Shear centre eccentricity in SCIA Engineer

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Preface

This thesis is the final project for the Bachelor Civil Engineering at the Delft University of Technology, where I investigated an error in the framework analysis software *SICA engineer* regarding shear centre eccentricity.

I want to thank dr. ir. P.C.J. Hoogenboom for his support throughout this project. His experience and knowledge were extremely helpful. I also want to thank dr. M. Veljkovic for helping with the assessment. Finally, I want to thank Simon Top for turning my sketches into wonderful illustrations.

Delft, June 13, 2021
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Summary

Torsional moments and shear stresses are not calculated correctly in the framework analysis software *SCIA Engineer* for cross-sections with a shear centre eccentricity. The aim of this research is to analyze and solve this problem. This thesis is composed of five chapters. Chapter 1 is introductory and describes the solution strategy and observations done by others. Chapter 2 compares hand calculations with SCIA and also with the engineering software Ansys. All calculations are executed using a cantilever beam with a channel section. Chapter 3 dives deeper into the calculation approach in SCIA to get a better understanding of where the error arises within the program. Chapter 4 is an analysis of the safety risks. Conclusions are drawn in Chapter 5.

It was established that Ansys calculates the correct results for the cantilever beam with a shear centre eccentricity, whereas SCIA does not. SCIA has an option called 'consider torsion due to shear centre eccentricity' which was found to be working correctly, also for statically indeterminate beams. However, the option can be confusing and is hidden within the calculation results. Also, it can only report shear stresses and no torsional moments or rotations. The author suggests that SCIA should move the system line to the normal force centre. The self weight has to be applied in the normal force centre as well.

Contents

Chapter 1

Introduction

Beams and columns in constructions can be subjected to torsional moments. It was found that these moments are not calculated correctly in the framework analysis software *SCIA Engineer* (Drillenburg, 2017). The software engineers were notified, but the error was not removed. In this project we will continue the research on this problem.

1.1 The problem

We want to investigate what exactly goes wrong with the calculations in SCIA. Therefore, we first need to look into the problem in more detail.

Gravity acts through the normal force centre of a cross-section[1]. In cross-sections with only one axis of symmetry the normal force centre does not coincide with the shear centre. Forces which do not act through the shear centre will result in torsion. A cantilever with a channel section will therefore rotate around its axis (Welleman, 2021). In SCIA however, this does not happen.

To investigate where the error arises within the program, Drillenburg considered three parameters to check for possible discrepancies: geometry of cross-sections, different boundary conditions and different load cases. It was established that the problems with torsion only occur if there is an eccentricity of the shear centre.

1.2 Solution strategy

We will consider a channel section acting as a cantilever beam for every calculation throughout this research. The dimensions are given in Figure 1.1 and are based on an UPE 220 steel profile, the length of the beam is 5 meters. The material properties are listed in Table 1.1. In the next chapter we will compare hand calculations with SCIA and another software package *Ansys*. Chapter 3 will revolve around the computational method SCIA uses and offers possible solutions for the error. Finally we will investigate if this error could potentially lead to dangerous situations.

[1]Note that the normal force centre differs from the centre of gravity for an inhomogeneous cross-section. It is defined as the location where the normal force only causes strains and no curvatures.

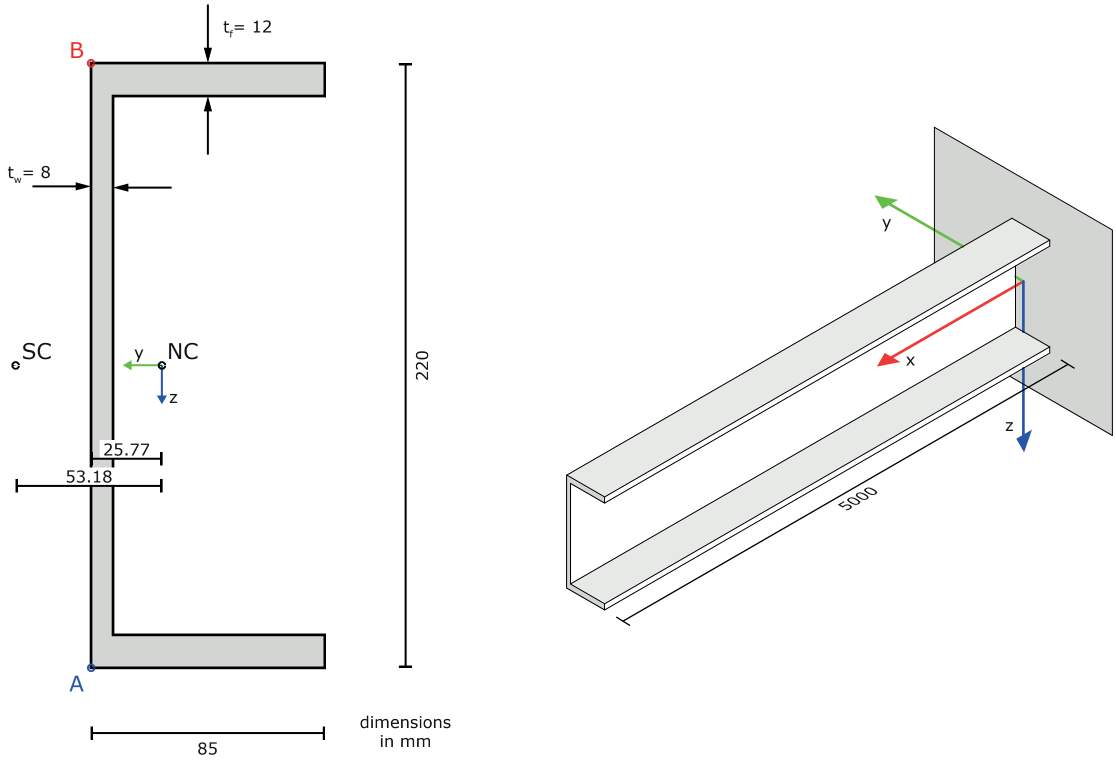


Figure 1.1: Dimensions of a channel section

Table 1.1: Material properties

Parameter	Symbol	Value	Unit
Density	ρ	7850	kg/m^3
Young's Modulus	E	210	GPa
Shear Modulus	G	80769	MPa
Yield Strength	σ_y	235	MPa
Ultimate Tensile Strength	σ_{ult}	360	MPa

1.3 Work by others

Drillenburg used an L-shaped cross-section for his research. He looked at SCIA's response to different geometries, boundary conditions and load cases. His findings are summarized and later on used for comparison.

SCIA's behavior according to Drillenburg is summarized below. In the next chapter we will check if these observations are valid or not.

Geometry

- Cross-sections which are symmetrical in two axes show no discrepancies compared with the expected values.
- Cross-sections which are symmetrical in one axis show no significant discrepancies compared with the expected values, only the position of the shear centre differs slightly.

Boundary Conditions

- A single clamped cantilever beam loaded in pure torsion shows no discrepancies compared with the expected values.
- A double clamped beam loaded in pure torsion shows no discrepancies compared with the expected values. This is also true if the torsional moment is applied at different positions along the beam.

Loading

- A beam loaded in pure bending shows no discrepancies compared with the expected values.
- A beam loaded with a point load in the normal force centre shows no torsional deformation and no internal moment around the x-axis. The shear stress in this situation did not match the expected value.
- A beam loaded with a point load in the shear centre shows torsional deformation, whereas pure bending is expected.

Additional

- Using the option 'consider torsion due to shear centre eccentricity' does not give the correct magnitude for the torsional shear stress.

Chapter 2

Comparison

In this chapter we will compare hand calculations with SICA's output and calculations made with the engineering simulation software *Ansys*. Two situations will be considered: one where the self weight acts through the normal force centre and one where an equivalent line load acts through the shear centre of the channel section. All hand calculations are made using Maple and can be found in Appendix A.

2.1 Cross-sectional properties

From Appendix A.1 we obtain the cross-sectional properties. These properties are also reported in Table 2.1. The cross-sectional properties that are obtained using SCIA and Ansys are listed in this table as well. The location of the normal force centre is given with respect to **point A** in Figure 1.1 using a coordinate system as indicated in the same figure. The location of the shear centre is given with respect to the normal force centre and is obtained using Appendix A.2.

Table 2.1: Cross-sectional properties

	Hand calculation	**SCIA**	**Ansys**
NC	(25.77 mm; -110 mm)	(25.77 mm; 110 mm)	(25.77 mm; 110 mm)
A	3608 mm^2	3608 mm^2	3608 mm^2
I_{yy}	$2.55 \cdot 10^6$ mm^4	$2.55 \cdot 10^6$ mm^4	$2.55 \cdot 10^6$ mm^4
I_{zz}	$2.71 \cdot 10^7$ mm^4	$2.71 \cdot 10^7$ mm^4	$2.71 \cdot 10^7$ mm^4
I_{yz}	0	0	0
I_t	$1.24 \cdot 10^5$ mm^4	$1.25 \cdot 10^5$ mm^4	$1.26 \cdot 10^5$ mm^4
SC	(53.18 mm; 0)	(52.89 mm; 0)	(52.87 mm; 0)

The only noticeable differences in the table are the values for the torsional constant and the location of the shear centre. The torsional constant I_t for a channel section can be calculated using approximation equations. Applying a simple thin-walled approach would result in a value of $1.35 \cdot 10^5$ mm^4 which is rather high compared to the results of SCIA and Ansys. Using an extensive formula from the book *Roark's formulas for stress and strain* gives a more pleasant result as can be seen in the table. The difference in the location of the shear centre is a result of a different calculation approach. SCIA and Ansys calculate this location using the Finite Element method. For the hand calculation, a simple case of equilibrium is assumed (Appendix A.2).

2.2 Hand calculation

The hand calculation starts with the determination of the cross-sectional properties which has been done in the previous section. Additional to the cross-sectional properties from Table 2.1, the self weight of the beam is needed which can be calculated using Equation 2.1.

$$q_z = A \cdot \rho \cdot g = 3608 \cdot 7850 \cdot 9.81 \cdot 10^{-9} = 0.278\,\text{N/mm} \tag{2.1}$$

If the self weight would act through the shear centre, no torsional moment and rotation can occur. The deflection of the beam can then be calculated using one of the *vergeet-mij-nietjes* ('forget-me-nots') for a single clamped beam, since it will only deflect in the z-direction (Hartsuijker, 2016).

$$u_z = \frac{q_z \cdot l^4}{8EI_{zz}} = \frac{0.278 \cdot 5000^4}{8 \cdot 5.69 \cdot 10^{12}} = 3.81\,\text{mm} \tag{2.2}$$

In reality, the self weight of the beam acts through the centroid which coincides with the normal force centre in this case. We know that for the channel section the normal force centre and shear centre do not coincide, therefore a torsional moment will act on the beam. The maximum torsional moment at the clamped edge can be calculated by multiplying the self weight with the length of the beam and subsequently with the shear centre eccentricity. The beam will now rotate around the x-axis due to the presence of the torsional moment. The maximum rotation of a cantilever beam loaded with an uniform torque can be calculated using Equation 2.3 (James F. Lincoln Arc Welding Foundation, 2021).

$$\varphi_x = \frac{M_x \cdot l}{2 \cdot GI_t} = \frac{-7.39 \cdot 10^4 \cdot 5000}{2 \cdot 80769 \cdot 1.24 \cdot 10^5} = -18.45\,\text{mrad} \tag{2.3}$$

The beam will now also deflect in the y-direction because of the rotation. The total deflection in the y- and z-direction can be calculated by adding the contribution of the rotation to the earlier calculated deflections for pure bending. Small rotations are assumed and axial deformation is neglected.

