Adaptive strategies for platooning

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Abstract

Automated driving, one of the rapidly growing research topics in the field of smart traffic, has proved to be a recognized solution for potentially improving road throughput and reducing vehicles' energy consumption by grouping individual vehicles into platoons controlled by one leading vehicle.

The advances in distributed inter-vehicle communication networks have stimulated a fruitful line of research in Cooperative Adaptive Cruise Control (CACC). In CACC, individual vehicles, grouped into platoons, must automatically adjust their own speed using on-board sensors and communication with the preceding vehicle so as to maintain a safe inter-vehicle distance. The importance of CACC lies in the fact that it enables small inter-vehicle time gaps, which leads to a major reduction of the aerodynamical drag force applied on vehicles in such a driving pattern. Consequently, vehicle emissions, which play a main role in trucks and heavy automobiles, are expected to be highly reduced.

However, a crucial limitation of the state-of-the-art research in this control scheme is that the string stability of the platoon can be proven only when the vehicles in the platoon have identical driveline dynamics and perfect engine performance (homogeneous platoon), and possibly an ideal communication channel.

Thus, the objective of this MSc thesis is to address the problem of CACC for heterogeneous platoons under realistic inter-vehicle network conditions. In the first part, we propose a novel CACC strategy that overcomes the homogeneity assumption and that is able to adapt its action and achieve string stability for uncertain heterogeneous platoons under ideal inter-vehicle network conditions. In the second part, in order to handle the inevitable communication losses, we formulate an extended average dwell-time framework and design an adaptive switched control strategy which activates an augmented CACC or an augmented Adaptive Cruise Control strategy depending on communication reliability.

Stability of the proposed control strategies is proven analytically and simulations are conducted to validate the theoretical analysis.

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List of Notation

The notations used in the MSc thesis are standard:

$\mathbb{R}$: the set of real numbers;

$\mathbb{N}$: the set of natural numbers;

$\mathbb{N}^+$: the set of positive natural numbers;

$\|x\| = \sqrt{\sum_{i=1}^{n}|x_i|^2}$: the Euclidean norm of a vector $x \in \mathbb{R}^n$;

$P = P^T > 0$: a symmetric positive definite matrix P, where the superscript T represents the transpose of a matrix;

$I_{n \times n}$: identity matrix of dimension $n \times n$;

$\sup|f|$: the least upper bound of a function f;

$tr[X]$: the trace of a square matrix X;

$\mathscr{L}_2$ class: a vector signal $x \in \mathbb{R}^n$ is said to belong to $\mathscr{L}_2$ class ($x \in \mathscr{L}_2$), if $\sqrt{x^T(t)x(t)} < \infty$, $\forall t \geq 0$;

$\mathscr{L}_\infty$ class: a vector signal $x \in \mathbb{R}^n$ is said to belong to $\mathscr{L}_\infty$ class ($x \in \mathscr{L}_\infty$), if $\max_{t \geq 0}\|x(t)\| < \infty$, $\forall t \geq 0$.

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It is a great pleasure to acknowledge my deepest thanks and gratitude to my supervisor Dr. Simone Baldi, Assistant Professor at the Delft Center for Systems and Control – Delft University of Technology, for suggesting this research topic and for his continuous support and guidance.

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Chapter 1

Introduction

Automated driving is an active area of research striving to increase road safety, manage traffic congestion, and reduce vehicles' emissions by introducing automation into road traffic [2]. In this chapter, we introduce the typical features of a particular aspect of automated driving, namely, vehicle platooning, we identify the important research done in this field, and we describe its common practical challenges. This chapter is organized as follows: Section 1-1 provides a definition of vehicle platooning along with the main adopted control strategies and stability criteria. Section 1-2 presents the state-of-the-art studies conducted by the research community in this field and the important practical challenges of cooperative and heterogeneous platooning. Section 1-3 summarizes the research objectives and contributions of this MSc thesis and finally Section 1-4 concludes this chapter by presenting the outline of the proposed work.

1-1 Vehicle platooning

Platooning is an automated driving method in which vehicles are grouped into platoons, where the speed of each vehicle (except eventually the speed of the leading vehicle) is automatically adjusted so as to maintain a safe inter-vehicle distance [3]. The most celebrated technology to enable platooning is Cooperative Adaptive Cruise Control (CACC), an extension of Adaptive Cruise Control (ACC) [4] where platooning is enabled by inter-vehicle communication in addition to on-board sensors as shown in Fig. 1-1.
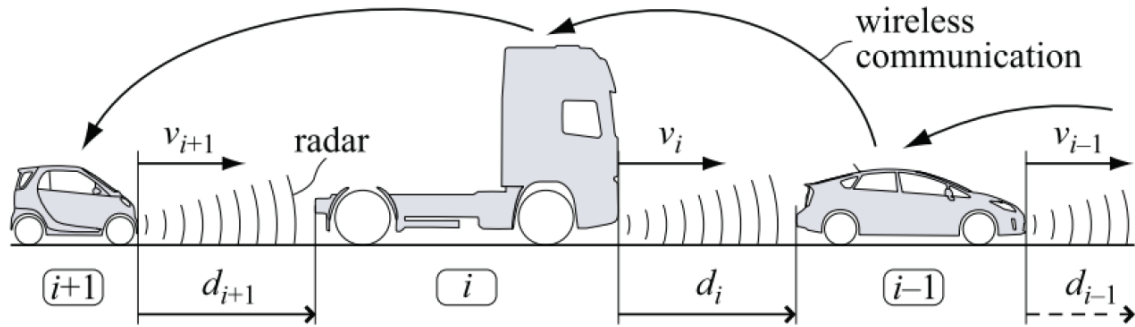
Figure 1-1: CACC-equipped vehicle platoon [1]

CACC systems have overcome ACC systems in view of their better string stability properties [5]: string stability implies that disturbances which are introduced into a traffic flow by braking and accelerating vehicles are not amplified in the upstream direction. In fact, while string stability in ACC strategies cannot be guaranteed for inter-vehicle time gaps smaller than 1 second [6], CACC was shown to guarantee string stability for time gaps significantly smaller than 1 second [1]. This directly leads to improved road throughput [7], reduced aerodynamic drag, and reduced fuel consumption [8] over ACC systems.

1-2 State of the art

Despite this potential, state-of-the-art studies and demonstrations of CACC crucially rely on the assumption of a homogeneous platoon (vehicle-independent driveline dynamics), shown in Fig. 1-2.
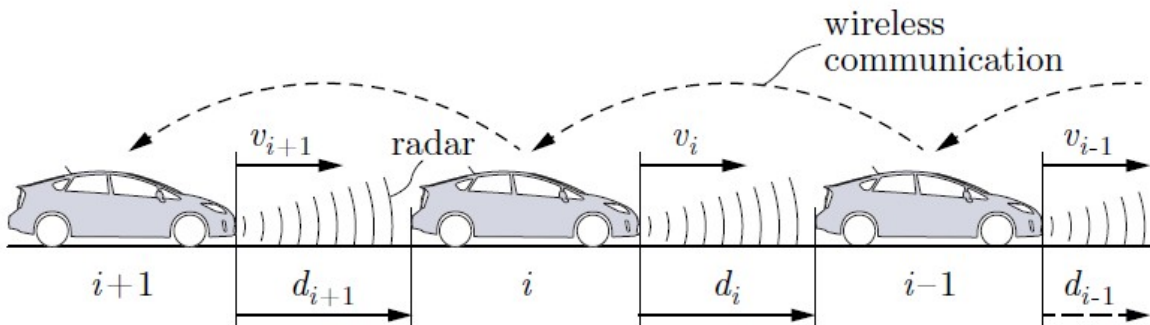
Figure 1-2: CACC-equipped homogeneous vehicle platoon [1]

Under this assumption, a one-vehicle look-ahead cooperative adaptive cruise controller was synthesized in [1], by using a performance oriented approach to define string stability. An adaptive bidirectional platoon-control method was derived in [9] which utilized a coupled sliding mode controller to enhance the string stability characteristics of the bidirectional platoon topology. A longitudinal controller based on a constant spacing policy was developed in [10],

showing that string stability can be achieved by broadcasting the leading vehicle's acceleration and velocity to all vehicles in the platoon. In [11], a linear controller was augmented by a model predictive control strategy to maintain the platoon's stability while integrating safety and physical constraints. In addition, for a platoon composed of identical agents with different controllers, [12] assessed the performance and challenges, in terms of string stability, of unidirectional and asymmetric bidirectional control strategies.

Communication is an important ingredient of CACC systems: the work [13] reviews the practical challenges of CACC and highlights the importance of robust wireless communication. Fig. 1-3 illustrates the wireless networked control system underlying a CACC strategy, and stresses on some of the wireless network's realistic characteristics.
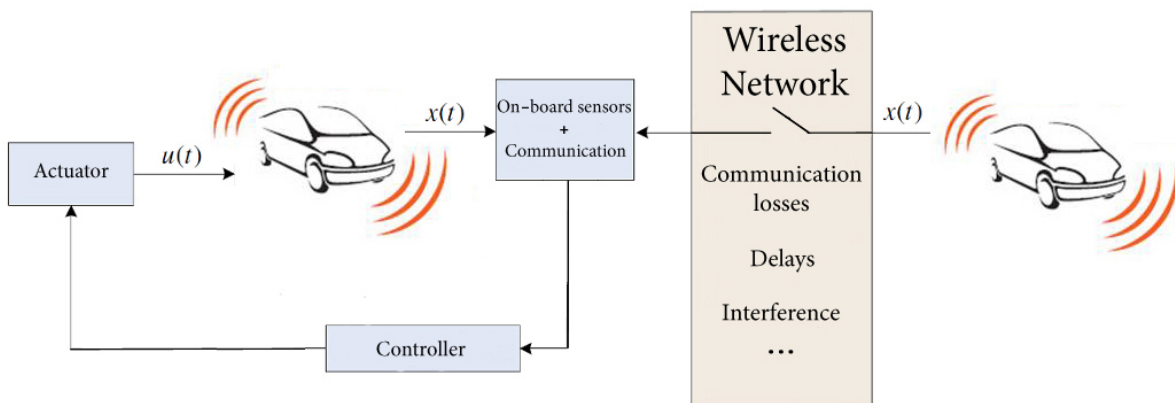
Figure 1-3: Wireless networked control system

From here a series of studies aiming at addressing the effect of non-ideal communication on CACC performance: in order to account for network delays and packet losses caused by the wireless network, an H_∞ controller was synthesized in [14], guaranteeing string stability criteria and robustness for some small parametric uncertainty. The authors in [15] derived a controller that integrates inter-vehicle communication over different realistic network conditions which models time delays, packet losses, and interferences. Random packet dropouts were modeled as independent Bernoulli processes in [16] in order to derive a scheduling algorithm and design a controller for vehicular platoons with inter-vehicle network capacity limitation that guarantees string stability and zero steady state spacing errors.

All the aforementioned works rely on the crucial platoon's homogeneity assumption. However, in practice, having a homogeneous platoon is not feasible: there will always be some heterogeneity among the vehicles in the platoon (e.g. different driveline dynamics, parametric and networked-induced uncertainties). A study conducted in [17] assessed the causes for heterogeneity of vehicles in a platoon and their effects on string stability. A distributed adaptive sliding mode controller for a heterogeneous vehicle platoon was derived in [18] to guarantee string stability and adaptive compensation of disturbances based on constant spacing policy. While addressing heterogeneous platoons to some extent, the aforementioned work neglects the effect of wireless communication, as pointed out by [13].

1-3   Research objectives and contributions

The brief overview of the state-of-the-art reveals the need to develop CACC with new functionalities, that can handle platoons of heterogeneous vehicles, and guarantee string stability while adapting to changing conditions and unreliable communication.

Consequently, the following thesis objectives are defined:

- **Heterogeneity:** Address the problem of CACC for heterogeneous platoons where the heterogeneity of the platoon is represented by different (and uncertain) time constants for the driveline dynamics and possibly different (and uncertain) engine performance coefficients through an adaptive control strategy.

- **Unreliable communication:** Model communication losses via an extended average dwell-time framework and synthesize an adaptive switched control strategy to overcome the loss of information.

- **Validation:** Validate the control strategies in software by simulating the controllers on a CACC-equipped heterogeneous platoon.

These objectives are addressed by means of the following contributions:

- Using a Model Reference Adaptive Control (MRAC) augmentation method, we prove analytically the asymptotic convergence of the heterogeneous platoon to an appropriately defined string stable reference platoon.

- Furthermore, inter-vehicle communication losses are handled by switching the control strategy of the vehicle at issue to a string stable ACC strategy with a different reference model. For this adaptive switching control scheme, stability with bounded state tracking error is proven under realistic switching conditions that match the Packet Error Rate of the two most widely adopted vehicular wireless communication standards, namely IEEE 802.11p/wireless access in vehicular environment (WAVE) and long-term evolution (LTE) [19],[20].

- Finally, both control strategies are tested in MATLAB/Simulink [21] on a heterogeneous platoon of 5+1 vehicles. To model realistic vehicular ad hoc networks (IEEE 802.11p, WAVE), we use the TrueTime2.0 [22] wireless network simulator.

1-4   Thesis outline

This thesis is organized as follows.

In **Chapter 2**, the system structure and control objectives of a heterogeneous vehicle platoon with engine performance losses are presented.

Chapter 3 presents a MRAC augmentation of a CACC strategy to stabilize the platoon under an ideal inter-vehicle communication network.

Chapter 4 presents an adaptive switched control strategy to stabilize the platoon in the heterogeneous scenario with engine performance losses while coping with an unreliable inter-vehicle communication network.

Chapter 5 aims at validating the theoretical analysis by simulating the proposed control strategies on a CACC-equipped heterogeneous vehicle platoon under reliable and unreliable inter-vehicle communication networks.

Chapter 6 summarizes the thesis contributions and presents some concluding remarks as well as suggestions for future works.

Chapter 2

System structure and control objectives

This chapter is organized as follows: Section 2-1 presents a mathematical model for a heterogeneous platoon and states the platooning stability objectives. Section 2-2 redefines the platoon model as a uncertain linear time invariant cascaded system, and finally Section 2-3 formulates the heterogeneous platooning control problem.

2-1 Heterogeneous platoon dynamics and stability objectives

Consider a heterogeneous platoon with M vehicles. Fig. 2-1 shows the platoon where v_i represents the velocity (m/s) of vehicle i, and d_i the distance (m) between vehicle i and its preceding vehicle $i-1$. This distance is measured using a radar mounted on the front bumper of each vehicle. Furthermore, each vehicle in the platoon can communicate with its preceding vehicle via wireless communication.
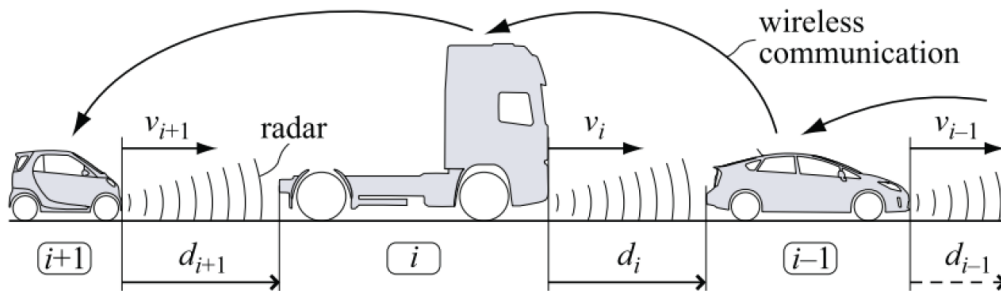

Figure 2-1: CACC-equiped heterogeneous vehicle platoon [1]

The main goal of every vehicle in the platoon, except the leading vehicle, is to maintain a desired distance $d_{\mathrm{r},i}$ between itself and its preceding vehicle. Define the set $S_M = \{i \in \mathbb{N} | \ 1 \leq$

$i \leq M$} with the index $i = 0$ reserved for the platoon's leader (leading vehicle). A constant time headway (CTH) spacing policy will be adopted to regulate the spacing between the vehicles [23]. The CTH is implemented by defining the desired distance as:

$$d_{\mathrm{r},i}(t) = r_i + h_i v_i(t) \ , \ \ i \in S_M \tag{2-1}$$

where r_i is the standstill distance (m) and h_i the time headway (s) (or time gap). It is now possible to define the spacing error (m) of the ith vehicle as:

$$e_i(t) = d_i(t) - d_{\mathrm{r},i}(t) \tag{2-2}$$
$$= (q_{i-1}(t) - q_i(t) - L_i) - (r_i + h_i v_i(t)) \tag{2-3}$$

with q_i and L_i representing the rear-bumper position (m) and length (m) of vehicle i, respectively.

A desired behavior of the platoon is instantiated when the effect of disturbances (e.g. emergency braking) introduced along the platoon is attenuated as they propagate in the upstream direction [1]. Such behavior is denoted with the term string stability. A standard definition of string stability considered in this work is given as follows.

