An overview of structural systems theory

Ramos, Guilherme; Aguiar, A. Pedro; Pequito, Sérgio

DOI
10.1016/j.automatica.2022.110229

Publication date
2022

Document Version
Final published version

Published in
Automatica

Citation (APA)
Ramos, G., Aguiar, A. P., & Pequito, S. (2022). An overview of structural systems theory. *Automatica*, *140*, Article 110229. https://doi.org/10.1016/j.automatica.2022.110229

Automatica 140 (2022) 110229

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Survey paper

An overview of structural systems theory ☆

Guilherme Ramos [a], A. Pedro Aguiar [b], Sérgio Pequito [c,*]

[a] LASIGE, Faculdade de Ciências, Universidade de Lisboa, Portugal
[b] Department of Electrical and Computer Engineering, Faculty of Engineering, University of Porto, Portugal
[c] Delft Center for Systems and Control, Delft University of Technology, 2600 AA Delft, The Netherlands

ARTICLE INFO

Article history:
Received 25 August 2020
Received in revised form 27 January 2022
Accepted 8 February 2022
Available online 22 March 2022

Keywords:
Structural/structured systems
Controllability
Decentralized control
Combinatorial optimization
Graph theory
Generic systems properties

ABSTRACT

This paper provides an overview of the research conducted in the context of structural (or structured) systems. These are parametrized models used to assess and design system theoretical properties without considering a specific realization of the parameters (which could be uncertain or unknown). The research in structural systems led to a principled approach to a variety of problems, into what is known as structural systems theory. Hereafter, we perform a systematic overview of the problems and methodologies used in structural systems theory since the latest survey by Dion et al. in 2003. During this period, most of the focus seems to be on structural system's properties related to controllability/observability and decentralized control, in the context of linear time-invariant systems, under the classic assumption that the parameters are independent and belonging to infinite fields. Notwithstanding, it is notable an increase in research in topics that go beyond such scope and underlying assumptions, as well as applications in a variety of domains. Lastly, we provide a compilation of open questions on several settings and we discuss future directions in this field.

1. Introduction

Structural (or, structured) systems theory consists of the principled study of system-theoretic properties of parametrized dynamical systems, which parameters capture the structure of such systems. Therefore, a given parameter indicates if a state contributes to the dynamics of another state. In other words, if a parameter is identically zero, then there is no such contribution. Thus, the parameters establish an interdependency pattern between state variables, which is often referred to as a structural (or, structured) pattern.

A variety of system-theoretical properties (e.g., controllability and observability) can be assessed upon the possible numerical realization (i.e., the concrete values and, therefore, interdependencies). To make it concrete, consider the following example in the context of linear time-invariant (LTI) systems.

☆ This work was supported in part by projects: RELIABLE (PTDC/EEI-AUT/3522/2020) funded by FCT/MCTES; and DynamiCITY (NORTE-01-0145-FEDER-000073), funded by NORTE2020/PORTUGAL2020, through the European Regional Development Fund. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by EditorSonia Martinez.

* Corresponding author.
E-mail addresses: ghramos@fc.ul.pt (G. Ramos), pedro.aguiar@fe.up.pt (A.P. Aguiar), sergio.pequito@tudelft.nl (S. Pequito).

1.1. An illustrative example

Consider a series resistance–inductance–capacitance (RLC) circuit with a power generator, depicted in Fig. 1. This circuit has a resistance of R Ohm (Ω), an inductor with inductance of L Henry (H), and a capacitor with capacitance of C Farad (F). Using the Kirchhoff's circuits laws, we can model the system's evolution in continuous-time with the following equations:

$$\begin{cases} Ri_L(t) + L\dot{i}_L(t) + V_C(t) = V_s(t) \\ C\dot{V}_C(t) = i_C(t) = i_L(t) \implies \dot{V}_C(t) = \frac{1}{C} i_L(t), \end{cases}$$

where $i_C(t) \in \mathbb{R}$ and $i_L(t) \in \mathbb{R}$ denote the current through the capacitor and the inductor, respectively, and $V_C(t) \in \mathbb{R}$ and $V_s(t) \in \mathbb{R}$ denote the capacitor and energy source voltages, respectively. Now, we can write the state–space model, as a continuous-time LTI system, modeled as $\dot{x}(t) = Ax(t) + Bu(t)$, $y(t) = Cx(t)$, $t \geq 0$, with state variables

$$x(t) = \begin{bmatrix} V_C(t) & i_L(t) \end{bmatrix},$$

dynamics, input and output matrices given by

$$A = \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix}, B = \begin{bmatrix} 0 & \frac{1}{L} \end{bmatrix}^{\intercal}, \text{ and } C = \begin{bmatrix} 0 & R \end{bmatrix}, \quad (1)$$

respectively, where $u(t) = V_s(t)$, and in this case $y(t)$ is the voltage at the terminals of the resistor.

Notice that we can identify properties about the structure of the physical model presented above:

https://doi.org/10.1016/j.automatica.2022.110229

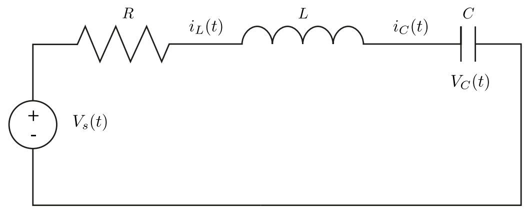


Fig. 1. A series resistance–inductance–capacitance (RLC) circuit with a voltage source.

- zero entries representing that some state variables do not have direct action on others variables' dynamics;
- some entries that depend on the system's parameters, related via algebraic relationships.

A standard approach to study various properties of the system implies obtaining the values of the physical parameters. We may obtain these values numerically, for instance, via experimentally measurements, or the components' physical values may be given by the constructor. Once we get a standard state–space representation including numerical data, we can resort to state–space theory to study some system-theoretical properties (e.g., controllability and observability).

Another possibility is to study the properties using symbolic calculus, i.e., without setting the parameter values. Such an approach can state powerful properties exploring relations between parameters. Notwithstanding, following this path becomes prohibitive as the dimension of the system grows, and, even when it is possible, we may get cumbersome conditions in terms of the parameters.

An approach that bridges the gap between the two aforementioned ones is to consider solely the structure of the system. More specifically, the focus is only on the *structural pattern* of the parametric model in (3) (i.e., matrices A, B, and C). A structural pattern is characterized by the parameters that are always equal to zero – referred to as *fixed zeros* – and denoted by 0, and the free parameters that take values (possibly including zero) in a given field — referred to as a *free* parameter. In the example in (1), we would have

$$\bar{A} = \begin{bmatrix} 0 & \lambda_1 \\ \lambda_2 & \lambda_3 \end{bmatrix}, \bar{B} = \begin{bmatrix} 0 \\ \lambda_4 \end{bmatrix}, \text{ and } \bar{C} = \begin{bmatrix} 0 & \lambda_5 \end{bmatrix}, \tag{2}$$

with $\lambda_1, \ldots, \lambda_5 \in \mathbb{R}$.

In 1974, Lin Lin (1974) provided the first results of what would grow to be structural systems theory. Specifically, he introduced the notion of structural controllability (i.e., the structural counterpart of controllability) for single-input LTI systems, where the parameters were assumed to be real and independent of each other. An interesting aspect of structural controllability, and other structural properties (e.g., observability and fixed modes in decentralized control), is that they hold for almost all possible realizations of the independent parameters belonging to infinite fields. Yet, despite this asset of structural systems, this characterization exists since at least 1962 (Markus & Lee, 1962). Structural properties allow an assessment of systems' necessary conditions, but in general, from them one cannot quantify such system properties (e.g., obtain the controllability/observability measures through the respective Grammians). Nonetheless, in many cases, this limitation is relative, since such assessment might be prohibitive when dealing with large scale systems — see Remark 1.

In this overview, we position the research in perspective to the latest survey in the field conducted by Dion, Commault, and Van Der Woude (2003) — see Table 1.

Most recent research was done in the context of LTI systems, and the characterization of structural system-theoretical properties suitable to perform the design of structural systems attaining such properties. Yet, structural counterparts of system-theoretical properties for several dynamical systems, beyond LTI systems, and classical (implicit) assumptions on the independence of the parameters were proposed. Therefore, emerging topics have not been studied in greater depth as others and, naturally, lead to smaller-in-length sections — see Table of Content below for an overview of the different topics.

Briefly, we start with an overview in the context of structural systems theory for LTI systems, where a large body of research was done in the context of structural controllability/observability, decentralized control, and fault detection and isolation. In fact, it is remarkable the amount of applications where structural systems theory is used to assess systems' properties. Thus, giving evidence of its usefulness in the practice of control engineering, as well as a scientific tool to make discoveries (e.g., in the field of network science).

We wrap up this overview with some envisioned directions towards structural-convex optimization. That is, the use of discrete-convex optimization – a term coined by Murota (2009b) that refers to a combination of a first step of discrete optimization (solvable with polynomial time complexity) to cover discrete analogues of the fundamental concepts such as conjugacy, subgradients, the Fenchel rain-max duality, separation theorems and the Lagrange duality framework, plus a second stage of convex (or convexification) optimization — in the context of control systems, where structural systems can guarantee the feasibility of the corresponding optimization problems.

2. Linear time-invariant systems

A variety of dynamical systems including mechanical and electrical systems can be modeled/described by a *linear time-invariant* (LTI) system. For example, multi-agent systems, social networks, and biological systems, just to name a few. These can be described by a *linear time-invariant* (LTI) system, whose (discrete-time) state–space representation is given by

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) + Du(t), \quad t = 0, 1, \ldots, \end{aligned} \tag{3}$$

where $x(t) \in \mathbb{R}^n$ is the state, $x(0) = x_0$ is the initial condition, $u(t) \in \mathbb{R}^p$ is the input, and $y(t) \in \mathbb{R}^q$ is the output.

With some abuse of notation, it is common to denote the free parameters with the symbol $\star$, where "$\star$'s" in different locations correspond to different free parameters (Shields & Pearson, 1976). For our illustrative example, the structural (i.e., the structural pattern) version of the RLC circuit in (2) is represented as

$$\bar{A} = \begin{bmatrix} 0 & \star \\ \star & \star \end{bmatrix}, \bar{B} = \begin{bmatrix} 0 \\ \star \end{bmatrix}, \text{ and } \bar{C} = \begin{bmatrix} 0 & \star \end{bmatrix}. \tag{4}$$

Structural patterns play a key role in structural systems as they admit a representation as a directed graph (digraph) (Cormen, Leiserson, Rivest, & Stein, 2009), and several structural properties (e.g., structural controllability, structural observability, and structurally fixed modes) can be characterized by properties of such digraphs. The digraph consists of a set of vertices (or nodes) and directed edges. The representation of the systems dynamics is provided by the *state digraph* $\mathcal{G}(\bar{A}) = (\mathcal{X}, \mathcal{E})$, where $\mathcal{X} = \{x_1, \ldots, x_n\}$ denote vertices labeled by the states of the system, and $\mathcal{E} = \{(x_i, x_j) : \bar{A}_{ij} \neq 0\}$ are the edges capturing the dependencies between the states. For the RLC example, the state digraph $\mathcal{G}(\bar{A})$ is depicted in Fig. 2(a).

Input(-state) and output(-state) digraphs can be similarly obtained by considering, together with the dynamics matrix, the

Table 1
Summary of structural systems research directions before and after Dion et al. survey (Dion et al., 2003). A "–" means that the previous survey did not specifically focus on the subtopic.

	Structural systems theory	
Research directions	Before Dion et al. survey	After Dion et al. suvery
Structural controllability	Basile and Marro (1969), Glover and Silverman (1976), Lin (1974), Markus and Lee (1962), Shields and Pearson (1976) Aling and Schumacher (1984), Dion and Commault (1982), Reinschke and Reinschke (1988), Suda, Wan, and Ueno (1989), Wonham (1985) Commault, Dion, and Perez (1991), Van Der Woude (1991)	Sections 2.1–2.7
Decoupling and disturbance rejection	Commault et al. (1991), Descusse and Dion (1982), Dion and Commault (1993), Linnemann (1981), Schumacher (1980) Commault, Dion, and Hovelaque (1997), Dion, Commault, and Montoya (1994), Van Der Woude (1996)	Sections 2.8 and 5.1
Decentralized control	Kobayashi and Nakamizo (1987), Kobayashi and Yoshikawa (1982), Linnemann (1983), Sezer and Šiljak (1981) Kong and Seo (1996), Reinschke, Jantzen, and Evans (1992), Trave, Titli, and Tarras (1989)	Section 5.3
Fault detection and isolation (FDI)	Commault, Dion, Sename, and Motyeian (2002), Frank (1996), Roberts (2001)	Sections 2.9–5.2
Other Subclasses of Structural Systems	–	Section 3
Variations on Structural Systems Theory	–	Section 4
Applications	–	Section 5

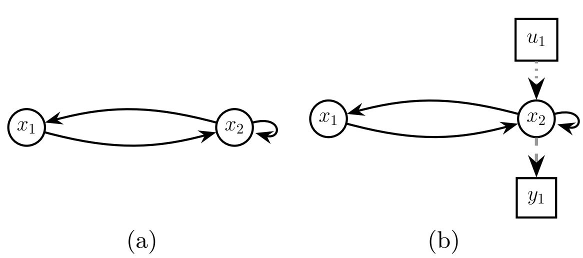


Fig. 2. Digraph representations from (4) of the structural matrix (state digraph) $\mathcal{G}(\bar{A})$, in (a), and input- and output-state digraph, $\mathcal{G}(\bar{A}, \bar{B}, \bar{C})$, in (b).

input and output matrices, respectively. Specifically, the input vertices only have outgoing edges to the states, whereas the output vertices only have incoming edges from the states. In these cases, the edges from the inputs to the states represent which states are under the direct influence of the actuators/controller, whereas the edges to the output vertices encode the observations/measurements of the states obtained by the sensors.

In summary, the digraph representation yields a simple way to visualize the dependencies between the system's variables. Furthermore, it enables us to study (structural) systems properties as graphical properties for which there might exist efficient algorithms to either assess them or design systems having such properties. In Fig. 2(b), we depict the input and output state digraph of the RLC example, $\mathcal{G}(\bar{A}, \bar{B}, \bar{C})$.

In what follows, we focus on the discrete-time LTI system. Notwithstanding, most results readily apply to continuous-time LTI, and we will specify when this is not the case.

2.1. Structural controllability

Structural controllability of the LTI system described by the dynamics and input matrices with structural pattern $(\bar{A}, \bar{B})$ is attained if and only if there exists a controllable system described by (A, B) with the same structural pattern as $(\bar{A}, \bar{B})$. It is possible invoke measure theoretical arguments to establish that structural controllability is a generic property when parameters belong to an infinite field (Lin, 1974). More precisely, a pair of matrices (A_0, B_0) is structurally controllable *if and only if* for every $\varepsilon > 0$ there exists a controllable pair (A_1, B_1) with the same structure of (A_0, B_0) – i.e., $(\bar{A}_0, \bar{B}_0) = (\bar{A}_1, \bar{B}_1)$ – such that $\|A_1 - A_0\| \leq \varepsilon$ and $\|B_1 - B_0\| \leq \varepsilon$, where $\|\cdot\|$ is any matrix norm. In other words, if there is a pair (A_1, B_1) with the same structural pattern as $(\bar{A}_0, \bar{B}_0)$, then almost all pairs (A', B') with the same structural pattern lead to controllable systems. From this point on, we say that a pair $(\bar{A}, \bar{B})$ is structurally controllable, as the property depends only on the structural pattern, meaning that for any (A_0, B_0) with the structure of $(\bar{A}, \bar{B})$, the previous property holds.

In particular, it follows that if one replaces the nonzero entries in $(\bar{A}, \bar{B})$ at random by entries drawn from a continuous distribution whose measure has support[1] in a nonzero Lebesgue measure set, such pair will be almost surely controllable. Note also that if a pair $(\bar{A}, \bar{B})$ is not structurally controllable, then there exists no realization of a controllable pair (A, B) with such a structural pattern. Thus, structural controllability entails a necessary condition for controllability.

To assess structural controllability, we can use a variety of graph theoretical tools like an auxiliary *(undirected) bipartite*

1 The support of a probability distribution function $f : \mathbb{R} \to \mathbb{R}$ is the set of points such that f is not zero.

graph on which we can find *maximum matchings* (Dion et al., 2003). The undirected bipartite graph, $\mathcal{B}(\bar{A}) = (\mathcal{X}_L \cup \mathcal{X}_R, \mathcal{E}_{\mathcal{X}_L,\mathcal{X}_R})$, is built from the system digraph, $\mathcal{G}(\bar{A}) = (\mathcal{X}, \mathcal{E}_{\mathcal{X},\mathcal{X}})$, with $\bar{A} \in \{0, \star\}^{n\times n}$, as follows. The disjoint sets of (left and right) vertices $\mathcal{X}_L$ and $\mathcal{X}_R$ are created as $\mathcal{X}_L = \{x_i^L : i = 1, \dots, n\}$ and $\mathcal{X}_R = \{x_i^R : i = 1, \dots, n\}$, respectively. The set of edges is $\mathcal{E}_{\mathcal{X}_L,\mathcal{X}_R} = \{(x_i^L, x_j^R) : (x_i, x_j) \in \mathcal{E}_{\mathcal{X},\mathcal{X}}\}$. A maximum matching of $\mathcal{B}(\bar{A}) = (\mathcal{X}_L \cup \mathcal{X}_R, \mathcal{E}_{\mathcal{X}_L,\mathcal{X}_R})$ is a set of edges $\mathcal{M}^*$ that is a solution of the optimization problem

$$\begin{aligned} \mathcal{M}^* &= \arg\max_{\mathcal{M}\subseteq\mathcal{E}_{\mathcal{X}_L,\mathcal{X}_R}} |\mathcal{M}| \\ \text{such that} &\quad (u, v), (u', v') \in \mathcal{M} \textit{ iff } u \neq u', v \neq v'. \end{aligned}$$

Finding a maximum matching in a bipartite graph leads to a decomposition of the digraph into disjoint paths and cycles, see the example in Fig. 5. Notice that an undirected edge in the bipartite graph corresponds to a directed edge in the state digraph. For example, in Fig. 5(c) the undirected edge (x_1^L, x_2^R) corresponds to the edge (x_1, x_2) in Fig. 5(a). In fact, such a decomposition has the minimum number of paths (possibly degenerated, i.e., single vertices) and an arbitrary number of cycles (see Lemma 3 of Pequito, Kar, & Aguiar, 2015a for more details). Such decomposition in paths and cycles of the digraph plays a critical role in the notion of structural controllability (Dion et al., 2003). More generally, all maximum matchings can be described through the so-called Dulmage–Mendelsohn decomposition that plays a role in characterizing the fixed controllable subspace (Commault, van der Woude, & Boukhobza, 2017). The Dulmage–Mendelsohn decomposition encodes the partition of the vertices of a bipartite graph into subsets that satisfy the following property: any two adjacent vertices belong to the same subset *if and only if* they are paired in a maximum matching of the graph. Intuitively, such encoding can be done through a matrix in a block diagonal form (after proper left and right permutations), where the following hold: (i) the maximum number of nonzero entries in the diagonal that identify the edges in a maximum matching, and (ii) any permutation on the columns and rows of the block yields the same number of nonzero entries in the diagonal that characterize all the maximum matchings.

Nonetheless, it is possible to assess structural controllability by resorting to other methods, e.g., dynamic graph properties (Van Der Woude, Commault, & Boukhobza, 2019). In this work, the authors studied linear structural systems and focused on the fixed part of the controllable subspace of such systems. To do so, they use the concept of dynamic graphs, which not only represent the structure of the system but also its evolution in time. To further represent the evolution in time, the dynamic graph has $n(n+m)$ nodes and nk edges, where n and m denote the number of states and inputs, respectively, and k the number of nonzeros in the system matrices. Hence, it is a different interpretation, whose advantage concerning the Dulmage–Mendelsohn decomposition is yet to be established.

It is worth noticing that the study of the structural observability of a pair $(\bar{A}, \bar{C})$ is equivalent (by duality) to the study of the structural controllability of the pair $(\bar{A}^\intercal, \bar{C}^\intercal)$.

It is important to stress, however, that the representation choice leads to different (discrete) combinatorial optimization problems — some of which can be efficiently solved, whereas others enable us only to obtain efficient approximate solutions with possible optimality guarantees, as we explore and detail in the subsequent sections. While the computational efficiency of different representation choices for assessing structural systems properties may not differ much, these representations can make the difference when designing systems that possess such structural properties. To make a parallel that is more familiar for this paper's possible audience, a (classical) optimization problem may seem nonconvex, yet it might admit a convex representation that ensures the use of computationally efficient numerical algorithms to determine its solution. Nonetheless, different convex representations might be possible for the same problem (i.e., linear program, quadratic program, geometric, second-order cone program, or semi-definite program), but they require numerical algorithms that have different computational efficiency.

2.2. Designing systems to attain structural controllability

In the context of designing structural systems that attain structural controllability, there are two main classes of problems: *(i)* actuator placement (or input/actuator selection) problem; and *(ii)* structural dynamics design problem (i.e., addition of inter-dependencies between state variables).

Let $\Theta \equiv \Theta(A, B)$ be a collection of the interesting parameters in A and/or B and $\mathcal{L}_\Theta$ be an objective function that assigns a cost to each combination of interesting parameters in Θ, $\mathcal{L}_\Theta : \mathbb{R}^{n\times n} \times \mathbb{R}^{n\times p} \to \mathbb{R}$ parametrized by Θ, and where (A, B) describes the model (3). Therefore, from a structural perspective, we seek to determine the solution to the following problem in terms of the generic controllability

$$\begin{aligned} &\min_{\Theta} \quad \mathcal{L}_\Theta(A, B) \\ &\text{s.t. } \underset{\Theta}{grank}(\mathcal{C}(A, B)) = n, \end{aligned} \tag{5}$$

where *grank* is the *generic rank* of $\mathcal{C}(A, B)$, i.e., the maximum rank that can be achieved with matrices (A, B) that possess the same structural pattern as $(\bar{A}, \bar{B})$, and $\mathcal{C}(A, B) = [B \ AB \ A^2B \ \dots \ A^{n-1}B]$ is the *controllability matrix* used to establish the controllability of a system described by (A, B) (i.e., the rank of the controllability matrix equals the dimension n of the state). Notice that, for large matrices, when we analyze the rank of the controllability matrix up to a certain computational precision, we may get more linearly dependent column (or row) vectors than the real number. This issue leads to a matrix rank that is smaller than the real value. Furthermore, it is interesting to notice that structural controllability also emerges as a way to assess controllability under numerical instability caused by computational precision issues that we can find in the finite-horizon Grammian — see Remark 1.

