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Radiation from the Open-ended Over-moded Cylindrical Waveguide

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Abstract

The theory of radiation from an open-ended circular cross-section waveguide is extended by including the excitation of all possible higher-order waveguide modes at the waveguide-to- free-space boundary. The theoretical expressions are formulated using spectral-domain techniques. The validation is performed using the existing commercial tool FEKO with the *Method of Moments* (MoM) solver for a cylindrical waveguide with the fundamental mode excitation (TE_{11}) and the corresponding higher-order mode, which has the same azimuthal variation as of the fundamental mode, which is (TE_{12}).

1 Introduction

The demand for *wideband single-pixel feeds* (WBSPFs) is increasing in radio astronomy applications [1] [2]. Designing a wideband feed for a reflector dish is a challenging task given the requirements of the dish-feed system, like the aperture efficiency, return loss, and cross-polarization levels. Numerical optimizations of horn geometry via numerous parameter sweeps in commercially available full-wave simulators like FEKO and CST are used to design such feeding structures. These parameter sweeps can take hours and days to compute and optimize a horn antenna feed for a wide range of frequencies with a risk that the numerical tool will find a sub-optimal solution.

In this paper, we propose a computationally efficient technique to compute the reflection coefficient from the aperture of a conical horn. The smooth wall conical horn antennas are usually discretized with cylindrical waveguides with varying cross-sections. The generalized scattering matrix (GSM) of each waveguide junction (at the boundary of two successive cylindrical waveguides) is computed first with the *mode-matching* (MM) technique. The GSM of a three-waveguide problem is then solved using two consecutive junctions. This process is then iteratively carried out till the final waveguide junction problem is solved. The generalized scattering matrix computation needs the computation of double integrals across the two-dimensional circular cross-section [3]. These double integrals are derived analytically with closed-form solutions in [4, Ch.3, 4]. Due to the varying cross-sections of the cylindrical waveguides in the conical profiles, higher-order waveguide modes are excited. Therefore, the aperture of such antennas is typically over-moded when some higher-order modes can be excited locally at the boundary [5]. Hence, there is a need to compute the reflection coefficients of all such modes at the waveguide to free space boundary.

In the aperture admittance method with Fredholm integral equations (Implemented for cylindrical waveguides with a metal flange at the aperture in [6], [7] and [8]), a Green's function approach is used to find the aperture admittance of the waveguide free-space boundary. However, in [6], [7] and [8], the *locally excited higher-order modes* (LHM) excitation at the aperture is not considered due to the awkwardness of the expressions of the magnetic current at the aperture. The shortcomings of this model are that the exact expressions are known only for the fundamental mode for cylindrical waveguides when no other higher-order modes are expected to be excited at the boundary locally.

In this paper, we extend the approach presented in [8] (referred to below as Mishustin integrals) to consider the higher order modes and simplify the expressions for the mode reflection coefficients by transferring double integrals to single ones and introducing Lommel's integrals for spectral representations. The LHMs considered excited at the boundary are chosen based on the coupling among the modes and have the same azimuthal variations as the fundamental mode. A similar approach has been applied in the case of rectangular waveguides [9]. The reflection coefficients are derived by equating Rumsey's reaction integral [10] on both media (the waveguide and free space). The fields of the free space are expressed in terms of the spectral domain Green's functions of the potentials. The proposed method is the first attempt at applying Rumsey's reaction integral method with cylindrical waveguides to compute the reflection coefficients by considering LHM excitation at the boundary to free space transition.

2 Theoretical Derivation

The section explains the aperture reflection with Rumsey's reaction integral method mentioned in [10]. Using the same principles, parameters such as the aperture admittance, the reflection coefficient, and the excitation coefficients for the higher-order modes at the aperture are derived for cylindrical waveguides. Let us consider a semi-infinite cylindrical

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waveguide with an infinite ground plane at the free-space boundary to ensure the absence of free-space radiation in the left half-space of Fig. 1 outside the waveguide. A cylindrical waveguide's modes are generally expressed in cylindrical coordinates for mathematical simplicity. The tangential component of the electric field here has both ρ and ϕ components, which can be represented as,

