the third image, the red and blue circles respectively denote bars of the right and left side of the system.

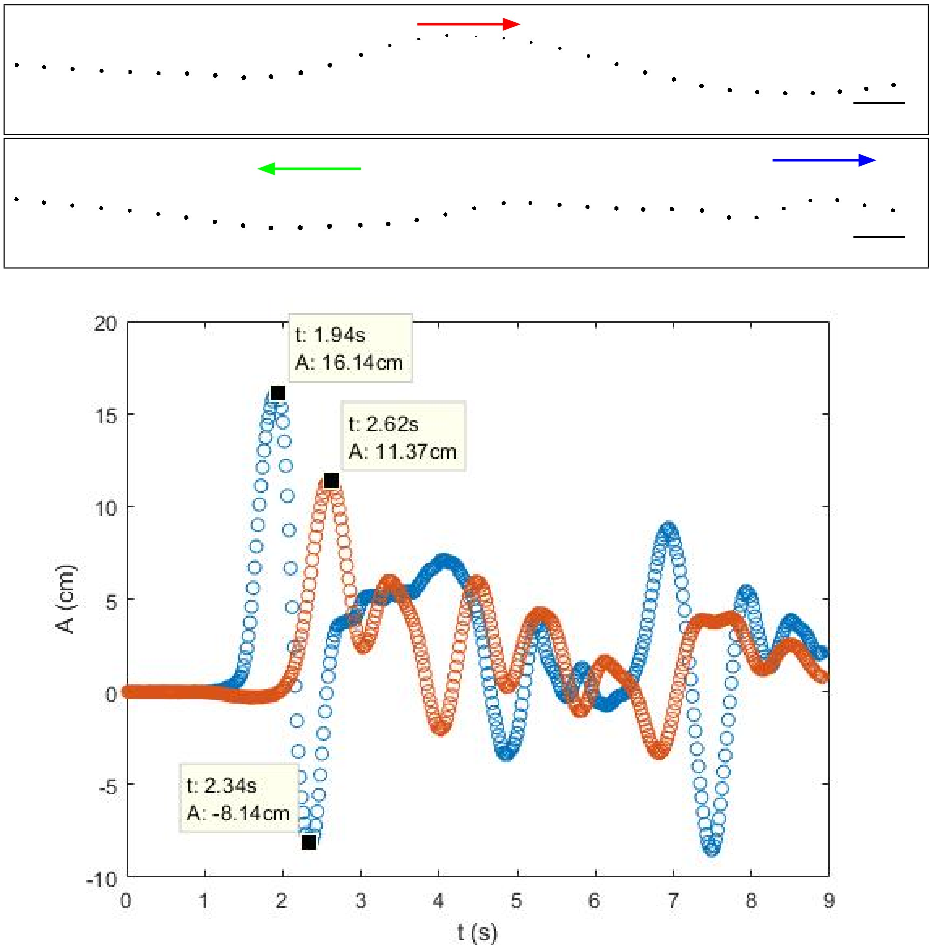

Figure 4.8: The binary frames (first two images) and the plot of the amplitudes (third image) of a wave through system 3. In the first two images, the red, green and blue arrow respectively denote the direction of the initial, reflected and transmitted wave, and the black bar represents a distance of 20cm. In the third image, the red and blue circles respectively denote bars of the right and left side of the system.

