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Convex Modeling of Pumps in Order to Optimize Their Energy Use

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Abstract This study presents convex modeling of drainage pumps so that real-time control systems can be implemented to minimize their energy use. A convex model is built based on pump curves and then used in mixed-integer optimization to allow pumps to be turned on or off. It is implemented as an extension to the open source software package RTC-Tools. The formulation is such that the continuous relaxations of the mixed-integer problem are convex, hence branch-and-bound techniques may be used to find a global optimum. The formulation can be used for variable-speed and constant-speed pumps. There are several possible applications, such as optimization of polder systems, pumped-storage systems, or certain water distribution networks. Finally, an example of the drainage pump is presented to compare the method to current methods and show that energy can be saved by using the proposed method.

Plain Language Summary Pumps are present in several water systems, like drinking water networks or drainage pumps. Drainage pumps are one of the greatest energy consumers in low-lying countries. Water managers decide when to turn them on depending on the sea level, weather prediction, and other factors. The operation costs depend on the time of operation. This article explains a method to decide when to turn these pumps on to use the least amount of energy, cheaper, and more sustainable drainage possible. The method can be further applied to any water systems containing pumps.

1. Introduction

Freshwater is a basic resource. However, freshwater is often distributed unevenly. To correct this distribution and prevent the problems originating from it, operational water management ensures the right amount of water at the right place and time. The distribution of water is carried out by hydraulic structures, such as overshot gates, undershot gates, and pumps. The decision about the operation of these structures determines the distribution of water. These principles hold for different water systems, such as irrigation and drainage systems or water distribution networks. To decide on the operation, mathematical optimization is used to enable water managers to take into account the future water demands and weather forecasts at the time of the decision making. For example, for a drainage canal, the task of the optimization is to determine the pump operation such that the water level in the canal stays in the prescribed bounds while the nearby areas are drained into that canal. In doing so, disturbances should be taken into account such as rain. Mathematical optimization of water systems requires a suitable model for each part of such systems. This work focuses on the modeling of the pumps to optimize their energy use. The approach is demonstrated on a drainage system, but it can be applied to other systems, such as ground water pumping (Ahlfeld & Laverty, 2011) or to water distribution networks.

Modeling and optimization of pump operation can be found in the literature mainly in the context of water distribution networks. Mala-Jetmarova et al. (2017) provides an overview of this topic. The reason why optimization is studied mainly in connection with water distribution systems is that the costs of such a network are strongly related to pumping costs (Ormsbee & Lansey, 1994). Pump operation can be classified into two big categories: constant-speed pumps, whose speed does not vary, and variable-speed pumps, whose speed varies depending on the discharge and the head.

Constant-speed pumps are often used in water distribution networks. For such pumps, the optimization algorithm only has to determine when they should be turned on and off. This is a mixed-integer optimization

problem; hence, it can be very time-consuming to find the global optimum compared to the continuous version of the problem. For this reason, pumps in water distribution networks are often optimized by faster algorithms providing only suboptimal solutions. In Marchi et al. (2017), for instance, a genetic algorithm is used combined with real pump dynamics. Heuristics can be used to increase the solution speed (Fatemi et al., 2017); however, they also result in suboptimal solutions.

There are few methods found in the literature that can provide the global optimum within acceptable time frames. Cembrano et al. (2000) avoids using mixed-integer programming by calculating the volume to be pumped in the formulation (instead of the on-off state of the pump) resulting in a continuous problem, and then the pump-schedule is obtained as post processing. However, in this case the number of pump-switching times must be known a priori, which is not the case most of the time. Lansey and Awumah (1994) developed simplified hydraulic models for each possible pump combination, but the applicability of the model is limited by the number of pumps in the system. Dorini et al. (2012) describes convex modeling and optimization of pumps, supposing they are always on. Another way to reach the optimum without having to solve a mixed-integer problem is to use time as the optimization variable to determine the start and length of the pump operation (Dekens et al., 2014; Price & Ostfeld, 2014). This approach provides a mathematical solution to the problem that leads to global optimum; however, it becomes very complex when it is combined with other elements of the water system, because the objective of the other elements might not be expressible using time as optimization variable. Moreover, similarly to the previous case, the number of pump switches should be set a priori. Although the last two methods are able to reach a global optimum (even having their own limitations), they can only be applied to constant-speed pumps. They are not suitable to minimize the energy use of variable-speed pumps.

The optimization of variable-speed pumps is a challenging task. Similar to constant-speed pumps, variable-speed pumps can be seen as discrete systems—as they have the on and off position—and hence their optimization is a mixed-integer problem. As opposed to constant-speed pumps, not only the time of operation should be determined but also the shaft speed. Due to their complexity, several writers have used heuristic methods to solve the optimization problem. The currently used heuristic optimization methods can only achieve a suboptimal solution, for example, ant-colony optimization (Hashemi et al., 2014), stochastic optimization techniques such as Particle Swarm Optimization (Wegley et al., 2000), or genetic algorithm (Olszewski, 2016). There are several examples of these methods applied in water distribution networks, for instance in Bagloee et al. (2018), Candelieri et al. (2018), Marchi et al. (2016), and Oikonomou et al. (2018), machine learning techniques are combined with optimization methods and also other pump systems such as pumped-storage hydropower plants (Alizadeh-Mousavi & Nick, 2016; Schmidt et al., 2017) or optimizing pumps used as turbines (Fecarotta et al., 2016). Malrait et al. (2017) minimized the motor losses of the pump for a given hydraulic operation point (i.e., pump speed); thus, it does not optimize the whole hydraulic system. Another way to tackle the problem of finding the global optimum of the discrete system is using the solution of the continuous system: Ulanicki and Kennedy (1994) is solving a nonconvex continuous problem and rounds the solution of the continuous optimization to integers. This method also leads to suboptimal solutions.

