Using Model Predictive Control to model the Role of the Government in the US Economy

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***Abstract*—This research paper describes the design and implementation of a Model Predictive Controller, using economic engineering principles, on a model of the US economy. The purpose of the Model Predictive Controller is to mimic the US government policy by maximizing the Net Domestic Product. The Model Predictive Controller is integrated with a bond graph model which is modeled based on macroeconomic principles and used to simulate the US economy. In all cases, the Model Predictive Controller was able to successfully stabilize the US economy, while maximizing the Net Domestic Product e.g. the economic output of the economy. The model described in this paper provides a promising new way of generating more accurate prospects of future market movements.**

I. Introduction

This research paper aims to design a controller that models the role of the government in the US economy. To simulate the US government policy, the controller needs to maximize an economy's economic output, being the Net Domestic Product (NDP). This paper will focus on the design process and analyze the performance of the controller. The controller interacts with a bond graph model (Appendix A) designed by Gilbert Kruimer [1] based on macroeconomic theory. A custom cost function will be designed to maximize the NDP. Multiple economic scenarios will be modeled to observe the response of the system when exposed to disturbances. In the following Subsections, relevant background information to this research will be discussed.

A. *Socio-economic Impact of an automated Government*

Companies that are active in financial markets manage the risks and uncertainties of the markets by predicting the market movement. Their current models are engineered by extracting historic data, identifying patterns, and using them to create forecasts of the economy. This way of generating economic forecasts presents its limitations since one is restricted to historic events for identifying patterns and uncertainty increases when increasing the prediction horizon. A more promising way of generating economic forecasts is through the use of economic engineering. Making use of Gilbert Kruimer's model [1] and the controller designed in this paper, companies active in financial markets could gain valuable insights, improve their forecasts and therefore increase their profitability. Examples of how this technology could improve forecasts are anticipating the effect of changes made in government policies and responding to them, or making better predictions of how changes in politics, technology, or climate affect government policies. Relating this to a current-day problem, one could predict the economic consequences of the COVID-19 pandemic and how to respond to such a crisis.

Another possible application of using a controller to govern the US economy is the possibility to test the effectiveness of future policies. The US government often changes from government policies because of changing politics, technology, or climate for instance. In the long run, these changes can cause policies to be contradicting and inefficient [2]. This often happens, because politicians dissociate themselves from government policy revisions due to a fear of being blamed for an inefficient policy and losing political credibility [3]. Using a controller to implement policy decisions will give more insight into government policies in the short and the long run and could result in a more effective government policy.

B. *Macroeconomics*

Macroeconomics is the study that deals with the economy as a whole [4]. It takes interest in the functioning, composition, and performance of economic systems. Examples of economic systems are a state or a nation, but also the entire world economy. Through observations of economic quantities, macroeconomists try to formulate theories about how an economy behaves and make predictions on how an economy will react to change. In macroeconomics these theories are used in combination with static models, while in this paper these theories are translated to apply to dynamic models, forming the foundation for modeling the US economy.

C. *Economic Engineering*

In each domain of engineering, energy is the most fundamental unit used. A similar form of energy can be applied in economics, allowing economic engineers to translate problems from the economic domain to problems in the engineering domain. Then, engineering principles are applied to solve these problems. Figure 1 shows the similarities between different engineering domains by comparing the variables corresponding to the Flow, Effort, Displacement, and Momentum in the different domains.

Bond graphs are graphical representations of dynamical systems and are often used in economic engineering [5]. Every bond within a bond graph has a specific flow and effort. A bond graph consists of various elements connected through zero-type or one-type junctions. Elements found in bond graphs are resistors: elements that dissipate energy; capacitive elements: elements that store energy in the form of flow; inertia elements: elements that store energy in the form of effort, transformers, gyrators, sources, and sinks [6]. Together, all these elements and junctions model a dynamic system.

Domain	Flow f(t)	Effort e(t)	Displacement q(t)	Momentum p(t)
LinearMechanical	Velocity(m/s)	Force (N)	Displacement (m)	Momentum (N*s)
Electromagnetic	Current(A)	Voltage (V)	Charge (C)	Flux (V*s)
Economic	Product per time (#/year)	Benefit per time ($/#*year)	Quantity (#)	Price ($/#)

Figure 1: Comparison of units from the different engineering domains, illustrating similarities between the different engineering domains.

The functional relationships between the variables in a bond graph [7] are displayed in a tetrahedron of state. In Figure 2, the tetrahedron of state is shown, containing the four domain variables, $e(t), p(t), f(t)$ and $q(t)$, in a generalized form. The tetrahedron of state works by following the direction of the arrows and multiplying the variable by the constant on the arrow.

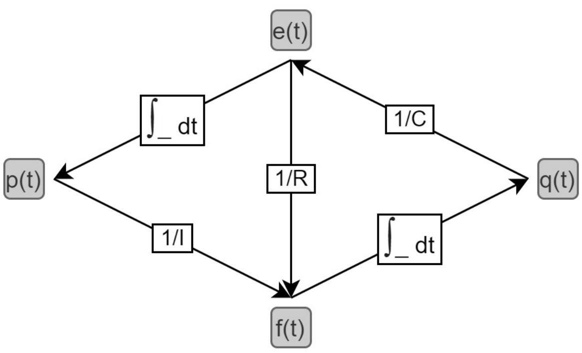

Figure 2: Tetrahedron equation of state in generalized form. Connects the different domain variables and shows the basic relations used in bond graph modeling.

Using this tetrahedron of state, five relations between the variables are derived. The relation between the momentum and the effort is defined as:

$$p(t) = \int e(t)\, dt \tag{1}$$

The relation between the flow and the momentum, with I as an inertia element, is defined as:

$$f(t) = \frac{1}{I} \cdot p(t) \tag{2}$$

The relation between the displacement and the flow is defined as:

$$q(t) = \int f(t)\, dt \tag{3}$$

The relation between the effort and the displacement, with C as a capacitive element, is defined as:

$$e(t) = \frac{1}{C} \cdot q(t) \tag{4}$$

The relation between the flow and the effort, with R as a resistance element, is defined as:

$$f(t) = \frac{1}{R} \cdot e(t) \tag{5}$$

These relations will be used to determine the cost and constraint function. The paper will proceed as follows: In Section II the model of the US economy is introduced together with the role of a government within an economy. Then, Section III will elaborate on the controller selection and explain how the NDP is being maximized. Section IV will then continue with an analysis of the performance of the controller, followed by Section V that concludes the results. Finally, Section VI discusses suggestions for further research.

II. Modeling the US Economy

This Section introduces the model of the US economy. Then the function of the government in a macroeconomic context is discussed and how the role of a government is translated to a control problem. Finally, the selection of the model variables and parameters will be explained.

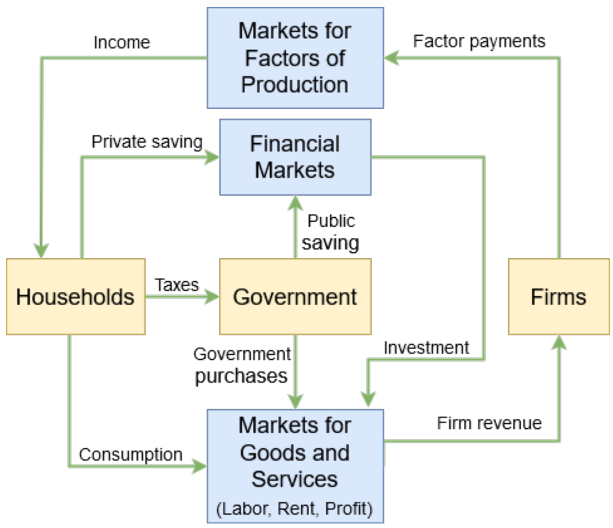

Figure 3: Cash flows through a circular economy, connecting the economic actors (Households, Government, and Firms) to the economic markets (Factors of Production, Financial, and Goods and Services). Directly copied from Mankiw [4].

A. Model of the US Economy

The foundation for the model of the US Economy is a circular flow diagram provided by N. Gregory Mankiw (Figure 3) in his book on Macroeconomics [4]. The flow diagram depicts how economic actors interact with economic markets. There are three economic actors in the economy, namely: the households, the firms, and the government. Likewise, the three types of economic markets are the markets for Goods and Services, Financial markets, and the markets for Factors of Production, which are subdivided into the Labor market, the Rent market, and the Profit market.

Figure 3 only depicts flows while a bond graph is a combination of flows and efforts. The first step is to alter the system, resulting in Figure 4. Important to note is the inclusion of a new economic actor "Rest of World" in the model since the circular flow diagram represents a closed economy.

