Hybrid Population Based MVMO for Solving CEC 2018 Test Bed of Single-Objective Problems

Rueda, José; Erlich, Istvan

DOI
10.1109/CEC.2018.8477822

Publication date
2018

Document Version
Accepted author manuscript

Published in
2018 IEEE Congress on Evolutionary Computation, CEC 2018

Citation (APA)
Rueda, J., & Erlich, I. (2018). Hybrid Population Based MVMO for Solving CEC 2018 Test Bed of Single-Objective Problems. In *2018 IEEE Congress on Evolutionary Computation, CEC 2018 : Proceedings* (pp. 1-8). Article 8477822 IEEE. https://doi.org/10.1109/CEC.2018.8477822

Hybrid Population Based MVMO for Solving CEC 2018 Test Bed of Single-Objective Problems

José Rueda, Senior Member, IEEE
Department of Electrical Sustainable Energy
Delft University of Technology
Delft, The Netherlands
j.l.ruedatorres@tudelft.nl

István Erlich, Senior Member, IEEE
Institute of Electrical Power Systems
University Duisburg-Essen
Duisburg, Germany
istvan.erlich@uni-due.de

***Abstract*—The MVMO algorithm (Mean-Variance Mapping Optimization) has two main features: i) normalized search range for each dimension (associated to each optimization variable); ii) use of a mapping function to generate a new value of a selected optimization variable based on the mean and variance derived from the best solutions achieved so far. The current version of MVMO offers several alternatives. The single parent-offspring version is designed for use in case the evaluation budget is small and the optimization task is not too challenging. The population based MVMO requires more function evaluations, but the results are usually better. Both variants of MVMO can be improved considerably if additionally separate local search algorithms are incorporated. In this case, MVMO is basically responsible for the initial global search. This paper presents the results of a study on the use of the hybrid version of MVMO, called MVMO-PH (population based, hybrid), to solve the IEEE-CEC 2018 test suite for single objective optimization with continuous (real-number) decision variables. Additionally, two new mapping functions representing the unique feature of MVMO are presented.**

Keywords—Stochastic optimization; MVMO algorithm; single objective problems; CEC2018 test functions.

I. Introduction

Modern metaheuristic algorithms find widespread application in different engineering problems. They have the ability to deal with complex and hard-to-solve optimization problems (e.g. non-convex, discontinuous, a multimodal search space). The competitions organized in the context of IEEE-CEC (World Congress on Evolutionary Computation) [1] provide various groups of test functions. These sets of problems offer a common reference for testing and conducting comparative performance assessment of different types of stochastic optimization algorithms. This practice allows comparing different heuristic optimization algorithms with one another, and help the emergence of more powerful optimization solvers.

The MVMO algorithm (Mean-Variance Mapping Optimization), which belongs to the family of evolutionary algorithms, is one of the emerging algorithms. The first published algorithmic framework of MVMO is essentially a single parent-child approach (i.e. a single solution is generated throughout each function evaluation). The evolutionary mechanism of MVMO is conceived to perform within a search space in the range [0,1] regardless of the type of the optimization variable [2]. Additionally, a solution archive is adopted in MVMO to continuously record a small set of best solutions found so far throughout the function evaluations. To generate a new (child) solution, MVMO selects the best ranked solution in the archive as a parent solution. The numerical value of some dimensions of the new solution are assigned from the parent, and the remainder dimensions are generated by using the mapping function, which accounts for the mean and variance computed from the numerical values of these dimension stored in the solution archive. The output of the mapping function is defined in the [0, 1] range. The shape of the mapping function adapts throughout the search process to automatically swap the search priority between exploration and exploitation.

Recently, MVMO was modified to be able to also perform as a population-based approach. This variant of MVMO has the option of calling a local search strategy. The new version with the additional features is referred to as MVMO-PH, where P refers to population based approach, and H refers to hybridization to account for local search. Broadly speaking, the evolution of each solution is similar to the first variant of MVMO (evolution of a single solution). However, MVMO-PH uses rules to merge information from different parents to generate new solution. In this way, it is possible to reorient the search of unsuccessful solutions (with worse fitness) towards more attractive search space sub-regions [3]. This paper addresses, on the one hand, the adoption of a new mapping function, and on the other, the implementation of an improved crossover strategy. Both modifications aim to achieve higher diversification of the generation of solutions across the problem search space. MVMO-PH is applied to the IEEE-CEC 2018 test suite of single objective optimization problems with continuous (real parameter) decision variables. The numerical performance of MVMO-PH is tested on each problem of the test suite tested with different dimensions (10D, 30D, 50D, and 100D). The settings defined by the organizers of the IEEE-CEC 2018 competition are considered in the application of MVMO-PH [4].

The remainder of the paper has the following structure: Section II overviews the principles of MVMO-PH. The experimental setup and discussion on numerical are presented in Section III. Section IV summarizes the concluding remarks.

II. PROCEDURE OF MVMO-PH

The calculation steps of MVMO-PH are depicted in Fig. 1.

