Modeling the Price Dynamics of Competition using Economic Engineering

A solution for regulators and hedge funds

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Master of Science Thesis

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Abstract

Although competition is a dynamic phenomenon, currently used competition models do not take price dynamics into account, and some models are not even quantitative. This poses a problem for regulators or hedge funds and M&A departments who rely on these models to either quantify price and demand movements, or to determine the cost of competition.

This thesis solves that problem by using Economic Engineering to build on the existing game-theoretic models of competition to include price dynamics. A bond-graph model of a competitive market is developed, from which the price dynamics are derived. Model-predictive controllers are used to model profit-maximizing companies within this model, and to simulate competitive behavior and its effects on prices and demand flows.

Finally, this thesis shows how control engineering tools in both the time and the frequency domain can then be exploited by regulators and hedge funds. Time-domain simulations enable regulators to quantify the effects of competition on prices and demands, and analyses in the frequency domain enable hedge funds to determine the change in company valuations due to changes in competition, i.e., the cost of competition.

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Preface

I notice it feels quite strange to write this part, as I'm about to hand in the final version of this thesis. It certainly took longer than I originally expected, but it was definitely time well spent. I learned a lot, both inside and outside of academics, and I'm very grateful I was given the freedom to explore and learn a great many things during my thesis period.

I would like to thank prof. Max Mendel for the many meetings - often even outside of business hours - and for never shying away from giving honest feedback on my work. It lead to a lot of improvements and reminded me of the importance of being continuously critical of my own work. Additionally, I want to thank Coen Hutters and the students within the Economic-Engineering group. The weekly meetings were a great opportunity to discuss and improve my research and they put me on the right track to completing this thesis.

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Chapter 1

Introduction

Within the field of economics, competition is a dynamic phenomenon. Industry structure, market shares and competitor pricing change over time in almost any industry. Consequently, if a company wants to generate higher revenues or obtain a bigger market share, it will need to take its competition into account when it optimizes its strategy [5]. The different types of competition and its (economic) background - including the concept of supply and demand - are reviewed in Section 2-2.

However, the majority of the competition models in literature is of a qualitative nature [6, 7, 8]. Models like Porter's Five Forces Framework are very popular within business and are useful to get a good general understanding of competition [9]. This framework and other qualitative models are reviewed in Section 2-3, which shows that since these models are qualitative, they are often vaguely formulated and lack formulas. Consequently, they can not be used to model competition in a dynamic and quantitative way.

This poses a problem for regulators and hedge funds. Regulators need to be able to estimate how prices will change when companies merge or when a new competitor enters a market. Less competition typically increases prices, which might incentivize a regulator to stop a merger from happening [10]. In order to do so effectively, they would need a quantitative model. On the other side, hedge funds and M&A departments need to assess by how much the total valuation of two companies would increase when they get merged [11]. Again, they need quantitative models to do so.

Various quantitative models of competition have already been developed [2, 12, 13, 14, 15], but they currently do not contain price dynamics. Moreover, the level of detail and the possibilities to adjust these quantitative models are often limited [16, 17, 18]. Game-theoretic competition models provide more detail and various model additions, but typically only consider steady-state solutions [19]. This is reviewed in more detail in Section 2-4. The impact of all the limitations of the current models on regulators and hedge funds - including the lack of price dynamics - is assessed in Section 2-5.

Chapter 3 shows how Economic Engineering can be used to introduce price dynamics to a competitive market model. This model is based on the existing game-theoretic Bertrand model of competition [19], but uses bond graphs to include price dynamics. The subsystems of this modular model are described in Section 3-2, after which Section 3-3 presents the full model.

Model-Predictive Control (MPC) can subsequently be used by regulators to simulate and quantify (the effects of) competitive behaviour in the time domain. This is described in Section 3-4. Within a model-predictive controller, the objective functions represent the different strategies of the rivaling companies, and determine if a firm optimizes for profit, revenue or market share. The results of the model simulations and the effects of competition on prices and demand flows are shown in Chapter 4. The economically interpretable parameters are assigned illustrative values in Section 4-2. Thereafter, both competitive and cooperative scenarios are simulated in Sections 4-3 and 4-4.

The differences between the competitive and the cooperative scenario can subsequently be used by hedge funds to quantify the effects of competition on company valuations. This is described in Sections 4-5 and 4-6, where the cost of competition, i.e., the total value that has evaporated due to competition, is calculated by using a control engineering tool in the frequency domain - the Laplace transform.

Chapter 2

Review of the Models of Competition

2-1 Introduction

This Chapter first surveys the relevant literature on the (economic) background of competition. It provides a definition of competition, and assesses the different types that economists distinguish between. Monopolies are discussed first (Section 2-2-1), followed by oligopolies (Section 2-2-2), monopolistic competition (Section 2-2-3), and lastly perfect competition (Section 2-2-4). The concept of supply and demand and related ideas are introduced in Section 2-2-5.

Subsequently, the various existing competition models and its differences and limitations are reviewed. Most competition models in the literature are qualitative in nature. Some of the most important ones and its limitations are discussed in Section 2-3. Section 2-4 subsequently reviews several quantitative models, starting with a model based on the work of Pareto in Section 2-4-1. Several analytical competition models are reviewed in Section 2-4-2. Finally, the field of game theory is described in Section 2-4-3, in combination with two different competition models; Cournot competition and Bertrand competition. Additionally, common additions to these models are discussed.

The impact of the model limitations on regulators and hedge funds is assessed in Section 2-5. The differences between the models are reviewed in Section 2-6, followed by the conclusion. Section 2-7 states the objectives of this thesis.

2-2 Competition Types and Economic Background

"The activity or condition of striving to gain or win something by defeating or establishing superiority over others" [20]. This accurately defines competition. It is a dynamic phenomenon within the field of economics, and it is present in almost any industry.

Generally, economists distinguish four different kinds of competition, each with its own models and underlying assumptions [21]. The main differences include the level of price control, the profit potential of an industry and the number of companies selling a similar good, i.e., the number of competitors.

There generally exists an inverse relationship between the number of competitors and other industry characteristics, which means that as more companies enter the market, both individual profits and the profit potential of the industry will go down. Lower individual profits lead to lower company valuations, and this decrease in value as a result of competition can be seen as the "cost of competition".