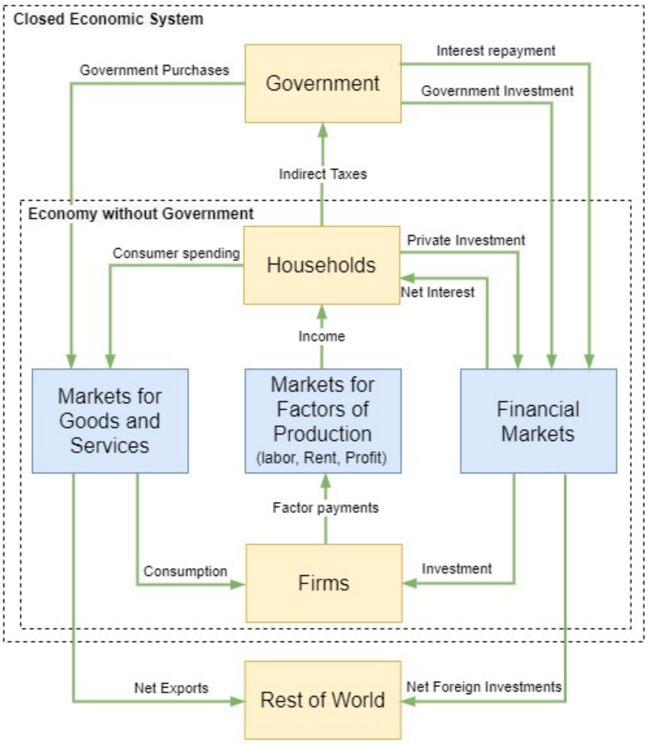

Figure 4: Cash flows through a circular economy, visualizing the plant (Economy without Government), the controller (Government), and the interaction with a new economic actor: Rest of World. Based on the work of Mankiw [4].

The last step is to convert Figure 4 to a bond graph (Appendix A). The economic interpretation of the bond graph elements is listed in Table I. Because the markets represent different sorts of products, they are expressed in different sets of economic variables. Therefore each of the markets is connected by transformers and gyrators.

The dynamics of the bond graph are captured in a state-space model that connects the states ($\mathbf{x}$) and inputs ($\mathbf{u}$) of the model to the change in states ($\dot{\mathbf{x}}$) (Appendix B). The state vector ($\mathbf{x}$) thus contains:

1) $x_1 = p_1$, Wage $[\$/FTE^1]$
2) $x_2 = q_1$, Unemployment $[FTE]$
3) $x_3 = p_2$, Return on Equity $[\%]$
4) $x_4 = q_2$, Profit $[\$]$
5) $x_5 = p_3$, Rent Level $[\$/\#]$
6) $x_6 = q_3$, Available Stock $[\#]$
7) $x_7 = p_4$, Price Level $[\$/\#]$
8) $x_8 = q_4$, Inventory Stock $[\#]$
9) $x_9 = p_5$, Short Term Bond Index $[\%]$
10) $x_{10} = q_5$, Outstanding T-Bill $[\$]$
11) $x_{11} = p_6$, Long Term Bond Index $[\$]$
12) $x_{12} = q_6$, Outstanding T-Notes $[\$]$
13) $x_{13} = p_7$, Capital Price Index $[\%]$
14) $x_{14} = q_7$, Investment $[\$]$

In macroeconomics, the Net Exports and Net Foreign Investments have to cancel each other out and can thus be set to zero to simplify the dynamics, resulting in a closed economy (Figure 4) with a total of eight inputs. The inputs can then be divided into manipulated variables and model disturbances. The Technological Advancement (S_{f_5}), Discovery National Resources (S_{f_4}), and Immigration (S_{f_1}), make up the model disturbances. The input vector $\mathbf{u}$ then contains:

1) $u_1 = S_{f_2}$, Government Purchases $[\#/year]$
2) $u_2 = S_{f_3}$, Government Investment $[\%/year]$
3) $u_3 = S_{e_1}$, Coupon Rate Short Term $[\%/year^2]$
4) $u_4 = S_{e_2}$, Coupon Rate Long Term $[\%/year^2]$
5) $u_5 = S_{e_3}$, Indirect Taxes $[\$/year^2]$
6) $u_6 = S_{f_1}$, Immigration $[FTE/year]$
7) $u_7 = S_{f_5}$, Technological Advancement $[\$/year]$
8) $u_8 = S_{f_4}$, Discovery National Resources $[\#/year]$

The simplification of the model to an economy without growth yields another advantage: the principle of national accounting. The principle of national accounting defines a way of computing the GDP of a country and states that the income that a country receives needs to be equal to the expenditure of a country. This approach of income and expenditure can be thought of as the balance sheet of a company and can also be located in the bond graph. The '1' junction in the households represents the income approach consisting of the following:

1) Compensation of Employees $(Wage)$
2) Proprietors Income and Corporate Profits $(Profit)$
3) Rental Income $(Rent)$
4) Net Interest $(Interest)$
5) Indirect Business Taxes $(Indir.Tax)$

Since the Depreciation is dissipated, the NDP is computed, not the GDP. The same is done for the '1' junction of the firms that represents the expenditure approach:

1) Consumption (C)
2) Investment[2] $(G_i + I_{rest} + I_{inv})$
3) Net Exports (NX)
4) Government Purchases (G_{es})

In an economy without growth, the net cash flow through these two junctions has to be equal. The principle of national accounting gives us an idea of how the economy will react to

[1] FTE (Full-Time Equivalent) indicates the workload of one full-time employed person.

[2] G_i = Government Investments,, I_{inv} = Inventory Investments, I_{rest} = Remaining Investments

shifts in cash flows and thus how the system dynamics will behave.

B. The US Government as Controller

In Figure 3, the government interacts in three ways with the system: through taxes, government purchases, and public savings. The government behaves similarly to the households. The government receives an income (taxes), and balance their spending between consumption (government purchases) and saving (public saving/government investment). The big difference between households and the government is that the government can change its income and expenditure to influence the economic state. Translating this concept to the field of economic engineering, one sees the possibilities of modeling the US government as a controller.

In the bond graph (Appendix A), the government has been modeled as several source flows and effort flows. The five manipulated variables are Indirect Taxes (S_{e_3}), Government Purchases (S_{f_2}), Government investment (S_{f_3}), Coupon Rate Short Term (S_{e_2}), and Coupon Rate Long Term (S_{e_3}). The addition of the Short and Long Term Coupon Rate is because of how the financial market is modeled in the bond graph. Issued debt in the shape of bonds dominates the financial markets [8], and is divided into short term (T-Bill) and long term (T-Note) debt. Since the issued debt is government-controlled, the Coupon Rate Short Term and Coupon Rate Long Term are also taken as manipulated variables.

There are limitations to what a government can do for instance a government cannot double or triple taxes in one moment, and change it back the other. The most general limitation is that a government can not spend more than it receives as income. Its government expenditures can exceed the income received from taxes, but then a government must compensate for this budget deficit by taking out loans. Next to limitations, each government executes a policy, which can vary depending on the current economical situation or because of political ideologies. This policy has to be simplified to apply to the control problem. Looking at the US economy in general it could be said a capitalistic view would apply as a government policy: maximizing a nation's economic output, being the NDP [9].

In Figure 5 a graphical representation is given of how the US government will function as a controller with the model of the US economy. The model of the US economy produces the states as output, which are fed into the controller. The controller then calculates the manipulated variables which are directed towards the model of the US economy. Together with the model disturbances and the dynamics of the model of the US economy, the new states of the model are determined. Looking at Figure 4, a more detailed representation of the plant (Economy without Government) and the controller (Government) is shown.

C. Initializing the Model Parameters

The model parameters cover all the different elements in the bond graph (Appendix A) and define the behavior of the system. The economic interpretation of these parameters can be found in Table I:

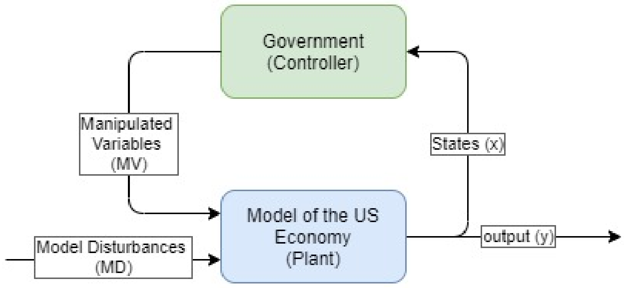

Figure 5: The model of the US economy produces output (y), which are the same as the states (x), and feeds these into the controller. The controller outputs the manipulated variables (MV) impacting the US economy. Together with the model disturbances (MD), the dynamics of the model of the US economy determine the new states of the system.