The results of the hand calculation are listed in Table 2.2. The displacements are given with respect to **point B** in Figure 1.1. The reader is encouraged to consult Appendix A for more detailed calculations.

Table 2.2: Hand calculation results

	Loading in NC	**Loading in SC**
u_y	-2.03 mm	0
u_z	4.32 mm	3.81 mm
φ_x	-18.45 mrad	0
M_x	$-7.39 \cdot 10^4$ Nmm	0
τ_{tor}	7.15 N/mm^2	0

2.3 Results in SCIA

We construct the channel section in SCIA and change the local coordinate system to the same coordinate system that has been used for the hand calculation. We then add the self weight as a line load to the beam so the position of this load can be changed. We emulate the load in the shear centre by disabling the self weight of the structure and moving the line load to the shear centre using an offset. SCIA can now calculate and graphically display the displacements and stresses. For calculating the shear stress, the option 'consider torsion due to shear centre eccentricity' is left unchecked for both load cases. The maximum value of the torsional shear stress with this option enabled is given between parenthesis in Table 2.3.

Table 2.3: SCIA results

	Loading in NC	**Loading in SC**
u_y	0	2.01 mm
u_z	3.84 mm	4.33 mm
φ_x	0	18.20 mrad
M_x	0	$7.35 \cdot 10^4$ Nmm
τ_{tor}	0 (-7.06 N/mm^2)	7.06 N/mm^2 (0)

We want to check Drillenburg's statement that using the option 'consider torsion due to shear centre eccentricity' gives incorrect torsional shear stresses. By comparing Table 2.2 with Table 2.3 we can conclude that this is not the case for a cantilever beam with a channel section. We will now add a fixed support at the other end of the beam to make it statically indeterminate and redo the calculation in SCIA.

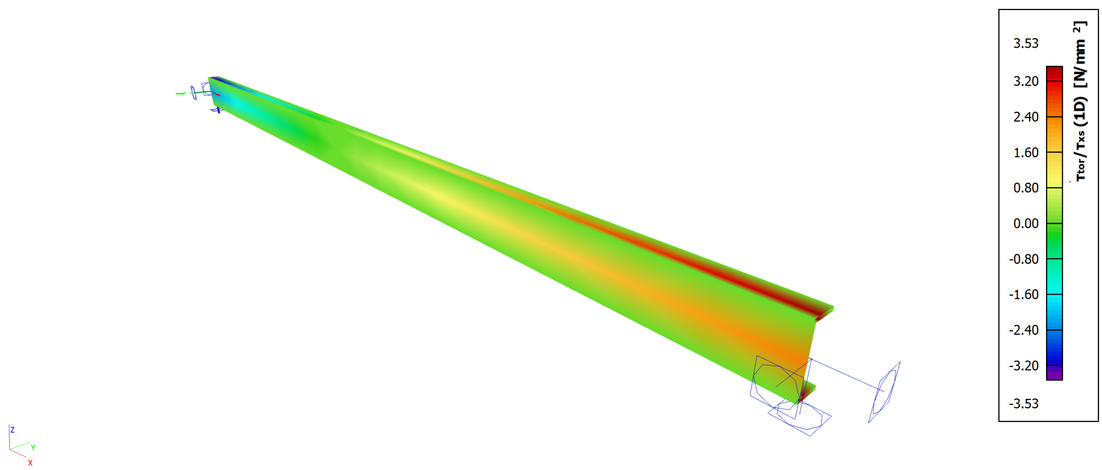


Figure 2.1: Torsional shear stress due to the self weight with the option 'consider torsion due to shear centre eccentricity' enabled

From Figure 2.1 we can observe that the maximum torsional shear stress at the flange equals 3.53 N/mm^2 and at the centre of the web 2.35 N/mm^2.

To obtain the torsional stress at the clamped edges using a hand calculation, we use Equation A.13 and A.14 from Appendix A.2 and take $\frac{1}{2}M_x$ for M_t.

$$\tau_{f;max} = \left|\frac{\frac{1}{2}M_x \cdot t_f}{I_t}\right| = \left|\frac{\frac{1}{2} \cdot -7.39 \cdot 10^4 \cdot 12}{1.24 \cdot 10^5}\right| = 3.58\,\mathrm{N/mm^2} \tag{2.4}$$

$$\tau_{w;max} = \left|\frac{\frac{1}{2}M_x \cdot t_w}{I_t}\right| = \left|\frac{\frac{1}{2} \cdot -7.39 \cdot 10^4 \cdot 8}{1.24 \cdot 10^5}\right| = 2.38\,\mathrm{N/mm^2} \tag{2.5}$$

These values therefore match with SCIA's output[2]. This strongly suggests that Drillenburg's statement is incorrect.

We can now conclude the following regarding SCIA's calculations:

- SCIA reports no torsional moment and no rotation when the system is loaded in the normal force centre.
- SCIA reports a torsional moment and a rotation when the system is loaded in the shear centre whereas pure bending is expected.
- The reported rotation is positive which indicates that the system rotates anti-clockwise, see Figure 2.2. This is due to the fact that the line load is now physically moved to the shear centre and is treated as an eccentric load.
- Using the option 'consider torsion due to shear centre eccentricity' when loading the system in the normal force centre results in the same torsional shear stresses as can be found when moving a line load to the shear centre, but with an opposite sign. Using this option when loading the system in the shear centre results in zero stress. It was established that the option also works correctly for a statically indeterminate beam.

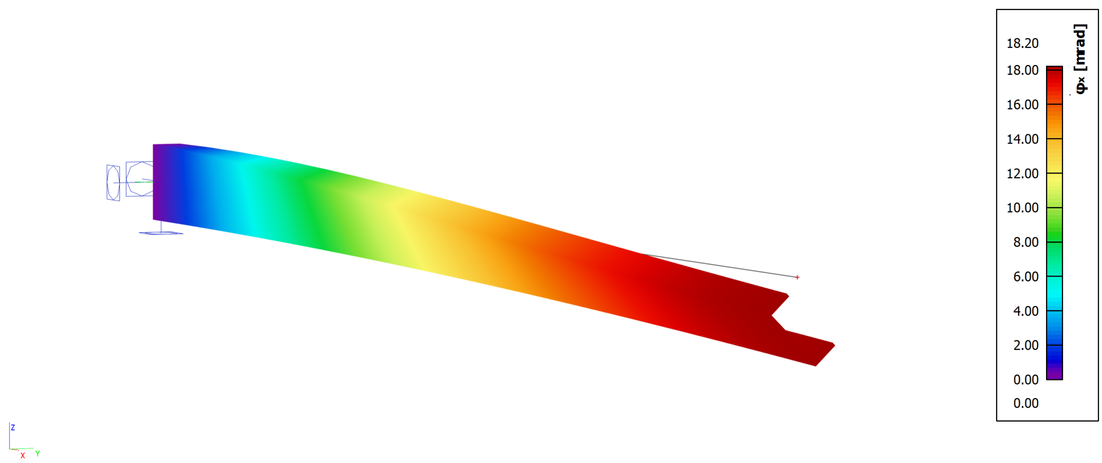


Figure 2.2: Axial rotation due to a line load in the shear centre in SCIA

[2]Note that the decimals are slightly different, because the torsional constant and shear centre coordinates differ as well (Table 2.1).

One last point to mention about the option 'consider torsion due to shear centre eccentricity' is that it is only applicable for finding stresses and strains and not displacements or rotations. In Chapter 3 we will try to solve this problem, but we will first look at the results in Ansys.

2.4 Results in Ansys

We will now analyse the same cantilever beam with Ansys Workbench. Ansys can model solids, but also beam elements which was used for this calculation. The same approach as with SCIA is applied. In order to have Ansys report the wanted results, it is important to enable the post-processing option 'Beam Section Results'. The maximum rotation is obtained by using a rotational probe at the end of the beam, the outcome is then converted from degrees to radians.

Table 2.4: Ansys results

	Loading in NC	**Loading in SC**
u_y	-1.95 mm	0
u_z	4.32 mm	3.84 mm
φ_x	-17.72 mrad	0
M_x	$-7.26 \cdot 10^4$ Nmm	0
τ_{tor}	7.00 N/mm^2	0

From Table 2.4 the following can be concluded:

- Ansys reports the displacement, rotation, torsional moment and torsional shear stress as expected for both load cases.
- Ansys reports a smaller torsional moment, rotation and horizontal displacement if the self weight acts through the normal force centre compared to previous calculations. This is due to the fact that the torsional constant and shear centre coordinates are slightly different.

From Figure 2.3 it can be seen that Ansys does report a rotation due to loading in the normal force centre. The beam rotates clockwise which is as expected.

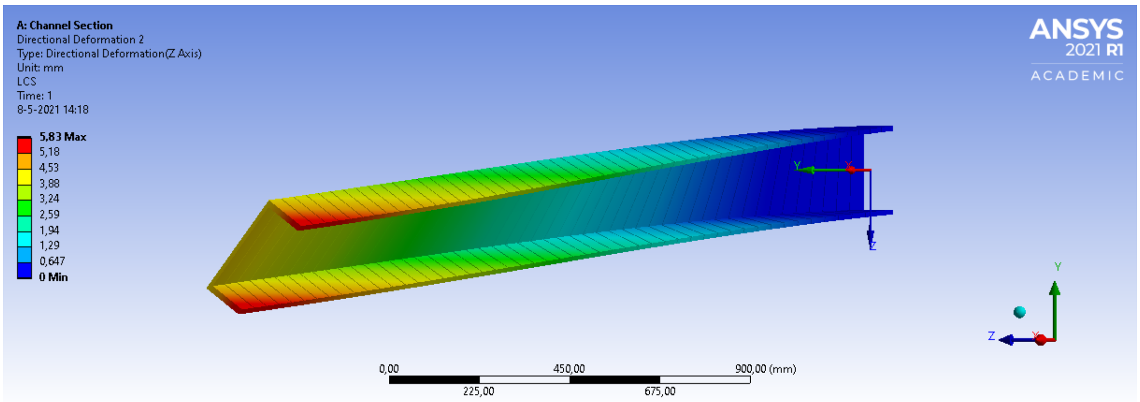


Figure 2.3: Vertical displacement due to the self weight in Ansys

2.5 Conclusion

We looked at the channel section cantilever from Figure 1.1 loaded due to its self weight and compared the calculation results from a hand calculation, SCIA and Ansys. A summary of the results for the normal situation (i.e. self weight through NC) is given in Table 2.5, a summary of the results for an equivalent line load in the shear centre is given in Table 2.6. Again, the output with the option 'consider torsion due to shear centre eccentricity' enabled is given between parentheses.