A brief description of the different competition types is given below and Table 2-1 provides an overview. Additionally, the mechanisms behind supply and demand are introduced.

2-2-1 Monopoly

The situation in which a company has the highest level of price control, is the one of monopoly. In that situation, the company is the only one on the market selling a certain product or service, and competition is absent. The monopolist is able to exert control over the prices, and is therefore a price maker. This is the polar opposite of the perfect competition scenario, which is discussed later.

Several types of monopoly exist. In a simple monopoly, firms charge their customers equal prices. When prices vary per customer, there is a discriminating monopoly [22]. In a perfect monopoly, the company's product lacks even remote substitutes. In practice, this is rarely the case. More often, a firm has an imperfect monopoly, in which closer substitutes are available. Consequently, this puts a limit on their pricing power. Finally, natural reserves and patents can result in respectively natural and legal monopolies. An example of the former can be found in oil producing gulf states, whereas the latter is common within the pharmaceutical industry. Additionally, high entry barriers are often present or are imposed by monopolists. In the case of e.g., Facebook, significant network effects prevent competitors from easily entering the market [23].

2-2-2 Oligopoly

In an oligopoly - derived from the Greek *ὀλὶγοι* ("few") and *πωλέω* ("to sell") [24], a large share of the market is comprised of only a few companies, which is the case in e.g., the airline industry. As a group, they have a lot of (pricing) power, similar to the monopolist. However, they need to take into account that when they adjust their prices, their competitors will likely react to that and follow, since their products are relatively similar.

2-2-3 Monopolistic competition

Under monopolistic competition, a lot of competitors are present in an industry, and their products can be slightly differentiated from each other, although they serve a comparable purpose. As a result of these small (perceived) differences, they can still possess some control over the price, by marketing themselves e.g., as a brand with superior quality products or by having a geographical advantage. Nevertheless, this level of control is significantly lower than in the aforementioned scenarios, since they will lose business to their competition when they raise their prices too much.

Table 2-1: Competition types overview and comparison.

	Monopoly	Oligopoly	Mon. comp.	Perfect comp.
Number of competitors	*None*	*Low*	*High*	*Very high*
Pricing control	*High*	*Medium*	*Low*	*None*
Profit potential	*Very high*	*High*	*Low*	*Minimal*
Differentiation	*High*	*Medium*	*Low*	*Minimal*
Typical company size	*Large*	*Large*	*Medium*	*Small*
Entry barriers	*Very high*	*High*	*Low*	*None*
Information	*Imperfect*	*Imperfect*	*Imperfect*	*Perfect*

2-2-4 Perfect competition

In the case of perfect competition, a lot of sellers exist, customers have full access to all the information they need to make decisions (perfect information), and product differentiation is either missing or irrelevant, i.e., the product is standardized. The last two characteristics are the main differences with monopolistic competition. Additionally, new firms can easily enter the market and no company has a market share or influence large enough to affect prices; they simply have to accept the price for which they can sell their product or service, i.e., they are price takers. In practice, no industry is completely perfect, but near-perfect markets can be analyzed using this idealized structure [25]. Although neoclassical economists believe that perfect competition would be the best scenario from a society or consumer point of view, there exists a lot of debate around this idea. Many economists, including Friedrich Hayek and Joseph Schumpeter, argue that the framework of perfect competition is either rendered unusable due to its unrealistic underlying assumptions, or is actually less efficient than imperfect competition in the long term [26].

2-2-5 Supply and demand

Crucial to the theories behind various types of competition, is the concept and the law of supply and demand, which is illustrated graphically in Figure 2-1. This theory describes the interaction between the buyers and the sellers of a good, i.e., the relationship between the price of a good and the quantity they are willing to respectively buy or sell per unit of time [27]. Behind this lies the assumption that suppliers attempt to maximize their profits and customers aim to maximize their utility [28]. Section A-2 elaborates on how supply and demand can be modeled as a control system.

Figure 2-1a shows a downward-sloping demand curve. The downward slope reflects that - given that other factors remain equal, i.e., *ceteris paribus* - there exists an inverse relationship between the price of a good and the quantity demanded of that good by customers. This means that at a higher price, customers buy less products. This phenomenon is known as the law of demand [1].

In contrast, the supply curve - also shown in Figure 2-1a - runs in the opposite direction and is sloped upwards. This upward slope means that at a higher price, the quantity supplied will also be higher, since the suppliers will aim to maximize their profits by increasing the amount of products to sell. This is known as the law of supply [27].

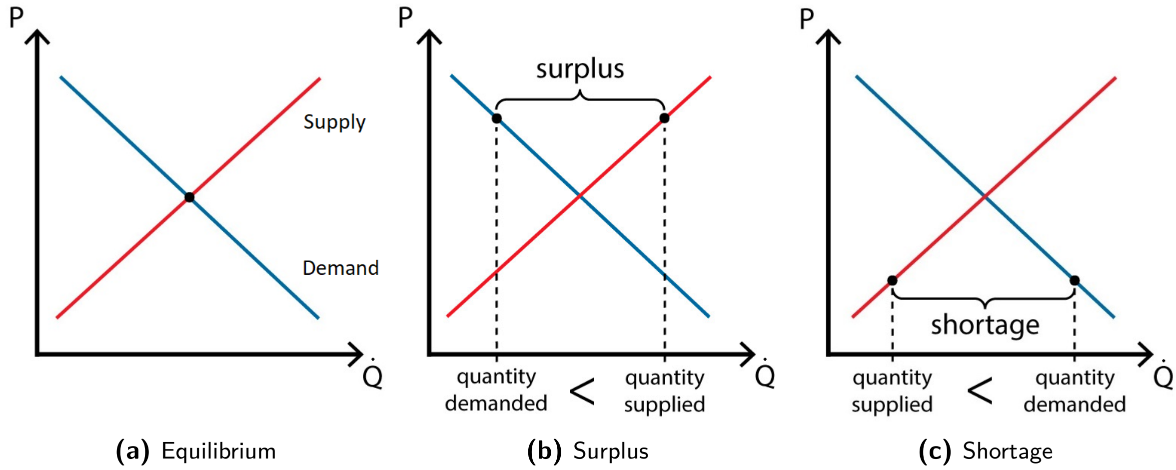
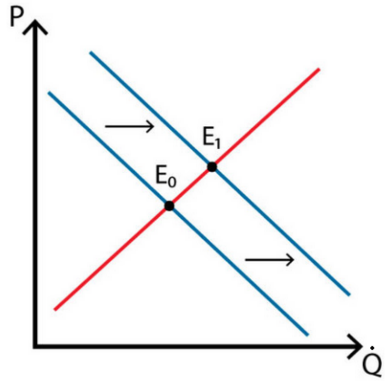


Figure 2-1: Law of supply and demand. Figure adjusted from [1].

