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Passenger-centered vulnerability assessment of railway networks

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ABSTRACT

The performance and behaviour of critical infrastructure in case of disruptions is an important topic and we are still lacking of insights. Due to disruptions, infrastructure becomes unavailable and may force the trains and passengers to adapt. In this paper, we introduce a problem of railway network vulnerability from the perspective of passenger flows and train operations. We propose a new Railway Network Vulnerability Model (RNVM) to assess the vulnerability of the system by finding the critical combination of links, which cause the most adverse consequences to passengers and trains. To solve this challenging problem, we present a RNVM framework, which combines two heuristics based on column and row generation with mixed integer linear programming, to efficiently model alternative passenger flows and infrastructure constraints. The developed framework provides the critical combination of links, the corresponding passenger flows, train routes and timetables. We demonstrate the performance of the RNVM framework on the real-world instance of a part of the Dutch railway network. The results show that the RNVM framework can efficiently reassign passenger flows and reroute trains during disruptions. The results also reveal that the critical links are highly demand dependent rather than a static feature of the networks topology. Finally, the computation times remain small when increasing the number of disrupted links as well as the size of the passenger demand, which allows fast and efficient network vulnerability assessment.

1. Introduction

1.1. Situation

Critical infrastructure networks form the backbone for the functioning of our economy, society and technology. With the increasing importance of sustainable and environmentally friendly transport, Public Transport (PT), and railways in particular, are getting even more public interest and desire towards providing high quality services. For fully using advantages of PT and railways, transport services need to be attractive and beneficial for passengers and potential users. Understanding the network behaviour during normal conditions is highly relevant and rather known in current literature. However, their particular performance and behaviour in case of eventual disruptions is an equally important topic (see Candelieri et al., 2019)

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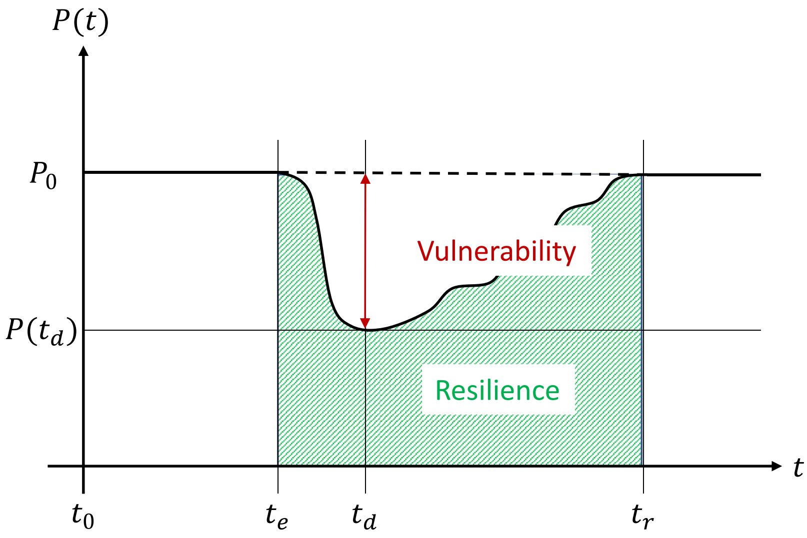

Fig. 1. Schematic network performance during disruption in reference to Wan et al. (2017).

and we are still lacking any significant insights. Therefore, a deeper understanding of network performances of transport networks during disruptions is of major importance.

Generally speaking, a disruption is an event, causing a major adverse change in the networks behaviour and performance. Compared to disturbances (i.e. small everyday variations), disruptions are occurring less frequently but cause a higher performance deterioration. A typical disruption can be a removal of one or multiple components in the network, such as infrastructure elements or vehicles. Furthermore, disruptions can originate within or even outside the systems boundaries and can cause infrastructural/operational blockages or even direct or indirect injuries and fatalities (Mattsson and Jenelius, 2015; Wan et al., 2017). Focusing on transport networks, disruption effects can be increased travel times, cancelled trips and consequently increased cost and even fewer reachable activities and locations (Mattsson and Jenelius, 2015; Yap et al., 2018).

Understanding performance and behaviour during disruptions is of utmost importance for the multiple interdependent strategical and tactical planning steps in the railway planning process, e.g. for motivating future network investments, allocating spare-, emergency or maintenance resources and designing resilient traffic operations. In a broader perspective, this may lead towards the goal of seamless mobility, increasing the attraction of railways as part of the public transport and therefore contributes to effective mobility but also economy, environment and equity (see van Oort et al., 2017).

1.2. Resilience and vulnerability

To assess the performance of networks during disruptions, two closely related concepts exist – resilience and vulnerability. To distinguish between them, we introduce them both and to this purpose, we give a schematic representation of a network performance P during a disruption in Fig. 1. During regular operations a system performs at state P_0. A disruption starts at time t_e and causes deviations from P_0. The system may gradually deteriorate until it reaches the most disrupted point $P(t_d)$ and stays at this state for a certain time. Eventually, after the disruption ends, the system performance returns to its original state at time t_r.

Resilience is a comprehensive metric for assessing a systems performance under disruptions which captures the system's ability to maintain its function under disruptions, including its capability and the process of recovering to its original state (Mattsson and Jenelius, 2015). Hence, it covers the whole process, as displayed in Fig. 1, consisting of multiple performance aspects, starting with beginning of a disruption t_e until reaching its resolution t_r. Depending on the field of research e.g. economy or ecology, there exist even broader definitions of resilience. The reader may be referred to Hosseini et al. (2016) and Bešinović (2020) for further reading.

Vulnerability is the system's "susceptibility to [disruptions] that can result in considerable reductions in [..] network serviceability" (Berdica, 2002). Similar definitions can be found in Candelieri et al. (2019), Zhou et al. (2019) and Wan et al. (2017). Sometimes, the probability of the occurrence of a certain scenario is taken into account, following the commonly used definition of risk (Kaplan and Garrick, 1981). Vulnerability of a railway network represents the decreased network performance $P(t_d)$ in a stable disrupted system state (see t_d in Fig. 1). So, the duration of the disruption, transition phases to the degraded service state as well as during recovery processes are not taken into account. It can be said that vulnerability is one of the (static) elements of resilience. For a further overview of different definitions of vulnerability, the reader may be referred to Mattsson and Jenelius (2015) and Yap et al. (2018).

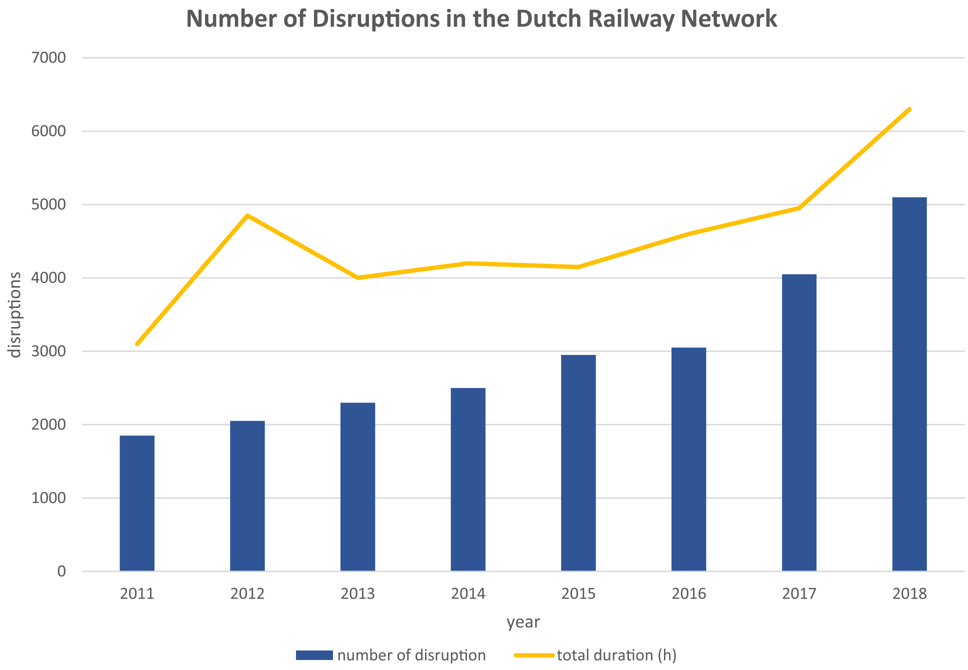

Fig. 2. Evolution of number and total duration of disruptions in the Dutch railway network, data source: https://www.rijdendetreinen.nl/en/statistics.

1.3. This paper

In this research, we are investigating the vulnerability of railway networks. Based on a typically sparse structure and lower spatial and temporal connectivity of railway networks, they are considered to be very dependent on a few critical network elements (Cats and Jenelius, 2014). Accordingly, their performance does not only depend on their physical topology, but also on their service characteristics (e.g. timetables, passenger demand). In addition, the number of disruptions (infrastructure failures, broken/blocked vehicles) in highly utilized networks such as the Dutch railway network has been steadily increasing in recent years (see Fig. 2). Therefore, the ability of providing disrupted services and their quality is an essential component for the vulnerability assessment of railway networks. From a modelling perspective, we consider a disruption to be a failure of one or multiple components in the network.

In this paper, we propose a new Railway Network Vulnerability Model (RNVM) to assess the vulnerability of the system by finding the critical combination of links which cause the most adverse consequences to passengers and trains. Vulnerability is measured based on total passenger travel cost, the number of disconnected passengers, i.e. passengers not being able to reach their destination, and also the cost for adjustments of train services. A disruption represents a complete link failure (single or multiple) and lasts the whole considered time period. During a disruption, trains may be cancelled, rerouted, short-turned or retimed; while passengers need to take alternative paths through the network following their shortest path in terms of travel time, and in some cases cannot even reach their destination. To solve this challenging problem, we present a RNVM framework, which combines two heuristics based on column generation and row generation with mixed integer linear programming, to efficiently model alternative passenger flows and infrastructure constraints. The developed framework provides the critical combination of links, the corresponding passenger flows, train routes and timetables.

We demonstrate the performance of the RNVM framework on the real-world instance of a part of the Dutch railway network, and show that critical links directly depend on the provided passenger demand and their overall impact on network performance can be even larger than that of the accumulated individual ones.

The main contributions of the paper are the following:

- A new passenger-centered vulnerability measure to capture both stranded passengers as well as rerouted ones due to disruptions,
- A new mathematical formulation for assessing vulnerability of passenger railway networks,
- Combination of three different network levels, i.e. infrastructure, train services and passengers,
- A new framework for solving RNVM combining column generation and row generation based heuristics,
- Real-life experiments on a part of Dutch railway network.

The remainder of the paper is structured as follows. Section 2 gives an overview of the existing approaches in the field of railway vulnerability assessment. Section 3 defines the problem of assessing the vulnerability of railway networks and Section 4 introduces a mathematical formulation of RNVM. Section 5 presents the RNVM assessment framework. The framework is applied on a case study in Section 6, where we demonstrate the framework's features, its general and computational performance and discuss our findings. Section 7 gives the final conclusions and future research directions. Finally, extended results of the experiments in Section 6 can be found in Appendix A.

2. Current research

In the following section, we give an overview about the current literature. First, we present the state-of-the-art modelling techniques for railway operations (Section 2.1). Second, we give an overview of the commonly used vulnerability assessment methods (Section 2.2) and approaches (Section 2.3). Third, we focus on optimization models for assessing vulnerability and summarize the different approaches highlighting the railway-related ones. (Section 2.4). Concluding the section, the gaps in current research are identified and the scope of our research is specified (Section 2.5).

2.1. Models for railway operations

For modelling railway operations, there are mainly three approaches. The most commonly used approach to model periodic timetabling is the Periodic Event Scheduling Problem (PESP), introduced by Serafini and Ukovich (1989). For aperiodic timetabling, the models rely on either a path- or an arc-based formulation of the train operations in the infrastructure network arcs, and it is mostly used in a time-space network framework (see e.g. Cacchiani et al., 2007 for a path-based approach and Carey and Lockwood, 1995 for an arc-based approach). Typical objectives are efficiency of the journey times, timetable feasibility, timetable stability, robustness and energy consumption (see Goverde et al., 2016). For a detailed overview of models and solution techniques, the reader may be referred to Caimi et al. (2017) and Cacchiani and Toth (2012).

2.2. Vulnerability assessment methods and metrics

The assessment of vulnerability in railways is done following one of the two methods: graph theory or system-based (Mattsson and Jenelius, 2015). The first is based on graph-topological measures, originating in theory of complex networks. Assuming a failure of single components in the disrupted network, the vulnerability is assessed by analyzing the structure of the systems graph model, disregarding dynamic effects on the performance within the system. The second approach follows a system-based analysis, which overcomes the limitation of the graph methods and represents the demand and supply of the system as well as responses to the disruption. The system-based methods focus on either measuring selected aspects for a specific operating state, or the performance for all operational states from the disruption until the complete recovery of the system (Taylor, 2017; Zhou et al., 2019). Commonly, the former is used for vulnerability assessment, while the latter is applied in the resilience context.

Multiple vulnerability measures are used in literature to capture different aspects. For example, to capture the importance of element measures such as accounting traffic flow, link capacities, travel times and route alternatives, weighted topological measures (e.g. betweenness or closeness) are used, while to measure the societal welfare(losses) – generalized cost indicators are applied. For a comprehensive introduction to measures, the reader may be referred to Mattsson and Jenelius (2015) and Zhou et al. (2019).

2.3. Vulnerability assessment approaches

The three mainly used approaches for assessing vulnerability of railway networks are topological, simulation and optimization. The topological ones use the metrics developed in complex network theory and are based on graph properties. They are then applied on different graph representations, mainly modelling either the PT infrastructure or the service network (see Berche et al., 2010, Zhang et al., 2015, Shanmukhappa et al., 2018). While removing single links either randomly or following a certain strategy, the evolution of the indicators is tracked and their vulnerability is analyzed. This follows the approach of complete enumeration and finds the critical (single) elements in the network. When it comes to the removal of multiple elements and the corresponding effects on the network, topological approaches suffer from prohibitively high computation times due to the exponential growth of combinations with increasing number of elements (Wang et al., 2016).

Simulation approaches usually use similar metrics as in the topological approach as well as PT performance indicators (e.g. delay, passenger loads) to evaluate the network performance in a stochastic environment. Most commonly, simulation describes the vulnerability evaluation based on theoretical and/or real-life disruption distributions and modelling the network impacts and system reactions accordingly. Such methods are used for e.g. identifying link vulnerability from a passenger perspective (Yap et al., 2018), evaluating network performance using a discrete event simulation (Meesit and Andrews, 2018) and assessing the supply-demand interactions in a dynamic stochastic setting given certain disruption scenarios (Cats and Jenelius, 2014). While being able to catch the dynamic effects of a disruption in a better way, these approaches still suffer from 1) the exponential growth of possible combinations when multi-component failures are considered, and 2) computationally demanding simulation models. To overcome this, heuristics procedures such as the evaluation of pre-defined scenarios (Cats and Jenelius, 2014) and pre-selection of promising candidate sets (Yap et al., 2018) are applied.

To overcome limitations of topological and simulation approaches, optimization approaches gain more attention due to their capability to solve extreme scenarios without the complete enumeration of all other scenarios. As scenarios, particularly including multiple simultaneous disruptions, with the greatest impact may not be easy to find and might be strongly

Table 1
Current optimization-based vulnerability assessment approaches.