Parameters	Economic Interpretation
m_1	Transformer Constant
m_2	Transformer Constant
m_3	Transformer Constant
m_4	Transformer Constant
m_5	Transformer Constant
m_6	Transformer Constant
r_1	Gyrator Constant
r_2	Gyrator Constant
cr_1	Modulated Gyrator Constant
cm_1	Modulated Transformer Constant
cm_2	Modulated Transformer Constant
cm_3	Modulated Transformer Constant
CM	Constant, $(cr_1)/(cr_1 - r_1)$
R_1	Miscellaneous
R_2	Depreciation
I_1	Free Market Force of the Labor Market
C_1	Labor Productivity
I_2	Free Market Force of the Profit Market
C_2	Return on Revenue
I_3	Free Market Force of the Rent Market
C_3	Economic Rent
I_4	Free Market Force of the Market of Goods and Services
C_4	Force of Supply
I_5	Change in Coupon Rate Short Term
C_5	Coupon Short Term Yield
I_6	Change in Coupon Rate Long Term
C_6	Coupon Long Term Yield
I_7	Free Market Force of the Financial Market
C_7	Return on Invested Capital

Table I: Economic interpretation of parameters used to model the US economy.

To control the model, the system has been chosen to be stable, requiring that the real parts of the eigenvalues of the $\mathbf{A}$-matrix have to be negative. To achieve this, a MATLAB-script (Appendix C) assigns a random value to all of the parameters until the eigenvalues of the $\mathbf{A}$-matrix are stable. The resulting model parameters are defined in Table II (Appendix C). These parameters however do not represent the actual US economy since the economic interpretations of the elements are not taken into account. But, because defining each of the elements realistically is outside the scope of this paper, the method described above is used to define the model parameters.

III. Maximize NDP with a nonlinear MPC

This Section explains why a Nonlinear Model Predictive Controller is used as the controller in this research. Then the properties of a nonlinear MPC are explained, after which it is described how to configure these properties in such a way that it achieves the desired optimization. Then the input constraints are defined accordingly to the economic limitations of a government. Finally, a formula for the NDP is derived and it is explained how the MPC uses this formula to maximize the NDP.

A. Nonlinear MPC

To get a realistic simulation, the controller used to model the US government has to have the following properties: it has to be multi-variable and able to anticipate future events. Following these requirements, two different controllers are applicable: the Linear-Quadratic Regulator (LQR) and the MPC. However, the MPC is the better option, since the LQR can not deal with hard constraints. As stated before, a government has limitations when executing policies and because of these constraints, a MPC is the better option to model the US government. The particular choice for a nonlinear MPC is because MATLAB only allows for a custom cost function to be defined when using a nonlinear MPC.

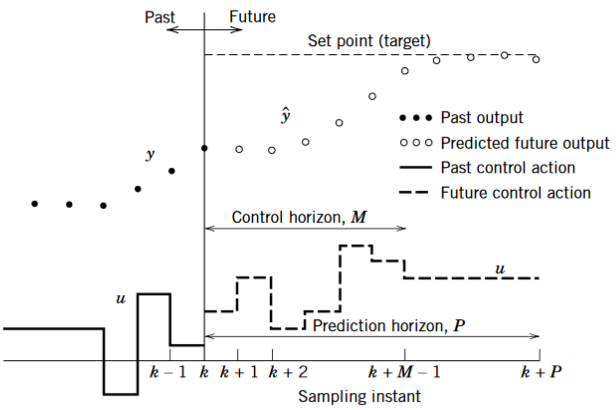

Figure 6: Characteristics of the receding horizon approach used by the MPC. [10]

The optimization of the MPC works with a receding horizon approach (Figure 6). The horizon contains the prediction horizon and the control horizon. The prediction horizon p is the number of future control intervals which the MPC must predict when optimizing its manipulated variables. The control horizon M calculates certain control moves to minimize the cost over the trajectory of the predicted future outputs within the prediction horizon. These control moves correspond to the actions the government takes to achieve the desired goal. So the sampling instants $u(k), u(k+1), ..., u(k+M-1)$ are calculated at every time step until the cost function is minimized.

There are constraints on the control and prediction horizon to gain a good performance of the MPC [11]. The prediction horizon p should not be too big otherwise the computational effort gets heavy. Also, $M < p$ is necessary otherwise some of the manipulated variables will not affect the outputs of the plant. The control horizon should not be too small otherwise this will result in aggressive control actions. The recommended setting is that, if there is a response time of T, to choose p such that $T \approx p \cdot T_s$ where T_s is the control interval. Also, increasing M makes the MPC more aggressive and results in more computational effort.

Modeling the US government as the controller, the prediction horizon could be interpreted as a presidential term of four years and the control horizon can be set based on how fast a government wants its policy to take effect. A small control horizon could cause fast and drastic changes in certain places in the economy, while a bigger control horizon causes a more fluent transition to the effect of new policies.

B. Defining the Constraints of the Controller

As discussed in Section II-B, governments are limited in their actions, and in this case, control actions. One of the advantages of using a MPC is the possibility of adding a custom constraint function and adding constraints to the states and manipulated variables. To constrain the government, the net power outflow of the government as a function of the inputs is set to zero, equating the expenditure to the income. In bond graph modeling power P is defined as the multiplication of the flow f and the effort e:

$$P = f \cdot e \tag{6}$$

At every flow or effort source/sink, either the effort or the flow is known. To then determine the matching flow or effort, the flow or effort is written out as a function of the states and inputs. The flows and efforts corresponding to the manipulated variables are given in equations 7-11.

$$f_{u_1} = u_1,\ e_{u_1} = \frac{CM}{cm_4} \cdot \left(\frac{C_3 x_5}{m_3} + \frac{C_2 x_3}{m_2} + \frac{C_1 x_1}{m_1} - u_5\right) \tag{7}$$

$$f_{u_2} = u_2, \qquad e_{u_2} = C_7 \cdot x_{14} \tag{8}$$

$$e_{u_3} = u_3, \qquad f_{u_3} = \frac{x_9}{I_5} \tag{9}$$

$$e_{u_4} = u_4, \qquad f_{u_4} = \frac{x_{11}}{I_6} \tag{10}$$

$$e_{u_5} = u_5,\ f_{u_5} = \frac{cr_1}{cr_1 + r_1} \cdot \Bigg(\bigg(\Big(\big(\frac{x_7}{I_4} - u_1\big) \cdot \frac{1}{cm_4} \Big) + \frac{CM}{R_1} \cdot \Big(\frac{C_3 \cdot x_5}{m_3} + \frac{C_2 \cdot x_3}{m_2} + \frac{C_1 \cdot x_1}{m_1} - u_5 \Big) + \frac{C_7 \cdot x_{14}}{cr_1} + \frac{m_4}{cr_1 \cdot C_5 \cdot x_{10}} + \frac{m_5}{cr_1 \cdot C_6 \cdot x_{12}} \bigg) \tag{11}$$

Using equation 6 and equations 7-11, the power of the manipulated variables is determined. To formulate the custom constraint function, the direction of the power flows in the bond graph has to be established. Government Purchases (u_1) and Government Investment (u_2) must both be positive, while the Coupon Rate Short Term (u_3), Coupon Rate Long Term (u_4), and Indirect Taxes (u_5) must all be negative (Appendix B). Especially the Indirect Taxes have to be negative, as it is part of the income approach (see Section II-A). The custom constraint function, expressed in power flows at the different inputs, then becomes:

$$P_{u_1} + P_{u_2} - P_{u_3} - P_{u_4} - P_{u_5} = 0 \tag{12}$$

To prevent the MPC from violating the directions of the power flows, additional constraints are imposed on the manipulated variables. There are two kinds of constraints: hard and soft. Hard constraints may never be violated, whereas soft constraints may be violated to satisfy hard constraints. If the direction of the power flow of the Coupon Rate Short Term and Coupon Rate Long Term faces is reversed, this results in government saving, in contrast to lending, so these constraints may be soft. Government Purchases, Government Investment, and Indirect Taxes are all constrained hard since the reverse power flow is very rare.

C. NPD as Cost Function

To manage what control actions will lead to a maximization of the NDP, the MPC makes use of a cost function. The MPC uses an optimization algorithm that minimizes the cost function by manipulating the input variables. Since the MPC can only minimize the cost function, the cost function J will be defined as:

$$J(\mathbf{x}, \mathbf{u}) = \frac{1}{NDP} \tag{13}$$

Minimizing this equation will therefore result in a maximized NDP.

The NDP is defined by the total energy flow through the '1' junction (Appendix A) of the firms, also defined as the expenditure approach [4]. Excluding the Net Exports, the expenditure approach then defines the NDP as:

$$NDP = C + I_{inv} + G_{cs} - Depreciation \tag{14}$$

The flow of the GDP is expressed in state variables, which are obtained out of the bond graph model (Appendix A), and model parameters (see Section II-C):

$$f_{GDP} = \frac{x_1}{cm_1 \cdot I_1} + \frac{x_3}{cm_2 \cdot I_2} + \frac{x_5}{cm_3 \cdot I_3} \tag{15}$$

The energy flow of a bond is defined as:

$$E = \int f \cdot e \ dt \tag{16}$$

The effort flowing out is divided between the Resistive element R_2 and the bond going to the markets for Factors of Production.

$$e = -f_{GDP} \cdot R_2 \tag{17}$$

The energy flow of the bond thus simplifies to:

$$E = \int f_{GDP} \cdot (1 - R_2) \ dt \tag{18}$$

This results in the following function for the NDP:

$$NDP = (1 - R_2) \cdot \Big(\frac{x_1^2}{2\ cm_1 I_1} + \frac{x_3^2}{2\ cm_2 I_2} + \frac{x_5^2}{2\ cm_3 I_3} \Big) \tag{19}$$

The dominant variables in this equation are the states x_1, x_3, and x_5, representing the markets for Factors of Production. Because the total energy flowing through the markets for Factors of Production equals the NDP, it is expected that these variables dominate the cost function. However, the NDP does not only depend on these states.