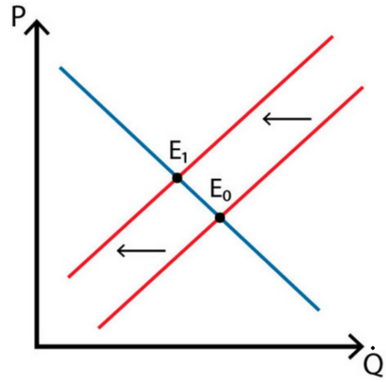
Under perfect competition, the two curves intersect at the equilibrium price and quantity. Subsequently, *market clearing* takes place, meaning that the quantity demanded equals the quantity supplied. Movements along these curves and curve shifts (or *shocks*) can take place. Whereas curve shifts affect this equilibrium point, movements along the curves do not change it [1]. Moving along the curve simply means that the price changes and the quantity demanded or supplied changes accordingly, or vice versa. Put differently, such a movement occurs when a change is caused only by a change in the other variable, i.e., the demand or supply relationship remains consistent [27]. In the case of the demand curve, the percentage change in the quantity demanded as a result of a 1% change in the price is known as the elasticity of demand [29]. In Figure 2-1, the elasticity of demand is the inverse of the demand curve slope. When the price of a different product affects the quantity demanded of the original product, this relationship can be described by the cross elasticity of demand. The cross elasticity of demand can be calculated as the percentage change in the quantity demanded of product A as a result of a 1% change in the price of product B [29].

On the other hand, a curve shift changes the relationship between the quantity demanded or supplied and the price, meaning that other factors are involved as well. At the same price, this will result in different quantities compared to before. This can be seen in Figure 2-2. The demand shift to the right in Figure 2-2a shows that consumers are now willing to pay more for the product, or purchase more products at the same price. This could be the result of e.g., an increase in income or an increase in the prices of competitors. Since the equilibrium has shifted upwards and to the right, total revenue will increase. The supply shift to the left in Figure 2-2b could indicate e.g., higher prices of raw materials or the occurrence of a natural disaster, which consequently affects the product price [1]. In contrast to a demand shift, the effects on total revenue can vary.

Several ideas have been developed by economists as to how the equilibrium point - where market clearing takes place, is reached. Léon Walras describes the price as the factor driving the dynamics towards equilibrium. When there is a surplus of supplied goods, which is shown in Figure 2-1b, prices are lowered. In the case of a shortage of goods, as in Figure 2-1c, prices are increased in order to move towards the equilibrium point. Eventually, the (general) equilibrium point is reached, through a process he called *tâtonnement* - French for "trial and



(a) Demand shift



(b) Supply shift

Figure 2-2: Supply and demand shocks. Figure adjusted from [1].

error". This approach generally makes most sense in a market that has perfect information and is easily modified, e.g., in auctions [30]. Whether transactions can take place already before or after arriving at equilibrium depends on which of his theories is used. Walras developed two; his *pledges* model uses a static approach aimed at finding the equilibrium values, in which transactions can only be made after reaching the equilibrium point. His *disequilibrium-production* model, which among others Pareto and Edgeworth adhered to, does not include this limitation. It is more dynamic and generally believed to be a better representation of both real competitive markets and Walras' overall work [31].

On the contrary, Alfred Marshall argues that the quantity will effectuate this adjustment towards equilibrium. The quantity will thus be lowered when the price of demand is less than the price of supply and vice versa. Transactions can subsequently take place at different prices, depending on what consumers are willing to pay, until the (partial) equilibrium is reached [32]. This approach would be relevant when inventories are high and prices cannot be adjusted easily [30].

However, contemporary economists see the price as the independent variable and the flow of goods as a result of the set price, despite the counter-intuitive standard choice of axes in supply and demand figures. Consequently, this mechanism will also be adhered to in the later proposed model.

2-3 Qualitative Competition Models and its Limitations

A large number of qualitative models and frameworks of competition exist in the literature, and they take different approaches towards explaining the differences between firms and the factors that drive its success. This Section illustrates the advantages and the criticisms behind some of the most influential models.

Five Forces framework

The Five Forces Framework uses five competitive forces to analyze the level of competition that is present within an industry [9]. It thus takes an external point of view. It is a popular framework for determining how to position your firm or product in an strategically optimal way.

The Five Forces Framework is qualitative in nature; although the so called forces are often referred to as being high or low, there are typically no (meaningful) numbers assigned to them. Also, it typically assumes that the industry is in equilibrium and differences between companies (or competitive advantages) are sustainable [6]. Consequently, this static approach has led to criticism [7].

Resource-Based View

The Resource-Based View aims to explain the persisting differences between firms by looking at the differences in resources and capabilities they possess or have access to, i.e., which resources they can use to create a sustained competitive advantage [33, 34]. These resources can be tangible, like a production plant, or intangible in the form of e.g., knowledge and experience. This framework thus takes an internal point of view. Consequently, it ignores industry aspects, in contrast to e.g., Porter's Five Forces framework. Another disadvantage is that it is often unclear which exact resources could lead a firm to higher profits or which combination of resources are the reason for its current success.

The framework is static in nature, which is a common criticism of the Resource-Based View. The field of *dynamic capabilities* aims to adjust this framework to deal with changing environments and the (external) industry [35, 6]. Nevertheless, both frameworks are still qualitative in nature.

Structure-Conduct-Performance paradigm

The Structure-Conduct-Performance paradigm is a framework that arose from *Industrial Organization* economics [36]. It describes the factors that drive competition in an industry by analyzing the causal relationships between market structure, firm conduct (which includes strategy) and firm performance. It is at the basis of various other models, including Porter's Five Force framework.