Source	Vulnerability measure	Domain *(Railway Constraints)*	Disruption type
Murray et al. (2007)	Interdicted Flow	Generic/Internet network *(None)*	multiple nodes, complete failure
Peterson and Church (2008)	Additional impedance and distance	Freight railway *(Track capacity)*	single arc, complete failure
Babick (2009)	Network operating cost	Freight railway *(Track capacity)*	multiple arcs, complete failure
Santos et al. (2010)	Average increase of travel cost	Road network *(None)*	single element, complete failure
Lou and Zhang (2011)	Maximum system travel time, Unsatisfied demand, Transport capacity	Generic/Road network *(None)*	multiple arcs, complete and partial failure
Khaled et al. (2015)	Total (generalized) cost	Freight railway *(Track/station capacity, Train length/weight, No. of shunting stops, Flow-dependent travel times)*	single element, complete failure
Gedik et al. (2014)	Operating cost and delay	Freight railway *(Track/station capacity)*	multiple nodes, complete failure
Bababeik et al. (2017a)	Minimal routing (generalized) cost	Freight railway *(Track capacity)*	multiple arcs, complete failure
Wang et al. (2016)	Maximum travel time	Road network *(None)*	multiple arcs, complete failure
Whitman et al. (2017)	Unmet demand	Freight railway *(Commodity-specific/general track capacity)*	single arc, complete failure
This paper	Disconnected passengers, passenger detours	Passenger railway *(Track capacity, train capacity, timetable, short-turning, rerouting, cancellation)*	multiple arcs, complete failure

related to the system behaviour as a whole rather than single facilities, optimization models are even more applicable in the network context (Murray et al., 2008).

Beside these three most common approaches, there are probability theory models evaluating stochastic performance reductions or applying Bayesian networks for quantifying resilience as well as fuzzy logic and data driven models, which have been used (see Zhou et al., 2019).

2.4. Vulnerability assessment using optimization

The most common way to assess the element criticality in networks using optimization models is to use an interdiction approach, where one or multiple elements are interdicted, representing disruptions. Murray et al. (2007) use a path-based interdiction model to identify critical system infrastructure by analysing multiple disruptions and regarding the network flows. Peterson and Church (2008) use the Multi-Commodity-Flow (MCF) model including link capacities to asses a freight railway network before and after the removal of single links to assess network vulnerability. Babick (2009) introduces a modified MCF model to evaluate the critical infrastructure resilience. He develops a three-level framework to model mutual attacks on multiple links and defense reactions of a fictitious attacker and defender of the network, obeying the link capacities and allowing certain network modifications.

A Network Design Problem (NDP) approach is also used to model network modifications and assessing infrastructure resilience. Santos et al. (2010) tackles the NDP by simultaneously optimizing accessibility and different robustness measures of road networks, using a non-linear optimization model. Lou and Zhang (2011) extends the NDP to incorporate congestion aspects and assess impacts of different types of multiple disruptions such as capacity reductions and complete link closures. The routing of the network flows is computed using the user-equilibrium Transit Assignment Problem (TAP) approach. Khaled et al. (2015) presented a detailed complex model to evaluate the criticality of railway infrastructures using the single-element removal approach. Vulnerability is analysed based on the increased cost per disrupted element. Freight trains are rerouted in the network, according to the disruptions. The complexity of the model results from considering multiple important, railway-specific operational constraints such as link and station capacity, length and weight of the trains, congestion, coupling and decoupling processes.

Gedik et al. (2014) uses the interdiction model to analyse the failure of multiple elements and assess the most critical combination of disruptions. The rerouting of the train flows in the disrupted network is done by using a dynamic network formulation, considering capacity restrictions and congestion. Bababeik et al. (2017a) applies a bi-level interdiction problem, using the minimum cost flow problem with a partial enumeration of network components which is combined with an additional time-space network model to retime trains. Bababeik et al. (2017b) proposes a heuristic to analyse critical multi-link blockages, considering capacity constraints and congestion effects.

Following a different approach, Wang et al. (2016) develops an equilibrium-based TAP, regarding flow dependent travel times, to identify the critical combination of link closures in the network. Whitman et al. (2017) analyses the single element removal using the MCF model. The model is able to compare the vulnerability over different commodities.

Table 1 gives an overview of used approaches in transport domain, indicating railway applications, vulnerability measures, considered disruptions and in particular highlights the contributions of our paper.

2.5. Gap

The current railway-related optimization papers on vulnerability assessment are mostly motivated by solving economical/freight-related problems and thus, only train routing and station/track capacity has been introduced, while rescheduling measures such as short-turning, cancelling and rescheduling freight trains (except Bababeik et al., 2017b) has not been addressed. Instead, in passenger railway networks, trains operate according to strictly defined timetables, and thus, timetabling aspects as well as rescheduling measures need to be considered, which make the problem significantly more complex to tackle. In addition, passengers routing through the network can be independent of scheduled trains, so to evaluate disruption effects from passenger perspective, passenger transport demand needs to be addressed. To the best of authors knowledge, there are no optimization-based passenger-centered vulnerability measures and studies in the railway context. The RNVM follows a system-based approach that uses mathematical optimization modelling and the vulnerability measure of unsatisfied passenger demand and passenger rerouting costs. Such measure represents the impacts and effort to cope with disruptions for a given number of disrupted links, while still ensuring reasonable passenger and train routes.

3. Problem description

In our problem, the vulnerability of railway networks is assessed by determining the most critical combination of multiple links. Critical links are considered to be the links which cause most adverse consequences in terms of network performance. In order to capture the network performance, three perspectives are taken into account: the perspective of infrastructure manager, subsequently considered as topological/physical network, the perspective of the railway operator, subsequently considered as train services and the perspective of the passengers, subsequently considered as passenger network. In this section, the problem and the measures are introduced. In the beginning, the underlying infrastructure network and the formulation of disruptions is introduced (Section 3.1). Then, the train services formulation (Section 3.2), the passenger network formulation (Section 3.3) and the objective function of our problem (Section 3.4) are presented. Finally, the notation is summarized in a notational glossary (Section 3.5).

Fig. 3 shows the interdependent structure of the problem where for each network level an example layer is provided. The bottom layer represents the complete infrastructure network. The middle layer shows the route for two trains on the simplified infrastructure network (trains are given in red and blue with corresponding origin and destination, traversed stations in black, the train routes are displayed in solid lines). The top layer shows the passenger paths for a certain Origin-Destination (OD) pair, including two different paths (origin and destination are shown in red, traversed stations in black, the paths are displayed in bold lines).

3.1. Infrastructure network and disruptions

The infrastructure network is the base of our model as it represents the physical infrastructure where train services are running and passengers are transported on. It is modelled using a graph $G^I = (N, A)$ consisting of nodes N and undirected arcs A. The nodes represent stations in the railway network and arcs represent the railway corridors which connect the stations. Each arc has a capacity CAP_{ij}, representing the maximum operable number of trains on the arc per time-period. This parameter represents an upper bound of the arcs capacity, which in reality is usually achieved by operating homogeneous services.

To model the disruption of an arc, this capacity concept is used. The binary decision variable v_{ij} sets the arc capacity to zero, if an arc is disrupted ($v_{ij} = 1$). Otherwise, the undisrupted arc capacity is available ($v_{ij} = 0$). Only complete disruptions of arcs are considered. The parameter n limits the number of simultaneous disruptions. Furthermore, the parameter $h_{ij}^{t,m}$ represents the safety restrictions of the infrastructure as the minimal headway time on arc $(i, j) \in A$ between two successive trains $t, m \in T$.

3.2. Train services

Train services are modelled by a service network and the corresponding timetable. The service network is modelled by routing trains $t \in T$ on the infrastructure network G^I. A train route consists of the traversed arcs and nodes connecting one origin and one destination node. The subsets A^t and N^t represent the arcs and nodes, each train's route is originally scheduled along, in the order of traversing. The origin and destination nodes $N(O_t)$ and $N(D_t)$ are used to model the terminals of each train. The trains are modelled as arc-based flows on the infrastructure network. Each train is represented by the binary decision variable x_{ij}^t, modelling the flow of train $t \in T$ on arc $(i, j) \in A$: if the train is using the corresponding arc $x_{ij}^t = 1$, else $x_{ij}^t = 0$. In that way, a train route of train $t \in T$ can be defined as the set of arcs $(i, j) \in A$ with $x_{ij}^t = 1$.

For modelling rescheduling responses of the operators to disruptions, the following four measures are considered: rerouting, short-turning, cancellation and retiming.

To model rerouting and short-turning, different train types are distinguished. The subset $T^{RR} \subset T$ consists of trains which can be rerouted. The subset $T^{ST} \subset T$ contains trains which can be short-turned. The two sets T^{RR} and T^{ST} are disjoint, meaning that trains which are considered to be rerouted cannot be short-turned and vice versa. In practice, long distance trains such

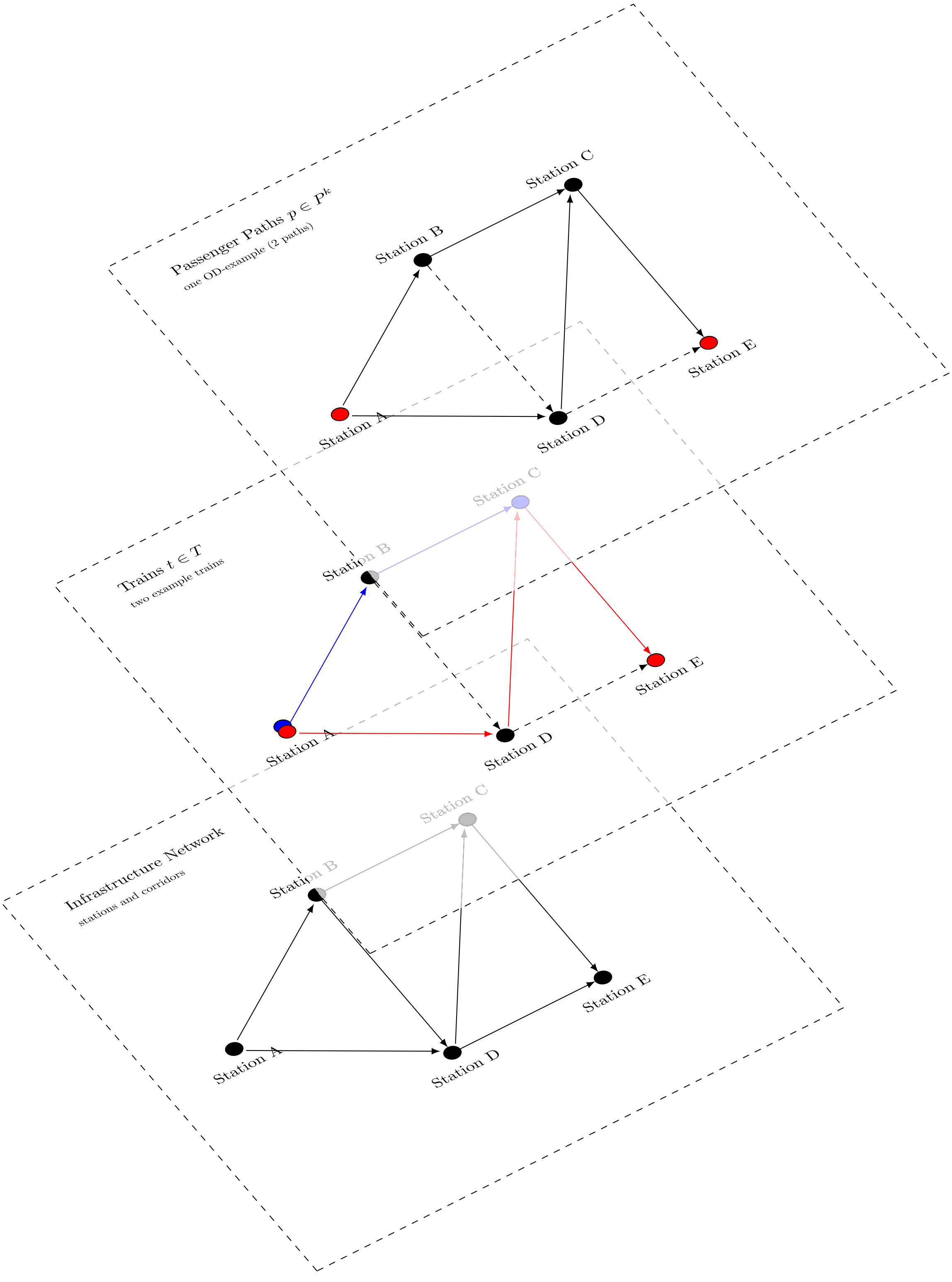

Fig. 3. Scheme of the different modelling levels.

as freight or international trains may be rerouted in case of disruptions as their main objective is to reach their destination, e.g. port, marshalling yard or border crossing, while the exact route is less relevant. Instead, for passenger local and intercity trains, it is more important to operate on their route in order to serve the existing passenger demand during a disruption as much as possible. In that case, rerouting is prohibited, and trains can only be short-turned (close to the disrupted link).

The cancellation of trains is always possible. To model rerouting and short-turning, the decision variables x^t_{ij} for $t \in T^{RR}$ are modelled for the whole network $(i, j) \in A$ to allow network-wide rerouting, whilst for $t \in T^{ST}$, x^t_{ij} is only modelled for $(i, j) \in A^t$ to limit to predetermined train routes. The binary decision variables o^t_i and d^t_i model the actual origins and destinations of train $t \in T$. If train t originates (terminates) in node $i \in N^t$, $o^t_i = 1$ ($d^t_i = 1$), else $o^t_i = 0$ ($d^t_i = 0$). In order to allow train origins and destinations only in nodes of the original train route, these decision variables are only introduced for the nodes $i \in N^t$.

To perform rerouting, the fastest undisrupted route is searched in the network. The parameter c^t_{ij} is used to model the routing cost between nodes $i, j \in N$, representing the cost per minute of running time of train $t \in T^{RR}$. In order to avoid unrealistically long routes, the trains can get cancelled as well. The parameter c^t_{cancel} represents the cancelling cost of train $t \in T^{RR}$. If this parameter is set to a certain maximum amount of train cost, it will limit the length of the alternative routes for the corresponding train. If this maximum cost is exceeded, the train will be rather cancelled than rerouted.

The short-turning is based on the disruption v_{ij} of links $(i, j) \in A$. The trains $t \in T^{ST}$ will be short-turned due to disrupted links and will change their actual origins and destinations o^t_i/d^t_i accordingly. Assuming an operator's behaviour, targeting for services as similar to the undisrupted services as possible, the operating cost for short-turned trains are not taken into account. Furthermore, we assume spare vehicles to be available in order to operate the potentially emerging multiple parts of the former single train route.

In order to model feasible railway operations in the disrupted network, retiming is performed. Therefore, a feasible timetable is needed as an input. The parameters $T^t_{D,i}$ represent the departure/through times of train $t \in T$ at node $i \in N$ of the input timetable. The parameter t^t_i captures the scheduled (or minimal) dwell time of train $t \in T$ at station $i \in N$. In order to enable different train types and speeds, the arc weights, representing the running times, are train specific; representing the running time τ^t_{ij} between the nodes $i, j \in N$ of train $t \in T$. The retiming is mainly used to ensure the feasibility of the disrupted operations e.g. obeying the headway times/arc occupation and minimal dwell times. The decision variables $T^t_{\tilde{A},i}$ and $T^t_{\tilde{D},i}$ represent the retimed arrival and departure times of train $t \in T$ in node $i \in N$ respectively.

3.3. Passenger network

The passenger network is modelled by routing passenger flows $k \in K$ on corresponding paths $p \in P^k$ on the network. The determination of paths allows for determining passenger detours as well as completely disconnected OD relations. A passenger path is a sequence of nodes $n \in N$ between the origin and the destination of the passenger flow $k \in K$. The parameter d_k represents the corresponding demand of passengers.