Remark 1 (*Inadequacy of Finite-Horizon Grammian to Evaluate Controllability*)**.** Consider the use of the finite horizon controllability Gramian associated with (A, B) and a finite horizon value $k > 0$, given by

$$W = \sum_{i=0}^{k-1} A^i BB^\intercal (A^\intercal)^i,$$

which describes the controllability energy through its eigenvalues' sum. Additionally, consider the matrix A as the adjacency matrix of the digraph depicted in Fig. 3(a), and <u>add</u> a uniformly generated random noise <u>solely</u> to the entry $A_{21} = 1$, where the noise is uniformly generated from the interval $]-\varepsilon, \varepsilon[$ to simulate computational precision errors, for different $\varepsilon > 0$ values ranging from 0 up to 0.0001. If $\varepsilon = 0$ then we consider the absence of noise (no computational precision errors). In Fig. 3(b), for $k = 10$, we illustrate the absolute value of the Gramian eigenvalues' sum variation for different values of ε, defined as $\Delta_{tr(\sigma(W))} = |tr(\sigma(W)) - tr(\sigma(W_\varepsilon))|$, where W_ε is the finite horizon Gramian with $k = 10$ and matrix A with noise ε added to the entry A_{21}. Observe that a minimal perturbation to a single entry of matrix A produces substantial changes in the Gramian eigenvalues' sum. Therefore, quantitative assessment of controllability energy might be misleading, and we should pursue other methods that enable us to assess controllability among other properties for large scale systems. ∘

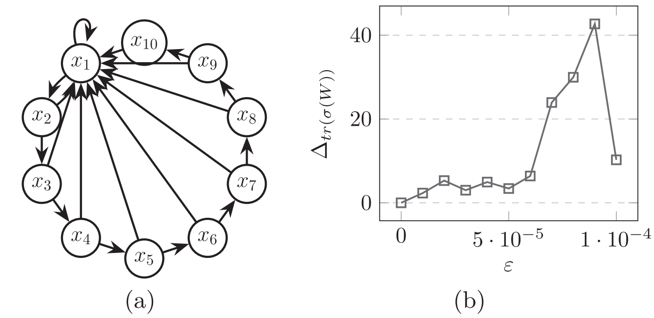


Fig. 3. In (a), a digraph with 10 vertices is depicted, $\mathcal{G} \equiv \mathcal{G}(A)$. In (b), we depict the variation of the eigenvalues trace using the finite horizon Gramian matrix, with $k = 10$, when we add uniformly generated noise with radius ε only to the entry A_{21}. Notice that a small ε value can reflect in a $\Delta_{tr(\sigma(W))}$ in the order of $10^5 \times \varepsilon$.

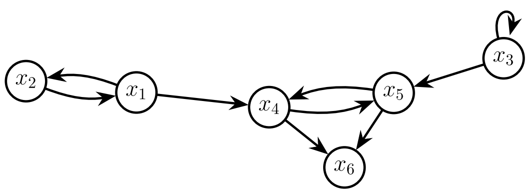


Fig. 4. Digraph representation of the structural matrix $\bar{A}$, $\mathcal{G}(\bar{A})$.

We refer to the pair $(\bar{A}, \bar{B})$ as being structurally controllable if and only if $grank(\mathcal{C}(A, B)) = n$, where (A, B) have the structural pattern $(\bar{A}, \bar{B})$. Thus, we can reformulate the latter problem (5), as

$$\begin{aligned} \min_{\substack{\bar{A} \in \{0,\star\}^{n \times n} \\ \bar{B} \in \{0,\star\}^{n \times p}}} \quad & \mathcal{L}_{\Theta}(\bar{A}, \bar{B}) \\ \text{s.t.} \quad & (\bar{A}, \bar{B}) \text{ structurally controllable,} \end{aligned} \tag{6}$$

where $\mathcal{L}_{\Theta} : \{0, \star\}^{n \times n} \times \{0, \star\}^{n \times p} \rightarrow \mathbb{R}$, and the combinatorial nature of the problem becomes apparent.

2.2.1. Actuator placement

In the actuator placement (or, input) design context, given $\bar{A}$ one often seeks to determine $\bar{B}$ that minimizes a given objective function. For instance, the objective could be $\|\bar{B}\|_0$, which implies that one seeks to determine the minimum number of state variables that need to be actuated to ensure structural controllability. Briefly, given $\bar{A}$, find $\bar{B}^* \in \{0, \star\}^{n \times p}$ that solves

$$\begin{aligned} \min_{\bar{B} \in \{0,\star\}^{n \times p}} \quad & \|\bar{B}\|_0 \\ \text{s.t. } & (\bar{A}, \bar{B}) \text{ structurally controllable.} \end{aligned} \tag{7}$$

Problem (7), and some of its variations, can be addressed efficiently (using polynomial-time algorithms) in terms of a combination of graph-theoretical procedures, see Pequito et al. (2015a). The solution consists of two steps: (i) identifying the maximum matchings on the state bipartite graph, and (ii) computing the unique decomposition of the state digraph into its *strongly connected components* (SCC), i.e., disjoint subgraphs with the property that there exists a path between any two vertices in each subgraph. As such, if we consider the additional constraint that at most one nonzero entry per column in $\bar{B}$ (i.e., the inputs are *dedicated*, meaning that each input actuates only one state variable), then the minimum number of dedicated inputs (i.e., nonzero columns) is given by

$$m = \max\{1, r + \beta - \alpha\},$$

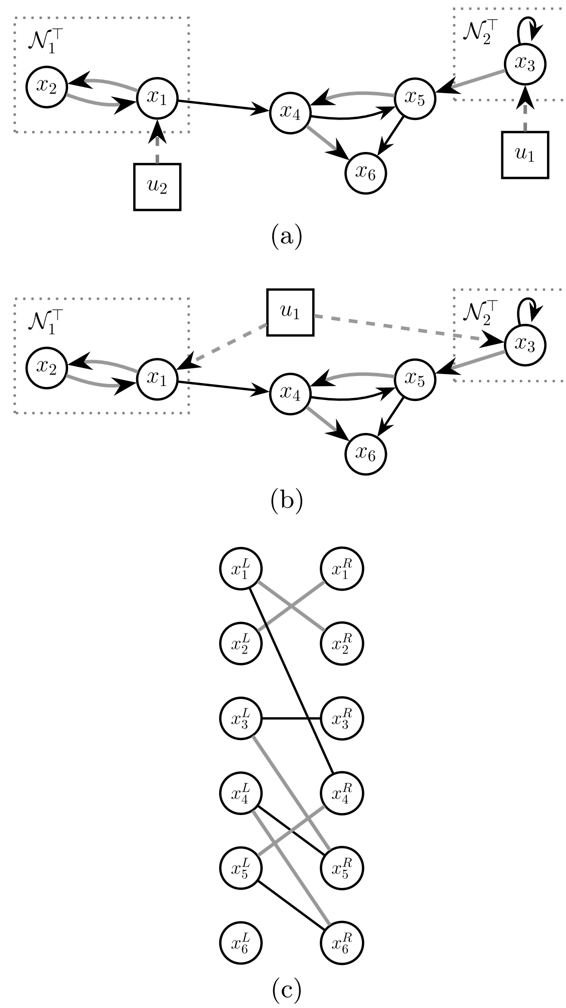


Fig. 5. Digraph representation $\mathcal{G}(\bar{A}, \bar{B})$ of two possible solutions to the problem stated in (7). A solution with dedicated inputs is depicted in (a), and a non-dedicated input scenario is depicted in (b). The edges in gray are the path and cycle decomposition obtained from a maximum matching of $\mathcal{B}(\bar{A})$, depicted by the gray edges in (c), with maximum assignability index. The sets of vertices inside the dotted gray boxes are the source-SCCs. Finally, the square vertices depict input variables.

where r is the number of right-unmatched vertices of the associated bipartite graph (i.e., the starting vertices of the edges that do not belong to a maximum matching), β is the number of source-SCC (i.e., connected components without incoming edges into their vertices, denoted by $\mathcal{N}_1^\top, \ldots, \mathcal{N}_k^\top$), and α is the maximum assignability index of the network. The maximum assignability index, α, is the maximum number of source-SCC that contains right-unmatched vertices among all maximum matchings of the state bipartite graph. Besides, it is possible to compute a computationally efficient solution by reformulating the problem as a *weighed* maximum matching problem that can be solve with the Hungarian matching algorithm (Cormen et al., 2009). In other words, a solution with maximum assignability index α can be constructed by properly defining a cost for each edge in the bipartite graph, and computing a maximum matching with maximum cost (where the cost is the sum of each edge cost in the maximum matching). This idea of reducing the actuator placement to a maximum matching problem is further explored in variation of the previous problem, that we detail next.

To exemplify the above results, consider the structural matrix

$$\bar{A} = \begin{bmatrix} 0 & \star & 0 & 0 & 0 & 0 \\ \star & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \star & 0 & 0 & 0 \\ \star & 0 & 0 & 0 & \star & 0 \\ 0 & 0 & \star & \star & 0 & 0 \\ 0 & 0 & 0 & \star & \star & 0 \end{bmatrix},$$

with digraph representation depicted in Fig. 4.

Two possible solutions to the problem in (7) are illustrated in Fig. 5(a)–(b), where $\bar{A}$ is associated with the state digraph depicted in Fig. 4. Similarly, in Commault and Dion (2015), the authors provide a polynomial complexity algorithm (i.e., an algorithm that can be efficiently computed, and whose computation-time is bounded by a polynomial that depends on the size of the algorithm inputs (Arora & Barak, 2009)) to achieve structural controllability for single-input systems. In this case, we simply have to check if it is possible to have a maximum matching that leads to placing the single input in the source-SCC (a single input is only possible when there is only one source-SCC). Specifically, we just need consider an input to each state variable in the source-SCC and verify if there exists a maximum matching satisfying the required conditions.

Other variants of the actuator placement problems have been suggested and addressed in the literature. An example is the computation of the minimum number of inputs to ensure structural controllability, without restricting to single-input systems. The solution equals the number r of right-unmatched vertices of the state bipartite graph, which can be solved with time complexity given by $\mathcal{O}(n^3)$. Alternatively, one can consider the minimum actuation cost on either the inputs (Pequito, Kar, & Pappas, 2015) or the state variables actuated (Pequito, Kar, & Aguiar, 2016). These problems can be solved by resorting to computationally efficient (i.e., $\mathcal{O}(n + m\sqrt{n})$, where m denotes the number of nonzero entries of A) (Olshevsky, 2015). In Doostmohammadian and Khan (2019), the authors look into the particular case of self-damping systems, where the use of maximum matchings is waived, and computational complexity reduces to that of finding the SCCs of the state digraph (i.e., $\mathcal{O}(n^2)$).

Notwithstanding, such computational complexities might still be prohibitive when dealing with large-scale systems and, therefore, in Shirani Faradonbeh, Tewari, and Michailidis (2017), the authors propose using fast maximum matching algorithms with optimality guarantees. Specifically, instead of incurring $\mathcal{O}(m\sqrt{n})$ time complexity to compute a maximum matching, it can be approximated with fast methods that hold linear time complexity (i.e., $\mathcal{O}(n)$). The author proposes the use of the Karp–Sipser algorithm (Karp & Sipser, 1981) and presents a generalization called the one-sided Karp–Sipser algorithm. Alternatively, random sampling schemes can be considered (also with certain approximation guarantees) (Jia & Barabási, 2013; Ravandi, Ansari, & Mili, 2019). In Ravandi et al. (2019), the authors study the statistical characteristics of the actuator placement problem by randomly selecting and assessing the resultant controllability properties of complex networks, whereas in Jia and Barabási (2013), a random sampling algorithm is developed to address the actuator placement problem. The algorithm not only provides a statistical estimate of the control capacity, but also to bridge the gap between multiple microscopic control configurations and macroscopic properties of the underlying network — these results for the actuator placement problem complement some available heuristics to find a maximum matching for directed networks (Chatterjee, Das, Naskar, Pal, & Mukherjee, 2013).

Remark 2. In contrast with the sparsest minimum structural controllability problem in (7) that is polynomially solvable, the (sparsest) minimum controllability problem (i.e., the non-structural counterpart) is NP-hard[2] (Olshevsky, 2014). Interestingly, by invoking the fact that structural controllability holds generically when the parameters belong to infinite fields, it follows that given an arbitrary matrix A with the structure $\bar{A}$, the (sparsest) minimum controllability problem is almost surely polynomially solvable. ∘

Remark 3. An important application of the actuator placement problem is that of *leader selection* in the context of multi-agent systems (Blackhall & Hill, 2010; Commault & Dion, 2013). In Commault and Dion (2013) the authors present a unified framework for controllability via different input addition settings for dynamical graph-based systems. The paper investigates the structural mechanisms of controllability resulting from an input addition (or leader selection) under different assumptions, presenting a structural characterization of all the solutions. The authors in Blackhall and Hill (2010) extended the notion of network controllability of structural linear systems, providing a criterion to assess the structural controllability of large networks, using only local information about each system in the network and the network interconnection structure.

In this context, it is possible to assess the selection of leaders to ensure resilience as well as analytical properties that ensure the network to be resilient (Jafari, Ajorlou, & Aghdam, 2011). In addition, it is also possible to determine an efficient solution to the *distributed* leader selection that builds upon structural controllability, yet it guarantees (non-structural) controllability by determining proper weights in a fully distributed fashion (Pequito, Preciado, & Pappas, 2015; Tsiamis, Pequito, & Pappas, 2017). The work in Pequito, Preciado, and Pappas (2015) determines the minimum subset of agents that should act as leaders. Also, when the communication graph is time-invariant, the authors design a communication protocol (i.e., select the weights for the agents' linear updates) to ensure some control-theoretic specifications and/or performance guarantees. In Tsiamis et al. (2017), the problem is extended and addressed for switching networks of high-order integrators. ∘

It is worth noticing that changes in the assumptions of the actuator placement problem might lead to a NP-hard problem (Pequito, Kar, & Aguiar, 2015b). For instance, consider the following optimization problem: given $\bar{A}$ and $\bar{B}$, determine the smallest subcollection of inputs $B(\mathcal{I})$ ensuring structural controllability, where $\mathcal{I} \subset \{1, \ldots, p\}$ denotes the labels of the inputs described by the columns of the matrix $\bar{B}$, i.e.,

$$\begin{aligned} \min_{\mathcal{I} \subset \{1,\ldots,p\}} \quad & |\mathcal{I}| \\ \text{s.t. } & (\bar{A}, \bar{B}(\mathcal{I})) \text{ structurally controllable.} \end{aligned} \tag{8}$$

In fact, the decision version of the problem in (8) is NP-complete[3] (Assadi, Khanna, Li, & Preciado, 2015). Briefly speaking, this problem is as difficulty as several of the other well-known NP-complete problems. Nonetheless, it is possible to determine efficient approximations with provable optimality guarantees (Moothedath, Chaporkar, & Belur, 2018a) by formulating a new graph-theoretic necessary and sufficient condition for checking structural controllability using flow-networks, and proposing a polynomial reduction of the problem to the *minimum-cost fixed-flow problem* – an NP-hard problem for which polynomial approximation algorithms exist.

Additionally, as systems increase their dimension, it is of interest to determine the conditions that allow us to assess structural controllability efficiently by possibly considering composite

[2] A problem $\mathcal{P}$ is in NP if, given a candidate solution to the problem, it can be verified if it is indeed a solution in polynomial time. A problem $\mathcal{P}$ is NP-hard when every problem $\mathcal{P}'$ in NP can be reduced in polynomial time to $\mathcal{P}$.

[3] A problem is NP-complete if it is both in NP and NP-hard.

systems or leveraging distributed algorithms (Carvalho, Pequito, Aguiar, Kar, & Johansson, 2017).

2.2.2. Sensor placement

The sensor placement problem seeks to determine the sensors to be deployed, or sensing capabilities,

$$y(t) = Cx(t), \tag{9}$$

with $y(t) \in \mathbb{R}^m$, required to attain *structurally observability* – a system is structurally observable if and only if there exists an observable (A, C) with the structural pattern $(\bar{A}, \bar{C})$. By invoking the duality between controllability and observability in the context of LTI systems, it readily follows that all the actuator placement problems can be posed as sensor placement problems and vice-versa.

Notwithstanding, a variety of sensor placement problems have been proposed in Boukhobza, Hamelin, and Martinez-Martinez (2007) and Commault and Dion (2007). Specifically, the authors of Commault and Dion (2007) considered observer-based FDI problems for linear structural systems with disturbances, focusing on when, using the measurements available on the system, the problem has no solution. The authors considered that new sensors could be placed on the system at some cost. They studied solutions with a minimal number of additional sensors with minimal cost, presented in simple and intuitive graph terms. In Boukhobza et al. (2007), the authors propose a graph-theoretic tool to analyze the state and input structural observability of structural linear systems. They present necessary and sufficient conditions for state and unknown input structural observability, providing a method with polynomial-time computational complexity.

In Liu and Sinopoli (2018), the authors propose to determine a collection of sensors to ensure that a certain observability subspace allows the retrieval of certain state variables of interest. Additionally, sensor placement can be achieved under possible cost constraints (Liu, Niu, & Ren, 2019; Liu, Zhang, & Tian, 2017). More specifically, Liu et al. (2017) tackle the optimal control configuration design problem of multi-agent systems, where the costs to control or measure are nonuniform among agents. The authors reduce the problem to the structural controllability problem of a bilinear system with minimum cost and present a polynomial-time optimization algorithm to determine the controlled nodes and measured nodes. In Liu et al. (2019), the control configuration design of bilinear networks is studied. Under the assumptions that the costs to actuate or measure are nonuniform and the dynamics matrix has independent entries, the authors devise a method with polynomial-time complexity to address this design problem with heterogeneous costs. In Doostmohammadian, Rabiee, Zarrabi, and Khan (2018), the authors propose to leverage the structure of the parametric models by performing contractions of the graph (i.e., to obtain equivalence classes of parametric models). Two states, x_i and x_j, are in the same equivalence class whenever

$$grank\left(\begin{bmatrix} A \\ C_i \end{bmatrix}\right) = grank\left(\begin{bmatrix} A \\ C_j \end{bmatrix}\right) = grank\,(A) + 1,$$

where C_i and C_j denote the observability matrices of states x_i and x_j, respectively.

In Commault, Dion, and Do (2011), sensor placement is considered to achieve disturbance rejection by measurement feedback. Additionally, sensor placement can be considered to attain resilience/robustness with respect to sensor failure that maintain structural observability (Boukhobza, 2010; Boukhobza & Hamelin, 2009; Commault, Dion, et al., 2008). Using a graph-theoretical approach, in Commault, Dion, et al. (2008), the observability preservation under sensor failure is studied, classifying the sensors concerning their critical nature concerning observability. The work in Boukhobza and Hamelin (2009) deals with the problem of additional sensor location to recover the state and input observability for structural linear systems. To recover the generic state and input observability of structural linear systems, Boukhobza (2010) provides the minimal number of the required extra sensors and either their location or necessary and sufficient conditions to be satisfied by any permitted location.

Lastly, in Kruzick, Pequito, Kar, Moura, and Aguiar (2018), the authors address the sensor placement when backbone nodes (e.g., routers) are considered in the context of sensor networks.

In the context of learning the model parameters (i.e., to perform system identification), it would be interesting to find where to place sensors that would allow *structural learning* – i.e., sensor placement to attain structural identifiability in the sense of Bellman and Åström (1970). Some aspects towards structural learning have been addressed in Jacquez and Greif (1985), but necessary and sufficient conditions are lacking. That said, some necessary or sufficient conditions are available, see for instance (Agbi & Krogh, 2014; Cantó, Coll, & Sánchez, 2009). In Agbi and Krogh (2014), the problem of identifying large building models is addressed by developing a theoretical framework for decentralized identification of such models, using a novel heuristic to decompose large building models into zone models using graph theory. The work in citecanto2009structural presents a mathematical model to describe a dialysis process based on linear dynamic systems and addresses the problem of estimating the system parameters to obtain conditions to assure their uniqueness.

2.2.3. Dynamics design problem to attain structural controllability

In contrast with the actuator placement problem, where the goal is to add actuation capabilities, the dynamics design problem focus on changing the structural pattern of the dynamics to ensure structural controllability under given actuation capabilities. Specifically, given $(\bar{A}, \bar{B})$, find the sparsest structural perturbation $\bar{\Delta}^* \in \{0, \star\}^{n\times n}$ that is a solution to the following problem:

$$\begin{aligned} \min_{\bar{\Delta}\in\{0,\star\}^{n\times n}} \quad & \|\bar{\Delta}\|_0 \\ \text{s.t. } & (\bar{A}+\bar{\Delta}, \bar{B}) \text{ structurally controllable.} \end{aligned} \tag{10}$$

The problem in (10) can be efficiently solved (Chen, Pequito, Pappas, & Preciado, 2018; Mu, Li, Zou, & Li, 2019; Zhang & Zhou, 2019b). More specifically, in Chen et al. (2018), the authors address the problem of optimally modifying the topology of a directed dynamical network to ensure structural controllability. They propose a framework with polynomial-time complexity to find the minimum number of directed edges that need to be added to the network topology to generate a structurally controllable system. The problem of selecting the minimal number of subsystem interconnection links is studied in Zhang and Zhou (2019b), under the requirement of constructing a structurally controllable networked dynamic system. The authors derive a heuristic method with provable approximation bounds and low computational complexity. In Mu et al. (2019), through some proper adjustments, the authors address the minimum input/edge addition problem of newly obtained networked systems by reducing it to a maximum matching problem. They present effective criteria to assure structural controllability for the networked systems via adding minimum extra control inputs or minimum extra edges.

Furthermore, the problem in (10) is associated with the resilience and security of structural systems, where the problem is that of determining the impact of edges failure/removal in the structural controllability (Alcaraz & Wolthusen, 2014; Jafari, Ajorlou, Aghdam, & Tafazoli, 2010; Rahimian & Aghdam, 2013; Ramos, Pequito, Aguiar, & Kar, 2015; Zhang, Xia, Zhang, & Shang, 2020). In Ramos et al. (2015), the authors define p-robustness to ensure the proper functioning of the electric power grids, in the sense of guaranteeing generic controllability of the associated dynamical system, under arbitrary p transmission line failures. They provide conditions to ensure p-robustness and an algorithm that determines the minimum number of transmission lines to add to transform a non-robust (0-robust) electric power grid into a 1-robust electric power grid.

