Compensator Design

Extending Van der Woude-Jeltsema's orthogonal projection approach

by

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Abstract

The aim of this Bachelor Thesis is to extend the orthogonal projection method described in 'An Orthogonal Projection Method for Computing Active, Reactive and Scattered Power and its Application to Compensator Design' by Jacob van der Woude and Dimitri Jeltsema [10]. In the aforementioned article, a method is described to calculate the active, reactive and scattered power of a RLC-network. It uses an orthogonal projection of the current on the space of (anti)derivatives of the voltage. This method can be used to improve the power factor of the network, as it also shows how to design a compensator for the reactive power. The compensator consists of a parallel connection of an electromagnetic coil and a capacitor. Two working examples are given.

This research shows that Van der Woude and Jeltsema's method is not always applicable. It often results in negative values for the coil and the capacitor, although this has no physical meaning. When the given voltage consists of more than two frequencies, the method is also inapplicable. This research gives a condition that ensures an applicable execution of this method. It also shows when these conditions are met.

We show how the method can be extended to a working method for any RLC-network, provided the voltage consists of two frequencies. It uses a second Foster canonical form to expand the initially proposed compensator. Depending on the coefficients given by the orthogonal projection, a different type of compensator is needed. It is also described how a fitting compensator can be chosen, and how the coefficients for the coils and capacitors can be determined.

Preface

This research would not have been possible without the help of Jacob van der Woude, my supervisor. His enthusiasm for the subject and his patience were indispensable to thoroughly understand his previous work on this topic and have a good basis to continue with his remarks. My thanks also goes out to Jeroen Spandaw, who is not only on my bachelor thesis committee, but also helped me expand my presentation skills during an earlier presentation course. He was helped in this by Regina Tange-Hoffman, to whom I am also deeply grateful. My thanks also goes out to Neil Budko.

My parents, Annehieke and Patrick, have supported me all the way through, re-reading my report and giving me good advice. Thank you for this. In the category of good advice, the conversations with my friend Julia can not be underestimated. Her look on things helped me put everything I was doing into perspective. Karen's help is also difficult to overestimate. On multiple occasions she took the time to give me a Electrical Engineering 101 course. Considering my total lack of knowledge on this subject upon beginning this research, this was extremely helpful. She lent me her books for freshmen electronics courses, which have also been very helpful. And of course: thank you for taking time during your vacation to read my thesis.

Finally, I can not thank Daan enough, putting up with me when I was stressed and cooking me dinner. Rereading my report when he was just three days away from his Master's Thesis presentation. Thank you all very much.

C. A. Bryan
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Contents

1 Introduction

Energy transfer optimisation is an interesting problem in the field of electronic engineering. It entails many different challenges and the associated solutions are equally diverse [7] [5] [3]. This report will discuss a certain approach to optimise the so-called power factor of a circuit. The power factor is the ratio between the power that a resistance in a circuit consumes (the active or real power P) and the power that is supplied by a power source (the apparent power S). In an ideal situation, the power factor is equal to 1 and all the power that is supplied is also consumed by the load.

However, when a network becomes more complex, for example when capacitors and coils are added to a circuit, a different type of power called the reactive power is generated. This power is not used by the load and is returned to the source, causing the power factor to become less than 1. This reactive power is caused by a phase difference between the current and the voltage. More on this will follow in chapter 2.

Reactive power seems useless, as it does not actively contribute to 'useful work'. Magnetic fields and capacitor cause reactive current. For many loads, such as motors, these elements are necessary for the circuit to function properly. However, from a power factor point of view, the reactive current is undesired. By installing decent compensators, consisting of coils and capacitors, parallel to the circuit, the reactive power needed for the functioning of the load can be generated. This reduces the reactive power demand so more of the apparent power is transferred to active power, increasing the power factor. We choose coils and capacitors as they are lossless elements. They do not dissipate any active power.

Energy transfer optimisation is a problem that is getting more complex as we switch towards alternative energy sources. Our energy sources become more diverse, and consequently have more frequencies, creating complicated functions to describe the power. As a result, decomposing the apparent power into active and reactive power also becomes more complicated. Nowadays, efficiency is becoming increasingly important, and so creating a decent compensator is crucial. A good compensator can fully compensate the reactive current, so the apparent power can be converted entirely into active power, or useful work. However, a poorly designed compensator can cause the reactive power to be needlessly large, causing the load to require a superfluous amount of apparent power.

Decomposing power has been thoroughly researched by multiple authors. Czarnecki wrote a detailed work, formulating the Current's Physical Components (CPC) method [4]. This method describes how apparent power can be decomposed into three specific types of energy conversion: active current (permanent energy conversion), scattered current (change of the load conductance with harmonic order) and reactive current (phase-shift between voltage and current). Czarnecki also describes how the design of the compensator is achieved.

Other authors have also designed methods for this problem, some taking a fundamentally mathematical approach. A notable research was conducted by Menti, Zacharias and Milias-Argitis [8]. Geometric algebra was used as a tool for representing the different powers.

Another mathematical approach was pursued by Jacob van der Woude and Dimitri Jeltsema [10]. Their approach uses an orthogonal projection to calculate the active, reactive and scattered current in a RLC-network. [1] A RLC network is a network consisting of a series connection of a resistor (R), a capacitor (C) and an electromagnetic coil (L). Using this orthogonal projection, a compensator consisting of a parallel coil and capacitor is looked for to compensate the reactive current of the network, as seen in figure 1.1. This report focuses on the Van der Woude-Jeltsema approach and extends it to make it more broadly applicable. The following research questions will be considered:

1. In which cases does the orthogonal projection method provide relevant values for C_c and L_c?

2. How can the orthogonal projection method be extended to give relevant values for all RLC networks with two-frequency voltages?

Chapter 2 focuses on the theoretical background of reactive current and the orthogonal projection method of Van der Woude and Jeltsema. The concept of the second Foster canonical form is also explained. The following two chapters elaborate on the orthogonal projection method. Chapter 3 explains where this method falls short. Chapter 4 proposes an extension to the method, so that it is more widely applicable.

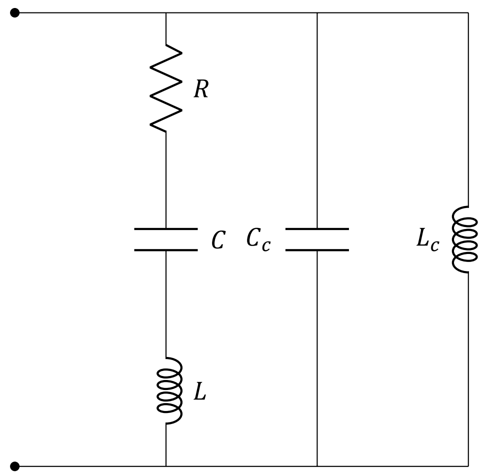


Figure 1.1: RLC network with parallel LC compensator described in the work of Van der Woude and Jeltsema.

[1] The article focuses on RLC networks, but the method can also be used for more general networks.

2

Theoretical Background

2.1. Reactive Power

In this section, a simple explanation on reactive power will be given. Simple circuits, where the load consists only of a resistor, do not generate any reactive power. For example, this is the case for a simple lamp running on a battery. However, when capacitors and electromagnetic coils are added to the circuit, a phase difference is generated, i.e. a phase difference between the current and the voltage. When the current is ahead of the voltage, we call the network leading. When the voltage is ahead of the current, the network is called lagging. A capacitor will cause the network to lead and this can be remedied by adding a coil to the network. The other way around, when a coil causes a circuit to lag, it can be remedied by adding a capacitor. A lag or lead is called a phase difference.