The passengers are routed according to the train capacity in the service network. The parameter s^t models the number of seats, available in train $t \in T$, which can be used to model the actually provided service capacity. The passenger flows are represented by the decision variable f^k_p as the demand share of the total demand of path $p \in P^k$ of the OD pair k. This variable represents the actual passenger flows in the network. The parameter c^k_p, containing the travel times per arc of path p, allows to calculate the travelling costs per passenger flow. The decision variable y^k_p is introduced for detecting if a path $p \in P^k$ is impacted (or remains unaffected) by the networks disruption. If a path p is unaffected, $y^k_p = 1$, else $y^k_p = 0$. For considering the unsatisfied demand, the decision variable z_k is used to determine if the corresponding OD pair k is disconnected ($z_k = 1$) or remains connected ($z_k = 0$). The parameter c_k models the cost per disconnected passenger on OD relation $k \in K$.

For routing the passengers, the application of shortest path routing is assumed. This is based on the assumption, that the passenger flows during a disruption can be influenced by the railway operator, which aims for the shortest path for the passengers within the network, regarding operational and capacity constraints. This assumption can be justified, regarding the fact that in case of disruptions, the operator usually has – unlike the passengers – (almost) complete information of the network, its states and available remaining connections within. Therefore, using real-time information to inform the passengers (e.g. apps, announcements at platforms and in trains), the passengers will follow the given advice as they trust and rely on the information provided by the operator. This results in the use of a simplistic shortest (available) path routing.

3.4. Objective

The model idea for detecting the critical links is based on the combination of the approaches of Murray et al. (2007) and Shen et al. (2013) which are doing so by measuring the impact regarding the disconnection and detours of trains in the network. For measuring these consequences in our case, passenger related effects are used: the number of disconnected passengers and the accumulated passenger travel cost. Therefore, the links to be identified will maximize the number of disconnected passengers and the passengers cost to reach their destinations accordingly.

Table 2
Notation overview.

Symbol		Description	Symbol		Description
		Sets Passengers			**Sets Trains**
K	:	set of OD pairs	N^t	:	set of nodes on train route t
P	:	set of passenger paths	$N(O_t)$	:	origin node of train route t
P^k	:	set of paths of the OD pair	$N(D_t)$	:	destination node of train route t
A^p	:	set of arcs contained in path p	A^t	:	set of arcs on train route t
$N(O_k)$	:	origin node of OD pair k	T	:	set of all trains
			T^{RR}	:	subset of trains which can be rerouted
			T^{ST}	:	subset of trains, which can be short-turned
		Parameters Passengers			**Parameters Trains**
s^t	:	number of seats per train t	c^t	:	cost per seat of train t
c_p^k	:	cost of path p for passenger flow k	c_t	:	cost per minute of time
d_k	:	demand of OD pair k	t_i^s	:	dwell time of train s in node i
δ_{ij}^p	:	$\begin{cases} 1, \text{ arc } (i,j) \text{ is included in path } p. \\ 0, \text{ else} \end{cases}$	τ_{ij}^t	:	minimum running time on link (i, j) of train t
			$T_{D,i}^t$	:	scheduled departure time of train t in node i
			$T_{A,i}^t$	:	scheduled arrival time of train t in node i
			M	:	sufficiently large value
		Decision Variables Passengers			**Decision Variables Trains**
y_p^k	:	$\begin{cases} 1, \text{ path } p \text{ of train } t \text{ is unaffected} \\ 0, \text{ else} \end{cases}$	o_i^t	:	$\begin{cases} 1, \text{ if node } i \text{ is origin of train } t \\ 0, \text{ else} \end{cases}$
z_k	:	$\begin{cases} 1, \text{ OD-pair } k \text{ is disconnected.} \\ 0, \text{ else} \end{cases}$	d_i^t	:	$\begin{cases} 1, \text{ if node } i \text{ is the destination of train } t \\ 0, \text{ else} \end{cases}$
f_p^k	:	share of the demand of OD pair k, transported via path p	x_{ij}^t	:	train flow from node i to node j of train t
			$\tilde{T}_{D,i}^t$	:	actual departure time of train t in node i
			$\tilde{T}_{A,i}^t$	:	actual arrival time of train t in node i
					Sets Network
			G^I	:	infrastructure network
			N	:	set of nodes
			A	:	set of arcs
					Parameters Network
			CAP_{ij}	:	capacity in arc (i, j)
			$h_{ij}^{t,m}$	:	headway time between train t and m on arc (i, j)
					Decision Variables Network
			v_{ij}	:	$\begin{cases} 1, \text{ if link } (i,j) \text{ is disrupted} \\ 0, \text{ else} \end{cases}$

At the same time, the formulation also needs to capture realistic reactions of passengers and operators during disruptions in order to realistically measure the disruption impact. Thereof, the model simultaneously needs to minimize the passenger travel cost, resulting in reasonable passenger paths, and to minimize rerouting and cancellation cost of the trains.

These aspects can be achieved by using different terms in the objective, each standing for a separate objective. The variables z_k and y_p^k are used for the maximisation of the disruption consequences. The variables f_p^k, x_{ij}^t and d_i^t are used to achieve the realistic passenger and operator behaviour.

3.5. Notation

Table 2 summarizes the notations introduced in this section.

4. Model development

In this section, a new mathematical formulation – the Railway Network Vulnerability Model (RNVM) – is developed to find the most critical combination of links in the railway network. The passenger- and train-related effects of a disruption are explicitly captured by modelling the passenger travel times via rerouting, cancelling or short-turning the trains and also considering rescheduling. In order to deal with the specific properties of railway- and passenger-related effects and also taking their adaptations into account, the model relies on two different network flows: the train flows x_{ij}^t based on the infrastructure and train services to capture railway performance aspect, and the passenger flows f_p^k to account for the passenger impacts (see Fig. 3). For our modelling, the following assumptions hold:

- Every stop in the network can be used for short-turning of trains and provides enough capacity to allow as much short-turns as necessary,
- The original timetable is feasible and does not exceed capacities or violate other operational constraints,
- Trains are short-turned only if they are directly affected by a disruption,
- Passengers are routed along the shortest paths within the network.

The RNVM formulation consists of two building parts. It combines arc-based and path-based formulations to model network infrastructure, train services and passenger flows. First, the train services and the underlying infrastructure are closely related and highly interdependent; therefore, they are modelled together applying an arc-based formulation where trains are modelled as flows in an infrastructure network. Second, the passenger flows depend on the resulting provided capacities (i.e. trains running in the network); thus, they are modelled using a path-based formulation. This formulation could be solved more efficiently, when expecting a large number of alternative passenger paths for real-life problem instances. Related to Fig. 3, the infrastructure and service levels are aggregated into the first building part, and the passenger level into the second one. Section 4.1 presents modelling of infrastructure and train services. Section 4.2 introduces passenger modelling. Section 4.3 gives a mathematical formulation of the objective function.

4.1. Arc-based formulation of the infrastructure network and train services

The trains in the network are modelled using an arc-based MCF formulation. Hence, the trains are routed as flows x_{ij}^t between the nodes $i, j \in N$ in the infrastructure network. In order to enable short-turning (partial cancellation) and rerouting as an operators reaction to disruptions, this way of modelling has been chosen as the arc-based structure seems to represent the needs of these, locally and globally applied, measures in a better way than a path-based formulation. Arc-based formulation further enables an easy implementation of rerouting in railway networks, which is generally not the case in event-time network models. Unfortunately, this also causes a large number of constraints for all possible flow combinations per each arc, which significantly increases the size of the problem, when it comes to scheduling constraints. The train routing, cancelling and short-turning part of the model then reads as:

$$\sum_{j \in N} x_{ij}^t - \sum_{j \in N} x_{ji}^t = \begin{cases} -o_i^t, & \text{if node i is a starting node} \\ d_i^t, & \text{if node i is an end node} \\ 0, & \text{else} \end{cases} \qquad \forall t \in T, i \in N^t \tag{1}$$

$$\sum_{i \in N^t} o_i^t = \sum_{i \in N^t} d_i^t \qquad \forall t \in T \tag{2}$$

$$o_i^t = 1 \qquad \forall t \in T, i = N(O_t) \tag{3}$$

$$d_i^t = 1 \qquad \forall t \in T, i = N(D_t) \tag{4}$$

$$o_i^t = 0 \qquad \forall t \in T^{RR}, i \neq N(O_t), N(D_t) \tag{5}$$

$$d_i^t = 0 \qquad \forall t \in T^{RR}, i \neq N(O_t), N(D_t) \tag{6}$$

$$o_j^t \geq v_{ij} \qquad \forall t \in T^{ST}, i \in N^t, (i, j) \in A^t \tag{7}$$

$$d_i^t \geq v_{ij} \qquad \forall t \in T^{ST}, j \in N^t, (i, j) \in A^t \tag{8}$$

$$\sum_{i \in N^t} o_i^t \leq \sum_{(i,j) \in A^t} v_{ij} + 1 \qquad \forall t \in T \tag{9}$$

$$\sum_{j \in N^t} d_j^t \leq \sum_{(i,j) \in A^t} v_{ij} + 1 \qquad \forall t \in T \tag{10}$$

$$\sum_{t \in T} x_{ij}^t \leq CAP_{ij}(1 - v_{ij}) \qquad \forall (i, j) \in A^t \tag{11}$$

$$\sum_{(i,j) \in A} v_{ij} \leq n \tag{12}$$

$$x_{ij}^t, o_i^t, d_i^t, v_{ij} \in \{0, 1\} \qquad \forall t \in T, n \in N^t \tag{13}$$

Eq. (1) ensures the flow continuity and that the trains are only allowed to start or end at origin and destination nodes o_i^t and d_i^t, respectively. Eq. (2) ensures that there are always as many origins as destinations on each train route. Eq. (3) and Eq. (4) ensure that the originally scheduled origins and destinations at the terminals of the trains are always kept as sources and sinks for the train flows in order to maintain the original train services in the undisrupted case. Eqs. (5) and (6) prevent rerouted trains of being short turned. Eq. (7) and (8) link the short-turn location selection to the disruption of a link. If a link is disrupted, trains are forced to short turn at the station right next to the disrupted link. This constraint deals with short-turning due to disrupted links in the original train route only. Eq. (9) and (10) ensure that trains are only short-turning according to the number of disrupted links at the original route, in order to prevent unnecessary short-turning, e.g. on undisrupted part of a corridor. Additionally, it allows the existence of at least one origin and destination for representing the undisrupted state. Eq. (11) guarantees that the train flows on each link do not exceed the link capacity. The disruption of a link is modelled by setting its capacity to zero. Eq. (12) limits the maximum number of simultaneous disruptions. Eq. (13) sets the range of the decision variables of the problem.

In order to create an adjusted feasible timetable for the disrupted network and also capture the passenger impacts of a disruption more precisely, additional constraints are needed to model the timetable of the introduced trains (Eqs. (14)–(20)). Therefore, departure and arrival times of each train are defined. Obeying the original scheduled departure (arrival) times $T_{D,i}^t$ ($T_{A,i}^t$), the trains are retimed for all stations i, while not allowing early departures. The timetable is considered to be non-periodic. The RNVM model can be adapted to consider periodic timetables, as it is not essential for vulnerability assessment. The timetabling constraints are listed below:

$$T_{\tilde{D},i}^t + \tau_{ij}^t \leq T_{\tilde{A},j}^t + M(1 - x_{ij}^t) \qquad \forall t \in T^{RR}, (i,j) \in A \tag{14}$$

$$T_{\tilde{D},i}^t + \tau_{ij}^t x_{ij}^t \leq T_{\tilde{A},j}^t \qquad \forall t \in T^{ST}, (i,j) \in A^t \tag{15}$$

$$T_{\tilde{D},i}^t \geq \sum_{j \in N} T_{D,i}^t x_{ij}^t \qquad \forall t \in T, i \in N^t \tag{16}$$

$$T_{\tilde{D},i}^t - T_{\tilde{A},i}^t \geq t_i^t - M \cdot (1 - x_{ij}^t) \qquad \forall t \in T^{RR}, (i,j) \in A \tag{17}$$

$$T_{\tilde{D},i}^t - T_{\tilde{A},i}^t \geq t_i^t x_{ij}^t \qquad \forall t \in T^{ST}, (i,j) \in A^t \tag{18}$$

$$T_{\tilde{D},i}^t \geq T_{\tilde{D},i}^m + h_{ij}^{t,m} - M(1 - x_{ij}^t) - M(1 - x_{ij}^m) \qquad \forall t \in T, m \in T, i \in N^t, m \neq t \tag{19}$$

$$T_{\tilde{A},i}, T_{\tilde{D},i} \geq 0 \qquad \forall t \in T, i \in N^t \tag{20}$$

Eqs. (14) and (15) guarantee the minimal running times between two stations for rerouting and short-turning trains in case of train operations on link (i, j). Eq. (16) ensures that the actual departure cannot be rescheduled earlier than the originally scheduled one. The dwell time for rerouted and short-turned trains is constrained in Eqs. (17) and (18), respectively, which only needs to hold if there are trains running. The minimal headway times are ensured in Eq. (19), only if both involved trains are running. Eq. (20) guarantees the non-negativity of all time instances.

4.2. Path-based formulation of the passenger flows

In order to route the passenger flows, we use a path-based formulation. For given passenger paths P^k per OD pair k, passengers are routed, i.e. assigned, to trains running in the disrupted network. The constraints for modelling the passenger flows are shown below:

$$\sum_{p \in P^k} y_p^k + z_k \geq 1 \qquad \forall k \in K \tag{21}$$

$$z_k \leq 1 - y_p^k \qquad \forall k \in K, p \in P^k \tag{22}$$

$$y_p^k \geq 1 - \sum_{(i,j) \in A^p} v_{ij} \qquad \forall k \in K, p \in P^k \tag{23}$$

$$\sum_{k \in K} \sum_{p \in P^k} \delta_{i,j}^p y_p^k \leq \sum_{t \in T} M \cdot x_{ij}^t \qquad (i,j) \in A \tag{24}$$

$$\sum_{p\in P^k} f_p^k \le (1 - z_k) \qquad \forall k \in K \tag{25}$$

$$\sum_{k\in K}\sum_{p\in P^k} \delta_{i,j}^p d_k f_p^k \le \sum_{t\in T} s^t x_{ij}^t \qquad \forall (i,j) \in A \tag{26}$$

$$z_k, y_p^k = \{0, 1\} \qquad \forall k \in K, p \in P^k \tag{27}$$

$$f_p^k = [0, 1] \qquad \forall k \in K, p \in P^k \tag{28}$$

The OD disconnection of an OD pair k is represented by the decision variable z_k. Eqs. (21) and (22) define that a passenger flow can only be disconnected by disruptions, if all of its paths are affected. If any path remains unaffected, the OD pair can not be disconnected, and passengers may still be rerouted. Eqs. (23) and (24) state that a path can only be affected, if at least one of the used train arcs is disrupted or no trains are running on it, respectively. Eq. (25) ensures that if the passenger flows are disconnected, all path shares for each disconnected OD pair need to be zero, representing the disconnection also in the flow perspective. Eq. (26) models the actual capacity on an arc (provided by the arc-based train flow part) limits the cumulative passenger flows at each arc. The parameter $\delta_{i,j}^p$ is used to determine whether the path $p \in P^k$ of passenger flow $k \in K$ traverses the arc. Eqs. (27) and (28) restrict the range of the decision variables.

4.3. Objective function

The objective function for solving our multi-objective problem contains three terms: first, for passengers disconnections, second, for passengers detours (when disrupting the network) and third, for adapting passengers behaviour and railway services accordingly. Of these terms, the first two are dominant.

The first and the second term are of the disrupting type. As defined in Section 3.4, they are maximising the adverse impact of the disruptions by maximizing the number of disconnected passengers and the passenger cost to reach their destinations. First, the disconnection cost C_z^k is calculated in Eq. (29), regarding the disconnected passenger demand d_k and a certain amount of disconnection cost represented by the factor c_k. Second, the passengers travel cost is calculated based on the passengers using unaffected passenger paths in the network. Therefore, the passenger flows to be disrupted need to be estimated. This estimation is based on the logit model of probabilistic route choice (see Lohse and Schnabel, 2011) using the travel times c_p^k of a path and the parameter β. Although the estimated paths may not be independent and disjoint alternatives, the caused error using the logit model is small enough to allow good estimates of the travel time dependent flows in the network. The passenger travel cost $C_y^{p,k}$ is obtained calculating the travel cost of the estimated demand d_k. The applied formulation is shown in Eq. (30).

