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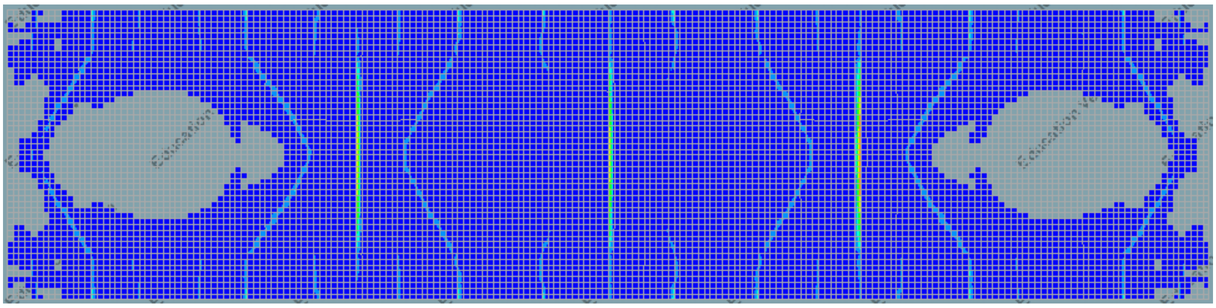

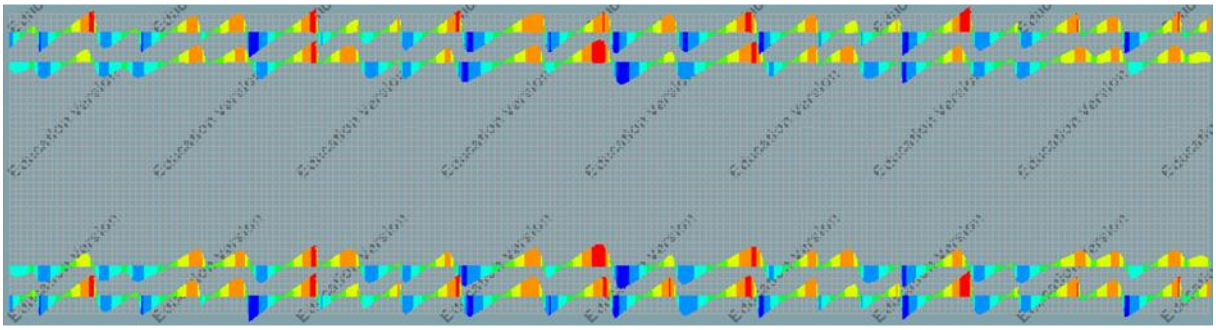

Effective concrete tension zone

COMPARISON BETWEEN FINITE ELEMENT ANALYSIS USING DIANA AND EUROCODE 2 AND JONES METHOD

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VolkerInfra

TU Delft

Effective concrete tension zone

A thesis on the comparison of the effective concrete tension zone between finite element analyses using DIANA and Eurocode 2 and Jones method.

By

Richard Verschuur

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This thesis is confidential and cannot be made public until the date of presentation

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PREFACE

In order to complete the master Civil Engineering in the direction of structural engineering with a specialization in Concrete structures at Delft University of Technology, one is obliged to write a master thesis. Within this report the master thesis is elaborated. This thesis is about the comparison of the effective concrete tension zone height between finite element analyses performed with DIANA and the method described by Eurocode 2 and Jones method. The topic of this research has been provided by and performed at VolkerInfra.

I would like to thank my committee for the support, expertise and guidance throughout the completion of the master thesis which led me to this end result. Furthermore I would like to thank my colleagues at VolkerInfra for their support, expertise, feedback and facilities which they let me use during the completion of the master thesis.

Yours sincerely,

Richard Verschuur

ABSTRACT

When engineers at VolkerInfra have to determine the crack width of massive concrete structures with multiple layers of reinforcement and edge restraint the question arises, on how to determine the effective concrete tension zone. In Eurocode 2 the determination of the effective concrete tension zone height for multiple layers of reinforcement is not straightforward. This raised the question of how to determine the effective concrete tension zone height for multiple layers of reinforcement and if the effective concrete tension zone height is comparable with results from finite element analyses. Furthermore, what will be the influence of Jones method on the determination of the effective concrete tension zone height? With these questions the subject of the master thesis was obtained.

Are the effective concrete tension zone heights determined with either Eurocode 2 or Jones Method comparable with the effective concrete tension zone heights determined with finite element models using DIANA?

In this thesis the effective concrete tension zone height from Eurocode 2 and Jones method is compared to finite element models. First the influence of the different parameters on the effective concrete tension zone height has been investigated by using parameters which are common in practice. Where after a comparison is performed between Eurocode 2 and Jones method to see which method results in a smaller effective concrete tension zone height for models loaded by tension. In the situations investigated with one layer of reinforcement, the effective concrete tension zone height of Jones method is equal to Eurocode 2. This is due to the fact that the formulas for the determination of the effective concrete tension zone height are identical. For the situations investigated with two layers of reinforcement, Jones method results in a smaller effective concrete tension zone height of 24%. For situations with three layers of reinforcement, Jones method results in smaller effective concrete tension zone height but the differences between both methods varies from 6% to 50%. For situations with vertical bar spacings varying from 125mm till 200mm the difference between the methods is in the upper region.

To make a comparison between Eurocode 2, Jones method and finite element models a finite element program needs to be used. The program which is used is called DIANA. First the possibilities and functions of DIANA have been investigated after the study of the effects of the parameters on the determination of the effective concrete tension zone height. With the specific material models, 3D tensile member models have been investigated to check if the parameters, with which the effective concrete tension zone can be calculated, are correct. These parameters are the bond stress and the transfer length which is depending on the crack spacing. It turned out that the parameters can be derived from DIANA to calculate the effective concrete tension zone.

After the tests performed using 3D models, 2D models have been used for the variation study. The use of 2D models is permissible because the crack distribution and bond stress gradient, which are key parameters for this research, do not change between 2D and 3D models. The variation study has been performed on models loaded by tension in which the cover, reinforcement diameter, width and height of the model varies. Two layers of reinforcement have been applied in the models in which the height varies, to make it possible to make a comparison between Eurocode 2 and Jones method.

In the variation studies of the cover, reinforcement diameter and width the effective concrete tension zone height of Eurocode 2 is consecutively 16%, 29% and 33% lower than the finite element models. For the models which vary in height different comparisons are made between Eurocode 2 and finite element models, Jones method and finite element models and Eurocode 2 and Jones method. Eurocode 2 is on average 14% higher, according to the first comparison. The finite element models output is on average 10 % higher, according to the second comparison. Eurocode 2 is on average 28% higher, according to the third comparison.

The conclusion of the variation study on the models loaded by tension is that the effective concrete tension zone heights determined with finite element models are in most cases larger than those determined with Eurocode 2 or Jones method. Therefore Eurocode 2 underestimates the effective concrete tension zone which does have an effect on the crack width calculation. Furthermore Jones

method results in smaller effective concrete tension zone heights than Eurocode 2 if multiple layers of reinforcement have been applied.

Three 2D bending models have been investigated following the models loaded by tension. The conclusion for the bending models is that the effective concrete tension zone height of the finite element models is almost equal to those determined by Eurocode 2 and Jones method for one layer of reinforcement. For two layers of reinforcement the result of the finite element model is almost equal to the result given by Jones method. The effective concrete tension zone height of Eurocode 2 is 32% higher than Jones method. These conclusions are based on only three models therefore the conclusion is a provisional conclusion. More bending models need to be investigated to make a sound conclusion.

The overall conclusion of the thesis is that the effective concrete tension zone can be calculated from finite element models, but for models loaded by tension the effective concrete tension zone height is not comparable to those determined with Eurocode 2. For multiple layers of reinforcement the effective concrete tension zone height is comparable between Jones method and finite element models. For bending models the effective concrete tension zone height is in the same order for finite element models, Eurocode 2 and Jones method if one layer of reinforcement is applied. If multiple layers of reinforcement were to be used, the effective concrete tension zone height is equal for finite element models and Jones method while Eurocode 2 gives larger heights. However this last conclusion is only based on three models which is why the last conclusion is a provisional conclusion. More research needs to be undertaken to give a sound conclusion about the bending models.

TABLE OF CONTENTS

LIST OF SYMBOLS

$h_{c,eff}$	effective concrete tension zone height
h	height of the structure
x	height of the concrete compression zone
d	effective height
c	cover of the longitudinal reinforcement
$\emptyset_{sl}$	diameter of the longitudinal reinforcement
$\emptyset_{w}$	diameter of the stirrup
$\emptyset_{slf}$	diameter of the fictitious longitudinal reinforcement
ssv	vertical bar spacing
A_s	cross sectional area of the reinforcement
A_c	cross sectional area of the concrete
$A_{c,eff}$	cross sectional area of the effective concrete tension zone
$\rho_{p,eff}$	effective reinforcement ratio
ϵ	total strain
ϵ_e	elastic strain
ϵ_{cr}	crack strain
G_f	fracture energy
h	crack bandwidth
A	cross sectional area
V	volume
f_{cm}	mean compressive concrete strength
f_{ck}	characteristic concrete cube strength
f_{ctm}	concrete tensile strength
f_b	bond stress
E_s	young's modulus of the reinforcement
E_c	young's modulus of the concrete
E_{bs}	shear stiffness of the bond interface around the reinforcement
E_{bn}	normal stiffness of the bond interface around the reinforcement

w_k	crack width
$h_{c,efftot}$	total effective concrete tension zone height
$h_{c,effEC}$	effective concrete tension zone height according to Eurocode 2
$h_{c,effJM}$	effective concrete tension zone height according to Jones method
N_{cr}	cracking force
$N_{cr,ave}$	average cracking force
B	width of the structure
H	height of the structure
L	length of the structure
Δu_t	slip or relative displacement
t_t	traction or bond stress
$\tau_{b\max}$	maximum bond stress according to fib Model Code 2010
τ_{bf}	ultimate bond stress
σ_c	concrete stress
E_0	initial young's modulus of the concrete
f_y	yield strength of the reinforcement
ν	poisson's ratio
L_{st}	transfer length
U_s	perimeter of the reinforcement
$\tau_{b,ave}$	average bond stress
σ_s	steel stress
$\Delta\sigma_s$	increase in steel stress due to cracking of the concrete
$\Delta\sigma_{sl}$	increase in steel stress due to cracking of the concrete on the left side of the crack
$\Delta\sigma_{sr}$	increase in steel stress due to cracking of the concrete on the right side of the crack
F_{sl}	force due to the increase in steel stress on the left side of the crack
F_{sr}	force due to the increase in steel stress on the right side of the crack
F_{bl}	force due to the bond stress on the left side of the crack
F_{br}	force due to the bond stress on the right side of the crack

n number of cracks

s horizontal bar spacing

1 INTRODUCTION

Concrete is a building material which is often used for structures due to its freedom of shape, availability, price and ease of use. Concrete is an inhomogeneous building material which means that it has different properties in different directions. Some of these properties varying are the compressive and tensile strength and the young's modulus. The concrete compression strength is on average ten times larger than the concrete tensile strength. Therefore concrete should not be loaded in tension however this is often unavoidable in structures. The stress in the concrete builds up until the tensile strength of the concrete is exceeded. This results in a crack in the concrete. Concrete can crack in different patterns due to the type of loading and geometry of the structure. Cracking of concrete is a brittle failure mechanism, this means failure without a warning. To add redundancy in the structure, steel reinforcement is added. When the concrete cracks, the stresses are transferred to the reinforcement via bond stress.

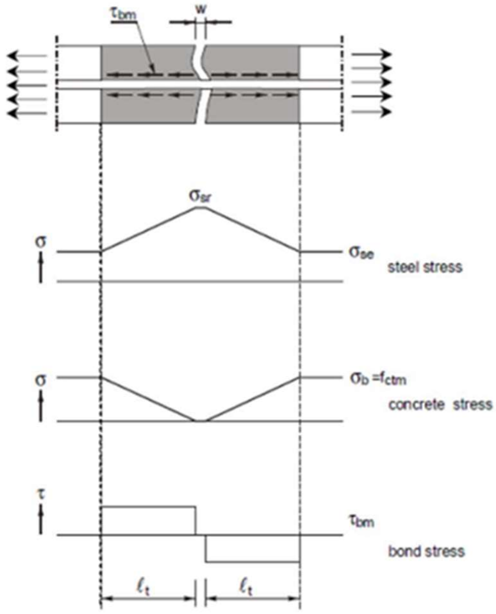


Figure 1 Cracking behavior of the concrete

For large structures the reinforcement is concentrated at the edges of the structure to withstand cracking at the edges. The area in which the reinforcement is concentrated is called a hidden tensile member, here the tension forces are transferred to the reinforcement. Therefore this part of the structure is usually under tension. The cracks which form in structures with a hidden tensile member are primary cracks which run through the whole structure and secondary cracks which are small and concentrated in the tensile member, some of these run towards the primary cracks.

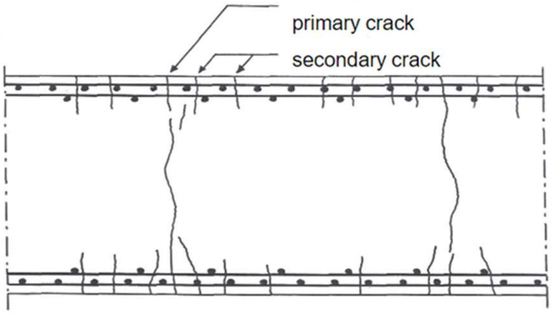


Figure 2 Construction with hidden tensile member with primary and secondary cracks

If cracks in the concrete become too large the reinforcement is no longer passivated by the concrete surrounding it which can affect the durability of the concrete structure. The reinforcement will corrode when the passivation is lost. When reinforcement corrodes its strength decreases which can result in failure of the structure. Therefore cracks need to be within certain limits to prevent corrosion of the reinforcement. The limits for the crack width are given in Eurocode 2 and depend on the environment of the structure. An aggressive environment will result in a smaller crack width tolerance. By adding reinforcement to the structure the crack widths can be controlled. Another way of preventing cracks is by using different types of concrete or to use cooling pipes. These topics will not be examined in this thesis.

The crack width can be determined by the multiplication of the crack spacing and the difference in strain between the steel reinforcement and the concrete. The maximum crack spacing according to Eurocode 2 is determined by different parameters like cover, reinforcement diameter, effective reinforcement ratio and statistical Eurocode 2 parameters depending on bond and the type of loading.

The difference in strain is called the local tension stiffening effect and is dependent on the stress in the reinforcement, young's modulus of the reinforcement and concrete, concrete tensile strength and effective reinforcement ratio. The effective reinforcement ratio has an influence in both formulas. An increase in effective reinforcement ratio will result in a decrease in crack spacing but an increase in strain difference. So the effective reinforcement ratio has a positive and negative effect on the crack width calculation but which formula will be decisive? To investigate this the effective reinforcement ratio needs to be determined. The effective reinforcement ratio is depending on the cross sectional area of the reinforcement and on the effective concrete tension zone. The effective concrete tension zone is an average fictitious area in which the concrete tensile strength is exceeded. Thus this whole area will fictitiously crack. This fictitious area is determined make it possible to calculate crack width with. The effective concrete tension zone is dependent on the effective concrete tension zone height and the width.

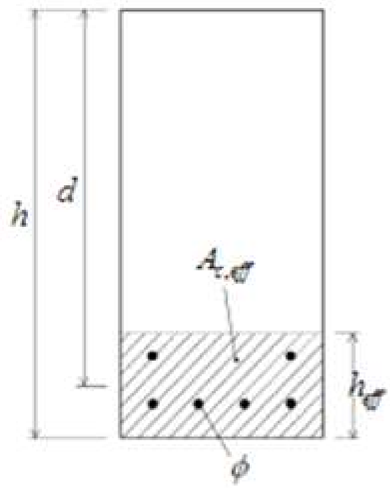


Figure 3 Effective concrete tension zone

Different interpretations are used to determine the effective concrete tension zone height. These interpretations are written in different papers and regulations. In this thesis the effective concrete tension zone is determined by using Eurocode 2. Eurocode 2 is the legally required standard in European countries and calculates the effective concrete tension zone for all reinforcement in the cross section. The effective concrete tension zone height is determined by the minimum of four different formulas, in which two are almost identical. These formulas depend on different parameters like cover, height, reinforcement diameter, compression zone height and effective height. For circular cross sections, irregular reinforcement lay-outs and columns loaded by double bending the approach given by Eurocode 2 is difficult to implement because it is written for rectangular cross sections with linear stress distributions. A new method has been written by Jones to cope with these cross sections and load configurations. With this method, called "Jones method" in this thesis, the effective concrete tension zone is determined around a single reinforcement bar. The bar which is considered is the bar at which cracking is most certain to occur due to the stress distribution in the cross section. This method will be used to make a comparison between the effective concrete tension zone height of Eurocode 2 and Jones method.

Maximum crack width [$W_{k,max}$]

=

Maximum crack spacing $*$ Local tension stiffening

$$S_{r,\max} = k_3 * c + \frac{k_1 * k_2 * k_4 * \emptyset_{sl}}{\boxed{\rho_{p,eff}}} \quad * \quad \epsilon_{sm} - \epsilon_{cm} = \frac{\sigma_s - k_t * \left(\frac{f_{ct,eff}}{\boxed{\rho_{p,eff}}}\right) * (1 + \alpha_e * \boxed{\rho_{p,eff}})}{E_s}$$

Effective reinforcement ratio

$$\rho_{p,eff} = \frac{A_s}{A_{c,eff}}$$

Effective concrete tension zone

$$A_{c,eff} = b * h_{c,eff}$$

Effective concrete tension zone height

$h_{c,eff}$

Eurocode 2:

$$h_{c,effEC2} = MIN \begin{cases} \le \dfrac{h-x}{3} & (Bending) \quad 1 \\ 2{,}5 * (h-d) & 2 \\ \le \dfrac{h}{2} & (Axial\ tension) \quad 3 \\ 2{,}5 * \left(c + \dfrac{\phi_{sl}}{2}\right) & 4 \end{cases}$$

Jones method:

$$h_{c,effMJ} = c + \frac{\phi_{sl}}{2} + \min \begin{cases} \dfrac{ssv}{2} \\ 1{,}5 * \left(c + \dfrac{\phi_{sl}}{2}\right) \end{cases}$$

Figure 4 Flowchart on how to determine crack width

The situations which have been considered are different reinforcement lay-outs with single or multiple layers of reinforcement. The effective concrete tension zone height and effective reinforcement ratio are compared with each other by looking at the way in which these parameters are defined using the Eurocode 2 and Jones Method. Following this chapter a conclusion will be drawn about which method should be used for the determination of the effective concrete tension height. These methods will be validated using finite element models to see if the formulas used in Eurocode 2 and the Jones Method are correct.

The preliminary designs made with DIANA are used to understand how DIANA computes its models and to know what the possibilities are within DIANA. Firstly a brief introduction about DIANA is given. Secondly the different models and possibilities which will be used for this master thesis are investigated and commented on. Some of these models and possibilities are crack models, bond-slip behavior, types of reinforcement and tension softening. After that, different concrete tensile beam models are constructed and calculated to check the influence of the different input parameters. When the correct input is known, different models will be created to check the influence of the mesh on the crack width and crack spacing. The crack spacing needs to be known to determine the effective concrete tension zone by using the bond-slip properties.

When the input for the most suitable situation is known, the variation study can begin. Within this variation study the influence of the following parameters will be investigated: the cover, bar diameter, width of the model, number of layers of reinforcement and height of the model. The output of these parameters will be reviewed and commented on after which a comparison with the Eurocode 2 and Jones Method will be made to check whether Eurocode 2 and Jones Method give reliable values in terms of the effective concrete tension zone. The variation study is based on models loaded by tension.

After the variation study on the models loaded by tension, some bending models will be elaborated and commented on. The results of the bending models can be used for further investigation. With the results of both tension and bending models, a conclusion will be given to the problem statement to complete the thesis.

1.1 Note to the literature study:

At the start of the master thesis a literature study has been carried out. The literature study is added to the appendix and is about crack width control on imposed deformations due to edge restraint in massive concrete structures determined by different methods. The methods which are investigated are Eurocode 2, CIRIA C660, ICE706 and the Jones method. CIRIA C660 is about early age thermal cracking. ICE706 is about crack width control of edge and end restraint concrete structures. The Jones method is a method about crack width control in which the effective concrete tension zone is determined differently than with Eurocode 2. Within every method the crack width formulas have been investigated to see which method will be the most beneficial to calculate the crack width with, by making a comparison between the different methods for different geometries with a varying factor of restraint.

During further investigation the scope of the master thesis changed, from crack width control of different methods by using structures which are massive and edge restrained, to the determination and comparison of the effective concrete tension zone height for different configurations by making a comparison between finite element models using DIANA and Eurocode 2 and Jones method.

Therefore the part of the literature study about CIRIA C660 and ICE706 will not be used because in these methods the effective concrete tension zone is determined in the same way as in Eurocode 2. These methods will be less relevant to the remaining part of the thesis. The parts about Eurocode 2 and Jones method are still relevant to the thesis. Subsequently the section about imposed deformations and massive concrete structures is left for further investigation.

1.2 Main objective thesis

As is written earlier the effective concrete tension zone height is an important parameter for the determination of the crack width. But is the determination of the effective concrete tension zone height correct? Or are there differences between different norms and regulations which describe on how to determine the effective concrete tension zone height? To investigate these questions a comparison needs to be made. This comparison is the main objective of this master thesis. The comparison is between the effective concrete tension zone height of finite element models using DIANA and Eurocode 2 or Jones method. By making this comparison, the answer to the problem statement and several sub questions should be given. The problem statement is as follows:

- ***Are the effective concrete tension zone heights determined with either Eurocode 2 or Jones Method comparable with the effective concrete tension zone heights determined with finite element models using DIANA?***

Sub questions:

- How is the effective concrete tension zone determined?
- How can the effective concrete tension zone be determined in finite element models?
- Which method lies closer to the results given by finite element models?
- Can the cracking forces be traced back within the finite element models?
- Are other factors of the crack spacing formula visible within the finite element models?

Goals:

- Use DIANA to model reinforced concrete structures;
- Check the differences between the determination of the effective concrete tension height;
- Learn the principles of the cracking behavior of concrete;
- Reducing the amount of reinforcement needed by using finite element models for the determination of the effective concrete tension zone.

1.3 Guide

The thesis is divided into four different sections as follows:

Literature study:
First the literature study is performed on the determination of the crack width according to different regulations and papers. As is written earlier, the literature study does not comply with the rest of the thesis because over time the scope of the thesis changed, therefore the literature study is added to appendix A because some parts of the literature study are still applicable.

The effective concrete tension zone:
The second part of the thesis is about the effective concrete tension zone and the difference in the approach of the effective concrete tension zone between Eurocode 2 and Jones method. A conclusion is given on which method is preferable at the end of the chapter.

DIANA, the program and models:
The third part is about the program DIANA and about the possibilities within this program. Furthermore the variation study which is carried out for 2D models loaded by tension or bending is given in this section.

Conclusions and recommendations:
The fourth part covers the conclusions and the recommendations obtained by the research performed for this thesis.

2 COMPARISON BETWEEN EUROCODE 2 AND JONES METHOD

2.1 Effective tension zone of the concrete

The effective concrete tension zone is depending on the effective concrete tension zone height. These will be determine for Eurocode 2 and Jones method in this chapter. In Eurocode 2 clause 7.3.2 (3) [1] the effective concrete tension zone height is determined by the following formula:

$$h_{c,eff} = MIN \begin{cases} \leq \dfrac{h-x}{3} & (Bending) \quad 1 \\ 2{,}5 * (h-d) & \quad 2 \\ \leq \dfrac{h}{2} & (Axial\ tension) \quad 3 \\ 2{,}5 * \left(c + \dfrac{\phi_{sl}}{2}\right) & \quad 4 \end{cases}$$

Where:

h is the height of the cross section

x is the height of the concrete compression zone of the cross section

d is the effective height of the cross section determined by $d = h - c - {}^{\phi_{sl}}/_{2} - \phi_w$

c is the cover applied to the reinforcement

ϕ_{sl} is the main reinforcement

The formulas given above are valid for horizontal bar spacings till:

$$s = 5 * \left(c + \frac{\emptyset_{sl}}{2}\right)$$

The effective concrete tension zone height is the minimum value of the four given formulas. In the formula given above formula 2 and formula 4 are almost identical. The difference between both formulas is the stirrup. In the calculations stirrups are not included. Therefore formula 4 will be used for situations with one layer of reinforcement. And formula 2 will be used for multiple layers of reinforcement. In Figure 5 an effective concrete tension zone is given for a situation with multiple layers of reinforcement, the dimensions for the input are given in the figure.

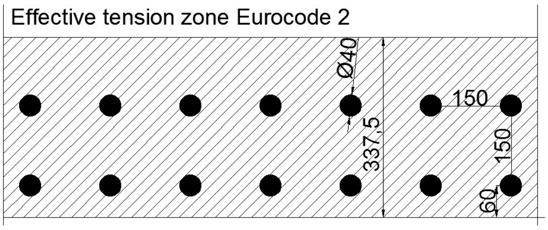


Figure 5 Effective concrete tension zone height according to Eurocode 2 with multiple layers of reinforcement

From practice, difficulties due to the implementation of the formula for the determination of the effective concrete tension zone arises. This is because the determination is not clear for constructions with varying reinforcement lay outs, irregular cross sections and construction which are loaded by double bending. Jones [5] came up with a new method on how to determine the effective concrete tension zone. This method is called “Jones method” in this thesis. With this method, the effective concrete tension zone is calculated for each single bar at which cracking is expected to occur. The area around each individual reinforcement bar will be appointed as one effective concrete tension zone. By using this method it is possible to take account for irregular cross sections, multiple layers of reinforcement or varying reinforcement spacings. This is an advantage in contrast to the approach given in Eurocode 2. But is this new approach advantageous when looking at the required reinforcement in each cross section to control the crack width and satisfy the requirements of cracking? This is something that has to be investigated and will be investigated in this thesis.

The formula for Jones method to determine the effective concrete tension zone height is given by:

$$h_{c,effJM} = c + \frac{\phi_{sl}}{2} + \min \begin{cases} \frac{ssv}{2} \\ 1{,}5 * \left(c + \frac{\phi_{sl}}{2}\right) \end{cases}$$

Where:
ssv is the vertical bar spacing

The effective concrete tension zone height of the cross section which has been considered for the approach given by Eurocode 2 is also determined for Jones Method and shown in Figure 6.

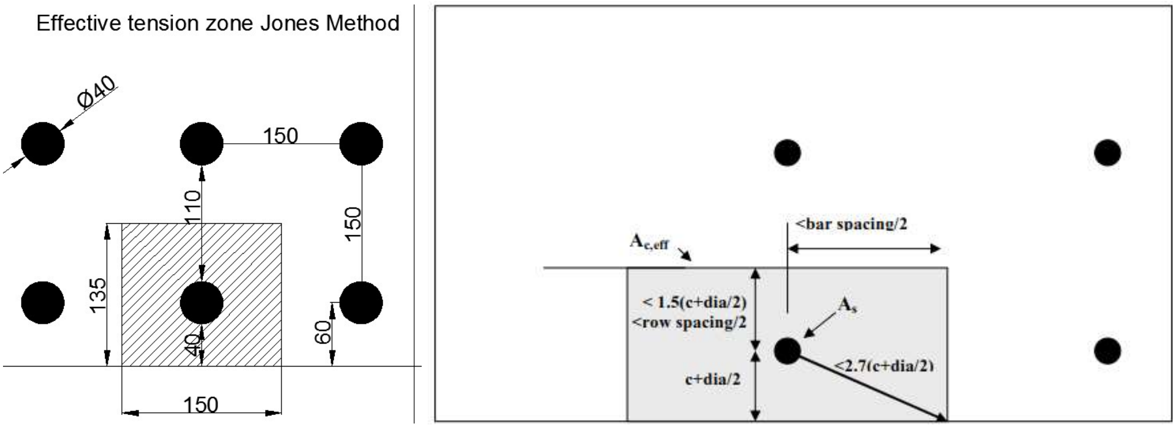


Figure 6 Effective tension zone determined with the Jones Method

By using the different formulas of Eurocode 2 and Jones method for the determination of the effective concrete tension zone height some questions arise; what will be the optimum and when will which formula be decisive? To investigate these questions a study has been carried out to come up with limits to the formulas. In this study different layers of reinforcement have been investigated with varying bar distances, bar diameters and covers.

The boundary conditions which have been considered in the test are:

- The cross section is in full tension which implies x=0→ no bending formula;
- Height cross section (h) variation: 1000mm to 3000mm;
- Diameter reinforcement (ϕ_{sl}): 16-40mm;
- Cover (c) variation: 30-100mm;
- Number of layers of reinforcement: 1-3;
- Vertical bar spacing (ssv): 50-200mm.

These boundary conditions have been chosen because these are common in practice. First the models with a single layer of reinforcement will be elaborated to determine what the influence of each individual boundary condition is. Secondly the models with two layers of reinforcement will be elaborated to determine which method is most beneficial for multiple layers of reinforcement. In appendix D models with three layers of reinforcement have been elaborated.

For Jones method two graphs will be given to see the effect of the parameters. The formula for the effective concrete tension zone height is determined by the vertical bar spacing and/or by the cover and main reinforcement. When this formula is applied a certain limit can be shown with certain input parameters. This limit is given in Figure 7:

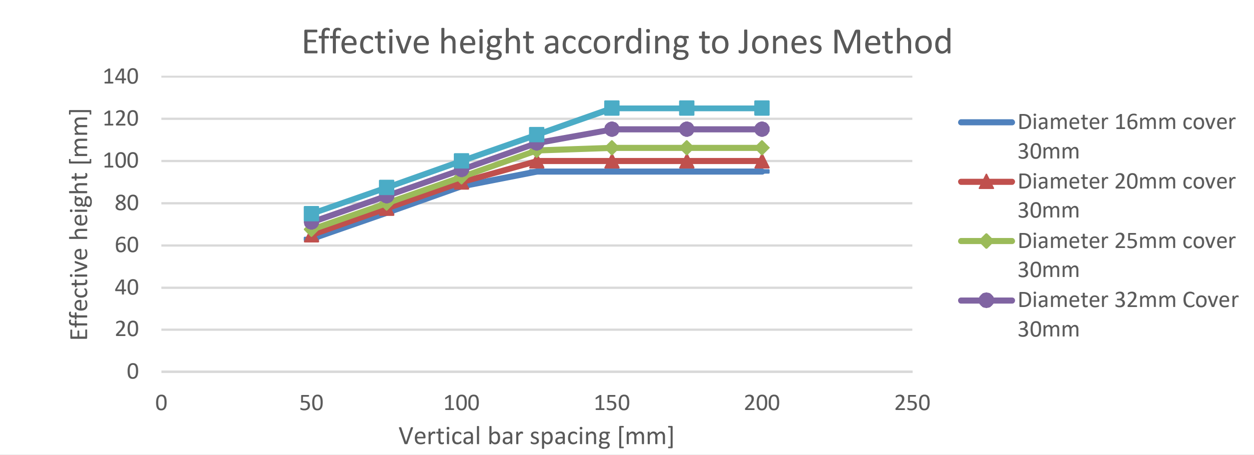


Figure 7 Effective concrete tension zone height vs vertical bar spacing according to Jones method with a limit value

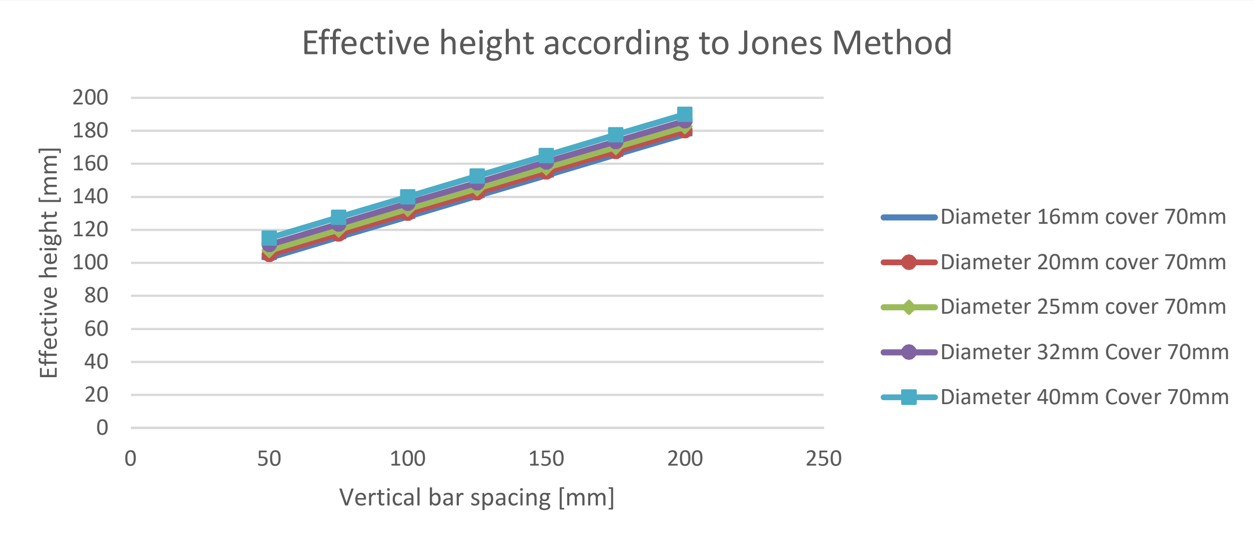


Figure 8 Effective concrete tension zone height vs vertical bar spacing according to Jones Method without a limit value

In Figure 7 a limit is visible this limit is due to the determination of the effective concrete tension zone height by Jones method. After the kink the effective concrete tension zone height is determined by the cover and diameter because this will stay constant by increasing vertical bar spacing. In the second figure the effective concrete tension zone height is determined by the vertical bar spacing. The lines will keep increasing linearly till the point where the cover and diameter term is normative. The point of the kink lies outside the scope of this figure.

2.1.1 Single layer of reinforcement calculated with Eurocode 2

What formula is governing when using a rectangular cross section with a single layer of reinforcement? For the given input parameters: thickness, cover and diameter the results have been examined.

- Thickness: 1000, 1250, 1500, 1750, 2000, 2500, 3000mm;
- Cover: 30, 40, 50, 60, 70, 80, 90, 100mm;
- Bar diameter: 16, 20, 25, 32, 40mm;

With these input parameters the effective concrete tension zone height has been calculated according to the four different formulas and in all cases formula 4 has the lowest value. This formula is normative in all different calculations when only one layer of reinforcement is applied. Some figures will be given in which the dependency on the different parameters is shown. To begin with, the dependency on the thickness of the element:

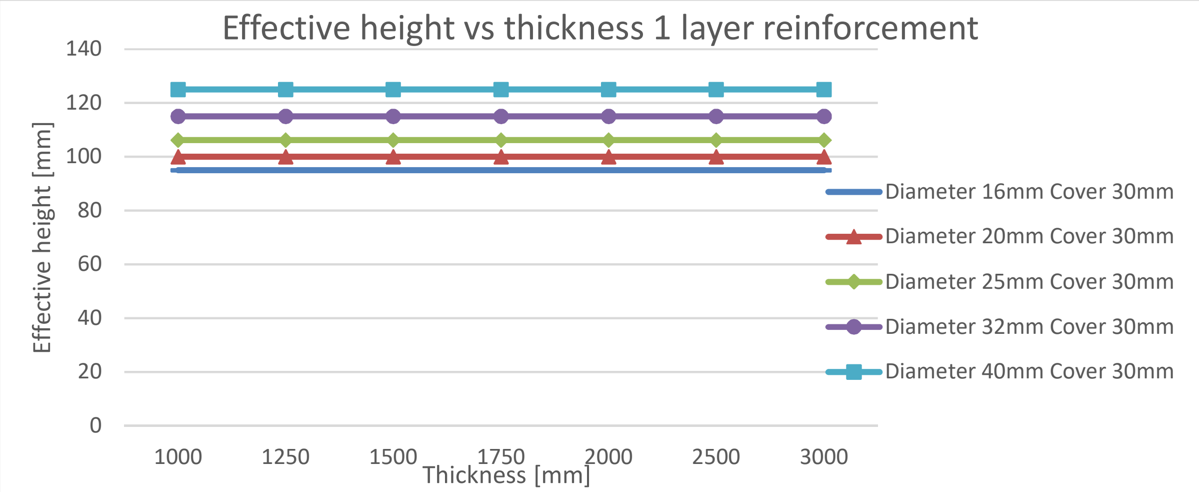


Figure 9 Effective concrete tension zone height of 1 layer of reinforcement according to Eurocode 2

As can be seen in Figure 9, the effective concrete tension zone height is not dependent on the thickness for the situation of one layer of reinforcement. The cover is kept constant and the increase in effective concrete tension zone height is only possible due to the increase of the diameter of the reinforcement. If the effective concrete tension zone height is not dependent on the thickness or height of the cross section the three formulas in which the height is included can be ignored so it has to be formula 4.
The influence of the cover term will be showed in Figure 10:

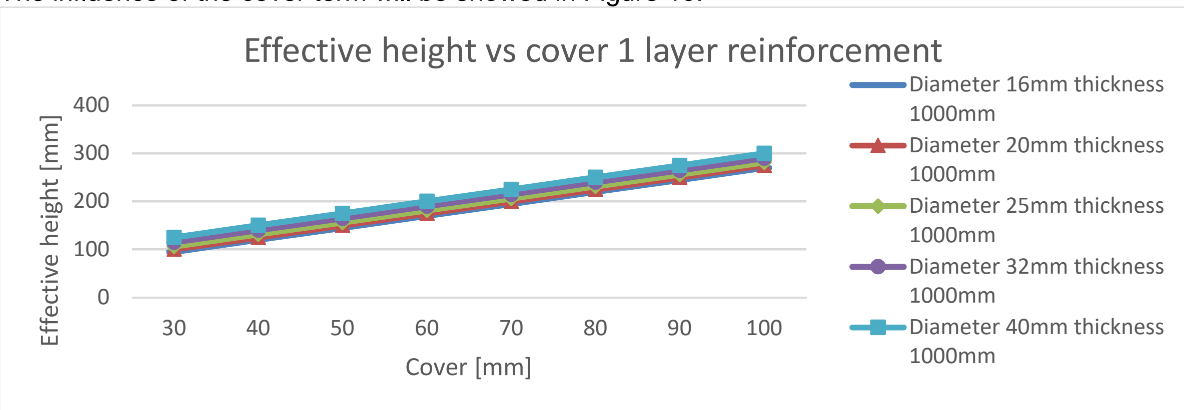


Figure 10 Effective concrete tension zone height of 1 layer of reinforcement according to Eurocode 2

In Figure 10, the situation for a 1000mm thick section has been showed because the thickness does not have an effect on the increase of the effective concrete tension zone height. As can be seen in the figure, linear lines are showed, these lines are parallel to each other which implies a constant increase of a certain amount, 25mm in this case, in every line. The amount of 25mm is due to the increase of 10mm in cover at each step. Again the results show that formula 4 is the normative one.

2.1.2 Model with two layers of reinforcement calculated with Eurocode 2

The calculation of the effective concrete tension zone height is somewhat more difficult if multiple layers of reinforcement have been used, because the center of gravity of the reinforcement needs to be determined and used in the height formula's. Even so, the dependency of the formula for the effective concrete tension zone height is changed from three input parameters to four input parameters (thickness, cover, vertical bar spacing and diameter). This has an impact on the amount of output and as a result it is harder to make simple plots to show the effects of the different parameters on the effective concrete tension zone height formula's. In the tables below the formulas with which the effective concrete tension zone height needs to be determined are given, with the accompanying parameters. The calculations are performed for models loaded by tension.

Table 1 Conditions for the determination of the effective concrete tension zone height for 2 layers of reinforcement according to Eurocode 2

Thickness is kept at t=1000mm		$h_{c,eff} = 2{,}5 * (h - d)$	$h_{c,eff} = h/2$
Cover c [mm]	Vertical bar spacing ssv [mm]	Governing diameter ϕ_{sl} [mm]	Governing diameter ϕ_{sl} [mm]
100	150	16-32	40→
100	175	16	20→
90	175	16-25	32→
80	200	16-25	32→

With a thickness larger or equal to t=1250mm formula 2 is always governing when two layers of reinforcement are applied. The cross section is in full tension.

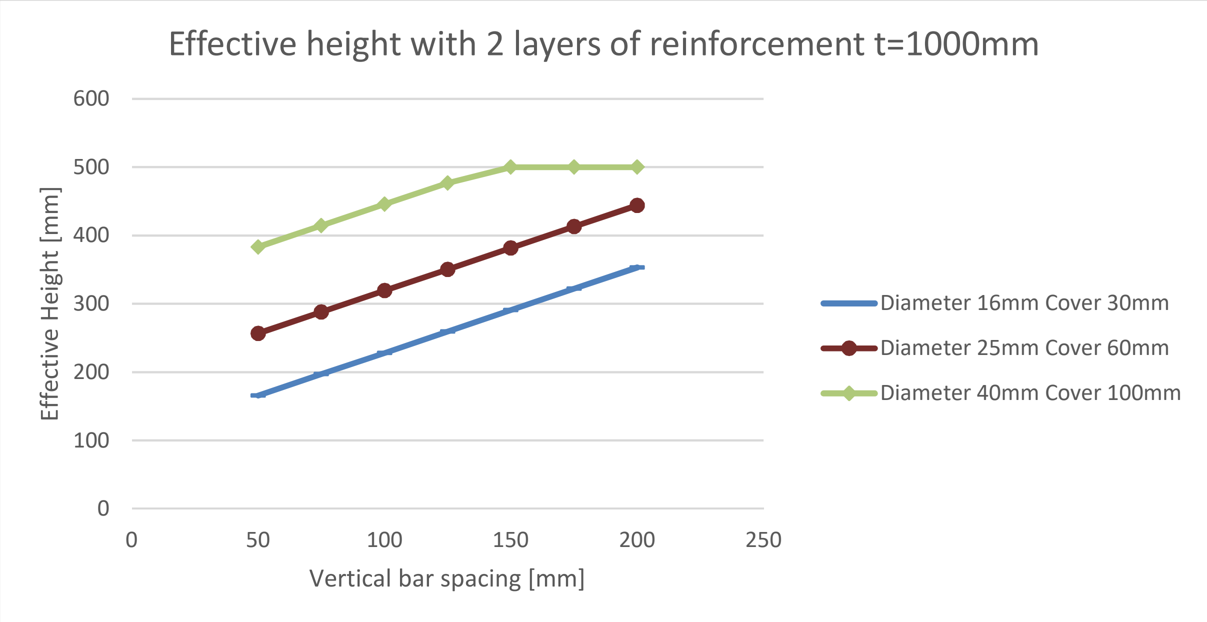


Figure 11 Effective concrete tension zone height of 2 layers of reinforcement according to Eurocode 2

In Figure 11, the influence of the cover, bar spacing and reinforcement diameter is presented for a certain thickness of 1000mm. There is a limit noticeable at 500mm, this limit is due to formula 3 which becomes normative at higher vertical bar spacings. If the thickness of the structure will increase this limit will lie higher, for the situations investigated here the limit will no longer be visible. For most situations formula 2 is decisive. The upper limit is given by the line which represents a model with 40mm reinforcement and a cover of 100mm. The bottom limit is given by a model with 16mm reinforcement and a cover of 30mm.

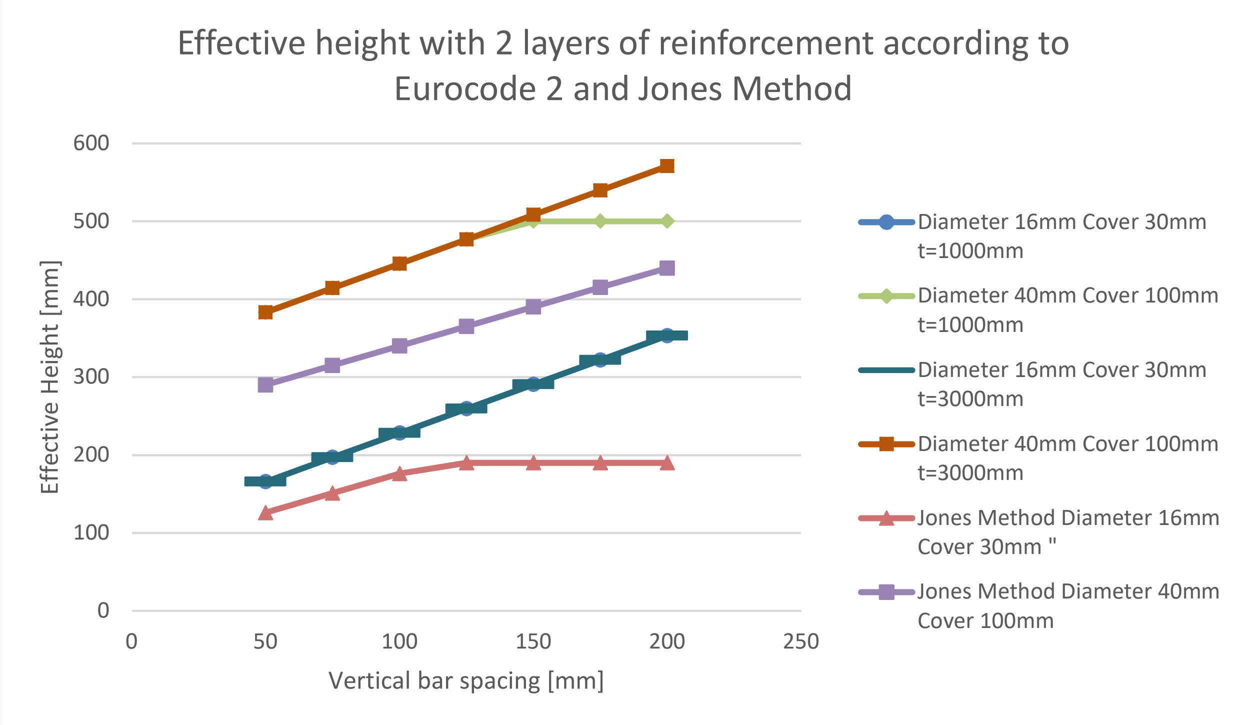


Figure 12 Effective concrete tension zone height with 2 layers of reinforcement according to Eurocode 2 and Jones Method

In Figure 12, the limits of the effective concrete tension zone height for two layers of reinforcement according to Eurocode 2 and Jones method are given, in which the input parameters are used from the previous paragraph. The horizonal line represent the limit given by formula 3 of Eurocode 2 because this value does not increase by an increase in vertical bar spacing. The linearly increasing limits are given by formula 2 because the cover term will increase linearly by a linear increase of vertical bar spacing. The bottom limit of Eurocode 2 is given by the smallest diameter ($\phi_{sl} = 16mm$, cover ($c = 30mm$ and thickness. The upper limit of Eurocode 2 is given by diameter ($\phi_{sl} = 40mm$), cover ($c = 100mm$ and thickness. The highest input parameters result in the largest effective concrete tension zone height.

Jones method has been added here to see the effect of the method in comparison with Eurocode 2. The effective concrete tension zone height of Jones Method is lower for both situations which implies that the effective concrete tension zone is smaller, this is beneficial for the determination of the maximum crack spacing. The lower limit is given by the smallest diameter and cover. This line does not increase after a certain point by increasing vertical bar spacing because after this point the effective concrete tension zone height is determined by the cover and diameter which are kept constant. The upper limit of Jones method is in between the limits of Eurocode 2. This effective concrete tension zone height is determined by the vertical bar spacing because the cover and diameter are large and thus not normative. The conclusion which can be drawn from this figure is that by using Jones Method for two layers of reinforcement the effective concrete tension height decreases substantially which is beneficial for the maximum crack spacing calculation.

See appendix D for the calculation of the effective concrete tension zone height for three layers of reinforcement.

2.2 Effective reinforcement ratio

As is explained earlier the effective reinforcement ratio is needed for the determination of the crack width and is depending on the effective concrete tension zone height. Therefore a comparison is made between the effective reinforcement ratio of Eurocode 2 and Jones method.

$$\rho_{p,eff} = \frac{A_s}{A_{ct,eff}}$$

In the figures below some trends are given in which Jones method and Eurocode 2 are elaborated. With the Eurocode 2 approach a difference has been made in the amount of layers, varying from 1 to 3. At each figure the thickness, vertical bar spacing, cover or diameter has been varied, while the other parameters remain the same.

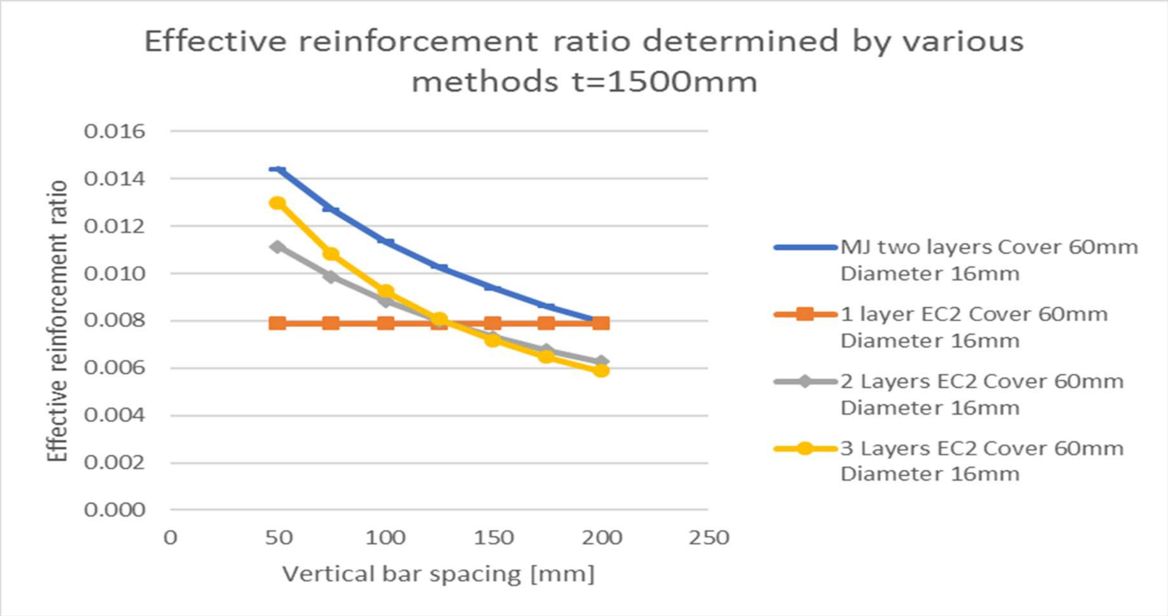


Figure 13 Effective reinforcement ratio of various methods and input parameters

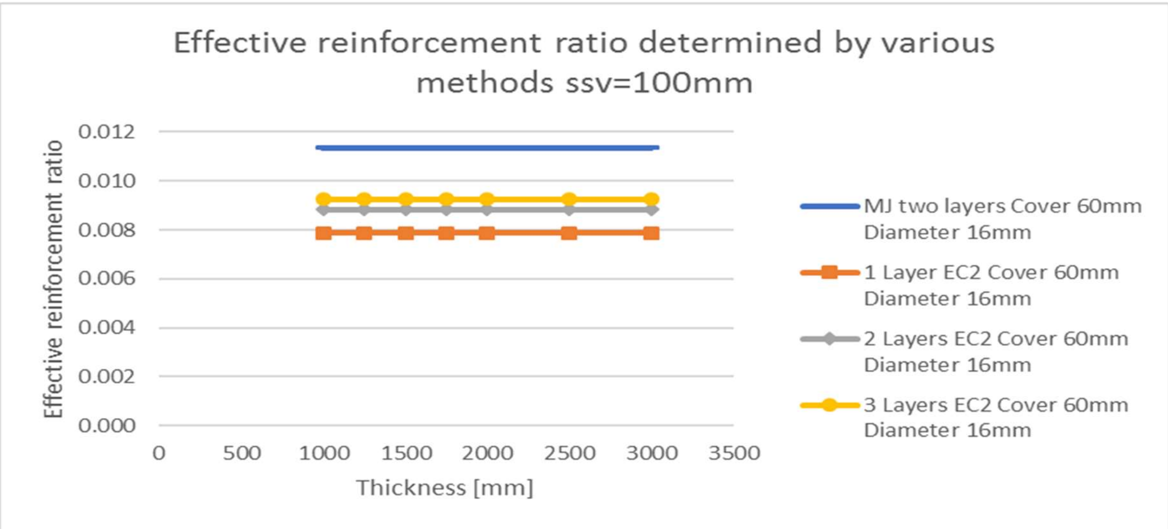


Figure 14 Effective reinforcement ratio of various methods and input parameters

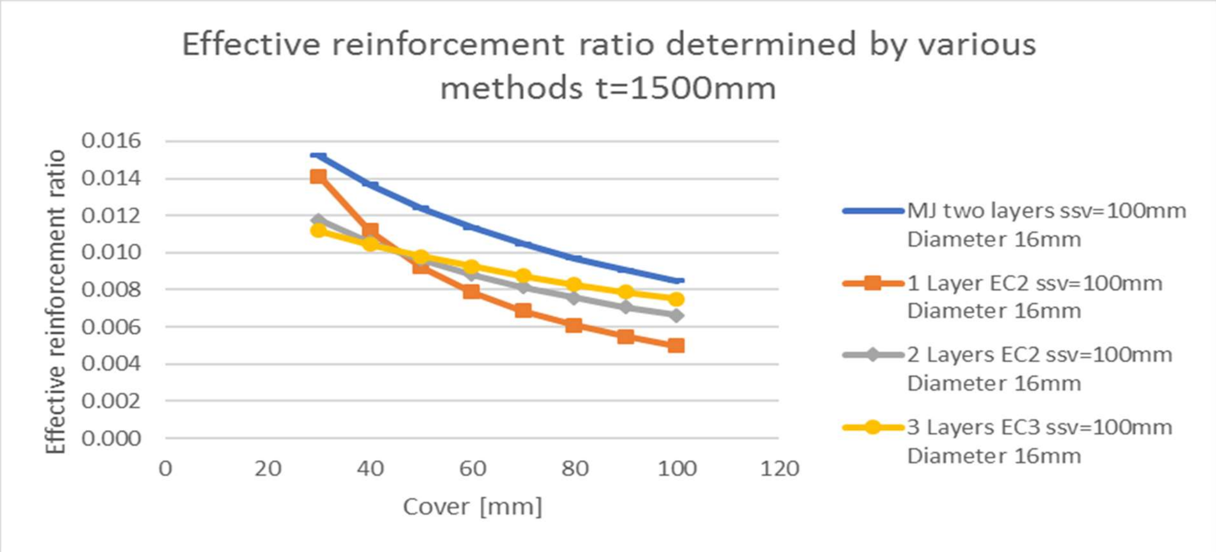


Figure 15 Effective reinforcement ratio of various methods and input parameters

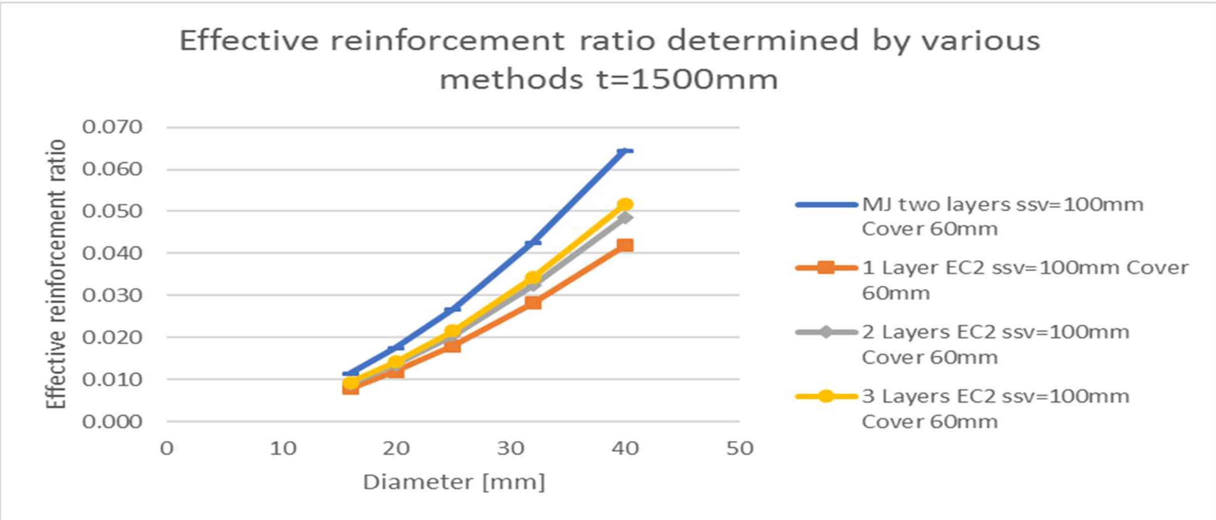


Figure 16 Effective reinforcement ratio of various methods and input parameters

In Figure 13 a pattern is shown that by increasing the vertical bar spacing the effective reinforcement ratio decreases till a certain limit. This limit is given by the limit of the effective concrete tension zone height because when the effective concrete tension zone height remains the same, the effective reinforcement ratio will remain the same with the unchanged cross sectional area of the reinforcement. For multiple layers of reinforcement the effective concrete tension zone height increases which implies a decrease in effective reinforcement ratio. Due to the fact that the effective concrete area increases more than the cross sectional area of reinforcement.

In Figure 14 the effective reinforcement ratio is a straight line with no increase by increasing the thickness of the structure. This is due to the fact that the effective concrete tension zone height is determined by formula 2 which is independent of the thickness.

In Figure 15 it is shown that the effective reinforcement ratio decreases by an increase of cover, because the effective concrete tension zone increases while the cross sectional area of the reinforcement remains the same. The lines of Eurocode 2 decrease in a different order because the cross sectional area of the reinforcement for multiple layers of reinforcement increases more than with a single layer in reinforcement. Therefore the effective reinforcement ratio decreases slower for three layers of reinforcement.

In Figure 16 it is shown that the effective reinforcement ratio increases by an increase in diameter, thus formula 2 is decisive. This is due to the fact that the cross sectional area of the reinforcement increases. This has a positive effect on the effective reinforcement ratio. Here the effective reinforcement ratio for three layers of reinforcement increases faster because the cross sectional area of the reinforcement increases faster.

In most of the figures displayed above, Jones method (blue line with a stripe marker), gives the highest effective reinforcement ratio. Therefore Jones method is the most favorable method in terms of maximum crack spacing but unfavorable in terms of local tension stiffening. These are needed to determine the crack width. The influence of the effective reinforcement ratio will be investigated into the following chapters.

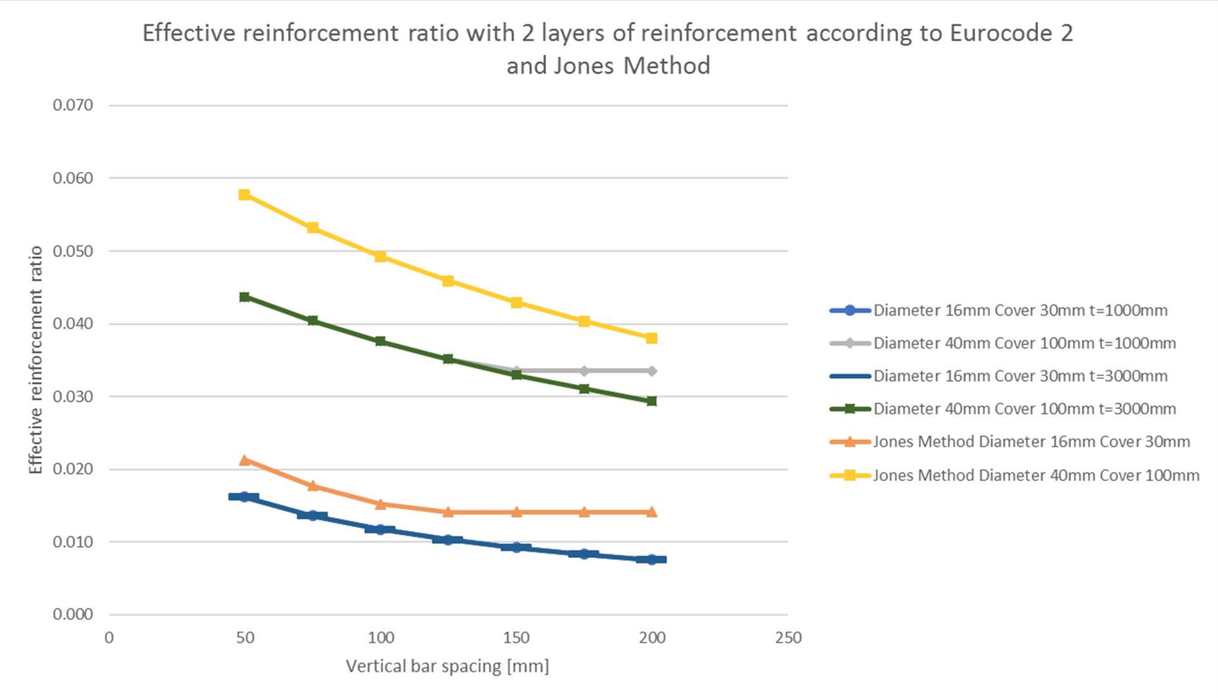


Figure 17 Effective reinforcement ratio with 2 layers of reinforcement according to Eurocode 2 and Jones Method

In Figure 17 the limits of the effective reinforcement ratio of a model loaded by tension with two layers of reinforcement are given according to Eurocode 2 and Jones Method. There is a clear distinction noticeable between the lines of Eurocode 2 with 16mm diameter and 30mm cover (lower bound values) and the lines of 40mm and 100mm cover (higher bound values). The outcomes of different thicknesses lie on the exact same spot. Which implies that effective reinforcement ratio is determined by the effective concrete tension zone height formula 2. The results of different input parameters will lie between the lower and upper limit of Eurocode 2 which are given in this figure. The horizontal line is an upper limit of an unrealistic situation with a small thickness and a large bar diameter. This line can be ignored because it shows a situation which is not used in practice.

Both lines given by Jones Method give a higher effective reinforcement ratio, this is due to the fact that the effective concrete tension zone height of both situations is lower than the effective concrete tension zone height given by Eurocode 2. For the situation with the smallest parameters, the line does not increase after a certain point (ssv=125mm). This is because the line is not dependent on the vertical bar spacing after this point. But on the cover and bar diameter which remains the same.

See appendix D for the determination of the effective reinforcement ratio of the model with three layers of reinforcement.

2.3 Conclusion

The effective concrete tension zone height according to Eurocode 2 for a single layer of reinforcement give results which are the same results as given by Jones method. This is because the formulas which are used are the same. If multiple layers of reinforcement have been used the formulas for the determination of the effective concrete tension zone height change. Jones method results in a smaller effective concrete tension zone height than Eurocode 2. Therefore the effective reinforcement ratio of Jones method is higher than given by Eurocode 2. This results in a smaller crack spacing which is beneficial for the crack width calculation. But due to the larger effective reinforcement ratio the local tension stiffening effect increases which will result in an increase in crack width. The results and conclusions obtained in this chapter will be investigated by finite element models using DIANA in the following chapters.

3 INTRODUCTION TO DIANA WITH DIFFERENT MODELS AND POSSIBILITIES

3.1 Introduction DIANA

DIANA (DIsplacement ANAlyzer) is an extensive multi-purpose finite element software package that is dedicated, but not exclusive, to a wide range of problems arising in Civil engineering including structural, geotechnical, tunnelling, earthquake disciplines and oil & gas engineering. DIANA is a well proven and tested software package that has been used on various landmark projects all over the world. The program's robust functionality includes extensive material models, element libraries and analysis procedures, which are based on the latest and the most advanced finite element analysis techniques.[1]

DIANA is chosen as the FEA-modelling program in this master thesis due to the possibilities and the freedom in modeling. The cracks due loads can be modelled by using various cracking models. The reinforcement can be modelled in various ways by implementing different phenomena like plasticity and bond-slip behavior etc. This makes DIANA an appropriate tool for this master thesis.

When starting with this master thesis, the knowledge of DIANA was limited. Various examples about DIANA where performed to understand the program and it's possibilities. Various options from DIANA are important for this thesis, such as: cracking behavior and the various cracking models, bond models and bond properties and the influence of the mesh.

3.2 Cracking models

In DIANA there are two main cracking models: the discrete cracking model and the smeared cracking model. These cracking models will be explained briefly, additional information about the models can be found in the DIANA manual [7].

3.2.1 Discrete cracking model:

The discrete crack model has been investigated to get to know if it is possible to model the cracks using this crack model. The discrete cracking model computes a crack in a predefined interface if the load exceeds the properties of the interface. With the discrete crack model the crack width can be determined correctly but a disadvantage of this model is that it only cracks at the interface. The interface is a membrane with certain stiffness properties of which the position is already determined by the user. The stiffness properties in the normal and shear direction are about 1000 times larger than the stiffness of the mother element in which the interface is positioned.

The test is performed in a displacement controlled manner, because this gives the most accurate results. If the test is performed in a force controlled manner, the force will be increased by a certain amount. The model will fail(crack) if the stresses in the model exceed the strength of the model. If the model fails the force in the model decreases but the force exerted on the model keeps increasing. Therefore the model cannot show any results because there cannot be an equilibrium between the force exerted on the model and the force within the model. By using a displacement controlled model the forces can be displayed even if the model is cracked because the model is loaded by a displacement. With every displacement the model has one solution in terms of force within the model. This is shown in the force/displacement graph for reinforced concrete. The test case showed a crack in the beam at the predefined location which was predicted in advance but this method is unusable for this thesis. Due to the fact that the location of the cracks are not known in advance. Therefore the location of the interface which will crack cannot be defined properly. The model can be used to model experimental results in which the crack pattern is defined. This is not the case here so this model will not be used.

[1] Source: www.dianafea.com

3.2.2 Smeared cracking model:

The smeared cracking model is a cracking model in which every element of the model can crack. These cracks can form in different directions. Within the smeared cracking model there are different cracking models possible which are dependent on the strains and directions of the crack. The information about these cracking models is given in the DIANA manual. Two models will be described here:

Multi-directional fixed crack model:
In the multi-directional fixed crack model, cracking is specified as a combination of tension cut-off, tension softening and shear retention. The fundamental feature of the multi directional fixed crack model is the decomposition of the total strain ϵ into an elastic strain ϵ_e and a crack strain ϵ_{cr} as
$\epsilon = \epsilon_e + \epsilon_{cr}$
Modeling a number of cracks that simultaneously occur is made possible due to the sub-decomposition of the crack strain ϵ_{cr}. The basis feature of this multi-directional fixed crack concept is that the stress and strain rotate in the direction of the crack so that they are aligned with the crack, see the figure below for detail.

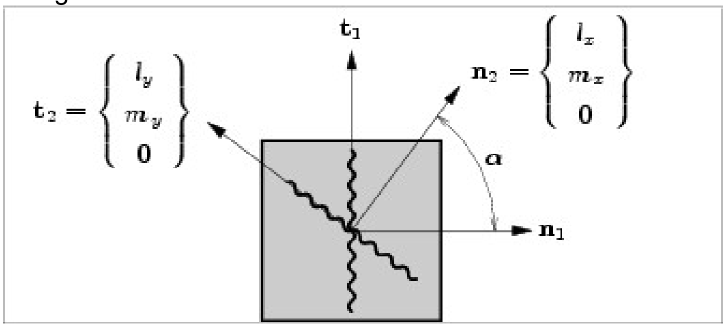


Figure 18 Multi-directional fixed crack model

Total strain crack model:
This constitutive model is based on total strain which describes the stress as a function of the strain. If the stress-strain relationship is evaluated in the principal direction of the strain vector, the model is called the rotating crack model. This model is well suited for reinforced concrete structures. For a more physical nature crack model, the fixed crack model can be used, which determines the cracks in a fixed coordinate system.

The total strain based crack model with a rotating crack orientation is chosen for this master thesis because this model is best applicable for reinforced concrete structures which this master thesis is about.

3.3 Tensile behavior

Concrete behaves differently in tension than in compression. In the elastic phase the concrete behaves linearly till the point when the concrete cracks. The tensile behavior of the cracked concrete can be determined by different predefined tension softening curves. The softening functions are based on the fracture energy (G_f). For a smeared cracking model there are different tension softening functions which can be used which are related to a crack bandwidth (h). The crack bandwidth is the size of the element defined by $h = \sqrt[3]{V}$ in which V is the volume of the element for 3D models. For 2D models $h = \sqrt{A}$ in which A is the area of the element. Some examples of tension softening curves are shown in Figure 19. For this master thesis the tension softening model of Hordijk (g) will be used. Additional information about the tension softening functions can be found in the DIANA manual [7].

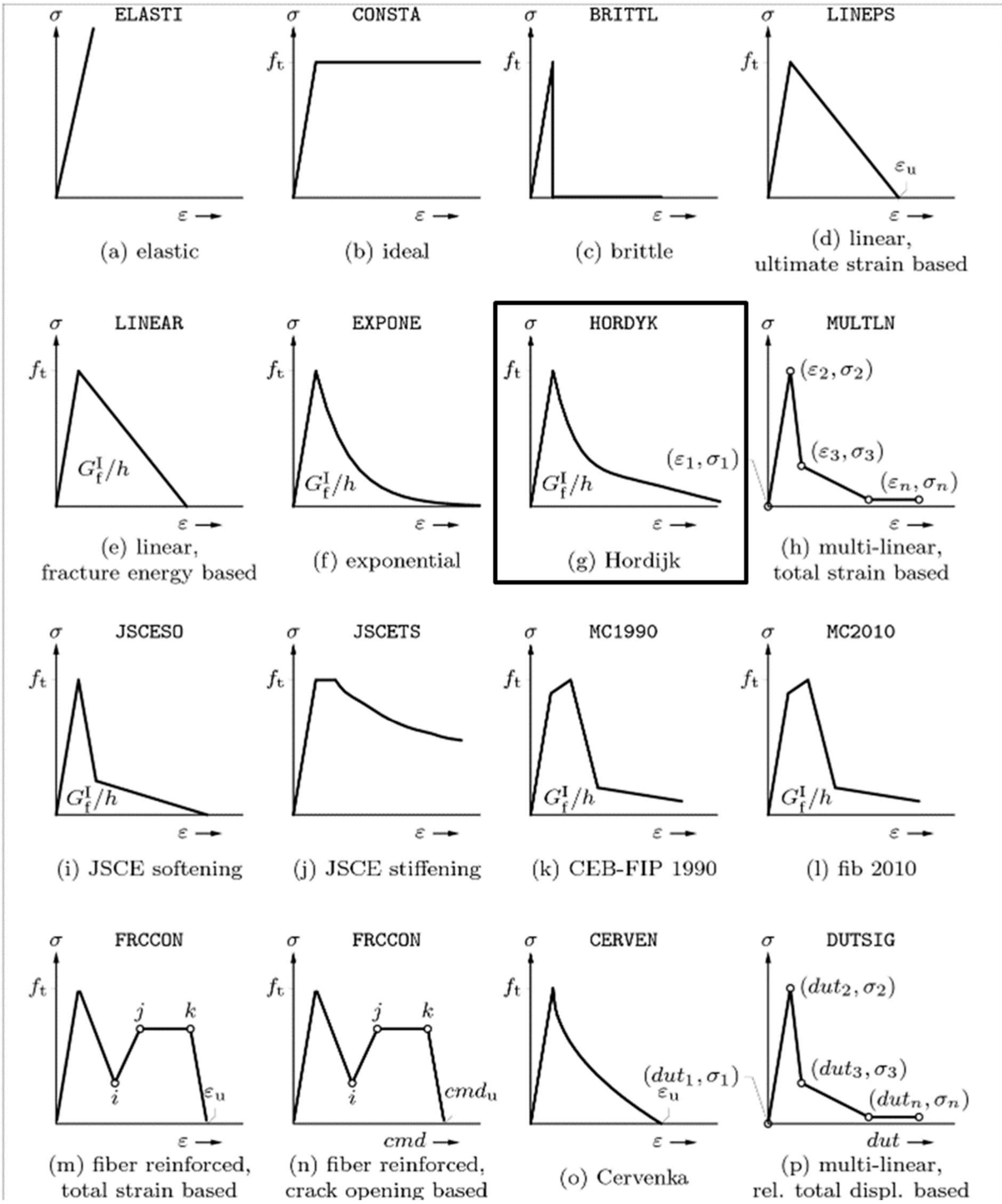


Figure 19 Tension softening models, the tension softening model of Hordijk has been applied

3.3.1 Fracture energy and Poisson's reduction:

The fracture energy is determined according to the FIB model code 2010, in which it is stated that the fracture energy is $G_f = 73 * fcm^{0,18} = 73 * 38^{0,18} = 140{,}50 N/m$ for C30/37

Where:

fcm= the mean compressive strength according to the eq. 5.1-1 from the model code [Mpa].

$fcm = fck + \Delta f$

$\Delta f = 8$

When the damage based Poisson's ratio reduction option is selected, the Poisson's ratio decreases to 0 when the concrete cracks. This is used to prevent unrealistic compression strains in the direction perpendicular to the crack, when large crack strains arise in the vicinity of the crack.

3.4 Compressive behavior concrete

The compressive behavior of the concrete can be determined by different predefined models within DIANA. Most of the models have some kind of softening after the ultimate compression strength has been reached. For the calculations the elastic (a) compressive behavior will be used, because most of the test performed are pure tension tests in which the ultimate concrete compression strength will not be reached. Additional information about the compressive behavior functions can be found in the DIANA manual [7].

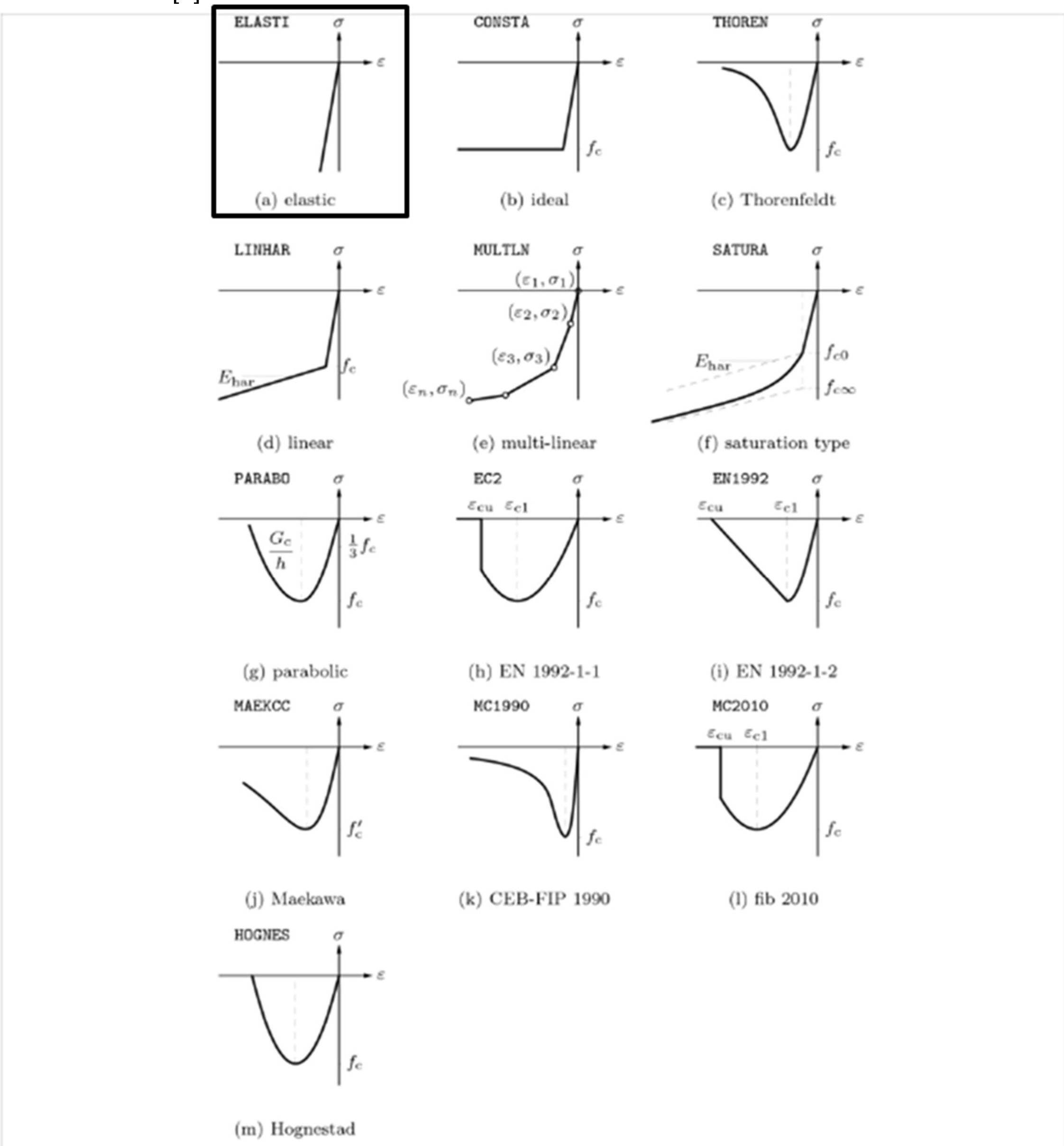


Figure 20 Compression behavior curves according to DIANA, the elastic compressive behavior has been applied

3.5 Bond-slip behavior

The bond mechanism between the steel ribbed reinforcement and concrete is given by three phenomena, namely: adhesion, friction and mechanical interlocking. Adhesion is the least important of these three phenomena. This is because the adhesion is lost when the concrete is loaded. The mechanical interlocking provides inclined bond stresses which can be subdivided into two different types of forces, namely longitudinal bond stress and radial splitting stress [10].

The inclined forces are balanced by ring tension forces which can result in cracks in the concrete. Two failure modes can be distinguished: splitting failure and pulling out failure. Splitting failure is failure which comes forth out of the ring tension forces, these forces result in cracks at the surface of the concrete. This type of failure occurs in structures which have a small cover, because the normal stresses between the concrete and the rebar cannot build up in the small cover which results in a lower bond stress transfer. Pull-out failure occurs when splitting failure does not occur and the load keeps increasing until a point that the concrete shears off the reinforcement. The bond stress which can build up is higher for pull out failure than for splitting failure. This is only possible with a large cover or large bar distances, because splitting failure will occur when the cover is small or when the bar distance is too small.

