Wave Propagation In Alternating Fluid-Solid Layered Media Via The Thin-Layer Method

Tsetas, Athanasios; Tsouvalas, Apostolos; Metrikine, Andrei

Publication date
2024

Document Version
Final published version

Published in
Proceedings of the 30th International Congress on Sound and Vibration, ICSV 2024

ICSV30 AMSTERDAM 8-11 JULY 2024

WAVE PROPAGATION IN ALTERNATING FLUID-SOLID LAYERED MEDIA VIA THE THIN-LAYER METHOD

Athanasios Tsetas
Faculty of Civil Engineering and Geosciences, Delft University of Technology, Delft, The Netherlands
E-mail: a.tsetas@tudelft.nl
Apostolos Tsouvalas
Faculty of Civil Engineering and Geosciences, Delft University of Technology, Delft, The Netherlands
E-mail: a.tsouvalas@tudelft.nl
Andrei Metrikine
Faculty of Civil Engineering and Geosciences, Delft University of Technology, Delft, The Netherlands
E-mail: a.metrikine@tudelft.nl

Periodic fluid-solid layered media exhibit distinctive features that can be utilized in various engineering disciplines, such as selective transmission of guided waves, omnidirectional band-gaps and Fano resonances, depending on the spatial configuration and the material properties of the fluid and solid layers involved. This work utilizes the Thin-Layer Method (TLM) in the study of layered fluid-solid media, by extending the original normal modes-based method to acousto-elastic problems. By means of this development, the band-gap structure of these periodic systems can be analysed and their effect when incorporated in fluid or solid full-/half-spaces can be investigated seamlessly for different periodic arrangements. Conclusively, this study presents a framework to analyse these systems, serving as a basis for non-local continuum theories, as well as unveiling desirable dispersion characteristics for the design of metamaterials.

Keywords: wave propagation, periodic waveguides, alternating fluid-solid layers, Thin-Layer Method

1. Introduction

Wave propagation through periodic and quasi-periodic systems has been gaining traction recently in various engineering disciplines, owing to the unique dynamic properties and wave phenomena exhibited by these structures [1]. An interesting example of such a system corresponds to waveguides composed of alternating fluid and solid layers, addressed in topics ranging from geophysics [2] and poroelasticity [3] to ultrasonic waves in biomedical applications [4]. Periodic fluid-solid layered media possess features such as selective transmission of guided waves, interface wave modes, and pass/stop bands of large variability, depending on the stacking arrangement considered.

In this paper, the Thin-Layer Method (TLM) is utilised to study the layered fluid-solid media, by extending the original normal modes-based approach to acousto-elastic problems. Furthermore, the inclusion of multiple fluid and solid layers - arranged periodically - is addressed, to facilitate the study of dispersive properties of such waveguides. The preceding developments facilitate the analysis of the modal structure of such periodic waveguides and the effect of their inclusion in fluid or solid full-/half-spaces. Conclusively, a case study is performed to demonstrate the applicability of the method to the problem of alternating fluid-solid media and the effect of porosity on the dispersion curves is considered.

2. The Thin-Layer Method in alternating fluid-solid layered media

In the following, the Thin-Layer Method (TLM) is presented for the study of alternating fluid-solid layered media. The TLM is a superbly efficient computational method to analyse wave motion in 2-D and 3-D layered media [5]. Its fundamental concept lies in the partial discretization of the problem only along the direction of layering. Specifically, a finite element (FE) discretization is used along one spatial coordinate combined with exact analytical solutions for the remaining directions. Therefore, the TLM corresponds to the discrete version of the normal modes approach, which is widely used in acoustic, elastic and acousto-elastic problems [6, 7]. By virtue of its discrete nature, this approach leads to a quadratic eigenvalue problem, circumventing the need for complex search techniques. In the few decades of its application, the TLM has been employed to study fluid [8], poro-elastic [9] and anisotropic media [10], and has been coupled to other numerical frameworks in the context of pile driving[11, 12] and seismic scenarios with topographic features [13].

To deal with the problem of a fluid-solid layered medium, the first step is to derive the TLM expressions for single fluid and solid layers. The physical domain (fluid or solid) is discretized into thin horizontal layers of infinite lateral extent, whereas the response is approximated along the vertical (layering) direction with Lagrange polynomials. The remaining dependencies in the radial and circumferential directions (3-D medium in cylindrical coordinates) are based on the exact solutions for cylindrical wave propagation in a homogeneous medium [14]. It is remarked that for a solid layer, the displacement field is the approximated response quantity. In the case of a fluid layer, various choices can be made regarding the response quantity to be computed; in the ensuing developments, the velocity potential is chosen as the approximated response quantity for the ideal fluid.

