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Modelling turbulent heat transfer in rough channels using phenomenological theory

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Abstract. Rough walls are often encountered in industrial heat transfer equipment. Even though it is well known that a rough wall affects velocity fields and thermal fields differently (and therefore also skin friction factors and Stanton or Nusselt numbers), predicting the effect of rough walls on turbulent heat transfer remains difficult. A relation between the scalar spectrum and the Stanton number is derived for channels with both smooth and rough walls. It is shown that the new relation agrees reasonably well with recent DNS experiments for wall roughness sizes of $k^+ < 150$ and when $Pr = 0.7 - 1.0$. Under these conditions, a thermal analogue of Moody's diagram can be created using the newly developed relation.

1. Introduction

Hydrodynamically rough walls are often encountered in heat exchangers, condensers and turbines. Rough walls are created through machining, fouling and corrosion. However, surface enhancements such as ribs, turbulators, and grooves can also be regarded as roughness. While the effect of roughness on turbulence is extensively investigated, our understanding of roughness effects on heat transfer are lacking in comparison. Recently, it was shown by different researchers that the wall roughness functions for scalars $\Delta\Theta^+$ and velocity ΔU^+ are not the same, [1, 9, 8]. These wall roughness functions describe the downward shift of the logarithmic profile of the nondimensional temperature Θ or velocity U due to rough walls. DNS data suggests that $\Delta\Theta^+$ 'levels off' at larger values of the wall roughness parameter k^+ , whereas ΔU^+ does not [9, 8]. The parametrization of the wall roughness function is important as there is a direct relation between $\Delta\Theta^+$ and the Stanton number [9]:

$$\Delta\Theta^+ = \sqrt{\frac{C_{f,s}}{2}} \left(\frac{1}{St_s} - \frac{Pr_t}{c_\kappa^2} \right) - \sqrt{\frac{C_{f,r}}{2}} \left(\frac{1}{St_r} - \frac{Pr_t}{c_\kappa^2} \right), \quad (1)$$

where C_f is the skin friction coefficient, St the Stanton number, Pr_t the turbulent Prandtl number, c_κ the von Karman constant and where subscripts 'r' and 's' refer to rough and smooth conditions respectively. Eq. (1) effectively means that if $\Delta\Theta^+$ is known, so is St_r .

While a relation between the skin friction factor and the turbulent spectrum exists [2], a similar relation between the Stanton number St and the scalar spectrum does not. Such a relation is derived in this paper by considering a mathematical model of the scalar spectrum as was introduced by [4].



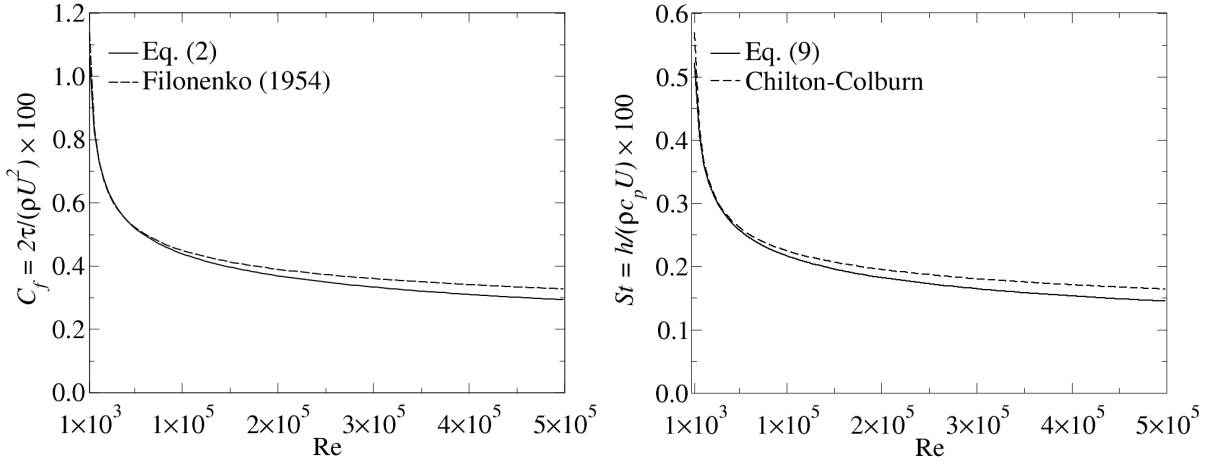


Figure 1. Left: skin friction factor as predicted by Eq. (2) and the Filonenko correlation [7]. Right: the Stanton number as predicted by Eq. (9) and the Chilton-Colburn analogy.