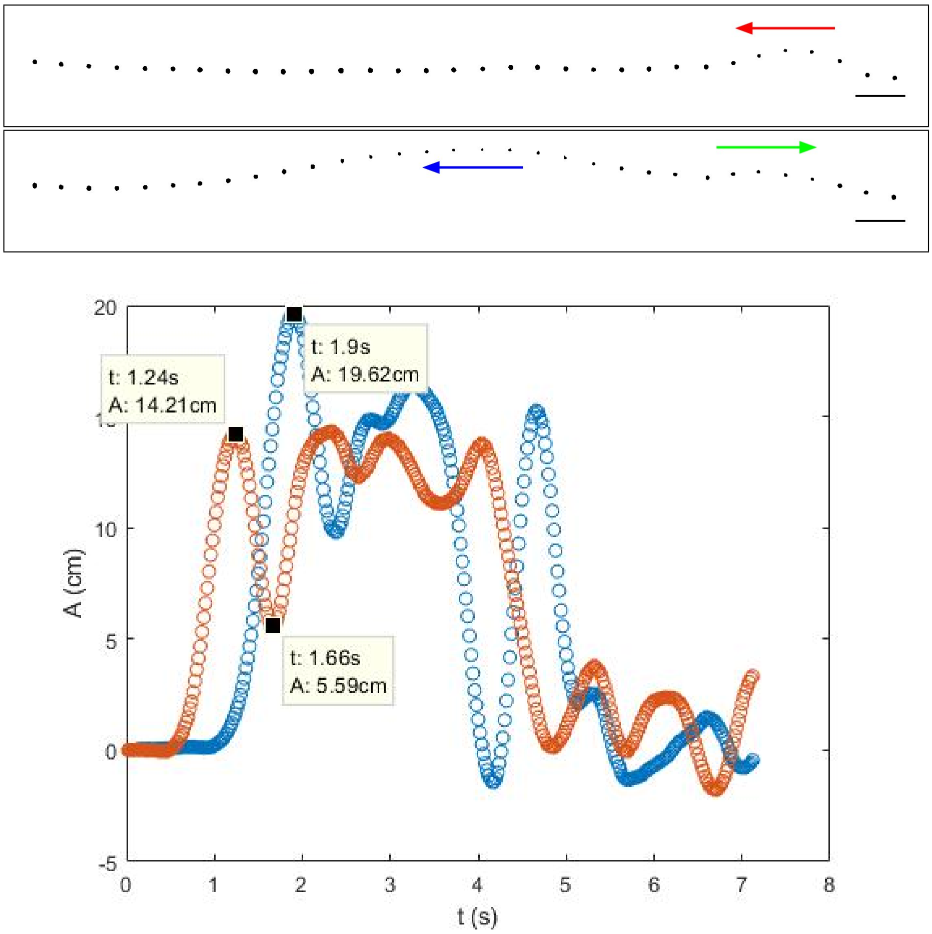

Figure 4.9: The binary frames (first two images) and the plot of the amplitudes (third image) of a wave through system 4. In the first two images, the red, green and blue arrow respectively denote the direction of the initial, reflected and transmitted wave, and the black bar represents a distance of 20cm. In the third image, the red and blue circles respectively denote bars of the right and left side of the system.