Without further delay, the normal modes of a layered fluid medium can be found by the following generalized linear eigenvalue problem:

$$\left(k^2\mathbf{A}_\mathrm{f} + \mathbf{G}_\mathrm{f} - \omega^2\mathbf{M}_\mathrm{f}\right)\boldsymbol{\phi}_\mathrm{f} = \mathbf{0} \tag{1}$$

where the matrices $\mathbf{A}_\mathrm{f}$, $\mathbf{G}_\mathrm{f}$ and $\mathbf{M}_\mathrm{f}$ can be derived analytically from the principle of virtual work for an ideal fluid layer, $\boldsymbol{\phi}_\mathrm{f}$ is the vector of velocity potential at the different elevations and k is the radial wavenumber of cylindrical wave modes.

Similarly, the normal modes of a layered solid medium can be obtained by the following quadratic eigenvalue problem:

$$\left(k^2\mathbf{A}_\mathrm{s} + k\mathbf{B}_\mathrm{s} + \mathbf{G}_\mathrm{s} - \omega^2\mathbf{M}_\mathrm{s}\right)\boldsymbol{\phi}_\mathrm{s} = \mathbf{0} \tag{2}$$

where the matrices $\mathbf{A}_\mathrm{s}$, $\mathbf{B}_\mathrm{s}$, $\mathbf{G}_\mathrm{s}$ and $\mathbf{M}_\mathrm{s}$ can be derived analytically from the principle of virtual work for a linear elastic solid layer [14], $\boldsymbol{\phi}_\mathrm{s}$ is the vector of modal displacements at the different elevations and k is the radial wavenumber of cylindrical wave modes. It is remarked that Eq. (7) encompasses both the SV-P and the SH eigenvalue problems.

Given the FE-based formulation of the TLM, an acousto-elastic medium can be readily described by overlapping the corresponding TLM matrices in a classical FE fashion. However, the coupling between fluid and solid layers is not yet addressed and it is imposed by incorporating the appropriate interface conditions in the system of equations. In particular, the following interface conditions hold:

$$\tau_{zr}^{(i)} = 0 \tag{3}$$

$$\tau_{z\theta}^{(i)} = 0 \tag{4}$$

where $\tau_{zr}^{(i)}$ and $\tau_{z\theta}^{(i)}$ are the shear stresses on the i-th (fluid-solid) interface along the radial and circumferential directions. These stress components vanish as the ideal fluid cannot support shear stresses. Furthermore, we have continuity of vertical displacements and normal stresses:

$$u_{z,\mathrm{f}}^{(i)} = u_{z,\mathrm{s}}^{(i)} \tag{5}$$

$$-\sigma_{z,\mathrm{s}}^{(i)} = p_{\mathrm{f}}^{(i)} \tag{6}$$

where $u_{z,\mathrm{f}}^{(i)}$ and $u_{z,\mathrm{s}}^{(i)}$ are the vertical displacements of the fluid and the solid along i-th (fluid-solid) interface, respectively, $\sigma_{z,\mathrm{s}}^{(i)}$ is the normal stress of the solid and $p_{\mathrm{f}}^{(i)}$ is the fluid acoustic pressure.

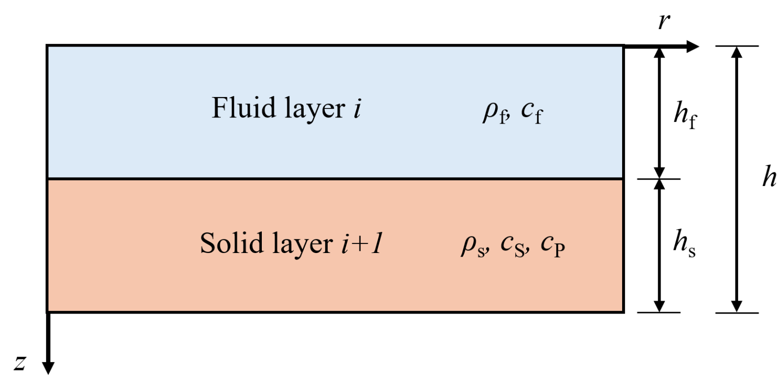

Figure 1: A unit cell of a periodically layered fluid-solid medium.

