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Implications of evanescent waves for the Marchenko method through the lens of the transfer-scattering matrix relation

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Introduction

The Marchenko method (Wapenaar et al., 2021) appears to be the only one capable of correctly handling multiples caused by Earth layering below seismic resolution limit (Dukalski et al., 2019; Elison et al., 2020; Peng et al., 2021). It is therefore of great importance in practice to not only understand how this (suite of) method(s) is performs for wavefields propagating *in*, but also *whether* this approach works for waves tunneling as evanescent modes *through* high velocity thin beds. Wapenaar (2020) proposed to modify the representation theories such that they separately address the travelling and the evanescent modes on the method's integration boundaries. Diekmann and Vasconcelos (2021a,b), on the other hand argued that the "standard" Marchenko approach is capable of correctly handling the evanescent modes, while Kiraz et al. (2021) argued that only the evanescent components that are provided on input (initial condition first arrival Green's or focusing function) are recoverable by the standard method, and the other ones are not.

In this work, using a different formalism, we further expand on the work by aforementioned authors and show that in order to be able to handle the full evanescent wave spectrum the "standard" Marchenko approach needs to be appended by additional, in practice unavailable in single-sided illumination, wavefields. It appears, however, that waves that tunnel as evanescent modes through a high velocity layer (i.e. evanescent in bulk), the standard scattering relations appears to be sufficient. In the process we relate the scattering matrix formalism to that of the so-called transfer matrix, such that we can derive a form of "energy conservation" which simultaneously encompasses both traveling and evanescent waves, effectively replacing time-reversal (critical for all convolution-correlation based methods, not just the Marchenko equation related one) with what we refer to as path reversal. To keep the argument exact and easy to follow, we restrict this discussion to a case of 1.5-D acoustic media under the assumption of one-way wavefield decomposition. We will attempt to assess and quantify the degree to which the absence of additional wavefield affects the final result in follow-up research.

Transfer-scattering matrix relation

In a unified representation (Wapenaar, 2019), the wave equation can be formulated in the space, $\mathbf{x} = (x,y,z)$, frequency ω domain as, $\partial_z \mathbf{q} - \mathscr{A}\mathbf{q} = \mathbf{d}$, where the vectors, $\mathbf{q}$ and $\mathbf{d}$, contain the wavefield and source components, respectively, and where we arbitrarily choose the z-axis as preferential direction. We can define operators $\mathscr{L}$, which translate the physical wavefields into those with a preferential propagation direction

$$\mathbf{q} = \mathscr{L}\mathbf{p} = \mathscr{L}\begin{pmatrix}\mathbf{p}^+ \\ \mathbf{p}^-\end{pmatrix}, \quad \text{and} \quad \mathscr{A} = \mathscr{L}\mathscr{H}\mathscr{L}^{-1}, \tag{1}$$

such that $\mathscr{H}$ is a diagonal operator (matrix) and where $\mathbf{p}^\pm$ are the up (-) or down (+) -ward propagating wave components. Fields $\mathbf{p}^\pm$ propagating in the first ($\mathbf{p}_1^\pm$) and N^{th} ($\mathbf{p}_N^\pm$) layer of an $N-$layer medium (see Figure 1a) can be related to each other in two ways

$$\begin{pmatrix}\mathbf{p}_N^+ \\ \mathbf{p}_1^-\end{pmatrix} = \begin{pmatrix}T^{\downarrow} & R^{\cap} \\ R^{\cup} & T^{\uparrow}\end{pmatrix}\begin{pmatrix}\mathbf{p}_1^+ \\ \mathbf{p}_N^-\end{pmatrix} = S\begin{pmatrix}\mathbf{p}_1^+ \\ \mathbf{p}_N^-\end{pmatrix}, \quad \text{and} \quad \begin{pmatrix}\mathbf{p}_N^+ \\ \mathbf{p}_N^-\end{pmatrix} = \prod_{k=1}^{N}\left(\mathscr{L}_{k+1}^{-1}\mathscr{L}_k\mathscr{W}_k\right)\begin{pmatrix}\mathbf{p}_1^+ \\ \mathbf{p}_1^-\end{pmatrix} = \begin{pmatrix}\mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D}\end{pmatrix}\begin{pmatrix}\mathbf{p}_1^+ \\ \mathbf{p}_1^-\end{pmatrix}, \tag{2}$$