For the crack width calculations, the bond stress is an important factor because the force which can build up due to bond determines the next crack in the concrete. Bond is determined by the concrete parameters and can vary in the concrete structure just like the tension strength. The bond stress is determined by the concrete strength class and slip. The force due to the bond stress, the perimeter of the reinforcement bar and the transfer length is equal to the effective concrete tension zone times the concrete tensile strength when a crack occurs. This is because a crack occurs if the concrete tensile strength is exceeded. This will imply that the force given by bond exceeded the resistance given by the effective concrete tension zone multiplied by the concrete tensile strength. These cracks usually start at the ribs of the reinforcement and occur at the weakest spot in the concrete structure. In reality this can occur at any position due to the inhomogeneity of the concrete, but for this thesis the concrete has a uniform tensile strength. At first these cracks are micro cracks and do not harm the structure in terms of durability. The durability is affected when these cracks expand towards the surface of the concrete.

In DIANA there are different models which describe the bond-slip behavior. These models are shown below in which the nonlinear relation between shear traction(bond stress) and shear slip(relative displacement) is given. The bond-slip mechanism is based on a total deformation theory. In which the bond stress is expressed as a function of the total relative displacements. DIANA constructs an interface with zero thickness around the reinforcement bars to model the bond behavior. This interface has certain normal and shear stiffness. For this thesis three different bond-slip models have been considered. Firstly the bi-linear bond-slip curve (c), secondly the fib Model Code 2010 bond-slip curve (f) [6] and thirdly the cubic function of Dörr (a). For more information about each model see the DIANA manual [7].

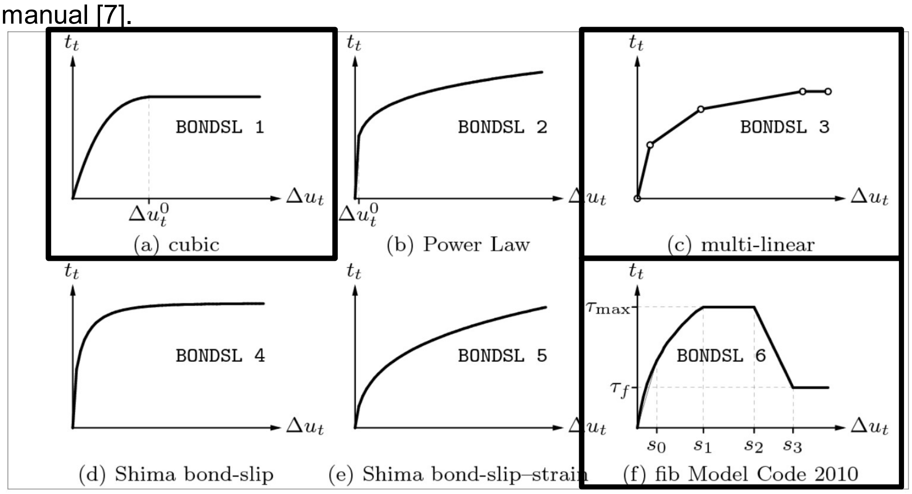


Figure 21 Bond-slip behavior curves, the curves which have been used are framed

Reinforcement is of importance for the determination of the bond-slip behavior. In DIANA there are different options for reinforcement [7], these are embedded reinforcement and bond-slip reinforcement. The reinforcement will be put in as 2D lines which are given specific reinforcement properties like tension strength, Young's modulus, strain capacity and mass density. Embedded reinforcement is reinforcement which is fully embedded in the element in which it is located and is fully coupled. This means that both elements are fully bonded and that there is no slip possible between the elements. Embedded reinforcement will not be used in this master thesis because the bond-slip behavior is of importance. Embedded reinforcement cannot slip so the bond-slip cannot be measured.

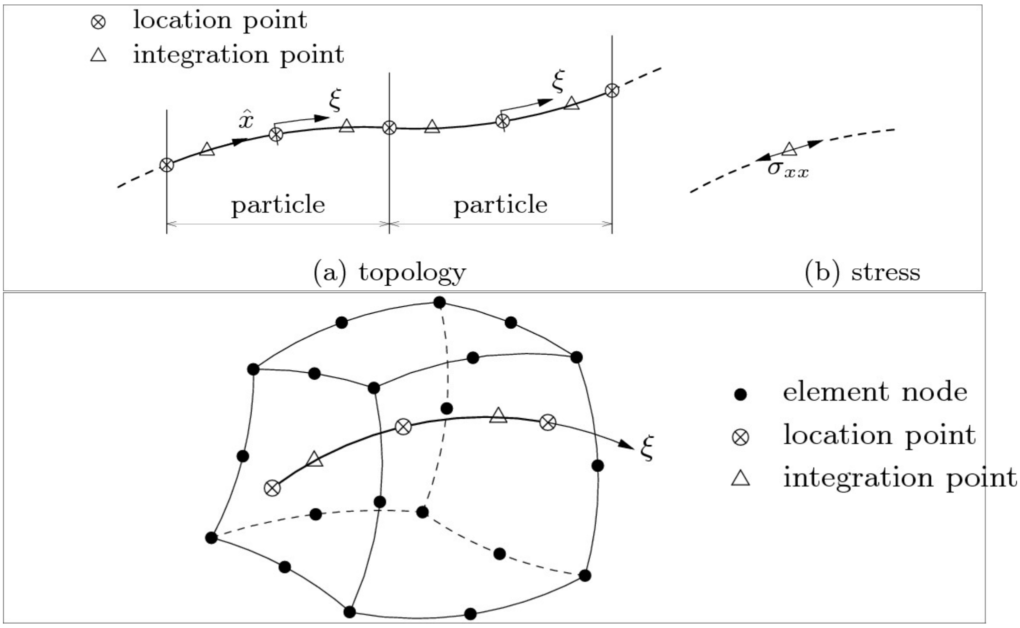


Figure 22 Embedded reinforcement

With bond-slip reinforcement it is possible to model the bond-slip behavior which is needed for this master thesis. Bond-slip reinforcement takes account of the relative displacement of the reinforcement in contrast to the element in which it is located.

3.6 Elements used for the analyses

DIANA splits the model into elements when the mesh is constructed. These elements have different properties for different situations. The 2D elements are different from the 3D elements in terms of displacements/rotations and amount of nodes. The elements used in this thesis are given in the figures below. For 2D applications the CQ16M element is used. This is an 8 noded quadrilateral isoparametric plane stress element with two degrees of freedom.

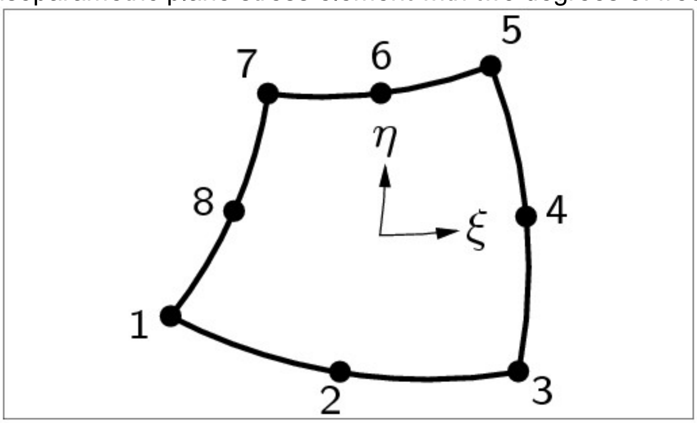


Figure 23 2D plane stress element CQ16M

For 3D applications the CHX60 elements are used. These elements are twenty-node isoparametric solid brick elements. The element has three degrees of freedom and is based on quadratic interpolation and Gauss integration.

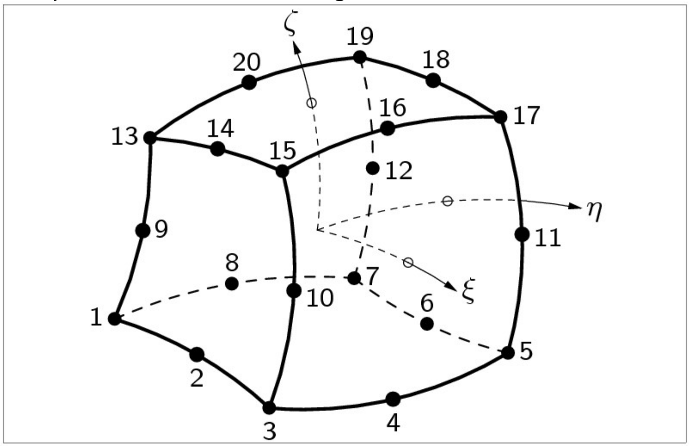


Figure 24 3D brick element CHX60

The bond-slip reinforcement is either a truss or a beam 2D line element with no further element type because it is implemented in the concrete beams. Additional information about the element types can be found in the DIANA manual [7].

4 TEST MODELS

Now that the different options have been explained and chosen, small tests can be performed to see the influences on the models of the choices made. These models consist of pieces of the total model. At first a bond-slip test will be performed. Hereafter a tensile member will be modelled to check whether the mesh has an influence on the determination of the crack widths. After that the influence of the mesh on crack spacing will be investigated. The guideline for nonlinear finite element analysis of concrete structures is used for dimensions of the mesh and other parameters [8].

4.1 3D (CHX60) Tensile member model

4.1.1 Bond test model

To understand the bond-slip behavior in DIANA, a small test model has been developed to reduce the influences of other phenomena within the model. A small model also improves the computational time which is beneficial if improvements to the model have to be made. The model consists of a 50mm CHX60 cubic block with a reinforcement bar of 4mm in the middle of the block. The block is loaded by a prescribed deformation in the Y-direction of 0.1mm per load step. The prescribed deformation is applied to the reinforcement bar in the direction of the reinforcement to model the bond-slip behavior of the concrete and reinforcement. The model is supported in the Y-direction at the tip of the reinforcement. In the vertical Z-direction the model is supported at both ends of the reinforcement. The X-direction is also supported at both ends of the reinforcement. The 3D block has been modelled as C30/37 concrete using the total strain based crack model. The reinforcement bar is modelled as a truss bond-slip bar which implies that the bar can only transfer normal forces.

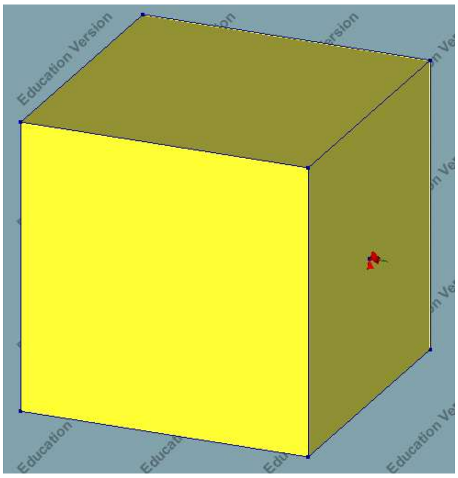


Figure 25 Model to test the bond-slip stiffness

In DIANA a special data set needs to be used in order to calculate the bond-slip behavior, this option is called the INTERF option. Within this option one can choose for either TRUSS reinforcement or BEAM reinforcement. TRUSS reinforcement cannot bend and can only transfer normal forces. BEAM reinforcement can bend and can transfer normal forces. The INTERF option models an interface around the reinforcement which represents the bond-slip behavior. The interface has a normal and shear stiffness input which can be specified as part of the reinforcement properties. At first the normal stiffness is set at 20000N/mm³. The shear stiffness is set at 250N/mm³. These stiffness values have been obtained from experimental data [10]. For the bond-slip behavior, a bi-linear curve has been used. The bond stress is set at 5,22N/mm² which is 1,8 times the mean tensile strength (f_{ctm}). The curve is given in Figure 26.

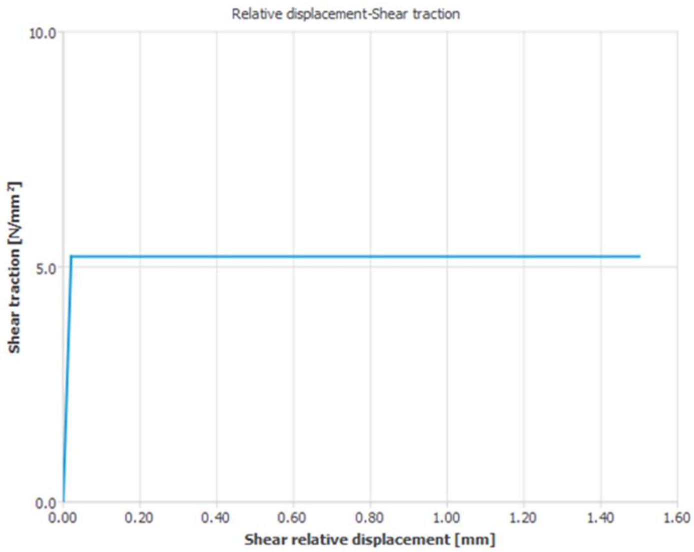


Figure 26 Bi-linear bond-slip curve

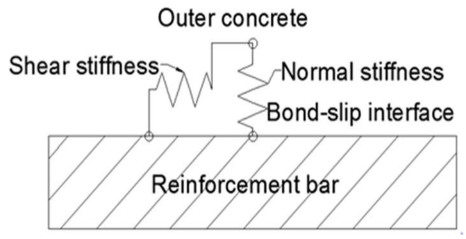


Figure 27 Bond-slip stiffness

With the aforementioned properties non-linear calculations have been performed to calculate the relative displacement (slip) between the reinforcement and the concrete. As well as the interface traction (bond stress) between concrete and reinforcement. The effect of the normal and shear stiffness on the bond-slip behavior has been investigated. By using different stiffness properties, the relative displacement and interface tractions did not change. This implies that these stiffness properties do not have an effect on the bond-slip behavior while these stiffnesses should have an effect on the slip and bond stress.

With this test the bi linear curve was not changed. Therefore the influence of the change of the shear stiffness property was not noticeable because DIANA uses the shear stiffness from the bi linear curve. Thus DIANA overrules the shear stiffness which is put in as parameter, by the shear stiffness given by the shear stiffness of the bi linear curve. The normal stiffness should be high because the reinforcement cannot be pushed in the concrete perpendicular to the length axis of the reinforcement.

By changing the young's modulus of the reinforcement and the concrete, the relative interface displacement and traction did change. This implies that the stiffness of the reinforcement and concrete do have an influence on the relative displacement and bond stress between concrete and reinforcement.

4.1.2 Influence of mesh size on crack width

The influence of the element size is investigated to check whether the crack width increases or decreases by a decrease in element size. A 3D tensile member model is used to check this influence. At first the mesh is set at 100mm, so each element has a length of 100mm. With the model the crack widths are determined at certain points at which the tensile member cracks. These crack widths are compared with the results given by the model in which the mesh is refined to 50mm. The difference between the models is very large. This implies that the crack width is dependent on the element size. According to DIANA the crack width is determined by:

$$w_k = h * \left(\epsilon - \frac{\sigma_c}{E_o}\right)$$

Where:

h is the crack band width = $\sqrt[3]{V}$ in which V is the volume of the element so it's equal to the element size according to the Rots method

ϵ is the total strain

σ_c is the stress in the concrete

E_o is the initial young's modulus

According to DIANA the crack width is determined by the crack band width and the crack strains within one element. This study shows that this approach does not seem reasonable and there are some doubts about this method. This is because the crack width should not be depending on the crack band width. For the determination of the crack width different options should be used. An option is to consider the crack strains in a certain range close to the smeared crack and add these strains. The crack width is given by the total crack strain over the area multiplied by that area. Another option is to check the differences in displacements in the length direction of the structure over the complete crack spacing. The difference is the crack width[13].

4.1.3 Determination of the crack spacing

The crack spacing, which is the length in between cracks, is of importance because this length is determined by the transfer length. The transfer length is the length over which the bond stress builds up till the force at which the concrete cracks. As was written before, the effective concrete tension zone multiplied by the concrete tensile strength is equal to the force which builds up due to the bond stress. To determine the effective concrete tensile area, the crack spacing needs to be known. To determine the crack spacing a tensile bar model has been created. This model is the same model as the one used for the determination of the influence of the stiffness on the bond-slip behavior. The only difference is that the model is also supported in the Z-direction on the edges at (x,y,z)[mm]=(0;0;25), (50;0;25), (0;L;25) and (50;L;25) these supports are added to inhibit rotation. The model is loaded by a prescribed deformation at the tip of the reinforcement to transfer forces from the reinforcement to allow the concrete to crack. The transferred force can result in cracks which can result in pull out failure of a concrete cone at the tip of the reinforcement.

For the next crack to develop, the amount of reinforcement needs to be sufficient to withstand yielding of the reinforcement. Yielding can be prevented if the cross sectional area of the concrete, multiplied by the concrete tensile strength is lower than the cross sectional area of the reinforcement, multiplied by the yield strength of the reinforcement (minimum required reinforcement).

$$A_c * f_{ctm} < f_y * As \rightarrow 2500 * 2{,}9 = 7250N > 500 * \frac{\pi}{4} * 4^2 = 6283N$$

$$A_c * f_{ctm} < f_y * As \rightarrow 2500 * 2{,}9 = 7250N < 500 * \frac{\pi}{4} * 6^2 = 14137N$$

For this example the model will undergo yielding because the cross sectional area of the reinforcement is too low. To overcome this problem the reinforcement needs to be 6mm in diameter instead of the current 4mm. Now the model has the possibility to crack and yielding will not occur. This implies that the model needs to crack in different cross sections what will result in multiple cracks.

The model will be extended to check where and how many cracks will develop by using the same loading procedure, see Figure 28. The total length of the model is 450mm. The results show that the second crack originates at different positions depending on the mesh size. So the mesh could have an influence on the crack spacing. The results of this model are compared. The load displacement diagram and the maximum crack width and crack distances are given in Figure 30 & Figure 31.
For the determination of the effective concrete tension zone, which is the main purpose of this master thesis, the crack spacing or transfer zone of the bond stress is important because these are interlinked. So the bond-slip behavior is important to say something useful about the effective concrete tension zone.

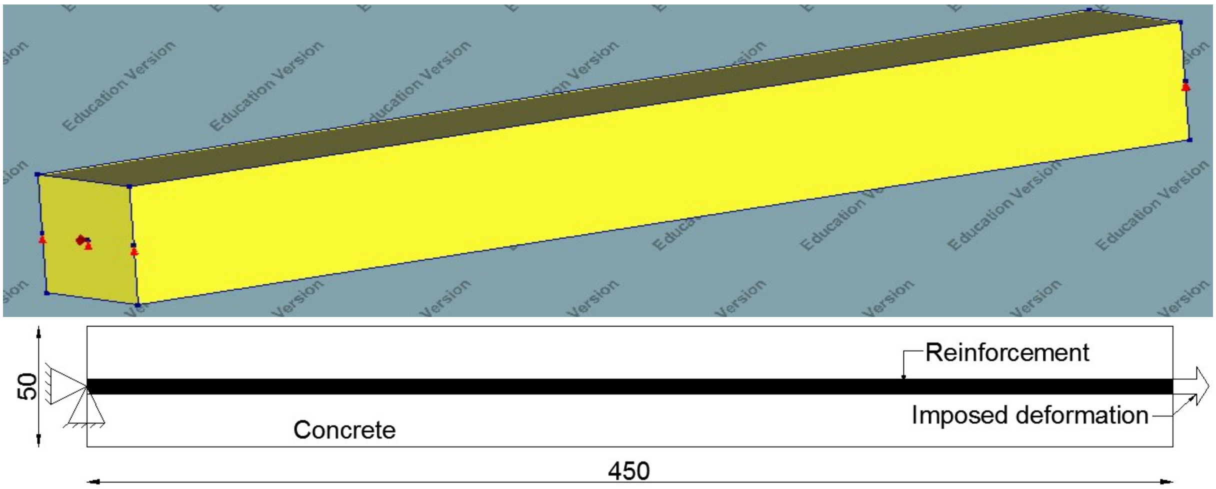


Figure 28 Tension bar model

Figure 29 Crack pattern

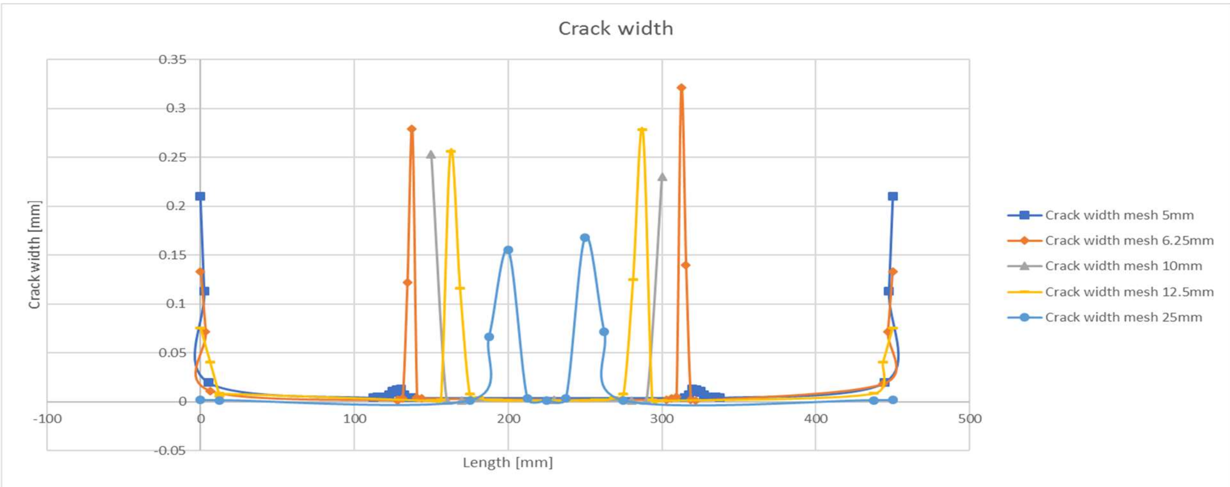


Figure 30 Crack spacing vs mesh size

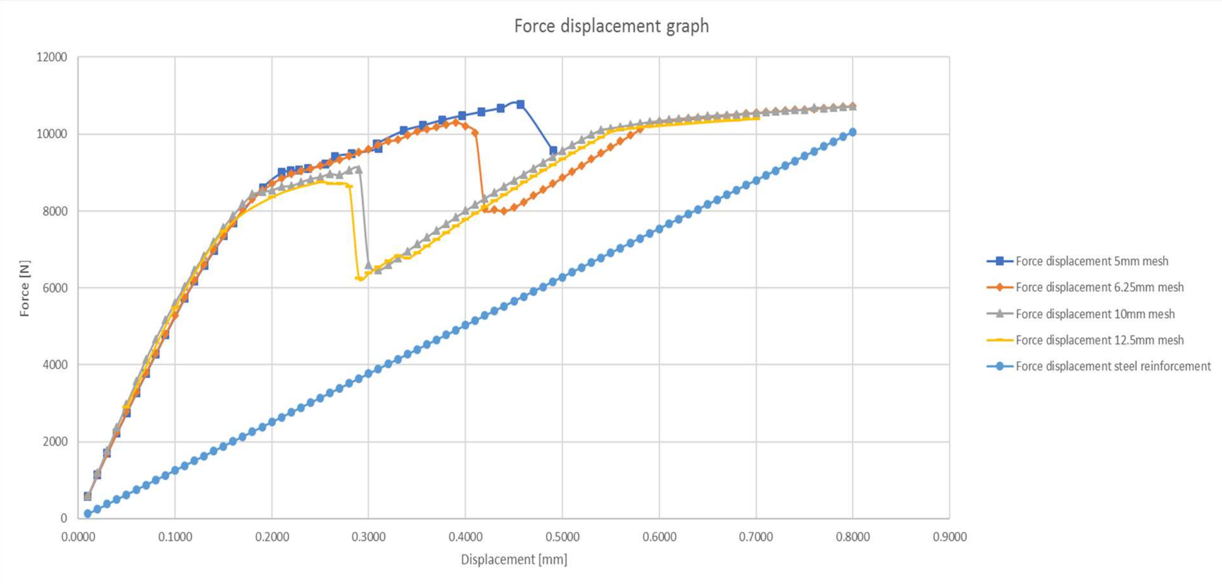


Figure 31 Force displacement diagram mesh dependent

In Figure 29, the crack pattern of the beam is shown. The mesh size used is 10mm. In the figure it is clearly visible that the beam cracks at the tips of the reinforcement and at the middle part of the beam. In this part the stress in the concrete exceeded the concrete tensile stress which results in a crack. A symmetry line is visible in the middle of the model because both cracks have the same distance from the tips of the reinforcement. In the model a fully developed crack pattern is visible because the length which is needed for a next crack to appear is too small.

In Figure 30, the crack spacing is given on the horizontal axis and the crack width is given on the vertical axis. The lines represent the cracks given by different mesh sizes. It is shown that the crack spacing decreases at an increase in mesh size. This implies that the crack spacing could be dependent on the mesh size. So for the determination of the transfer length of the bond stress a mesh size and crack spacing should be assumed which seems reasonable. The crack width given in this plot are not realistic but are shown to see the difference in between the mesh sizes.

In Figure 31, the force displacement diagram at the tip of the reinforcement is given. When a crack occurs the force suddenly decreases. Again the different lines represent the different meshes. As can be seen the smallest mesh size will crack at the highest force and vice versa. The lines intersect at different points which shows that the overall force displacement diagram has good conformity. The tension stiffening effect is visible by the difference between the lines of the different meshes and the line representing the reinforcement bar.

The overall conclusion about the model is that the model produces a crack pattern which will be hardly observed in reality, because the crack pattern is symmetrical with only two cracks in the model if the pull out failure of the cone is ignored. This implies that the crack spacing is the same for the cracks while in reality the crack spacing varies in length. The pull out failure cracks are induced by the buildup of the bond stress force from the point of prescribed deformation. The reinforcement is supported at both ends therefore the force exerted on the model is symmetrical. The model is homogeneous because a constant concrete strength results in less randomness in the results if the transfer length needs to be determined. In reality concrete is an inhomogeneous material which will result in different crack patterns and different force displacement diagrams. The crack pattern is fully developed in this case. To get a better understanding about the crack spacing the model needs to crack in several positions to get a more realistic crack pattern. The model needs to be extended and the prescribed deformation needs to be increased to get more cracks.

The same model with the same procedure will be reviewed, but this time the model will have a length of 1000mm. To check whether the mesh still has an influence on the crack spacing and to check whether the crack pattern will differ from the model described before. By lengthening the model it will be possible to model more cracks within the model. In the model of 1000mm length there are four distinct cracks and two cracks due to the pulling out failure of the cone. The four distinct cracks are shown in Figure 32. The positions of the cracks differ significantly, this is also shown in the figure in which the mesh is plotted against the crack spacing (see Figure 34). The crack spacing which is used in Figure 34 is the crack spacing in between the two distinct cracks. The crack spacing between the crack at the tip of the reinforcement and the first crack in the model is disturbed by the fact that the cone at the tip around the reinforcement cracks. This will result in a smaller crack spacing than the one given by the two distinct cracks, this is shown in Figure 33. If the prescribed deformation were to be increased it could be possible that more cracks will form in the uncracked section between the middle two cracks, because the force due to bond can build up in this part of the model which can result in a crack. Due to the fact that the length which is still available is larger than the length of the crack spacing.

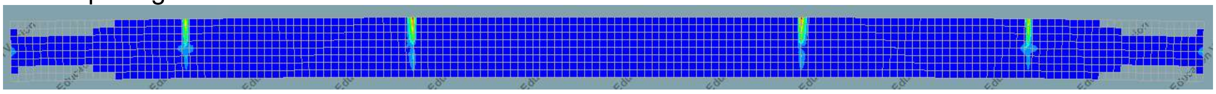


Figure 32 Cracks within the 1000mm model 6.25mm mesh

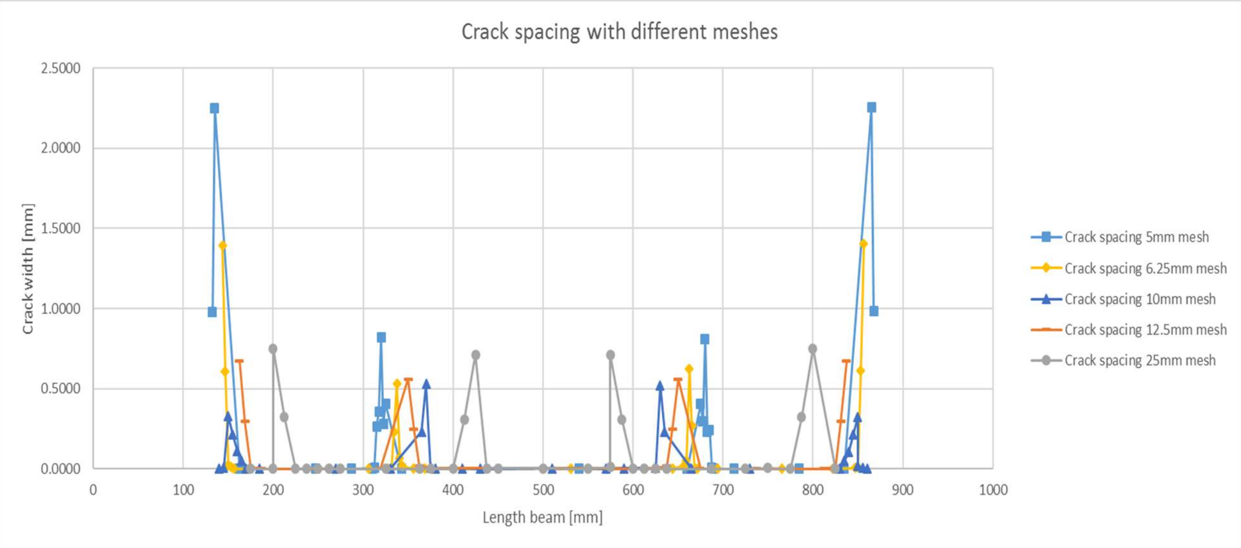


Figure 33 Crack spacing 1000mm length model

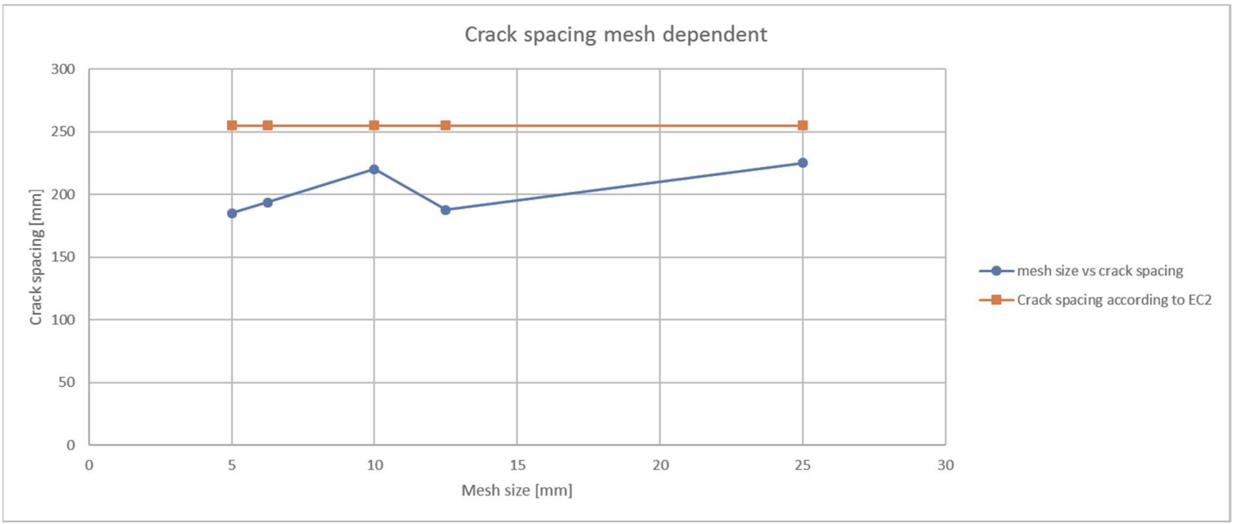


Figure 34 Mesh size vs crack spacing 1000mm length model

In Figure 34 the crack spacing is plotted versus the size of the elements as is stated before. Within this plot it is visible that the mesh size does not have a huge influence on the crack spacing, because all crack spacings are in the range of 185mm and 225mm. These minimum and maximum values are close to each other. In reality the crack spacing is also variable and varies between 1 and 2 times the transfer length. Another possibility of the variation in crack size is the element size itself because the cracks are determined in the nodes. The nodes are positioned in the corners and middle of the element. If the element size is large the length in between the nodes is larger as well. So an exact position of the crack is hard to determine and there is some variation possible in the crack spacing. It is shown that all crack spacings given by the finite element models are smaller when the crack spacings are compared to the maximum crack spacing given by Eurocode 2.

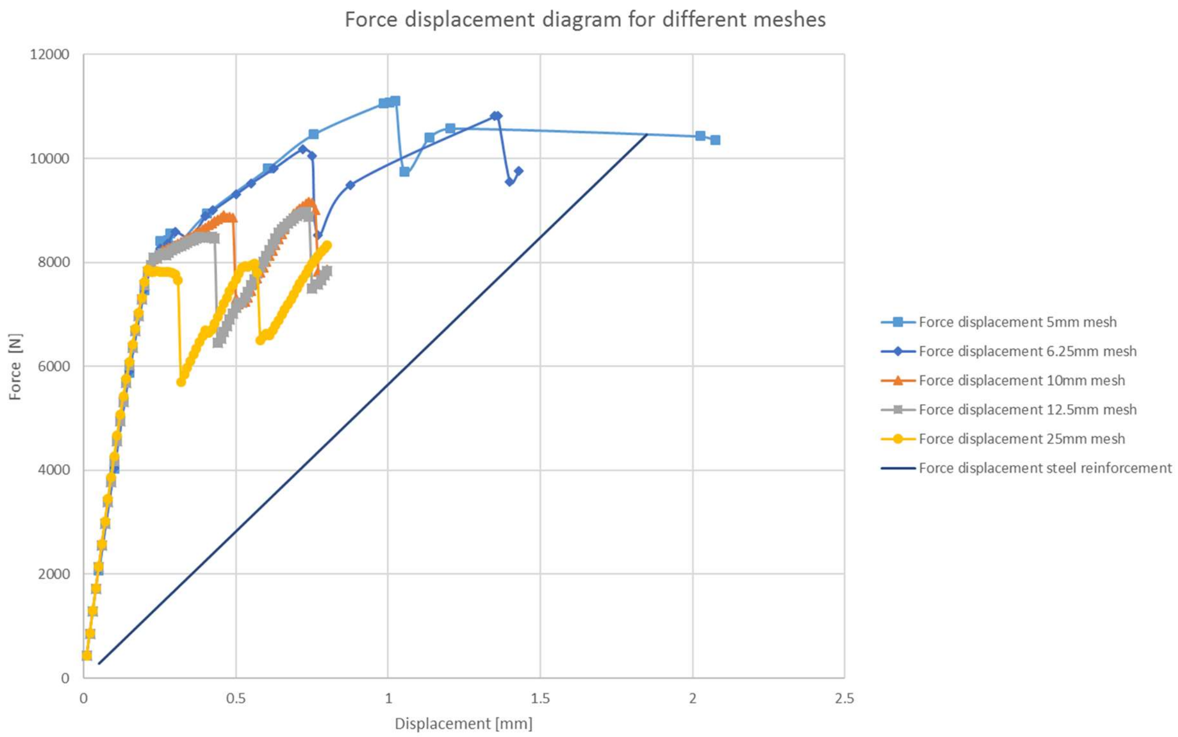


Figure 35 Force displacement diagram mesh dependent of the 1000mm length model

In Figure 35 the force displacement diagram is given for the 1000mm model. It is shown that the model has a higher cracking force with a decrease in mesh size. This also holds for the 450mm model. Before the first crack occurs all mesh sizes have the same elastic stiffness which should be correct because the concrete stiffness is the same for every model.

To check the influence of the length of the model on the crack spacing, a model has been investigated by using different beam lengths with the same mesh and bar diameter. The models which have been investigated are: L=1000, 1100, 1150, 1200 and 1250mm. The element size which is used is 12.5mm and the bar diameter is 6mm. This mesh has been chosen because the computational time will be less with this mesh and the mesh does not have a huge influence on the crack spacing as shown earlier. The different lengths of the model have been plotted on the x-axis of the graph. On the y-axis the average cracks spacing is given. As can be seen in Figure 36 the average cracks spacing is approximately the same for all lengths. Furthermore the crack spacing given by the finite element models is smaller than the maximum crack spacing according to Eurocode 2.

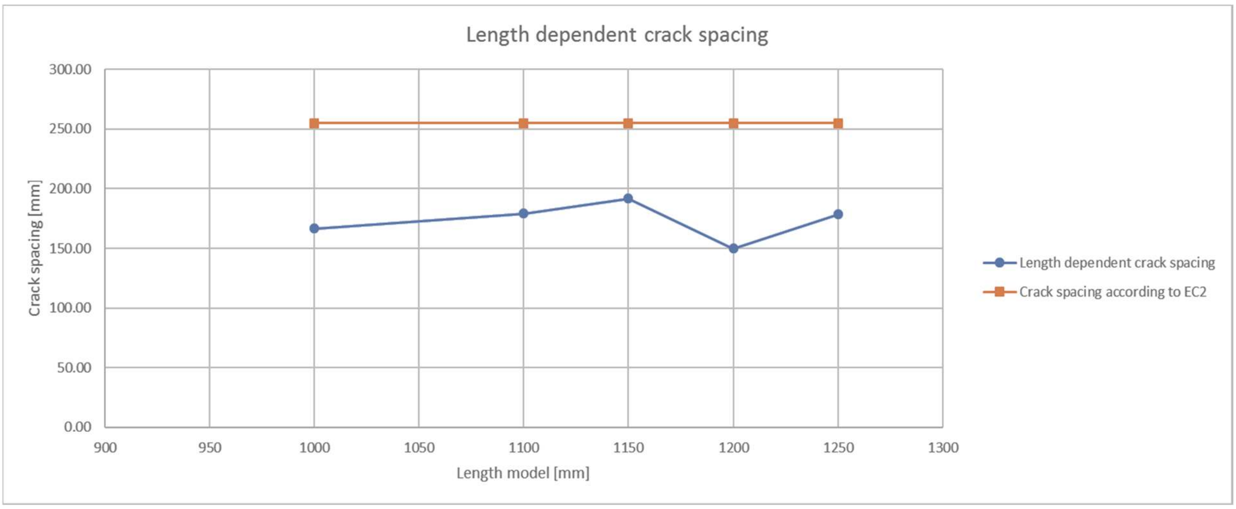


Figure 36 Length dependent crack spacing

The influence of the reinforcement diameter has also been checked by taking a look at the difference in the force-displacement diagram. When a crack occurs in the model the force in the force-displacement diagram decreases significantly by an increase in deformation. By increasing the prescribed deformation the force increases as well. If the prescribed deformation is large enough, then the model will crack at several locations. These cracks are showed in the force displacement diagram. The influence of the diameter have been checked by using a model with 1200mm length 50mm height and width and varying diameters of 6, 8, 10, 12mm.

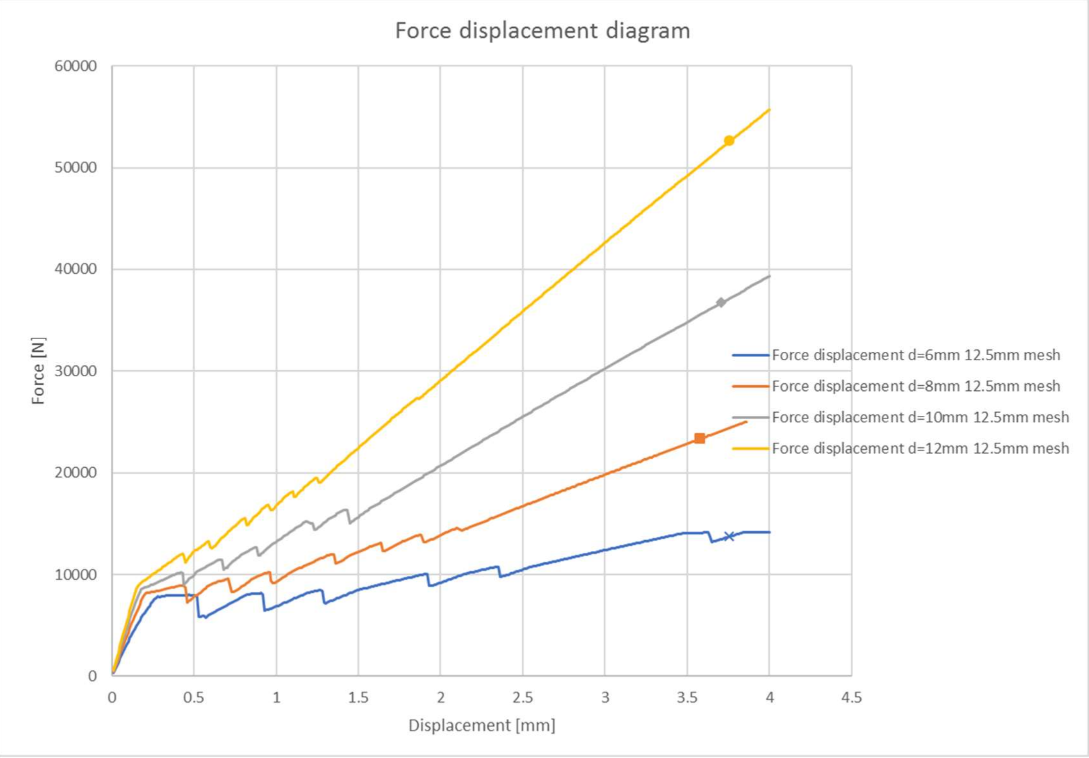


Figure 37 Force displacement diagram different reinforcement diameters

As can be seen in the force displacement diagram the model with the highest diameter (12mm) has the steepest line in the force displacement diagram. The decrease in force due to cracks is small for this configuration. This is due to the fact that the model almost behaves like a plain reinforcement bar under tension without concrete. A plain reinforcement bar has a higher average stiffness than a tension bar model with reinforcement and concrete. The tension stiffening effect is small for this configuration.

If the diameter of the reinforcement decreases the cracks will be more pronounced in the force-displacement diagram. The stiffness, which is given by the steepness of the line in the diagram, will decrease by a decrease in reinforcement diameter. The tension stiffening effect is larger for situations with smaller diameters. With a smaller diameter the model behaves as a tensile bar model instead of a model which almost behaves as a plain reinforcement bar. This was the case for the model with the largest diameter.

The model cracks in two different phases. First the model cracks symmetrically in certain locations, this is shown in Figure 38. After the first cracks, the model cracks between the already existing cracks, this is shown in Figure 39. A fully developed crack pattern is given in Figure 39, this can also be observed in the force-displacement diagram because the lines keep increasing linearly after the last cracks. No new cracks can be formed and the force is taken up by the reinforcement when the crack pattern is fully developed.

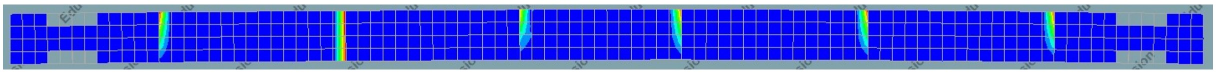


Figure 38 First cracking stage

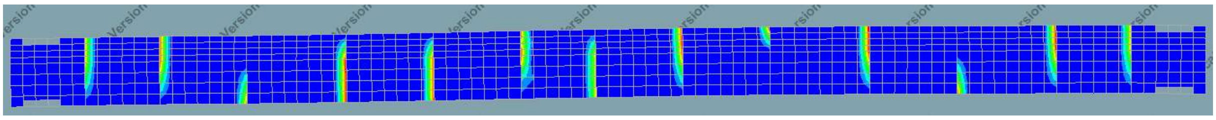


Figure 39 Second cracking stage

Concrete cracks when the concrete tensile strength is reached. In the figures above it is shown that the stress can built up in between the already existing cracks, because new cracks form in between the existing cracks. The buildup of stress is a result of bond stresses between the concrete and reinforcement. The bond stress in between the cracks is given in Figure 40. The model which is used is a model with 1200mm length, 50mm width, 50mm height and a diameter of the reinforcement of 8mm. Only a part of the model has been plotted but this part shows the gradient of the bond stress in good conformity. Within the graph the crack width is also plotted to show how the bond stress behaves in contrast on the crack width. The crack width has been exaggerated by a factor 10 otherwise it would be a flat line in the graph. When the crack width is at its maximum, the bond stress passes the horizontal y=0 axis which should be the case when looking at the literature about cracking [9]. The horizontal branches in the bond stress show the maximum bond stress, due to the fact that the bond stress is implemented as a bi-linear curve, the bond stress has a horizontal maximum. The gradient of the bond-slip diagram can be observed in the figure with the bond stress.

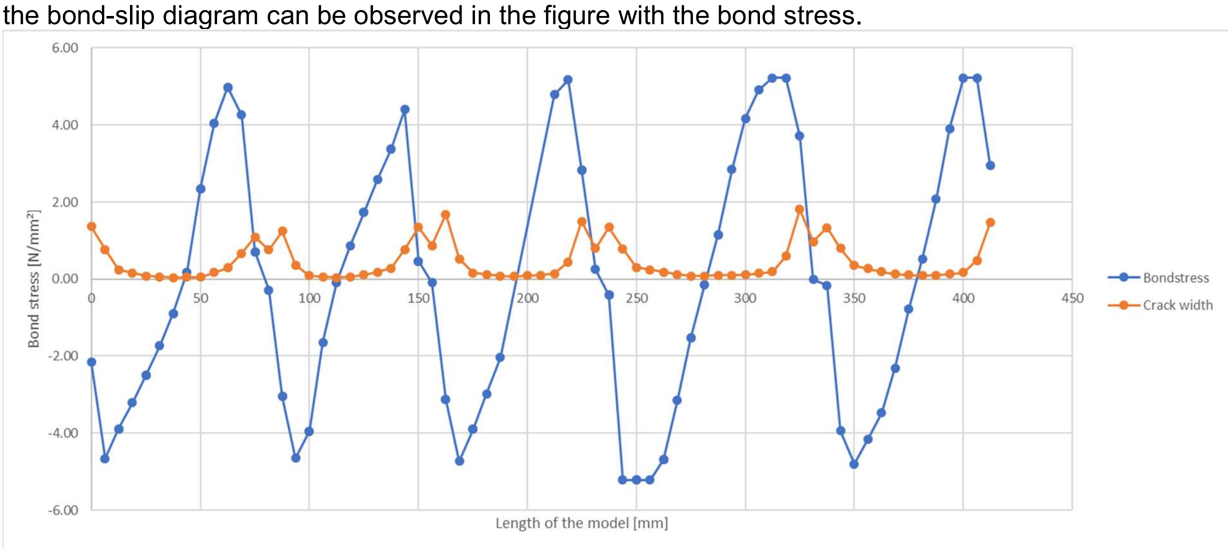


Figure 40 Bond stress gradient of a part of the model

The bond stress depends on the slip (relative displacement). This behavior is given by the bond stress-slip diagram. In DIANA the slip of the interface surrounding the reinforcement can be obtained as well. This slip is equal to the displacement in the length direction of the reinforcement. The slip is shown in Figure 41. The model which is considered here is a smeared cracking model. In the smeared cracking model the elements lose stiffness when they crack. If the stiffness decreases the displacement decreases as well. Therefore the slip is close to zero in the center of the crack. The slip at the crack is negative on one side of the crack and positive on the other side of the crack. This shows that in a crack the concrete moves in both directions which is reasonable because the crack opens. Due to the positive and negative slip, the bond stress is positive and negative as well because the bond stress is depending on the slip.

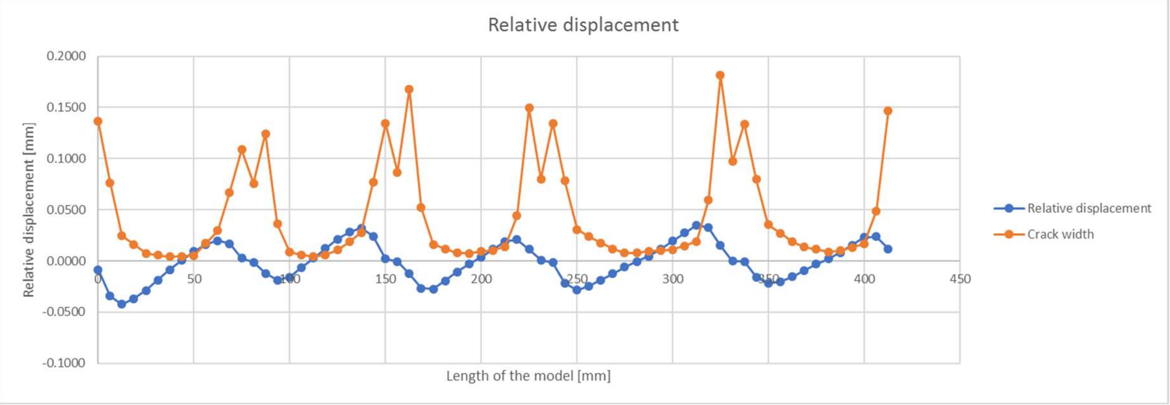


Figure 41 Relative displacement of the interface around the reinforcement

In Figure 42 the stress in the Y-direction and the crack widths are plotted as a function of the length of the model. The model consists of a concrete tensile member with a length of 1200mm, an height and width of 50mm and a reinforcement bar placed in the middle of the model with a diameter of 6mm. The stresses and crack widths are measured at the position of the reinforcement. In the diagram the contour plots of the stresses and crack widths are given as well. In these contour plots it is clearly visible where the cracks originate and what the influence of a crack on the stress is. The concrete stress decreases to zero at the position of the crack.

At two positions (shown by the arrows) new cracks can originate because at these positions the stress in the concrete reached the tensile concrete strength. The concrete tensile strength is given by the horizontal grey line. According to the literature the distance in between two cracks at which the stress can build up till the concrete tensile strength should be the maximum crack spacing. The maximum crack spacing is equal to two times the transfer length. With the transfer length the effective concrete tension zone can be calculated. By using this method it is possible to calculate the transfer length.

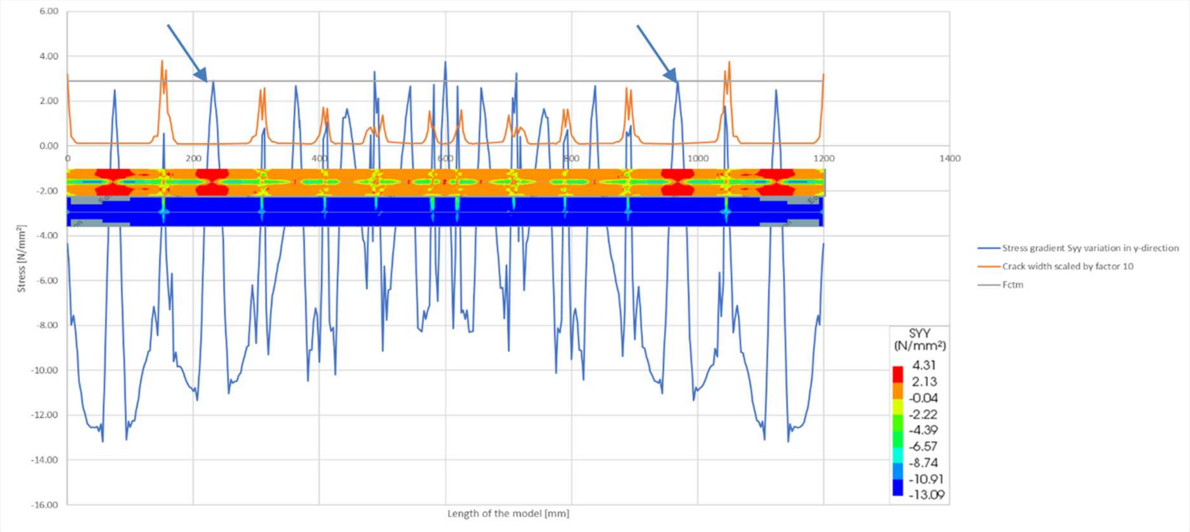


Figure 42 Stress in Y-direction and crack width plotted over the length of the model

4.2 Improvements in terms of bond model and convergence

The 3D models which have been considered in the previous paragraph experience difficulties with the convergence tolerances. One of the reasons for the problems is the bi-linear bond-slip curve. This is a curve with a linear part and a horizontal constant part as is shown in an earlier paragraph. The horizontal constant part causes the model to have different solutions for the same bond stress. Multiple solutions for one outcome result in not meeting the convergence criteria.

Converging of the model is very important because the solution is useless if the solutions are not within certain tolerances. Due to the non-linear behavior of the models there are different solution paths possible but the model has to convergence to know if the solution path chosen is the right one. In some of the load steps described above, the solution does not always converge which suggests that some parts of the solution are not completely correct however overall the model is converged. The model has difficulty converging predominantly when cracks occur. This is due to the fact that the concrete material is homogeneous, which implies a constant tension strength over the whole model. When the model is loaded by pure tension, which is the case here, the model can crack at every position. This makes it hard for the finite element model to converge during cracking. After cracking, the model is able to converge again at different load steps, so the solutions given after cracking are useable solutions.

In DIANA there are different bond-slip models applicable within the program as discussed before. One of these models is the Model Code 2010 bond-slip model. This model will be used in further calculations because this model gives less difficulties in terms of the convergence criteria. The model will have a different bond stress for different slip values up to a slip of 1mm. For the models used in this master thesis the slip of 1mm will not be reached, so the part after the 1mm slip is not relevant. The bond-slip model is shown below up to a slip of 2mm. A concrete class of C30/37 is used in the thesis and calculations.

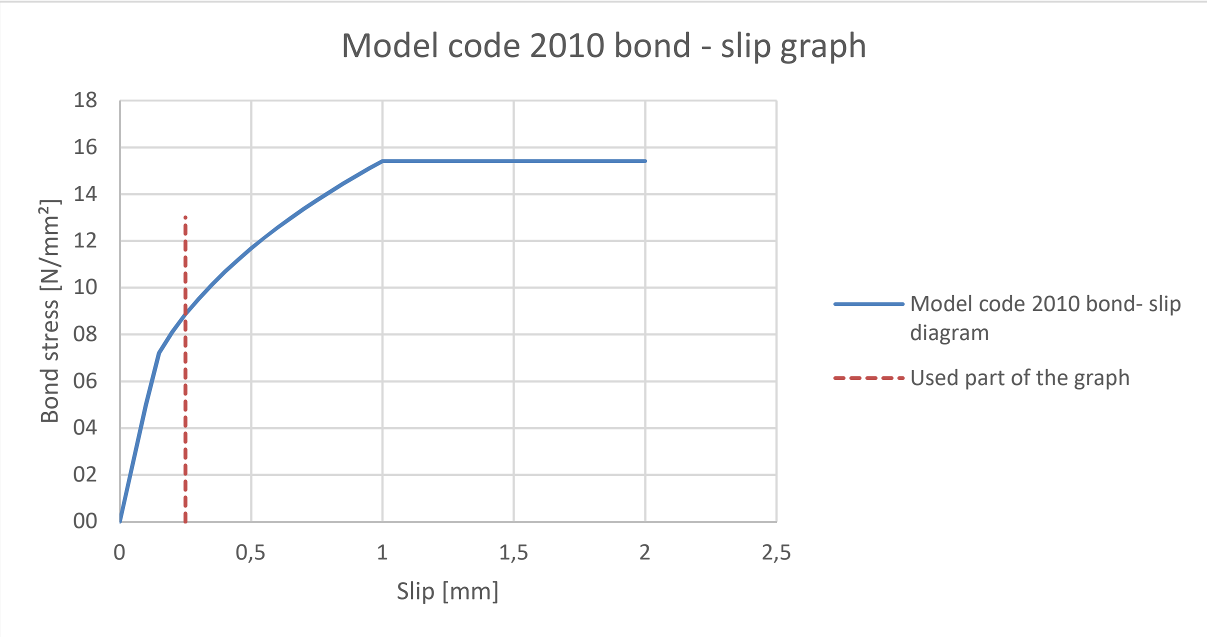


Figure 43 Bond-slip diagram according to fib Model Code 2010

The linear branch of the bond-slip model is given by a shear stiffness of 50 N/mm³. The stiffness is lowered because a too stiff bond-slip interface gives numerical errors in the finite element model. These numerical errors will result in not meeting the convergence criteria. Furthermore the stiffness represents the unloading and reloading branch of the bond-slip model, if this stiffness is too high the residual slip will be too high. Which again causes problems with the convergence. The shear stiffness of the model does have an effect on the crack spacing because the maximum bond stress is reached faster with a smaller slip. If the average bond stress is higher the transfer length can be smaller because the force which is needed for cracking remains the same.

To check whether the model did convergence during the maximum amount of iteration steps a graph can be plotted. In this graph the iteration steps needed for the model to converge and the convergence criteria are plotted for every load step.

2D models will be used in further calculations because the models which will be investigated are larger and consist of too many nodes and elements, if performed in 3D. Therefore the time required to run a model is too long. Furthermore, the output and data files of 3D models require much more disk space. The available disk space was not sufficient to save the 3D model files. With 2D models the time needed for a model to run was still significant but it was doable in contrast to the 3D models. With 2D models it is possible to use a finer mesh because the amount of elements is decreased in comparison with 3D models. Furthermore the bond stress behavior and the crack pattern, which are key parameters for this thesis, do not vary between a 2D and a 3D model. Therefore it is allowed to use 2D models instead of 3D models.

5 VARIATION STUDY OF THE 2D (CQ16M) TENSILE MEMBER MODEL

For a variation study of the influence of the cover, width and reinforcement diameter on the effective concrete tension zone, different 2D models will be investigated. The model which is investigated consists of a beam with two reinforcement bars, one at the top of the model and one at the bottom of the model see Figure 44. The dimensions of the model are: length=4000mm, height=500mm, width=200mm(normal situation), cover=60mm(normal situation), reinforcement diameter=40mm (normal situation). The bar diameter varies from 20mm till 40mm with intermediate diameters of 25 and 32mm. The width varies from 100mm till 250mm with intermediate steps of 50mm. A maximum width of 250mm is used because the influence zone of a reinforcement bar in the width direction is given by $5 * \left(C + {}^{\varnothing_{sl}}/_{2}\right) = 5 * \left(40 + {}^{40}/_{2}\right) = 300mm$. The cover varies from 50mm till 100mm with intermediate steps of 10mm. Cover in Eurocode 2 is defined by the distance between the perimeter of the reinforcement and the outer fiber of the beam. In this thesis the term cover means the distance from the outer fiber of the beam to the center of the reinforcement. So there is a difference of the radius of the reinforcement between the cover term of Eurocode 2 and the cover term used in this thesis. This is due to the fact that the center of the reinforcement needs to be defined in DIANA.

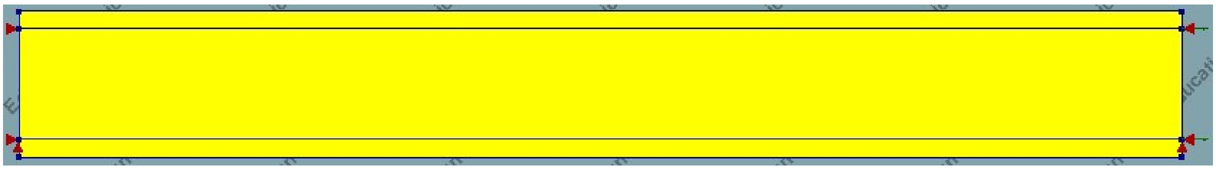
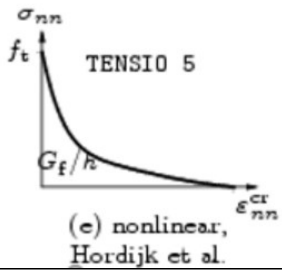


Figure 44 2D model of the variation study

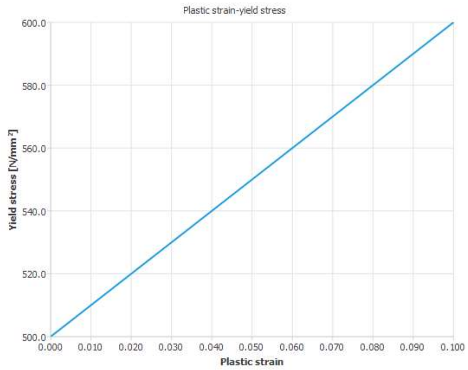
The concrete and steel properties which are used are given in the tables below. The concrete properties are given for concrete class C30/37. The steel properties are given for B500 reinforcement.

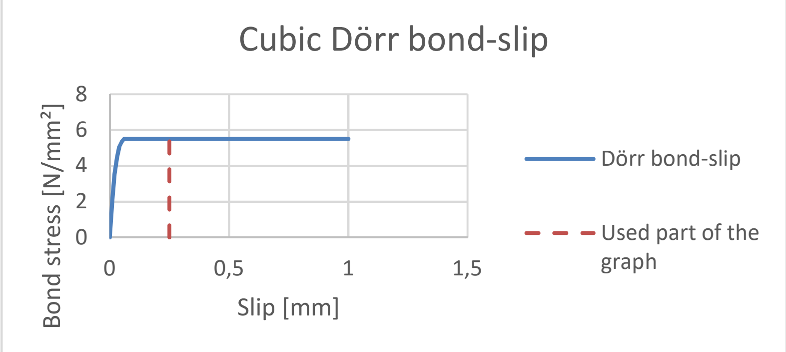
Table 2 Concrete material properties 2D model

The concrete material properties:	
Material class	Concrete and masonry
Material model	Total strain based crack model
Young's modulus	33.000 N/mm²
Poisson's ratio	0.15
Mass density	2400 kg/m³
Crack orientation	Rotating
Tensile curve	Hordijk 
Tensile strength	2.9 N/mm²
Mode-I tensile fracture energy	0.1405 N/mm
Crack bandwidth specification	Rots
Reduction model	Damage based
Compression curve	Elastic

The reinforcement material properties:

Table 3 Reinforcement material properties 2D model

Material class	Reinforcements and pile foundations
Material model	Bond-slip reinforcement
Young's modulus	210000 N/mm²
Mass density	7850 kg/m³
Plasticity model	Von Mises plasticity
Hardening function	Plastic strain-yield stress
Hardening hypothesis	Strain hardening
Plastic strain-yield stress	
Hardening type	Isotropic hardening
Normal stiffness model	1000 N/mm³
Shear stiffness model	50 N/mm³
Bond-slip interface failure model	CEB-fib Model Code 2010 Bondslip function
Maximum shear stress τmax	15.41 N/mm²
Ultimate shear stress τf	6.164 N/mm²
Linearized initial shear slip s0	0.1 mm
Relative shear slip s1	1 mm
Relative shear slip s2	2 mm
Relative shear slip s3	5 mm
Exponent alpha	0.4
Bond-slip curve MC2010	

Bond-slip curve Doerr	

First an introduction will be given about how to determine the effective concrete tension zone height. Thereafter the variation study of the 2D models will be given. Not all models will be shown here, for further information about all models a reference is made to Appendix B. One model of every type of variation will be given and examined. The overall conclusion of the models will be given after the examination of the model.

5.1 Introduction to the determination of the effective concrete tension zone height

The determination of the effective concrete tension zone height is done in a specific way, which will be explained in this chapter. The effective concrete tension zone is the part of the structure which is under tension and will crack when the tensile strength is exceeded. When the concrete cracks, the stress within the concrete in the crack is zero. This implies that the reinforcement should bear the force within the crack. So at a crack, the stress within the reinforcement increases. The force due to the increase in steel stress is equal to the force which build up due to bond stress. This force is called the bond force and can be determined by the average bond stress over the transfer length and the perimeter of the reinforcement.

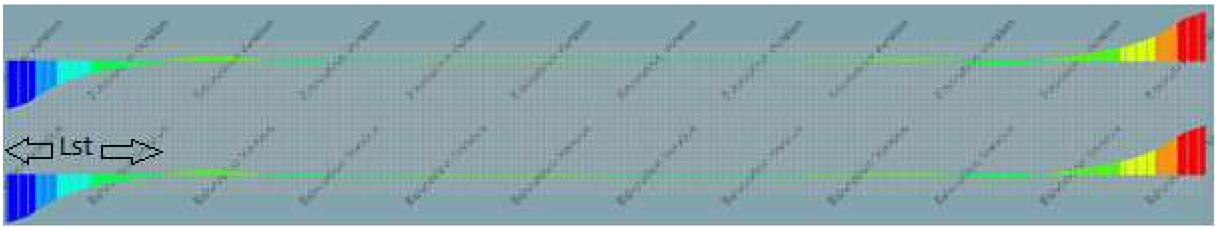
$Bond\ forc\quad \Delta\sigma_s * A_s$

$Bond\ force = averag\quad bond\ stress * transfer\ length * perimeter\ reinforcement$

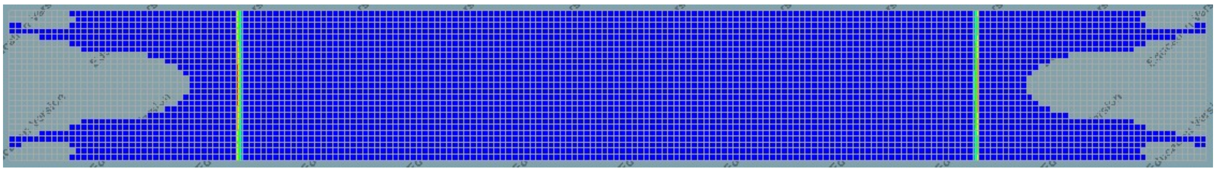
$Bond\ force = \tau_{b,ave} * Lst * U_s$

Just before the crack the bond force reaches its maximum. This force needs to be taken into account to determine the effective concrete tension zone. The force can be determined using finite element models however the positions and load steps of the cracks first need to be determined. The load steps at which a crack occur can be obtained by looking at the convergence graph because the steps at which the model did not converge represent the load steps in which the model cracks. In DIANA the load steps which did not converge are checked to see if the cracks form in these steps. If the model cracks, the load steps just before the crack occurs will be used to determine the average bond stress, with which the bond force will be determined. The average bond stress is determined by integrating the area under the bond stress curve and dividing this area by the transfer length. The transfer length is the length over which the bond stress passes the y=0 line twice.

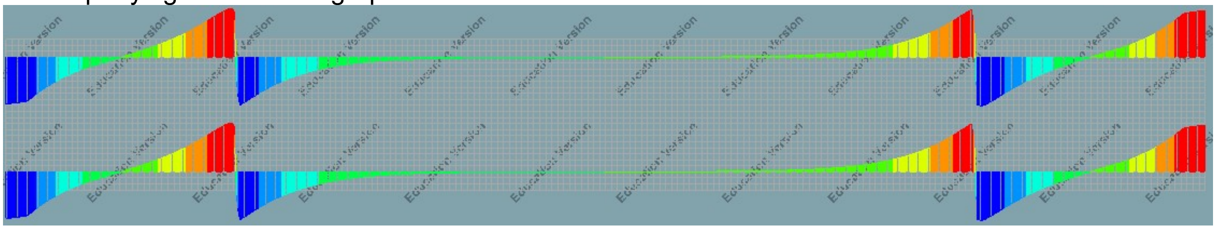
The steps in which the model cracks are given in the figures below. For every model the load steps and positions are different but the method on how the determine the bond force is the same.
Before the first crack:



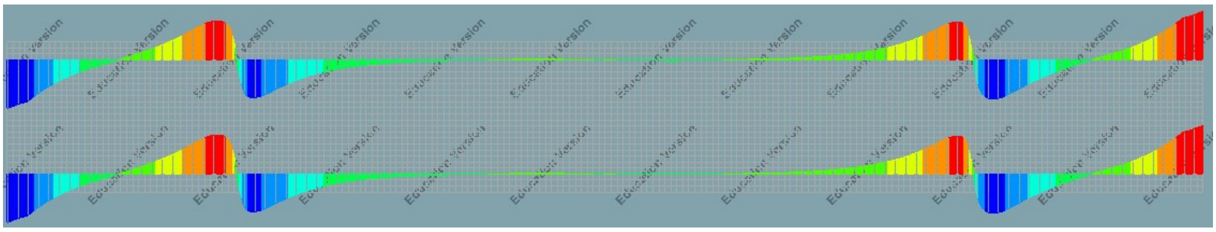
First crack:



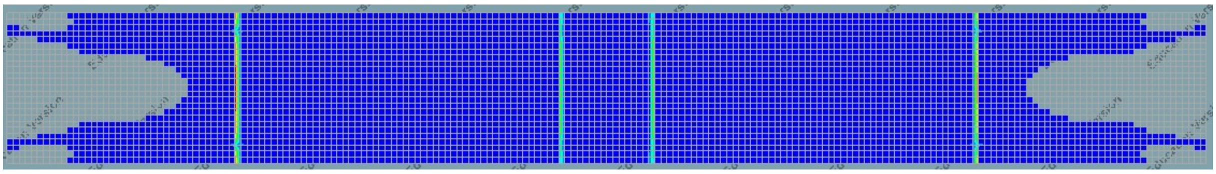
Accompanying bond stress graph:



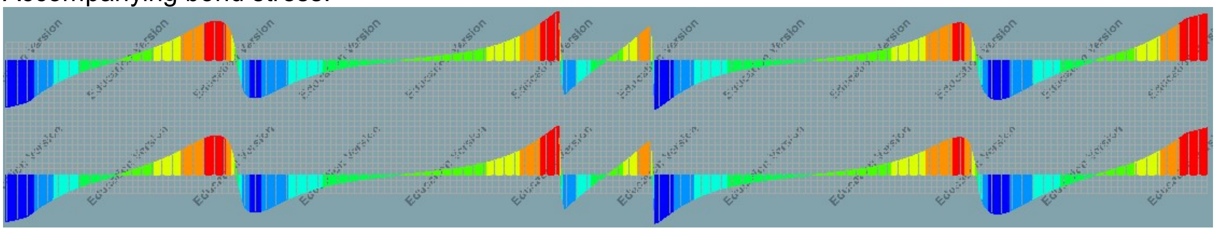
Before the second crack:



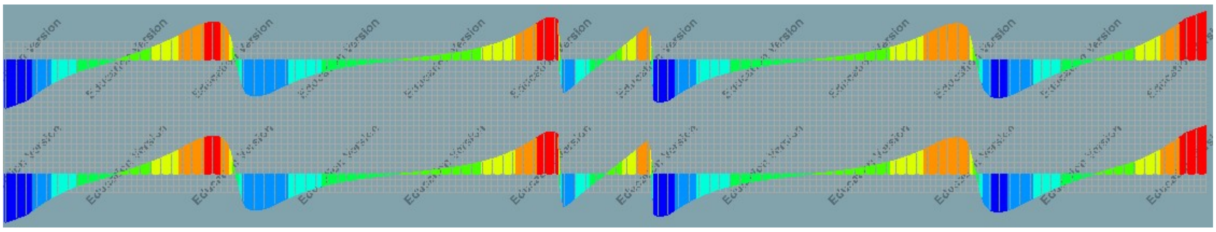
Second crack:



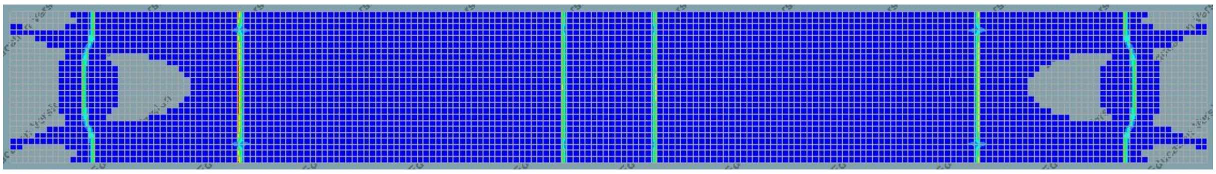
Accompanying bond stress:



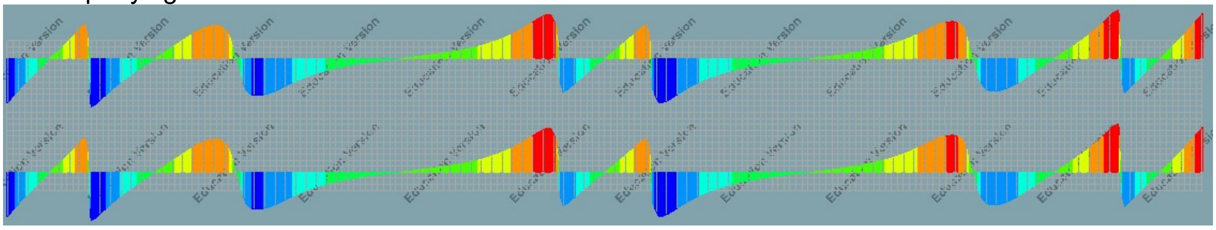
Before the third crack:



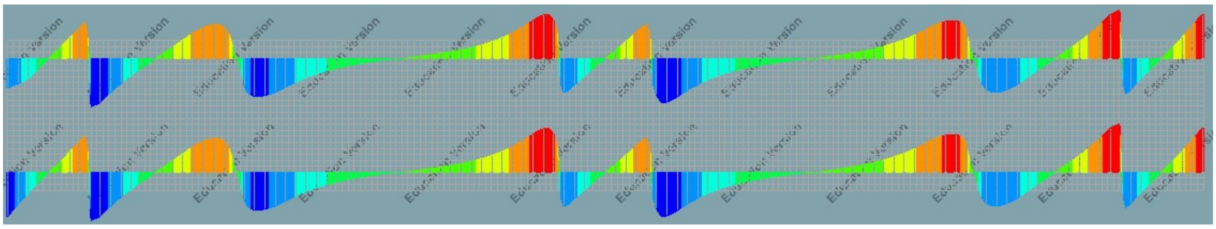
The third crack:



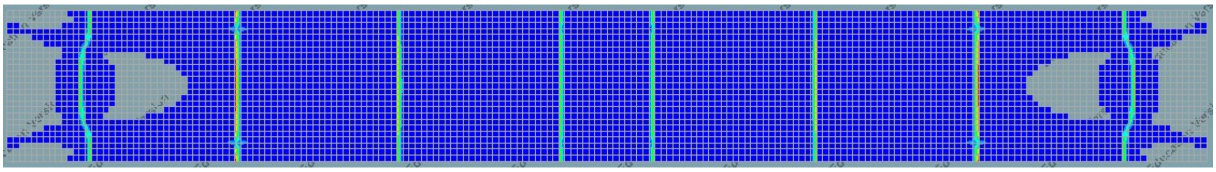
Accompanying bond stress:



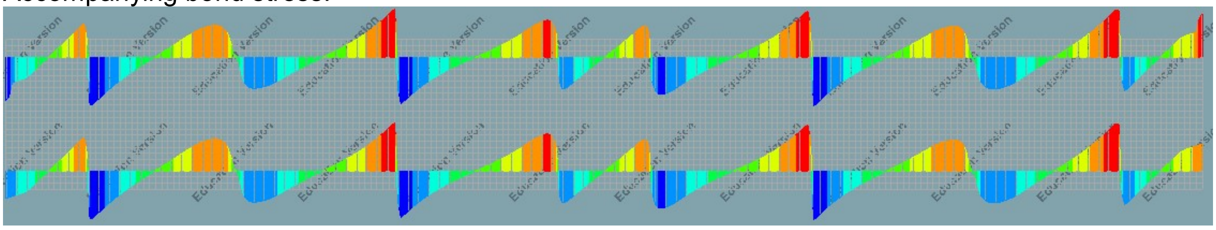
Before the fourth crack:



The fourth crack:



Accompanying bond stress:



This process is repeated for every crack until the point were no new cracks form in the model. With the bond forces or cracking forces, the average cracking force will be calculated. This will be done by adding the bond forces together and dividing the sum of the bond forces by the number of bond forces. With the average cracking force the effective concrete tension zone height can be determined by dividing the average cracking force with the concrete tensile strength and the width of the model. This effective concrete tension zone height will be compared to the effective tension zone height given by Eurocode 2 or Jones method.

5.2 Variation in cover

The first model which will be examined is the model in which the cover is changed from 50mm to 100mm. In these examples the cover is the distance from the outer fiber of the concrete beam to the center of the reinforcement as was described before. The model has the following outer dimensions.

Table 4 Dimensions of the 2D model with 70mm cover

Variables:	Dimension [mm]
Length	4000
Width	200
Height	500
Diameter reinforcement	40
Cover	70
Mesh size	20

Figure 45 2D model with 70mm cover

In Figure 45 the 2D model is shown. On the right hand side the prescribed deformations are given as small arrows. The prescribed deformation is 0.01mm per load step in the positive Y-direction of the model. The triangles display the supports which are used. The bottom reinforcement is supported in both horizontal x-direction and in vertical y-direction. The top reinforcement is only supported in horizontal x-direction because the model undergoes transverse contraction due to the tension force. The transverse contraction will result in stresses in the model in the transverse direction if the model is supported on both reinforcement bars. The stresses will result in parallel cracks around the reinforcement, these cracks need to be prevented to occur because they influence the results needed for this master thesis.

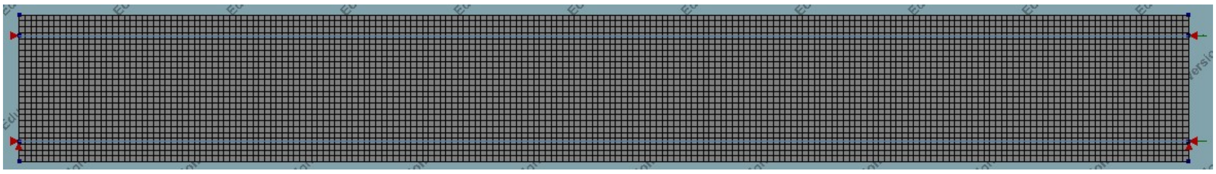


Figure 46 Mesh of the 2D model with 70mm cover

In Figure 46 the mesh of the 2D model is given the mesh size used is 20mm. The mesh is fine because the position of the cracks, bond stress and transfer length need to be accurately determined.

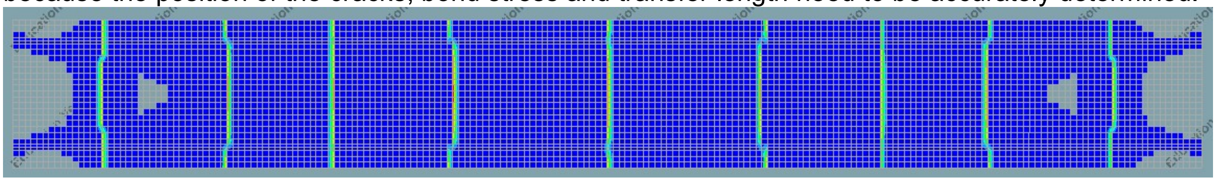


Figure 47 Crack pattern of the 2D model with a cover of 70mm

In Figure 47 the crack pattern is shown for the 2D model with a cover of 70mm. The crack spacing varies over the length of the model. In the middle of the model a symmetry line can be observed. This effect is obtained because the model is loaded on the right side of the model and supported on the left side of the model. In total there are 9 distinct crack visible within the model. Some cracks run perfectly perpendicular to the reinforcement while other cracks bend when they passed the reinforcement. A clear reason for this phenomena is not obtained, but it has to do something with the height of the model. If the height of the models is increased the cracks tend to bend more.