The Industrial Organization foundation upon which it is based is often criticized for ignoring dynamical behavior. Furthermore, it has been proven difficult to use the SCP framework in combination with actual data, due to both a shortage in data and difficulties in defining the exact boundaries of a market [6, 8].

Competitive dynamics

Competitive dynamics is a newer field that studies markets in a more dynamic way, i.e., it is interested more in the movements towards equilibrium and it therefore typically does not deal with equilibrium conditions itself. Similar to *Game Theory*, which is discussed in Section 2-4-3, it considers inter-firm behavior, which includes the reactions of competitors to a companies' strategic decisions, their motivations, and its outcomes.

Nevertheless, in contrast to game theory, it is still a qualitative framework, which means it mostly deals with "how" and "why" types of questions [37, 38, 6].

2-4 Quantitative Competition Models and its Limitations

This Section describes various quantitative competition models and its limitations. A model based on the work of Pareto has been created within the Economic Engineering group. This model will be discussed first in Section 2-4-1. Subsequently, analytical models are reviewed in Section 2-4-2, and finally game theory competition models and model additions are described in Section 2-4-3. Section 2-6 reviews the most important differences between both qualitative and quantitative models and describes why game theory competition models will be used as the basis of the dynamic model proposed in Chapter 3.

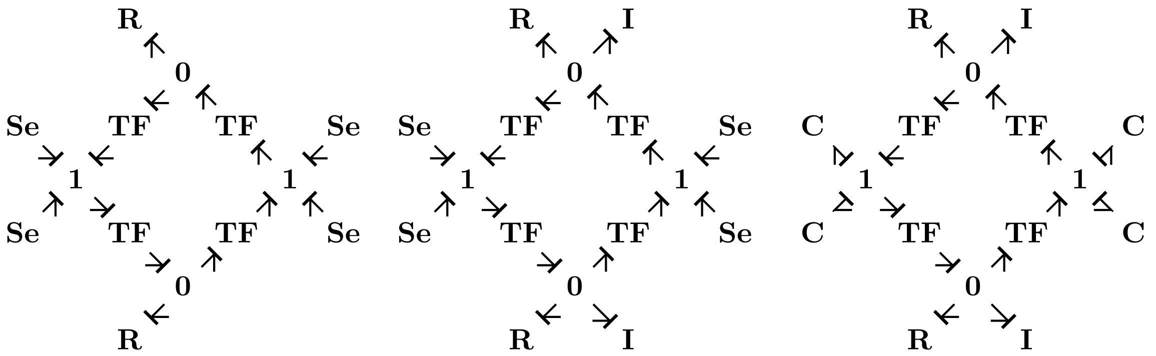
2-4-1 Pareto

In his 1906 Manual of Political Economy, Pareto draws an analogy between rational mechanics and pure economics [39]. Whereas the former reduces bodies to point masses for abstraction, the latter portrays men as rational, self-serving economic agents. This *homo economicus* [40] acts to maximize his utility, which serves as an underlying principle of subsequent economic mechanisms suggested by Pareto.

Pareto's work has been used within the Economic Engineering research group (more on this can be found in Appendix B) to construct a model of a two-sector economy. Individuals are assumed to have no influence on the price, i.e., they are price takers. In contrast, companies or governments can exert influence on the price; they are thus price makers. This is seen as analogous to the existence of a certain system, comprised of low-influence individuals, in which the larger companies are the controllers, i.e., they can influence the system [2, 41]. In the basic scenario, as described in [41], there are two price-taking households (i.e., the price is inelastic) - which supply two factors of production, capital and labor, with inelastic factor supply, and two firms - which produce two different products. The firms then compete with each other for the households' factors of production, while the consumers compete for the products. This basic scenario can be modeled with a bond graph, with is shown in Figure 2-3a. More information on bond graphs and how it relates to Economic Engineering can be found in Appendix B-2. In this model, the prices are relative and the price ratio of the two products equals the output marginal utilities ratio [41].

A pricing mechanism - more aligned with Economic Engineering modeling - has been included with an I-element in Figure 2-3b. The prices in the model vary based on the incoming factors of production from the households. Figure 2-3c shows how the effort sources can be replaced by C-elements to model elastic factor supply by the households. In contrast to the basic model, these additional elements introduce stock variables - in the form of quantities or prices - to the system. The value of the R-elements - representing the product losses or consumption by the firms - is typically assumed very high, so that the power dissipation is approximately zero. This can also be observed in Eq. (B-9). The energy in the system then arises from its initial conditions, in the form of non-zero quantity and price stock variables.

However, this model has its limitations. By using only utility functions, it does not use the supply and demand curves common in both economic and Economic Engineering models. Expanding the model to e.g., include multiple firms is not a straightforward process, due to its narrow focus on only two different firms and two products. Moreover, Pareto's famous concept of *Pareto efficiency* - in which no one can be better off without making someone else



(a) Price-taking market **(b)** Added price dynamics **(c)** Elastic factor supply

Figure 2-3: Bond graph models based on Pareto. Figures based on [2].

worse off - only applies to perfectly competitive markets [42]. Additionally, a competitive scenario might intuitively be described better by a so called *Nash equilibrium* [43], in which someone cannot get a better outcome by choosing a different strategy, unless their competitors also change their strategy. This concept is borrowed from *Game Theory*, which is discussed in Section 2-4-3.

2-4-2 Analytical models

A variety of analytical models of competition exist as well. They generally take a marketing point of view and study competitive interaction in relation to optimal advertising strategies [12]. Most models aim to maximize solely (discounted) profits [16, 17, 18, 13], although Deal's model for advertising strategies in an industry with two firms [14] additionally takes market share as an objective that it aims to maximize at the end of its time horizon. The different objectives are then assigned fixed weights, which can differ for each firm, and the overall objective becomes a linear function of the two individual objectives. This is similar to the concept of conflicting objective functions, which is discussed in Section 3-4. Moreover, others take the maximization of revenues as their objective [15]. Some models only use advertising expenses as the control variable [18, 14], whereas others also allow for the adjustment of prices [16, 17].