Power is the product of current and voltage. As can be seen in figure 2.1, when the current and voltage are perfectly synchronised, their product is completely positive, coinciding with a purely active power. However, when we introduce a phase difference as in figure 2.2, this creates a partially negative power, which in turn indicates the presence of reactive power. It can be calculated that when the phase difference is equal to π, we achieve a completely negative power. In this case, we only have reactive power and no active power.

These examples are relatively simple and hold true for only one frequency. When current and power with more frequencies get involved, it is more difficult to describe the active and reactive power. This is where more complicated methods such Czarnecki's CPC method or Van der Woude and Jeltsema's orthogonal projection method get useful. More information on the second method can be found in section 2.3.

2.2. Second Foster Canonical Form

Within the subject of synthesis of LC-networks, the Foster and Cauer canonical forms are of interest. These are repetitive forms of LC networks. Their strength lies in the fact that the coefficients for the coils and capacitors can be calculated by rewriting the admittance or impedance functions in their partial fraction form. This research uses the second Foster canonical form. The network corresponding to this form can be seen in figure 2.3. To find the correct values for the coefficients, the admittance function $Y(s)$ must be rewritten to a partial fraction of the following form:

$$Y(s) = \frac{I(s)}{V(s)} = Hs + \frac{K_0}{s} + \sum_{i=1}^{n} \frac{2K_i s}{s^2 + \omega^2}$$

As we can see, the compensator proposed by Van der Woude and Jeltsema can be seen as a second Foster canonical form with only two elements. The corresponding admittance

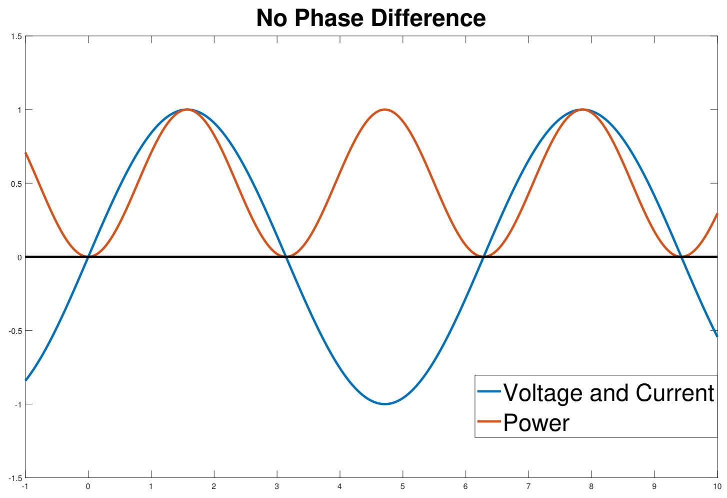


Figure 2.1: Voltage, current and power without phase difference between voltage and current.

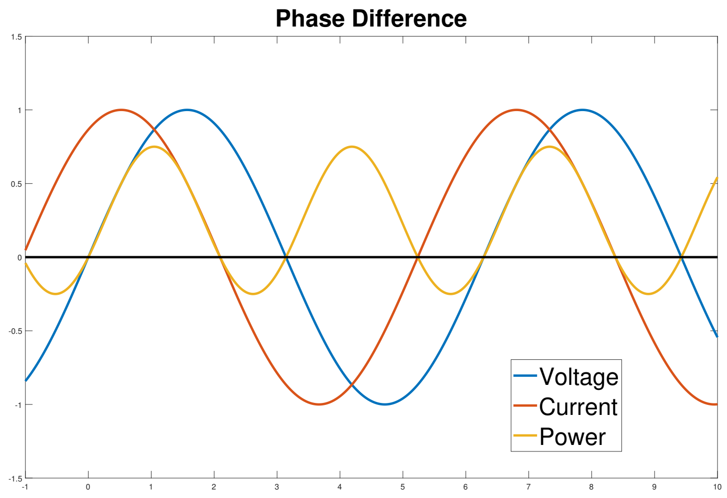


Figure 2.2: Voltage, current and power with phase difference between voltage and current.

function then becomes:

$$Y(s) = Hs + \frac{K_0}{s}$$

Further information on different types of Foster and Cauer canonical forms can be found in the work of Guillemin and Chen [6] [2].

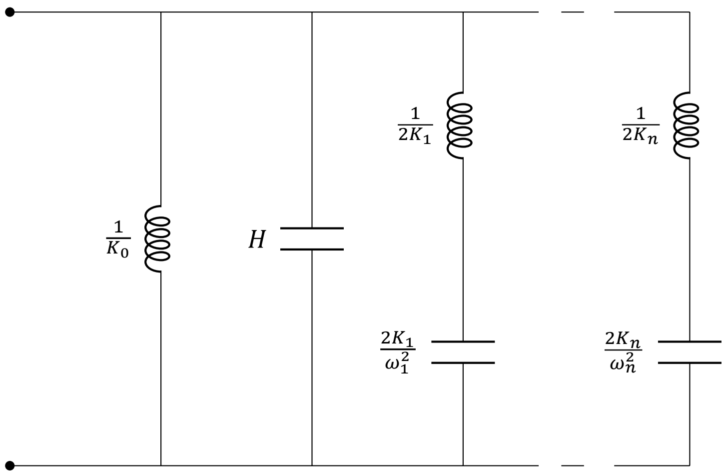


Figure 2.3: Second Foster canonical form.

2.3. Orthogonal Projection Method

As mentioned in the introduction, this research focuses on a method for computing a LC compensator for RLC networks, described in the article 'An Orthogonal Projection Method for Computing Active, Reactive, and Scattered Power and its Application to Compensator Design' by Jacob van der Woude and Dimitri Jeltsema [10]. The method aims to compute an optimal compensator, consisting of a capacitor with coefficient C_c and a coil with coefficient L_c placed parallel to the network.

This is achieved by calculating the reactive current using orthogonal projections on the space of odd derivatives and antiderivatives of the voltage. The source voltage is given as a finite number of harmonics:

$$v(t) = \sum_{l=1}^{L} \alpha_l \cos(lt) + \beta_l \sin(lt)$$

$v^{(k)}$ with k a positive integer represents the k-th derivative of the voltage. Inversely, $v^{(k)}$ with k a negative integer denotes the k-th antiderivative of $v(t)$.

Furthermore, an inner product is introduced for functions that have specific properties, such as being represented by a finite Fourier series (for example, the voltage $v(t)$):

$$\langle f, g \rangle := \frac{1}{2\pi} \int_0^{2\pi} f(t)g(t)\mathrm{d}t$$

It can be shown that

$$\langle v^{(k)}, v^{(m)} \rangle = 0 \tag{2.1}$$

with k, m integers and $k - m$ odd. For the proof, the article of Van der Woude and Jeltsema should be consulted.

The current of the circuit is denoted by $i(t)$. It can be written as a linear combination of the (anti)derivatives of $v(t)$. The following paragraph will elaborate on this statement.

Taking R, L and C as the respective values for the load, the coil and the capacitor, the equation for a RLC circuit is given by:

$$Ri(t) + Li^{(1)}(t) + \frac{1}{C} i^{(-1)}(t) = v(t)$$

When solving this differential equation for $i(t)$, using the method undetermined coefficients, the solution is indeed given by a linear combination of (anti)derivatives of $v(t)$.
This means it is possible to decompose $i(t)$ in an even and an odd part:

$$i(t) = i_e(t) + i_o(t)$$

with

$$i_e(t) := \sum_{k=-N}^{N} \gamma_k v^{(k)}(t) \text{ for k even}$$

$$i_o(t) := \sum_{k=-N}^{N} \gamma_k v^{(k)}(t) \text{ for k odd}$$

An important property following from equation (2.1) is that $\langle i_e, i_o \rangle = 0$, which means that i_e and i_o are orthogonal, making $i(t) = i_e(t) + i_o(t)$ an orthogonal decomposition. As a consequence, $\langle v, i_o \rangle = 0$, which means there is no loss of energy in the interaction between $v(t)$ and $i_o(t)$. This coincides with the reactive current $i_r(t)$ discussed in section 2.1. To summarise, when we project the given voltage $v(t)$ of a circuit on the space of odd (anti)derivatives of the current $i(t)$, we find the reactive current.