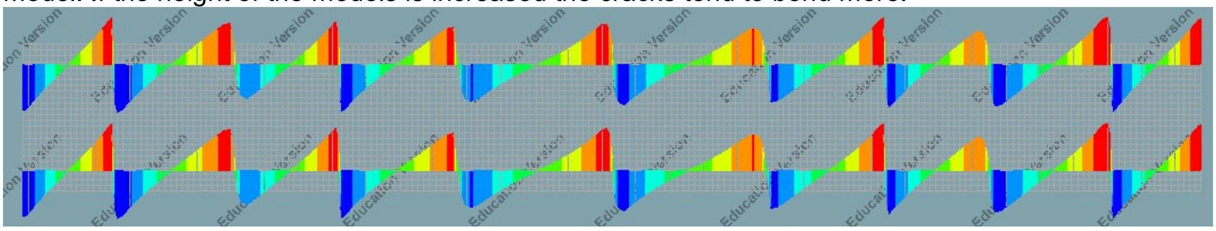


Figure 48 Bond stress gradient of the 2D model with a cover of 70mm

In Figure 48 the bond stress gradient of the 2D model with a cover of 70mm is given. The positions of the cracks can be observed because at the positions of the cracks, the bond stress changes from a maximum positive value to a maximum negative value. At these positions the slope of the curve is steep. Between the cracks, the bond stress runs from a maximum negative value to a maximum positive value with a nearly constant linear slope. The buildup of the bond stress is clearly visible. Furthermore the shape of the bond-slip graph is visible because the bond stress has a linear gradient which is also the case for the bond-slip graph. The slip within the models is too small to get to the horizontal part of the bond-slip graph.

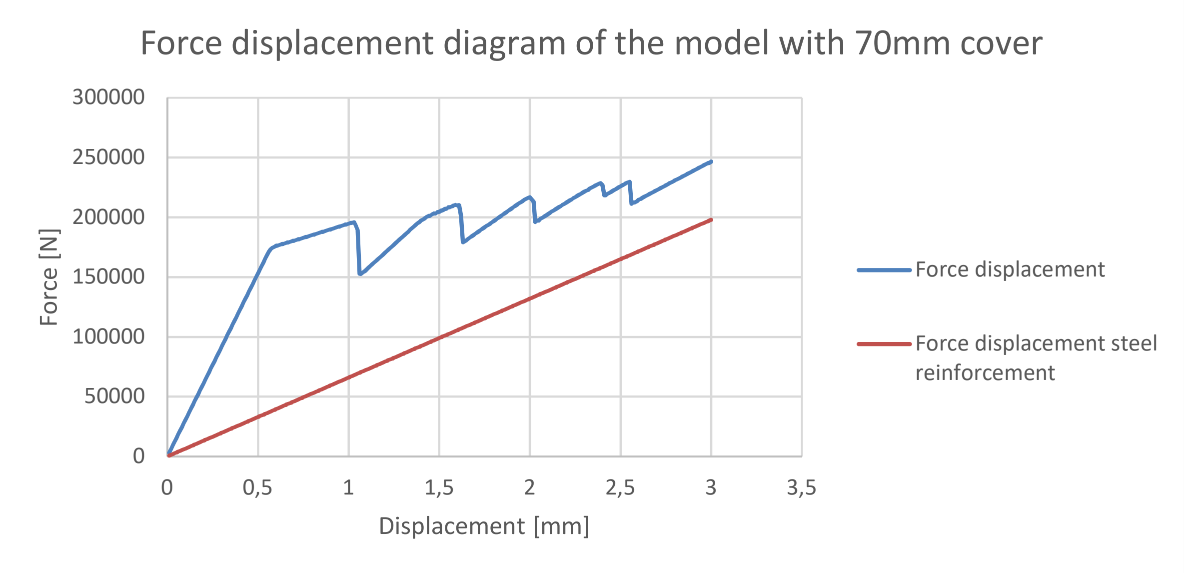


Figure 49 The force displacement diagram of the 2D model with a cover of 70mm

In Figure 49 the force-displacement diagram of the 2D model with a cover of 70mm is given. Within this figure, the elastic linear part of the force-displacement diagram can be observed. After the elastic part the model reaches the cracking force. Due to the fact that the concrete is homogeneous and loaded by tension a large part of the structure can crack at the same time. This is because the tensile strength is exceed in many elements at the same time. The large area which can crack, gets smaller by an increase in load till a distinct crack occurs within this area. Before the distinct crack occurs, the model is already out of the elastic branch which will result in a loss of stiffness. The decrease of stiffness can be observed in the force displacement diagram because the force increase is smaller by an increase in displacement. When the distinct crack occurs, the force decreases substantially, after which the force builds up until a new crack occurs. This process repeats itself until a fully developed crack pattern is reached. A fully developed crack pattern is given in the force-displacement diagram. The number of cracks for half of the model equals 5. This amount of cracks can also be observed in Figure 47 because the model has a symmetry line in the middle of the model. In the model a total of 9 cracks can be observed, one of these cracks is exactly in the middle of the model. The linearly increasing line represents the behavior of the plain reinforcement. The tension stiffening effect is obtained by the difference between the plain reinforcement and the tensile model.

In the table below the average bond stress, transfer length, perimeter of the reinforcement, cracking force, concrete tensile strength and the effective concrete tension zone are given. These are the values for only half of the model but due to symmetry in the middle of the model all these values can be multiplied by two to get the values for the whole model. With these values the average cracking force can be calculated.

Table 5 Summary of the parameters of the 2D model with a cover of 70mm

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
2.40	490	125.7	147939	2.9	51014
1.94	540	125.7	131434	2.9	45322
3.05	350	125.7	133966	2.9	46195
2.77	370	125.7	128947	2.9	44465
1.86	520	125.7	121589	2.9	41927
2.50	380	125.7	119398	2.9	41172
2.49	380	125.7	118851	3.9	30475

$$\sum cracking\ force = 1804248N$$
$$number\ of\ cracking\ forces = n = 14$$
$$Average\ cracking\ force = N_{cr,ave} = \frac{\sum cracking\ force}{n} = \frac{1804248}{14} = 128875N$$

With an average cracking force of 128875N, the effective concrete tensile zone becomes: $A_{c,eff} = {N_{cr,ave}}/{f_{ctm}} = 128875/2.9 = 44440mm^2$. This implies an effective concrete tension zone height of:

$$h_{c,eff} = \frac{A_{c,eff}}{b} = \frac{44440}{200} = 222mm$$

The effective concrete tension zone height for this case is, according to Eurocode 2, the minimum of: $h/2$ and $2{,}5 * (c + Ø_{sl}/2)$.

$$EC2: \min\left[\frac{h}{2} = \frac{500}{2} = 250mm, \quad 2{,}5 * \left(50 + \frac{40}{2}\right) = 175mm\right] = 175mm$$

The value determined by the finite element models is larger than the value given by Eurocode 2. The difference in effective concrete tension zone height is 47mm. For this configuration it is therefore not possible to get a reduction of the effective reinforcement ratio, which would have a positive effect on the crack width formula. The difference in crack width is 0.03mm. Here, one example has been elaborated in order to give a conclusion about the effect of the cover on the effective concrete tension zone the models in which the cover varies need to be compared with each other. In Appendix B all models with a varying cover have been elaborated. The results are presented here to make a comparison.

Table 6 Average values of the results of the 2D models with a varying cover

Cover c [mm]	Average cracking force $N_{cr,ave}$ [N]	Concrete tensile strength f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]	Width of the model B [mm]	Effective concrete tension zone height $h_{c,eff}$ [mm]	Effective concrete tension zone height EC2 $h_{c,effEC2}$ [mm]
50	113527	2.9	39147	200	196	125
60	132006	2.9	45519	200	228	150
70	128875	2.9	44440	200	222	175
80	136860	2.9	47193	200	236	200
90	133567	2.9	46058	200	230	225
100	130619	2.9	45041	200	225	250

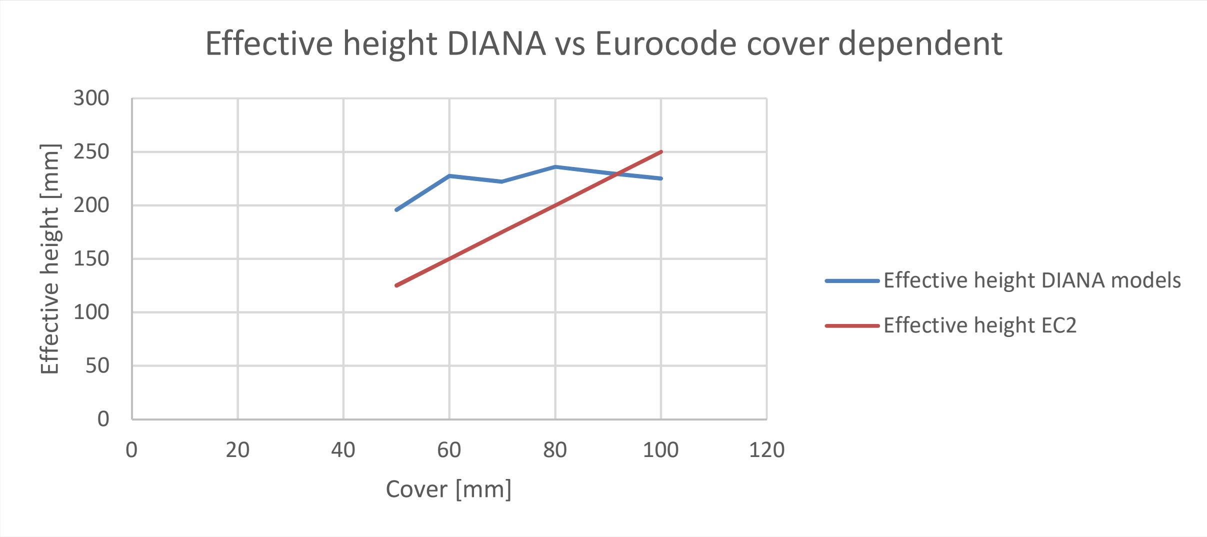


Figure 50 Comparison of the effective concrete tension zone height between the FEM and the Eurocode 2

In Figure 50 a comparison of the effective concrete tension zone height is made between the finite element models and Eurocode 2. The Jones method is not considered here because it gives the same results as Eurocode 2 for structures with one layer of reinforcement. In most of the cases the average values of the finite element models suggest a higher effective concrete tension zone height than given by Eurocode 2. The average values are needed because the maximum crack spacing is determined with two times the average transfer length. So to make a good comparison with Eurocode 2, the average values should be used. The average difference of the effective concrete tension zone height between the finite element models and Eurocode 2 is 35mm. This implies a difference of 0.03mm in crack width. The trendline of the finite element models has a smaller slope than the line given by Eurocode 2. This is due to the fact that the differences between the cracking forces determined with the finite element models are small. In some of the cases the effective concrete tension zone height decreases with an increase of cover. This should not be the case because experiments have proven that the effective concrete tension zone height should increase by an increase in cover. After the cover of 100mm Eurocode 2 has a limit of 250mm due to the fact that the height of the model is given as 500mm. The conclusion which can be drawn here is that Eurocode 2 gives smaller effective concrete tension zone heights in comparison with the finite element models. Furthermore some of the results do not seem correct due to the decrease in effective concrete tension zone height with an increase in cover.

5.3 Variation in width

The second variation study has been carried out with a varying width of the model. The width varies from 100 to 250mm. One model will be elaborated here the other models have been elaborated in Appendix B. The dimensions of the model are given in the table below.

Table 7 Dimensions of the 2D model with 200mm width

Variables:	Dimension [mm]
Length	4000
Width	200
Height	500
Diameter reinforcement	40
Cover	60
Mesh size	20

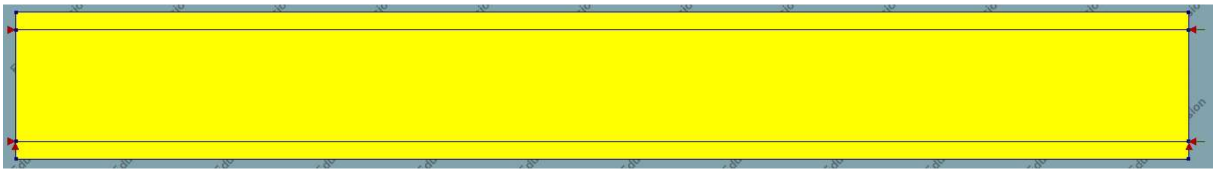


Figure 51 2D model with a width of 200mm

In Figure 51 the geometry of the model is given. For this model the same boundary conditions apply as for the model with a cover of 70mm.

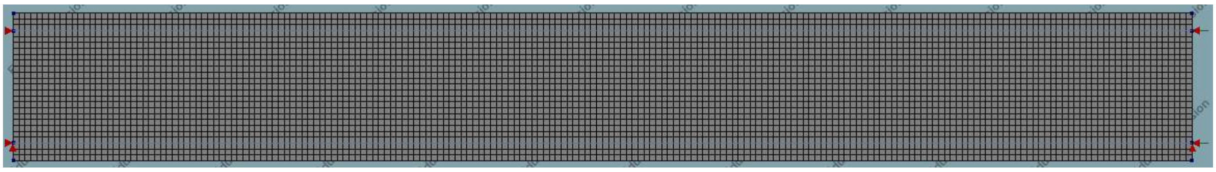


Figure 52 Mesh of the 2D model with a width of 200mm

In Figure 52 the mesh of the model is given. This model has the same mesh as the model described with a cover of 70mm.

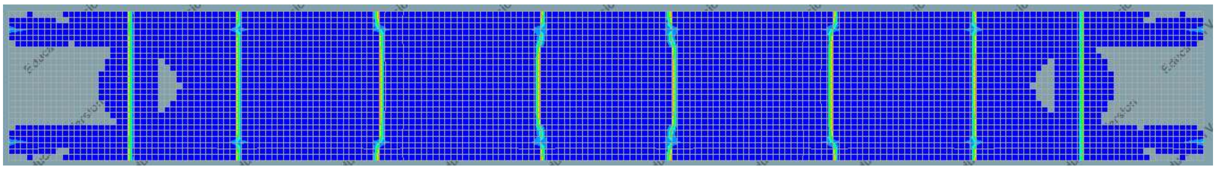


Figure 53 Crack pattern of the 2D model with a width of 200mm

In Figure 53 the crack pattern of the 2D model with a width of 200mm is given. This model also has a symmetry line in the middle of the model. This holds for every model and is due to the loading and supports on the models. In total there are 8 cracks within the model with varying crack spacing. Within this model some cracks run perpendicular to the reinforcement while others bend just like the model in which the cover is varied. The bending of the cracks will be more pronounced if the height of the model will be increased.

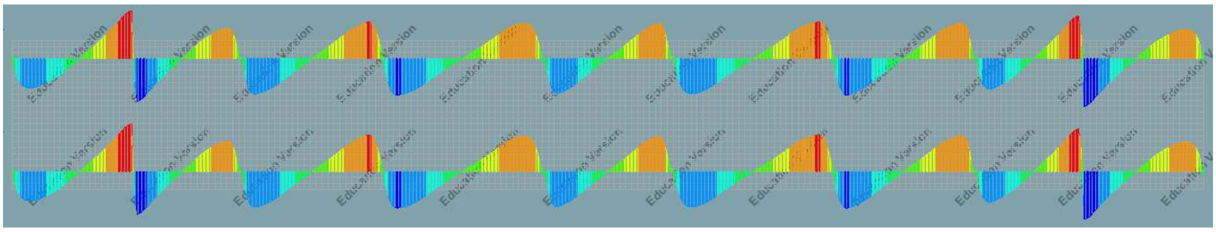


Figure 54 Bond stress gradient of the 2D model with a width of 200mm

In Figure 54 the bond stress gradient is given. In this figure the positions of the cracks are visible and the lengths over which the bond stress varies. Even so, the symmetry of the model is visible when looking at the gradient of the bond stress. For this model the same applies as for the model in which the cover is varied. The gradient of the bond-slip graph is visible.

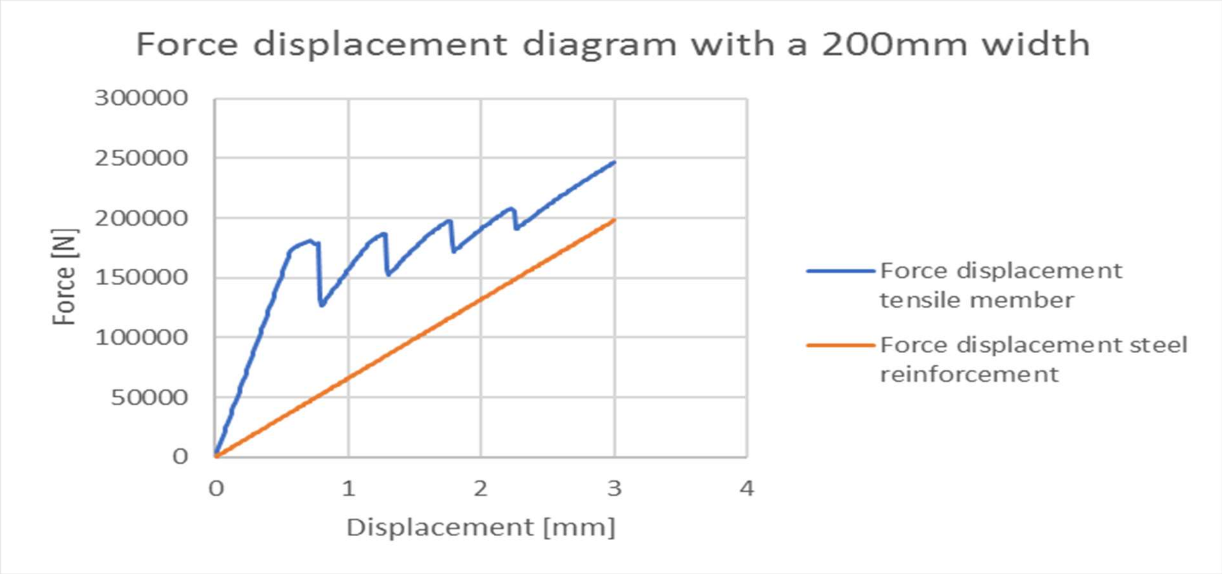


Figure 55 Force displacement diagram of the 2D model with a width of 200mm

In Figure 55 the force displacement diagram is given. In this diagram the force needed for the next crack to appear is clearly visible by the sudden drops in the force displacement curve. In the graph a fully developed crack pattern is given. The maximum amount of cracks per half of the model is 4. Furthermore the way of cracking is equal to the way of cracking as is written about the model in which the cover varies. The only difference between the models is the cracking force which is needed for a next crack to appear. This is due to the different geometry properties.

In the table below, the parameters for the determination of the effective concrete tension zone height are given. These are the values for only half of the model but due to symmetry in the middle of the model all these values can be multiplied by two to get the values for the whole model. With these values the average cracking force can be calculated.

Table 8 Summary of the parameters from the 2D model with a width of 200mm

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
1.86	610	125.7	142230	2.9	49045
1.36	820	125.7	140308	2.9	48382
2.09	490	125.7	128836	2.9	44426
2.05	510	125.7	131336	2.9	45288

$$\sum crackin \quad force = 1085420N$$

$$n = 8$$

$$Average\ cracking\ force = N_{cr,ave} = \frac{\sum cracking\ force}{n} = \frac{1085420}{8} = 135678N$$

With an average cracking force of 132006N, the effective concrete tensile zone becomes:

$A_{c,eff} = {N_{cr,ave}}/{f_{ctm}} = {135678}/{2.9} = 46785mm^2$. This implies an effective concrete tension zone height of:

$$h_{c,eff} = \frac{A_{c,eff}}{b} = \frac{46785}{200} = 234mm$$

The effective concrete tension zone height for this case is, according to Eurocode 2, the minimum of:

$^{h}/_{2}$ and $2{,}5 * (c + {}^{\emptyset_{sl}}/_{2})$.

$$EC2: \min\left[\frac{h}{2} = \frac{500}{2} = 250mm, \quad 2{,}5 * \left(40 + \frac{40}{2}\right) = 150mm\right]$$

In this configuration Eurocode 2 gives a smaller effective concrete tension zone height than the output of the finite element model. The difference between both methods is 84mm. This will imply a difference of 0.06mm in crack width. To get a clear view on the effect of the width on the effective concrete tension zone height, the other examples need to be considered as well. The other examples have been elaborated in Appendix B and the results are presented in Table 9.

Table 9 Average values of the 2D model with a varying width

Width B [mm]	Average cracking force $N_{cr,ave}$ [N]	Concrete tensile strength f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]	Effective concrete tension zone height $h_{c,eff}$ [mm]	Effective concrete tension zone height EC2 $h_{c,effEC2}$ [mm]
100	61817	2.9	21316	213	150
150	97552	2.9	33639	224	150
200	135678	2.9	46785	234	150
250	162074	2.9	55887	224	150

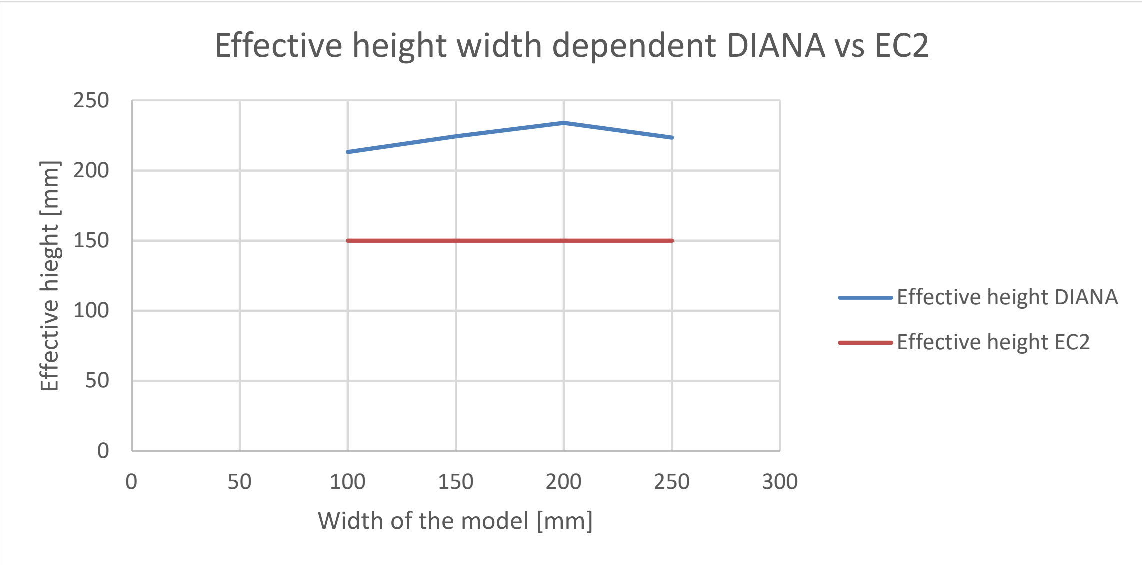


Figure 56 Comparison of the effective concrete tension zone height between finite element models and EC2 in which the width of the model varies

In Figure 56 a comparison of the effective concrete tension zone height is made between the finite element models and Eurocode 2. Again Jones method is not considered here because it will give the same effective concrete tension zone height as Eurocode 2. The slope of the trendline of the average values is almost zero, which is the same for Eurocode 2 because the effective concrete tension zone height is independent of the width of the model. The effective concrete tension zone height of the finite element models is on average 74mm higher than Eurocode 2. This difference is large and results in a difference of 0.04mm in crack width. The conclusion about these models is that the determination of the effective concrete tension height according to Eurocode 2 should be reviewed, because the differences are too large. Eurocode 2 gives a smaller effective concrete tension zone which is beneficial to the determination of the crack width, but these examples show that the Eurocode 2 underestimates the effective concrete tension zone height for tension models.

5.4 Variation in reinforcement diameter

The third variation study has been carried out with a varying reinforcement diameter. The diameter varies from 20 to 40mm with intermediate steps of 25mm and 32mm. One model will be elaborated here, the other models have been elaborated in Appendix B. The dimensions of the model are given in Table 10.

Table 10 Dimensions of the 2D model with 32mm reinforcement

Variables:	Dimension [mm]
Length	4000
Width	200
Height	500
Diameter reinforcement	32
Cover	60
Mesh size	20

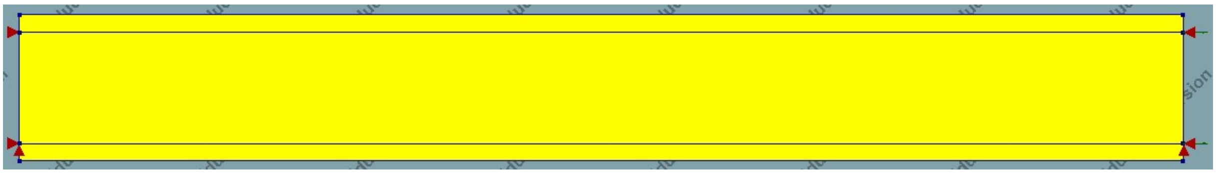


Figure 57 2D model with a 32mm diameter reinforcement

In Figure 57 the 2D model with a 32mm diameter reinforcement is given. For this model the same boundary conditions apply as for the model with a cover of 70mm.

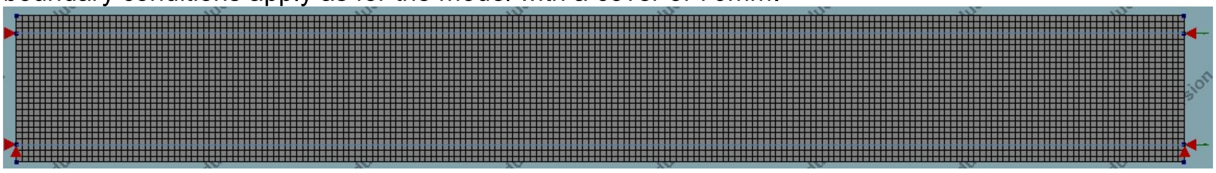


Figure 58 Mesh of the 2D model with a 32mm diameter reinforcement

In Figure 58 the mesh of the 2D model is given. This model has the same mesh as the model described with a cover of 70mm.

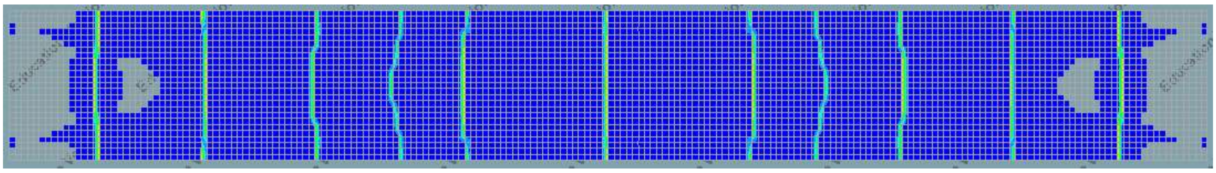


Figure 59 Crack pattern of the 2D model with a 32mm diameter reinforcement

In Figure 59 the crack pattern is given. A symmetry line can be observed in the middle of the model. The fully developed crack pattern consists of 11 cracks. The crack spacing varies within this model. Again some of the cracks run perpendicular while others bend when passing the reinforcement.

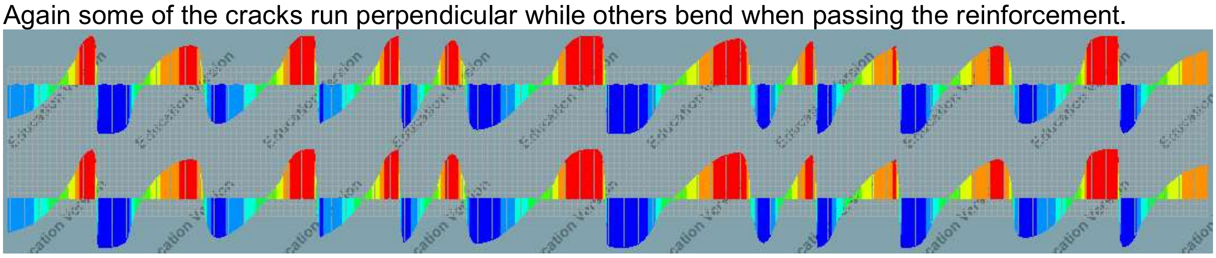


Figure 60 Bond stress gradient of the 2D model with a 32mm diameter reinforcement

In Figure 60 the bond stress gradient is given. In this figure the positions of the cracks are visible and the lengths over which the bond stress varies. Even so, the symmetry of the model is visible when looking at the gradient of the bond stress. For this model and other models in which the diameter is varied another bond-slip diagram is used to obtain converged results. The used bond-slip diagram is the cubic function of Dörr, the graph is given at the start of chapter 4. This model reaches its maximum value with a smaller slip. In Figure 60 some parts reached the maximum values. The gradient of the bond-slip curve can be obtained when looking at the bond-stress gradient.

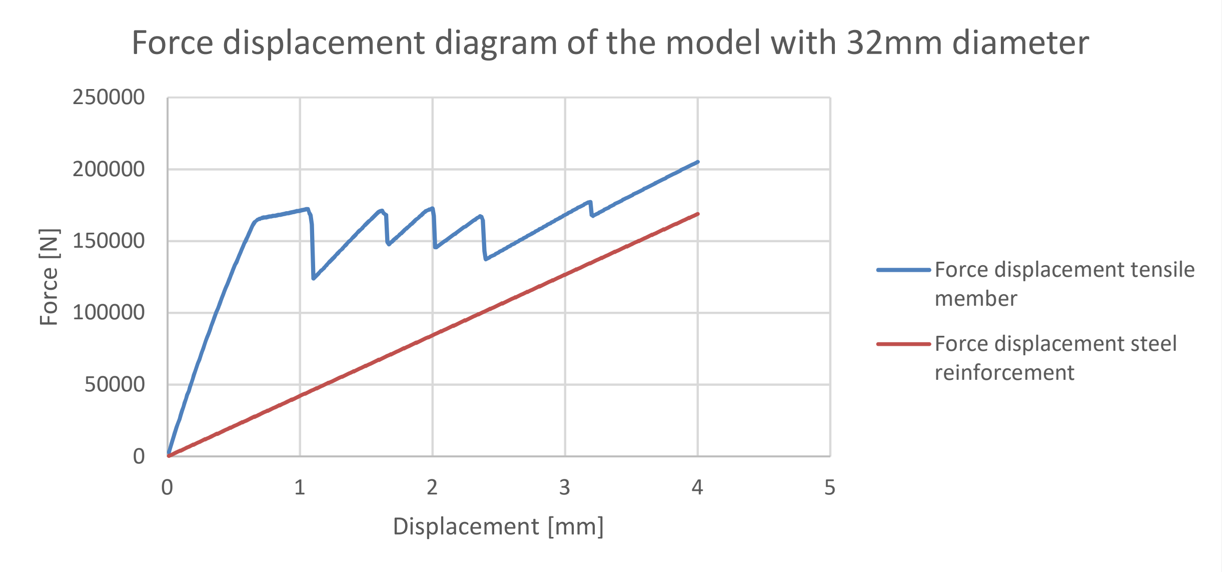


Figure 61 Force displacement diagram of the 2D model with a 32mm diameter reinforcement

In Figure 61 the force displacement diagram is given. In this diagram the force needed for the next crack to appear is clearly visible by the sudden drops in the force displacement curve. In the graph a fully developed crack pattern is given, the maximum number of cracks is 11 for the full model. In here 5 distinct cracks can be observed these are from half of the model. This model cracks in the same way as the models described by the variation study in width and cover. Furthermore the tension stiffening effect is shown.

In the table below, the parameters for the determination of the effective concrete tension zone are given. These are the values for only half of the model but due to symmetry in the middle of the model all these values can be multiplied by two to get the values for the whole model. With these values, the average cracking force can be calculated.

Table 11 Summary of the parameters from the 2D model with a 32mm diameter reinforcement

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
2.65	550	100.5	146747	2.9	50602
3.75	290	100.5	109452	2.9	37742
3.31	340	100.5	113086	2.9	38995
3.44	360	100.5	124544	2.9	42946
2.71	440	100.5	119992	2.9	41376
2.71	460	100.5	125517	2.9	43282
3.63	220	100.5	80282	2.9	27684
3.47	230	100.5	80134	2.9	27632

$$\sum cracking\ force = 1799508N$$

$n = 16$

$$Average\ cracking\ force = N_{cr,ave} = \frac{\sum cracking\ force}{n} = \frac{1799508}{16} = 112469N$$

With an average cracking force of 112469N, the effective concrete tensile zone becomes:

$$A_{c,eff} = {N_{cr,ave}}/{f_{ctm}} = {112469}/{2.9} = 38782mm^2.$$

This implies an effective concrete tension zone height of:

$$h_{c,eff} = \frac{A_{c,eff}}{b} = \frac{38782}{200} = 194mm$$

The effective concrete tension zone height for this case, is according to Eurocode 2, the minimum of: ${h}/{2}$ and $2{,}5 * (c + {\emptyset_{sl}}/{2})$.

$$EC2: \min\left[\frac{h}{2} = \frac{500}{2} = 250mm, \qquad 2{,}5 * \left(40 + \frac{32}{2}\right) = 140mm\right]$$

In this configuration, Eurocode 2 gives lower results than the finite element model calculation. The difference in effective concrete tension zone height is 54mm. The difference in crack width is 0.04mm. So the procedure described by Eurocode 2 underestimates the effective concrete tension zone according to this calculation. Other models with different diameters are examined to check whether the effective concrete tension zone height of these models varies as well. The output of these models is given in Appendix B. The results of the models are given in Table 12.

Table 12 Average values of the models with varying reinforcement diameter

Diameter $\emptyset_{sl}$ [mm]	Average cracking force $N_{cr,ave}$ [N]	Concrete tensile strength f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]	Width of the model B [mm]	Effective concrete tension zone height $h_{c,eff}$ [mm]	Effective concrete tension zone height EC2 $h_{c,effEC2}$ [mm]
20	111369	2.9	38403	200	192	125
25	117995	2.9	40688	200	203	131.25
32	112469	2.9	38783	200	194	140
40	106866	2.9	36850	200	184	150

With the values given in the table above, graphs can be made to represent the differences in a graphical way.

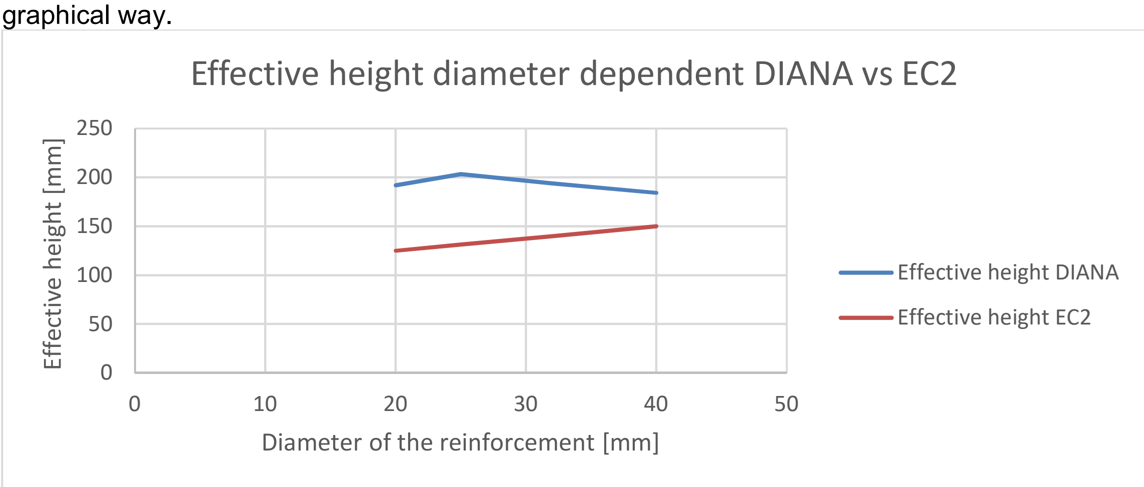


Figure 62 Effective concrete tension zone height of the models with a varying reinforcement diameter finite element models vs Eurocode 2

In Figure 62 the average effective concrete tension zone heights of the models with a varying reinforcement diameter are given. A comparison is made between Eurocode 2 and the average values according to finite element models. Again Jones method is not considered. The average values given by the finite element models result in larger effective concrete tension zone heights than the ones determined by Eurocode 2, so Eurocode 2 underestimates the effective concrete tension zone height. The average difference in effective concrete tension zone height is 57mm. The average difference in crack width is 0.03mm. Another strange phenomena is visible within the graph, this phenomena is the decrease of the effective concrete tension zone height by an increase in reinforcement diameter for the finite element models. The effective concrete tension zone height should increase with an increase in reinforcement diameter. In Eurocode 2 the effective tension zone height increase with an increase in reinforcement diameter. This phenomena can be the result of the number of cracks because the bond force usually decreases when the number of cracks increase. In the model with a 40mm diameter, the number of cracks is larger and thus the last bond forces are lower. This results in a lower average bond force and thus in a smaller effective concrete tension zone height.

5.5 Variation in height

To check whether the effective concrete tension zones intersect with each other, a variation test has been carried out in which the height of the structure varies. In the previous chapter it was shown that with an increase in cover, the effective concrete tension zone height increases as well. Therefore the cover term will be a constant in this variation study. The distance between the center of the reinforcement and the outer fiber of the beam will be 80mm(cover as specified in earlier examples). This will imply a cover of 60mm according to Eurocode 2. The different diameters which have been considered are 32mm and 40mm. Due to the fact that multiple layers of reinforcement will be used, the diameters are large, otherwise one reinforcement bar of, for example, 40mm would be sufficient. This is because one bar of 40mm has a higher cross sectional area than two bars of 25mm. The reinforcement has to have a large distance from the natural axis to be most effective therefore the vertical bar spacing has a maximum of 200mm. The vertical bar spacing cannot be too small because the concrete needs to be able to flow in between the reinforcement bars to ensure good bond and to prevent air holes. The width of the model is dependent on the maximum influence zone of the reinforcement bars given by both Eurocode 2 and Jones Method. The maximum influence zone of the reinforcement bar in the width direction is given by $5 * \left(C + {}^{\emptyset_{sl}}/_{2}\right) = 5 * \left(60 + {}^{40}/_{2}\right) = 400mm$. So the width of the model should be within this limit because the model will consist of one reinforcement bar in the width direction. The width of the model will be a constant as well and will be 200mm.

The heights of the models are 500mm, 1000mm and 1500mm. With these models the same test has been carried out as for the other variation studies. The vertical bar spacing is set at 100mm, with 40mm diameter reinforcement bars. For an overview of the models see Figure 63.

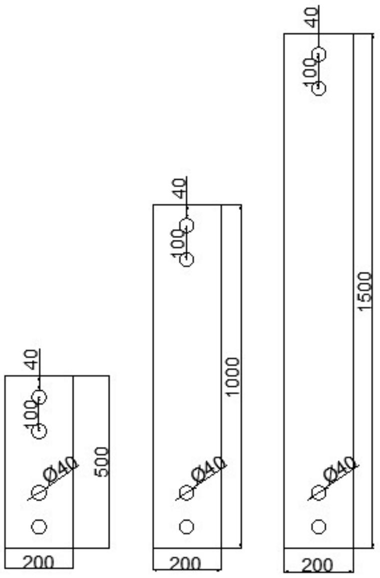


Figure 63 Cross section of the models which vary in height dimensions given in mm

One of the models will be elaborated here, the other models are elaborated in Appendix B. The comparison between the models will be given after the elaboration of the model. The model which will be presented here is the model with a height of 1000mm. The dimensions of the model are given in Table 13.

Table 13 Dimensions of the 2D model with a height of 1000mm

Variables:	Dimension [mm]
Length	4000
Width	200
Height	1000
Diameter reinforcement	40
Cover	60
Mesh size	20
Vertical bar spacing	100

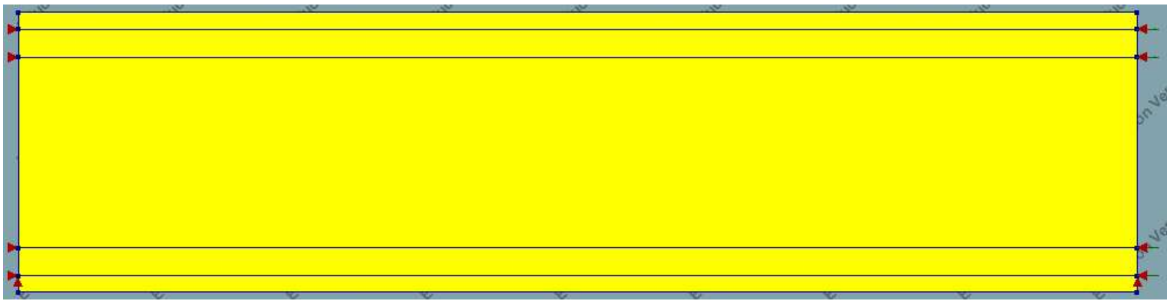


Figure 64 2D model with a height of 1000mm and two layers of reinforcement

In Figure 64 the 2D model with a height of 1000mm and two layers of reinforcement is given. The horizontal lines represent the reinforcement. The prescribed deformation is 0.01mm in X-direction per load step, depicted by the arrows on the right side of the model. The supports are given by the red triangles the horizontal x-direction is supported on both ends of both reinforcements. The vertical y-direction is supported by the bottom reinforcement. In order to prevent transverse contraction of the model, the upper reinforcement is not supported in the y-direction. If the transverse contraction of the model is restrained by the vertical supports, tension stresses in y-direction arise in the model which can result in cracks parallel to the reinforcement. This has to be prevented because parallel cracks have a negative influence on the bond stress gradient.

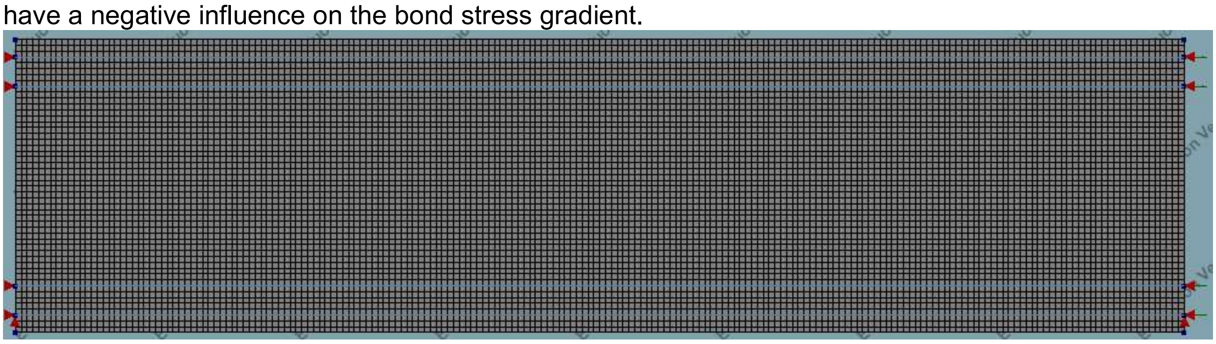


Figure 65 Mesh of the 2D model with a height of 1000mm and two layers of reinforcement

In Figure 65 the mesh of the 2D model is given. The mesh has a size of 20mm per element. It is a fine mesh to get accurate results for the crack spacing and bond stresses.

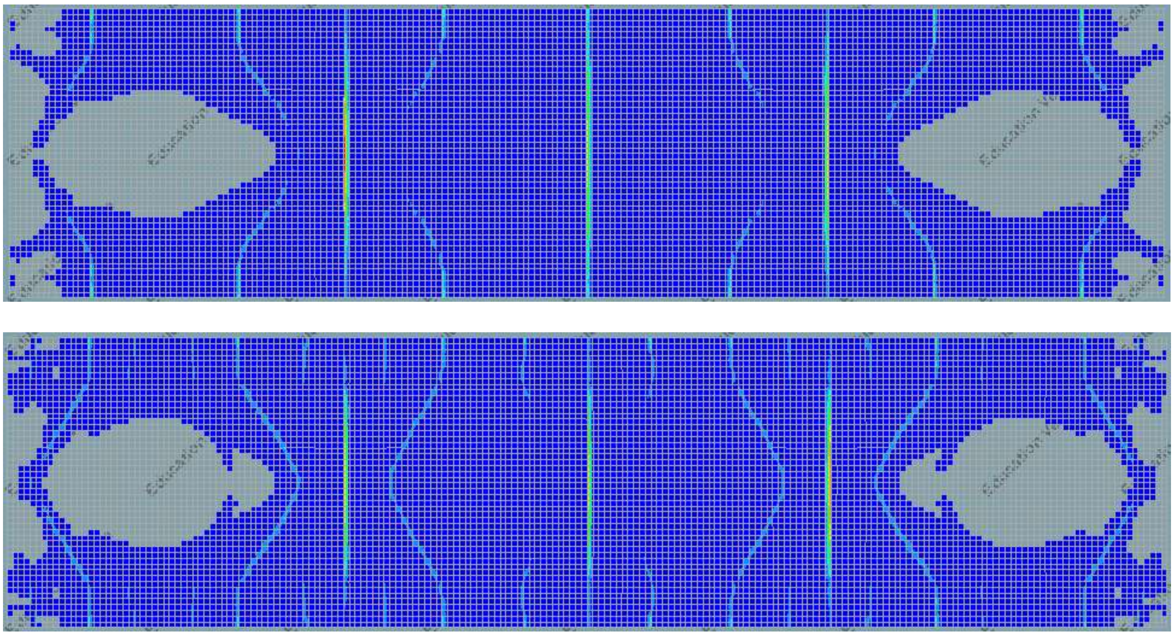


Figure 66 Crack pattern of the 2D model with a height of 1000mm and two layers of reinforcement

In Figure 66 the crack pattern is given. In the first stage, at load step 200, the model is cracked at the maximum crack spacing. Three (primary) cracks run perpendicular to the length axis through the model while the other cracks bend towards the primary cracks. When the load keeps increasing, the model will crack at positions in between the already consisting cracks to get a fully developed crack pattern(load step 400). The cracks which form are secondary cracks and do not run through the height of the model. The fully developed crack pattern consists of 21 cracks. Due to the increase in height the secondary cracks bend towards the primary cracks. The primary cracks are most pronounced between the inner reinforcement bars. The effective concrete tension zone is visible here when looking at the secondary cracks because this is the area in which the secondary cracks are located.

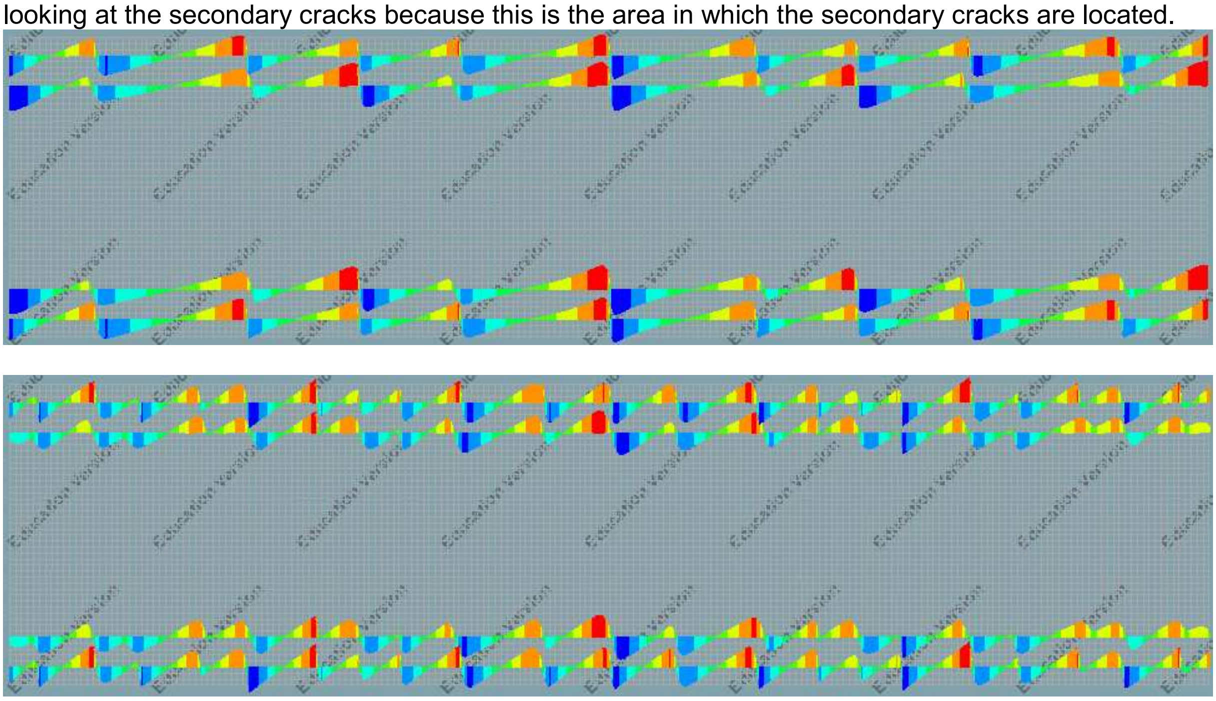


Figure 67 Bond stress gradient of the 2D model with a height of 1000mm and two layers of reinforcement

In Figure 67 the bond stress gradient is given. In this figure the positions of the cracks and the lengths over which the bond stress varies are visible. Even so, the symmetry of the model is visible when looking at the gradient of the bond stress. Both load phases are shown to see the difference in bond stress for both load steps. The length over which the bond stress varies decreases by an increase in the amount of cracks. The bond-slip graph gradient is visible within the bond stress gradient plots.

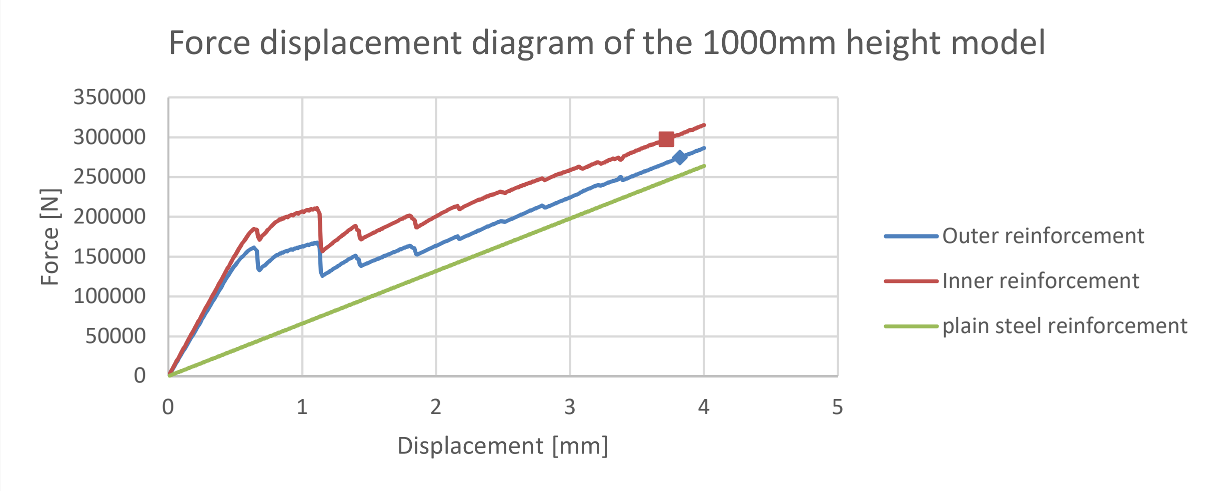


Figure 68 Force displacement diagram of the 2D model with a height of 1000mm and two layers of reinforcement

In Figure 68 the force displacement diagram is given. In this diagram the force needed for the next crack to appear is clearly visible by the sudden drops in the force displacement curve. In the graph, a fully developed crack pattern is given, the maximum amount of cracks is 21 for the full model. In here, 5 distinct cracks can be observed the other cracks result in small drops which are barely visible in the force displacement diagram. The lines represent both reinforcement bars; the outer reinforcement bar has the lowest cracking force(blue line), the inner reinforcement bars has the highest cracking force(red line). The plain reinforcement is given by the linear line. This line is added to show the tension stiffening effect.

In the table below, the parameters to determine the effective concrete tension zone per reinforcement bar are given. These are the values for only half of the model but due to symmetry in the middle of the model all these values can be multiplied by two to get the values for the whole model. With these values, the average cracking force can be calculated. The values are given for the inner reinforcement first where after the values of the outer reinforcement are given.

Table 14 Summary of the parameters from the 2D model with a height of 1000mm and two layers reinforcement

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
3.39	270	125.7	115130	2.9	39700
1.46	560	125.7	102543	2.9	35360
2.70	310	125.7	105254	2.9	36295
1.95	480	125.7	117646	2.9	40567
1.54	390	125.7	75274	2.9	25957
1.22	510	125.7	78218	2.9	26972
2.50	270	125.7	84903	2.9	29277
1.61	230	125.7	46563	2.9	16056
2.23	250	125.7	69983	2.9	24132
1.93	260	125.7	63220	2.9	21800
2.96	200	125.7	74351	2.9	25638
1.86	140	125.7	32705	2.9	11278
2.88	190	125.7	68665	2.9	23677

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
2.45	290	125.7	89424	2.9	30836
1.48	620	125.7	115082	2.9	39683
2.00	330	125.7	82853	2.9	28570
1.84	370	125.7	85544	2.9	29498
1.46	510	125.7	93356	2.9	32192
1.41	490	125.7	86926	2.9	29975
2.19	250	125.7	68713	2.9	23694
1.95	230	125.7	56353	2.9	19432
2.60	240	125.7	78270	2.9	26990
2.25	180	125.7	50847	2.9	17533
2.54	170	125.7	54330	2.9	18735
2.29	170	125.7	48825	2.9	16836

$$\sum cracking\ force\ inner = 2068908N$$

$n = 26$

$$Average\ cracking\ force = N_{cr,ave} = \frac{\sum cracking\ force}{n} = \frac{2068908}{26} = 79573N$$

With an average cracking force of 79573N, the effective concrete tensile zone becomes:

$A_{c,eff} = {N_{cr,ave}}/{f_{ctm}} = {79573}/{2.9} = 27439mm^2$. This implies an effective concrete tension zone height of:

$$h_{c,eff\ inner} = \frac{A_{c,eff}}{b} = \frac{27439}{200} = 137mm$$

$$\sum cracking\ force\ outer = 1821047N$$

$n = 24$

$$Average\ cracking\ force = N_{cr,ave} = \frac{\sum cracking\ force}{n} = \frac{1821047}{24} = 75877N$$

With an average cracking force of 75877N, the effective concrete tensile zone becomes:

$A_{c,eff} = {N_{cr,ave}}/{f_{ctm}} = {75877}/{2.9} = 26164mm^2$. This implies an effective concrete tension zone height of:

$$h_{c,eff\ outer} = \frac{A_{c,eff}}{b} = \frac{26164}{200} = 131mm$$

Now both effective concrete tension zone heights will be added to get the effective concrete tension zone height of both reinforcement bars:

$Effective\ concrete\ tension\ zone\ height = 137 + 131 = 268mm$

The effective concrete tension zone height for this case, is according to Eurocode 2, the minimum of: ${h}/{2}$ and $2{,}5 * (c + {\emptyset_{sl}}/{2})$.

$$EC2: \min\left[\frac{h}{2} = \frac{1000}{2} = 500mm, \quad 2{,}5 * \left(\frac{100}{2} + 40 + \sqrt{2} * \frac{40}{2}\right) = 296mm\right.$$

In this configuration Eurocode 2 gives an higher effective concrete tension zone height than the output of the finite element model. So for this configuration it can be beneficial to use the finite element model approach to determine the effective concrete tension zone height. The difference in effective concrete tension zone height is 28mm it is a small difference compared to the other models. The difference in crack width is 0.02mm. Within this model there are a lot of cracks visible. When multiple cracks occur, the average cracking force decreases, which makes the effective concrete tension zone height decrease as well.

The results of the 500mm height model and the 1500mm height model will be given here as well to make a comparison between the values. With this comparison a conclusion can be given about the determination of the effective concrete tension zone height according to Eurocode 2 and Jones Method. The results will be given in the table below.

Table 15 Determination of the effective concrete tension zone height for varying height models

Height H [mm]	Reinforcement [-]	Average cracking force $N_{cr,ave}$ [N]	Concrete tensile strength f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]	Width of the model B [mm]	Effective concrete tension zone height $h_{c,eff}$ [mm]	Total effective concrete tension zone height $h_{c,efftot}$ [mm]	Effective concrete tension zone height EC2 $h_{c,effEC2}$ [mm]	Effective concrete tension zone height JM $h_{c,effJM}$ [mm]
500	Inner	62829	2.9	21665	200	108	210	250	110
500	Outer	58832	2.9	20287	200	101		250	110
1000	Inner	79573	2.9	27439	200	137	268	296	110
1000	Outer	75877	2.9	26164	200	131		296	110
1500	Inner	69759	2.9	24055	200	120	247	296	110
1500	Outer	73459	2.9	25331	200	127		296	110

With the effective concrete tension zone heights given in the table above, graphs can be plotted. These graphs are shown in Figure 69 and Figure 70. As can be seen in the figures, Eurocode 2 gives an upper limit of the effective concrete tension zone height while Jones method gives a bottom limit. The values determined by the finite element models are in between the values given by either Eurocode 2 or Jones method. The model with a height of 1000mm gives the highest effective concrete tension zone height. An upper limit of the effective concrete tension zone height determined with the finite element models is not visible within the graph, however this was to be expected because with a higher model the effective concrete tension zone height will not intersect with each other. This limit should be visible within this graph but it is not because the effective concrete tension height decreases with an increase in model height. The effective concrete tension zone heights of the inner and outer rebars are larger than those determined with Jones method but the differences are small. Jones method underestimates the effective concrete tension zone height this is shown in the figure. The average differences of the effective concrete tension zone height between Eurocode 2 and finite element models, Jones method and finite element models and Eurocode 2 and Jones method are consecutively 39mm, 11mm and 61mm. The average differences in crack width are consecutively 0.02mm, 0.01mm and 0.03mm.

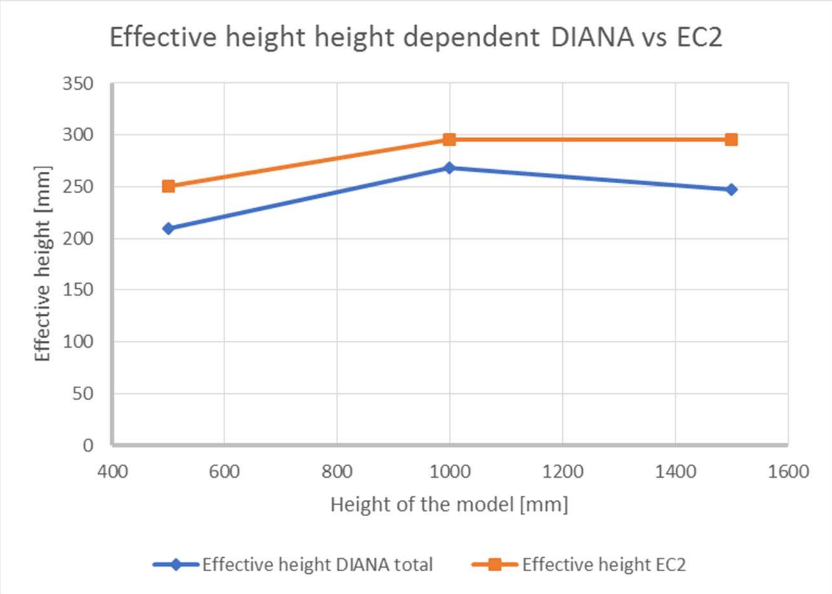


Figure 69 Effective concrete tension zone heights of EC2 and FEM with varying height

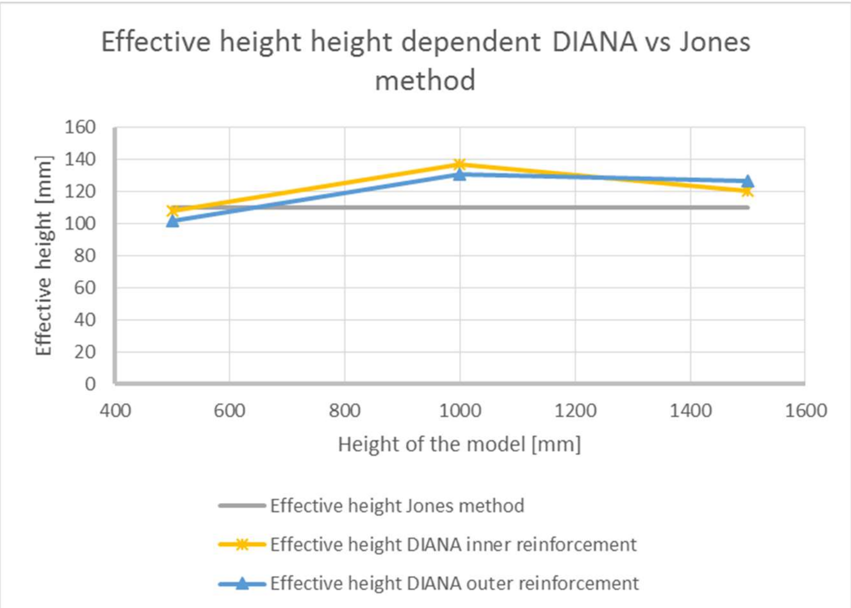


Figure 70 Effective concrete tension zone heights of FEM and Jones method

5.6 Equilibrium between the forces

The cracking force given in the force displacement diagram determines the force needed for a crack to appear. This force needs to be equal to the force taken up by the reinforcement and the concrete. In a crack the force in the concrete is almost equal to zero, so the total force needs to be carried by the reinforcement. To check whether this is true for the finite element models, a test has been undertaken. The models do not convergence when a crack occurs, so the load step which will be used for this test will be just after the moment that the crack occurred. The test has been carried out on the model in which the cover is 70mm.

The first crack in the model occurs at load step 105, this step is not converged so load step 110 will be used. The crack occurs at x=720mm, the position which will be considered is x=700mm.This position is given by the black line in Figure 71. At this position the stresses in the concrete in the x-direction are investigated, see Figure 72 for the stress gradient over the height of the structure. The peaks in the stress are given at the positions of the reinforcement. Close to the reinforcement and next to the crack the force exerted on the model spreads into the concrete. At these position the stress is high. The force in the concrete is obtained by multiplying the concrete stress by the area of the concrete section. The cross sectional area of the concrete is 500mm in height and 200mm in width. The total force of the concrete in the section at x=700mm is: 30730N.

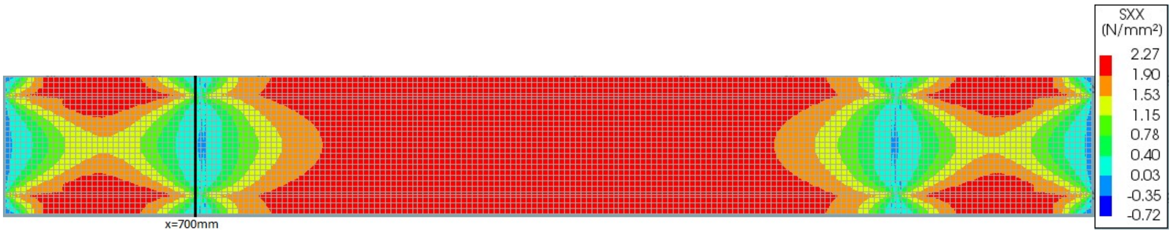


Figure 71 Stresses in the concrete in X-direction

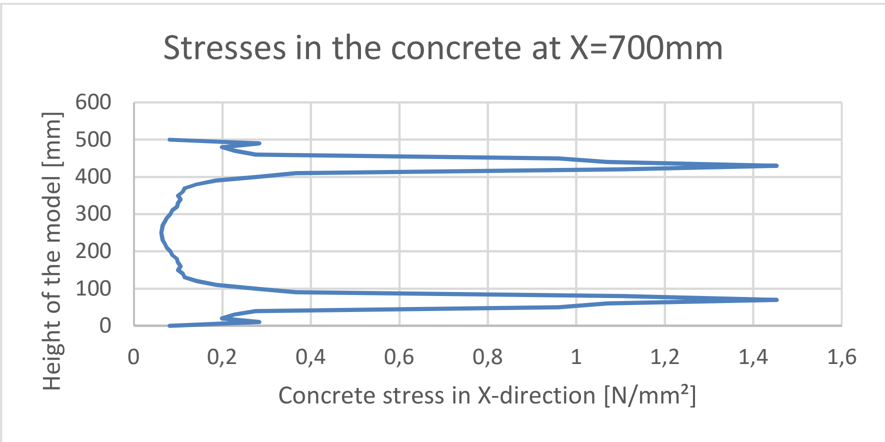


Figure 72 Concrete stress gradient

The force in the reinforcement is obtained by multiplying the cross sectional area of the reinforcement by the stress in the reinforcement at x=700mm. The cross sectional area of the reinforcement is given by two 40mm reinforcement bars: $A_s = 2 * 40^2 * {}^{\pi}/_{4} = 2513mm^2$.

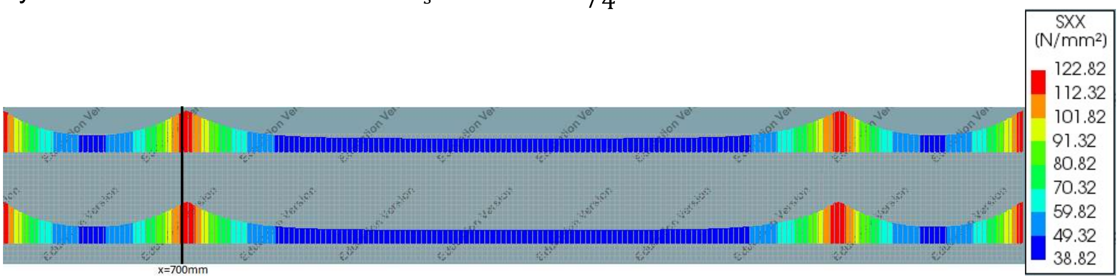


Figure 73 Stresses in the reinforcement in the X-direction

The stress in the reinforcement at x=700 is equal to $\sigma_s = 113.7\ N/mm^2$. The total force for both reinforcement bars is given by: 285728N. The force given at load step 110 is: 156471N. This force has to be multiplied by two because the force is exerted on both reinforcement bars, thus total force exerted on the model is 312942N.

$Force\ in\ reinforcement + Force\ in\ concrete \quad 285727 + 30730 = 316458N$

$Total\ force = 312942N$

$Difference = Total\ force - Forc\ \ in\ reinforcement - Force\ in\ concrete = 312942 - 31645$
$= -3516N$
The difference between the forces is 3516N. This difference is small compared to the values and is a result of rounding off errors. This difference can also be a result of the way in which the results are obtained. Because the results of the finite element model are determined in the integration points these results are extrapolated to the nodes in which the results are displayed as output. Each node has results of different integration points therefore there are different results in each single node. The results used here are averaged around the node.

When the concrete cracks, the stress in the reinforcement increases and the stress in the concrete decreases. The bond stress represents the increases of the reinforcement stress and the decrease of the concrete stress. The increase of reinforcement stress is determined by the maximum value in the crack and the minimum value in between cracks. For load step 110 the difference of the maximum and the minimum reinforcement stress differs for the left and right side of the crack. So the bond stress needs to have different values on both sides of the crack as well. This is clearly visible in Figure 74

Figure 74 Bond stress gradient

To check whether the force on the left side of the crack, given by the bond stress, is equal to the force, given by the increase in reinforcement stress, a comparison between the forces is made. The difference of the steel stress on the left side of the crack is equal to $\Delta\sigma_{sl} = 74.02N/mm^2$. The increase of force in the reinforcement on the left side of the crack is given by $F_{sl} = \Delta\sigma_s * A_s = 74.02 * 1257 = 93043N$. The force given by the bond stress on the left side of the crack is determined in the same way as is explained in the previous paragraphs and is given by:$F_{bl} = 92641N$. The difference of these forces is:
$Difference = F_{sl} - F_{bl} = 93043 - 92641 = 402N$

The difference is less than 0.5% so the forces are in good comparison. The difference is a result of rounding off errors.

The right hand side of the crack needs to be checked as well. The increase of stress in the reinforcement on the right side of the crack is: $\Delta\sigma_{sr} = 83.2N/mm^2$. The force given by the increase of stress is:$F_{sr} = \Delta\sigma_{sr} * A_s = 83.2 * 1257 = 104582N$. The force given by the bond stress on the right side of the crack is given by:$F_{br} = 104233N$. The difference between these forces is:
$Difference = F_{sr} - F_{br} = 104582 - 104233 = 349N$
The forces are in good comparison because the difference is small compared to the values. The difference is a result of rounding off errors.

5.7 Cone shaped cracks close to the reinforcement

In the crack spacing formula of Eurocode 2, a term is used which represents pulled out cones next to a crack. This term is 3.4 multiplied by the cover of the reinforcement. The cones have the size of the cover of the reinforcement. This would imply that the total length of both cones equals two times the cover. In Eurocode 2 a safety factor of 1.7 is used to get to the 3.4 value. This master thesis is about cracking and the effective concrete tension zone which is used for the determination of the crack width. The phenomena of the pulling out of the cones is important for the crack spacing formula and thus for the determination of the crack width. Different models have been investigated for the determination of the effective concrete tension zone, but is it possible to see the phenomena of the pulling out of the cones close to the crack within these models?

At first sight, the cracks close to the reinforcement run perpendicular to the direction of the reinforcement. These cracks are the primary cracks and have the largest width. Due to the auto scale option in DIANA, the smaller cracks are not visible because these cracks would fall in the lowest scale and thus in the same color, see Figure 75. If the auto scale option is changed to specified values, it is possible to specify the values of the crack width with maximum and minimum values. By using 0.2mm crack width as a maximum value the color scale of the crack width plot changes. Now the pulled out cones can be visible, see Figure 76.

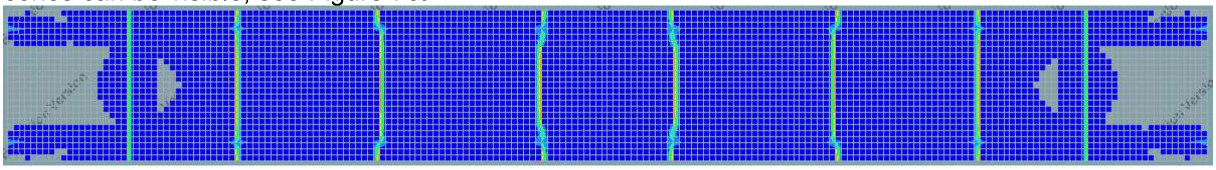


Figure 75 Crack widths with auto scale

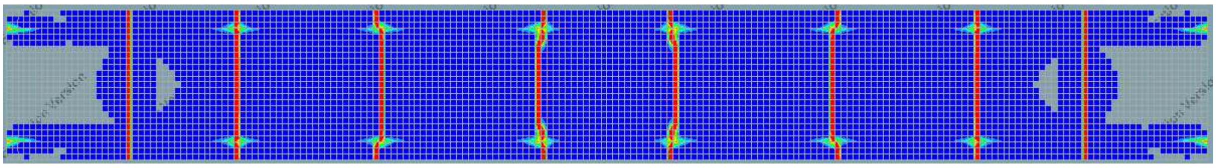


Figure 76 Crack width with specified values

Due to the fact that the size of the cone is dependent on the cover, the models in which the cover is varied will be used. The figures which are used are given by the model with a cover of 60mm(40mm cover according to Eurocode 2). In the model with 50mm cover there were no cone cracks visible even when the specified values option was selected with a maximum crack size of 0.1mm. In the table below the maximum and average cone size will be given for the models in which the cover varies. The cones at the positions of the initiation of the force are not taken into account.

Table 16 Dimensions of the pulled out cones close to a crack

Cover model [mm]	Cover Eurocode 2 [mm]	Maximum cone length [mm]	Average cone length [mm]	Eurocode 2 value 3.4*c [mm]	FEM model value 3.4*ave [mm]
50	30	0	0	102	0
60	40	60	40	136	136
70	50	60	40	170	136
80	60	100	60	204	204
90	70	40	20	238	68
100	80	140	80	272	272

In Table 16, the even values of cover give exactly the same results while the uneven values give smaller cone dimensions. A reason for this outcome could be that the element size is equal to 20mm. So when the cover is uneven the reinforcement bars run through the middle of the element which could be the reason that the element experiences difficulties with cracking.

5.8 Conclusion variation study

In the previous chapters and in Appendix B the different models have been elaborated for the variation study in which the effective concrete tension zone height, for models under tension needs to be determined. The comparison of the effective concrete tension zone height between either Eurocode 2 or Jones method with the finite element models is the main objective of this master thesis. In this chapter the conclusion of the total variation study will be given. The models which have been investigated are models in which the cover, width, reinforcement diameter and height of the model vary. In the height variation models, two layers of reinforcement have been used to check whether there is a limit to the effective concrete tension zone height as is the case according to Eurocode 2.

Firstly the variation in cover has been elaborated. The conclusions about these models are:

- Generally the effective concrete tension zone height is larger for the finite element models than determined using Eurocode 2 when looking at the variation in cover. This is unfavorable for the determination of the crack spacing thus for the crack width calculations. But it is favorable for the local tension stiffening effect thus for the crack width calculations. The average difference in effective concrete tension zone height of all models is 35mm. This difference results in an average crack width difference of 0.03mm. The crack spacing formula is normative because the difference in crack width is unfavorable. A reason for the increase in effective concrete tension height is the amount of cracks and the accompanying bond stress and transfer length. With these values, the bond force is determined with which the effective concrete tension zone height is determined. With fewer cracks, the bond force is high because the transfer lengths are large and the average bond stresses are high. When more cracks occur in the model the bond force decreases, which results in a smaller average cracking force.
- The slope of the average effective concrete tension zone height given by the finite element models is smaller than the slope given by Eurocode 2, which means that with an increase in cover, the increase of the effective concrete tension zone height of the finite element models is smaller than the one given by Eurocode 2. On average the slope of Eurocode 2 increases with 25mm per 10mm cover increase while the finite elements models increase with 6mm per 10mm cover increase. This implies that the dependency of the cover on the effective concrete tension zone height is smaller according to the finite element models than assumed by Eurocode 2.
- The crack spacing between the cracks is usually in the same order for each individual model. With an increase in cover the crack spacing of the models increase. This is also assumed by Eurocode 2 because the crack spacing formula is depending on the cover.

Secondly the variation in width was elaborated. The conclusions about these models are:

- The effective concrete tension zone height determined with the finite element models is considerably higher than determined using Eurocode 2. The average difference of the effective concrete tension zone height between both methods is 74mm. This is unfavorable for the determination of the crack spacing and thus for the determination of the crack width. But it is favorable for the determination of the crack width in terms of the local tension stiffening effect. The average crack width difference is 0.04mm. This implies that the crack spacing formula is normative in the determination of the crack width when the effective reinforcement ratio is the variable. A reason for these differences is the higher cracking forces due to the amount of cracks. The cover used for these examples is small, and according to the examples in which the cover term is varied, the difference between Eurocode 2 and the finite element models is larger for models with a small cover.
- The slope of the average values obtained by the finite element models is nearly zero which is the same for Eurocode 2, due to the fact that the effective concrete tension zone height is not influenced by the width according to Eurocode 2. This is shown by these examples.
- The cracks are evenly distributed over the model. With an increase in width the distance in between the cracks increase. This is due to the fact that the force needed for the model to crack is larger, thus the length needed to build up the bond force is larger.

Thirdly the variation in reinforcement diameter was elaborated. The conclusions about these models are:

- The effective concrete tension zone height determined with the finite element models is larger than determined using Eurocode 2. The average difference of the effective concrete tension zone height is 57mm. The average difference in crack width is 0.03mm. According to earlier conclusions and the conclusion made here the crack spacing term is normative. For these models the cover term is small which causes a large difference according to the variation study done on cover. The number of cracks is small in the models with a small diameter because a fully developed crack pattern is reached with fewer cracks. This results in high cracking forces which result in a large effective concrete tension zone.
- The slope of the line obtained by the average values decrease with an increase in diameter according to the finite element models. According to Eurocode 2 calculations, the effective concrete tension zone height should increase with an increase in diameter. This is also observed in reality. Therefore the results given by these models are questionable.

Fourthly the variation in height of the model was elaborated to see if the effective concrete tension zones intersect with each other. The conclusions about these models are:

- In these models two layers of reinforcement have been used. Therefore the determination of the effective concrete tension zone height changes. The effective concrete tension zone height of the finite element models is determined for both reinforcement bars individually and added. The effective concrete tension zone height of Jones method is determined for each individual bar and compared to the finite element models. For Eurocode 2 the effective concrete tension zone height is determined for both reinforcement bars. Due to the determination of the effective concrete tension zone height differences occur. These differences are consecutively between Eurocode 2 and FEM, Jones method and FEM and Eurocode 2 and Jones method: 39mm, 11mm and 61mm. The differences between Eurocode 2 and Jones method are large because Eurocode 2 gives an upper limit while Jones method gives a bottom limit.
- The differences in crack width between the methods considered are 0.02mm between Eurocode 2 and FEM, 0.01 between Jones method and FEM and 0.03mm between Eurocode 2 and Jones method. Here the crack spacing formula is normative because an increase in effective concrete tension zone height increases the crack width.
- There is no limit visible in the effective concrete tension zone height which was to be expected because at a certain point the effective concrete tension zone height would not intersect with each other. With an increase in height of the model, the effective concrete tension zone height decreases between a height of 1000mm and 1500mm. One would expect an increase with an increase in height of the model. A reason for this phenomena could be that in the model with a height of 1500mm the number of cracks is larger than the number of cracks in the 1000mm height model. More cracks result in a smaller average cracking force, so in a smaller effective concrete tension zone height. This is a reason that the line decreases between 1000mm and 1500mm height.

6 MODEL LOADED BY BENDING

In addition to the tensile member models, some bending models have been investigated as well. The amount of bending models is smaller because the time needed to perform a lot of bending models was no longer available. The bending models are performed to see if it could be possible to determine the effective concrete tension zone height in the same way as was performed for the tensile member models. Furthermore the question of if it is possible to reduce the amount of reinforcement, by taking a closer look at the determination of the effective concrete tension zone, also holds for the bending models. The bending models are elaborated in DIANA and are loaded in the same way as the tensile member models, by applying a prescribed deformation. The prescribed deformation is still applied on the top and bottom reinforcement but the deformation on the top reinforcement is now in the opposite direction, it pushes the reinforcement instead of pulling it. Due to the reversed deformation on the top reinforcement a compression force is exerted on the model, while the bottom reinforcement is still pulled which results in a tension force in the bottom side of the model. With this configuration, a moment is applied to the model.