2. Theory

According to Gioia and Chakraborty [2], the velocity u_s of an eddy of size s can be written in the form of $u_s^2 = \int_0^s E(\sigma)\sigma^{-2}d\sigma$ where $E = B\sigma^{5/3}c_d(\eta/\sigma)c_e(\sigma/R_h)$ is the turbulent spectrum and $\sigma = 1/\kappa$ is a length scale corresponding to wavenumber κ , B a dimensionless constant, η the Kolmogorov length scale, c_d a function that models the dissipative range of the spectrum and c_e a function that describes the energetic range. Considering a rough wall consisting of successive elements of size k , it is reasoned that the dominant eddy contributing to the wall shear stress is an eddy of size $s/R_h = k/R_h + a\eta/R_h$, where R_h is the hydraulic radius. As a result, the skin friction coefficient $C_f = \tau/(1/2\rho U_b^2)$ can be modelled as:

$$C_f = 2c_\tau c_u \left(\frac{2}{3}\right)^{1/2} \int_0^{\frac{k}{R_h} + ab'Re^{-3/4}} x^{-1/3} c_d \left(\frac{b'Re^{-3/4}}{x}\right) c_e(x) dx \quad (2)$$

In which $x = \sigma/R_h$, $c_u = 0.036$, $a = 3$ and $b' = 11.4 \times (1/2)^{-3/4}$, k/R_h the relative roughness height and Re the Reynolds number based on the hydraulic diameter. It should be noted that Eq. (2) is slightly different from the original relation, in which the Reynolds number was defined using the radius. Setting $c_\tau = 0.46$, equation (2) is in reasonable agreement with well known friction correlations for smooth pipes, such as the Filonenko correlation [7], as is shown in figure 1.

A thermal analogue of equation (2), i.e. a direct relation between the heat transfer coefficient (in the form of the Stanton number) and the turbulent scalar spectrum, is hereafter derived by first considering the turbulent scalar spectrum. In the inertial-convective range, the scalar spectrum scales as $E_\phi(\kappa) \sim \langle\chi\rangle\langle\epsilon\rangle^{-1/3}\kappa^{-5/3}$ (where $\langle\epsilon\rangle$ is the mean energy dissipation rate, $\langle\chi\rangle$ is the mean scalar dissipation rate and where τ_η is the Kolmogorov time scale) [10, 11], but when $Pr \gg 1$ for $1/\eta \ll \kappa \ll 1/\eta_B$, the scalar spectrum follows $E_\phi(\kappa) \sim \langle\chi\rangle\tau_\eta\kappa^{-1}$. Several mathematical models of the scalar spectrum exist in the literature that can reproduce both the $\kappa^{-5/3}$ law in the inertial-convective range and the κ^{-1} law in the dissipative range, when $Pr \gg 1$ [4]. The simplest of these models is written as

$$E_\phi(k) = \frac{\langle\chi\rangle\eta^3}{\nu} \beta Q^{\frac{5}{2}} y^{-\frac{5}{3}} \left(1 + y^{\frac{2}{3}}\right) \exp\left\{-A\left(\frac{3}{2}y^{\frac{4}{3}} + y^2\right)\right\} c_e(\sigma/R_h), \quad (3)$$

where $y \equiv Q^{\frac{3}{2}}k\eta$, $A \equiv \beta Pr^{-1}Q^{-2}$ and where it is assumed that the scalar spectrum and energy spectrum behave similarly in the energetic range. $\beta \approx 0.7$ while $Q \approx 2.5$. Analogous to an eddy of size s , a thermal structure t_s of size s can be considered:

$$t_s^2 = \int_0^s E_\phi(\sigma)\sigma^{-2}d\sigma, \quad (4)$$

Using Taylor's scaling $\langle \epsilon \rangle = c_\epsilon U_R^3/R_h$ (where U_R is the length scale of the largest eddy, R_h the hydraulic radius and $c_\epsilon = 5/4$), $u_R = c_u V$ (with V being the bulk velocity), and changing variables $x = \sigma/R_h$, y can be rewritten as $Q^{\frac{3}{2}}\eta/\sigma = Q^{\frac{3}{2}}b'Re^{-3/4}x^{-1}$, where $b' = \frac{1}{2}^{-3/4} \times (c_\epsilon c_u^3)^{-1/4}$. A thermal analogue of Taylor's scaling may be written as $\langle \chi \rangle = c_\chi \Delta T_R^2 U_R/R_h/Pr$, while $\Delta T_R = c_T \Delta T$ lead to the following scaling arguments:

$$\langle \epsilon \rangle \sim c_\epsilon \frac{U_R^3}{R_h} = c_\epsilon c_u^3 \frac{V^3}{R_h} \quad \text{and} \quad \langle \chi \rangle \sim c_\chi Pr^{-1} \Delta T^2 \frac{U_R}{R_h} = c_\chi c_u c_T^2 Pr^{-1} \Delta T^2 \frac{V}{R_h}, \quad (5)$$

Applying Eq. (5) to Eq. (4) yields:

$$t_s^2 = \beta Q^{\frac{5}{2}} \langle \chi \rangle \eta^3 \nu^{-1} R_h^{-1} \int_0^{s/R_h} \tilde{E}_\phi(\kappa) x^{-2} dx = \frac{c_\chi c_T^2}{c_\epsilon^{3/4} c_u^{5/4}} \left(\frac{V R_h}{\nu} \right)^{-5/4} \frac{\Delta T^2}{Pr} \int_0^{s/R_h} \tilde{E}_\phi(x) x^{-2} dx, \quad (6)$$

where

$$\tilde{E}_\phi(x) = y^{-\frac{5}{3}} \left(1 + y^{\frac{2}{3}} \right) \exp \left\{ -A \left(\frac{3}{2} y^{\frac{4}{3}} + y^2 \right) \right\} c_e(x), \quad (7)$$

and $y = Q^{\frac{3}{2}}k\eta = Q^{\frac{3}{2}}b'Re^{-3/4}/x$. Analogous with the relation for the shear stress, the heat flux is modelled as $q = \rho c_p V t_s$. Then, by its definition, $St \equiv \frac{q}{\rho c_p V \Delta T}$, the Stanton number can be written as

$$St = \frac{t_s}{\Delta T} = \sqrt{\beta Q^{\frac{5}{2}} \frac{c_\chi c_T^2}{c_\epsilon^{3/4} c_u^{5/4}} \left[Pr^{-1} \left(\frac{V R_h}{\nu} \right)^{-5/4} \int_0^{s/R_h} \tilde{E}_\phi(x) x^{-2} dx \right]^{\frac{1}{2}}} \quad (8)$$

Rewriting Eq. (8) using $Re = VD_h/\nu$ the following relation is obtained

$$St = K_T \left[Pr^{-1} Re^{-5/4} \int_0^{s/R_h} \tilde{E}_\phi(x) x^{-2} dx \right]^{\frac{1}{2}}, \quad (9)$$

where $K_T = \sqrt{\beta Q^{\frac{5}{2}} \frac{c_\chi c_T^2}{c_\epsilon^{3/4} c_u^{5/4}} \left(\frac{1}{2} \right)^{-5/4}}$ and if $s/R_h = k/R_h + a\eta/R_h$ is the size of the eddy that fits between successive roughness elements, then the thermal structure fitting between the same elements is assumed to be $s/R_h = k/R_h + a\eta_{OC}/R_h$. Comparing Eq. (9) to well known experimental correlations, it is found that $K_T \approx 0.09$. A comparison between Eq. (9) and the well known Chilton-Colburn analogy is shown in figure 1.

3. Results

Eq. (9) represents a correlation between the Stanton number, the Reynolds and Prandtl number and the relative roughness height k/R . It is assumed that for successive roughness elements $k \approx k_s$ (where 's' means sand grain equivalent); therefore $k_s^+ = \frac{1}{2} \sqrt{\frac{C_f}{2}} Re \frac{k}{R_h}$. Together with Eq. (1), the theory that was presented above can now be compared with literature results for the wall roughness function $\Delta\Theta^+$. Figure 2 shows DNS results for two different Prandtl numbers. The theory agrees reasonably well with the literature results. However, while the DNS results show that $\Delta\Theta^+$ 'levels off' for approximately $k_s^+ > 150$ when $Pr = 0.71$, the theoretical predictions do not.