Table 2.5: Comparison - loading in NC

	Hand calculation	**Ansys**	**SCIA**
u_y	-2.03 mm	-1.95 mm	0
u_z	4.32 mm	4.32 mm	3.84 mm
φ_x	-18.45 mrad	-17.72 mrad	0
M_x	$-7.39 \cdot 10^4$ Nmm	$-7.26 \cdot 10^4$ Nmm	0
τ_{tor}	7.15 N/mm^2	7.00 N/mm^2	0 (-7.06 N/mm^2)

Table 2.6: Comparison - loading in SC

	Hand calculation	**Ansys**	**SCIA**
u_y	0	0	2.01 mm
u_z	3.81 mm	3.84 mm	4.33 mm
φ_x	0	0	18.20 mrad
M_x	0	0	$7.35 \cdot 10^4$ Nmm
τ_{tor}	0	0	7.06 N/mm^2 (0)

The results of Ansys are somewhat different compared to the hand calculation, but this is mainly due to the use of a slightly different torsional constant. The program does show the correct behavior for the system with the axial rotation being clockwise. SCIA however, shows remarkable behavior and this has to be investigated further.

We can observe the following:

- It is possible that SCIA makes the assumption that shear forces always act through the shear centre. This could explain the discrepancies for the considered channel section, because its shear centre does not coincide with the normal force centre.
- Drillenburg concluded that using the option 'consider torsion due to shear centre eccentricity' reports incorrect magnitudes for the torsional shear stresses. This statement is not true for a channel section, as the magnitude for the torsional shear stress matches with the expectation. The option also works correctly for a statically indeterminate beam.

We will use these observations in the next chapter to search for the origin of the problem and the possible solutions.

Chapter 3

Stiffness Method

SCIA incorporates Finite Element technology to create a global stiffness matrix $\overline{K}$ from elemental stiffness matrices. The coefficients in a stiffness matrix link the forces and moments to the displacements and rotations. In this chapter we will construct the stiffness matrix for a three dimensional beam element and use it to compute the unknown displacements and rotations for the cantilever with the channel section. We call this solution procedure the *Stiffness Method*.

3.1 Stiffness matrix

For the establishment of the stiffness matrix, the Euler-Bernoulli beam theory is considered and expanded with the theory of Saint-Venant for torsion. This means that linear elastic behavior is expected and shear deformation and restrained warping are neglected. In SCIA we made the same assumptions when comparing the output to the hand calculation and Ansys.

We will use the following eight steps to construct and solve the stiffness matrix:

1. Define a coordinate system for internal and external forces for a 3D beam element.
2. Define the equations for the beam properties.
3. Set up the differential equations and boundary conditions.
4. Solve the system.
5. Substitute the solution back into the previously defined equations.
6. Set up the forces and moments on both ends of the beam element.
7. Determine the stiffness matrix coefficients by differentiating the loading at both ends of the beam element to the unknown displacements and rotations.
8. Solve the system for the unknown displacements and rotations by applying known forces, moments, displacements and rotations.

A detailed elaboration on this solution strategy is given in Appendix B.1. The corresponding Maple calculation file can be found in Appendix B.2.

3.2 Results

The cross-sectional properties and loading are entered in the Maple calculation file (Appendix B.2). To model the line load, it is split into equivalent nodal loads. This is explained in more detail in Appendix B.1. The results are stored in a table and can be compared with the results from Chapter 2.

Table 3.1: Hand calculation and Stiffness Method

	Hand calculation	**Stiffness Method**
u_y	-2.03 mm	-2.03 mm
u_z	4.32 mm	4.32 mm
φ_x	-18.45 mrad	-18.45 mrad
M_x	$-7.39 \cdot 10^4$ Nmm	$-7.39 \cdot 10^4$ Nmm

Table 3.2: SCIA and Stiffness Method

	SCIA - NC	**SCIA - SC**	**Stiffness Method**
u_y	0	2.01 mm	-2.00 mm
u_z	3.84 mm	4.33 mm	4.31 mm
φ_x	0	18.20 mrad	-18.20 mrad
M_x	0	$7.35 \cdot 10^4$ Nmm	$-7.35 \cdot 10^4$ Nmm

Table 3.1 shows that the hand calculation and stiffness method bring about the same result. When comparing the results of the stiffness method with the SCIA output, it becomes once again clear that SCIA does not report the correct results. Based on this comparison, the following hypothesis is formulated:

"SCIA's stiffness matrix is correct for a system line through the shear centre."

In the maple file we can change the positions of n_y and s_y which are the y-coordinates of the normal force centre and shear centre respectively. The normal force centre is moved with respect to the system line and the shear centre is moved with respect to the normal force centre. However, we need one extra parameter in order to move the load. This is the load eccentricity parameter e_y. The load eccentricity has to be added to the torsional moment equations in the nodes.

$$M_{x1} = -M_x - V_z \cdot (s_y - e_y) + V_y \cdot (s_z - e_z) \tag{3.1}$$

$$M_{x2} = M_x + V_z \cdot (s_y - e_y) - V_y \cdot (s_z - e_z) \tag{3.2}$$

The load eccentricity also has to be added to the loading in step 8. The complete maple file can be found in Appendix B.3.

System line through the normal force centre
We will first look at the standard situation where the system line passes through the normal force centre. This is represented in Figure 3.1 where situation A is the correct situation with the self weight passing through the normal force centre. Situation B is the situation where an equivalent line load is moved to the shear centre. The positions for n_y, s_y and e_y that are entered in the maple file are listed in Table 3.3.

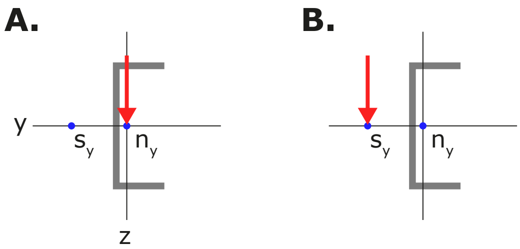


Figure 3.1: System line through the normal force centre

Table 3.3: Coordinates for situation A and B

Coordinate	Situation A	Situation B
n_y	0	0
s_y	52.89	52.89
e_y	0	52.89

The results from the Stiffness Method calculation are given in Table 3.4. We can observe the correct behavior for situation A with the rotation being clockwise. Situation B results in pure bending which is also as expected.

Table 3.4: Situation A and B results

	Situation A	Situation B
u_y	-2.00 mm	0
u_z	4.31 mm	3.81 mm
φ_x	-18.20 mrad	0
M_x	$-7.35 \cdot 10^4$ Nmm	0

System line through the shear centre

It is suspected that SCIA has its system line passing through the shear centre. We alter again the position of n_y, s_y and e_y to model this situation. The self weight now passes through the shear centre in situation C. In situation D, the equivalent line load is given an eccentricity equal to the shear centre eccentricity. These situations are shown in Figure 3.2 with their corresponding coordinates in Table 3.5.

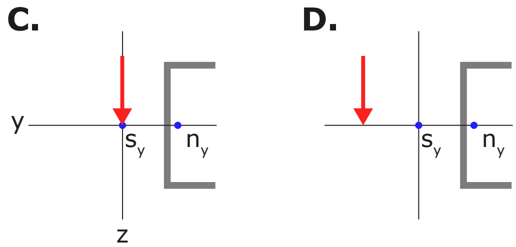


Figure 3.2: System line through the shear centre

Table 3.5: Coordinates for situation C and D

Coordinate	**Situation C**	**Situation D**
n_y	-52.89	-52.89
s_y	52.89	52.89
e_y	52.89	105.78

The results from the Stiffness Method calculation are given in Table 3.6. We can observe pure bending in situation C which is also the case with SCIA's calculation for the cantilever beam loaded due to self weight. Situation D results in an anti-clockwise rotation which also matches with SCIA when placing the equivalent line load over a distance equal to the shear centre eccentricity.

Table 3.6: Situation C and D results

	Situation C	**Situation D**
u_y	0	2.00 mm
u_z	3.81 mm	4.28 mm
φ_x	0	18.20 mrad
M_x	0	$7.35 \cdot 10^4$ Nmm

Load to the right of the shear centre

It is likely that the problem in SCIA originates from the position of the system line. We can do one final check by placing the load on the other side of the shear centre and compare the results with SCIA's output. This situation, situation E, is shown in Figure 3.3. The corresponding coordinates are listed in Table 3.7.

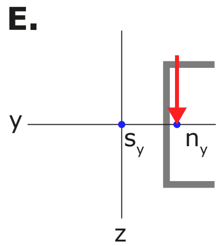


Figure 3.3: System line through the shear centre

Table 3.7: Coordinates for situation E

Coordinate	Situation E
n_y	-52.89
s_y	52.89
e_y	0

We place the equivalent line load in SCIA to the right of the shear centre and report the results for the upper left fibre (point B in Figure 1.1). Comparing the result with situation E shows that they are almost equal. The value for the vertical displacement from SCIA is shown in Figure 3.4[3] with the upper left fibre called 'Linksboven'.

Table 3.8: Situation E and SCIA

	Situation E	SCIA
u_y	-2.00 mm	-2.01 mm
u_z	3.34 mm	3.39 mm
φ_x	-18.20 mrad	-18.20 mrad
M_x	$-7.35 \cdot 10^4$ Nmm	$-7.35 \cdot 10^4$ Nmm

	Name	dx [m]	Fibre	Case	ux [mm]	uy [mm]	uz [mm]	φx [mra...	φy [mra...	φz [mra...	Utotal [m...
333	B1	5,000	11	LC2	0,11	1,99	**-4,93**	-18,20	1,02	0,00	5,32
334	B1	5,000	12	LC2	0,11	2,00	**-3,86**	-18,20	1,02	0,00	4,35
335	B1	5,000	Linksboven	LC2	0,11	2,01	**-3,39**	-18,20	1,02	0,00	3,94
336	B1	5,000	14	LC2	0,00	0,00	**-3,37**	-18,20	1,02	0,00	3,37

Figure 3.4: Tabulated results in SCIA

[3]Note that some signs are different in this table, this has to do with SCIA's sign convention. A negative vertical displacement is for instance downwards.

3.3 Conclusion

SCIA assumes that the system line passes through the shear centre which is remarkable, but theoretically not incorrect. Engineers who know this, could take it into account when working with the software. However, SCIA makes the self weight pass through the shear centre as well. This is probably a mistake caused by the definition of the system line. The error will stay unnoticed if the self weight is small in relation to the loading.

When a beam is assumed to be loaded in the shear centre, it is actually loaded with an offset from the shear centre. This is why the equivalent line load applied at a distance equal to the shear centre eccentricity results in torsion (situation D), whereas pure bending is expected (situation C). To compensate for the error, the option 'consider torsion due to shear centre eccentricity' can be enabled which is a post-processing fix.

Chapter 4

Safety

Now that we know more about the shear centre eccentricity error in SCIA and its possible solution, one question remains: could there be a potentially dangerous situation in the future if the error is not fixed?

4.1 Dangers

There are two problems that arise with the error:

- SCIA only reports pure bending when there is an eccentricity of the shear centre.