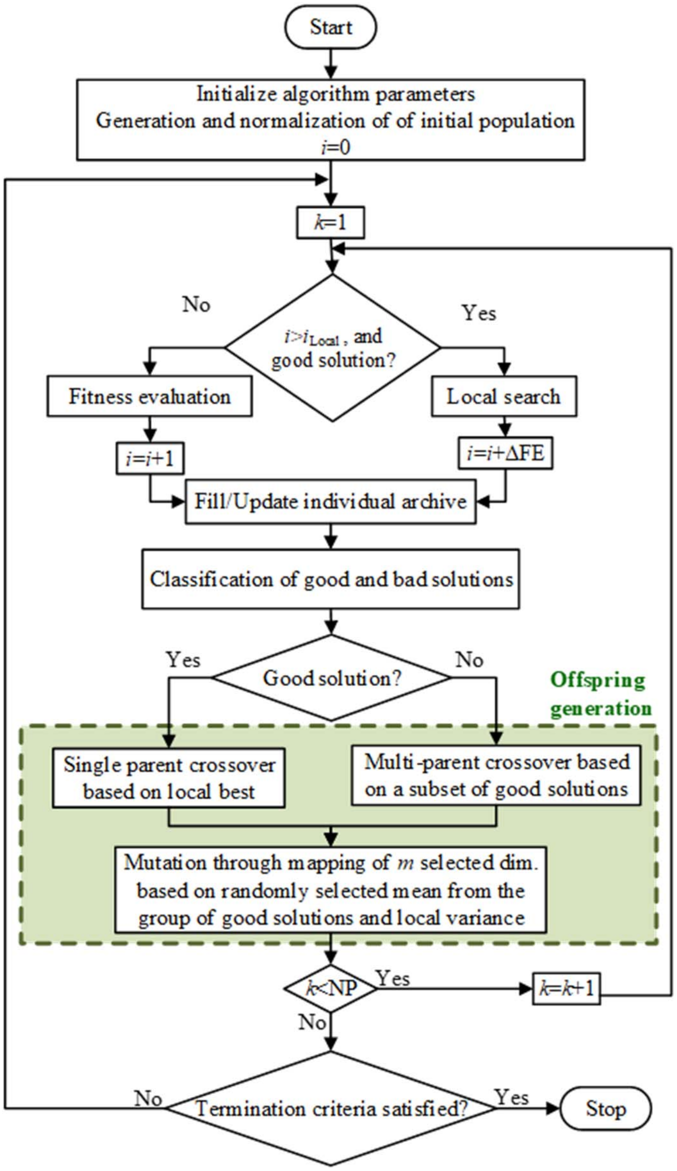

Fig. 1. Procedure of MVMO-PH. The variable i and k constitute counters of number of fitness evaluation and candidate solution, respectively [2].

Initially, MVMO-PH reads the settings defined by the user and a set (population) of N_P trial solutions is generated randomly. The trial solutions are normalized from [min, max] original boundaries to [0, 1] boundaries. All operations performed to generate a new solution are done by considering [0, 1] boundaries. This entails that transformation back to [min, max] original boundaries is done before performing calculation of fitness or local search. Next, each solution is evolved by considering the best positions achieved so far (stored in the solution archive). Successful solutions are evolved in a different way depending on the classification of the first ranked solution of each archive. This means, single or multi-parent criteria is used. MVMO-PH can be configured to stop once the maximum number of function evaluations is met.

A. Calculation of Fitness/Launch of local search

Before calculating the fitness or executing local search, the optimization variables are transformed back from [0, 1] range into the original [min, max] range. Considering that the optimization problems of the IEEE-CEC 2018 test suite are only subjected to bound constraints, there is no need to implement any constraint handling strategy. In this case, objective function value corresponds with fitness value..

MVMO-PH can be set to call Interior-Point algorithm or Active-Set algorithm once a predefined amount of fitness evaluations (i_{local}) has been reached. For the application to the IEEE-CEC 2018 test suite, this option is used to intensify the search around a new generated solution that is judged as good. Local search is allowed to perform in the range

$$\alpha_{LS\_min} < \alpha < \alpha_{LS\_max}, \quad \alpha = i / i_{max} \tag{1}$$

where the counter of function evaluations is denoted by i. The local search algorithm to be used is selected by the MVMO-PH based on the fitness achieved so far.

B. Relevance of the solution archive

A solution archive is defined to track the success throughout the evolution of each solution of the population. This archive is conceived to save the n best positions of the evolved solutions, ranking them according to a descending order of fitness.(cf. Fig. 2) [3].

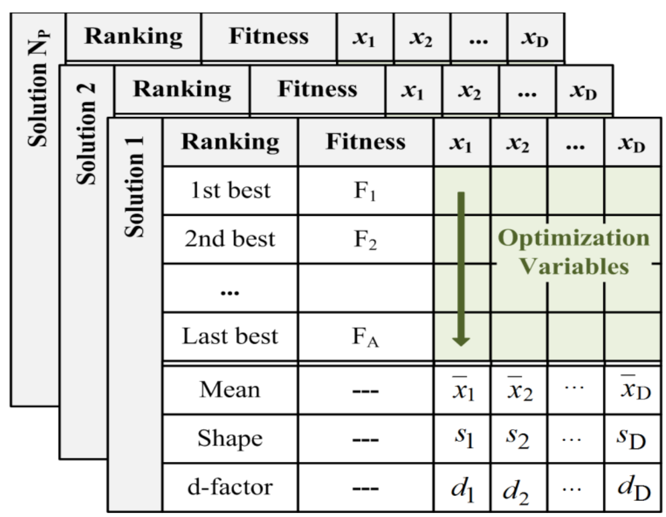

Fig. 2. Solution archives of MVMO-PH [3].

The size of the archive is defined in the initial stage of MVMO-PH and is kept fixed in the whole optimization procedure. In addition, the archive is updated only if a newly generated solution is found to be more successful (i.e. better fitness value) than those solutions already stored in the preceding function evaluation number.

In every update of the archive, the mean value $\bar{x}_i$ and the shape factor s_i corresponding to every optimization variable are computed. In the calculation of the mean value and the shape factor, the following minimum distance between the variables saved in the archive is assumed:

$$\Delta x = 10^{-(3.5+5\cdot\alpha)} \quad (2)$$

With the progress of the iteration (increasing alpha), the value of the distance becomes continuously smaller. In this way, a more global search is made possible at the beginning (Δx is still large) while towards the end of the iteration Δx becomes small and so the emphasis is on local search.