The determination of the effective concrete tension zone height is different for bending models. The effective concrete tension zone height according to Eurocode 2 is determined by:

$$EC2\!: \min\left[\frac{h-x}{3}, \quad 2{,}5 * \left(c + \frac{\emptyset_{sl}}{2}\right)\right]$$

In the first formula, the x is the height of the compression zone in the concrete. This value can be obtained from the finite element models by looking at the stresses in the model.

The effective concrete tension zone height according to Jones method is given by:

$$h_{c,effJM} = c + \frac{\emptyset_{sl}}{2} + \min\left[\frac{ssv}{2}; 1.5 * \left(c + \frac{\emptyset_{sl}}{2}\right)\right]$$

The investigated models vary in height and have the following heights: 500mm, 1000mm and 3000mm. The model with 3000mm height is used to see if secondary cracks can occur in the model, secondary crack were usually not seen in the tensile member models. A large distance between the compression zone and the point of gravity of the reinforcement is necessary to get a model with secondary cracks [12].

With the 500mm and 1000mm model, the distance between the compression zone and the center of gravity of the reinforcement is too small to get secondary cracks. Secondary cracks are small cracks which do not run through the whole cross section of the structure and run towards the primary cracks. Primary cracks are cracks which run through the whole thickness of the structure.

The model with a height of 3000mm will be elaborated here to show the crack patterns and the results of a model which is loaded by bending. The other models are elaborated in Appendix C. After the elaboration of the model, the results of the models will be compared to the results given by Eurocode 2 and Jones method. The dimensions of the model are given in the table below.

Table 17 Dimensions of the 2D bending model with a height of 3000mm

Variables:	Dimension [mm]
Length	4000
Width	200
Height	3000
Diameter reinforcement	40
Cover	80
Mesh size	20

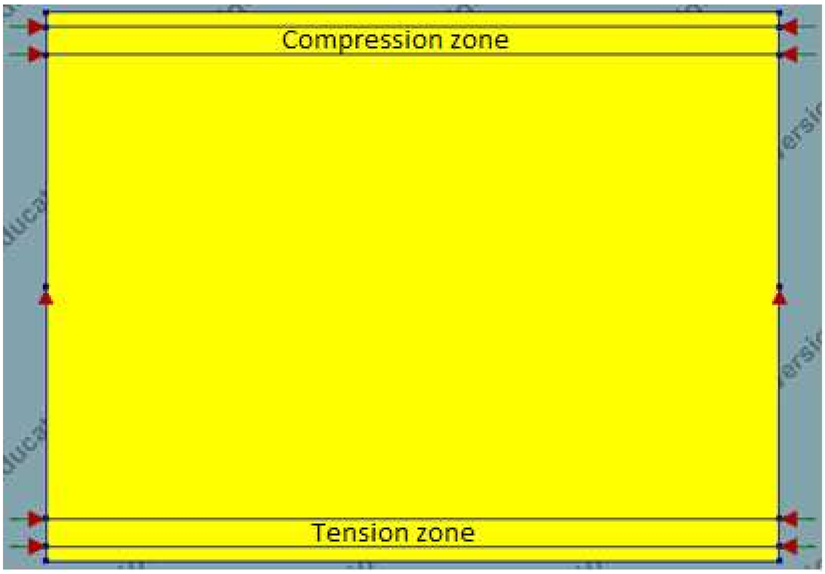


Figure 77 2D bending model with a height of 3000mm

In Figure 77 the 2D bending model is shown. This model is supported at half of the height in the vertical Y-direction. At this position the model rotates around the z-axis. The ends of the reinforcement are supported in horizontal x-direction because the finite element model needs to have a support in the direction of the prescribed deformation. Furthermore two reinforcement bars are applied at the top and bottom of the model to ensure structural safety. A prescribed deformation towards the middle of the model (pushing) is applied on both ends of the top reinforcement. A pulling prescribed deformation is applied on both ends of the bottom reinforcement. These prescribed deformations are applied to get a bending moment onto the model.

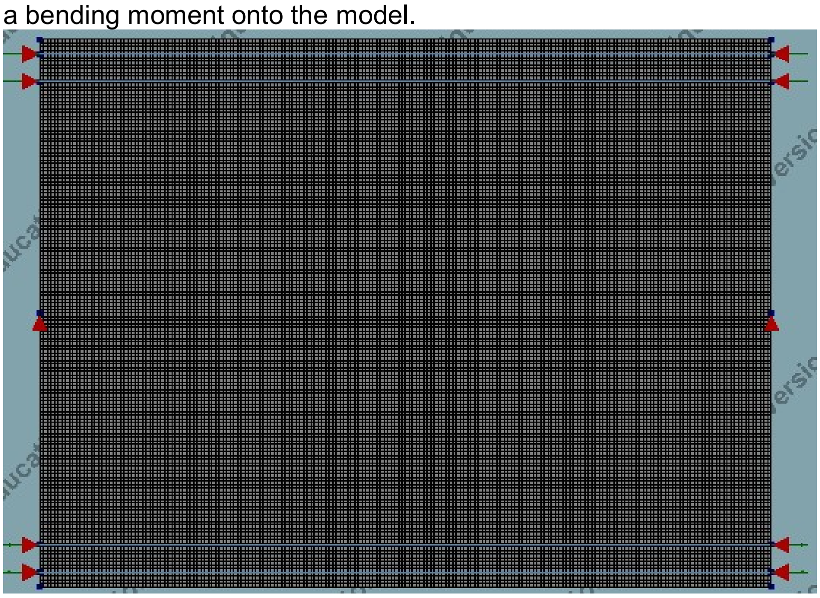


Figure 78 Mesh of the 2D bending model with a height of 3000mm

In Figure 78 the mesh of the 2D bending model is given, the mesh size used is 20mm. This mesh is a fine mesh to get clear crack patterns and stress gradients.

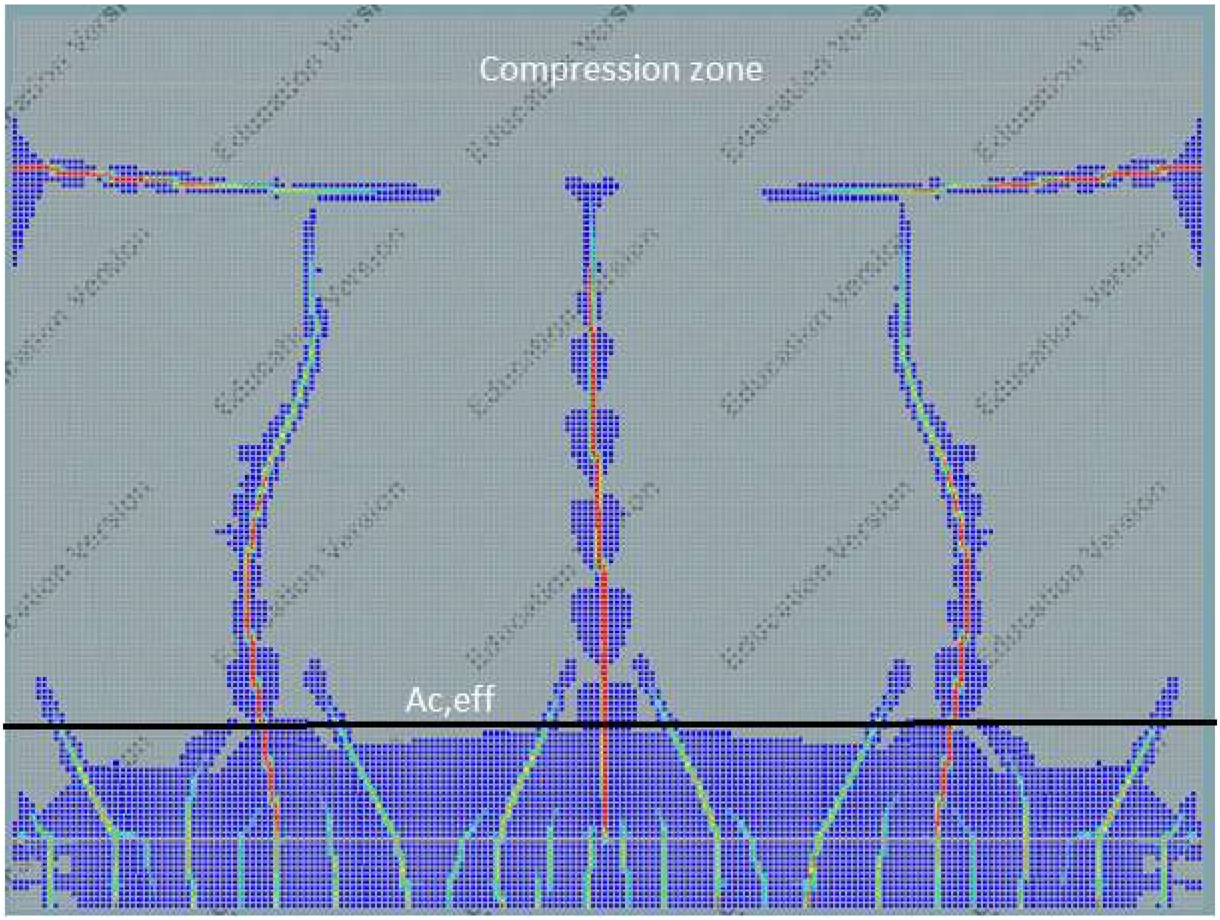


Figure 79 Crack pattern of the 2D bending model with a height of 3000mm

In Figure 79 the crack pattern of load step 300 is shown. The crack spacing varies over the length of the model. In the middle of the model a symmetry line can be observed. This effect is obtained because the model is loaded by the prescribed deformations on both ends of the model. The crack pattern shows 3 primary cracks which run towards the compression zone and 21 secondary cracks which mostly run towards the primary cracks. Along the base of the model, a tensile beam model can be observed in which the secondary cracks originate. This is shown by the area under the black horizontal line depicted by Ac,eff. The scale at which the cracks are presented is modified to see the cracks clearly. The horizontal cracks at the top of the model are due to the way the model is loaded on the top. The prescribed deformation on top results in a small beam model at the top of the model. This part cracks off the total model. In reality this phenomena would not occur, so these cracks can be ignored. The horizontal cracks are not important for the determination of the effective concrete tension zone height because these cracks do not hinder the effective concrete tension zone. Furthermore the compression stresses in the top of the model do not spread towards the effective concrete tension zone because the length of the model is small. Therefore the compression stress does not have an influence on the effective concrete tension zone.

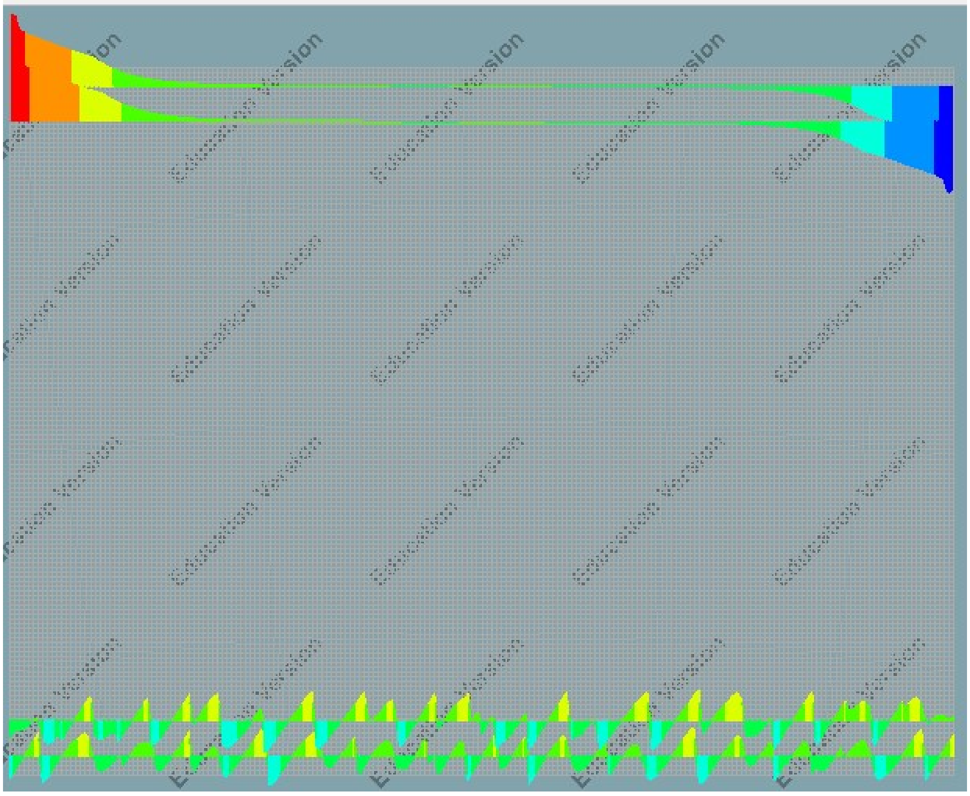


Figure 80 Bond stress gradient of the 2D bending model with a height of 3000mm

In Figure 80 the bond stress gradient for load step 300 is given. The positions of the cracks can be observed because at the positions of the cracks the bond stress suddenly changes sign from a positive value to a negative value. At these positions the slope of the curve is steep. Between the cracks, the bond stress runs from a large negative value to a large positive value with a nearly constant linear slope. This slope is given by the bond-slip curve of fib Model code 2010. With the slip, the bond stress is determined. The total slip of the top reinforcement is located at the ends of the reinforcement, therefore the bond stress is large at these positions. Furthermore the model is under compression at the top reinforcement, therefore no cracks occur at the top reinforcement, which can influence the bond-slip behavior. Due to the cracks in the bottom reinforcement the bond stress decreases because the slip is spread out over the full length of the model.

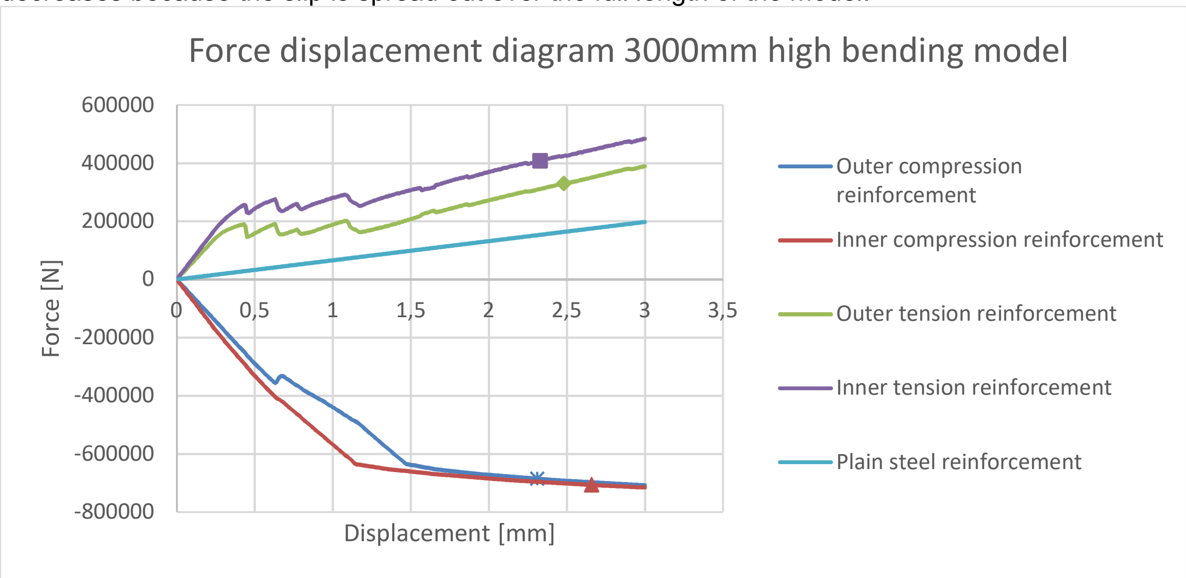


Figure 81 Force displacement diagram of the 2D bending model with a height of 3000mm

In Figure 81 the force-displacement diagram is given. The green and purple lines represent the tension force in the inner and outer reinforcement. After the elastic part of the tension force, the load steps at which a crack occurs are clearly visible by drops in the line. The red and blue lines represent the compression forces in the inner and outer reinforcement. The elastic and plastic branches are clearly visible in the compression lines.

In the table below, the parameters for determination of the effective concrete tension zone are given. These are the values for only half of the model but due to symmetry in the middle of the model all these values can be multiplied by two to get the values for the whole model. With these values, the average cracking force can be calculated. First the results of the outer reinforcement will be given in a table, thereafter the results of the inner reinforcement will be given.

Table 18 Summary of the parameters from the outer reinforcement of the 2D bending model with a height of 3000mm

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
2.23	350	125.7	98142	2.9	33842
2.28	520	125.7	149270	2.9	51472
2.36	430	125.7	127584	2.9	43995
1.91	600	125.7	143645	2.9	49533
2.76	220	125.7	76441	2.9	26359
2.45	290	125.7	89190	2.9	30755
2.49	280	125.7	87787	2.9	30271
2.18	310	125.7	85094	2.9	29343
2.88	150	125.7	54211	2.9	18693
3.13	210	125.7	82582	2.9	28476
3.04	210	125.7	80190	2.9	27652
3.25	140	125.7	57173	2.9	19715
3.10	100	125.7	38998	2.9	13448
2.64	130	125.7	43170	2.9	14886
3.43	130	125.7	56113	2.9	19349
3.46	130	125.7	56517	2.9	19488

$$\sum cracking\ force\ oute \qquad 2652211N$$

$$n = 32$$

$$Average\ cracking\ force = N_{cr,ave} = \frac{\sum cracking\ force}{n} = \frac{2652211}{32} = 82882N$$

With an average cracking force of 82882N, the effective concrete tensile zone becomes:

$$A_{c,eff} = \frac{N_{cr,ave}}{f_{ctm}} = \frac{82882}{2.9} = 28580mm^2$$

This implies an effective concrete tension zone height of:

$$h_{c,eff\ outer} = \frac{A_{c,eff}}{b} = \frac{28580}{200} = 143mm$$

Table 19 Summary of the parameters from the inner reinforcement of the 2D bending model with a height of 3000mm

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
3.99	340	125.7	170583	2.9	58822
2.26	530	125.7	150561	2.9	51918
2.57	400	125.7	129341	2.9	44600
1.69	660	125.7	140209	2.9	48348
3.05	240	125.7	92109	2.9	31762
2.60	280	125.7	91481	2.9	31545
2.60	340	125.7	110892	2.9	38239
2.19	310	125.7	85411	2.9	29452
3.06	160	125.7	61507	2.9	21209
3.59	230	125.7	103810	2.9	35796
4.07	180	125.7	91982	2.9	31718
3.20	120	125.7	48281	2.9	16648
3.99	180	125.7	90336	2.9	31150
3.42	160	125.7	68758	2.9	23710
4.74	160	125.7	95382	2.9	32890
4.09	120	125.7	61705	2.9	21278

$$\sum crackin \quad force\ inner = 3184693N$$

$$n = 32$$

$$Average\ cracking\ for \qquad {}_{cr,ave} = \frac{\sum cracking\ force}{n} = \frac{3184693}{32} = 99522N$$

With an average cracking force of 99522N, the effective concrete tensile zone becomes:

$$A_{c,eff} = \frac{N_{cr,ave}}{f_{ctm}} = \frac{99522}{2.9} = 34318mm^2$$

This implies an effective concrete tension zone height of:

$$h_{c,eff\ inner} = \frac{A_{c,eff}}{b} = \frac{34318}{200} = 172mm$$

The total effective concrete tension zone height is the inner and outer value added together.

$$h_{c,eff} = h_{c,eff\ inner} + h_{c,eff\ outer} = 172 + 143 = 315mm$$

The effective concrete tension zone height according to Eurocode 2 is given by:

$$h_{c,effE} \quad = \min\left[\frac{h-x}{3}; 2.5 * \left(c + \frac{\emptyset_{sl}}{2}\right)\right]$$

$$\frac{h-x}{3} = \frac{3000-560}{3} = 813mm, \qquad 2{,}5 * \left(\frac{150}{2} + 60 + \sqrt{2} * \frac{40}{2}\right) = 408mm$$

The effective concrete tension zone height according to Jones method is given by:

$$h_{c,effJM} = c + \frac{\emptyset_{sl}}{2} + \min\left[\frac{ssv}{2}; 1.5 * \left(c + \frac{\emptyset_{sl}}{2}\right)\right]$$

$$h_{c,effJM} = 60 + \frac{40}{2} + \min\left[\frac{150}{2}; 1.5 * \left(60 + \frac{40}{2}\right)\right] = 80 + \min[75; 120] = 155mm$$

The effective concrete tension zone height according to Jones method is given for a single reinforcement bar because the reinforcement bar at which a crack will most certainly occur, needs to be considered.

In this configuration, the output of the finite element model results in a smaller effective concrete tension zone height than given by Eurocode 2. The difference between both methods is 94mm. This results in a difference in crack width of 0.06mm. For the bending models the crack spacing formula is normative for the determination of the crack width. If both rebars were to be considered, the effective concrete tension zone height given by Jones method is in between the values of the finite element model. The difference between both methods is 4mm. The difference in crack width is 0.00mm.

The results of the 500mm and the 1000mm height model will be presented here to make a comparison between the values of the bending models. With this comparison a conclusion can be given about the determination of the effective concrete tension zone height according to Eurocode 2 and Jones Method. The results will be presented both in a table and graphically.

Table 20 Determination of the effective concrete tension zone height of the bending models

Height H [mm]	Reinforcement [-]	Average cracking force $N_{cr,ave}$ [N]	Concrete tensile strength f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]	Width of the model B [mm]	Effective concrete tension zone height $h_{c,eff}$ [mm]	Total effective concrete tension zone height $h_{c,efftot}$ [mm]	Effective concrete tension zone height EC2 $h_{c,effEC2}$ [mm]	Effective concrete tension zone height JM $h_{c,effJM}$ [mm]
500	bottom	53863	2.9	18574	200	93	93	107	107
1000	bottom	118885	2.9	40995	200	205	205	200	200
3000	Both	182403	2.9	62898	200	314	314	408	310
3000	Upper	99522	2.9	34318	200	172	172	408	155
3000	Bottom	82882	2.9	28580	200	143	143	408	155

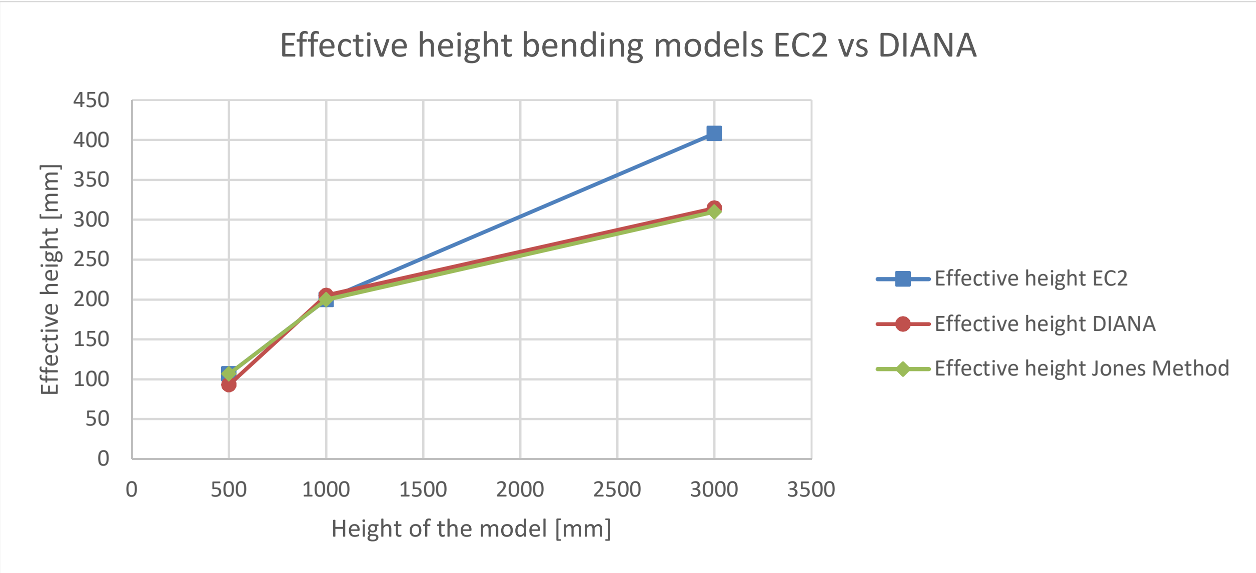


Figure 82 Comparison of the effective concrete tension zone heights of the bending models between EC2 and finite element models

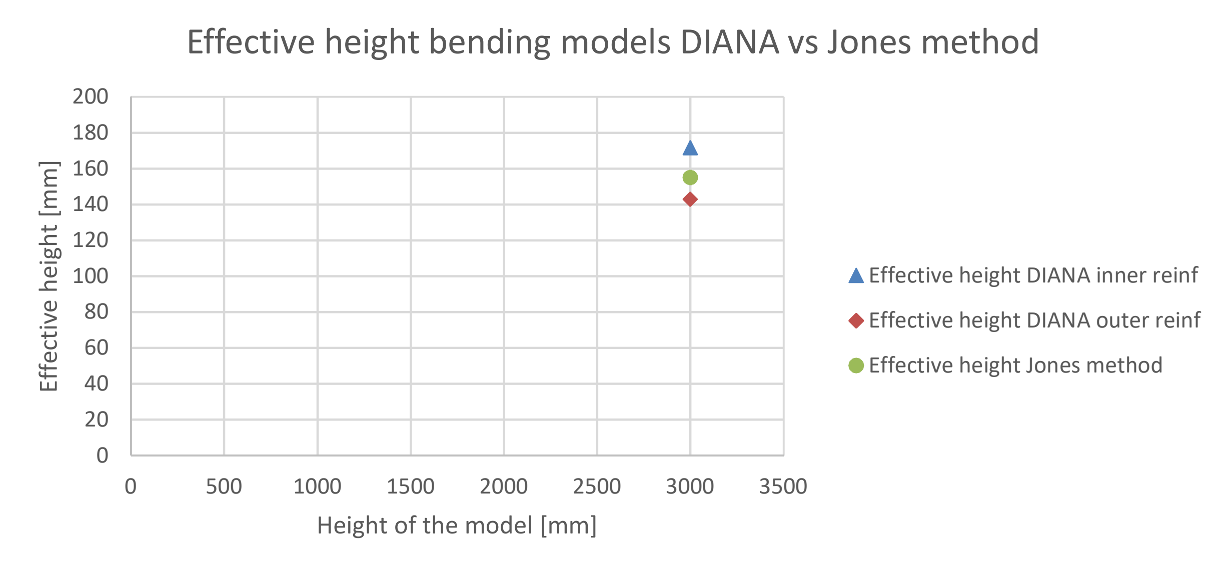


Figure 83 Comparison of the effective concrete tension zone heights of the bending models between DIANA and Jones method

The effective concrete tension zone height of the bending models with a single layer of reinforcement (500mm and 1000mm height) are almost equal for the finite element models, Eurocode 2 and Jones method, as can be seen in Figure 82. The average difference in effective concrete tension zone height between Eurocode 2 and the finite element models is 10mm. This difference results in a crack width difference of 0.01mm. Eurocode 2 gives a higher effective concrete tension zone for the model with a height of 3000mm. This difference is 94mm which results in a crack width difference of 0.06mm. This difference is due to the fact that the determination of the effective concrete tension zone is different for Eurocode 2 and Jones method for multiple layers of reinforcement. The effective concrete tension zone height in the finite element model for the outer and inner reinforcement bars is lower and higher respectively than the effective concrete tension zone height given by Jones method, as can be seen in Figure 83. On average, the effective concrete tension zone is almost equal to the value obtained by Jones method. The difference between Jones method and the finite element models is 4mm. This results in a crack width difference of 0.00mm. These results show that the Jones method is in good conformity with the finite element models. This conclusion is based on one situation, thus a solid conclusion about the effective concrete tension zone height of an individual beam with multiple layers of reinforcement cannot be given here.

The results given here are promising because the effective concrete tension zone height of the finite element models is smaller than the ones obtained by using Eurocode 2 for the investigated situations. Additional models should be investigated to see if the conclusion drawn up here is correct. These models should contain different reinforcement lay-outs and dimensions of the model.

7 CONCLUSIONS

In this chapter the conclusions to the problem statement and the sub questions will be given.

The main question of the thesis is:
Are the effective concrete tension zone heights determined with either Eurocode 2 or Jones Method comparable with the effective concrete tension zone heights determined with finite element models using DIANA?

For a single layer of reinforcement in models loaded by tension, Eurocode 2 underestimates the effective concrete tension zone according to the finite element models. For multiple layers of reinforcement the effective concrete tension zone height of Jones method is comparable with finite element models loaded by tension. Eurocode 2 overestimates the effective concrete tension zone height for multiple layers of reinforcement according to the finite element models loaded by tension. Therefore the effective concrete tension zone heights of Eurocode 2 are not comparable to finite element models while Jones method is comparable if two layers of reinforcement will be used.

The variation study on the models loaded by tension show that the effective concrete tension zone height determined with finite element models is larger than determined with Eurocode 2 for situations with one layer of reinforcement. Eurocode 2 underestimates the effective concrete tension zone height according to these models which results in an underestimation of the crack width. The effective concrete tension zone heights are not comparable according to these models. The variation study of the cover has an average difference of 35mm between Eurocode 2 and FEM. The variation study on the width of the model has an average difference of 74mm in effective concrete tension zone height. According to the variation study on the diameter this difference is on average 57mm. These differences result in an average difference in crack width of consecutively 0.03mm, 0.04mm and 0.03mm. The crack spacing formula is normative for the determination of the crack width when the effective reinforcement ratio is the variable according to this variation study. An increase of the effective concrete tension zone height results in an increase of the crack spacing formula. This is negative for the crack width calculation. An increase in effective concrete tension zone would increase the local tension stiffening effect. This would decrease the crack width, which is positive, but this is not observed. Therefore the influence of the crack spacing formula is larger.

Furthermore the slope of the effective concrete tension zone height of the different variation studies is smaller than the slope given by Eurocode 2. A reason for these differences is the way in which the effective concrete tension zone height is determined with the finite element models by using the cracking forces. The cracking forces are in the same range for the different models therefore the effective concrete tension zone height does not change substantially with varying model properties.

The variation study of the models loaded by tension with two layers of reinforcement in which the height is varied show that the effective concrete tension zone height of the finite element models is smaller than determined with Eurocode 2. Therefore the effective concrete tension zone heights are not comparable according to these models. The average difference between both methods is 39mm. This results in an average difference in crack width of 0.02mm. Jones method results in smaller effective concrete tension zone heights than the finite elements models the difference is on average 11mm. This results in a difference of 0.01mm in crack width. Jones method also results in smaller effective concrete tension zone heights than Eurocode 2. The difference in between these methods is on average 61mm. This results in a crack width difference of 0.03mm on average. For this variation study the crack spacing formula is normative for the determination of the crack width as is explained in the conclusion about the models with one layer of reinforcement.

The results of the models loaded by bending show that the effective concrete tension zone height determined with the finite element models, Eurocode 2 or Jones method are almost equal for models with one layer of reinforcement. The difference in effective concrete tension zone height between the models is 10mm on average. This results in a difference of 0.01mm in crack width. If two layers of reinforcement are used the results of the finite element models and Jones Method are almost equal, the difference is 4mm, while Eurocode 2 overestimates the effective concrete tension zone height with 98mm. The differences in crack width are consecutively 0.00mm and 0.06mm. A reason for the overestimation of Eurocode 2 is the way in which the effective concrete tension zone height is determined. This is due to the fact that when the effective concrete tension zone height of structures with multiple layers of reinforcement of Eurocode 2 and Jones method will be compared to each other, Jones method will always result in smaller or equal effective concrete tension zone heights than given by Eurocode 2. This is due to the formula which is used for the determination of the effective concrete tension zone height, as is explained in this thesis. The results of the bending models are promising but are determined with only three models therefore the conclusion about these model is a provisional conclusion. To give a sound conclusion more models with different geometry parameters should be investigated.

Answers to the sub questions:

- The determination of the effective concrete tension zone height for either Eurocode 2 and Jones method are described in the codes itself and are elaborated in this thesis.

- The effective concrete tension zone height can be determined in finite element models by using the average bond stress, transfer length, perimeter of the reinforcement, the concrete tensile strength and width of the model investigated. Calculations performed with these parameters are given in the thesis.

- Jones method results in effective concrete tension zone heights which are closer to the heights resulting from the finite element models.

- In the thesis it has been shown that there is equilibrium between the force exerted on the model and the stresses in the finite element models. Therefore the cracking forces can be traced back within the finite element models.

- The crack spacing formula of Eurocode 2 consists of two parts, first the part which is determined by the cover and second the part which depends on the effective reinforcement ratio and factors regarding loading and bond properties. The first part represents the pulling out of cones next to a crack. The size of the cone is as large as the cover multiplied by a safety factor of 1.7. This phenomenon is also observed in DIANA for some of the finite element models and came close to the values given by Eurocode 2.

Some other remarkable phenomena and conclusions are found when the thesis was elaborated. These are given below.

- By increasing the bond shear stiffness in the finite element model, the crack spacing decreases, which will result in more cracks in the model. The increase in bond shear stiffness results in a faster buildup of bond stress which will increase the bond force. The bond force, which is needed for a next crack to appear, remains the same. The bond force is determined by bond stress, transfer length and perimeter of the reinforcement, so an increase in bond stress results in a decrease of the transfer length when the bond force remains constant.

- When a bi-linear bond-slip graph is used in the finite element model, the shear stiffness, which is entered as a constant factor, is overruled by the shear stiffness of the bi-linear bond-slip graph.

- Within the 3D models, the pull out of cones at the positions of the prescribed deformation is more visible than in the 2D models. Furthermore the tearing off of the reinforcement within the 2D models is not found in the 3D models. Therefore the conclusion can be drawn that there are some 3D effects which have been underestimated by using 2D models but these effects do not hinder the results represented in this thesis.

- The tension stiffening effect is larger for models with a larger cross sectional area due to the fact that the concrete area is larger. The concrete area can take up more force if it is larger because the stress remains the same.

- The finite element models crack in two different phases. First the model cracks with a large crack spacing after which the model produces new cracks in between the already existing cracks. When this phenomena occurs the difference in steel stress decreases which results in a decrease in bond force because these forces are equal. The effective concrete tension zone height decreases as well when the bond force decreases because the effective concrete tension zone height is determined with the bond force in this thesis.

- The force-displacement diagram of the bending models runs smoother than the diagram of the models loaded by tension. A reason could be that the cracks form in a more gradual manner when the model is loaded by bending instead of tension. The sudden drops in the force displacement diagram are smaller as well.

- In bending models, the convergence criteria are achieved better than in models loaded by tension due to the fact that the stress distribution in bending models vary over the height. This makes it an inhomogeneous stress distribution which is easier for the finite element models because the positions in which the tensile strength is exceeded is lower. For the models loaded by tension, the positions in which the tensile strength is exceeded is larger for a model with homogeneous tensile strength. This makes it harder for the finite element model to converge.

- The variation study in which the width of the model is varied shows that the effective concrete tension height is not dependent on the width of the model. This is also suggested in Eurocode 2.

- The crack widths in the finite element models are smaller closer to the reinforcement than at a distance away from the reinforcement. This also holds for cracks in reality because cracks tend to open from the reinforcement to the surface.

8 RECOMMENDATIONS

A homogeneous concrete tensile strength is used in this master thesis. To make the model more realistic it is also possible to use a random field approach. A random field approach is a script in which it is possible to manipulate the concrete tensile strength. The concrete tensile strength can be varied in every direction and can be varied over certain distances. It is also possible to make the concrete tensile strength dependent on the concrete tensile strength of the neighboring elements.

Experiments need to be performed to check whether all the assumptions made in this master thesis are correct. Especially the shear stiffness which has been used for the bond-slip because the shear stiffness has an influence on the bond stress and the crack spacing.

The variation study is based on 2D models without a 3D effect. To get a more realistic model the model needs to be 3D to get 3D effects. The differences between 2D and 3D models need to be investigated as well.

The models investigated in this master thesis are fictitious models. To get a better understanding of the effective concrete tension zone height of realistic models, it is advisable to use the approach of this master thesis on realistic models, to check whether the results are comparable.

The models investigated in this thesis are mostly loaded by pure tension. Some bending models have been investigated as well. It is advisable to use the approach described here on more bending models and models with a certain degree of restraint, to see if the approach is still applicable for these kind of loading configurations.

To use the approach given in this master thesis to see if it is possible to reduce the amount of reinforcement needed. By using a smaller effective concrete tension zone height which will reduce the crack spacing and thus the crack width.

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10 APPENDIX

A APPENDIX A – LITERATURE STUDY

A.1 Summary

This master thesis is about the calculation method of the crack width of concrete structures which are subjected to imposed deformations due to certain types of restraint. The concrete structures which have been investigated are massive concrete structures with single or multiple layers of reinforcement. For concrete structures the Eurocode 2 is the normative code to calculate crack widths. The crack width formulas are dependent on different parameters, these parameters can be determined by formulas which are stated in the Eurocode 2. The determination of the effective height for concrete cross sections with multiple layers of reinforcement or irregular cross sections is not clearly written in the Eurocode. With the effective height the effective reinforcement ratio has to be determined, which is a parameter which influences the crack width. Engineers experience difficulties with the determination of the effective height. Jones, which is an engineer at ARUP, wrote a paper about the determination of the effective height. This paper is called the Jones Method in this master thesis. The influence on the crack width of the Jones Method has been investigated and compared with the crack widths given by the Eurocode in this literature study.

There are also other methods or papers written about the determination of the crack width. These papers are called the CIRIA C660, which is about early age thermal cracking, and the ICE706 which gives a better understanding about the crack width calculations for structures who are subjected to edge restraint. These papers are also investigated within this literature study to get a better understanding of the cracking behavior in different situations. And to come up with a calculation method in which the crack width can be determined for massive concrete structures with multiple layers of reinforcement.

The CIRIA C660 is about early age thermal cracking as is written before. Early age thermal cracking is a result of temperature differentials, shrinkage and the degree of restraint. If the structure is not restrained the strains can result in free deformations which implies that there will not be any stresses due to the deformations. The degree of edge restraint can be calculated according to a formula which is given in the CIRIA C660. The formula consists of the cross section and modulus of elasticity of the new and old concrete. Furthermore the CIRIA C660 gives ways to improve the concrete mix composition to decrease the temperature during hardening. And the way in which the cracking strains are determined differs from the way in which they are determined by the Eurocode.

In the Eurocode 2-3 two different types of external restraint are considered. These types of restraint differ in the determination of the crack strain. For edge restraint the crack strain is determined by the free strain and the degree of restraint. For end restraint the crack strain is determined by the tensile concrete strength, the modular ratio, the effective reinforcement ratio, modulus of elasticity and coefficients. The influencing factors differ in every way while someone would suggest that the influencing factors should be somewhat the same. This has been investigated in the ICE706. The crack strain for edge restraint situations is determined by two stages. The crack strain of the first stage is dependent on the strain in the reinforcement and the strain of the cracked concrete. The second stage depends on the continuing contraction of the cracked concrete in contrast to the reinforcement.

The different methods which have been described in this summary are compared to each other in an Excel sheet for different situations. The results are given in chapter 7. Overall the ICE706 method gives the best results for both types of external restraint.

A.2 Introduction

Concrete has to crack to make sure that the reinforcement takes over the forces which are generated inside the concrete element. But what makes the concrete crack and how can these cracks be prevented or controlled? The answer to this question is given within this literature study. The literature study focusses on the crack width control of massive concrete structures which are externally restraint and have single or multiple layers of reinforcement. The structure is loaded by imposed deformations due to the restraint. The crack width control is investigated by looking at different methods which describe cracking behavior for different situations and cross sections. The normative method to control crack width is given in the Eurocode 2. But other methods have been investigated as well to get a better understanding of the cracking behavior. And to come up with realistic values in situations in which the Eurocode lacks in clear descriptions of the determination of the crack widths. These methods are CIRIA C660, ICE706 and the Method of Jones. At first the phenomena of cracking is described. Secondly the crack width determination according to the Eurocode 2 is given, after that the method of the CIRIA C660 is described. Followed by the method given in the ICE706 and finally the method described by Jones is given. All methods have their own improvement in contrast to the Eurocode 2 that is why these methods will be investigated within this literature study. In chapter 7 calculations have been performed to show the differences in the methods. And to come up with a method which will be investigated in the next phase of the master thesis.

A.3 Introduction to cracking

When concrete is poured the concrete is still fluid and workable and has low strength and stiffness. During the hardening of the concrete, which is a reaction between cement, water, aggregates and additives, the stiffness and strength of the concrete increases by time. The chemical reaction of concrete is a exothermic reaction which implies that heat is produced during this process. The heat which is produced increases the speed of the reaction process. The temperature within the structure keeps rising if enough reaction material is available. At this stage the wooden formwork is still around the structure what implies that the heat cannot be transferred to the environment. If steel formwork would be applied the heat can be transferred to the environment, because the steel formwork has a higher heat conduction coefficient. The core of the structure heats up if no heat can be transferred to the environment and reaches the peak temperature. In massive concrete structures this peak temperature is higher than the peak temperature for thin structures because the structures have a smaller core and there is less reaction material available to increase the temperature.

The structure wants to expands due to the increase in temperature within the structure. The expansion of the structure can be referred to as strain. When the structure can expand freely and does not have a restraining element the strain is called the free strain or free expansion of the structure. The free expansion of the structure can be reduced or hindered due to a restraining element. A restraining element is an already poured element which is already hardened and cooled down to ambient temperature. The type of restraint what belongs to these elements is called external restraint because the structure is restrained by an external element. External restraint has two different types the first type is end restraint which implies a restraint at the ends of the structure for instance a slab cast between two already cast walls. The second type of external restraint is edge restraint this type of restraint is given by a restraining element on the edge of the structure a typical example is a wall cast on a foundation slab. For massive concrete structures a third type of restraint is also possible this type of restraint is called internal restraint. Which implies that the structure itself restrains the expansion of the core. The behavior of the different types of restraint will be explained in the next chapter. It is expected that the degree of restraint varies over the height of the structure because the restraint at the joint of the old and new poured concrete would be the highest. The restraining element is closest by at the joint that is why this would be expected. Even so the geometry of the newly poured concrete could have an effect on the variety of the degree of restraint.

Stresses will arise within the structure if the structure is restrained, if the structure can deform freely there will not be any stresses within the structure. Due to the restraint the structure cannot deform as it wants to deform which will imply stresses within the element. This type of loading is called imposed deformations. The stresses inside the structure, due to temperature rise from the hardening process of the concrete, change from compression in the heating phase to tension in the cooling phase. The compression stresses in the heating phase are small due to the fact that the concrete has low stiffness because the stiffness increases by time. The stress, stiffness and strain are interconnected due to Hooke's law ($\epsilon = \frac{\sigma}{E}$). When the structure starts to cool down the structure wants to shrink, but the shrinkage of the structure is restrained due to the restraining element. The compression stresses which were present in the structure will change to tension stresses. The tension stresses will grow due to the fact that the stiffness of the concrete has increased. Concrete has a 10 times weaker strength when it comes to tension instead of compression. So cracks can form if the tensile stress due to the imposed deformations are higher than the concrete tensile strength which develops over time.

The process which is described above is given in the figure on the next page. In the previous indention it has been written that the degree of restraint varies over the height of the structure. The degree of restraint has an influence on the stresses which will result in cracks. This will imply that the cracks will change when the height of the structure increases, because the degree of restraint varies over the height of the structure.

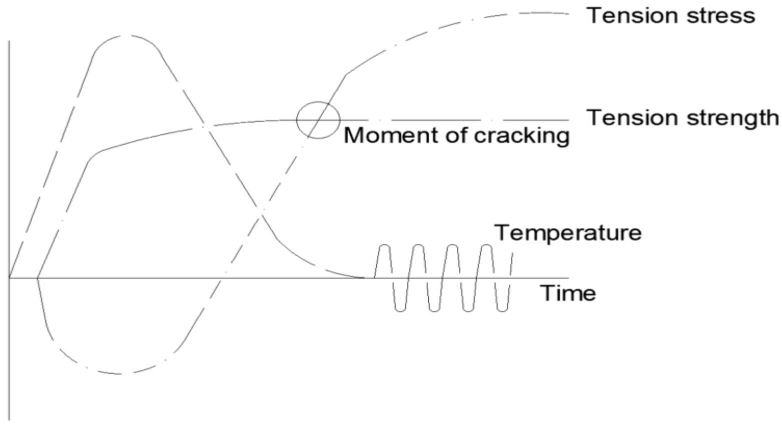


Figure 1 Temperature, tension stress and strength development in time

The tension stresses are equally spread over a part of the cross section where the restraint remains the same. The first crack will form at the weakest location within the concrete structure. Concrete is an inhomogeneous material so the concrete tensile strength varies within the structure. When a crack occurs the stress in the concrete is relieved and is transferred to the reinforcement through bond stresses. A second crack can develop due to increasing stresses in the concrete when the stresses rise beyond the point of the concrete tensile strength.

There are also other types of strain which are time dependent and have an effect on the crack formation. These phenomena are called: shrinkage, creep and relaxation. At first two different types of shrinkage are described, these are autogenous shrinkage and drying shrinkage. Autogenous shrinkage [12] is a type of shrinkage which comes fourth out of the hardening process of the concrete. Research is still done to the phenomena of autogenous shrinkage because it is not quite clear what the influencing factors for the determination of autogenous shrinkage are. The autogenous shrinkage is dependent on the type of cement because in concrete where blast furnace slag cement have been used a higher autogenous shrinkage is noticeable. Concrete in which Portland cement have been used has a lower autogenous shrinkage. Autogenous shrinkage is most present in the early age concrete.

Drying shrinkage is a type of shrinkage what is determined by the drying of the concrete during its lifetime. The concrete shrinks due to drying. The drying shrinkage is determined by the exposure area of the structure and the environment conditions like relative humidity. Drying shrinkage can result in early-age cracks if the concrete structure is not cured after pouring or if the formwork is taken off to fast. Curing means that the concrete is kept wet at the drying surfaces to prevent drying out. Curing can be done by applying wet blankets on the concrete or protect the concrete with plastic foil. Sunlight or wind can speed up the drying process to increase the chance of early age cracks. When both types of shrinkage are added up for the different types of cement which are described here the total shrinkage is almost the same. Which implies that Portland cement has a higher drying shrinkage than blast furnace slag cement.

Creep is the time dependent effect that with constant stress the strain of the structure increases. Relaxation is the phenomena that with constant strain the stress in the structure decreases. These time dependent effects are showed in the graphs onto the next page.

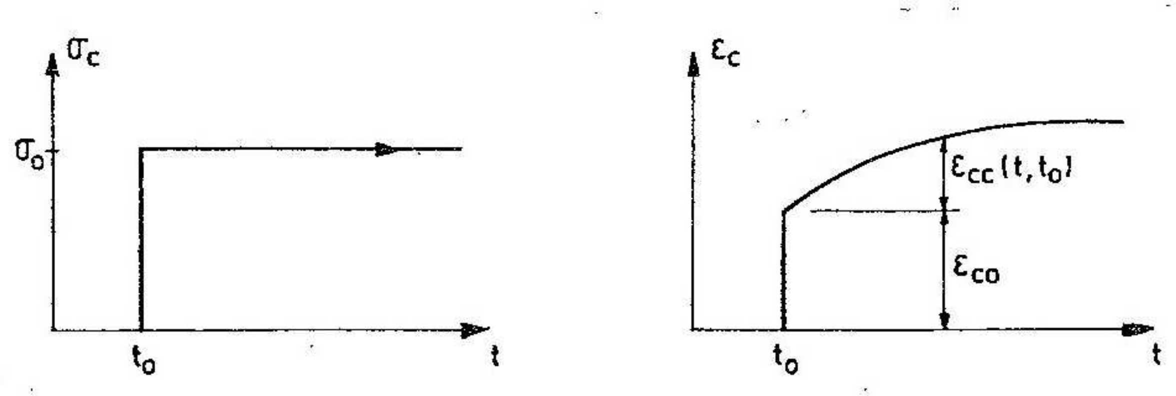
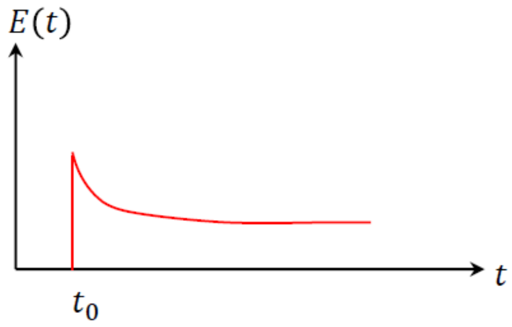


Figure 2 Creep of the concrete   Source: www.google.nl kruip van beton grafiek



Relaxation function

$$E_{(t)} = \frac{1}{J(t, t_0)}$$

Figure 3 Relaxation of concrete   Source: DIANA manual

There are two different types of crack patterns a not fully developed crack pattern and a fully developed crack pattern. In a not fully developed crack pattern new cracks can form due to the increase in load. In a fully developed crack pattern there cannot form any new cracks due to the fact that the additional stress cannot be transferred through the concrete to the steel. The stress between the cracks cannot reach the concrete tensile strength because the length over which the stress is transferred via bond stress is too small. This length is called the transfer length. The spacing between the cracks is between one and two times the transfer length. In a fully developed crack pattern the additional load is directly taken up by the reinforcement. The structure fails if the stresses in the steel reaches the yield stress of the steel.

The cracks which form in the lifetime of the structure have to be controlled and/or prevented. This can be achieved by using reinforcement or by taking other measurements by manipulating the concrete mix composition, change geometry, pouring sequence etc. Using reinforcement is a way to control cracking, because if the concrete cracks the reinforcement takes over the tensile forces. By using smaller diameters and a finer mesh of reinforcement crack widths will get smaller and the crack spacing will reduce as well. Cracks can be prevented by reducing the strains which will develop due to temperature rise, restraining components and imposed deformations. By reducing the effects described here the strains will reduce and this will reduce the stresses. If the occurring stresses reduce the chance of cracking also reduces.

The cracks which form need to be in a certain range depending on the environment in which the structure is. These ranges are given within the Eurocode 2-1-1. If it is an aggressive environment the cracks need to be smaller than when the environment is not aggressive. This is due to the fact that in an aggressive environment the chance of rebar corrosion is higher. Rebar corrosion has to be prevented to insure the durability of a structure. In water retaining structures different classes are given to ensure liquid tightness. These classes are given in EN1992-3 and depend on the crack width. Class 3 is the highest level of liquid tightness. In this class no leaking is permitted which implies that there may not be any cracks. For durability reasons the crack width of structures needs to be below 0,3mm or less depending on the type of structure.

In the figure below a flow chart is given with the influence parameters of crack width control.

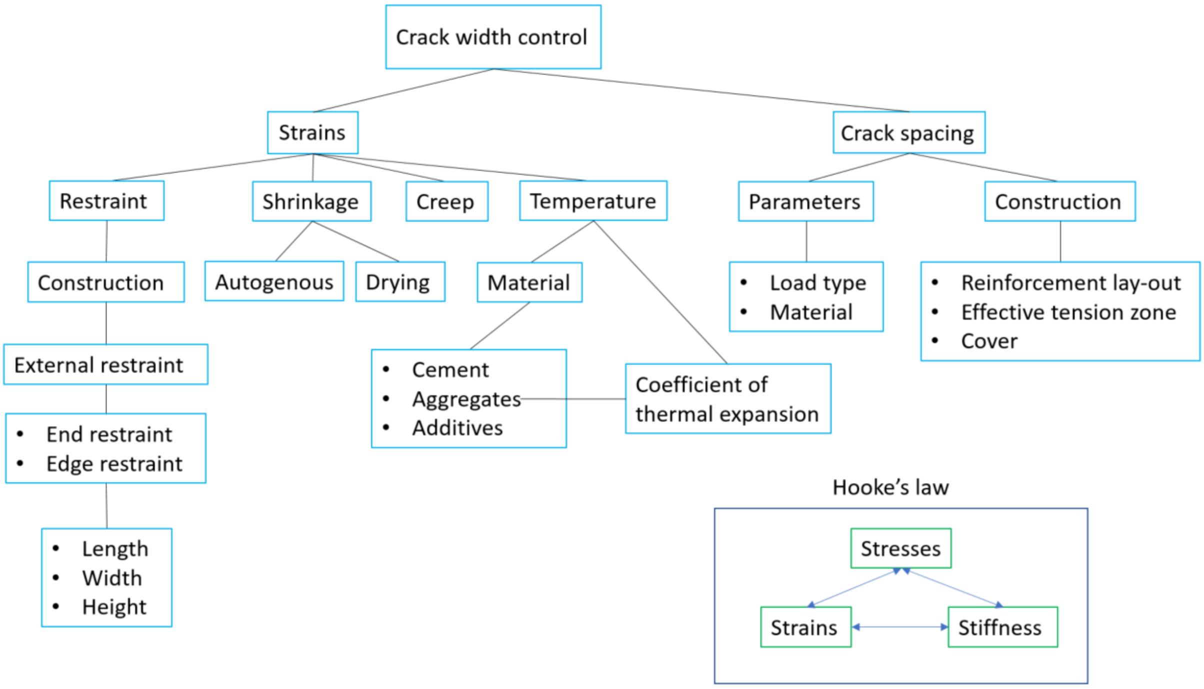


Figure 4 Flow chart crack width control

A.4 Eurocode EN1992-1-1 [1] and EN1992-3 [2]

A.4.1 Introduction

The Eurocode is the standardization for the calculation of all kinds of different structures and components of structures. These codes are mandatory for all calculations. The Eurocodes which have been looked at for this thesis are EN1992-1-1 and EN1992-3. These Eurocodes are for the general calculation of concrete structures and for concrete liquid retaining structures. The topic of the thesis is about crack width control of massive concrete structures with single or multiple rows of reinforcement. Massive concrete structures [11] are concrete structures with a thickness of at least 800mm. The parts of the Eurocode which have been investigated are about crack width control and their formulas and calculations.

A.4.2 Influencing factors of crack width according to the Eurocode

In EN1992-3 different types of restraint are given these types of restraint can be subdivided in internal restraint, external end restraint and external continuous edge restraint. Internal restraint is a type of restraint which is only taken into account at massive structures. Because internal restraint is dependent on the temperature rise and decrease within the cross section. When the concrete hardens heat is produced as is described before. The temperature of the core of the cross section will increase which will result in an expansion of the core. When the core expands the surface of the element will undergo tension stresses which can result in cracks at the surface but in most of the cases these stresses will not result in cracks during hardening. If the core cools down the core wants to shrink but this will be restrained by the surface which will induce cracks in the core. The surface will undergo compression stresses at this moment and the surface cracks which formed earlier will close due to the compression stresses. Usually internal restraint does not have an influence on crack width calculations because the surface cracks, which are important, are closed due to the compression stresses in the surface. A picture of internal restraint is given below:

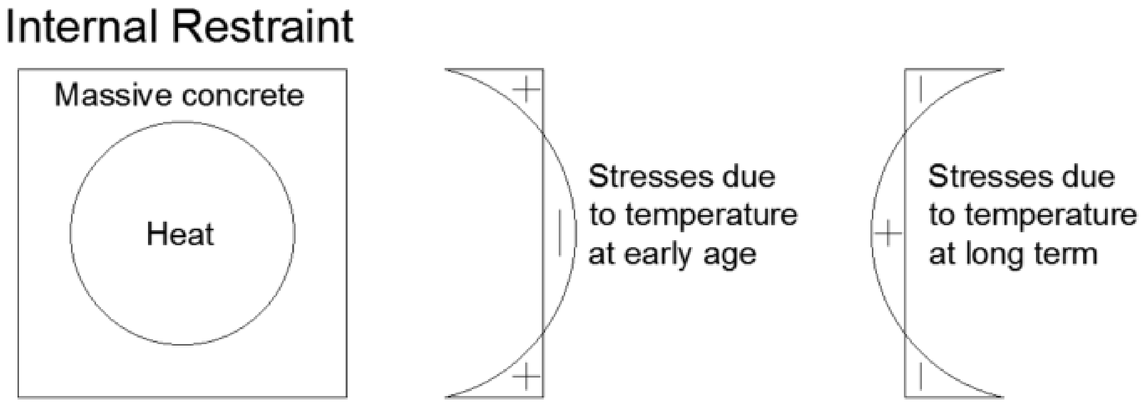


Figure 5 Internal restraint

External end restraint is a type of restraint which comes forth out of floors and/or walls which are casted between rigid walls or columns that already cooled down to ambient temperature and provide restraint. The ends of the casted element are restraint in this form of restraint. Due to the temperature rise in the casted element the element expands which will result in compression stresses. These stresses are low because the E-modulus is still low. The strains during hardening can be taken up by the concrete. When the element cools down the E-modulus is higher and the strains change from sign because the element wants to shrink if it cools down. Due to the higher E-modulus the stresses in the cooling phase will be higher than the stresses in the heating phase. The stresses in the cooling phase are tension stresses. And due to the fact that the element is restraint at its ends the tension stresses will increase and will result in a crack at the weakest point of the cross section. The crack width of these type of cracks is larger than cracks from continuous edge restraint. The amount of cracks is usually smaller compared to continuous edge restraint. The cracks which will form run through the whole cross section. A picture of the situation of end restraint is given on the next page.

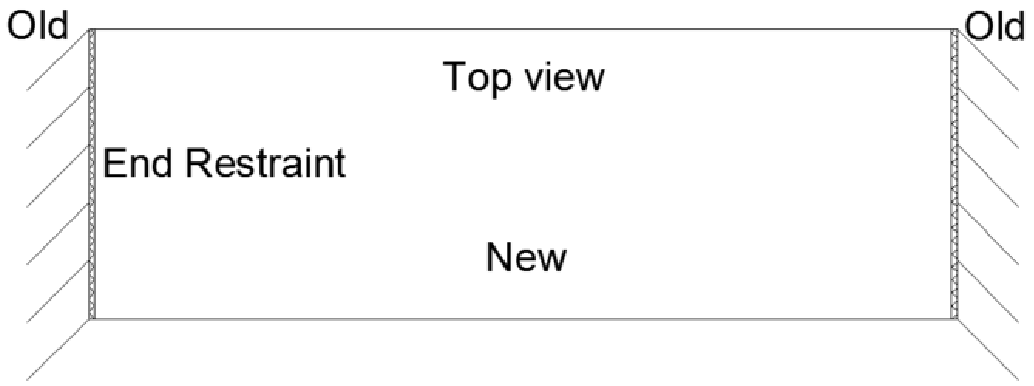


Figure 6 End restraint

External continuous edge restraint is a type of restraint which comes forth out of walls which are casted on floor slabs that already cooled down to ambient temperature. The principle is the same as with the end restraint except that the restraining element, which is the floor slab, takes up a part of the force that is why the crack width and the crack spacing reduces. Usually the maximum crack width is given at an height of 0,1*L above the joint, where L is the length of the wall. Pictures of edge restraint situations are given below with an explanation of the phenomena:

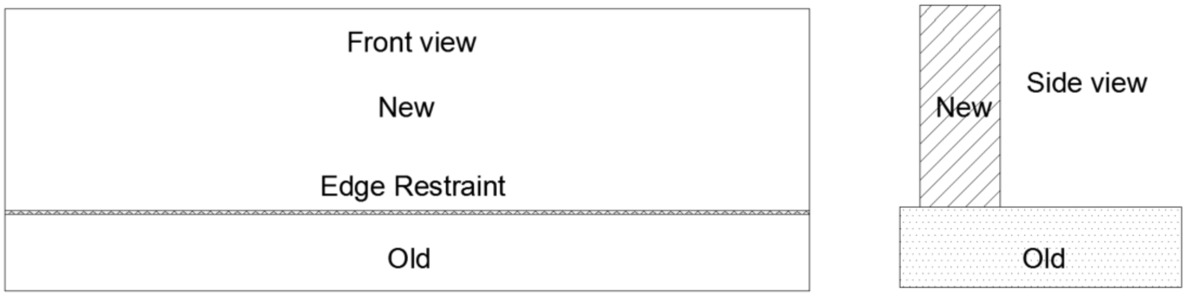


Figure 7 Edge restraint

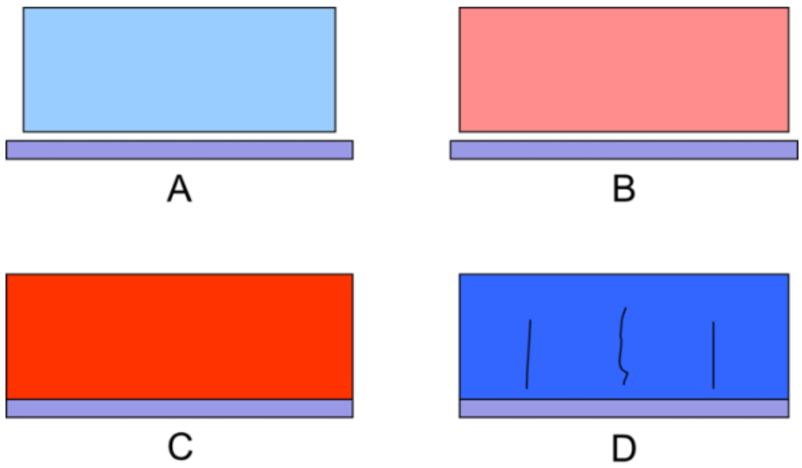


Figure 8 Edge restraint phases   Source: Stufib-rapport 16 [11]

In figure 7 the phases of edge restraint of pouring a wall casted on a slab are given.
A= The wall is casted on the slab the wall starts to hydrate and the temperature rises

B= During hydration the wall starts to heat up, the floor slab does not increase in temperature. The wall expands while the floor slab does not expand. In the wall small compression stresses arise because it heats up but these stresses are small because the wall has a low stiffness.
C= The temperature in the wall increased to the peak temperature. Because the stiffness of the wall is still low small compression stresses are present in the bottom side of the wall.
D= During cooling down the wall wants to shrink, but this is restraint due to the floor slab. The stiffness of the wall is higher at this point and the stresses which are present now are tension stresses. If the tension stresses are higher than the concrete tensile strength the concrete cracks. These cracks run through the whole cross section.

In EN1992-3 different situations are given with the factor of restraint which has to be used in the calculation. It is written that the degree of restraint varies over the height of the structure. The variation depends on the Length/Height ratio. If the length/height ratio is two the restraint at the top equals zero. If the length/height ratio is eight or higher the restraint at the top equals the restraint at the joint.

For the calculation of the crack width control the tension area of the concrete has to be known. The Eurocode makes a difference between the tension area in the concrete (A_{ct}) and the effective concrete tension zone $(A_{ct,eff})$. The tension area in the concrete is given by that part of the cross section which is in tension. The effective concrete tension zone is the zone which is in tension and is around the reinforcement. The effective concrete tension zone is calculated with the effective height. In the Eurocode different effective heights are given which have to be calculated in order to determine the effective concrete tension area. The effective height which has to be used is the smallest height of the four different calculated effective heights. The calculations are given below:

$$\text{Effective height: } h_{c,eff} \leq \begin{cases} 2{,}5(h-d) \\ \frac{h-x}{3} \quad (Bending) \\ \frac{h}{2} \quad (Axial\ tension) \\ 2{,}5\left(c+\frac{\phi_{mr}}{2}\right) \end{cases}$$

Where:
h is the height of the cross section
d is the length between the center of gravity of the reinforcement and the outer compression fiber
x is the height of the compression zone
c is the height of the concrete cover
ϕ_{mr} is the diameter of the reinforcement

In most cases the term $2{,}5(c+\frac{\phi_{mr}}{2})$ is the governing one when massive concrete structures are investigated. This term is almost equal to the $2{,}5(h-d)$ term the only difference is the diameter of the stirrup which has been taken into account in the $2{,}5(h-d)$ term. When the effective height is known it can be multiplied by the width to get to the effective concrete tension area which is given by $A_{ct,eff}$. With this effective concrete tension area the effective reinforcement ratio can be calculated by dividing the reinforcement area by the effective concrete tension area. This is given by:

$$\rho_{,eff} = \frac{A_s}{A_{ct,eff}}$$

The concrete strength class is also important for the determination of the concrete tensile strength. The concrete tensile strength develops in time and in 28 days the strength is at the level of the strength which is used in calculations. In the Eurocode a time function is given in which the tensile strength can be determined for a time which differs from the 28 days strength. The strength which has to be used is the effective strength at the time in which a crack will form. For example if the crack is expected to occur at day 8 the 8th day tensile strength has to be used in the calculation and can be calculated by using the time dependent function from the Eurocode. An increase in shrinkage will result in an increase in crack width.

A.4.3 Crack width formulae according to the Eurocode

To calculate the crack width the crack spacing and the strain difference of the steel and concrete has to be known. The crack spacing can be calculated according to the following formula:

$$S_{r,max} = k_3 c + \frac{k_1 k_2 k_4 \phi_{mr}}{\rho_{,eff}}$$

Where:

k_i are different parameters with $i = 1,2,3,4$
c is the concrete cover
ϕ_{mr} is the diameter of the reinforcement
$\rho_{,eff}$ is the effective reinforcement ratio

The different parameters (k_i) for the calculation of the maximum crack spacing are given in the EN1992-1-1. The value of:

k_1 is determined by the bond properties of the reinforcement. $k_1 = 0{,}8$ for good bond properties and $k_1 = 1{,}6$ for bad bond properties like reinforcement with smooth surfaces.
k_2 is dependent on the type of load. $k_2 = 1$ for pure tension and $k_2 = 0{,}5$ for pure bending. Intermediate values can be calculated by $k_2 = \frac{\epsilon_1+\epsilon_2}{2\epsilon_1}$ in which ϵ_1 is the largest strain at the edge of the cross section and ϵ_2 is the smallest strain at the edge of the cross section.
k_3 is 3,4 according to the Eurocode and National Annex
k_4 is 0,425 according to the Eurocode and National Annex

The difference in the strain of the steel and the concrete gives the strain in the cracked zone this value has to be multiplied by the crack spacing to get the crack width. The difference in strain is given by the following formula given in [2]:

$$\epsilon_{sm} - \epsilon_{cm} = \frac{0{,}5 k k_c f_{ct,eff}}{E_s}\left(\frac{1}{\alpha_e \rho} + 1\right)$$

Where:

k is a parameter to cope with the height and width of the element which is considered $k = 1$ for $h \leq 300mm$ and $b \leq 300mm$. $k = 0{,}65$ for $h \geq 800mm$ and $b \geq 800mm$. Intermediate values can be linearly interpolated.
k_c is depending on the type of load. $k_c = 1$ for pure tension. For bending of rectangular cross sections $k_c = 0{,}4\left[1 - \frac{\sigma_c}{k_1\left(\frac{h}{h^*}\right)f_{ct,eff}}\right] \leq 1$
α_e is the modular ratio between E_s/E_{cm}
$f_{ct,eff}$ is the effective concrete tension strength
E_s is the modulus of elasticity of the reinforcement
ρ is the ratio of reinforcement of the cross section A_s/A_{ct}
σ_c is the average concrete stress $\frac{N_{Ed}}{bh}$
h^* is $h \; for \; h < 1{,}0m \; and \; 1{,}0 \; for \; h > 1{,}0m$
k_1 is 1,5 if N_{Ed} is a compression force and $\frac{2h^*}{3h}$ if N_{Ed} is a tension force

With these formulas the characteristic crack width can be determined by:

$$w_k = S_{r,max} * (\epsilon_{sm} - \epsilon_{cm})$$

A.4.4 Shortcomings Eurocode and improvement methods

The characteristic crack width and maximum crack spacing can be calculated following the approach of the Eurocode. But the approach of the Eurocode is vague and not consistent when irregular and/or massive cross sections are used. For irregular cross sections and cross sections with multiple layers or varying reinforcement lay out the Eurocode does not give a consistent approach on how to determine the effective tension zone in the cross section. And this is a key parameter in the determination of the crack width and crack spacing. A better and clear definition of the effective concrete tension zone is needed to improve the crack width control calculations.

For the improvement of the crack width control calculations different papers have been written which will be discussed in the following chapters. At first the CIRIA C660 will be discussed this paper is more about the influence of the degree of restraint and the temperature differentials due to different cement types, aggregates and additives and next to that the time dependency is also treated. Second the ICE706 paper will be discussed this paper is about the influence of continuous edge restraint on the crack width and crack spacing control. At last the paper of Jones is discussed in which a method is described about how to determine the effective concrete tension zone for irregular cross sections with or without multiple layers of reinforcement. By using all these different papers and approaches a better understanding of the crack width control is given to come up with solutions to the problem of the Eurocode.

A.5 CIRIA C660 [3]

A.5.1 Introduction

In the CIRIA C660 a design approach for the estimation of early-age and long-term thermal cracks is given. The focus is on early-age cracking. Most of the known influencing factors have been investigated for the estimation of thermal cracks. A good design about early-age thermal cracking can be made when all of the influencing factors are determined and taken into account in the calculations. But in most of the times not all influencing factors are known in advance so a simplified method is given as well. Influencing factors are:

- Temperatures
- Coefficient of thermal expansion
- Degree/type of restraint
- Shrinkage and creep
- Tensile concrete strength

A.5.2 Influencing factors of crack width according to CIRIA C660

The influence on early-age cracking of these influencing factors is described below.

Temperature:
In the CIRIA C660 different types of temperatures are described like the difference between the peak temperature of the cross section and the mean ambient temperature T_1, temperature between mean ambient temperature and minimum ambient temperature T_2, peak temperature T_p and the mean temperature of the cross section T_m. The data of these temperatures is given by statistical data and by using the input of the type of aggregates, cement, formwork and additives. The ambient and placing temperature is also important in determining the temperatures which are needed for the design of early-age thermal cracks. The temperatures can be influenced by using ice or by cooling down the aggregates in order to reduce the placing temperature, which will result in a lower peak temperature. It is also possible to cool the concrete from the inside by installing cooling pipes and let water or chilled air run through these pipes. When all factors are known the temperatures can be derived. The type of cement has an influence on the temperature rise and the hardening process of the concrete. If for example blast furnace slag cement is used the peak temperature of the concrete is lower than the peak temperature of Portland cement. And the more cement is used the higher the peak temperature of the concrete will be.

Coefficient of thermal expansion:
The coefficient of thermal expansion is a coefficient which describes the free thermal expansion of a material by a given temperature change. A low coefficient of thermal expansion results in a small strain which can reduce the chance of early-age cracking, because a small strain will result in small stresses. The type of aggregate has an influence on the coefficient of thermal expansion of concrete, because the biggest volumetric part of the concrete consists out of aggregates. In the CIRIA C660 different coefficients have been given for different types of aggregates these values are on the high end of the observed range and represent safe values. These values can be used in the calculations to get a better understanding of the strain due to temperature changes. When there is no data available the coefficient of thermal expansion according to CIRIA C660 is $12\mu\epsilon/℃$.

Shrinkage and creep:
Shrinkage and creep have an influence on the early-age thermal cracking. The shrinkage of the concrete results in strains in the concrete what will result in stresses which can result in cracks. The different types of shrinkage which have been considered in the CIRIA C660 are autogenous shrinkage and drying shrinkage. Both of these shrinkage types are discussed in the Eurocode and there are no changes in the calculation of both types in the CIRIA C660. The creep influence is dealt with by using a factor for the creep effect this factor is equal to $K_1 = 0{,}65$.

Restraint:
The types of restraint which are investigated in the CIRIA C660 are the same as stated in the Eurocode. But the values differ in a way that in the CIRIA C660 a creep coefficient is used in the calculations. In the Eurocode the creep coefficient has already been taken into account in the values which are given in the figures and tables. For external edge restraint a formula is given in the CIRIA C660 which describes the external restraint factor by using the cross sectional area and the modulus of elasticity of the old and new concrete element.

$$R_j = \frac{1}{1 + \left(\frac{A_n}{A_o}\frac{E_n}{E_o}\right)}$$

Where
A_n is the cross sectional area of the new (restrained) concrete
A_o is the cross sectional area of the old (restraining) concrete
E_n is the modulus of elasticity of the new concrete
E_o is the modulus of elasticity of the old concrete

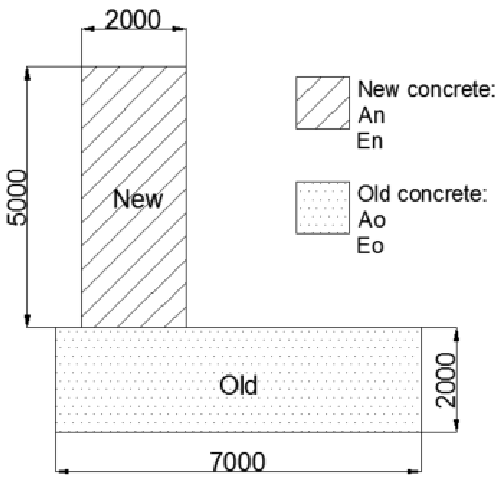


Figure 9 Determination of the degree of restraint

The calculation of the end restraint uses the same approach as the approach of the Eurocode 2-3. In the CIRIA C660 it has been stated that the way of cracking for the different types of external restraint differ just like is stated in the Eurocode 2-3 approach. In thick sections the internal restraint is dominant while in thinner sections the external restraint is governing. In some situations both types of restraint need to be considered. The calculation of the restraint factor is sometimes difficult when difficult geometries have been used. If this is the case it is possible to fall back on simple geometries because there is no data available for difficult geometries. This is up to further investigation.

The tensile concrete strength:
The tensile concrete strength is an important factor because when the stresses inside the concrete are higher than the concrete tensile strength a crack occurs. In most of the times this occurs at the weakest point in the concrete element. The tensile strength is dependent on the time because the strength increases by time. In the Eurocode a formula is given to calculate the tensile concrete strength of a certain concrete strength class at a given time in days. This value has to be used in the CIRIA C660. With this value it is possible to calculate the tensile strain capacity of the concrete. The tensile strain capacity is also of importance because when this concrete strain is exceeded the concrete cracks and the reinforcement has to take over the strain of the concrete. The concrete tensile strain capacity is given by the concrete tensile strength divided by the modulus of elasticity. The formula used in the CIRIA C660 is the same formula as the formula given in the Eurocode.

A.5.3 Crack width formulae according to CIRIA C660

Now that the different influencing factors have been described in more detail the calculation method according to CIRIA C660 will be described in detail. According to CIRIA C660 there are three different types of calculations namely: Wall on a rigid foundation (continuous edge restraint), slab casted between core wall and columns (end restraint), massive foundation slab (internal restraint). These different types of restraint have been described earlier with figures of the configuration of the type of restraint. In the calculations the difference in restraint is important. The calculation method and the formulas which will be used will be described.

The crack strain for internal restraint is given by:

$$\epsilon_{cr} = \epsilon_r - 0{,}5\epsilon_{ctu}$$

With:
$\epsilon_r = K_1 \Delta T \alpha_c R$ the restraint strain depending on: creep coefficient, temperature differential, coefficient of thermal expansion and the degree of restraint.
$\epsilon_{ctu} =$ concrete tensile strain capacity

The maximum crack spacing is given by:

$$S_{r,max} = 3{,}4c + 0{,}425 \frac{k_1 \phi_{mr}}{\rho_{,eff}}$$

The formula of the maximum crack spacing equals the formula for maximum crack spacing given by the Eurocode 2-1-1. The only difference is the value of $k_1 = 1{,}14$ which is higher in the CIRIA C660 because full bond is harder to be achieved in most practical situations according to the writers. They suggest that full bond properties are only possible in laboratory tests.
For internal restraint the crack width is given by a multiplication of the maximum crack spacing and the crack strain:

$$w_k = S_{r,max} * \epsilon_{cr}$$

The crack strain under edge restrain for the long term is given by:

$$\epsilon_{cr} = K_1\left((\alpha_c T_1 + \epsilon_{ca})R_1 + \alpha_c T_2 R_2 + \epsilon_{cd} R_3\right) - 0{,}5\epsilon_{ctu}$$

The first part of the equation before the minus sign can be seen as the restrained strain. The restrained strain is not fully relieved when a crack occurs because there is still some residual tensile strain within the concrete. On average the residual strain within the concrete is equal to 0,5 of the total tensile strain capacity. The crack strain is given by the restrained strain minus the residual strain.

The crack strain formula reduces in length if the early age cracking has to be determined then the formula becomes:

$$\epsilon_{cr} = K_1(\alpha_c T_1 + \epsilon_{ca})R_1 - 0{,}5\epsilon_{ctu}$$

The characteristic crack width under edge restraint is given by the multiplication of the crack strain and the maximum crack spacing:

$$w_k = \epsilon_{cr} * S_{r,max}$$

The characteristic crack width under end restraint is given by:

$$w_k = \frac{0{,}5\alpha_e k_c k f_{ct,eff}}{E_s}\left(1 + \frac{1}{\alpha_e \rho}\right) S_{r,max}$$

The parameters which have been used have the same meaning as the parameters given in the Eurocode [1], [2]. The determination of the parameters can differ in some cases.

A.5.4 Conclusion

In the CIRIA C660 the influencing factors and the background of them have been investigated to come up with more accurate values for the calculation of early-age and long-term cracking. This is an improvement in contrast to the Eurocode. Even though most of the parameters of the formulas are the same. The biggest difference is in the determination of the crack inducing strains. There is also a difference noticeable in the maximum crack spacing due to the k_1 factor which is smaller in the Eurocode. A difference in crack width may be noticeable when the results will be compared see chapter 7 for a comparison between the different methods.

A.6 ICE 706 [4]

A.6.1 Introduction

In the ICE706 the theory for the different types of external restraint as assumed by the Eurocode 2-3 and CIRIA C660 will be investigated. In the ICE706 it is stated that the phenomena of edge restraint does not differ from end restraint the only difference about these two is the boundary condition. To make an assumption about the differences between the two conditions a new approach method had to be investigated. The ICE 706 proposes a unified approach that assumes that the maximum potential crack width may only occur on conditions of end restraint, for the condition of edge restraint a reduction in crack width is assumed because of:

- A part of the load in the restraint element is taken up by the restraining element
- The edge restraint will inhibit the extent to which the cracks can open. The higher the restraint the smaller the crack width.
- A new crack may be influenced by the other cracks which can determine the stress relaxation between the cracks.

Due to the edge restrained the first crack cannot grow to its full potential because it will be restrained on the sides which will result in a distributed crack pattern with smaller cracks. This is the basis for the revised unified approach which is assessed in the ICE706.

According to the Eurocode the strain to calculate the crack width of edge restraint is only dependent on the degree of restraint times the free strain $(\epsilon_{sm} - \epsilon_{cm}) = R_{ax}\epsilon_{free}$. And the cross sectional area or lay-out of the reinforcement has no impact. As is stated before this formula lacks a certain amount of reliability.