  This can be potentially dangerous, because torsional moments can lead to high stresses. Also, the axial rotation will increase the deflection for some parts of the system. If there is an unity check with little capacity left then this could be exceeded due to the extra displacement. In case of the channel section in this thesis, the increase in vertical displacement was more than 10 percent alongside horizontal displacement which was not present at all in case of pure bending.

- Torsional stresses are only reported with the option 'consider torsion due to shear centre eccentricity' enabled.

  One of the biggest issues comes with the shear stresses. In Appendix A.2 the shear stresses are elaborated for the channel section. Here we can see that the torsional shear stresses at the flanges are almost seventeen times bigger than the shear stresses due to the shear force. These stresses will increase with an increase in shear force, but the torsional shear stresses then also increase. This can definitely be problematic as the shear stresses are now much higher than expected which decreases the section's shear capacity. Although enabling the option 'consider torsion due to shear centre eccentricity' will return correct results, the button itself can be easily overlooked.

Apart form the problems mentioned above, showing the incorrect behavior for a system with shear centre eccentricity is a bit careless as students and engineers who work with the program can misinterpret the output.

4.2 Practical examples

Asymmetrical cross-sections or cross-sections with one axis of symmetry like channel sections are not encountered as often as symmetrical cross-section like I-beams. In addition, those cross-sections are most of the time not loaded in torsion or only subjected to low loading. Because of these reasons, the chances of a structure failing due to the error in SCIA are exceptionally low. A few examples of locations where these cross-sections can be encountered are purlins and staircases.

Purlins
Purlins are sometimes shaped as channel sections as can be seen in Figure 4.1. They are mainly subjected to transverse loading perpendicular to the roof, therefore torsional moments will occur. The roof stabilizes the purlins which has a favourable effect on the warping of the cross-sections. A research was conducted on whether the unfavourable effect of the warping stesses is compensated by the favourable effects of the lateral stiffness and the torsional restraint caused by corrugated sheeting. It turned out that the warping stresses can be neglected if the sheeting is attached to the top flanges of the purlins (Lidner, 1988).

Figure 4.1: C-Purlins (Steeline, 2021)

Staircases
Another usage for a channel section are staircases. On top of these channel sections are the hand rails and in between are the stairs. It is therefore possible to load the profiles eccentrically when climbing the stairs.

Figure 4.2: Staircase at the TU Delft Faculty of Architecture

Figure 4.2 gives an example of a staircase with channel sections. There are no real stairs in this figure, it is a ramp used for fire emergencies. The bottom beams of this ramp are also made out of channel sections with their backs towards the stairs. The channel sections are supported at continuous intervals and welded all around resulting in a fixed connection. Therefore, the effect of torsion due to shear centre eccentricity is again small.

4.3 Conclusion

The chances of coming across a situation where asymmetrical cross-sections or cross-sections with one axis of symmetry are loaded in such a way that they become unsafe are low. These profiles can for example be encountered in purlins and staircases. SICA's calculation error will therefore most likely not cause any dangerous situations.

Chapter 5

Conclusion and recommendations

It was established that SCIA Engineer calculates no torsion if a cross-section has a shear centre eccentricity. Drillenburg investigated this problem in 2017 by considering different boundary conditions and load cases. We looked specifically at a cantilever beam with a channel section loaded due to its self weight (Figure 1.1) in order to figure out how the error could possibly be fixed.

5.1 Calculation comparison

We compared hand calculations with SCIA and Ansys Workbench, summarized the results in tables and concluded the following:

- The results of Ansys match with the expectations, but the magnitudes of the displacements and rotations differ slightly from the hand calculation. This can be explained by the use of slightly different torsional constant.
- SCIA reports no torsional moment, rotation or horizontal displacement under normal loading circumstances. When moving a line load to the shear centre, SCIA does calculate a torsional moment which is not the correct behavior.
- The option 'consider torsion due to shear centre eccentricity' reports the correct magnitudes for the torsional shear stresses for both statically determinate and indeterminate beams, therefore Drillenburgs statement about the option not working properly is at least not true for a channel section.

5.2 Solution using the Stiffness Method

SCIA uses stiffness matrices to link forces and moments to displacements and rotations. To investigate where the error arises within the program, we created a stiffness matrix for a 3D beam element that accounts for the shear centre eccentricity using Maple. All the steps are carefully explained in Appendix B.1. With the use of this model we came to the following hypothesis:

"SCIA's stiffness matrix is correct for a system line through the shear centre."

By looking at different positions for the system line, normal force centre and shear centre in the code, we concluded where the error originated from. In a normal situation, the system line would pass through the normal force centre. The self weight of the channel section will then result in torsion. By placing an equivalent line load in the shear centre, only pure bending will be observed. This is illustrated in Figure 5.1.

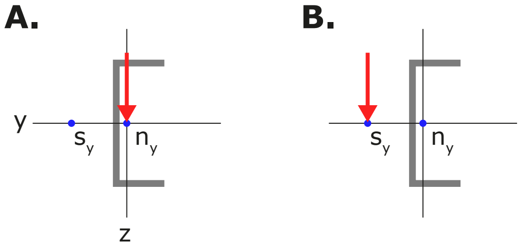


Figure 5.1: System line in the normal force centre

If SCIA assumes that the system line passes through the shear centre, situations A and B will change to situation C and D in Figure 5.2. The self weight now acts through the shear centre resulting in pure bending. Situation D is obtained by applying an equivalent line load at a distance equal to the shear centre eccentricity.

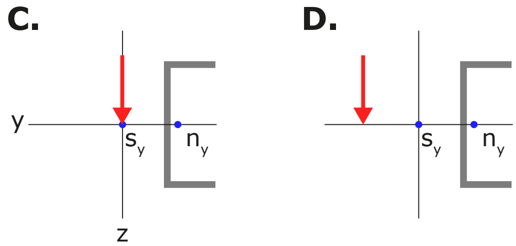


Figure 5.2: System line in the normal force centre

This assumption is remarkable, but theoretically not incorrect. Engineers who are aware of this, can take it into account when working with the software. Having the self weight act through the shear centre however, is a mistake. This error will stay unnoticed if the self weight is small in relation to the loading.

5.3 Recommendations

SCIA

Even though the chances of encountering a situation were the error could be dangerous are exceptionally low, it is better to avoid the risks. SCIA is therefore strongly advised to move the system line to the normal force centre to avoid confusion. The self weight must also act through the normal force centre instead of through the shear centre. If this is implemented correctly, the post-processing option 'consider torsion due to shear centre eccentricity' can be removed.

Engineers

Special attention is asked for cross-sections with a shear centre eccentricity, like channel sections. If a system line through the normal force centre is preferred, the option 'consider torsion due to shear centre eccentricity' can be enabled in the results tab to display the correct torsional shear stresses.

Follow-up research

Calculations for buckling are always made using a system line that passes through the normal force centre. It was established that SCIA uses a system line that passes through the shear centre. This will have consequences regarding the construction of the stiffness matrix for buckling. It is therefore strongly advised to check if the buckling of profiles with a shear centre eccentricity is calculated correctly in SCIA.

Bibliography

Drillenburg, M. (2017). *Torsion and shear stresses due to shear centre eccentricity in SCIA Engineer* (Bachelor's Thesis). Delft University of Technology.

Hartsuijker, C. (2016). *Spanningen, vervormingen en verplaatsingen* (2nd ed.). Boom uitgevers Amsterdam.

Hoogenboom, P. (2008). *Aantekeningen over wringing.* (pdf-edition)

James F. Lincoln Arc Welding Foundation. (2021). *Torque equations.* Retrieved from `https://www.sprecace.com/node/61`

Lidner, J. (1988). *To the load carrying capacity of cold-formed purlins.* Missouri University of Science and Technology. (pdf-edition)

Moaveni, S. (2011). *Finite element analysis theory and application with ANSYS* (3rd ed.). Pearson. (table 4.2)

Steeline. (2021). *Purlin-C.* Retrieved from `https://www.steeline.com.au/wp-content/uploads/2019/03/IMG_6921-2000x1224.jpg`

Welleman, H. (2021). *Non-symmetrical and inhomogeneous cross sections.* (pdf-edition)

Young, W. C., Budynas, R. G., & Sadegh, A. M. (2002). *Roark's formulas for stress and strain* (7th ed.). McGraw-Hill Education. (pg. 407)

Appendices

Appendix A

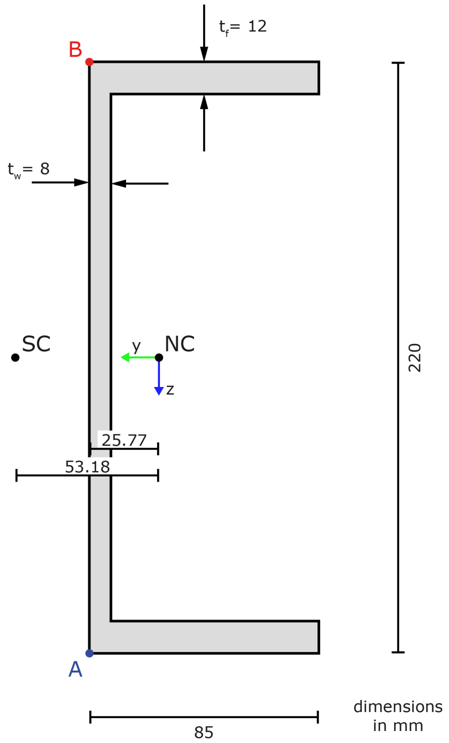
Hand calculations

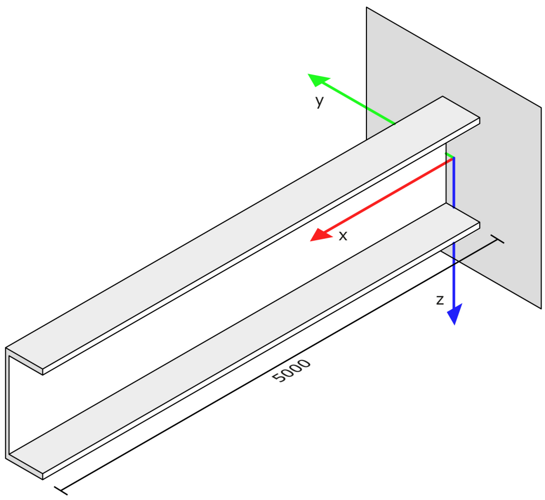
A.1 Cross-sectional properties

Hand calculations

> $restart;$

> $with(linalg):$





Roarks Method

> $$Iw := a \cdot b^3 \cdot \left(\frac{1}{3} - 0.21 \cdot \frac{b}{a} \cdot \left(1 - \frac{b^4}{12 \cdot a^4} \right) \right) + c \cdot d^3 \cdot \left(\frac{1}{3} - 0.105 \cdot \frac{d}{c} \cdot \left(1 - \frac{d^4}{192 \cdot c^4} \right) \right) + Dw^4 \cdot \frac{d}{b} \cdot \left(0.07 + 0.076 \cdot \frac{r}{b} \right) :$$