The initial value $\bar{x}_{\mathrm{ini}}$ is defined as 0.5, i.e. located in the middle of [min, max] boundaries. For other applications, it is possible to change this rule, e.g. randomly sampled value. Besides, the selection of the initial values for s_i can be done by following a random approach. Nevertheless, for the application of MVMO-PH to the IEEE CEC2018 test suite, it was preferred to select $s_i = 0.001$, and then updating the value based on the stored information in the archive for each dimension. Both $\bar{x}_{\mathrm{ini}}$ and s_i change the shape of the mapping function. This helps in prioritizing the search emphasis (i.e. swapping exploration and exploitation).

C. Parent assignment

In the beginning of the MVMO-PH search, trial solutions are randomly and independently sampled for a couple of function evaluations. Afterwards, the procedure illustrated in Fig. 3 is followed to discriminate between a group of *GP* "good solutions" and a group of *Np-GP* "bad solutions". This is done by comparing the first ranked solution among all archives. The good particles are evolved by considering their first ranked solution in the corresponding archive. The bad particles are evolved by considering a multi-parent crossover as defined in (3) [3].

$$\mathbf{x}_k^{\mathrm{parent}} = \mathbf{x}_{\mathrm{RG}}^{\mathrm{best}} + \beta\left(\mathbf{x}_{\mathrm{GB}}^{\mathrm{best}} - \mathbf{x}_{\mathrm{LG}}^{\mathrm{best}}\right) \quad (3)$$

where $\mathbf{x}_{\mathrm{GB}}^{\mathrm{best}}$, $\mathbf{x}_{\mathrm{LG}}^{\mathrm{best}}$ and $\mathbf{x}_{\mathrm{RG}}^{\mathrm{best}}$ stand for first (global best), the last ranked good solution, and a randomly selected good solution. The mean values associated to $\mathbf{x}_k$ are inputs to the mapping function, and are randomly picked up from the group of good solutions. The parameter β is derived as follows [3]:

$$b = 1.1 + (rand - 0.5)\cdot 2.0$$
$$\beta = \delta\cdot b\cdot\left(rand - \left(1-\alpha^2\right)\cdot 0.9\right)\cdot rand \quad (4)$$

where δ constitutes a tuning parameter defined in the range [0.1 – 10]. The parameters calculated using (3) are limited to within the range [0, 1] as necessary.

In the evolution of either a "good" or a "bad" solution, the mean value passed as input of the mapping function is picked up randomly among the "good" solutions. The shape variables s_i and d_i are determined from the information stored in the corresponding solution archive.

The number of *GP* "good" solutions is recalculated after every function evaluation as follows [3]:

$$GP = \mathrm{round}\left(\mathrm{N_p}\cdot g_p^{\,*}\right) \quad (5)$$

$$g_p^{\,*} = \mathrm{g_{p\_ini}} - \alpha\cdot\left(\mathrm{g_{p\_final}} - \mathrm{g_{p\_ini}}\right) \quad (6)$$

It is worth mentioning that (6) is not calculated in beginning of MVMO-PH, since every solution is evaluated on an individual basis. It can be observed from (5) and (6) that *GP* is linearly decreased from $\mathrm{g_{p\_ini}}$ to $\mathrm{g_{p\_final}}$.

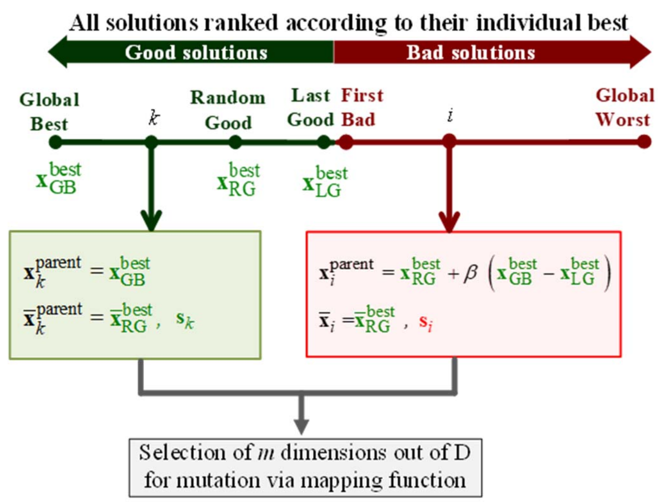

Fig. 3. Parent selection approach used in MVMO-PH [3].

D. Generation of new solution: features from parent

After each function evaluation, a new (child) solution $\mathbf{x}^{\mathrm{new}} = [x_1, x_2, x_3, \ldots, x_D]$, where D stands for dimension, is created by directly inheritance of the values of D-*m* optimization variables belonging to $\mathbf{x}_{\mathrm{p}}^{\mathrm{parent}}$, whereas the values of the remaining m dimensions are generated by using the mapping function. The mapping function uses the actual values of $\bar{x}_i$, s_i. The parameter *m* is decreased throughout function evaluations according to (7) and (8) [3].

$$m = \mathrm{round}\left(\mathrm{m_{final}} + irand\left(m^* - \mathrm{m_{final}}\right)\right) \quad (7)$$

$$m^* = \mathrm{round}\left(\mathrm{m_{ini}} - \alpha^4\left(\mathrm{m_{ini}} - \mathrm{m_{final}}\right)\right) \quad (8)$$

where *irand* denotes a random integer between 0 and the value indicated in the brackets of (7). The *m* variables, in which the mapping function is applied, are selected by using one of the strategies presented in [8].

E. Generation of new solution: mapping function

The classical mapping function (denoted as Mapping #1 hereafter), cf. (9), is applied to generate new values of selected variables x_r of $\mathbf{x}^{\mathrm{new}}$.

$$x_r = h_x + (1 - h_1 + h_0)\cdot x_r^* - h_0 \tag{9}$$

where x_r^* denotes a uniform random number generated in the [0, 1] range. The term h denotes the mapping, and h_x, h_1 and h_0 are computed according to (10) [3].

$$h_x = h(x = x_r^*),\quad h_0 = h(x = 0),\quad h_1 = h(x = 1) \tag{10}$$

where

$$h(\bar{x}, s_1, s_2, x) = \bar{x}\cdot(1 - e^{-x\cdot s_1}) + (1-\bar{x})\cdot e^{-(1-x)\cdot s_2} \tag{11}$$

It follows from (9), (10), and (11), that x_r is guaranteed to be within [0, 1]. The shape factor s_r is computed based on (12).

$$s_r = -\ln(v_r)\cdot f_s \tag{12}$$

where v_r is the variance associated to x_r and f_s constitutes a scaling factor which allows to control the shape of the mapping function.