However, these models have various limitations as well. The effect of competition is only indirectly modeled in some cases [17, 14], i.e., the revenues of a company are not directly affected by variables pertaining to its competitors. This limits the extent to which competitive effects can be simulated within the model. Additionally, some models assume that the largest competitor can simply set the market price [17], or only consider very specific scenarios [16, 18].

2-4-3 Game theory

Section 2-2 has introduced several kinds of competition, ranging from a monopoly to perfect competition. In the case of an oligopoly, only a few companies control most of the market. The field of Game Theory presents a popular framework that is typically used to analyze these kinds of scenarios. The field arose after John von Neumann's 1928 publication [44] and has since become a popular tool for analyzing simplified competitive scenarios in business.

Within game theory, competition can be seen as a *non-cooperative game*, i.e., a game in which independent agents or players compete with each other and are only interested in their own payoffs. As such, it allows for the modeling of the interactions between firms.

Two different forms of competition exist within game theory; Bertrand competition and Cournot competition. They are quite similar in the standard assumptions that they make; firms aim to maximize their profits and consumers are assumed to act rationally, i.e., they have full information and maximize their utility. However, their underlying mechanisms are distinctly different. Consequently, also its outcomes - game theorists typically look at the resulting equilibria - are very different [19].

Bertrand competition

In its most basic scenario, the Bertrand competition model considers an oligopoly with only two companies, i.e., a *duopoly*. It is typically assumed that these companies are not allowed to collude. Both firms produce and sell a homogeneous product, which means there is no differentiation.

The firms each determine the price for which they sell their products, and the rational consumer picks the product with the lowest price. Similar to Walras as described in Section 2-2-5, Bertrand competition thus treats the price as the independent variable, which is in line with contemporary economic theory. Consequently, the company with the lowest price will receive a 100% market share. Alternatively, when prices are exactly equal, both companies receive half of the market. The price is then negatively (and typically linearly) related to the resulting quantity demanded, i.e., the higher the price, the lower the quantity demanded, which is in line with the law of supply and demand described in Section 2-2-5. Although game theorists typically refer to quantities, the mechanism can easily be adapted for flows of goods, i.e., quantities per unit of time.

Given an assumed constant unit production cost $c > 0$ (i.e., the marginal cost) and excluding fixed costs, this mechanism will result in both companies charging a price $p = c$. Consequently, they will receive zero profit, but it is the only possible equilibrium, since at any other point, companies could simply undercut their competitors and capture the whole market. Within game theory, this is known as a Nash equilibrium, since no company could improve their profits after considering their competitors' choices without making themselves worse off [43]. The zero profit outcome stands in stark contrast to the monopoly situation, in which the profit potential of the market can be maximized. Simply adding one company would make all profits evaporate.

A profit of zero is not always realistic. Consumers might not actually have full information, which means they cannot always adjust their choice of product based on small price differences. Additionally, it is very rare that no amount of differentiation is present, especially in an oligopoly scenario, which was described in Table 2-1. At the end of this subsection, it is illustrated how concepts like differentiation can be included in the models to overcome this problem.

Cournot competition

The Cournot duopoly competition model is based on assumptions similar to the Bertrand model. However, instead of the price they want to charge their customers, firms each decide on the quantities (per unit of time) they want to produce and sell. The total produced quantities are then negatively related to the price, which will be equal for both products. Companies can then calculate their profits by using the product of the market price and their individually produced quantities, and subtracting the product of these same quantities and their unit production cost c. The monopoly scenario would - similarly to the Bertrand competition case - lead to the highest profit potential of the market.

In the case of duopoly, companies can use their individual profit functions, and differentiate them simultaneously with respect to their own quantities produced to maximize their profits. This will results in a slightly lower market profit potential, which is now shared between two competitors. Total produced quantities will be higher than in the case of a monopoly. This is exactly in line with Table 2-1.

The more competitors are present in the model, the lower both the profit potential and the individual profits will be. However, the total produced quantities will be higher. The resulting lower price indicates that consumers are thus better off (i.e., they pay less) in a highly competitive market.

In some scenarios this seems like a fitting market mechanism, e.g., when a country opens its oil taps and floods the markets with oil. This will consequently lead to a decrease in prices. However, this mechanism - similar to Marshall as described in Section 2-2-5 - is not in line with the typical supply and demand causality. In Cournot competition, the quantities are taken as the independent variable, instead of the price. Consequently, the concept of Cournot competition will not be used in this thesis.

Model additions

As described previously, it is unlikely that there is no differentiation at all between the products in an oligopoly scenario. Fortunately, the models can easily be adjusted to include the effect of differentiation [19]. In the Bertrand competition duopoly scenario, rational consumers won't buy the more expensive product, which means the total quantity produced depends only on the lowest price P, i.e., $\dot{Q}(P) = (a-P)^+$, in which a is an arbitrary constant greater than the unit production cost. The "+"-sign indicates that no goods will be produced when $P > a$. A parameter b can then be used to introduce differentiation as follows:

$$\dot{q}_1(p_1, p_2) = (a - p_1 + bp_2)^+ \tag{2-1}$$

$$\dot{q}_2(p_1, p_2) = (a - p_2 + bp_1)^+ \tag{2-2}$$

In this case, the second company will produce more products (because there is a higher demand for them) when the price of their competitors' product increases and vice versa. Differentiation is then high for a low value of b - i.e., an increase in your competitors' price barely affects your production quantities, and there is little product differentiation when b is high. In the latter case, customers will be more price sensitive. In equilibrium, the two prices will again be equal.

These basic scenarios assume situations in which the companies choose their prices or produced quantities simultaneously. The *Stackelberg model* allows for the modeling of a dominant or leading company. This means that they will move first, and their competitors can then observe the move and react to it. This is similar to an industry in which one company has significantly more influence than their competitors. The influential company will receive a *first-mover advantage* and generate more profits, at the expense of its competitors. The industry as a whole will produce more, at a lower price, which means consumers will be better off.

Both competition models can also be adjusted to include more competing companies. Additionally, differentiation and sequential movements can be built into these more advanced models as well. In the case of Bertrand competition, it is typically assumed that the companies can always meet demand. The Bertrand-Edgeworth model allows to model competitors with constraints on their production capacity [45]. Furthermore, many papers describe how both Bertrand and Cournot competition models can be combined [46, 47, 48, 49].