None of the above-mentioned methods for variable-speed pumps is able to reach global optimum due to the use of heuristic optimization methods. Using heuristics can decrease the robustness of the solution: A small perturbation in the initial conditions can lead to a different local optimum. This is not a desirable behavior in real operational systems, as the system may provide completely different solutions when a slightly different initial condition is used. This also makes it difficult for the human operator to trust the algorithm. Therefore, for robustness, it is necessary to use an algorithm that is able to find the global optimum. One such algorithm was presented in Menke et al. (2015, 2016). The authors use simplified modeling of pumps which enables the use of mixed-integer convex optimization to reach the global optimum. Moreover, they show that linear approximations outperform nonlinear approximations in computation time without significant loss of accuracy. They use convex approximation of the pump curves combined with a relatively high (5%) optimality gap in the branch and bound algorithm to be able to find the optimal solution within affordable computation time. The power consumption of the fixed-speed pump is assumed to be constant and for variable-speed pumps depends only on the discharge. However, the actual power consumption of the pump highly depends on discharge and head. It introduces a large error in power approximation, thus in the estimated energy consumption.
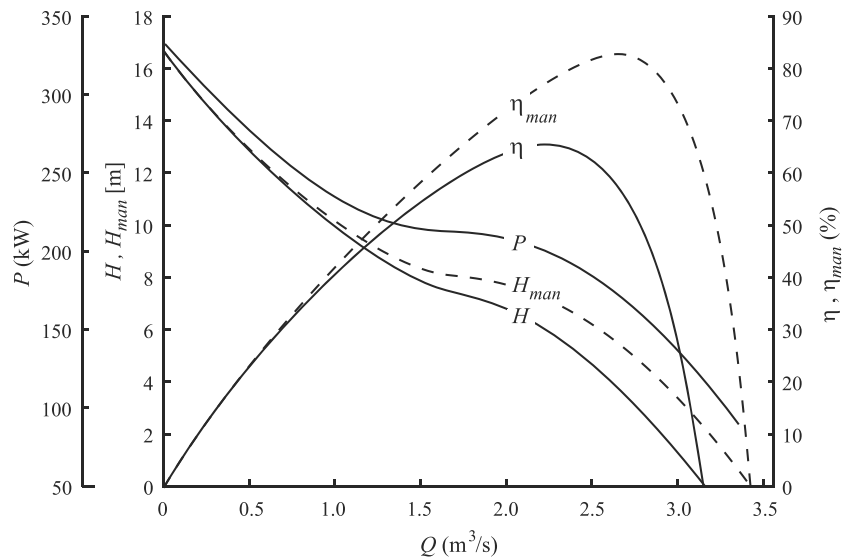
Figure 1. Pump characteristic curves for an axial flow pump with impeller diameter 1,040 mm running at 356 rpm. Shown are characteristics for manometric head H_{man}, total or static head H, power P, manometric efficiency η_{man}, and total efficiency η.

In this work, we present a convex modeling of pumps which results in reaching the global optimum while minimizing energy use by taking into account its dependence on head and flow conditions.

2. Methodology

In this section the convex modeling of pumps is described. First, the operation of variable-speed pumps is recalled. Next, the case is described when the pump is always turned on and formulated as continuous convex optimization problem. Then, the possibility to turn off the pump is included. This requires mixed-integer modeling of pumps with convex relaxed problems, which accounts for the main contribution of this article. Finally, the formulation is extended to constant-speed pumps.

2.1. Description of Variable-Speed Pumps

In this section the operation of variable-speed pumps is described. A pump is a dynamic device that increases the pressure of a liquid by transferring the mechanical energy of the rotating impeller to the liquid (Wright & Gerhart, 2009). The pressure head generated by the pump at a certain shaft speed depends on the discharge. This dependency is presented graphically by the so-called QH curve of the pump. Figure 1 presents an example of a large axial flow pump used for polder dewatering, running at 356 rpm. Apart from the QH curves, characteristic curves for power and efficiency are also included in the figure and are seen to depend on discharge as well. The manometric head, H_{man}, generated by the pump is partially used to compensate the pressure losses in the pumping station and partially to overcome the total, static head H, that is, the difference in water levels between the two sides of the pumping station. For every shaft speed, there is a corresponding QH curve. Thus, the relationship between discharge, head, and shaft speed is unique, any two determine the third. This relationship is described by the affinity laws. The performance of an axial flow pump is given in Figure 2 for a range of different shaft speeds, showing how the pump can be used for a range of discharges and heads. However, in practice, the operating range is limited by several factors, like minimum discharge Q_{min} to prevent unstable operation, cavitation if available net positive suction head (NPSH) is not sufficient, and maximum power P_{max} of the motor. All these curves together define an area where the pump can be operated: This is called working area in the following. In case of constant-speed pumps, there is only one QH curve, and the working area is reduced to a section of the curve.

2.2. Convex Modeling and Optimization of Pumps That Are Always Turned On

This section describes how the operation of the variable-speed pump can be modeled with convex functions in case the pump is turned on. An optimization problem is called convex if a convex function is minimized
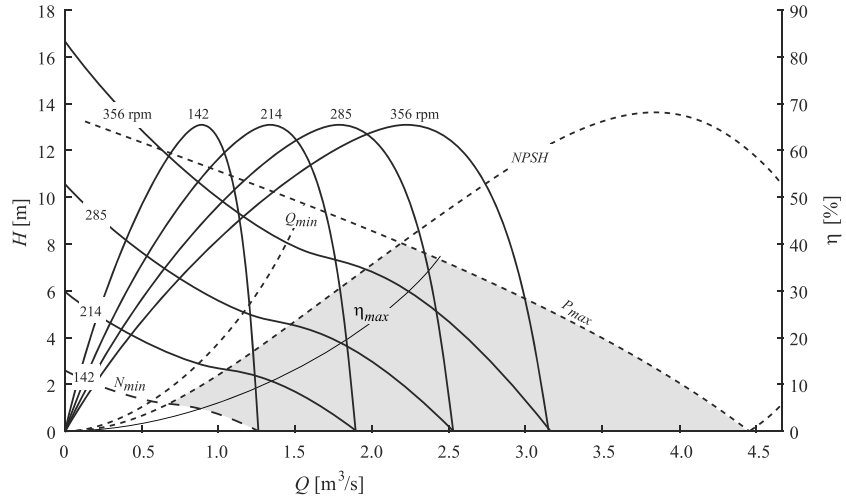
Figure 2. Performance of the axial flow pump for different shaft speeds. The working area is bounded by minimum discharge Q_{min}, the minimum shaft speed N_{min}, cavitation NPSH, and maximum motor power P_{max} and is depicted as a gray area. NPSH = net positive suction head.