In Jafari et al. (2010), the structural controllability of a leader–follower multi-agent system is considered. The authors introduce the notions of p-link and q-agent controllability. They measure the controllability of the system in the presence of failures in communication links or agents. Necessary and sufficient conditions for the system to remain structurally controllable in these scenarios are presented, and polynomial-time algorithms developed to determine the maximum number of each type of failure can occur while keeping the structural controllability. The same problem is studied in Rahimian and Aghdam (2013). The authors introduce indices and importance measures that help characterize and quantify the role of individual links and agents in the controllability of the overall network. Moreover, the authors identify a class of digraphs where joint the introduced measure is a necessary and sufficient condition for the system to remain controllable after the failure of any set with bounded number of links and agents.

The paper in Alcaraz and Wolthusen (2014) studies strategies for the efficient restoration of structural controllability following attacks and attacker–defender interactions in power-law networks. The authors propose three strategies to this end: re-linking without restrictions, re-linking with constrained network diameter, and the use of pre-computed instances of driver nodes. In Zhang et al. (2020), the authors present algebraic conditions for the network topology failures detection and isolation. They give necessary and sufficient graph-theoretic conditions for generic detectability and isolability. They address minimal sensor placement problems for a given failure (set) generically detectable and isolable, via reducing them to hitting set problems and then using greedy algorithms to approximate them with guaranteed performances.

A natural extension of the problem (10) is that of considering the dynamics parametric model design in the context of networked dynamical systems. Specifically, consider a set of N subsystems $\{\bar{A}_i, \bar{B}_i\}_{i=1}^N$ that are interconnected through $\bar{A}_{i,j}$ with possible restrictions on its own as they represent outputs from subsystem j that serve as inputs in the subsystem i. In this context, if all the systems are alike, then we have *homogeneous* networked dynamical systems. Otherwise, we are in the presence of *heterogeneous* networked dynamical systems. In both cases, as these systems increase their size, it is of interest to determine easy to verify conditions for structural controllability that depend only on the interconnection structure and possibly on subsystems general properties (Carvalho et al., 2017). In fact, due to the geographically distributed nature of these systems, it is often desirable that structural controllability is assessed in a distributed fashion (Carvalho et al., 2017). In Doostmohammadian (2020), conditions to achieve the minimum number of sensors attaining structural observability are provided leveraging Cartesian product representation.

Subsequently, we can pose the problem of determining structural changes $\bar{\Delta}_{i,j}$ (with the same dimensions of $\bar{A}_{i,j}$) that will change the interconnection between the different subsystems yielding structural controllability. Unfortunately, in its most general formulation the problem is NP-hard, which then require the development of approaches that will efficiently provide an approximate solution with provably optimality guarantees or, alternatively, efficient solutions to restricted versions of the problem. For instance, under the assumption that all subsystems are irreducible, in Moothedath, Chaporkar, and Belur (2019), the authors identify a minimum cardinality set of interconnection edges the subsystems should establish amongst each other such that the composite system is structurally controllable. The authors of Wang and Xiang (2021) propose a new method to optimize the network topology to ensure structural controllability via a minimum number of edge additions. The authors obtain all the edge-addition possible configurations and, after, determine the optimal edge-addition configuration with minimum cost.

2.2.4. Joint dynamics and input design to attain structural controllability

In Zhang and Zhou (2019a) it is proven that it is NP-hard to find $\bar{\Delta}_x \in \{0, \star\}^{n\times n}$ and $\bar{\Delta}_u \in \{0, \star\}^{n\times p}$ with costs associated costs $M_x \in \left(\mathbb{R}_0^+\right)^{n\times n}$ and $M_u \in \left(\mathbb{R}_0^+\right)^{n\times p}$, such that the sum of the entry-wise costs associated with the nonzero entries of $(\bar{\Delta}_x, \bar{\Delta}_u)$ is minimized and ensures that $(\bar{A} + \bar{\Delta}_x, \bar{B} + \bar{\Delta}_u)$ is structurally controllable. Similarly, it is NP-hard to determine the minimal cost of the entries of $(\bar{\Delta}_x, \bar{\Delta}_u)$ whose deletions (i.e., $(\bar{A} - \bar{\Delta}_x, \bar{B} - \bar{\Delta}_u)$) compromise structural controllability of the system (Dey, Balachandran, & Chatterjee, 2018; Zhang & Zhou, 2019a).

2.2.5. Open questions

Some of the open questions regarding the topics overviewed in this section are as follows:

Open question 2.2.1. *Is there a (reasonable) approximation algorithm with linear computational complexity to determine the minimum number of actuation capabilities ensuring structural controllability? Thus, enabling the design for large-scale dynamical systems.*

Open question 2.2.2. *How to devise distributed algorithms to allocate actuation capabilities to ensure structural controllability? This would be suitable to determine the actuator placement in the context of geographically distributed systems.*

Open question 2.2.3. *What are and how to determine the joint minimum actuation/sensing capabilities to ensure structural controllability/observability such that the minimum number of state variables is considered? This problem is relevant in the context of multi-agent systems, where the smallest number of agents would need to have actuation/sensing capabilities.*

Open question 2.2.4. *Assuming different costs of actuation (respectively, sensing) capabilities, as well as adding inter-dependencies between state variables, what would be the lowest cost solution ensuring structural controllability (respectively, observability)? This is a general question that will allow us to understand the trade-offs between the costs of selecting the different modalities.*

Open question 2.2.5. *Are there necessary and sufficient conditions for sensor placement to attain structural learning?*

Open question 2.2.6. *What is the minimum sensor placement to ensure structural learning? We can also think about a dual problem where we seek to design sensor placement such that, in the context of dynamical networks, agents cannot retrieve the network structure (e.g., in social networks, it would correspond to determine who is a friend with who).*

2.3. Structural stabilizability and decentralized control

In this section, we provide an overview on the efforts in the context of structural stabilizability and decentralized control.

2.3.1. Structural stabilizability

A system is said to be structurally stable if the dynamics described by A is stable (i.e., its eigenvalues lie within the unit circle in the complex plane) for almost all matrices that have a given structural pattern $\bar{A}$. This problem has been addressed in Belabbas (2013) and Kirkoryan and Belabbas (2014), where the authors study the patterns of the matrices that are stable. In Belabbas (2013), the authors derive a set of necessary conditions and of sufficient conditions for the existence of stable matrices in a vector space of sparse (structural) matrices. In the same research line, in Kirkoryan and Belabbas (2014), a complete characterization of the symmetric sparse matrix spaces that contain stable matrices is presented. In particular, to guarantee such properties, it is necessary to ensure that the minors satisfy some requirements. That said, it is important to notice that, in contrast with structural controllability, structural stabilizability is not a generic property.

2.3.2. Control configuration design problem (or, structural feedback selection)

In the context of decentralized control, it often happens that the outputs of the system (9) are not available for feedback to all the actuators of the system. Specifically, when we consider *static output feedback*, the control law takes the form

$$u(t) = Ky(t), \tag{11}$$

where the *gain* $K \in \mathbb{R}^{p\times m}$ is time-invariant. In this case, the gain satisfies an *information (structural) pattern* $\bar{K}$, where an entry $\bar{K}_{ij} = \star$ if the output (or sensor) j is available to the input (or controller) i. In this context, it suffices one entry $\bar{K}_{ij} = 0$ to be in the scenario of decentralized control. Notice that this generalizes the initial notion of decentralized control where the information pattern is taken to be diagonal or block-diagonal.

When designing closed-loop control laws, we often seek to shape the performance of the system through the change in the spectrum of the closed-loop dynamics. When we consider *state feedback* (or alternatively, when the output matrix equals the identity), it is possible to attain arbitrary pole placement when the system is controllable. Nonetheless, when we consider output feedback, arbitrary pole placement may not be possible to attain. In fact, in the context of decentralized control, we often seek to guarantee that the system does not have (decentralized) *fixed modes*, that result in eigenvalues of the closed-loop system that do not change regardless of the gain chosen (satisfying a specified information pattern). Specifically, let $\mathcal{K} = \{K : K_{ij} = 0 \text{ if } \bar{K}_{ij} = 0\}$, then the system described by (A, B, C) has fixed modes with respect to the information pattern $\bar{K}$ if and only if

$$\bigcap_{K\in\mathcal{K}} \sigma(A + BKC) \neq \emptyset,$$

where $\sigma(M)$ denotes the spectrum of the square matrix M. It is also possible to consider fixed modes in the context of structural systems, which are referred to as *structurally fixed modes*, and are defined as follows (Sezer & Šiljak, 1981): a system $(\bar{A}, \bar{B}, \bar{C})$ has structurally fixed modes with respect to the information pattern $\bar{K}$ if there exists a realization (A, B, C) with the same structural pattern of $(\bar{A}, \bar{B}, \bar{C})$ that has fixed modes with respect to the information pattern $\bar{K}$. In other words, the system has no structurally fixed modes if there is no realization satisfying the structural pattern of the state space matrices that yields fixed modes with respect to a given information pattern.

In particular, it is easy to see that if a system is not controllable, then the uncontrollable modes are also fixed modes — and similar consequences can be drawn in the context of structural systems. Yet, controllability (structural controllability, respectively) do not guarantee the non existence of fixed modes (structurally fixed modes, respectively) with respect to a specific information pattern. In fact, in Lee (2017b), the authors show that structural fixed modes cannot co-exist with quadratically invariant or partially nested information structures (Mahajan, Martins, Rotkowitz, & Yüksel, 2012).

Therefore, the *control configuration design problem* seeks to find the solution to the following optimization problem: given the system's structure $(\bar{A}, \bar{B}, \bar{C})$, find the sparsest information pattern $\bar{K}^* \in \{0, \star\}^{p\times m}$

$$\begin{aligned} \min_{\bar{K}\in\{0,\star\}^{p\times m}} \quad & \|\bar{K}\|_0 \\ \text{s.t.} \quad & (\bar{A}, \bar{B}, \bar{C}, \bar{K}) \text{ has no structurally fixed modes.} \end{aligned} \tag{12}$$

The interest on the problems of the form (12) goes back to Trave, Tarras, and Titli (1987), where different possible costs could be considered. Nonetheless, it was only recently that their complexity was established, i.e., NP-hard. Therefore, efficient algorithms to compute approximations with provable guarantees or under restricted settings are addressed in Moothedath, Chaporkar, and Belur (2018b), Pequito et al. (2015a) and Pequito, Kar, and Pappas (2015). Specifically, the authors in Moothedath, Chaporkar, and Belur (2019) proposed polynomial-time algorithms to design the sparsest feedback gain for cyclic systems, and in Pequito, Khorrami, Krishnamurthy, and Pappas (2018), the authors describe efficient algorithms for the sparsest robust control configuration design for cyclic systems.

The work in Moothedath et al. (2018b) addresses the optimal feedback selection for arbitrary pole placement of structural systems with costs associated with each feedback edge. In Moothedath et al. (2019), the authors solve the sparsest feedback selection problem for LTI structural systems for structurally cyclic systems with dedicated inputs and outputs. In contrast, in Pequito et al. (2018), the authors consider the analysis and design of resilient/robust decentralized control systems, assessing how the pairing of sensors and actuators are resilient to attacks /hacks for industrial control systems and other complex cyber–physical systems.

The work in Pequito et al. (2015a) proposes an efficient and unified framework to determine the minimum number of manipulated/ measured variables to achieve structural controllability/ observability of the system and to select the minimum number of feedback interconnections between measured and manipulated variables such that the closed-loop system has no structural fixed modes. Moreover, in Pequito, Kar, and Pappas (2015), given a linear time-invariant plant where a collection of possible inputs and outputs is known *a priori*, the authors determine the collection of inputs, outputs, and communication between them incurring in the minimum cost, ensuring the desired control performance, measured in terms of the arbitrary pole-placement capability of the closed-loop system.

It is worth mentioning that there are two natural extensions to the above problems: (i) in practice, it could be possible that one seeks to guarantee that the system only has stable fixed modes. In other words, we seek to determine structural feedback links that ensure that the system has no unstable structural fixed modes; and (ii) instead of a static (memoryless) feedback controller, we can equip the controller with memory and in that case it might be possible to ascertain structural stability without the need to guarantee that the closed-loop system is structurally stable — see Section 2.3.1. Towards this direction, the authors in Pajic, Mangharam, Pappas, and Sundaram (2013) and Pajic, Sundaram, Pappas, and Mangharam (2011), leverage the sensor network that has it own dynamics and, subsequently, behaves as a controller with memory, which in closed-loop is able to attain stability conditions of the composed system — see also Section 5.4 for the application in wireless sensor networks. The work in Pajic et al. (2011) introduced the concept of a Wireless Control Network

(WCN), where the network itself acts as a controller for the plant. Each node in a WCN executes a simple procedure by updating its state as a linear combination of its neighbors' states. The authors propose a procedure to design the linear combinations to stabilize the closed-loop system, which can be made robust to link failures. In Pajic et al. (2013), the authors address the problem of stabilizing a given dynamical system over a network. They propose an approach that relies on inducing carefully chosen dynamics on the network (via a simple distributed algorithm) and using those dynamics to stabilize the plant. Lastly, in Kalaimani, Belur, and Sivasubramanian (2013), the authors explore the minimal controller structure for generic pole placement.

Remark 4 (*Connection Between Structural and Non-Structural*). It is also possible to connect some of the above ideas originated in structural systems theory with non-structural results. Specifically, on the connection between the structure and the results for a parametrized system, in Torres and Roy (2015), the authors propose conditions of single input–output feedback stabilization. In Lee (2017a), the authors provide a simple design mechanism to obtain a gain to accomplish pole placement with dynamic output feedback with a given structure. ◦

2.3.3. Co-design of actuator/sensor placement and control configuration

Due to the inter-dependency between the actuator/sensor placement problem and the control configuration problem, it is critical to address the *co-design* problem of actuator/sensor placement and control configuration, where the goal is as follows: given $\bar{A}$, determine $(\bar{B}^*, \bar{C}^*, \bar{K}^*)$ such that

$$\begin{aligned} \min_{\bar{B},\bar{C},\bar{K}\in\{0,\star\}^{n\times n}} \quad & \|\bar{B}\|_0 + \|\bar{C}\|_0 + \|\bar{K}\|_0 \\ \text{s.t.} \quad & (\bar{A}, \bar{B}, \bar{C}, \bar{K}) \text{ has no structurally fixed modes.} \end{aligned} \tag{13}$$

The problem in (13) is addressed in Pequito et al. (2015a), which can be polynomially solvable. Extensions that consider heterogeneous costs on both the actuator/sensor placement and the control configuration lead to NP-hard problems, but some subclasses of systems can still be solvable polynomially (Pequito, Kar, & Pappas, 2015). Similarly to the actuator placements, if we seek to determine the sparsest information pattern that uses the smallest collection of inputs/outputs from a specified collection of possible inputs/outputs, then the problem is also NP-hard, and subsequently, several schemes should be considered to efficiently obtain a suboptimal solution with provable optimality guarantees (Moothedath et al., 2018b).

2.3.4. Open questions

Open question 2.3.1. *Is it possible to find composition rules yielding structural stabilizability in interconnected dynamical systems? This would provide us with elementary building blocks to ensure that large-scale interconnected dynamical systems ensure structural stabilizability by design.*

Open question 2.3.2. *Noticing that actuation-sensing-communication capabilities can replace physical interconnection between state variables and that these can incur different costs, what would be the best trade-off between structural dynamics changes and actuation-sensing-communication co-design that would incur in the minimum cost? This would allow us to make decisions about the design of cyber–physical systems.*

2.4. Controllability index

The controllability index $k^* \in \mathbb{N}$ is defined as

$$k^* = \arg\min\{k \in \mathbb{N} : rank\ \mathcal{C}_k(A, B) = n\}, \tag{14}$$

where $\mathcal{C}_k(A, B) = [B\ AB\ A^2B\ \ldots\ A^{k-1}B]$ is the partial controllability matrix. It plays a role in assessing the minimum number of time steps that lead to the existence of a control law capable of steering the system state towards a desired goal, which we refer to as the time-to-control.

In Pequito, Preciado, Barabási, and Pappas (2017), the authors provide a characterization of the structural controllability index, for which they give a near-optimal approximation scheme to discover the minimum number of inputs required to attain a given controllability index. Later, in Imae and Cai (2021), the authors provide a characterization and algorithms to attain the minimum number of inputs in the context of scale-free networks. In Ding, Tan, and Lu (2016), the authors propose to minimize the controllability index upon a budget on the number of inputs. More recently, in Ramos and Pequito (2020), the authors propose a generative model to attain a specific *actuation spectrum* that captures the minimum number of state variables required to ensure structural controllability in a given number of time-steps.

2.4.1. Open questions

Open question 2.4.1. *What is and how to compute the minimum number of inputs required to ensure that different structural controllable subspaces have a structural controllability index equal to a pre-specified k'? This may play an important role in the recovery of systems by guaranteeing that some state variables are steered towards a desirable goal faster than others.*

Open question 2.4.2. *What are and how to determine conditions under which the composition of heterogeneous networked dynamical systems with a given controllability index systems change the structural controllability index of the overall system? This would enable the design of modular systems that guarantee by design that it is possible to steer the system towards a desirable goal within a specific time horizon.*

2.5. Target controllability (output controllability)

Consider a target node set $\mathcal{T} = \{c_1, \ldots, c_s\}$, and denote by $C(\mathcal{T})$ the matrix that comprises the s rows of the $n \times n$ identity matrix indexed by $\mathcal{T}$. The triple $(A, B, C(\mathcal{T}))$ is *target controllable* if

$$rank([\ C(\mathcal{T})B \quad C(\mathcal{T})AB \quad \ldots \quad C(\mathcal{T})A^{n-1}B\ ]) = s.$$

In Gao, Liu, D'souza, and Barabási (2014), the authors introduce the structural target controllability problem and propose a k-walk theory to address it for directed-tree like networks with a single input. In Van Waarde, Camlibel, and Trentelman (2017), using the structural output controllability properties, the authors study target controllability of dynamic networks (Monshizadeh, Camlibel, & Trentelman, 2015). In Czeizler, Wu, Gratie, Kanhaiya, and Petre (2018), the authors provide more efficient algorithms than the ones introduced in Gao et al. (2014) and illustrate some of the limitations of the k-walk approach proposed in Gao et al. (2014).

In Li, Tang, et al. (2020), a method to allocate a minimum number of external control sources that ensure structural target controllability is presented, by locating a set of directed paths and cycles that cover the target set. In Moothedath, Yashashwi, Chaporkar, and Belur (2019), the target controllability of structural systems is generally addressed by providing a bipartite matching-based necessary condition. Notwithstanding, necessary and sufficient conditions on which state variables should be controlled to attain target controllability is proved to be NP-hard (Gao et al., 2014). Furthermore, if the selection of variables is constrained to a pre-defined set, the problem is also NP-hard (Guo et al., 2017). Remarkably, the former problem is polynomially solvable for symmetric state matrices (Li, Chen, Pequito, Pappas, & Preciado, 2018; Li, Chen, et al., 2020). The work in Li et al. (2018), extended in Li, Chen, et al. (2020), addresses the structural output controllability problem in structural systems with symmetric state matrices, such as undirected networks. The authors derive necessary and sufficient conditions for structural target controllability and structural output controllability of undirected networks that can be assessed in polynomial time. Furthermore, in Guan and Wang (2019), the authors propose sufficient conditions for target controllability under possible switches in the parametric model.

In Czeizler, Popa, and Popescu (2018), the authors show that the structural target controllability problem admits a polynomial-time complexity algorithm when parametrized by the number of target nodes. In general, they show that it is hard to approximate at a factor better than $\mathcal{O}(\log n)$. The work in Commault, Van der Woude, and Frasca (2019) considers a stronger controllability notion, when only a small set of state variables can be actuated. In particular, instead of using the Kalman controllability, the authors require the ability to drive the target variables as time functions. In this setup, they solve the corresponding problem using a functional approach.

2.5.1. Open questions

Open question 2.5.1. *Given a desirable target set, what would be the sparsest structural perturbation in the dynamics that ensures structural target controllability? This problem would provide us with insights into how the inter-dependencies between the state variables contribute to the structural target controllability. It is worth noticing that a change in the dynamics pattern may be the only alternative when actuation capabilities cannot be added or are too expensive.*

Open question 2.5.2. *What is the role of time-varying (or switching) systems on structural target controllability, i.e., when the structural pattern dynamics change over time? May we guarantee structural target controllability by considering switching between different structural dynamic patterns, thus avoiding adding actuation capabilities?*

Open question 2.5.3. *Some of the fixed modes with respect to a given information pattern may be avoided using time-varying output feedback, whereas others remain fixed (even when using the latter) and are referred to as* quotient fixed modes *(Gong & Aldeen, 1997). What is the role of time-varying output feedback in removing structurally fixed modes, and the existence of the structural counterpart of quotient fixed modes?*

2.6. Edge dynamics

In the context of flow-networks, the dynamics occur at the edges of a systems' digraph representation, where in- and out-flow rules impose the constraints. In this context, it is possible to formulate the flow-network actuation placement problem that seeks to determine the minimum number of actuators on the edges required to ensure that the flow across different network sectors is steered towards a specified goal. This problem can be reduced to that of the actuator placement when the dynamics occur at the nodes by considering the *dual graph*.[4] (Nepusz & Vicsek, 2012; Pequito, Khambhati, et al., 2016; Shen, Ji, & Yu, 2018) The work in Nepusz and Vicsek (2012) introduces and evaluates a dynamical process defined on the network's edges and demonstrates that the controllability properties of this process significantly differ from simple nodal dynamics. In Pequito, Khambhati, et al. (2016), the authors propose the generic controllability for dynamic-flow networks notion, where some edges' weights can be constant over time. They devise necessary and sufficient conditions to ensure structural controllability, and they address the problem of minimum actuator placement with minimum. In Shen et al. (2018), a theoretical framework to determine the controllable subspace and its generic dimension for the edge dynamic systems is presented together with methods to analyze the structural controllability of the system.

[4] https://mathworld.wolfram.com/DualGraph.html

2.6.1. Open questions

Open question 2.6.1. *Networks can have hybrid dynamics (i.e., on the nodes and on the edges), and the actuation capabilities deployed in the nodes can change the dynamics on the edges, and vice-versa. Therefore, a natural question is what is the minimum number of actuators required such that both nodes' states and edges' are driven towards a specified goal?*

Open question 2.6.2. *Due to the inter-dependency between nodal and edge dynamics, would it be possible to have only nodal inputs and change the dynamics pattern (hence, the flow-network) to guarantee structural controllability of both nodes and edges dynamics?*

Open question 2.6.3. *Can we develop efficient algorithms capable of handling actuator placement in large-scale networks?*

2.7. Structural non-minimum phase

A system is so-called minimum phase if it has not zero dynamics or asymptotically stable zero dynamics, and *non-minimum phase* otherwise. This notion can also have its structural counterpart, i.e., *structural non-minimum phase* if almost all of the numerical realizations of the structural matrices are non-minimum phase. In Daasch, Schultalbers, and Svaricek (2016), the authors provide necessary and sufficient graphical conditions to verify if a system has the structural non-minimum phase property.