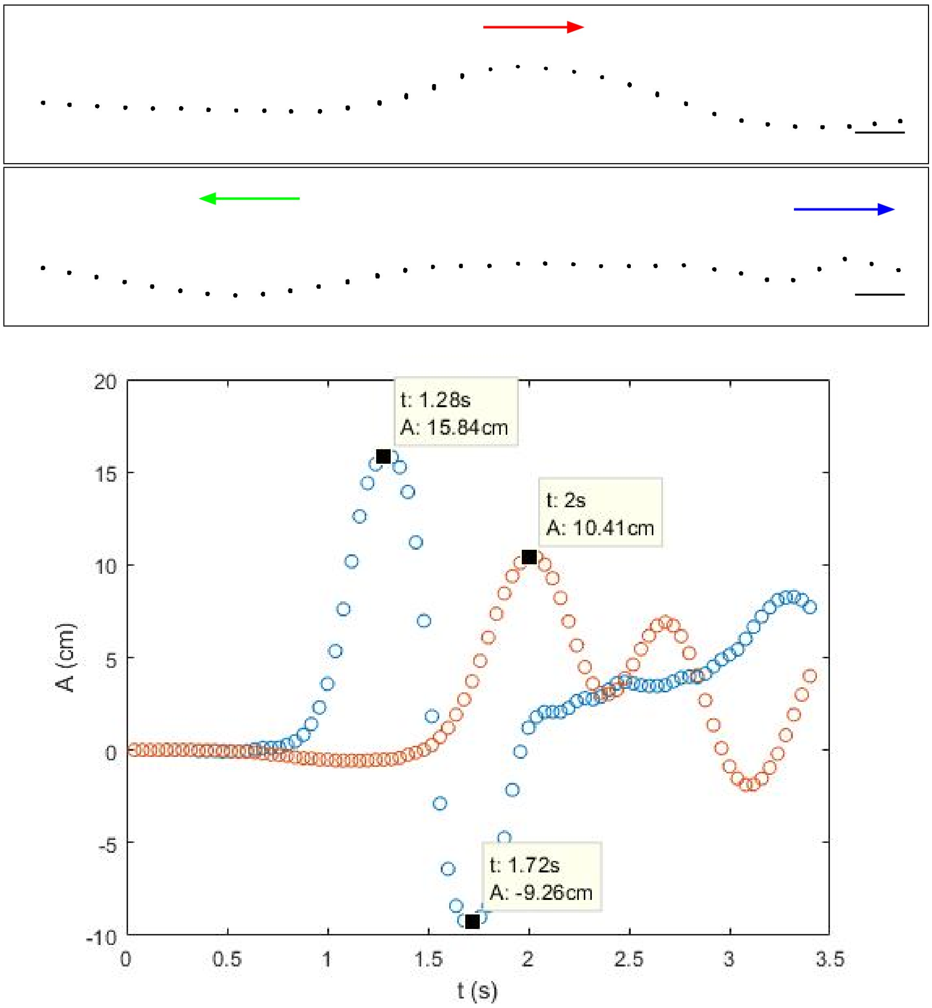

Figure 4.10: The binary frames (first two images) and the plot of the amplitudes (third image) of a wave through system 5. In the first two images, the red, green and blue arrow respectively denote the direction of the initial, reflected and transmitted wave, and the black bar represents a distance of 20cm. In the third image, the red and blue circles respectively denote bars of the right and left side of the system.