The article of Van der Woude and Jeltsema also shows that the active and scattered current can be calculated using orthogonal projections. When the current is projected on the space of even (anti)derivatives of $v(t)$, the active current $i_a(t)$ is found. It is also shown that the scattered current $i_s(t)$ is the difference between the even current and the active current: $i_s = i_e - i_a$. The scattered current and active current are also orthogonal, meaning $\langle i_a, i_s \rangle = 0$. As these two currents are also orthogonal to the reactive current, their relation can be descirbed as such:

$$||i||^2 = ||i_a||^2 + ||i_r||^2 + ||i_s||^2$$

This leads us to conclude that the apparent power S, the active power P, the reactive power Q_r and the scattered power Q_s have the following relation:

$$S^2 = P^2 + Q_r^2 + Q_s^2$$

Two examples are described in the article of Van der Woude and Jeltsema, and both give promising results. It is shown that in each case, when the current is projected on the space of odd (anti)derivatives of $v(t)$, the reactive current can be described in the following form:

$$i_r(t) = -c_1 v^{(1)}(t) + -c_2 v^{(-1)}(t)$$

This current can then be compensated by the compensator that generates a current of the form:

$$i_c(t) = c_1 v^{(1)}(t) + c_2 v^{(-1)}(t) \tag{2.2}$$

that is implemented in parallel to the RLC network.

The compensator that is suggested as decent in this situation is shown in figure 1.1. We want to know if this type of compensator indeed generates a current of the form in equation 2.2. In other words, can the admittance function of the compensator be rewritten to a form that matches equation 2.2? If so, what values should the coefficients have? This can be shown as follows.

We only consider the compensator, as shown in figure 2.4. This can be seen as a second Foster canonical form, without any series elements, which was elaborated on in section 2.2.

This form is shown in figure 2.5. The admittance function for this circuit is given by:

$$Y(s) = \frac{I(s)}{V(s)} = Hs + \frac{K_0}{s}$$
$$\Leftrightarrow \; I(s) = (Hs + \frac{K_0}{s})V(s)$$

To find the current generated by this circuit, we take the inverse Laplace transform. We then find:

$$\begin{aligned}\mathcal{L}^{-1}\left[I(s)\right] &= \mathcal{L}^{-1}\left[(Hs + \frac{K_0}{s})V(s)\right] \\ \Leftrightarrow \; i(t) &= H \cdot \mathcal{L}^{-1}\left[s \cdot V(s)\right] + K_0 \cdot \mathcal{L}^{-1}\left[s^{-1} \cdot V(s)\right] \\ &= H \cdot v^{(1)}(t) + K_0 \cdot v^{(-1)}(t) \end{aligned} \tag{2.3}$$

When we compare equation (2.2) and equation (2.3), they indeed have the same form. The only thing that is left to examine are the coefficients for the coil and the capacitor. The differential equations relating the current and the voltage for capacitors and coils respectively, are as follows [9]:

$$Cv^{(1)}(t) = i(t) \qquad\qquad v(t) = Li^{(1)}(t)$$
$$\Leftrightarrow \; Cv^{(1)}(t) = i(t) \qquad\qquad \Leftrightarrow \; \frac{1}{L}v^{(-1)}(t) = i(t)$$

Thus, if we set $C_c = c_1 = H$ and $L_c = \frac{1}{c_2} = \frac{1}{K_0}$, we have found a compensator that completely compensates the reactive current. This leads to an improved power factor of the circuit and, depending on the presence of scattered power, it could improve the power dissipation in such a way that the power factor becomes 1. In this case, all apparent power is converted to active power. This shows that the compensator in figure 2.4 generates a current of the form shown in equation (2.2).

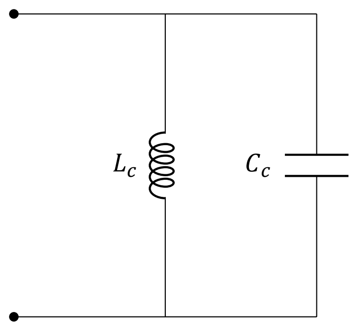


Figure 2.4: LC compensator described in Van der Woude and Jeltsema's work.

This result seems promising, as the article goes on to show that when we examine the complex Fourier series of the voltage and the current and use the coefficients in their vector form, the computation of the reactive, active and scattered current and the power factor becomes much easier. Another advantage is that the vector notation allows a rapid implementation in mathematical programming languages such as Matlab. This allows us to quickly determine a compensator for any RLC circuit with a given voltage and current. When applied to two examples in the article, the power factor is greatly improved, and in one example even becomes 1.

Indeed, this method is easy in calculation, and works well for certain values of R, L and C. However, after testing different values for R, L and C, the method frequently gives negative values for C_c and L_c. These are not relative coefficients, as they have no physical meaning.

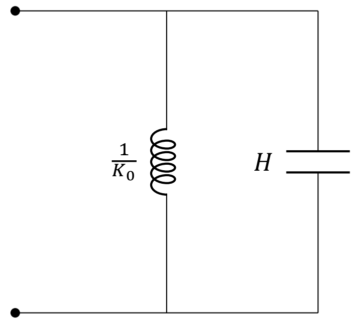


Figure 2.5: Second Foster canonical form without any series elements.

Also, the examples used in the article use a voltage with two frequencies. When trying currents with more frequencies, the projection of the current on the space of the (anti)derivatives of the voltage, gives us a linear combination of the following form:

$$i_r(t) = -c_1 v^{(1)}(t) - c_2 v^{(-1)}(t) - c_3 v^{(3)}(t) - c_4 v^{(-3)}(t) - c_5 v^{(5)}(t) - c_6 v^{(-5)}(t) - \dots$$

The amount of basis vectors depends on the number of frequencies introduced. As shown above, the compensator shown in figure 2.4 does not coincide with a current of this form. It raises the question if a different type of compensator is needed. If so, what form should this compensator have?

The following chapter will focus on answering the first research question: In which cases does the orthogonal projection method provide relevant values for C_c and L_c? In other words: when does the projection method give us positive values for L_c and C_c?

3

Limitations of the Orthogonal Projection Method

As mentioned in the previous chapter, the orthogonal projection method has some very useful properties, such as the ease of implementation. However, the method also has some serious limitations.
In the article by Van der Woude and Jeltsema, two examples are shown for which the method succeeds. However, after trying a couple of other values for R, L and C and trying different voltages, it is soon discovered that this method often does not succeed at giving two useful values for C_c and L_c. There are two common problems that occur: on one hand C_c and L_c are often negative and on the other hand the orthogonal projection method sometimes generates more than two coefficients. These issues will be discussed in the following paragraphs.

3.1. Negative values for coefficients

When preforming the orthogonal projection method, certain combinations of R, L and C in combination with a certain voltage give negative values for L_c and C_c. This has no physical meaning for coils and capacitors, and is thus an inapplicable practical result. The occurrence of negative values of either L_c, C_c or both is illustrated in figure 3.1. Every combination for R, L and C that yielded a negative value for either C_c or L_c is indicated with a blue marker. The tested values for L and C range between 0 and 3 (excluding 0 in the case of C), taking steps of 0.05. R was also tested between 0 and 3, but to improve visibility of the image, the steps were chosen at 0.5. However, the pattern that can clearly be seen in the image continues when testing other values for R.