Definition 2-1.1. *String stability* [1]: *Let the acceleration of vehicle i be denoted with $a_i(t)$. Then a platoon is considered string stable if,*

$$\sup_{\omega} |\Gamma_i(j\omega)| = \sup_{\omega} \left| \frac{a_i(j\omega)}{a_{i-1}(j\omega)} \right| \leq 1, \ \ 1 \leq i \leq M \tag{2-4}$$

where, $a_i(s)$ is the Laplace transform of the acceleration $a_i(t)$ of vehicle i.

The control objective is to regulate e_i to zero $\forall i \in S_M$, while ensuring the string stability of the platoon.

The following model is used to represent the dynamics of the vehicles

$$\begin{pmatrix} \dot{e}_i \\ \dot{v}_i \\ \dot{a}_i \end{pmatrix} = \begin{pmatrix} 0 & -1 & -h_i \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau_i} \end{pmatrix} \begin{pmatrix} e_i \\ v_i \\ a_i \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} v_{i-1} + \begin{pmatrix} 0 \\ 0 \\ \frac{\Lambda_i}{\tau_i} \end{pmatrix} u_i \tag{2-5}$$

where a_i and u_i are respectively the acceleration (m/s^2) and control input (m/s^2) of vehicle i. Moreover, τ_i represents each vehicle's **unknown** driveline time constant (s) and Λ_i represents the **unknown** engine's performance: for the nominal performance we have $\Lambda_i = 1$, while performance might decrease below 1 due to wear or wind gusts, or increase above 1 due to wind in the tail; Λ_i can also be affected by the slope of the road. Model (2-5) was proposed in [1] for the special case of $\Lambda_i = 1$, $\forall i \in S_M$.

The leading vehicle's model is defined as:

$$\begin{pmatrix} \dot{e}_0 \\ \dot{v}_0 \\ \dot{a}_0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau_0} \end{pmatrix} \begin{pmatrix} e_0 \\ v_0 \\ a_0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{\tau_0} \end{pmatrix} u_0. \tag{2-6}$$

2-2 Uncertain heterogeneous platoon model

Note that, under the assumption of a homogeneous platoon with perfect engine performance, we have $\tau_i = \tau_0$ and $\Lambda_i = 1$, $\forall i \in S_M$. In this work, we remove the homogeneous assumption by considering that $\forall i \in S_M$, τ_i can be represented as the sum of two terms:

$$\tau_i = \tau_0 + \Delta\tau_i \tag{2-7}$$

where τ_0 is a known constant representing the driveline dynamics of the leading vehicle and $\Delta\tau_i$ is an unknown constant deviation of the driveline dynamics of vehicle i from τ_0. In fact, $\Delta\tau_i$ acts as an unknown parametric uncertainty. In addition, we remove the perfect engine performance assumption by considering Λ_i as an unknown input uncertainty. Substituting (2-7) into the third differential equation of (2-5) we obtain

$$\tau_i \dot{a}_i = -a_i + \Lambda_i u_i \tag{2-8}$$

$$\dot{a}_i = -\frac{1}{\tau_0} a_i + \frac{1}{\tau_0} \Lambda_i^* [u_i + \Omega_i^* \phi_i] \tag{2-9}$$

where $\Lambda_i^* = \frac{\Lambda_i \tau_0}{\tau_i}$, $\Omega_i^* = -\frac{\Delta\tau_i}{\Lambda_i \tau_0}$, and $\phi_i = -a_i$.

Substituting (2-8) in (2-5), the vehicle model in a heterogeneous platoon with engine performance loss under spacing policy (2-1) can be defined as the uncertain linear-time invariant system:

$$\begin{pmatrix} \dot{e}_i \\ \dot{v}_i \\ \dot{a}_i \end{pmatrix} = \begin{pmatrix} 0 & -1 & -h_i \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau_0} \end{pmatrix} \begin{pmatrix} e_i \\ v_i \\ a_i \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} v_{i-1} \tag{2-10}$$

$$+ \begin{pmatrix} 0 \\ 0 \\ \frac{1}{\tau_0} \end{pmatrix} \Lambda_i^* [u_i + \Omega_i^* \phi_i], \ \forall i \in S_M. \tag{2-11}$$

2-3 Problem formulation

We can now formulate the control objective for the heterogeneous platoon as follows:

Problem 1. *Heterogeneous platooning: Design the control input $u_i(t)$, $\forall i \in S_M$, such that the heterogeneous platoon described by (2-6) and (2-10) asymptotically tracks the behavior of a string stable platoon for any possible vehicles' parametric uncertainty.*

Having this control problem at hand, in the following Chapters we will subsequently design two control strategies to tackle it:

- In Chapter 3 an adaptive control strategy that assumes an ideal inter-vehicle communication network between the vehicles is designed. Meaning the control input $u_i(t)$ has always available, at any time, any significant information that is received from its the preceding vehicle.

- In Chapter 4 a switched adaptive control strategy is developed that models the inter-vehicle communication network as a switching signal characterized by a mode dependent average dwell time (MDADT).

Adaptive heterogeneous platooning

This chapter is organized as follows: Section 3-1 defines the platooning control problem under an ideal inter-vehicle communication network. Moreover, in order to design the control input, Section 3-2 presents string stable reference dynamics for the vehicles in the platoon. Section 3-3 defines a stabilizing $u_i(t)$ through a MRAC augmentation approach, while Section 3-4 presents the main stability and convergence results.

3-1 Problem statement

To design the control input $u_i(t)$ $\forall i \in S_M$ such that the dynamics of the platoon converge to string stable dynamics, it is essential to know what information about the dynamics of the vehicle i and its preceding one $(i-1)$ is available and reliable for utilization. Therefore, Section 3-1-1 defines the adopted model for the wireless communication and consequently Section 3-1-2 formulates the control problem.

3-1-1 Ideal inter-vehicle communication network

In this first part of the thesis, we assume an ideal wireless network between consecutive vehicles. Fig. 3-1 illustrates this scenario where any information sent by the preceding vehicle is received with no losses, no delays, and no distortions.
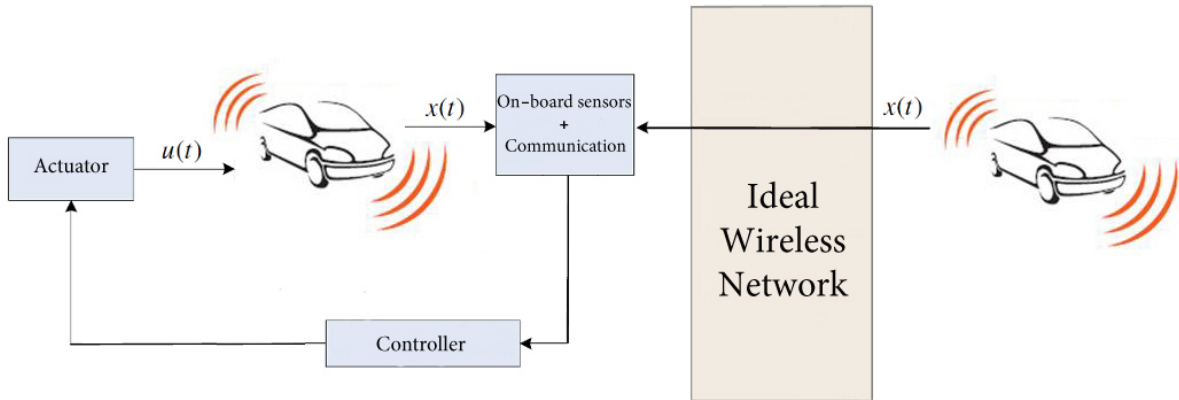
Figure 3-1: Networked control system with an ideal communication network

Under this assumption, the following section formulates the resultant platooning control problem.

3-1-2   Control problem formulation

In the presence of an ideal inter-vehicle communication network, the following problem is defined:

Problem 2. *Adaptive heterogeneous platooning: Design the adaptive control input $u_i(t)$, $\forall i \in S_M$, such that the heterogeneous platoon described by (2-6) and (2-10) asymptotically tracks the behavior of a string stable platoon for any possible vehicles' parametric uncertainty under an ideal communication network between all consecutive vehicles.*

This adaptive control system is outlined in Fig. 3-2.
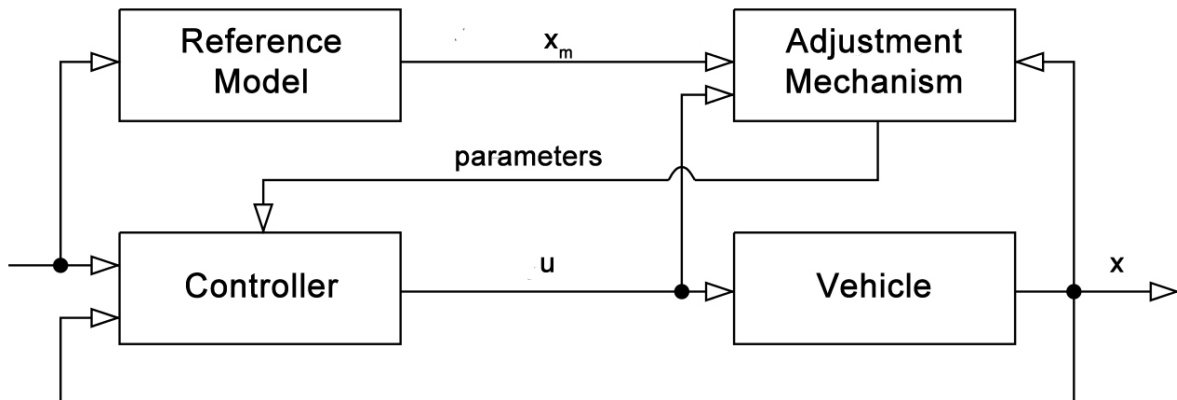
Figure 3-2: Model reference adaptive control

In order to synthesize a MRAC strategy for the platoon we first need to define string stable reference dynamics which are the subject of the following section.

3-2 CACC reference model

Under the baseline conditions of identical vehicles, perfect engine performance, and no communication losses between any consecutive vehicles, [1] derived, using a CACC strategy, a controller and spacing policy which proved to guarantee the string stability of the platoon.

The time headway constant of the spacing policy (2-1) is set as $h_i = h^C$, $\forall i \in S_M$, where the superscript C indicates that communication is maintained between the vehicle and its preceding one. Moreover, the CACC baseline controller is defined as:

$$h^C \dot{u}_{bl,i}^C = -u_{bl,i}^C + K_p^C e_i + K_d^C \dot{e}_i + u_{bl,i-1}^C \ , \ \ i \in S_M \tag{3-1}$$

In addition, the leading vehicle control input is defined as:

$$h_0 \dot{u}_0 = -u_0 + u_r \tag{3-2}$$

where u_r is the platoon's input representing the desired acceleration (m/s^2) of the leading vehicle, and h_0 a positive design parameter denoting the nominal time headway. The initial condition of (3-2) is $u_0(0) = 0$. The cooperative aspect of (3-1) resides in $u_{bl,i-1}^C$, which is received over the wireless communication link between vehicle i and $i - 1$.

We can now define the reference dynamics for (2-10) as: the dynamics of system (2-10) with $\Omega_i^* = 0$, $\Lambda_i^* = 1$, and control input $u_{i,m} = u_{bl,i}^C$.

The reference model can be therefore described by:

$$
\begin{pmatrix} \dot{e}_{i,m} \\ \dot{v}_{i,m} \\ \dot{a}_{i,m} \\ \dot{u}_{i,m} \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & -1 & -h^C & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{\tau_0} & \frac{1}{\tau_0} \\ \frac{K_p^C}{h^C} & -\frac{K_d^C}{h^C} & -K_d^C & -\frac{1}{h^C} \end{pmatrix}}_{A_m^C} \underbrace{\begin{pmatrix} e_{i,m} \\ v_{i,m} \\ a_{i,m} \\ u_{i,m} \end{pmatrix}}_{x_{i,m}}
$$

$$
+ \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{K_d^C}{h^C} & \frac{1}{h^C} \end{pmatrix}}_{B_w^C} \underbrace{\begin{pmatrix} v_{i-1} \\ u_{bl,i-1}^C \end{pmatrix}}_{w_i}, \ \forall i \in S_M \tag{3-3}
$$

where $x_{i,m}$ and w_i are vehicle i's reference state vector and exogenous input vector, respectively.

Consequently, (3-3) is of the following form:

$$\dot{x}_{i,m} = A_m^C x_{i,m} + B_w^C w_i, \ \forall i \in S_M. \tag{3-4}$$

Furthermore, using (3-2), the leading vehicle's model becomes

$$
\begin{pmatrix} \dot{e}_0 \\ \dot{v}_0 \\ \dot{a}_0 \\ \dot{u}_0 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{\tau_0} & \frac{1}{\tau_0} \\ 0 & 0 & 0 & -\frac{1}{h_0} \end{pmatrix}}_{A_r} \underbrace{\begin{pmatrix} e_0 \\ v_0 \\ a_0 \\ u_0 \end{pmatrix}}_{x_0} + \underbrace{\begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{h_0} \end{pmatrix}}_{B_r} u_r.
\tag{3-5}
$$

Reference model (3-4) has been proven in [1] to be asymptotically stable around the equilibrium point

$$
x_{i,m,eq} = \begin{pmatrix} 0 & \overline{v}_0 & 0 & 0 \end{pmatrix}^T \text{ for } x_0 = x_{i,m,eq} \text{ and } u_r = 0
\tag{3-6}
$$

where $\overline{v}_0$ is a constant velocity, provided that the following Routh-Hurwitz conditions are satisfied

$$
h^C > 0, \quad K_p^C, K_d^C > 0, \quad K_d^C > \tau_0 K_p^C.
\tag{3-7}
$$

To assess the string stability of the reference platoon dynamics, it is found that

$$
\Gamma_i(s) = \frac{1}{h^C s + 1}, \quad \forall i \in S_M
\tag{3-8}
$$

Therefore, we conclude that (3-8) satisfies the string stability condition (2-4) of Definition 2-1.1 for any $h^C > 0$, and thus the defined reference platoon dynamics (3-4) are string stable.

3-3   Adaptive augmentation of a baseline strategy

In this Section, reference model (3-4) will be used to design the control input $u_i(t)$ such that the uncertain platoon's dynamics described by (2-6) and (2-10) converge to string stable dynamics. With this scope in mind, we will augment a baseline controller with an adaptive term using an architecture as in Fig. 3-3, similar to the one as proposed in [24].
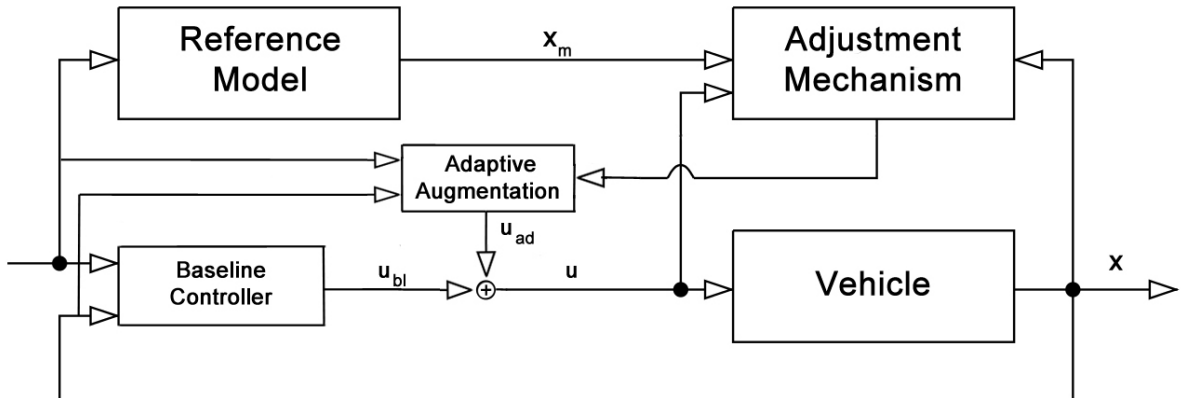
Figure 3-3: MRAC augmentation of a baseline strategy

To include the adaptive augmentation, the input $u_i(t)$ is split, $\forall i \in S_M$, into two different inputs:

$$u_i(t) = u_{bl,i}(t) + u_{ad,i}(t) \tag{3-9}$$

where $u_{bl,i}$ and $u_{ad,i}$ are the baseline controller and the adaptive augmentation controller (to be constructed), respectively.

First, define the control input of the leading vehicle $u_0(t)$ as in (3-2). Moreover, define $u_{bl,i}(t) = u_{bl,i}^C(t)$. Substituting (3-9) into (2-10), we get the uncertain vehicle model

$$\dot{x}_i = A_m^C x_i + B_w^C w_i + B_u \Lambda_i^* \left[u_{ad,i} + \Theta_i^{*T} \Phi_i \right], \ \forall i \in S_M \tag{3-10}$$

where $x_i = \begin{pmatrix} e_i & v_i & a_i & u_{bl,i} \end{pmatrix}^T$, and the matrices A_m^C and B_w^C are known and defined in (3-4), and $B_u = \begin{pmatrix} 0 & 0 & \frac{1}{\tau_0} & 0 \end{pmatrix}^T$.