> $Dw := 2 \cdot (d + b + 3 \cdot r - \mathrm{sqrt}(2 \cdot (2 \cdot r + b) \cdot (2 \cdot r + d))):$

> $a := 85 : b := 12 : c := 98 : d := 8 : r := 0 :$

> $Itr := 2 \cdot Iw \cdot mm^4 : evalf(\%)$

$$123929.0629\ mm^4 \tag{1}$$

Simplified approach (thin-walled)

> $h := 220 \cdot mm : b := 85 \cdot mm : tf := 12 \cdot mm : tw := 8 \cdot mm : l := 5000 \cdot mm :$

> $It := \left(\frac{1}{3} \cdot h \cdot tw^3 + \frac{2}{3} \cdot b \cdot tf^3 \right) : evalf(\%)$

$$135466.6667\ mm^4 \tag{2}$$

Cross-sectional properties

> $A1 := (h - 2\cdot tf)\cdot tw : A2 := b\cdot tf : A := A1 + 2\cdot A2 :$

Normal force centre

> $yNC := \dfrac{A1\cdot\left(\frac{tw}{2}\right) + 2\cdot A2\cdot\left(\frac{b}{2}\right)}{A} : evalf(\%)$

$$25.76829268\ mm \tag{3}$$

> $zNC := \dfrac{A1\cdot\left(\frac{h}{2}\right) + A2\cdot\left(h - \frac{tf}{2}\right) + A2\cdot\left(\frac{tf}{2}\right)}{A} : evalf(\%)$

$$110.\ mm \tag{4}$$

> $Iyy := \left(\frac{1}{12}\cdot(h - 2\cdot tf)\cdot tw^3 + A1\cdot\left(yNC - \frac{tw}{2}\right)^2 + 2\cdot\left(\frac{1}{12}\cdot tf\cdot b^3 + A2\cdot\left(\frac{b}{2} - yNC\right)^2\right)\right) :$
> $evalf(\%)$

$$2.550720959\ 10^6\ mm^4 \tag{5}$$

> $Izz := \left(\frac{1}{12}\cdot tw\cdot(h - 2\cdot tf)^3 + 2\cdot\left(\frac{1}{12}\cdot b\cdot tf^3 + A2\cdot\left(zNC - \frac{tf}{2}\right)^2\right)\right) : evalf(\%)$

$$2.710881067\ 10^7\ mm^4 \tag{6}$$

> $Iyz := \left(A2\cdot\left(-\frac{b}{2} + yNC\right)\cdot\left(zNC - \frac{tf}{2}\right) + A2\cdot\left(-\frac{b}{2} + yNC\right)\cdot\left(-zNC + \frac{tf}{2}\right)\right)\cdot mm^4 :$
> $evalf(\%)$

$$0. \tag{7}$$

Shear centre

> $ySC := \left(\left(yNC - \frac{tw}{2}\right) + \dfrac{\left(\left(b - \frac{tw}{2}\right)^2\cdot(h - tf)^2\cdot tf\right)}{(4\cdot Izz)}\right) : evalf(\%)$

$$53.18115408\ mm \tag{8}$$

Loading, displacements and rotations

> $g := \dfrac{9.81\ N}{kg} : \text{rho} := \dfrac{7850e-9\ kg}{mm^3} :$

> $Mz := A\cdot g\cdot \text{rho}\cdot\left(\frac{l^2}{2}\right) : evalf(\%)$

$$3.473083350\ 10^6\ mm\ N \tag{9}$$

> $qz := A\cdot \text{rho}\cdot g : evalf(\%)$

$$\frac{0.2778466680\ N}{mm} \tag{10}$$

> $My := 0:$

> $E := \frac{210000 \cdot N}{mm^2} : G := \frac{80769\ N}{mm^2} :$

> $EI := \langle E \cdot Iyy, E \cdot Iyz | E \cdot Iyz, E \cdot Izz \rangle : evalf(\%)$

$$\begin{bmatrix} 5.356514015\ 10^{11}\ N\ mm^2 & 0. \\ 0. & 5.692850240\ 10^{12}\ N\ mm^2 \end{bmatrix} \quad (11)$$

> $Mx := -ySC \cdot qz \cdot l : evalf(\%)$

$$-73881.03230\ mm\ N \quad (12)$$

> $phix := \frac{Mx \cdot l \cdot 0.5}{G \cdot Itr} \cdot 1000 \cdot mrad : evalf(\%)$

$$-18.45249480\ mrad \quad (13)$$

> $\frac{phix \cdot 360}{2 \cdot \text{Pi} \cdot 1000} \cdot \frac{degrees}{mrad} : evalf(\%)$

$$-1.057250073\ degrees \quad (14)$$

> $uy := \frac{phix \cdot zNC}{1000 \cdot mrad} : evalf(\%)$

$$-2.029774427\ mm \quad (15)$$

> $uz\_bending := \frac{qz \cdot l^4}{8 \cdot E \cdot Izz} : evalf(\%)$

$$3.812988226\ mm \quad (16)$$

> $uz := \left(uz\_bending - \frac{(ySC - yNC) \cdot phix}{1000 \cdot mrad} \right) : evalf(\%)$

$$4.318823908\ mm \quad (17)$$

Torsional shear stress

> $tau\_max\_flens := \frac{(-Mx \cdot 0.5 \cdot tf)}{0.5 \cdot Itr} : evalf(\%)$

$$\frac{7.153869854\ N}{mm^2} \quad (18)$$

> $tau\_max\_lijf := \frac{(-Mx \cdot 0.5 \cdot tw)}{0.5 \cdot Itr} : evalf(\%)$

$$\frac{4.769246569\ N}{mm^2} \quad (19)$$

A.2 Shear centre and shear stresses

The shear centre and shear stresses for the channel section can be found using the following procedure:

Define the general equation for the shear stress due to a shear force.

$$\sigma_{xm} = -\frac{V_z \cdot S_z^a}{b^a \cdot I_{zz}} \tag{A.1}$$

where:

V_z = shear force

S_z^a = static moment of area

b^a = thickness of the area perpendicular to the shear

I_{zz} = moment of inertia in the x-z plane

Find the shear stress in the lower flange.

$$S_{z,1}^a = b \cdot t_f \cdot \frac{1}{2}h \tag{A.2}$$

$$\tau_1 = \left| -\frac{V_z \cdot S_{z,1}^a}{t_f \cdot I_{zz}} \right| \tag{A.3}$$

Use the principle 'inflow is outflow' to determine the shear stress in the web.

$$\tau_1 \cdot t_f = \tau_2 \cdot t_w \tag{A.4}$$

Determine the maximum shear stress in the web.

$$S_{z,2}^a = S_{z,1}^a + t_w \cdot \frac{1}{2}h \cdot \frac{1}{4}h \tag{A.5}$$

$$\tau_3 = \tau_{max} = \left| -\frac{V_z \cdot S_{z,2}^a}{t_w \cdot I_{zz}} \right| \tag{A.6}$$

Take the sum of moments around the centre of the web.

$$V_z \cdot e = H \cdot \frac{1}{2}h + H \cdot \frac{1}{2}h \tag{A.7}$$

$$e = \frac{H \cdot h}{V_z} \tag{A.8}$$

Find H by combining Equation A.2 and A.3.
The horizontal force H is equal to the area of the lower triangle in Figure A.1 multiplied with the thickness.

$$\tau_1 = \left| -\frac{V_z \cdot b \cdot \frac{1}{2}h}{I_{zz}} \right| \tag{A.9}$$

$$H = \frac{1}{2}b \cdot \tau_1 \cdot t_f = \frac{1}{4} \cdot \frac{V_z}{I_{zz}} \cdot b^2 \cdot h \cdot t_f \tag{A.10}$$

Combine Equation A.8 and A.10 to obtain the shear centre eccentricity with respect to the centre of the web.

$$e = \frac{b^2 \cdot h^2 \cdot t_f}{4I_{zz}} \tag{A.11}$$

Equation A.11 can also be used for a thick-walled channel section by accounting for the thicknesses. The validation of this equation is left to the reader.

$$e = \frac{(b - \frac{t_w}{2})^2 \cdot (h - t_f)^2 \cdot t_f}{4I_{zz}} \tag{A.12}$$

Torsional shear stress
The torsional shear stresses are assumed to be linear across the wall thickness. The maximum torsional shear stress then occurs at $\frac{1}{2}t$. Using this in combination with the previously defined equations for the shear stress, the following equations are obtained for the maximum torsional shear stress:

$$\tau_{f;max} = \frac{M_t \cdot t_f}{I_t} \tag{A.13}$$

$$\tau_{w;max} = \frac{M_t \cdot t_w}{I_t} \tag{A.14}$$

The total maximum shear stress can be obtained by using the superposition principle: adding the torsional shear stress and the shear stress due to the shear force. It is important to note that the shear stress due to the shear force is assumed to be constant across the wall thickness. In Figure A.1 τ_1 is calculated using Equation A.3, τ_2 is calculated using Equation A.4 and τ_3 using Equation A.6.