3.1.1. Influence of R

We see an interesting relationship appear between the values for L and C. A white space means that the method is successful for these values for R, L and C. When examining a cross-section of figure 3.1, we get plots such as those shown in figure 3.2. As the resistor increases in value, more and more combinations with L and C lead to a successful execution of the method. Successful in this context means that both L_c and C_c are positive.

This makes sense when looking at the situation from a physical point of view. When the load of a network becomes much more influential than the coils and the capacitors, much of the power in the network is already being used as active power.

3.1.2. Boundary

An interesting question to ask is: what is the boundary for success? How do the voltage and the values for R, L and C combine to form a condition ?

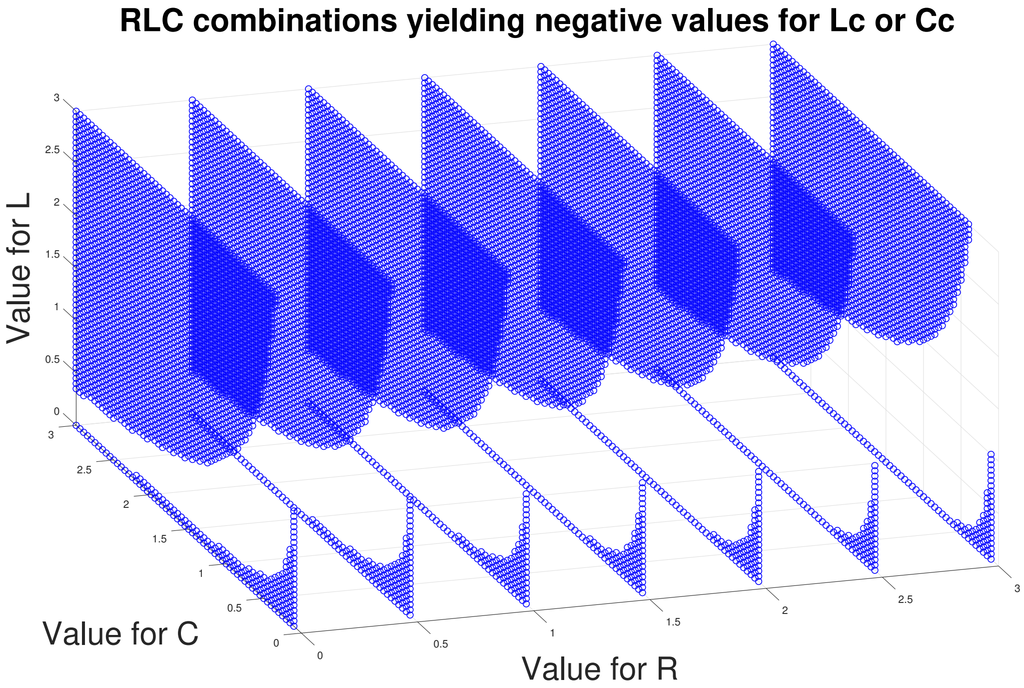


Figure 3.1: The blue dots indicate the combinations for R, L and C for which negative coefficients were calculated. The used voltage is $v(t) = \sqrt{2}(100\cos(3t) + 100\sin(t))$.

To examine this, we perform the orthogonal method again, assuming a random value for R, L and C and a random alternating voltage with two frequencies of the form:

$$v(t) = b_1 \cdot \cos(at) + b_2 \cdot \sin(bt) \tag{3.1}$$

Due to the complexity of these calculations, they were performed in Matlab.
All variables (R, L, C, b_1, b_2, b and a) were assumed to be unknown. Following the examples of chapter 7 in the article of Van der Woude and Jeltsema, V and W were defined. These are vectors containing the coefficients of Fourier representation of the voltage and the frequencies respectively. The complex notation for the current, given by I was then calculated by solving the equation

$$(RI_4 + LW + \frac{1}{C}W^{-1}) = V$$

The basis for the orthogonal projection was defined in a matrix as such:

$$A = \left[WV, WV^{-1}\right]$$

Finally, the reactive current I_r was calculated using the orthogonal projection of I on A:

$$I_r = A(A^T A)^{-1} A^T \cdot I$$

Rewriting the values of the coefficients given by this projection, We find the following values for C_c and L_c:

$$C_c = \frac{C(a^2b^2C^3LR^2 - a^2b^2C^2L^2 + a^2CL + b^2CL - 1)}{(C^2R^2a^2 + (1 - CLa^2)^2) \cdot (C^2R^2b^2 + (1 - b^2CL)^2)}$$

$$L_c = \frac{(C^2R^2a^2 + (1 - CLa^2)^2) \cdot ((CLb^2 - 1)^2 + C^2R^2b^2)}{a^2b^2C^2(-a^2b^2C^2L^3 + a^2CL^2 + b^2CL^2 + CR^2 - L)}$$

As we can see, the values are not dependent on the amplitudes of the voltage, only on the frequencies and the values for R, L and C. This can be expected, as the amplitude is only a constant that is multiplied with the voltage. When using the voltage and its (anti)derivatives as a basis, the coefficients of the linear combination of the basis elements will not be influenced by any constant multiplied with the voltage.

The boundary for the areas of success is given by the following conditions:

$$C_c = 0 \qquad\qquad L_c = 0$$

As can be seen in figure 3.3, these lines are indeed the boundaries between the areas of success and of failure. $C_c = 0$ is plotted in red and $L_c = 0$ is plotted in blue. These plots also make a distinction between the RLC combinations for which C_c is negative (red) and for which L_c is negative (blue). In these images, these plots do not seem to overlap. However, this is not a correct assumption, as we will see in the next paragraph.

3.1.3. Influence of frequencies

Remember the equation for the voltage given in equation (3.1). To find out what influence the frequencies have, the values for a and b are varied. As we have two frequencies, it is more difficult to summarise the results. It is safe to say that, when a and b both become larger, there are increasingly less RLC combinations for which the method succeeds. However, if we keep b small and increase the value of a, we will see that we get an increase in successful combinations. Keeping a small and increasing the value of b does not seem to have this effect. This can also be seen in figure 3.4.
Something interesting we see in these images as well, is the appearance of a green zone. Remember the red dots coincide with the RLC combinations for which C_c is negative, and the blue dots have an negative L_c value. The green dots are the combinations for which both L_c and C_c are negative. Indeed, the line $C_c = 0$ crosses through the area of negative L_c values, creating a large subset of combinations yielding two negative coefficients.

As the previous paragraphs show, there are many cases for which the orthogonal projection method is not applicable yet. Chapter 4 will focus on a solution to extend the method in such a way that the method does work for any RLC network with a voltage with two frequencies.