The third term is used to ensure reasonable disruption adapting reactions, which consist of the rerouting of passengers and trains according to the available shortest paths and train cancellation. It consists of the three sub-terms. The first sub-term is used to ensure that passengers are routed as much as possible along the shortest available paths. In order to do so, the factor $C_f^{p,k}$ is calculated, using the inverted travel cost $\frac{1}{c_p^k}$. This ensures that the shortest paths are preferred over longer ones. Since the corresponding decision variable represents the demand shares, the OD demand d_k needs to be included as well. The resulting generalized routing cost $C_f^{p,k}$ is shown in Eq. (31). In order to simplify routing, the travel cost c_p^k is calculated per traversed arc using averaged travel cost over all passenger trains running on this arc. This allows to connect passenger paths to served relations rather than single trains. The second sub-term is used to ensure reasonable routing of the rerouted trains by minimizing the routing cost. To do so, the generalized rerouting cost $C_x^{i,j,t}$ is calculated using the train-related travel times per arc t_{ij}^t and the corresponding cost per time unit c_t. The resulting generalized rerouting cost are shown in Eq. (32). The last sub-term is used to limit the rerouting cost to a certain threshold. Above this threshold, the rerouted train will be cancelled. To do so, the generalized cancellation cost C_c^t is calculated based on a certain detour factor u and the cost of the originally scheduled train route c_{sched}^t. The calculation of the generalized cancellation cost is shown in Eq. (33).

$$C_z^k = c_k d_k \tag{29}$$

$$C_y^{p,k} = \frac{e^{-\beta \cdot c_p^k}}{\sum_{p\in P^k} e^{-\beta \cdot c_p^k}} d_k \tag{30}$$

$$C_f^{p,k} = \frac{1}{c_p^k} d_k \tag{31}$$

$$C_x^{i,j,t} = c_t t_{ij}^t \tag{32}$$

$$C_c^t = u \cdot c_{sched}^t \tag{33}$$

As the objective goal is maximization, the model preferably tries to disconnect OD pairs and/or deactivate the shortest and (estimated) frequently used paths in order to maximize the objective. Simultaneously, passengers will be routed on the shortest paths as the inverted cost per passenger flow are maximized, while the train rerouting and cancellation cost are minimized. In order to obtain pareto solutions in terms of disconnecting and detouring passengers, the two first terms are weighted using the factor λ and the term $1 - \lambda$ respectively. In order to find solutions which are dominated by the disrupting effects, the last term is weighted by a factor ω, which needs to be set regarding $\omega << \min\{\lambda, (1-\lambda)\}$.

The objective function for RNVM results then in:

$$\max \lambda \sum_{k\in K} C_z^k z_k - (1-\lambda) \sum_{k\in K} \sum_{p\in P^k} C_y^{p,k} y_p^k + \omega \Big[\sum_{k\in K} \sum_{p\in P^k} C_f^{p,k} f_p^k - \sum_{t\in T^{RR}} \sum_{i\in N} \sum_{j\in N} C_x^{i,j,t} x_{ij}^t - \sum_{t\in T^{RR}} C_c^t d_{N(O_t)} \Big] \tag{34}$$

5. Solution framework

We introduce a RNVM solution framework for tackling large-scale challenges of real-life railway networks. First, passengers typically have extremely large numbers of travelling path alternatives. And so, considering them all in the RNVM at once may render the problem impossible to solve. In addition, many of those alternative paths may not be actually used due to their unattractiveness, e.g. represent excessively long detours. Second, large-scale railway problems can have a great number of headway constraints to satisfy infrastructure dependencies to guarantee operational feasibility of computed solutions. As we start with an existing timetable that satisfies all headway constraints already, when adjusting train services due to disrupted critical links, only a limited number of extra headway constraints may be needed additionally. In order to keep our model computationally solvable, we develop a two component heuristics framework to efficiently handle 1) a big number of passenger paths and 2) a great amount of headway constraints. Within the first component, we propose a column generation inspired heuristic to identify new beneficial passenger paths that can be added iteratively to the optimization model (Section 5.1). The output of this component is the ideal RNVM solution, containing all beneficial paths. Within the second component, we introduce a row generation based heuristic to achieve the operational feasibility by adding headway constraints to prevent eventual timetabling conflicts (Section 5.2). The output of this component is the real RNVM solution. The final output of the RNVM framework is the most critical combination of disrupted links, the actual passenger paths and flows and the feasible adjusted train routes and timetable.

5.1. Heuristic for generating new passenger paths

For generating new passenger paths, a heuristic is used to obtain and evaluate the paths to be added. The procedure follows a column generation approach, which allows to tackle large scale problems without considering all (probably unnecessary) decision variables. Therefore the problem is solved applying two optimization problems. The master optimization problem is restricted to a feasible initial solution, containing only a subset of all possible decision variables. The pricing problem is based on a relaxed optimization problem, which is used to identify the variables (columns) to be added in order to improve the solution of the master problem. This procedure is executed until no new variables can be found by the pricing problem (see Lusby et al., 2012). In our case the master problem is the RNVM, defined by Eq. (1)–(34). Since we only consider a certain amount of paths, the corresponding problem is called the restricted master problem and it contains a subset $\bar{P}^k$ of all possible passenger paths P^k. For applying column generation, we also need to relax the master problem. After solving the relaxed version, the results are used to feed the pricing problem in order to identify the passenger paths to be added. The component's details are shown in Fig. 4 and explained below.

1. **Initialize Model:** The model is initialized by assigning every OD pair the shortest path between its origin and its destination on the undisrupted network.
2. **Solve relaxed RNVM:** In order to perform the pricing, the introduced model needs to be modified. Hence the RNVM needs to be relaxed to obtain a linear problem: the so called relaxed RNVM. The relaxation is done by relaxing Eq. (27):

$$z_k, y_p^k \in [0,1] \qquad \forall k \in K, p \in \bar{P}^k, t \in T, (i,j) \in A \tag{35}$$

The relaxed master problem is now defined by Eqs. (1)-(26), Eqs. (34) and (35). The important constraints for adding new paths and their corresponding dual variable are shown in Table 3. By solving the relaxed RNVM the values of the dual variables are obtained. These are used to calculate reduced cost on which the identification of new path candidates is based.

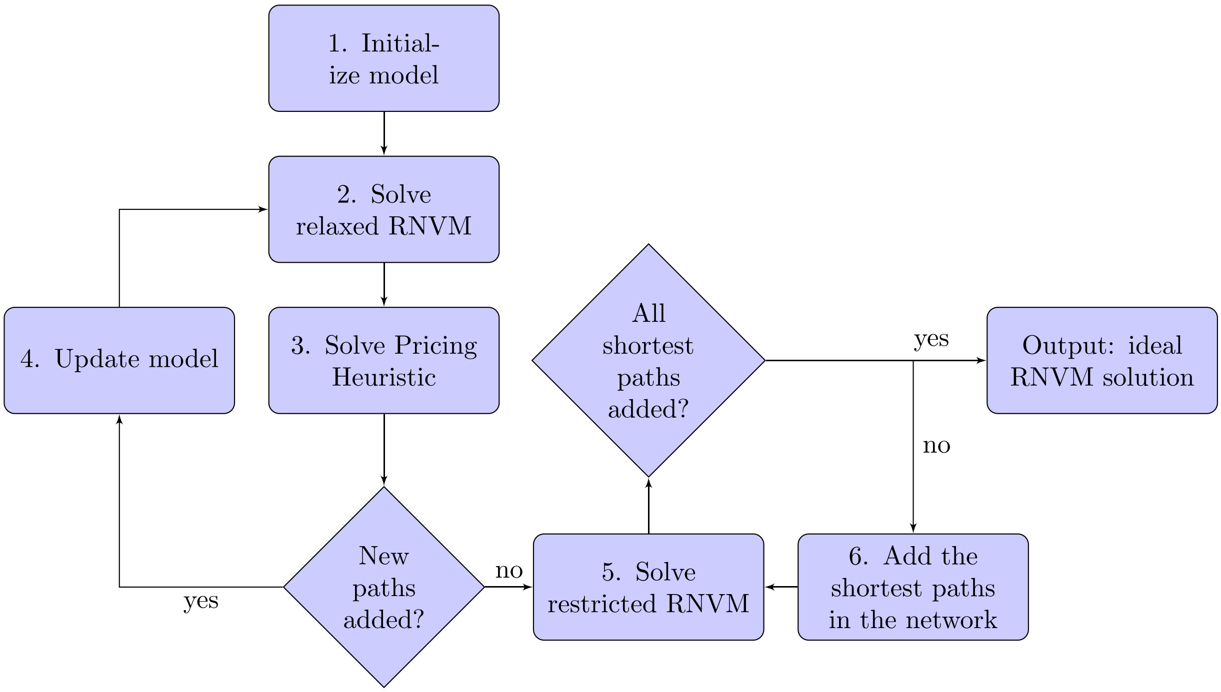

Fig. 4. Heuristic for generating new passenger paths.

Table 3
Constraints and dual variables of the relaxed RNVM.

Constraint	Dual variable
25	σ_k
26	π_{ij}

3. **Pricing Heuristic:** For finding the new passenger paths, we use the dualisation of the relaxed master problem for generating “new routes”, i.e. reactions on the disruptions. Routing in a modified service network (where currently trains are running), results in potential new path candidates which can be added. The calculation of the reduced cost for the identification of potential candidates is given as:

$$\sum_{(i,j)\in A} \delta^p_{i,j}\pi_{ij} + \sigma_k - \frac{\omega}{c^k_p} \cdot d_k \geq 0 \qquad \forall k \in K \tag{36}$$

Eq. (36) is used to assess the quality of potential path candidates. In order to find a potential new beneficial path, this dual constraint needs to be violated. Thus, if the reduced cost of the path candidate is negative, it is added to $\bar{P}^k$. Recall that to search for shortest paths per OD pair $k \in K$ in the network (due to the passenger routing), the path lengths are minimized in the objective function of the primal problem (see Eq. (34)). For finding the most promising candidates, the strongest violation of the dual constraint Eq. (36) needs to be found. Therefore, the left-hand-side of Eq. (36), i.e. the reduced cost, has to be minimized. This can be formulated as:

$$min\Big[\sum_{(i,j)\in A} \delta^p_{i,j}\pi_{ij} + \sigma_k - \frac{\omega}{c^k_p} \cdot d_k\Big] \qquad \forall k \in K \tag{37}$$

Hence, the shortest path in terms of $\sum_{(i,j)\in A} \delta^p_{i,j}\pi_{ij}$ and $\frac{1}{c^k_p}$ needs to be found. Since the path costs c^k_p are inverted, the reduced cost for finding new paths needs to be estimated. The shortest path is calculated in a network with modified arc weights $\bar{c_{ij}}$, which are used as an estimate for finding the constraint violation ones for Eq. (37), regarding the values of the dual variables and the requirements of reasonable passenger routing, while the unserved arcs are additionally deleted to respect the actual train flows, to prevent paths of running on arcs without train services:

$$\bar{c_{ij}} = \begin{cases} 0 & ; \ \sum_{t\in T} x^t_{ij} = 0, \forall (i,j) \in A \\ c_{ij}\frac{(1+\pi_{ij})}{\omega} & ; \ \sum_{t\in T} x^t_{ij} > 0, \forall (i,j) \in A \end{cases} \tag{38}$$

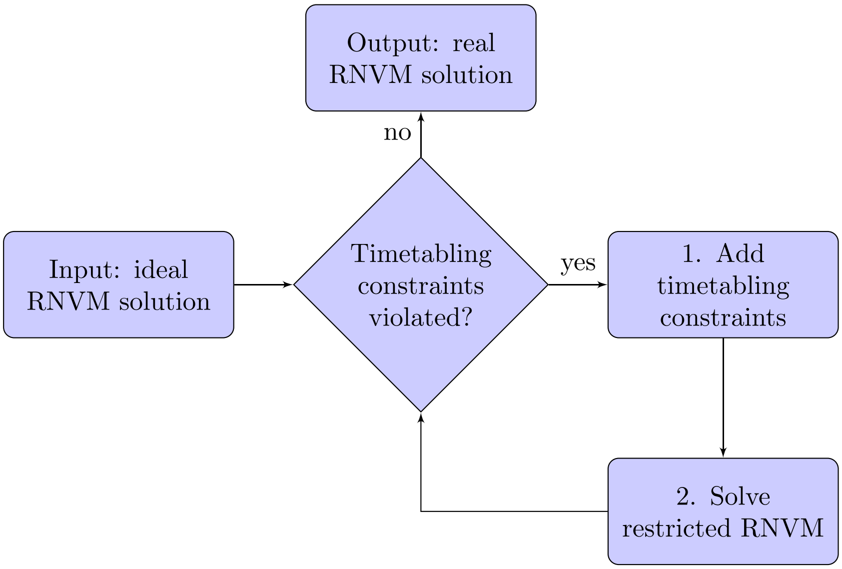

Fig. 5. Heuristic for ensuring timetable feasibility.

The resulting pricing heuristics objective function is defined as:

$$min \sum_{(i,j)\in A^p} \delta^p_{i,j} \bar{c_{ij}} \tag{39}$$

For determining the paths to be added, the sums are calculated and compared according to Eq. (36) and added if violating the inequality. The actual calculation of the cost is executed based on unmodified network weights and dual variable values of the primal problem. The other dual variables of the primal problem remain unused for the pricing since they capture the detour, disruption or train routing perspective of our problem while not leading towards the minimization of the passenger path length.

4. **Update the model:** If the reduced cost is lower than zero, the additional path should be considered as an additional column and added to the relaxed master problem. Conversely, if the reduced cost is higher or equal to zero, then no additional column can improve the solution in the restricted master problem. If new beneficial paths have been found, they are added to $\bar{P}^k$ and the procedure continues to step 2.
5. **Solve the restricted RNVM:** If no paths have been added, the resulting solution is used to solve the corresponding restricted RNVM problem.
6. **Add the shortest Paths:** Since the relaxed master problem does not have integrality constraints, it allows for incomplete (partial) disruptions. Since we only consider complete disruptions in the restricted master problem, the shortest paths in both, the relaxed and the restricted problem, might differ as the shortest paths in the relaxed problem might use arcs which are unavailable in the restricted problem. Hence, the paths in the restricted problem might not fulfill the requirement of the shortest path routing after solving the restricted problem and the new shortest paths need to be added to $\bar{P}^k$. In that case, another optimization run is necessary and the procedure continues to step 5.

After the first component has been carried out, we obtain a solution which contains the critical combination of links, all necessary passenger paths and the adapted train services. Since the infrastructural feasibility has not been checked nor ensured yet, we refer to it as the ideal RNVM solution.

5.2. Heuristic for generating infrastructure constraints

In order to reduce the total number of constraints, the infrastructure constraints are created only if they are necessary. A heuristic based on row generation is used to check feasibility of the obtained ideal RNVM solution after all promising passenger paths have been added. Fig. 5 outlines this procedure. If there is a violation of the headway constraints, the corresponding constraint(s) is introduced (Step 1) and the resulting modified problem is solved again (Step 2). This procedure iterates until no constraints are violated and the final result – the real RNVM solution – is obtained. This procedure has been applied similarly as in van Aken et al. (2017).

6. Computational experiments

In this section, we demonstrate the performance of the developed RNVM framework on the Dutch railway network. In the beginning, the input data and considered network used in our experiments are presented (Section 6.1). We perform

Table 4
Experimental parameter settings.