2-5 Implications of the Model Limitations for Regulators and Hedge Funds

As described in Sections 2-3 and 2-4, several competition models already exist, but they still lack price dynamics. For various agents, this poses a significant problem.

Section 2-2 described how a scenario with fewer competitors in a market tends to result in higher prices. Consequently, consumers are likely worse off after a merger [10, 50]. Therefore, it is crucial for regulators to have access to models that include price dynamics, so that they can accurately determine to what extent market prices will change as a result of companies merging. If the effect is expected to be too great, they could decide to not allow the merger to take place [51]. On the other hand, if additional competition is very likely to benefit consumers, regulators could promote more competition and aim to reduce existing industry entry barriers. This will then result in lower market prices; a scenario in which consumers are better off.

On the other hand, hedge funds and M&A departments also directly benefit from the ability to model price dynamics. Higher market prices will result in higher profits, which will be reflected in higher company valuations [11]. By modeling the price dynamics, they are thus able to estimate by how much the combined valuation of two companies will increase when they merge, which decreases the amount of competition in the market. Consequently, hedge funds can decide based on that number e.g., how much they are willing to bid in order to acquire a competitor in a market where they are already present. Conversely, if they expect other competitors to enter their market, they can determine with the price dynamics how much value would evaporate from the combined valuation, i.e., the cost of competition.

2-6 Conclusions

The current models of competition are either not quantifiable or do not include price dynamics, which poses a problem for regulators and hedge funds. Out of all the quantitative models of competition, game-theoretic models present the most options for modeling additions and the least limitations. Consequently, the Bertrand model of competition is used as the basis for constructing a price-dynamic Economic-Engineering model of a competitive market, as is described in the next Chapter.

2-7 Thesis Objectives

To solve the problems described in Section 2-5, the objectives of this thesis are twofold. The first objective is to:

1. Use Economic Engineering to build on the existing game-theoretic competition models to create a new model of competition that includes price dynamics.

Chapter A elaborates on how Economic Engineering has been used to model supply and demand, and Appendix B provides more information on the background behind it. The new model is described in Chapter 3.

The second objective is to:

2. Use control engineering tools in both the time and frequency domain to quantify the effects of competition.

This will allow regulators to model the price dynamics of competition, e.g., to model the effect of a merger on market prices and demand flows. Additionally, it will enable hedge funds to determine the additional value a potential merger will generate. The analyses performed in both the time and the frequency domain are described in Chapter 4.

Chapter 3

Development of a Price-Dynamic Model of a Competitive Market

3-1 Introduction

This Chapter first describes how a dynamic time-domain model of a competitive market can be constructed with bond graphs. This is done with Economic Engineering, which is used to build price dynamics into the game-theoretic competition models. Section 3-2 describes how suppliers and demanders can be modeled as modular bond-graph subsystems. The assumptions behind these subsystems are discussed, and the complete model of a competitive market in constructed in Section 3-3.

Section 3-4 subsequently shows how model-predictive controllers (representing companies) can be used to simulate competitive behaviour. Sections 3-4-1 and 3-4-2 discuss how various types of competition can be modeled with Model-Predictive Control (MPC). The complete bond-graph model is then discretized in Section 3-4-3, after which the MPC objective functions and constraints are described in Sections 3-4-4 and 3-4-5. MPC tuning and stability are discussed in Sections 3-4-6 and 3-4-7, after which Section 3-5 states the conclusion.

3-2 Suppliers & Demanders as Subsystems

This Section elaborates on how the various subsystems of the model can be constructed and describes the modeling assumptions that are made in the process. This is done in a modular way, meaning that additional elements can easily be added to adjust the model. The supply and demand mechanism incorporated in the model is similar to the one described in Section A-2. For the basic model, a system with two competing suppliers is developed. These suppliers can alter the market prices of their products and decide how much they want to produce, similar to the game-theoretic Bertrand competition model.

3-2-1 Suppliers and supply prices

As illustrated in Section 2-2-2, at least two companies need to be present in order for competition to exist. Consequently, the most basic competition scenario will be an oligopoly with only two competitors, a *duopoly*. These competitors both want to maximize their profits [28], and they do so by selling their products to their customers. Therefore, these competitors are both suppliers.

Section 2-2 describes how suppliers have a supply curve. This upward-sloping curve shows that at a higher market price, the total quantity supplied in a market will also be higher, as in Figure 2-1. This makes intuitive sense, since an increase in market prices would incentivize other companies to enter a market. Typically, this supply curve represent an aggregation of the entire industry under perfect competition, i.e., a large group of suppliers all taken together. Also, it is assumed that only one price determines both the quantity supplied and demanded. This price is a reservation price, meaning that for suppliers it is the lowest price for which they would be willing to sell the corresponding amount of goods. For a higher total amount, the suppliers would need to receive a higher price.

For my model, I assume that different prices can determine the quantities demanded and supplied, which is illustrated in Figure 3-2b. Additionally, I assume that supply curves can also be used to model individual suppliers - in this case in a duopoly scenario. The reasoning behind this is that suppliers are assumed to be in control of how much they supply, regardless of whether that is the most financially sound strategy. Additionally, they would need to incur additional costs when they would be (almost) exceeding their capacity, since they would need to acquire additional personnel and resources. These additional costs - or increasing marginal costs - are in line with an upward-sloping supply curve. Consequently, these suppliers can each determine their own *supply price*, and based on those prices they each supply a certain amount of goods (per unit of time). This additionally allows to model suppliers that can supply or produce more efficiently [52]. Furthermore, it implies that there are two different products - one supplied by supplier A and one by supplier B - with potentially different supply prices.

Model Assumptions

- The quantities supplied and demanded are based on distinct prices.
- Supply curves can be used to model individual suppliers.