where S is the scattering matrix of the medium between the 1^{st} and N^{th} layer, and the transfer matrix $\mathscr{T}$ is the one with the $\mathbf{A}-\mathbf{D}$ components. Here we have also introduced $R^{\cup}$ ($R^{\cap}$) and $T^{\downarrow}$ $(T^{\uparrow})$ – the reflection and transmission responses due to sources above (below) the medium. In 1.5-D acoustic media we can study the relations above for each value of ray parameter p separately, such that, S and $\mathscr{T}$ can be written as 2×2 matrices, and $\mathbf{A}-\mathbf{D}$ are scalar functions of p, ω and medium parameters. Moreover, we have used that for the $\mathbf{p}_k$ inside the k^{th} layer inside the medium, is continuous across the interface $\mathscr{L}_{k+1}\mathbf{p}_{k+1} = \mathscr{L}_k\mathbf{p}_k$ and that $\mathscr{W}_k = \exp\left[\mathscr{H}_k(z_k - z_{k-1})\right]$, is the wavefield extrapolation operator between two interfaces, $k-1$ and k, located at z_{k-1} and z_k, respectively, $\mathscr{H}_k = \text{diag}\,[1,-1]\, i\omega\sqrt{c_k^{-2} - p^2}$, where c_k is the propagation velocity in the k^{th} layer. The derivation that follows holds provided that the $\mathscr{L}_k$ is non-singular, which is true if p does not correspond to a critical angle at any of the interfaces. Rearranging the two expressions in equation 2 one can show that

$$\begin{pmatrix} \mathbf{p}_N^+ \\ \mathbf{p}_0^- \end{pmatrix} = \begin{pmatrix} T^{\downarrow} & R^{\cap} \\ R^{\cup} & T^{\uparrow} \end{pmatrix} \begin{pmatrix} \mathbf{p}_0^+ \\ \mathbf{p}_N^- \end{pmatrix} \equiv \begin{pmatrix} \mathbf{D}^{-1}\det \mathscr{T} & \mathbf{B}\mathbf{D}^{-1} \\ -\mathbf{D}^{-1}\mathbf{C} & \mathbf{D}^{-1} \end{pmatrix} \begin{pmatrix} \mathbf{p}_0^+ \\ \mathbf{p}_N^- \end{pmatrix}, \tag{3}$$

which is a relation also used in other areas of physics (e.g., Katsidis and Siapkas, 2002). Using the form of $\mathscr{L}$ (see e.g., Wapenaar, 1998), it is easy to show that

$$\mathbf{A} = \mathbf{t}_+(-\vec{z}), \qquad \mathbf{B} = \mathbf{t}_-(\vec{z}), \qquad \mathbf{C} = \mathbf{t}_-(-\vec{z}), \qquad \text{and} \qquad \mathbf{D} = \mathbf{t}_+(\vec{z}) \tag{4}$$

where $\vec{z}$ is a vector of all layer thicknesses and $\mathbf{t}_\pm$ are fields which will study later. We define *path* reversal operation $\mathscr{P}[\cdots]$, such that $\mathscr{P}[h(\vec{z})] = h(-\vec{z})$, $\mathscr{P}[\mathscr{T}] = \sigma_x \mathscr{T} \sigma_x$, where σ_x is the real off-diagonal Pauli matrix, and

$$\mathscr{P}[\mathscr{L}_j] = \mathscr{L}_j, \quad \mathscr{P}\left[\mathscr{L}_j^{-1}\right] = \mathscr{L}_j^{-1}, \quad \mathscr{P}[\mathscr{W}_j] = \mathscr{W}_j^{-1}, \quad \mathscr{P}[ab] = \mathscr{P}[a]\,\mathscr{P}[b] \text{ and } \mathscr{P}\left[a^{-1}\right] = \mathscr{P}[a]^{-1},$$

where a and b are some scalar functions. Equation 3 shows that $\mathbf{t}_+(\vec{z}) = T^{\uparrow -1} \equiv f_2^-$, and $\mathbf{t}_-(\vec{z}) = R^{\cap} T^{\uparrow -1} \equiv f_2^+$, where $f_2^{\pm}$ are the one-way components of the Marchenko focusing function f_2 , and hence

$$\mathscr{T} = \begin{pmatrix} \mathscr{P}\left[f_2^-\right] & f_2^+ \\ \mathscr{P}\left[f_2^+\right] & f_2^- \end{pmatrix} \qquad \text{and} \qquad \mathscr{T}^{-1} = \begin{pmatrix} f_1 + & \mathscr{P}\left[f_1^-\right] \\ f_1^- & \mathscr{P}\left[f_1^+\right] \end{pmatrix} \tag{5}$$

where the expression for $\mathscr{T}^{-1}$ was found by following the same sort of argument as we did for $\mathscr{T}$, and where $f_1^+ = T^{\downarrow -1}$, and $f_1^- = R^{\cup} T^{\downarrow -1}$ is the other set of focusing functions. A similar relationship between the *propagator* matrix $\left(\mathscr{L}_k \mathscr{T} \mathscr{L}_1^{-1}\right)$ and two-way focusing function was recently derived by Wapenaar and de Ridder (2022). The exact relationship between that result and the one in equation 5 and the underlying assumptions used in each derivation require careful investigation.