The uncertain ideal parameter vector is defined as

$$\Theta_i^* = \begin{pmatrix} K_{u,i}^* & \Omega_i^* \end{pmatrix}^T$$

where $K_{u,i}^* = 1 - \Lambda_i^{*-1}$. The regressor vector is defined as

$$\Phi_i = \begin{pmatrix} u_{bl,i} & \phi_i \end{pmatrix}^T.$$

Therefore, the heterogeneous platoon with engine performance loss and control input (3-9) can be defined as system (3-5)-(3-10).

Furthermore, taking (3-4) as the vehicle reference model, the adaptive control input is defined as

$$u_{ad,i} = -\Theta_i^T \Phi_i \tag{3-11}$$

where Θ_i is the estimate of Θ_i^*.

Define the state tracking error as

$$\tilde{x}_i = x_i - x_{i,m}, \ \forall i \in S_M. \tag{3-12}$$

Replacing (3-11) in (3-10) and subtracting (3-4) results in the following state tracking error dynamics

$$\dot{\tilde{x}}_i = A_m^C \tilde{x}_i - B_u \Lambda_i^* \tilde{\Theta}_i^T \Phi_i \tag{3-13}$$

where $\tilde{\Theta}_i = \Theta_i - \Theta_i^*$.

Since A_m^C is stable, there exists a unique symmetric positive define matrix $P_m = P_m^T > 0$ such that

$$(A_m^C)^T P_m + P_m A_m^C + Q_m = 0$$

where $Q_m = Q_m^T > 0$ is a designed matrix. Define the adaptive law

$$\dot{\Theta}_i = \Gamma_\Theta \Phi_i \tilde{x}_i^T P_m B_u \tag{3-14}$$

with $\Gamma_\Theta = \Gamma_\Theta^T > 0$ being the adaptive gain.

3-4   Main results

Using the previously presented design, the following stability and convergence results can be stated:

Theorem 3-4.1. *Consider the heterogeneous platoon model (2-10) with reference model (3-4). Then, the adaptive input (3-11) with adaptive law (3-14) makes the platoon's dynamics asymptotically converge to string stable dynamics. Consequently,*

$$\lim_{t \to \infty} [x_i(t) - x_{i,m}(t)] = 0, \ \forall i \in S_M$$

and

$$\lim_{t \to \infty} \|\Theta_i^T(t)\Phi_i(t)\| = 0, \ \forall i \in S_M.$$

Proof. Define a radially unbounded quadratic Lyapunov candidate function as:

$$V_i(t) = \tilde{x}_i^T P_m \tilde{x}_i + \text{tr}(\tilde{\Theta}_i \Gamma_\Theta^{-1} \tilde{\Theta}_i \Lambda_i^*).$$

Taking the time derivative of $V_i(t)$ and substituting the error dynamics into (3-13) results in:

$$\dot{V}_i(t) = -\tilde{x}_i^T Q_m \tilde{x}_i - 2\tilde{x}_i^T P_m B_u \Lambda_i^* \tilde{\Theta}_i \Phi_i + 2\text{tr}(\tilde{\Theta}_i \Gamma_\Theta^{-1} \dot{\tilde{\Theta}}_i \Lambda_i^*).$$

When calculating the time derivative we have used the fact that the extra input from system $i-1$ in (3-3) to reference model i is canceled by the last term in (3-1). In such a way we can proceed showing that this interconnection does not destroy stability.

Using the identity

$$a^T b = \text{tr}(ba^T)$$

results in:

$$\dot{V}_i(t) = -\tilde{x}_i^T Q_m \tilde{x}_i + 2\text{tr}(\tilde{\Theta}_i^T \{\Gamma_\Theta^{-1} \dot{\tilde{\Theta}}_i - \Phi_i \tilde{x}_i^T P_m B_u\}\Lambda_i^*). \tag{3-15}$$

Choosing the adaptive law as in (3-14) reduces (3-15) to:

$$\dot{V}_i(t) = -\tilde{x}_i^T Q_m \tilde{x}_i \leq 0 \tag{3-16}$$

which proves the uniform ultimate boundedness of $(\tilde{x}_i, \tilde{\Theta}_i)$. Furthermore, it can be concluded from (3-16) that $\tilde{x}_i \in L_2$. In addition, since $w_i(t)$ is bounded, then $x_{i,m} \in L_\infty$ and consequently, $x_i \in L_\infty$ and $u_{bl,i} \in L_\infty$. Moreover, since Θ_i^* is constant then the estimated value is also bounded, $\tilde{\Theta}_i \in L_\infty$. Since $(x_i, u_{bl,i}) \in L_\infty$ and the components of the regressor vector Φ_i are locally Lipschitz continuous, then the regressor's components are bounded. Therefore, $u_i \in L_\infty$ and $\dot{x}_i \in L_\infty$. Hence, $\dot{\tilde{x}}_i \in L_\infty$, which implies that $\ddot{V}_i \in L_\infty$. Thus, $\dot{V}_i$ is a uniformly continuous function of time.

In addition, since

       1) $V_i(t)$ has a lower bound

       2) $\dot{V}_i(t) \leq 0$

       3) $\dot{V}_i(t)$ is uniformly

then by Barbalat's Lemma:

$$\text{1) } V_i(t) \text{ tends to a limit}$$

$$\text{2) } \lim_{t \to \infty} \dot{V}_i(t) = 0$$

Hence, the tracking error $\tilde{x}_i$ tends asymptotically to zero as $t \to \infty$:

$$\lim_{t \to \infty} \tilde{x}_i(t) = 0$$

Furthermore, since V_i is radially unbounded, then $\tilde{x}_i$ globally asymptotically tends to zero as $t \to \infty$. This means that the tracking error dynamics are globally asymptotically stable. From (3-13), it can be deduced that $\ddot{\tilde{x}}_i \in L_\infty$ which indicates that $\dot{\tilde{x}}_i$ is uniformly continuous. Moreover, since $\tilde{x}_i \to 0$ as $t \to \infty$ then using Barbalat's lemma:

$$\lim_{t \to \infty} \|\dot{\tilde{x}}_i(t)\| = 0.$$

Which leads to:

$$\lim_{t \to \infty} \|\tilde{\Theta}_i^T(t)\Phi_i(t)\| = 0 \ , \ \forall i \in S_M.$$

This proves that for any bounded w_i, the closed-loop system globally asymptotically tracks the reference model as $t \to \infty$.

This completes the proof.

$\square$

Remark 1. *We have shown that the augmented control input*

$$u_i = u_{bl,i} + u_{ad,i}$$
$$= u_{bl,i} - \Theta_i^T \Phi_i$$

guarantees the global asymptotic convergence of the vehicle's states to the reference ones $\forall i \in S_M$. Since the reference model was chosen as the string stable controlled vehicle model in the homogeneous scenario (3-4), then the heterogeneous platoon converges asymptotically to a string stable platoon, and the spacing error goes asymptotically to zero, i.e. $\lim_{t \to \infty} e_i = 0$, $\forall i \in S_M$.

3-5   Summary

A novel adaptive control strategy to stabilize a platoon with non-identical vehicle dynamics and engine performance losses has been derived in this chapter under the assumption of an ideal inter-vehicle communication network. Furthermore, the asymptotic convergence of the platoon's dynamics to string stable dynamics has been analytically proven through a MRAC augmentation approach. However, communication losses are always present in practice and coping with them is the subject of the next chapter.

Chapter 4

Adaptive switched heterogeneous platooning

This chapter is organized as follows: Section 4-1 defines the platooning control problem under a non-ideal inter-vehicle communication network. Moreover, in order to design the control input, Section 4-2 presents mixed CACC-ACC string stable reference dynamics for the vehicles in the platoon. Section 4-3 defines a stabilizing $u_i(t)$ through an adaptive switched control approach, while Section 4-4 presents the main stability and convergence results. Finally, Section 4-5 assesses the designed control strategy's robustness to network induced delays.

4-1 Problem statement

To design the control input $u_i(t)$ $\forall i \in S_M$ such that the dynamics of the platoon converge to string stable dynamics, it is essential to know what information about the dynamics of the vehicle i and its preceding one $(i-1)$ is available and reliable for utilization. Therefore, Section 4-1-1 defines the adopted model for the wireless communication and consequently Section 4-1-2 formulates the control problem.

4-1-1 Non-ideal inter-vehicle communication network

In this second part of the thesis, we assume an non-ideal wireless network between consecutive vehicles. More specifically we consider that the wireless network suffers from communication losses (or packet dropouts). Fig. 4-1 illustrates this scenario where information sent by the preceding vehicle is not guaranteed to arrive to the receiving end.
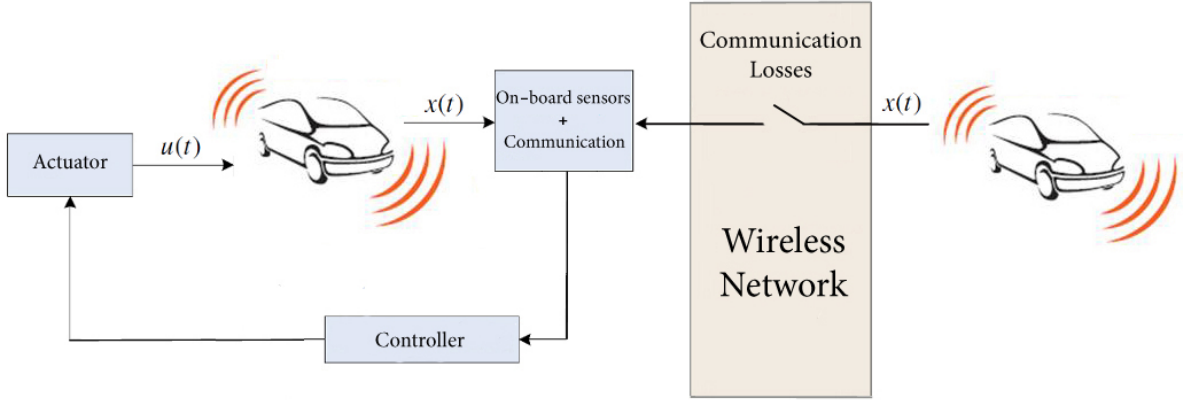
Figure 4-1: Networked control system with communication losses

One way of handling the unavoidable communication losses is by switching between CACC and ACC depending on the network's state at each single communication link. In this aim, an adaptive switched control method is presented for the scenario with joint heterogeneous dynamics and inter-vehicle communication losses. Note that ACC does not require inter-vehicle communication, but as a drawback it requires to increase the time gap in order to guarantee string stability [1]. So, the switched control system also takes into account that a different spacing policy might be active in the CACC case (indicated with h^C) and in the ACC case (indicated with h^L), where the superscript L stands for communication loss. The adaptive switched controller is based on a Mode-Dependent Average Dwell Time (MDADT) which is used to characterize the network switching behavior as a consequence of communication losses.

Definition 4-1.1. *Mode-Dependent Average Dwell Time [25]: For a switched system with S subsystems, a switching signal $\sigma(\cdot)$, taking values in $\{1, 2, 3, ..., S\} = \mathscr{M}$, and for $s \geq t \geq 0$ and $k \in \mathscr{M}$, let $N_{\sigma k}(t, s)$ denote the number of times subsystem k is activated in the interval $[t, s)$, and let $T_k(t, s)$ be the total time subsystem k is active in the interval $[t, s)$. The switching signal $\sigma(\cdot)$ is said to have a MDADT τ_{ak} if there exist positive numbers N_{0k}, called mode-dependent chatter bounds, and τ_{ak} such that:*

$$N_{\sigma k}(t, s) \leq N_{0k} + \frac{T_k(t, s)}{\tau_{ak}}, \ \forall s \geq t \geq 0. \tag{4-1}$$

Moreover, the general notions of stability under slowly switching conditions, and the definitions of dwell time and average dwell time are introduced in Appendix A. Furthermore, an example of a switching signal with MDADTs $\tau_{a1} = 2\ s$ and $\tau_{a2} = 3\ s$; and mode-dependent chatter bounds $N_{01} = N_{02} = 2$ is shown in Fig. 4-2 to illustrate Definition 4-1.1.
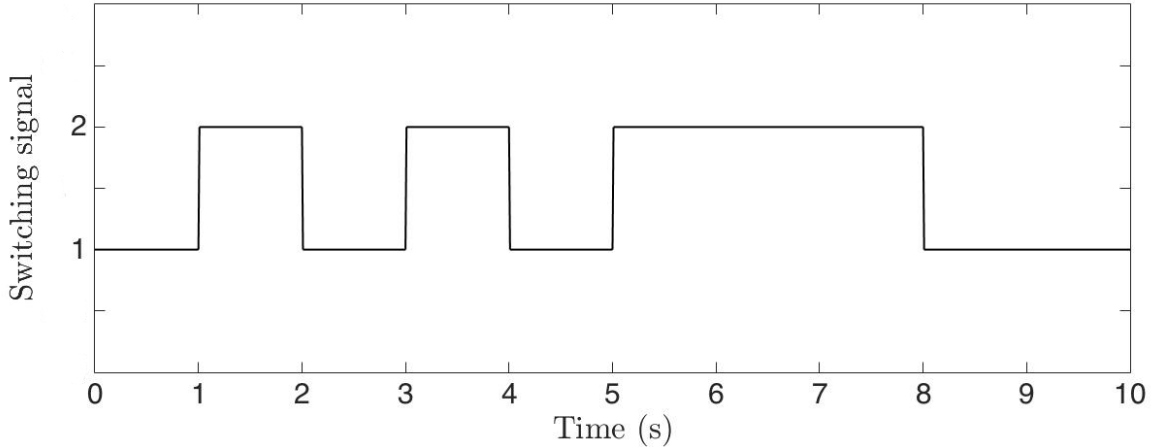
Figure 4-2: A MDADT switching signal

Note that $T_1 = T_2 = 5$ s and that $N_{\sigma 1} = 4$ and $N_{\sigma 2} = 3$. Therefore, (4-1) is satisfied for $k \in \{1, 2\}$.

Furthermore, in the presence of communication losses, the following notion of stability must be introduced.

Definition 4-1.2. *Global uniform ultimate boundedness [26]: A signal $\phi(t)$ is said to be globally uniformly ultimately bounded (GUUB) with ultimate bound if there exists a positive constant b, and for arbitrarily large $a \geq 0$, there is a time instant $T = T(a, b)$, where b and T are independent of t_0, such that*

$$\|\phi(t_0)\| \leq a \Rightarrow \|\phi(t)\| \leq b, \ \forall \ t \geq t_0 + T. \tag{4-2}$$

By extension, we say that a system is GUUB when its trajectories are GUUB.

4-1-2 Control problem formulation

In the presence of inter-vehicle communication losses, the following problem is defined:

Problem 3. *Adaptive switched heterogeneous platooning: Design the adaptive switched control input $u_i(t)$ and the switching parameters τ_{ak} and N_{0k} as in (4-1) such that for any MDADT switching signal satisfying (4-1) and in the presence of vehicles' parametric uncertainties, the heterogeneous platoon, described by (2-6) and (2-10), with communication losses tracks the behavior of a string stable platoon with GUUB error.*

Remark 2. *The reason for seeking GUUB stability (in place of asymptotic stability) is that asymptotic stability of switched systems with large uncertainties and average dwell time is a big open problem in control theory [27].*

This networked switched control system is outlined in Fig. 4-3.