on a convex domain, in the presence of convex inequality and affine equality constraints:

$$\underset{x}{\text{minimize}} \quad f_0(x)$$
$$\text{subject to} \quad f_i(x) \leq 0, \ i = 1, \dots, m,$$
$$h_i(x) = 0, \ i = 1, \dots, p, \tag{1}$$

where $x \in \mathbb{R}^n$ is the optimization variable, $f_0 : \mathbb{R}^n \mapsto \mathbb{R}$ is the convex objective function, $f_i : \mathbb{R}^n \mapsto \mathbb{R}$ are the convex inequality constraints, and $h_i : \mathbb{R}^n \mapsto \mathbb{R}$ are the affine equality constraints. The next step is to describe the operation of the pump in such form.

The optimization problem for pumps is the following: How to choose the pump speed for the instantaneous head during a time period t_{dn} so that the least amount of energy is used. It could be expressed as

$$\underset{N}{\text{minimize}} \quad \sum_{t_d=0}^{t_{dn}} P(N(t_d), H(t_d))\Delta t$$
$$\text{subject to} \quad h_1(Q(N, t_d), H(t_d)) = 0,$$
$$f_i(Q(N, t_d), H(t_d)) \leq 0, \ i = 1, \dots, m, \tag{2}$$

where N is the pump speed, Q is the pump discharge, P is the power, H is the static head, Δt is the time step of the discretization, and t_{dn} is the horizon of the optimization. The first constraint expresses the QH characteristic of the pump, which depends on the pump speed N following affinity theory. The inequality constraints can be divided into two groups. The first group of constraints ensure that the pump operates inside the working area. The second group describes the hydraulic system where the pump is used, for example, the volume to be pumped should be within a certain range for the water levels to remain within their bounds. Based on the example above, the function of the power should be convex, and the QH characteristics should be affine. However, none of these relationships is even convex. Therefore, we reformulate equation (2) to become a convex optimization problem: For a pump that is always turned on, the final convex optimization problem is now formulated as

$$\underset{Q}{\text{minimize}} \quad \sum_{t_d=0}^{t_{dn}} P_{\text{app}}(Q(t_d), H(t_d))\Delta t$$
$$\text{subject to} \quad f_i(Q(t_d), H(t_d)) \leq 0, \ i = 1, \dots, n. \tag{3}$$

There are three main characteristics of the convex optimization (equation (3)) that are not present in the nonconvex one (equation (2)): (1) The optimization variable is the discharge and not the pump speed; (2) the equality constraint and the inequality constraints are reformulated to convex inequality constraints; (3)
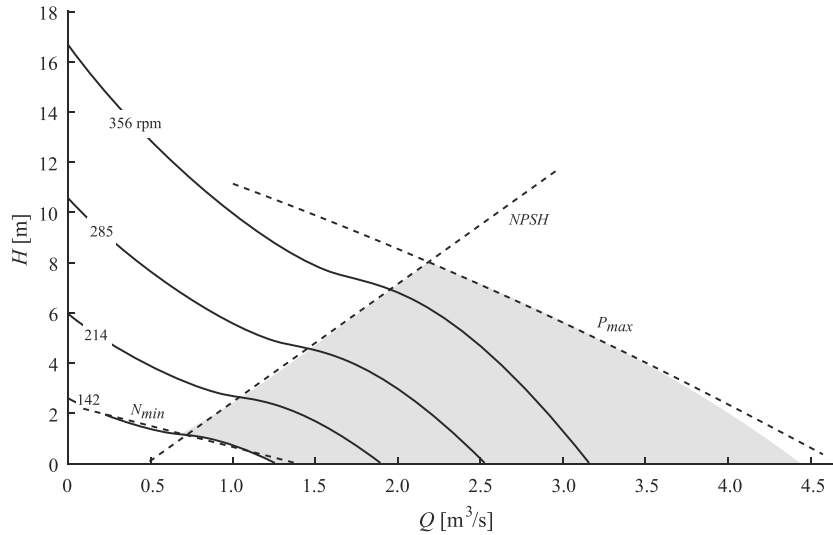
Figure 3. The working area of the pump. The original area is shaded with gray, and the convex approximation of the edges is shown with dashed lines. NPSH = net positive suction head.

the approximated power (P_{app}) is used instead of the power. In what follows, these changes are explained one by one.

The first change, using discharge instead of pump speed as optimization variable, helps to express to build a convex continuous objective function. The shaft speed is no longer required during the optimization, therefore its calculation is postponed until the post-processing. It is calculated by using the unique relationship $N(Q, H)$ stated by the affinity rules:

$$\frac{Q_b}{Q_a} = \frac{N_b}{N_a}, \tag{4}$$

$$\frac{H_b}{H_a} = \left(\frac{N_b}{N_a}\right)^2, \tag{5}$$

where N_a and N_b are different shaft speeds, and Q and H are the corresponding discharge and head. A consequence of these affinity rules is that each QH point in the working area is related to pump speeds via a parabolic relation. An example for such calculation is given in Appendix A.