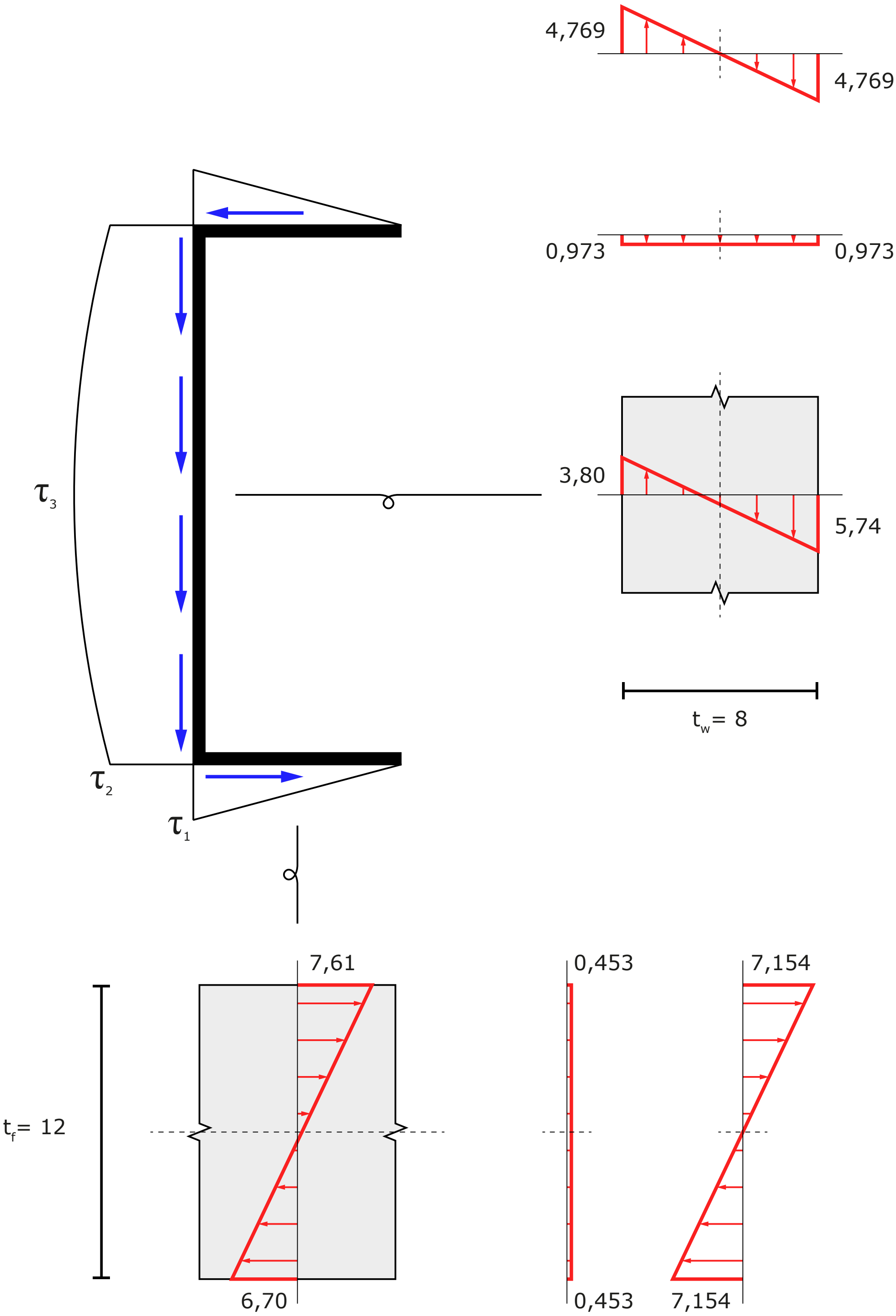


Figure A.1: Shear stresses at the support

Appendix B

Stiffness Method

B.1 Solution strategy

1. Define a coordinate system for internal and external forces for a 3D beam element.

The positive definitions for the forces and moments follow from the equilibrium of a beam element for the given coordinate system. In Figure B.1 the positive directions for the internal forces are illustrated.

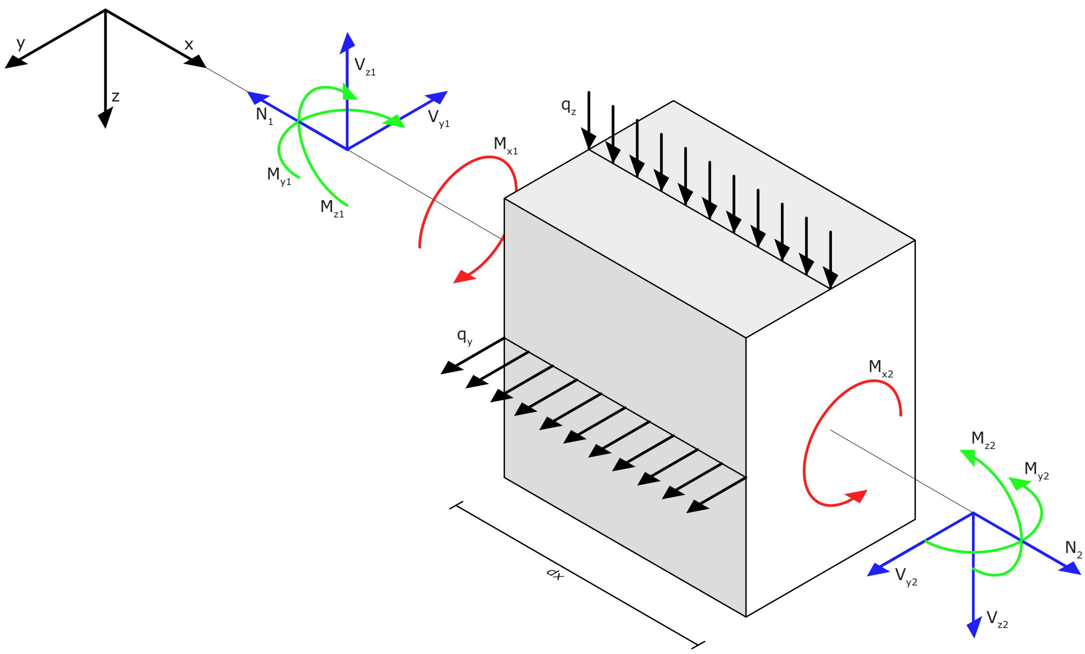


Figure B.1: Equilibrium of a beam element

2. Define the equations for the beam properties.

Elongation

The elongation of the beam is defined as the change of displacement in x-direction.

$$e = \frac{\mathrm{d}u_x}{\mathrm{d}x} \tag{B.1}$$

Curvatures

The curvatures of the beam in y- and z-direction are defined as the change in rotation in the y- and z-direction.

$$\kappa_y = -\frac{\mathrm{d}\varphi_y}{\mathrm{d}x} \tag{B.2}$$

$$\kappa_z = -\frac{\mathrm{d}\varphi_z}{\mathrm{d}x} \tag{B.3}$$

Forces and Moments

$$N = EA \cdot e + EA \cdot n_y \cdot \kappa_y + EA \cdot n_z \cdot \kappa_z \tag{B.4}$$

$$M_y = EA \cdot n_y \cdot e + EI_{yy} \cdot \kappa_y + EI_{yz} \cdot \kappa_z \tag{B.5}$$

$$M_z = EA \cdot n_z \cdot e + EI_{yz} \cdot \kappa_y + EI_{zz} \cdot \kappa_z \tag{B.6}$$

$$M_x = GI_t \cdot \frac{\mathrm{d}\varphi_x}{\mathrm{d}x} \tag{B.7}$$

$$V_y = \frac{\mathrm{d}M_y}{\mathrm{d}x} \tag{B.8}$$

$$V_z = \frac{\mathrm{d}M_z}{\mathrm{d}x} \tag{B.9}$$

3. Set up the differential equations and boundary conditions.

Differential Equations

$$\frac{\mathrm{d}N}{\mathrm{d}x} = 0 \tag{B.10}$$

$$\frac{\mathrm{d}^2 M_y}{\mathrm{d}x^2} = 0 \tag{B.11}$$

$$\frac{\mathrm{d}^2 M_z}{\mathrm{d}x^2} = 0 \tag{B.12}$$

$$\frac{\mathrm{d}^2 M_x}{\mathrm{d}x^2} = 0 \tag{B.13}$$

$$\varphi_y - \frac{\mathrm{d}u_y}{\mathrm{d}x} = 0 \tag{B.14}$$

$$\varphi_z - \frac{\mathrm{d}u_z}{\mathrm{d}x} = 0 \tag{B.15}$$

Boundary Conditions
n_y and n_z are the coordinates of the normal force centre, s_y and s_z are the coordinates of the shear centre with respect to the normal force centre and p_y and p_z are the coordinates of a point on the cross-section.

$$U = \begin{cases} u_{x1} = u_x(0) \\ u_{x2} = u_x(L) \\ u_{y1} = u_y(0) + \varphi_x(0) \cdot (s_z - p_z + n_z) \\ u_{y2} = u_y(L) + \varphi_x(L) \cdot (s_z - p_z + n_z) \\ u_{z1} = u_z(0) - \varphi_x(0) \cdot (s_y - p_y + n_y) \\ u_{z2} = u_z(L) - \varphi_x(L) \cdot (s_y - p_y + n_y) \end{cases} \tag{B.16}$$

$$\Phi = \begin{cases} \varphi_{x1} = \varphi_x(0) \\ \varphi_{x2} = \varphi_x(L) \\ \varphi_{y1} = -\varphi_z(0) \\ \varphi_{y2} = -\varphi_z(L) \\ \varphi_{z1} = \varphi_y(0) \\ \varphi_{z2} = \varphi_y(L) \end{cases} \tag{B.17}$$

4. Solve the system.

Solving the system results in expressions for the displacements and rotations as functions of x.

5. Substitute the solution back into the previously defined equations.

The results from step 4 are substituted back into the equations from step 2.

6. Set up the forces and moments on both ends of the beam element. The forces and moments in the nodes are according to the positive directions in Figure B.1.

$$F_1 = \begin{cases} F_{x1} = -N \\ F_{y1} = -V_y \\ F_{z1} = -V_z \\ M_{x1} = -M_x - V_z \cdot s_y + V_y \cdot s_z \\ M_{y1} = -M_z \\ M_{z1} = M_y \end{cases} \tag{B.18}$$

$$F_2 = \begin{cases} F_{x2} = N \\ F_{y2} = V_y \\ F_{z2} = V_z \\ M_{x2} = M_x + V_z \cdot s_y - V_y \cdot s_z \\ M_{y2} = M_z \\ M_{z2} = -M_y \end{cases} \tag{B.19}$$

7. Determine the stiffness matrix coefficients by differentiating the loading at both ends of the beam element to the unknown displacements and rotations.

In Figure B.2 the stiffness matrix is given for a beam element where EI_{yz}, n_y, n_z, p_y and p_z all equal zero. The complete stiffness matrix is otherwise too large to print on a page.

$$\begin{bmatrix}
\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 & -\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{12\,EIyy}{L^3} & 0 & -\frac{12\,EIyy\,sz}{L^3} & 0 & \frac{6\,EIyy}{L^2} & 0 & -\frac{12\,EIyy}{L^3} & 0 & \frac{12\,EIyy\,sz}{L^3} & 0 & \frac{6\,EIyy}{L^2} \\
0 & 0 & \frac{12\,EIzz}{L^3} & \frac{12\,EIzz\,sy}{L^3} & -\frac{6\,EIzz}{L^2} & 0 & 0 & 0 & -\frac{12\,EIzz}{L^3} & -\frac{12\,EIzz\,sy}{L^3} & -\frac{6\,EIzz}{L^2} & 0 \\
0 & -\frac{12\,EIyy\,sz}{L^3} & \frac{12\,EIzz\,sy}{L^3} & \frac{12\,EIyy\,sz^2 + 12\,EIzz\,sy^2 + GIw\,L^2}{L^3} & -\frac{6\,EIzz\,sy}{L^2} & -\frac{6\,EIyy\,sz}{L^2} & 0 & \frac{12\,EIyy\,sz}{L^3} & -\frac{12\,EIzz\,sy}{L^3} & \frac{-12\,EIyy\,sz^2 - 12\,EIzz\,sy^2 - GIw\,L^2}{L^3} & -\frac{6\,EIzz\,sy}{L^2} & -\frac{6\,EIyy\,sz}{L^2} \\
0 & 0 & -\frac{6\,EIzz}{L^2} & -\frac{6\,EIzz\,sy}{L^2} & \frac{4\,EIzz}{L} & 0 & 0 & 0 & \frac{6\,EIzz}{L^2} & \frac{6\,EIzz\,sy}{L^2} & \frac{2\,EIzz}{L} & 0 \\
0 & \frac{6\,EIyy}{L^2} & 0 & -\frac{6\,EIyy\,sz}{L^2} & 0 & \frac{4\,EIyy}{L} & 0 & -\frac{6\,EIyy}{L^2} & 0 & \frac{6\,EIyy\,sz}{L^2} & 0 & \frac{2\,EIyy}{L} \\
-\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 & \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{12\,EIyy}{L^3} & 0 & \frac{12\,EIyy\,sz}{L^3} & 0 & -\frac{6\,EIyy}{L^2} & 0 & \frac{12\,EIyy}{L^3} & 0 & -\frac{12\,EIyy\,sz}{L^3} & 0 & -\frac{6\,EIyy}{L^2} \\
0 & 0 & -\frac{12\,EIzz}{L^3} & -\frac{12\,EIzz\,sy}{L^3} & \frac{6\,EIzz}{L^2} & 0 & 0 & 0 & \frac{12\,EIzz}{L^3} & \frac{12\,EIzz\,sy}{L^3} & \frac{6\,EIzz}{L^2} & 0 \\
0 & \frac{12\,EIyy\,sz}{L^3} & -\frac{12\,EIzz\,sy}{L^3} & \frac{-12\,EIyy\,sz^2 - 12\,EIzz\,sy^2 - GIw\,L^2}{L^3} & \frac{6\,EIzz\,sy}{L^2} & \frac{6\,EIyy\,sz}{L^2} & 0 & -\frac{12\,EIyy\,sz}{L^3} & \frac{12\,EIzz\,sy}{L^3} & \frac{12\,EIyy\,sz^2 + 12\,EIzz\,sy^2 + GIw\,L^2}{L^3} & \frac{6\,EIzz\,sy}{L^2} & \frac{6\,EIyy\,sz}{L^2} \\
0 & 0 & -\frac{6\,EIzz}{L^2} & -\frac{6\,EIzz\,sy}{L^2} & \frac{2\,EIzz}{L} & 0 & 0 & 0 & \frac{6\,EIzz}{L^2} & \frac{6\,EIzz\,sy}{L^2} & \frac{4\,EIzz}{L} & 0 \\
0 & \frac{6\,EIyy}{L^2} & 0 & -\frac{6\,EIyy\,sz}{L^2} & 0 & \frac{2\,EIyy}{L} & 0 & -\frac{6\,EIyy}{L^2} & 0 & \frac{6\,EIyy\,sz}{L^2} & 0 & \frac{4\,EIyy}{L}
\end{bmatrix}$$