Similarly, this supply price now reflects the lowest price for which one supplier would supply a certain amount of goods. Alternatively, the supply price represents the costs that would be incurred when one additional product would be supplied. In order for a supplier to supply more products, he would at least want those costs to be covered by the market price it receives for its products. This is illustrated in Figure 3-2a, where the area under the red supply curve represents the *supply expenses*. Figure 3-2b shows how a difference between the supply price and the market price (for this example assuming a monopoly situation) would benefit the supplier. Due to the lower supply price, less goods would be supplied, but the costs per product of supplying those goods - the *marginal cost* - would be less, and the products could be sold at a higher price. Consequently, the supplier surplus has increased, i.e., $III > I$ in Figure 3-2b. Put differently, the supplier originally incurred more supply expenses (area II)

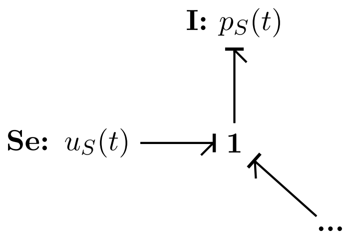


Figure 3-1: Supplier with a supply price as a bond graph subsystem.

and received some surplus for it (area I), but consequently missed out on a higher amount of surplus he could have taken away from the consumers (area III, or the area under the blue demand curve in Figure 3-2a). Effectively, this graphically displays why consumers are best off under perfect competition (consumer surplus - as in Figure 3-2a - is maximized there), whereas suppliers would generate the most profits in a monopoly scenario [21].

Within Economic Engineering bond graph modeling, the supply curve can be modeled with an I-element, representing inertia. As described in Eq. (B-11) and Figure B-3d, incoming price adjustments (forces) are accumulated into a (supply) price (a momentum), which is stored in the I-element. Based on the parameter value of this I-element, it then returns a quantity supplied (a flow). As mentioned before, I assume that suppliers can set their own supply price. Within the model, this means they would need to be able to control that price directly.

Within bond graph modeling, price control can be modeled with an effort source that the supplier controls (described in Appendix B-2-1). The force originating from this effort source then represents the price adjustments that are being made by the supplier. Figure 3-1 shows the bond graph subsystem that models a supplier that sets its own supply price. Quite literally, a supplier can thus put in (more) effort with the controlled input $u_S(t)$, to increase its own supply price $p_S(t)$. As a result of this higher supply price, its quantity supplied will go up.

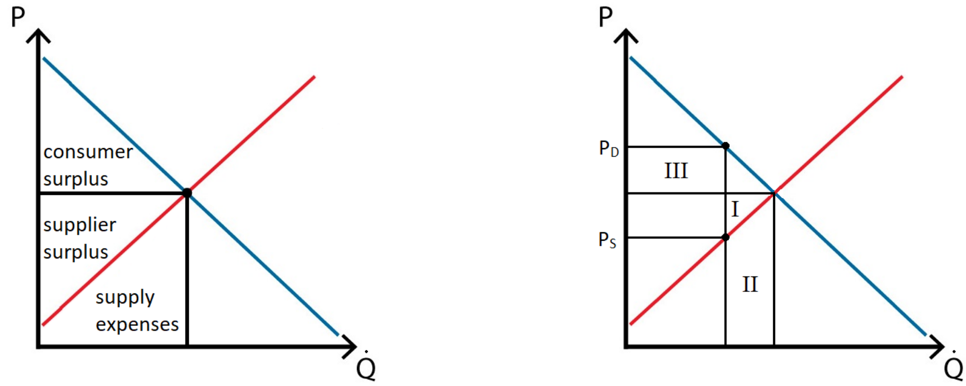
Model Assumption

- Suppliers can control their own supply price.

Consequently, modeling a supplier and its supply price with Economic Engineering allows for a physical interpretation of the supplier's surplus and expenses, as shown in Figure 3-2a. The supplier surplus (noting that the independent variable - price - is on the vertical axis) now represents the *kinetic energy* stored in the I-element, whereas the supply expenses are now analogous to the *kinetic co-energy*. The consumer surplus is related to the demand, which is covered in the next Section.

3-2-2 Demanders and market prices

Section 3-2-1 describes how both suppliers can set their own supply price. Consequently, this leads to the supply of two different products, possibly in different quantities. These supply prices do not need to be equal to the prices for which these products are actually sold, i.e.,



(a) Economic interpretation **(b)** Distinct supply and market price

Figure 3-2: Supply and demand curve areas.

the *market prices*. Typically, a supplier would not be willing to supply his products if he only gets a minimal amount of surplus in return; he would want at least a decent minimum *profit margin*, which is defined as the proportion of the profit per one dollar of sales. Typically, companies with a high profit margin also have higher overall profit levels, which is obviously beneficial to that company [53]. Thus, suppliers would most likely want some margin or difference between the market price and their supply price, i.e., $p_D(t) - p_S(t) > 0$.

These two market prices then determine the quantities demanded for both products. Similar to the supply prices, the market prices also represent *reservation prices*. That means that a market price is ideally equal to the maximum price a consumer is willing to pay for a product, based on the consumer's utility and all substitute products. Put differently, at this price, a consumer is indifferent between buying and not buying a specific product [54, 55]. The actual market price could theoretically be below what a consumer is willing to pay, but this is obviously not ideal from the perspective of a supplier, since there is easy profit left on the table.

Several pricing mechanisms exist, and generally these are categorized into three distinct groups [56]. With *cost-based pricing*, a business uses its costs as a guideline to what the market prices for its products should be. It is a relatively straight-forward and very popular approach that only requires data that is readily internally available for most companies, but it does not take competition into account [57]. Consequently, the risk exists that a company is charging a price that is either too high or too low. *Competition-based pricing* is the next alternative, which observes or estimates what competitors are doing to optimally set prices [58]. Although this is already an improvement over cost-based pricing, neither takes the actual customer into account. This is typically only the preferred pricing approach for commodities [56]. Finally, with *value-based pricing*, market prices are set based on what customers believe a product is worth.

According to the literature, the value-based pricing method is considered superior to the rest [59, 60], especially when profit maximization is the primary objective [56, 61], since value-based pricing has the largest profit potential of these three pricing mechanisms [62]. Additionally, value-based pricing is most closely related to the demand curve described in Section 2-2. Consequently, I will adhere to value-based pricing for my model.

Consequently, I assume in my model that suppliers can individually set the market prices of their products. This means they have full control over how much they supply and at what market price they aim to sell it, and partial control over the demand for their product, which is based on both their market price and the market price of their competitor. This means they need to have at least a decent estimate of both the individual demand for their products, and of their competitor's prices and products. Subsequently, they would need to estimate how these prices affect the demand for their own product, i.e., the *cross elasticity of demand*, as described in Section 2-2-5. I believe that for relatively large companies, especially in more mature markets, this is a valid assumption to make. Suppliers can perform e.g., data analysis based on observable market prices or conduct experiments to obtain the required demand data.