2.7.1. Open questions

Open question 2.7.1. *What are and how to design the minimum actuation-sensing-communication capabilities that ensures the structural non-minimum phase property?*

Open question 2.7.2. *What are the necessary and sufficient conditions for the structural non-minimum phase property of non-linear systems?*

Open question 2.7.3. *What are and how to obtain the necessary and sufficient conditions for the structural non-minimum phase property to sign non-minimum phase systems?*

2.8. Decoupling and disturbance rejection

The problem of state and/or output feedback decoupling and disturbance rejection consists of using feedback to obtain a system whose transfer function is diagonal and nonsingular. An important contribution is the work in Dion et al. (2003), where necessary and sufficient conditions to guarantee both state and/or output feedback decoupling and disturbance rejection are known. Despite being a fundamental problem in control theory, and an active area of research until the last survey, in the last two decades, there has not been much research done on the topic in the context of structural systems.

Some exceptions include the work of Abad Torres and Roy (2014) which finds a relationship between the network's graph parametric model and the infinite-zero and finite-zero structures of an input–output network's dynamics. The authors express the zero-dynamics state matrix as a perturbation of the reduced graph matrix (a sub-matrix of the network dynamics state matrix). In Conte, Perdon, Zattoni, and Moog (2019), the authors propose new graph-theoretic notions (e.g., invariance, controlled invariance, conditioned invariance, and essential feedback) to address the disturbance decoupling in the context of structural systems.

2.8.1. Open questions

Open question 2.8.1. *How to obtain the minimum perturbation in the structural dynamics pattern and/or the actuation-sensing-communication capabilities ensuring that state/output feedback decoupling and disturbance rejection hold?*

Open question 2.8.2. *What are the composability conditions required to ensure that the networked dynamical systems yield state/output feedback decoupling and disturbance rejection?*

2.9. Fault detection and isolation (FDI)

The fault detection and isolation problem (FDI) consists of designing a set of signals, for instance, via observers, called residuals. This set of signals should be such that the transfer matrix from the disturbance and control inputs to the residuals is zero. Moreover, the transfer matrix from the faults to the residuals must have a specific form, e.g., it should be diagonal or triangular. These residuals are insensitive to controls and disturbances but sensitive to faults. This property allows them to detect and isolate the faults. For an overview of the different approaches to tackle the FDI problem, we refer the reader to the work in Simon, Boukhobza, and Hamelin (2013).

In Commault et al. (2002), the authors provide necessary and sufficient conditions under which the FDI problem has a solution for almost any values of the free parameters. These conditions are expressed in terms of input–output paths of the directed graph associated with the original LTI system (not the closed-loop LTI system associated with the residual error). Alternatively, in Commault, Dion, and Agha (2008), the authors use structural systems to determine the minimal number of required extra sensors and the sets of internal variables that need to be measured for solving the FDI problem. Decentralized FDI is studied in Sauter, Boukhobza, and Hamelin (2006), and the authors use structural control to draw necessary and sufficient conditions for detectability and isolation. In Chamseddine, Noura, and Theilliol (2009) and Commault, Dion, Trinh, and Do (2011), the authors address the problem of sensor placement to ensure that FDI can be performed, when disturbances may exist. In Commault, Dion, Trinh, and Do (2011), the authors assume that the disturbance to residual transfer matrix is null and the fault to residual transfer matrix is non-singular, proper and diagonal. The authors of Chamseddine et al. (2009) propose a strategy to formulate the problem of sensor location for diagnosis in structural systems as an optimization problem, without assumptions on the type of disturbances. The main drawback of the proposed solution is that it becomes untractable (i.e., it has no polynomial-time complexity) to achieve as the dimension of the system grows.

Additionally, in Boukhobza, Hamelin, and Canitrot (2008), the authors study the FDI problem but for the case of bilinear systems. Lastly, in Staroswiecki (2007b), the authors present a structural view of FDI (see more details in Section 3.11). The authors show that, by resorting to structural analysis, it is possible to establish a link between critical faults and reliability.

2.9.1. Open questions

Open question 2.9.1. *What is the minimum sensor placement that ensures that FDI can occur within at most k time steps?*

Open question 2.9.2. *How to develop distributed FDI mechanisms?*

3. Other subclasses of structural systems

In what follows, we present an overview of recent results in the context of classes that contain LTI systems, to which several structural conditions have been proposed, as well as the corresponding design problems.

3.1. Composite linear-time invariant systems

Consider r continuous-time LTI systems described as

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t),\ i = 1, \ldots, r,$$

where the state $x_i(t) \in \mathbb{R}^{n_i}$ and the input $u_i(t) \in \mathbb{R}^{p_i}$. By considering the interconnection between subsystem i and j, for all possible subsystems, we collect the interconnected dynamical system represented as follows:

$$\dot{x}(t) = \underbrace{\begin{bmatrix} A_1 & E_{1,2} & \cdots & E_{1,r} \\ E_{2,1} & A_2 & \cdots & E_{2,r} \\ \vdots & \ddots & \ddots & \vdots \\ E_{r,1} & \cdots & E_{r,r-1} & A_r \end{bmatrix}}_{A} x(t) + \underbrace{\begin{bmatrix} B_1 & 0 & \cdots & 0 \\ 0 & B_2 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & B_r \end{bmatrix}}_{B} u(t),$$

where the state is $x(t) = [x_1^\intercal(t)I \ldots x_r^\intercal(t)] \in \mathbb{R}^n$, $n = \sum_{i=1}^r n_i$, and $u(t) = [u_1^\intercal(t) \ldots u_r^\intercal(t)] \in \mathbb{R}^p$, $p = \sum_{i=1}^r p_i$. Additionally, $E_{i,j} \in \mathbb{R}^{n_i \times n_j}$ is the connection matrix from the jth to the ith subsystems.

Although these are linear systems, due to their wide applicability, we consider it as a class on its own as the tools and characterizations differ from those used in general linear time-invariant systems. For instance, in Carvalho et al. (2017) and Xue and Roy (2019), the authors leverage the similarity of the graph to derive easy to verify necessary and sufficient conditions for structural controllability for homogeneous systems (i.e., the similar structure of subsystems) and serial systems to be structurally controllable. They design distributed algorithms to verify necessary and sufficient conditions to assure structural controllability for any interconnected dynamical system, consisting of LTI subsystems, in a distributed fashion. In Xue and Roy (2019), it is shown that structural controllability for networks involving homogeneous subsystems may not decompose into subsystem-level and network-level conditions since the system can have structural network invariant modes. Similarly, in Doostmohammadian, Rabiee, and Khan (2018), the authors analyze the computational cost of sensor networks optimization monitoring structurally full-rank systems under distributed observability constraints. In Moothedath et al. (2019) and Zhang, Xia, and Zhai (2021), the authors introduce methods to identify a minimum cardinality set of interconnection edges that the subsystems of a heterogeneous system should ascertain between them to yield a structurally controllable composite system.

In Commault (2019), Commault and van der Woude (2019), Doostmohammadian (2019) and Wang, Jiang, and Wu (2017), the authors render a classification of subsystem nodes based on their role in the overall structural controllability. The authors of Commault and van der Woude (2019), with the goal of achieving structural controllability, present a classification of the associated steering nodes as being essential (always required to be present), useful (present in certain configurations), and useless (never necessary in whatever configuration). The authors in Commault (2019) extend the previous work for the case when the nodes are dynamical structural nodes. In Doostmohammadian (2019), a graph-theoretic approach is adopted to solve the problem of minimal driver nodes for controllability. The authors identify two types of driver nodes: Type-I to recover input connectivity of state nodes and Type-II to recover rank condition for structural controllability. The work in Wang et al. (2017) investigates the structural controllability of general complex dynamical networks with multidimensional node dynamics. The authors classify driver nodes into fully or partially controlled and use this classification to divide the problem into two subproblems. In Ramos, Silvestre, and Silvestre (2021), the authors propose a framework to study the resistance to bribery of nodes in a network via average consensus. The proposed framework evaluates quantitatively how much an external entity needs to drive the state of an agent away from its current state to change the final consensus value, either only using the structure of the system or using a concrete model with parameters.

Moreover, we can assess the minimum number of dedicated inputs required to accomplish structural controllability properties for subclasses of systems such as bipartite networks (Nacher & Akutsu, 2013), and in multiplex networks (Nacher, Ishitsuka, Miyazaki, & Akutsu, 2019). More recently, in Bai, Li, Zou, and Yin (2019), the authors provide a divide and conquer strategy that divides the system into different blocks to address the minimum input design problem for structural controllability for large-scale systems. In Moothedath, Chaporkar, and Belur (2020), the authors explore the problem of designing composite systems by introducing two indices (i.e., maximum commonality index and dilation index) that explore the trade-offs on the size of subsystems and connections among these.

3.1.1. Open questions

Open question 3.1.1. *Is there a distributed solution to the minimum actuator placement that ensures structural controllability of the interconnected dynamical system?*

Open question 3.1.2. *What are the necessary and sufficient conditions, for different subclasses of heterogeneous subsystems, based on the system's interconnections yielding structural controllability and (possibly) resilient to actuators faults?*

3.2. Descriptor linear time-invariant systems

Descriptor linear time-invariant systems (also referred to as implicit LTI systems) are those of the form:

$$Ex(t+1) = Ax(t) + Bu(t), \ \ t = 0, 1, \ldots,$$

where the matrix E has appropriate dimensions, and the structural description of the systems is given by $(\bar{E}, \bar{A}, \bar{B})$.

Although they represent a small modification concerning the linear time-invariant systems, their descriptive power is much broader. The analytical tools to assess these systems' properties (e.g., controllability) are different from LTI systems. Nevertheless, some cases are more straightforward and resemble those of linear time-invariant systems, as it is the case of regular descriptor systems (Lewis, 1992). Subsequently, all the problems overviewed in the context of linear time-invariant systems can be posed in the context of descriptor time-invariant systems. Remarkably, only a few papers address some of these problems. For instance, in Boukhobza, Hamelin, and Sauter (2006), the authors provide conditions for structural observability of descriptor systems. In Clark, Alomair, Bushnell, and Poovendran (2017) and Terasaki and Sato (2021), the authors address the actuator placement problem to ensure structural controllability. Lastly, in Mathur and Datta (2018), the closed-loop properties are explored for low dimensional descriptor systems.

3.2.1. Open questions

Open question 3.2.1. *What is the minimum perturbation in the dynamics pattern and/or actuation capabilities that yield structural controllability, with a possible pre-specified controllability index?*

Open question 3.2.2. *How to design minimum actuation-sensing-communication capabilities that are robust to actuators/sensors failures?*

Open question 3.2.3. *What are the minimum actuation-sensing-communication capabilities that yield a structurally fixed mode-free decentralized control system?*

3.3. Linear time-invariant systems with delays

In this case, the main focus has been on obtaining conditions of structural controllability when dealing with linear time-invariant systems with known delays (Qi, Ju, Zhang, & Chen, 2016; van der Woude, Boukhobza, & Commault, 2018). The authors consider constant in time delays bounded by a maximum which affect the state variables in Qi et al. (2016), and both the state variables and input variables in van der Woude et al. (2018).

3.3.1. Open questions

Open question 3.3.1. *What is the role of unknown delays in the characterization of structural controllability?*

Open question 3.3.2. *What is the solution to the minimum actuator and sensor placement?*

3.4. Linear time-invariant systems with unknown inputs

Another class of linear time-invariant systems for which structural systems properties have been explored is that with unknown inputs (Boukhobza, Hamelin, & Simon, 2014; Commault, Dion, Sename, & Motyeian, 2001). Specifically, in Commault et al. (2001), the authors explore the unknown input observers design analysis from a structural systems perspective. In Boukhobza et al. (2014), necessary and sufficient conditions that guarantee that a given set of unknown parameters describing the system's model is structurally identifiable are drawn in graphical terms.

In Sundaram and Hadjicostis (2006), the authors present necessary and sufficient conditions to ensure structural state and input observability for discrete-time systems under unknown inputs. The counterpart for continuous-time switched linear-time invariant systems under unknown inputs is considered in Boukhobza (2012), Boukhobza, Hamelin, Kabadi, and Aberkane (2011) and Boukhobza and Hamelin (2011). In particular, Boukhobza (2012), Boukhobza and Hamelin (2011) analyzes the graph-theoretic necessary and sufficient conditions for the generic discrete mode observability of a continuous-time switched linear systems with unknown inputs. The designed method to verify such conditions has a computational method of $O(n^6)$, where n is the number of states. The works of Boukhobza (2012), Boukhobza et al. (2011) display sufficient conditions for the generic observability of the discrete mode of continuous-time switched linear systems with unknown inputs and find an exhaustive location set to place sensors, when these conditions are not satisfied, with a computational complexity of $O(n^4)$.

3.4.1. Open questions

Open question 3.4.1. *How to unveil state-input structural observability characterizations under restricted assumptions on the nature of the unknown inputs (e.g., constant, or linear-time invariant switching)?*

Open question 3.4.2. *How to develop efficient algorithms to determine the sensor placement to ensure state-input structural observability by possibly measuring some of the unknown input sources (i.e., by also designing the structural pattern of the feedforward matrix)?*

3.5. Bilinear systems

A *bilinear system* can be formally described as (Isidori, D'Alessandro, & Ruberti, 1974; Mohler, Kolodziej, et al., 1980)

$$x(t+1) = Ax(t) + Nx(t)u(t) + Bu(t),\ \ t = 0, 1, \ldots,$$

where N is a matrix with appropriate dimensions and $u(t)$ is a scalar input.

Structural controllability properties are addressed in the context of homogeneous (no linear component of the input, i.e., $B = 0$) rank-1 bilinear systems (Ghosh & Ruths, 2016; Ghosh, Ruths, & Yeo, 2017). Algebraic and graph-theoretic conditions for the structural controllability of a class of bilinear systems with a single control where the input matrix is rank one are presented in Ghosh and Ruths (2016). Moreover, given a system state graph, the authors develop an algorithm to design the location of controlled edges such that the system is structurally controllable. In Ghosh et al. (2017), we can find a graphical algorithm to guarantee the existence of coprime walks via producing a cyclic partition of the state graph. This approach has computational advantages and further theoretical insight by identifying an equivalence between the cyclic partitions and the existence of controllably invariant subspaces in the state–space.

In Liu and Tie (2021) and Tsopelakos, Belabbas, and Gharesifard (2019), the authors address the structural controllability of driftless bilinear control systems. Additionally, they study the accessibility of these with a drift. More specifically, the work in Tsopelakos et al. (2019) introduces and studies the structural controllability of driftless bilinear control systems. The authors provide algorithms to compute the minimum number of matrices required for the structural controllability of driftless bilinear systems. In Liu and Tie (2021), the structural controllability problems of a class of driftless discrete-time bilinear systems that have nearly the same structure as linear systems are explored. The authors obtain algebraic and graph-theoretic conditions for structural controllability of such systems with single-input and multi-input. Also, they introduce the notion of structural near controllability, providing necessary and sufficient conditions for structural near-controllability of the bilinear systems.

In Boukhobza (2008), Boukhobza and Hamelin (2007) and Canitrot, Boukhobza, and Hamelin (2008), the authors scrutinize the structural observability of bilinear systems, and in Boukhobza et al. (2008), the authors propose the study of sensor selection in the context of FDI. In Boukhobza and Hamelin (2007), the authors express in graphic terms the necessary and sufficient conditions for the generic observability of structural bilinear systems. The work in Boukhobza (2008) studies generic uniform observability for structural bilinear systems, providing necessary and sufficient conditions expressed in graphic terms. Furthermore, sensor placement for the observability of structural bilinear systems is studied in Canitrot et al. (2008). The authors provide solutions for the minimal number of sensors and their placement to recover the system observability. Lastly, in Svaricek (2006), the author presents an interesting discussion on the use of structural systems to assess uniform observability of bilinear systems.

3.5.1. Open questions

Open question 3.5.1. *What are the necessary and sufficient conditions for structural controllability of bilinear systems for arbitrary heterogeneous systems?*

Open question 3.5.2. *What is the solution to the minimum actuator placement to ensure structural controllability in bilinear systems?*

3.6. Discrete-time fractional-order systems

Fractional-order systems are successfully used to model different physiological processes (e.g., trains of spikes, local field potentials, and electroencephalograms) (Klaus, Yu, & Plenz, 2011; Xue, Pequito, Coelho, Bogdan, & Pappas, 2016). The following dynamics describe these systems

$$\Delta^{\alpha} x(t+1) = Ax(t) + Bu(t),\ \ t = 0, 1, \ldots,$$

where $\alpha = [\alpha_1, \ldots, \alpha_n]^\top \in \mathbb{R}^n_+$ are the fractional exponents, and where $\Delta^{\alpha} x_k = \sum_{j=0}^{k} D(\alpha, j) x_{k-j}$ is the fractional derivative given by

$$\begin{aligned} D(\alpha, j) &= \operatorname{diag}(\psi(\alpha_1, j), \ldots, \psi(\alpha_n, j)),\ \text{and}, \\ \psi(\alpha, j) &= \frac{\Gamma(j-\alpha)}{\Gamma(-\alpha)\Gamma(j+1)} = \prod_{\ell=1}^{j} \left(\frac{j-\ell-\alpha}{j+1-\ell} \right) \end{aligned}$$

where $\Gamma(\cdot)$ denotes the gamma function defined as $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt$ for $z \in \mathbb{C}$. It is worth noticing that if $\alpha = \mathbf{1}_n$ (i.e., the n-vector of ones), then we obtain the description of an LTI. Thus, fractional-order systems are inherently nonlinear with infinite memory but require only compact parametric descriptions. Structural observability properties of these systems, as well as sensor placement, has been addressed in Pequito, Bogdan, and Pappas (2015).

3.6.1. Open questions

Open question 3.6.1. *What are the necessary and sufficient conditions for structural controllability and observability when the fractional coefficient is assumed as a structural parameter, which we refer to as structural fractional controllability and observability?*

Open question 3.6.2. *How to provide the solution to the minimum actuator and sensor placement to ensure structural fractional controllability and observability, respectively?*

3.7. Switching systems

In what follows, we consider two classes of switching systems: (i) temporal networks, i.e., the same structural pattern across switches but with different realizations over time; and (ii) linear time-invariant switching systems.

3.7.1. Temporal networks

In recent years, a particular focus from the networks science community has been on temporal networks (Holme & Saramäki, 2012). In this context, structural controllability of temporal networks considers those modeled by linear time-varying systems whose structural pattern remains unchanged, and their realization may vary over time (Pósfai & Hövel, 2014; Srighakollapu, Kalaimani, & Pasumarthy, 2021). The authors of Pósfai and Hövel (2014) investigate the controllability of systems with of the dynamics' timescale comparable with the changes in the network timescale. They present analytical and computational tools to study controllability based on temporal network characteristics. The work in Srighakollapu et al. (2021) presents conditions for structural controllability of temporal networks that change topology and edge weights with time.

In Hou, Li, and Chen (2016), the authors consider the case where there is a finite number of possible switches (i.e., a finite number of realizations of a given structural pattern), yet it can be repeated multiple times. In Yao, Hou, Pan, and Li (2017), the authors propose using a switching controller to increase the dimension of the structural controllable subspace.

3.7.2. Linear time-invariant switching systems

Conceptually, we can see a linear time-invariant switching (LTIS) system as a set of LTI systems, where each element of the set is called a *mode*, together with a set of discrete events that cause the system to switch between modes. Subsequently, an LTIS may be described as follows:

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t), \tag{15}$$

where $\sigma : \mathbb{R}^+ \rightarrow \mathbb{M} = \{1, \ldots, m\}$ is a piecewise switching signal, that only switches once in a given dwell-time, $x(t) \in \mathbb{R}^n$ the state of the system, and $u(t) \in \mathbb{R}^p$ is a piecewise continuous input signal. As we may expect, to find a set of the sparsest input matrices $\{B_{\sigma(t)}\}$ that ensures each mode of the system to be controllable is an NP-complete problem (Ramos, Pequito, & Caleiro, 2018).

The necessary and sufficient conditions to ensure structural controllability of linear time-invariant switching systems have been proposed in Liu, Lin, and Chen (2013a, 2013b) and Ramos, Pequito, Aguiar, Ramos, and Kar (2013), with possible robustness characterization (Ramos, 2013; Ramos et al., 2015) (see more details in Section 3.10). Afterward, the actuator placement problem is addressed in Pequito and Pappas (2017). Specifically, in Ramos et al. (2013), the authors propose the concept of structural hybrid systems to address the model checking problem of switching linear time-invariant systems. They provide the necessary conditions to guarantee controllability at each time, which can be verified with polynomial complexity. Subsequently, Liu et al. (2013b) study the structural controllability of a class of uncertain switched linear systems. In Liu et al. (2013a), a graph-theoretic characterization of structural controllability for multi-agent systems with switching topologies is presented.

In Boukhobza (2012) and Boukhobza and Hamelin (2011) the authors propose conditions for sensor placement in linear time-invariant switching systems with unknown inputs (see more details in Section 3.4). Later, they generalize these conditions to linear time-invariant switching descriptor systems (Boukhobza & Hamelin, 2013; Gracy, Garin, & Kibangou, 2018). In Boukhobza and Hamelin (2013), the authors present necessary and sufficient graphical conditions which ensure the generic discrete mode observability of structural switching descriptor systems. They generate a new approach that builds a new type of digraph dedicated to the discrete mode observability study. Moreover, Gracy et al. (2018) study the problem of reconstructing the initial state as well as the sequence of unknown inputs for linear network systems having a time-varying topology.

3.8. Linear time-varying systems

A linear time-varying (LTV) system can be seen as the system in (3), where the matrices A, B, C, and D are time-dependent, i.e., vary with time.

In Lichiardopol and Sueur (2007), the authors propose conditions to evaluate the structural controllability of linear time-varying systems. See also Hartung, Reißig, and Svaricek (2013) that extends these results and compare their implications in different controllability contexts. In Gracy, Garin, and Kibangou (2018), the authors present conditions on the parametric model required to assure FDI for linear time-varying systems with unknown inputs, which allow them to retrieve both the initial state and the unknown inputs over long time windows.