Figure B.2: Stiffness matrix

The stiffness matrix is singular, which means its determinant is zero. Therefore, known forces, moments, displacements and rotations are needed to solve the system.

8. Solve the system for the unknown displacements and rotations by applying known forces, moments, displacements and rotations.

The distributed load must be split in equivalent nodal loads. This can be accomplished by placing a downward vertical force in both nodes and simultaneously placing a moment in both nodes around the y-axis. The magnitude of these forces and moments can be determined using the 'forget-me-nots'. Figure B.3 reports different equivalent nodal loading for different load cases.

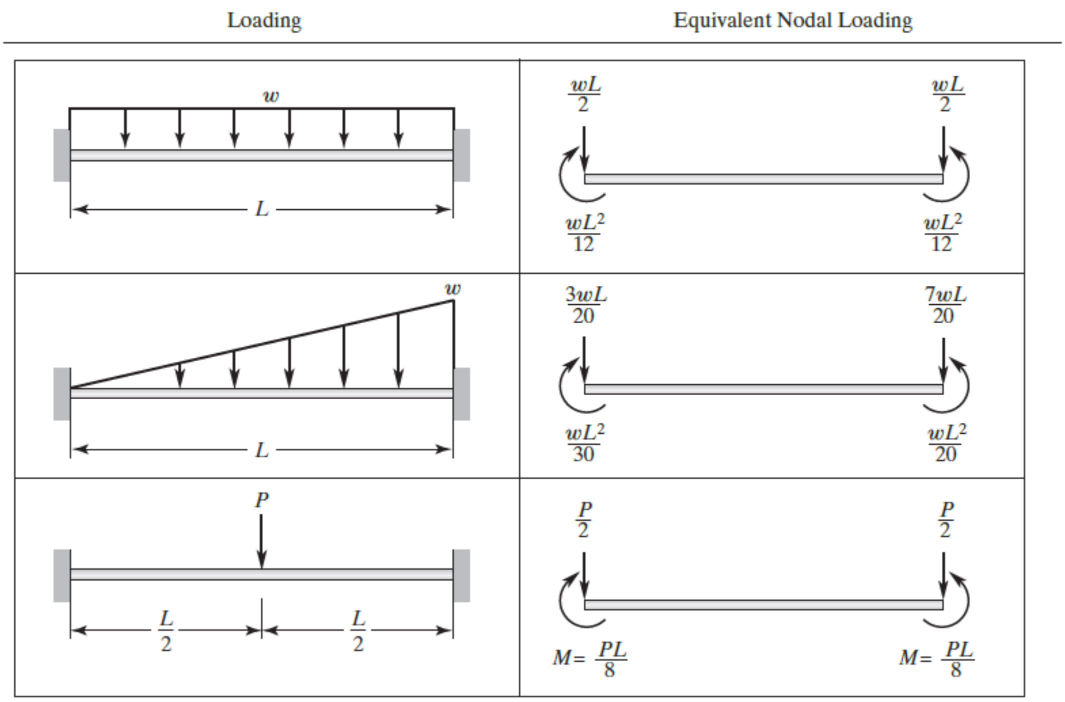


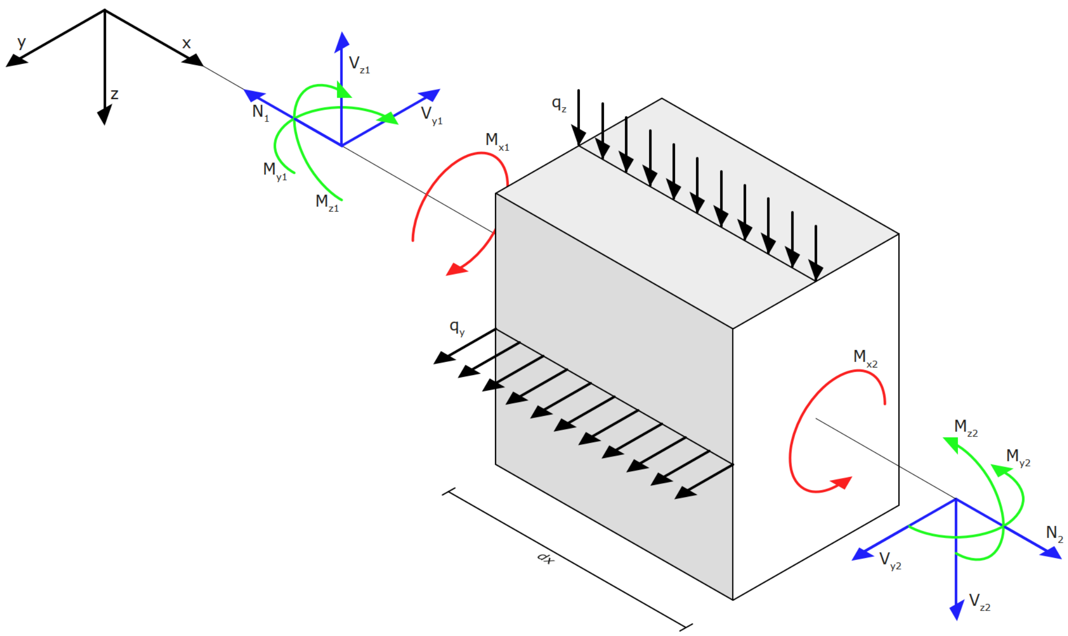
Figure B.3: Equivalent nodal loading (Moaveni, 2011)

B.2 Maple file - hand calculation properties

Stiffness Method

(properties: hand calculation)

> $restart; with(LinearAlgebra):$



Beam Equations

> $e := diff(ux(x), x):$

> $Ky := diff(-phiy(x), x):$

> $Kz := diff(-phiz(x), x):$

> $N := EA \cdot e + EA \cdot ny \cdot Ky + EA \cdot nz \cdot Kz:$

> $My := EA \cdot ny \cdot e + EIyy \cdot Ky + EIyz \cdot Kz:$

> $Mz := EA \cdot nz \cdot e + EIyz \cdot Ky + EIzz \cdot Kz:$

> $Vy := diff(My, x):$

> $Vz := diff(Mz, x):$

> $Mx := GIw \cdot diff(\mathrm{phi}(x), x):$

Differential Equations

> $DE1 := diff(N, x) = 0:$

> $DE2 := diff(My, x, x) = 0:$

> $DE3 := diff(Mz, x, x) = 0:$

> $DE4 := diff(Mx, x) = 0:$

> $DE5 := phiy(x) - diff(uy(x), x) = 0:$

> $DE6 := phiz(x) - diff(uz(x), x) = 0:$

Boundary Conditions

> $BC1 := ux1 = ux(0), ux2 = ux(L):$

> $BC2 := uy1 = uy(0) + \mathrm{phi}(0)\cdot(sy - py + ny),\ uy2 = uy(L) + \mathrm{phi}(L)\cdot(sy - py + ny):$

> $BC3 := uz1 = uz(0) - \mathrm{phi}(0)\cdot(sy - py + ny),\ uz2 = uz(L) - \mathrm{phi}(L)\cdot(sy - py + ny):$

> $BC4 := phix1 = \mathrm{phi}(0),\ phix2 = \mathrm{phi}(L),\ phiy1 = -phiz(0),\ phiy2 = -phiz(L),\ phiz1 = phiy(0),\ phiz2 = phiy(L):$

Solve

> $sol := dsolve(\{DE1, DE2, DE3, DE4, DE5, DE6, BC1, BC2, BC3, BC4\}, \{ux(x), uy(x), uz(x), \mathrm{phi}(x), phiy(x), phiz(x)\}) : assign(sol):$

Back Substitution

> $e := diff(ux(x), x):$

> $Ky := diff(-phiy(x), x):$

> $Kz := diff(-phiz(x), x):$

> $N := EA\cdot e + EA\cdot ny\cdot Ky + EA\cdot nz\cdot Kz:$

> $My := EA\cdot ny\cdot e + EIyy\cdot Ky + EIyz\cdot Kz:$

> $Mz := EA\cdot nz\cdot e + EIyz\cdot Ky + EIzz\cdot Kz:$

> $Vy := diff(My, x):$

> $Vz := diff(Mz, x):$

> $Mx := GIw\cdot diff(\mathrm{phi}(x), x):$

Forces and Moments at beam ends

> $x := 0 : Fx1 := -N : Fy1 := -Vy : Fz1 := -Vz : Mx1 := -Mx - Vz\cdot sy + Vy\cdot sz : My1 := -Mz : Mz1 := My : x := 'x':$

> $x := L : Fx2 := N : Fy2 := Vy : Fz2 := Vz : Mx2 := Mx + Vz\cdot sy - Vy\cdot sz : My2 := Mz : Mz2 := -My : x := 'x':$

Stiffness Coefficiens

> $K[1, 1] := simplify(diff(Fx1, ux1)):$

> $K[1, 2] := simplify(diff(Fx1, uy1)):$

...

> $K[12, 12] := simplify(diff(Mz2, phiz2)):$

etc.