Optimizing the NDP requires equation 19 to be an expression of all manipulated variables (and possibly state variables). This expression is obtained by writing out the state-space equation of the US economy in $\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$ (Appendix B). To get all five of the manipulated variables, matrix manipulations have to be performed on rows 2, 9, 11, and 14, which results in an expression for x_1 containing all manipulated variables.

Substituting the derived equation of x_1 into equation 19 and then into equation 13 results in an expression for the cost function containing all the manipulated variables. Equation 20 is depended on state derivatives($\dot{\mathbf{x}}$) which are acquired from the state space equation $\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$ (Appendix B).

$$J(\mathbf{x}, \mathbf{u}) = \Bigg[(1 - R_2) \cdot \Bigg(\frac{1}{2 \cdot cm_1 \cdot I_1^3} \cdot \Bigg(\dot{x_2} - \frac{CM \cdot C_1}{m_1^2 \cdot R_1} \cdot \bigg(\Big(\dot{x_{14}} - \frac{CM \cdot C_2 \cdot x_4}{cm_1 \cdot m_2} - \frac{CM \cdot C_3 \cdot x_6}{cm_1 \cdot m_3} + I_7 \cdot x_{13} - u_2 + \frac{CM \cdot u_5}{cm_1} \Big) \cdot \frac{cm_1 \cdot m_1}{CM \cdot C_1} \bigg) - \frac{CM \cdot C_2 \cdot x_4}{m_1 \cdot m_2 \cdot R_1} - \frac{CM \cdot C_3 \cdot x_6}{m_1 \cdot m_3 \cdot R_1} - \frac{I_4 \cdot x_7}{m_1 \cdot cm_4} - \frac{m_4 \cdot C_5 (u_3 - \dot{x_9})}{m_1 \cdot cr_1 \cdot C_5} - \frac{m_5 \cdot C_6 (u_4 - \dot{x_{11}})}{m_1 \cdot cr_1 \cdot C_6} - \frac{C_7 \cdot x_{14}}{m_1 \cdot cr_1} + \frac{u_1}{m_1 \cdot cm_4} + \frac{CM \cdot u_5}{m_1 \cdot R_1} \Bigg)^2 + \frac{x_3^2}{2 \cdot cm_2 \cdot I_2} + \frac{x_5^2}{2 \cdot cm_3 \cdot I_3} \Bigg) \Bigg]^{-1} \tag{20}$$

IV. Analyzing MPC Performance

This Section discusses the performance of the MPC. First, the response of the model without MPC is analyzed, followed by the model with MPC. Finally, the system behavior under influence of different sorts of disturbances will be discussed.

When analyzing the states of the system, states x_1, x_3, and x_5, are identified as dominant in influencing the value of the NDP. This Section chooses to elaborate on the relation between these three states and the NDP.

A. US Economy without MPC

Before analyzing the performance of the model with MPC, it is interesting to analyze the behavior of the model without manipulated variables. To achieve this, the model states are set to an initial value, while keeping all manipulated variables zero. In Figure 7 it is observed that all states display an oscillating behavior, with different amplitudes and frequencies.

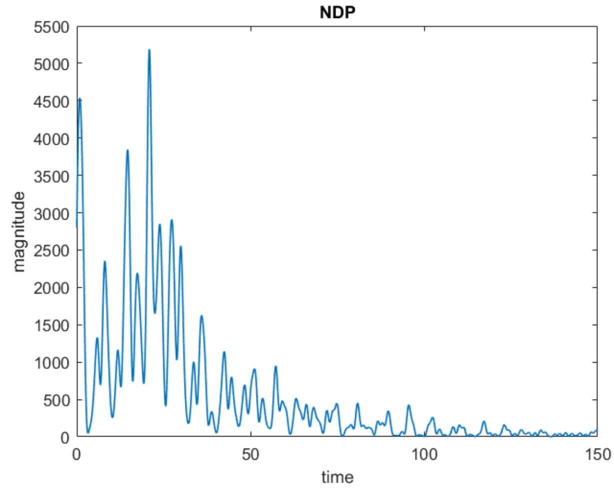

Figure 8: The NDP without MPC as government, displaying oscillating and decreasing behaviour over time.

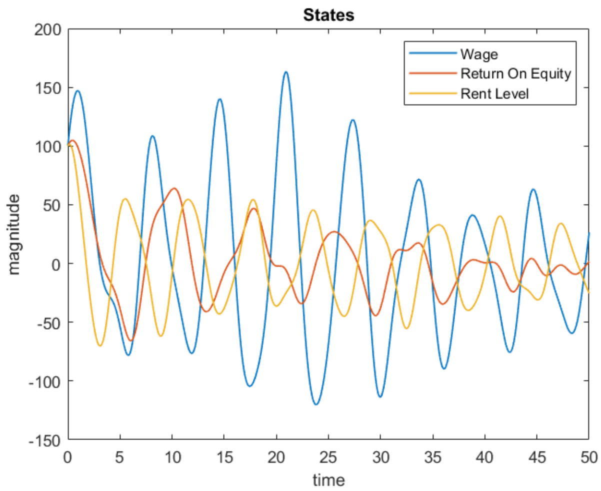

Figure 7: The states x_1 (Wage), x_2 (Return on Equity), and x_3 (Rent Level) over time without MPC as government. All states display an oscillating behavior, with different amplitudes and frequencies.

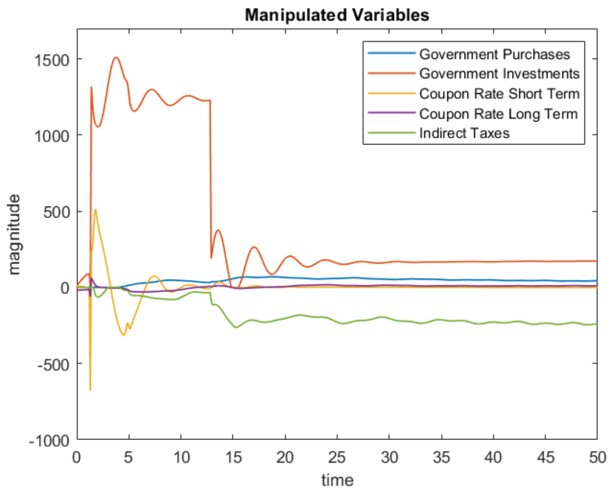

Figure 9: The manipulated variables over time representing government policy. At first, the inputs vary greatly after which they stabilize and converge to a steady-state value.

Over time, the states are slowly decreasing, which is caused by the Resistive elements in the bond graph. The Resistive elements in the bond graph representing "Miscellaneous" and "Depreciation", dissipate energy, e.g. wealth out of the economy. In an economy without growth, over time, capital will wear out and labor will diminish because of mortality, also known as depreciation. Similarly, some wealth will diminish through consumption in, for instance, shadow markets, which are not included in the NDP. This consumption is defined as miscellaneous.

The oscillation of the different states is a natural phenomenon in economics and can be explained by the circular structure of the bond graph. The oscillation of the NDP over time, known as the business cycle, is caused by the oscillation of the states (see Figure 8).

B. US Economy with MPC

In Figure 9 the inputs, representing the control actions of the MPC, are depicted over a duration of time. At first, the manipulated variables vary greatly after which they stabilize and converge to a steady-state value. Because the initial value of the inputs is zero, the initial response is relatively violent. Since the model of the US Economy does not account for growth, the expectation is that there is one optimal control input that maximizes the NDP. This explains why the manipulated variables converge to a steady-state value after initialization.

Looking at the manipulated variables, it can also be seen that the Government Purchases and Government Expenditure are positive, the Indirect Taxes are negative and the Short and Long Term Coupon Rate vary around zero. This illustrates that the custom constraint function limits the control actions of the government.

As a result of the manipulated variables, the states are changing over time (see Figure 10). Again, after initialization the states converge to a steady-state value, indicating an optimal state of the US economy.

The maximization of the NDP can best be seen in Figure 11. It depicts the NDP over time, which shows that the US economy achieves a maximum economic output given its resources. Comparing the NDP of the model with MPC (Figure 11) to the NDP of the model without (Figure 8), it is clear that the economic output with MPC is much higher, asserting the optimal economic output of the US economy.