3.2. More than 2 frequencies.

As soon as we add more than two frequencies, we see that the projection obtains the following form:

Three frequencies: $i_r(t) = -c_1v^{(1)}(t) - c_2v^{(-1)}(t) - c_3v^{(3)}(t)$

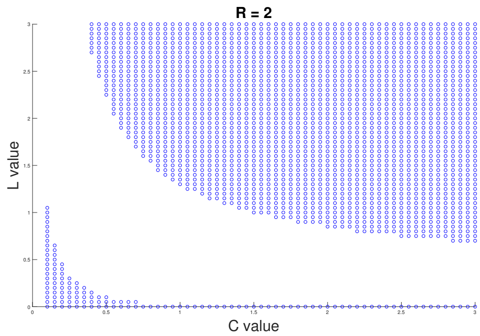
Four frequencies: $i_r(t) = -c_1v^{(1)}(t) - c_2v^{(-1)}(t) - c_3v^{(3)}(t) - c_4v^{(-3)}(t)$

Five frequencies: $i_r(t) = -c_1v^{(1)}(t) - c_2v^{(-1)}(t) - c_3v^{(3)}(t) - c_4v^{(-3)}(t) - c_5v^{(5)}(t)$

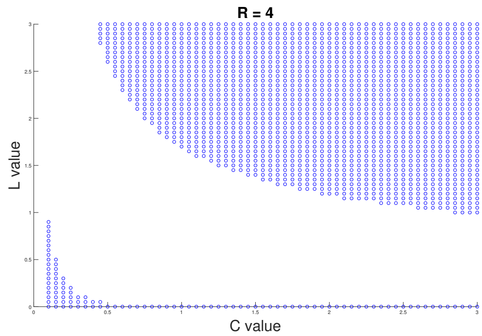
Six frequencies: $i_r(t) = -c_1v^{(1)}(t) - c_2v^{(-1)}(t) - c_3v^{(3)}(t) - c_4v^{(-3)}(t) - c_5v^{(5)}(t) - c_6v^{(-5)}(t)$

...

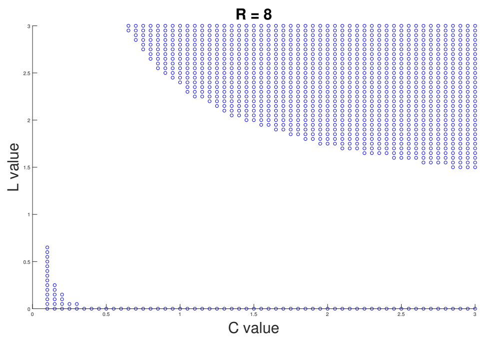
Every added frequency generates a separate base element. Remember the basis in this situation is the space of odd (anti)derivatives of the source voltage $v(t)$. As shown in chapter 2, the proposed compensator only generates a current with two base elements. Therefore, the orthogonal projection method must be extended to compensate a reactive current of the form shown above.
This research will not focus on this problem, but the conclusion includes a recommendation in what way this problem can be tackled.



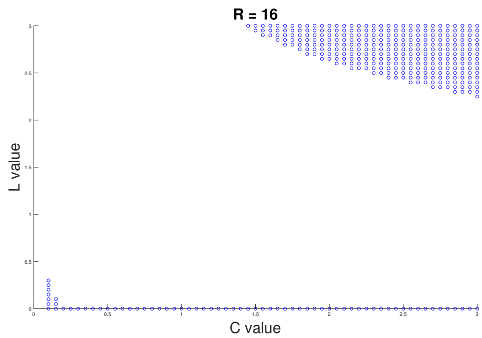
(a) Result for R=2



(b) Result for R=4

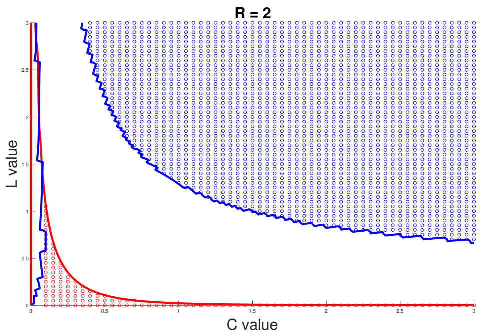


(c) Result for R=8

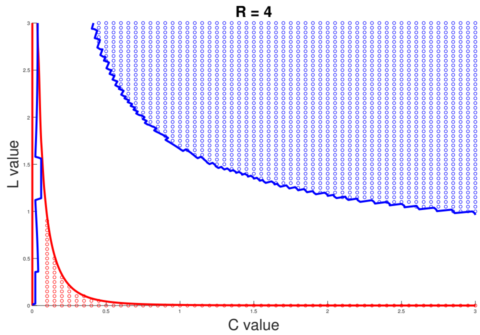


(d) Result for R=16

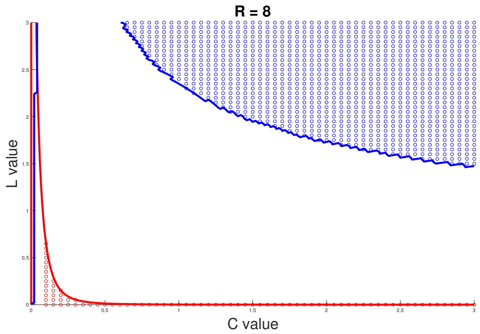
Figure 3.2: The blue dots indicate the combinations for L and C for which negative coefficients were calculated. The value of R is varied. The used voltage is $v(t) = \sqrt{2}(100\cos(3t) + 100\sin(t))$.



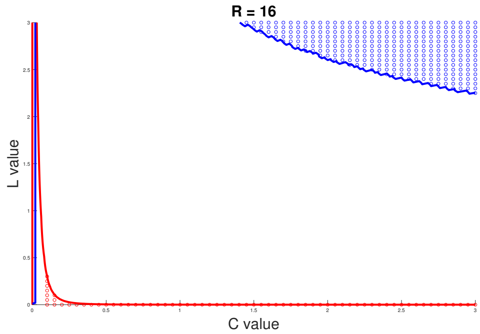
(a) Result for R=2



(b) Result for R=4

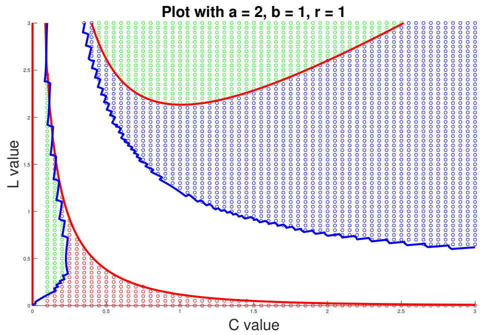


(c) Result for R=8

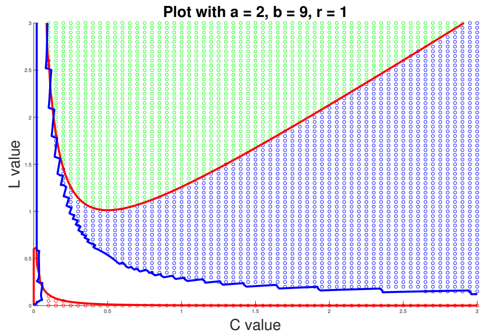


(d) Result for R=16

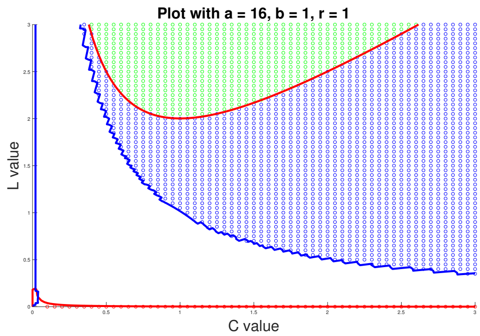
Figure 3.3: The blue dots indicate the combinations for L and C for which L_c was negative. Red dots indicate combinations for which C_c is negative. The red line indicates $C_c = 0$. The blue line indicates $L_c = 0$. The value of R is varied. The used voltage is $v(t) = \sqrt{2}(100\cos(3t) + 100\sin(t))$.



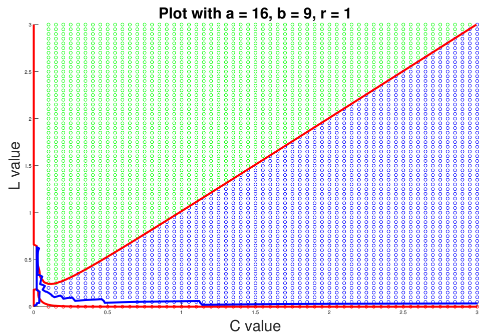
(a) Result for a=2 and b=1.