The second change is to reformulate all constraints as convex inequality constraints. The equality constraint is removed. This step is possible, because of the first change, since the discharge is the optimization variable and the shaft speed is calculated as post-processing, the QH characteristic is no longer required as equality constraint. The next step is to ensure that all inequality constraints are convex. In case of the constraints related to operating points, it means to ensure that the bounding curves enclose a convex working area. This is implemented in the following way: The bounding curves are approximated by a second-order function. If the area is not convex because of a curve, then this curve is approximated with a linear function. The final curves are used as inequality constraints. An example is shown in Figure 3: The dashed lines are approximating the boundaries of the working area: the maximum power curve, the NPSH curve, and the minimum shaft speed curve. The maximum power curve is approximated with a second-order function, which is convex in Q. The NPSH and the minimum shaft speed curve should be concave in Q, and thus, they are approximated with a linear function. The convex working area (dashed lines) is only slightly different than the original one (gray area). That means that the optimization has some different choice of Q-H points at the boundaries, than it would have using a nonconvex working area.

The third change is the convex approximation of the power. The power is a function of two variables: the discharge and the head. It can be approximated as a second-order function, which makes verifying its convexity straightforward.

$$P_{app} = P_{c11} + P_{c12}Q + P_{c13}Q^2 + P_{c21}H + P_{c23}H^2 + P_{c22}QH. \tag{6}$$

This function is convex if the Hessian is positive semidefinite.

The approximation of the power is calculated as follows: First, the power is determined in different Q-H points in the working area. Then the coefficients P_{cij} are obtained through the following optimization:

$$
\begin{aligned}
\underset{P_c}{\text{minimize}} \quad & \sum_{Q,H} (P_{c11} + P_{c12}Q + P_{c13}Q^2 + P_{c21}H + P_{c23}H^2 + P_{c22}QH - P)^2 \\
\text{subject to} \quad & -P_{c13} - P_{c23} \leq 0, \\
& P_{c22}^2 - 4P_{c13}P_{c23} \leq 0.
\end{aligned}
\tag{7}
$$

The convexity of the power approximation is guaranteed by the constraints that are equivalent to the Hessian being positive semidefinite. The optimization problem in equation (7) itself is not necessarily convex, while the resulting power approximation is convex. This means that the solution might be a local optimum and the power approximation might not be the best possible one. Therefore, the error of the resulting power approximation is calculated and assessed. Not only the difference between the original and the approximated values of the power is calculated but also the difference between the original values and approximated values of the gradient of the power with respect to the discharge and the head; which plays an important role in the optimization. Another solution for this nonconvex optimization problem could be to use homotopy methods to find the global optimum (Baayen et al., 2018). The error between the power and the approximated power did not exceed 10% for the case considered in this study.

2.3. Mixed-Integer Optimization: Including the Possibility to Turn Off the Pump

In this section, the optimization problem is extended to the case when the pump is turned off. This can be described as mixed-integer optimization, where at least one of the variables can only take integer values. A possible solution of such a problem involves relaxation to a continuous problem. In other words, the integers are allowed to take noninteger values. In order to reach global optimum, these relaxed problems should be convex. In order to find a global optimum of a mixed-integer problem, several relaxed problems should be solved, which can lead to high computation times. Note that the convexity of the continuous relaxations of the mixed-integer problem will be guaranteed by making sure that the integer variables enter into the constraints linearly, as explained below.

In order to describe the pump optimization as a true mixed-integer problem, we introduce δ, a boolean variable indicating if the pump is on ($\delta = 1$) or off ($\delta = 0$). Suppose the power is approximated with a convex polynomial $P_{\text{app}}(Q, H)$ which is not necessarily zero at zero discharge. Therefore, it can occur when the discharge is zero, and there is a head the consumed power is estimated to be positive. If the pump is allowed to be turned off, P_{app} should be multiplied by δ, so that it is zero when the pump is off. However, the product of these two variables does not necessary lead to a convex-relaxed problem. The optimization problem can be written as

$$
\begin{aligned}
\underset{Q}{\text{minimize}} \quad & \sum_{t_d=0}^{t_{dn}} \delta P_{\text{app}}(Q(t_d), H(t_d))\Delta t \\
\text{subject to} \quad & f_i(Q,H)\delta \leq 0, \ i = 1, \ldots, m \\
& \delta = sgn(Q) \\
& \delta \in \{0,1\} \text{ and } \delta \in \mathbb{Z}, \\
& f(Q(t_d)) \leq 0,
\end{aligned}
\tag{8}
$$

where δ indicates the status of the pump. In order to transform equation (8) to a convex optimization problem, the objective function and the constraints should be replaced by convex ones.

First, the transformation of the working area constraints is described ($f_i(Q, H) \leq 0, i = 1, \ldots, m$) and then the objective function ($P_{\text{app}}(Q, H, t_d)$). The working area constraints define a boundary for the working area of the pump when it is on: The working area is shaded with purple and its boundaries are shown with blue lines in Figure 4. If the pump is off, the discharge is zero, but the head is not, therefore the Q-H points will be on the y axis. However, this is outside the working area. In order to allow the pump to be off, the working area should be extended to contain the physically possible portion of the y axis when the pump is off. This is achieved by shifting the left boundaries of the working area. The constraints should be shifted such that they include the y axis with the minimum and maximum possible heads. These head values can be obtained from
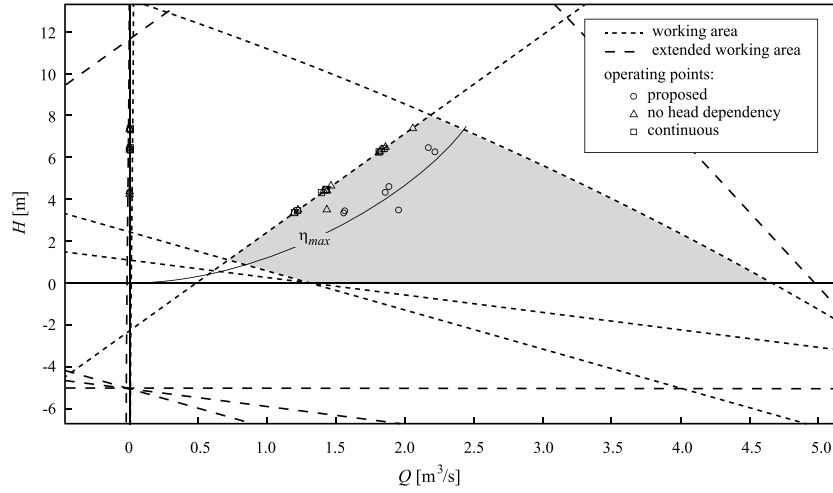
Figure 4. Definition of the extended working area (dashed lines). The extended working area is shown in case the pump is turned off. The markers show the actual operating points of the pump during the presented scenarios.