The theory of the ICE706 that edge restraint inhibits crack opening is examined by: comparing predicted and observed crack development, by investigating theories on crack development and by doing FE analysis.

The degree of restraint is also dependent on the geometry of the wall according to the studies of Kheder and Anson. Kheder reported data that the primary crack spacing was dependent on the height and that the maximum crack width was given at a height of 0,1L above the joint. Anson has found maximum restrained strain at about the same height.

Restraint is a function of wall height in relation to both cross sectional area and length/height ratio. It has also been found that by increasing the restraint factor a reduction in crack spacing is achieved this results in more cracks which are smaller in width.

By using FE analysis results have been found that the Poisson's ratio and the thickness of the wall have little effect on the crack width. The primary effect is due to crack spacing and the height of the wall for the crack width. This is showed in the graph below:

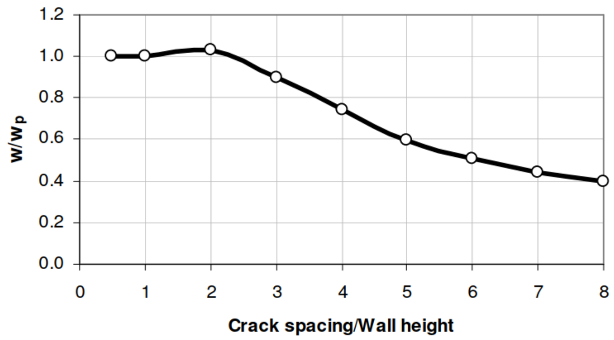


Figure 10 Dependency of the crack width to the geometry source: ICE706

These results show a relationship between the restraint factor, which decreases for shorter and heigher walls, and the reduction in crack width. By integrating the restraint along the top of the wall the reduction in crack width(Δw) could be derived. This will result in the following formula:

$$\Delta w = R_{ax} \epsilon_{free} s_r$$

A.6.2 Limitations of the current approach to design

The assumption of EN1992 that cracks are independent is unlikely because there is strain relief in the area beyond the crack while this is suggested to be not present. Independency of cracks may only be possible in cases for full restraint while full restraint is not present in most of the cases. The strain relief behind the maximum crack spacing has an effect on the crack width and the position of the following crack. So cracking may result in stress relief over much larger areas than assumed by EN1992.

EN1992 assumes that cracks and crack spacing, which are equally distributed, are determined by the reinforcement. A report and observations of Kheder have shown that the primary crack spacing is mostly influenced by the geometry and especially the height of the wall. While reinforcement plays a secondary role in primary cracks it plays a more consistent role in secondary cracks. Primary cracks are cracks which run through the whole cross section and run from the top to the bottom of the height. While secondary cracks are mostly centered around the reinforcement and do not run through the whole cross section. The spacing between primary cracks is often larger than the spacing between secondary cracks. The secondary cracks which are centered around the reinforcement have a smaller height and width as is shown in the picture below.

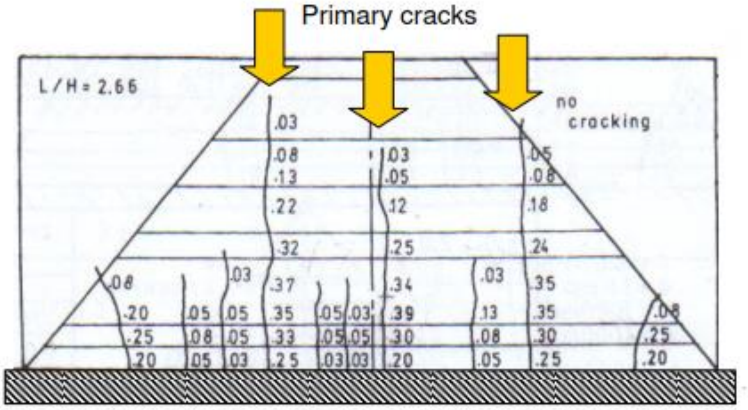


Figure 11 Primary and secondary cracks within the crack pattern in an edge restraint situation Source: ICE706

In the EN1992-3 an increase in area of reinforcement is not taken into account for continuous edge restraint. This is shown by two examples:

Example 1:
The continuous edge restraint in EN1992-3 is determined solely by restrained strain ($R_{ax}\epsilon_{free}$). This will imply that an increase in area of reinforcement (A_s) will not have an effect on the steel strain after cracking. While this seems in contradiction to mechanisms of cracking.

Example 2:
EN1992 requires a minimum amount of reinforcement $A_{s,\min}$ to keep the steel stress below the yield strength. For stresses generated by the restraint the value of $A_{s,\min}$ can be reduced because even in a severe case the steel stress is about 32% of the yield strength.
The expression to get the minimum area of reinforcement is given by:

$$A_{s,\min} = k_c k A_{ct} \left(\frac{f_{ct,eff}}{f_{yk}}\right)$$

For continuous edge restraint this formula can be reduced to:

$$A_{s,\min} \quad = (1 - R_{edge}) k_c k A_{ct} (\frac{f_{ct,eff}}{f_{yk}})$$

This is due to the fact that a part of the load is tranfered to the restraining member. The higher the factor of restraint the more load the restraining member takes and the lower the area of reinforcement for the new member has to be.

Especially for thinner sections it is hard to achieve efficient conforming designs in the EN1992. For cracks widths of 0,2 to 0,3mm the requirement of minimum reinforcement or spacing is not met and impractical designs have to be used.

A.6.3 Development of the revised unified method of design

The simple method describes the crack width with the following formula in which the maximum potential crack width(w_p) is multiplied by the factor of restraint for a continuous edge restraint(R_{edge}) situation.

$$w_k = w_p(1 - R_{edge})$$

This formula is a simple expression which reliably predicts crack widths in normal circumstances. It is less reliable with low steel ratios. The simple version is not applicable to a wide range of subjects and is worst in comparison with existing techniques That is why the simple version of the revised approach have been investigated in more detail to come to a useable version. This resulted in a two stage cracking process investigation.

A.6.3.1 The two stage process

In the two stage process there are two different stages of cracking which are derived separately. In the first stage it is assumed that the crack opens instantaneously to a value of w_{k1} at this time the load is transferred from the concrete to the steel. At the second stage the crack gets wider by w_{k2} as the concrete is assumed to contract relative to the reinforcement in a continue manner. The full crack width formula is:

$$w_k = w_{k1} + w_{k2}$$

Stage 1 crack width $w_{k1}$is estimated by using the current method for end restraint with a modification factor for edge restraint in both attracting load and preventing crack opening. The relative lengths of the cracked zone S(debonding zone) and the uncracked zone(strain relief zone after cracking) are taken into account in the revision.

Stage 2 cracking w_{k2} is continued opening of the crack which implies the residual contraction of the concrete within the cracked zone relative to the reinforcement. The restraint also has an influence on stage 2 cracking because it inhibits the contraction and hence the extent to which a crack may open.

A.6.3.2 Development of expressions for stage 1 cracking

Stage 1 cracking is estimated using a revised end restraint formula given by EN1992-3.

$$\epsilon_{sm} - \epsilon_{cm} = \frac{0{,}5 k k_c f_{ct,eff} \alpha_e}{E_s} \left(\frac{1}{\alpha_e \rho} + 1 \right)$$

Where:

k, k_c	are coefficients defined in EN1992-1-1 which take account of the stress distribution in the concrete and self-equilibrating effects
$f_{ct,eff}$	is the mean design tensile strength of the concrete at the time of cracking $=\alpha_{ct} f_{ctm}(t)$. $\alpha_{ct} = 0{,}8$ with the revised method.
E_s	is the modulus of elasticity of the reinforcement
α_e	is the modular ratio $\frac{E_s}{E_{c,eff}}$ and $E_{c,eff}$ is the modulus of elasticity of the concrete at the time of cracking
ρ	is the ratio A_s/A_{ct} based on the full section thickness
A_s	is the (total) area of reinforcement
A_{ct}	is the concrete cross section in tension

The crack strain expression of an element loaded in direct tension is given above this formula is used for situations of end restraint. It assumes that the concrete load is transferred to the steel after cracking. A revised expression is proposed, this expression has a little difference(about 1%) with the expression given in the EN1992-3. For lower strength concrete classes the difference can be 2% but this is still marginally small. The expression of EN1992-3 is still applicable.

The revised expression is given by:

$$\epsilon_{sm} - \epsilon_{cm} = \frac{0{,}5\alpha_e f_{ct,eff}}{E_s}\left(\frac{kk_c}{\alpha_e \rho} + 1\right)$$

Effect of element length:
In the uncracked zone stress relief occurs that is why the load which is generated by restrained contraction and stress transferred to the steel immediately after cracking cannot be sustained after cracking. The element, theoretically, needs to be infinitelly long to maintain the load transfer to the steel imediatelly after cracking. To calculate the mean residual strain in the steel after cracking the following formula can be used:

$$\epsilon_{smr} = \frac{0{,}5\epsilon_{ctu}(B+1)}{1 + 0{,}5\left(\frac{S_{r,\max}}{L}\right)(B-1)}$$

Where:

$$B = \left(\frac{kk_c}{\alpha_e \rho} + 1\right)$$

By filling in L→∞ $\quad \epsilon_{smr} \rightarrow \epsilon_{smr} = 0{,}5\epsilon_{ctu}(B+1)$

$$\epsilon_{smr} - \epsilon_{cm} = \epsilon_{sm} - 0{,}5\epsilon_{ctu} = 0{,}5\epsilon_{ctu}(B+1) - 0{,}5\epsilon_{ctu} = 0{,}5\epsilon_{ctu}B$$

As $\epsilon_{ctu} = \frac{f_{ct,eff}}{E_{c,eff}}$ and the modular ratio $\alpha_e = E_s/E_{c,eff}$ then $\epsilon_{ctu} = \frac{\alpha_e f_{ct,eff}}{E_s}$ and

$$\epsilon_{smr} - \epsilon_{cm} = 0{,}5\epsilon_{ctu}B = \frac{0{,}5\alpha_e f_{ct,eff}}{E_s}\left(\frac{kk_c}{\alpha_e \rho} + 1\right) = \epsilon_{sm} - \epsilon_{cm}$$

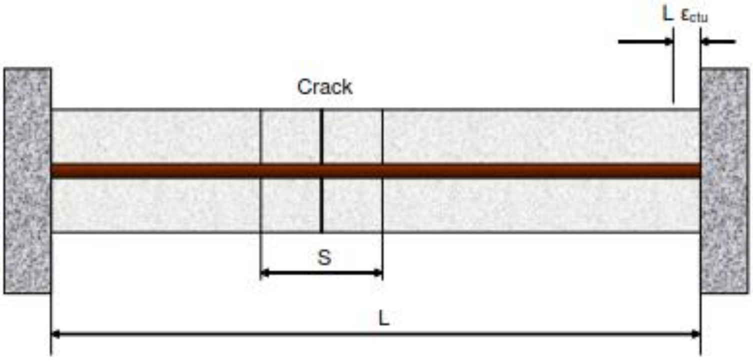


Figure 12 Element subjected to end restraint Source: ICE706

The length will be the limiting factor for most practical conditions in determining $\epsilon_{sm} - \epsilon_{cm}$ and the crack width immediately after cracking will be(with ϵ_{ctr} the residual tensile strain):

$$\epsilon_{smr} - 0{,}5\epsilon_{ctr} = \frac{0{,}5L\epsilon_{ctu}B}{L + 0{,}5S(B-1)}$$

$$\epsilon_{smr} - 0{,}5\epsilon_{ctr} = \frac{\epsilon_{sm} - \epsilon_{cm}}{1 + \frac{0{,}5S}{L}\left(\frac{kk_c}{\alpha_e \rho}\right)}$$

The effect of the length will be implemented by dividing the result by the factor $1 + \frac{S}{L}\left(\frac{kk_c}{\alpha_e \rho}\right)$ for the estimation of the maximum crack width for end restraint using the expression from EN1992-3.
A second crack occurs after continuing contraction of the concrete and an increase in steel stress which widens the crack. This process will keep repeating itself until a fully developed crack pattern develops.

Application to continuous edge restraint:
With edge restraint there are two essential differences.

- The restraining element will partially carry the load while the residual force in the steel at the crack will be balanced by the residual force in the un-cracked section.

The elongation of the concrete in the un-cracked section $(L_{eff} - S)$ and the sum of the elongation of the steel in the cracked section S relative to the free (unrestrained) contraction must equal the potential contraction of the element$(L_{eff}(1 - R_{edge})\epsilon_{ctu})$ prior to cracking.

The expression for the crack strain with edge restraint is given below. The strain relief in the un-cracked zone when a crack occurs will be limited by the degree of restraint of the restraining element.

$$\epsilon_{smr} = \frac{0{,}5\epsilon_{ctu}\left[\left(1 - R_{edge}\right)B + 1\right]}{1 - \frac{S}{L_{eff}}\left[1 - 0{,}5\left(B + \frac{1}{1 - R_{edge}}\right)\right]}$$

When the edge restraint in the formula above will be equal to zero the equation will be the same as the equation in which the edge restraint does not have an effect on the formula. This equation is given in the previous paragraph.

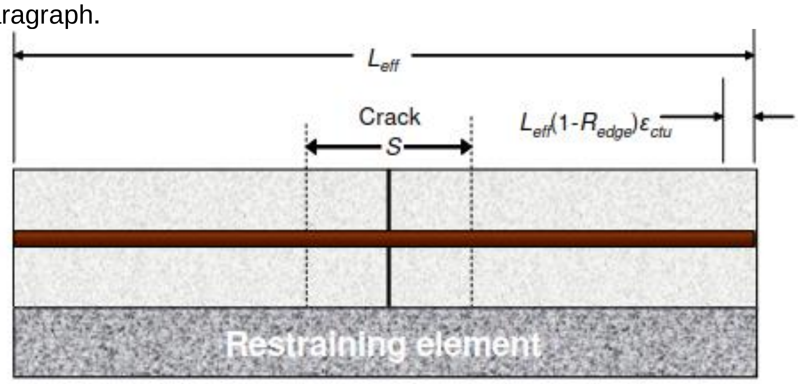


Figure 13 An element subjected to edge restraint Source: ICE706

The degree of edge restraint and the natural crack spacing S_n(crack spacing for under/unreinforced concrete members) have an influence on the effective length which has to be used in the equation. The natural crack spacing is given to be between one and two times the wall height. Eurocode 2 recommends a value of 1.3 times the wall height.

The length over which strain relaxation occurs is less if the degree of restraint is high and vice versa.. So the following is assumed:

$$L_{,eff} = \frac{S_n}{R_{edge}} = \frac{k_L H}{R_{edge}}$$

In which k_L is the length coefficient which value lies between 1 and 2. When these formulas are applied the formula of crack strain for edge restraint is given by:

$$\epsilon_{smr} = \frac{0{,}5\epsilon_{ctu}\left[\left(1 - R_{edge}\right)B + 1\right]}{1 - \frac{SR_{edge}}{k_L H}\left[1 - 0{,}5\left(B + \frac{1}{1 - R_{edge}}\right)\right]}$$

The stage 1 crack strain can be estimated by:

$$\epsilon_{sm} - \epsilon_{cm} = \frac{0{,}5\epsilon_{ctu}\left(1 - R_{edge}\right)B}{1 - \frac{SR_{edge}}{k_L H}\left[1 - 0{,}5\left(B + \frac{1}{1 - R_{edge}}\right)\right]}$$

This formula assumes that there is a residual strain in the concrete equal to half the strain capacity of the concrete after the crack has occurred.

A.6.3.3 Development of expressions for stage 2 cracking
The continuing contraction of the concrete in the cracked zone is called stage 2 cracking. Due to the contraction of the concrete outside the cracked zone the steel inside the cracked zone will maintain its stress. Between the cracks, within the range of the points of zero displacement, the concrete will be restrained from contraction. So the concrete in the cracked zone will keep contracting relative to the steel which implies crack growth.

The first stage of cracking occurs when the tensile strain capacity of the concrete is exceeded. The contraction required to cause a crack is $\frac{\epsilon_{ctu}}{R_{edge}K_1}$ by taking into account creep and the factor of edge restraint. Stage 2 cracking will be given by the residual contraction given by:

$$\epsilon_{res} = \epsilon_{free} - \frac{\epsilon_{ctu}}{R_{edge}K_1}$$

The restraint local to the crack prevents contraction of the concrete in the cracked zone. The restraint to contraction of the concrete builds up linearly from zero at the crack to the pre-cracked value beyond the zone of cracking. The restraint will be $0{,}5R_{edge}$ in the cracked zone by average. So the additional crack width due to contraction of the concrete in the cracked zone will be:

$$W_{k2} = S_{r,\max} \ (1 - 0{,}5R_{edge})K_1\left(\epsilon_{free} - \frac{\epsilon_{ctu}}{R_{edge}K_1}\right)$$

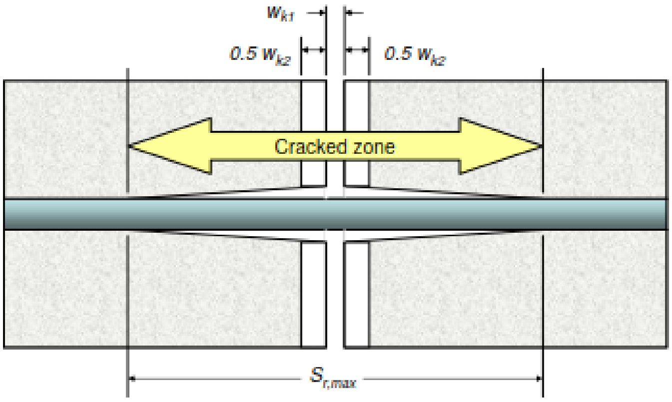


Figure 14 Stage one and two cracking   Source: ICE706

A.6.4 Critical parameters for predicting crack width

For end and edge restraint the parameters which are assumed to control the restrained strain and the crack width are completely different. For edge restraint the contraction and restraint influence are of importance while for end restraint the area of reinforcement, the tensile strength and modulus of elasticity are dominant. The restraint factor does not have an influence on the crack width for end restraint according to EN1992-3. The unified approach is based upon the EN1992-3 approach for end restraint so the parameters which have to be used have to be of appropriate values.

A.6.4.1 Tensile strength
The effective tensile strength at the time of cracking is needed for the calculation. The first crack is likely to occur at the weakest location so the lowest 5 percentile is likely to be used here. However the cracks which will form later will occur with a higher tensile strength because the concrete strength increases over time. Furthermore the forces will be balanced between the steel and concrete. The stress in the steel has to be equal at every crack if the reinforcement ratio, crack spacing and crack width is constant for the entire section. So the cracks will be equal of width when debonding is assumed to be constant. The cracks will be determined by the in situ tensile strength at the location of cracking.

The value of the tensile strength which has to be used can be calculated according to EN1992-1-1. This value can be the mean value and/or can be determined by the time to which cracking is expected to occur. It is also affected by the curing and drying conditions as well of the dimensions of the structure. When the development of the tensile strength is important tests should be carried out.

The in situ tensile strength should represent a safe value. A study has been carried out to check whether the design value which is given in the codes represent a safe value. The results from this study were that the estimated in situ values were lower than the values given in the EN1992-1-1. The values given in the Eurocode represent conservative values. A safe value for design according to this study will be $0{,}8 f_{ctm}(t)$.

A.6.4.2 Modulus of elasticity and creep

The modulus of elasticity is an important parameter because it is used for the modular ratio and for the determination of the tensile strain capacity of the concrete by using the tensile strength. Different values of the modulus of elasticity are used to estimate the risk of cracking and the calculation of crack inducing strain.

Estimation of the risk of cracking

If the restrained strain $\epsilon_r = R_{ax}\epsilon_{free} > \epsilon_{ctu}$ a crack will occur. The tensile strain is affected by the modulus of elasticity and the effect of creep also has to be taken into account in the modulus of elasticity if the effective modulus of elasticity has to be used. $\epsilon_{ctu} = \frac{f_{ct,eff}}{\frac{E_{cm}(t)}{K_1}}$ $K_1 = 0{,}65$

Estimating $(\epsilon_{sm} - \epsilon_{cm})$

The load transfer from the concrete to the steel is instantaneous when a crack is formed. This implies that there is no effect of creep and the modulus of elasticity is given by the mean value for calculations of the crack width for end restraint.

A.6.4.3 Estimating continuous edge restraint

The influence of restraint on the crack width from the currently used method of EN1992-3 differs significantly from the revised approach. It is assumed that the continuous edge restraint acts in the same way as the reinforcement within the revised approach. So a high restraint limits the crack width to a certain amount and lead to more smaller cracks. Therefore it is important to estimate the degree of restraint with reliable values.

The most known example of continuous edge restraint is the wall cast on a slab. In this example the factor of restraint lies in the range of 0,3 to 0,7. In the EN1992-3 the value of 0,5 is used as this is the average value. But a difference in restraint factor of 0,1 will affect the restrained strain by 20%. If the crack width is the governing criteria this will result in an increase of reinforcement. So a good estimation of the factor of restraint can save reinforcement and thus money. Different research has been undertaken in this subject and an expression is made to calculate the restraint factor. This expression which is the same as the expression given in CIRIA C660 is given by:

$$R_j = \frac{1}{1 + \left(\frac{A_n}{A_o}\frac{E_n}{E_o}\right)}$$

Where:

A_n is the cross sectional area of the new restrained pour
A_o is the cross sectional area of the old restraining pour
E_n is the modulus of elasticity of the new pour
E_o is the modulus of elasticity of the old pour

The modulus of elasticity of the old pour will remain the same while the modulus of elasticity of the new pour will increase rapidly within the first few days. The value of the ratio between the modulus of elasticity of the new and old concrete lies between 0,7 and 0,8.

The factor of restraint varies over the height of the element. The higher the ratio of length/height the lesser the factor of restraint decreases by increasing height. The factor of restraint is determined at the position of the joint. At the position of the joint a crack can only open if there is debonding, but at the joint debonding is not possible. So the factor of restraint can be decreased which is beneficial for the calculation. Different studies have been carried out to determine the position of maximum restraint which has to be used for the calculations. The height at which this factor occurs is at the position of 10% of the length of the wall from the joint up. So at this position the restraint is the highest and decreases when moving to the top of the wall. The rate of the decrease in restraint is determined by the geometry of the wall. The higher and shorter the wall the faster the degree of restraint decreases with increasing height. If the wall is long and has a low height the degree of restraint decreases slowly with increasing height. If the ratio between the new and old cross sectional area remains the same(=1) the length does not have an effect on the restraint factor.

A.6.5 The effect of edge restraint on crack widths calculated using both methods

An example has been used to show the effect of edge restraint between the different methods. The CIRIA C660 (almost the same as EN1992-3) predicts a higher crack width with increasing restraint while the revised method predicts a reducing crack width while increasing restraint which assumes that the edge restraint delivers some degree of crack control. The range of the factor of restraint lies between 0,33 and 0,77. The example is shown in the graph below.

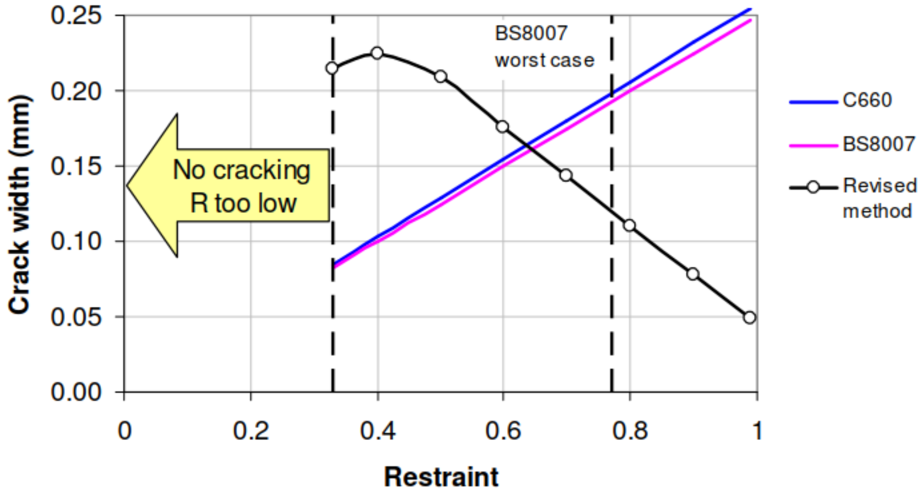


Figure 15 Crack width of different methods with edge restraint Source: ICE706

A.6.6 Combining crack widths due to early-age thermal restraint and other actions

The revised method is developed to model early age thermal cracking due to imposed deformations and does not take account of the effects of imposed actions. Is it possible to add a increment of crack width to take account of the imposed action effects?

In the EN1992-1-1 different load combinations are described and have to be checked although the combination of early age and load induced crack effects are not explicitly required. But it can be necessary to consider the potential of such an effect.

It is necessary to know the steel stresses of the imposed deformations before the stresses of the imposed actions can be added in order to get the total steel stress. But for continuous edge restraint the current methods do not take account of the steel stress after cracking for the estimation of the crack width. In contrast the revised unified approach assumes that a part of the load from the cracked concrete is transferred to the restraining element, because it is based on estimating the residual strain in the steel after cracking. The revised approach enables a combination of imposed deformation cracking and cracking due to imposed actions by using the stresses in the steel at each stage and add these up.

Bond determines the rate at which strains can develop between the concrete and reinforcement. Therefore the distribution of strain in the concrete is limited with increasing distance from the crack position. The concrete strain at the crack is zero and if the strain increase is limited the additional strain of the imposed load will not result in additional stain in the concrete. This will imply that there is no tension stiffening effect by the imposed action. See the figure on the next page. The additional crack width can be calculated with:

$$\Delta w_k = S_{r,\max} \quad \Delta \epsilon_s$$

The increase in crack width is given by the maximum crack spacing times the increase of reinforcement strain due to the imposed action. This is a conservative approach due to the fact that the crack spacing will be smaller than the maximum crack spacing and some of the strain of the imposed action will be transferred to the concrete via shear.

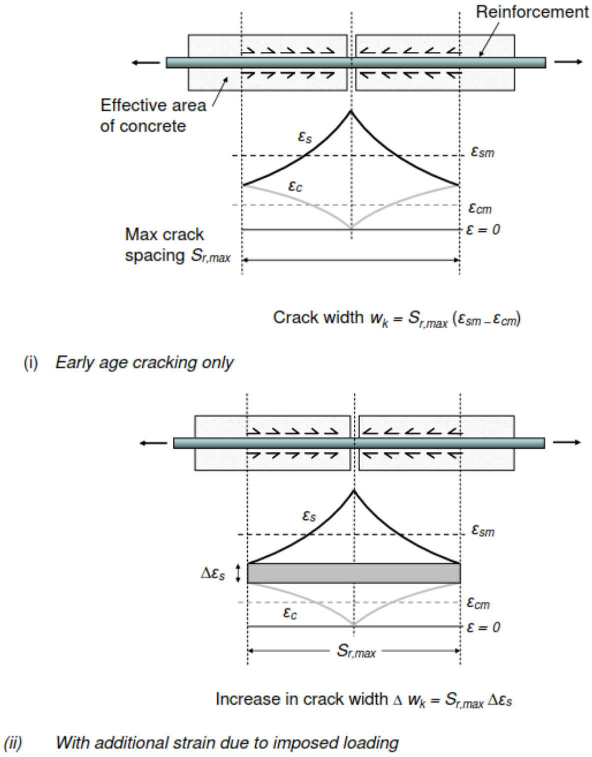


Figure 16 Imposed deformation + imposed action strains   Source: ICE706

A.6.7 Conclusion

The ICE706, as described here, distinguishes itself from the Eurocode in the approach to calculate the early-age cracks due to continuous edge restraint. The formula from the EN1992-3 for end restraint has been rewritten to a formula which can be used for continuous edge restraint. This formula consists of a two stage cracking process. At first stage one cracking is determined by the formula for end restraint with some modifications to take account for the restraint and the length effect. And secondly stage two cracking is assumed to be the contraction of the cracked concrete around the reinforcement causing the crack to grow. By adding stage one and stage two cracking the resulting early age crack width can be calculated. The parameters to calculate the different stages of cracking have also been improved by using the effective values which are improved by adding a time factor or by comparing different tests results. A conservative approach is also described to include loads different from early-age thermal restraint in the calculation of the crack width. This can be included by adding the additional strains in the steel times the maximum crack spacing.

The approach described here is an improvement to the current method used by the Eurocode, because it provides a better formulation of crack width formula for continuous edge restraint. It can be used in accordance to the CIRIA C660 to come up with a better understanding of the edge restraint calculations and predict values which are closer to reality than values given by the Eurocode. In chapter 7 a comparison is given for the difference in crack width between EN1992-3, CIRIA C660 and ICE706.

A.7 Jones method [5]

A.7.1 Introduction

The method of Jones has been developed to help engineers by the interpretation of the crack formula and the determination of the effective concrete tension zone. The determination of the concrete effective tension area given in the Eurocode is not straight forward and engineers need to make decisions on how to implement the rules. This can result in different calculation results. Especially when the method has to be implemented in software. And for situations in which the cross section is irregular, and/or the reinforcement consists of multiple rows.

Solutions given in this paper have been found in the form of:

- The use of the tension strain ϵ_m for two spacing formulae
- Two different physical models represent the crack width calculation namely: Cover vs bar slip model.
- Geometry models

The crack width is calculated by multiplication of the crack spacing s_{rmax} times the strain ϵ_m. The strain which is used here is less than the strain calculated from a fully cracked section. This is due to the use of a tension stiffening model. The crack width calculations follow the principle that crack spacing is governed by the reinforcement: close to a bar, the neutral axis or at some distance from the bars. Two different approaches to calculate the crack width are given for these two situations.

The Eurocode gives two different models when crack spacing is controlled by reinforcement namely: crack width model based on slip at the bar surface and a tapered crack in the cover zone model. The expression for the crack spacing is given by:

$$s_{rmax} = k_3 c + \frac{k_1 k_2 k_4 \phi_{mr}}{\rho_{,eff}}$$

k_3 is recommended to be 3,4 according to the Eurocode. This 'cover term' value is the dominant value in the crack spacing formula because it is about 50-80% of the total value of s_{rmax}. The second term represents the crack width at the bar through slip this term is about 20-50% of the crack spacing.

A.7.2 Tension stiffness.

The tension stiffness model used to calculate crack width given in the Eurocode is called the local tension stiffness model within this method. It gives accurate values for pure tension situations, but it requires interpretation when used for general bending cases. The model is given by:

$$\epsilon_{sm} - \epsilon_{cm} = \frac{\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{,eff}}\left(1 + \alpha_e \rho_{,eff}\right)}{E_s} \geq 0{,}6 \frac{\sigma_s}{E_s}$$

Where:

ϵ_{sm} is the mean strain in the reinforcement under the relevant combination of loads, including imposed deformations and the effect of tension stiffening

ϵ_{cm} is the mean strain of the concrete between cracks

The local tension model is given by a rectangular tension stress block of height h_{ceff} and a constant stress of $k_t f_{ctm} = 0{,}4\ f_{ctm}$ located at the base of the section. The tension stiffness strain remains constant and the formula is based on the centroid of the tension block which coincides with the reinforcement. This model seems a clear model but according to test results the model gives a variety of answers and has a higher tension stiffness than other models.

The interpolated tension stiffness model is modeled by a triangular stress block. This model is used to calculate deflections. It can be rewritten to get the predicted strain values by adding a ζ factor.
The interpolated tension stiffness model gives a better fit to other models. The local tension model is a simple model for pure tension but the adaptation for flexure has not been successful in all situations.

By using the local tension model for flexure two difficulties are associated with the interpretation of $\rho_{,eff}$ and k_t. If $\rho_{,eff}$ is increased the tension stiffness should decrease which will result in an increase in crack width. While by increasing $\rho_{,eff}$ the crack spacing decreases which results in a decrease in crack width. This implies that the effects cancel each other out what means that the formula is insensitive to bar layout. This differs from current understanding.

If the cover is increased and the value of 'd' remains constant the local crack strain model will reduce the crack strain ϵ_m. And the zone of influence of the bar will increase both externally and internally. What implies that the change in tension stiffening will be almost commensurable to the change in cover. The local model is a simple model which is also applicable to hand calculations and gives good results for pure tension and deep bending elements with small covers and high tension strains. It will represent other tension models better if the value of $k_t = 0{,}4$ will be lowered to 0,3 or 0,2. If these values are applied it will reduce the problem of sensitivity to bar lay-out which will imply insensitivity to bar lay-out.

The limits to the effective height ($\frac{h-x}{3}, 2{,}5(h-d), h/2$) are implemented in the Eurocode for general flexure situations because it was known that the formula of the local tension model would not fit all results given by this formula.. The limits cause problems of interpretations because there is no clarity about how to derivate these limits in the code. $\frac{h-x}{3}$ is the governing limit in most practical situations.

A typical problem for this limit is a situation in which reinforcement is distributed at the side of the element, which implies that a certain area of concrete contributes to the tension zone while there is no reinforcement present inside this area. For circular sections there is no clear distinction what the tension zone is this has to be improved as well.

For situations described above the behavior of the entire tension zone has to be calculated this is determined with the interpolated tension stiffness model. If this model is adopted for all cases it would prevent inconsistencies in crack width calculations.

A.7.3 Crack strain & crack width position

The code does not determine at which position the crack strain and crack width should be calculated. For the steel strain it is better to consider the bar surface and for concrete strain it can be better to consider the surface of the concrete. The positions of the crack width and crack strain do not have to coincide, adjustments can be used to compensate this difference. If tension stiffness has to be calculated the position of the crack strain needs to be known.

The crack spacing formula is calculated by including a combination of slip at the bar and crack opening in the cover zone. The crack spacing which is calculated occurs at the surface of the concrete. But it is not clear if the parameters which are needed to calculated the crack spacing are calibrated at the surface of the concrete. Guidance on this problem is needed since the studies done for this method encountered crack widths which were much higher than calculated.
A following interpretation has been done for within this method:

The strain ϵ_m for the crack spacing controlled by reinforcement is measured at the bar. For crack spacing not controlled by reinforcement the strain ϵ_m is measured at the surface. And the crack widths predicted are at the surface. The Eurocode uses the same strain for both situations which will result in significantly different values.

The discussion above and the problems for achieving acceptable crack widths for sections with large covers, led to the investigation into the two terms in the crack spacing formula:

$$s_{rmax} = k_3 c + \frac{k_1 k_2 k_4 \phi_{mr}}{\rho_{peff}}$$
$$= term\ A + term\ B$$
$$= cover\ zone\ cracking + cracking\ at\ bar$$
$$crack\ width = \epsilon_m * (cover\ zone\ cracking + cracking\ at\ bar)$$

The two components of the crack spacing where studied in a test and taken individually to see if they correlated with the crack width associated by the term. This study showed that the correlation between crack width and cover was stronger than used in the code.

The results of this comparison of test results and the Eurocode are given below:

- The measured crack width at the bar was lower than the calculated crack width
- The calculated growth of the crack width from the bar to the concrete surface approaches the average data but the values where lower than the maximum values

The study also shows that the cover term is greater by the measured data than calculated, for the bar slip it is the opposite. It seems that at least a part of Term A is related to the difference between the strain at the surface and at the bar. So it may be appropriate to consider it in the calculation of ϵ_m instead of s_{rmax}.

For sections with excess cover the crack width limits and calculations are subject of much debate. It is not clear if the models which are proposed have been calibrated for the use of very large covers, or if the models can predict cracks at fictitious positions between bar and surface. By increasing the cover it will:

- Increase the crack spacing due to an increase in the term $k_3 c$, thus increasing crack width
- Increase the crack spacing by reducing $\rho_{,eff}$, thus increasing crack width
- Reduce the crack strain ϵ_m in exp. 7.9 by reducing $\rho_{,eff}$ thus reducing crack width

If $\frac{h-x}{3}$ governs the calculation the las two points will not occur. The result of increasing cover will lead to an increase in crack width. The implementation of using or excluding excess cover has been investigated in the Method of Jones, but the results were that the difference was too small to be included because it stretched the cracking formula beyond their intended use. It is better to use a factor to change the limiting or calculated crack width by C_{nom}/C_{dur}.

A.7.4 Geometry

The code need to give clarification for problems which arise for sections with multiple layers of reinforcement, reinforcement on the sides, uneven spacing and circular/irregular sections. The code now results in:

- Incompatibility between the definitions of h_{ceff}
- Incompatibility between different sections
- Lack of clarity of definition of ‘d’
- No definition of ‘width’ for M sections and sections with tapering sides
- Lack of clear definitions for situations with variable bar diameters, cover and sections where the neutral axis, the surface and/or rows of reinforcement are not parallel.

A.7.4.1 Two rows of reinforcement

There are two problems which occur when two or more rows of reinforcement have been used and the effective height is governed by $h_{ceff} = \frac{h-x}{3}$

First the inner row can fall just inside or outside the A_{ceff} this can result in an increase or decrease of ρ_{peff} which will have an influence on the crack spacing. And second the effective concrete area has to be adjusted to get to the same reinforcement ratio this will imply that $h_{ceff} = (h-d) + \frac{ssv}{2}$.

A.7.4.2 Local crack width checks

If Eurocode 2 crack formulas are used every section which is different from the simplest section should be treated as irregular. A simplified method for hand calculations is given below in two steps.

- The first step is to calculate the stress and strain distribution by performing a full section analyses.
- The second part is to consider an equivalent rectangular part of the section perpendicular to the neutral axis in which the maximum crack width is likely to occur. The crack spacing formula can be used by using the bars which are closest to the surface if there are more rows.

A.7.5 Generalized Local crack-widths checks

The method described above can be used in general situations and software implementations. First a section analyses will be carried for the whole section. The tension stiffness can be implemented by using a flexural tension stiffness model which is useable for full section analyses.

Each bar is associated by a concrete area which is a portion of the section surface after the full section analyses has been performed. The area is determined by the spacing formula. The crack spacing for each bar can be found by using the formulas of the Eurocode 2. If the concrete areas are outside the influence of a bar $> 2{,}7(c + \frac{\phi_{mr}}{2})$ then the expression of crack spacing not controlled by reinforcement have to be used. The crack width is calculated by using the crack spacing and strain.

In the figure below a configuration is given for the area of concrete in influence of the bar.

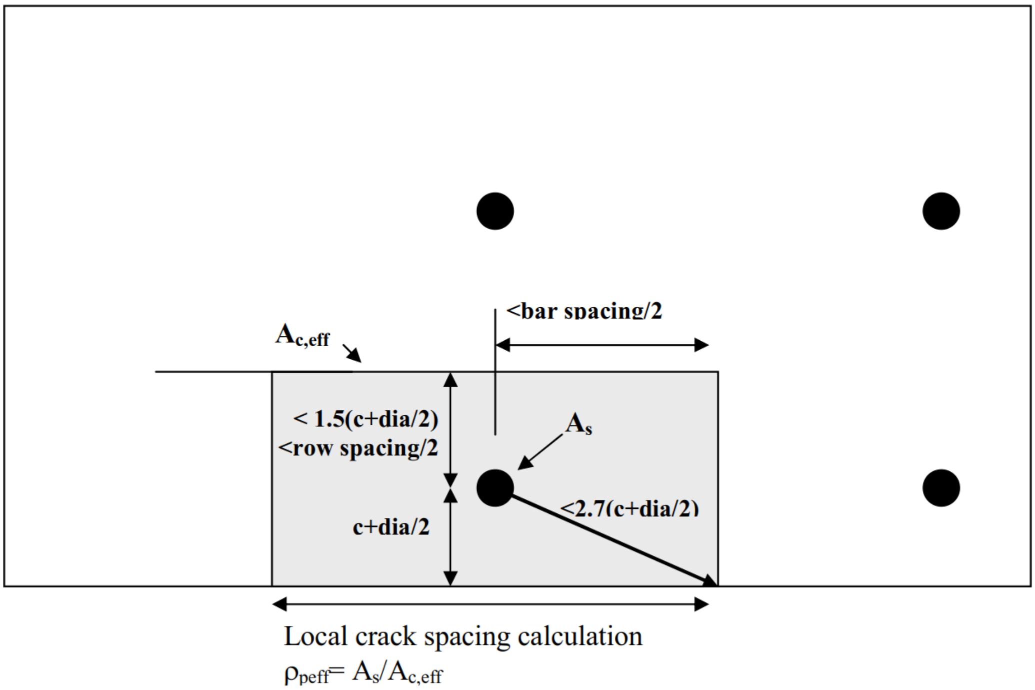


Figure 17 Effective concrete tension area defined by Jones Source: Jones Method

All parameters are given in relation to the bar, so it is possible to change the reinforcement layout(diameter, spacing, cover, rows) in different ways.

The crack spacing of a pure tension test is higher than the crack spacing of a bending test. That is why in the Eurocode the limit of $\frac{h-x}{3}$ is used. In a general case this limit have to be replaced by a factor on the effective tension zone. Sometimes crack width calculations may be required above the height of $\frac{h-x}{3}$ or in situations in which the angle of the area under consideration is not parallel to the neutral axis. For these situations it is proposed that $A_{c,eff}$ is multiplied by a factor ($\beta = h_t(7{,}5a) \leq 1$) to overcome this problem.

Where:
h_t is the distance perpendicular from the neutral axis to the outermost tension fiber of the section
a is the shortest distance from the center of the bar to the perimeter
In case of a single layer this will meet the code and limit the crack spacing for shallow bending elements. For all other cases it does not have an effect except if the cover of the bar is large than the slip term will be reduced.

The generalized local tension area is developed for crack spacing only. The full section geometry is taken into account for the bar strain calculations in the tension stiffness model. The local tension stiffening model may not be applicable if the area of concrete under consideration is not at the extreme tension fiber of the section.

A.7.6 Conclusion

The method described by Jones for the estimation of the crack width can be applied to the currently used Eurocode. As is stated here the Eurocode is often vague and inconsistent for different types of geometry. By using the method described by Jones this problem can be overcome to get to a better understanding of the way how to determine the effective concrete tension area. The effective concrete tension area is an important parameter for the determination of the crack width. By using the effective concrete tension area a reduction in crack width may be possible which implies a reduction in reinforcement which is beneficial for the costs of the project. The method can be used in accordance to the already given CIRIA C660 and ICE706. But it is also possible to compare the different approaches and see where the differences between these different approaches are given. In chapter 7 the differences between the different approaches is given with one varying variable.

A.8 Comparison different methods

In this chapter the different approaches have been investigated by using different input parameters. In every situation only one input parameters is changed to get to the results. The formulas which are used by the different approaches have been worked out in Excel and are shown for each case.
The input parameters which have been used are given below only one parameters has been changed in every situation:

Restraint:
Edge restraint: 0,667
End restraint: 0,45

Concrete class: C30/37

Element geometry:
Thickness new: 2m
Thickness old: 2m
Height new: 5m
Height/Width old: 7m
Length new= length old= 20m

Time at cracking: 7 days

Reinforcement lay out:
Diameter reinforcement: 40mm
Number of layers reinforcement: 2

Horizontal reinforcement spacing: 150mm
Vertical reinforcement spacing: 150mm
Concrete cover: 60mm

Temperature differential: 40℃

The cross section is under pure tension.
A picture of the lay out has been given here:

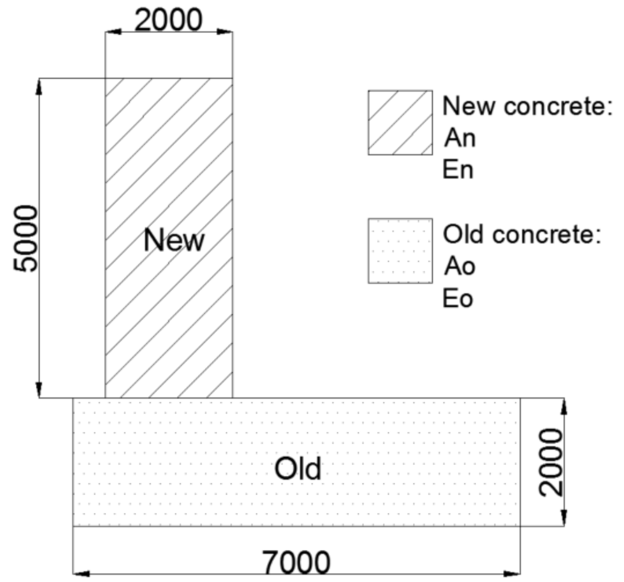


Figure 18 Considered geometry

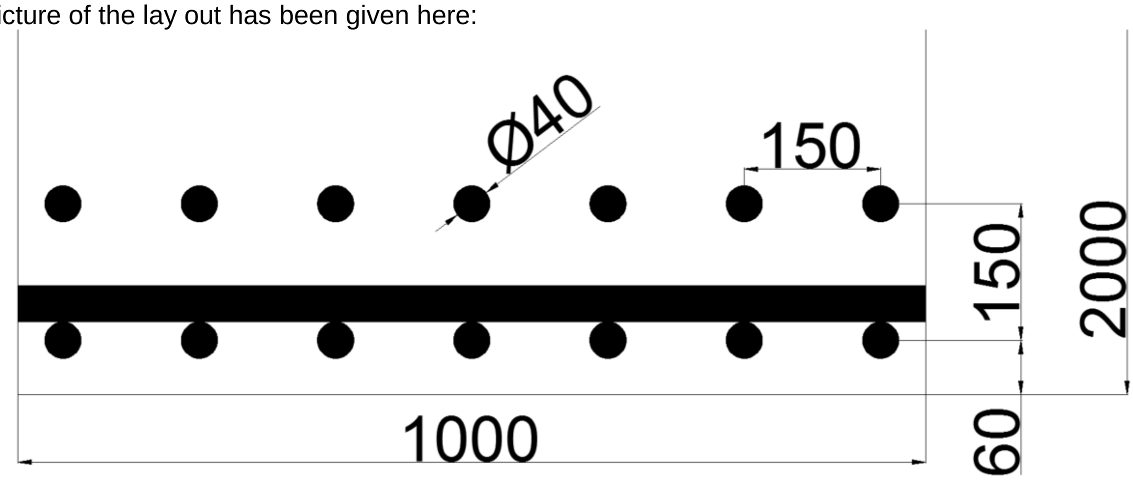
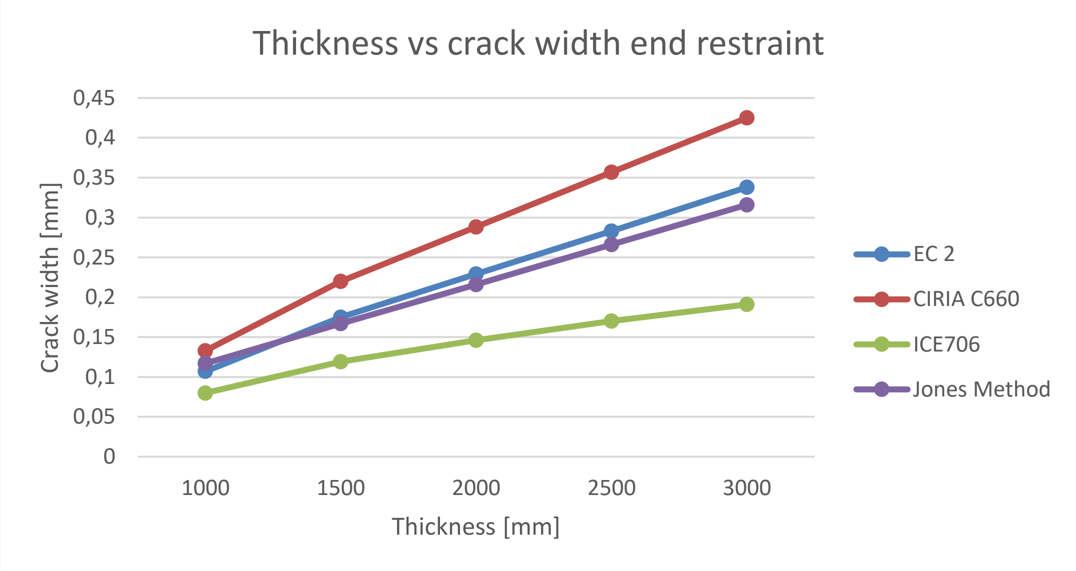
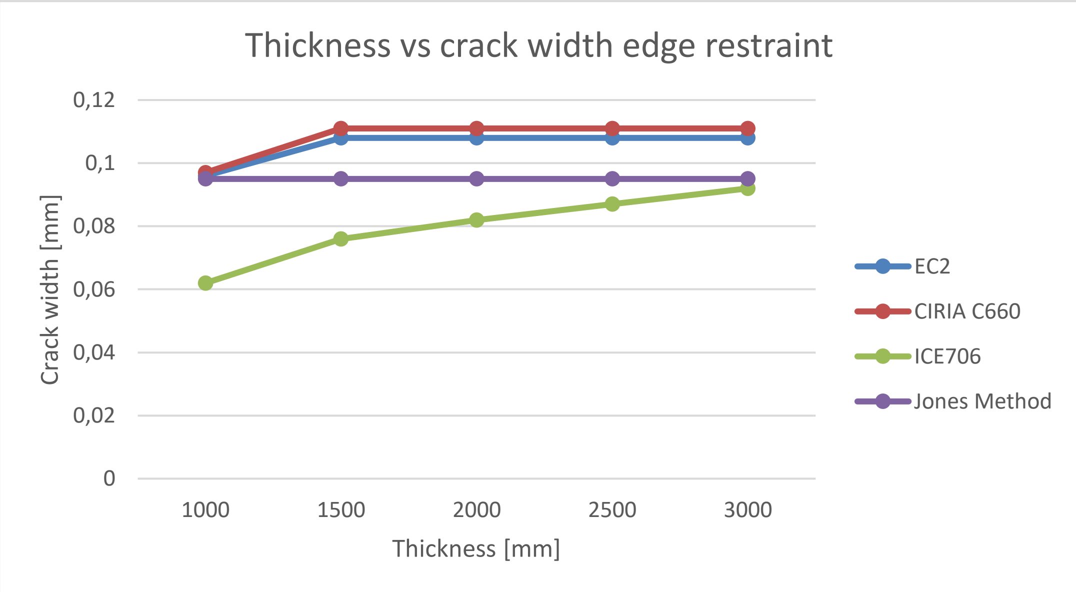


Figure 19 Considered reinforcement lay-out

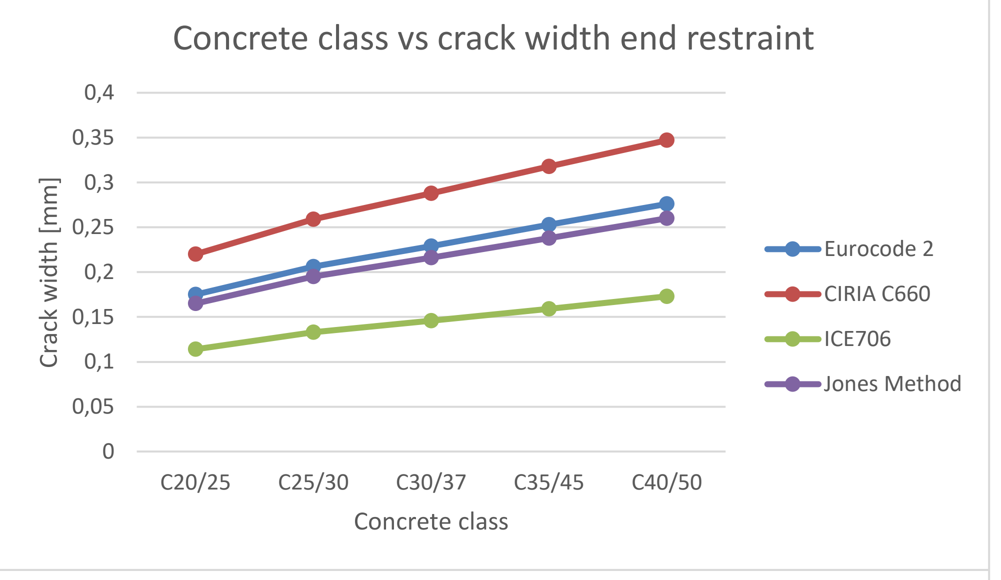
Variable thickness:
At first the thickness of the elements have been considered. The thickness will be varied from 1000mm to 3000mm with steps of 500mm. To maintain the same degree of restraint the thickness of the old concrete is varied as well.

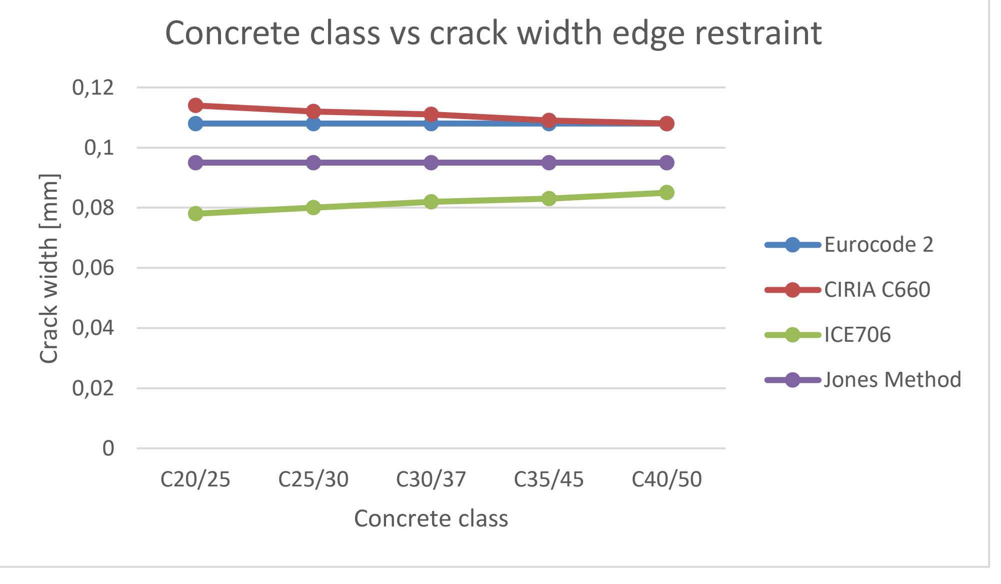




As can be seen in the graphs the influence of the thickness is small in situations with edge restraint. This is due to the fact that the thickness has a small influence on the crack spacing formula for the crack width due to edge restraint. The influence of the thickness is most noticeable in the ICE 706 approach because the B-factor which is used in this approach is dependent on the reinforcement ratio which is affected by the thickness. For situations with end restraint the variation of thickness is noticeable due to the fact that the reinforcement ratio decreases with increasing thickness. Due to the decrease in reinforcement ratio the crack strain increases what will result in an increase in crack width.

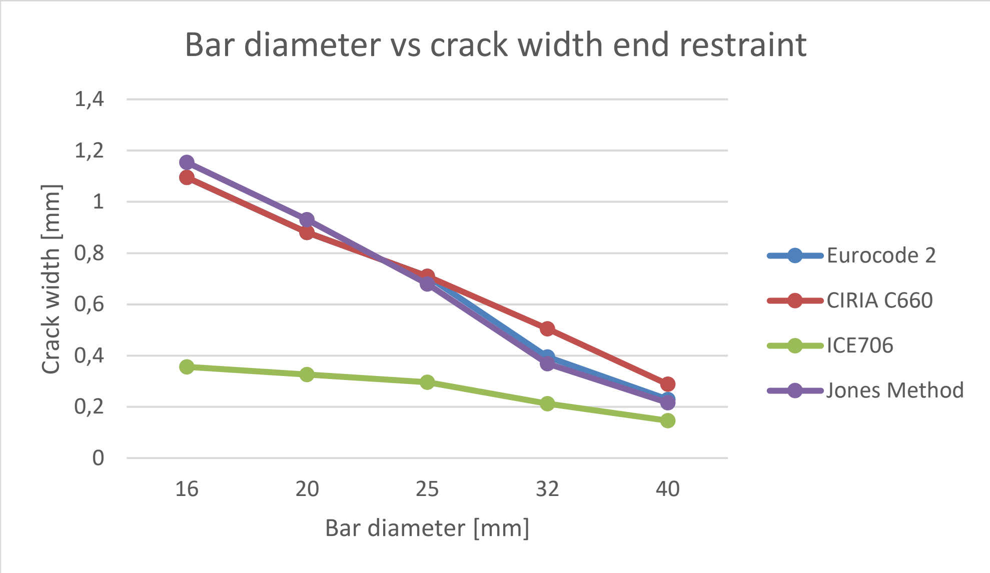
Variable concrete class:
Secondly the effect of the concrete strength class will be investigated as can be seen in the graph the following concrete strength classes have been investigated C20/25, C25/30, C30/37, C35/45, C40/50. See the following graphs for the influence of the concrete strength classes:

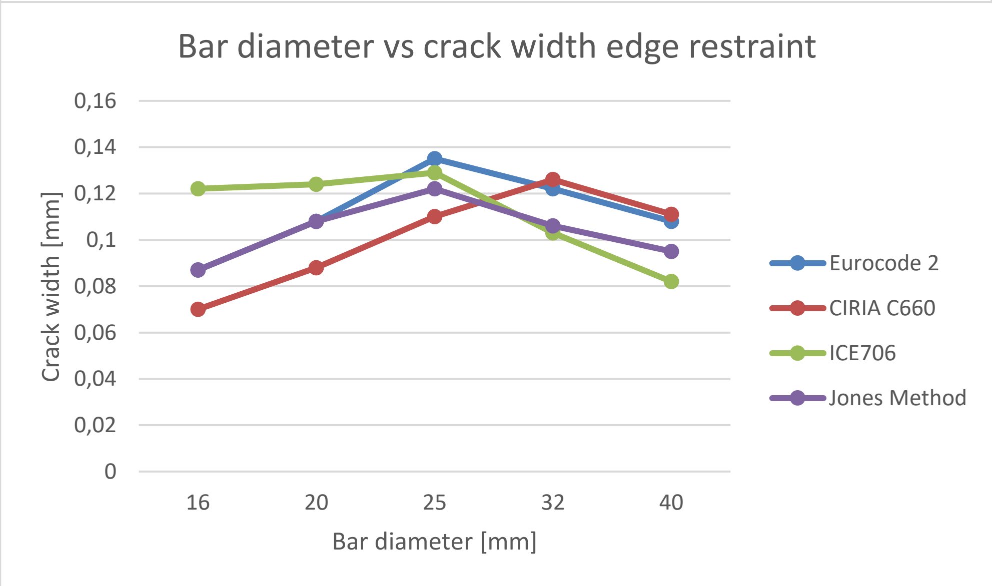




In the first graph it can be seen that with increasing concrete strength class the crack width increases. This due to the fact that the concrete tensile strength increases which implies that the crack strain increases, what results in an increase in crack width. For the second graph it can be concluded that the increase of concrete strength class does not have an effect on the increase in crack width for a situation with edge restraint. Because the concrete tensile strength is no input parameter for the edge restraint situation except for the CIRIA C660 and ICE706 because here the concrete tensile strain capacity is included in the crack strain formula.

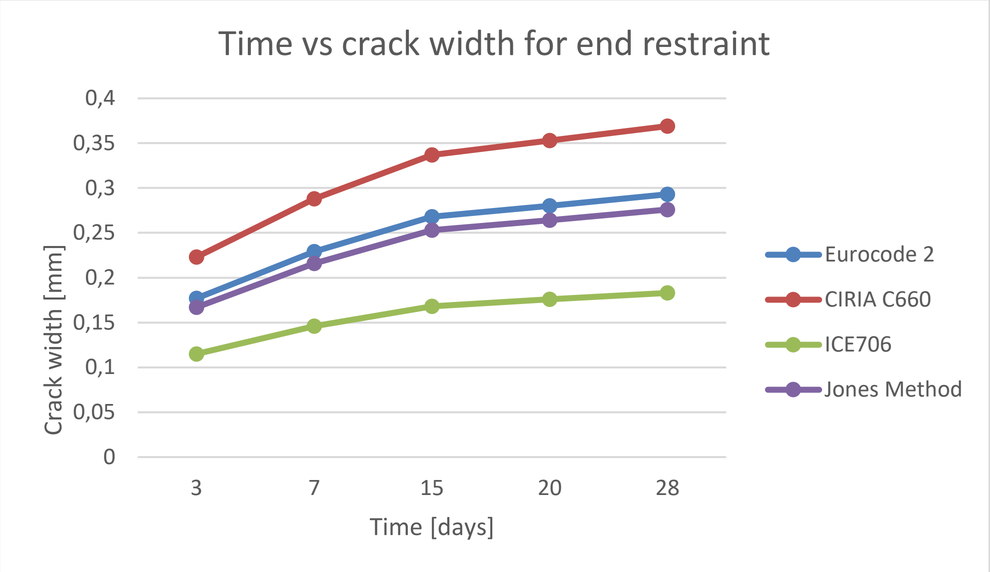
Variable bar diameter:
The influence of the diameter of the reinforcement on the crack width control for both types of restraint has also been investigated the results are shown below:

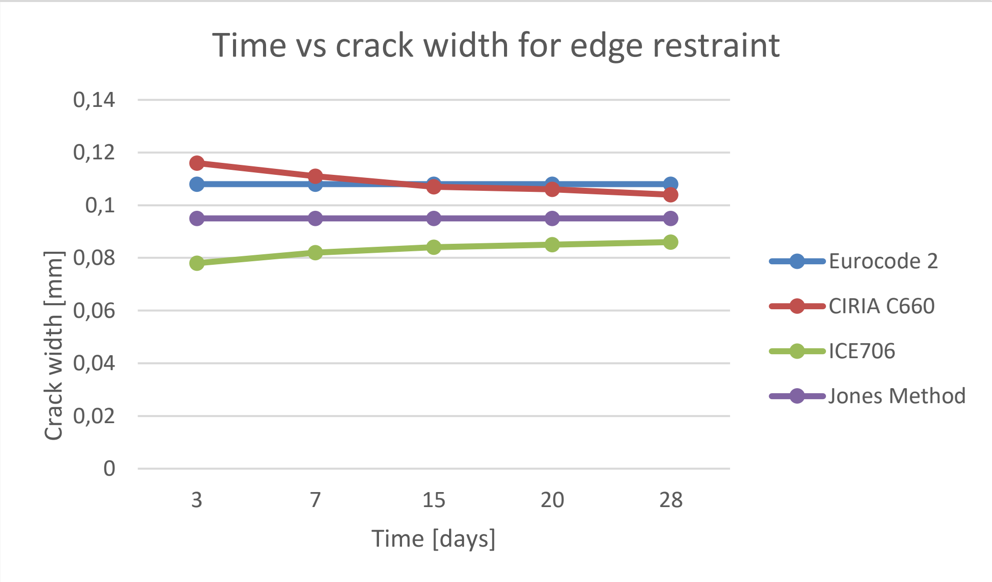




The bar diameter has for both situations an influence in the determination of the crack width. Because the diameter has an influence in the cross sectional area of the steel and has an influence on the effective concrete tensile zone. Both have an effect on the (effective) reinforcement ratio. The reinforcement ratio comes back in both crack width calculation formulas. In the first graph the values decrease rapidly by increasing bar diameter except for the ICE706 this value is already low and decreases gradually this is due to the method of the ICE706. In the graph of the edge restraint situation a peak value is given for the different methods. The increase in crack width is determined by the maximum limit of the crack spacing formula which is given by either $15 * \emptyset$ or by $(50 - 0{,}8 * f_{ck}) * \emptyset$. For the smaller diameter these terms are normative, after 25mm or 32mm the normal crack spacing formula is normative. And the lines decrease by an increase in diameter which is due to the fact that the (effective) reinforcement ratio increases.

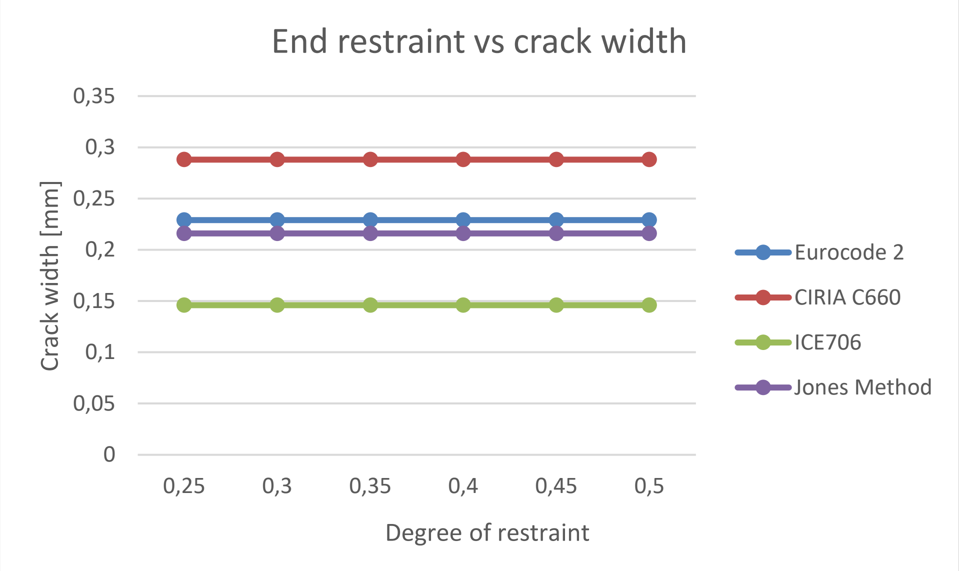
Variable time of cracking:
The influence of the time of cracking has also been investigated by using the time at 3, 7, 15, 20 and 28 days after pouring. This is showed in the graphs below:

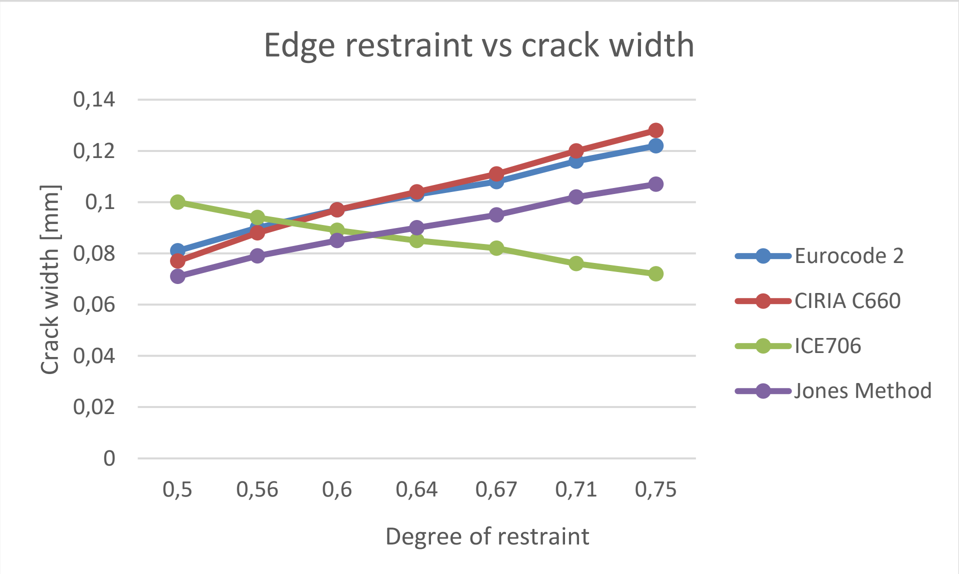




The time of cracking has an influence on the crack width for a situation in which end restraint is governing. Because when the time of cracking increases the concrete tensile strength increases as well. By increasing the concrete tensile strength the cracking strain increases which will result in an increase in crack width. This is almost the same situation as with increasing the concrete strength. The time does not have an effect on the crack width when an edge restraint situation is governing. Except for the CIRIA C660 and the ICE 706 because the crack width due to edge restraint is dependent on the strength which is influenced by the time, although the influence is small.

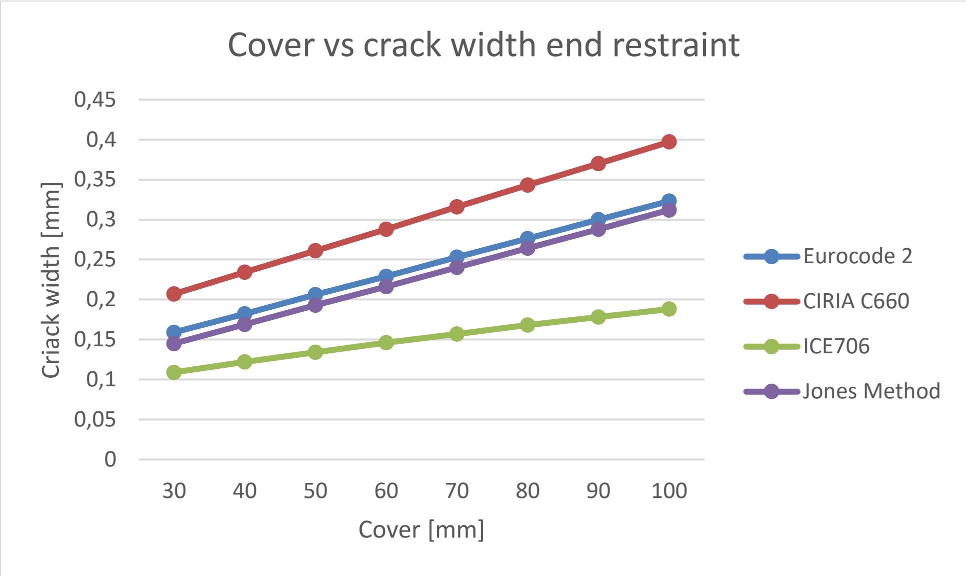
Variable degree of restraint:
The degree of restraint is also an influencing factor for the crack width control because it is used in most of the strain calculations. The effect of restraint is shown below for end restraint and edge restraint. By changing the edge restraint the ratio between the new and old cross sectional area had to be changed so the thickness of the section has been changed. This is done in the old section so only one parameter is variable for the calculation of the crack width. The degree of edge restraint is variable due to changing the thickness of the old concrete from a thickness of 1250mm to 3000mm.

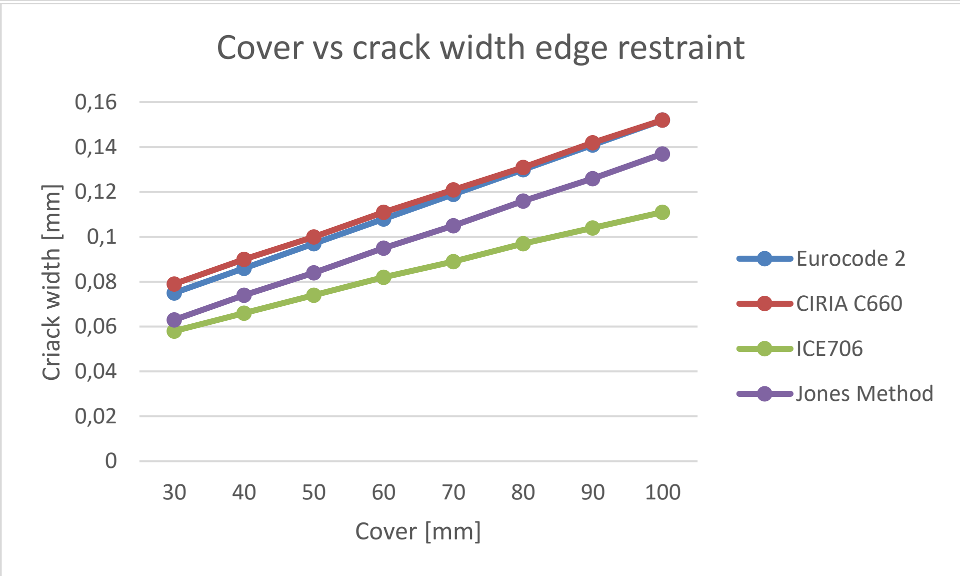




The end restraint situation does not change by increasing end restraint. As can be seen at the graph of the end restraint situation. This is due to the fact that the degree of restraint does not contribute to the strain and crack width calculation. For the edge restraint situation a typical pattern is observed when looking at the results. The crack width increases by increasing degree of restraint for every approach except for the ICE706. This is exactly what the ICE706 describes because this approach suggest that by increasing restraint the restraining element takes up some of the forces and obstructs cracks from growing.

Variable cover:
The last influencing factor which have been looked at is the influence of the cover. The cover have been changed from 30 to 100mm with steps of 10mm. The cover has influence on the effective concrete tension zone and on the crack spacing formula. So the influence of the cover will be visible in both types of restraint crack width formulas. The results are shown below:





For the end restraint situation all four approaches give an increase in crack width by increasing the cover. This is due to the fact that the crack spacing formula is determined partly by the cover term. So by increasing the cover the crack spacing increases which will affect the crack width because the crack width is determined by the crack spacing. The crack spacing formula is also determined by the effective reinforcement ratio. The effective reinforcement ratio is determined by the effective concrete tension height which is in most of the cases partly determined by the cover. By increasing the effective height due to increasing the cover, the reinforcement ratio decreases and the crack spacing increases. These principles hold for both restraint situations.

A.9 Conclusion

The different methods which have been looked at for this literature study all have their special feature in improving the calculation of the crack width and crack spacing formulas given by the Eurocode. The biggest improvements are:

- Better understanding of the effects of the concrete mix on the crack width(CIRIA C660)
- A formula to determine the factor of restraint (CIRIA C660/ICE706)
- Improvement of the crack width formula for continuous edge restraint situations (ICE706)
- Improvement of the method to determine the effective concrete tension zone (method of Jones)

The calculations methods differ in the way of the determination of the cracking strain. This is due to the parameters which are used for each calculation method. For example the cracking strain of the CIRIA C660 is calculated differently in comparison with the Eurocode and the ICE706 or Method of Jones. The parameters used for the ICE706 and method of Jones are based on the Eurocode while the CIRIA C660 uses parameters like temperature and autogenous shrinkage.

The graphs in chapter 7 indicate the differences in the different methods with varying parameters. The results from the graphs indicate that the smallest crack width for both situations of restraint are given by the approach of the ICE 706 in most of the cases which are considered. The ICE 706 has the lowest crack width values due to the smaller concrete tensile strength which is considered. In the ICE 706 a factor $\alpha_{ct} = 0{,}8$ is given to decrease the concrete tensile strength by this factor. By lowering the concrete tensile strength the concrete tensile strain capacity decreases as well. And the crack strain decreases due to the decrease in concrete tensile strength. This results in a decrease of the crack width. The Jones method is the second best method.

B. APPENDIX B – VARIATION STUDY PURE TENSION TESTS

In this appendix all models of the variation study are elaborated to make a comparison between the models and to give a conclusion about the effect of the cover, width, diameter and height on the effective concrete tension zone. The comparison and the conclusion about these models is given in the thesis. The models which have been elaborated here are models with a cover of 50mm till a cover of 100mm with intermediate steps of 10mm, models with a varying width from 100mm till 250mm with intermediate steps of 50mm, models with a varying diameter from 20mm till 40mm with intermediate steps 25mm and 32mm and model with two layers of reinforcement with a varying height of 500, 1000 and 1500mm. The term cover means the distance between the outer fiber of the concrete and the center of the reinforcement bar. In Eurocode 2 the cover is defined differently as the distance between the outer fiber and the edge of the reinforcement bar. The difference between these two terms is the radius of the reinforcement bar. In these examples the radius of the reinforcement bar is 20mm, so the difference between the cover used here and the cover used in Eurocode is 20mm. This difference will be used in the comparison between the effective height according to DIANA and the effective height according to Eurocode 2 formulas. For the models with a varying width a standard diameter of 40mm is used. The standard cover which is applied here is 60mm (40mm according to the Eurocode). For the models with a varying diameter the standard width is 200mm and the cover is 60mm. The length of the models is equal for every model and has a length of 4000mm.

B.1 Variation in cover of the reinforcement

The first elaborated example is the model in which the cover is 50mm (30mm according to Eurocode 2). The dimensions of the model are given in the table below.

Table 1 Dimensions of the 2D model with 50mm cover

Variables:	Dimension [mm]
Length	4000
Width	200
Height	500
Diameter reinforcement	40
Cover	50
Mesh size	20

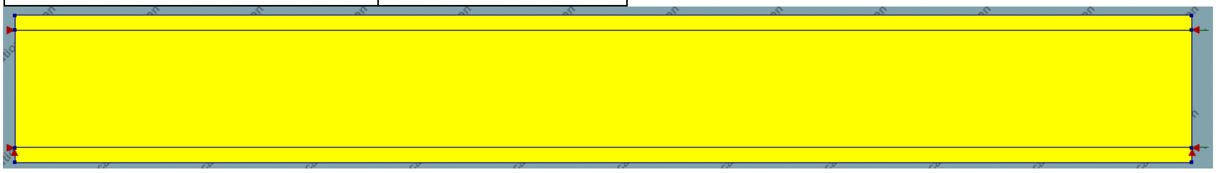


Figure 1 2D model with 50mm cover

In Figure 1 the 2D model is shown. On the right hand side the prescribed deformations are given as small arrows. The triangles display the supports which are used. The bottom reinforcement is supported in both horizontal x-direction as in vertical y-direction. The top reinforcement is only supported in horizontal x direction, because the model undergoes transverse contraction due to the tension force. The transverse contraction will result in stresses in the model in the transverse direction if the model is supported on both reinforcement bars. The stresses will result in parallel cracks around the reinforcement, these cracks need to be prevented because they influence the results needed for this master thesis. Every model is supported in the same way the only difference of the models is the cover.

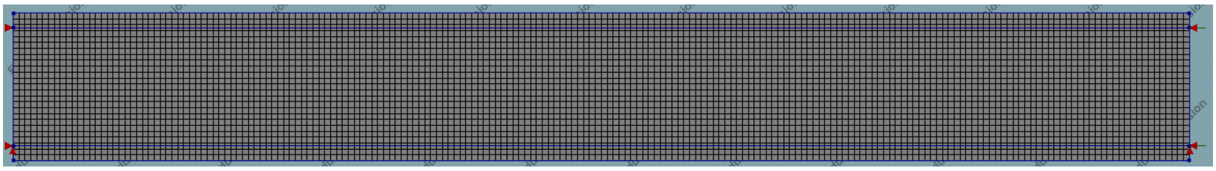


Figure 2 Mesh of the 2D model with 50mm cover

In Figure 2 the mesh of the 2D model is given the mesh size used is 20mm. The mesh is fine because the position of the cracks, bond stress and transfer length need to be accurately determined. The mesh size is 20mm for every model.

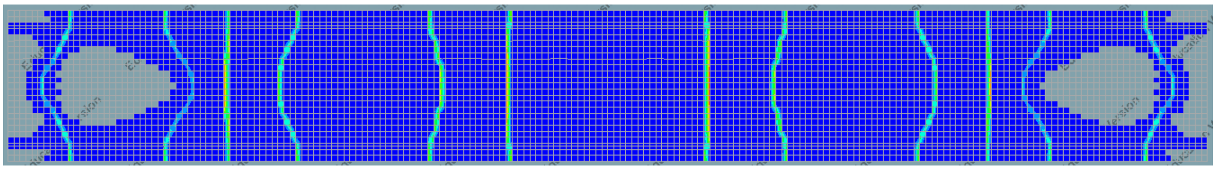


Figure 3 Crack pattern of the 2D model with 50mm cover

In Figure 3 the crack pattern of load step 300 is shown for the 2D model with a cover of 50mm. The crack spacing varies over the length of the model. In the middle of the model a symmetry line can be observed. This effect is obtained because the model is loaded on both ends of the model due to the supports on both sides and the prescribed deformation on the right side of the model. The crack pattern shows 4 distinct cracks and 8 smaller cracks.