Parameter	Value
λ	0.5
ω	0.0001
β	0.3
$\forall s^t$	500
M	100,000
$\forall h^t_{ij}$	3 min
u	2

three types of experiments. First, we test the model capabilities to adapt passenger flows (Section 6.2) and train rerouting (Section 6.3). Second, we perform a sensitivity analysis of the framework in respect of parameter λ (Section 6.4). In essence, we solve our multi-objective problem using a weighted sum approach to support selection of λ values. Third, we present a real-life demonstration on the case network to determine the most critical combination of disrupted links in the network (Section 6.5) and we give computational performance (Section 6.6). The model and the framework are implemented in Matlab and solved using CPLEX version 12.8.0.

6.1. Input data

The Dutch passenger railway network, one of the most utilized railway networks in Europe, has been chosen for performing our experiments. In particular, a sub-network of the national network has been selected, consisting of the whole eastern part of the Netherlands (Fig. 6).

This network includes more than 500 infrastructure arcs and 250 stations, with the operated timetable from a working day in 2019, consisting of around 330 trains within one hour of operation. The input consists of train- and passenger-related data. The train-related data is the General Transit Feed Specification (GTFS) data provided by the Dutch railway operator which consists of the operated train lines, routes and the scheduled arrival and departure times. For trains not originating in the considered network, i.e. entering or leaving the case study network, train routes were modified to operate only within the considered boundaries by deleting all stops outside the case study network for each train route. This results in trains, originating/terminating at the considered border stations. As the timetable of the passenger services in the Netherlands is periodic, considering a certain limited time period (instead of the complete day) is sufficient to capture the patterns and interdependencies of the railway operations.

The input includes three types of passenger trains such as local, intercity trains and international trains. For all the following experiments, the following train-specific assumptions are made 1) only international trains can be rerouted, and 2) trains dwell only at originally scheduled stops.

The passenger demand is based on real demand data from the Dutch railway network. In particular, OD pairs with a demand larger than 100 passengers are considered in our experiments to include only more important passenger flows. Table 4 defines the model parameters used in the experiments, unless stated differently in a particular section.

6.2. Experiment 1: Passenger routing

For testing the functionality of the our framework, we consider the complete network with all train services, but focus on a small triangular part of our case-study network consisting of the most northern part of the Dutch railway network between three different stations: Groningen (Gn), Leeuwarden (Lw) and Zwolle (Zl) (see Fig. 7). For demonstration purposes, we use an artificial uniform demand of 100 passengers per OD pair between each of these three stations, resulting in 600 passengers, while all other demand in the network is set to 0. The train services of the whole network remain unchanged. Rerouting of trains is not considered for now.

Fig. 8 shows the network visualisation of the passenger flows in the undisrupted triangular network on the left, and the corresponding time-distance diagram for the corridor Lw-Gn on the right. The network visualisation consists of nodes and arcs. The nodes represent stations in the network while arcs represent the single-direction infrastructure connecting the stations. Each node and arc are assigned a unique ID (only node IDs are displayed). The passenger flows are highlighted with bold lines at the corresponding arc. The corresponding flow magnitude is indicated at each arc, showing the number of passengers travelling on that arc. The disrupted arcs are shown in red. Note that one disrupted link is represented with a pair of single-direction arcs. Additionally, the total numbers of transported and disconnected passengers are displayed in the header of the figure. The total path cost (in [PAXmin]) is displayed in the upper right corner. The time-distance diagram shows the train services on one particular corridor, with the distance (in [km]) on the horizontal and the time (in [min]) at the vertical axis. For the sake of simplicity only one direction is shown in the time-distance diagram.

For the first experiment, we vary the number of disruptions from 0 to 3 and follow the network performance in terms of passenger flows and train services. Table 5 shows the results of the first experimental setting with an increasing number of disruptions, the resulting passenger detour cost, the transported and disconnected number of passengers and the value of

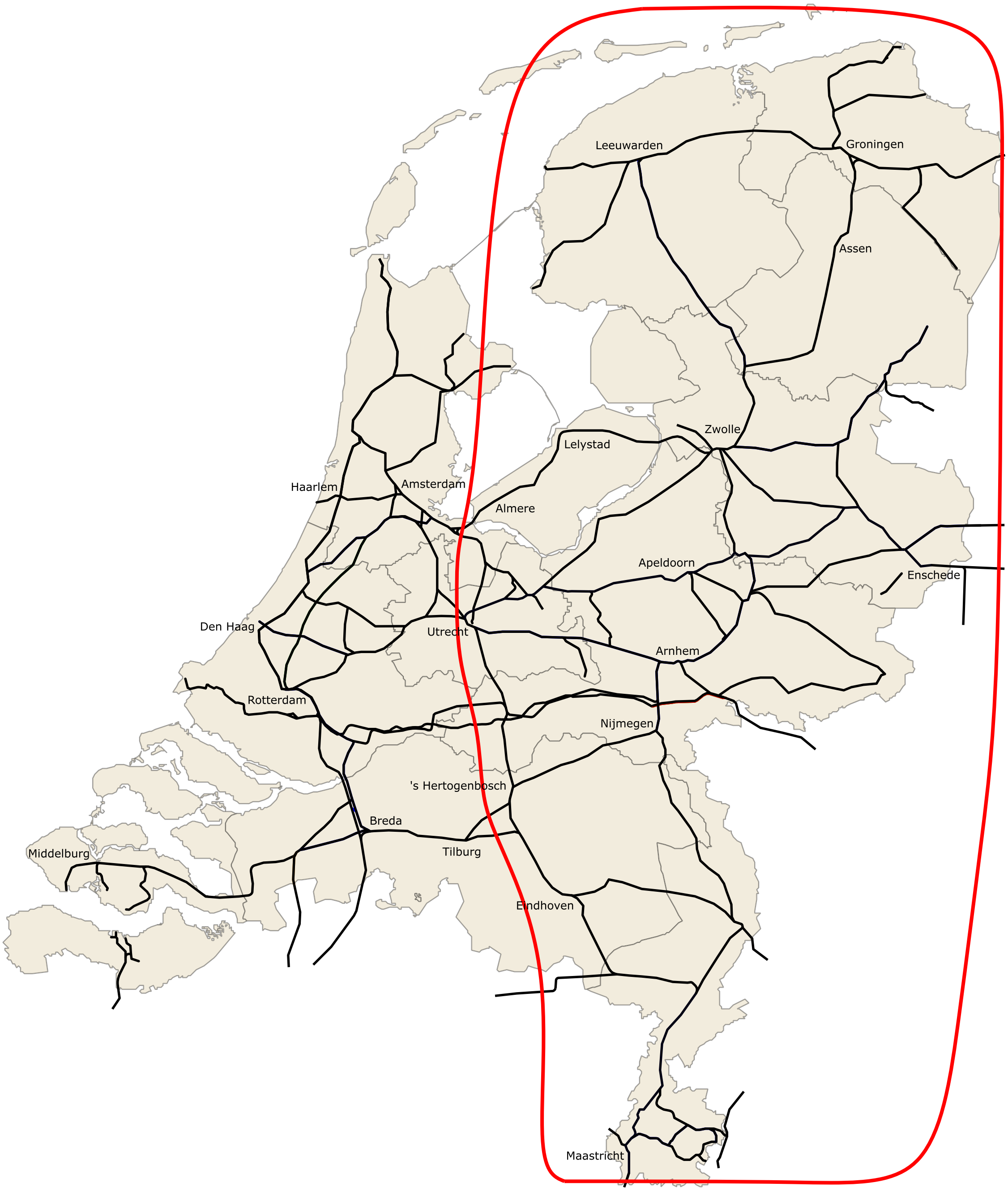

Fig. 6. Case Study Network, map adapted from wikimedia (2017).

Table 5
Results Experiment 1.

No. Disturbed Links	Passenger Cost [PAXmin]	Transported / Disconnected Passengers	Objective Value
0	1,766,010	600/0	−3000.00
1	400,612.5	200/400	18,999.98
2	1,035,158	200/400	19,157.30
3	0	0/600	29,999.98

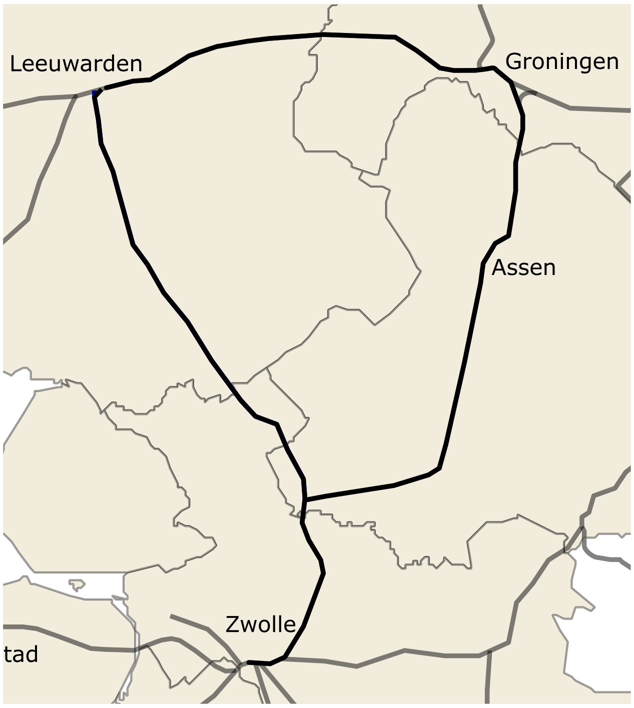

Fig. 7. The triangular part in the north of the network between three stations: Groningen (Gn), Leeuwarden (Lw) and Zwolle (Zl), map adapted from wikimedia (2017).

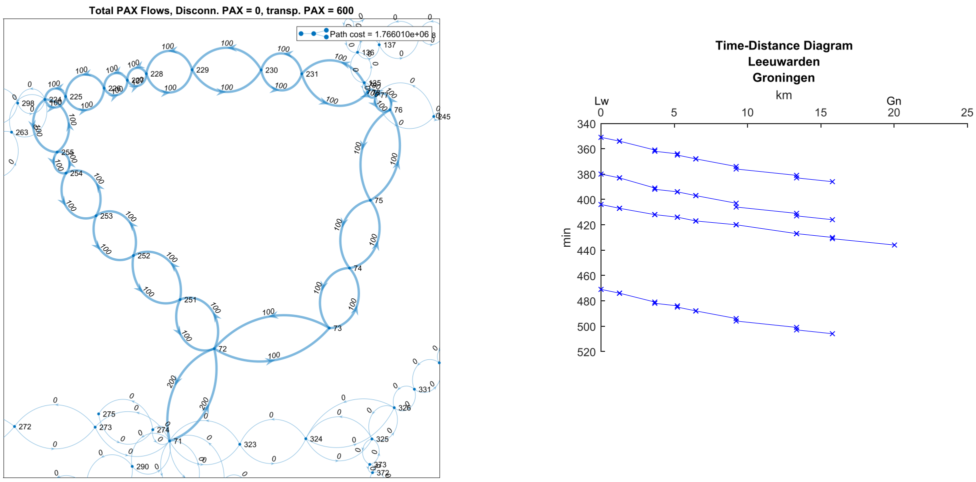

Fig. 8. Experiment 1: Undisrupted Passenger Flows (left) and timetable Leuwarden – Groningen, undisrupted (right).

the objective function. The corresponding passenger flows and time-distance diagrams with 1, 2, and 3 critical disruptions are shown in Figs. 9–11, respectively.

For the undisrupted case, all 600 passengers follow their original flows which correspond to the shortest available paths in the network (6 OD pairs in total). The passenger cost represent the total effort of passengers in the network, multiplying the travel time of each path with the number of passengers travelling on it. This results in 1,766,010 passenger minutes. The negative OF value of −3000 represents the absence of adverse consequences, since the models objective is to maximize disruption effects of the network but the number of available disruptions has been set to zero.

For the case with one disruption (Fig. 9), the model places the disruption in the south, disconnecting the most southern station of Zwolle. This results in 4 of 6 disconnected OD pairs, since all passenger paths from and to this station are cut off, leaving 200 passengers unaffected. This results in a decrease of the passenger cost to 400,612.5 passenger minutes,

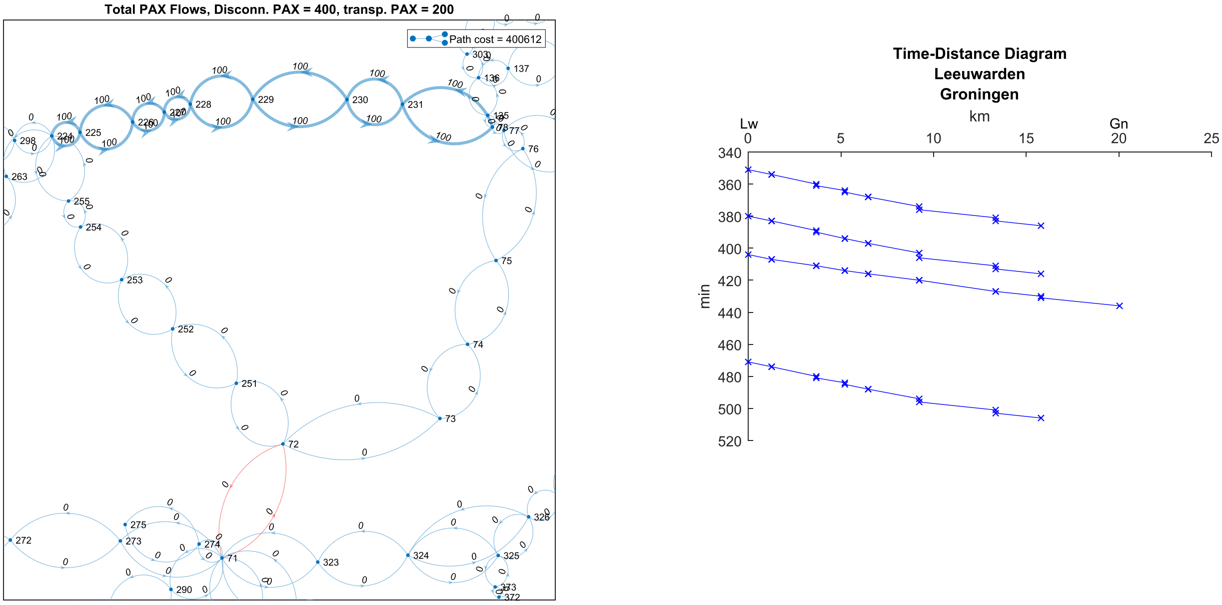

Fig. 9. Experiment 1: 1 Disruption – Disrupted Passenger Flows (left), Timetable Leuwarden – Groningen (right).

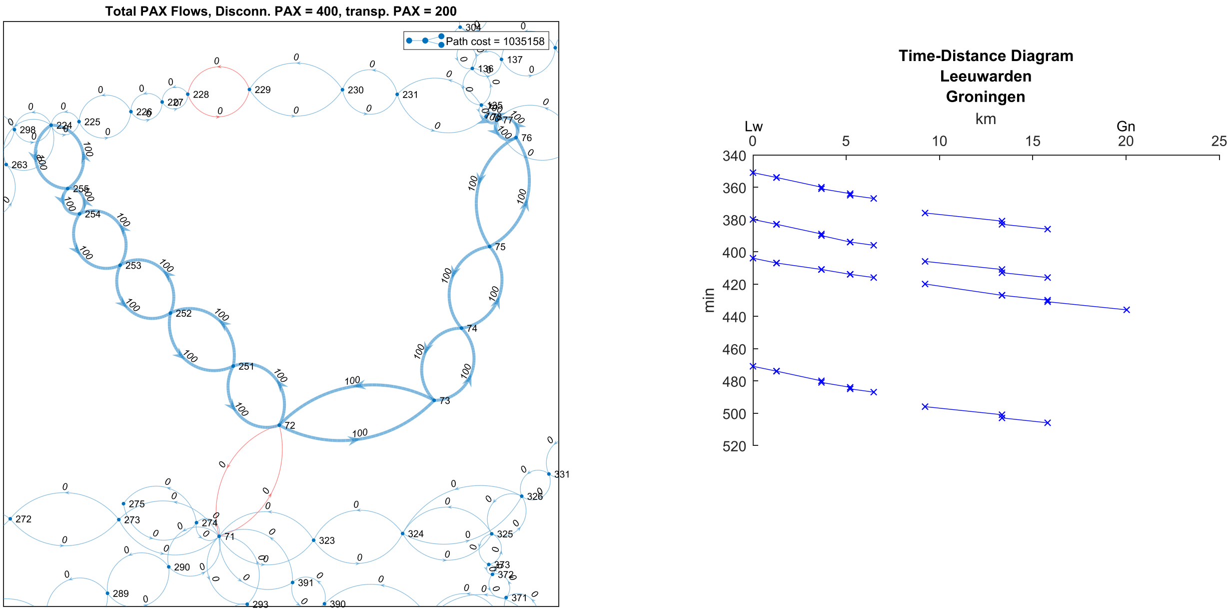

Fig. 10. Experiment 1: 2 Disruptions – Disrupted Passenger Flows (left), Timetable Leuwarden – Groningen (right).

since fewer passengers are travelling in the network. The increased objective value of 18,999.98 represents the disconnected passenger paths in the network.