Unitarity relation and implication for the Marchenko method

By evaluating $\mathscr{T}\mathscr{T}^{-1} = 1$ written in terms of reflection and inverse transmission responses one e.g. gets $\mathscr{P}\left[T^{\uparrow}\right] T^{\downarrow} + \mathscr{P}[R^{\cup}] R^{\cup} = 1$. Also from equation 3 and the form of $\mathscr{T}$ in equation 5 we get $R^{\cap} = -\mathscr{P}\left[T^{\downarrow -1} R^{\cup}\right] T^{\uparrow}$. These and two more relations can be recast as a *very familiar* "unitarity" relation $\sigma_x \mathscr{P}[S]\,\sigma_x S = 1$, which means that the scattering matrix has an inverse which is composed of the same path reversed terms. Path reversal is a generalization of time reversal which allows us to cover both cases of when $\mathscr{H}_k$ is imaginary or real, i.e. when waves are propagating or evanescent. For waves which are propagating in every layer of the medium, reflection and transmission coefficients are real, and thus path reversal is equivalent to complex conjugation (*) in the frequency domain or time reversal. In this case $\sigma_x \mathscr{P}[S]\,\sigma_x S = 1$ becomes the familiar energy conservation (diagonal term) and phase (off-diagonal terms) laws.

The result $\sigma_x \mathscr{P}[S]\,\sigma_x S = 1$ and the fact that it forms the root for the derivation of the Marchenko method Dukalski and de Vos (2022), suggest that one needs $\mathscr{P}[R^{\cup}]$ which is not guaranteed to equation to $R^{\cup *}$ if evanescent modes are present. One can show that

$$\begin{array}{llll} (a) & G^- + f_1^- = R^{\cup} f_1^+, & (b) & \mathscr{P}[G^-] + \mathscr{P}\left[f_1^-\right] = \mathscr{P}[R^{\cup}]\,\mathscr{P}\left[f_1^+\right], \\ (c) & -\mathscr{P}[G^+] + f_1^+ = \mathscr{P}[R^{\cup}]\, f_1^-, & (d) & -G^+ + \mathscr{P}\left[f_1^+\right] = R^{\cup}\,\mathscr{P}\left[f_1^-\right] \end{array} \tag{6}$$

is the relevant set of equations to solve in the presence of evanescent waves. These two pairs of equations are related to each other by applying the path reversal operation to both sides, which for traveling waves is always replaceable with complex conjugation (time reversal), meaning that the set is reduced to two equation with four unknowns. In fact, this is very similar to the work of Slob (2016) in the context of a dissipative medium, where the time reversal combined with a change of the nature of the medium from dissipative for effectual, could be replaced with a path reversal. In order to retrieve $\mathscr{P}[R^{\cup}]$ (but only when $\neq R^{\cup *}$), one has to measure all of S components and invert $\sigma_x \mathscr{P}[S]\,\sigma_x S = 1$ for $\mathscr{P}[S]$. This means that one has to be able to illuminate and record the wavefields on both sides of the medium - a significant limitation in practice for seismic exploration. Alternatively, one can calculate theinverse of the transfer matrix and use $\mathscr{P}[R^{\cup}] = \mathscr{P}\left[R^{\cup}T^{\downarrow -1}\right]/\mathscr{P}\left[T^{\downarrow -1}\right]$, however, this would require detailed knowledge of the medium parameters which are what we are trying to determine in the first place.