Upon stacking the TLM matrices of all fluid and solid layers of the medium, following the layering along the vertical direction z, we can formulate the coupled eigenvalue problem of the alternating fluid-solid layered system as follows:

$$\left(k^2\mathbf{A}^* + k\mathbf{B}^* + \mathbf{G}^* - \omega^2\mathbf{M}^*\right)\boldsymbol{\phi} = \mathbf{0} \tag{7}$$

where the matrices $\mathbf{A}^*$, $\mathbf{B}^*$, $\mathbf{G}^*$ and $\mathbf{M}^*$ describe the coupled layered fluid-solid medium. To fix ideas without increasing the complexity of presentation, we consider a single unit cell composed of a fluid layer overlying a solid layer (see Fig. 1). The fluid layer is characterized by mass density ρ_f and sound velocity c_f, whereas the linear elastic solid layer is characterized by mass density ρ_s, S-wave velocity c_S and P-wave velocity c_P. In that case, the above matrices for the considered unit cell read:

$$\mathbf{A}^* = \begin{bmatrix} \mathbf{A}_\mathrm{f} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\rho_\mathrm{f}^{-1}\mathbf{A}_r & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\rho_\mathrm{f}^{-1}\mathbf{A}_z \end{bmatrix}, \ \mathbf{B}^* = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\rho_\mathrm{f}^{-1}\mathbf{B}_{rz} \\ \mathbf{0} & -\rho_\mathrm{f}^{-1}\mathbf{B}_{zr} & \mathbf{0} \end{bmatrix} \tag{8}$$

$$\mathbf{G}^* = \begin{bmatrix} \mathbf{G}_\mathrm{f} & \mathbf{0} & \mathbf{G}_{\mathrm{f}z} \\ \mathbf{0} & -\rho_\mathrm{f}^{-1}\mathbf{G}_r & \mathbf{0} \\ -\rho_\mathrm{f}^{-1}\mathbf{G}_{z\mathrm{f}} & \mathbf{0} & -\rho_\mathrm{f}^{-1}\mathbf{G}_z \end{bmatrix}, \ \mathbf{M}^* = \begin{bmatrix} \mathbf{M}_\mathrm{f} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\rho_\mathrm{f}^{-1}\mathbf{M}_r & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\rho_\mathrm{f}^{-1}\mathbf{M}_z \end{bmatrix} \tag{9}$$

where all the sub-matrices introduced above stem from the TLM formulation of the individual fluid and solid layers. The fluid-solid coupling is enabled through the sub-matrices $\mathbf{G}_{\mathrm{f}z}$ and $\mathbf{G}_{z\mathrm{f}}$, which are derived by the numerical implementation of Eqs. (3), (4), (5) and (6). It is remarked that in the presence of more fluid-solid interfaces, the matrices $\mathbf{G}_{\mathrm{f}z}$ and $\mathbf{G}_{z\mathrm{f}}$ will couple each fluid (or solid) layer with the adjacent solid (or fluid) layers (implying slight modification of terms). Finally, the factor $-\rho_{\mathrm{f}}^{-1}$ is introduced to certain sub-matrices to render the matrices $\mathbf{A}^{*}$, $\mathbf{B}^{*}$, $\mathbf{G}^{*}$ and $\mathbf{M}^{*}$ symmetric.

3. Numerical results

In this section, we study the specific case of a finite set of fluid-solid cells (see Fig. 2). As regards the material properties, the fluid medium is water with mass density $\rho_{\mathrm{f}} = 1000$ kg/m^3 and sound velocity $c_{\mathrm{f}} = 1500$ m/s, whereas the solid layers are composed of steel with $\rho_{\mathrm{f}} = 7850$ kg/m^3 and S-wave velocity $c_{\mathrm{S}} = 3192$ m/s and $c_{\mathrm{P}} = 5972$ m/s.