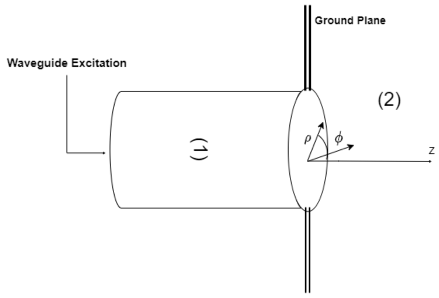

Figure 1. Geometry of cylindrical waveguide to free-space transition

$$E_\rho^{(1)} = \sqrt{N_{11}^{TE}}(1+\Gamma)\frac{1}{\rho}J_1(\beta_{\rho,(1,1)}\rho)\sin(\phi) + \sum_{n=2,3,..}^{\infty}\sqrt{N_{1n}^{TE}}(1+\Gamma)\frac{1}{\rho}J_1(\beta_{\rho,(1,n)}\rho)\sin(\phi)D_n^{TE} \tag{1}$$

$$E_\phi^{(1)} = \sqrt{N_{11}^{TE}}(1+\Gamma)\beta_{\rho,(1,1)}J_1^{'}(\beta_{\rho,(1,1)}\rho)\cos(\phi) + \sum_{n=2,3,..}^{\infty}\sqrt{N_{1n}^{TE}}(1+\Gamma)\beta_{\rho,(1,n)}J_1^{'}(\beta_{\rho,(1,n)}\rho)\cos(\phi) \tag{2}$$

Here, only the modes TE_{1n} are considered because they have the same azimuthal variation with TE_{11} mode and have considerable coupling with TE_{11} mode at the aperture. The term N_{1n}^{TE} is a normalization constant for mode $TE_{1,n}$ mode [4, Ch. 3, eq. 3.63]. The function J is the Bessel function of the first kind. The superscript $'$ refers to the derivative. The term $\beta_{\rho,(1,n)}$ is the radial wavenumber of the mode $TE_{1,n}$. The term Γ is the reflection coefficient. Similarly, the magnetic fields are

$$H_\rho^{(1)} = -\sqrt{N_{11}^{TE}}(1-\Gamma)Y_{11}^{TE}\beta_{\rho,(1,1)}J_1^{'}(\beta_{\rho,(1,1)}\rho)\cos(\phi) + \sum_{n=2,3,..}^{\infty}\sqrt{N_{1n}^{TE}}(1+\Gamma)\beta_{\rho,(1,n)}J_1^{'}(\beta_{\rho,(1,n)}\rho)\cos(\phi)Y_{1n}^{TE}D_n^{TE} \tag{3}$$

$$H_\phi^{(1)} = \sqrt{N_{11}^{TE}}(1-\Gamma)Y_{11}^{TE}\frac{1}{\rho}J_1(\beta_{\rho,(1,1)}\rho)\sin(\phi) - \sum_{n=2,3,..}^{\infty}\sqrt{N_{1n}^{TE}}(1+\Gamma)\frac{1}{\rho}J_1(\beta_{\rho,(1,n)}\rho)\sin(\phi)Y_{1n}^{TE}D_n^{TE} \tag{4}$$

Therefore, the reaction integral [10] can be written as,

$$<1,1>= \int\int_{S_{ap}}(E_\rho H_\phi - H_\rho E_\phi)dS \tag{5}$$

where all field components are considered in the medium (1) ($z<0$). This integral can be reduced to the following form using Bessel function properties and Lommel's integrals [11, p. 101].

$$<1,1>= \frac{\pi}{2}(1+\Gamma)^2\left[\frac{1-\Gamma}{1+\Gamma}(N_{11}^{TE})Y_{11}^{TE}J_1^2(\chi_{1,1}^{'})(\chi_{1,1}^{'2}-1)\right] - \sum_{n=1,2,...}^{\infty}\frac{\pi}{2}(1+\Gamma)^2\left[(N_{1n}^{TE})Y_{1n}^{TE}J_1^2(\chi_{1,n}^{'})(\chi_{1,n}^{'2}-1)D_n^2\right] \tag{6}$$

$$<1,1>= (1+\Gamma)^2\left[\frac{1-\Gamma}{1+\Gamma}Y_{11}^{TE}\right] - \sum_{n=1,2,...}^{\infty}(1+\Gamma)^2\left[Y_{1n}^{TE}D_n^2\right] \tag{7}$$