3.9. Petri nets

Petri nets consist of a framework that allows the modeling and analysis of discrete event dynamic systems. Petri nets can be represented as bipartite graphs, where the state variables are called *places*, and the transformations on the states are referred to as *transitions*. Such places and transitions are connected through pre-incidence (i.e., inputs) and post-incidence (i.e., outputs), under possible constraints indicating the resources required. Due to the graphical nature of these networks, it is possible to leverage the notion of structural observability to retrieve the places (i.e., state variables) under known transitioning models (Silva, 2013). To assess structural observability, we must consider a suitable transformation of the original graph (Mahulea, Recalde, & Silva, 2010; Silva, Júlvez, Mahulea, & Vázquez, 2011). In Silva et al. (2011), we can find a survey of fluid views or approximations of Petri nets that introduces some new ideas and techniques. In Mahulea et al. (2010), the authors introduce the notion of redundant modes and give a necessary and sufficient condition for a mode to be redundant in the context of continuous-time Petri nets. Then, they devise an observability criterion, a structural observability criterion, and an intermediate concept between the previous two called weak structural observability.

3.10. Hybrid systems

A hybrid system is a dynamical system that manifests both continuous and discrete dynamic behavior. A special case of hybrid systems is when the continuous dynamics is given by an LTI system, which yields the class of linear time-invariant switching systems.

In Ramos (2013), the authors introduce a tool for the design and verification of structural controllability for hybrid systems. Later they consider it in the context of the analysis and design of electric power grids with robustness guarantees on the link failures (Ramos et al., 2015).

3.11. Nonlinear systems

The authors, in Stefani (1985), revealed a local controllability condition for nonlinear systems. Nevertheless, there is limited research about structural controllability for nonlinear systems. The first work in the line of structural controllability of such systems is the one in Fradellos, Rapanakis, and Evans (1977), where changes concerning controllability or uncontrollability behavior of perturbed linear and non-linear systems are examined.

In Qiang (2010), the conception of structural controllability is extended to nonlinear systems utilizing Lie algebra theory. This extension is then used to analyze the structural properties of nonlinear systems. In Ma (2010), the author proposes to assess the structural controllability of the nonlinear system through the system transfer function. In Zañudo, Yang, and Albert (2017), the authors propose to use feedback vertex sets in relation to structural properties to assess the controllability of nonlinear systems. The work in Angulo, Aparicio, and Moog (2019) leverages structural properties to enable nonlinear assessment of controllability and observability properties. In Kawano and Cao (2019), necessary conditions for the structural controllability and observability of complex nonlinear networks are presented. These conditions, which are based on refined notions of structural controllability and observability, can be used for networks governed by nonlinear balance equations to develop a systematic actuator/sensor placement.

In Woude (2018), the author explores the controllability conditions of nonlinear systems performing its linearization, and in the same lines (Staroswiecki, 2007a, 2007b) suggest to explore FDI settings. In both Staroswiecki (2007a, 2007b), the author presents a structural view of fault-tolerant estimation algorithms, identifying the minimal submodels by which unknown variables can be estimated, both in healthy and in faulty conditions, connecting to critical faults and reliability.

3.11.1. Open questions

Open question 3.11.1. *How to render necessary and sufficient conditions for structural controllability and observability for the different subclasses, as well as how to derive subclasses where such conditions can be efficiently verified?*

Open question 3.11.2. *How to provide efficient (and possibly distributed) algorithms to obtain the solution to the minimum actuator and sensor placement, where the latter might be in the context of unknown inputs and FDI?*

Open question 3.11.3. *How the inter-dependencies between state variables affect structural stabilizability, controllability, and observability?*

4. Extensions of structural systems theory

As previously emphasized, structural systems theory deals with parametric models, under the classic assumption that parameters belong to an infinite field (e.g., the reals) and are independent of each other. In what follows, we provide a brief description of results that built upon (classic) structural systems theory to obtain methodologies to handle cases under a different set of assumptions.

4.1. Positive systems

A discrete-time linear time-invariant system is a *positive system* if, for any initial condition and any nonnegative input sequence, the state vector entries remain positive over time. In Commault (2004), the authors address the reachability of discrete-time linear time-invariant systems to assess when the state can lie on the positive octant that has broad applications in practice (Rantzer & Valcher, 2018). This problem also motivates the study presented in Ruf, Egerstedt, and Shamma (2018) and She and Kan (2020), where the authors address the input selection to achieve the reachability property. In Lindmark and Altafini (2016), the authors build upon structural systems to explore the parametric dependencies that lead to the controllability of positive systems. Along the same lines, the authors in Bru, Cacetta, and Rumchev (2005) explore related digraph properties to study the controllability of positive systems. As an extension to the use of structural systems theory to positive systems, the authors in Hartung and Svaricek (2014) leverage the former systems to render necessary conditions for signed systems' properties, specifically to attain sign stabilizability.

4.2. Parameter-dependent structural systems

In Murota (2009a, 2012), the parametric dependencies have been accounted for, using the notion of *mixed matrices*, where the entries could be zero/nonzero or fixed constant, often capturing the network dependencies (or, generally speaking, losses of degrees of freedom). More recently, in Menara, Bassett, and Pasqualetti (2019), Mousavi, Haeri, and Mesbahi (2018), Whalen, Brennan, Sauer, and Schiff (2015) and Whalen, Brennan, Sauer, and Schiff (2016), the authors examine the structural controllability criterion when the dependency of the parameters is given by the symmetry of the system's autonomous matrix. In Whalen et al. (2015), we can find a numerical and group representational framework to quantify the observability and controllability of nonlinear networks with explicit symmetries, showing the relationship between symmetries and nonlinear measures of observability and controllability. The authors apply this work to neural networks in Whalen et al. (2016). In Menara et al. (2019), the authors show that (symmetric) structural controllability can be assessed by graph-theoretic elements similar to those previously proposed to verify (classic) structural controllability. Last, the note in Mousavi et al. (2018) studies the controllability analysis of certain families of undirected networks via combinatorial constructions.

In Romero and Pequito (2018), the authors use the latter criterion to address the actuator placement problem in this context under possible cost constraints. In Zhang and Zhou (2019b), the authors investigate conditions when subsystems satisfy fractional parametrizations. In Liu and Morse (2019), the authors propose a graphic–theoretic characterization to attain structural controllability when arbitrary linear dependencies exist between the system's dynamics parameters.

Lastly, it is worth mentioning that matroid theory and algorithms to solve problems related to these can be used to approximate the solutions to design problems (e.g., actuator placement). In particular, even when optimality cannot be guaranteed in several of the design problems, it is often the case that some suboptimality guarantees may be ensured and improved by considering the system's structure. For instance, some structural systems properties are *submodular*, for which efficient greedy algorithms are available (Clark, Alomair, Bushnell, & Poovendran, 2015; Francis Bach, 2013; Guo, Karaca, Summers, & Kamgarpour, 2020). For example, structural controllability problems can be posed as matroid optimization problems that can be solved exactly under certain assumptions (Clark et al., 2017; Rocha, 2014). Such developments should be complemented with recent research that unveils new insights on additional properties (e.g., submodular ratio and curvature); thus, tightening the suboptimality guarantees (Bian, Buhmann, Krause, & Tschiatschek, 2017; Gupta, Pequito, & Bogdan, 2018; Iyer, Jegelka, & Bilmes, 2013).

4.3. Structural theory on finite fields

In Feng, Liu, and Lu (2008) and Sundaram and Hadjicostis (2012), the authors propose to assess structural controllability properties when the parameters are taken to be independent but take values on a finite field. In this context, properties are no longer valid generically but rather with a certain likelihood. In Yuan, Lu, and Yan (2016), the authors provide conditions for the analysis and design of such systems in the frequency domain.

4.4. Strong structural theory

In contrast with structural theory, the (classical) strong structural theory (Mayeda & Yamada, 1979) seeks to guarantee properties for any set of parameters considered for the realization of the nonzero entries of the structural pattern except when the parameters are zero. Notwithstanding, it is possible to extend to scenarios where some of the entries could be either zero, nonzero (i.e., a real scalar different from zero), and possibly a real scalar that could be either a zero or a nonzero (Jia, Van Waarde, Trentelman, & Camlibel, 2020; Popli, Pequito, Kar, Aguiar, & Ilić, 2019). It is worth noticing that several of the problems discussed in this review can be posed in the strong structural controllability scenario, but their computational complexity often changes. For instance, the strong structural controllability of the sparsest minimal controllability problem is NP-hard (Trefois & Delvenne, 2015). This result contrasts with the existing polynomial solution when the goal is to attain structural controllability.

4.5. Bond-graphs

The concept of bond graphs (Paynter, 1961) seeks to describe the dynamic behavior of physical systems based on energy and energy exchange (Thoma, 2016). The *basic units* can be seen as concepts and/or objects, enabling object-oriented physical systems' modeling. Bond graphs are labeled directed graphs, in which the vertices represent basic units, and the edges represent an ideal energy connection between them, and they are referred to as bonds. As such, each basic unit could be seen, in particular, as a linear time-invariant system that interconnected through the others using bonds. Structural systems theory has been leveraged to assess several system properties of these systems and to address similar problems as those overviewed in this survey, seeking to design the systems to attain such properties — see, for example, Alem and Benazzouz (2014) and Sueur and Dauphin-Tanguy (1989, 1991).

5. Applications

In this section, we provide an overview of different applications and domains where structural systems theory made its footprint.

5.1. Security and resilience

Distributed control systems (DCS) usually rely on different components that can be exposed to malicious attacks. Henceforth, the use of DCS in critical infrastructures makes their security a topic of utmost importance. Nefarious incidents connected to the security of DCS include the Stuxnet uranium plant attack (Langner, 2011) and the Maroochy Shire (Abrams & Weiss, 2008) episode. Consequently, there has been a growing effort to mitigate DCS from being exposed to undetectable attacks.

In Sundaram and Hadjicostis (2008), the authors design a scheme that allows nodes of time-invariant connected networks to attain consensus on any arbitrary function of the initial nodes' state in a finite number of steps for almost all weight matrices with the same structure. In Liu, Ning, and Reiter (2011), for power grids' design, the authors render algebraic conditions that allow an adversary to generate state estimation errors. The work in Sandberg, Teixeira, and Johansson (2010) suggests multiple security indices for sensors, allowing a system operator to identify sparse power grids attacks. In Mo and Sinopoli (2010), the bias that an undetectable adversary may introduce into the state estimation error of control systems and sensor networks is studied. Afterward, Sundaram and Hadjicostis (2010) and Pasqualetti, Bicchi, and Bullo (2011) address the resilience of consensus-based algorithms. Specifically, in Sundaram and Hadjicostis (2010), the authors determine graphical conditions under which a set of agents can compute a function of their initial states in the presence of malicious nodes. In Pasqualetti et al. (2011), using connectivity and left-invertibility, delineates attack identifiability and detectability. Additionally, in Sundaram, Pajic, Hadjicostis, Mangharam, and Pappas (2010), the authors study the design of an intrusion detection scheme for DCS, which identifies malicious agents and can recover from the attacks.

In the context of structural observability, we also have to keep in mind that the properties hold generically. Simply speaking, structural observability is only a necessary condition. Consequently, we can explore the set with zero Lebesgue measure to design attacks that will not be identifiable from the observer perspective (Pasqualetti, Dörfler, & Bullo, 2013) (through the left-invertibility of regular descriptor systems). In the same line, in Weerakkody, Liu, Son, and Sinopoli (2016), it is considered the secure design problem in the context of distributed control systems to guarantee the detection of stealthy integrity attacks. In Milošević, Sandberg, and Johansson (2018), the authors propose a security index upon the previous definitions. Also, in the same research direction, in Weerakkody, Liu, and Sinopoli (2017), the authors propose to explore structural observability properties to prevent zero dynamics attacks.

In Milošević, Teixeira, Johansson, and Sandberg (2020), the authors leverage structural systems theory to derive a robustness index. In Jafari et al. (2011), the authors perform the assessment of robust control in the context of resilient leader selection, and analytical properties that ensure the network is resilient. The work in Zhang and Wolthusen (2019) proposes to evaluate the minimum number of additional actuation capabilities needed to ensure structural controllability under possible failures in the form of a security index. In Alcaraz and Lopez (2017) and Alcaraz, Lopez, and Choo (2017), the authors explore self-healing properties to guarantee structural controllability through various centrality measures and severity degrees. In Alcaraz et al. (2017), the authors present an optimal reachability-based restoration approach for interconnection in cyber–physical control systems, which can restore the structural control in linear times, using structural controllability, supernode theory, the IEC- 62351 standard, and the contextual conditions. A checkpoint model based on a cooperative cyber–physical network composed of trustworthy elements (auditors) is presented in Alcaraz and Lopez (2017). The approach manages distributed warning replicas that help produce sufficient data redundancy for fault and intrusion detection, providing resilience to the network when the control structures can be seriously threatened by attackers or perturbed by the dynamic changes caused by entering or leaving nodes.

The sensor placement problem can be considered to attain resilience/robustness with respect to sensor failure that maintain structural observability (Boukhobza, 2010; Boukhobza & Hamelin, 2009; Commault, Dion, et al., 2008) (see more details in Section 2.2.3). In Pequito et al. (2018), the authors provide efficient algorithms for the sparsest robust feedback design for cyclic systems. In a similar manner, Ramasubramanian, Rajan, and Chandra (2016) assess security properties using several criteria that serve as a denial of service. Additionally, in Jafari et al. (2010), the authors assess the impact of link failures on structural controllability (i.e., zeroing the free parameters of the autonomous system matrix). In Dakil, Boukhobza, and Simon (2015) and Maza, Simon, and Boukhobza (2012), the authors address the issue of ensuring structural controllability under the scenario where the actuators can fail with known probabilities. Similarly, in Dakil, Simon, and Boukhobza (2015) and Liu, Weerakkody, and Sinopoli (2016), a methodology is proposed to determine the sensors which ensure structural observability within a certain chance. In Guan and Wang (2019), the authors present sufficient conditions for target controllability under possible switches in the parametric model.

In Zhang and Wolthusen (2017, 2018), the authors assess the impact of node removal on structural controllability. They propose a strategy to recalculate the minimum number of inputs to guarantee structural controllability. The structure of dynamical systems might change for different reasons. For this setting, in Zhang and Wolthusen (2017), the authors present a method to efficiently obtain a maximum matching of each incremental digraph in linear time, to reduce the complexity of iterative computations of maximum matchings. Following the same line, in Zhang and Wolthusen (2018), the authors address the problem of efficient control recovery after removing a known system vertex by finding the minimum number of inputs.

In Shoukry et al. (2015), the authors introduce the notion of structural abstraction to address formal guarantees in the context of the security index of systems. In the context of hybrid systems, the work in Ramos et al. (2015) introduces a tool for the design and verification of structural controllability of these systems, and they utilize this tool in the context of the analysis and design of electric power grids, ensuring robustness to the link failures (Ramos et al., 2015). The authors in Alcaraz and Wolthusen (2014), Jafari et al. (2010) and Rahimian and Aghdam (2013) also consider the impact of edge failure/removal to ensure resilient structural controllability (see more details in Section 2.2.3).

5.2. Privacy

In the context of dynamical systems, privacy can be posed as an observability problem. Specifically, states are referred to as private if they do not belong to the observability subspace. Subsequently, increasing sensing capabilities is more likely to increase the observability subspace's dimension, and it readily follows that the systems parametric model constraints the latter.

In Yan, Sundaram, Vishwanathan, and Qi (2012), the authors explore the concept of observability as a way to retrieve the objective function used by different agents in a network that leverages the knowledge of the different iterations performed by an iterative algorithm. In Pequito, Kar, et al. (2014), the authors propose to design the parametric model structural pattern to ensure that some nodes' state cannot be retrieved by some of the agents in the network. Some of these insights are further considered in the context of wireless sensor networks (Pequito & Pappas, 2015). In a different direction, in Lin and Ling (2015), the authors consider privacy-preserving decentralized matrix completion, in which a network of agents collaborate to complete a low-rank matrix that is the collection of multiple local data matrices.

5.3. Distributed/decentralized estimation and optimization

In Doostmohammadian and Khan (2013, 2014) and Sundaram and Hadjicostis (2008), the authors use the notion of structural observability to ensure that each node in the network is capable of retrieving the neighbors' states towards computing the solution to an optimization problem in a distributed fashion. In contrast, in Alexandru, Pequito, Jadbabaie, and Pappas (2016, 2017), the authors propose the design of the sensor network parametric model to guarantee that every sensor can retrieve the state of the entire system under possibly link failures. Lastly, in Khan and Jadbabaie (2011), the authors investigate generic observability properties to infer the stability of distributed estimation schemes, possibly under distributed settings (Doostmohammadian et al., 2018).

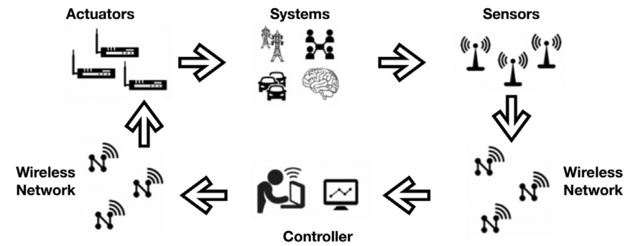


Fig. 6. Ongoing and future applications of structural systems (e.g., cyber–physical systems).

5.4. Wireless networks

In Pajic et al. (2013), Pajic et al. (2011) and Sundaram et al. (2010), the authors consider sensor networks as an extension to the state space, connected through feedback with the plant. Due to the freedom in the design of the sensor network, it is possible to guarantee enough redundancy, in terms of paths between the states of the plant and the sensors, such that proper monitoring is secured (see Section 5.1, for more details). In Pajic et al. (2011) the authors aim to assess the controllability and stability properties of these joint plant-sensor network systems. Some privacy properties have also been considered (Pequito & Pappas, 2015). Additionally, extensions of this setting considered the scenario of potential communication link failures (Sundaram, Revzen, & Pappas, 2012). In Martinez-Martinez, Hashemi-Nejad, and Sauter (2010), the authors explore how the transmission sequence (through a networked system over wireless networks) should be designed to ensure controllability and observability properties. In Kruzick et al. (2018), the authors address the observability problem when backbone nodes (e.g., routers) are considered in the context of sensor networks.

Alternatively, if the sensors include a queuing capability (i.e., WirelessHART, see details in D'Innocenzo, 2018), then the authors in D'Innocenzo, Smarra, and Di Benedetto (2016) were able to derive necessary conditions for observability and stabilizability of the network, in the context of state feedback.

5.5. Networked control systems

A networked control system is a control system where the control loops are closed through a communication network — see Fig. 6.

In Sauter et al. (2006), the authors propose an FDI scheme (see Section FDI) for networked control systems. More recently, the work in Doostmohammadian, Rabiee, and Khan (2020) leverages the structural systems' results to propose a cyber–social system framework. Lastly, in Zhang and Zhou (2019b), the authors explore conditions when subsystems satisfy fractional parametrizations.

5.6. Network neurosciences

The works in Pasqualetti, Gu, and Bassett (2019), Tang and Bassett (2018) and Tu, Rocha, Corbetta, Zampieri, Zorzi, and Suweis (2018) assess the structural controllability aspects of brain networks. In Tu et al. (2018), the authors contrast results on brain structural controllability through the analysis of five different datasets and numerical simulations. We find that brain networks are not controllable (in a statistically significant form) by one single region. In response to the previous work, in Pasqualetti et al. (2019), the authors show that brain networks are controllable from a single region, require large control energy, and feature distinctive controllability properties concerning a class of random network models. Finally, the colloquium in Tang and Bassett (2018) overviews the control of dynamics in brain networks. Also, a work-related to brain networks, in the context of using discrete-time fractional-order systems, is presented in Pequito, Kar, and Pappas (2015) (see Section 3.11).

5.7. Multi-agent systems

In the context of multi-agent systems, due to the finite memory and number of tasks required to perform, it turns out that their dynamics are modeled using finite fields. In Sundaram and Hadjicostis (2012, 2013), the authors leverage the notions of structural controllability and extended the dynamics in the context of finite fields realizations — see also Section 4.3.

Alternatively, under (regular) structural systems theory (i.e., infinite field realizations), a multitude of approaches were proposed. For instance, in Zamani and Lin (2009), the authors address the structural controllability of multi-agent networks driven by a single agent. In Ouyang, Pati, Wang, and Lu (2018), the authors consider the minimum number of leaders that ensure structural controllability (i.e., the *leader selection problem*), which may be recast as an input selection problem. The authors in Partovi, Lin, and Ji (2010) study the structural controllability of high-order dynamic multi-agent systems, and in Guan and Wang (2017) and Liu et al. (2013a) results for switching parametric models are presented. In Liu et al. (2017), the authors address the actuator and sensor placement under possible cost constraints for multi-agent bilinear systems (see Section 3.5 for more bilinear systems related work). In Pequito, Rego, et al. (2014), the authors address the design of a communication parametric model dynamics that should be observable from each agent, and that achieves minimal overall transmission cost.

In the context of (fully) distributed leader selection, in Pequito, Preciado, and Pappas (2015) and Tsiamis et al. (2017), the authors propose a two-level approach. First, the agents determine with which agents they need to interact with to ensure structural controllability. Then they pick weights locally that ensure controllability of the overall network. In the same spirit, the authors in Mehrabadi, Zamani, and Chen (2019) leverage structural controllability properties to propose a parametrization technique to attain consensus from a centralized perspective, with a collection of multiple leaders.

5.7.1. Consensus and agreement protocols

In Goldin and Raisch (2013), the authors characterize generic controllability properties for dynamics that implement consensus algorithms. It is worth emphasizing that the direct application of structural systems results does not ensure the system's desired assessment. In these dynamics, the diagonal entries depend on the remaining row entries, thus violating the assumption that all nonzero parameters are independent. As an alternative, one can consider the settings discussed in Section 4.2.

5.8. Power grids

The work in Bhela, Kekatos, Zhang, and Veeramachaneni (2017) ensures that rank conditions are achieved in the context of algebraic differential equations as part of the power-flow optimization. In Bhela, Kekatos, and Veeramachaneni (2017) and Bhela et al. (2017), the authors assess the identifiability of linearized power grids systems by leveraging structural systems theory. In Luo, Li, and Jiang (2018), the authors leverage structural systems theory to assess its vulnerability. Lastly, in Xiaoyu (2012), the authors evaluate the structural controllability of electrical networks using rational function matrices.

5.9. Medical applications

Structural learning is proposed to be used in the context of a model for dialysis (Cantó et al., 2009). In contrast, in Pequito, Kar, and Pappas (2015), the authors propose to assess the minimum number of electrodes required for monitoring electroencephalographic data (i.e., sensor placement in the brain).