For the case with two disruptions (Fig. 10), the model places the second disruption on the short connection between the two remaining stations in the north. No additional passengers are disconnected due to the triangular structure of the network. Instead, this extra disruption creates passengers detours and thus increased passenger cost of 1,035,158 PAXmin (see Table 5), which are also reflected in the increased objective value of 19,157.3.

For the last case of 3 disrupted links, the model disconnects all passenger flows, resulting in 600 disconnected and 0 transported passengers (see Table 5). Therefore the model keeps the disconnection of the two-disruption case and additionally disrupts one link in the remaining connection of the last detoured OD pair. Hence the resulting passenger cost decrease to 0 due to no travelling passengers, while the objective value further increases. Since the artificial demand is between the three stations only, there exist multiple similar solutions, consisting of the same disconnected OD pairs but differing in

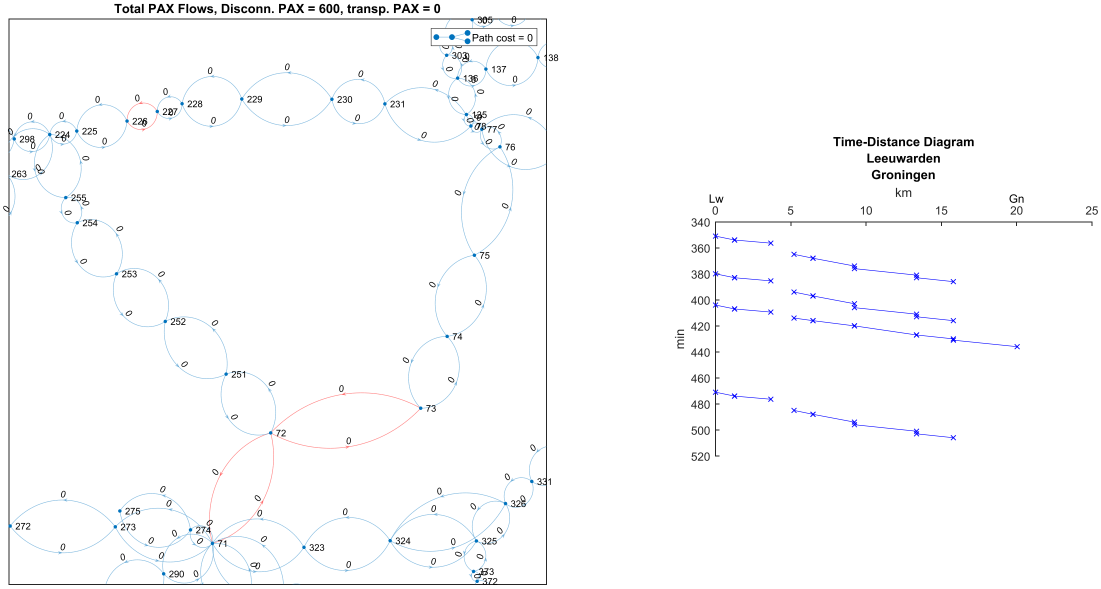

Fig. 11. Experiment 1: 3 Disruptions – Disrupted Passenger Flows (left), Timetable Leuwarden – Groningen (right).

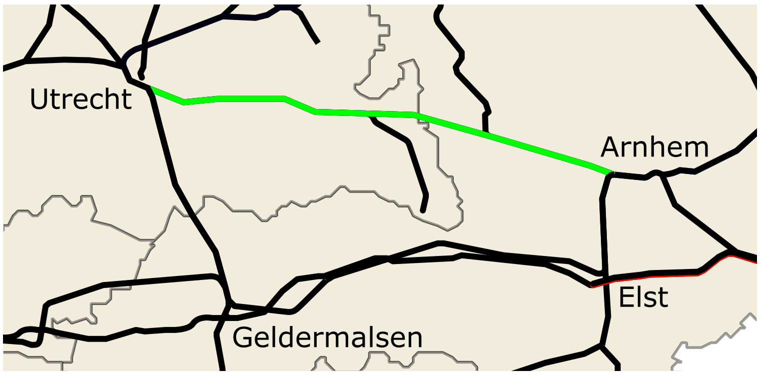

Fig. 12. Experiment 2: Rerouting Train, Original Route, map adapted from wikimedia (2017).

the specific location of the disrupted arcs. As can be seen by comparing the case of 2 and 3 disrupted links, the northern connection between Lw and Gn is disrupted in both cases, using different specific arcs, providing the same result.

The corresponding time-distance diagrams show that the disruptions on the critical links cause trains short-turning at the adjacent nodes, causing interrupted train paths. The operations on the undisrupted links remain unaffected. As it can be seen in the network visualisations, the model generally aims for disconnecting OD pairs. Placing the first disruption in the south, close to Zwolle, all passenger connections to and from this station are cut off, not causing any detours. Since with two disruptions a triangular network can not be disconnected, the second disruption causes detours, while three disruptions fully disconnect the given network. Regarding the results of this first experiment, it can be shown that the model behaves as expected.

6.3. *Experiment 2: Train rerouting*

The second experiment is used to illustrate the train rerouting capabilities of the model. The corridor between Utrecht (Ut) and Arnhem (Ah) is used as an example (see green line, Fig. 12). The time-distance diagram of the undisrupted case is shown in Fig. 13, where the rerouting train is highlighted in green. As can be seen in this figure, there are train paths overlapping each other. Since the original timetable allows the overlap in dedicated points, i.e. due to multiple railway tracks, these parts are excluded when introducing infrastructure constraints by our model. For all remaining network segments, the infrastructure constraints are checked and fulfilled.

For showing the rerouting of this particular train, the original route of the train gets disrupted and the train is rerouted accordingly (Fig. 14). The disruption (marked with red cross) occurs on the original train route (dashed green line) close to

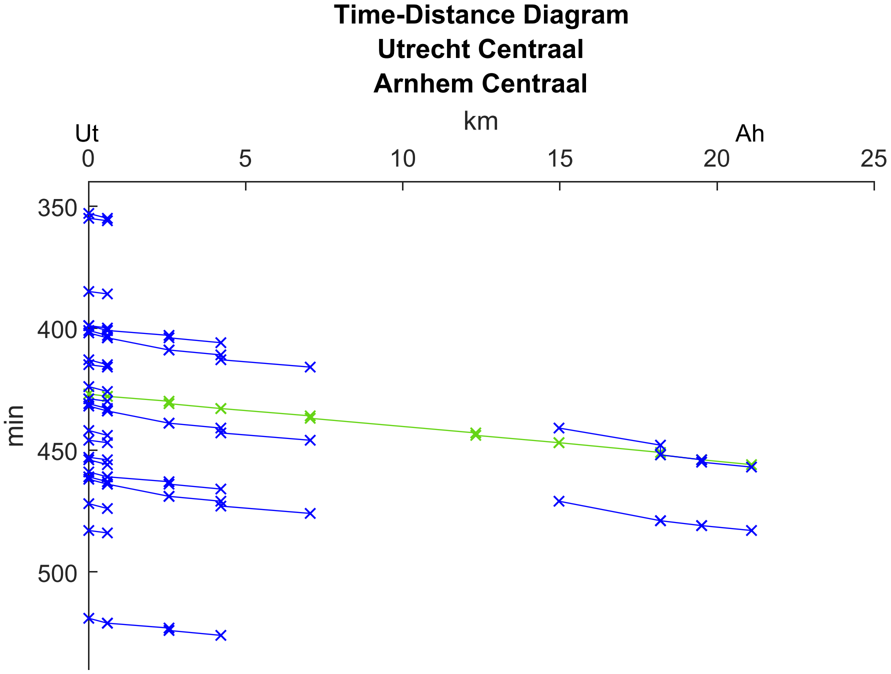

Fig. 13. Experiment 2: Original Timetable of the Rerouting Train.

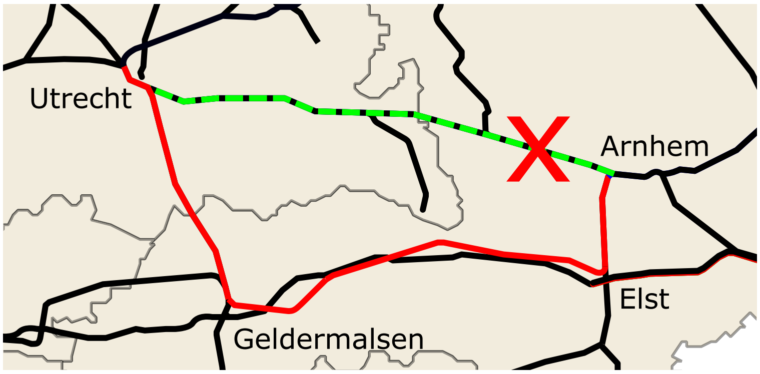

Fig. 14. Experiment 2: Rerouted Train Route, map adapted from wikimedia (2017).

Arnhem. Hence the train is rerouted via Geldermalsen (Gdm) and Elst (Est), as can be seen by the red route in the figure, which is the shortest available path, enforced by the objective function. The timetable of the route segments between the stations Utrecht – Geldermalsen, Geldermalsen – Elst and Elst – Arnhem is shown in Figs. 15–17 – for the undisrupted (left) and disrupted case (right). As it can be seen, the train is rerouted, while satisfying the restrictions of the initial timetable.

It can be concluded from Figs. 15–17, disruptions on the original corridor of the train necessitates to redirect the train over an alternative route. Only if the rerouting cost exceeds a certain level, the entire train trip would be cancelled (see Section 4.3).

6.4. Experiment 3: Parameter λ variations

The RNVM formulation allows to tune the model aiming either more towards disconnecting passenger OD relations or causing passenger detours by varying parameter λ. Experiment 3 is used to illustrate the models performance under different settings of this parameter. In order to show the resulting effects, all parameters except λ has been kept constant. For this experiment, we consider at most 1 disruption to be identified in the complete case study network, using the entire passenger demand and the real-life OD patterns.

Fig. 18 shows solutions for different values of λ. The horizontal axis shows the partial objective values for disconnecting passengers (first term of the objective function (34), disconnection-term). The vertical axis shows the partial objective

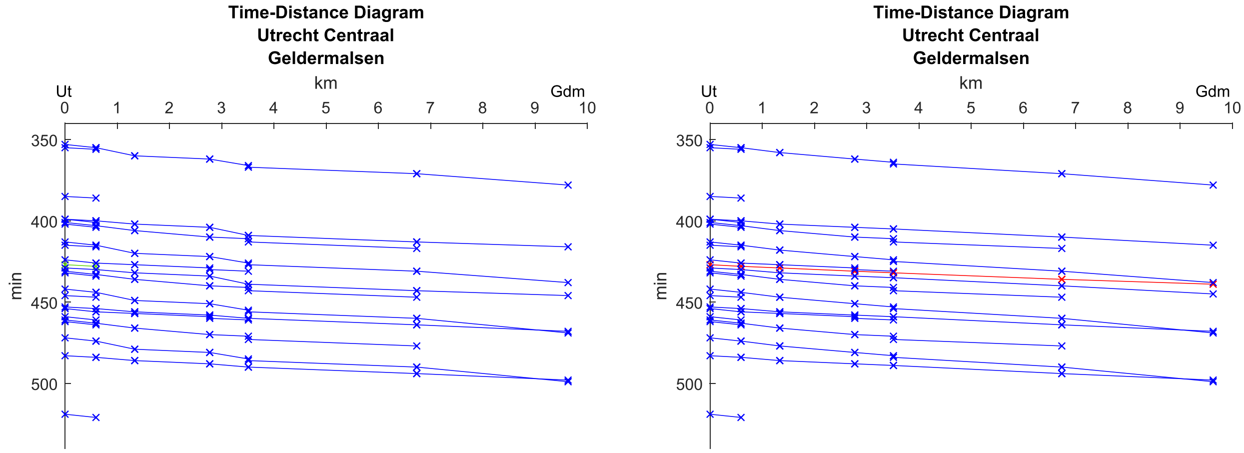

Fig. 15. Experiment 2: Original (left) and Rerouted (right) Train Paths, Stations Utrecht – Geldermalsen.

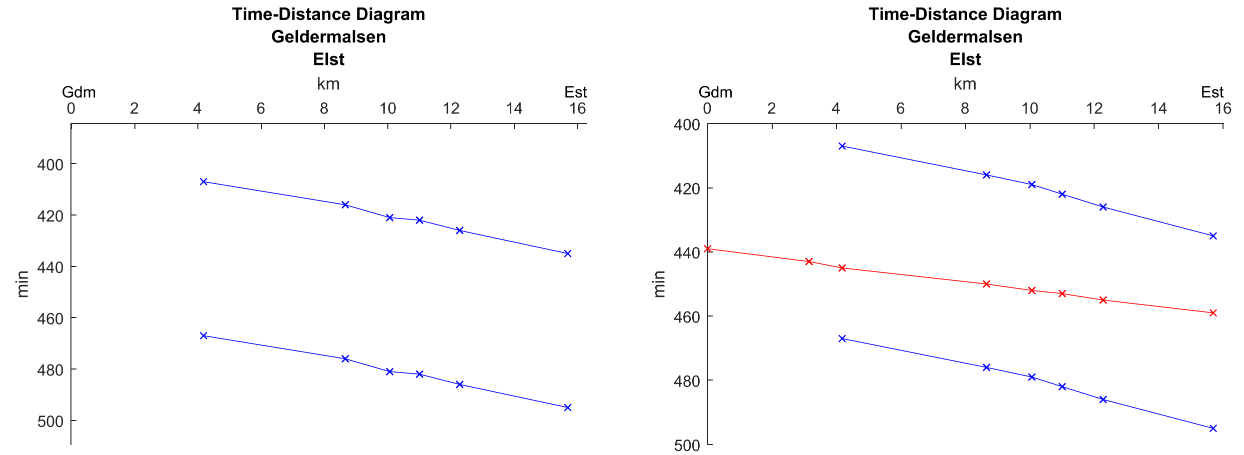

Fig. 16. Experiment 2: Original (left) and Rerouted (right) Train Paths, Stations Geldermalsen – Elst.

values for the passengers detours (second term of the objective function (34), detour-term). The corresponding λ values are indicated at the data points.

As it can be seen from Fig. 18, the objective function tends to aim for disconnecting passengers rather than causing detours, since the detour-term value decreases with increasing disconnection-term values for all $\lambda \geq 0.25$. Only when $\lambda < 0.25$, detouring passengers is the dominating objective. This might result from the cost-factor of the decision variables z_k. Since they are weighted with the total demand of the corresponding OD pair and the path variables y_p^k only with a demand share, z_k tends to have higher weights and can dominate the objective in that way. As can be seen from the graph, this effect can successfully encountered by tuning parameter λ. Therefore, railway decision makers can adjust this parameter to their preference. Since we care more for disconnecting passengers, i.e. they represent more excessive harm in railway networks, our selection of $\lambda = 0.5$ is supported.