For the application of MVMP-PH to the CEC2018 test suite, two alternative mapping functions are considered. The first one is named as Mapping #2. Essentially, it is derived from Mapping #1. Nevertheless, as shown in (13), the mathematical function is formulated differently around the mid-point of the random variable x_r^*. This ensures that the mapping function reaches $\bar{x}$ always at $x_r^* = 0.5$. Therefore, there is equal probability that the optimization variables can increase or decrease from the mid-point towards the maximum and minimum boundaries.

$$\begin{array}{ll}
\underline{for\quad x_r^* < 0.5} & \underline{for\quad x_r^* \geq 0.5} \\
s_1^* = s_1 / (1-\bar{x}) & s_2^* = s_2 / \bar{x} \\
h_m = \bar{x}\cdot\left(1 - e^{-0.5\cdot s_1^*}\right) & h_m = (1-\bar{x})\cdot e^{-0.5\cdot s_2^*} \\
h_f = \bar{x}\cdot\left(1 - e^{-x_r^*\cdot s_1^*}\right) & h_b = (1-\bar{x})\cdot e^{-(1-x_r^*)\cdot s_2^*} + \bar{x} \\
h_c = (\bar{x} - h_m)\cdot 2\cdot x_r^* & h_c = h_m\cdot 2\cdot(1 - x_r^*) \\
x_r = h_f + h_c & x_r = h_b - h_c
\end{array} \tag{13}$$

Another mapping function, named as Mapping #3, is developed in a similar fashion as shown in (14). Unlike Mapping #1 and Mapping #2, hyperbolic functions are used instead of exponential functions.

Fig. 4 shows the resulting shapes for the three mapping functions, when different values of mean and shape factors are considered. It can be seen that Mapping #2 and Mapping #3 ensure that the mean value is reached at exactly 0.5 at the horizontal axis. This entails that there is equal probability of generating a value of x_r that is smaller of bigger than the mean value. By contrast, this probability cannot be ensured when using Mapping #1. Note also in Fig. 4 that when having the same values of s_{r1} and s_{r2}, the slope Mapping #3 is more pronounced than the slope of Mapping #1 and Mapping #2. This entails that Mapping #3 is more attractive to pursue global search more efficiently. It is also important to highlight that Mapping #2 is more attractive when local search is of higher probability. In view of this, Mapping #3 was chosen for the application of MVMO-PH to the IEEE CEC2019 test suite.

$$\begin{array}{ll}
\underline{if\quad x_r^* < 0.5} & \underline{if\quad x_r^* \geq 0.5} \\
s_1^* = s_1 / (1-\bar{x}) & s_2^* = s_2 / \bar{x} \\
h_m = \bar{x} - \dfrac{\bar{x}}{(0.5\cdot s_1^* + 1)} & h_m = \dfrac{(1-\bar{x})}{(0.5\cdot s_2^* + 1)} \\
h_f = \dfrac{-\bar{x}}{(x_r^*\cdot s_1^* + 1)} + \bar{x} & h_b = \dfrac{1-\bar{x}}{((1-x_r^*)\cdot s_2^* + 1)} + \bar{x} \\
h_c = (\bar{x} - h_m)\cdot 2\cdot x_r^* & h_c = h_m\cdot 2\cdot(1 - x_r^*) \\
x_r = h_f + h_c & x_r = h_b - h_c
\end{array} \tag{14}$$

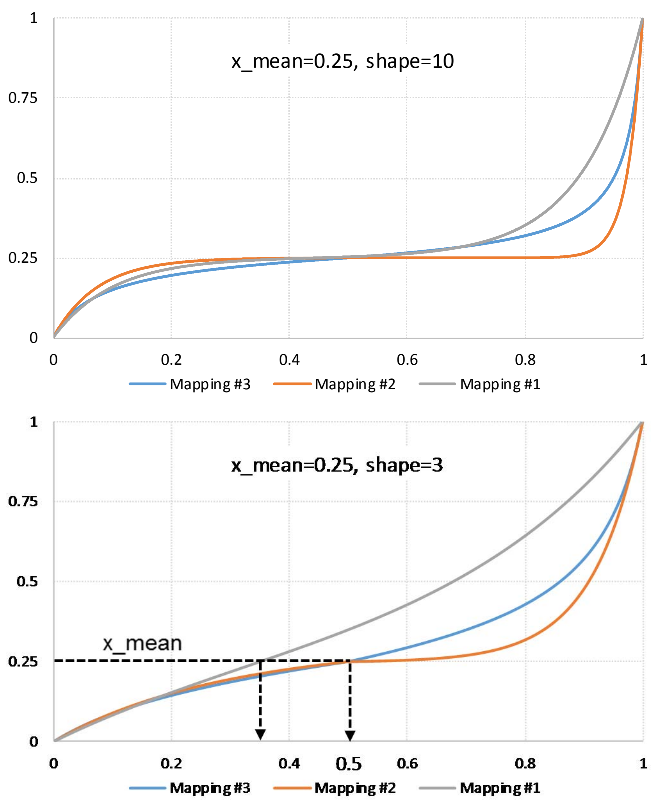

Fig. 4. Shape of mapping functions that can be used in MVMO-PH.