5.10. Network coding

In Campobello, Leonardi, and Palazzo (2009), the authors leverage structural systems to impose conditions for network coding in the context of state–space representation in the spirit of Koetter and Medard (2003). In Sundaram and Hadjicostis (2009), some of these ideas are used in the context of sensor networks.

5.11. Science exploration tools

In Liu and Barabási (2016), the authors overview some of the different applications of structural control as a tool to unveil features in the context of network science. This application has attracted interest since 2011 when the paper (Liu, Slotine, & Barabási, 2011) was featured on the cover of Nature. In the latter, the authors explore the actuation placement problem and provide evidence of its correlation with the degree distribution when nodal dynamics were not considered (Cowan, Chastain, Vilhena, Freudenberg, & Bergstrom, 2012). More recently, in Liu, Slotine, and Barabási (2012), the notion of control centrality is introduced.

Among the possible applications, structural systems theory enables the characterization of networks in classes. For instance, in Ruths and Ruths (2014), the characterization is upon the partition of dilations in a network. This characterization can also be used to construct models to attain such characterizations (Campbell, Ruths, Ruths, Shea, & Albert, 2015). More recently, in Chung, Ruths, and Ruths (2021), the role of dilations is further studied to understand their implications in the context of structural controllability of networks.

In Pequito et al. (2017), the authors introduce the notion of *actuation spectrum*, which captures the trade-offs between the minimum number of state variables required to attain structural controllability in a given number of time-steps. Lastly, in Ramos and Pequito (2020), the authors provide evidence that several of the generative models and centrality measures fail to capture the actuation spectrum of real networks. Therefore, they propose a novel generative model that builds upon the notion of structural time-to-control communities.

5.12. Gas turbine and water distribution applications

Fault detection and identification – see Section 2.9 – applications are considered in Verde and Sánchez-Parra (2007) and Veldman - de Roo, Tejada, van Waarde, and Trentelman (2015) for monitoring gas turbines and water distribution networks, respectively.

5.13. Software routines

Besides the code available by the different authors at their personal websites and file exchange platforms (e.g., Mathworks), there are MATLAB toolboxes such as (Geisel & Svaricek, 2019), C++ (Martinez-Martinez, Mader, Boukhobza, & Hamelin, 2007), and Modelica and Python (Perera, Lie, & Pfeiffer, 2015).

6. Conclusions and future research directions

This paper overviewed the ongoing research in structural systems theory, its extensions, and its applications since the latest survey conducted by Dion et al. (2003). Most of the problems deal with linear-time invariant systems, for which we presented several key definitions and concepts and provided a glance at the graph-theoretic tools used to address such problems. We also overviewed open research directions within the scope of (classic) structural systems theory for each of the topics covered. Some of these can also be studied in the context of other classes of systems (e.g., nonlinear and hybrid systems). Additionally, we provided a connection with research thrusts that build upon classic structural systems (e.g., parameters belong to finite fields and can be linearly dependent). Lastly, we presented different focused areas where structural systems have been used to assess systems properties or to design systems that yield desirable guarantees (e.g., privacy and security). Ultimately, structural systems can play a crucial role in the design of cyber–physical systems.

Besides the fundamental research questions in the context of classic structural systems theory that we pinpointed in our overview, there are two directions towards a *discrete-convex optimization in the context of control systems*, which we believe to be fruitful and where structural systems can play a crucial role.

Firstly, structural systems theory can be used as a tool to aid in finding a solution to optimization algorithms. Specifically, we can consider algorithms that aim to attain structural systems properties (e.g., structural controllability/observability/stability), which ascertain the feasibility of the solutions for almost all sets of parameters. Then, in a second step, the set of parameters are determined such that they minimize/maximize a desirable objective. For instance, consider the following three possible applications: (i) in Becker, Pequito, Pappas, and Preciado (2020), the authors use structural systems to ensure that for almost all set of parameters, controllability would be guaranteed to determine a set of perturbations in the dynamics that improve the controllability energy (see Section 2.2); (ii) in Pequito et al. (2018), the authors guarantee generic stabilizability properties for the decentralized control, and then an iterative procedure is considered to find a set of parameters that stabilize the plant — see Remark 4; and (iii) in Pequito, Preciado, and Pappas (2015), the authors determine a fully distributed leader selection to attain controllability. In this case, the problem is decomposed into two procedures: (1) determination of the leaders that attain structural controllability; and (2) computation of a set of parameters for the local interactions that ensure (non-structural) controllability of the network.

Secondly, we envision a *structural-convex optimization* framework, where several discrete mathematics algorithms could be intertwined with the convex optimization tools already in use in discrete-convex optimization — term coined by Murota (2009b) that refers to a combination of a first step of discrete optimization (solvable with polynomial time complexity) to cover discrete analogues of the fundamental concepts such as conjugacy, subgradients, the Fenchel rain-max duality, separation theorems and the Lagrange duality framework, plus a second stage of convex (or convexification) optimization. A particular example arises in the process of obtaining suitable convex relaxations that often involve performing a set of operations that lead to a description of the optimization problem where the only nonlinear constraint is that of a rank constraint on a matrix of interest (Boyd, Boyd, & Vandenberghe, 2004). In this context, we suggest the use of the generic rank instead of the rank constraint that is often dropped from the optimization problem. In this way, it is possible to guarantee an upper bound on the rank of matrices with some structure. Lastly, it might be possible to leverage the structural similarity between the rank of a transfer function and the Schur complement to perform some of the algebraic transformations and ensure some graph-theoretical properties on the generic rank. Specifically, consider the matrix $M(s) = \begin{bmatrix} A - sI & B \\ C & 0 \end{bmatrix}$ for which the following holds: rank $C(sI - A)^{-1}B$ = rank $M(s) - n$, where A is an $n \times n$ matrix. Now, notice that the grank $C(sI-A)^{-1}B$ equals the number of vertex-disjoint paths from the inputs to the outputs in the system digraph $\mathcal{G}(\bar{A}, \bar{B}, \bar{C})$ (see Van der Woude, 1991). Consequently, we may potentially design the structure of the matrices $(\bar{A}, \bar{B}, \bar{C})$ such that almost all realizations would attain a desirable rank, after which we can use conventional convex optimization tools.

References

Abad Torres, Jackeline, & Roy, Sandip (2014). Graph-theoretic characterisations of zeros for the input–output dynamics of complex network processes. *International Journal of Control*, *87*(5), 940–950.

Abrams, Marshall, & Weiss, Joe (2008). *Malicious control system cyber security attack case study–Maroochy water services, Australia*. McLean, VA: The MITRE Corporation.

Agbi, C., & Krogh, B. (2014). Decentralized identification of building models. In *2014 American control conference* (pp. 1070–1075). IEEE.

Alcaraz, Cristina, & Lopez, Javier (2017). A cyber-physical systems-based checkpoint model for structural controllability. *IEEE Systems Journal*, *12*(4), 3543–3554.

Alcaraz, Cristina, Lopez, Javier, & Choo, Kim-Kwang Raymond (2017). Resilient interconnection in cyber-physical control systems. *Computers & Security*, *71*, 2–14.

Alcaraz, Cristina, & Wolthusen, Stephen (2014). Recovery of structural controllability for control systems. In Jonathan Butts, & Sujeet Shenoi (Eds.), *Critical infrastructure protection VIII* (pp. 47–63). Berlin, Heidelberg: Springer Berlin Heidelberg, Springer.

Alem, Saïd, & Benazzouz, Djamel (2014). Bond graph to digraph conversion: A sensor placement optimization for fault detection and isolation by a structural approach. *Sadhana*, *39*(5), 1151–1164.

Alexandru, A. B., Pequito, S., Jadbabaie, A., & Pappas, G. J. (2016). Decentralized observability with limited communication between sensors. In *2016 IEEE 55th conference on decision and control* (pp. 885–890). IEEE.

Alexandru, A. B., Pequito, S., Jadbabaie, A., & Pappas, G. J. (2017). On the limited communication analysis and design for decentralized estimation. In *2017 IEEE 56th annual conference on decision and control* (pp. 1713–1718). IEEE.

Aling, H., & Schumacher, Johannes M. (1984). A nine-fold canonical decomposition for linear systems. *International Journal of Control*, *39*(4), 779–805.

Angulo, M. T., Aparicio, A., & Moog, C. H. (2019). Structural accessibility and structural observability of nonlinear networked systems. *IEEE Transactions on Network Science and Engineering*, 1.

Arora, Sanjeev, & Barak, Boaz (2009). *Computational complexity: a modern approach*. Cambridge University Press.

Assadi, Sepehr, Khanna, Sanjeev, Li, Yang, & Preciado, Victor M. (2015). Complexity of the minimum input selection problem for structural controllability. *IFAC-PapersOnline*, *48*(22), 70–75, 5th IFAC Workshop on Distributed Estimation and Control in Networked Systems NecSys 2015.

Bai, Ting, Li, Shaoyuan, Zou, Yuanyuan, & Yin, Xiang (2019). Block-based minimum input design for the structural controllability of complex networks. *Automatica*, *107*, 68–76.

Basile, Giuseppe, & Marro, Giovanni (1969). Controlled and conditioned invariant subspaces in linear system theory. *Journal of Optimization Theory and Applications*, *3*(5), 306–315.

Becker, C. O., Pequito, S., Pappas, G. J., & Preciado, V. M. (2020). Network design for controllability metrics. *IEEE Transactions on Control of Network Systems*, 1.

Belabbas, M.-A. (2013). Sparse stable systems. *Systems & Control Letters*, *62*(10), 981–987.

Bellman, Ror, & Åström, Karl Johan (1970). On structural identifiability. *Mathematical Biosciences*, *7*(3–4), 329–339.

Bhela, Siddharth, Kekatos, Vassilis, & Veeramachaneni, Sriharsha (2017). Power distribution system observability with smart meter data. In *2017 IEEE global conference on signal and information processing* (pp. 1070–1074). IEEE.

Bhela, S., Kekatos, V., Zhang, L., & Veeramachaneni, S. (2017). Enhancing observability in power distribution grids. In *2017 IEEE international conference on acoustics, speech and signal processing* (pp. 4551–4555). IEEE.

Bian, Andrew An, Buhmann, Joachim M., Krause, Andreas, & Tschiatschek, Sebastian (2017). Guarantees for greedy maximization of non-submodular functions with applications. In Doina Precup, & Yee Whye Teh (Eds.), *Proceedings of machine learning research*: *vol. 70*, *Proceedings of the 34th international conference on machine learning* (pp. 498–507). International Convention Centre, Sydney, Australia: PMLR.

Blackhall, Lachlan, & Hill, David J. (2010). On the structural controllability of networks of linear systems. *IFAC Proceedings Volumes*, *43*(19), 245–250.

Boukhobza, Taha (2008). Generic uniform observability analysis for bilinear systems. *Automatica*, *44*(12), 3133–3138.

Boukhobza, T. (2010). Partial state and input observability recovering by additional sensor implementation: a graph–theoretic approach. *International Journal of Systems Science*, *41*(11), 1281–1291.

Boukhobza, T. (2012). Sensor location for discrete mode observability of switching linear systems with unknown inputs. *Automatica*, *48*(7), 1262–1272.

Boukhobza, T., & Hamelin, F. (2007). Observability analysis for structured bilinear systems: A graph-theoretic approach. *Automatica*, *43*(11), 1968–1974.

Boukhobza, T., & Hamelin, F. (2009). State and input observability recovering by additional sensor implementation: A graph-theoretic approach. *Automatica*, *45*(7), 1737–1742.

Boukhobza, T., & Hamelin, F. (2011). Observability analysis and sensor location study for structured linear systems in descriptor form with unknown inputs. *Automatica*, *47*(12), 2678–2683.

Boukhobza, Taha, & Hamelin, Frédéric (2013). Discrete mode observability of structured switching descriptor linear systems: A graph-theoretic approach. *Automatica*, *49*(10), 3042–3048.

Boukhobza, T., Hamelin, F., & Canitrot, S. (2008). A graph-theoretic approach to fault detection and isolation for structured bilinear systems. *International Journal of Control*, *81*(4), 661–678.

Boukhobza, Taha, Hamelin, Frédéric, Kabadi, G, & Aberkane, Samir (2011). Discrete mode observability of switching linear systems with unknown inputs. a graph-theoretic approach. *IFAC Proceedings Volumes*, *44*(1), 6616–6621.

Boukhobza, Taha, Hamelin, Frédéric, & Martinez-Martinez, Sinuhé (2007). State and input observability for structured linear systems: A graph-theoretic approach. *Automatica*, *43*(7), 1204–1210.

Boukhobza, T., Hamelin, F., & Sauter, D. (2006). Observability of structured linear systems in descriptor form: A graph-theoretic approach. *Automatica*, *42*(4), 629–635.

Boukhobza, T., Hamelin, F., & Simon, C. (2014). A graph theoretical approach to the parameters identifiability characterisation. *International Journal of Control*, *87*(4), 751–763.

Boyd, Stephen, Boyd, Stephen P., & Vandenberghe, Lieven (2004). *Convex optimization*. Cambridge University Press.

Bru, Rafael, Cacetta, L., & Rumchev, Ventsi G. (2005). Monomial subdigraphs of reachable and controllable positive discrete-time systems. *International Journal of Applied Mathematics and Computer Science*, *15*, 159–166.

Campbell, Colin, Ruths, Justin, Ruths, Derek, Shea, Katriona, & Albert, Réka (2015). Topological constraints on network control profiles. *Scientific Reports*, *5*, 18693.

Campobello, G., Leonardi, A., & Palazzo, S. (2009). A distributed framework for network coding based on a novel state space approach. In *2009 6th IEEE annual communications society conference on sensor, mesh and Ad Hoc communications and networks workshops* (pp. 1–6). IEEE.

Canitrot, Sébastien, Boukhobza, Taha, & Hamelin, Frédéric (2008). Observability recovering by additional sensor implementation in structured bilinear systems. *IET Control Theory & Applications*, *2*(10), 860–865.

Cantó, Begoña, Coll, Carmen, & Sánchez, Elena (2009). Structural identifiability of a model of dialysis. *Mathematical and Computer Modelling*, *50*(5), 733–737, Mathematical Models in Medicine & Engineering.

Carvalho, J. Frederico, Pequito, Sérgio, Aguiar, A. Pedro, Kar, Soummya, & Johansson, Karl H. (2017). Composability and controllability of structural linear time-invariant systems: Distributed verification. *Automatica*, *78*, 123–134.

Chamseddine, A., Noura, H., & Theilliol, D. (2009). Optimal sensor network for fault diagnosis using structural analysis. In *2009 International conference on advances in computational tools for engineering applications* (pp. 492–497). IEEE.

Chatterjee, A., Das, D., Naskar, M. K., Pal, N., & Mukherjee, A. (2013). Heuristic for maximum matching in directed complex networks. In *2013 International conference on advances in computing, communications and informatics* (pp. 1146–1151). IEEE.

Chen, Ximing, Pequito, Sérgio, Pappas, George J, & Preciado, Victor M (2018). Minimal edge addition for network controllability. *IEEE Transactions on Control of Network Systems*, *6*(1), 312–323.

Chung, Liam, Ruths, Derek, & Ruths, Justin (2021). Dilations and degeneracy in network controllability. *Scientific Reports*, *11*(1), 1–10.

Clark, Andrew, Alomair, Basel, Bushnell, Linda, & Poovendran, Radha (2015). *Submodularity in dynamics and control of networked systems*. Springer.

Clark, Andrew, Alomair, Basel, Bushnell, Linda, & Poovendran, Radha (2017). Input selection for performance and controllability of structured linear descriptor systems. *SIAM Journal on Control and Optimization*, *55*(1), 457–485.

Commault, Christian (2004). A simple graph theoretic characterization of reachability for positive linear systems. *Systems & Control Letters*, *52*(3–4), 275–282.

Commault, Christian (2019). Structural controllability of networks with dynamical structured nodes. *IEEE Transactions on Automatic Control*.

Commault, Christian, & Dion, Jean-Michel (2007). Sensor location for diagnosis in linear systems: A structural analysis. *IEEE Transactions on Automatic Control*, *52*(2), 155–169.

Commault, Christian, & Dion, Jean-Michel (2013). Input addition and leader selection for the controllability of graph-based systems. *Automatica*, *49*(11), 3322–3328.

Commault, Christian, & Dion, Jean-Michel (2015). The single-input minimal controllability problem for structured systems. *Systems & Control Letters*, *80*, 50–55.

Commault, Christian, Dion, Jean-Michel, & Agha, Sameh Yacoub (2008). Structural analysis for the sensor location problem in fault detection and isolation. *Automatica*, *44*(8), 2074–2080.

Commault, Christian, Dion, Jean-Michel, & Do, Trong Hieu (2011). Sensor location and classification for disturbance rejection by measurement feedback. *Automatica*, *47*(12), 2584–2594.

Commault, Christian, Dion, Jean-Michel, & Hovelaque, V. (1997). A geometric approach for structured systems: Application to disturbance decoupling. *Automatica*, *33*(3), 403–409.

Commault, C., Dion, J.-M., & Perez, A. (1991). Disturbance rejection for structured systems. *IEEE Transactions on Automatic Control*, *36*(7), 884–887.

Commault, C., Dion, J., Sename, O., & Motyeian, R. (2001). Unknown input observer — A structural approach. In *2001 European control conference* (pp. 888–893).

Commault, Christian, Dion, Jean-Michel, Sename, Olivier, & Motyeian, Reza (2002). Observer-based fault detection and isolation for structured systems. *IEEE Transactions on Automatic Control*, *47*(12), 2074–2079.

Commault, Christian, Dion, Jean-Michel, Trinh, D. H., & Do, T. H. (2011). Sensor classification for the fault detection and isolation, a structural approach. *International Journal of Adaptive Control and Signal Processing*, *25*(1), 1–17.

Commault, Christian, Dion, Jean-Michel, et al. (2008). Observability preservation under sensor failure. *IEEE Transactions on Automatic Control*, *53*(6), 1554–1559.

Commault, C., & van der Woude, J. (2019). A classification of nodes for structural controllability. *IEEE Transactions on Automatic Control*, *64*(9), 3877–3882.

Commault, Christian, van der Woude, Jacob, & Boukhobza, Taha (2017). On the fixed controllable subspace in linear structured systems. *Systems & Control Letters*, *102*, 42–47.

Commault, Christian, Van der Woude, Jacb, & Frasca, Paolo (2019). Functional target controllability of networks: structural properties and efficient algorithms. *IEEE Transactions on Network Science and Engineering*.

Conte, G., Perdon, A. M., Zattoni, E., & Moog, Claude H. (2019). Invariance, controlled invariance and conditioned invariance in structured systems and applications to disturbance decoupling. *IOP Conference Series: Materials Science and Engineering*, *707*, Article 012010.

Cormen, Thomas H, Leiserson, Charles E, Rivest, Ronald L, & Stein, Clifford (2009). *Introduction to algorithms*. MIT Press.

Cowan, Noah J, Chastain, Erick J, Vilhena, Daril A, Freudenberg, James S, & Bergstrom, Carl T (2012). Nodal dynamics, not degree distributions, determine the structural controllability of complex networks. *PLoS One*, *7*(6).

Czeizler, Eugen, Popa, Alexandru, & Popescu, Victor (2018). Fixed parameter algorithms and hardness of approximation results for the structural target controllability problem. In Jesper Jansson, Carlos Martín-Vide, & Miguel A. Vega-Rodríguez (Eds.), *Algorithms for computational biology* (pp. 103–114). Cham: Springer International Publishing, Springer.

Czeizler, E., Wu, K., Gratie, C., Kanhaiya, K., & Petre, I. (2018). Structural target controllability of linear networks. *IEEE/ACM Transactions on Computational Biology and Bioinformatics*, *15*(4), 1217–1228.

Daasch, Andreas, Schultalbers, Matthias, & Svaricek, Ferdinand (2016). Structural non-minimum phase systems. In *2016 American control conference* (pp. 3758–3763). IEEE.

Dakil, Manal, Boukhobza, Taha, & Simon, Christophe (2015). Disturbance rejection problem solvability: From structural approach to reliability/availability analysis. *European Journal of Control*, *21*, 36–44.

Dakil, Manal, Simon, Christophe, & Boukhobza, Taha (2015). Generic methodology for the probabilistic reliability assessment of some structural properties: a graph theoretical approach. *International Journal of Systems Science*, *46*(10), 1825–1838.

Descusse, J., & Dion, J. (1982). On the structure at infinity of linear square decoupled systems. *IEEE Transactions on Automatic Control*, *27*(4), 971–974.

Dey, Priyanka, Balachandran, Niranjan, & Chatterjee, Debasish (2018). On minimum cost sparsest input-connectivity for controllability of linear systems. In *2018 57th Annual conference of the society of instrument and control engineers of Japan* (pp. 409–414). IEEE.

Ding, Jin, Tan, Ping, & Lu, Yong-Zai (2016). Optimizing the controllability index of directed networks with the fixed number of control nodes. *Neurocomputing*, *171*, 1524–1532.