(b) Result for a=2 and b=9.



(c) Result for a=16 and b=1



(d) Result for a=16 and b=9

Figure 3.4: The blue dots indicate the combinations for L and C for which L_c was negative. Red dots indicate combinations for which C_c is negative. Green dots indicate combinations for which both coefficients are negative. The red line indicates $C_c = 0$. The blue line indicates $L_c = 0$. The value of R is varied. The used voltage is $v(t) = \sqrt{2}(100\cos(3t) + 100\sin(t))$.

4
Possible Solution: Second Foster Canonical Form

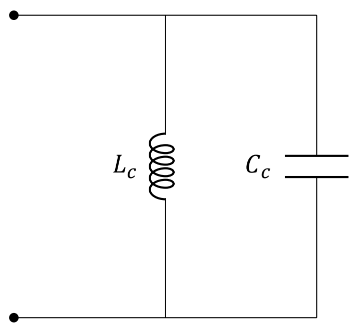


Figure 4.1: Compensator configuration as suggested by Van der Woude and Jeltsema [10].

As was shown in the previous chapter, a compensator as the one depicted in figure 4.1 is a good option for certain RLC networks. However, many combinations of R, L and C yield negative values for C_c and/or L_c. Therefore, the method must be extended, so that it gives us a solution for all combinations of R, L and C. This study will focus on the case of a voltage with two frequencies.

As mentioned in chapter 2, the compensator in figure 4.1 is an example of a second Foster form with admittance function:

$$Y(s) = \frac{I(s)}{V(s)} = Hs + \frac{K_0}{s}$$

A possible solution to finding a compensator that works for more RLC networks is to expand the second Foster form with one step, as shown in figure 4.2. The goal is to evaluate if this realisation meets our requirements to act as a compensator for any RLC network with a voltage of two frequencies. To achieve this, the admittance function must be assessed. The key is to find out how it can be rewritten to a linear combination of odd (anti)derivatives of the network's voltage.

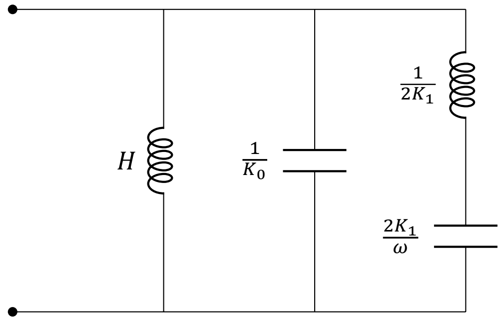


Figure 4.2: A configuration of a second Foster canonical form.

4.1. Admittance Function

As explained by W. Chen [2], the admittance function belonging to the realisation in figure 4.2 is given by:

$$Y_c(s) = \frac{I_c}{V_c} = Hs + \frac{K_0}{s} + \frac{2K_1 s}{s^2 + \omega^2}$$
$$\Leftrightarrow\ I_c = (Hs + \frac{K_0}{s} + \frac{2K_1 s}{s^2 + \omega^2})V_c$$

The first two terms correspond to the original compensator in figure 4.1, and can be rewritten easily to a linear combination as such:

$$\mathcal{L}^{-1}\left[(Hs + \frac{K_0}{s})V_c(s)\right]$$
$$= H \cdot \mathcal{L}^{-1}\left[s \cdot V_c(s)\right] + K_0 \cdot \mathcal{L}^{-1}\left[s^{-1} \cdot V_c(s)\right]$$
$$= H \cdot v^{(1)}(t) + K_0 \cdot v^{(-1)}(t)$$

The difficulty lies in the addition of the LC element. We must rewrite the last term $\frac{2K_1 s}{s^2+\omega^2}$ to a combination of $v^{(k)}$ with k odd.

$$\mathcal{L}^{-1}\left[\frac{2K_1 s}{s^2 + \omega^2} \cdot V_c(s)\right]$$
$$= 2K_1(\cos(\omega t) * v(t))$$
$$= 2K_1 \int_0^t \cos(\omega(t-u)) \cdot v(u)\mathrm{d}u \tag{4.1}$$

4.2. Integral Assessment

To assess this integral we must first find an equation for $v(t)$. A voltage with two frequencies can be represented as:

$$v(t) = b_1 \cos(at) + b_2 \sin(bt)$$

In this expression, b_1 and b_2 are constants associated with the amplitude, and a and b are variables that allow for different frequency combinations.

Continuing with expression 4.1, we find:

$$
\begin{aligned}
&2K_1 \int_0^t \cos(\omega(t-u)) \cdot v(u)\mathrm{d}u \\
=&2K_1 \int_0^t \cos(\omega(t-u)) \cdot A \cdot (c_1 \cos(at) + c_2 \sin(bt))\mathrm{d}u \\
=&2K_1 \cdot A \cdot \left[c_1 \int_0^t \cos(\omega(t-u)) \cdot \cos(au)\mathrm{d}u + c_2 \int_0^t \cos(\omega(t-u)) \cdot \sin(bu)\mathrm{d}u \right] \\
=&2K_1 \cdot A \cdot \left[c_1 \frac{a \cdot \sin(at) - \omega \sin(\omega t)}{a^2 - \omega^2} + c_2 \frac{b\cos(\omega t) - b\cos(bt)}{b^2 - \omega^2} \right] \\
=&\frac{2K_1 A}{(a^2-\omega^2)(b^2-\omega^2)} \left[c_1(b^2-\omega^2)(a \cdot \sin(at) - \omega \cdot \sin(\omega t)) + c_2(a^2-\omega^2)(b \cdot \cos(\omega t) - b \cdot \cos(bt) \right] \\
=&\frac{2K_1 a^2 b^2}{(a^2-\omega^2)(b^2-\omega^2)} \cdot v^{(-1)}(t) && (4.2) \\
&+ \frac{2K_1 \omega^2}{(a^2-\omega^2)(b^2-\omega^2)} \cdot v^{(1)}(t) && (4.3) \\
&+ \frac{2K_1 A}{(a^2-\omega^2)(b^2-\omega^2)} \left[-c_1 b^2 \omega \sin(\omega t) + c_1 \omega^3 \sin(\omega t) + c_2 a^2 b \cos(b\omega t) - c_2 \omega^2 b \cos(\omega t) \right] && (4.4)
\end{aligned}
$$

These results are very promising, as we have already found a linear combination of $v^{(1)}$ (equation (4.3)) and $v^{(-1)}$ (equation (4.2)). However, a last term remains that seems more difficult to handle. It contains terms of the form $\sin(\omega)$ and $\cos(\omega)$. These can not be rewritten to an odd (anti)derivative of $v(t)$, as setting ω equal to a or b leads to a division by 0 in the coefficient. However, these terms can be explained by a phenomenon called *resonance* [1].

Resonance can be interpreted as an internal impedance. Any network with at least one coil and one capacitor has its own resonance. It can act as a filter to select frequencies and it has many practical applications. However, these frequencies do not coincide with the frequencies caused by the driving voltage of the network. They can be seen as homogeneous solutions of the differential equation of this circuit. In other words: these are the frequencies we perceive when the circuit is not driven by a voltage. This means they have no influence on the reactive current fed by the voltage. They can be omitted from this equation.

We have now found a linear combination of $v^{(1)}(t)$ and $v^{(-1)}(t)$.