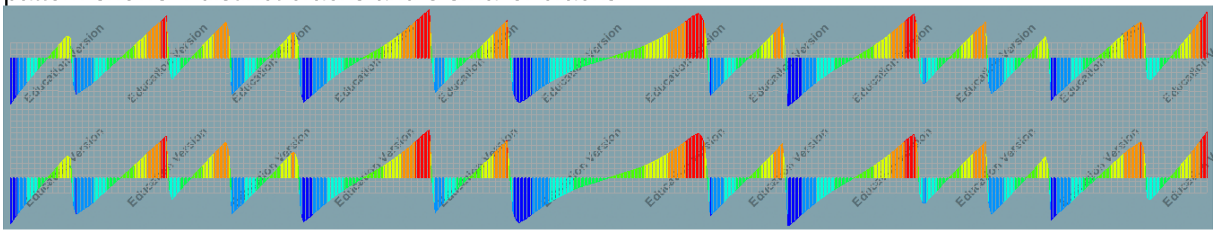


Figure 4 Bond stress gradient of the 2D model with a cover of 50mm

In Figure 4 the bond stress gradient for load step 300 of the 2D model with a cover of 50mm is given. The positions of the cracks can be observed, because at the positions of the cracks the bond stress suddenly changes sign from a positive value to a negative value. At these positions the slope of the curve is steep. In between the cracks the bond stress runs from a large negative value to a large positive value with a nearly constant linear slope. Due to the fact that the smaller cracks are close to the distinct cracks the bond stress in between these cracks is low. The transfer length is also small in between those cracks.

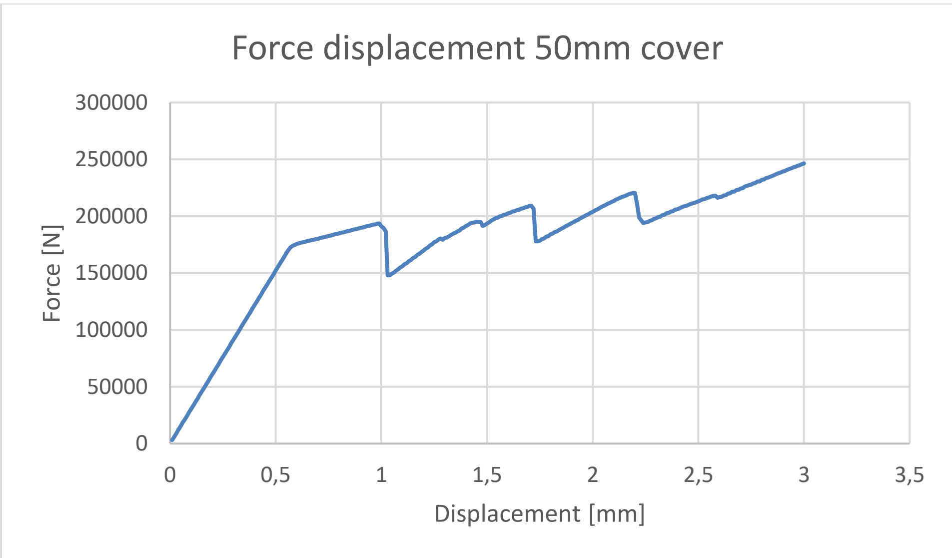


Figure 5 Force displacement diagram of the 2D model with a 50mm cover

In Figure 5 the force-displacement diagram of the 2D model with a cover of 50mm is given. Within the figure the elastic linear part of the force-displacement diagram can be observed. After the elastic part the model reaches the cracking force. When the model cracks the force decreases substantially after which the force build up till a new crack. This process repeats itself until a fully developed crack pattern. A fully developed crack pattern is given in the force-displacement diagram. The amount of cracks equals 6 this amount of cracks can also be observed in Figure 3, in half of the model. Some of the cracks are better visible in the force displacement diagram than other cracks. Three of the six cracks are clearly visible.

The results which are necessary for the determination of the effective concrete tension zone height are obtained from DIANA. The cracking forces are obtained by multiplying the average bond stress times the transfer length times the perimeter of the reinforcement. The average cracking force is determined by adding all cracking forces and divide it by the amount of cracking forces.

In the table below, the parameters for the determination of the effective concrete tension zone are given. These are the values for only half of the model but due to symmetry in the middle of the model all these values can be multiplied by 2 to get the values for the whole model. With these values the average cracking force can be calculated.

Table 2 Summary of the parameters from the 2D model with a 50mm cover

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
1.75	510	125.7	111956	2.9	38606
1.86	580	125.7	135422	2.9	46697
2.31	440	125.7	127894	2.9	44101
2.05	490	125.7	126522	2.9	43628
2.64	230	125.7	76250	2.9	26293
2.83	290	125.7	103120	2.9	35558

$$\sum cracking\ force = 1362328N$$
$$n = 12$$
$$Average\ cracking\ force = N_{cr,ave} = \frac{\sum cracking\ force}{n} = \frac{1362328}{12} = 113527N$$

With an average cracking force of 113527N, the effective concrete tensile zone becomes: $A_{c,eff} = {N_{cr,ave}}/{f_{ctm}} = {113527}/{2.9} = 39147mm^2$. This implies an effective concrete tension zone height of:

$$h_{c,eff} = \frac{A_{c,eff}}{b} = \frac{39147}{200} = 196mm$$

The effective concrete tension zone height for this case is, according to Eurocode 2, the minimum of: ${h}/{2}$ and $2{,}5 * (c + {\emptyset_{sl}}/{2})$.

$$\frac{h}{2} = \frac{500}{2} = 250mm, \qquad 2{,}5 * \left(30 + \frac{40}{2}\right) = 125mm$$

In this configuration Eurocode 2 gives a smaller effective concrete tension zone height than the finite element models. The difference between both values is 71mm. This difference is large due to the fact that the cracking forces of the model are high. Eurocode 2 results in a smaller crack width of 0.06mm. This implies that the crack spacing formula is normative in the crack width formula because a higher effective concrete tension zone height has a negative influence on the crack spacing. The influence displayed here is negative therefore the crack spacing formula is normative.

The second elaborated example is the model in which the cover is 60mm (40mm according to Eurocode 2). The dimensions of the model are given in the table below.

Table 3 Dimensions of the 2D model with 60mm cover

Variables:	Dimension [mm]
Length	4000
Width	200
Height	500
Diameter reinforcement	40
Cover	60
Mesh size	20

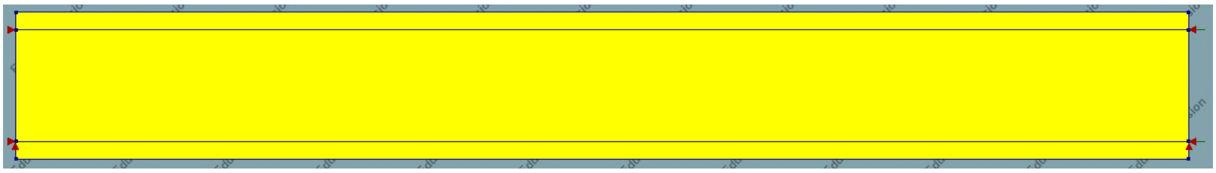


Figure 6 2D model with a cover of 60mm

For this model the same conditions apply as the 50mm model.

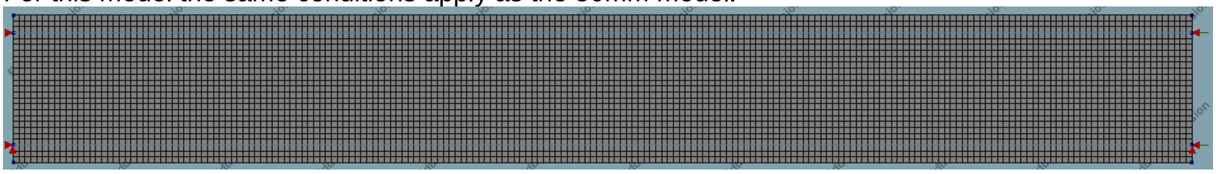


Figure 7 Mesh of the 2D model with a cover of 60mm

For this model the same conditions apply as the 50mm model.

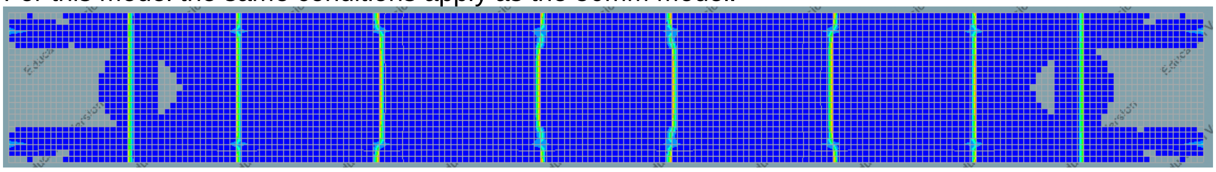


Figure 8 Crack pattern of the 2D model with a cover of 60mm

In Figure 8 the crack pattern of the 2D model with a cover of 60mm is given. This model also has a symmetry line in the middle of the model. This holds for every model and is due to the loading and supports of the models. In total there are 8 cracks within the model with a varying crack spacing.

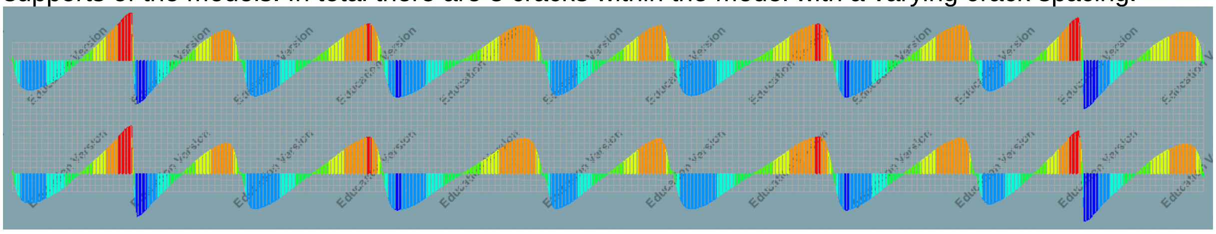


Figure 9 Bond stress gradient of the 2D model with a cover of 60mm

In Figure 9 the bond stress gradient is given for the 2D with a cover of 60mm. In this figure the positions of the cracks are visible and the lengths over which the bond stress varies. Even so the symmetry of the model is visible when looking at the gradient of the bond stress. The gradient of the bond-slip curve can be observed in the bond-stress plot.

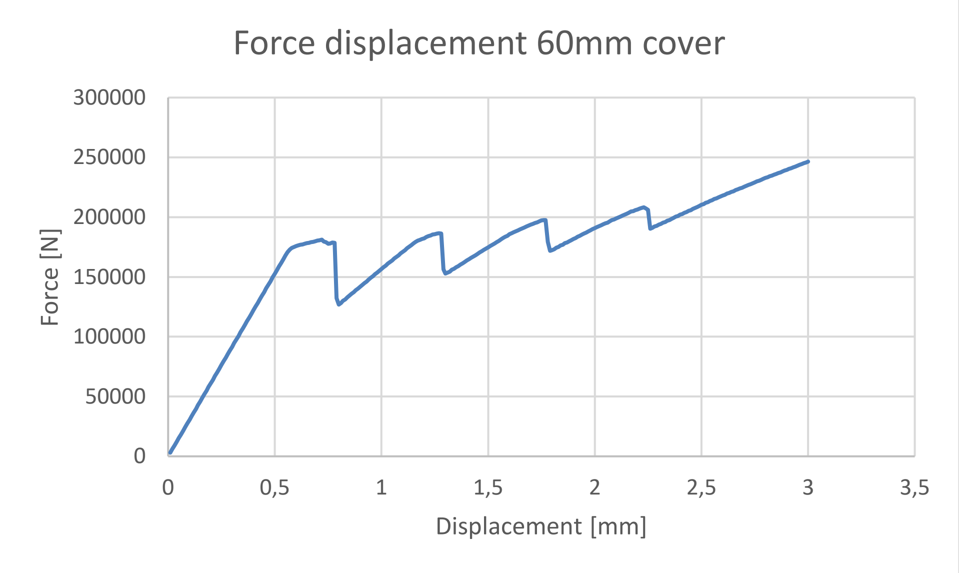


Figure 10 Force displacement diagram of the 2D model with a cover of 60mm

In Figure 10 the force displacement diagram of the 2D model with a cover of 60mm is given. In this diagram the force needed for a next crack to appear is clearly visible by the sudden drops in the force displacement curve. In the graph a fully developed crack pattern is given the maximum amount of cracks per half of the model is 4.

In the table below, the parameters for the determination of the effective concrete tension zone are given. These are the values for only half of the model but due to symmetry in the middle of the model all these values can be multiplied by two to get the values for the whole model. With these values the average cracking force can be calculated.

Table 4 Summary of the parameters from the 2D model with a 60mm cover

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
1.85	630	125.7	146281	2.9	50442
1.52	730	125.7	139033	2.9	47942
2.09	500	125.7	131064	2.9	45194
2.01	510	125.7	129066	2.9	44506
2.59	380	125.7	123640	2.9	42635
2.57	380	125.7	122954	2.9	42398

$$\sum cracking\ force = 1584077N$$

$$n = 12$$

$$Average\ cracking\ force = N_{cr,ave} = \frac{\sum cracking\ force}{n} = \frac{1584077}{12} = 132006N$$

With the average cracking force of 132006N, the effective concrete tensile zone becomes:

$A_{c,eff} = {N_{cr,ave}}/{f_{ctm}} = {132006}/{2.9} = 45519mm^2$. This implies an effective concrete tension zone height of:

$$h_{c,eff} = \frac{A_{c,eff}}{b} = \frac{45519}{200} = 228mm$$

The effective concrete tension zone height for this case is, according to Eurocode 2, the minimum of

${h}/{2}$ and $2{,}5 * (c + {\emptyset_{sl}}/{2})$.

$$\frac{h}{2} = \frac{500}{2} = 250mm, \qquad 2{,}5 * \left(40 + \frac{40}{2}\right) = 150mm$$

In this configuration Eurocode 2 gives a smaller effective concrete tension zone height than the finite element models. The difference is 78mm. This results in a crack width difference of 0.06mm. So the procedure described by Eurocode 2 underestimates the effective concrete tension zone height according to this calculation.

The third model, with a cover of 70mm is elaborated here.

Table 5 Dimensions of the 2D model with 70mm cover

Variables:	Dimension [mm]
Length	4000
Width	200
Height	500
Diameter reinforcement	40
Cover	70
Mesh size	20

Figure 11 2D model with 70mm cover

For this model the same conditions apply as for the 50mm model.

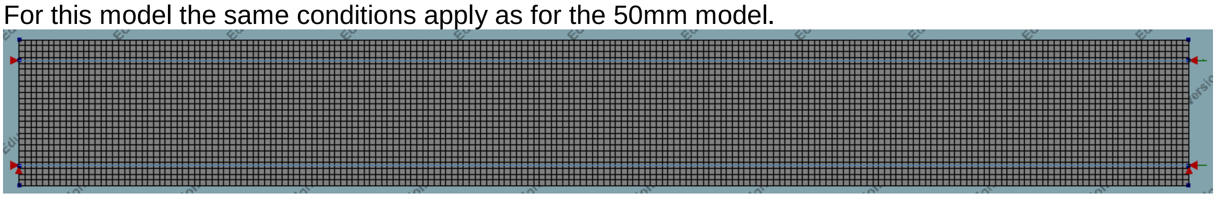


Figure 12 Mesh of the 2D model with 70mm cover

In Figure 12 the mesh of the 2D model is given the mesh size used is 20mm. For this model the same conditions apply as for the 50mm cover model.

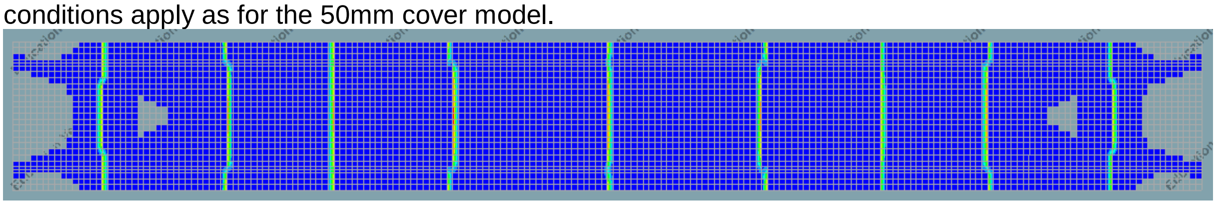


Figure 13 Crack pattern of the 2D model with a cover of 70mm

In Figure 13 the crack pattern is shown for the 2D model with a cover of 70mm. The crack spacing varies over the length of the model. In the middle of the model a symmetry line can be observed. This effect is obtained because the model is loaded on both ends of the model due to the supports on both sides and the prescribed deformation on the right side of the model.

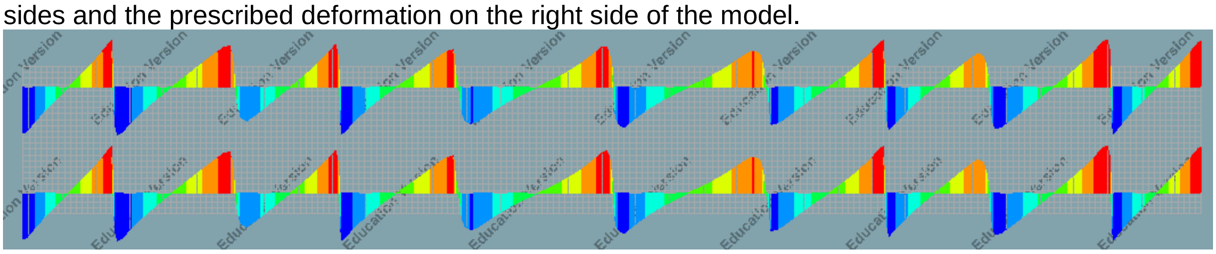


Figure 14 Bond stress gradient of the 2D model with a cover of 70mm

In Figure 14 the bond stress gradient of the 2D model with a cover of 70mm is given. The positions of the cracks can be observed, because at the positions of the cracks the bond stress changes from a maximum positive value to a maximum negative value. At these positions the slope of the curve is steep. In between the cracks the bond stress runs from a maximum negative value to a maximum positive value with a nearly constant linear slope.

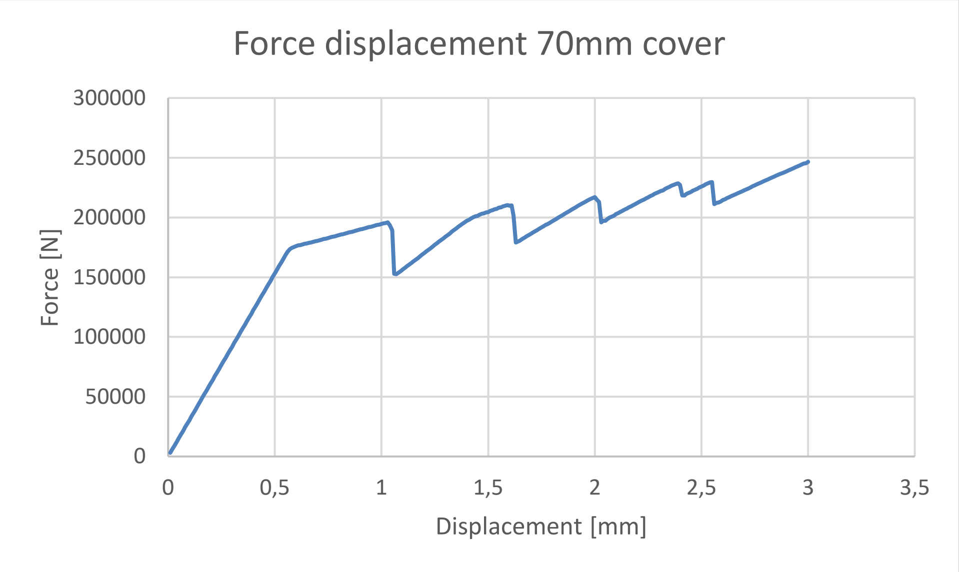


Figure 15 The force displacement diagram of the 2D model with a cover of 70mm

In Figure 15 the force-displacement diagram of the 2D model with a cover of 70mm is given. Within the figure the elastic linear part of the force-displacement diagram can be observed. After the elastic part the model reaches the cracking force. When the model cracks the force decreases substantially after which the force built up till a new crack. This process repeats itself until a fully developed crack pattern. A fully developed crack pattern is given in the force-displacement diagram. The amount of cracks equals 5 this amount of cracks can also be observed in Figure 13, because the model has a symmetry line in the middle of the model. In the model a total of 9 cracks can be observed one of these cracks is exactly in the middle of the model.

In the table below, the parameters for the determination of the effective concrete tension zone are given. These are the values for only half of the model but due to symmetry in the middle of the model all these values can be multiplied by two to get the values for the whole model. With these values the average cracking force can be calculated.

Table 6 Summary of the parameters from the 2D with a cover of 70mm

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
2.40	490	125.7	147939	2.9	51014
1.94	540	125.7	131434	2.9	45322
3.05	350	125.7	133966	2.9	46195
2.77	370	125.7	128947	2.9	44465
1.86	520	125.7	121589	2.9	41927
2.50	380	125.7	119398	2.9	41172
2.49	380	125.7	118851	2.9	40983

$$\sum cracking\ force = 1804248N$$
$$number\ of\ cracking\ forces = n = 14$$
$$Average\ cracking\ force = N_{cr,ave} = \frac{\sum cracking\ force}{n} = \frac{1804248}{14} = 128875N$$

With the average cracking force of 128875N, the effective concrete tensile zone becomes: $A_{c,eff} = {}^{N_{cr,ave}}/_{f_{ctm}} = {}^{128875}/_{2.9} = 44440mm^2$. This implies an effective concrete tension zone height of:

$$h_{,eff} = \frac{A_{c,eff}}{b} = \frac{44440}{200} = 222mm$$

The effective concrete tension zone height for this case is, according to Eurocode 2, the minimum of ${}^{h}/_{2}$ and $2{,}5 * (c + {}^{\emptyset_{sl}}/_{2})$.

$$EC2\text{: min}\left[\frac{h}{2} = \frac{500}{2} = 250mm, \quad 2{,}5 * \left(50 + \frac{40}{2}\right) = 175mm\right] = 175mm$$

The effective concrete tension zone height determined with the finite element models is larger than Eurocode 2. The difference between both methods is 47mm. This results in a difference in crack width of 0.03mm. For this situation the effective concrete tension zone heights are not comparable.

The forth elaborated example is the model in which the cover is 80mm (60mm according to Eurocode 2). The dimensions of the model are given in the table below.

Table 7 Dimensions of the 2D model with 80mm cover

Variables:	Dimension [mm]
Length	4000
Width	200
Height	500
Diameter reinforcement	40
Cover	80
Mesh size	20

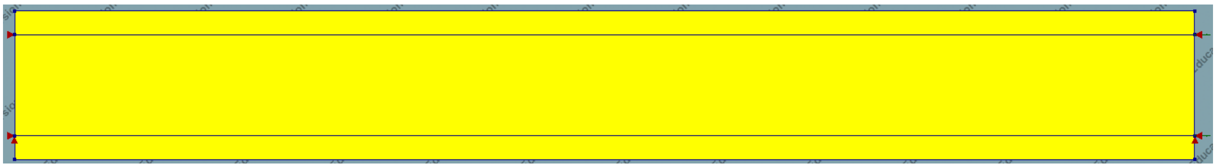


Figure 16 2D model with a cover of 80mm

For this model the same conditions apply as the 50mm model.

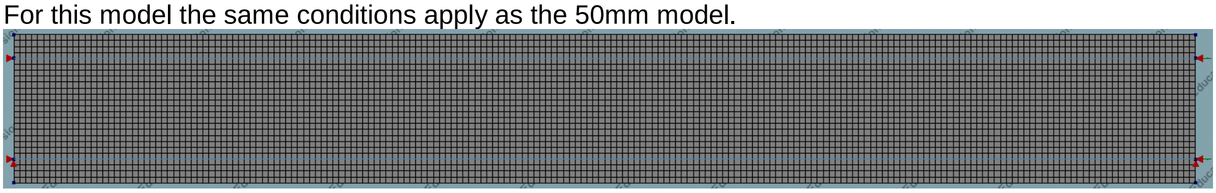


Figure 17 Mesh of the 2D model with a cover of 80mm

For this model the same conditions apply as the 50mm model.

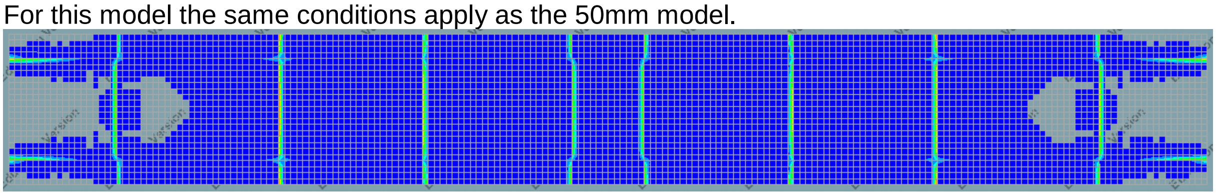


Figure 18 Crack pattern of the 2D model with a cover of 80mm

In Figure 18 the crack pattern of the 2D model with a cover of 80mm is given. In total there are 8 cracks within the model with a varying crack spacing. At the position of the supports cracks run parallel to the reinforcement these cracks have an influence on the bond stress gradient.

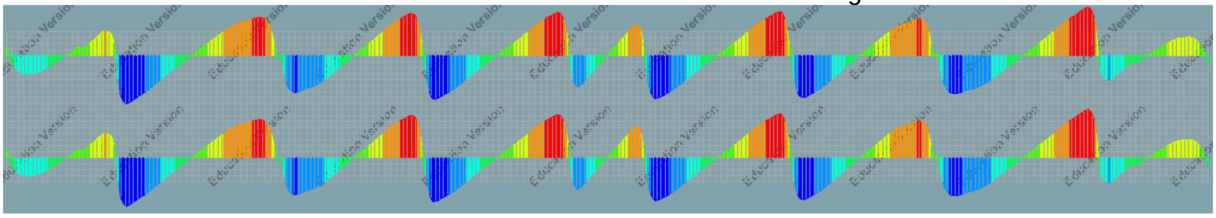


Figure 19 Bond stress gradient of the 2D model with a cover of 80mm

In Figure 19 the bond stress gradient is given for the 2D model with a cover of 80mm. In this figure the positions of the cracks are visible and the lengths over which the bond stress varies. Even so the symmetry of the model is visible when looking at the gradient of the bond stress. The parallel cracks at the positions of the supports let the bond stress decrease. The length in between the cracks in the middle of the model is in reality too small to result in a crack but due to the symmetry the distance in between the cracks is possible. Because the stress which results in a crack built up from the edges who are supported.

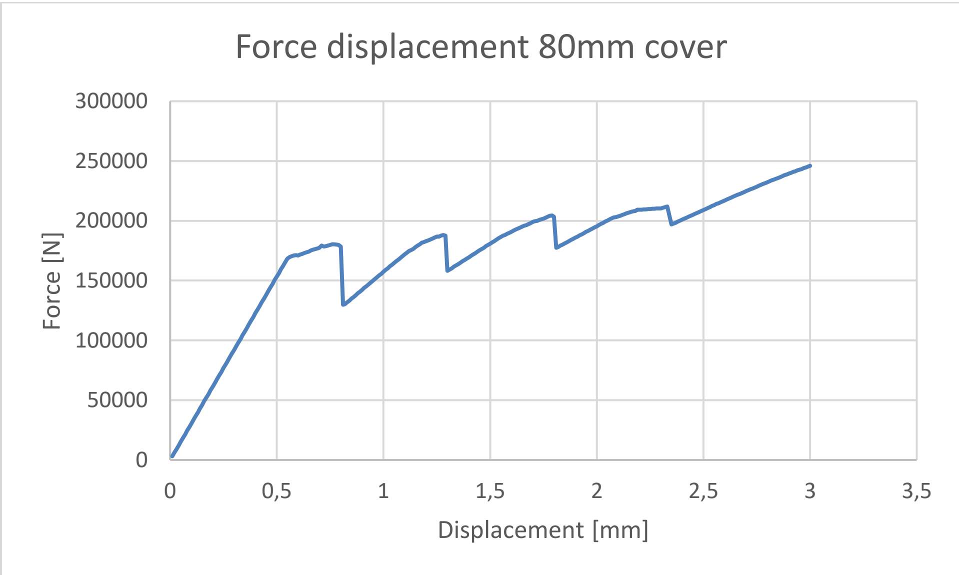


Figure 20 Force displacement diagram of the 2D model with a cover of 80mm

In Figure 20 the force displacement diagram of the 2D model with a cover of 80mm is given. In this diagram the force needed for a next crack to appear is clearly visible by the sudden drops in the force displacement curve. In the graph a fully developed crack pattern is given the maximum amount of cracks per half of the model is 4.

In the table below, the parameters for the determination of the effective concrete tension zone are given. These are the values for only half of the model but due to symmetry in the middle of the model all these values can be multiplied by 2 to get the values for the whole model. With these values the average cracking force can be calculated.

Table 8 Summary of the parameters from the 2D model with a cover of 80mm

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
1.72	680	125.7	146679	2.9	50579
1.32	840	125.7	139575	2.9	48129
2.16	490	125.7	132928	2.9	45837
2.15	480	125.7	129971	2.9	44818
2.30	470	125.7	135858	2.9	46848
2.52	430	125.7	136152	2.9	46949

$$\sum cracking\ force = 1642325N$$
$$n = 12$$
$$Average\ cracking\ force = N_{cr,ave} = \frac{\sum cracking\ force}{n} = \frac{1642325}{12} = 136860N$$
With the average cracking force of 136860N, the effective concrete tensile zone becomes:
$A_{c,eff} = {N_{cr,ave}}/{f_{ctm}} = {136860}/{2.9} = 47193mm^2$. This implies an effective concrete tension zone height of:
$$h_{,eff} = \frac{A_{c,eff}}{b} = \frac{47193}{200} = 235mm$$
The effective concrete tension zone height for this case is, according to Eurocode 2, the minimum of ${h}/{2}$ and $2{,}5 * (c + {\emptyset_{sl}}/{2})$.
$$\frac{h}{2} = \frac{500}{2} = 250mm, \qquad 2{,}5 * \left(60 + \frac{40}{2}\right) = 200mm$$
In this configuration Eurocode 2 gives a smaller effective height than the output of the finite element models. The difference is 36mm which will result in a difference of crack width of 0.02mm. So the procedure described by Eurocode 2 underestimates the effective concrete tension zone according to this calculation.

The fifth elaborated example is the model in which the cover is 90mm (70mm according to Eurocode 2). The dimensions of the model are given in the table below.

Table 9 Dimensions of the 2D model with 90mm cover

Variables:	Dimension [mm]
Length	4000
Width	200
Height	500
Diameter reinforcement	40
Cover	90
Mesh size	20

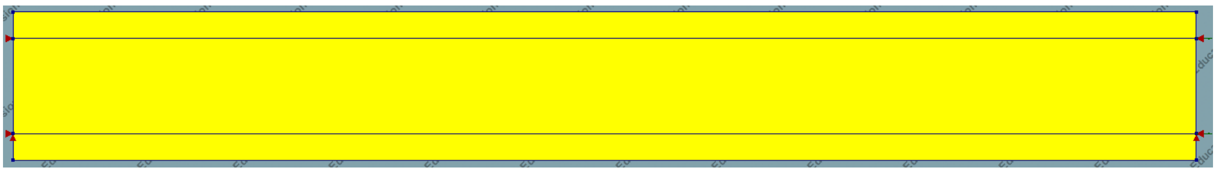


Figure 21 2D model with a cover of 90mm

For this model the same conditions apply as the 50mm model.

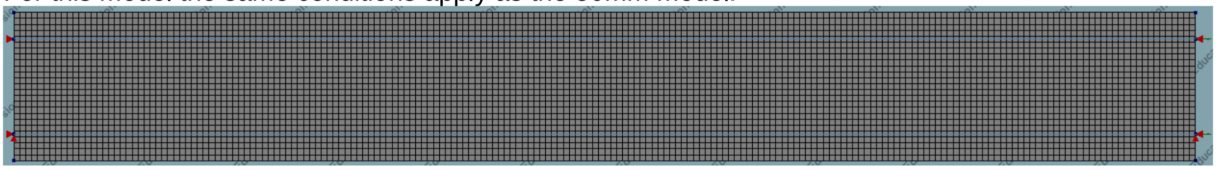


Figure 22 Mesh of the 2D model with a cover of 90mm

For this model the same conditions apply as the 50mm model.

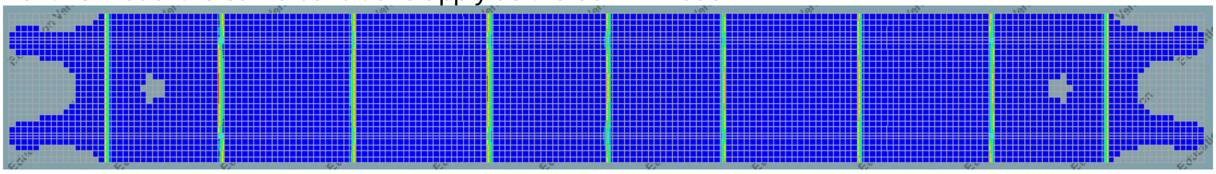


Figure 23 Crack pattern of the 2D model with a cover of 90mm

In Figure 23 the crack pattern of the 2D model with a cover of 90mm is given. In total there are 9 cracks within the model with a varying crack spacing. In this example the model is also cracked in the middle of the model (symmetry line).

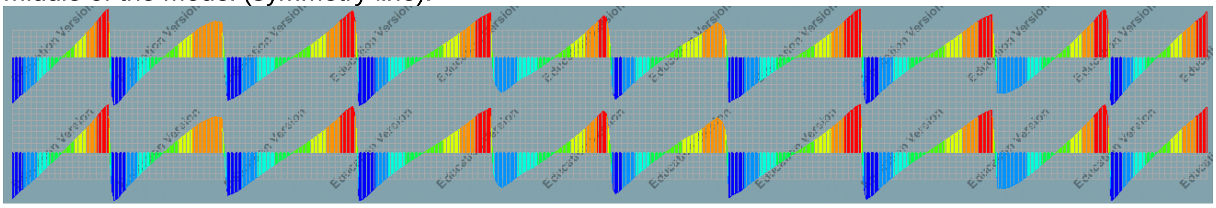


Figure 24 Bond stress gradient of the 2D model with a cover of 90mm

In Figure 24Figure 9 the bond stress gradient is given for the 2D model with a cover of 90mm. In this figure the positions of the cracks are visible and the lengths over which the bond stress varies. Even so the symmetry of the model is visible when looking at the gradient of the bond stress. The bond stress gradient is not hindered by parallel cracks or small/large crack spacings. This is why the gradient runs smooth and has almost similar peaks.

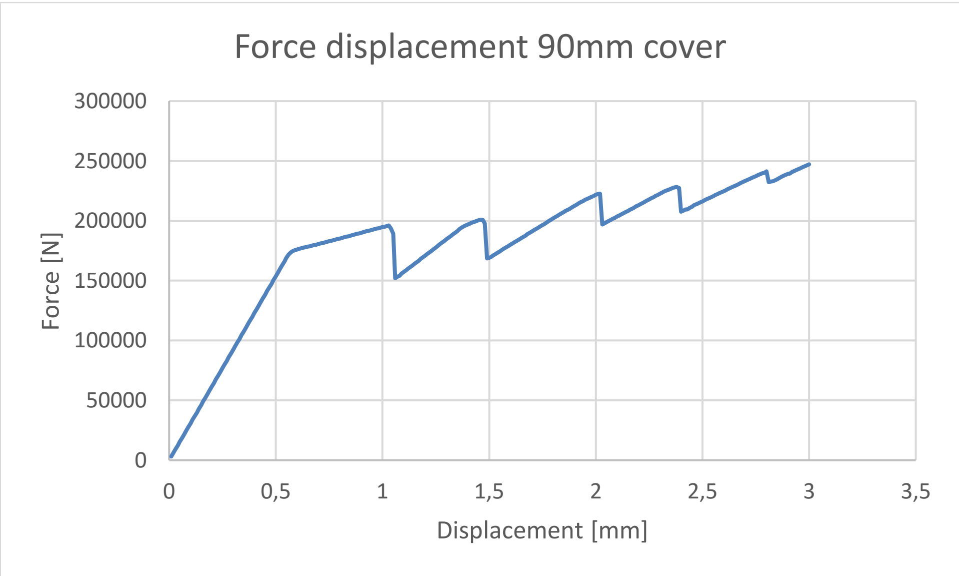


Figure 25 Force displacement diagram of the 2D model with a cover of 90mm

In Figure 25 the force displacement diagram of the 2D model with a cover of 90mm is given. In this diagram the force needed for a next crack to appear is clearly visible by the sudden drops in the force displacement curve. In the graph a fully developed crack pattern is given the maximum amount of cracks per half of the model is 5 with the last crack in the middle of the model.

In the table below, the parameters for the determination of the effective concrete tension zone are given. These are the values for only half of the model but due to symmetry in the middle of the model all these values can be multiplied by two to get the values for the whole model. With these values the average cracking force can be calculated.

Table 10 Summary of the parameters from the 2D model with a cover of 90mm

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
2.41	490	125.7	148566	2.9	51230
1.39	770	125.7	134274	2.9	46301
2.37	440	125.7	131248	2.9	45258
2.24	460	125.7	129494	2.9	44653
3.27	340	125.7	139622	2.9	48145
2.95	370	125.7	137043	2.9	47256
2.55	390	125.7	125161	2.9	43159
2.51	390	125.7	123131	2.9	42459

$$\sum cracking\ force = 2137077N$$

$$n = 16$$

$$Average\ cracking\ force = N_{cr,ave} = \frac{\sum cracking\ force}{n} = \frac{2137077}{16} = 133567N$$

With the average cracking force of 133567N, the effective concrete tensile zone becomes:

$A_{c,eff} = {N_{cr,ave}}/{f_{ctm}} = {133567}/{2.9} = 46058mm^2$. This implies a effective concrete tension zone height of:

$$h_{c,eff} = \frac{A_{c,eff}}{b} = \frac{46058}{200} = 230mm$$

The effective concrete tension zone height for this case is, according to Eurocode 2, the minimum of ${h}/{2}$ and $2{,}5 * (c + {\emptyset_{sl}}/{2})$.

$$\frac{h}{2} = \frac{500}{2} = 250mm, \qquad 2{,}5 * \left(70 + \frac{40}{2}\right) = 225mm$$

In this configuration Eurocode 2 gives a smaller effective concrete tension zone height than the output of the finite element models. The difference between both methods is 5mm. The difference in crack width due to the difference in effective concrete tension zone is 0.00mm. So the procedure described by Eurocode 2 underestimates the effective concrete tension zone according to this calculation but the difference between both models is small.

The sixth and last elaborated example is the model in which the cover is 100mm (80mm according to Eurocode 2). The dimensions of the model are given in the table below.

Table 11 Dimensions of the 2D model with 100mm cover

Variables:	Dimension [mm]
Length	4000
Width	200
Height	500
Diameter reinforcement	40
Cover	100
Mesh size	20

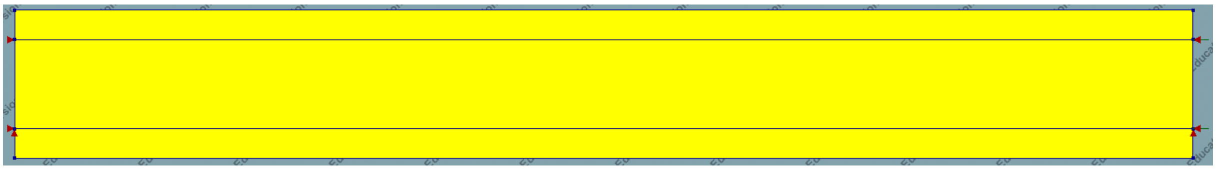


Figure 26 2D model with a cover of 100mm

For this model the same conditions apply as the 50mm model.

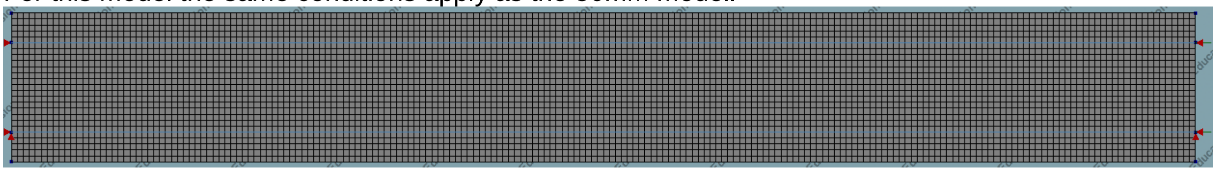


Figure 27 Mesh of the 2D model with a cover of 100mm

For this model the same conditions apply as the 50mm model.

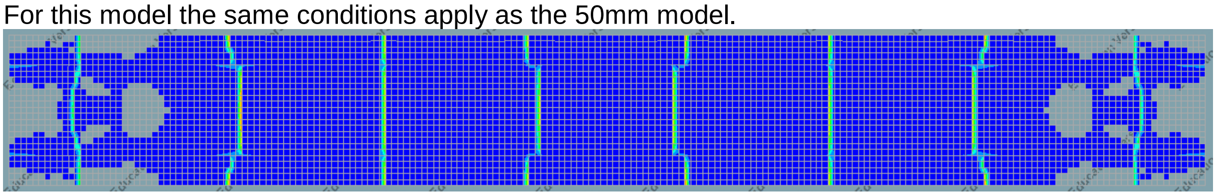


Figure 28 Crack pattern of the 2D model with a cover of 100mm

In Figure 28 the crack pattern of the 2D model with a cover of 100mm is given. In total there are 8 cracks within the model with a varying crack spacing. Some cracks make a jump when they cross the reinforcement this is due to numerical errors.

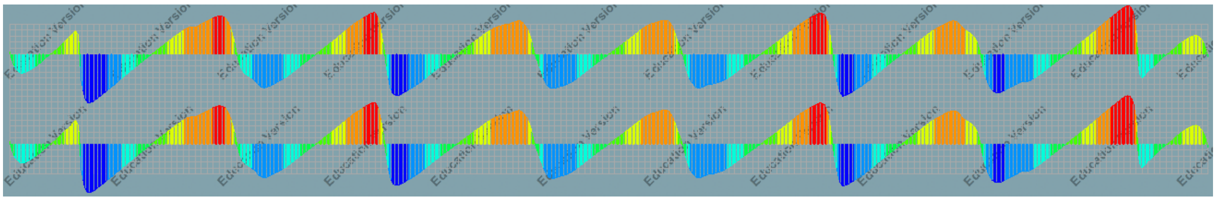


Figure 29 Bond stress gradient of the 2D model with a cover of 100mm

In Figure 29 the bond stress gradient is given for the 2D model with a cover of 100mm. In this figure the positions of the cracks are visible and the lengths over which the bond stress varies. Even so the symmetry of the model is visible when looking at the gradient of the bond stress. The bond stress gradient is hindered by parallel cracks at the edges and due to the jumps in the cracks. That is why the peaks and the gradient do not look the same for the individual parts of the model.

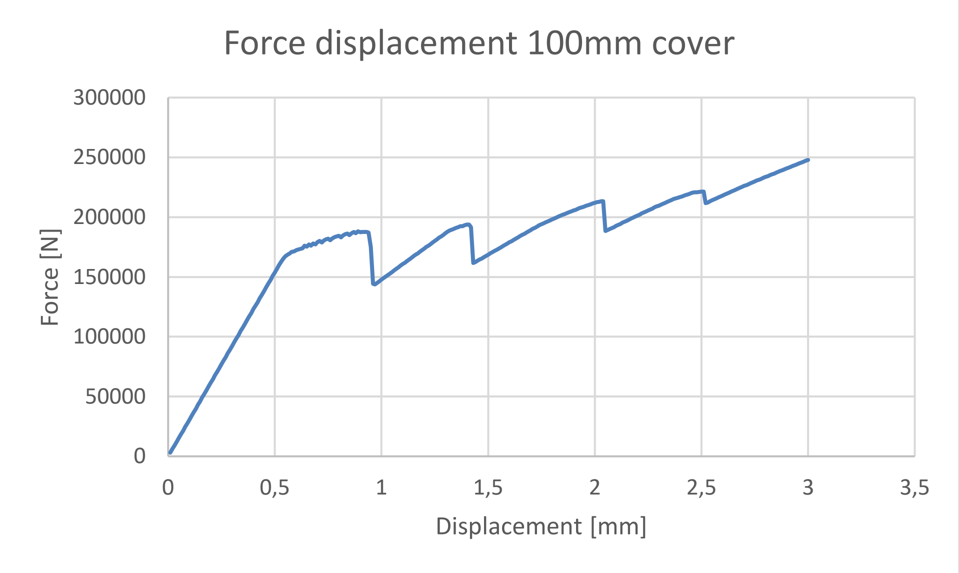


Figure 30 Force displacement diagram of the 2D model with a cover of 100mm

In Figure 30 the force displacement diagram of the 2D model with a cover of 100mm is given. In this diagram the force needed for a next crack to appear is clearly visible by the sudden drops in the force displacement curve. In the graph a fully developed crack pattern is given the maximum amount of cracks per half of the model is 4.

In the table below, the parameters for the determination of the effective concrete tension zone are given. These are the values for only half of the model but due to symmetry in the middle of the model all these values can be multiplied by 2 to get the values for the whole model. With these values the average cracking force can be calculated.

Table 12 Summary of the parameters from the 2D model with a cover of 100mm

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
2.08	560	125.7	146331	2.9	50459
1.22	860	125.7	131923	2.9	45491
2.09	500	125.7	131108	2.9	45210
2.03	500	125.7	127372	2.9	43921
2.68	370	125.7	124432	2.9	42908
2.64	370	125.7	122550	2.9	42258

$$\sum cracking\ force = 1567430N$$

$$n = 12$$

$$Average\ cracking\ force = N_{cr,ave} = \frac{\sum cracking\ force}{n} = \frac{1567430}{12} = 130619N$$

With the average cracking force of 130619N, the effective concrete tensile zone becomes:

$A_{c,eff} = {N_{cr,ave}}/{f_{ctm}} = {130619}/{2.9} = 45041mm^2$. This implies a effective concrete tension zone height of:

$$h_{c,eff} = \frac{A_{c,eff}}{b} = \frac{45041}{200} = 225mm$$

The effective concrete tension zone height for this case is, according to Eurocode 2, the minimum of:

$h/_2$ and $2{,}5 * (c + {}^{\emptyset_{sl}}/_2)$.

$\frac{h}{2} = \frac{500}{2} = 250mm, \quad 2{,}5 * \left(80 + \frac{40}{2}\right) = 250mm$

Both of the values of Eurocode 2 are higher than the value determined by the finite element models. The difference between both models is 25mm. The crack width due to this difference differs 0.01mm. This model is the only model in which the effective concrete tension zone height of the finite element models is smaller than in Eurocode 2, for situations in which the cover is varied. So this result is not in the line of expectations.

B.2 Variation in width of the model

The first elaborated example is the model in which the width is 100mm. The dimensions of the model are given in the table below.

Table 13 Dimensions of the 2D model with 100mm width

Variables:	Dimension [mm]
Length	4000
Width	100
Height	500
Diameter reinforcement	40
Cover	60
Mesh size	20

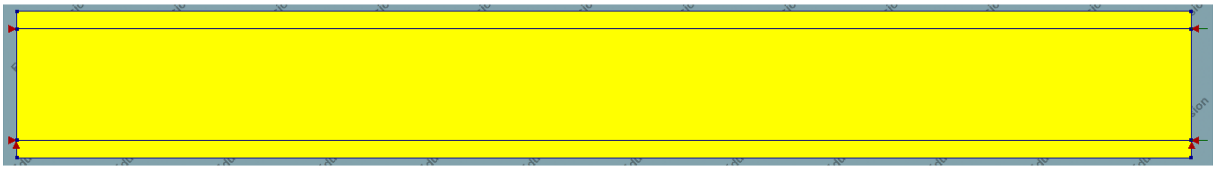


Figure 31 2D model with a width of 100mm

In Figure 31 the 2D model with a width of 100mm is given. The horizontal lines represent the reinforcement. The prescribed deformation is given by the arrows on the right side of the model. The supports are given by the red triangles the horizontal x-direction is supported on both ends of both reinforcements. The vertical y-direction is supported by the bottom reinforcement. In order to prevent transverse contraction of the model the upper reinforcement is not supported in the y-direction. If the transverse contraction of the model is restrained by the vertical supports tension stresses in y-direction arise in the model which can result in cracks parallel to the reinforcement. This has to be prevented because parallel cracks have a negative influence on the bond stress gradient.

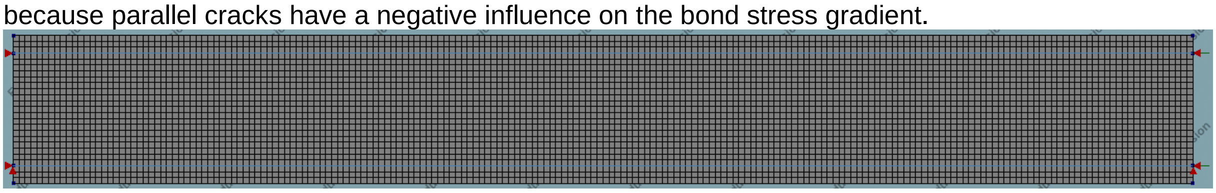


Figure 32 Mesh of the 2D model with a width of 100mm

In Figure 32 the mesh of the 2D model with a width of 100mm is given. The mesh has a size of 20mm per element. It is a fine mesh to get accurate results for the crack spacing and bond stresses. The mesh is evenly distributed over the model without errors in the mesh.

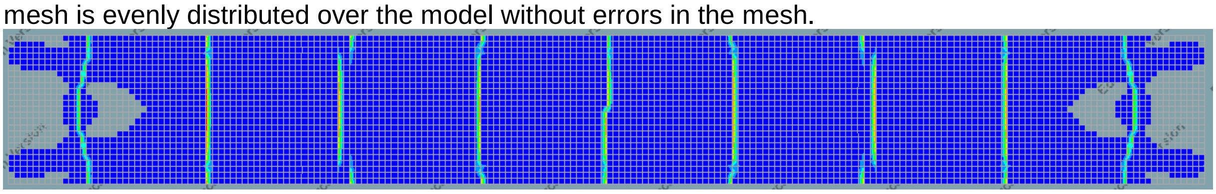


Figure 33 Crack pattern of the 2D model with a width of 100mm

In Figure 33 the crack pattern of the 2D model with a width of 100mm is given. In total there are 9 cracks within the model with a varying crack spacing. The crack pattern is symmetrical with the symmetry line in the middle of the model. This model has a crack in the symmetry line.

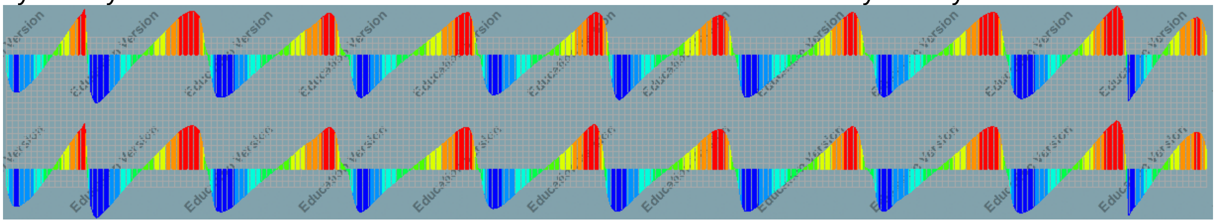


Figure 34 Bond stress gradient of the 2D model with a width of 100mm

In Figure 34 the bond stress gradient is given for the 2D model with a width of 100mm. In this figure the positions of the cracks are visible and the lengths over which the bond stress varies. Even so the symmetry of the model is visible when looking at the gradient of the bond stress.

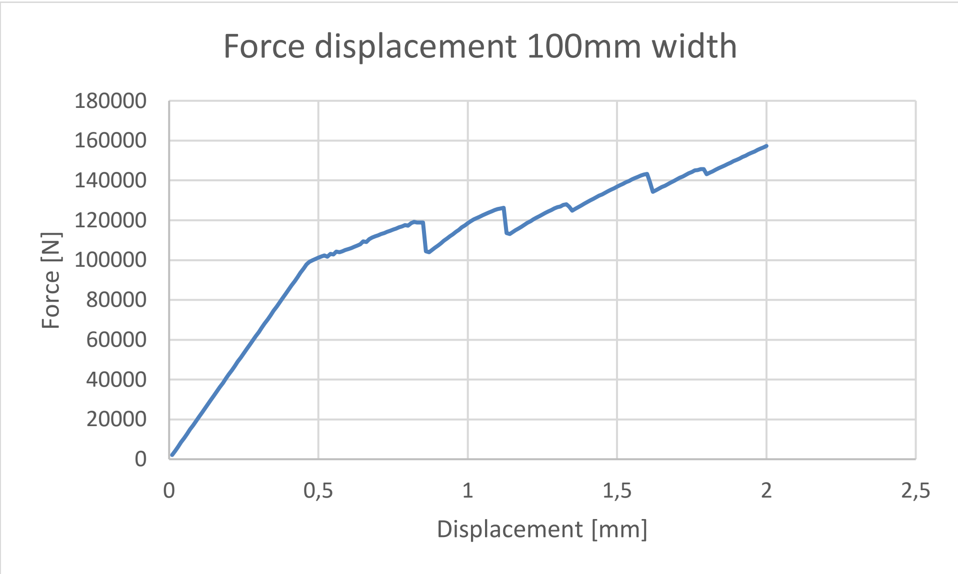


Figure 35 Force displacement diagram of the 2D model with a width of 100mm

In Figure 35 the force displacement diagram of the 2D model with a width of 100mm is given. In this diagram the force needed for a next crack to appear is clearly visible by the sudden drops in the force displacement curve. In the graph a fully developed crack pattern is given the maximum amount of crack per half of the model is 5. In which the fifth crack is the crack in the symmetry line so that one counts for both halves.

In the table below, the parameters for the determination of the effective concrete tension zone are given. These are the values for only half of the model but due to symmetry in the middle of the model all these values can be multiplied by two to get the values for the whole model. With these values the average cracking force can be calculated.

Table 14 Summary of the parameters from the 2D model with a width of 100mm

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
1.19	490	125.7	73448	2.9	25327
0.79	660	125.7	65682	2.9	22649
1.44	310	125.7	56142	2.9	19359
1.39	340	125.7	59592	2.9	20549
1.11	450	125.7	62505	2.9	21554
1.16	430	125.7	62798	2.9	21654
1.03	440	125.7	57192	2.9	19721
1.03	440	125.7	57178	2.9	19717

$$\sum cracking\ force = 989073N$$
$$n = 16$$
$$Average\ cracking\ force = N_{cr,ave} = \frac{\sum cracking\ force}{n} = \frac{989073}{16} = 61817N$$
With the average cracking force of 61817N, the effective concrete tensile zone becomes:
$A_{c,eff} = {N_{cr,ave}}/{f_{ctm}} = {61817}/{2.9} = 21316mm^2$. This implies a effective concrete tension height of:
$$h_{c,eff} = \frac{A_{c,eff}}{b} = \frac{21316}{100} = 213mm$$
The effective concrete tension zone height for this case is, according to Eurocode 2, the minimum of:
${h}/{2}$ and $2{,}5 * (c + {\emptyset_{sl}}/{2})$.
$$\frac{h}{2} = \frac{500}{2} = 250mm, \qquad 2{,}5 * \left(40 + \frac{40}{2}\right) = 150mm$$
In this configuration Eurocode 2 gives a smaller effective concrete tension zone height than the output of DIANA. The difference between both methods is 63mm. This results in a difference in crack width of 0.02mm. So the procedure described by Eurocode 2 underestimates the effective concrete tension zone according to this calculation.

The second elaborated example is the model in which the width is 150mm. The dimensions of the model are given in the table below.

Table 15 Dimensions of the 2D model with 150mm width

Variables:	Dimension [mm]
Length	4000
Width	150
Height	500
Diameter reinforcement	40
Cover	60
Mesh size	20

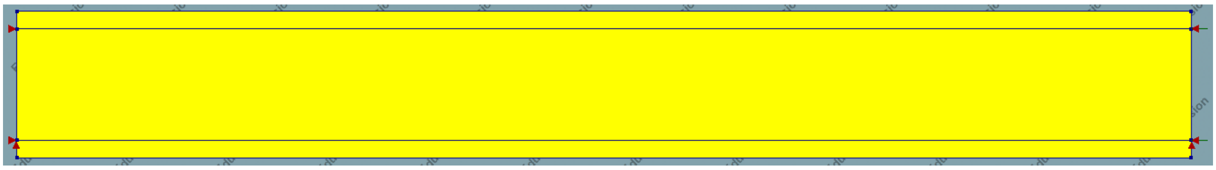


Figure 36 2D model with a width of 150mm

For this model the same conditions apply in term of geometry as for the model in which the width is 100mm.

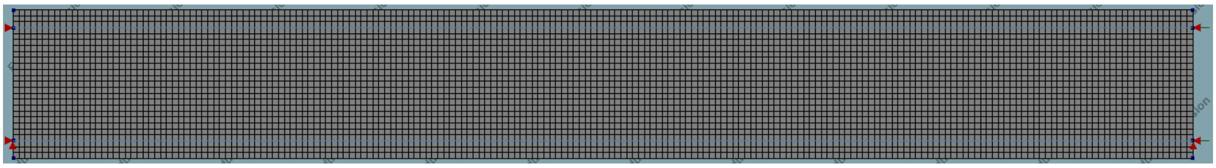


Figure 37 Mesh of the 2D model with a width of 150mm

For this model the same conditions apply in terms of mesh as for the model in which the width is 100mm.

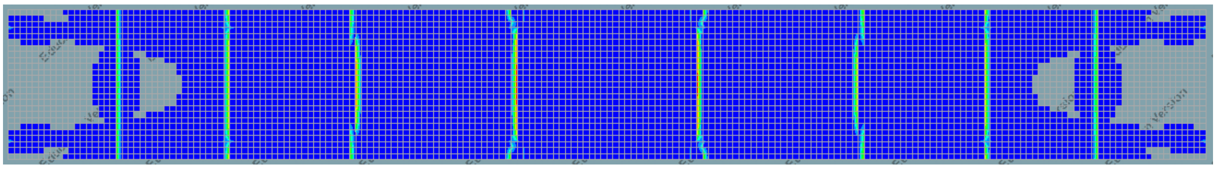


Figure 38 Crack pattern of the 2D model with a width of 150mm

In Figure 38 the crack pattern of the 2D model with a width of 150mm is given. In total there are 8 cracks within the model with a varying crack spacing. The crack pattern is symmetrical with the symmetry line in the middle of the model.

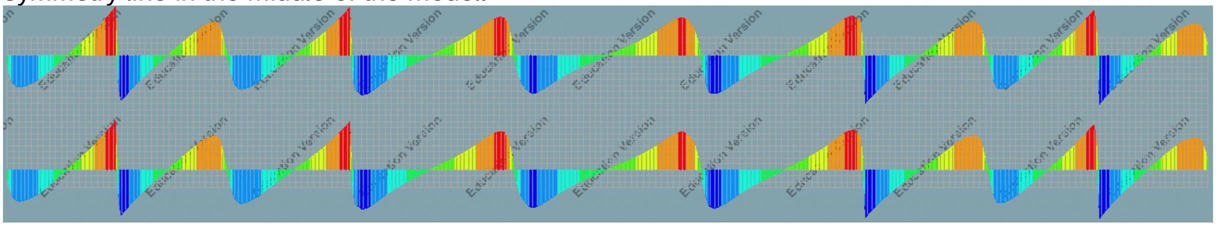


Figure 39 Bond stress gradient of the 2D model with a width of 150mm

In Figure 39 the bond stress gradient is given for the 2D model with a width of 150mm. In this figure the positions of the cracks are visible and the lengths over which the bond stress varies. Even so the symmetry of the model is visible when looking at the gradient of the bond stress. Due to the variation in the crack spacing the bond stress gradient differs in between the cracks.

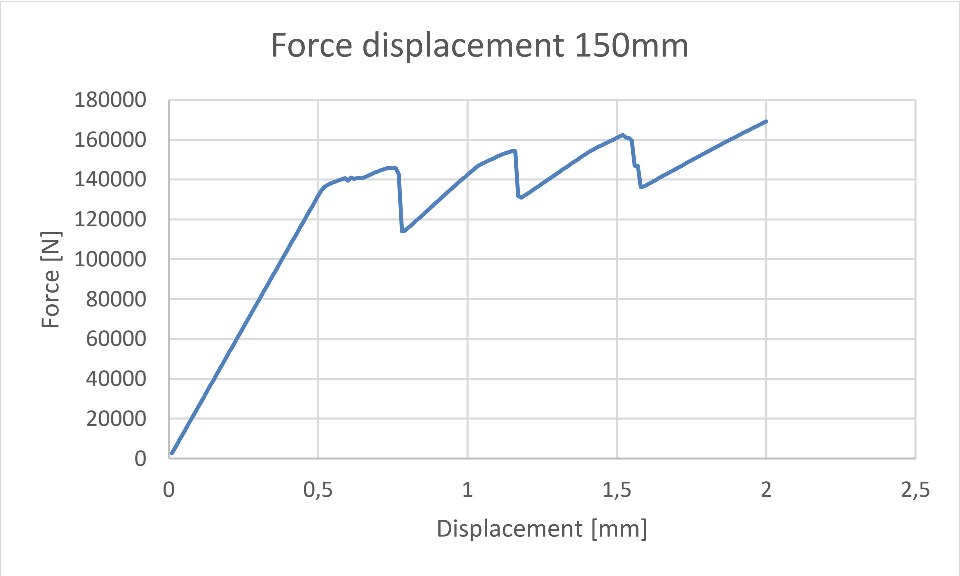


Figure 40 Force displacement diagram of the 2D model with a width of 150mm

In Figure 40 the force displacement diagram of the 2D model with a width of 150mm is given. In this diagram the force needed for a next crack to appear is clearly visible by the sudden drops in the force displacement curve. In the graph a fully developed crack pattern is given the maximum amount of crack per half of the model is 4.

In the table below, the parameters for the determination of the effective concrete tension zone are given. These are the values for only half of the model but due to symmetry in the middle of the model all these values can be multiplied by two to get the values for the whole model. With these values the average cracking force can be calculated.

Table 16 Summary of the parameters from the 2D model with a width of 150mm

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
1.42	610	125.7	108918	2.9	37558
1.11	740	125.7	103609	2.9	35727
2.04	350	125.7	89873	2.9	30991
1.96	370	125.7	91074	2.9	31405
1.63	460	125.7	94429	2.9	32562
1.65	470	125.7	97407	2.9	33589

$$\sum cracking\ force = 1170621N$$

$$n = 12$$

$$Average\ cracking\ force = N_{cr,ave} = \frac{\sum cracking\ force}{n} = \frac{1170621}{12} = 97552N$$

With the average cracking force of 97552N, the effective concrete tensile zone becomes: $A_{c,eff} = \frac{N_{cr,ave}}{f_{ctm}} = \frac{97552}{2.9} = 33639mm^2$. This implies a effective concrete tension height of:

$$h_{c,eff} = \frac{A_{c,eff}}{b} = \frac{33639}{150} = 224mm$$

The effective concrete tension zone height for this case is, according to Eurocode 2, the minimum of: $h/_2$ and 2,5 $* (c + {}^{\emptyset_{sl}}/_2)$.

$$\frac{h}{2} = \frac{500}{2} = 250mm, \quad 2{,}5 * \left(40 + \frac{40}{2}\right) = 150mm$$

In this configuration the Eurocode 2 gives a smaller effective concrete tension zone height than the output of DIANA. The difference between both methods is 74mm. This results in a difference in crack width of 0.02mm. So the procedure described by Eurocode 2 underestimates the effective concrete tension zone according to this calculation.

The third elaborated example is the model in which the width is 200mm. The dimensions of the model are given in the table below.

Table 17 Dimensions of the 2D model with 200mm width

Variables:	Dimension [mm]
Length	4000
Width	200
Height	500
Diameter reinforcement	40
Cover	60
Mesh size	20

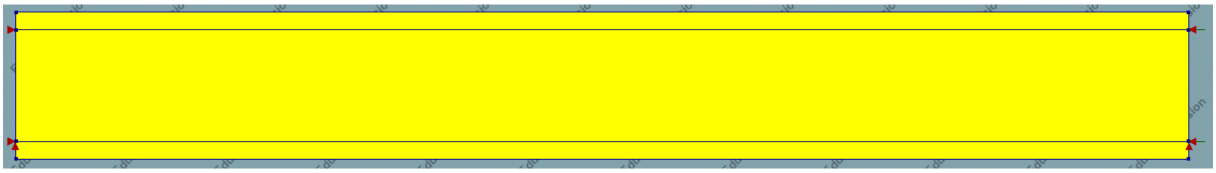


Figure 41 2D model with a width of 200mm

For this model the same conditions apply as the 100mm width model.

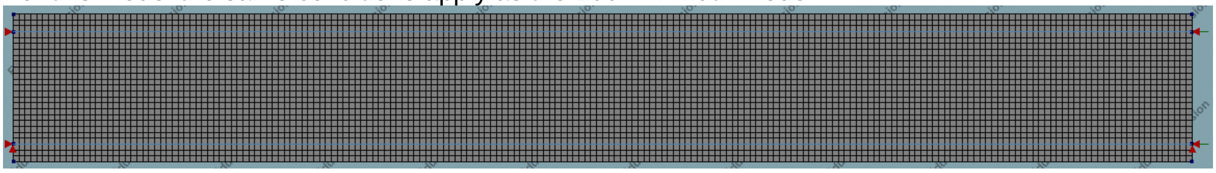


Figure 42 Mesh of the 2D model with a width of 200mm

For this model the same conditions apply as the 100mm width model.

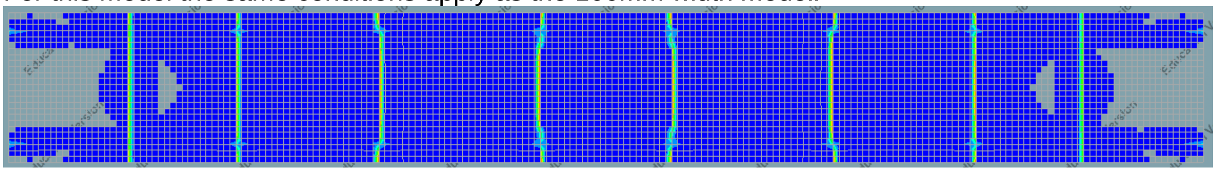


Figure 43 Crack pattern of the 2D model with a width of 200mm

In Figure 43 the crack pattern of the 2D model with a width of 200mm is given. This model also has a symmetry line in the middle of the model. This holds for every model and is due to the loading and supports of the models. In total there are 8 cracks within the model with a varying crack spacing.

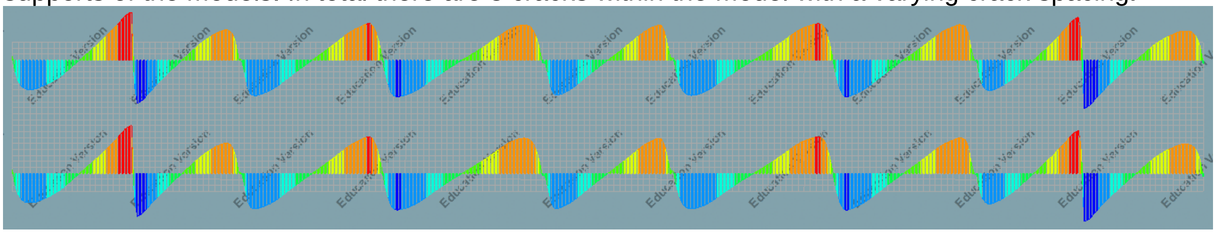


Figure 44 Bond stress gradient of the 2D model with a width of 200mm

In Figure 44 the bond stress gradient is given for the 2D with a width of 200mm. In this figure the positions of the cracks are visible and the lengths over which the bond stress varies. Even so the symmetry of the model is visible when looking at the gradient of the bond stress.

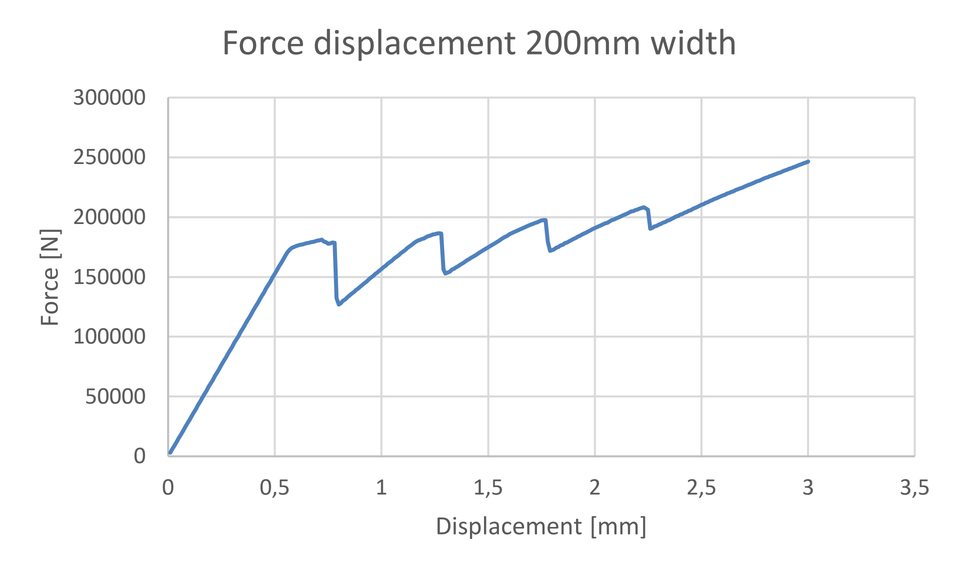


Figure 45 Force displacement diagram of the 2D model with a width of 200mm

In Figure 45 the force displacement diagram of the 2D model with a width of 200mm is given. In this diagram the force needed for a next crack to appear is clearly visible by the sudden drops in the force displacement curve. In the graph a fully developed crack pattern is given the maximum amount of cracks per half of the model is 4.

In the table below, the parameters for the determination of the effective concrete tension zone are given. These are the values for only half of the model but due to symmetry in the middle of the model all these values can be multiplied by two to get the values for the whole model. With these values the average cracking force can be calculated.

Table 18 Summary of the parameters from the 2D model with a width of 200mm

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
1.85	630	125.7	146281	2.9	50442
1.52	730	125.7	139033	2.9	47942
2.09	500	125.7	131064	2.9	45194
2.01	510	125.7	129066	2.9	44506
2.59	380	125.7	123640	2.9	42635
2.57	380	125.7	122954	2.9	42398

$$\sum cracking\ force = 1584077N$$

$n = 12$

$$Average\ cracking\ force = N_{cr,ave} = \frac{\sum cracking\ force}{n} = \frac{1584077}{12} = 132006N$$

With the average cracking force of 132006N, the effective concrete tensile zone becomes:

$A_{c,eff} = {N_{cr,ave}}/{f_{ctm}} = {132006}/{2.9} = 45519mm^2$. This implies a effective concrete tension zone height of:

$$h_{c,eff} = \frac{A_{c,eff}}{b} = \frac{45519}{200} = 228mm$$

The effective concrete tension zone height for this case is, according to Eurocode 2, the minimum of:

${h}/{2}$ and $2{,}5 * (c + {\emptyset_{sl}}/{2})$.

$$\frac{h}{2} = \frac{500}{2} = 250mm, \qquad 2{,}5 * \left(40 + \frac{40}{2}\right) = 150mm$$

In this configuration the Eurocode 2 gives a smaller effective concrete tension zone height than the output of the finite element models. The difference between both methods is 84mm. This results in a difference in crack width of 0.06mm. The difference between both methods is large. The procedure described by Eurocode 2 underestimates the effective concrete tension zone according to this calculation.

The fourth elaborated example is the model in which the width is 250mm. The dimensions of the model are given in the table below.

Table 19 Dimensions of the 2D model with 250mm width

Variables:	Dimension [mm]
Length	4000
Width	250
Height	500
Diameter reinforcement	40
Cover	60
Mesh size	20

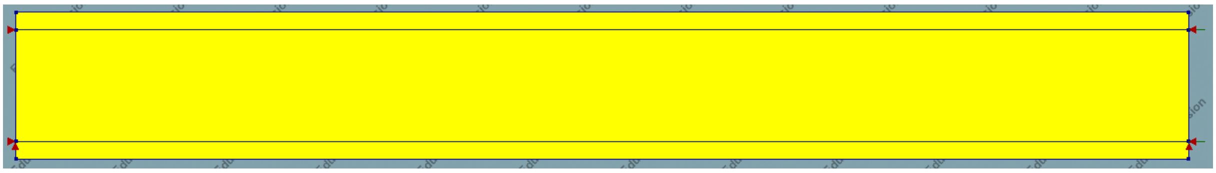


Figure 46 2D model with a width of 250mm

For this model the same conditions apply as the 100mm width model.

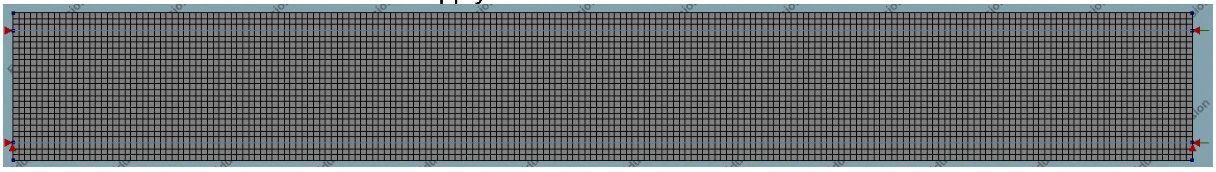


Figure 47 Mesh of the 2D model with a width of 250mm

For this model the same conditions apply as the 100mm width model.

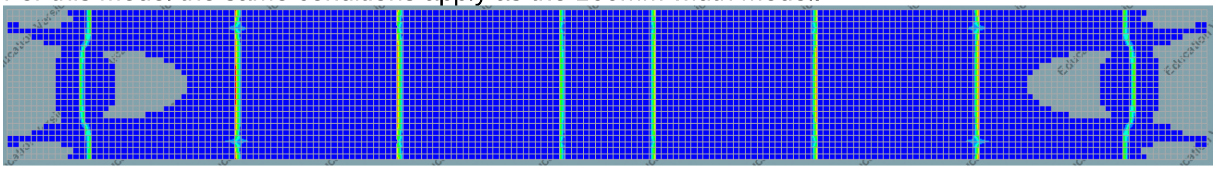


Figure 48 Crack pattern of the 2D model with a width of 250mm

In Figure 48 the crack pattern of the 2D model with a width of 250mm is given. This model also has a symmetry line in the middle of the model. This holds for every model and is due to the loading and supports of the models. In total there are 8 cracks within the model with a varying crack spacing. The spacing in between the two middle crack is rather small. In reality these cracks cannot lie this close to each other. The error here is due to the loading of the model, because the force in introduced from the edges.

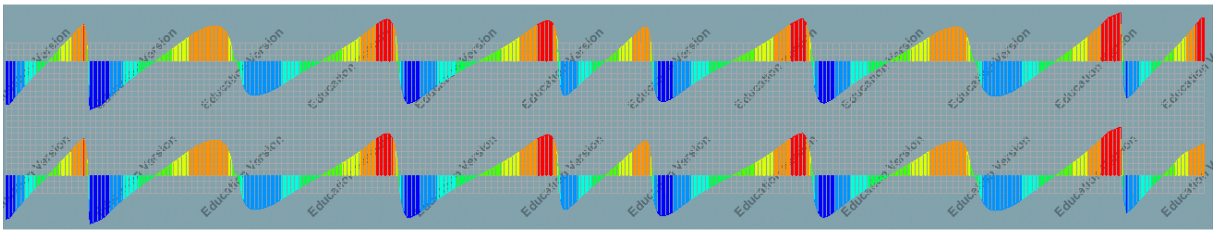


Figure 49 Bond stress gradient of the 2D model with a width of 250mm

In Figure 49 the bond stress gradient is given for the 2D with a width of 250mm. In this figure the positions of the cracks are visible and the lengths over which the bond stress varies. Even so the symmetry of the model is visible when looking at the gradient of the bond stress. Due to the small crack spacing in the middle of the model the bond stress in this region is smaller.

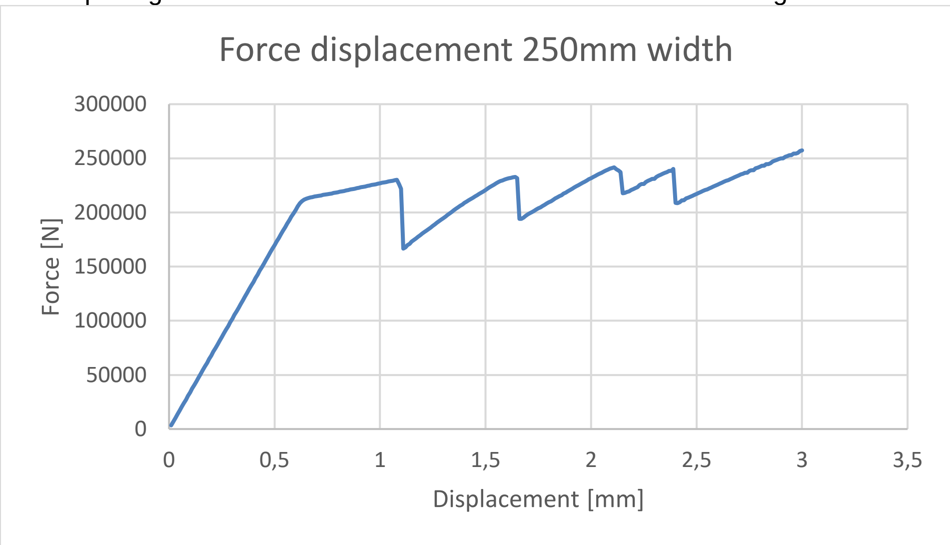


Figure 50 Force displacement diagram of the 2D model with a width of 250mm

In Figure 50 the force displacement diagram of the 2D model with a width of 250mm is given. In this diagram the force needed for a next crack to appear is clearly visible by the sudden drops in the force displacement curve. In the graph a fully developed crack pattern is given the maximum amount of cracks per half of the model is 4.