Figure 4-3: Networked switched control system

4-2   Mixed CACC-ACC reference model

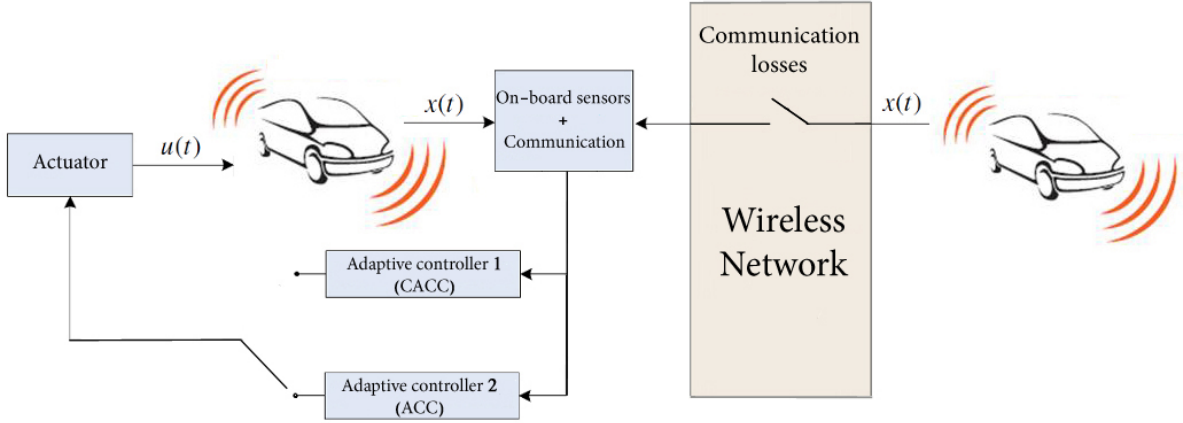In order to design the switched adaptive control input, we present in this section mixed CACC-ACC string stable dynamics which serve as reference dynamics of the vehicles in the platoon. Let S_M^L be the subset of S_M containing the indices of the vehicles that lose communication with their preceding vehicle. In addition, let S_M^C be the subset of S_M containing the indices of the vehicles with maintained communication with their preceding vehicle. In the presence of inter-vehicle communication losses, reference dynamics (3-4) fail in general to guarantee the string stability of the platoon since, $u_{bl,i-1}^C$ is now no longer present for measurement $\forall i \in S_M^L$, and (2-4) might be violated. In this case, the time headway constant of the spacing policy (2-1) is set as $h_i = h^L$, $\forall i \in S_M^L$, with h^L to be determined in order to recover string stability. To do so, we define a new ACC baseline controller as follows

$$h^L \dot{u}_{bl,i}^L = -u_{bl,i}^L + K_p^L e_i + K_d^L \dot{e}_i, \ \forall i \in S_M^L \tag{4-3}$$

where K_p^L and K_d^L are the design parameters of the controller, and $u_{bl,i}^C(0) = 0$, $\forall i \in S_M^L$, without loss of generality. Similar to the CACC case, the ACC reference model is defined as system (2-10) with $\Omega_i^* = 0$, $\Lambda_i^* = 1$, and control input $u_{i,m} = u_{bl,i}^L$. Thus, the reference model can be described by

$$
\begin{aligned}
\begin{pmatrix} \dot{e}_{i,m} \\ \dot{v}_{i,m} \\ \dot{a}_{i,m} \\ \dot{u}_{i,m} \end{pmatrix} &= \underbrace{\begin{pmatrix} 0 & -1 & -h^L & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{\tau_0} & \frac{1}{\tau_0} \\ \frac{K_p^L}{h^L} & -\frac{K_d^L}{h^L} & -K_d^L & -\frac{1}{h^L} \end{pmatrix}}_{A_m^L} \begin{pmatrix} e_{i,m} \\ v_{i,m} \\ a_{i,m} \\ u_{i,m} \end{pmatrix} \\[2mm]
&+ \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{K_d^L}{h^L} & 0 \end{pmatrix}}_{B_w^L} \underbrace{\begin{pmatrix} v_{i-1} \\ u_{bl,i-1}^C \end{pmatrix}}_{w_i}, \ \ \forall i \in S_M^L
\end{aligned}
\tag{4-4}
$$

which is of the form

$$\dot{x}_{i,m} = A_m^L x_{i,m} + B_w^L w_i, \quad \forall i \in S_M^L \tag{4-5}$$

The asymptotic stability of the reference model (4-5) around equilibrium point (3-6) can be guaranteed by deriving conditions on K_p^L and K_d^L through the Routh-Hurwitz stability criteria. These conditions were found to be the same as (3-7).

String stability of (4-5) can be additionally guaranteed by deriving sufficient conditions on the gains of controller (4-3) using condition (2-4) of Definition 2-1.1; when vehicle i is operating under ACC ($i \in S_M^L$), $\Gamma_i(s)$ is

$$\Gamma_i(s) = \frac{K_p^L + K_d^L s}{(\tau_0 s^3 + s^2 + K_d^L s + K_p^L)(h^L s + 1)}, \quad \forall i \in S_M^L. \tag{4-6}$$

It gives,

$$|\Gamma_i(j\omega)| = \frac{\sqrt{(K_d^L \omega)^2 + K_p^{L^2}}}{\sqrt{(h^L \omega)^2 + 1}\sqrt{(K_p^L - \omega^2)^2 + (K_d^L \omega - \tau_0 \omega^3)^2}}. \tag{4-7}$$

For a defined h^L, $\sup_w |\Gamma_i| \leq 1$, $\forall i \in S_M^L$, is verified by choosing K_p^L and K_d^L such that, $\forall \omega > 0$,

$$
\begin{aligned}
(h^L \tau_0)^2 \omega^6 &+ ((h^L)^2 - 2K_d^L \tau_0 (h^L)^2 + \tau_0^2)\omega^4 + (1 - 2K_p^L h^L \\
&+ (h^L K_d^L)^2 - 2K_d^L \tau_0)\omega^2 + ((h^L K_p^L)^2 - 2K_p^L) \geq 0.
\end{aligned}
\tag{4-8}
$$

Note. *A summary on the string stability conditions on the baseline CACC and ACC strategies is presented in Appendix B.*

Thus, for a homogeneous platoon with no engine performance loss, when a communication link is lost, one can switch, for that link, from a string stable CACC strategy designed via (3-12), to a string stable ACC strategy designed via (4-6).

The resulting string stable mixed CACC-ACC reference dynamics can be described by

$$\dot{x}_0 = A_r x_0 + B_r u_r \tag{4-9}$$

$$\dot{x}_{i,m} = A_m^C x_{i,m} + B_w^C w_i, \quad \forall i \in S_M^C \tag{4-10}$$

$$\dot{x}_{i,m} = A_m^L x_{i,m} + B_w^L w_i, \quad \forall i \in S_M^L. \tag{4-11}$$

4-3 Adaptive augmentation of a switched baseline strategy

In this section, reference models (4-10) and (4-11) will be used to design the piecewise continuous control input $u_i(t)$ such that the uncertain platoon's dynamics described by (2-6) and (2-10) track with a bounded error string stable dynamics even in the presence of communication losses.

We define a new switched control input as

$$u_i(t) = u_{bl,i}(t) + u_{ad,i}(t), \quad \forall i \in S_M \tag{4-12}$$

where

$$u_{bl,i}(t) = \begin{cases} u_{bl,i}^C, & \text{when communication is present} \\ u_{bl,i}^L, & \text{when communication is lost} \end{cases} \quad (4\text{-}13)$$

First, defining the control input of the leading vehicle $u_0(t)$ as in (3-2), results in a lead vehicle model as in (4-9). Then substituting (3-9) into (2-10), the uncertain switched linear system vehicle model becomes, $\forall i \in S_M$ and $\sigma_i(t) \in \mathscr{M} := \{1, 2\}$, as:

$$\dot{x}_i = A_{m,\sigma_i(t)} x_i + B_{w,\sigma_i(t)} w_i + B_u \Lambda_i^* \Big[u_{ad,i} + \Theta_i^{*T} \Phi_i \Big], \quad (4\text{-}14)$$

where $\sigma_i(\cdot)$ is the switching law of vehicle i (defined at the single link level), and $A_{m,\sigma_i(t)}$ and $B_{w,\sigma_i(t)}$ are time variant matrices taking values, depending on the activated subsystem, as the known matrices $A_{m,k}$ and $B_{w,k}$ respectively, defined in (4-10) and (4-11), with $k \in \mathscr{M}$ representing the two subsystems in our system. In fact, subsystem $k = 1$ is activated by $\sigma_i(\cdot)$ when communication is maintained between vehicle i and its preceding one (when $i \in S_M^C$), and subsystem $k = 2$ is activated by $\sigma_i(\cdot)$ otherwise (when $i \in S_M^L$).

Therefore, the heterogeneous platoon with engine performance loss under the control input $u_i(t) = u_{bl,i}(t) + u_{ad,i}(t)$ can be described by (4-9) and (4-14).

Furthermore, define the group of reference models representing the desired behavior of each subsystem as:

$$\dot{x}_{m,i}(t) = A_{m,\sigma_i(t)} x_{m,i}(t) + B_{w,\sigma_i(t)} w_i(t), \ \forall i \in S_M, \ \sigma_i(t) \in \mathscr{M} \quad (4\text{-}15)$$

where $x_{m,i} = \begin{pmatrix} e_{m,i} & v_{m,i} & a_{m,i} & u_{i,m} \end{pmatrix}^T$. Note that (4-15) is of the form (4-10) for $\sigma_i(t) = 1$ (when $i \in S_M^C$) and (4-10) for $\sigma_i(t) = 2$ (when $i \in S_M^L$).

The adaptive control input is defined as:

$$u_{ad,i}(t) = -\Theta_{i,\sigma_i(t)}^T \Phi_i \quad (4\text{-}16)$$

where $\Theta_{i,k}$ is the estimate of Θ_i^* of subsystem k. Moreover, the state tracking error is defined as in (3-12). Replacing (4-16) in (4-14) and subtracting (4-15) we obtain, $\forall i \in S_M$ and $\sigma_i(t) \in \mathscr{M} = \{1, 2\}$, the following state tracking error dynamics

$$\dot{\tilde{x}}_i = A_{m,\sigma_i(t)} \tilde{x}_i - B_u \Lambda_i^* \tilde{\Theta}_{i,\sigma_i(t)}^T \Phi_i \quad (4\text{-}17)$$

where $\tilde{\Theta}_{i,k} = \Theta_{i,k} - \Theta_i^*$.

Moreover, define (t_{k_l}, t_{k_l+1}) as the switch-in and switch-out instant pair of subsystem k, with $k \in \mathscr{M}$ and $l \in \mathbb{N}^+$.

Since $A_{m,k}$ is stable, there exist symmetric positive definite matrices $P_k = P_k^T > 0$ for every subsystem $k \in \{1, 2\}$ such that

$$A_{m,k}^T P_k + P_k A_{m,k} + \gamma_k P_k \leq 0.$$

Define $\overline{\lambda}_k$ and $\underline{\lambda}_k$ as the maximum and the minimum eigenvalue of P_k respectively, and $\beta = \min_{k \in \mathscr{M}} \{\underline{\lambda}_k\}$. Furthermore, assume known upper and lower bounds for Θ^* such that $\Theta^* \in [\underline{\Theta}, \overline{\Theta}]$, and assume $\Lambda_i^* \geq 0$ with a known upper bound such that $0 \leq \Lambda_i^* \leq \overline{\Lambda}$.

Moreover, define the adaptive law for any $S_k = S_k^T > 0$ as

$$\dot{\Theta}_{i,k}^T(t) = S_k^T B_u^T P_k \tilde{x}_i(t) \Phi_i^T + F_{i,k}^T(t). \; \forall k \in \{1, 2\} \tag{4-18}$$

where $F_{i,k}(t)$ is a parameter projection term, defined in [28], that acts component-wise and guarantees the boundedness of the estimated parameters in $[\underline{\Theta}, \overline{\Theta}]$. In particular, $F_{i,k}$ is zero whenever the corresponding component of $\Theta_{i,k}$ is within the prescribed uncertainty bounds; otherwise, $F_{i,k}$ is set to guarantee that the corresponding time derivative of $\Theta_{i,k}$ is zero.

Furthermore, we define the switching law $\sigma_i(t)$ based on a MDADT strategy as follows

$$\tau_{ak} > \frac{1 + \zeta}{\gamma_k} \ln(\mu_k) \tag{4-19}$$

with $\zeta > 0$ is a user-defined positive constant, and μ_k, $k \in \mathcal{M}$ defined as $\mu_1 = \frac{\overline{\lambda}_2}{\underline{\lambda}_1}$ and $\mu_2 = \frac{\overline{\lambda}_1}{\underline{\lambda}_2}$.

4-4   Main results

The following stability and convergence results can be guaranteed by (4-18)-(4-19):

Theorem 4-4.1. *Consider the heterogeneous platoon model (2-10) with reference models (4-10) and (4-11) in the CACC and ACC mode respectively. Then, the adaptive input (4-16) with adaptive laws (4-18) makes the error dynamics (4-17) GUUB, provided that the switching between CACC and ACC satisfies the MDADT (4-19). Furthermore, the following state tracking error upper bound is derived*

$$\|\tilde{x}_i(t)\|^2 \le \frac{1}{\beta} exp\left\{ \sum_{k=1}^{2} N_{0k} ln\mu_k \right\} M \; , \quad \forall i \in S_M \tag{4-20}$$

where

$$M = \max_{k \in \mathcal{M}} \left\{ \|\tilde{x}_i(t_0)\|^2 + c_1 + c_2, \kappa \frac{(1+\zeta)}{\zeta}(c_1 + c_2) \right\}$$

$$c_k = tr\left[(\overline{\Theta} - \underline{\Theta}) S_k^{-1} (\overline{\Theta} - \underline{\Theta})^T \overline{\Lambda} \right] > 0$$

and

$$\kappa = \max_{k \in \mathcal{M}} \{\mu_k\}.$$

Finally, the ultimate bound b on the norm of the state tracking error is found to be

$$b \in \left[0, \sqrt{\exp\left(\sum_{k=1}^{2} N_{0k} \ln \mu_k \right) \frac{\kappa \overline{B}}{\beta}} \right]. \tag{4-21}$$

with

$$\overline{B} = (c_1 + c_2)\frac{1 + \zeta}{\zeta} > 0 \tag{4-22}$$

Proof. The stability proof is based on two Lyapunov functions, one active when communication is present and one active when it is lost. An appropriate MDADT will be constructed in such a way that switching among the Lyapunov functions guarantees GUUB. Define the following Lyapunov function:

$$V_i(t) = \tilde{x}_i^T(t) P_{\sigma_i(t)} \tilde{x}_i(t) + \sum_{k=1}^{2} \mathrm{tr}(\tilde{\Theta}_{i,k}(t) S_k^{-1} \tilde{\Theta}_{i,k}^T(t) \Lambda_i^*), \ \forall i \in S_M. \tag{4-23}$$

Using the switched adaptive law (4-18), the derivative of $V_i(t)$ with respect to time between two consecutive discontinuities (i.e. $t \in [t_l, t_{l+1})$) is

$$\begin{aligned}
\dot{V}_i(t) = {} & \tilde{x}_i^T(t) (A_{m\sigma_i(t_{l+1}^-)}^T P_{\sigma_i(t_{l+1}^-)} + P_{\sigma_i(t_{l+1}^-)} A_{m\sigma_i(t_{l+1}^-)}) \tilde{x}_i(t) \\
& + 2\mathrm{tr}[\tilde{\Theta}_{i,\sigma_i(t_{l+1}^-)} S_{\sigma_i(t_{l+1}^-)}^{-1} F_{i,\sigma_i(t_{l+1}^-)}^T \Lambda_i^*] \\
\leq {} & -\gamma_{\sigma_i(t_{l+1}^-)} \tilde{x}_i^T(t) P_{\sigma_i(t_{l+1}^-)} \tilde{x}_i(t) \\
& + 2\mathrm{tr}[\tilde{\Theta}_{i,\sigma_i(t_{l+1}^-)} S_{\sigma_i(t_{l+1}^-)}^{-1} F_{i,\sigma_i(t_{l+1}^-)}^T \Lambda_i^*].
\end{aligned}$$

In fact the following two inequalities hold [28]

$$\begin{aligned}
\tilde{\Theta}_{i,\sigma_i(t_{l+1}^-)} S_{\sigma_i(t_{l+1}^-)}^{-1} F_{i,\sigma_i(t_{l+1}^-)}^T \Lambda_i^* \leq 0 \\
\sum_{k=1}^{2} \mathrm{tr}(\tilde{\Theta}_{i,k}(t) S_k^{-1} \tilde{\Theta}_{i,k}^T(t) \Lambda_i^*) \leq c_1 + c_2
\end{aligned} \tag{4-24}$$

where $c_k = \mathrm{tr}\left[(\overline{\Theta} - \underline{\Theta}) S_k^{-1} (\overline{\Theta} - \underline{\Theta})^T \overline{\Lambda}\right]$ is a finite positive constant. This results in, for any $\zeta > 0$

$$\begin{aligned}
\dot{V}_i(t) \leq {} & -\gamma_{\sigma_i(t_{l+1}^-)} \tilde{x}_i^T(t) P_{\sigma_i(t_{l+1}^-)} \tilde{x}_i(t) \\
& + \gamma_{\sigma_i(t_{l+1}^-)} (c_1 + c_2) - \gamma_{\sigma_i(t_{l+1}^-)} (c_1 + c_2) \\
\leq {} & -\frac{\gamma_{\sigma_i(t_{l+1}^-)}}{1 + \zeta} V_i(t) + \frac{\gamma_{\sigma_i(t_{l+1}^-)}}{1 + \zeta} \left[(1 + \zeta)(c_1 + c_2) - \zeta V_i(t)\right].
\end{aligned} \tag{4-25}$$

Let us define a finite positive constant $\overline{B}$ as in (4-22). Then using (4-22) and (4-25) we can conclude that, between two consecutive discontinuities, $V_i(t)$ is

- decreasing at an exponential rate when $V_i(t) > \overline{B}$ since $\dot{V}_i(t) \leq -\frac{\gamma_{\sigma_i(t_{l+1}^-)}}{1+\zeta} V_i(t)$