Let us introduce the aperture admittance as $y_{ap}=\frac{1-\Gamma}{1+\Gamma}$,

$$y_{ap} = \frac{<1,1>}{(1+\Gamma)^2 Y_{11}^{TE}} + \sum_{n=1,2,...}^{\infty}\left[D_n^2\frac{Y_{1n}^{TE}}{Y_{11}^{TE}}\right] \tag{8}$$

Using Green's function spectral domain representation of EM fields explained in [9], the magnetic field components of the second medium can be written as,

$$H_x^{(2)}(k_x,k_y) = -\frac{1}{\omega\mu k_z}(k_xk_yE_x^{(2)}(k_x,k_y) + (k^2-k_x^2)E_y^{(2)}(k_x,k_y)) \tag{9}$$

$$H_y^{(2)}(k_x,k_y) = \frac{1}{\omega\mu k_z}(k_xk_yE_y^{(2)}(k_x,k_y) + (k^2-k_y^2)E_x^{(2)}(k_x,k_y)) \tag{10}$$

The terms k_x, k_y are the spectral domain wavenumbers in x and y directions, respectively. The k is the total wavenumber in spectral domain $k^2=k_x^2+k_y^2+k_z^2$. Therefore, the reaction integral in the second medium is given by,

$$<2,2>= \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(E_x^{(1)}(k_x,k_y)H_y^{(1)}(k_x,k_y) - E_y^{(1)}(k_x,k_y)H_x^{(2)}(k_x,k_y))dk_xdk_y \tag{11}$$

The superscript (2) is replaced with (1) because the reaction integrals are found at the aperture, and the boundary conditions suggest that the tangential fields should be continuous at the aperture. The expression can be transformed to the cylindrical domain (k_Ω,Θ) from the Cartesian domain (k_x,k_y):

$$E_x^{TE}(k_\Omega,\Theta) = \sqrt{N_{11}^{TE}}(1+\Gamma)o_{11}^{TE}(k_\Omega,\Theta) + \sum_{n=1,2,..}^{\infty}\sqrt{N_{1n}^{TE}}(1+\Gamma)D_n o_{1n}^{TE}(k_\Omega,\Theta), \tag{12}$$

where $k_\Omega=\sqrt{(k_x^2+k_y^2)}$, $\Theta=\arccos(\frac{k_x}{k_\Omega})=\arcsin(\frac{k_y}{k_\Omega})$ and,

$$o_{1n}^{TE}(k_\Omega,\Theta) = \int_0^R\left(\left(\frac{1}{\rho}J_1(\beta_{\rho,(1,n)}\rho) - \beta_{\rho,(1,n)}J_1^{'}(\beta_{\rho,(1,n)}\rho)\right)\left(\int_0^{2\pi}\sin\phi\cos\phi e^{k_\Omega\rho\cos(\phi-\Theta)}d\phi\right)\right)\rho d\rho \tag{13}$$

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The integral (13) can be analytically reduced to a single integral by using the Bessel function properties, and the detailed derivation is given in the appendix [4, eq. C.12] for TE modes. Similarly the spectrum function of $E_y(x,y)$ can be found as

$$E_y^{TE}(k_x,k_y) = \sqrt{N_{11}^{TE}}(1+\Gamma)u_{11}^{TE}(k_\Omega,\Theta) + \sum_{n=1,2,..}^{\infty}\sqrt{N_{1n}^{TE}}D_n(1+\Gamma)u_{1n}^{TE}(k_\Omega,\Theta) \quad (14)$$

The term $u_{1n}^{TE}(k_\Omega,\Theta) = \int\int_{S_{ap}}\left[\left(\frac{1}{\rho}J_1(\beta_{\rho,(1,n)}\rho)\sin^2\phi + \beta_{\rho,(1,n)}J_1'(\beta_{\rho,(1,n)}\rho)\cos^2\phi\right)\right]e^{k_\Omega\rho\cos(\phi-\Theta)}\rho d\rho d\phi$. The integral can be found using a similar approach as o_{1n}^{TE} using Bessel function properties [4, eq. C.17]. Applying equations of the form (in terms of k_Ω and Θ) (12) and (14) into the expression of $<2,2>$ and finally using it in the equation (8) for the aperture admittance, we have:

$$y_{ap}Y_{11}^{TE} = \tau_{1,1} + 2\mathbf{d}^{\mathrm{T}}\mathbf{t} + \mathbf{d}^{\mathrm{T}}\mathbf{T}\mathbf{d}, \quad (15)$$

where $\mathbf{d}$ vector contains all the excitations of the LHMs ($\mathbf{d} = \begin{bmatrix}D_2, & D_3, & D_4, & \cdots, D_\infty\end{bmatrix}^{\mathrm{T}}$), and $\mathbf{t}$ has all the mutual admittances between the primary mode at the aperture and all the LHMs ($\mathbf{t} = \begin{bmatrix}\tau_{1,2}, \tau_{1,3}, \tau_{1,4}, \cdots, \tau_{1,\infty}\end{bmatrix}^{\mathrm{T}}$), and the matrix $\mathbf{T}$ has the entries:

$$\mathbf{T} = \begin{bmatrix} \tau_{2,2}+Y_{1,2}^{TE} & \tau_{2,3} & \dots & \tau_{2,\infty} \\ \tau_{3,2} & \tau_{3,3}+Y_{1,3}^{TE} & \dots & \tau_{3,\inf} \\ . & & & \\ \tau_{\infty,2} & \tau_{\infty,3} & \dots & \tau_{\infty,\infty}+Y_{1,\infty}^{TE} \end{bmatrix} \quad (16)$$

For small variations of the fields at the aperture, it is assumed that the changes of y_{ap} are small with respect to D_n. Therefore using $\frac{\partial y_{ap}}{\partial D_n} = 0$, we have:

$$\mathbf{d} = \mathbf{T}^{-1}\mathbf{t} \quad (17)$$

$$y_{ap}Y_{11}^{TE} = \tau_{1,1} + \mathbf{d}^{\mathrm{T}}\mathbf{t} \quad (18)$$

Using the y_{ap} the reflection coefficient Γ can be found using $y_{ap} = \frac{1-\Gamma}{1+\Gamma}$. The mutual admittances $\tau_{m,n}$ have double integrals. After separating the expressions with Θ and integrating it from 0 to 2π, we can find the expression for $\tau_{m,n}$ as an integral in terms of only k_Ω as,

$$\tau_{m,n} = \int_0^\infty \frac{\sqrt{N_{1,n}^{TE}N_{1,m}^{TE}}\beta_{\rho(1,n)}\beta_{\rho(1,m)}\pi}{4\omega\mu k_z(k_\Omega)} \Bigg((2k_0^2-k_\Omega^2)(I_{00}^{1,n}(k_\Omega)I_{00}^{1,m}(k_\Omega)+ I_{22}^{1,n}(k_\Omega)I_{22}^{1,m}(k_\Omega))- k_\Omega^2(I_{00}^{1,n}(k_\Omega)I_{22}^{1,m}(k_\Omega)+ I_{22}^{1,n}(k_\Omega)I_{00}^{1,m}(k_\Omega))\Bigg)k_\Omega dk_\Omega \quad (19)$$

where,

$$k_z(k_\Omega) = -j\sqrt{-(k^2-k_\Omega^2)} \quad (20)$$

The analytical forms of Lommel's integrals I_{22} and I_{00} have two different forms ([4, eq.3.19]) and ([4, eq.3.85]): one when the arguments are the same and one when the arguments are different. Therefore, Lommel's integrals with the same arguments can be used at the vicinity of the pole locations in the above integral expression of $\tau_{m,n}$ (19) to avoid having the singularities. While computing the integral, it is also important not to cross the branch cut of the square root function $k_z(k_\Omega)$. The real part has a branch cut because there is an immediate sign change when the real line is crossed. The imaginary part is always negative. In the Riemann sheet convention, with the expression $k_z(k_\Omega) = -j\sqrt{-(k^2-k_\Omega^2)}$, the top Riemann sheet is considered for the integration path ($\Re(-j\sqrt{-(k^2-k_\Omega^2)}) > 0$, and $\Im(-j\sqrt{-(k^2-k_\Omega^2)}) < 0$) as the integral is from 0 till ∞. The branch cut and the integral path are shown in Fig. 2. Interestingly, the integral from 0 to k_0 yields the real part of the admittance, whereas the integral from k_0 to ∞ yields the imaginary part. The K space integrals have advantages over the Mishustin integrals as they also can compute the mutual admittance between different e-modes in the waveguide. Furthermore, they are mathematically more elegant than Mishustin's integral because the poles at $|k_\Omega| = \beta_{\rho(1,1)}$ (same as $\eta = \frac{\chi'(1,1)}{k_0 R}$ in Mishustin's case) can be avoided by using the different versions of the Lommel's integrals as a step function. Similarly, in the case of mutual admittance when ($m \neq n$), the poles can be avoided at the points $|k_\Omega| = \beta_{\rho(1,m)}$ and $|k_\Omega| = \beta_{\rho(1,n)}$.