D'Innocenzo, A. (2018). Modeling and co-design of control tasks over wireless networking protocols. In *Control subject to computational and communication constraints* (pp. 261–285). Springer.
D'Innocenzo, Alessandro, Smarra, Francesco, & Di Benedetto, Maria Domenica (2016). Resilient stabilization of multi-hop control networks subject to malicious attacks. *Automatica*, *71*, 1–9.
Dion, J. M., & Commault, C. (1982). Smith-mcmillan factorizations at infinity of rational matrix functions and their control interpretation. *Systems & Control Letters*, *1*(5), 312–320.
Dion, J. M., & Commault, C. (1993). Feedback decoupling of structured systems. *IEEE Transactions on Automatic Control*, *38*(7), 1132–1135.
Dion, J. M., Commault, C., & Montoya, J. (1994). Simultaneous decoupling and disturbance rejection—a structural approach. *International Journal of Control*, *59*(5), 1325–1344.
Dion, Jean-Michel, Commault, Christian, & Van Der Woude, Jacob (2003). Generic properties and control of linear structured systems: a survey. *Automatica*, *39*(7), 1125–1144.
Doostmohammadian, M. (2019). Minimal driver nodes for structural controllability of large-scale dynamical systems: Node classification. *IEEE Systems Journal*, 1–8.
Doostmohammadian, Mohammadreza (2020). Recovering the structural observability of composite networks via cartesian product. *IEEE Transactions on Signal and Information Processing over Networks*, *6*, 133–139.
Doostmohammadian, Mohammadreza, & Khan, Usman A. (2013). On the genericity properties in distributed estimation: Topology design and sensor placement. *IEEE Journal of Selected Topics in Signal Processing*, *7*(2), 195–204.
Doostmohammadian, Mohammadreza, & Khan, Usman A. (2014). Graph-theoretic distributed inference in social networks. *IEEE Journal of Selected Topics in Signal Processing*, *8*(4), 613–623.
Doostmohammadian, M., & Khan, U. (2019). On the complexity of minimum-cost networked estimation of self-damped dynamical systems. *IEEE Transactions on Network Science and Engineering*, 1.
Doostmohammadian, Mohammadreza, Rabiee, Hamid R., & Khan, Usman A. (2018). Structural cost-optimal design of sensor networks for distributed estimation. *IEEE Signal Processing Letters*, *25*(6), 793–797.
Doostmohammadian, M., Rabiee, H. R., & Khan, U. A. (2020). Cyber-social systems: Modeling, inference, and optimal design. *IEEE Systems Journal*, *14*(1), 73–83.
Doostmohammadian, M., Rabiee, H. R., Zarrabi, H., & Khan, U. (2018). Observational equivalence in system estimation: Contractions in complex networks. *IEEE Transactions on Network Science and Engineering*, *5*(3), 212–224.
Feng, Xiao-Yu, Liu, Bi-Gang, & Lu, Kai-Sheng (2008). The structural conditions of controllability for RLCM networks over F(z). In *2008 IEEE international conference on industrial technology* (pp. 1–6). IEEE.
Fradellos, G., Rapanakis, M., & Evans, P. J. (1977). Structural controllability in non-linear systems. *International Journal of Systems and Science*, *8*(8), 915–932.
Francis Bach (2013). Learning with submodular functions: A convex optimization perspective. *Foundations and Trends® in Machine Learning*, *6*(2–3), 145–373. http://dx.doi.org/10.1561/2200000039.
Frank, Paul M. (1996). Analytical and qualitative model-based fault diagnosis–a survey and some new results. *European Journal of Control*, *2*(1), 6–28.
Gao, Jianxi, Liu, Yang-Yu, D'souza, Raissa M, & Barabási, Albert-László (2014). Target control of complex networks. *Nature communications*, *5*(1), 1–8.
Geisel, Alfred, & Svaricek, Ferdinand (2019). A MATLAB toolbox for structural analysis of linear systems. *IFAC-PapersOnline*, *52*(17), 7–12.
Ghosh, Supratim, & Ruths, Justin (2016). Structural control of single-input rank one bilinear systems. *Automatica*, *64*, 8–17.
Ghosh, Supratim, Ruths, Justin, & Yeo, Anders (2017). Graphical coprime walk algorithm for structural controllability of discrete-time rank-one bilinear systems. *Automatica*, *86*, 166–173.
Glover, K., & Silverman, L. (1976). Characterization of structural controllability. *IEEE Transactions on Automatic Control*, *21*(4), 534–537.
Goldin, Darina, & Raisch, Jörg (2013). On the weight controllability of consensus algorithms. In *2013 European control conference* (pp. 233–238). IEEE.
Gong, Zhiming, & Aldeen, Mohammad (1997). Stabilization of decentralized control systems. *Journal of Mathematical Systems Estimation and Control*, *7*, 111–114.
Gracy, Sebin, Garin, Federica, & Kibangou, Alain Y. (2018). Input and state observability of network systems with time-varying topology. *IEEE Transactions on Control of Network Systems*, *6*(2), 897–905.
Gracy, S., Garin, F., & Kibangou, A. Y. (2018). Structural and strongly structural input and state observability of linear network systems. *IEEE Transactions on Control of Network Systems*, *5*(4), 2062–2072.
Guan, Yongqiang, & Wang, Long (2017). Structural controllability of multi-agent systems with absolute protocol under fixed and switching topologies. *Science China. Information Sciences*, *60*(9), Article 092203.
Guan, Yongqiang, & Wang, Long (2019). Target controllability of multiagent systems under fixed and switching topologies. *International Journal of Robust and Nonlinear Control*, *29*(9), 2725–2741.
Guo, Baiwei, Karaca, Orcun, Summers, Tyler H, & Kamgarpour, Maryam (2020). Actuator placement under structural controllability using forward and reverse greedy algorithms. *IEEE Transactions on Automatic Control*.
Guo, Wei-Feng, Zhang, Shao-Wu, Wei, Ze-Gang, Zeng, Tao, Liu, Fei, Zhang, Jingsong, et al. (2017). Constrained target controllability of complex networks. *Journal of Statistical Mechanics: Theory and Experiment*, *2017*(6), Article 063402.
Gupta, Gaurav, Pequito, Sérgio, & Bogdan, Paul (2018). Approximate submodular functions and performance guarantees. arXiv preprint arXiv:1806.06323.
Hartung, Christoph, Reißig, Gunther, & Svaricek, Ferdinand (2013). Necessary conditions for structural and strong structural controllability of linear time-varying systems. In *2013 European control conference* (pp. 1335–1340). IEEE.
Hartung, C., & Svaricek, F. (2014). Sign stabilizability. In *22nd Mediterranean conference on control and automation* (pp. 145–150). IEEE.
Holme, Petter, & Saramäki, Jari (2012). Temporal networks. *Physics Reports*, *519*(3), 97–125.
Hou, Baoyu, Li, Xiang, & Chen, Guanrong (2016). Structural controllability of temporally switching networks. *IEEE Transactions on Circuits and Systems I: Regular Papers*, *63*(10), 1771–1781.
Imae, Takanobu, & Cai, Kai (2021). Structural controllability and time-to-control of directed scale-free networks with minimum number of driver nodes. *Systems & Control Letters*, *156*, Article 105025.
Isidori, Alberto, D'Alessandro, Paolo, & Ruberti, Antonio (1974). Realization and structure theory of bilinear dynamical systems. *SIAM Journal on Control*, *12*(3), 517–535.
Iyer, Rishabh K., Jegelka, Stefanie, & Bilmes, Jeff A. (2013). Curvature and optimal algorithms for learning and minimizing submodular functions. In *Advances in neural information processing systems* (pp. 2742–2750).
Jacquez, John A., & Greif, Peter (1985). Numerical parameter identifiability and estimability: Integrating identifiability, estimability, and optimal sampling design. *Mathematical Biosciences*, *77*(1–2), 201–227.
Jafari, Saeid, Ajorlou, Amir, & Aghdam, Amir G. (2011). Leader localization in multi-agent systems subject to failure: A graph-theoretic approach. *Automatica*, *47*(8), 1744–1750.
Jafari, S., Ajorlou, A., Aghdam, A. G., & Tafazoli, S. (2010). On the structural controllability of multi-agent systems subject to failure: A graph-theoretic approach. In *49th IEEE conference on decision and control* (pp. 4565–4570). IEEE.
Jia, Tao, & Barabási, Albert-László (2013). Control capacity and a random sampling method in exploring controllability of complex networks. *Scientific Reports*, *3*(1), 1–6.
Jia, Jiajia, Van Waarde, Henk J, Trentelman, Harry L, & Camlibel, M Kanat (2020). A unifying framework for strong structural controllability. *IEEE Transactions on Automatic Control*.
Kalaimani, Rachel K, Belur, Madhu N, & Sivasubramanian, Sivaramakrishnan (2013). Generic pole assignability, structurally constrained controllers and unimodular completion. *Linear Algebra and its Applications*, *439*(12), 4003–4022.
Karp, Richard M., & Sipser, Michael (1981). Maximum matching in sparse random graphs. In *22nd Annual symposium on foundations of computer science* (pp. 364–375). IEEE.
Kawano, Yu, & Cao, Ming (2019). Structural accessibility and its applications to complex networks governed by nonlinear balance equations. *IEEE Transactions on Automatic Control*, *64*(11), 4607–4614.
Khan, U. A., & Jadbabaie, A. (2011). Coordinated networked estimation strategies using structured systems theory. In *2011 50th IEEE conference on decision and control and european control conference* (pp. 2112–2117). IEEE.
Kirkoryan, A., & Belabbas, M. (2014). Decentralized stabilization with symmetric topologies. In *53rd IEEE conference on decision and control* (pp. 1347–1352). IEEE.
Klaus, Andreas, Yu, Shan, & Plenz, Dietmar (2011). Statistical analyses support power law distributions found in neuronal avalanches. *PLoS One*, *6*(5), Article e19779.
Kobayashi, Nobuaki, & Nakamizo, Takayoshi (1987). A disturbance rejection problem in structural aspects. *Transactions of the Society of Instrument and Control Engineers*, *23*(9), 928–934.
Kobayashi, Hiroaki, & Yoshikawa, Tsuneo (1982). Graph-theoretic approach to controllability and localizability of decentralized control systems. *IEEE Transactions on Automatic Control*, *27*(5), 1096–1108.
Koetter, R., & Medard, M. (2003). An algebraic approach to network coding. *IEEE/ACM Transactions on Networking*, *11*(5), 782–795.
Kong, Jaesop, & Seo, Jin H. (1996). Graph-theoretic characterization of fixed modes in frequency domain. *Automatica*, *32*(7), 1057–1060.
Kruzick, S., Pequito, S., Kar, S., Moura, J. M. F., & Aguiar, A. P. (2018). Structurally observable distributed networks of agents under cost and robustness constraints. *IEEE Transactions on Signal and Information Processing over Networks*, *4*(2), 236–247.
Langner, Ralph (2011). Stuxnet: Dissecting a cyberwarfare weapon. *IEEE Security & Privacy*, *9*(3), 49–51.

Lee, Ji-Woong (2017a). Decentralized pole placement controller design for networks of linear systems. In *2017 IEEE 56th annual conference on decision and control* (pp. 463–468). IEEE.

Lee, Ji-Woong (2017b). Structurally sound networks of control systems. In *2017 American control conference* (pp. 3353–3358). IEEE.

Lewis, Frank L. (1992). A tutorial on the geometric analysis of linear time-invariant implicit systems. *Automatica*, *28*(1), 119–137.

Li, Jingqi, Chen, Ximing, Pequito, Sérgio, Pappas, George J, & Preciado, Victor M (2018). Structural target controllability of undirected networks. In *2018 IEEE conference on decision and control* (pp. 6656–6661). IEEE.

Li, Jingqi, Chen, Ximing, Pequito, Sergio, Pappas, George J, & Preciado, Victor M (2020). On the structural target controllability of undirected networks. *IEEE Transactions on Automatic Control*.

Li, Guoqi, Tang, Pei, Chen, Xumin, Xiao, Gaoxi, Meng, Min, Ma, Cheng, et al. (2020). Target control and expandable target control of complex networks. *Journal of the Franklin Institute*, *357*(6), 3541–3564.

Lichiardopol, S., & Sueur, C. (2007). Linear time-varying structured systems: Part i. graphical representation and analysis. *IFAC Proceedings Volumes*, *40*(20), 554–559.

Lin, Ching-Tai (1974). Structural controllability. *IEEE Transactions on Automatic Control*, *19*(3), 201–208.

Lin, An-Ya, & Ling, Qing (2015). Decentralized and privacy-preserving low-rank matrix completion. *Journal of the Operations Research Society of China*, *3*(2), 189–205.

Lindmark, G., & Altafini, C. (2016). Positive controllability of large-scale networks. In *2016 European control conference* (pp. 819–824). IEEE.

Linnemann, Arno (1981). Decoupling of structured systems. *Systems & Control Letters*, *1*(2), 79–86.

Linnemann, Arno (1983). Fixed modes in parametrized systems. *International Journal of Control*, *38*(2), 319–335.

Liu, Yang-Yu, & Barabási, Albert-László (2016). Control principles of complex systems. *Reviews of Modern Physics*, *88*(3), Article 035006.

Liu, Xiaomeng, Lin, Hai, & Chen, Ben M. (2013a). Graph-theoretic characterisations of structural controllability for multi-agent system with switching topology. *International Journal of Control*, *86*(2), 222–231.

Liu, Xiaomeng, Lin, Hai, & Chen, Ben M. (2013b). Structural controllability of switched linear systems. *Automatica*, *49*(12), 3531–3537.

Liu, F., & Morse, A. S. (2019). A graphical characterization of structurally controllable linear systems with dependent parameters. *IEEE Transactions on Automatic Control*, *64*(11), 4484–4495.

Liu, Yao, Ning, Peng, & Reiter, Michael K. (2011). False data injection attacks against state estimation in electric power grids. *ACM Transactions on Information and System Security*, *14*(1), 1–33.

Liu, P., Niu, X., & Ren, Y. (2019). Configuration design of bilinear networked systems with minimum cost constraints. In *2019 Chinese control conference* (pp. 5237–5242). IEEE.

Liu, Xiaofei, & Sinopoli, Bruno (2018). On partial observability of large scale linear systems: A structured systems approach. In *2018 IEEE conference on decision and control* (pp. 4655–4661). IEEE.

Liu, Yang-Yu, Slotine, Jean-Jacques, & Barabási, Albert-László (2011). Controllability of complex networks. *Nature*, *473*(7346), 167–173.

Liu, Yang-Yu, Slotine, Jean-Jacques, & Barabási, Albert-László (2012). Control centrality and hierarchical structure in complex networks. *PLoS One*, *7*(9), Article e44459.

Liu, Jiapeng, & Tie, Lin (2021). Structural controllability and structural near-controllability of a class of discrete-time bilinear systems. *International Journal of Control*, 1–13.

Liu, X., Weerakkody, S., & Sinopoli, B. (2016). Sensor placement for reliable observability: a structured systems approach. In *2016 IEEE 55th conference on decision and control* (pp. 5414–5421). IEEE.

Liu, P., Zhang, Y., & Tian, Y. (2017). Scheduling algorithm of observation and controlling for multi-agent systems to guarantee structural controllability. In *2017 13th IEEE international conference on control automation* (pp. 672–677). IEEE.

Luo, Xiaoyuan, Li, Jianjin, & Jiang, Zhongping (2018). Structural detectability analysis of cyber attacks for power grids via graph theory. *IET Cyber-Physical Systems: Theory & Applications*, *3*(3), 158–166.

Ma, Q. (2010). Structural controllability of nonlinear systems: A polynomial method. In *2010 2nd International workshop on database technology and applications* (pp. 1–4). IEEE.

Mahajan, Aditya, Martins, Nuno C, Rotkowitz, Michael C, & Yüksel, Serdar (2012). Information structures in optimal decentralized control. In *2012 51st IEEE conference on decision and control* (pp. 1291–1306). IEEE.

Mahulea, Cristian, Recalde, Laura, & Silva, Manuel (2010). Observability of continuous Petri nets with infinite server semantics. *Nonlinear Analysis. Hybrid Systems*, *4*(2), 219–232, IFAC World Congress 2008.

Markus, L., & Lee, E. B. (1962). On the existence of optimal controls. *Journal of Basic Engineering*, *84*(1), 13–20.

Martinez-Martinez, Sinuhe, Hashemi-Nejad, Hossein, & Sauter, Dominique (2010). Communication sequence design in networked control systems with communication constraints: a graphic approach. In *ACD 2010* (p. 314).

Martinez-Martinez, Sinuhé, Mader, Theodor, Boukhobza, Taha, & Hamelin, Frédéric (2007). LISA: a linear structured system analysis program. In IFAC (Ed.), *3rd IFAC symposium on system, structure and control* (p. CDROM). Foz do Iguaçu, Brazil: Elsevier.

Mathur, H., & Datta, S. (2018). Design of state feedback control for a class of small scale structured descriptor systems. In *2018 IEEE conference on control technology and applications* (pp. 1307–1312). IEEE.

Mayeda, Hirokazu, & Yamada, Takashi (1979). Strong structural controllability. *SIAM Journal on Control and Optimization*, *17*(1), 123–138.

Maza, S., Simon, C., & Boukhobza, T. (2012). Impact of the actuator failures on the structural controllability of linear systems: a graph theoretical approach. *IET Control Theory & Applications*, *6*, 412–419(7).

Mehrabadi, Milad Kazemi, Zamani, Mohsen, & Chen, Zhiyong (2019). Structural controllability of a consensus network with multiple leaders. *IEEE Transactions on Automatic Control*, *64*(12), 5101–5107.

Menara, T., Bassett, D. S., & Pasqualetti, F. (2019). Structural controllability of symmetric networks. *IEEE Transactions on Automatic Control*, *64*(9), 3740–3747.

Milošević, Jezdimir, Teixeira, André, Johansson, Karl H, & Sandberg, Henrik (2020). Actuator security indices based on perfect undetectability: Computation, robustness, and sensor placement. *IEEE Transactions on Automatic Control*, 1.

Milošević, J., Sandberg, H., & Johansson, K. H. (2018). A security index for actuators based on perfect undetectability: Properties and approximation. In *2018 56th Annual allerton conference on communication, control, and computing* (pp. 235–241). IEEE.

Mo, Yilin, & Sinopoli, Bruno (2010). False data injection attacks in control systems. In *Preprints of the 1st workshop on secure control systems* (pp. 1–6).

Mohler, Ronald R., Kolodziej, W. J., et al. (1980). An overview of bilinear system theory and applications. *IEEE Transactions on Systems, Man and Cybernetics*, *10*(10), 683–688.

Monshizadeh, Nima, Camlibel, Kanat, & Trentelman, Harry (2015). Strong targeted controllability of dynamical networks. In *2015 54th IEEE conference on decision and control* (pp. 4782–4787). IEEE.

Moothedath, Shana, Chaporkar, Prasanna, & Belur, Madhu N. (2018a). A flow-network-based polynomial-time approximation algorithm for the minimum constrained input structural controllability problem. *IEEE Transactions on Automatic Control*, *63*(9), 3151–3158.

Moothedath, Shana, Chaporkar, Prasanna, & Belur, Madhu N. (2018b). Minimum cost feedback selection for arbitrary pole placement in structured systems. *IEEE Transactions on Automatic Control*, *63*(11), 3881–3888.

Moothedath, Shana, Chaporkar, Prasanna, & Belur, Madhu N. (2019). Optimal selection of essential interconnections for structural controllability in heterogeneous subsystems. *Automatica*, *103*, 424–434.

Moothedath, S., Chaporkar, P., & Belur, M. N. (2019). Sparsest feedback selection for structurally cyclic systems with dedicated actuators and sensors in polynomial time. *IEEE Transactions on Automatic Control*, *64*(9), 3956–3963.

Moothedath, S., Chaporkar, P., & Belur, M. N. (2020). Optimal network topology design in composite systems for structural controllability. *IEEE Transactions on Control of Network Systems*, 1.

Moothedath, Shana, Yashashwi, Kumar, Chaporkar, Prasanna, & Belur, Madhu N (2019). Target controllability of structured systems. In *2019 18th European control conference* (pp. 3484–3489). IEEE.

Mousavi, S. S., Haeri, M., & Mesbahi, M. (2018). On the structural and strong structural controllability of undirected networks. *IEEE Transactions on Automatic Control*, *63*(7), 2234–2241.

Mu, Jianbin, Li, Shaoyuan, Zou, Yuanyuan, & Li, Ning (2019). Guaranteed structural controllability for networked systems with minimum input/edge addition. In *2019 Chinese control conference* (pp. 5374–5380). IEEE.

Murota, Kazuo (2009a). *Matrices and matroids for systems analysis. Vol. 20*. Springer Science & Business Media.

Murota, Kazuo (2009b). Recent developments in discrete convex analysis. In *Research trends in combinatorial optimization* (pp. 219–260). Springer.

Murota, Kazuo (2012). *Systems analysis by graphs and matroids: structural solvability and controllability. Vol. 3*. Springer Science & Business Media.

Nacher, Jose C., & Akutsu, Tatsuya (2013). Structural controllability of unidirectional bipartite networks. *Scientific Reports*, *3*, 1647.

Nacher, Jose C, Ishitsuka, Masayuki, Miyazaki, Shuichi, & Akutsu, Tatsuya (2019). Finding and analysing the minimum set of driver nodes required to control multilayer networks. *Scientific Reports*, *9*(1), 1–12.

Nepusz, Tamás, & Vicsek, Tamás (2012). Controlling edge dynamics in complex networks. *Nature Physics*, *8*(7), 568–573.

Olshevsky, Alex (2014). Minimal controllability problems. *IEEE Transactions on Control of Network Systems*, *1*(3), 249–258.

Olshevsky, Alex (2015). Minimum input selection for structural controllability. In *2015 American control conference* (pp. 2218–2223). IEEE.

Ouyang, Bo, Pati, Siddharth, Wang, Cong, & Lu, Lu (2018). Controllability of networks with multivariable agents. In *2018 Annual American control conference* (pp. 4918–4924). IEEE.

Pajic, M., Mangharam, R., Pappas, G. J., & Sundaram, S. (2013). Topological conditions for in-network stabilization of dynamical systems. *IEEE Journal on Selected Areas in Communications*, *31*(4), 794–807.

Pajic, Miroslav, Sundaram, Shreyas, Pappas, George J, & Mangharam, Rahul (2011). The wireless control network: A new approach for control over networks. *IEEE Transactions on Automatic Control*, *56*(10), 2305–2318.

Partovi, A., Lin, H., & Ji, Z. (2010). Structural controllability of high order dynamic multi-agent systems. In *2010 IEEE conference on robotics, automation and mechatronics* (pp. 327–332). IEEE.

Pasqualetti, Fabio, Bicchi, Antonio, & Bullo, Francesco (2011). Consensus computation in unreliable networks: A system theoretic approach. *IEEE Transactions on Automatic Control*, *57*(1), 90–104.

Pasqualetti, Fabio, Dörfler, Florian, & Bullo, Francesco (2013). Attack detection and identification in cyber-physical systems. *IEEE Transactions on Automatic Control*, *58*(11), 2715–2729.

Pasqualetti, Fabio, Gu, Shi, & Bassett, Danielle S. (2019). RE: Warnings and caveats in brain controllability. *NeuroIMage*, *197*, 586–588.

Paynter, Henry M. (1961). *Analysis and design of engineering systems*. MIT Press.

Pequito, S., Bogdan, P., & Pappas, G. J. (2015). Minimum number of probes for brain dynamics observability. In *2015 54th IEEE conference on decision and control* (pp. 306–311). IEEE.

Pequito, Sergio, Kar, Soummya, & Aguiar, A. Pedro (2015a). A framework for structural input/output and control configuration selection in large-scale systems. *IEEE Transactions on Automatic Control*, *61*(2), 303–318.

Pequito, Sérgio, Kar, Soummya, & Aguiar, A. Pedro (2015b). On the complexity of the constrained input selection problem for structural linear systems. *Automatica*, *62*, 193–199.

Pequito, Sergio, Kar, Soummya, & Aguiar, A. Pedro (2016). Minimum cost input/output design for large-scale linear structural systems. *Automatica*, *68*, 384–391.