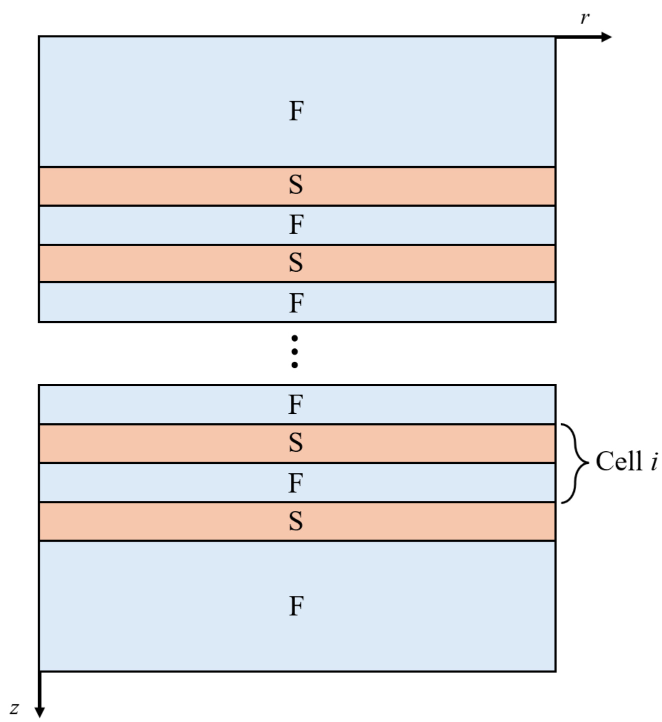

Figure 2: An alternating fluid-solid layered medium with finite number of cells.

In Fig. 3, we present the dispersion curves for the two different 2-cell waveguides, differing in the fluid-to-solid thickness ratio of the alternating layers (i.e. porosity). In particular, the radial wavenumber of the propagating wave modes are plotted against the frequency. As can be seen, the high porosity case ($h_{\mathrm{f}} = 10h_{\mathrm{s}} = 1$ m) leads to the formation of partial band-gaps in the lower wavenumber range throughout the frequencies, whereas for the low porosity case such behaviour is not present. Furthermore, the combination of an alternating fluid-solid system with an end solid layer of dissimilar material properties can transform such partial band-gaps to omnidirectional band-gaps, given that appropriate thickness ratio, number of cells and material properties are considered [15]. Finally, one of the interesting phenomena that arise in systems similar to the one at hand, is the formation of the so-called Kraklis wave [16]. This special wave mode is a slow dispersive guided wave that propagates in fluid layers bounded by elastic layers and can be distinguished in the high porosity . It results from the interference of the two Scholte waves associated with the solid-fluid interfaces bounding the fluid layer [16].

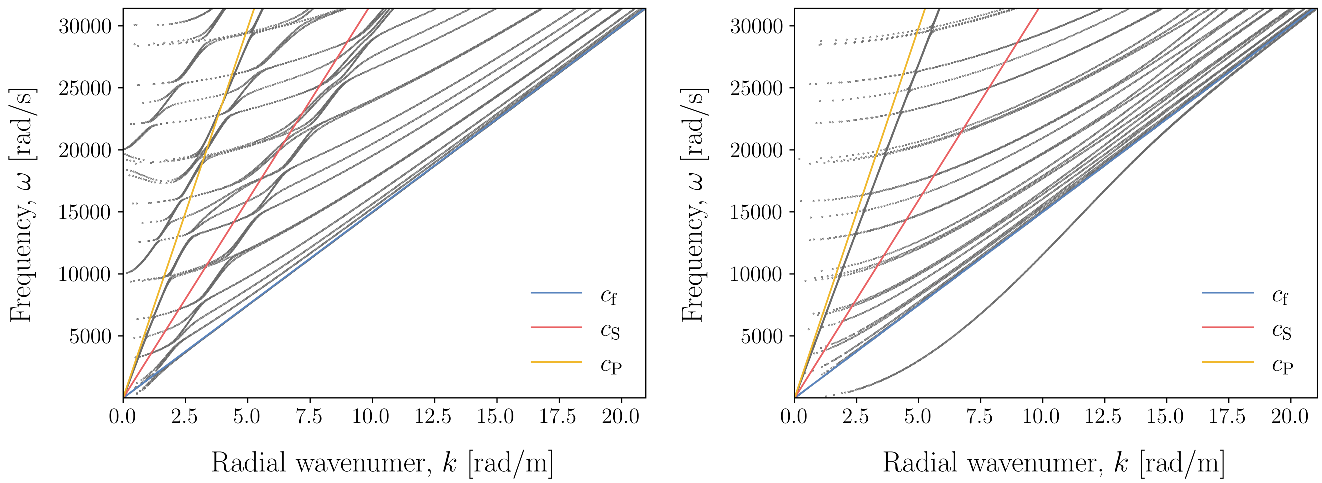

Figure 3: Dispersion curves for a 2-cell fluid-solid waveguide with (a) $h_{\mathrm{f}} = h_{\mathrm{s}} = 1$ m and (b) $h_{\mathrm{f}} = 10h_{\mathrm{s}} = 1$ m.