4.3. Linear Combination

As shown in the previous section, the admittance function of the compensator in figure 4.2 can be rewritten to a linear combination of the following form:

$$i_c(t) = (H + \frac{2K_1\omega^2}{(a^2-\omega^2)(b^2-\omega^2)})v^{(1)}(t) + (K_0 + \frac{2K_1 a^2 b^2}{(a^2-\omega^2)(b^2-\omega^2)})v^{(-1)}(t) \qquad (4.5)$$

The added rational terms generates an extra degree of freedom. We have seen in section 3 that the reactive current, when projected on the space of odd (anti)derivatives, yields a compensator with a current of the form:

$$i_c(t) = c_1 v^{(1)}(t) + c_2 v^{-1}(t)$$

In some cases, both coefficients are positive, in which case the method is successful at generating a compensator. However, in some cases either c_1 or c_2 are negative and sometimes both are negative. We must therefore assess if the coefficients of equation 4.5 can become simultaneously positive and negative, negative and positive, or both, keeping in mind that H, K_0, K_1 and ω must remain positive. If these coefficients are not positive, they loose their physical meaning.

First, we rename the coefficients:

$$c_1 = C_{v_1} = H + t_1 \qquad \text{with} \qquad t_1 = \frac{2K_1\omega^2}{(a^2-\omega^2)(b^2-\omega^2)}$$

and

$$c_2 = C_{v_{-1}} = K_0 + t_2 \qquad \text{with} \qquad t_2 = \frac{2K_1 a^2 b^2}{(a^2-\omega^2)(b^2-\omega^2)}$$

Let us first assume that C_{v_1} has to be negative and $C_{v_{-1}}$ has to be positive. In order to get C_{v_1} negative, t_1 has to be negative, as H has to be positive. As the numerator of t_1 already consists of purely positive terms, it is up to the denominator to make sure this happens.

$$(a^2-\omega^2)(b^2-\omega^2) < 0$$
$$\Leftrightarrow\ (a^2-\omega^2) < 0 \wedge (b^2-\omega^2) > 0 \quad \text{or} \quad (a^2-\omega^2) > 0 \wedge (b^2-\omega^2) < 0$$
$$\Leftrightarrow\ a < \omega < b \quad \text{or} \quad b < \omega < a$$

As we see, when ω is chosen between a and b, t_1 becomes negative. An interesting result is that the freedom given in choosing K_1 is enough to make sure that t_1 equals c_1. H is not needed as extra degree of freedom to influence C_{v_1}. Therefore, the capacitor is not needed in the original network. For $C_{v_{-1}}$, K_0 can then be used to make sure that $C_{v_{-1}}$ equals $K_0 + t_2$.

The same result is achieved when we assume that C_{v_1} has to be positive, and $C_{v_{-1}}$ has to be negative. In that case, we do not need K_0 as an extra degree of freedom, and we can omit the coil from the network. So, if the coefficient of $v^{(1)}$ is negative, the network in figure 4.3 can be used as a compensator. If the coefficient of $v^{(-1)}$ is negative, we can use the network in figure 4.4 as a compensator.

Finally, when both coefficients are negative, we can take a similar approach as in the above case. However, it is important to note that $\omega < ab$, as a and b are both positive real integers and ω is smaller than either a or b (but not both). This means that, if we want t_1 and t_2 to be negative, t_2 will be more negative than t_1. We then have two cases:

1. $C_{v_1} < C_{v_{-1}}$
   In this case, we only need to choose ω and K_1 in such a way that $t_1 = C_{v_1}$. K_0 can then be chosen in such a way that $C_{v_{-1}} = K_0 + t_2$.

2. $C_{v_{-1}} < C_{v_1}$
   In this case, choosing the proper coefficients becomes more difficult. First, K_1 and ω have to be chosen so that $C_{v_{-1}} = t_2$. If we find that these values for K_1 and ω give a value for t_1, so that $t_1 < C_{v_1}$, then we only need H to make sure that $C_{v_1} = H + t_1$. However, should we find that $C_{v_1} < t_1$, then there is no H for which $C_{v_1} = H + t_1$ is valid. In that case, we need to re-evaluate the values for ω and K_1. We need to re-choose these values so that $t_2 < C_{v_{-1}}$ and $t_2 < C_{v_1}$. Then we need to introduce both H and K_0 to ensure that $C_{v_{-1}} = K_0 + t_2$ and $C_{v_1} = H + t_1$. In that case, a compensator such as the compensator in figure 4.2 is needed.

4.4. Choosing omega

As shown before, if we want to choose an ω so that t_1 and t_2 become negative, ω must satisfy $a < \omega < b$. K_1 makes $t_{1,2}$ small 'enough'. However, is there an optimal ω?
In theory, all values for ω between a and b are valid. However, from a practical perspective, a smaller K_1 is a more economical choice, as it means the coil and capacitor do not need to be very strong. K_1 is minimal when the denominator of t_1 and t_2 is minimal. For this, we

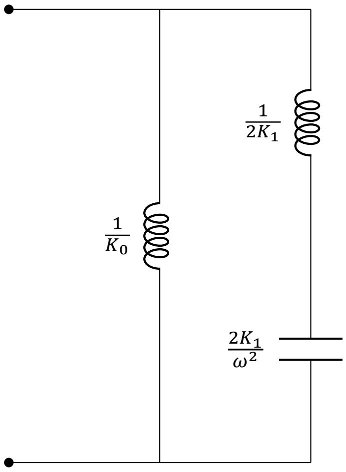


Figure 4.3: Compensator in the case of a negative C_{v_1}

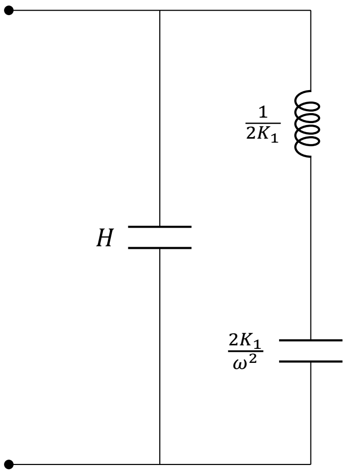


Figure 4.4: Compensator in the case of a negative $C_{v_{-1}}$

differentiate the denominator for ω.

$$
\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\left[(a^2-\omega^2)(b^2-\omega^2)\right]=0\\
\Leftrightarrow\ &\frac{\mathrm{d}}{\mathrm{d}t}\left[a^2b^2-a^2\omega^2-b^2\omega^2+\omega^4\right]=0\\
\Leftrightarrow\ &-2a^2\omega-2b^2\omega+4\omega^3=0\\
\Leftrightarrow\ &\cancel{\omega=0}\text{ or }-a^2-b^2+2\omega^2=0\\
\Leftrightarrow\ &\omega^2=\frac{a^2+b^2}{2}\ \Leftrightarrow\ \omega=\sqrt{\frac{a^2+b^2}{2}}
\end{aligned}
$$

If we choose this value for ω, ω is still in between a and b, as ω squared is the average of the squares of a and b. When examining graphs of functions of this form, we find that this value for ω is indeed a minimum, and not a maximum.

4.5. Results

Following this method, an expansion to Van der Woude and Jeltsema's method is found, making this method applicable to any RLC network and any two-frequency voltage. When using the orthogonal projection method proposed in their paper, a linear combination is found of the form:

$$i_r(t) = -c_1 \cdot v^{(1)} - c_2 \cdot v^{(-1)}$$

This can be compensated by an LC-network generating the current:

$$i_c(t) = c_1 \cdot v^{(1)} + c_2 \cdot v^{(-1)}$$

Depending on the sign of c_1 and c_2, a different type of network is needed.

We rename $c_1 = C_{v_1}$ and $c_2 = C_{v_{-1}}$. The different situations are summarised below:

- $C_{v_1} > 0$ and $C_{v_{-1}} > 0$
  In this case, the original method of Van der Woude an Jeltsema can be used. A compensator of the form in figure 4.1 can be used, with:

  $$C_{v_1} = C_c \qquad \text{and} \qquad C_{v_{-1}} = \frac{1}{L_c}$$

  with $C_c > 0$ and $L_c > 0$.