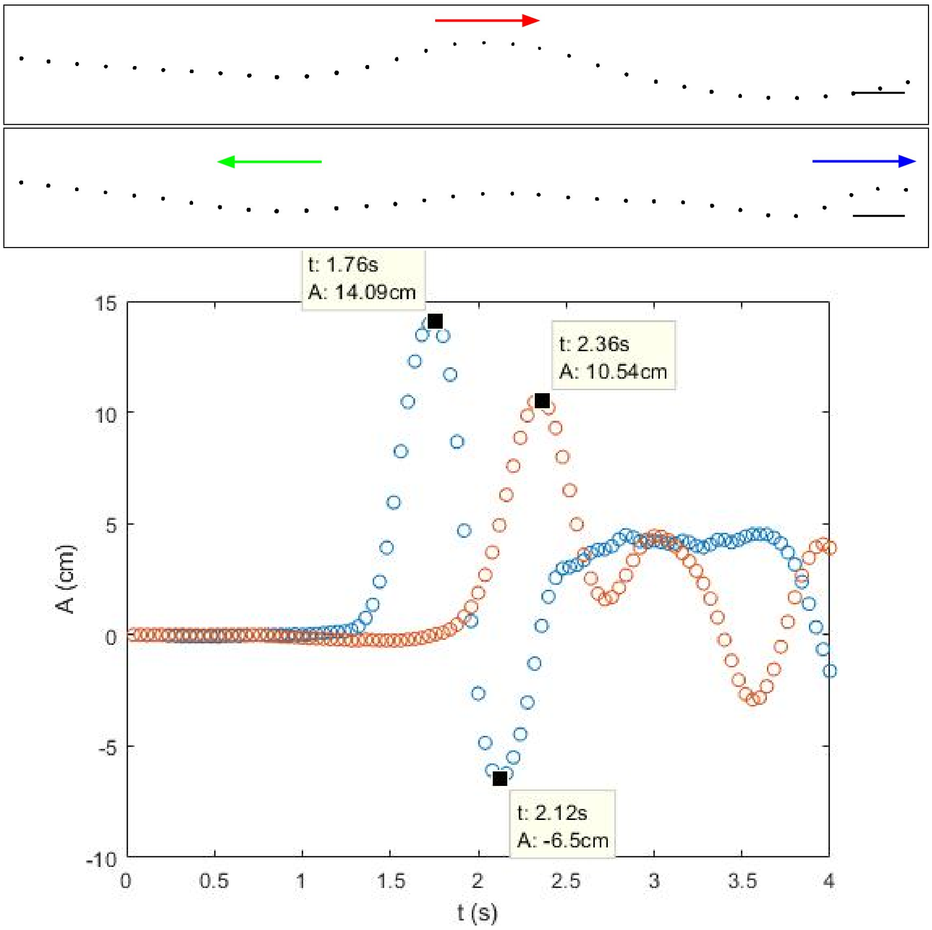

Figure 4.11: The binary frames (first two images) and the plot of the amplitudes (third image) of a wave through system 6. In the first two images, the red, green and blue arrow respectively denote the direction of the initial, reflected and transmitted wave, and the black bar represents a distance of 20cm. In the third image, the red and blue circles respectively denote bars of the right and left side of the system.

Out of the Matlab-plots, the reflection R and transmission T can be calculated via formulas 3.18 and 3.19, and the uncertainties in f, R and T via formulas 3.13, 3.14 and 3.15. The measured results for the amplitude and moment of inertia are given in B. The measured and expected values for R and T are shown in table 4.3:

Table 4.3: The results of the measurements by changing I throughout the Shive wave machine. The theoretically expected values of R and T are also given.

f(-)	R_{cal}(-)	T_{cal}(-)	R_{meas}(-)	T_{meas}(-)
0.51 ± 0.01	-0.32 ± 0.01	0.68± 0.01	-0.49 ±0.04	0.76 ± 0.04
1.94 ± 0.03	0.32 ± 0.01	1.32± 0.01	0.20 ±0.03	1.23 ± 0.05
0.40 ± 0.01	-0.43 ± 0.01	0.57± 0.01	-0.50 ±0.03	0.70 ± 0.04
2.48 ± 0.05	0.43 ± 0.01	1.43± 0.01	0.39 ±0.04	1.38 ± 0.06
0.38 ± 0.01	-0.45 ± 0.01	0.55± 0.01	-0.58 ±0.04	0.66 ± 0.04
0.47 ± 0.01	-0.36 ± 0.01	0.64± 0.01	-0.46 ±0.04	0.75 ± 0.04

4.2.3 Plot of data points

The results for R and T in the Shive wave machine for media that differ in their value of r and I respectively are given in tables 4.2 and 4.3. These results are plotted against the factor f in figure 4.12. In this figure, the red data points denote R and T and their uncertainties for changing r, and the green data points denote R and T and their uncertainties for changing I. Furthermore, the blue and black line are the theoretical relations between R and f or T and f of formulas 2.22 and 2.23. The results will be discussed in section 4.3.