the physical characteristics from the system, they are denoted as f_{offset} (see the yellow circles on Figure 4). The offset is calculated in the positive head direction as

$$f_{\text{offsetmax, i}} = f_i(Q, H_{\text{max}}), \tag{9}$$

and

$$f_{\text{offsetmin, i}} = f_i(Q, H_{\text{min}}). \tag{10}$$

Then $f_{\text{offset, i}}$ can be calculated as follows:

$$f_{\text{offset, i}} = \max(|f_{\text{offsetmin, i}}|, |f_{\text{offsetmin, i}}|). \tag{11}$$

Hence, the working area constraints from equation (8) can be written as

$$f_i(Q, H) - (1 - \delta)f_{\text{offset, i}} \leq 0, \ i = 1, \ldots, m. \tag{12}$$

This equation is convex. If the pump is on, it is equivalent to the original convex constraint (equation (3)). If the pump is off ($\delta = 0$), the equation simply states that the head is less than the maximum possible head. This formulation makes it possible to extend the working area to the situation when the discharge is zero and the pump is off.

Now the transformation of the objective function from nonconvex to convex is described. Currently, it is a convex function multiplied with a boolean, which is not necessarily convex. As a first step, we move the approximated power to the constraints by replacing P_{app} the objective with a helper variable P_{help}. The sum of this single variable over time is going to be minimized, and a new constraint is added stating $P_{\text{app}}(Q, H, t_d) \leq P_{\text{help}}$. As P_{help} is minimized, its possible smallest value will be equal to P_{app}. The objective function in equation (8) is multiplied with a boolean. This step was necessary to express that the power is zero when the pump is off, while the power approximation function is not zero in that case. In order to create a convex constraint, the function P_{app} should not be multiplied with a boolean, but we still need to get zero P_{help} when Q is zero. The following inequalities help to describe this. Let us denote the minimum and maximum of the approximated power by m and M, respectively. These values can be obtained from the characteristics of the pump. The power constraint from equation (8) is replaced by the following five equations:

$$0 \leq Q, \tag{13}$$

$$Q - \delta Q_{\text{max}} \leq 0, \tag{14}$$

$$m\delta - P_{\text{help}} \leq 0, \tag{15}$$

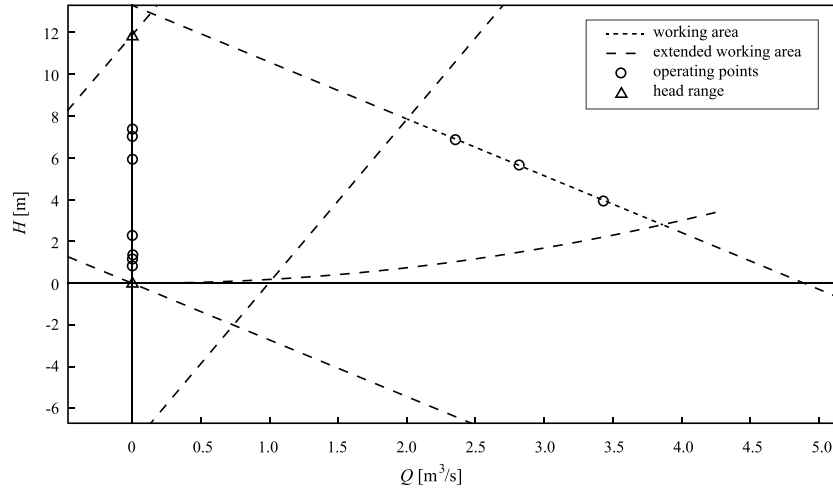$$P_{\text{help}} - M\delta \leq 0, \tag{16}$$

Figure 5. Definition of the extended working area of a constant speed pump. The extended working area is shown in case the pump is turned off. The red crosses show the actual working point of the pump during the presented scenario.

and

$$P_{\text{app}}(Q, H) - M(1 - \delta) \leq P_{\text{help}}. \tag{17}$$

According to these constraints, if δ is zero, pump is off, $0 \leq P_{\text{help}} \leq 0$, so $P_{\text{help}} = 0$. If δ is 1, pump is on, $m \leq P_{\text{help}} \leq M$ and $P_{\text{app}} \leq P_{\text{help}}$ is equivalent to the original constraint.

The final optimization problem for a variable-speed pump, valid for all conditions, is

$$
\begin{aligned}
\underset{Q}{\text{minimize}} \quad & \sum_{t_d=0}^{t_{dn}} P_{\text{help}}(t_d)\Delta t \\
\text{subject to} \quad & f_i(Q, H) - (1 - \delta)H_{\text{offset},i} \leq 0, \ i = 1, \dots, m, \\
& P_{\text{app}}(Q, H) - M(1 - \delta) \leq P_{\text{help}}(t_d), \\
& m\delta - P_{\text{help}}(t_d) \leq 0, \quad P_{\text{help}}(t_d) - M\delta \leq 0, \\
& Q - \delta Q_{\text{max}} \leq 0, \quad 0 \leq Q, \\
& \delta \in [0, 1] \text{ and } \delta \in \mathbb{Z}, \quad f(Q(t_d)) \leq 0.
\end{aligned}
\tag{18}
$$

Finally, we discuss the errors between the continuous and the mixed-integer approximation. The mixed-integer approach does not introduce new errors; however, two aspects should be pointed out. Several helper variables are needed (like m, M, and Q_{max}) that should be deduced from the physical characteristics of the system. By underestimating them, errors are introduced to the solution. In case of overestimation, the computation time and even the accuracy of the solution is affected. The second aspect is the difference in computation time: A mixed-integer optimization is several orders of magnitude slower than the continuous one. Having a mixed-integer problem with convex-relaxed problems allows RTC-Tools 2 to find the global optimum. RTC-Tools is using standard mixed-integer solvers such as CPLEX that, given enough time, probably reach the global optimum of mixed-integer problems with convex relaxations (Floudas, 1995).