6.5. Experiment 4: Number of disruptions variations

In the fourth experiment, we use the original demand patterns, and assess the critical combination of links in the network. For analysing the changing locations of the critical links, the number of maximum disrupted links n is varied. Due to different effects according to their total number in the network, the location may depend on the number of disrupted links. Therefore, we track the ID of the disturbed link(s) as well as the passenger cost due to disruption and the number of (un)affected passengers. Table 6 reports the number of disrupted links, the value of the objective function, passenger cost, number of transported and disconnected passengers, train cost, rerouted and cancelled trains and short-turned trains. Since one link consists of two arcs, the disrupted arc-pairs per disruption-scenario are shown in Table 7. The corresponding figures of the passenger flows in the network for 1 disruption is in Fig. 19, while for 2,3,5 and 10 disruptions are displayed in Appendix A.

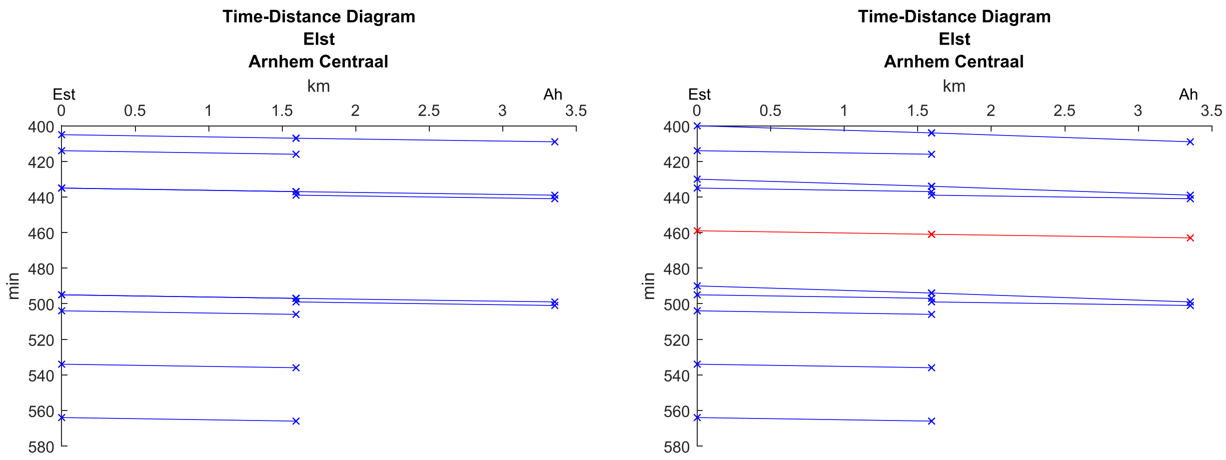

Fig. 17. Experiment 2: Original (left) and Rerouted (right) Train Paths, Stations Elst – Arnhem.

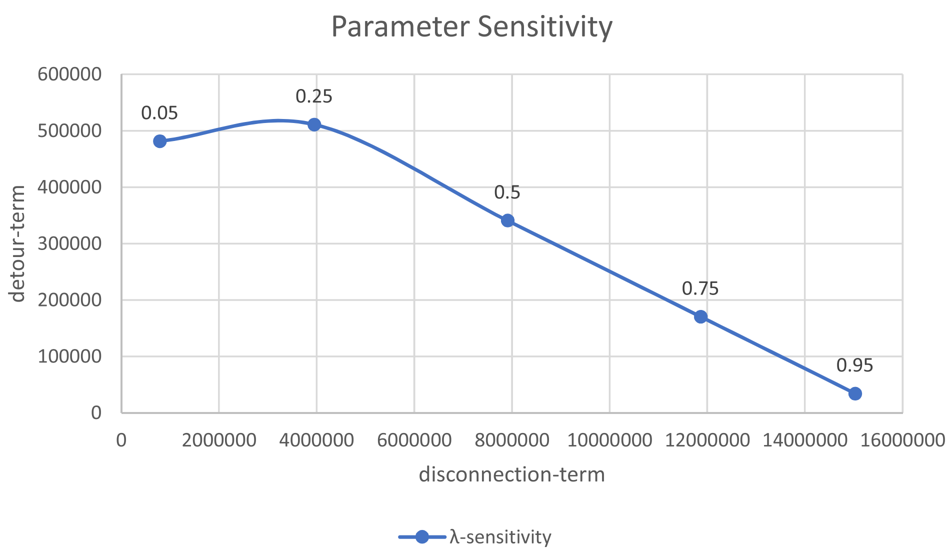

Fig. 18. Sensitivity to parameter λ.

Table 6
Results Experiment 4 – Disruptions.

No. Disrupted Links	Objective Value	Passenger Cost [10^6-PAXmin]	Transported / Disconnected Passengers	Train Cost [10^3-Euro]	Rerouted/ Cancelled Trains	Short-turned Trains
0	−1131674.40	5400438	125876/0	6165.21	0/0	0
1	3694080.55	4016297	104553/21323	6165.21	0/0	13
2	5446215.53	3111848	88979/36897	6165.21	0/0	24
3	7569820.39	1458818	55219/70657	6165.21	0/0	56
5	8670189.03	1146296	45045/80831	8158.20	1/0	69
10	10047693.02	555725	22923/102953	8158.20	1/0	113

As it can be seen from Table 6, the value of the objective function increases with the number of disruptions. Since more possible disruptions result in more adverse consequences, this is a reasonable and expected behaviour. Due to the chosen setting of λ in the objective function, the model aims preferably to disconnect OD pairs. This results in the continuous increase of the number of disconnected passengers with increasing number of disruptions. Since disconnected passengers are not transported anymore, the passenger cost decrease with increasing disconnection. For cases with 5 and more disruptions, train rerouting was applied. Note that we allowed rerouting only to international trains. In our experiments, we have in total 3 international trains, mostly crossing the central part of the network. Hence, they are not affected by the first three dis-

Table 7
Disrupted links combinations.

No. Disrupted Links	Disrupted Arc ID														
	11010 11264	3112 13018	20994 21248	8962 9216	8809 13058	13312 35225	18206 25572	17528 47754	1794 2048	12034 12288	28443 35047	35330 35584	35586 35840	52226 52480	54285 57333
1	x														
2	x	x													
3	x		x	x											
5	x				x	x	x	x							
10	x						x	x	x	x	x	x	x	x	x

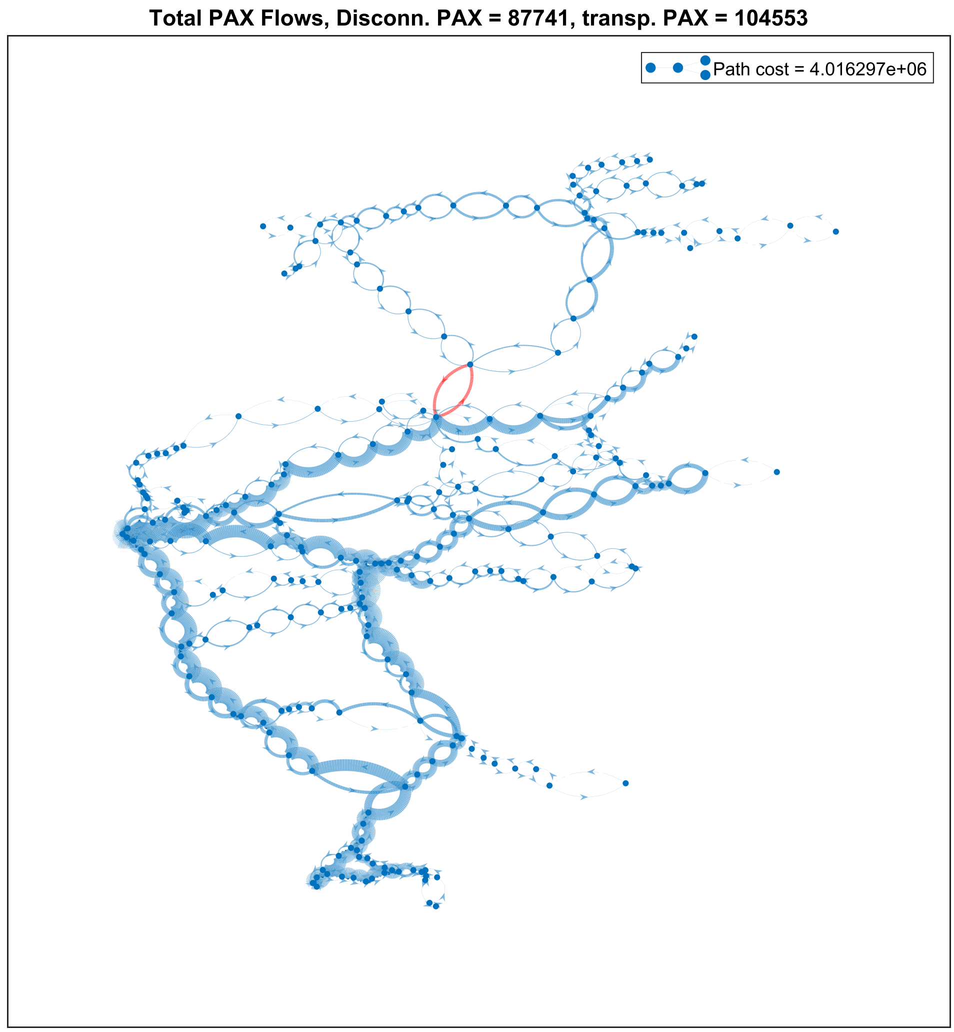

Fig. 19. Experiment 3: Passenger Flows, 1 Disruption.

ruption cases, since these disruptions occur more towards the rims of the network (see Figs. A.20 and A.21 in Appendix A). The higher the number of disruptions gets, the more arcs get disrupted also in the central parts of the network, which results in trains being rerouted and the thus increasing the train cost (see Figs. A.22 and A.23 in Appendix A). No trains get cancelled in all cases. The number of short-turned trains increases continuously with the number of disrupted links, since each disruption causes additionally disrupted train paths.

Table 8
Results Experiment 4 – Computation Time.

No. Disrupted Links	No. Iterations Passenger Paths	No. Iterations Infrastructure Constraints	Solving time final problem [s]
0	1	0	0.25
1	1	0	2.17
2	3	0	3.83
3	2	0	2.39
5	5	1	3.72
10	6	1	35.30

In particular, the link described by arcs 11010 and 11264 exists in all disruption scenarios. This link is by experience considered to be the most vulnerable link in the Dutch railway network, as it disconnects the network in two independent ones (see Fig. 19). Since the model was capable of reproducing this result for the known case, it may confirm its ability to provide meaningful results. Thus, the other disrupted links (in multiple links combinations) may be sometimes less trivial, although always reasonable and comprehensible. The critical links of a certain number of disruptions are not necessarily part of the set of links corresponding to a bigger number of disruptions, as can be seen in Table 7. Hence, the function of the link, and thereof the impact of a potential disruption, changes according to the flows in the network. Therefore, the dynamic rerouting and adaptation processes (of both trains and passengers) play a major role to assessing network vulnerability in complex networks and cause the effect, that the impact of a combination of links differs from the impact of single elements. The dynamic character of disruptions of the most critical combinations of links shows the importance of considering infrastructure, train services and passenger flows jointly in the assessment, and thus points out the usefulness and applicability of optimization approaches.

6.6. Experiment 5: Computation times

We report the computational performance of the RNVM framework against the number of disrupted links and the passenger demand. First, we give the computational results for the Experiment 4 and, in particular, the computation times for solving the final restricted RNVM model; and then, the computation times of the complete RNVM framework for variable passenger demand.

Table 8 shows the performance for Experiment 4 containing the number of disruptions, the number of iterations of the two heuristics, for adding passenger paths and for infrastructure constraints, and the solving time for the final restricted RNVM, i.e. with all passenger paths and infrastructure constraints included. For passenger paths, the number of iterations varies over different number of disruptions. As the number of iterations reflects adding passenger flow paths, it strongly depends on the location of the disruptions of the previous iteration step and the structure of the network. The more possibilities for alternative paths exist, the more iterations are likely to happen. Hence, the number of iterations generally increases with the number of disruptions, as the number of impacted passenger paths increases as well. For up to three disrupted links, there are no alternatives available which results in only a few iterations. The more disruptions occur simultaneously, the more they affect central parts of the network, which offers significantly more alternatives for rerouting passengers, and thus, results in a higher number of iterations. For infrastructure constraints, typically, one iteration was sufficient to check if infrastructure constraints are satisfied and, since no conflicts existed, confirm the feasibility of the solution. Instead, for 5 and 10 disrupted links, an additional iteration was needed. This coincides with train rerouting in these two cases, as conflicts tend to occur involving the rerouted train.

Table 8 also shows that the solving time of the model still remains low even if a high number of disruptions needs to be determined. The model solves each given problem quite efficiently, since the lower numbers of disruptions are all solved within 5 s, while only for 10 disrupted links it took 35 s. Since multiple iterations of the (modified) RNVM problem are carried out within our framework, it was important to compute a single problem fast. Hence, the RNVM model seems to be scalable regarding different number of disruptions and their increasing combinatorial complexity.

Looking at the whole RNVM framework, an average computation time per iteration was approx. 14 min for the column generation heuristic for adding passenger paths, and approx. 11 min for the row generation heuristic for adding infrastructure constraints. Note that each iteration includes, for the first heuristic, several optimization runs (i.e. relaxed RNVM, restricted RNVM), pricing computation, and model updating, and for the second heuristic, conflict detection, model updating and resolving restricted RNVM. Hence, the computation time of the restricted RNVM model remains low, which allows the proposed RNVM framework to efficiently assess vulnerability of dense railway networks.

To evaluate the effect of different passenger demands on the computational performance, we use ODs matrices with different number of OD pairs. In particular, we compare three passenger demands with different demand thresholds: the original demand including OD pairs with more than 100 passengers (total of 628 OD pairs), the one with more than 200 passengers (total of 333 OD pairs) and the one with more than 300 passengers (total of 214 OD pairs). The train services and demand pattern remain unchanged. The experiment has been executed with at most one disruption in the network. Table 9 gives the resulting computation times for the heuristics for adding passenger paths, the heuristics for infrastructure constraints and the total run time.

Table 9
Computation time comparison.

No. OD pairs	Time Passenger Paths [min]	Time Infrastructure Constraints [min]	Time Total [min]
214	4	3	7
333	5	4	9
628	14	11	25

The computation time for 628 OD pairs equals 25 min while the times decrease with reducing the number of OD pairs. For the most limiting OD case, with 214 OD pairs the total time is as small as 7 min. Also, in all cases, the column generation heuristic contributes more to the total run time than the row generation one. Therefore, we show that the framework is capable of dealing with real-life instances within limited computational effort.

In general, when dealing with (even) larger and/or real-life instances, omitting the OD pairs with relatively low demand could be a remedy and a potential approach for reducing computation times. Although this might come with a loss of detail, it still enables the application of our framework since railway networks are generally likely to be dominated by the larger demands. Hence, our framework is capable of providing practically relevant solutions even for large-scale railway networks with complex demand patterns.

6.7. Discussion

The experiments are used to show multiple aspects of the RNVM including the general working principle, the behaviour, the computational performance and the practical applicability. Finding the critical combination of links in the network requires the identification of the links, which cause the biggest impacts on passengers in case of a disruption. Hence, a realistic passenger behaviour is crucial for obtaining meaningful and interpretable results from the framework. The experiment 1 shows that the RNVM selects the links, which cause the most adverse consequences to passengers, given a certain number of disruptions and passenger demand. Responding on the resulting service changes, the passengers adapt their routing correspondingly, which results in detours or even disconnected passenger journeys. The resulting passenger routing causes changes in the passenger cost. Passengers behaviour in experiment 1 shows the expected properties of the passenger routing, which causes increased passenger effort or even disconnected OD relations resulting from disrupted links while aiming for shortest paths in the network. Additionally, the experiment visualizes the resulting selection of critical links, hence it proves the general passenger-centered behaviour of the RNVM.