In the table below, the average bond stress, transfer length, perimeter of the reinforcement, cracking force, concrete tensile strength and the effective concrete tension zone are given. These are the values for only half of the model but due to symmetry in the middle of the model all these values can be multiplied by two to get the values for the whole model. With these values the average cracking force can be calculated.

Table 20 Summary of the parameters from the 2D model with a width of 250mm

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
2.76	530	125.7	183718	2.9	63351
1.59	840	125.7	167481	2.9	57752
3.21	380	125.7	153193	2.9	52825
3.39	370	125.7	157478	2.9	54303
1.15	1070	125.7	155295	2.9	53550
2.25	550	125.7	155276	2.9	53543

$$\sum cracking\ force = 1944883N$$
$$n = 12$$
$$Average\ cracking\ force = N_{cr,ave} = \frac{\sum cracking\ force}{n} = \frac{1944883}{12} = 162074N$$
With the average cracking force of 162074N, the effective concrete tensile zone becomes: $A_{c,eff} = {N_{cr,ave}}/{f_{ctm}} = {162074}/{2.9} = 55888mm^2$. This implies a effective concrete tension zone height of:
$$h_{c,eff} = \frac{A_{c,eff}}{b} = \frac{55888}{250} = 224mm$$
The effective concrete tension zone height for this case is, according to Eurocode 2, the minimum of: ${h}/{2}$ and $2{,}5 * (c + {\emptyset_{sl}}/{2})$.
$$\frac{h}{2} = \frac{500}{2} = 250mm, \quad 2{,}5 * \left(40 + \frac{40}{2}\right) = 150mm$$
In this configuration Eurocode 2 gives a smaller effective concrete tension zone height than the finite element models. The difference between both methods is 74mm. This results in a difference in crack width of 0.07mm. So the procedure described by Eurocode 2 underestimates the effective concrete tension zone according to this calculation.

B.3 Variation in diameter of the reinforcement

The first elaborated example is the model in which the diameter of the reinforcement is 20mm. The dimensions of the model are given in the table below.

Table 21 Dimensions of the 2D model with 20mm reinforcement

Variables:	Dimension [mm]
Length	4000
Width	200
Height	500
Diameter reinforcement	20
Cover	60
Mesh size	20

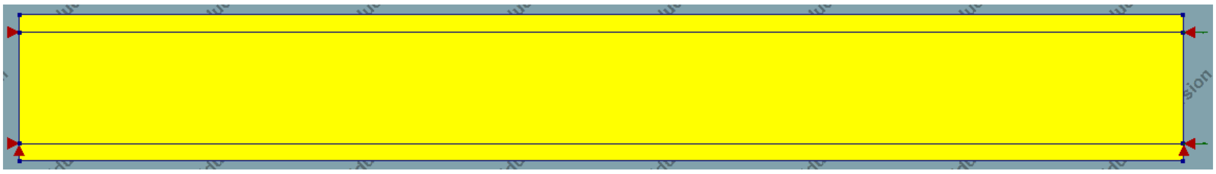


Figure 51 2D model with a 20mm diameter reinforcement

In Figure 51 the 2D model with a 20mm diameter reinforcement is given. The horizontal lines represent the reinforcement. The prescribed deformation is given by the arrows on the right side of the model. The supports are given by the red triangles the horizontal x-direction is supported on both ends of both reinforcements. The vertical y-direction is supported by the bottom reinforcement. In order to prevent transverse contraction of the model the upper reinforcement is not supported in the y-direction. If the transverse contraction of the model is restrained by the vertical supports tension stresses in y-direction arise in the model which can result in cracks parallel to the reinforcement. This has to be prevented because parallel cracks have a negative influence on the bond stress gradient.

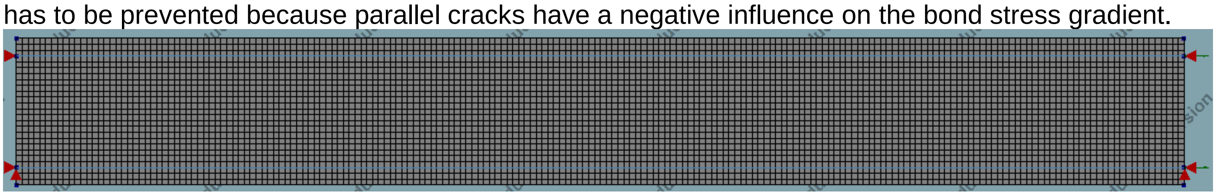


Figure 52 Mesh of the 2D model with a 20mm diameter reinforcement

In Figure 52 the mesh of the 2D model with a 20mm diameter reinforcement is given. The mesh has a size of 20mm per element. It is a fine mesh to get accurate results for the crack spacing and bond stresses. The mesh is evenly distributed over the model without errors in the mesh.

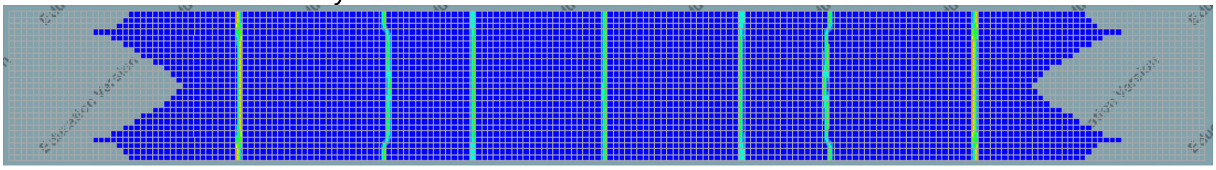


Figure 53 Crack pattern of the 2D model with a 20mm diameter reinforcement

In Figure 53 the crack pattern of the 2D model with a 20mm diameter reinforcement is given. The fully developed crack pattern consists of 7 cracks.

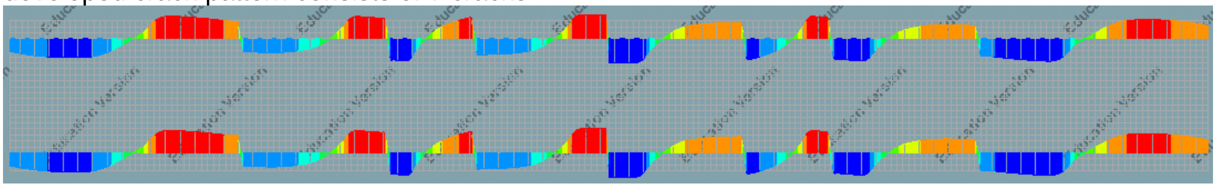


Figure 54 Bond stress gradient of the 2D model with a 20mm diameter reinforcement

In Figure 54 the bond stress gradient is given for the 2D model with a 20mm diameter reinforcement. In this figure the positions of the cracks are visible and the lengths over which the bond stress varies. Even so the symmetry of the model is visible when looking at the gradient of the bond stress.

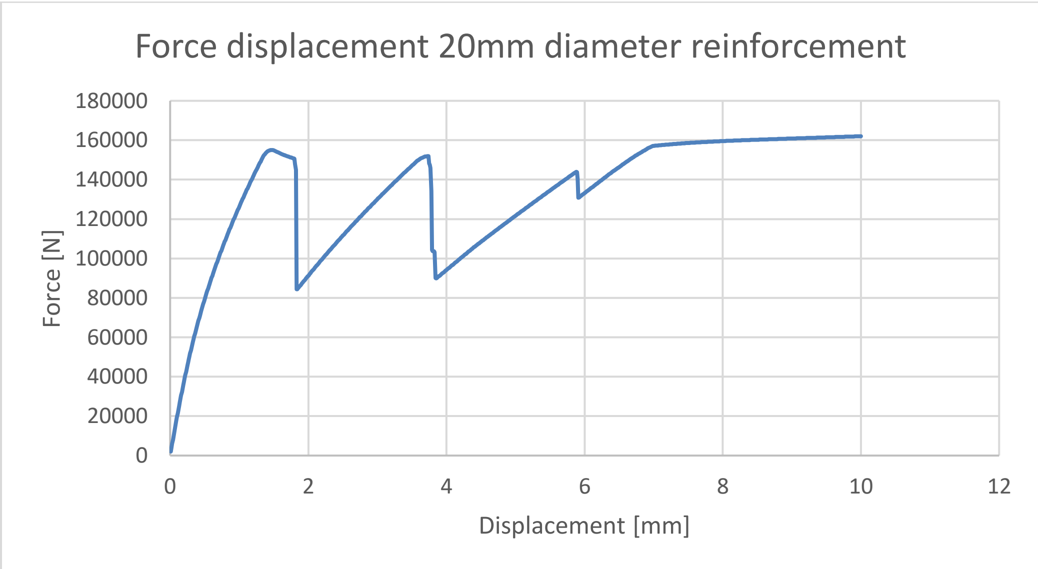


Figure 55 Force displacement diagram of the 2D model with a 20mm diameter reinforcement

In Figure 55 the force displacement diagram of the 2D model with a 20mm diameter reinforcement is given. In this diagram the force needed for a next crack to appear is clearly visible by the sudden drops in the force displacement curve. In the graph a fully developed crack pattern is given the maximum amount of cracks is 7 for the full model. In here 3 distinct cracks can be observed. The outer left and right cracks are given by the first drop. The second and third left and right crack are given by the second drop and the middle crack is given by the third drop.

In the table below, the parameters for the determination of the effective concrete tension zone are given. These are the values for only half of the model but due to symmetry in the middle of the model all these values can be multiplied by two to get the values for the whole model. With these values the average cracking force can be calculated.

Table 22 Summary of the parameters from the 2D model with a 20mm diameter reinforcement

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
3.65	610	62.8	140080	2.9	48303
3.20	660	62.8	132791	2.9	45790
4.14	240	62.8	62428	2.9	21527
3.90	450	62.8	110179	2.9	37993

$$\sum cracking\ force = 890955N$$
$$n = 8$$
$$Average\ cracking\ force = N_{cr,ave} = \frac{\sum cracking\ force}{n} = \frac{890955}{8} = 111369N$$
With the average cracking force of 111369N, the effective concrete tensile zone becomes:
$A_{c,eff} = {N_{cr,ave}}/{f_{ctm}} = {111369}/{2.9} = 38403mm^2$. This implies a effective concrete tension zone height of:
$$h_{c,eff} = \frac{A_{c,eff}}{b} = \frac{38403}{200} = 192mm$$
The effective concrete tension zone height for this case is, according to Eurocode 2, the minimum of: ${h}/{2}$ and $2{,}5 * (c + {\emptyset_{sl}}/{2})$.
$$\frac{h}{2} = \frac{500}{2} = 250mm, \qquad 2{,}5 * \left(40 + \frac{20}{2}\right) = 125mm$$
In this configuration Eurocode 2 gives a smaller effective concrete tension zone height than the finite element models. The difference between both methods is 67mm. This results in a difference in crack width of 0.02mm. So the procedure described by the Eurocode underestimates the effective concrete tension zone according to this calculation.

The second elaborated example is the model in which the diameter of the reinforcement is 25mm. The dimensions of the model are given in the table below.

Table 23 Dimensions of the 2D model with 25mm reinforcement

Variables:	Dimension [mm]
Length	4000
Width	200
Height	500
Diameter reinforcement	25
Cover	60
Mesh size	20

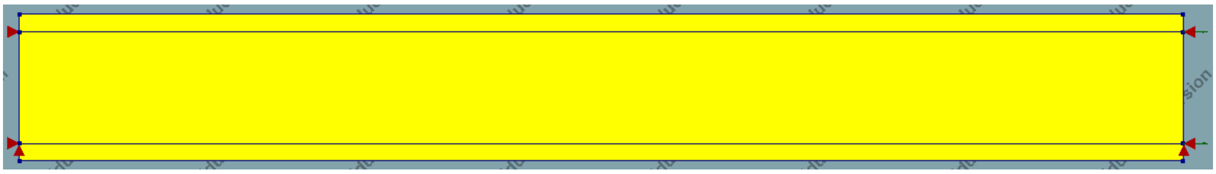


Figure 56 2D model with a 25mm diameter reinforcement

In Figure 56 the 2D model with a 25mm diameter reinforcement is given. The same conditions apply as for the model with a 20mm diameter

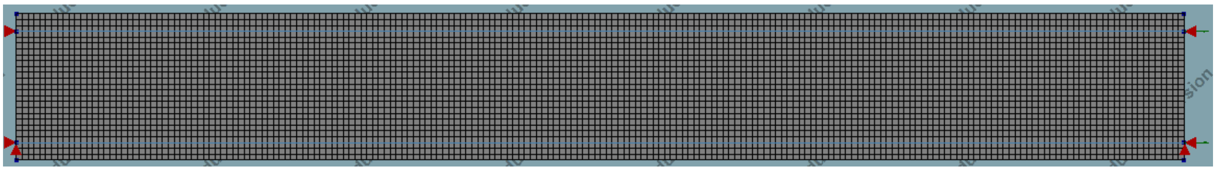


Figure 57 Mesh of the 2D model with a 25mm diameter reinforcement

In Figure 57 the mesh of the 2D model with a 25mm diameter reinforcement is given. The same conditions apply as for the 20mm diameter model.

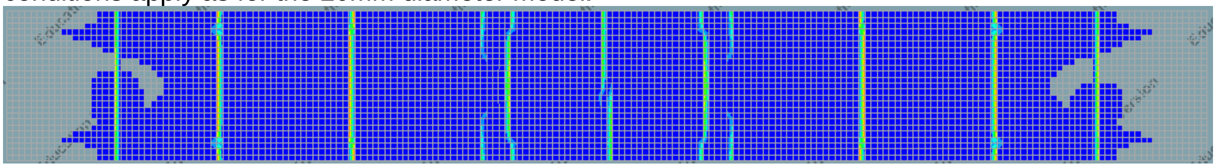


Figure 58 Crack pattern of the 2D model with a 25mm diameter reinforcement

In Figure 58 the crack pattern of the 2D model with a 25mm diameter reinforcement is given. The fully developed crack pattern consists of 9 cracks. Close to the middle crack secondary cracks can be observed.

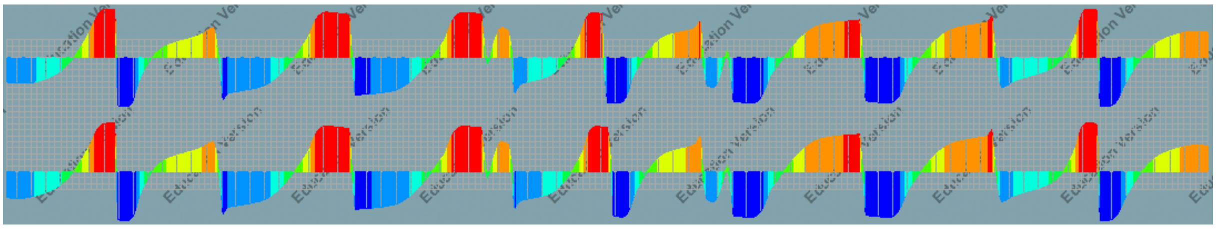


Figure 59 Bond stress gradient of the 2D model with a 25mm diameter reinforcement

In Figure 59 the bond stress gradient is given for the 2D model with a 25mm diameter reinforcement. In this figure the positions of the cracks are visible and the lengths over which the bond stress varies. Even so the symmetry of the model is visible when looking at the gradient of the bond stress. Due to the secondary cracks the bond stress varies in between the primary and secondary crack over a smaller distance.

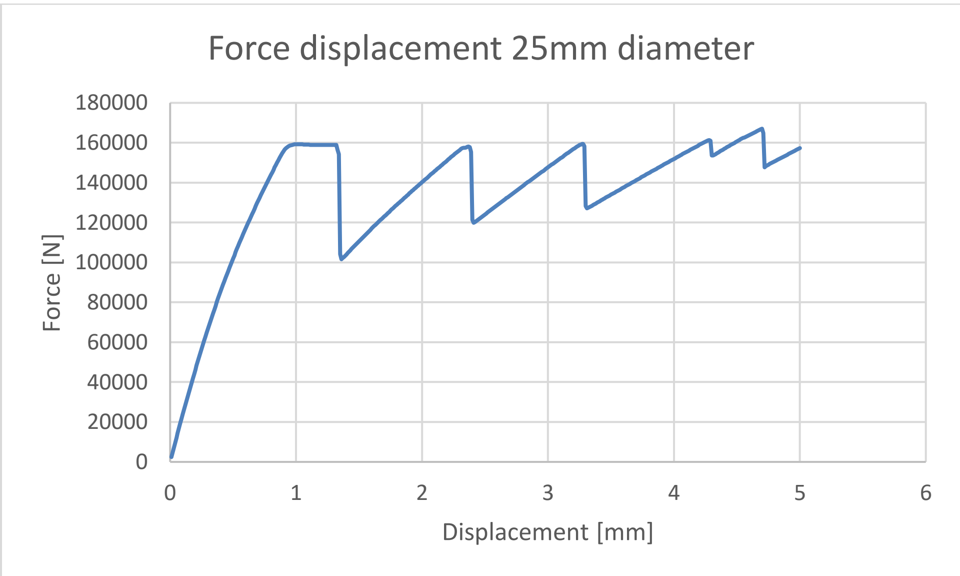


Figure 60 Force displacement diagram of the 2D model with a 25mm diameter reinforcement

In Figure 60 the force displacement diagram of the 2D model with a 25mm diameter reinforcement is given. In this diagram the force needed for a next crack to appear is clearly visible by the sudden drops in the force displacement curve. In the graph a fully developed crack pattern is given the maximum amount of cracks is 9 for the full model. In here 5 distinct cracks can be observed.

In the table below, the parameters for the determination of the effective concrete tension zone are given. These are the values for only half of the model but due to symmetry in the middle of the model all these values can be multiplied by two to get the values for the whole model. With these values the average cracking force can be calculated.

Table 24 Summary of the parameters from the 2D model with a 25mm diameter reinforcement

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
3.36	540	78.5	142595	2.9	49171
3.81	430	78.5	128600	2.9	44345
3.08	530	78.5	128104	2.9	44174
4.15	300	78.5	97841	2.9	33738
4.11	310	78.5	99970	2.9	34472
4.38	330	78.5	113424	2.9	39112
4.08	360	78.5	115431	2.9	39804

$$\sum cracking\ force = 1651930N$$

$$n = 14$$

$$Average\ cracking\ force = N_{cr,ave} = \frac{\sum cracking\ force}{n} = \frac{1651930}{14} = 117995N$$

With the average cracking force of 117995N the effective concrete tensile zone becomes:

$A_{c,eff} = {}^{N_{cr,ave}}/_{f_{ctm}} = {}^{117995}/_{2.9} = 40688mm^2$. This implies an effective concrete tension zone height of:

$$h_{c,eff} = \frac{A_{c,eff}}{b} = \frac{40688}{200} = 203mm$$

The effective concrete tension zone height for this case is, according to Eurocode 2, the minimum of ${}^{h}/_{2}$ and $2{,}5 * (c + {}^{\emptyset_{sl}}/_{2})$.

$$\frac{h}{2} = \frac{500}{2} = 250mm, \qquad 2{,}5 * \left(40 + \frac{25}{2}\right) = 131.25mm$$

In this configuration Eurocode 2 gives a smaller effective concrete tension zone height than the finite element models. The difference between both methods is 72mm. This results in a difference in crack width of 0.04mm. So the procedure described by Eurocode 2 underestimates the effective concrete tension zone according to this calculation.

The third elaborated example is the model in which the diameter of the reinforcement is 32mm. The dimensions of the model are given in the table below.

Table 25 Dimensions of the 2D model with 32mm reinforcement

Variables:	Dimension [mm]
Length	4000
Width	200
Height	500
Diameter reinforcement	32
Cover	60
Mesh size	20

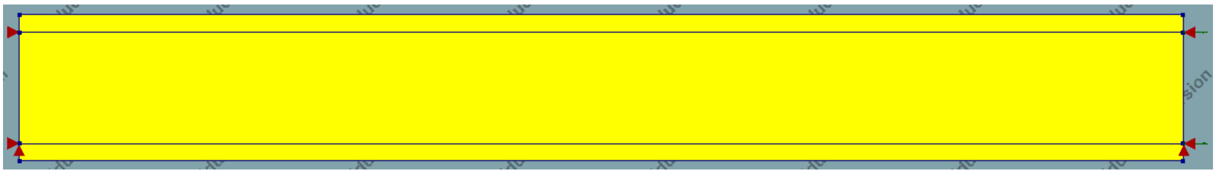


Figure 61 2D model with a 32mm diameter reinforcement

In Figure 61 the 2D model with a 32mm diameter reinforcement is given. The same conditions apply as for the model with a 20mm diameter

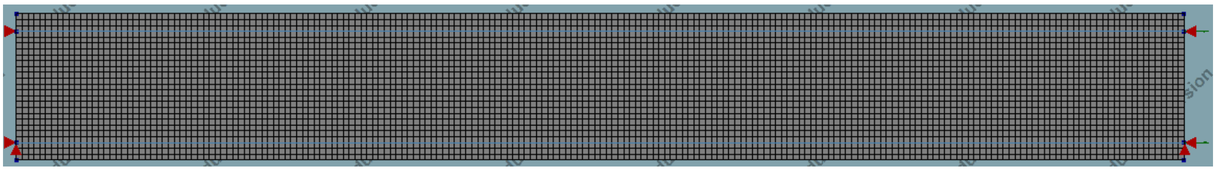


Figure 62 Mesh of the 2D model with a 32mm diameter reinforcement

In Figure 62 the mesh of the 2D model with a 32mm diameter reinforcement is given. The same conditions apply as for the 20mm diameter model.

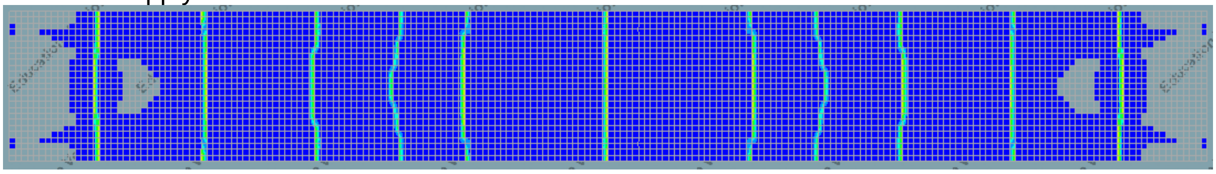


Figure 63 Crack pattern of the 2D model with a 32mm diameter reinforcement

In Figure 63 the crack pattern of the 2D model with a 32mm diameter reinforcement is given. The fully developed crack pattern consists of 11 cracks.

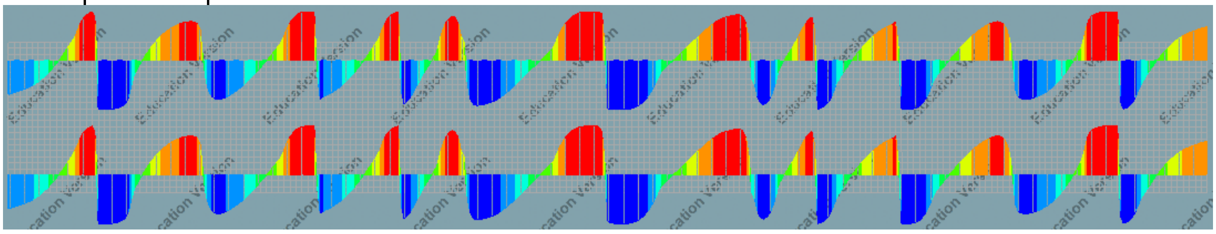


Figure 64 Bond stress gradient of the 2D model with a 32mm diameter reinforcement

In Figure 64 the bond stress gradient is given for the 2D model with a 32mm diameter reinforcement. In this figure the positions of the cracks are visible and the lengths over which the bond stress varies. Even so the symmetry of the model is visible when looking at the gradient of the bond stress.

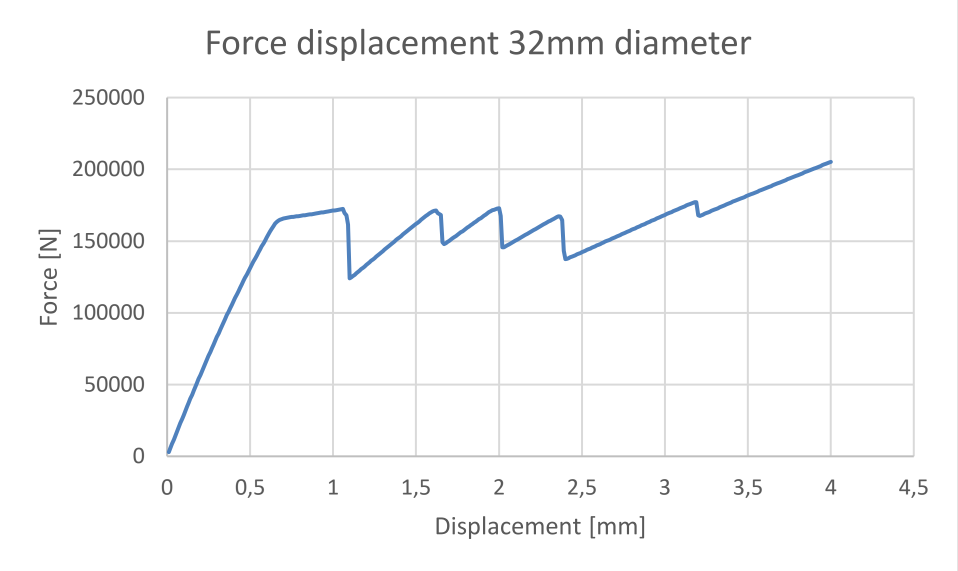


Figure 65 Force displacement diagram of the 2D model with a 32mm diameter reinforcement

In Figure 65 the force displacement diagram of the 2D model with a 32mm diameter reinforcement is given. In this diagram the force needed for a next crack to appear is clearly visible by the sudden drops in the force displacement curve. In the graph a fully developed crack pattern is given the maximum amount of cracks is 11 for the full model. In here 5 distinct cracks can be observed.

In the table below, the parameters for the determination of the effective concrete tension zone are given. These are the values for only half of the model but due to symmetry in the middle of the model all these values can be multiplied by two to get the values for the whole model. With these values the average cracking force can be calculated.

Table 26 Summary of the parameters from the 2D model with a 32mm diameter reinforcement

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
2.65	550	100.5	146747	2.9	50602
3.75	290	100.5	109452	2.9	37742
3.31	340	100.5	113086	2.9	38995
3.44	360	100.5	124544	2.9	42946
2.71	440	100.5	119992	2.9	41376
2.71	460	100.5	125517	2.9	43282
3.63	220	100.5	80282	2.9	27684
3.47	230	100.5	80134	2.9	27632

$$\sum cracking\ force = 1799508N$$
$$n = 16$$
$$Average\ cracking\ force = N_{cr,ave} = \frac{\sum cracking\ force}{n} = \frac{1799508}{16} = 112469N$$

With the average cracking force of 112469N, the effective concrete tensile zone becomes:
$A_{c,eff} = {N_{cr,ave}}/{f_{ctm}} = {112469}/{2.9} = 38782mm^2$. This implies an effective concrete tension zone height of:

$h_{c,eff} = \frac{A_{c,eff}}{b} = \frac{38782}{200} = 194mm$

The effective concrete tension zone height for this case is, according to Eurocode 2, the minimum of $h/_2$ and $2{,}5 * (c + {}^{\emptyset_{sl}}/_2)$.

$\frac{h}{2} = \frac{500}{2} = 250mm, \quad 2{,}5 * \left(40 + \frac{32}{2}\right) = 140mm$

In this configuration Eurocode 2 gives lower results than the finite element models. The difference between both methods is 54mm. This results in a difference in crack width of 0.02mm. So the procedure described by Eurocode 2 underestimates the effective concrete tension zone according to this calculation.

The forth elaborated example is the model in which the diameter of the reinforcement is 40mm. The dimensions of the model are given in the table below.

Table 27 Dimensions of the 2D model with 40mm reinforcement

Variables:	Dimension [mm]
Length	4000
Width	200
Height	500
Diameter reinforcement	40
Cover	60
Mesh size	20

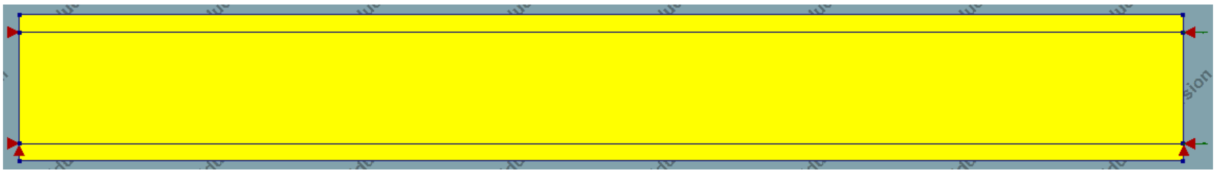


Figure 66 2D model with a 40mm diameter reinforcement

In Figure 66 the 2D model with a 40mm diameter reinforcement is given. The same conditions apply as for the model with a 20mm diameter

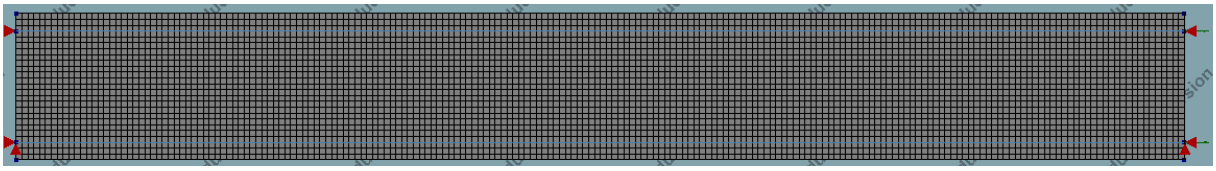


Figure 67 Mesh of the 2D model with a 40mm diameter reinforcement

In Figure 67 the mesh of the 2D model with a 40mm diameter reinforcement is given. The same conditions apply as for the 20mm diameter model.

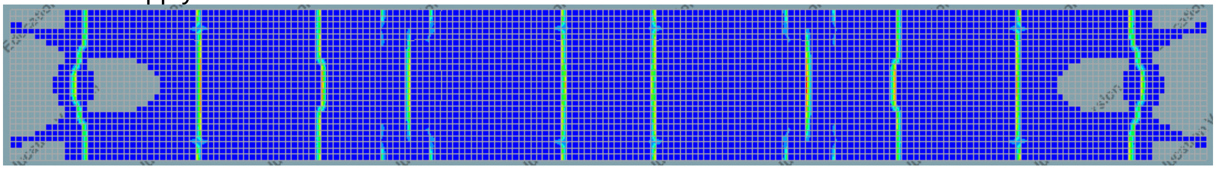


Figure 68 Crack pattern of the 2D model with a 40mm diameter reinforcement

In Figure 68 the crack pattern of the 2D model with a 40mm diameter reinforcement is given. The fully developed crack pattern consists of 10 cracks. Some secondary cracks are also observed in the figure.

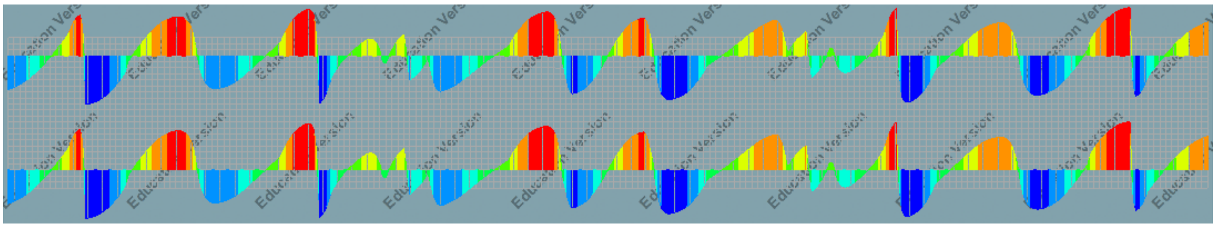


Figure 69 Bond stress gradient of the 2D model with a 40mm diameter reinforcement

In Figure 69 the bond stress gradient is given for the 2D model with a 40mm diameter reinforcement. In this figure the positions of the cracks are visible and the lengths over which the bond stress varies. Even so the symmetry of the model is visible when looking at the gradient of the bond stress. Due to the secondary cracks some parts of the bond stress gradient do not run smoothly.

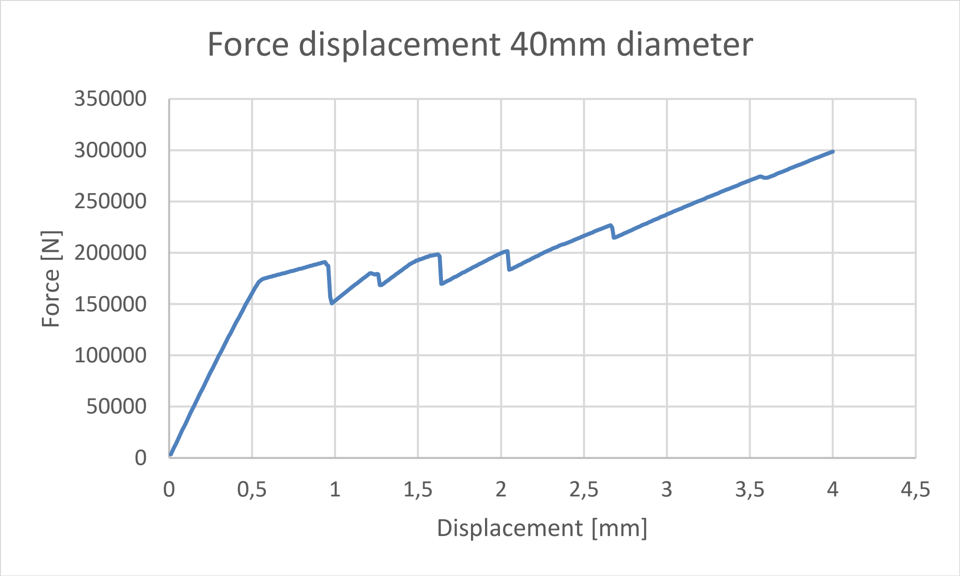


Figure 70 Force displacement diagram of the 2D model with a 40mm diameter reinforcement

In Figure 70 the force displacement diagram of the 2D model with a 40mm diameter reinforcement is given. In this diagram the force needed for a next crack to appear is clearly visible by the sudden drops in the force displacement curve. In the graph a fully developed crack pattern is given the maximum amount of cracks is 10 for the full model. In here 5 distinct cracks can be observed.

In the table below, the parameters of the determination of the effective concrete tension zone are given. These are the values for only half of the model but due to symmetry in the middle of the model all these values can be multiplied by two to get the values for the whole model. With these values the average cracking force can be calculated.

Table 28 Summary of the parameters from the 2D model with a 40mm diameter reinforcement

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
2.39	490	125.7	147119	2.9	50731
3.06	270	125.7	103803	2.9	35794
2.55	340	125.7	109103	2.9	37622
1.98	530	125.7	132109	2.9	45555
1.89	450	125.7	106675	2.9	36784
2.53	290	125.7	92275	2.9	31819
2.83	320	125.7	113671	2.9	39197
3.31	210	125.7	87255	2.9	30088
2.64	210	125.7	69780	2.9	24062

$$\sum cracking\ force = 1923580N$$
$$n = 18$$
$$Average\ cracking\ force = N_{cr,ave} = \frac{\sum cracking\ force}{n} = \frac{1923580}{18} = 106866N$$

With a cracking force of 106866N the effective concrete tensile zone becomes:

$A_{c,eff} = {N_{cr,ave}}/{f_{ctm}} = {106866}/{2.9} = 36850mm^2$. This implies an effective concrete tension zone height of:

$$h_{c,eff} = \frac{A_{c,eff}}{b} = \frac{36850}{200} = 184mm$$

The effective concrete tension zone height for this case is, according to Eurocode 2, the minimum of ${h}/{2}$ and $2{,}5 * (c + {\emptyset_{sl}}/{2})$.

$$\frac{h}{2} = \frac{500}{2} = 250mm, \qquad 2{,}5 * \left(40 + \frac{40}{2}\right) = 150mm$$

In this configuration Eurocode 2 gives lower results than the finite elements models. The difference between both methods is 34mm. This results in a difference in crack width of 0.03mm. So the procedure described by Eurocode 2 underestimates the effective concrete tension zone according to this calculation.

B.4 Variation in height of model

The first elaborated example is the model in which the height of the model is 500mm. The dimensions of the model are given in the table below.

Table 29 Dimensions of the 2D model with a height of 500mm

Variables:	Dimension [mm]
Length	4000
Width	200
Height	500
Diameter reinforcement	40
Cover	60
Mesh size	20
Vertical bar spacing	100

Figure 71 2D model with a height of 500mm and two layers of reinforcement

In Figure 71 the 2D model with a height of 500mm and two layers of reinforcement is given. The horizontal lines represent the reinforcement. The prescribed deformation is given by the arrows on the right side of the model. The supports are given by the red triangles the horizontal x-direction is supported on both ends of both reinforcements. The vertical y-direction is supported by the bottom reinforcement. In order to prevent transverse contraction of the model the upper reinforcement is not supported in the y-direction. If the transverse contraction of the model is restrained by the vertical supports tension stresses in y-direction arise in the model which can result in cracks parallel to the reinforcement. This has to be prevented because parallel cracks have a negative influence on the bond stress gradient.

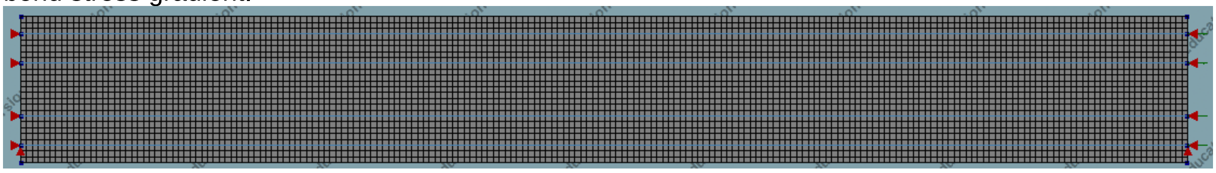


Figure 72 Mesh of the 2D model with a height of 500mm and two layers of reinforcement

In Figure 72 the mesh of the 2D model with a height of 500mm and two layers of reinforcement is given. The mesh has a size of 20mm per element. It is a fine mesh to get accurate results for the crack spacing and bond stresses. The mesh is evenly distributed over the model without errors in the mesh.

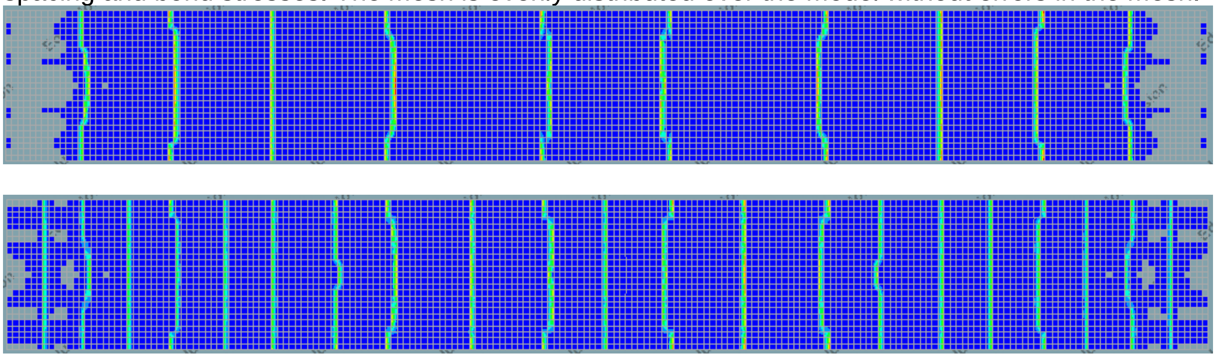


Figure 73 Crack pattern of the 2D model with a height of 500mm and two layers of reinforcement

In Figure 73 the crack pattern of the 2D model with a height of 500mm and two layers of reinforcement is given. In the first stage at load step 200 the model is cracked at the maximum crack spacing. When the load keeps increasing the model will crack at positions in between the already consisting cracks to get a fully developed crack pattern(load step 550). The fully developed crack pattern consists of 21 cracks.

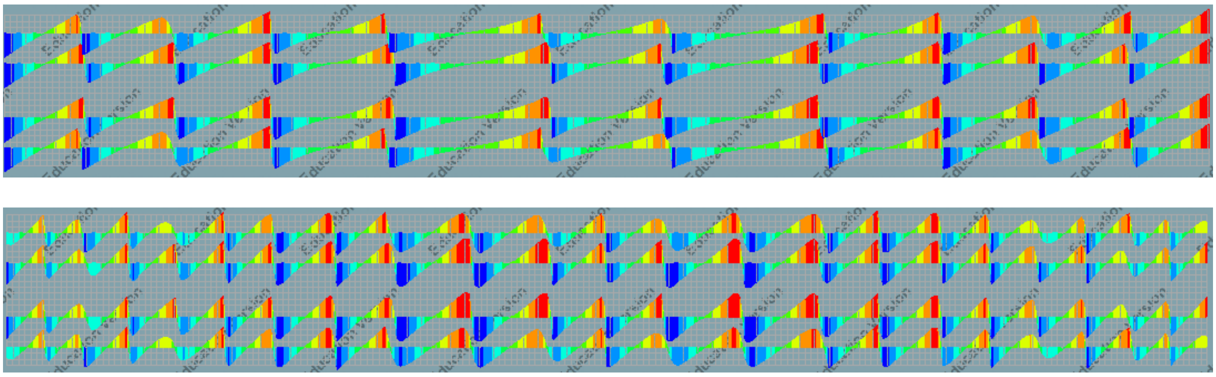


Figure 74 Bond stress gradient of the 2D model with a height of 500mm and two layers of reinforcement

In Figure 74 the bond stress gradient is given for the 2D model with a height of 500mm and two layers of reinforcement. In this figure the positions of the cracks are visible and the lengths over which the bond stress varies. Even so the symmetry of the model is visible when looking at the gradient of the bond stress. Both load phases are shown to see the difference in bond stress for both load steps.

Figure 75 Force displacement diagram of the 2D model with a height of 500mm and two layers of reinforcement

In Figure 75 the force displacement diagram of the 2D model with a height of 500mm and two layers of reinforcement is given. In this diagram the force needed for a next crack to appear is clearly visible by the sudden drops in the force displacement curve. In the graph a fully developed crack pattern is given the maximum amount of cracks is 21 for the full model. In here 6 distinct cracks can be observed. The green line represents the plain reinforcement bar. The difference in between the plain reinforcement bar and the inner and outer reinforcement is the tension stiffening effect. This effect represents the force which is taken up by the concrete.

In the tables below, the parameters of the determination the effective concrete tension zone are given. These are the values for only half of the model but due to symmetry in the middle of the model all these values can be multiplied by two to get the values for the whole model. With these values the average cracking force can be calculated. The first and second table represent the values for the inner and outer reinforcement.

Table 30 Summary of the parameters of the inner reinforcement from the 2D model with a height of 500mm

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
1.84	330	125.7	76365	2.9	26333
1.45	350	125.7	63688	2.9	21961
1.13	420	125.7	59745	2.9	20602
1.65	310	125.7	64416	2.9	22213
1.24	400	125.7	62239	2.9	21462
2.30	270	125.7	78107	2.9	26933
1.95	300	125.7	73381	2.9	25304
1.91	250	125.7	59890	2.9	20652
1.84	260	125.7	60172	2.9	20749
2.37	180	125.7	53680	2.9	18511
2.00	210	125.7	52899	2.9	18241
2.48	180	125.7	56095	2.9	19343
2.35	190	125.7	56095	2.9	19343

Table 31 Summary of the parameters of the outer reinforcement from the 2D model with a height of 500mm

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
1.80	330	125.7	74709	2.9	25762
1.33	350	125.7	58428	2.9	20148
1.08	420	125.7	57106	2.9	19692
1.53	310	125.7	59665	2.9	20574
1.18	400	125.7	59121	2.9	20386
2.31	260	125.7	75451	2.9	26017
1.75	300	125.7	65942	2.9	22738
1.85	250	125.7	58050	2.9	20017
1.67	260	125.7	54523	2.9	18801
2.24	180	125.7	50767	2.9	17506
1.87	200	125.7	47097	2.9	16240
2.08	200	125.7	52190	2.9	17997
2.06	200	125.7	51769	2.9	17851

$$\sum cracking\ force\ inner = 1633547N$$

$n = 26$

$$Average\ cracking\ force = N_{cr,ave} = \frac{\sum cracking\ force}{n} = \frac{1633547}{26} = 62829N$$

With the average cracking force of 62829N, the effective concrete tensile zone becomes:

$A_{c,eff} = {N_{cr,ave}}/{f_{ctm}} = {62829}/{2.9} = 21665mm^2$. This implies an effective concrete tension zone height of:

$$h_{c,eff\ inner} = \frac{A_{c,eff}}{b} = \frac{21665}{200} = 108mm$$

$$\sum cracking\ force\ outer = 1529634N$$
$$n = 26$$
$$Average\ cracking\ force = N_{cr,ave} = \frac{\sum cracking\ force}{n} = \frac{1529634}{26} = 58832N$$
With a cracking force of 58832N, the effective concrete tensile zone becomes:
$A_{c,eff} = {}^{N_{cr,ave}}/_{f_{ctm}} = {}^{58832}/_{2.9} = 20287mm^2$. This implies an effective concrete tension zone height of:
$$h_{c,eff\ outer} = \frac{A_{c,eff}}{b} = \frac{20287}{200} = 101mm$$
Now both effective concrete tension zone heights will be added to get the effective concrete tension zone height of both reinforcement bars:
$$Effective\ concrete\ tension\ zone\ height = 108 + 101 = 209mm$$
The effective concrete tension zone height for this case is, according to Eurocode 2, the minimum of ${}^{h}/_{2}$ and $2{,}5 * (c + {}^{\emptyset_{sl}}/_{2})$.
$$\frac{h}{2} = \frac{500}{2} = 250mm, \qquad 2{,}5 * \left(\frac{100}{2} + 40 + \sqrt{2} * \frac{40}{2}\right) = 296mm$$
In this configuration Eurocode 2 gives an higher effective concrete tension zone height than the finite element models. The difference between both methods is 40mm. This results in a difference in crack width of 0.02mm. So for this configuration the use of finite element models can be beneficial.

The second elaborated example is the model in which the height of the model is 1000mm. The dimensions of the model are given in the table below.

Table 32 Dimensions of the 2D model with a height of 1000mm

Variables:	Dimension [mm]
Length	4000
Width	200
Height	1000
Diameter reinforcement	40
Cover	60
Mesh size	20
Vertical bar spacing	100

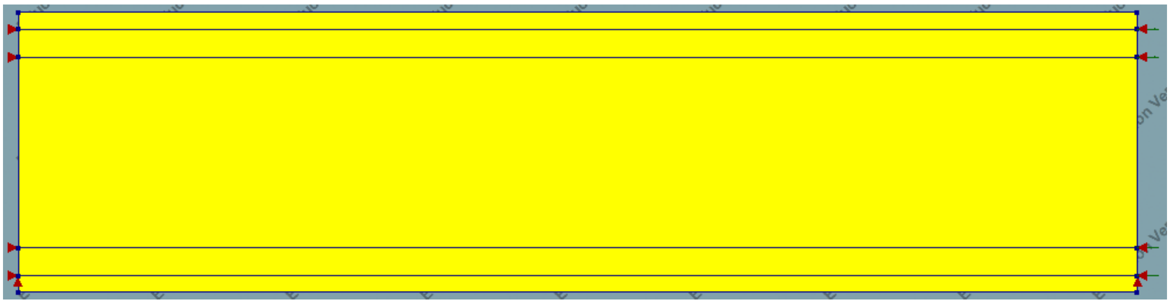


Figure 76 2D model with a height of 1000mm and two layers of reinforcement

In Figure 76 the 2D model with a height of 1000mm and two layers of reinforcement is given. For this model the same conditions apply as the 500mm height model

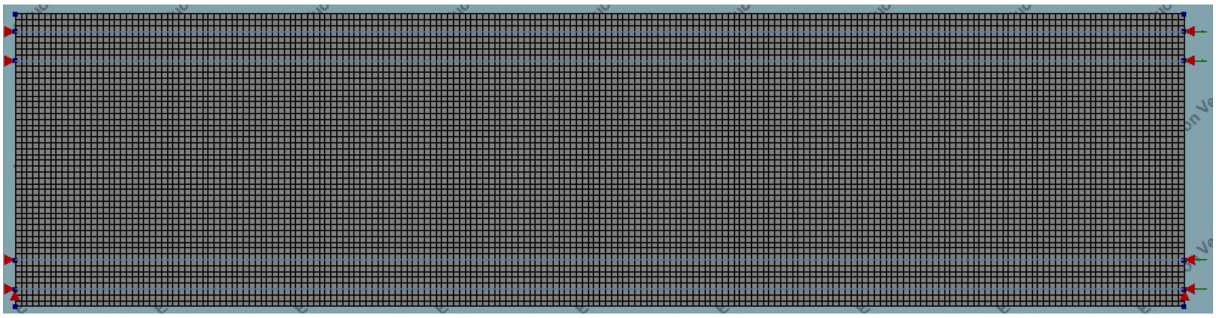


Figure 77 Mesh of the 2D model with a height of 1000mm and two layers of reinforcement

In Figure 77 the mesh of the 2D model with a height of 1000mm and two layers of reinforcement is given. For this model the same conditions apply as for the 500mm height model.

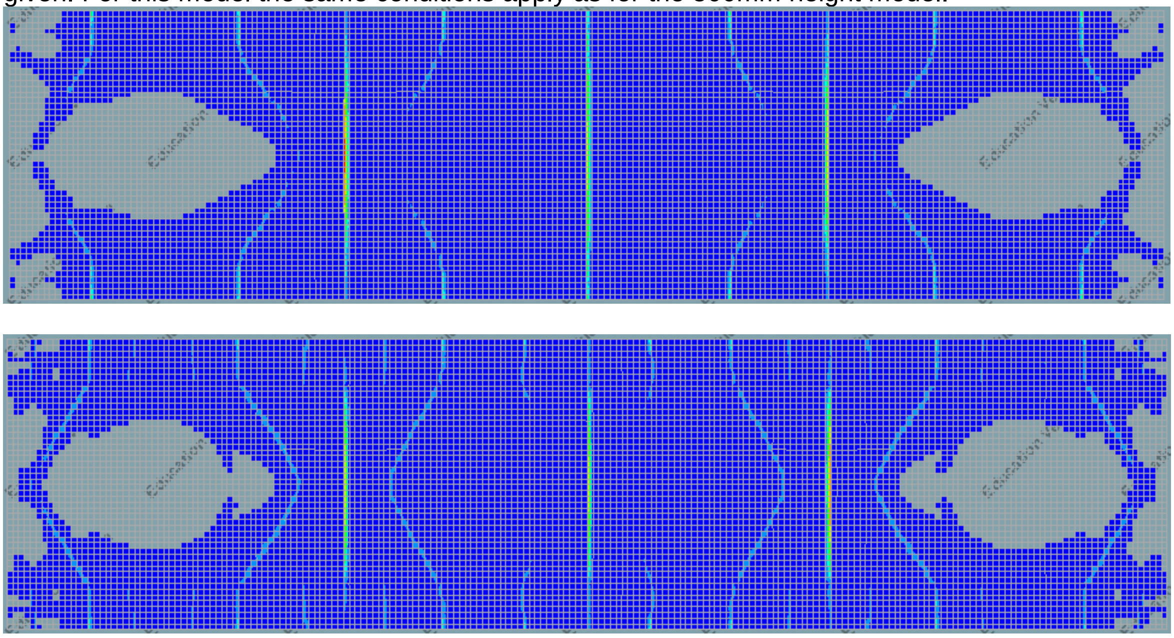
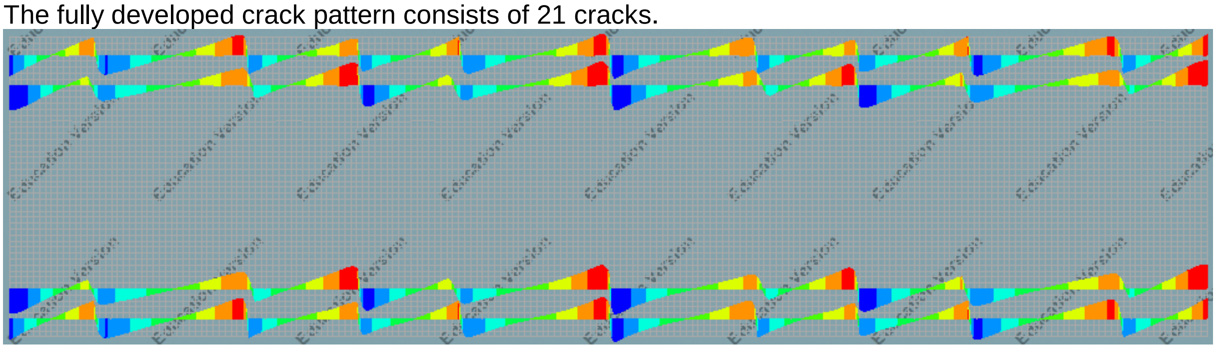


Figure 78 Crack pattern of the 2D model with a height of 1000mm and two layers of reinforcement

In Figure 78the crack pattern of the 2D model with a height of 1000mm and two layers of reinforcement is given. In the first stage at load step 200 the model is cracked at the maximum crack spacing. Three (primary) cracks run perpendicular to the length axis through the model while the other cracks bend towards the primary cracks. When the load keeps increasing the model will crack at positions in between the already consisting cracks to get a fully developed crack pattern(load step 400). The cracks which form are secondary cracks and do not run through the height of the model. The fully developed crack pattern consists of 21 cracks.



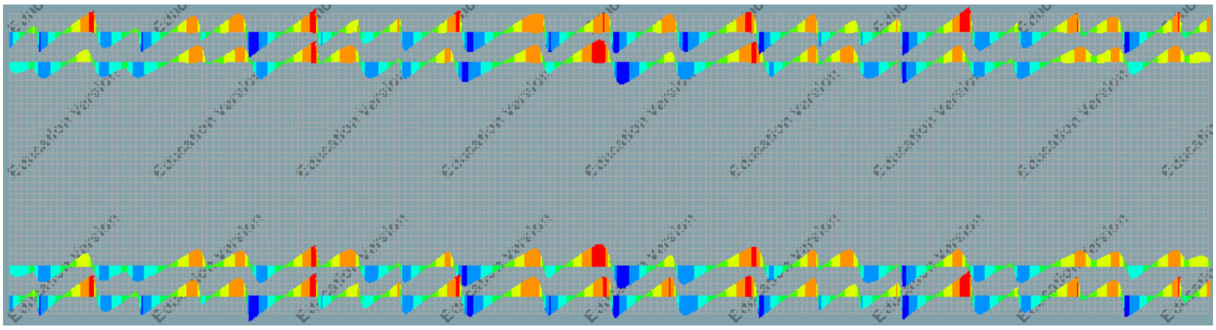


Figure 79 Bond stress gradient of the 2D model with a height of 1000mm and two layers of reinforcement

In Figure 79 the bond stress gradient is given for the 2D model with a height of 1000mm and two layers of reinforcement. In this figure the positions of the cracks are visible and the lengths over which the bond stress varies. Even so the symmetry of the model is visible when looking at the gradient of the bond stress. Both load phases are shown to see the difference in bond stress for both load steps.

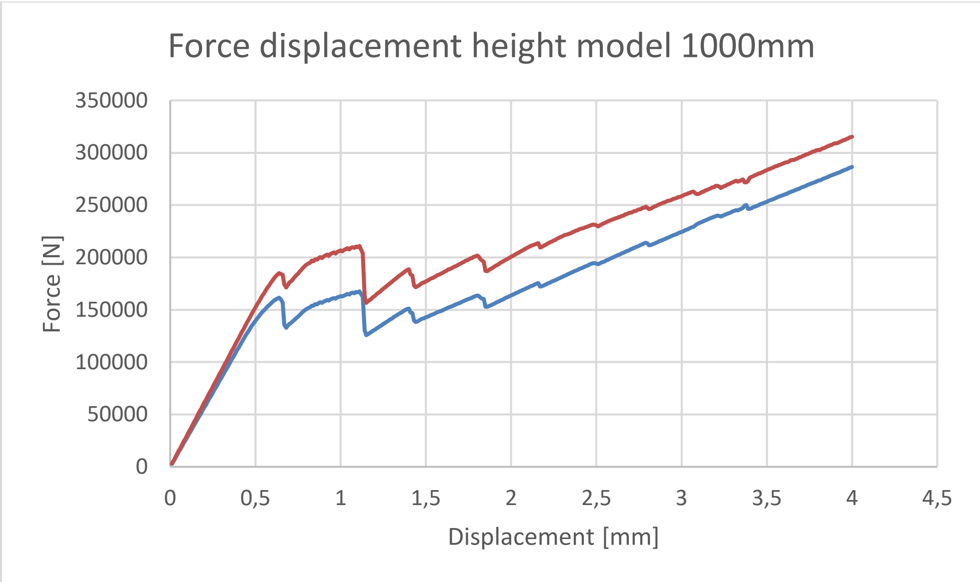


Figure 80 Force displacement diagram of the 2D model with a height of 1000mm and two layers of reinforcement

In Figure 80 the force displacement diagram of the 2D model with a height of 1000mm and two layers of reinforcement is given. In this diagram the force needed for a next crack to appear is clearly visible by the sudden drops in the force displacement curve. In the graph a fully developed crack pattern is given the maximum amount of cracks is 21 for the full model. In here 5 distinct cracks can be observed the other cracks result in small drops which a barely visible in the force displacement diagram. The lines represent both reinforcement bars the outer reinforcement bar has the lowest cracking force(blue line), the inner reinforcement bars have the highest cracking force(red line).

In the table below, the parameters of the determination of the effective concrete tension zone per reinforcement bar are given. These are the values for only half of the model but due to symmetry in the middle of the model all these values can be multiplied by two to get the values for the whole model. With these values the average cracking force can be calculated. The first and second table consists of the values for the inner and outer reinforcement.

Table 33 Summary of the parameters of the inner reinforcement from the 2D model with a height of 1000mm

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
3.39	270	125.7	115130	2.9	39700
1.46	560	125.7	102543	2.9	35360
2.70	310	125.7	105254	2.9	36295
1.95	480	125.7	117646	2.9	40567
1.54	390	125.7	75274	2.9	25957
1.22	510	125.7	78218	2.9	26972
2.50	270	125.7	84903	2.9	29277
1.61	230	125.7	46563	2.9	16056
2.23	250	125.7	69983	2.9	24132
1.93	260	125.7	63220	2.9	21800
2.96	200	125.7	74351	2.9	25638
1.86	140	125.7	32705	2.9	11278
2.88	190	125.7	68665	2.9	23677

Table 34 Summary of the parameters of the outer reinforcement from the 2D model with a height of 1000mm

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
2.45	290	125.7	89424	2.9	30836
1.48	620	125.7	115082	2.9	39683
2.00	330	125.7	82853	2.9	28570
1.84	370	125.7	85544	2.9	29498
1.46	510	125.7	93356	2.9	32192
1.41	490	125.7	86926	2.9	29975
2.19	250	125.7	68713	2.9	23694
1.95	230	125.7	56353	2.9	19432
2.60	240	125.7	78270	2.9	26990
2.25	180	125.7	50847	2.9	17533
2.54	170	125.7	54330	2.9	18735
2.29	170	125.7	48825	2.9	16836

$$\sum cracking\ force\ inner = 2068908N$$

$n = 26$

$$Average\ cracking\ force = N_{cr,ave} = \frac{\sum cracking\ force}{n} = \frac{2068908}{26} = 79573N$$

With the average cracking force of 79573N, the effective concrete tensile zone becomes:

$A_{c,eff} = {}^{N_{cr,ave}}/_{f_{ctm}} = {}^{79573}/_{2.9} = 27439mm^2$. This implies an effective concrete tension zone height of:

$$h_{c,eff\ inner} = \frac{A_{c,eff}}{b} = \frac{27439}{200} = 137mm$$

$$\sum cracking\ force\ outer = 1821047N$$
$$n = 24$$
$$Average\ cracking\ force = N_{cr,ave} = \frac{\sum cracking\ force}{n} = \frac{1821047}{24} = 75877N$$
With the average cracking force of 75877N, the effective concrete tensile zone becomes:
$A_{c,eff} = {N_{cr,ave}}/{f_{ctm}} = {75877}/{2.9} = 26164mm^2$. This implies an effective concrete tension zone height of:
$$h_{c,eff\ outer} = \frac{A_{c,eff}}{b} = \frac{26164}{200} = 131mm$$
Now both effective concrete tension zone heights will be added to get the effective height of both reinforcement bars:
$$Effective\ concrete\ tension\ zone\ height = 137 + 131 = 268mm$$

The effective concrete tension zone height for this case is, according to Eurocode 2, the minimum of ${h}/{2}$ and $2{,}5 * (c + {\emptyset_{sl}}/{2})$.
$$\frac{h}{2} = \frac{1000}{2} = 500mm, \qquad 2{,}5 * \left(\frac{100}{2} + 40 + \sqrt{2} * \frac{40}{2}\right) = 296mm$$
In this configuration Eurocode 2 gives an higher effective concrete tension zone height than the finite element models. The difference between both methods is 28mm. This results in a difference in crack width of 0.02mm. So for this configuration it can be beneficial to use finite element models to determine the effective concrete tension zone height.

The third elaborated example is the model in which the height of the model is 1500mm. The dimensions of the model are given in the table below.

Table 35 Dimensions of the 2D model with a height of 1500mm

Variables:	Dimension [mm]
Length	4000
Width	200
Height	1500
Diameter reinforcement	40
Cover	60
Mesh size	20
Vertical bar spacing	100

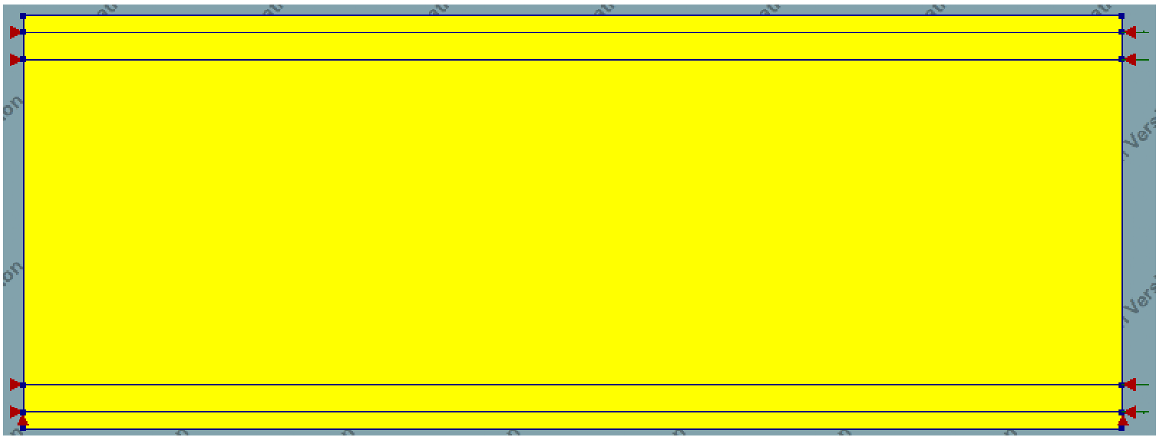


Figure 81 2D model with a height of 1500mm and two layers of reinforcement

In Figure 81 the 2D model with a height of 1500mm and two layers of reinforcement is given. For this model the same conditions apply as the 500mm height model

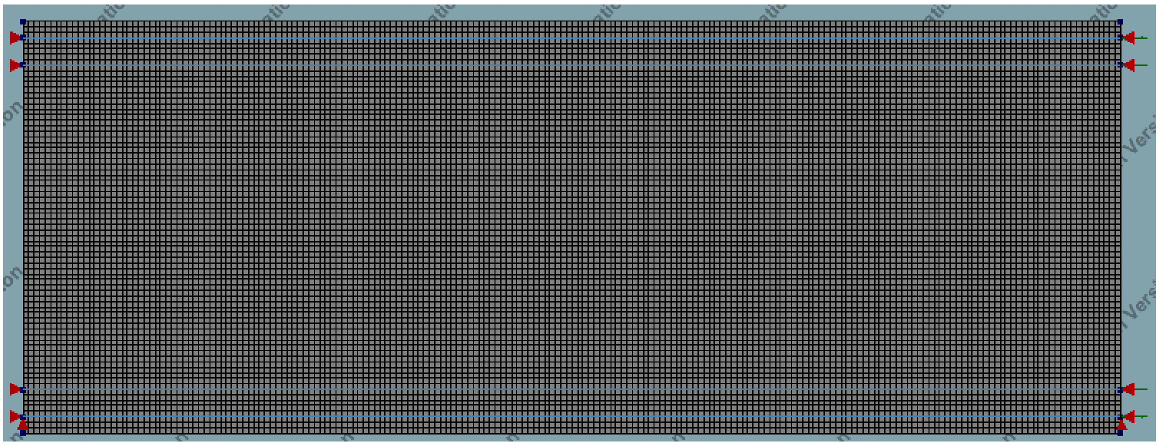
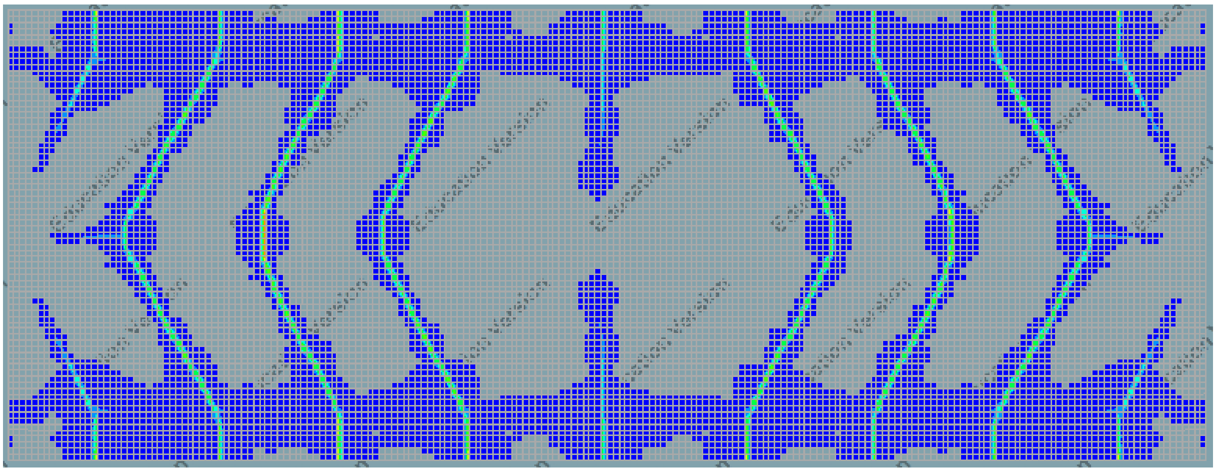


Figure 82 Mesh of the 2D model with a height of 1500mm and two layers of reinforcement

In Figure 82 the mesh of the 2D model with a height of 1500mm and two layers of reinforcement is given. For this model the same conditions apply as for the 500mm height model.



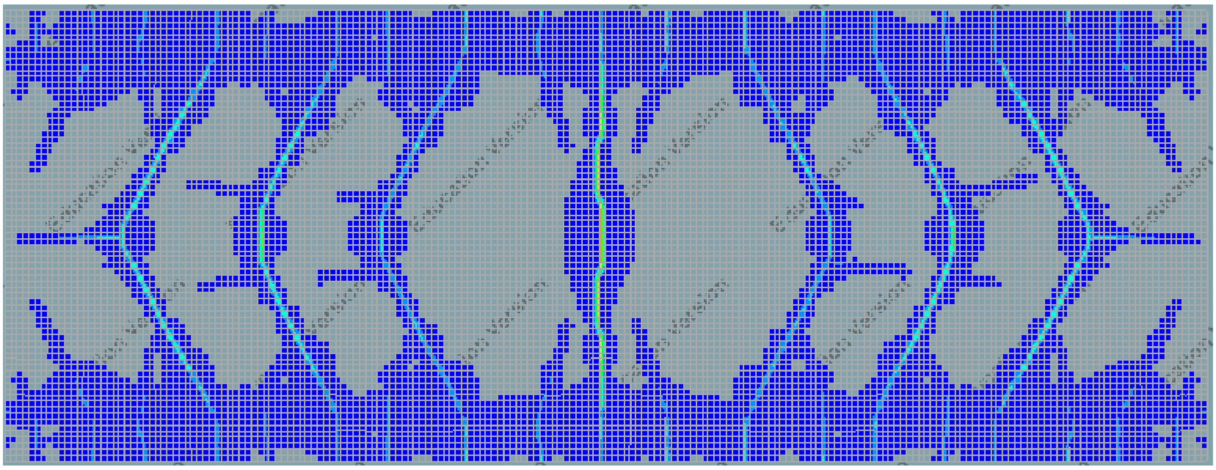


Figure 83 Crack pattern of the 2D model with a height of 1500mm and two layers of reinforcement

In Figure 83 the crack pattern of the 2D model with a height of 1500mm and two layers of reinforcement is given. In the first stage at load step 200 the model is cracked at the maximum crack spacing. Nine (primary) cracks run through the model and bend. When the load keeps increasing the model will crack at positions in between the already consisting cracks to get a fully developed crack pattern(load step 40). The cracks which form are secondary cracks and do not run through the height of the model. The fully developed crack pattern consists of 19 cracks.

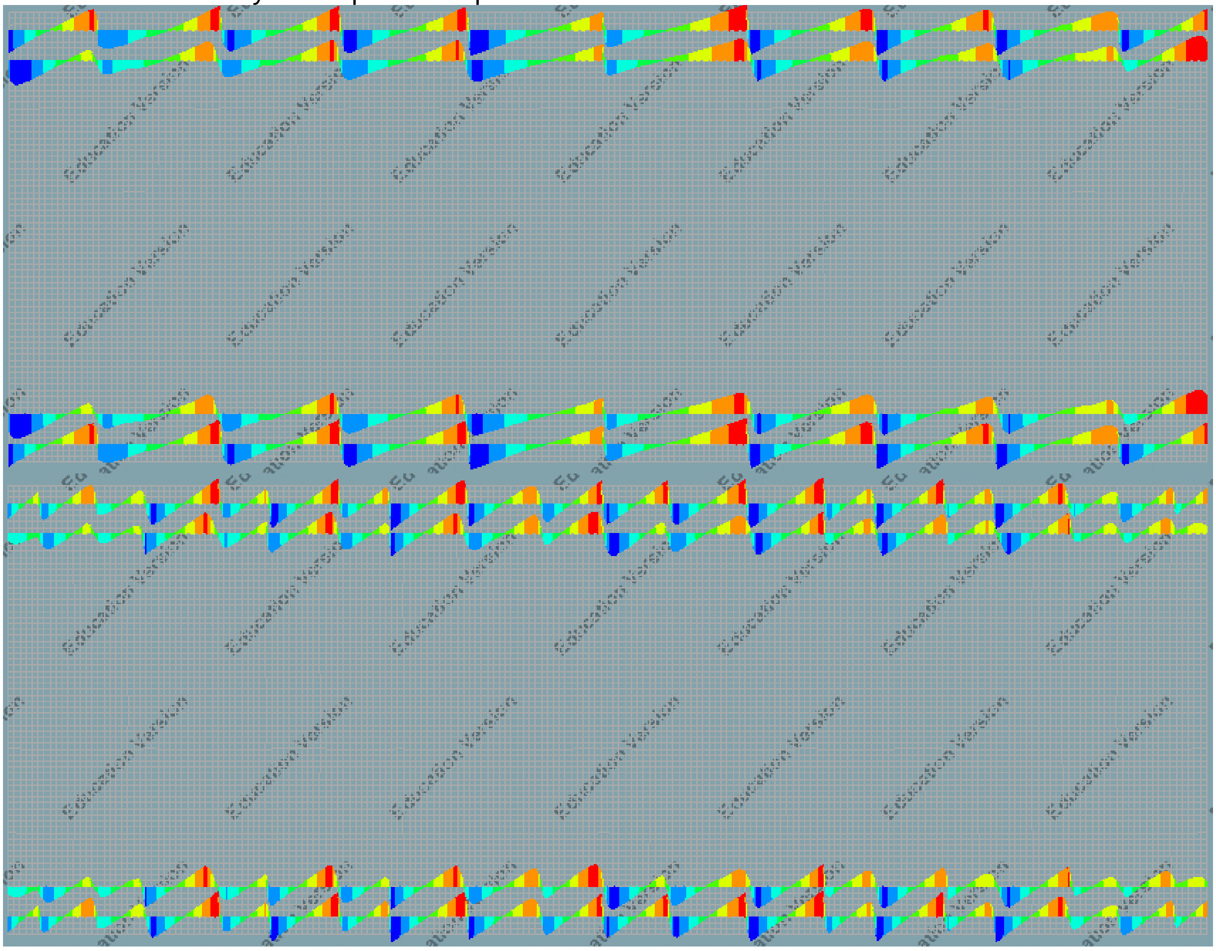


Figure 84 Bond stress gradient of the 2D model with a height of 1500mm and two layers of reinforcement

In Figure 84 the bond stress gradient is given for the 2D model with a height of 1500mm and two layers of reinforcement. In this figure the positions of the cracks are visible and the lengths over which the bond stress varies. Even so the symmetry of the model is visible when looking at the gradient of the bond stress. Both load phases are shown to see the difference in bond stress for both load steps.

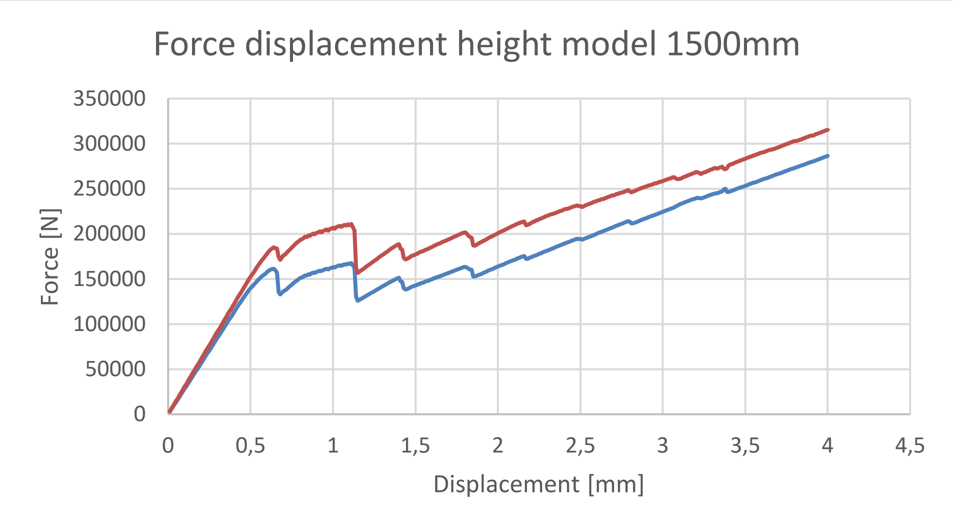


Figure 85 Force displacement diagram of the 2D model with a height of 1500mm and two layers of reinforcement

In Figure 85 the force displacement diagram of the 2D model with a height of 1500mm and two layers of reinforcement is given. In this diagram the force needed for a next crack to appear is clearly visible by the sudden drops in the force displacement curve. In the graph a fully developed crack pattern is given the maximum amount of cracks is 19 for the full model. In here 5 distinct cracks can be observed the other cracks result in small drops which a barely visible in the force displacement diagram. The lines represent both reinforcement bars the outer reinforcement bar has the lowest cracking force(blue line), the inner reinforcement bar has the highest cracking force(red line).

In the tables below, the parameters for the determination of the effective concrete tension zone per reinforcement bar are given. These are the values for only half of the model but due to symmetry in the middle of the model all these values can be multiplied by two to get the values for the whole model. With these values the average cracking force can be calculated. The first and second table consists of the values for the inner and outer reinforcement.

Table 36 Summary of the parameters from the 2D model with a height of 1500mm and two layers reinforcement

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
3.34	280	125.7	117656	2.9	40571
2.23	400	125.7	112029	2.9	38631
2.26	400	125.7	113630	2.9	39183
2.12	410	125.7	109482	2.9	37752
1.62	450	125.7	91674	2.9	31612
2.94	230	125.7	84851	2.9	29259
1.83	210	125.7	48301	2.9	16656
2.16	170	125.7	46124	2.9	15905
1.61	220	125.7	44567	2.9	15368
2.24	170	125.7	47857	2.9	16502
1.76	220	125.7	48630	2.9	16769
2.34	180	125.7	52864	2.9	18229
2.02	220	125.7	55872	2.9	19266
1.51	70	125.7	13308	2.9	4589
2.37	200	125.7	59541	2.9	20531

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
2.33	290	125.7	84759	2.9	29227
2.49	400	125.7	125289	2.9	43203
2.35	390	125.7	115378	2.9	39786
2.11	420	125.7	111565	2.9	38471
1.79	440	125.7	99153	2.9	34191
2.40	220	125.7	66265	2.9	22850
2.45	210	125.7	64655	2.9	22295
2.52	160	125.7	50724	2.9	17491
2.16	230	125.7	62494	2.9	21550
2.59	160	125.7	52107	2.9	17968
2.10	220	125.7	58115	2.9	20040
2.68	190	125.7	63954	2.9	22053
2.24	220	125.7	61966	2.9	21367
2.60	130	125.7	42530	2.9	14666
2.28	150	125.7	42926	2.9	14802

$\sum cracking\ force\ inner = 2092774N$

$n = 30$

$Average\ cracking\ force = N_{cr,ave} = \frac{\sum cracking\ force}{n} = \frac{2092774}{30} = 69759N$

With the average cracking force of 69759N, the effective concrete tensile zone becomes:

$A_{c,eff} = {}^{N_{cr,ave}}/_{f_{ctm}} = {}^{69759}/_{2.9} = 24055mm^2$. This implies an effective concrete tension zone height of:

$h_{c,eff\ inner} = \frac{A_{c,eff}}{b} = \frac{24055}{200} = 120mm$

$\sum cracking\ force\ outer = 2203760N$

$n = 30$

$Average\ cracking\ force = N_{cr,ave} = \frac{\sum cracking\ force}{n} = \frac{2203760}{30} = 73459N$

With the average cracking force of 73459N, the effective concrete tensile zone becomes:

$A_{c,eff} = {}^{N_{cr,ave}}/_{f_{ctm}} = {}^{73459}/_{2.9} = 25331mm^2$. This implies an effective concrete tension zone height of:

$h_{c,eff\ outer} = \frac{A_{c,eff}}{b} = \frac{25331}{200} = 127mm$

Now both effective concrete tension zone heights will be added to get the effective concrete tension zone height of both reinforcement bars:

$Effective\ concrete\ tension\ zone\ height = 120 + 127 = 247mm$

The effective concrete tension zone height for this case is, according to Eurocode 2, the minimum of ${}^{h}/_{2}$ and $2{,}5 * (c + {}^{\emptyset_{sl}}/_{2})$.