When evaluating each solution of the population for first time, $\bar{x}_r$ is set to 0.5 and v_r is set to 1.0 (other values could be chosen for other applications). This entails s_r=0. In subsequent function evaluations, the mean and variance are calculated again once solution archive has been updated. It is worth recalling that the input and output of the mapping function are defined in the [0, 1] range. By examining (11) and (14), it is pointed out that it is possible to change the shape of the mapping function by performing a change in $\bar{x}_r$ and s_r. This allows to achieve higher exploration. Motivated by this aspect, a scaling factor f_s is introduced to change the shape factors. Basically, f_s is designed to increase progressively after each function evaluation from a predefined start value (e.g. $f_{s\_ini} = 1$) until reaching a predefined (high) end value based on (15) and (16):.

$$f_s = \left| f_{s0} \left[4.0 + 1.65 \cdot (rand - 0.15) \right] \right| \tag{15}$$

$$f_{s0} = f_{s\_ini} + \alpha \left(f_{s\_final} - f_{s\_ini} \right) \tag{16}$$

F. Parameter tuning

In MVMO-PH algorithm used in this study, the following tuning parameters are available:

P1: Number of solutions to be evolved.
P2: Size of the archive.
P3: Initial value (f_{s_ini}) and final value (f_{s_final}) of the factors needed to control the shape of the mapping function.
P4: Initial (g_{p_ini}) and final (g_{p_final}) proportion of the population corresponding to good particles.
P5: Initial number (m_{ini}) and final number (m_{final}) of optimization variables selected for application of the mapping function.
P6: Probability threshold to launch local search (γ_{LS}).
P7: Initial range (α_{LS_min}) and final range (α_{LS_max}) of local search probability.

For tuning, the trial and error method was used. Due to restricted computing budget, 10 test runs were used to determine the parameters. These parameters were then used uniformly for all test functions. This determination was made in accordance with the rules of the competition. It is clear that an individual tuning for each function would have resulted in a better optimization result.

The following scores were used to evaluate the quality of the optimization [5]:

$$\text{Score1} = \sum_{1}^{30} mean(\text{OF})|_{\text{Dim}} + \sum_{1}^{30} median(\text{OF})|_{\text{Dim}} \tag{17}$$

$$\text{Score2} = \sum_{1}^{30} mean(f_a)|_{\text{Dim}} + \sum_{1}^{30} median(f_a)|_{\text{Dim}}$$
$$f_a = 0.5 \cdot \left(\text{OF}_{\text{MaxFEs}} + \text{OF}_{0.5\text{MaxFEs}} \right) \tag{18}$$

where Dim can be 10D or 30D, *OF* represents the error computed according to the guidelines given in [4], and the number 30 stands for the number of functions to be minimized in the competition. The abbreviation "MaxFE" stands for the allowed number of function evaluations.

III. Numerical Tests

MVMO-PH is evaluated by using the IEEE-CEC 2018 set of single objective problems with continuous (real number) optimization variables. The experiments were done by using a personal computer with the following features: Intel® Core™ i7-3770 CPU, 3.4 GHz and 16 GB RAM, Windows 8.1, 64 bit operating system. MVMO-PH was programmed by using the version R2017b of Matlab®. The Parallel Computing Toolbox was also used to distribute the optimization trials among seven cores.

A. CEC2018 test bed

TABLE I. shows the single objective problems of IEEE-CEC 2018 test bed. The definition of the problems, which are encrypted, are provided in [4]. Following the guidelines defined in [4], the numerical experiments performed for each problem of the test bed shall be conducted under the following conditions:

- Number of optimization variables: 10D, 30D, 50D, 100D.
- Search range: $[-100,100]^D$.
- Computing budget: 10000*D function evaluations.
- 51 optimization trials.
- Initial solution sampled by using a uniform distribution..
- The objective function computed as an error defined by $\text{OF}=\text{TF}_i(x) - \text{F}_i*$. The term F_i* constitutes the theoretical global optimum of each optimization problem (denoted as TF given in TABLE I.). If the obtained value of OF is lower than 1E-08, it is set to zero.

B. Analysis of results

The Appendix provides the statistics associated to OF. These statistical metrics were computed after running 51 runs of MVMO-PH. TABLES II and III show the results for all considered problem dimensions. From these tables, several conclusions are drawn:

- *Performance on unimodal functions:* MVMO-PH has a satisfactory performance in all dimensions of each function, and was able to find near zero error values (smaller than 1E-08) for OF in all runs. This is mainly attributed to the evolutionary mechanism of MVMO-PH, which is enhanced by the use of the new mapping function (to decide the search priority between exploration and exploitation), and the multi-parent crossover (to reorient the evolution of less successful solutions). Local search strategy supported MVMO-PH to effectively intensify the search around the global optimum. This evidences the ability of MVMO-PH to successfully solve all unimodal problems, without facing stagnation in any of the dimensions.

- *Performance on simple multimodal functions*: MVMO-PH was able to find zero and near zero (close to 1E-01) error values in all functions in 10D case. For 30D cases, the errors were in the order of 1E+01 for functions TF5, TF7 and TF8, whereas it was in the order of 1E+02 for TF10. The errors were in the order between 1E+02 and 1E+04 for the functions in the case of 50D and 100D. MVMO-PH found zero errors for all other multi modal functions in 30D.