The RNVM capabilities of adapting the train routing are illustrated in experiment 2. Since the offered train services are the base for the passenger flows in the network and the combination of critical links are identified from a passenger's perspective, the train rerouting capability is essential for a more realistic identification of the critical links. For a disrupted link, the corresponding trains are getting rerouted and rescheduled along the new train paths using the shortest path approach. The resulting detour paths are limited, preventing unrealistically long train detours. Therefore, the train services during disruptions are becoming more flexible and allow a close-to-reality modelling of the offered passenger services. However, the provided train capacity for the passenger flows on a link is defined by the resulting effectively provided capacities of all trains on that link, assuming all stops being served by all rerouted trains.

The network vulnerability also depends on the trade-off between disconnected and rerouted passengers. Hence, the impact of link disruptions on the objective function and consequently the results, depends on the user-specific priority of passenger detours versus disconnections. Illustrating these user-specific preferences, experiment 3 shows the behaviour of the model under different parameters of the objective function. The results show that tuning the parameters indeed leads to significantly different outcomes in terms of rerouting and disconnecting passengers. Since the precision of the results strongly depends on the priority of detours versus disconnections, the RNVM's behaviour under changing parameters shows its capability of realistically reproducing different user-specific scenarios.

The results of experiment 4 reveal new insights in the passenger-centered vulnerability measure. The varying locations of occurrence between scenarios indicate a strong dependence between critical links and the networks dynamic passenger flows. The passenger flows in the network do not only depend on the given demand patterns but also on the networks structure and the resulting offered services under different disruptions. Hence the resulting passenger flows seem to be a good measure for the vulnerability assessment. The passenger flows are a good indicator for quantifying the major disruption effects and showing the holistic disruption impact. Additionally, it can be seen that critical links are mostly links, where no passenger routing alternative exists (which causes passenger disconnection) or links with poor alternatives (not having available train capacities or causing long passenger detours). The RNVM effectively shows its capability of assessing the passenger-centered vulnerability based on the resulting passenger flows and thus, successfully identifies the critical combination of links from the passengers' point of view.

As it can be seen from the results of experiment 5, the task of assessing network vulnerability can be tackled within reasonable computation times, which shows the applicability of RNVM for real-life instances from the railway industry. In addition, RNVM could be used for assessing other transportation systems such as urban transit or air traffic.

7. Conclusion

This paper addressed the problem of vulnerability assessment in railway networks to identify the most critical combination of links, which in case of a disruption cause the most adverse consequences. We developed a new Railway Network Vulnerability Model (RNVM) to assess the passenger-centered vulnerability of railway networks combining three network levels: infrastructure, train operations and passenger behaviour. The new mathematical model provides explicit passenger-centered disruption impacts, dynamic passenger routing, and dynamic train rescheduling including train rerouting, short-turning, cancellation and retiming. By doing so, the model captures the interdependencies between the different network levels. To solve the RNVM efficiently, we designed a RNVM framework, which includes a column generation based approach to efficiently include passenger flows and a row generation based approach to model infrastructure constraints. The outcome of the RNVM framework is the most critical combination of links within a network, together with the resulting passenger flows, train routes and a feasible timetable.

The application on a part of the Dutch railway network revealed new insights in the behaviour of railway networks with passenger and train flows under simultaneous disruptions. We showed that the RNVM framework can efficiently reassign passenger flows and reroute trains during disruptions. We also demonstrated that the critical links are highly dependent on the dynamic properties of the network, i.e. trains and passengers, rather than the static properties of the underlying infrastructure network. Hence, the impact of a combination of multiple disruptions is not a simple summation of independent impacts of each disruption, but rather the outcome of changing interdependencies in a complex network which may typically result in even larger overall impacts. This results in varying locations of these critical links with a changing number of disruptions an thus changing remaining network loads. Finally, the computation times remain small when increasing the number of disrupted links as well as the size of the passenger demand, which allows fast and efficient network vulnerability assessment.

Several directions of future research can be envisioned. The passenger routing could be extended to also consider waiting and transfer times. Additionally, intelligent short turning (not only adjacent to disruptions) and the consideration of station capacity could further improve the results. For gaining deeper insights into the connection of the network flow dynamics and the network topology, the framework results could be analysed and compared with graph-theoretical measures (e.g. the spectral and geometric domain) to investigate potential structural dependencies. Furthermore, the interdependencies between link criticality, origins, destinations and demand could be analysed in greater detail to deepen our knowledge in this emerging field.

The developed framework can be used in the strategical and tactical planning stages of the railway planning process to evaluate the vulnerability of executed or future planned operations. Furthermore it can be also applied to assess and compare the vulnerability of different links in the network, providing detailed information about the consequences regarding passenger flows and train operations. As a result it may contribute to a deeper understanding of the service offered to the passengers and hence unveils the vulnerability-related consequences of planning decisions even at early planning stages. As such, it can become a building part of future decision support tools for designing more resilient railway operations, leading towards a more attractive and sustainable future transport system.

Appendix A. Additional network results for Experiment 4

This appendix shows the graphs for the passenger flows of experiment 4 (see Section 6), for 2,3,5 and 10 disruptions. The disrupted arcs are highlighted in red. The passenger flows are represented by the thickness of the corresponding arcs.

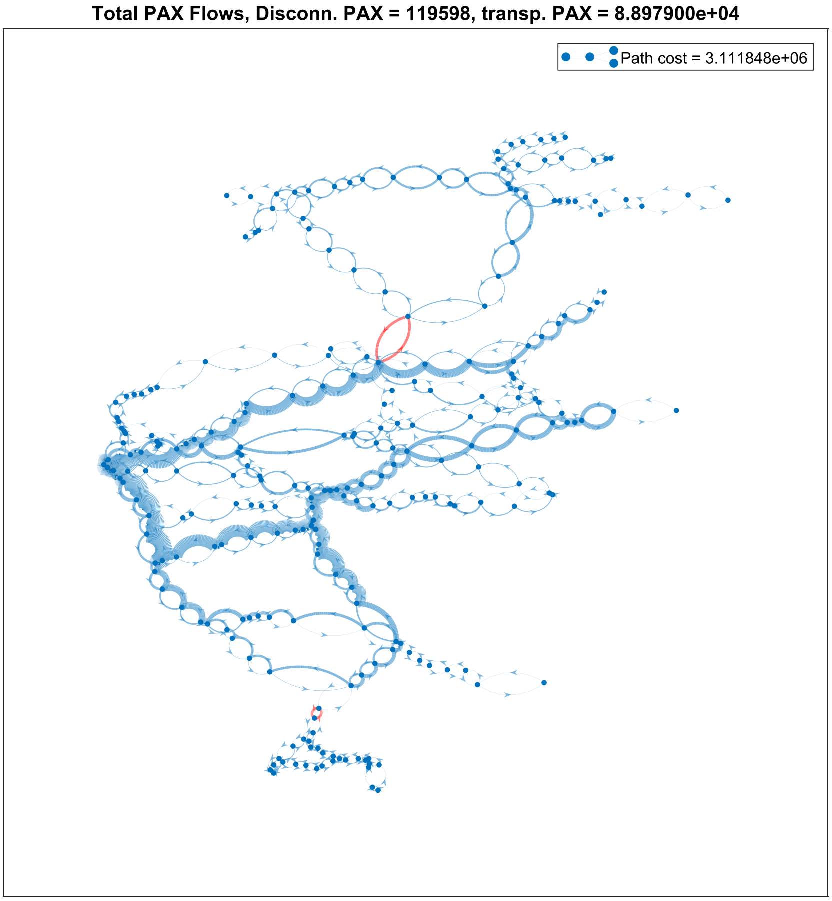

Fig. A.20. Experiment 4: Passenger Flows, 2 Disruptions.

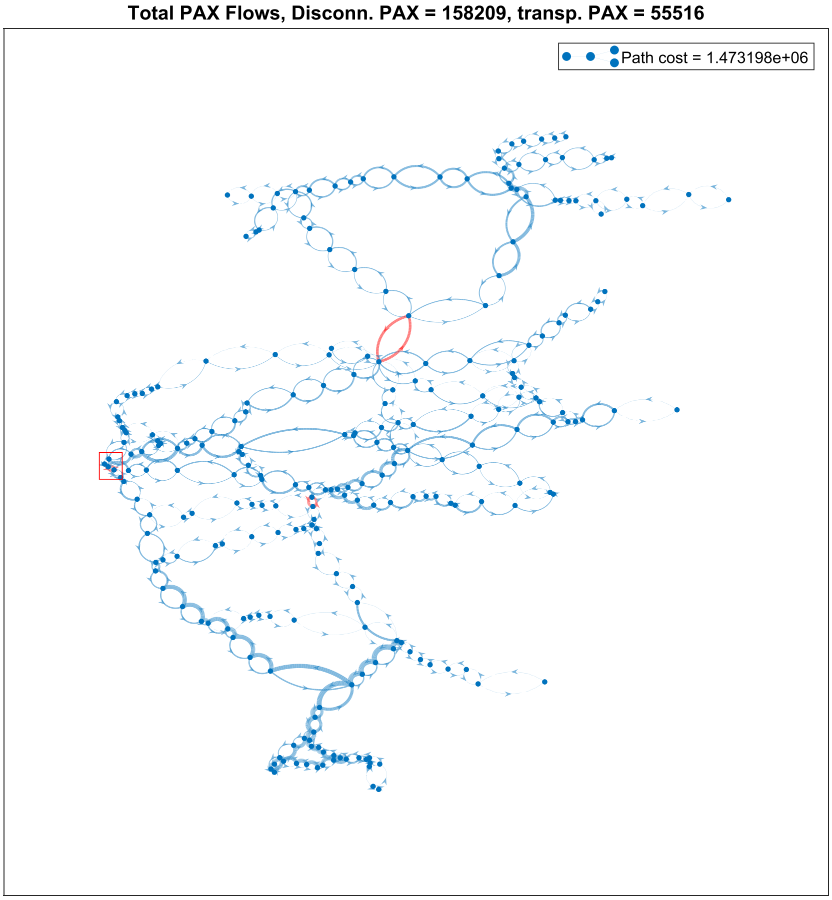

Fig. A.21. Experiment 4: Passenger Flows, 3 Disruptions.

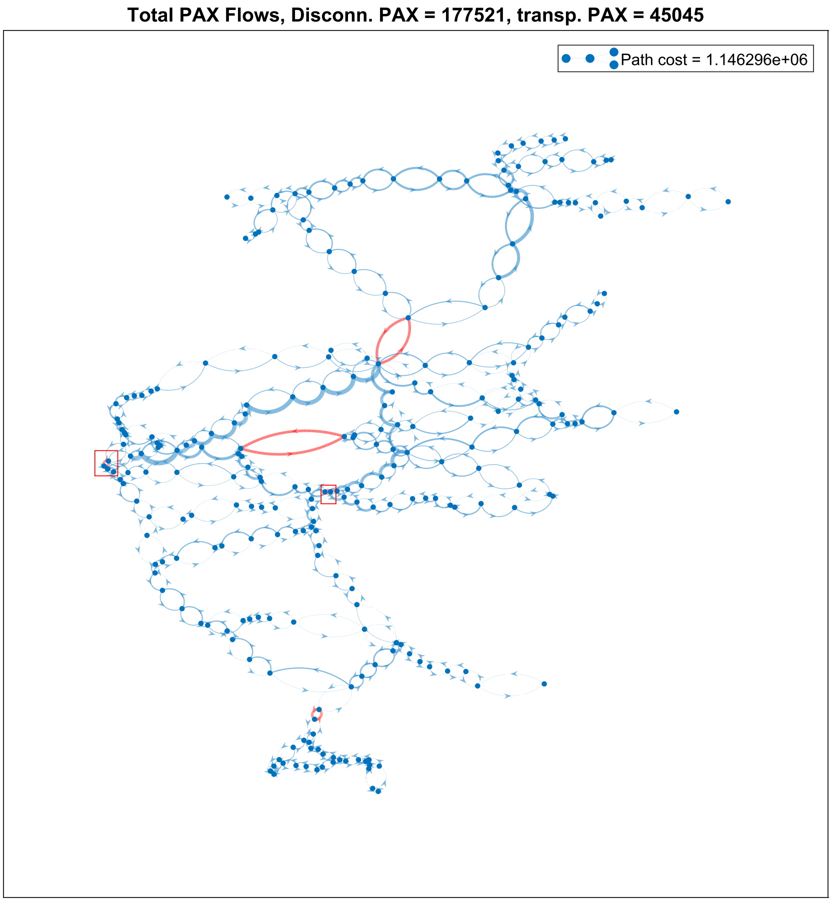

Fig. A.22. Experiment 4: Passenger Flows, 5 Disruptions.

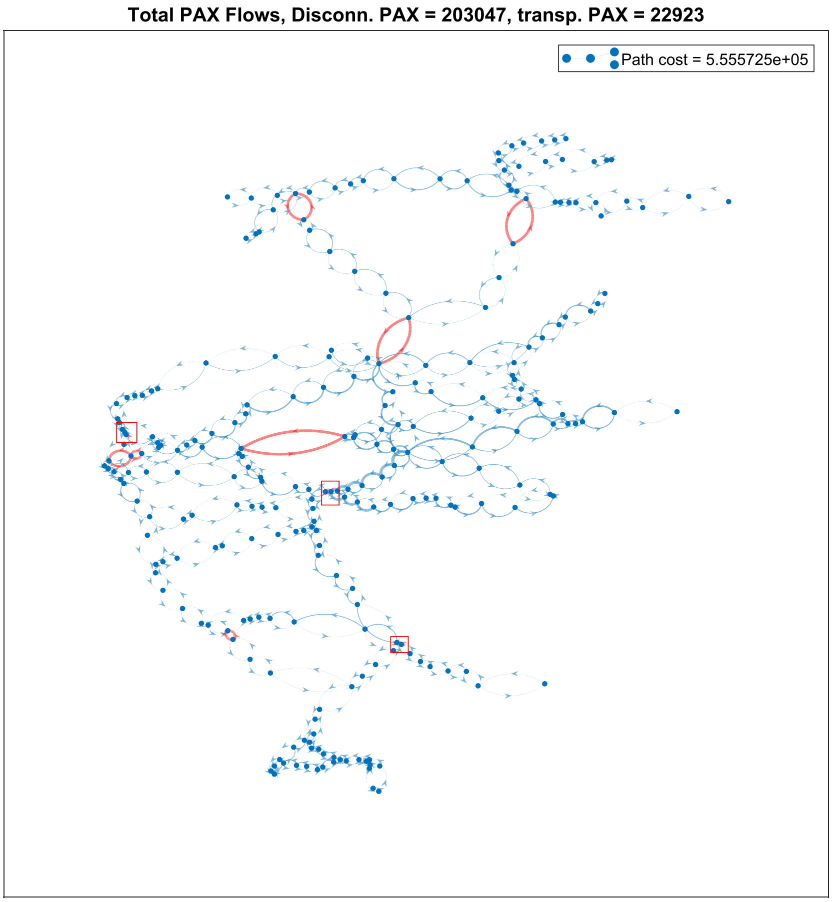

Fig. A.23. Experiment 4: Passenger Flows, 10 Disruptions.

CRediT authorship contribution statement

Christopher Szymula: Conceptualization, Methodology, Data curation, Formal analysis, Investigation, Visualization, Writing - original draft. **Nikola Bešinović:** Conceptualization, Methodology, Writing - review & editing.

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