$\frac{h}{2} = \frac{1500}{2} = 750mm, \qquad 2{,}5 * \left(\frac{100}{2} + 40 + \sqrt{2} * \frac{40}{2}\right) = 296mm$

In this configuration Eurocode 2 gives an higher effective concrete tension zone height than the finite element models. The difference between both methods is 49mm. This results in a difference in crack width of 0.03mm. By using this method it is shown that it could be possible to get a reduction in crack width.

C. APPENDIX C – VARIATION STUDY BENDING MODELS

In this appendix the models which are loaded by bending are elaborated. The models have a length of 4000mm. The height is variable with dimensions: 500mm, 1000mm and 3000mm. One layer of reinforcement is applied in the models with a height of 500 and 1000mm. Two layers of reinforcement are applied in the model with a height of 3000mm. The second layer is needed to ensure multiple crack within the model. With one layer of reinforcement the model would have a brittle failure. The diameter of the reinforcement is 40mm and the distance in between the reinforcement is 100mm. The cover is 60mm according to Eurocode 2. The distance in between the center of the reinforcement and the outer fiber of the beam is 80mm.

C.1 Bending models

Firstly the model with a height of 500mm will be elaborated. The dimensions of the model are given in the table below.

Table 1 Dimensions of the 2D bending model with a height of 500mm

Variables:	Dimension [mm]
Length	4000
Width	200
Height	500
Diameter reinforcement	40
Cover	80
Mesh size	20

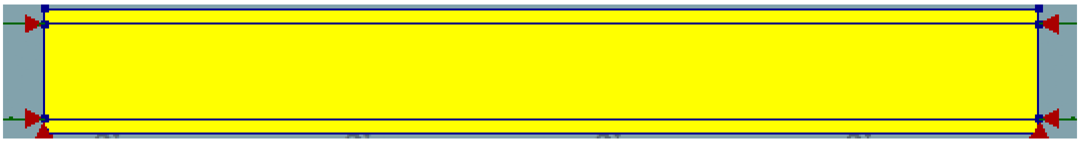


Figure 1 2D bending model with a height of 500mm

In Figure 1 the 2D bending model is shown. On the right and left hand side the prescribed deformations are given as small arrows. The prescribed deformations on the top reinforcement move towards the middle of the beam. They represent a compression force. The triangles display the supports which are used. The bottom reinforcement is supported in both horizontal x-direction as in vertical y-direction. The top reinforcement is only supported in horizontal x direction, because the model undergoes transverse contraction due to the tension force. The transverse contraction will result in stresses in the model in the transverse direction if the model is supported on both reinforcement bars. The stresses will result in parallel cracks around the reinforcement, these cracks need to be prevented because they influence the results needed for this master thesis. Every model is supported in the same way.

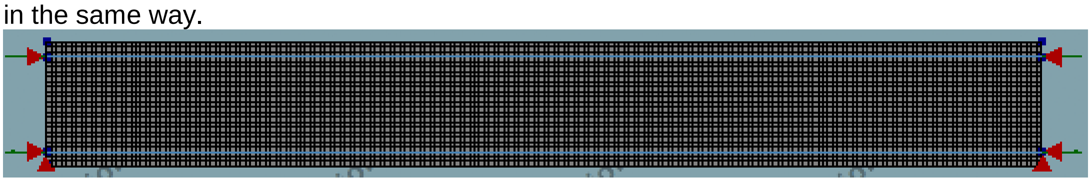


Figure 2 Mesh of the 2D bending model with a height of 500mm

In Figure 2 Mesh of the 2D bending model with a height of 500mm the mesh of the 2D bending model is given the mesh size used is 20mm. The mesh is fine because the position of the cracks, bond stress and transfer length need to be accurately determined. The mesh size is 20mm for every model.

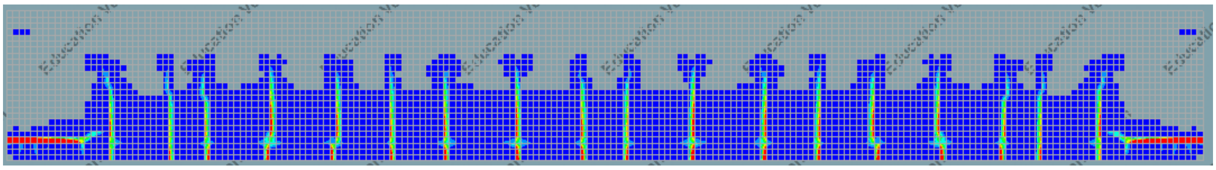


Figure 3 Crack pattern of the 2D bending model with a height of 500mm

In Figure 3 Crack pattern of the 2D bending model with a height of 500mmthe crack pattern of load step 400 is shown for the 2D bending model with a height of 500mm. The crack spacing varies over the length of the model. In the middle of the model a symmetry line can be observed. This effect is obtained because the model is loaded by the prescribed deformations on both ends of the model. The crack pattern shows 18 distinct cracks and 2 cracks parallel to the reinforcement at the position at which the tension force is introduced. The scale at which the cracks are presented is modified to see the cracks clearly.

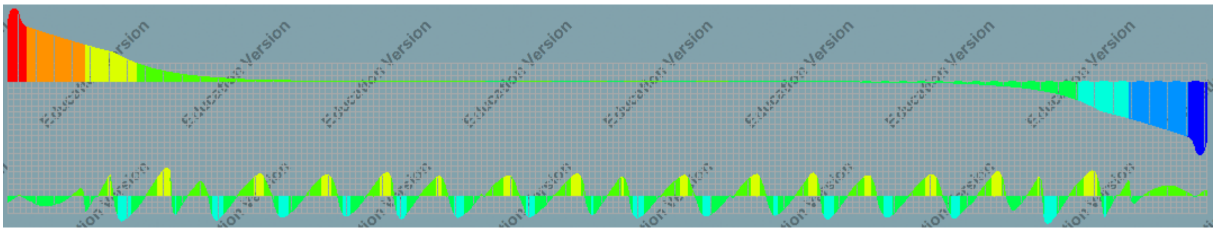


Figure 4 Bond stress gradient of the 2D bending model with a height of 500mm

In Figure 4 Bond stress gradient of the 2D bending model with a height of 500mmthe bond stress gradient for load step 400 of the 2D model with a height of 500mm is given. The positions of the cracks can be observed, because at the positions of the cracks the bond stress suddenly changes sign from a positive value to a negative value. At these positions the slope of the curve is steep. In between the cracks the bond stress runs from a large negative value to a large positive value with a nearly constant linear slope. The slip of the top reinforcement is positioned at the ends of the reinforcement, therefore the bond stress is large at these positions. Furthermore the model is under compression at the top reinforcement therefore no cracks occur at the top reinforcement, which can influence the bond-slip behavior at the top reinforcement.

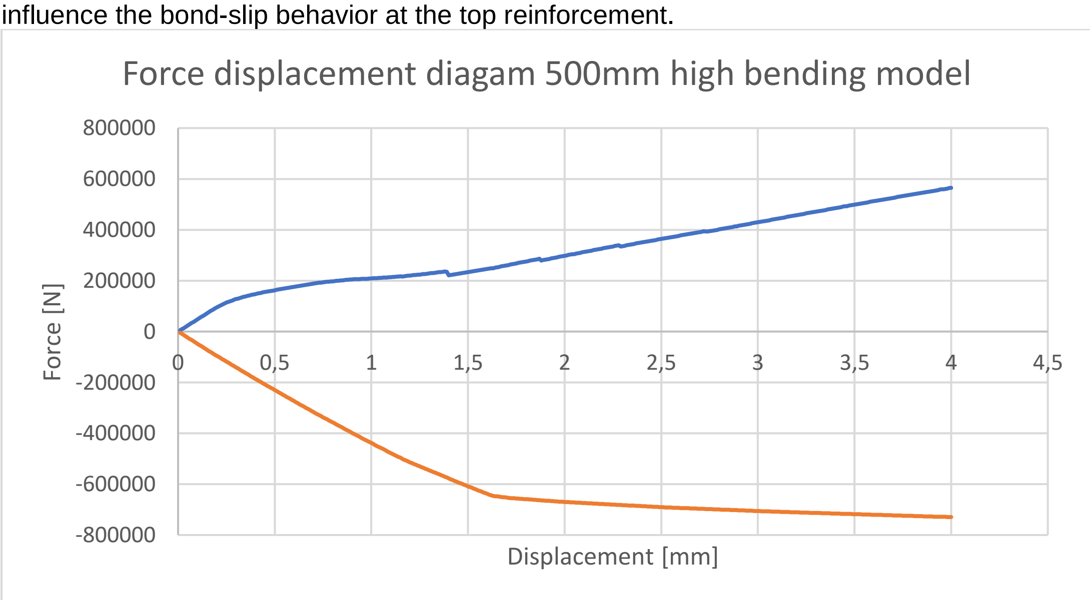


Figure 5 Force displacement diagram of the 2D bending model with a height of 500mm

In Figure 5 the force-displacement diagram of the 2D bending model with a height of 500mm is given. The blue line represent the tension force. After the elastic part of the tension force the load steps at which a crack occurs are not clearly visible but the model does crack. Usually the cracks are clearly visible by drops in the force displacement diagram. Just one crack is clearly visible in the force displacement diagram. The red line represents the compression force. The elastic and plastic branches are clearly visible in the compression line.

The results which are necessary for the determination of the effective concrete tension zone height are obtained from DIANA. The cracking forces are obtained by multiplying the average bond stress times the transfer length times the perimeter of the reinforcement. The average cracking force is determined by adding all cracking forces and divide it by the amount of cracking forces.

In the table below, the parameters for the determination of the effective concrete tension zone are given. These are the values for only half of the model but due to symmetry in the middle of the model all these values can be multiplied by two to get the values for the whole model. With these values the average cracking force can be calculated.

Table 2 Summary of the parameters from the 2D bending model with a height of 500mm

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
2.31	340	125.7	98518	2.9	33972
1.14	320	125.7	45988	2.9	15858
1.49	200	125.7	37554	2.9	12950
1.02	380	125.7	48557	2.9	16744
0.16	430	125.7	8582	2.9	2959
1.87	220	125.7	51607	2.9	17795
2.04	220	125.7	56410	2.9	19452
2.78	170	125.7	59344	2.9	20463
2.69	180	125.7	60825	2.9	20974
3.28	150	125.7	61860	2.9	21331
3.36	150	125.7	63252	2.9	21811

$$\sum cracking\ force = 1184992N$$

$$n = 22$$

$$Average\ cracking\ force = N_{cr,ave} = \frac{\sum cracking\ force}{n} = \frac{1184992}{22} = 53863N$$

With the average cracking force of 53863N, the effective concrete tensile zone becomes:

$$A_{c,eff} = \frac{N_{cr,ave}}{f_{ctm}} = \frac{53863}{2.9} = 18573mm^2$$

This implies an effective concrete tension zone height of:

$$h_{c,eff} = \frac{A_{c,eff}}{b} = \frac{18573}{200} = 93mm$$

The effective concrete tension zone height for this case is, according to Eurocode 2, the minimum of:

$$\min\left[\frac{h-x}{3}; 2.5*\left(c+\frac{\emptyset_{sl}}{2}\right)\right]$$

$$\frac{h-x}{3} = \frac{500-160}{3} = 113mm, \qquad 2,5*\left(60+\frac{40}{2}\right) = 200mm$$

In this configuration the output of the finite element models results in a smaller effective concrete tension zone height than given by Eurocode 2. The difference in effective concrete tension zone height between both methods is 14mm. This results in a difference in crack width of 0.02mm. For bending models it could be possible to get a reduction in effective concrete tension zone according to this model.

Secondly the model with a height of 1000mm will be elaborated. The dimensions of the model are given in the table below.

Table 3 Dimensions of the 2D bending model with a height of 1000mm

Variables:	Dimension [mm]
Length	4000
Width	200
Height	1000
Diameter reinforcement	40
Cover	80
Mesh size	20

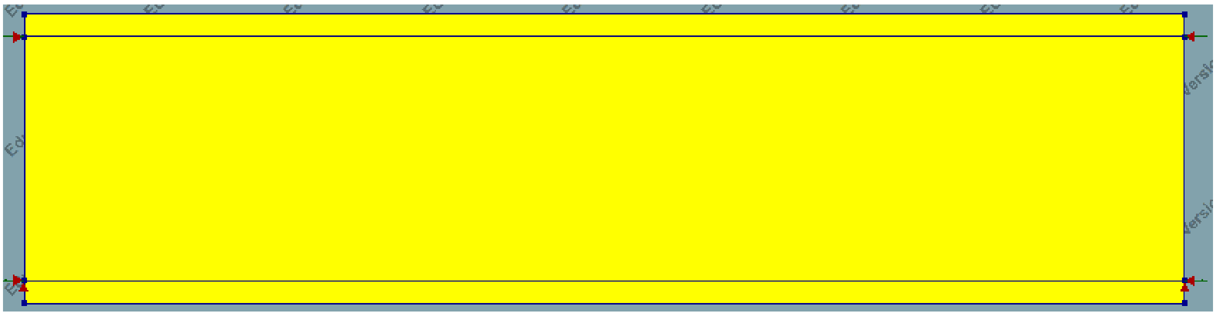


Figure 6 2D bending model with a height of 1000mm

In Figure 6 the 2D bending model is shown. The boundary conditions are just like the model with a height of 500mm.

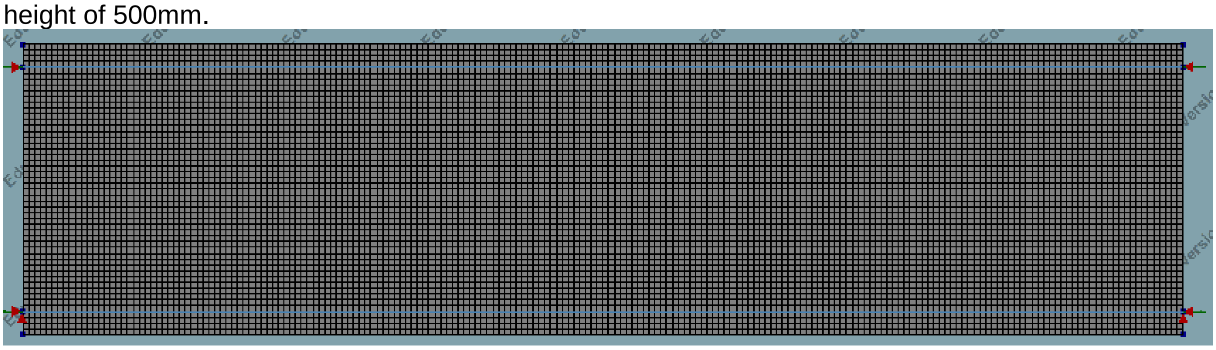


Figure 7 Mesh of the 2D bending model with a height of 1000mm

In Figure 7 the mesh of the 2D bending model is given the mesh size used is 20mm.

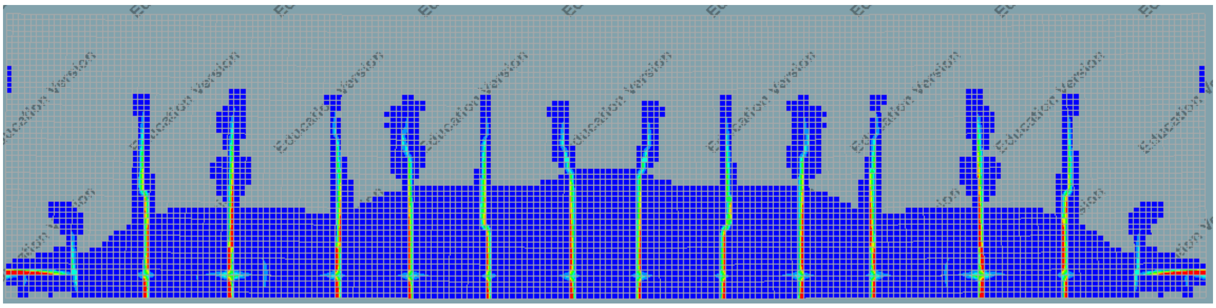


Figure 8 Crack pattern of the 2D bending model with a height of 1000mm

In Figure 8 the crack pattern of load step 300 is shown for the 2D bending model with a height of 1000mm. The crack spacing varies over the length of the model. In the middle of the model a symmetry line can be observed. This effect is obtained because the model is loaded by the prescribed deformations on both ends of the model. The crack pattern shows 14 distinct cracks and 2 cracks parallel to the reinforcement at the position at which the tension force is introduced. The scale at which the cracks are presented is modified to see the cracks clearly.

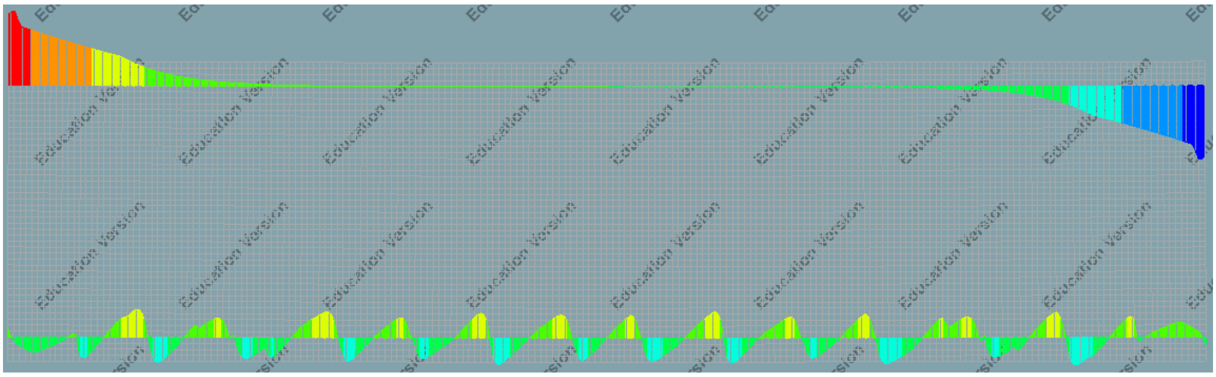


Figure 9 Bond stress gradient of the 2D bending model with a height of 1000mm

In Figure 9 the bond stress gradient for load step 300 of the 2D model with a height of 1000mm is given. The positions of the cracks can be observed, because at the positions of the cracks the bond stress suddenly changes sign from a positive value to a negative value. At these positions the slope of the curve is steep. In between the cracks the bond stress runs from a large negative value to a large positive value with a nearly constant linear slope. The slip of the top reinforcement is positioned at the ends of the reinforcement, therefore the bond stress is large at these positions. Furthermore the model is under compression at the top reinforcement therefore no cracks occur at the top reinforcement, which can influence the bond-slip behavior at the top reinforcement.

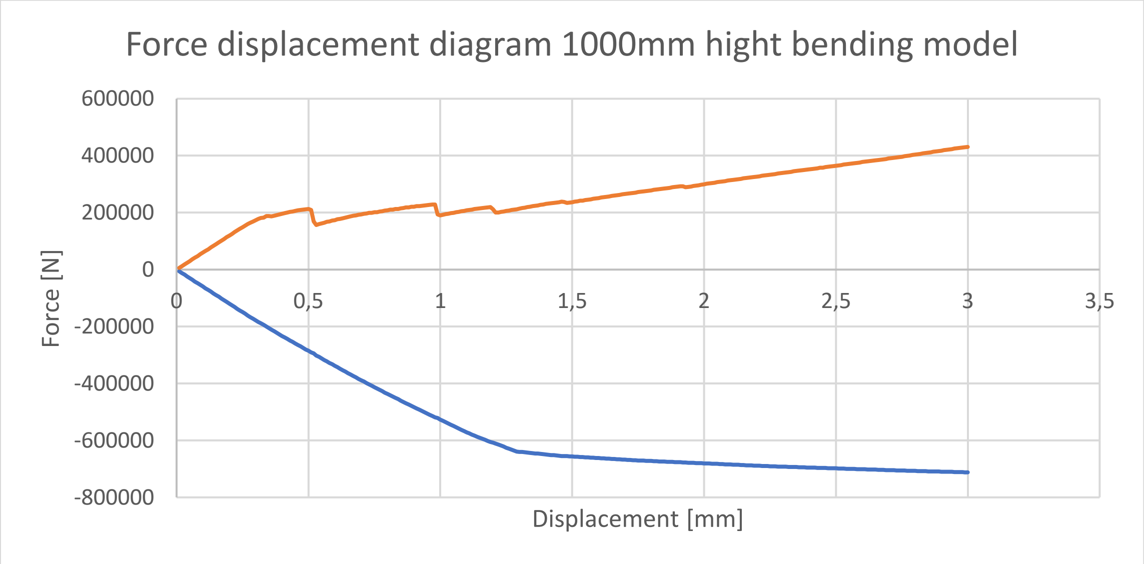


Figure 10 Force displacement diagram of the 2D bending model with a height of 1000mm

In Figure 10 the force-displacement diagram of the 2D bending model with a height of 1000mm is given. The blue line represent the tension force. After the elastic part of the tension force the load steps at which a crack occurs are clearly visible by drops in the line. The red line represents the compression force. The elastic and plastic branches are clearly visible in the compression line.

The results which are necessary for the determination of the effective concrete tension zone height are obtained from DIANA. The cracking forces are obtained by multiplying the average bond stress times the transfer length times the perimeter of the reinforcement. The average cracking force is determined by adding all cracking forces and divide it by the amount of cracking forces.

In the table below, the parameters for the determination of the effective concrete tension zone are given. These are the values for only half of the model but due to symmetry in the middle of the model all these values can be multiplied by two to get the values for the whole model. With these values the average cracking force can be calculated.

Table 4 Summary of the parameters from the 2D bending model with a height of 1000mm

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
2.63	460	125.7	151943	2.9	52394
1.93	590	125.7	143021	2.9	49318
1.95	530	125.7	129644	2.9	44705
2.80	300	125.7	105556	2.9	36399
3.00	280	125.7	105543	2.9	36394
3.14	250	125.7	98581	2.9	33993
3.00	260	125.7	97908	2.9	33761

$$\sum cracking\ force = 1664392N$$

$$n = 14$$

$$Average\ cracking\ force = \mu = \frac{\sum cracking\ force}{n} = \frac{1664392}{14} = 118885N$$

With the average cracking force of 118885N, the effective concrete tensile zone becomes:

$$A_{c,eff} = \frac{N_{cr,ave}}{f_{ctm}} = \frac{118885}{2.9} = 40995mm^2$$

This implies an effective concrete tension zone height of:

$$h_{c,eff} = \frac{A_{c,eff}}{b} = \frac{40995}{200} = 205mm$$

The effective concrete tension zone height for this case, is according to Eurocode 2, the minimum of:

$$\min\left[\frac{h-x}{3}; 2.5 * \left(c + \frac{\emptyset_{sl}}{2}\right)\right]$$

$$\frac{h-x}{3} = \frac{1000-300}{3} = 233mm, \qquad 2{,}5 * \left(60 + \frac{40}{2}\right) = 200mm$$

In this configuration the output of the finite element models is almost equal to the effective concrete tension zone height given by Eurocode 2. The difference in effective concrete tension zone height is 5mm. This results in a difference in crack width of 0.00mm. This example shows that Eurocode 2 represents a good method on how to determine the effective concrete tension zone height.

Thirdly the model with a height of 3000mm will be elaborated. The dimensions of the model are given in the table below.

Table 5 Dimensions of the 2D bending model with a height of 3000mm

Variables:	Dimension [mm]
Length	4000
Width	200
Height	500
Diameter reinforcement	40
Cover	80
Mesh size	20
Vertical bar spacing	150

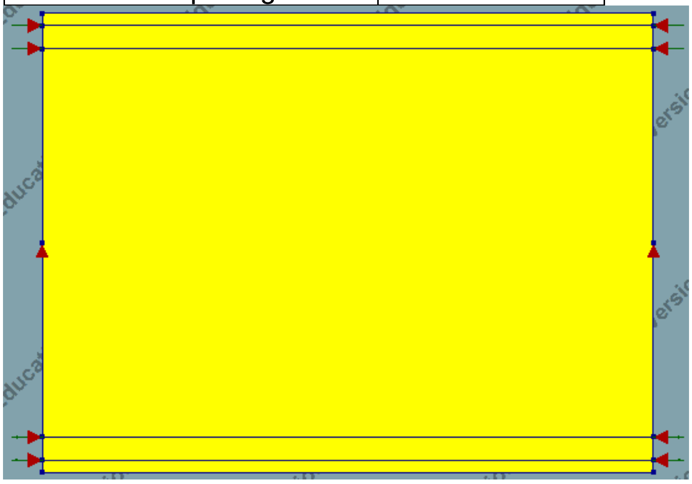


Figure 11 2D bending model with a height of 3000mm

In Figure 11 the 2D bending model is shown. This model is supported at half of the height in vertical direction. At this position the model rotates around the z-axis. The ends of the reinforcement are supported in horizontal x-direction because the finite element model needs to have a support in the direction of the prescribed deformation. Furthermore two reinforcement bars are applied in the top and bottom of the model to ensure structural safety. A prescribed deformation towards the middle of the model (pushing) is applied on the top reinforcement. A pulling prescribed deformation is applied to the bottom reinforcement. These deformations are applied to get a bending moment on the model.

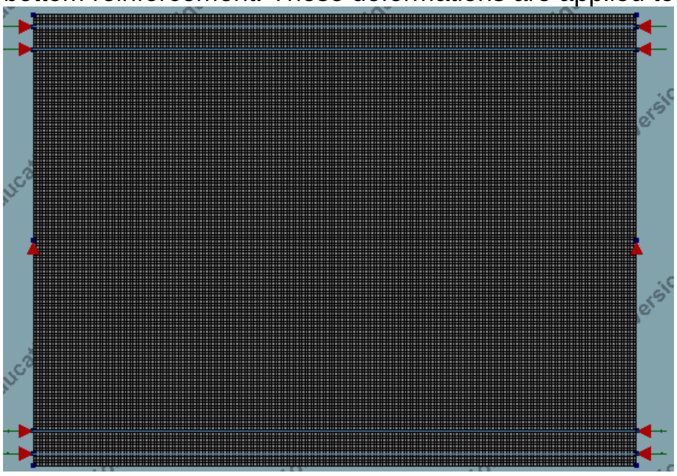


Figure 12 Mesh of the 2D bending model with a height of 3000mm

In Figure 12 the mesh of the 2D bending model is given the mesh size used is 20mm.

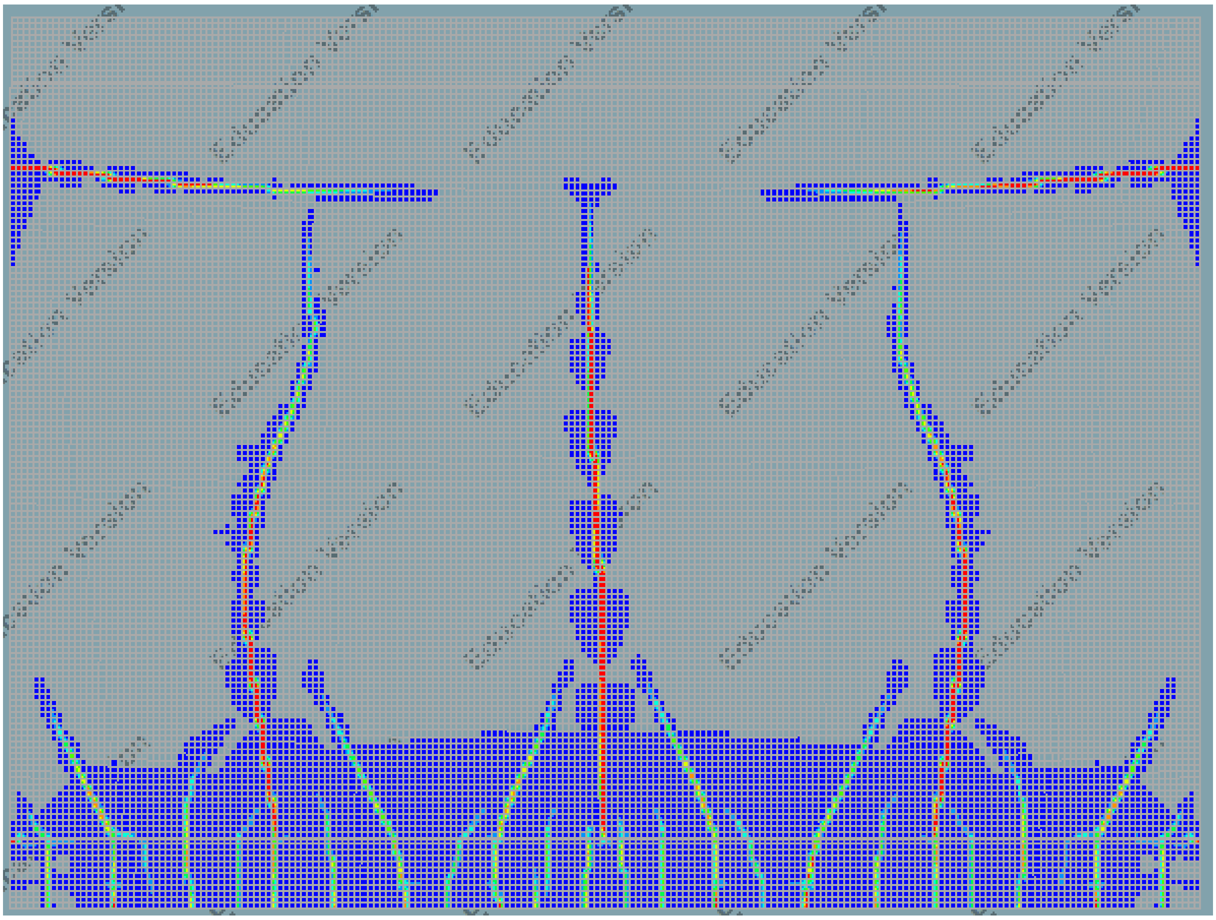


Figure 13 Crack pattern of the 2D bending model with a height of 3000mm

In Figure 13 the crack pattern of load step 300 is shown for the 2D bending model with a height of 3000mm. The crack spacing varies over the length of the model. In the middle of the model a symmetry line can be observed. This effect is obtained because the model is loaded by the prescribed deformations on both ends of the model. The crack pattern shows 3 primary cracks which run towards the compression zone and 21 secondary cracks which run towards the primary cracks. The scale at which the cracks are presented is modified to see the cracks clearly. The horizontal cracks in the top side of the model are due to the way the model is loaded on the top. The prescribed deformation on top results in a small beam model in the top of the model. This model cracks off the total model. In reality this phenomena would not occur, so these cracks can be ignored. The horizontal cracks are not important for the determination of the effective concrete tension zone height.

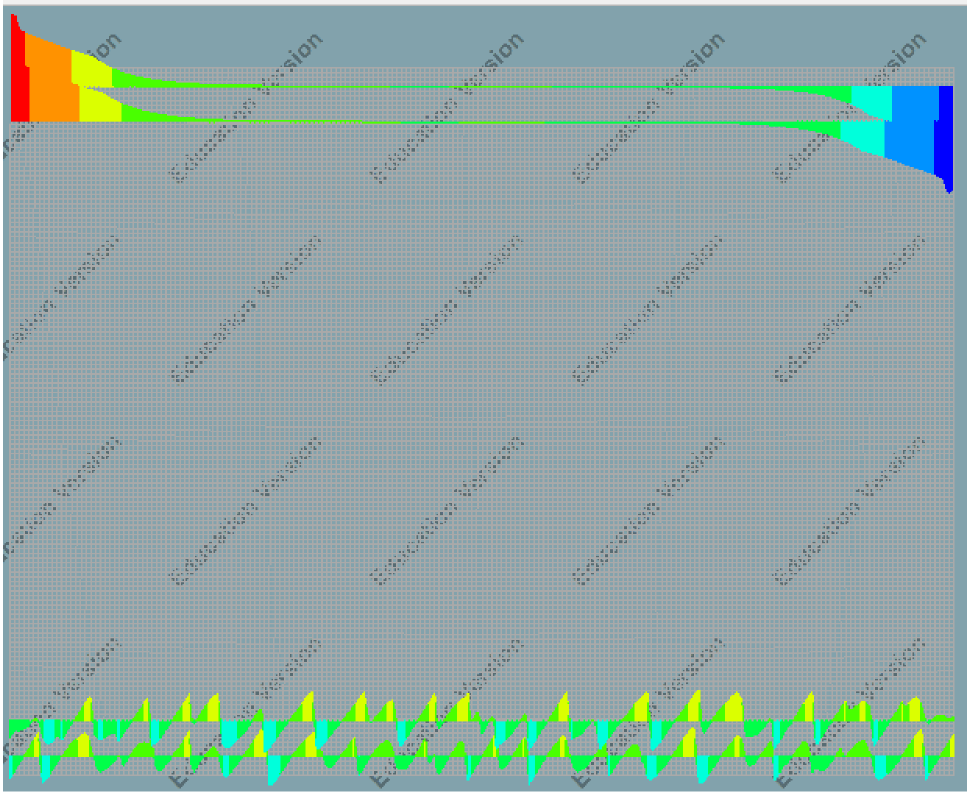


Figure 14 Bond stress gradient of the 2D bending model with a height of 3000mm

In Figure 14 the bond stress gradient for load step 300 of the 2D model with a height of 3000mm is given. The positions of the cracks can be observed, because at the positions of the cracks the bond stress suddenly changes sign from a positive value to a negative value. At these positions the slope of the curve is steep. In between the cracks the bond stress runs from a large negative value to a large positive value with a nearly constant linear slope. The slip of the top reinforcement is positioned at the ends of the reinforcement, therefore the bond stress is large at these positions. Furthermore the model is under compression at the top reinforcement therefore no cracks occur at the top reinforcement, which can influence the bond-slip behavior at the top reinforcement. Due to the cracks in the bottom reinforcement the bond stress decreases.

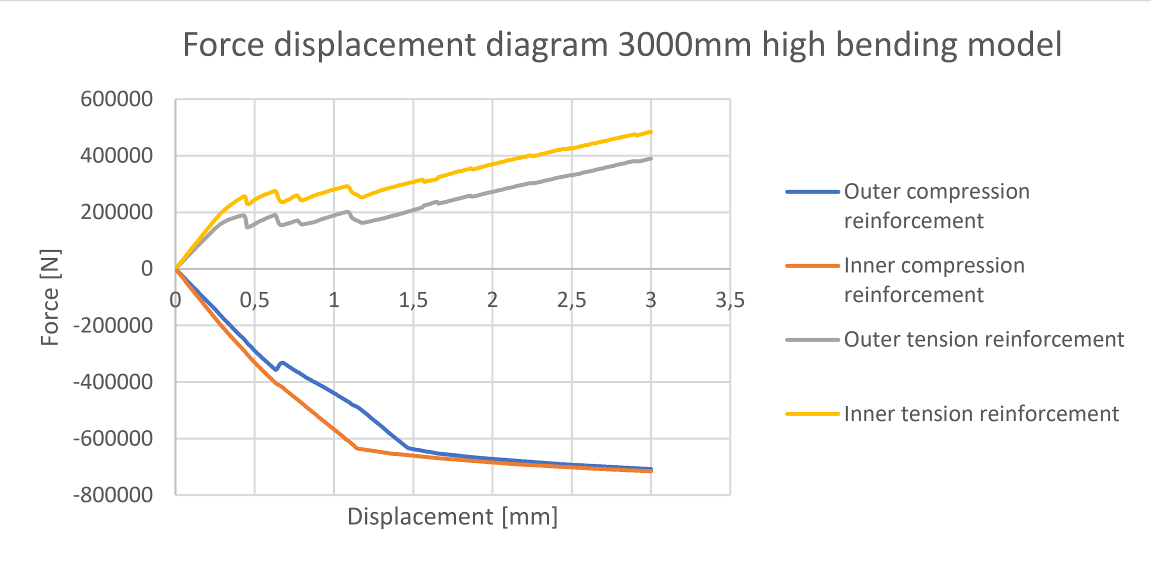


Figure 15 Force displacement diagram of the 2D bending model with a height of 3000mm

In Figure 15 the force-displacement diagram of the 2D bending model with a height of 3000mm is given. The yellow and grey line represent the tension force in the inner and outer reinforcement. After the elastic part of the tension force the load steps at which a crack occurs are clearly visible by drops in the line. The red and blue line represents the compression forces in the inner and outer reinforcement. The elastic and plastic branches are clearly visible in the compression lines.

The results which are necessary for the determination of the effective concrete tension zone height are obtained from DIANA. The cracking forces are obtained by multiplying the average bond stress times the transfer length times the perimeter of the reinforcement. The average cracking force is determined by adding all cracking forces and divide it by the amount of cracking forces.

In the table below, the average bond stress, transfer length, perimeter of the reinforcement, cracking force, concrete tensile strength and the effective concrete tension zone are given. These are the values for only half of the model but due to symmetry in the middle of the model all these values can be multiplied by two to get the values for the whole model. With these values the average cracking force can be calculated. First the results of the outer reinforcement will be given in a table thereafter the results of the inner reinforcement will be given.

Table 6 Summary of the parameters of the outer reinforcement from the 2D bending model with a height of 3000mm

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
2.23	350	125.7	98142	2.9	33842
2.28	520	125.7	149270	2.9	51472
2.36	430	125.7	127584	2.9	43995
1.91	600	125.7	143645	2.9	49533
2.76	220	125.7	76441	2.9	26359
2.45	290	125.7	89190	2.9	30755
2.49	280	125.7	87787	2.9	30271
2.18	310	125.7	85094	2.9	29343
2.88	150	125.7	54211	2.9	18693
3.13	210	125.7	82582	2.9	28476
3.04	210	125.7	80190	2.9	27652
3.25	140	125.7	57173	2.9	19715
3.10	100	125.7	38998	2.9	13448
2.64	130	125.7	43170	2.9	14886
3.43	130	125.7	56113	2.9	19349
3.46	130	125.7	56517	2.9	19488

Table 7 Summary of the parameters of the inner reinforcement from the 2D bending model with a height of 3000mm

Average bond stress $\tau_{b,ave}$ [MPa]	Transfer length Lst [mm]	Perimeter reinforcement U_s [mm]	Cracking Force N_{cr} [N]	Tensile strength concrete f_{ctm} [MPa]	Effective concrete tension zone $A_{c,eff}$ [mm²]
3.99	340	125.7	170583	2.9	58822
2.26	530	125.7	150561	2.9	51918
2.57	400	125.7	129341	2.9	44600
1.69	660	125.7	140209	2.9	48348
3.05	240	125.7	92109	2.9	31762
2.60	280	125.7	91481	2.9	31545
2.60	340	125.7	110892	2.9	38239
2.19	310	125.7	85411	2.9	29452
3.06	160	125.7	61507	2.9	21209
3.59	230	125.7	103810	2.9	35796
4.07	180	125.7	91982	2.9	31718
3.20	120	125.7	48281	2.9	16648
3.99	180	125.7	90336	2.9	31150
3.42	160	125.7	68758	2.9	23710
4.74	160	125.7	95382	2.9	32890
4.09	120	125.7	61705	2.9	21278

$$\sum cracking\ force\ outer = 2652211N$$
$$n = 32$$
$$Average\ cracking\ force = \mu = \frac{\sum cracking\ force}{n} = \frac{2652211}{32} = 82882N$$

With the average cracking force of 82882N, the effective concrete tensile zone becomes:

$$A_{c,eff} = \frac{N_{cr,ave}}{f_{ctm}} = \frac{82882}{2.9} = 28580mm^2$$

This implies an effective concrete tension zone height of:

$$h_{c,eff\ outer} = \frac{A_{c,eff}}{b} = \frac{28580}{200} = 143mm$$

$$\sum cracking\ force\ inner = 3184693N$$
$$n = 32$$
$$Average\ cracking\ force = \mu = \frac{\sum cracking\ force}{n} = \frac{3184693}{32} = 99522N$$

With the average cracking force of 99522N, the effective concrete tensile zone becomes:

$$A_{c,eff} = \frac{N_{cr,ave}}{f_{ctm}} = \frac{99522}{2.9} = 34318mm^2$$

This implies an effective concrete tension zone height of:

$$h_{c,eff\ inner} = \frac{A_{c,eff}}{b} = \frac{34318}{200} = 172mm$$

The total effective concrete tension zone height of the reinforcement is the inner and outer value added together.

$$h_{c,eff} = h_{c,eff\ inner} + h_{c,eff\ outer} = 172 + 143 = 315mm$$

The effective concrete tension zone height for this case is, according to Eurocode 2 the minimum of:

$$\min\left[\frac{h-x}{3}; 2.5 * \left(c + \frac{\emptyset_{sl}}{2}\right)\right]$$

$$\frac{h-x}{3} = \frac{3000-560}{3} = 813mm, \qquad 2,5 * \left(\frac{150}{2} + 60 + \sqrt{2} * \frac{40}{2}\right) = 408mm$$

The effective concrete tension zone height according to Jones method is given by:

$$h_{c,effJM} = c + \frac{\emptyset_{mr}}{2} + \min\left[\frac{ssv}{2}; 1.5 * \left(c + \frac{\emptyset_{mr}}{2}\right)\right]$$

$$h_{c,effJM} = 60 + \frac{40}{2} + \min\left[\frac{150}{2}; 1.5 * \left(60 + \frac{40}{2}\right)\right] = 80 + \min[75; 120] = 155mm$$

The effective concrete tension zone height according to Jones method is given for a single rebar because the rebar at which a crack will most certainly occur needs to be considered.

In this configuration the output of the finite element models results in a smaller effective concrete tension zone height than given by Eurocode 2. The difference in effective concrete tension zone height between both methods is 94mm. This results in a difference in crack width of 0.06mm. If both rebars will be considered the effective concrete tension zone height given by Jones method is in between the values of the finite element model. The difference in effective concrete tension zone height between both methods is 1mm. This results in a difference in crack width of 0.00mm. The difference in effective concrete tension zone height between Eurocode 2 and Jones method is 98mm. This results in a difference in crack width of 0.06mm. Therefore Jones method is comparable with the finite element models according to this model. Eurocode 2 overestimates the effective concrete tension zone height.

D APPENDIX D – EFFECTIVE CONCRETE TENSION ZONE HEIGHT

D.1 Three layers of reinforcement calculated with Eurocode 2:

For the estimation of the effective concrete tension zone height of the cross section, an analysis to the effect of three layers of reinforcement has been carried out, similar to that done for two layers of reinforcement. Here the same formulas apply as for the other models.
In Eurocode 2 clause 7.3.2 (3) [1] the effective concrete tension zone is determined by the effective concrete tension zone height. The effective concrete tension zone height is determined by the following formula:

$$h_{c,eff} = MIN \begin{cases} \frac{h-x}{3} & (Bending) \quad 1 \\ 2{,}5 * (h-d) & \quad 2 \\ \frac{h}{2} & (Axial\ tension) \quad 3 \\ 2{,}5 * \left(c + \frac{\phi_{sl}}{2}\right) & \quad 4 \end{cases}$$

Where:
h is the height of the cross section
x is the height of the concrete compression zone of the cross section
d is the effective height of the cross section determined by $d = h - c - {}^{\phi_{sl}}/_{2} - \phi_w$
c is the cover applied to the reinforcement
ϕ_{sl} is the main reinforcement
The formulas given above are valid for horizontal bar spacings till:

$$s = 5 * \left(c + \frac{\emptyset_{sl}}{2}\right)$$

The formula for Jones method to determine the effective concrete tension zone height is given by:

$$h_{c,effMJ} = c + \frac{\phi_{sl}}{2} + \min \begin{cases} \frac{ssv}{2} \\ 1{,}5 * \left(c + \frac{\phi_{sl}}{2}\right) \end{cases}$$

Where:
ssv is the vertical bar spacing
The difference of the multiple layers is that the effective height of the reinforcement will be higher due to the fact that three layers of reinforcement have been applied here. To show the effect of the different parameters, calculations of all the input parameters have been elaborated. For three layers of reinforcement models loaded by tension only have been investigated. For cross sections with a small height or elements with a small thickness (1000mm) formula 3 is normative. The effective concrete tension zone height of sections which are thicker than 1500mm will be mostly determined by formula 2. In the tables shown below the input parameters are given for the situation in which formula 2 is governing in the determination of the effective concrete tension zone height.

Table 1 Conditions for the determination of the effective concrete tension zone height for 3 layers of reinforcement according to Eurocode 2

Thickness is kept at t=1000mm		$h_{c,eff} = 2{,}5 * (h-d)$	$h_{c,eff} = h/2$
Cover c [mm]	Vertical bar spacing ssv [mm]	Governing diameter ϕ_{sl} [mm]	Governing diameter ϕ_{sl} [mm]
100	75	16-25	32→
80	100	16-20	25→
70	100	16-32	40→
60	125	16	20→
50	125	16-25	32→
30	150	16-20	25→

Thickness is kept at t=1250mm		$h_{c,eff} = 2{,}5 * (h - d)$	$h_{c,eff} = h/2$
Cover c [mm]	Vertical bar spacing ssv [mm]	Governing diameter ϕ_{sl} [mm]	Governing diameter ϕ_{sl} [mm]
100	125	16-25	32→
80	150	16-20	25→
70	150	16-32	40→
50	175	16-25	32→
30	200	16-20	25→

Thickness is kept at t=1500mm		$h_{c,eff} = 2{,}5 * (h - d)$	$h_{c,eff} = h/2$
Cover c [mm]	Vertical bar spacing ssv [mm]	Governing diameter ϕ_{sl} [mm]	Governing diameter ϕ_{sl} [mm]
100	175	16-25	32→
80	200	16-20	25→
70	200	16-32	40→

The conditions of the governing formulas of the effective height for the situations investigated are given in the tables above. For thicknesses of 1750mm or more, formula 2 always governs the equation for determining the effective concrete tension zone height.

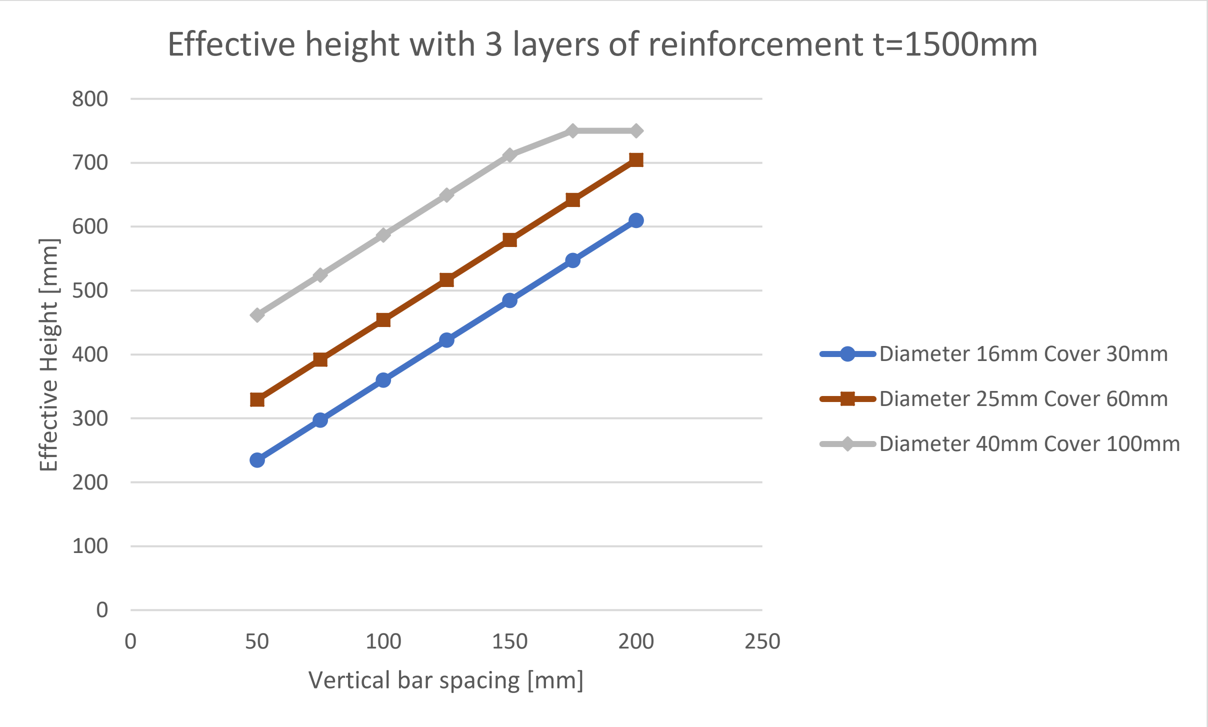


Figure 1 Effective concrete tension zone height with 3 layers of reinforcement according to Eurocode 2

In Figure 1, the effective concrete tension zone height of a model loaded by tension is given for a section thickness of 1500mm with different vertical bar spacings, diameters and covers. It can been seen that the figure has an upper limit at a value of 750mm this equals $^{h}/_{2} = {}^{1500}/_{2} = 750mm$. Below this limit formula 2 determines the effective concrete tension zone height. A trend can be observed for the dependency of the cover and diameter, by increasing cover and diameter the formula reaches the upper limit with a smaller vertical bar spacing.

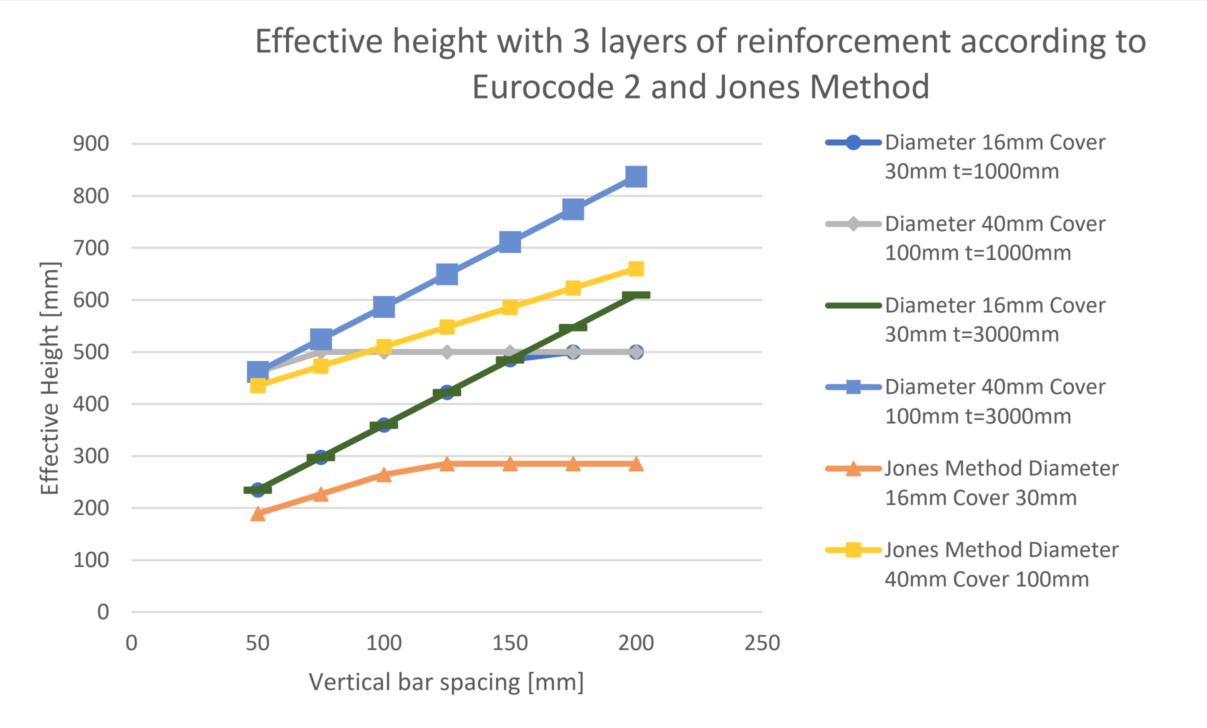


Figure 2 Effective concrete tension zone height with 3 layers of reinforcement according to Eurocode 2 and Jones Method

In Figure 2, the effective concrete tension zone height of a model loaded by tension is given for different situations with the limits of the input parameters of three reinforcement layers according to Eurocode 2 and Jones Method. A somewhat equal picture is given for two layers of reinforcement. The lower limit according to Eurocode 2 is given by the line which represent the input parameters of diameter $\phi_{sl} = 16mm$, cover $c = 30mm$ and a thickness of 1000mm. The upper limit is given by the input parameters of diameter $\phi_{sl} = 40mm$, cover $c = 100mm$ and a thickness of 3000mm. The horizontal line is given by formula 3 because this formula is normative for constructions with a small thickness and large diameter and cover. The output of the other parameters from Eurocode 2 lies between the lower and upper limit but this is not shown here, to see the limits of the effective concrete tension zone height for the given situations.

By using Jones Method the effective concrete tension zone height decreases significantly. The lower limit value of Jones Method is below the lower limit value of Eurocode 2 and the upper limit value lies between the upper and lower limits of Eurocode 2. A smaller effective concrete tension zone height is beneficial for the crack width calculation. This is because the effective concrete tension zone decreases in size and the effective reinforcement ratio increases.

D.1.1 Effective reinforcement ratio:

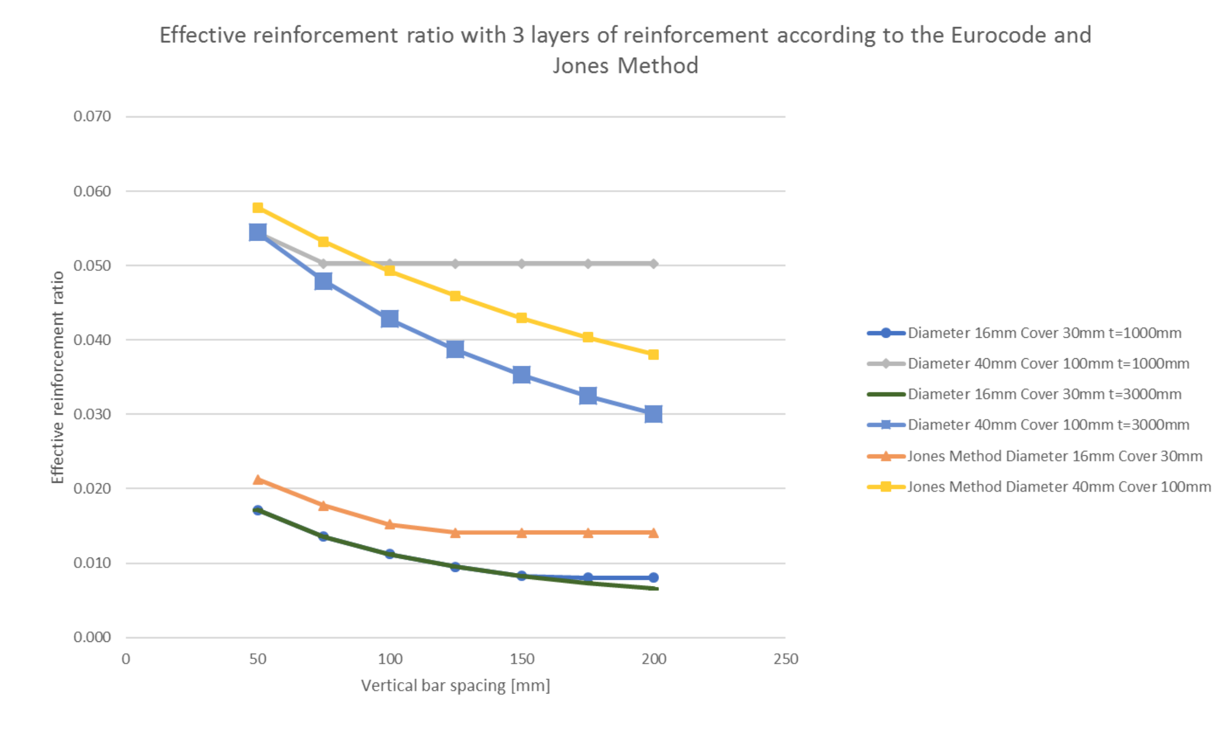


Figure 3 Effective reinforcement ratio with 3 layers of reinforcement according to Eurocode 2 and Jones Method

In Figure 3 the limits of the effective reinforcement ratio of a model loaded by tension with three layers of reinforcement are given according to Eurocode 2 and Jones Method. There is a clear distinction noticeable between the lines of Eurocode 2 of 16mm diameter and 30mm cover (lower bound values) and the lines of 40mm and 100mm cover (higher bound values). This is due to the fact that the outcomes of different thicknesses lie on the exact same spot. Which implies that effective reinforcement ratio is determined by the effective height formula which is dependent on the cover and diameter. The results of different input parameters will lie between the lower and higher limit of Eurocode 2 which are given in this figure. The horizontal lines are upper limits of unrealistic situations with small thicknesses and large bar diameters. These lines can be ignored because they show situations which are not used in practice. Most of the realistic values will lie between the two decreasing lines given by Eurocode 2 with the minimum and maximum parameters.

In addition the effective reinforcement ratio of Jones Method for the minimum and maximum diameter and cover term is added. The effective reinforcement ratio of Jones method is higher than the ones given by Eurocode 2 for both situations. This is due to the fact that the effective concrete tension zone height is smaller for Jones method therefore the effective reinforcement ratio decreases.

E APPENDIX E – PYTHON SCRIPT

In this appendix the python script which has been used for the tensile member models will be given. First different parameters are filled in which can be changed to change the dimensions of the model. Hereafter the script is given which builds the model with different material models, mesh, analysis and output.

```
newProject( 'name', 100 )
import math
Lb=4000
Hb=1000
Bz=200
Dia1=40
Dia2=40
nv=1
nh=0
nkop=1
Sv=100
Sh=0
cy=60
xv=680+Sv
a=20
deform1=0.1
LF="0.1(300)"
LR="ALL"


#Concrete parameters
Ecm=33000
Poison=0.15
Gewicht=2400
Gewicht1=Gewicht*10**-12
fctm=2.9
Gf=0.1405

#wapening parameters
Es=210000
Reku=0.1
Fu=600
NormalBSstiff=1000
ShearBSstiff=50
Taubmax=15.41
Tauf=0.4*Taubmax
s0=0.1
s3=5

setModelAnalysisAspects( [ "STRUCT" ] )
setModelDimension( "2D" )
setDefaultMeshOrder( "QUADRATIC" )
setDefaultMesherType( "HEXQUAD" )
setDefaultMidSideNodeLocation( "LINEAR" )
setUnit( "LENGTH", "MM" )
setUnit( "FORCE", "N" )

createSheet( "Beton", [[ 0, 0, 0 ],[ Lb, 0, 0 ],[ Lb, Hb, 0 ],[ 0, Hb, 0 ]]
)
```

```
addSet(SHAPESET, "Wapening")
setCurrentShapeSet ("Wapening")

createLine( "Wapening1", [ 0, cy, 0 ], [ Lb, cy, 0 ] )

addSet( GEOMETRYSUPPORTSET, "Vertical" )
createPointSupport( "Vertical", "Vertical" )
setParameter( GEOMETRYSUPPORT, "Vertical", "AXES", [ 1, 2 ] )
setParameter( GEOMETRYSUPPORT, "Vertical", "TRANSL", [ 0, 1, 0 ] )
setParameter( GEOMETRYSUPPORT, "Vertical", "ROTATI", [ 0, 0, 0 ] )
attach( GEOMETRYSUPPORT, "Vertical", "Wapening1", [[ 0, cy, 0 ],[ Lb, cy, 0
]] )

addSet( GEOMETRYSUPPORTSET, "Load support" )
createPointSupport( "Load support", "Load support" )
setParameter( GEOMETRYSUPPORT, "Load support", "AXES", [ 1, 2 ] )
setParameter( GEOMETRYSUPPORT, "Load support", "TRANSL", [ 1, 0, 0 ] )
setParameter( GEOMETRYSUPPORT, "Load support", "ROTATI", [ 0, 0, 1 ] )
attach( GEOMETRYSUPPORT, "Load support", "Wapening1", [[ Lb, cy, 0 ]] )

addSet( GEOMETRYSUPPORTSET, "Horizontal X" )
createPointSupport( "Horizontal X", "Horizontal X" )
setParameter( GEOMETRYSUPPORT, "Horizontal X", "AXES", [ 1, 2 ] )
setParameter( GEOMETRYSUPPORT, "Horizontal X", "TRANSL", [ 1, 0, 0 ] )
setParameter( GEOMETRYSUPPORT, "Horizontal X", "ROTATI", [ 0, 0, 1 ] )
attach( GEOMETRYSUPPORT, "Horizontal X", "Wapening1", [[ 0, cy, 0 ]] )

addSet( GEOMETRYLOADSET, "Deformation" )
createPointLoad( "Deformation", "Deformation" )
setParameter( GEOMETRYLOAD, "Deformation", "LODTYP", "DEFORM" )
setParameter( GEOMETRYLOAD, "Deformation", "DEFORM/TR/VALUE", 0.1 )
setParameter( GEOMETRYLOAD, "Deformation", "DEFORM/TR/DIRECT", 1 )
attach( GEOMETRYLOAD, "Deformation", "Wapening1", [[ Lb, cy, 0 ]] )

if nkop>0:
   arrayCopy(["Wapening1"], [0, xv, 0], [0, 0, 0], [0, 0, 0], 1)

   createPointSupport( "Load support4", "Load support" )
   setParameter( GEOMETRYSUPPORT, "Load support4", "AXES", [ 1, 2 ] )
   setParameter( GEOMETRYSUPPORT, "Load support4", "TRANSL", [ 1, 0, 0 ] )
   setParameter( GEOMETRYSUPPORT, "Load support4", "ROTATI", [ 0, 0, 1 ] )
   attach( GEOMETRYSUPPORT, "Load support4", "Wapening2", [[ Lb, cy+xv, 0
]] )

   createPointSupport( "Horizontal X4", "Horizontal X" )
   setParameter( GEOMETRYSUPPORT, "Horizontal X4", "AXES", [ 1, 2 ] )
   setParameter( GEOMETRYSUPPORT, "Horizontal X4", "TRANSL", [ 1, 0, 0 ] )
   setParameter( GEOMETRYSUPPORT, "Horizontal X4", "ROTATI", [ 0, 0, 1 ] )
   attach( GEOMETRYSUPPORT, "Horizontal X4", "Wapening2", [[ 0, cy+xv, 0 ]]
)

   attach( GEOMETRYLOAD, "Deformation", "Wapening2", [[ Lb, cy+xv, 0 ]] )
group_Wapening=[]
group_Wapening=namesIn (SHAPESET, "Wapening")

if nv>0:
   addSet(SHAPESET, "Wap2")
   setCurrentShapeSet ("Wap2")
```

```
    createLine( "Wap1", [ 0, cy+Sv, 0 ], [ Lb, cy+Sv, 0 ] )

    attach( GEOMETRYLOAD, "Deformation", "Wap1", [[ Lb, cy+Sv, 0 ]] )

    createPointSupport( "Load support2", "Load support" )
    setParameter( GEOMETRYSUPPORT, "Load support2", "AXES", [ 1, 2 ] )
    setParameter( GEOMETRYSUPPORT, "Load support2", "TRANSL", [ 1, 0, 0 ] )
    setParameter( GEOMETRYSUPPORT, "Load support2", "ROTATI", [ 0, 0, 1 ] )
    attach( GEOMETRYSUPPORT, "Load support2", "Wap1", [[ Lb, cy+Sv, 0 ]] )

    createPointSupport( "Horizontal X2", "Horizontal X" )
    setParameter( GEOMETRYSUPPORT, "Horizontal X2", "AXES", [ 1, 2 ] )
    setParameter( GEOMETRYSUPPORT, "Horizontal X2", "TRANSL", [ 1, 0, 0 ] )
    setParameter( GEOMETRYSUPPORT, "Horizontal X2", "ROTATI", [ 0, 0, 1 ] )
    attach( GEOMETRYSUPPORT, "Horizontal X2", "Wap1", [[ 0, cy+Sv, 0 ]] )


    if nkop > 0:
        arrayCopy(["Wap1"], [0, xv, 0], [0, 0, 0], [0, 0, 0], 1)
        attach(GEOMETRYLOAD, "Deformation", "Wap2", [[Lb, cy + Sv+xv, 0]])

        createPointSupport("Load support5", "Load support")
        setParameter(GEOMETRYSUPPORT, "Load support5", "AXES", [1, 2])
        setParameter(GEOMETRYSUPPORT, "Load support5", "TRANSL", [1, 0, 0])
        setParameter(GEOMETRYSUPPORT, "Load support5", "ROTATI", [0, 0, 1])
        attach(GEOMETRYSUPPORT, "Load support5", "Wap2", [[Lb, cy + Sv + xv,
0]])

        createPointSupport("Horizontal X5", "Horizontal X")
        setParameter(GEOMETRYSUPPORT, "Horizontal X5", "AXES", [1, 2])
        setParameter(GEOMETRYSUPPORT, "Horizontal X5", "TRANSL", [1, 0, 0])
        setParameter(GEOMETRYSUPPORT, "Horizontal X5", "ROTATI", [0, 0, 1])
        attach(GEOMETRYSUPPORT, "Horizontal X5", "Wap2", [[0, cy + Sv + xv,
0]])


    group_Wap=[]
    group_Wap=namesIn (SHAPESET, "Wap2")

if nv>1:
    addSet(SHAPESET, "Reinf3")
    setCurrentShapeSet ("Reinf3")

    createLine( "Reinf1", [ 0, cy+2*Sv, 0 ], [ Lb, cy+2*Sv, 0 ] )

    attach( GEOMETRYLOAD, "Deformation", "Reinf1", [[ Lb, cy+2*Sv, 0 ]] )

    createPointSupport( "Load support3", "Load support" )
    setParameter( GEOMETRYSUPPORT, "Load support3", "AXES", [ 1, 2 ] )
    setParameter( GEOMETRYSUPPORT, "Load support3", "TRANSL", [ 1, 0, 0 ] )
    setParameter( GEOMETRYSUPPORT, "Load support3", "ROTATI", [ 0, 0, 1 ] )
    attach( GEOMETRYSUPPORT, "Load support3", "Reinf1", [[ Lb, cy+2*Sv, 0 ]]
)

    createPointSupport( "Horizontal X3", "Horizontal X" )
    setParameter( GEOMETRYSUPPORT, "Horizontal X3", "AXES", [ 1, 2 ] )
    setParameter( GEOMETRYSUPPORT, "Horizontal X3", "TRANSL", [ 1, 0, 0 ] )
```

```
   setParameter( GEOMETRYSUPPORT, "Horizontal X3", "ROTATI", [ 0, 0, 1 ] )
   attach( GEOMETRYSUPPORT, "Horizontal X3", "Reinf1", [[ 0, cy+2*Sv, 0 ]]
)


   if nkop > 0:
      arrayCopy(["Reinf1"], [0, xv, 0], [0, 0, 0], [0, 0, 0], 1)

      attach(GEOMETRYLOAD, "Deformation", "Reinf2", [[Lb, cy + 2 * Sv, 0]])

      createPointSupport("Load support6", "Load support")
      setParameter(GEOMETRYSUPPORT, "Load support6", "AXES", [1, 2])
      setParameter(GEOMETRYSUPPORT, "Load support6", "TRANSL", [1, 0, 0])
      setParameter(GEOMETRYSUPPORT, "Load support6", "ROTATI", [0, 0, 1])
      attach(GEOMETRYSUPPORT, "Load support6", "Reinf2", [[Lb, xv + cy + 2
* Sv, 0]])

      createPointSupport("Horizontal X6", "Horizontal X")
      setParameter(GEOMETRYSUPPORT, "Horizontal X6", "AXES", [1, 2])
      setParameter(GEOMETRYSUPPORT, "Horizontal X6", "TRANSL", [1, 0, 0])
      setParameter(GEOMETRYSUPPORT, "Horizontal X6", "ROTATI", [0, 0, 1])
      attach(GEOMETRYSUPPORT, "Horizontal X6", "Reinf2", [[0, xv + cy + 2 *
Sv, 0]])


   group_Reinf=[]
   group_Reinf=namesIn (SHAPESET, "Reinf3")

addMaterial( "Beton", "CONCR", "TSCR", [] )
setParameter( MATERIAL, "Beton", "LINEAR/ELASTI/YOUNG", Ecm )
setParameter( MATERIAL, "Beton", "LINEAR/ELASTI/YOUNG", Ecm )
setParameter( MATERIAL, "Beton", "LINEAR/ELASTI/POISON", Poison )
setParameter( MATERIAL, "Beton", "LINEAR/ELASTI/POISON", Poison )

setUnit( "LENGTH", "M" )
setUnit( "MASS", "KG" )

setParameter( MATERIAL, "Beton", "LINEAR/MASS/DENSIT", Gewicht )

setUnit( "LENGTH", "MM" )
setUnit( "MASS", "T" )

setParameter( MATERIAL, "Beton", "LINEAR/MASS/DENSIT", Gewicht1 )
setParameter( MATERIAL, "Beton", "MODTYP/TOTCRK", "ROTATE" )
setParameter( MATERIAL, "Beton", "TENSIL/TENCRV", "HORDYK" )
setParameter( MATERIAL, "Beton", "TENSIL/TENSTR", fctm )
setParameter( MATERIAL, "Beton", "TENSIL/GF1", Gf )
setParameter( MATERIAL, "Beton", "TENSIL/POISRE/POIRED", "DAMAGE" )

addGeometry( "Element geometry 2", "SHEET", "MEMBRA", [] )
rename( GEOMET, "Element geometry 2", "geo_beton" )
setParameter( GEOMET, "geo_beton", "THICK", Bz )
setElementClassType( SHAPE, [ "Beton" ], "MEMBRA" )
assignGeometry( "geo_beton", SHAPE, [ "Beton" ] )
assignMaterial( "Beton", SHAPE, [ "Beton" ] )
addElementData( "Beton" )
assignElementData( "Beton", SHAPE, [ "Beton" ] )
```

```
addMaterial( "mat_Wapening", "REINFO", "REBOND", [] )
setParameter( MATERIAL, "mat_Wapening", "REBARS/ELASTI/YOUNG", Es )
setParameter( MATERIAL, "mat_Wapening", "REBARS/MASS/DENSIT", 7.85e-09 )
setParameter( MATERIAL, "mat_Wapening", "REBARS/PLATYP", "VMISES" )
setParameter( MATERIAL, "mat_Wapening", "REBARS/PLASTI/TRESSH", "KAPSIG" )
setParameter( MATERIAL, "mat_Wapening", "REBARS/PLATYP", "VMISES" )
setParameter( MATERIAL, "mat_Wapening", "REBARS/PLASTI/KAPSIG", [ 0, 500,
Reku, Fu] )
setParameter( MATERIAL, "mat_Wapening", "RESLIP/DSNY", NormalBSstiff )
setParameter( MATERIAL, "mat_Wapening", "RESLIP/DSSX", ShearBSstiff )
setParameter( MATERIAL, "mat_Wapening", "RESLIP/SHFTYP", "BONDS6" )
setParameter( MATERIAL, "mat_Wapening", "RESLIP/BONDS6/SLPVAL", [ Taubmax,
Tauf, s0, s1, 2, s3, 0.4 ] )

addGeometry( "Element geometry 1", "RELINE", "REBAR", [] )
rename( GEOMET, "Element geometry 1", "geo_Wapening" )
setParameter( GEOMET, "geo_Wapening", "REITYP", "CIRBEA" )
setParameter( GEOMET, "geo_Wapening", "CIRBEA/CIRCLE", Dia1 )
addElementData( "data_Wapening" )

setReinforcementAspects( group_Wapening )
assignMaterial( "mat_Wapening", SHAPE, group_Wapening )
assignGeometry( "geo_Wapening", SHAPE, group_Wapening )
assignElementData( "data_Wapening", SHAPE, group_Wapening )
setReinforcementDiscretization( group_Wapening , "ELEMENT" )

setParameter( DATA, "Beton", "INTEGR", "HIGH" )
setParameter( DATA, "data_Wapening", "INTERF", "BEAM" )


if nv>0:
   addGeometry( "Element geometry 2", "RELINE", "REBAR", [] )
   rename( GEOMET, "Element geometry 2", "geo_Wap2" )
   setParameter(GEOMET, "geo_Wap2", "REITYP", "CIRBEA")
   setParameter(GEOMET, "geo_Wap2", "CIRBEA/CIRCLE", Dia2)
   setReinforcementAspects( group_Wap )
   assignMaterial( "mat_Wapening", SHAPE, group_Wap )
   assignGeometry( "geo_Wap2", SHAPE, group_Wap )
   assignElementData( "data_Wapening", SHAPE, group_Wap )
   setReinforcementDiscretization( group_Wap, "ELEMENT" )

if nv>1:
   addGeometry( "Element geometry 3", "RELINE", "REBAR", [] )
   rename( GEOMET, "Element geometry 3", "geo_Reinf3" )
   setParameter( GEOMET, "geo_Reinf3", "REITYP", "REITRU" )
   setParameter( GEOMET, "geo_Reinf3", "REITRU/CROSSE", As1 )
   setParameter( GEOMET, "geo_Reinf3", "REITRU/PERIME", peri1 )
   setReinforcementAspects( group_Reinf )
   assignMaterial( "mat_Wapening", SHAPE, group_Reinf )
   assignGeometry( "geo_Reinf3", SHAPE, group_Reinf )
   assignElementData( "data_Wapening", SHAPE, group_Reinf )
   setReinforcementDiscretization( group_Reinf, "ELEMENT" )

setElementSize( [ "Beton"], a, -1, True )
setMesherType( [ "Beton"], "HEXQUAD" )
setMidSideNodeLocation( [ "Beton" ], "LINEAR" )
```

```
for i in group_Wapening:
   setElementSize( [ i], a, -1, True )
   setMesherType( [ i], "HEXQUAD" )
   setMidSideNodeLocation( [ i], "LINEAR" )
if nv>0:
   for i in group_Wap:
      setElementSize( [ i], a, -1, True )
      setMesherType( [ i], "HEXQUAD" )
      setMidSideNodeLocation( [ i], "LINEAR")
if nv>1:
   for i in group_Reinf:
      setElementSize( [ i], a, -1, True )
      setMesherType( [ i], "HEXQUAD" )
      setMidSideNodeLocation( [ i], "LINEAR" )

generateMesh( [] )
hideView( "GEOM" )
showView( "MESH" )

addAnalysis( "Analysis a mm" )

addAnalysisCommand( "Analysis a mm", "NONLIN", "Structural nonlinear" )
renameAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"EXECUT(1)", "Load" )
setAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"EXECUT(1)/LOAD/STEPS/EXPLIC/SIZES", LF )
addAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"EXECUT(1)/LOAD/LOADNR" )
setAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"EXECUT(1)/LOAD/LOADNR", 0 )
setAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"EXECUT(1)/LOAD/LOADNR", 1 )
setAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"EXECUT(1)/ITERAT/METHOD/METNAM", "NEWTON" )
setAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"EXECUT(1)/ITERAT/MAXITE", 30 )
addAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"EXECUT(1)/ITERAT/LINESE" )
setAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"EXECUT(1)/ITERAT/LINESE", True )
addAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"EXECUT(1)/ITERAT/CONVER/ENERGY" )
setAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"EXECUT(1)/ITERAT/CONVER/ENERGY", False )
setAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"EXECUT(1)/ITERAT/CONVER/DISPLA", True )
setAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"EXECUT(1)/ITERAT/CONVER/DISPLA/NOCONV", "CONTIN" )
setAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"EXECUT(1)/ITERAT/CONVER/FORCE/NOCONV", "CONTIN" )
setAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"OUTPUT(1)/SELTYP", "USER" )
addAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"OUTPUT(1)/USER" )
addAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"OUTPUT(1)/USER/DISPLA(1)/TOTAL/TRANSL/GLOBAL" )
addAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"OUTPUT(1)/USER/STRAIN(1)/TOTAL/GREEN/PRINCI" )
```

```
setAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"OUTPUT(1)/USER/STRAIN(1)/TOTAL/GREEN/PRINCI/LOCATI", "INTPNT" )
addAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"OUTPUT(1)/USER/STRAIN(2)/TOTAL/GREEN/GLOBAL" )
setAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"OUTPUT(1)/USER/STRAIN(2)/TOTAL/GREEN/GLOBAL/LOCATI", "INTPNT" )
addAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"OUTPUT(1)/USER/STRAIN(3)/TOTAL/TRACTI/LOCAL" )
setAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"OUTPUT(1)/USER/STRAIN(3)/TOTAL/TRACTI/LOCAL/LOCATI", "INTPNT" )
addAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"OUTPUT(1)/USER/STRAIN(6)/CRKWDT/GREEN/PRINCI" )
setAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"OUTPUT(1)/USER/STRAIN(6)/CRKWDT/GREEN/PRINCI/LOCATI", "INTPNT" )
addAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"OUTPUT(1)/USER/STRESS(1)/TOTAL/CAUCHY/GLOBAL" )
setAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"OUTPUT(1)/USER/STRESS(1)/TOTAL/CAUCHY/GLOBAL/LOCATI", "INTPNT" )
addAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"OUTPUT(1)/USER/STRESS(2)/TOTAL/CAUCHY/PRINCI" )
setAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"OUTPUT(1)/USER/STRESS(2)/TOTAL/CAUCHY/PRINCI/LOCATI", "INTPNT" )
addAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"OUTPUT(1)/USER/STRESS(4)/TOTAL/CAUCHY/LOCAL" )
setAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"OUTPUT(1)/USER/STRESS(4)/TOTAL/CAUCHY/LOCAL/LOCATI", "INTPNT" )
addAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"OUTPUT(1)/USER/STRESS(3)/TOTAL/TRACTI/LOCAL" )
setAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"OUTPUT(1)/USER/STRESS(3)/TOTAL/TRACTI/LOCAL/LOCATI", "INTPNT" )
addAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"OUTPUT(1)/USER/FORCE(1)/REACTI/TRANSL/GLOBAL" )
addAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"OUTPUT(1)/USER/STRAIN(8)/PLASTI/GREEN/GLOBAL" )
setAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"OUTPUT(1)/USER/STRAIN(8)/PLASTI/GREEN/GLOBAL/LOCATI", "INTPNT" )
setAnalysisCommandDetail( "Analysis a mm", "Structural nonlinear",
"OUTPUT(1)/SELECT/STEPS/RNGNRS", LR )

runSolver( "Analysis a mm" )
showView( "RESULT" )


saveProjectAs( 'name.dpf' )
```