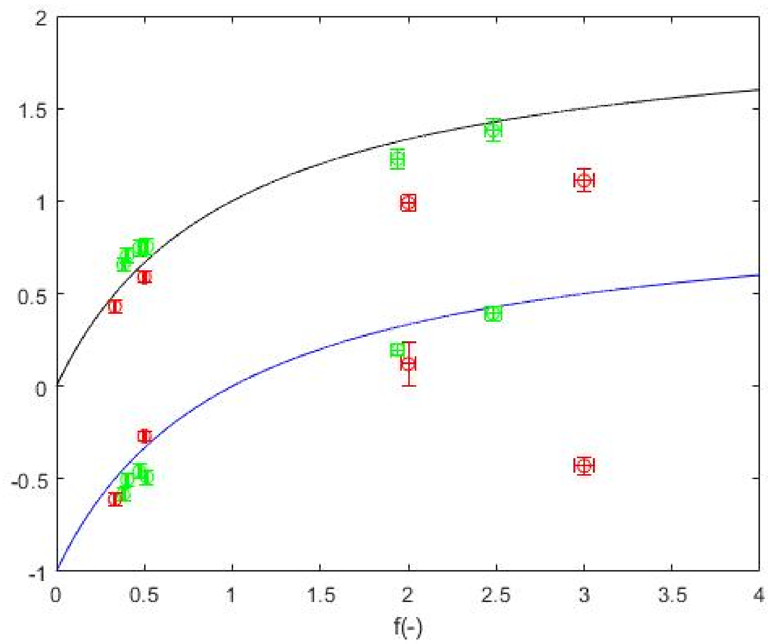

Figure 4.12: Matlab plot with the results of this research. The blue and black line denote respectively the theoretical relation between the reflection R and f, and the transmission T and f. The red data points denote the results of the system with changing r, and the green data points denote the results of the system with changing I.

4.3 Discussion

4.3.1 Validating the wave velocity theory

In table 4.1 and figure 4.1, the results of the validation of the Shive wave machine are given. I notice that all the measured velocities are greater than the calculated velocities. However, the calculated velocities have relatively large uncertainties and are therefore not in conflict with the calculated velocities.

As mentioned in section 3.1, the tension T is measured with a tension meter when the Shive wave machine is at rest. However, the tension increases when a wave is initiated. A wave results in an increase in tension because the tension wire stretches. In this research I chose to take this increase as uncertainty in T. For a more accurate result in both T and v, I recommend to measure the increased tension after a wave is initiated in the machine. The increase in tension could explain why the measured velocities are larger than the calculated velocities. I plotted the the measured velocities against the measured tension at rest in blue and against the measured tension plus increase in tension in red in figure 4.13. The red data points are closer to the expected relation than the blue points, but still too large.

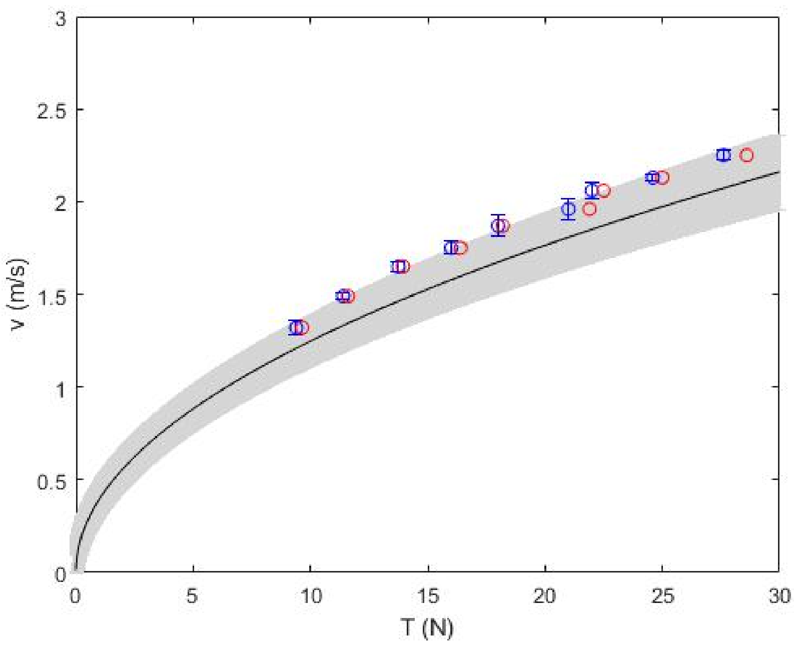

Figure 4.13: The blue points represent the measured velocities and their uncertainties against the tension measured at rest and the red points represent the measured velocities against the tension plus increase in tension. The black line represents the curve of formula 2.12, the gray area represents the uncertainty in the formula.