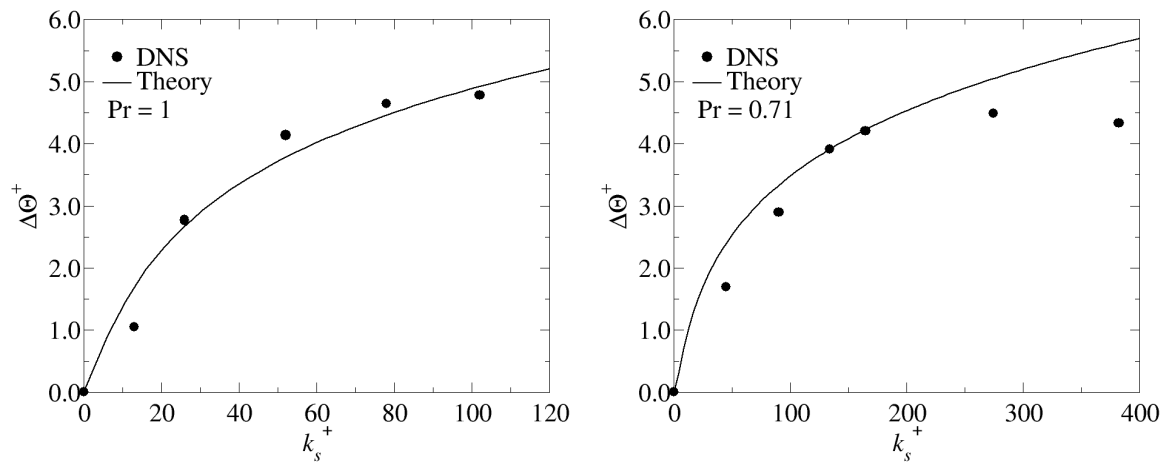


Figure 2. Left: $\Delta\Theta^+$ as predicted by Eq. (1) and (9) for $Pr = 1$ (left) and $Pr = 0.71$ (right) together with data from literature [8], [9].

4. Discussion

The discrepancy between the theoretical predictions for $\Delta\Theta^+$ and the DNS results from literature suggest that the presented theory should not be used in its current form for $k^+ > 150$. The origin of the discrepancy could be due to the fact that the skin friction factor is significantly affected by the pressure drag, while there is no corresponding mechanism for the Stanton number [12, 8]; this difference between scalar and momentum transport is unaccounted for in the model. Moreover, the theory is untested for values of the Prandtl number that are not at all close to unity. Furthermore, the theory is unable to reproduce experimental values of the skin friction factor and the Stanton number at large Reynolds numbers (even for smooth conditions). These facts limit the applicability of the presented theory. Thus, subsequent research is warranted.

References

- [1] Li, Q-X, Pan, M and Dong, Y-H, 2018, *Computers and fluids*, **165** 472–48544-42.
- [2] Gioia, G and Chakraborty, P, 2006, *Physical review letters*, **96** 044502.
- [3] Batchelor, G K, 1959, *Journal of fluid mechanics*, **5** 113–133.
- [4] Hill, R J, 1978, *Journal of fluid mechanics*, **88** 541–562.
- [5] Kraichnan, R H, 1968, *Physics of fluids*, **11** 945–953.
- [6] Kraichnan, R H, 1974, *Journal of fluid mechanics*, **64** 737–762.
- [7] Filonenko, G K, 1954, *Teploenergetica*, **4** 109–116.
- [8] Peeters, J W R and Sandham, N D, 2019, *International journal of heat and mass transfer*, **138** 454–467.
- [9] MacDonald, M, Hutchins, N., Chung, D, 2019, *Journal of fluid mechanics*, **861** 138–162.
- [10] Obukhov, A M, 1949, *Dokl. akad. nauk. SSSR*, **39** 391.
- [11] Corrsin, S, 1951, *Journal of applied physics*, **22** 469–473.
- [12] Bons, J, 2005, *Journal of heat transfer*, **127** 472–485.