2.4. Application of the Method to Constant-Speed Pumps

The above-described methodology can be applied to constant-speed pumps. The working area of a constant speed pump is a special case: The curves defining the minimum and maximum shaft-speed coincide, and the area is reduced to a line. An example of the working area of a constant-speed pump is shown in Figure 5. The curves bounding the working area define the beginning and end point of the segment, with other words the minimum and maximum possible discharges. The pump can operate either on this segment or, in case it is off, the vertical axes. The markers show examples of operation of a constant speed pump. As the minimum and maximum shaft speed lines coincide, the equation serves as minimum and maximum bounds, thus this constraint has to be affine.
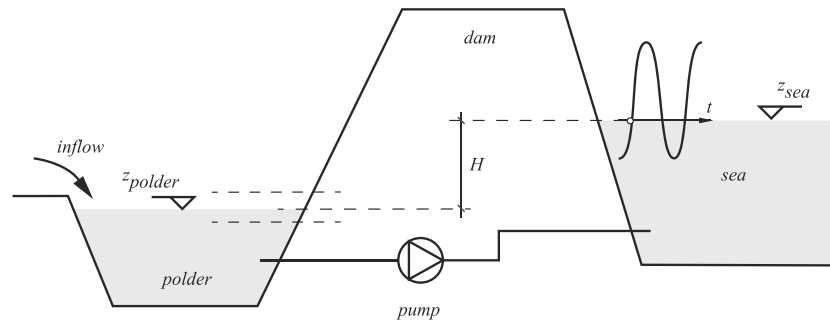
Figure 6. Schematics of the case study.

3. Results and Discussion

The proposed method will eventually be implemented in a model predictive control formulation with a receding horizon, as inflow boundary conditions are only known (to a certain degree of accuracy) in a time period up till the horizon. After each time step, the horizon shifts, and the optimization is performed again. In this paper we demonstrate one single optimization, as an indication of performance. The next step would be to test it with receding horizon formulation. The numerical example is shown and compared to two existing methods to demonstrate the advantages of the presented approach. First, the water system and the test scenario are introduced.

This water system consists of a lower-lying area (polder) and an area with higher level (sea). The goal is to keep the water level of the polder within some predefined bounds by pumping while water is flowing into the polder. A schematic view of the system is shown in Figure 6. The polder water level is at the midpoint between the lower and upper bounds the beginning of the period and water is flowing into the polder, while the outside (sea) water level also changes due to the tidal motion. A terminal constraint forces the water level to reach the midpoint between the lower and upper bounds at the end of the time period. In order to reach the goal, the excess water should be pumped out by using as little energy as possible.
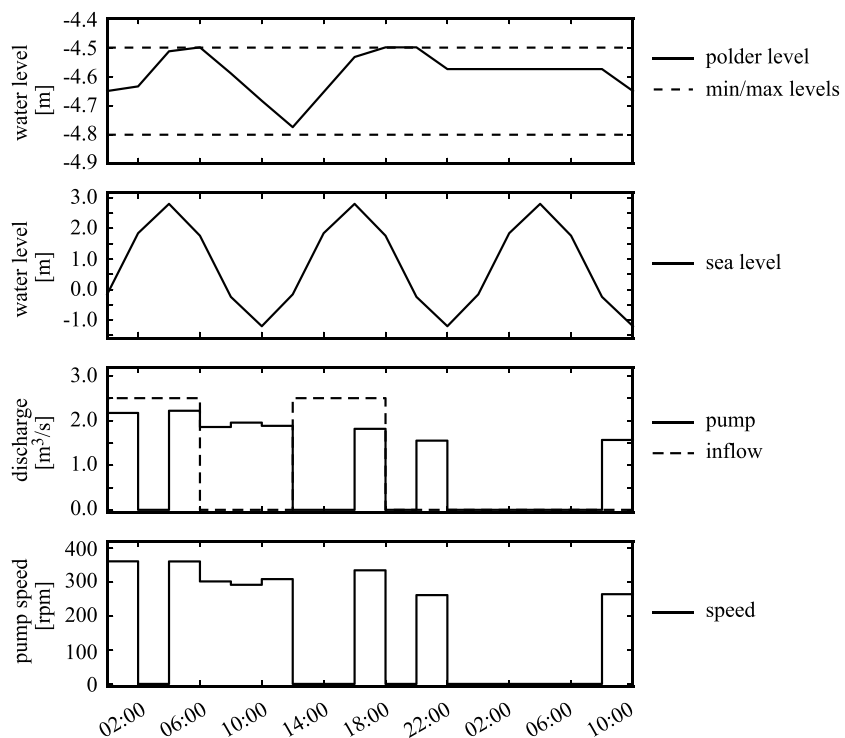

Figure 7. Results with the proposed optimization system.

Table 1
Energy Consumption and Computation Time of the Different Methods

	Proposed method	No head dependency	Continuous
Energy (kWh)	1,120	1,274	1,235
Time (s)	660	230	60

Without pumping, the water level would exceed the maximum polder level. It is favorable to wait for the polder water level to rise and/or the sea water level to drop (low tide) before starting pumping, as a lower head will lead to a lower energy consumption. However, letting the water level increase excessively in the polder might also risk exceeding the maximum bound. In what follows, the pump operation is calculated by optimization using three different methods: the method proposed in this paper, a method using head-independent power (Menke et al., 2016), and a method using continuous optimization (Ulanicki & Kennedy, 1994). The time horizon is 34 hr in all cases. The time step is 2 hr which is long enough for a pump to slowly change its shaft speed between consecutive time steps.