- non increasing when $V_i(t) \leq \overline{B}$ since $\dot{V}_i(t) \leq 0$

The next step is to assess the behavior of $V_i(t)$ at the discontinuous instants. We consider subsystem $\sigma_i(t_{l+1}^-)$ is active when $t \in [t_l, t_{l+1})$ and subsystem $\sigma_i(t_{l+1})$ is active when $t \in [t_{l+1}, t_{l+2})$. Therefore, before switching we have

$$\begin{aligned}
V_i(t_{l+1}^-) = {} & \tilde{x}_i^T(t_{l+1}^-) P_{\sigma_i(t_{l+1}^-)} \tilde{x}_i(t_{l+1}^-) \\
& + \sum_{k=1}^{2} \mathrm{tr}(\tilde{\Theta}_{i,k}(t_{l+1}^-) S_k^{-1} \tilde{\Theta}_{i,k}^T(t_{l+1}^-) \Lambda_i^*)
\end{aligned} \tag{4-26}$$

and after switching we have

$$
\begin{aligned}
V_i(t_{l+1}) = {}& \tilde{x}_i^T(t_{l+1}) P_{\sigma_i(t_{l+1})} \tilde{x}_i(t_{l+1}) \\
& + \sum_{k=1}^{2} \operatorname{tr}(\tilde{\Theta}_{i,k}(t_{l+1}) S_k^{-1} \tilde{\Theta}_{i,k}^T(t_{l+1}) \Lambda_i^*).
\end{aligned}
\tag{4-27}
$$

Since the tracking error $\tilde{x}_i(\cdot)$ and the parameter estimation error $\tilde{\Theta}_{i,k}(\cdot)$ are continuous, we have $\tilde{x}_i(t_{l+1}^-) = \tilde{x}_i(t_{l+1})$ and $\tilde{\Theta}_{i,k}(t_{l+1}^-) = \tilde{\Theta}_{i,k}(t_{l+1})$. Furthermore, we have the following properties:

- $\tilde{x}_i^T(t) P_{\sigma_i(t_{l+1})} \tilde{x}_i(t) \leq \overline{\lambda}_{\sigma_i(t_{l+1})} \tilde{x}_i^T(t) \tilde{x}_i(t)$

- $\tilde{x}_i^T(t) P_{\sigma_i(t_{l+1}^-)} \tilde{x}_i(t) \geq \underline{\lambda}_{\sigma_i(t_{l+1}^-)} \tilde{x}_i^T(t) \tilde{x}_i(t)$

where the first property is valid since we only have 2 subsystems and we know in advance to which subsystem we are switching to. Consequently, we get

$$
V_i(t_{l+1}) - V_i(t_{l+1}^-) = \tilde{x}_i^T(t)(P_{\sigma_i(t_{l+1})} - P_{\sigma_i(t_{l+1}^-)}) \tilde{x}_i(t)
$$

$$
V_i(t_{l+1}) - V_i(t_{l+1}^-) \leq \left(\frac{\overline{\lambda}_{\sigma_i(t_{l+1})}}{\underline{\lambda}_{\sigma_i(t_{l+1}^-)}} - 1 \right) V_i(t_{l+1}^-)
$$

$$
V_i(t_{l+1}) \leq \mu_{\sigma_i(t_{l+1})} V_i(t_{l+1}^-)
\tag{4-28}
$$

where $\mu_{\sigma_i(t_{l+1})} = \overline{\lambda}_{\sigma_i(t_{l+1})}/\underline{\lambda}_{\sigma_i(t_{l+1}^-)}$. The next step is to analyze the overall behavior of $V_i(t)$. Considering the initial condition, we have two cases: $a)$ $V_i(t_0) > \overline{B}$ and $b)$ $V_i(t_0) \leq \overline{B}$.

Case a) $V_i(t_0) > \overline{B}$. Since $V_i(t)$ is decreasing at an exponential rate between two consecutive discontinuities, there exists a finite time instant $t_0 + T_1$ such that $V_i(t_0 + T_1) \leq \overline{B}$. Denote the number of intervals that subsystem k, $k \in \mathcal{M}$, is active by N_{1k}. Therefore, it follows from (4-25) and (4-28) that, for $t \in [t_0, \ t_0 + T_1)$,

$$
\begin{aligned}
V_i(t) \leq {}& \prod_{k=1}^{2} \mu_k^{N_{1k}} \exp\left\{ -\sum_{k=1}^{2} \sum_{j=1}^{N_{1k}} (t_{k_j+1} - t_{k_j}) \frac{\gamma_k}{1+\zeta} \right\} V_i(t_0) \\
= {}& \exp\left(\sum_{k=1}^{2} N_{1k} \ln \mu_k \right) \exp\left(-\sum_{k=1}^{2} T_k \frac{\gamma_k}{1+\zeta} \right) V_i(t_0) \\
\leq {}& \exp\left\{ \sum_{k=1}^{2} \left[\left(N_{0k} + \frac{T_k}{\tau_{ak}} \right) \ln \mu_k - T_k \frac{\gamma_k}{1+\zeta} \right] \right\} V_i(t_0) \\
\leq {}& \exp\left(\sum_{k=1}^{2} N_{0k} \ln \mu_k \right) \exp\left\{ \sum_{k=1}^{2} \left(\frac{\ln \mu_k}{\tau_{ak}} - \frac{\gamma_k}{1+\zeta} \right) T_k \right\} V_i(t_0)
\end{aligned}
\tag{4-29}
$$

where T_k is the total time when subsystem k is active for $t \in [t_0, \ t_0 + T_1)$. By substituting MDADT in (4-19) to (4-29), $V_i(t)$ can be attracted into the interval $[0, \overline{B}]$ with sufficiently big $T_1 > 0$. To study the value of $V_i(t_0 + T_1)$, we consider the special case: when $t = t_0 + T_1$,

a switching is activated. Then, the interval $[0, \overline{B}]$ becomes $[0, \kappa\overline{B}]$, where the coefficient $\kappa := \max_{k \in \mathcal{M}} \mu_k$ is introduced by (4-28). Next, it is possible that $V_i(t)$ will diverge far away from the interval $[0, \kappa\overline{B}]$ due to fast switches when $t > t_0 + T_1$. By recursively performing the analysis above, we notice that it is possible that fast switches happen intermittently over the whole time horizon, which can only guarantee that the Lyapunov function enters and then exceeds the bound $\kappa\overline{B}$ intermittently over the whole time horizon. The worse scenario is that fast switches characterized by N_{0k} are initialized when the Lyapunov function exceeds the bound $\kappa\overline{B}$. This implies that only the following ultimate bound of the Lyapunov function can be guaranteed:

$$b_{\mathrm{V}} = \exp\left(\sum_{k=1}^{2} N_{0k} \ln \mu_k\right) \kappa\overline{B}. \tag{4-30}$$

Case b) $V_i(t_0) \le \overline{B}$. The Lyapunov function is non-decreasing at the beginning, and it might exceed the bound $\overline{B}$. Therefore, with a similar analysis as in *case a)*, the same ultimate bound b_{V} of the Lyapunov function can be guaranteed as in (4-30). Hence, it can be concluded that the switched system (4-14) is GUUB according to (4-30). Furthermore, using (4-29), we can easily obtain an upper bound on $V_i(t)$ with a switching law based on MDADT (4-19) as follows, $\forall t \ge t_0$,

$$V_i(t) \le \exp\left(\sum_{k=1}^{2} N_{0k} \ln \mu_k\right) \max\left\{V_i(t_0),\ \kappa\overline{B}\right\}.$$

Since

$$V_i(t) \ge \beta \|\tilde{x}_i(t)\|^2,$$

it follows, $\forall t \ge t_0$, the state tracking error upper bound is obtained as in (4-20). Moreover, using (4-30), an ultimate bound of the tracking error is obtained as

$$b \in \left[0,\ \sqrt{\exp\left(\sum_{k=1}^{2} N_{0k} \ln \mu_k\right) \frac{\kappa\overline{B}}{\beta}}\,\right]. \tag{4-31}$$

This completes the proof. $\square$

Remark 3. *The choice of ζ is based on the compromise between fast switching capabilities (4-19) (small ζ) and a small tracking error (4-20) upper bound (large ζ). Note that, as it is to be expected in any adaptive control setting [29], the error bounds are dependent on the size of the uncertainty set via c_1 and c_2.*

Remark 4. *Since reference models (4-15) were chosen to provide the desired string stable dynamics of the platoon under mixed network conditions as shown in Sections 3-2 and 4-2, then (4-18)-(4-19) guarantee that the heterogeneous platoon tracks, with a bounded tracking error, the behavior of a string stable platoon even in the presence of inter-vehicle communication losses.*

Remark 5. *The stability proof of Theorem 4-4.1 is based on two Lyapunov functions, one active when communication is present and one active when it is lost, cf. (4-23). Consequently, when communication is always maintained, only one Lyapunov function in (4-23) is active, from which we recover the asymptotic stability result as in Theorem 3-4.1.*

4-5   Robustness to network induced delays

So far, we have considered communication losses as the wireless networks' only imperfection. However, information sent over wireless networks always suffer in practice from queuing, contention, transmission, and propagation delays. Therefore, it is crucial to study their effect on the string stability of the platoon. For this reason, we assess in this section the robustness of the designed adaptive switched controller (4-12) to end-to-end network induced delays.

First, define ρ_i as the end-to-end delay of sending information from vehicle $i-1$ to vehicle i, when $i \in S_M^C$. In the Laplace domain, we can represent the network induced time delay as

$$D(s) = e^{-\rho_i s} \tag{4-32}$$

Consequently, under baseline conditions (i.e. homogeneous assumption) $\Gamma_i(s)$ is found to take the following form

$$\Gamma_i(s) = \frac{1}{h^C + 1} \frac{G(s)(K_p^C + K_d^C s) + D(s)}{G(s)(K_p^C + K_d^C s) + 1}, \quad \forall i \in S_M^C, \tag{4-33}$$

where

$$G(s) = \frac{1}{s^2(\tau_0 s + 1)}. \tag{4-34}$$

Remark 6. *When no communication delays are present (i.e. $D(s) = 1$) (4-33) reduces to (3-8) as found in the ideal communication scenario, where string stability was guaranteed for any choice of $h^C > 0$.*

In the presence of network induced delays, the string stability of the CACC reference models (4-10) is compromised as illustrated in Fig. 4-4. For $K_p^C = 0.2$, $K_d^C = 0.7$, $\tau_0 = 0.1$ s, and $h^C = 0.7$ s, Fig. 4-4 plots the gain of $|\Gamma_i(s)|$ for $\rho_i = 0$ s, $\rho_i = 0.4$ s, and $\rho_i = 0.7$ s. We can see that as the time delay increases, the upper bound of $|\Gamma_i(s)|$ increases. This means that after some threshold delay value, the system looses its string stable behavior.
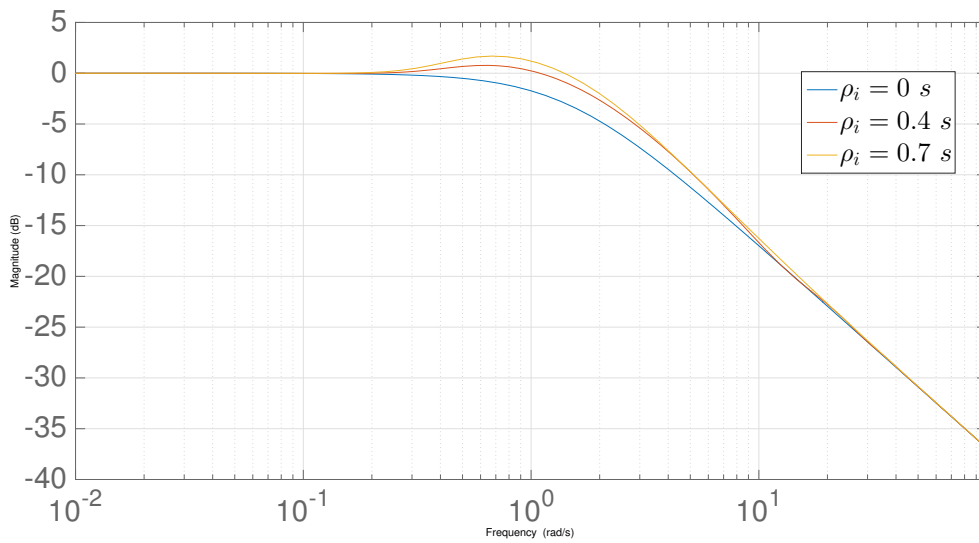

Figure 4-4: Time delay effect on string stability: Bode magnitude plot of $\Gamma_i(s)$ when $i \in S_M^C$.

Furthermore, (4-33) highlights an inverse relation between the headway constant h^C and the upper bound of $|\Gamma_i(s)|$. Therefore, if one has an insight on the maximum possible induced delay in a network ρ_{max}, one can design h^C such that string stability is guaranteed for any time delay $0 \leq \rho_i \leq \rho_{max}$.

For a given ρ_{max}, we denote the minimum required time headway constant for system string stability as h^C_{min}. The relationship between h^C_{min} and ρ_{max} is illustrated in Fig. 4-5 for $K_p^C = 0.2$, $K_d^C = 0.7$, and $\tau_0 = 0.1\ s$.
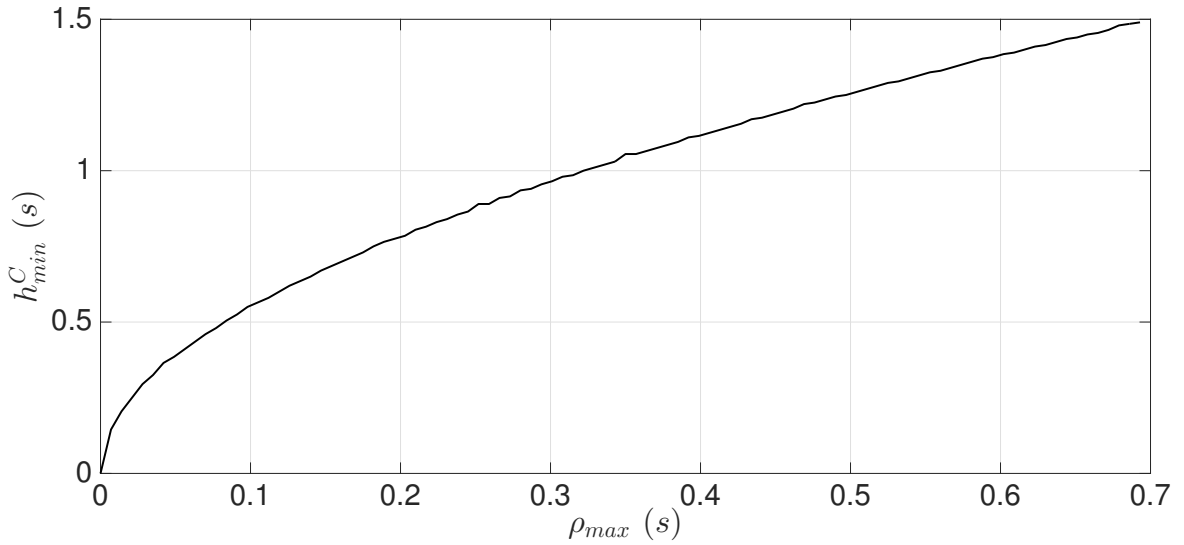

Figure 4-5: The minimum required time headway constant for system string stability h^C_{min} in function of ρ_{max}.

Therefore, we have seen in this section how to design the time headway constant h^C in such a way to guarantee the robustness of the CACC reference dynamics (4-10) to network induced delay up to a ρ_{max}. Under such a design, the mixed CACC-ACC reference dynamics (4-15) provide string stable dynamics for the adaptive switched control law (4-12) under bounded network time delays.

4-6   Summary

A novel adaptive switched control strategy to stabilize a platoon with non-identical vehicle dynamics, engine performance losses, communication losses, and network induced delays has been derived in this chapter. Furthermore, the convergence of the platoon's dynamics to string stable dynamics with a bounded error has been analytically proven through a MDADT approach. Finally, the robustness properties of the reference dynamics towards network induced delay have been analyzed along with a way to cope with the delays through spacing policy design has been discussed.

Chapter 5

Simulation results

To validate the different control strategies discussed earlier, we present in this chapter a simulation in Matlab/Simulink [21] of a heterogeneous platoon of 5+1 vehicles (including vehicle 0) with vehicles' engine performance loss. In the following, Section 5-1 describes the simulation setup, Section 5-2 details the spacing policies and controllers design procedure, and Section 5-3 presents the simulation results and performance analyses. Finally, Section 5-4 concludes this chapter with a few remarks on the practical improvements of the implemented control strategies.

5-1 Simulation setup

The platoon's characteristics are shown in Table 5-1, and are motivated by nominal values found and validated in the literature as in [1] and [30].