TABLE I. IEEE-CEC 2018 TEST SUITE

Type	No.	Description	F_i*
Unimodal functions	TF1	Shifted and Rotated Bent Cigar Function	100
	TF2	Shifted and Rotated Sum of Different Power Function	200
	TF3	Shifted and Rotated Zakharov Function	300
Simple Multimodal functions	TF4	Shifted and Rotated Rosenbrock's Function	400
	TF5	Shifted and Rotated Rastrigin's Function	500
	TF6	Shifted and Rotated Expanded Scaffer's F6 Function	600
	TF7	Shifted and Rotated Lunacek BiRastrigin Function	700
	TF8	Shifted and Rotated Non-Continuous Rastrigin's Function	800
	TF9	Shifted and Rotated Levy Function	900
	TF10	Shifted and Rotated Schwefel's Function	1000
Hybrid function	TF11	Hybrid Function 1 (N=3)	1100
	TF12	Hybrid Function 2 (N=3)	1200
	TF13	Hybrid Function 3 (N=3)	1300
	TF14	Hybrid Function 4 (N=4)	1400
	TF15	Hybrid Function 5 (N=4)	1500
	TF16	Hybrid Function 6 (N=4)	1600
	TF17	Hybrid Function 6 (N=5)	1700
	TF18	Hybrid Function 6 (N=5)	1800
	TF19	Hybrid Function 6 (N=5)	1900
	TF20	Hybrid Function 6 (N=6)	2000
Composition functions	TF21	Composition Function 1 (N=3)	2100
	TF22	Composition Function 2 (N=3)	2200
	TF23	Composition Function 3 (N=4)	2300
	TF24	Composition Function 4 (N=4)	2400
	TF25	Composition Function 5 (N=5)	2500
	TF26	Composition Function 6 (N=5)	2600
	TF27	Composition Function 7 (N=6)	2700
	TF28	Composition Function 8 (N=6)	2800
	TF29	Composition Function 9 (N=3)	2900
	TF30	Composition Function 10 (N=3)	3000

- *Performance on hybrid functions*: In 10D case, MVMO-PH was able to find zero and near zero (smaller than 1E-01). For 30D, 50D, and 100D cases, MVMO found errors (local minimum) in the orders of 1E+00, 1E+01, 1E+02 and 1E+03.
- *Performance on composition functions*: MVMO-PH was able to find near zero error (smaller than 1E-08) for TF1-TF23, TF26, and TF28 in 10D case. By contrast, it converged to error values (local minimum) in the order of 1E+02 for these functions in 30D case, and all other functions in both 10D, 30D, 50D, and 100D. For some functions the error was in the order of 1E+03 and 1E+05.

The score measures defined in (17) and (18) were calculated for 10 repetitions of the application of MVMO-PH to solve the IEEE-CEC 2018 IEEE-CEC2018 competition test bed of single objective optimization problems, with continuous (real-parameter) optimization variables, for 10D, 30D, 50D, and 100D. The calculated scores for each repetition are shown in Table IV. Note that the score measures have comparable values for 10D, 30D, and 50D, whereas a significant difference is observed for 100D.

IV. CONCLUSIONS

This paper presented the application of population based approach of MVMO-PH on 30 test functions of the IEEE-CEC2018 competition test bed of single objective optimization problems, with continuous (real-parameter) optimization variables. Simply put, MVMO-PH evolves a set of candidate solutions. Each solution has an archive to record and track the statistical success of the evolution of the corresponding solution throughout the iterations. From each archive, mean and variance are computed for selected dimensions (i.e. optimization variables). These statistical measures are fed into a newly proposed mapping function, which guides the generation of new variables for the selected dimensions. MVMO-PH also includes a fitness-based discrimination between successful (good) solutions, which entail smaller fitness value, and unsuccessful (bad) solutions, which show slow or no improvement of fitness. Concerning the good solutions, the starting point to generate a new solution is by choosing the first ranked solution from their solution archive as parent. By contrast, multi-parent crossover is adopted in MVMO-PH to guide the bad solutions to more attractive portions of the search space. MVMO-PH offers the option of calling a local search strategy based on Interior-Point algorithm or Active-Set algorithm. The tests performed on the 30 test functions evidenced a good performance of MVMO-PH, which is mainly attributed to the application of the newly proposed mapping function, which is the key factor to ensure efficient global search capability. Ongoing research effort is devoted to develop alternative strategies for local search, which can perform with significantly reduced computational budget. The implementation of MVMO-PH on high dimensional real-parameter and single objective problems in short-term operational planning of electrical power systems is also being investigated.

REFERENCES

[1] J. J. Liang, B. Y. Qu, P. N. Suganthan, Q. Chen, "Problem Definitions and Evaluation Criteria for the CEC 2015 Competition on Learning-based Real-Parameter Single Objective Optimization", Technical Report, Computational Intelligence Laboratory, Zhengzhou University, Zhengzhou, China and Technical Report, Nanyang Technological University, Singapore, Nov 2014. [Online]. Available at http://www.ntu.edu.sg/home/epnsugan/

[2] J.L. Rueda, and I. Erlich, "Testing MVMO on Learning-based Real-Parameter Single Objective Benchmark Optimization Problems", in Proc. 2015 IEEE Congress on evolutionary Computation, pp. 1-7, Sendai, Japan, May 2015.

[3] J.L. Rueda, and I. Erlich, "Solving the CEC2016 Real-Parameter Single Objective Optimization Problems through MVMO-PHM". 2016 IEEE World Congress on Computational Intelligence, Vancouver Canada, pp. 1-10, July 2016. [Online]. Available at http://www.ntu.edu.sg/home/epnsugan/.

[4] N. H. Awad, M. Z. Ali, J. J. Liang, B. Y. Qu and P. N. Suganthan, "Problem Definitions and Evaluation Criteria for the CEC 2017 Special Session and Competition on Single Objective Bound Constrained Real-Parameter Numerical Optimization," Technical Report, Nanyang Technological University, Singapore, November 2016..