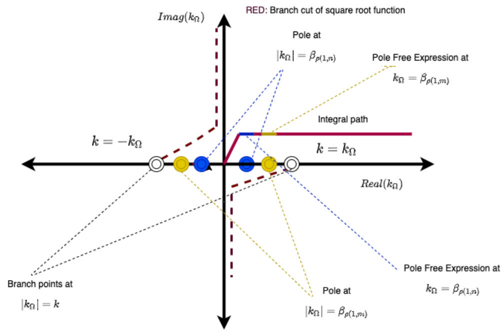

Figure 2. Integration path, branch cuts, branch points and poles to integrate K space integral

3 Results and Discussions

The simulation was carried out for the transition problem of Fig. 1 with the fundamental mode TE_{11}, and TE_{12} mode excitation using the proposed approach and the FEKO software with MoM approach. For TE_{11} mode, the reflection coefficient is shown in Fig. 3a. For TE_{12} mode, the reflection coefficient is shown in Fig. 3b. In the legend, HM stands for LHMs. Therefore, $0HM$ has no LHMs, and $3HM$ is with 3 LHMs.

For TE_{11} mode, the results proposed by this model are

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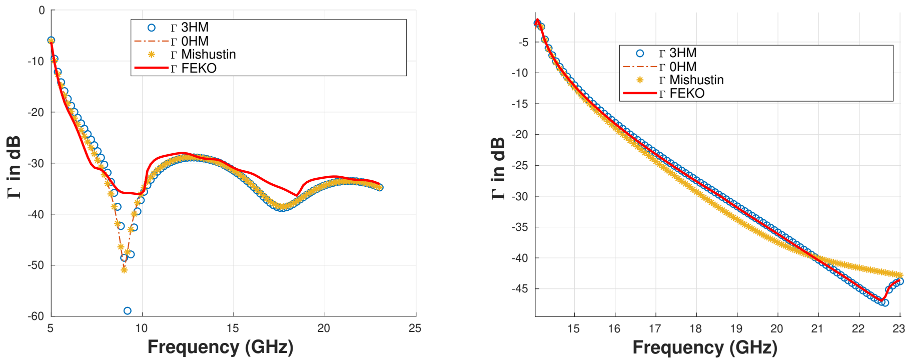

(a) (b)

Figure 3. (a) Reflection coefficient for fundamental mode TE_{11} mode excitation inside the waveguide (b) Reflection coefficient for TE_{12} mode excitation inside the waveguide.

very close to Mishustin's integral results and with FEKO. Certain ripples are observed in the commercial tool results for the reflection coefficient. This can be explained by the irregular approximate geometry considered in the meshed models of FEKO. For the TE_{12} mode excitation, the results are very close to the FEKO results. Supposing no higher-order modes are assumed to be excited at the boundary in the existing model, the results are very close to Mishustin's approach. It is also interesting to note that if higher-order modes are considered at the boundary, the proposed model is closer to the FEKO results than Mishustin's integrals.

4 Conclusions

This paper proposes a new computationally efficient approach to compute the reflection coefficient of an open-ended cylindrical waveguide using spectral domain techniques. The approach considers the excitation of higher-order waveguide modes at the aperture to free space transition. It is assumed that the higher-order modes excited at the boundary have the same azimuthal variation as the mode with which the waveguide is excited. The proposed technique computes the reactance and the reflection coefficients of modes of type $TE_{1,n}$. The integrals for computing the reactance have been reduced from double to single numerical integrals over the radial axis. To simplify further the expressions, Lommel's integrals are used to compute spectral integrals. This has a positive impact on computational time. The results obtained for a single cylindrical waveguide show that the reflection coefficients of TE modes agree with the FEKO results.

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