Table 5-1: Platoon parameters, $M=5$.

i	0	1	2	3	4	5
$\tau_i(s)$	0.1	0.5	0.7	0.3	0.7	0.9
Λ_i	-	0.5	0.7	0.75	0.7	0.7

In order to test the string stability of the heterogeneous platoon, the desired platoon acceleration $a_0(t)$, shown in Fig. 5-1, represents a stop-and-go scenario that undergoes a sudden disturbance at $t = 20$ s.
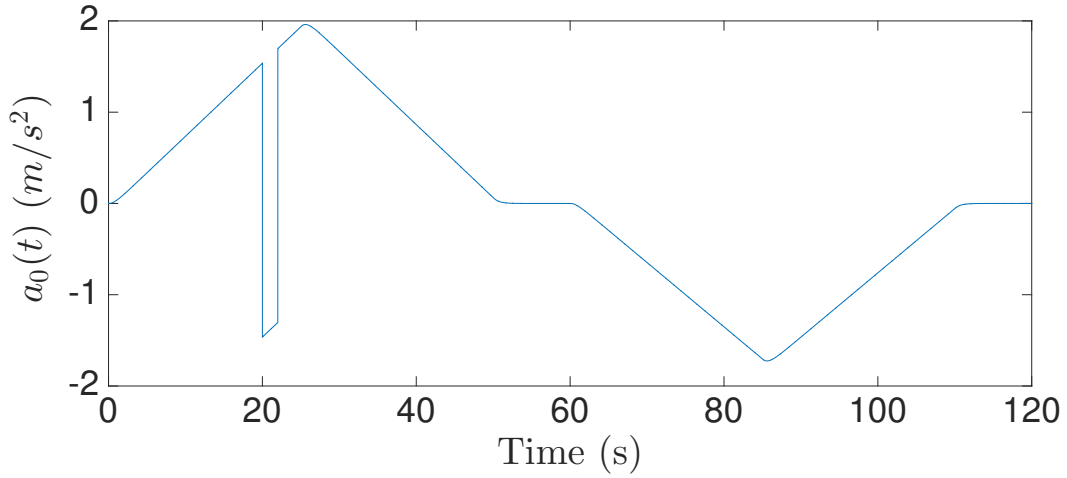
Figure 5-1: Desired platoon acceleration $a_0(t)$.

We define 4 experiments to showcase the performance and results of controllers (3-9) and (4-12)

- Experiment 1: (Perfect communication, no adaptation) Simulate the platoon under the control action of the CACC baseline controller (3-1) without adaptation.

- Experiment 2: (Perfect communication, adaptation) Simulate the platoon under the control action of the augmented adaptive CACC controller (3-9).

- Experiment 3: (Communication losses, no delay, adaptation) Simulate the platoon under the control action of the augmented adaptive switched controller (4-12) using True-Time2.0 [22] to model a realistic wireless communication network (IEEE 802.11p/WAVE) with update frequency of 10 Hz.

- Experiment 4: (Communication losses, delay ($\rho_{max} = 0.15$ s), adaptation) Simulate the platoon under the control action of the augmented adaptive switched controller (4-12) using TrueTime2.0 [22] to model a realistic wireless communication network (IEEE 802.11p/WAVE) with update frequency of 10 Hz and induced delay up to 0.15 seconds.

5-2   Controllers and spacing policies design

CACC reference model. The baseline controllers' gains are chosen (for all experiments) as $K_p^C = 0.2$ and $K_d^C = 0.7$ for $u_{bl,i}^C$ in order to respect the stability conditions (3-7). In terms of the spacing policy, CACC will operate in all experiments with a time headway constant $h^C = 0.7$ s. Even though string stability could be guaranteed for any $h^C > 0$ in Experiments 2 and 3, h^C was chosen based on desired network delay robustness properties for Experiment 4. Under such design parameters, the CACC reference model is guaranteed to provide string stable reference dynamics when $i \in S_M^C$ for Experiments 2, 3, and 4 as can be confirmed from the magnitude plot of $\Gamma_i(j\omega)$ in Fig. 5-2 and Fig. 5-3.
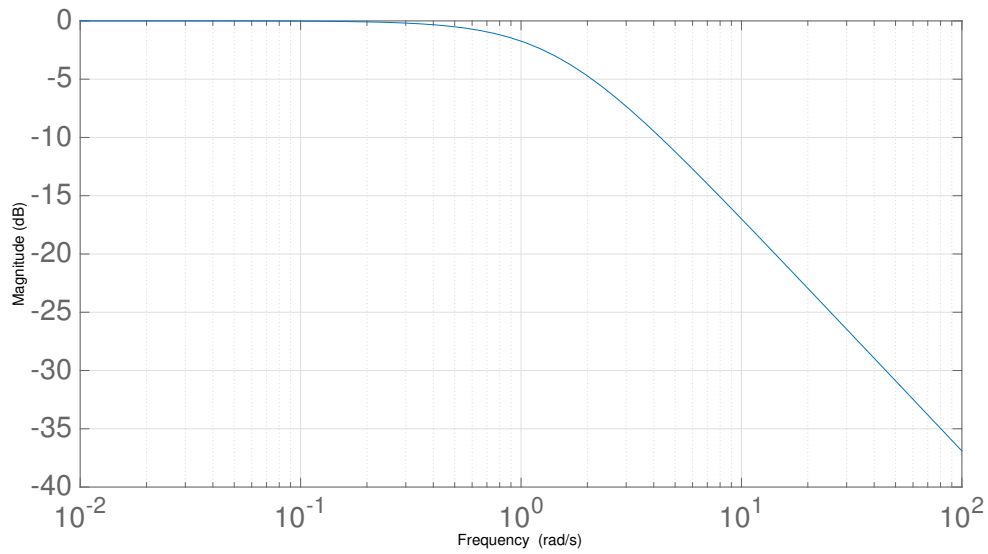
Figure 5-2: Experiments 2 and 3: Bode magnitude plot of $\Gamma_i(s)$ when $i \in S_M^C$
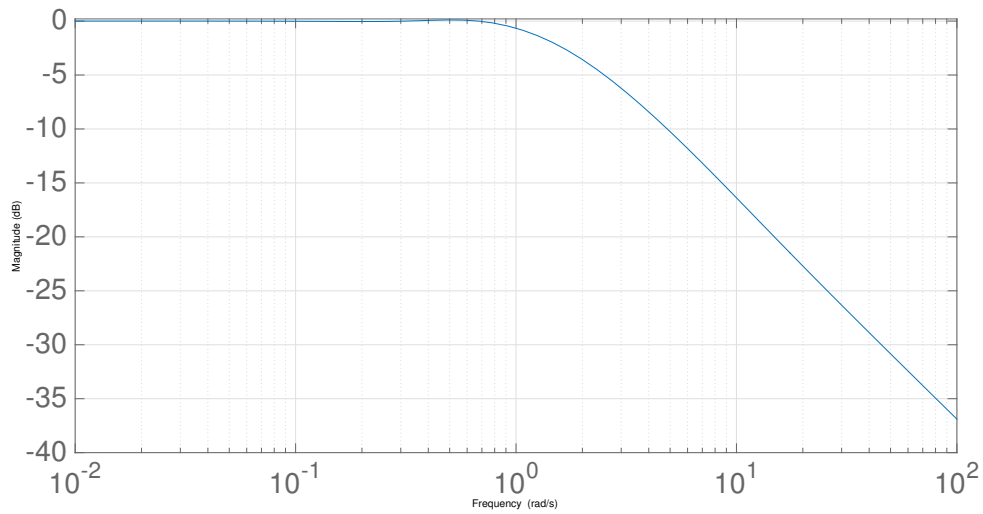

Figure 5-3: Experiment 4: Bode magnitude plot of $\Gamma_i(s)$ when $i \in S_M^C$

ACC reference model. For experiments 3 and 4, ACC will operate with the standardized minimum time gap of $h^L = 1$ s. Furthermore, the baseline controller's gains are chosen as $K_p^L = 2.5$ and $K_d^L = 2.3$ for $u_{bl,i}^L$ in order to respect the string stability conditions (4-8). Therefore, with such design parameters, the ACC reference model is guaranteed to provide string stable reference dynamics when $i \in S_M^L$ as can be confirmed from the magnitude plot of $\Gamma_i(s)$ in Fig. 5-4.
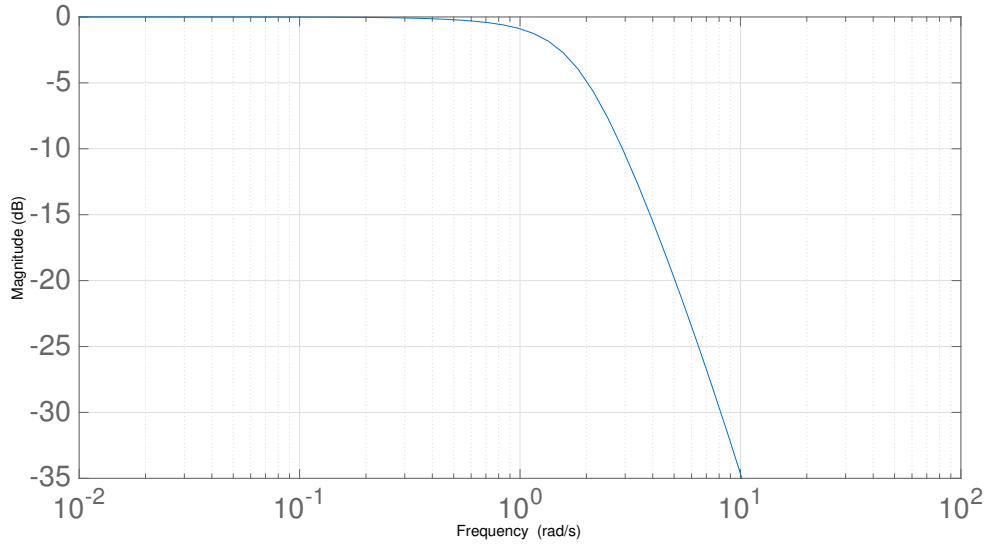
Figure 5-4: Experiments 3 and 4: Bode magnitude plot of $\Gamma_i(s)$ when $i \in S_M^L$

To summarize, we can confirm that for the ideal inter-vehicle communication network scenario (experiment 2: $S_M^L = \emptyset$), the CACC reference platoon offers string stable dynamics. Furthermore, under the non-ideal inter-vehicle communication scenario (experiments 3 and 4: S_M^L could be empty) the mixed CACC-ACC reference platoon offers string stable dynamics. Finally, for all experiments, the control input of the lead vehicle is defined by setting $h_0 = 0.7$.

Adaptive controller. In experiment 2, we designed the adaptive term (3-11) by setting $\Gamma_\Theta = 80I_{2\times 2}$ and $Q_m = 5I_{4\times 4}$.

Communication network characteristics. Furthermore, in order to design the adaptive input (4-16) for experiment 3, we need to quantify the loss of communication between vehicles which is represented as the switching signals $\sigma_i(t)$. In fact, for a velocity range of approximately $[0, 50]$ (m/s) and inter-vehicle distance range of approximately $[0, 40]$ (m), the Packet Error Rate between consecutive vehicles was measured in practice to be around 1% [20]. Therefore, since our operating conditions, characterized by the desired platoon acceleration and the headway constants, fall inside the previously defined intervals, and since the total experiment duration is 120 s, the expected average time of loss of communication can be calculated as 1% of 120 s for one inter-vehicle communication network. This results in an average total communication loss time of 1.2 s between consecutive vehicles during the total operating time of 120 s. Accounting for single packet loss and consecutive packet loss, we define the switching signals of the 5 vehicles, shown in Fig. 5-5, by the following MDADT characteristics $N_{01} = 2$, $N_{02} = 2$, $\tau_{a1} = 8.5$, and $\tau_{a2} = 0.7$, and a total communication loss time for one inter-vehicle communication link of 1.2 s. To implement the wireless network in MATLAB/Simulink, we use the TrueTime2.0 network simulator between consecutive vehicles due to its configurable settings to an IEEE 802.11p/WAVE network architecture.
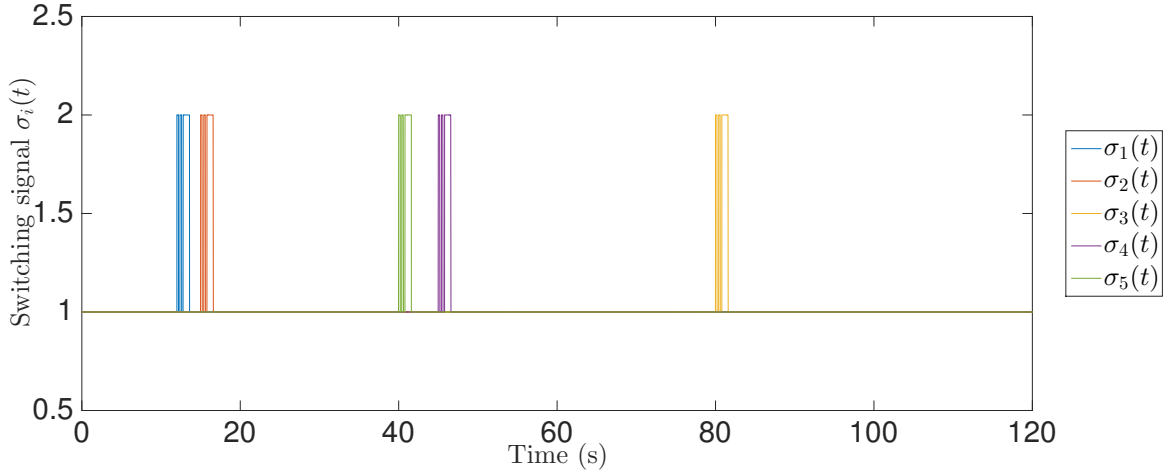
Figure 5-5: Experiments 3 and 4: Switching signals $\sigma_i(t)$ of vehicles 1-5: $\sigma_i(t)$, $i \in S_5$.

Network induced delay In Experiment 4, we test for the robustness of the control strategy towards network induced delays. It has been established in [31] that time delays can take values up to 0.15 seconds. Therefore, for string stability of the CACC reference dynamics: $h_{min}^C = 0.68$ (Fig. 4-5). Thus, our choice of $h^C = 0.7 > 0.68$ is now validated.

Adaptive switched controller. To keep the platoon stable when switching back and forth between control strategies, we need to design the adaptive term (4-16) such that the switching conditions for stability (4-19) are satisfied $\forall k \in \mathscr{M}$. In fact, by setting $\gamma_1 = 0.60$, $\gamma_2 = 1.00$, and $S_1 = S_2 = 100$, the following MDADT conditions are necessary to guarantee the overall stability of the switched system: $\tau_{a1} > 8.01$ and $\tau_{a2} > 0.66$. Therefore, since both conditions are satisfied by the switching signal's MDADT characteristics, then the switching controller in able to indeed guarantee the overall stability of the switched system.

5-3   Simulation results

From Fig. 5-6, it is clear that in Experiment 1, the CACC baseline controller (3-1) which guarantees the string stability of the platoon under the homogeneity and perfect engine assumptions, fails to maintain the platoon's stability when applied to the heterogeneous platoon. On the other hand, Fig. 5-7 shows that, in Experiment 2, the augmented CACC controller (3-9), under the same platoon desired acceleration $a_0(t)$, succeeds in maintaining the string stability of the platoon even-though the platoon is composed of unknown non-identical vehicles that suffer from unknown engine performance loss.
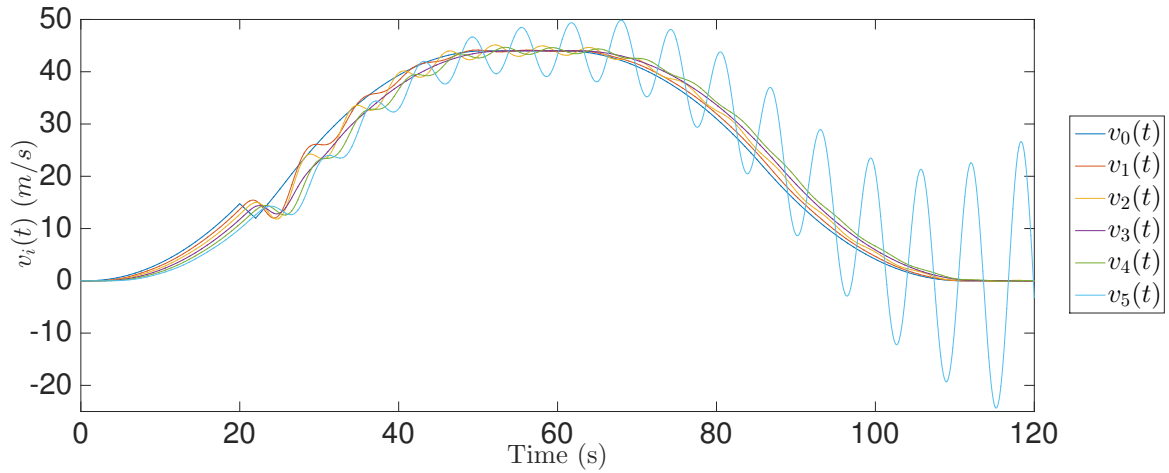
Figure 5-6: Experiment 1: Velocities of vehicles 0-5: $v_i(t)$, $i \in \{0, S_5\}$.
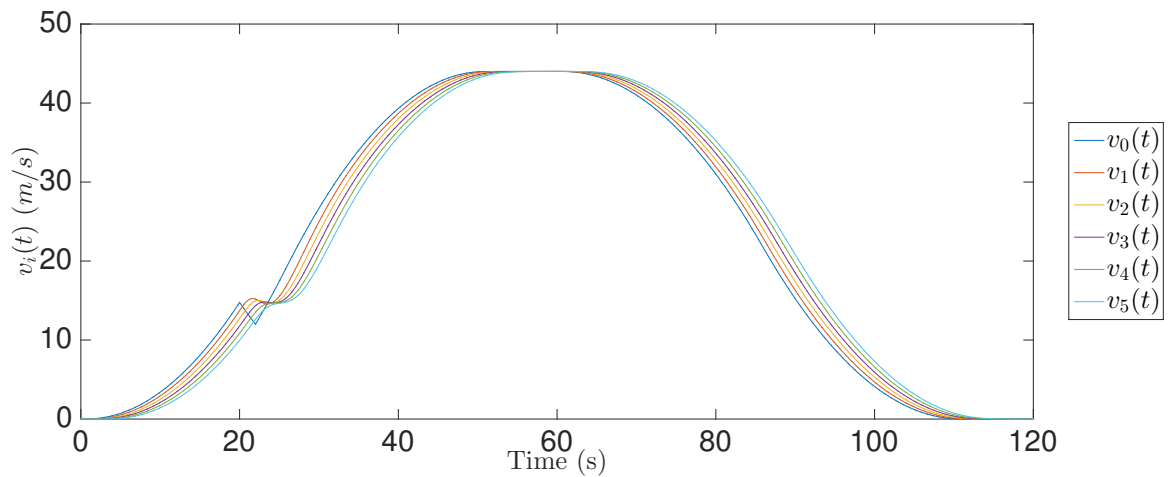

Figure 5-7: Experiment 2: Velocities of vehicles 0-5: $v_i(t)$, $i \in \{0, S_5\}$.