Pequito, S., Kar, S., & Pappas, G. J. (2015). Minimum cost constrained input-output and control configuration co-design problem: A structural systems approach. In *2015 American control conference* (pp. 4099–4105). IEEE.

Pequito, Sérgio, Kar, Soummya, Sundaram, Shreyas, & Aguiar, A Pedro (2014). Design of communication networks for distributed computation with privacy guarantees. In *53rd IEEE conference on decision and control* (pp. 1370–1376). IEEE.

Pequito, Sérgio, Khambhati, Ankit N, Pappas, George J, Bassett, Danielle, Litt, Brian, et al. (2016). Structural analysis and design of dynamic-flow networks: implications in the brain dynamics. In *2016 American control conference* (pp. 5758–5764). IEEE.

Pequito, Sérgio, Khorrami, Farshad, Krishnamurthy, Prashanth, & Pappas, George J (2018). Analysis and design of actuation–sensing–communication interconnection structures toward secured/resilient LTI closed-loop systems. *IEEE Transactions on Control of Network Systems*, *6*(2), 667–678.

Pequito, Sérgio, & Pappas, George J. (2015). Smart building: A private cyber-physical system approach. In *Proceedings of the second international workshop on the swarm at the edge of the cloud* (pp. 1–6).

Pequito, Sérgio, & Pappas, George J. (2017). Structural minimum controllability problem for switched linear continuous-time systems. *Automatica*, *78*, 216–222.

Pequito, Sérgio, Preciado, Victor M, Barabási, Albert-László, & Pappas, George J (2017). Trade-offs between driving nodes and time-to-control in complex networks. *Scientific Reports*, *7*, 39978.

Pequito, S., Preciado, V., & Pappas, G. J. (2015). Distributed leader selection. In *2015 54th IEEE conference on decision and control* (pp. 962–967). IEEE.

Pequito, Sérgio, Rego, Francisco, Kar, Soummya, Aguiar, A Pedro, Pascoal, Antonio, & Jones, Colin (2014). Optimal design of observable multi-agent networks: A structural system approach. In *2014 European control conference* (pp. 1536–1541). IEEE.

Perera, M. Anushka S., Lie, Bernt, & Pfeiffer, Carlos F. (2015). Structural observability analysis of large scale systems using modelica and python. *Modeling, Identification and Control*, *36*(1), 53–65.

Popli, Nipun, Pequito, Sérgio, Kar, Soummya, Aguiar, A Pedro, & Ilić, Marija (2019). Selective strong structural minimum-cost resilient co-design for regular descriptor linear systems. *Automatica*, *102*, 80–85.

Pósfai, Márton, & Hövel, Philipp (2014). Structural controllability of temporal networks. *New Journal of Physics*, *16*(12), Article 123055.

Qi, Ailing, Ju, Xuewei, Zhang, Qing, & Chen, Zengqiang (2016). Structural controllability of discrete-time linear control systems with time-delay: a delay node inserting approach. *Mathematical Problems in Engineering*, *2016*.

Qiang, Ma (2010). Some results on structural controllability of nonlinear systems. In *2010 International conference on intelligent system design and engineering application. Vol. 2* (pp. 391–394). IEEE.

Rahimian, Mohammad Amin, & Aghdam, Amir G. (2013). Structural controllability of multi-agent networks: Robustness against simultaneous failures. *Automatica*, *49*(11), 3149–3157.

Ramasubramanian, Bhaskar, Rajan, M. A., & Chandra, M. Girish (2016). Structural resilience of cyberphysical systems under attack. In *2016 American control conference* (pp. 283–289). IEEE.

Ramos, G. (2013). *Structural hybrid systems* (MSc Thesis), Instituto Superior Tecnico.

Ramos, Guilherme, & Pequito, Sérgio (2020). Generating complex networks with time-to-control communities. *PLoS One*, *15*(8), Article e0236753.

Ramos, G., Pequito, S., Aguiar, A. P., & Kar, S. (2015). Analysis and design of electric power grids with p-robustness guarantees using a structural hybrid system approach. In *2015 European control conference* (pp. 3542–3547). IEEE.

Ramos, G., Pequito, S., Aguiar, A. P., Ramos, J., & Kar, S. (2013). A model checking framework for linear time invariant switching systems using structural systems analysis. In *2013 51st Annual allerton conference on communication, control, and computing* (pp. 973–980). IEEE.

Ramos, Guilherme, Pequito, Sérgio, & Caleiro, Carlos (2018). The robust minimal controllability problem for switched linear continuous-time systems. In *2018 Annual American control conference* (pp. 210–215). IEEE.

Ramos, Guilherme, Silvestre, Daniel, & Silvestre, Carlos (2021). Node and network resistance to bribery in multi-agent systems. *Systems & Control Letters*, *147*, Article 104842.

Rantzer, Anders, & Valcher, Maria Elena (2018). A tutorial on positive systems and large scale control. In *2018 IEEE conference on decision and control* (pp. 3686–3697). IEEE.

Ravandi, Babak, Ansari, Forough S., & Mili, Fatma (2019). Controllability analysis of complex networks using statistical random sampling. *Advances in Complex Systems*, *22*(07n08), Article 1950012.

Reinschke, K. J., Jantzen, J., & Evans, F. J. (1992). CAD techniques for a structural approach to noninteraction and disturbance rejection. In *Design methods of control systems* (pp. 537–542). Elsevier.

Reinschke, Kurt Johannes, & Reinschke, Kurt (1988). *Multivariable control: a graph-theoretic approach. Vol. 41*. Springer.

Roberts, P. D. (2001). Robust model-based fault diagnosis for dynamic systems, Jie Chen and RJ Patton; Kluwer Academic Publishers, Boston, MA, USA, 1999, 354 pages, ISBN 0-7923-8411-3. *International Journal of Robust and Nonlinear Control: IFAC-Affiliated Journal*, *11*(14), 1400–1401.

Rocha, Pedro Manuel Sabino (2014). *Output selection for large scale structural systems: a matroid theory approach* (MSc Thesis), Department of Electrical and Computer Engineering, Faculty of Engineering, University of Porto, Porto, Portugal.

Romero, O., & Pequito, S. (2018). Actuator placement for symmetric structural controllability with heterogeneous costs. *IEEE Control Systems Letters*, *2*(4), 821–826.

Ruf, Sebastian F., Egerstedt, Magnus, & Shamma, Jeff S. (2018). Herdable systems over signed, directed graphs. In *2018 Annual American control conference* (pp. 1807–1812). IEEE.

Ruths, Justin, & Ruths, Derek (2014). Control profiles of complex networks. *Science*, *343*(6177), 1373–1376.

Sandberg, Henrik, Teixeira, André, & Johansson, Karl H. (2010). On security indices for state estimators in power networks. In *First workshop on secure control systems (SCS), Stockholm, 2010*.

Sauter, Dominique, Boukhobza, Taha, & Hamelin, Frédéric (2006). Decentralized and autonomous design for FDI/FTC of networked control systems. *IFAC Proceedings Volumes*, *39*(13), 138–143, 6th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes.

Schumacher, Johannes (1980). Compensator synthesis using (C, A, B)-pairs. *IEEE Transactions on Automatic Control*, *25*(6), 1133–1138.

Sezer, M. E., & Šiljak, D. D. (1981). Structurally fixed modes. *Systems & Control Letters*, *1*(1), 60–64.

She, Baike, & Kan, Zhen (2020). Characterizing controllable subspace and herdability of signed weighted networks via graph partition. *Automatica*, *115*, Article 108900.

Shen, Cong, Ji, Zhijian, & Yu, Haisheng (2018). The structural controllability of edge dynamics in complex networks. In *2018 Chinese control and decision conference* (pp. 5356–5360). IEEE.

Shields, Robert, & Pearson, J. (1976). Structural controllability of multiinput linear systems. *IEEE Transactions on Automatic Control*, *21*(2), 203–212.

Shirani Faradonbeh, M. K., Tewari, A., & Michailidis, G. (2017). Optimality of fast-matching algorithms for random networks with applications to structural controllability. *IEEE Transactions on Control of Network Systems*, *4*(4), 770–780.

Shoukry, Yasser, Nuzzo, Pierluigi, Puggelli, Alberto, Sangiovanni-Vincentelli, Alberto L, Seshia, Sanjit A, Srivastava, Mani, et al. (2015). Imhotep-SMT: A satisfiability modulo theory solver for secure state estimation. In *Proc. int. workshop satisfiability modulo theories* (pp. 3–13).

Silva, Manuel (2013). Half a century after carl adam Petri's Ph. D. thesis: A perspective on the field. *Annual Reviews in Control*, *37*(2), 191–219.

Silva, Manuel, Júlvez, Jorge, Mahulea, Cristian, & Vázquez, C Renato (2011). On fluidization of discrete event models: observation and control of continuous Petri nets. *Discrete Event Dynamic Systems*, *21*(4), 427.

Simon, Christophe, Boukhobza, Taha, & Hamelin, Frédéric (2013). Reliability assessment method for structural observer based FDI scheme by a graph theoretic approach. *Annual Reviews in Control*, *37*(1), 137–145.

Srighakollapu, Manikya Valli, Kalaimani, Rachel Kalpana, & Pasumarthy, Ramkrishna (2021). Optimizing driver nodes for structural controllability of temporal networks. *IEEE Transactions on Control of Network Systems*.

Staroswiecki, Marcel (2007a). Observability and the design of fault tolerant estimation using structural analysis. In Claudio Bonivento, Lorenzo Marconi, Carlo Rossi, & Alberto Isidori (Eds.), *Advances in control theory and applications* (pp. 257–278). Berlin, Heidelberg: Springer Berlin Heidelberg.

Staroswiecki, M. (2007b). A structural view of fault-tolerant estimation. *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, *221*(6), 905–914.

Stefani, Gianna (1985). Local controllability of non-linear systems: An example. *Systems & Control Letters*, *6*(2), 123–125.

Suda, Nobuhide, Wan, Baiping, & Ueno, Ichirou (1989). The orders of infinite zeros of structured systems. *Transactions of the Society of Instrument and Control Engineers*, *25*(10), 1062–1068.

Sueur, C., & Dauphin-Tanguy, G. (1989). Structural controllability/observability of linear systems represented by bond graphs. *Journal of the Franklin Institute*, *326*(6), 869–883.

Sueur, C., & Dauphin-Tanguy, G. (1991). Bond-graph approach for structural analysis of MIMO linear systems. *Journal of the Franklin Institute*, *328*(1), 55–70.

Sundaram, Shreyas, & Hadjicostis, Christoforos N. (2006). Designing stable inverters and state observers for switched linear systems with unknown inputs. In *Proceedings of the 45th IEEE conference on decision and control* (pp. 4105–4110). IEEE.

Sundaram, Shreyas, & Hadjicostis, Christoforos N. (2008). Distributed function calculation and consensus using linear iterative strategies. *IEEE Journal on Selected Areas in Communications*, *26*(4), 650–660.

Sundaram, S., & Hadjicostis, C. N. (2009). Linear iterative strategies for transmitting streams of values through sensor networks. In *2009 17th Mediterranean conference on control and automation* (pp. 106–111). IEEE.

Sundaram, Shreyas, & Hadjicostis, Christoforos N. (2010). Distributed function calculation via linear iterative strategies in the presence of malicious agents. *IEEE Transactions on Automatic Control*, *56*(7), 1495–1508.

Sundaram, Shreyas, & Hadjicostis, Christoforos N. (2012). Structural controllability and observability of linear systems over finite fields with applications to multi-agent systems. *IEEE Transactions on Automatic Control*, *58*(1), 60–73.

Sundaram, Shreyas, & Hadjicostis, Christoforos N. (2013). Control and estimation in finite state multi-agent systems: A finite field approach. *IEEE Transactions on Automatic Control*, *58*, 60–73.

Sundaram, Shreyas, Pajic, Miroslav, Hadjicostis, Christoforos N, Mangharam, Rahul, & Pappas, George J (2010). The wireless control network: Monitoring for malicious behavior. In *49th IEEE conference on decision and control* (pp. 5979–5984). IEEE.

Sundaram, Shreyas, Revzen, Shai, & Pappas, George (2012). A control-theoretic approach to disseminating values and overcoming malicious links in wireless networks. *Automatica*, *48*(11), 2894–2901.

Svaricek, Ferdinand (2006). Discussion on: "uniform observability analysis for structured bilinear systems. a graph-theoretic approach". *European Journal of Control*, *12*(5), 519–520.

Tang, Evelyn, & Bassett, Danielle S. (2018). Colloquium: Control of dynamics in brain networks. *Reviews of Modern Physics*, *90*(3), Article 031003.

Terasaki, Shun, & Sato, Kazuhiro (2021). Minimal controllability problems on linear structural descriptor systems. *IEEE Transactions on Automatic Control*.

Thoma, Jean U. (2016). *Introduction to bond graphs and their applications*. Elsevier.

Torres, Jackeline Abad, & Roy, Sandip (2015). Graph-theoretic analysis of network input–output processes: Zero structure and its implications on remote feedback control. *Automatica*, *61*, 73–79.

Trave, L., Tarras, A. M., & Titli, A. (1987). Minimal feedback structure avoiding structurally fixed modes. *International Journal of Control*, *46*(1), 313–325.

Trave, Louise, Titli, André, & Tarras, Ahmed (1989). *Large scale systems: decentralization, structure constraints and fixed modes. Vol. 120*. Springer.

Trefois, Maguy, & Delvenne, Jean-Charles (2015). Zero forcing number, constrained matchings and strong structural controllability. *Linear Algebra and its Applications*, *484*, 199–218.

Tsiamis, A., Pequito, S., & Pappas, G. J. (2017). Distributed leader selection in switching networks of high-order integrators. In *2017 55th Annual allerton conference on communication, control, and computing* (pp. 413–420). IEEE.

Tsopelakos, A., Belabbas, M., & Gharesifard, B. (2019). Classification of the structurally controllable zero-patterns for driftless bilinear control systems. *IEEE Transactions on Control of Network Systems*, *6*(1), 429–439.

Tu, Chengyi, Rocha, Rodrigo P, Corbetta, Maurizio, Zampieri, Sandro, Zorzi, Marco, & Suweis, Samir (2018). Warnings and caveats in brain controllability. *NeuroIMage*, *176*, 83–91.

Van der Woude, J. W. (1991). A graph-theoretic characterization for the rank of the transfer matrix of a structured system. *Mathematics of Control, Signals, and Systems*, *4*(1), 33–40.

Van Der Woude, J. W. (1991). On the structure at infinity of a structured system. *Linear Algebra and its Applications*, *148*, 145–169.

Van Der Woude, J. W. (1996). Disturbance decoupling by measurement feedback for structured transfer matrix systems. *Automatica*, *32*(3), 357–363.

van der Woude, Jacob, Boukhobza, Taha, & Commault, Christian (2018). On structural behavioural controllability of linear discrete time systems with delays. *Systems & Control Letters*, *119*, 31–38.

Van Der Woude, Jacob, Commault, Christian, & Boukhobza, Taha (2019). A dynamic graph characterisation of the fixed part of the controllable subspace of a linear structured system. *Systems & Control Letters*, *129*, 17–25.

Van Waarde, Henk J., Camlibel, M. Kanat, & Trentelman, Harry L. (2017). A distance-based approach to strong target control of dynamical networks. *IEEE Transactions on Automatic Control*, *62*(12), 6266–6277.

Veldman - de Roo, F., Tejada, A., van Waarde, H., & Trentelman, H. L. (2015). Towards observer-based fault detection and isolation for branched water distribution networks without cycles. In *2015 European control conference* (pp. 3280–3285). IEEE.

Verde, Cristina, & Sánchez-Parra, Marino (2007). Monitorability analysis for a gas turbine using structural analysis. In *Fault detection, supervision and safety of technical processes 2006* (pp. 675–680). Elsevier.

Wang, X., Jiang, G., & Wu, Xu (2017). Structural controllability of complex dynamical networks with nodes being multidimensional dynamics. In *2017 American control conference* (pp. 5013–5019). IEEE.

Wang, Xiao, & Xiang, Linying (2021). Optimizing network controllability with minimum cost. *Complexity*, *2021*.

Weerakkody, S., Liu, X., & Sinopoli, B. (2017). Robust structural analysis and design of distributed control systems to prevent zero dynamics attacks. In *2017 IEEE 56th annual conference on decision and control* (pp. 1356–1361). IEEE.

Weerakkody, Sean, Liu, Xiaofei, Son, Sang Hyuk, & Sinopoli, Bruno (2016). A graph-theoretic characterization of perfect attackability for secure design of distributed control systems. *IEEE Transactions on Control of Network Systems*, *4*(1), 60–70.

Whalen, Andrew J, Brennan, Sean N, Sauer, Timothy D, & Schiff, Steven J (2015). Observability and controllability of nonlinear networks: The role of symmetry. *Physical Review X*, *5*(1), Article 011005.

Whalen, Andrew J, Brennan, Sean N, Sauer, Timothy D, & Schiff, Steven J (2016). Effects of symmetry on the structural controllability of neural networks: A perspective. In *2016 American control conference* (pp. 5785–5790). IEEE.

Wonham, W. Murray (1985). Linear multivariable control: a geometric approach. *Applications of Mathematics*, *10*.

Woude, J. v. d. (2018). Zero controllability in discrete-time structured systems. In *2018 European control conference* (pp. 1851–1856). IEEE.

Xiaoyu, F. (2012). Investigation on structural controllability of electrical network by rational function matrices. In *Proceedings of the 31st Chinese control conference* (pp. 261–266). IEEE.

Xue, Yuankun, Pequito, Sergio, Coelho, Joana R, Bogdan, Paul, & Pappas, George J (2016). Minimum number of sensors to ensure observability of physiological systems: A case study. In *2016 54th Annual allerton conference on communication, control, and computing* (pp. 1181–1188). IEEE.

Xue, Mengran, & Roy, Sandip (2019). Structural controllability of linear dynamical networks with homogeneous subsystems. *IFAC-PapersOnline*, *52*(3), 25–30.

Yan, Feng, Sundaram, Shreyas, Vishwanathan, S. V. N., & Qi, Yuan (2012). Distributed autonomous online learning: Regrets and intrinsic privacy-preserving properties. *IEEE Transactions on Knowledge and Data Engineering*, *25*(11), 2483–2493.

Yao, Peng, Hou, Bao-Yu, Pan, Yu-Jian, & Li, Xiang (2017). Structural controllability of temporal networks with a single switching controller. *PLoS One*, *12*(1).

Yuan, Yupeng, Lu, Kaisheng, & Yan, Xinping (2016). A structural analysis and design of an engineering control system in the frequency domain over F(z). *International Journal of Control and Automation*, *9*(3), 277–288.

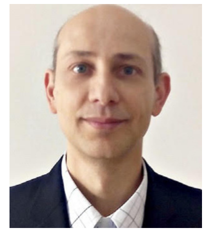
Zañudo, Jorge Gomez Tejeda, Yang, Gang, & Albert, Réka (2017). Structure-based control of complex networks with nonlinear dynamics. *Proceedings of the National Academy of Sciences*, *114*(28), 7234–7239.

Zamani, Mohsen, & Lin, Hai (2009). Structural controllability of multi-agent systems. In *2009 American control conference* (pp. 5743–5748). IEEE.

Zhang, Shuo, & Wolthusen, Stephen D. (2017). Iterative recovery of controllability via maximum matching. In *2017 13th IEEE conference on automation science and engineering* (pp. 328–333). IEEE.

Zhang, Shuo, & Wolthusen, Stephen D. (2018). Efficient control recovery for resilient control systems. In *2018 IEEE 15th international conference on networking, sensing and control* (pp. 1–6). IEEE.
Zhang, Shuo, & Wolthusen, Stephen D. (2019). Driver-node based security analysis for network controllability. In *2019 18th European control conference* (pp. 2246–2251). IEEE.
Zhang, Yuan, Xia, Yuanqing, & Zhai, Di-Hua (2021). Structural controllability of networked relative coupling systems. *Automatica*, *128*, Article 109547.
Zhang, Yuan, Xia, Yuanqing, Zhang, Jinhui, & Shang, Jun (2020). Generic detectability and isolability of topology failures in networked linear systems. *IEEE Transactions on Control of Network Systems*, *8*(1), 500–512.
Zhang, Yuan, & Zhou, Tong (2019a). Minimal structural perturbations for controllability of a networked system: Complexities and approximations. *International Journal of Robust and Nonlinear Control*, *29*(12), 4191–4208.
Zhang, Yuan, & Zhou, Tong (2019b). Structural controllability of an NDS with LFT parameterized subsystems. *IEEE Transactions on Automatic Control*, *64*(12), 4920–4935.

Guilherme Ramos received the B.Sc. (2011) and M.Sc. (2013) in applied mathematics, and Ph.D. in information security (2018) from the Instituto Superior Técnico, Lisbon, Portugal. From 2018 to 2020, he was a postdoctoral researcher with the Institute for Systems and Robotics at IST (ISR/IST). Between 2020 and 2022, he was a postdoctoral researcher at the Department of Electrical and Computer Engineering (DEEC), Faculty of Engineering, University of Porto (FEUP). Currently, he is an invited assistant professor with the computer science department at Faculdade de Ciências, Universidade de Lisboa. His areas of interest include structural systems, cryptography, information theory, ranking and recommendation Systems.



A. Pedro Aguiar received the Licenciatura, M.S. and Ph.D. in Electrical and Computer Engineering (ECE) from the Instituto Superior Técnico (IST), University of Lisbon, Portugal in 1994, 1998, and 2002, respectively. Currently, Dr. Aguiar is a Professor with the ECE Department at the Faculty of Engineering, University of Porto (FEUP). His research interests are in the area of Control Systems and Robotics with a particular focus on mobile networked cyber–physical systems; motion planning, guidance, navigation and control of single and multiple cooperative/coordinated autonomous robotic vehicles; nonlinear control and estimation theory; optimization-based and optimal control; networked control; the integration of machine learning with feedback control; and large-scale distributed systems.

Sergio Pequito is an assistant professor in the Delft Center for Systems & Control that is part of the Mechanical, Maritime and Materials Engineering faculty at Delft University of Technology. Pequito's research consists of understanding the global qualitative behavior of large-scale systems from their structural or parametric descriptions and provides a rigorous framework for the design, analysis, optimization, and control of large scale systems. Currently, his interests span to neuroscience and biomedicine, where dynamical systems and control theoretic tools can be leveraged to develop new analysis tools for brain dynamics towards effective personalized medicine and improve brain–computer and brain–machine–brain interfaces. Pequito was awarded the best student paper finalist in the 48th IEEE Conference on Decision and Control (2009) and the 2016 O. Hugo Schuck Award in the Theory Category by the American Automatic Control.