The measured velocities follow a similar curve as the calculated velocities which clearly depends on tension. This relation can be described as

$$v_{meas} = kT^c + d, \quad (4.1)$$

in which k, c and d are constant factors. To find the value of these constants, I recommend to increase the number of measurements and to make a fit through the results.

4.3.2 Simulating two media in the Shive wave machine

In tables 4.2 and 4.3, the results for the calculated and measured reflection R and transmission T are given. The data is also plotted in figure 4.12. What strikes is that whereas the green data points seem to agree with the theoretical curves, the red data points do not. In this discussion, I will first give general remarks, followed by specific remarks for the system with changing r or I.

A first general discussion point is that in section 2.2, I assumed that the slopes of the tension wires were continuous over the whole length of the wire. In practice, however, the wires are spanned from bar to bar and will make small angles inside the bars. To minimize this effect, the perturbation must be small and the wavelength large. In this way, the rise in amplitude between two bars will not be too large.

A second general point is that friction plays a role in using the Shive wave machine in practice. If a wave is initiated, its energy will decrease over time, and the wave will eventually have no amplitude left. Therefore, the energy of an initial wave will never be equal to the energy of the reflected and transmitted wave, so there will always be a small deviation in the measured R and T.

Distance of tension wire r_i

In section 2.2, I derived the formulas for the reflection and transmission by assuming the tension wires were 1-dimensional. This is possible because the tension wires normally only have components in the XZ-plane (the wires will twist a bit towards the pivot wire, but that is negligible for small angles of perturbation). In figure 3.1, a schematic close-up of a system with two different r's, r_1 and r_2, is given. We see that the tension wire now not only has components along the z-axis but also along the y-axis. Therefore, the theory can not be applied perfectly to this version of the Shive wave machine, and the measured data should conflict with the expected values for R and T out of formulas 2.22 and 2.23.

Moment of inertia I_i

In figure 3.2, a schematic overview of the applying of point masses is given. As can be seen, the point masses are actually no point masses but small weights. So, formula 3.9 is not completely accurate to calculate the added moment of inertia I_i of a mass i, since the masses are placed upon a small area instead of a point. Furthermore, the masses weigh 0.080kg and the bars weigh 0.16kg. If multiple masses are added to both ends of the bars, if the masses are not distributed equally and if the tension in the tension wires is not high enough to correct for the unequally distributed weight, the system will tend to tilt to one side, since there is a net force acting on one side of the bars. I recommend using bars of different materials or lengths to obtain the change in I. In this way, the weight is perfectly distributed over the bars, but there is still a difference in the moment of inertia of the different bars.

Chapter 5

Conclusion

The first part of this research was to validate the formula for the velocity of a wave in the Shive wave machine. The results show that the theoretical values are smaller than the measured values. My conclusion is that the machine is not useful yet for the researches about velocity of waves throughout different media because the behavior is not predictable enough. I recommend to enlarge the number of measurements and to make a fit through the measurements to obtain a formula which describes the experimental behavior of the machine. The second part of this research was to investigate the reflection and transmission of a wave traveling from medium 1 to medium 2. Whereas change in distance between the 'pivot' wire and 'tension' wires does not result in the predicted behavior, a change in the moment of inertia of the bars does. My first conclusion in this part of the research is that the behavior of the machine in which the distance between the wires changes needs more research, both theoretically and experimentally, to be able to predict the behavior. My second conclusion is that the behavior of the machine in which the moment of inertia changes can be predicted and is therefore useful in the study of acoustical wavefield imaging. However, more data points are needed to confirm the behavior. An improvement to the practical setup where the moment of inertia is changed would be to use bars of different materials and lengths instead of small masses.