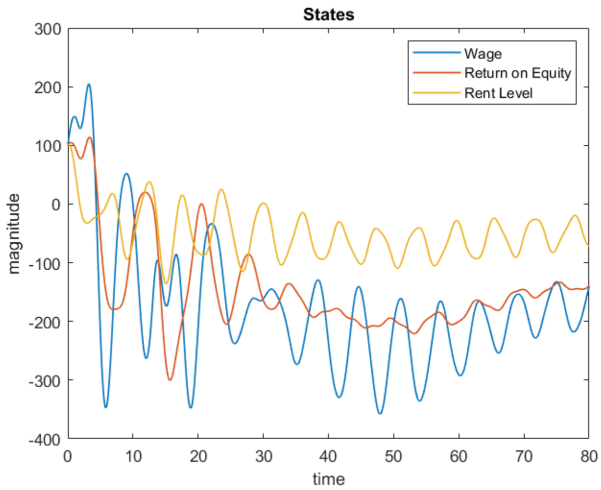

Figure 10: The states x_1 (Wage), x_2 (Return on Equity), x_3 (Rent Level) over time with MPC as government. After initialization the states converge to a steady-state value.

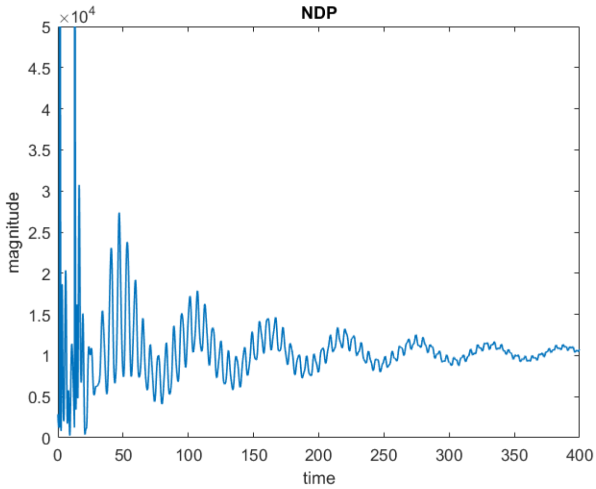

Figure 11: The NDP over time with MPC as government. After initialization the NDP converges to a steady-state value.

Interesting to see is that the NDP oscillates at two frequencies. One oscillation of a longer period consists of around 8 to 11 oscillations with a smaller period. A possible hypothesis is that the smaller oscillation is caused by seasonality (yearly), while the larger oscillation represents the business cycle, suggesting a period of around 8 to 11 years for the business cycle. Over time, both oscillations fade out, suggesting the government modeled by the MPC can fully stabilize the US economy.

C. Disturbance Modeling

In this Section, the model will be subjected to different sets of disturbances. First, the model will be subjected to three step functions of different magnitudes, at different times and different positions in the bond graph. These step functions represent shocks in the system. The shocks that are being modeled are an immigration shock: an influx of people into the country, which increases the labor force; a technological advancement: a disruptive technology that increases the profitability of the firms; and the discovery of natural resources: a new oil field with promising yields, that increases capital gains from the rent market. The three economic impacts on their respective markets including the magnitude and initialization time are:

- Immigration, Labour Market, 800, t = 50
- Technological Advancement, Profit Market, 300, t = 100
- Discovery Natural Resources, Rent Market, 100, t = 150

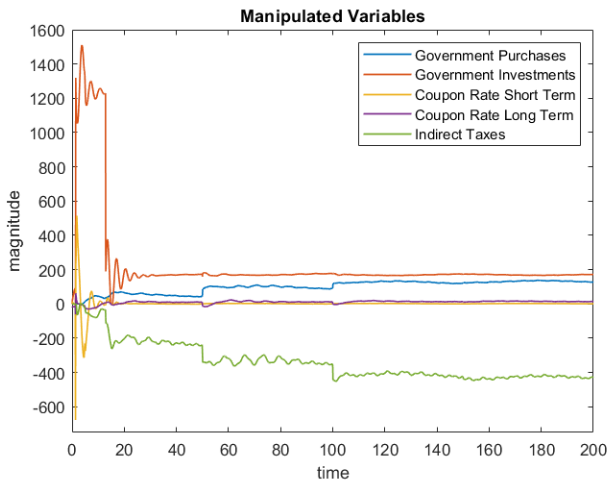

Figure 12: The manipulated variables over time representing government policy, reacting to an Immigration (t=50), Technological Advancement (t=100), and the Discovery of Natural Resources (t=150).

Figure 12 shows the manipulated variables of the MPC over time. At each economic impact, the MPC adjusts its manipulated variables proportional to the magnitude of the disturbance. After each disturbance, the MPC needs some time to adjust to the optimal control input, as was the case after initialization. The Government Purchases and Government Investments show an increase, while the Indirect Taxes show a decrease in its magnitude. The Short and Long Term Coupon Rates compensate for the change in net power outflow and this way satisfy the constraints.

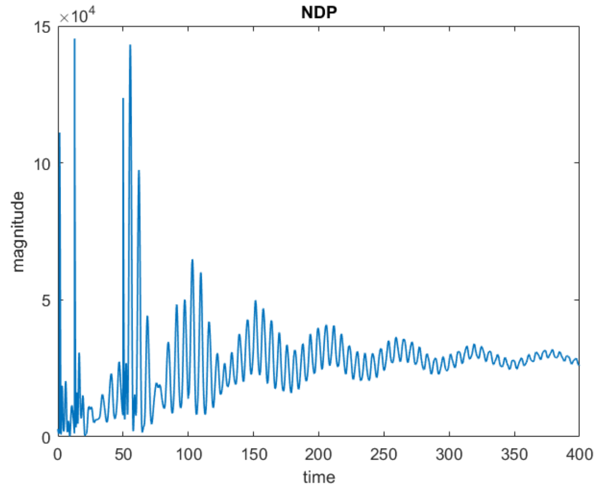

Figure 13: The NDP over time under the influence of an Immigration (t=50), Technological Advancement (t=100), and Discovery of Natural Resources (t=150).

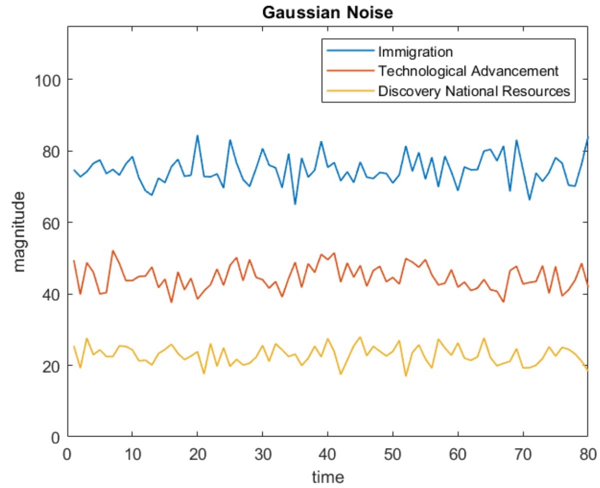

Figure 14: Gaussian Noise over time, representing continuous disturbances caused by Immigration, Technological Advancement, and Discovery of Natural Resources.

The principle of national accounting can also be used to explain the movement of the manipulated variables. Modeling a disturbance in the markets for Factors of Production increases the nation's income and thus increases revenues generated from indirect taxes. The government can then increase its expenditure, increasing the total expenditure of the economy. Together with the increase in consumption and investment of the households over time, the total expenditure equals the total income and thus satisfies the principle of national accounting.

Again applying the principle of national accounting, modeling disturbances must also increase the NDP of the US economy. In Figure 13, the NDP is set out over time, where an increase in NDP is observed each time an economic impact is imposed on the system. Interesting to see is that after each shock both frequencies of oscillations seem to increase, supporting the hypothesis that these oscillations are seasonality and the business cycle since a possible cause for cyclic behavior is a summation of random shocks with random causes. [12]

In reality, disturbances occur continuously at random points in time, with arbitrary magnitude. To simulate this, the three economic impacts, simulated as step functions earlier, are replaced by three normal distributions simulating Gaussian noise (see Figure 14). Looking at Figure 15, the manipulated variables show more jaggedness and a longer settling time compared to Figure 12. Eventually, the manipulated variables stabilize and the US economy achieves its maximum economic output, which is verified by Figure 16, showing the NDP over time.

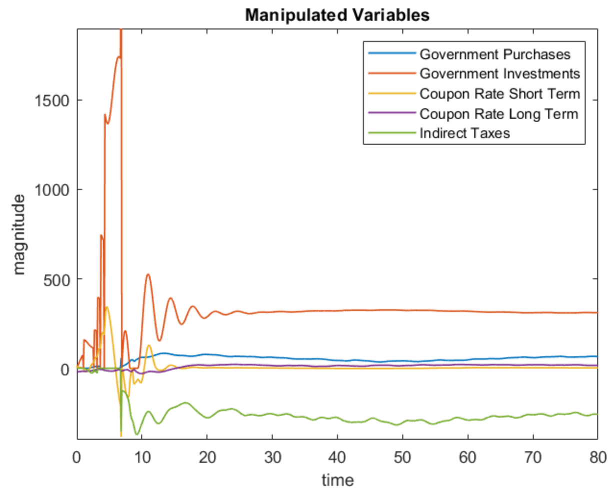

Figure 15: The manipulated variables over time representing government policy, reacting to Gaussian noise shown in Figure 14 as disturbance.