4. Conclusions

This paper presents an extension of the Thin-Layer Method that encompasses alternating fluid-solid media. In particular, the main equations that describe the wave motion in fluid and solid layers have been introduced, as a basis for the coupled problem of a waveguide comprised of alternating fluid-solid layers. Furthermore, the fluid-solid interface conditions have been translated into the relevant coupling sub-matrices, leading to a final quadratic eigenvalue problem for the alternating fluid-solid medium. Subsequently, the TLM is employed to study the case of an alternating fluid-solid medium with finite number of cells, showcasing the method's efficacy and demonstrating the effect of porosity on the modal structure. Conclusively, the developed framework can be readily employed to study the dispersion properties of periodically (or quasi-periodically) alternating fluid-solid media in a computationally efficient manner, allowing the exploration of a multitude of layering configurations towards desired dispersion features.

REFERENCES

1. Deymier, P. A., *Acoustic metamaterials and phononic crystals*, vol. 173, Springer Science & Business Media (2013).

2. Schoenberg, M. Wave propagation in alternating solid and fluid layers, *Wave motion*, **6** (3), 303–320, (1984).

3. Gurevich, B. and Ciz, R. Shear wave dispersion and attenuation in periodic systems of alternating solid and viscous fluid layers, *International Journal of Solids and Structures*, **43** (25-26), 7673–7683, (2006).

4. Hughes, E. R., Leighton, T. G., Petley, G. W. and White, P. R. Ultrasonic propagation in cancellous bone: A new stratified model, *Ultrasound in medicine & biology*, **25** (5), 811–821, (1999).

5. Kausel, E. Dynamic point sources in laminated media via the thin-layer method, *International Journal of Solids and Structures*, **36** (31-32), 4725–4742, (1999).

6. Aki, K. and Richards, P. G., *Quantitative seismology*, University Science Books (2002).

7. Jensen, F. B., Kuperman, W. A., Porter, M. B., Schmidt, H. and Tolstoy, A., *Computational ocean acoustics*, vol. 794, Springer (2011).

8. Lotfi, V., Roesset, J. M. and Tassoulas, J. L. A technique for the analysis of the response of dams to earthquakes, *Earthquake engineering & structural dynamics*, **15** (4), 463–489, (1987).

9. Bougacha, S., Roësset, J. M. and Tassoulas, J. L. Dynamic stiffness of foundations on fluid-filled poroelastic stratum, *Journal of engineering mechanics*, **119** (8), 1649–1662, (1993).

10. Kausel, E. Wave propagation in anisotropic layered media, *International Journal for Numerical Methods in Engineering*, **23** (8), 1567–1578, (1986).

11. Tsetas, A., Tsouvalas, A. and Metrikine, A. V. A non-linear three-dimensional pile–soil model for vibratory pile installation in layered media, *International Journal of Solids and Structures*, **269**, 112202, (2023).

12. Tsetas, A., Tsouvalas, A. and Metrikine, A. V. The mechanics of the gentle driving of piles, *International Journal of Solids and Structures*, **282**, 112466, (2023).

13. Nguyen, K. T., Kusanovic, D. S. and Asimaki, D. Three-dimensional nonlinear soil–structure interaction for rayleigh wave incidence in layered soils, *Earthquake Engineering & Structural Dynamics*, **51** (11), 2752–2770, (2022).

14. Kausel, E. and Roësset, J. M. Stiffness matrices for layered soils, *Bulletin of the seismological Society of America*, **71** (6), 1743–1761, (1981).

15. El Hassouani, Y., El Boudouti, E., Djafari-Rouhani, B. and Aynaou, H. Sagittal acoustic waves in finite solid-fluid superlattices: Band-gap structure, surface and confined modes, and omnidirectional reflection and selective transmission, *Physical Review B*, **78** (17), 174306, (2008).

16. Frehner, M. Krauklis wave initiation in fluid-filled fractures by seismic body waves, *Geophysics*, **79** (1), T27–T35, (2014).