Furthermore, Fig. 5-8 demonstrates the performance of the augmented adaptive switched controller (4-12) in Experiment 3 when communication loss is present in the platoon. We can see that controller (4-12) manages to maintain the string stability of the platoon while switching back and forth between control strategies to recover from the loss of communication throughout the platoon. Moreover, we can see that when a vehicle loses communication with its preceding one, it switches to a spacing policy characterized by a larger time gap. This is illustrated by the fact that the vehicle reduces its speed, for some time, in order to enlarge its inter-vehicle time gap and subsequently increases it speed again to match the platoon's speed. In turn, its following vehicles reduce their speeds in order to maintain their respective desired inter-vehicle spacing.
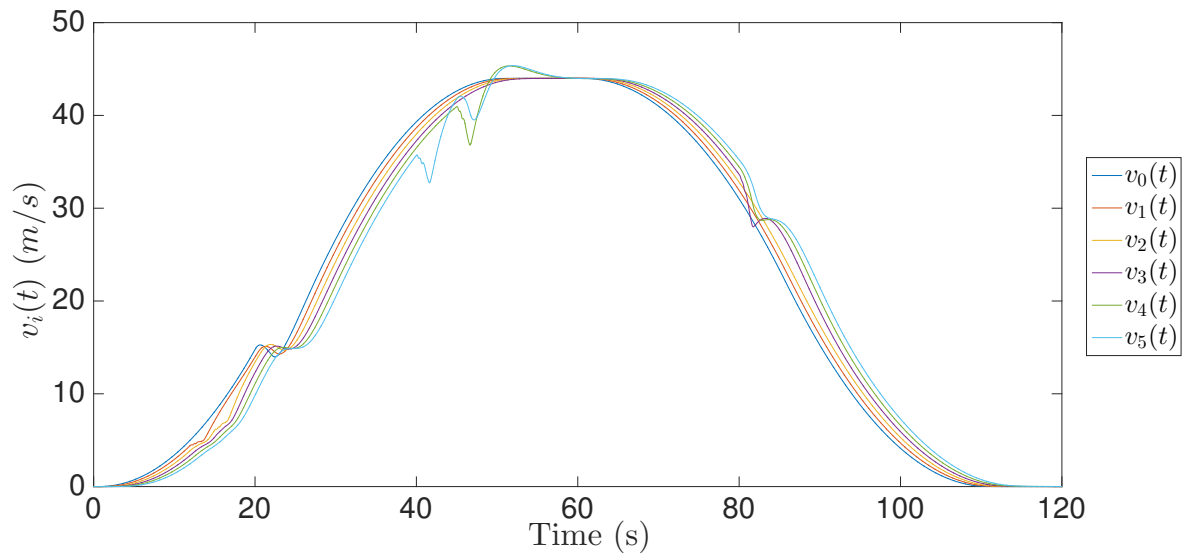
Figure 5-8: Experiment 3: Velocities of vehicles 0-5: $v_i(t)$, $i \in \{0, S_5\}$.

In terms of the norm of the state tracking error, Fig. 5-9 shows that when communication is always maintained, controller (3-9) regulates asymptotically the error to 0. Moreover, Fig. 5-10 shows that, under the action of controller (4-12), the platoon's dynamics track, with a bounded state tracking error, the dynamics of a string stable platoon even when communication loss is present in the system.
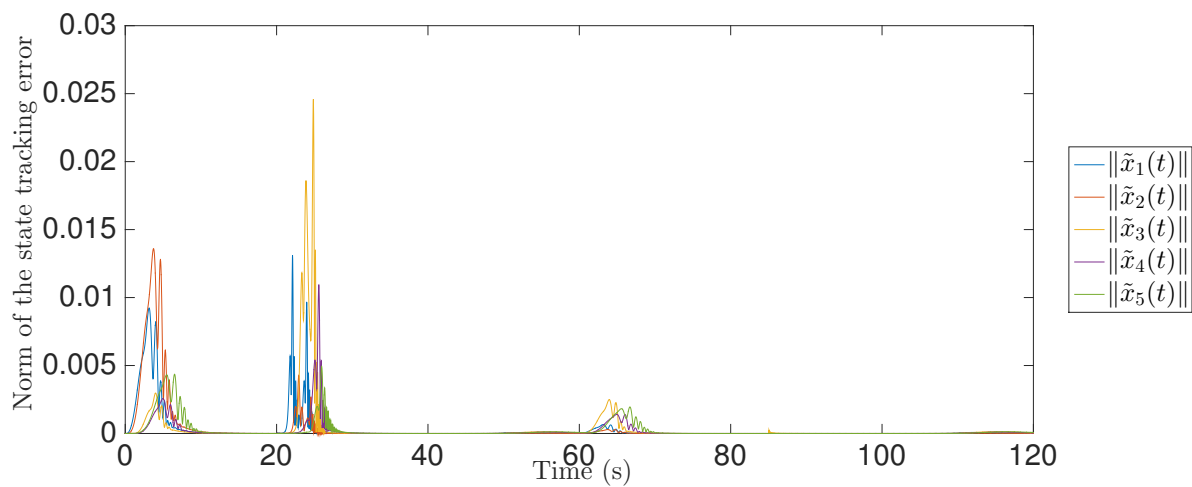

Figure 5-9: Experiment 2: Norm of the state tracking error of vehicles 1-5: $\|\tilde{x}_i(t)\|$, $i \in S_5$.
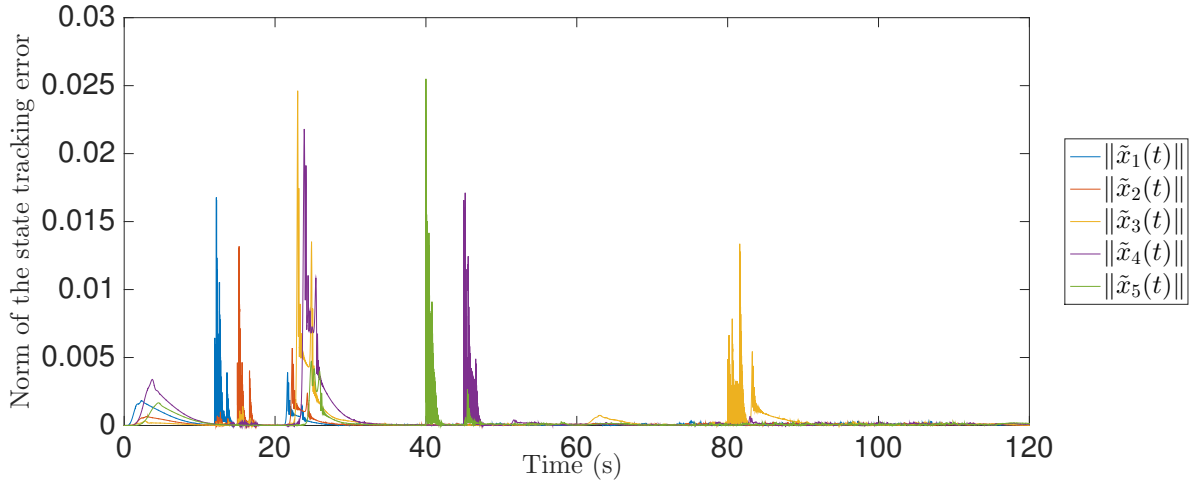
Figure 5-10: Experiment 3: Norm of the state tracking error of vehicles 1-5: $\|\tilde{x}_i(t)\|$, $i \in S_5$.

In Experiment 4, the inter-vehicle communication network is modeled to suffer from communication losses and network induced delays. Communication losses behave as the switching signals shown in Fig. 5-5. Moreover, the network induced delays $\rho_i(t)$ are chosen to be time varying with an upper bound of 0.15 seconds. For instance, Fig. 5-11 shows the network induced delay of vehicle 2 $\rho_2(t)$. Note: The delay for the remaining vehicles behaves in similar manner.
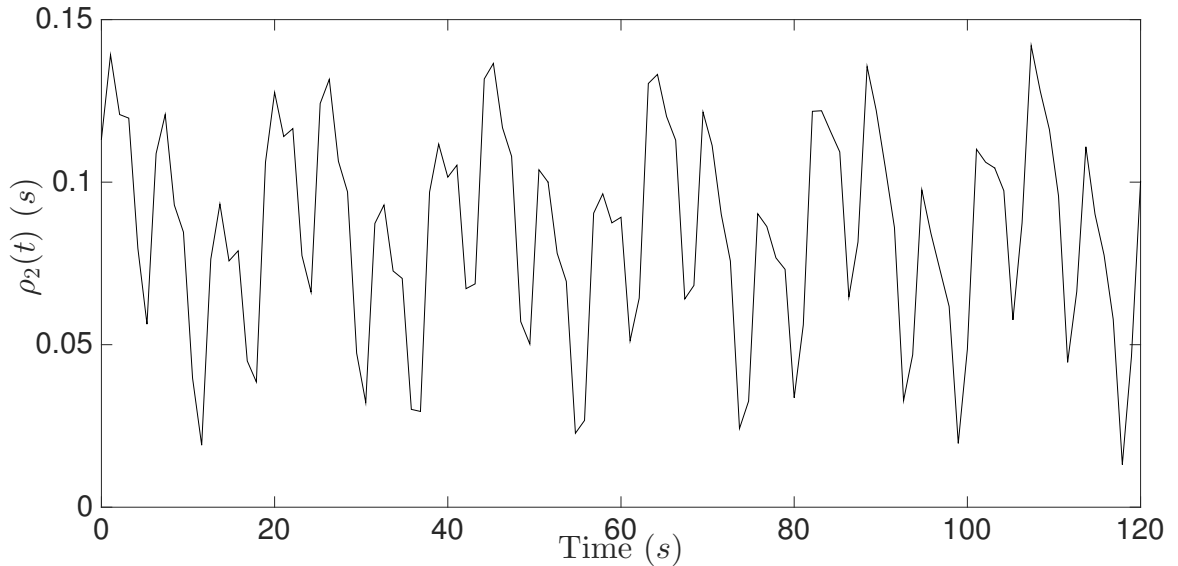
Figure 5-11: Experiment 4: Time delay of vehicle 2's wireless network.

When we introduce network induced delay to our wireless network model, we can see from Fig. 5-12 that the adaptive switched controller (4-12) is indeed able to cope with them and the resultant velocity response of the platoon illustrates a string stable behavior. Moreover, Fig. 5-13 confirms that controller (4-12) manages to track the mixed reference string stable

dynamics with a bounded state tracking error.
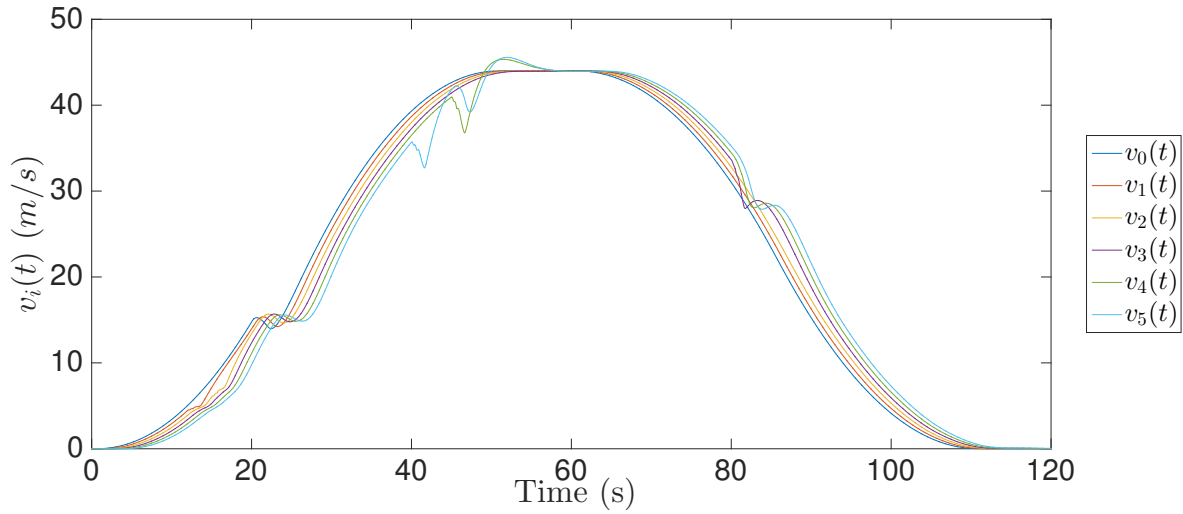
Figure 5-12: Experiment 4: Velocities of vehicles 0-5: $v_i(t)$, $i \in \{0, S_5\}$.
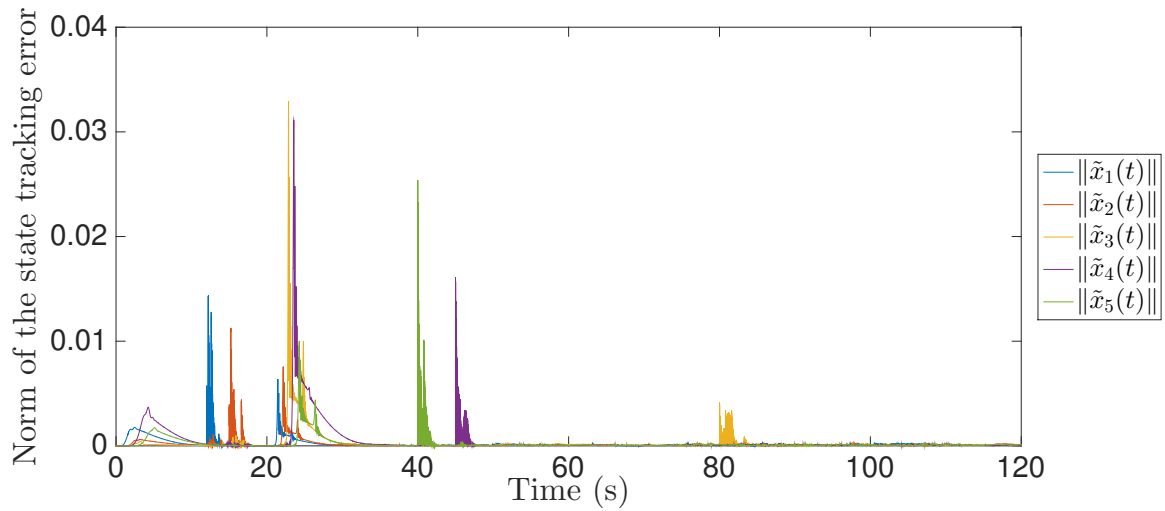
Figure 5-13: Experiment 4: Norm of the state tracking error of vehicles 1-5: $\|\tilde{x}_i(t)\|$, $i \in S_5$.

5-4   Practical improvements

Modeling the communication network losses by a switching signal with MDADT characteristics offers a major improvement over other modeling approaches such as switching signals with dwell time or average dwell time characteristics.

However, modeling the losses by a dwell time framework is not realistic since losses in wireless networks are best described by their average behavior. In fact, in practice, and as we have seen in Section 5-3, vehicular ad hoc networks have a total communication loss time around

1% to 5% on average depending on their operating conditions. This means that the total time spent in subsystem 2 is much less than the total time spent in subsystem 1. This motivates the use of a mode-dependent approach to be able to properly quantify the time in each communication state (or subsystem). Dwell time and average dwell time modeling fail to differentiate between the two.