- $C_{v_1} < 0$ and $C_{v_{-1}} > 0$
  When the orthogonal projection yields a negative C_{v_1}, a compensator of the form shown in figure 4.3 can be used. We then choose the coefficients so that we achieve:

  $$C_{v_1} = \frac{2K_1\omega^2}{(a^2-\omega^2)(b^2-\omega^2)} \qquad \text{and} \qquad C_{v_{-1}} = K_0 + \frac{2K_1a^2b^2}{(a^2-\omega^2)(b^2-\omega^2)}$$

  with $K_0, K_1 > 0$ and $a < \omega < b$ or $b < \omega < a$ (depending on whether a or b is larger).

- $C_{v_1} > 0$ and $C_{v_{-1}} < 0$
  In case of a negative $C_{v_{-1}}$, the compensator shown in figure 4.4 can be used. The coefficients must then be chosen in such a way that:

  $$C_{v_1} = H + \frac{2K_1\omega^2}{(a^2-\omega^2)(b^2-\omega^2)} \qquad \text{and} \qquad C_{v_{-1}} = \frac{2K_1a^2b^2}{(a^2-\omega^2)(b^2-\omega^2)}$$

  with $H, K_1 > 0$ and $a < \omega < b$ or $b < \omega < a$.

- $C_{v_1} < 0$ and $C_{v_{-1}} < 0$
  There are two cases.

  1. $C_{v_1} < C_{v_{-1}} < 0$
     In this case, a compensator as in figure 4.3 can be chosen, with

     $$C_{v_1} = \frac{2K_1\omega^2}{(a^2-\omega^2)(b^2-\omega^2)} \qquad \text{and} \qquad C_{v_{-1}} = K_0 + \frac{2K_1a^2b^2}{(a^2-\omega^2)(b^2-\omega^2)}$$

     with $H, K_1 > 0$ and $a < \omega < b$ or $b < \omega < a$.

  2. $C_{v_{-1}} < C_{v_1} < 0$
     In most cases, a compensator such as in figure 4.4 is a good choice with

     $$C_{v_1} = H + \frac{2K_1\omega^2}{(a^2-\omega^2)(b^2-\omega^2)} \qquad \text{and} \qquad C_{v_{-1}} = \frac{2K_1a^2b^2}{(a^2-\omega^2)(b^2-\omega^2)}$$

     with $H, K_1 > 0$ and $a < \omega < b$ or $b < \omega < a$. However, in some cases it will not be possible to find a valid ω and K_1. In that case, a compensator with an extra element is needed, such as the compensator in figure 4.2. We then have:

     $$C_{v_1} = H + \frac{2K_1\omega^2}{(a^2-\omega^2)(b^2-\omega^2)} \qquad \text{and} \qquad C_{v_{-1}} = K_0 + \frac{2K_1a^2b^2}{(a^2-\omega^2)(b^2-\omega^2)}$$

5
Conclusion

In their article 'An Orthogonal Projection Method for Computing Active, Reactive, and Scattered Power and its Application to Compensator Design' [10], Dimitri Jeltsema and Jacob van der Woude propose a method for power decomposition and compensator design in a RLC network. Their method is based on an orthogonal projection of the current on the space of the (anti)derivatives of the voltage.

By projecting the current on the space of odd (anti)derivatives, the reactive power can be calculated. Conversely, projecting the current on the space of even (anti)derivatives yields the active power of the circuit. Finally, the scattered power is represented by the difference between the space of even (anti)derivatives and the active power. A useful application of the method is the design of a lossless compensator, consisting of a parallel combination of a coil L_c and a capacitor C_c. This compensator is able to improve the power factor of the system.

Two examples are presented in van der Woude and Jeltsema's work. The projection of the current on the odd (anti)derivatives of the voltage yields the reactive current, taking on the following form:

$$i_r(t) = -c_1 v^{(1)} - c_2 v^{(-1)} \tag{5.1}$$

It is then shown that a compensator coil with magnitude $\frac{1}{c_2}$ and a capacitor with magnitude c_2 improves the power factor, in one case the power factor even become 1.

During this Bachelor Thesis, research was conducted in order to expand on the orthogonal projection method devised by Van der Woude and Jeltsema. After trying different values for the load R, the coil L, the capacitor C and the voltage, it becomes clear that this method often fails to give useful answers. The two main problems that occur are:

- The values for the coefficients of L_c and C_c are often negative. This has no physical meaning.
- When trying voltages with more than two frequencies, the reactive current does not obtain the form shown in equation 5.1. This does not coincide with the proposed compensator form.

The following research questions were posed:

1. In which cases does the orthogonal projection method give relevant values for C_c and L_c?

2. How can the orthogonal projection method be extended to give relevant values for all RLC networks with two-frequency voltages?

In order to answer the first research question, we found a condition for a successful execution of the method. This condition is twofold: C_c has to be larger than 0, as well as L_c. Our research constructed equations for these two coefficients, depending on R, L, C and the voltage frequencies. The following values were found:

$$C_c = \frac{C(a^2b^2C^3LR^2 - a^2b^2C^2L^2 + a^2CL + b^2CL - 1)}{(C^2R^2a^2 + (1 - CLa^2)^2) \cdot (C^2R^2b^2 + (1 - b^2CL)^2)}$$

$$L_c = \frac{(C^2R^2a^2 + (1 - CLa^2)^2) \cdot ((CLb^2 - 1)^2 + C^2R^2b^2)}{a^2b^2C^2(-a^2b^2C^2L^3 + a^2CL^2 + b^2CL^2 + CR^2 - L)}$$

When these two values give positive results, the values for C_c and L_c are relevant and the orthogonal projection method is applicable.

Calculating negative values is equivalent to an inapplicable execution of the method. Therefore, the research also revolved around examining the graphs for $C_c = 0$ and $L_c = 0$. These can be seen as a boundary for success/failure. Chapter 3 includes many plots showing the combinations for R, L and C for which this method does not provide a valid answer. It can be seen that a larger R yields more successful executions of the method. More RLC combinations lead to an applicable execution as the voltage frequencies simultaneously get bigger.

As seen in the first section, C_c and L_c often become negative. Therefore, a second contribution of this research is that it constructs a new type of compensator. This compensator also generates a current of the form:

$$i_c(t) = c_1 v^{(1)} + c_2 v^{(-1)}$$

However, an additional requirement is that c_1 and c_2 need to be able to become negative as well as positive. There are a few different cases for c_1 and c_2, that are summarised in section 4.5. This research shows that by adding an extra degree of freedom to the compensator, the reactive power of any RLC network with a two-frequency voltage can be fully compensated. This also answers the second reseach question that was posed.

The results of this research may inspire research in similar topics. A good follow-up research can be done in finding a compensator for circuits with voltages with more than two frequencies. As shown in the research, adding extra degrees of freedom (for example, by adding a larger second Foster canonical form) made the method more generally applicable. The same approach can be tried for more frequencies.

There are still a couple of question marks to be resolved. An example is the ragged appearance of the plot for $C_c = 0$. The reason for this is still unclear. Also, the assumption made in section 4.2 about the terms of the form $\sin(\omega t)$ and $\cos(\omega t)$ corresponding to the internal resonance can be further researched. Finally, it would also be interesting to test if the conclusions also work in an experimental setup.

This research is an addition to the orthogonal projection method devised by Van der Woude and Jeltsema. It makes the method generally applicable for any RLC network with a two-frequency voltage.

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