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Appendix A

Uncertainty in v, f R and T

A.1 $u(v)$

The uncertainty in v following from 2.12 is described in 2.13, as

$$u(v)^2 = (\frac{\partial v}{\partial r})^2(u(r))^2 + (\frac{\partial v}{\partial T})^2(u(T))^2 + (\frac{\partial v}{\partial Dz})^2(u(Dz))^2 + (\frac{\partial v}{\partial I})^2(u(I))^2. \tag{A.1}$$

In this equation, the velocity has to be differentiated with respect to each of its variables r, T, Dz and I, which results in:

$$\begin{aligned} \frac{\partial v}{\partial r} &= \sqrt{\frac{2TDz}{I}} = \frac{v}{r}, \\ \frac{\partial v}{\partial T} &= r\sqrt{\frac{Dz}{2TI}} = \frac{v}{2T}, \\ \frac{\partial v}{\partial Dz} &= r\sqrt{\frac{T}{2DzI}} = \frac{v}{2Dz}, \\ \frac{\partial v}{\partial I} &= -\frac{r}{I}\sqrt{\frac{TDz}{2I}} = -\frac{v}{2I}. \end{aligned} \tag{A.2}$$

Filling in and taking the square root of A.1, results in

$$u(v) = v\sqrt{(\frac{u(r)}{r})^2 + (\frac{u(T)}{2T})^2 + (\frac{u(Dz)}{2Dz})^2 + (\frac{u(I)}{2I})^2}. \tag{A.3}$$

A.2 $u(I_{bar})$

The formula for the moment of inertia of a bar is

$$I = 1/12m(b^2 + l^2). \tag{A.4}$$

Filling in $m = \rho V = \rho abl$ gives

$$I = 1/12\rho abl(b^2 + l^2). \tag{A.5}$$

Using again the calculus approach gives

$$\frac{\partial I}{\partial a} = 1/12\rho bl(b^2 + l^2),$$
$$\frac{\partial I}{\partial l} = 1/12a\rho(b^3 + 3bl^2), \tag{A.6}$$
$$\frac{\partial I}{\partial b} = 1/12a\rho(l^3 + 3lb^2),$$

in which $a = b = 0.01 \pm 0.005$m and $l = 0.60 \pm 0.005$m are the dimensions of the bar. The uncertainty in ρ is neglected, because the bars are made out of aluminum 6082, which ensures that the uncertainty is relatively very small [16]. The uncertainty $u(I)$ is equal to:

$$u(I) = \sqrt{\frac{\partial I}{\partial b}^2 u(b)^2 + \frac{\partial I}{\partial l}^2 u(l)^2 + \frac{\partial I}{\partial a}^2 u(a)^2}, \tag{A.7}$$

which results in $I = 0.0049 \pm 0.0004$kgm^2.

A.3 $u(f)$

The uncertainty in f, $u(f)$, follows from the relation $f = \frac{v_2}{v_1}$. The calculus approach claims that

$$u(f) = f\sqrt{(\frac{u(v_2)}{v_2})^2 + (\frac{u(v_1)}{v_1})^2}, \tag{A.8}$$

in which $\frac{u(v_{1,2})}{v_{1,2}}$ follows from A.3.

A.4 $u(R)$ and $u(T)$

Following the calculus approach, the uncertainty in $R = \frac{f-1}{f+1}$ will be equal to

$$(u(R))^2 = (\frac{2}{(f+1)^2})^2 u(f)^2, \tag{A.9}$$

or,

$$u(R) = \frac{2}{(f+1)^2} u(f). \tag{A.10}$$

The uncertainty in $T = \frac{2f}{f+1}$, $u(T)$, is equal to

$$(u(T))^2 = (\frac{2f}{f+1})^2 (u(f))^2, \tag{A.11}$$

or,

$$u(T) = \frac{2f}{f+1} u(f). \tag{A.12}$$

Appendix B

Results

B.1 Validating the wave velocity theory

In table B.1, all the results of the first part of this research are shown. The measured velocity is calculated out of the mean time, $\Delta\bar{t}$, and out of the traveled distance, Δz (m).