To illustrate this idea, we assessed the stability requirements for a switched adaptive controller strategy based on a dwell time approach (provided in [28]) when applied on our platooning system. The dwell time condition for system stability was found to be $T_{DT} = 1.67\ s$. This means that every time we switch control strategies, we have to wait, regardless of the state of the network, at least 1.67 seconds before we can switch again. This is highly inconvenient since communication losses in practice have a much shorter duration, which means that even though communication is active again, the controller cannot switch back to the CACC strategy. In our approach, the obtained MDADTs have a better representation of reality and the switching mechanism between strategies is more efficient in the sense that it maximizes the utilization of the CACC strategy which in turn maximizes the benefits of platooning.

In order to quantify this idea, we present the percent improvement of our approach over the dwell time approach, in two scenarios: the first scenario is when the communication losses in the links behave as in Fig. 5-5, and the second is when the communication losses in the links are more scattered in time, as shown in Fig 5-14.

Remark 7. *Note that in the MDADT case, the switching signals $\sigma_i(t)$ are used interchangeably as the communication losses and the controller's switching law. However, in the dwell time case, the switching signal's $\sigma_i(t)$ only describes the communication network's state.*
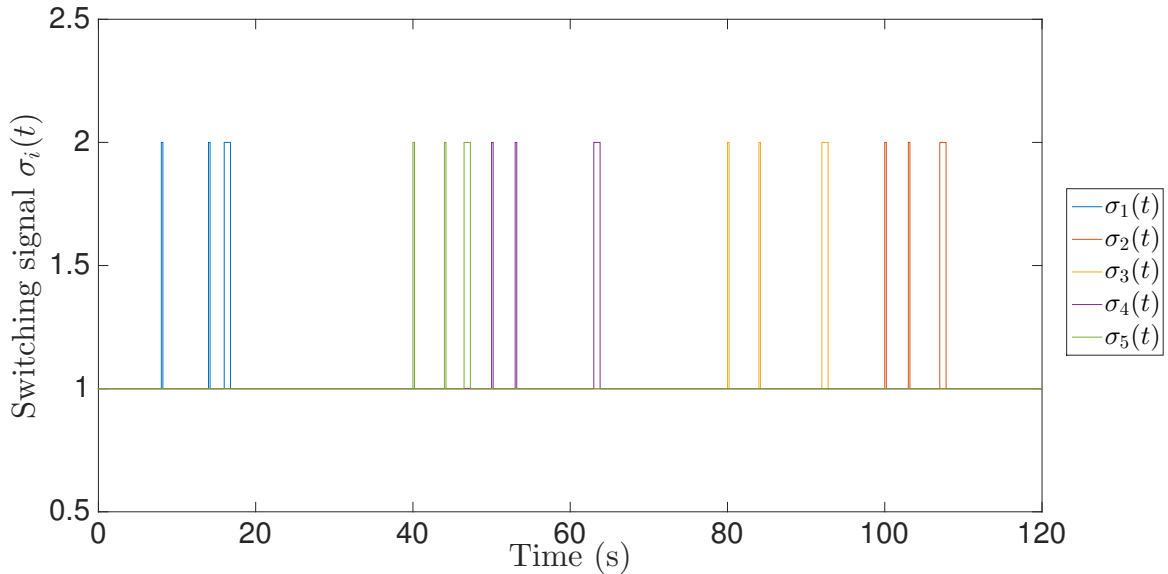

Figure 5-14: Scenario 2: Switching signals $\sigma_i(t)$ of vehicles 1-5: $\sigma_i(t)$, $i \in S_5$.

The time spent in the ACC strategy for a MDADT approach T_2, is the same in both scenarios. This is the case because such an approach exploits the average and mode-dependent behavior

of the communication losses. Moreover, in a dwell time approach, the time spent in an ACC strategy, denoted by T_2^{DT}, was found to triple in the second scenario. This is due to the communication losses being distant from each other and thus forcing the controller to wait at least 1.67 s at each switching instant when communication is lost (i.e. when subsytem 2 is activated). Table 5-2 summarizes these results.

Table 5-2: Total time spent in ACC strategy and percent reduction

	T_2^{DT}	T_2
Scenario 1	1.67 s	1.2 s
Scenario 2	5.01 s	1.2 s

In the first scenario, the total time spent in the ACC strategy for a vehicle in the platoon drops by 28.14% when the controller is designed based on a MDADT strategy rather than a dwell time strategy. Furthermore, when the communication losses are scattered (scenario 2), the total time spent in the ACC strategy is reduced by 76.04% when the control strategy is changed from a dwell time approach to a MDADT approach. Therefore, we can conclude on the importance of a MDADT strategy when dealing with a lossy communication network.

Reducing the time spent in the ACC strategy leads to an improved utilization of CACC which ultimately leads to an improved road capacity, improved road throughput and reduced vehicle emissions.

Chapter 6

Conclusions and recommendations

This chapter concludes the MSc thesis. Section 6-1 recalls the research objectives and summarize the main contributions of this study. Finally, Section 6-2 identifies topics for future research.

6-1   Conclusions

Vehicle platooning is a driving formation where individual vehicles, grouped into platoons, automatically adjust their own speeds as to maintain a desired inter-vehicle distance. Using only on-board sensors, ACC-equipped vehicles were able to achieve the desired driving pattern for restricted inter-vehicle time gaps greater than 1 second.

The need to maximize roads' capacity and throughput urged for novel control strategies that guarantee string stability for smaller time gaps. With the increase reliability of wireless communication networks, cooperative platooning has been a rapidly growing research topic. CACC strategies were able to guarantee string stability for platoons while allowing to considerably reduce inter-vehicle time gaps. A main limitation of these control systems is that they prove string stability only for homogeneous platoons with ideal communication networks.

The main contribution of this MSc thesis was to formulate a novel control strategy that achieves string stability for heterogeneous platoons with non-ideal communication networks. Therefore, an adaptive switched control strategy to stabilize a platoon with non-identical vehicle dynamics, engine performance losses, and communication losses has been considered. The proposed control scheme comprises a switched baseline controller (string stable under the homogeneous platoon with perfect engine performance assumption) augmented with a switched adaptive term (to compensate for heterogeneous dynamics and engine performance losses). The derivation of the string stable reference models and augmented switched controllers have been provided and their stability and string stability properties were analytically studied. When the switching respects a required mode-dependent average dwell time,

the closed-loop switched system is stable and signal boundedness is guaranteed. Numerical results have demonstrated the string stability of the heterogeneous platoon with engine performance losses under the designed control strategy.

6-2 Recommendations

In this thesis we modeled vehicle platoons as one dimensional systems controlled in the longitudinal direction. Evidently, this will not be the case in practice. In realistic platooning applications, vehicles in a platoon should be able to automatically adjust their speeds as well as their steering while maintaining a string stable behavior. Scenarios as lane changing or obstacle avoidance are of particular importance for platooning in order for it to be practically feasible. Therefore, extending the vehicle dynamics to include lateral motion is recommended. This requires the definition of lateral string stability which remains an open research topic.

Furthermore, the results of this thesis were obtained based on the assumption of linear time invariant vehicle models. This framework does not take into account sources of non-linearities such as input saturation, gear dynamics, and tire behavior. Even though tires might behave linearly at constant velocities, their behavior becomes highly non-linear during extreme decelerations. Due to the relevance of this scenario for obstacle avoidance or emergency braking, it is thus recommended to extend the notion of string stability of linear systems to non-linear systems.

In this work, we have only considered a one vehicle look ahead topology in terms of the cooperative aspect of the platoon. This topology could be extended to include communication between all vehicles so as to increase the robustness properties of the control strategies. Moreover, such a topology makes it possible to implement a platoon-level control mechanism for merging (or separating) equipped vehicles with the platoon. For instance, this could be accomplished by an automatic increase of the desired spacing between relevant vehicles in order to make room for the new vehicle to merge.

Appendix A

Preliminaries in stability of switched linear time invariant systems under a slowly switching signal

It is well known that a switched system is stable if all individual subsystems are stable and the switching is sufficiently slow, so as to allow the transient effects to dissipate after each switch. In this chapter we discuss how this property can be precisely formulated and justified using multiple Lyapunov function techniques.

A-1 Dwell time

The simplest way to specify slow switching is to introduce a number $\tau_d > 0$ and restrict the class of admissible switching signals to signals with the property that the switching times t_1, t_2,... satisfy the inequality $t_{i+1} - t_i \geq \tau_d$, for all i. This number τ_d is usually called the dwell time (because σ "dwells" on each of its values for at least τ_d units of time).

It is a well-known fact that when all linear systems in the family

$$\dot{x} = A_p x, \ \ p \in \mathscr{M} \tag{A-1}$$

are asymptotically stable, the switched linear system

$$\dot{x} = A_\sigma x \tag{A-2}$$

is asymptotically stable if the dwell time τ_d is sufficiently large. The required lower bound on τ_d can be explicitly calculated from the exponential decay bounds on the transition matrices of the individual subsystems.

Under suitable assumptions, a sufficiently large dwell time also guarantees asymptotic stability of the switched system in the nonlinear case. Probably the best way to prove most general

results of this kind is by using multiple Lyapunov functions. We now sketch the relevant argument.

Assume for simplicity that all systems in the family

$$\tag{A-3}$$

are globally exponentially stable. Then for each $p \in \mathcal{M}$ there exists a Lyapunov function V_p which for some positive constants a_p, b_p, and c_p satisfies

$$a_p|x|^2 \leq V_p(x) \leq b_p|x|^2 \tag{A-4}$$

and

$$\frac{\delta V_p}{\delta x} f_p(x) \leq -c_p|x|^2. \tag{A-5}$$

Combining (A-4) and (A-10), we obtain

$$\frac{\delta V_p}{\delta x} f_p(x) \leq -2\lambda_p V_p(x) \tag{A-6}$$

where

$$\lambda_p = \frac{c_p}{2b_p}, \quad p \in \mathcal{M}. \tag{A-7}$$

This implies that

$$V_p(x(t_0 + \tau_d)) \leq e^{-2\lambda_p \tau_p} V_p(x(t_0)) \tag{A-8}$$

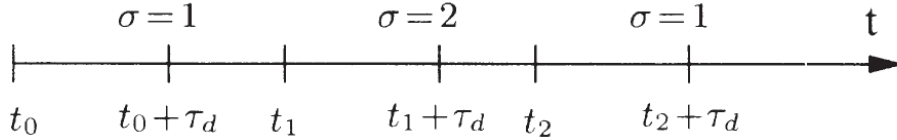provided that $\sigma(t) = p$ for $t \in [t_0, t_0 + \tau_d)$.

Figure A-1: A dwell time switching signal

To simplify the next calculation, let us consider the case when $\mathcal{M} = \{1, 2\}$ and σ takes the value 1 on $[t_0, t_1)$ and 2 on $[t_1, t_2)$, where $t_{i+1} - t_i > \tau_d$, $i = 0, 1$ (see Figure A-1). From the above inequalities we have

$$V_2(t_1) \leq \frac{b_2}{a_1} V_1(t_1) \leq \frac{b_2}{a_1} e^{-2\lambda_1 \tau_d} V_1(t_0) \tag{A-9}$$

and furthermore

$$V_1(t_2) \leq \frac{b_1}{a_2} V_2(t_2) \leq \frac{b_1}{a_2} e^{-2\lambda_2 \tau_d} V_2(t_1) \leq \frac{b_1 b_2}{a_1 a_2} e^{-2(\lambda_1 + \lambda_2)\tau_d} V_1(t_0). \tag{A-10}$$

It is now straightforward to compute an explicit lower bound on τ_d which guarantees that the switched system (A-2) is globally asymptotically stable. In fact, it is sufficient to ensure that

$$V_1(t_2) - V_1(t_0) \leq -\gamma|x(t_0)|^2 \tag{A-11}$$

for some $\gamma > 0$. In view of (A-10), this will be true if we have

$$(\frac{b_1 b_2}{a_1 a_2} e^{-2(\lambda_1 + \lambda_2)\tau_d} - 1)V_1(t_0) \leq -\gamma|x(t_0)|^2. \tag{A-12}$$

This will in turn hold, by virtue of (A-4), if

$$(\frac{b_1 b_2}{a_1 a_2} e^{-2(\lambda_1 + \lambda_2)\tau_d} - 1)a_1 \leq -\gamma \tag{A-13}$$

Since γ can be an arbitrary positive number, all we need to have is

$$\frac{b_1 b_2}{a_2} e^{-2(\lambda_1 + \lambda_2)\tau_d} \leq a_1 \tag{A-14}$$

which can be equivalently rewritten as

$$-2(\lambda_1 + \lambda_2)\tau_d < \log\frac{a_1 a_2}{b_1 b_2} \tag{A-15}$$

or finally as

$$\tau_d > \frac{1}{2(\lambda_1 + \lambda_2)}\log\frac{b_1 b_2}{a_1 a_2} \tag{A-16}$$

This is a desired lower bound on the dwell time.

We do not discuss possible extensions and refinements here because a more general result will be presented below. Note, however, that the above reasoning would still be valid if the quadratic estimates in (3.2) and (3.3) were replaced by, say, quartic ones. In essence, all we used was the fact that there exists a positive constant μ such that

$$V_p(x) \leq \mu V_q(x), \quad \forall x \in R^n, \quad \forall p, q \in \mathcal{M}. \tag{A-17}$$

If this inequality does not hold globally in the state space for any $\mu > 0$, then only local asymptotic stability can be established.

A-2   Average dwell time

In the context of controlled switching, specifying a dwell time may be too restrictive. If, after a switch occurs, there can be no more switches for the next τ_d units of time, then it is impossible to react to possible system fail- ures during that time interval. When the purpose of switching is to choose the subsystem whose behavior is the best according to some performance criterion, as is often the case, there are no guarantees that the performance of the currently active subsystem will not deteriorate to an unacceptable level before the next switch is permitted. Thus it is of interest to relax the concept of dwell time, allowing the possibility of switching fast when necessary and then compensating for it by switching sufficiently slowly later.

The concept of average dwell time serves this purpose. Let us denote the number of discontinuities of a switching signal σ on an interval (t, T) by $N_\sigma(T, t)$. We say that σ has average dwell time τ_a if there exist two positive numbers N_0 and τ_a such that

$$N_\sigma(T, t) \leq N_0 + \frac{T - t}{\tau_a} \quad \forall T \geq 0. \tag{A-18}$$

For example, if $N_0 = 1$ then (A-18) implies that σ cannot switch twice on any interval of length smaller than τ_a. Switching signals with this property are exactly the switching signals with dwell time τ_a. Note also that $N_0 = 0$ corresponds to the case of no switching, since a cannot switch at all on any interval of length smaller than τ_a. In general, if we discard the first N_0 switches (more precisely, the smallest integer greater than N_0), then the average time between consecutive switches is at least τ_a.

Besides being a natural extension of dwell time, the notion of average dwell time turns out to be very useful for analysis of the switching control algorithms. Our present goal is to show that the property discussed earlier — namely, that asymptotic stability is preserved under switching with a sufficiently large dwell time — extends to switching signals with average dwell time.

Theorem A-2.1. *Suppose that there exist functions $V_p : R^n \to R$, $p \in \mathcal{M}$, two class $\mathcal{K}_\infty$ functions α_1 and α_2, and a positive number λ_0 such that we have*

$$\alpha_1(|x|) \leq V_p(x) \leq \alpha_2(|x|) \quad \forall x, \forall p \in \mathcal{M} \tag{A-19}$$

and

$$\frac{\delta V_p}{\delta x} f_p(s) \leq -2\lambda_0 V_p(x) \quad \forall x, \forall p \in \mathcal{M}. \tag{A-20}$$

Suppose also that (A-17) holds. Then the switched system is globally asymptotically stable for every switching signal σ with average dwell time

$$\tau_a > \frac{log\mu}{2\lambda_0} \tag{A-21}$$

(and No arbitrary).

Proof. See [26]. □

<div align="right">

Appendix B

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String stability conditions for a homogeneous platoon

In this chapter we will summarize the CACC and ACC string stability conditions is Section B-1 and Section B-2, respectively.

B-1 CACC string stability conditions

In a homogeneous platoon, if vehicle i is operating under a CACC strategy, the vehicle's control parameters K_p^C and K_d^C and spacing policy h^C need to satisfy the following conditions to guarantee string stable dynamics:

$$h^C > 0, \quad K_p^C, K_d^C > 0, \quad K_d^C > \tau_0 K_p^C$$

B-2 ACC string stability conditions

In a homogeneous platoon, if vehicle i is operating under an ACC strategy, the vehicle's control parameters K_p^L and K_d^L and spacing policy h^L need to satisfy the following conditions to guarantee string stable dynamics:

$$h^L > 0, \quad K_p^L, K_d^L > 0, \quad K_d^L > \tau_0 K_p^L$$

and $\forall \omega > 0$

$$(h^L \tau_0)^2 \omega^6 + ((h^L)^2 - 2K_d^L \tau_0 (h^L)^2 + \tau_0^2)\omega^4 + (1 - 2K_p^L h^L$$
$$+ (h^L K_d^L)^2 - 2K_d^L \tau_0)\omega^2 + ((h^L K_p^L)^2 - 2K_p^L) \geq 0.$$

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