Stiffness Matrix

> $A := Matrix(12, 12, fill = 0):$

```
> printlevel := 2 :
  for i to 12 do
    for j to 12 do
      A[i,j] := K[i,j]
    end do:
  end do:
```

> $interface(rtablesize = 12):$

> $A:$

Parameters (hand calculation)

> $L := 5000 : EIyy := 210000 \cdot 2.550720959 \cdot 10^6 : EIzz := 210000 \cdot 2.710881067 \cdot 10^7 :$
> $EIyz := 0 : EA := 210000 \cdot 3608 : GIw := 80796 \cdot 123929.0629 : q := 0.2778466680 :$

> $ny := 0 : nz := 0 : sy := 53.18115408 : sz := 0 : py := 25.76829268 : pz := -110 :$

Loading

> $GenerateEquations\left(A, [ux\_1, uy\_1, uz\_1, phix\_1, phiy\_1, phiz\_1, ux\_2, uy\_2, uz\_2, phix\_2, phiy\_2, phiz\_2], \left\langle N\_1, Fy\_1, Fz\_1 + \frac{q \cdot L}{2}, Mx\_1 - Fz\_1 \cdot sy, My\_1 - \frac{q \cdot L^2}{12}, Mz\_1, N\_2, Fy\_2, Fz\_2, Mx\_2, My\_2, Mz\_2 \right\rangle\right) :$

> $equations := \{\%[1], \%[2], \%[3], \%[4], \%[5], \%[6], \%[7], \%[8], \%[9], \%[10], \%[11], \%[12]\} :$

> $ux\_1 := 0 : uy\_1 := 0 : uz\_1 := 0 : phix\_1 := 0 : phiy\_1 := 0 : phiz\_1 := 0 : ux\_2 := 0 :$
> $Mx\_2 := 0 : My\_2 := \frac{q \cdot L^2}{12} : Mz\_2 := 0 : Fy\_2 := 0 : Fz\_2 := \frac{q \cdot L}{2} :$

Solution

> $Solutions := solve(equations, \{N\_1, N\_2, Fy\_1, Fz\_1, Mx\_1, My\_1, Mz\_1, phix\_2, phiy\_2, phiz\_2, uy\_2, uz\_2\}) :$

> $assign(Solutions) :$

> $\langle uy, uz, phix, 'Mx' \rangle = \langle evalf[3](uy\_2), evalf[3](uz\_2), evalf[4](phix\_2), evalf[3](Mx\_1) \rangle$

$$\begin{bmatrix} uy \\ uz \\ phix \\ Mx \end{bmatrix} = \begin{bmatrix} -2.03 \\ 4.32 \\ -0.01845 \\ -73900. \end{bmatrix} \quad (1)$$

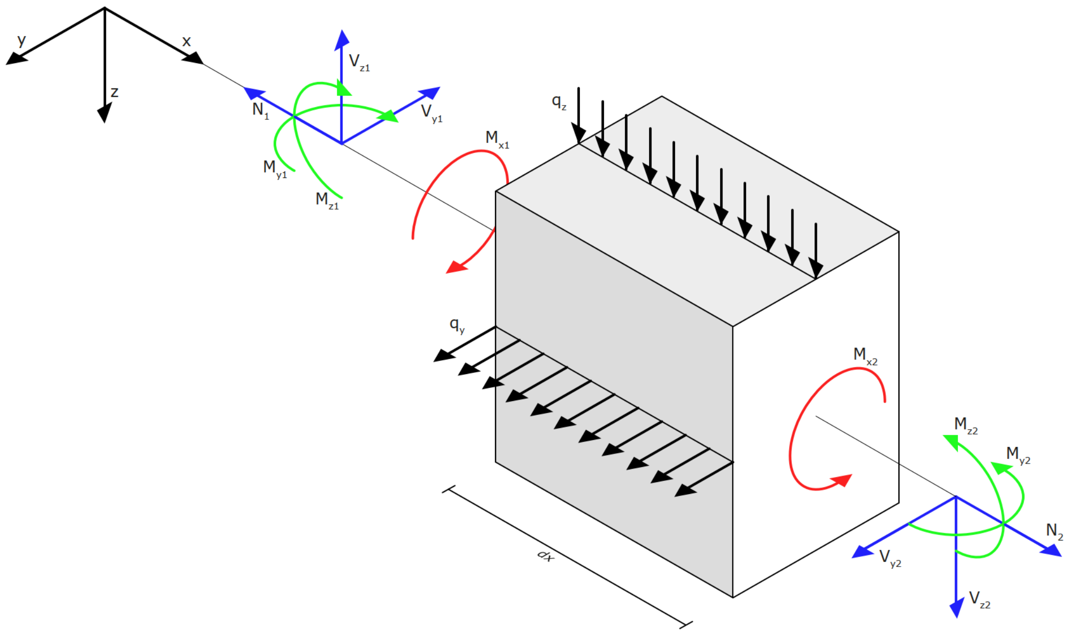
>

B.3 Maple file - SCIA properties

Stiffness Method

(properties: SCIA, situation E)

> $restart; with(LinearAlgebra):$



Beam Equations

> $e := diff(ux(x), x):$

> $Ky := diff(-phiy(x), x):$
> $Kz := diff(-phiz(x), x):$

> $N := EA \cdot e + EA \cdot ny \cdot Ky + EA \cdot nz \cdot Kz:$

> $My := EA \cdot ny \cdot e + EIyy \cdot Ky + EIyz \cdot Kz:$
> $Mz := EA \cdot nz \cdot e + EIyz \cdot Ky + EIzz \cdot Kz:$

> $Vy := diff(My, x):$
> $Vz := diff(Mz, x):$
> $Mx := GIw \cdot diff(\mathrm{phi}(x), x):$

Differential Equations

> $DE1 := diff(N, x) = 0:$
> $DE2 := diff(My, x, x) = 0:$
> $DE3 := diff(Mz, x, x) = 0:$
> $DE4 := diff(Mx, x) = 0:$
> $DE5 := phiy(x) - diff(uy(x), x) = 0:$
> $DE6 := phiz(x) - diff(uz(x), x) = 0:$

Boundary Conditions

> $BC1 := ux1 = ux(0), ux2 = ux(L)$:

> $BC2 := uy1 = uy(0) + \mathrm{phi}(0)\cdot(sz - pz + nz), uy2 = uy(L) + \mathrm{phi}(L)\cdot(sz - pz + nz)$:

> $BC3 := uz1 = uz(0) - \mathrm{phi}(0)\cdot(sy - py + ny), uz2 = uz(L) - \mathrm{phi}(L)\cdot(sy - py + ny)$:

> $BC4 := phix1 = \mathrm{phi}(0), phix2 = \mathrm{phi}(L), phiy1 = -phiz(0), phiy2 = -phiz(L), phiz1 = phiy(0), phiz2 = phiy(L)$:

Solve

> $sol := dsolve(\{DE1, DE2, DE3, DE4, DE5, DE6, BC1, BC2, BC3, BC4\}, \{ux(x), uy(x), uz(x), \mathrm{phi}(x), phiy(x), phiz(x)\})$: $assign(sol)$:

Back Substitution

> $e := diff(ux(x), x)$:

> $Ky := diff(-phiy(x), x)$:

> $Kz := diff(-phiz(x), x)$:

> $N := EA\cdot e + EA\cdot ny\cdot Ky + EA\cdot nz\cdot Kz$:

> $My := EA\cdot ny\cdot e + EIyy\cdot Ky + EIyz\cdot Kz$:

> $Mz := EA\cdot nz\cdot e + EIyz\cdot Ky + EIzz\cdot Kz$:

> $Vy := diff(My, x)$:

> $Vz := diff(Mz, x)$:

> $Mx := GIw\cdot diff(\mathrm{phi}(x), x)$:

Forces and Moments at beam ends

> $x := 0 : Fx1 := -N : Fy1 := -Vy : Fz1 := -Vz : Mx1 := -Mx - Vz\cdot(sy-ey) + Vy\cdot(sz-ez)$: $My1 := -Mz : Mz1 := My : x :=$'x':

> $x := L : Fx2 := N : Fy2 := Vy : Fz2 := Vz : Mx2 := Mx + Vz\cdot(sy-ey) - Vy\cdot(sz-ez)$: $My2 := Mz : Mz2 := -My : x :=$'x':

Stiffness Coefficiens

> $K[1, 1] := simplify(diff(Fx1, ux1))$:

> $K[1, 2] := simplify(diff(Fx1, uy1))$:

...

> $K[12, 12] := simplify(diff(Mz2, phiz2))$:

etc.

Stiffness Matrix

> $A := Matrix(12, 12, fill = 0)$:

```
> printlevel := 2 :
  for i to 12 do
    for j to 12 do
      A[i,j] := K[i,j]
    end do:
  end do:
```

$> interface(rtablesize = 12):$

$> A:$

Parameters (hand calculation)

$> L := 5000 : EIyy := 210000 \cdot 2.550721 \cdot 10^6 : EIzz := 210000 \cdot 2.710881 \cdot 10^7 : EIyz := 0 : EA := 210000 \cdot 3608 : GIw := 80796 \cdot 124919.8 : q := 0.2778466680 :$

$> ny := -52.8855 : nz := 0 : sy := 52.8855 : sz := 0 : py := 25.76829268 : pz := -110 : ey := 0 : ez := 0 :$

Loading

$$> GenerateEquations\left(A, [ux\_1, uy\_1, uz\_1, phix\_1, phiy\_1, phiz\_1, ux\_2, uy\_2, uz\_2, phix\_2, phiy\_2, phiz\_2], \left\langle N\_1, Fy\_1, Fz\_1 + \frac{q \cdot L}{2}, Mx\_1 - Fz\_1 \cdot (sy - ey), My\_1 - \frac{q \cdot L^2}{12}, Mz\_1, N\_2, Fy\_2, Fz\_2, Mx\_2, My\_2, Mz\_2 \right\rangle \right):$$

$> equations := \{\%[1], \%[2], \%[3], \%[4], \%[5], \%[6], \%[7], \%[8], \%[9], \%[10], \%[11], \%[12]\}:$

$$> ux\_1 := 0 : uy\_1 := 0 : uz\_1 := 0 : phix\_1 := 0 : phiy\_1 := 0 : phiz\_1 := 0 : ux\_2 := 0 : Mx\_2 := 0 : My\_2 := \frac{q \cdot L^2}{12} : Mz\_2 := 0 : Fy\_2 := 0 : Fz\_2 := \frac{q \cdot L}{2} :$$

Solution

$> Solutions := solve(equations, \{N\_1, N\_2, Fy\_1, Fz\_1, Mx\_1, My\_1, Mz\_1, phix\_2, phiy\_2, phiz\_2, uy\_2, uz\_2\}):$

$> assign(Solutions):$

$> \langle uy, uz, phix, 'Mx' \rangle = \langle evalf[3](uy\_2), evalf[3](uz\_2), evalf[4](phix\_2), evalf[3](Mx\_1) \rangle$

$$\begin{bmatrix} uy \\ uz \\ phix \\ Mx \end{bmatrix} = \begin{bmatrix} -2.00 \\ 3.34 \\ -0.01820 \\ -73500. \end{bmatrix} \tag{1}$$