[5] Q. Chen, B. Liu, Q. Zhang, J.J. Liang, P. N. Suganthan, and B.Y. Qu, "Problem Definition and Evaluation Criteria for CEC 2015 Special Session and Competition on Bound Constrained Single-Objective Computationally Expensive Numerical Optimization," Technical Report, Nov. 2014. [Online]. Available at: http://www.ntu.edu.sg/home/epnsugan/

APPENDIX

TABLE II. RESULTS FOR 10D AND 30D

Func.	Dimension 10D					Dimension 30D				
	Best	Worst	Median	Mean	Std.	Best	Worst	Median	Mean	Std.
TF1	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
TF2	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
TF3	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
TF4	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	3.987E+00	0.000E+00	5.473E-01	1.385E+00
TF5	9.950E-01	5.970E+00	3.980E+00	3.629E+00	1.201E+00	1.990E+01	5.373E+01	3.881E+01	3.812E+01	8.268E+00
TF6	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	1.543E-08	4.052E-06	3.943E-07	7.583E-07	8.540E-07
TF7	1.231E+01	1.691E+01	1.455E+01	1.449E+01	1.162E+00	6.099E+01	8.571E+01	7.280E+01	7.267E+01	5.885E+00
TF8	9.952E-01	7.948E+00	3.980E+00	4.060E+00	1.429E+00	2.388E+01	5.870E+01	4.278E+01	4.277E+01	7.787E+00
TF9	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	2.374E-08	8.953E-02	2.034E-07	5.267E-03	2.128E-02
TF10	1.249E-01	2.453E+02	3.170E+01	7.182E+01	6.826E+01	8.135E+02	2.055E+03	1.565E+03	1.549E+03	2.693E+02
TF11	0.000E+00	1.990E+00	1.873E-04	4.888E-01	5.412E-01	4.015E+00	2.687E+01	1.394E+01	1.419E+01	4.803E+00
TF12	0.000E+00	1.311E+02	3.394E-01	8.895E+00	2.941E+01	4.503E+00	7.149E+02	3.000E+02	3.101E+02	2.018E+02
TF13	6.177E-07	6.911E+00	1.990E+00	3.186E+00	2.437E+00	7.474E+00	4.129E+01	2.573E+01	2.502E+01	6.862E+00
TF14	0.000E+00	2.985E+00	9.950E-01	9.368E-01	7.812E-01	1.750E+01	5.913E+01	3.931E+01	3.878E+01	1.072E+01
TF15	1.764E-03	2.111E+00	7.928E-02	4.552E-01	6.105E-01	2.792E+00	1.892E+01	1.052E+01	1.065E+01	3.682E+00
TF16	2.037E-02	9.910E-01	5.707E-01	5.551E-01	1.915E-01	1.898E+00	6.183E+02	4.013E+02	3.906E+02	1.465E+02
TF17	0.000E+00	1.639E+00	3.319E-01	3.972E-01	3.924E-01	6.891E+00	1.735E+02	3.601E+01	4.423E+01	2.983E+01
TF18	1.606E-01	1.495E+00	4.983E-01	5.928E-01	3.363E-01	6.282E+00	3.292E+01	2.459E+01	2.442E+01	4.279E+00
TF19	3.697E-05	3.020E+00	3.916E-02	1.532E-01	4.847E-01	4.272E+00	2.476E+01	8.847E+00	9.494E+00	3.946E+00
TF20	0.000E+00	3.122E-01	0.000E+00	1.224E-02	6.120E-02	7.994E+00	1.813E+02	3.459E+01	8.022E+01	6.380E+01
TF21	0.000E+00	1.000E+02	1.000E+02	9.804E+01	1.400E+01	1.000E+02	2.585E+02	2.440E+02	2.191E+02	5.594E+01
TF22	0.000E+00	1.010E+02	4.600E+01	5.635E+01	3.409E+01	1.000E+02	1.000E+02	1.000E+02	1.000E+02	2.801E-06
TF23	0.000E+00	3.084E+02	3.046E+02	2.277E+02	1.345E+02	1.000E+02	4.016E+02	3.840E+02	3.017E+02	1.319E+02
TF24	1.000E+02	1.000E+02	1.000E+02	1.000E+02	0.000E+00	1.000E+02	5.307E+02	4.979E+02	4.762E+02	7.903E+01
TF25	1.000E+02	4.434E+02	3.977E+02	3.929E+02	4.231E+01	3.834E+02	3.870E+02	3.835E+02	3.850E+02	1.679E+00
TF26	0.000E+00	3.000E+02	0.000E+00	9.804E+00	5.002E+01	2.000E+02	3.000E+02	2.000E+02	2.020E+02	1.400E+01
TF27	3.869E+02	3.951E+02	3.920E+02	3.917E+02	2.417E+00	4.898E+02	5.125E+02	5.028E+02	5.016E+02	5.609E+00
TF28	0.000E+00	3.000E+02	3.000E+02	1.882E+02	1.465E+02	3.000E+02	3.000E+02	3.000E+02	3.000E+02	7.281E-05
TF29	2.318E+02	2.594E+02	2.422E+02	2.430E+02	5.831E+00	3.429E+02	5.771E+02	4.644E+02	4.583E+02	4.185E+01
TF30	3.946E+02	4.446E+02	4.010E+02	4.046E+02	1.009E+01	1.977E+03	5.548E+03	2.119E+03	2.189E+03	4.866E+02