The impact of the presence of evanescent waves on the standard Marchenko method can be analyzed for three separate cases, depending on whether the evanescent wave is present in: (1) the overburden, (2) the layer separating the latter and the target reflector, and (3) below that. Following the conventional procedure where we introduce appropriate mutes Θ^-,$\Theta^-_{\mathscr{P}}$,Θ^+, and $\Theta^+_{\mathscr{P}}$ (and assuming that such exist) to equations 6 (a-d) respectively we can *simultaneously* solve for $f_1^{\pm}$ and $\mathscr{P}\left[f_1^{\pm}\right]$ provided we have access to $R^{\cup}$ and $\mathscr{P}[R^{\cup}]$. In the process we need to provide the *overlap* between $-\mathscr{P}[G^+]$ and f_1^+ and between $-G^+$ and $\mathscr{P}\left[f_1^+\right]$. Subsequently, we use these solutions to find $G^{\pm}$ and $\mathscr{P}[G^{\pm}]$, which we can use to recover $G = G^+ + G^-$. To address impact (1), if we replace $\mathscr{P}[R^{\cup}] \to R^{\cup *}$ in the process above, and no evanescent waves are present in the overburden, then it is very probable that one would recover the correct $f_1^{\pm}$ because $\mathscr{P}[R^{\cup}] = R^{\cup *}$ *in the relevant to the solver interval* related to the choice of Θ^- and as a result $f_1^{\pm *} = \mathscr{P}\left[f_1^{\pm}\right]$. We should stress however, that we have been able to generate numerical examples where $\mathscr{P}[R^{\cup}] = R^{\cup *}$ (see Figure 1b), even when evanescent waves were present, albeit only if they were related to propagating waves in the lower and upper half-space. This suggests that tunneled waves could be correctly handled by the Marchenko method, in the parameter space where the wavefields are propagating at the target-overburden separating horizon. In case (2), i.e., if the evanescent waves are present in the redatuming layer, then $\mathscr{P}[R^{\cup}] \neq R^{\cup *}$ (see Figure 1c) inside the aforementioned interval, and then using $\mathscr{P}[R^{\cup}] \to R^{\cup *}$ will not yield the correct focusing functions $f_1^{\pm}$. In the third case, evanescence below the target, the reflection response due to the target-only will have $\mathscr{P}[R^{\cup}] \neq R^{\cup *}$, and hence the same will hold for the total reflection response Ű something that will manifest itself in the time interval outside of what is used in the Marchenko equation. In this case, however, we can probably use equation 6(d), provided $\mathscr{P}\left[f_1^{\pm}\right] = f_1^{\pm *}$, to recover the GreenŠs function. In any cases, we do not exclude the possibility of observing some events, which have evanescent-like features, but to what extend they approximate to true solutions is most likely medium-dependent, with greater differences linked to media with larger velocity contrasts. The exact impact and its relationship to path reversal introduced here are beyond scope of this work and require further investigation.

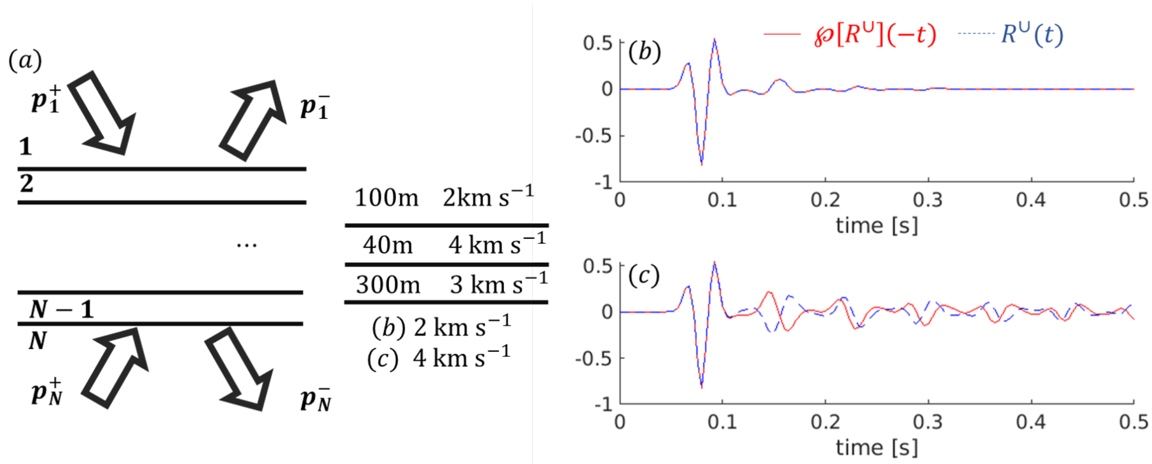

Figure 1 *(a) Schematic of the N-layer medium, (b) a $p = 1/3.5$ km$^{(}-1)$ s reflection response $\mathscr{P}[R^{\cup}] = R^{\cup *}$ for the 4-layer medium with a wave propagating in the top and bottom layer, but tunneling through the 40m layer. (c) Same as (b) but now we observe that $\mathscr{P}[R^{\cup}] \neq R^{\cup *}$ which appear to be related to the fact that the wave is evanescent in the bottom layer.*

Conclusion and Outlook

In this work we have studied a case of a 1.5-D acoustic medium, where we have shown that tunneled waves do not appear to be a challenge for the "standard" Marchenko method. In the process we have shown the relation between the transfer matrix and the Marchenko equation wavefields, and presented an argument how that the generalization of time reversal (complex conjugation in the frequency), which we dubbed "path reversal", is compatible with the Marchenko equation approach beyond propagating waves. Future research should establish the connection between this work and that presented in Wapenaar (2020) and investigation beyond acoustic and 1.5D media.

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