V. Conclusion

The aim of this research paper was to use economic engineering principles to design and implement a controller that models the government in the US economy. This was done by using a Model Predictive Controller that uses a custom cost function, defined as the inverse of the NDP, to maximize the economic output of the US economy. The manipulated variables of the MPC are constrained by a custom constraint function that caps the net power outflow of the MPC to zero, resulting in the simplification of the US economy without growth. The MPC can also deal with disturbances like Immigration, the Discovery of Natural Resources, and Technological Advancement, simulating a most basic form of economic growth. In all cases, the MPC was able to stabilize the economy and maximize the NDP.

As iterated before, there is no consensus among macroeconomists on what is the best way for policymakers to optimize the growth of an economy. A popular point of view is that the government should try to stabilize the economy

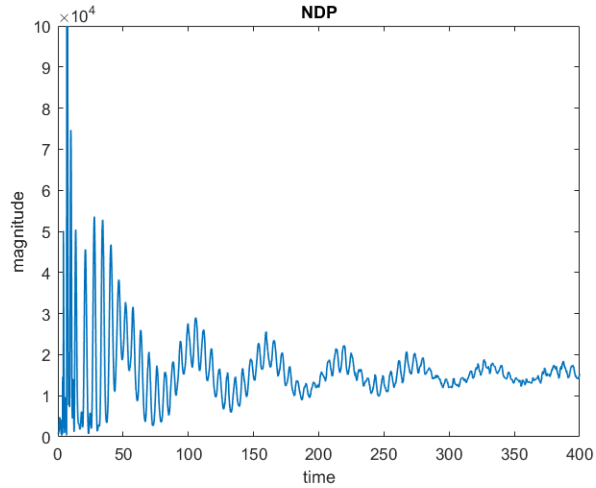

Figure 16: The NDP over time, influenced by Gaussian noise shown in Figure 14 as disturbance. After initialization, the NDP converges to a steady-state value.

[4]−(Mankiw, 2010, p.611), [13]. Comparing this point of view to the performance of the MPC, in all simulations, the MPC was able to successfully stabilize the US economy, while maximizing the NDP, e.g. the economic output of the economy. Over time, the controller was able to control the business cycle to some degree, even when subjected to disturbances. All results produced could be explained with macroeconomic theory, thus it can be concluded that the US government can be modeled as a controller.

The model described in this paper, like any model, is an approximation of reality but provides a promising new way of generating more accurate prospects of market trends and movements. Further development could improve the understanding of how an economy interacts and especially be useful to anticipate how changes in politics, technology, or climate affect government policy and thus the financial markets.

VI. Discussion

Based on the findings of this paper, keeping in mind the assumptions and approximations made, the following suggestions for future research in this field are discussed:

A. Historic Data

The parameters used in this research were selected in such a way that the dynamics of the model would be stable. By making use of historic data, the parameters could be fitted to the model so that the model output would result in realistic data. This would greatly increase the interpretability of the model, increasing the socio-economic impact of this research. The improved interpretability would also aid in refining the model, see Section VI-B, as unnatural model behavior could be eliminated faster.

B. Refinement of the Model

Parallel to this research, Gilbert Kruimer has refined the model of the US economy, which more accurately represents the behavior of the different economic actors and markets. Implementing improved iterations of the model would yield a more realistic model behavior and a better understanding of the impact of government policies on an economy. Similarly, the introduction of more inputs such as Net Export and Net Foreign Investments could result in a more detailed simulation. One could even go as far as to simulate multiple economies, resulting in a model representing the global economy.

C. Growth Modeling

One of the simplifications made in this paper was that the growth of the economy was not modeled. The modeling and prediction of growth is still one of the most active fields of research within macroeconomics. Including growth, and thus also recessions, could lead to more realistic results. A start has been made with the modeling of disturbances, but there is still a lot that can be improved on.

D. Verification of the Model

Although it is hard to perform economic experiments in reality, there is a wealth of historic events that could be used to verify the behavior of a model. For instance, an event that could be used in the future is the COVID-19 pandemic. Verification of the model could better map out the limitations and sensitivity of the model, increasing the value of the results produced by the model and thus increasing the socio-economic impact.

Acknowledgement

We would like to thank Ir. Coen Hutters and Gilbert Kruimer for their help and contribution to this research.

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Appendix A: Bond Graph Model