Table B.1: The results of the measurements by measuring the velocity for changing T.

T (N)	t_1 (s)	t_2 (s)	t_3 (s)	t_4 (s)	t_5 (s)	$\Delta\bar{t}$ (s)	Δz (m)	v (m/s)
9.40 ± 0.25	5.44	5.27	5.45	5.20	5.44	5.36 ± 0.16	7.08 ± 0.01	1.32 ± 0.04
11.40 ± 0.10	4.71	4.80	4.81	4.71	4.71	4.75 ± 0.06	7.08 ± 0.01	1.49 ± 0.02
13.75 ± 0.10	4.23	4.25	4.32	4.34	4.34	4.29 ± 0.06	7.08 ± 0.01	1.65 ± 0.02
16.00 ± 0.40	4.00	3.96	4.14	4.11	3.99	4.03 ± 0.08	7.08 ± 0.01	1.75 ± 0.03
18.00 ± 0.20	3.79	3.73	3.87	3.87	3.67	3.79 ± 0.12	7.08 ± 0.01	1.87 ± 0.06
21.00 ± 0.90	3.69	3.71	3.56	3.53	3.57	3.61 ± 0.10	7.08 ± 0.01	1.96 ± 0.05
22.00 ± 0.50	3.53	3.47	3.40	3.44	3.38	3.44 ± 0.07	7.08 ± 0.01	2.06 ± 0.04
24.60 ± 0.40	6.75	6.77	6.85	6.84	6.84	6.81 ± 0.06	14.50 ± 0.01	2.13 ± 0.02
27.60 ± 1.00	6.42	6.44	6.51	6.41	6.43	6.44 ± 0.07	14.50 ± 0.01	2.25 ± 0.02

B.2 Simulating two media in the Shive wave machine

B.2.1 Distance of tension wire r_i

In table B.2 the results of the amplitudes are given. The results for the amplitudes were measured out of the Matlab-plots. In this section, r was changed throughout the Shive wave machine:

Table B.2: The results of the measurements by changing r throughout the Shive wave machine. The uncertainty in r_1 and r_2 is constant and equal to 0.05cm.The uncertainties in the amplitude are constant equal to 0.50cm.

r_1(cm)	r_2(cm)	$A_{bar,i}$(cm)	$A_{bar,r}$(cm)	$A_{bar,t}$(cm)
9.00	3.00	12.07	-5.17	13.44
3.00	9.00	15.96	-9.70	6.80
6.00	3.00	17.03	2.04	16.92
3.00	6.00	19.72	-5.34	11.67

B.2.2 Moment of inertia I_i

In table B.3 the results of the amplitudes are given. The results for the amplitudes were measured out of the Matlab-plots. In this section, I was changed throughout the Shive wave machine:

Table B.3: The results of the measurements by changing I throughout the Shive wave machine. The uncertainties in the amplitude are constant equal to 0.50cm.

I_1(kgm^2)	I_2(kgm^2)	$A_{bar,i}$(cm)	$A_{bar,r}$(cm)	$A_{bar,t}$(cm)
0.0049 ± 0.0010	0.0183 ± 0.0010	14.08	-6.91	10.63
0.0183 ± 0.0010	0.0049 ± 0.0010	15.31	3.01	18.82
0.0049 ± 0.0010	0.0300 ± 0.0011	16.14	-8.14	11.37
0.0300 ± 0.0011	0.0049 ± 0.0010	14.21	5.59	19.62
0.0049 ± 0.0010	0.0336 ± 0.0013	15.84	-9.26	10.41
0.0049 ± 0.0010	0.0219 ± 0.0013	14.09	-6.50	10.54