The mixed-integer optimization problem in the proposed method is solved with RTC-Tools 2, an open-source toolbox for control and optimization of environmental systems (Deltares, 2018a, 2018b; Gijsbers et al., 2018). The results calculated by the proposed method are shown in Figure 7. It can be seen that the requirement is fulfilled: The polder water level stayed within the prescribed bounds, while the energy used by the pump is 1,120 kWh (Table 1). The pump started to work in the beginning of the period but stopped after 2 hr when the tide and therefore the head across the pump was the highest. Then it turned on again and pumped almost until the water level was reaching the lower edge of the bound, thus taking advantage of the entire low tide period. The final water level reaches the midpoint between the lower and upper bounds as a result of the terminal constraint. The operating points of the pump are shown in Figure 4 with diamonds. It can be seen that the pump operates close to the maximum efficiency when it is possible.

The scenario was also calculated with a mixed-integer optimization method similar to the proposed one, but in this case it is assumed that the power does not depend on the head, it does only depend on the discharge, and it is approximated as a second-order function of the discharge. Figure 8 shows that in this case the
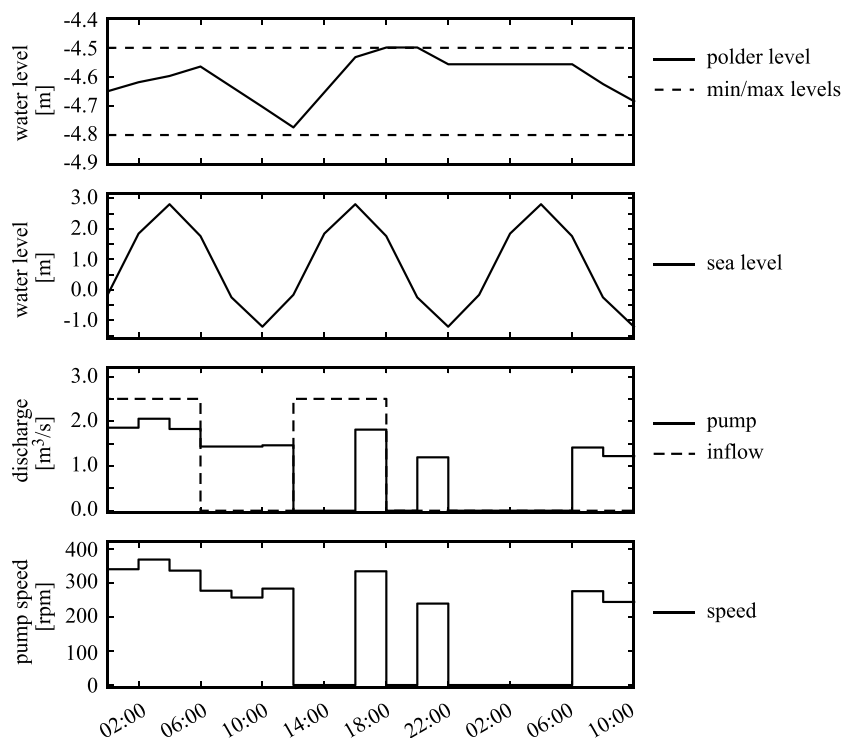

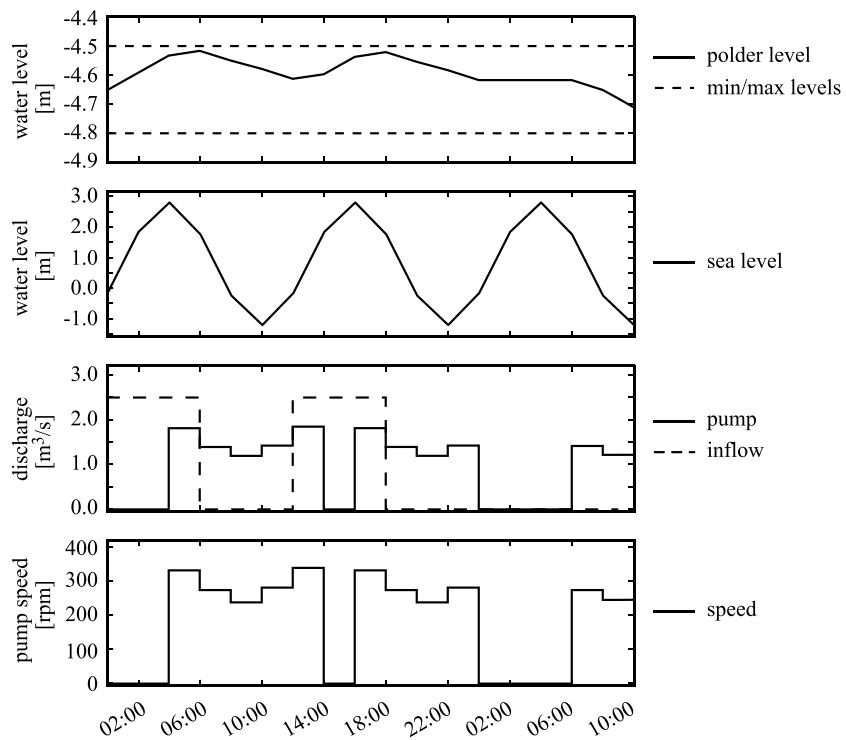**Figure 8.** Results without head dependency.

Figure 9. Results with continuous optimization.

pump started to work in the beginning of the period, and it kept on pumping even during the highest tide. It stopped when the water level was close to the lower bound, and it restarted when the polder was full. As in this formulation the power does not depend on the head; the pump rather pumps at low discharges instead of pumping at low head. This leads to less efficient pumping: It can be seen that the duty points of this pump (triangles in Figure 4) are further from the best efficiency line and predominantly at minimum discharge. The algorithm was able to reach the goal: The polder water level stayed within the prescribed bounds. However, the energy consumption was 1,274 kWh, which is about 14% higher than that of the proposed method (Table 1).