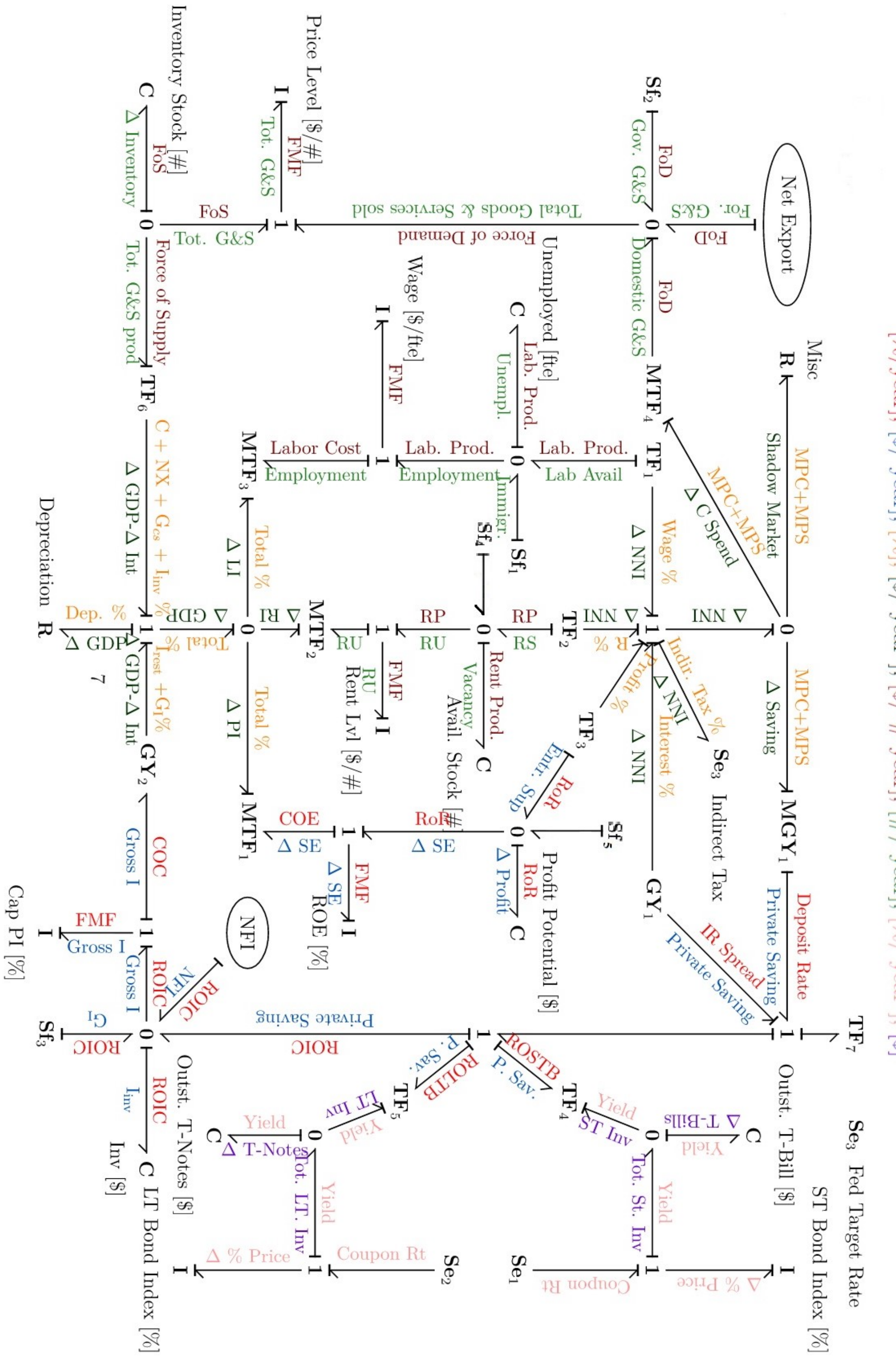

Figure 17: Bond graph of US Economy

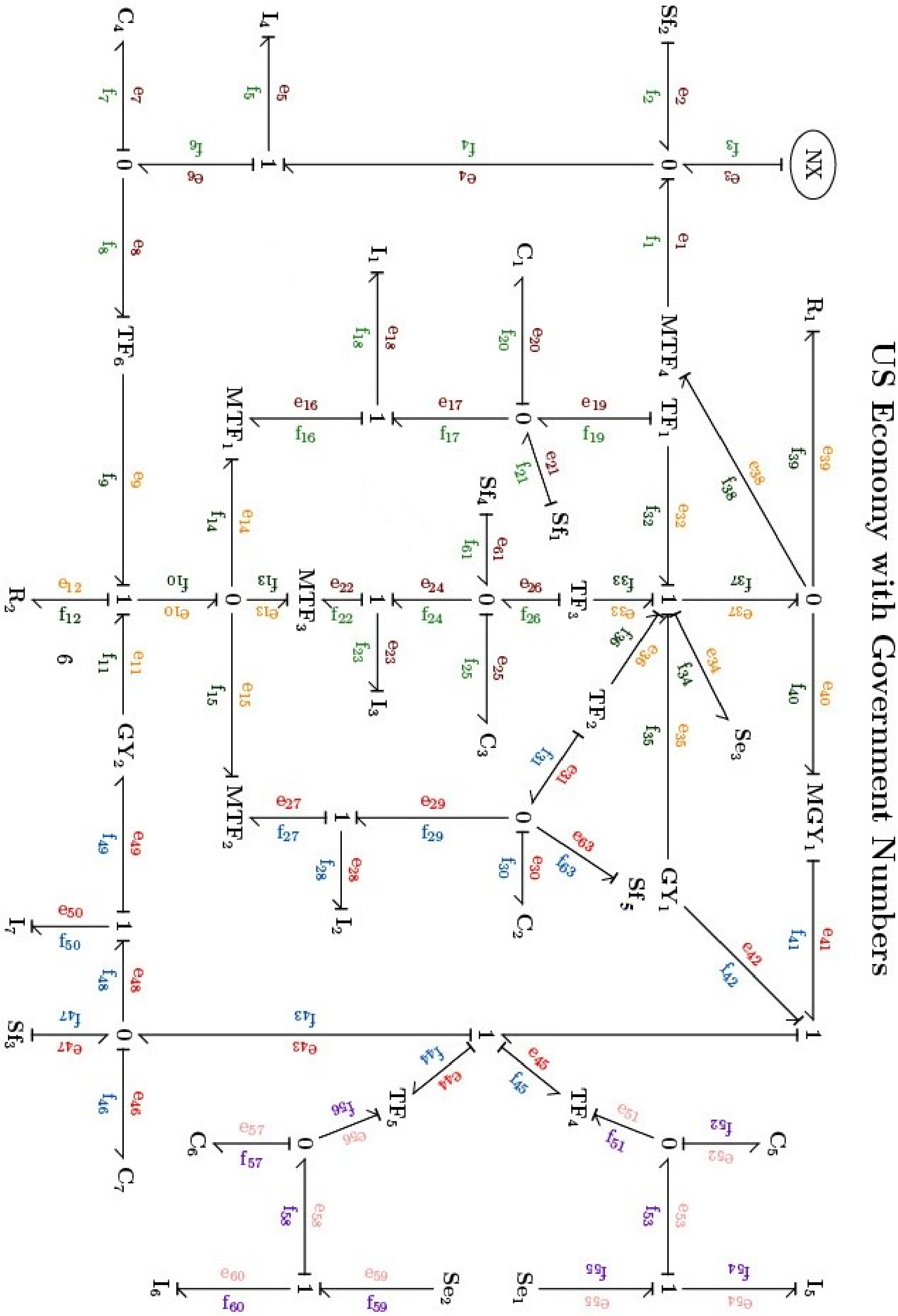

Figure 18: Numbered bond graph of US Economy

Appendix B: State-Space Representation

$$
A = \begin{pmatrix}
\frac{R2*I1}{(cm1^2)} & C1 & R2*\frac{I2}{cm2*cm1} & 0 & R2*\frac{I3}{cm3*cm1} & 0 & 0 & -\frac{C4}{cm1*m6} & 0 & 0 & 0 & 0 & -r2*\frac{I7}{cm1} & 0 \\
-I1 & CM*\frac{C1}{m1^2*R1} & 0 & CM*\frac{C2}{m1*m2*R1} & 0 & CM*\frac{C3}{m1*m3*R1} & \frac{I4}{m1*cm4} & 0 & 0 & m4*\frac{C5}{m1*cr1} & 0 & m5*\frac{C6}{m1*cr1} & 0 & \frac{C7}{m1*cr1} \\
R2*\frac{I1}{cm1*cm2} & 0 & R2*\frac{I2}{cm2^2} & C2 & R2*\frac{I3}{cm3*cm2} & 0 & 0 & -\frac{C4}{cm2*m6} & 0 & 0 & 0 & 0 & -r2*\frac{I7}{cm2} & 0 \\
0 & CM*\frac{C1}{m1*m2*R1} & -I2 & CM*\frac{C2}{m2^2*R1} & 0 & CM*\frac{C3}{m2*m3*R1} & \frac{I4}{m2*cm4} & 0 & 0 & m4*\frac{C5}{m2*cr1} & 0 & m5*\frac{C6}{m2*cr1} & 0 & \frac{C7}{m2*cr1} \\
R2*\frac{I1}{cm1*cm3} & 0 & R2*\frac{I2}{cm2*cm3} & 0 & R2*\frac{I3}{cm3*cm2} & C2 & 0 & -\frac{C4}{cm3*m6} & 0 & 0 & 0 & 0 & -r2*\frac{I7}{cm3} & 0 \\
0 & CM*\frac{C1}{m1*m3*R1} & 0 & CM*\frac{C2}{m2*m3*R1} & -I3 & CM*\frac{C3}{m3^2*R1} & \frac{I4}{m3*cm4} & 0 & 0 & m4*\frac{C5}{m3*cr1} & 0 & m5*\frac{C6}{m3*cr1} & 0 & C7/(m3*cr1 \\
0 & CM*\frac{C1}{cm4*m1} & 0 & CM*\frac{C2}{cm4*m2} & 0 & CM*\frac{C3}{cm4*m3} & 0 & -C4 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{I1}{m6*cm1} & 0 & -\frac{I2}{m6*cm2} & 0 & -\frac{I3}{m6*cm3} & 0 & I4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -C5 & 0 & 0 & 0 & 0 \\
0 & -\frac{m4*CM*C1}{cr1*m1} & 0 & -\frac{m4*CM*C2}{cr1*m2} & 0 & -\frac{m4*CM*C3}{cr1*m3} & 0 & 0 & I5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -C6 & 0 & 0 \\
0 & -\frac{m5*CM*C1}{cr1*m1} & 0 & -\frac{m5*CM*C2}{cr1*m2} & 0 & -\frac{m5*CM*C3}{cr1*m3} & 0 & 0 & 0 & 0 & I6 & 0 & 0 & 0 \\
-r2*\frac{I1}{cm1} & 0 & -r2*\frac{I2}{cm2} & 0 & -r2*\frac{I3}{cm3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C7 \\
0 & CM*\frac{C1}{cm1*m1} & 0 & CM*\frac{C2}{cm1*m2} & 0 & CM*\frac{C3}{cm1*m3} & 0 & 0 & 0 & 0 & 0 & 0 & -I7 & 0
\end{pmatrix}
$$

$$
\frac{dx}{dt} = Ax + Bu, \qquad y = Cx + Du
$$

$$
B = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{m1*cm4}, & 0 & 0 & 0 & -\frac{CM}{m1*R1} & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{m2*cm4} & 0 & 0 & 0 & -\frac{CM}{m2*R1} & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{m3*cm4} & 0 & 0 & 0 & -\frac{CM}{m3*R1} & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & -\frac{CM}{cm4} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & m4*\frac{CM}{cr1} & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & m5*\frac{CM}{cr1} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & -\frac{CM}{cm1} & 0 & 0 & 0
\end{pmatrix}, \quad
C = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}, \quad
D = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},
$$

$$
x = \begin{pmatrix} p_1 \\ q_1 \\ p_2 \\ q_2 \\ p_3 \\ q_3 \\ p_4 \\ q_4 \\ p_5 \\ q_5 \\ p_6 \\ q_6 \\ p_7 \\ q_7 \end{pmatrix}
\begin{array}{c} Wage \\ Unemployment \\ Return\ On\ Equity \\ Profit \\ Rent\ Level \\ Available\ Stock \\ Price\ Level \\ Inventory\ Stock \\ Short\ Term\ Bond\ Index \\ Outstanding\ T-Bill \\ Long\ Term\ Bond\ Index \\ Outstanding\ T-Notes \\ Capital\ Price\ Index \\ Investment \end{array}, \quad
u = \begin{pmatrix} Sf_2 \\ Sf_3 \\ Se_1 \\ Se_2 \\ Se_3 \\ Sf_1 \\ Sf_5 \\ Sf_4 \end{pmatrix}
\begin{array}{c} Government\ Purchases \\ Government\ Investments \\ Coupon\ Rate\ Short\ Term \\ Coupon\ Rate\ Long\ Term \\ Indirect\ Taxes \\ Immigration \\ Technological\ Advancement \\ Discovery\ National\ Recources \end{array}
$$