TABLE III. RESULTS FOR 50D AND 100D

Func.	Dimension 50D					Dimension 100D				
	Best	**Worst**	**Median**	**Mean**	**Std.**	**Best**	**Worst**	**Median**	**Mean**	**Std.**
TF1	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
TF2	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
TF3	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
TF4	0.000E+00	2.851E+01	0.000E+00	1.431E+00	5.630E+00	0.000E+00	3.987E+00	0.000E+00	7.059E-01	1.534E+00
TF5	6.467E+01	1.214E+02	9.353E+01	9.163E+01	1.195E+01	2.350E+02	3.930E+02	3.326E+02	3.316E+02	3.158E+01
TF6	3.760E-05	3.050E-04	1.112E-04	1.239E-04	5.691E-05	5.624E-03	3.893E-02	1.310E-02	1.439E-02	6.399E-03
TF7	1.282E+02	1.754E+02	1.513E+02	1.526E+02	1.013E+01	4.011E+02	5.168E+02	4.483E+02	4.519E+02	2.631E+01
TF8	6.169E+01	1.184E+02	9.651E+01	9.480E+01	1.202E+01	2.696E+02	4.109E+02	3.283E+02	3.324E+02	3.277E+01
TF9	9.037E-02	5.188E+00	1.183E+00	1.464E+00	1.122E+00	3.434E+02	3.076E+03	1.266E+03	1.409E+03	6.958E+02
TF10	1.548E+03	3.893E+03	3.370E+03	3.308E+03	4.071E+02	6.888E+03	1.033E+04	8.859E+03	8.882E+03	7.297E+02
TF11	2.823E+01	8.587E+01	5.616E+01	5.708E+01	1.257E+01	1.160E+02	3.302E+02	1.861E+02	1.901E+02	4.240E+01
TF12	3.051E+02	1.732E+03	1.113E+03	1.097E+03	3.442E+02	2.516E+03	4.205E+03	3.606E+03	3.522E+03	3.857E+02
TF13	1.992E+01	1.123E+02	5.917E+01	6.015E+01	2.269E+01	9.049E+01	2.693E+02	1.798E+02	1.801E+02	4.111E+01
TF14	6.264E+01	1.514E+02	9.915E+01	1.008E+02	1.914E+01	1.464E+02	2.548E+02	2.041E+02	2.042E+02	1.828E+01
TF15	1.968E+01	1.179E+02	4.833E+01	5.246E+01	1.653E+01	1.229E+02	2.353E+02	1.774E+02	1.778E+02	2.166E+01
TF16	3.246E+02	1.195E+03	8.630E+02	7.906E+02	2.285E+02	1.670E+03	2.940E+03	2.339E+03	2.340E+03	3.298E+02
TF17	2.995E+02	9.468E+02	5.643E+02	5.590E+02	1.465E+02	8.736E+02	2.442E+03	1.865E+03	1.856E+03	3.036E+02
TF18	2.622E+01	5.075E+01	3.252E+01	3.371E+01	4.860E+00	7.808E+01	1.558E+02	1.299E+02	1.268E+02	1.657E+01
TF19	1.805E+01	6.755E+01	3.516E+01	3.750E+01	1.071E+01	8.741E+01	1.672E+02	1.316E+02	1.283E+02	1.525E+01
TF20	1.704E+02	7.590E+02	4.184E+02	3.995E+02	1.162E+02	1.124E+03	2.335E+03	1.767E+03	1.740E+03	2.904E+02
TF21	2.691E+02	3.405E+02	3.023E+02	3.017E+02	1.370E+01	5.054E+02	6.044E+02	5.611E+02	5.594E+02	2.376E+01
TF22	1.000E+02	4.605E+03	1.000E+02	1.222E+03	1.764E+03	1.000E+02	1.140E+04	1.003E+04	9.563E+03	2.067E+03
TF23	4.185E+02	5.666E+02	5.397E+02	5.344E+02	2.733E+01	6.841E+02	7.520E+02	7.286E+02	7.265E+02	1.539E+01
TF24	5.744E+02	6.559E+02	6.140E+02	6.142E+02	1.826E+01	1.153E+03	1.274E+03	1.210E+03	1.208E+03	2.465E+01
TF25	4.288E+02	5.274E+02	4.802E+02	4.798E+02	1.947E+01	6.293E+02	7.619E+02	6.934E+02	6.867E+02	3.730E+01
TF26	3.000E+02	2.500E+03	2.215E+03	1.964E+03	6.860E+02	5.945E+03	7.229E+03	6.653E+03	6.656E+03	2.506E+02
TF27	5.255E+02	6.196E+02	5.675E+02	5.688E+02	2.022E+01	6.671E+02	7.677E+02	7.097E+02	7.119E+02	2.687E+01
TF28	4.534E+02	4.967E+02	4.588E+02	4.653E+02	1.289E+01	3.000E+02	5.404E+02	4.715E+02	4.119E+02	9.833E+01
TF29	3.943E+02	9.296E+02	6.763E+02	6.578E+02	1.340E+02	2.532E+03	4.208E+03	3.510E+03	3.487E+03	3.438E+02
TF30	5.794E+05	5.795E+05	5.794E+05	5.794E+05	1.880E+01	2.188E+03	2.684E+03	2.399E+03	2.418E+03	1.082E+02

TABLE IV. COMPARISON OF SCORES MEASURES

Run	D=10		D=30		D=50		D=100	
	Score1	**Score2**	**Score1**	**Score2**	**Score1**	**Score2**	**Score1**	**Score2**
1	4.580E+03	4.976E+03	1.584E+04	1.787E+04	1.186E+06	1.190E+06	9.708E+04	1.097E+05
2	4.699E+03	5.002E+03	1.580E+04	1.779E+04	1.186E+06	1.190E+06	9.737E+04	1.106E+05
3	4.594E+03	4.991E+03	1.561E+04	1.773E+04	1.185E+06	1.190E+06	9.722E+04	1.104E+05
4	4.624E+03	5.010E+03	1.565E+04	1.767E+04	1.186E+06	1.190E+06	9.668E+04	1.097E+05
5	4.565E+03	4.953E+03	1.564E+04	1.769E+04	1.186E+06	1.190E+06	9.682E+04	1.099E+05
6	4.598E+03	4.988E+03	1.570E+04	1.767E+04	1.186E+06	1.190E+06	9.721E+04	1.098E+05
7	4.701E+03	4.992E+03	1.579E+04	1.777E+04	1.186E+06	1.190E+06	9.733E+04	1.106E+05
8	4.691E+03	4.995E+03	1.559E+04	1.760E+04	1.186E+06	1.190E+06	9.674E+04	1.098E+05
9	4.605E+03	4.984E+03	1.572E+04	1.781E+04	1.186E+06	1.190E+06	9.715E+04	1.103E+05
10	4.596E+03	4.979E+03	1.559E+04	1.765E+04	1.186E+06	1.190E+06	9.723E+04	1.104E+05
Mean	4.625E+03	4.987E+03	1.569E+04	1.773E+04	1.186E+06	1.190E+06	9.708E+04	1.101E+05