Several practitioners suggest to use continuous optimization and then round the results off to integers to calculate when the pump is on or off as this approach results in faster computation. Therefore, we also tested the same scenario with a continuous algorithm. Note that the rounding off the solution took place step by step: The optimization was run once, then one point was rounded off, then it was run again. It was run several times, and the rounding method involves some heuristics, therefore it is not clear in all cases and cumbersome. In this case the pump did not turn on in the beginning of the period, only after 4 hr (Figure 9). Then it pumped until the second high tide came. The water level remained in the bound during the period, and in the end, it reaches the middle of the bound. The pump used 1,235-kWh energy, that is about 10% more than the proposed method. Note that the performance of this method is not optimal, and it depends on case by case and also on the way the user implements the rounding off.

Comparing the energy consumption of the three methods (Table 1), it is clear that the proposed method outperformed the existing methods for this example test scenario: The energy consumption is lower compared to the other two methods. All methods reached the desired goal of keeping the water level within the given bounds. It should be noted that the calculation time of the continuous method was almost an order of magnitude lower than in case of the other two mixed-integer methods. This faster performance of the continuous method was expected from the nature of the method; however, for this example, all times were acceptable for real-time implementation. It should be considered that by increasing the size of the problem, that is, using more pumps, the proposed method might lead to prohibitive computation times. As this example demonstrated, for problems consisting of small number of pumps, this is a promising solution.

The method can be used for different kinds of pumps and for different applications. It can be applied for water distribution networks with storage capacity, including water tanks or water towers. In that case the water levels would be replaced by pressure heads in the problem formulation. It can be applied for pumped storage where the driving parameter is energy price. The objective function can easily be extended with energy price.

4. Conclusion

In this paper a convex modeling to minimize energy use in pumps is introduced. The modeling method has several advantages compared to existing heuristic methods: It finds the global optimum, it includes pump switches, and it is applicable to variable-speed pumping. Due to these advantages, more energy can be saved by using this method when performing optimization. The methodology and its advantages were demonstrated on a numerical example of a drainage pump. In this example the energy use can be 10% to 13% lower compared to other optimization methods. Further implications of the method are that not only energy use can be optimized but also energy costs or sustainable energy use. The method can be used in several applications of pumps such as polder systems, ground-water pumping systems, pumped-storage systems, and water transmission or distribution networks with water towers or storage tanks. It is also possible to extend the approach to model different pump types.

Appendix A: Calculation of Pump Speed Using Affinity Rules

Here an example is given for calculating the speed of a pump in order to run at a specific working point (Q,H) using affinity rules. The pump has a QH curve as given in Table A1 for a reference speed of 356 rpm.

This performance curve is shown in Figure A1. It is the same pump as presented in the article, however with a lower number of points for simplicity. Suppose the result of the optimization is that the pump should operate at working point $(Q_{wp}, H_{wp}) = (2.39 \text{ m}^3/\text{s}, 4 \text{ m})$. The pump speed can be found as post-processing using the affinity rules as follows. First, the working point is scaled by using the following relation resulting from combining equations (4) and (5):

$$\frac{H_b}{H_a} = \left(\frac{Q_b}{Q_a}\right)^2. \tag{A1}$$

The result of the scaling is shown in Figure A1 as the dashed curve. From the graph, it shows that the corresponding working point at reference speed is at a head of 4.51 m and a discharge of 2.53 m^3/s. Now using equation (5) with the following input: $H_a = 4.51$ m, $H_b = 4$ m, and $N_a = 356$ rpm, the new shaft speed N_b is calculated as 335 rpm. The scaled pump performance curve at 335 rpm can be calculated using equations (4) and (5) and is presented in Table A2 and also shown in Figure A1.

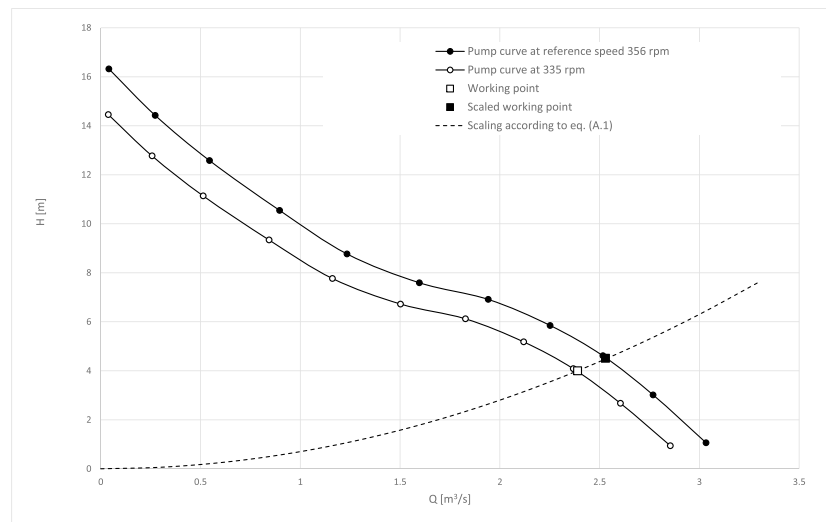| **Table A1**	
Pump Performance Curve at Reference Speed of 356 rpm	
Q	H
0.041	16.322
0.273	14.42
0.546	12.577
0.897	10.544
1.234	8.769
1.597	7.592
1.943	6.911
2.253	5.848
2.517	4.612
2.768	3.014
3.034	1.062

Figure A1. Scaling of reference pump curve for operation at the working point.

Table A2
Scaled Pump Performance Curve at 335 rpm

Q	H
0.038	14.453
0.257	12.769
0.513	11.137
0.844	9.337
1.161	7.765
1.503	6.723
1.828	6.12
2.12	5.179
2.368	4.084
2.605	2.669
2.855	0.94

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