Figure 19: State-Space US Economy

Appendix C: MATLAB Model Parameter Initialization

```
clear all
clc
rng('shuffle');

while 1
    m1 = abs(-5 +rand(1)*10); %TF1
    m2 = abs(-5 +rand(1)*10); %TF2
    m3 = abs(-5 +rand(1)*10); %TF3
    m4 = abs(-5 +rand(1)*10); %TF4
    m5 = abs(-5 +rand(1)*10); %TF5
    m6 = abs(-5 +rand(1)*10); %TF6
    r1 = abs(-5 +rand(1)*10); %GY1
    r2 = abs(-5 +rand(1)*10); %GY2
    cr1 = abs(-5 +rand(1)*10); %MGY1
    cm1 = abs(-5 +rand(1)*10); %MTF1
    cm2 = abs(-5 +rand(1)*10); %MTF2
    cm3 = abs(-5 +rand(1)*10); %MTF3
    cm4 = abs(-5 +rand(1)*10); %MTF4
    CM = cr1/(cr1-r1);
    R2 = 1/abs(0 +rand(1)*100);
    R1 = abs(0 +rand(1)*100);

    I1 = abs(-5 +rand(1)*10);
    C1 = abs(-5 +rand(1)*10);
    I2 = abs(-5 +rand(1)*10);
    C2 = abs(-5 +rand(1)*10);
    I3 = abs(-5 +rand(1)*10);
    C3 = abs(-5 +rand(1)*10);
    I4 = abs(-5 +rand(1)*10);
    C4 = abs(-5 +rand(1)*10);
    I5 = abs(-5 +rand(1)*10);
    C5 = abs(-5 +rand(1)*10);
    I6 = abs(-5 +rand(1)*10);
    C6 = abs(-5 +rand(1)*10);
    I7 = abs(-5 +rand(1)*10);
    C7 = abs(-5 +rand(1)*10);

A = [R2*I1/(cm1^2)  ,C1                    , R2*I2/(cm2*cm1), 0                  ...
    ,R2*I3/(cm3*cm1), 0                    , 0                , -C4/(cm1*m6)     , 0     , ...
    0               , 0     , 0            , -r2*I7/(cm1)     , 0;
     -I1            ,CM*C1/(m1^2*R1)       , 0                , CM*C2/(m1*m2*R1)   ,0 ...
                    , CM*C3/(m1*m3*R1)     , I4/(m1*cm4)      , 0                  , 0      ...
        , m4*C5/(m1*cr1)    , 0     , m5*C6/(m1*cr1)     , 0              , C7/(m1*cr1);
     R2*I1/(cm1*cm2),0                     , R2*I2/(cm2^2)   , C2                 ...
        ,R2*I3/(cm3*cm2), 0                , 0                , -C4/(cm2*m6)       , 0 ...
           , 0                , 0     , 0                , -r2*I7/(cm2)     , 0;
     0              ,CM*C1/(m1*m2*R1)      , -I2              , CM*C2/(m2^2*R1)    ,0 ...
                    , CM*C3/(m2*m3*R1)     , I4/(m2*cm4)      , 0                  , 0      ...
        , m4*C5/(m2*cr1)    , 0     , m5*C6/(m2*cr1)     , 0              , C7/(m2*cr1);
     R2*I1/(cm1*cm3),0                     , R2*I2/(cm2*cm3), 0                  ...
        ,R2*I3/(cm3*cm2), C2               , 0                , -C4/(cm3*m6)       , 0 ...
           , 0                , 0     , 0                , -r2*I7/(cm3)     , 0;
     0              ,CM*C1/(m1*m3*R1)      , 0                , CM*C2/(m2*m3*R1)   ,-I3 ...
                    , CM*C3/(m3^2*R1)      , I4/(m3*cm4)      , 0                  , 0     , ...
        m4*C5/(m3*cr1)    , 0     , m5*C6/(m3*cr1)     , 0              , C7/(m3*cr1);
     0              ,CM*C1/(cm4*m1)        , 0                , CM*C2/(cm4*m2)     , 0 ...
                    , CM*C3/(cm4*m3)       , 0                , -C4                , 0      ...
        , 0                 , 0     , 0                  , 0              , 0;
     -1*I1/(m6*cm1) ,0                     , -1*I2/(m6*cm2) , 0                  , ...
        -1*I3/(m6*cm3), 0                  , I4               , 0                  , 0 ...
           , 0                , 0     , 0                , 0                , 0;
     0              ,0                     , 0                , 0                  , 0 ...
                    , 0                    , 0                , 0                  , 0      ...
        , -C5               , 0     , 0                  , 0              , 0;
     0              ,-(m4*CM*C1)/(cr1*m1)  , 0                , -(m4*CM*C2)/(cr1*m2)  , 0 ...
                    , -(m4*CM*C3)/(cr1*m3) , 0                , 0                  , I5     ...
        , 0                 , 0     , 0                  , 0              , 0;
     0              ,0                     , 0                , 0                  , 0 ...
                    , 0                    , 0                , 0                  , 0      ...
        , 0                 , 0     , -C6                , 0              , 0;
```

```
    0              ,-(m5*CM*C1)/(cr1*m1)    , 0               , -(m5*CM*C2)/(cr1*m2)    , 0 ...
                    , -(m5*CM*C3)/(cr1*m3)      , 0                    , 0                    , 0      ...
        , 0                  , I6     , 0                    , 0                  , 0;
    -r2*I1/(cm1)    ,0                       , -r2*I2/(cm2)    , 0                    , ...
        -r2*I3/(cm3)   , 0                       , 0                    , 0                    , 0 ...
            , 0                  , 0      , 0                    , 0                  , C7;
    0              ,CM*C1/(cm1*m1)          , 0               , CM*C2/(cm1*m2)          , 0 ...
                    , CM*C3/(cm1*m3)            , 0                    , 0                    , 0      ...
        , 0                  , 0      , 0                    , -I7                , 0 ];

  if sum(eig(A)<0) == 14
      break
  end
end
B = [0, 0,            0, 0, 0           , 0, 0, 0;
     1, -1/(m1*cm4), 0, 0, -1/(m1*cm4), 0, 0, -CM/(m1*R1);
     0, 0,            0, 0, 0           , 0, 0, 0;
     0, -1/(m2*cm4), 0, 0, -1/(m2*cm4), 0, 0, -CM/(m2*R1);
     0, 0,            0, 0, 0           , 0, 0, 0;
     0, -1/(m3*cm4), 0, 1, -1/(m3*cm4), 0, 0, -CM/(m3*R1);
     0, 0,            0, 0, 0           , 0, 0, -CM/cm4;
     0, 0,            0, 0, 0           , 0, 0, 0;
     0, 0,            0, 0, 0           , 1, 0, 0;
     0, 0,            0, 0, 0           , 0, 0, m4*CM/cr1;
     0, 0,            0, 0, 0           , 0, 1, 0;
     0, 0,            0, 0, 0           , 0, 0, m5*CM/cr1;
     0, 0,            0, 0, 0           , 0, 0, 0;
     0, 0,            1, 0, 0           , 0, 0, -CM/cm1];

C = eye(14);
D = zeros(size(B));
sysc_us = ss(A,B,C,D);
eig(sysc_us.A)
```

MATLAB param.	Model param.	Value
m_1	TF1	4.603318989632962
m_2	TF2	2.605745165153566
m_3	TF3	3.660193266755863
m_4	TF4	1.095375304480562
m_5	TF5	2.587745553701714
m_6	TF6	4.213679124289397
r_1	GY1	4.247521199423449
r_2	GY2	0.157459777900391
cr_1	MGY1	1.142074463831221
cm_1	MTF12	2.785256890100966
cm_2	MTF2	4.592529284979094
cm_3	MTF3	3.828820498501003
cm_4	MTF4	3.060172788096613
CM	CM	-0.367764950125097
R_1	R_1	0.100000000000000
R_2	R_2	0.052383392033228
I_1	I_1	0.925641326812977
C_1	C_1	1.037233929655144
I_2	I_2	2.011276784067372
C_2	C_2	0.291644923812092
I_3	I_3	4.083628930127760
C_3	C_3	0.680717043470626
I_4	I_4	4.537206312115827
C_4	C_4	1.274510047206140
I_5	I_5	0.038699794731356
C_5	C_5	0.308856967038379
I_6	I_6	0.795427197294663
C_6	C_6	0.014835675551246
I_7	I_7	0.740348020329314
C_7	C_7	4.702597770920038

Table II: Model parameters used to simulate the US economy. The MATLAB parameters are used in the MATLAB model, the Model parameter are used in the bond graph.