Model Assumption

- Suppliers set the market price of their product using value-based pricing.

Therefore, similar to Bertrand competition models, which are discussed in Section 2-4-3, suppliers can now individually set their market price. I assume that this market price is equal for all customers, i.e., there is no *price discrimination*, and that customers are always aware of the market prices of both products. Since consumers are also assumed to behave like rational agents, they would always choose the product with the lowest price, when choosing between two identical products. In my model, I assume some *substitutability* is present, which means that the products are similar, but not identical. Consequently, price differences can exist for products that are *close substitutes*, although increasing prices for product A have a positive effect on the demand for product B [63].

Model Assumptions

- The two products are close substitutes.
- There is no price discrimination.

As described in Section 2-2, demanders have a downward-sloping demand curve, meaning that their quantities demanded are low when market prices are high. Since my model contains two different products, I assume a demand curve exists for each of them. Analogously to the case of suppliers, this demand curve can now be modeled as an I-element, meaning that incoming price adjustments from the supplier (forces) are accumulated into a market price (a momentum), which is stored in the I-element. Consequently, a quantity demanded for that product is returned as a flow. This scenario can thus be modeled as in Figure 3-3. This means the supplier can increase or decrease the market price $p_D(t)$ with the controlled input $u_D(t)$. Economically, this means the supplier can move the market price along the demand curve, which he will do in a way that generates him the most profit. The *consumer surplus*, as depicted in Figure 3-2a, consequently represents the *kinetic energy* stored in the I-element.

Model Assumption

- A demand curve exists for each product.

However, the subsystem depicted in Figure 3-3 can only be used to model the demand side of

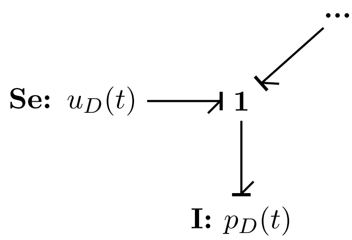


Figure 3-3: Supplier with a market price as a bond graph subsystem.

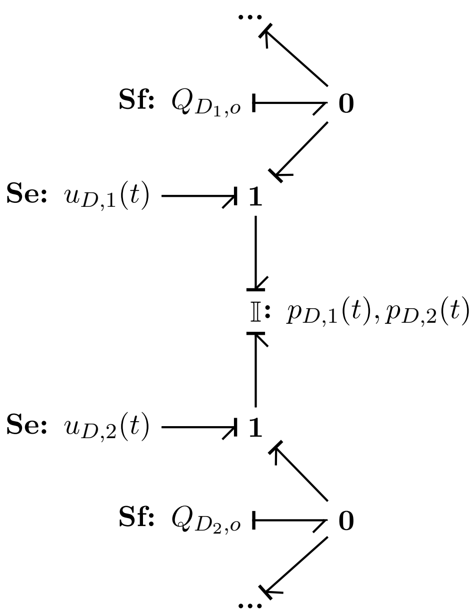


Figure 3-4: Multi-product demand bond graph system with a mass tensor and measured disturbances as demand offsets.

one product. As mentioned before, some degree of substitutability is present, meaning that the market prices of both products affect the demand of both. This can be modeled with a *mass tensor*, which essentially represents a matrix version of the price-demand relationship. This relationship can then be represented as:

$$\begin{bmatrix} Q_{D,1}(t) \\ Q_{D,2}(t) \end{bmatrix} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{21} & \varepsilon_{22} \end{bmatrix} \begin{bmatrix} p_{D,1}(t) \\ p_{D,2}(t) \end{bmatrix} + \begin{bmatrix} Q_{D_1,o}(t) \\ Q_{D_2,o}(t) \end{bmatrix} \tag{3-1}$$

The cross-diagonal terms ε_{11} and ε_{22} represent the price elasticities of demand for both products, whereas ε_{12} and ε_{21} represent the cross-elasticities of demand, i.e., how the market price of one product affects the demand for the other. The elasticities will have a negative value, whereas the cross-elasticities are assumed to be positive. For symmetry, the cross-elasticities are assumed to be equal.

Additionally, $Q_{D_1,o}(t)$ and $Q_{D_2,o}(t)$ represent demand *offsets*, i.e., curve shifts, as described in Section 2-2-5. Economically, this can be used to model the (assumed) quantities demanded for each individual product at a market price of zero. Within bond graph modeling, this curve shift can be represented as a measured disturbance, meaning it can be modeled with a *flow source*, as described in Appendix B-2-1. The complete demand side of the model is then displayed in Figure 3-4.

Whereas in the traditional economic supply and demand scenario only one price exists, several (supply and market) prices now exist in my model, governing both the quantities supplied and demanded. Physically, this means that in equilibrium, the momenta are constant but not necessarily equal, and the velocities or flows are equal. When the competitive market system is not in equilibrium, the flows - i.e., the quantities supplied and demanded - do not match, and either an *excess supply* or an *excess demand* is present, as described in Section A-2. This mismatch can then be modeled and stored with a *backlog* element, which is described in the next subsection.

3-2-3 Backlog

When the quantity demanded exceeds the quantity supplied - i.e., there is an *excess demand*, a *backlog* builds up. This refers to the orders from the market that are still remaining to be fulfilled [64], and can be seen as a negative storage. Much like a spring, the backlog actually describes a backlog level, meaning its "resting position" does not have to be the absolute zero point.

Within Economic-Engineering bond-graph modeling, a backlog can thus be modeled with a C-element. As described in Eq. (B-10), the difference between the quantity demanded and the quantity supplied (the *error flow*) is accumulated into a total backlog quantity q. Based on the parameter of the C-element, it then returns a convenience (a force) which drives up both the supply prices and the market prices. The reasoning behind this is twofold. First of all, a higher supply price incentivizes suppliers to supply more, in order to get rid of the excess demand. Secondly, in this scenario where there exists a shortage of goods, a higher market price also reduces the excess demand, since it basically filters out the customers with a higher willingness to pay (a higher reservation price). The complete model is shown in Figure 3-5.

Model Assumption

- Excess demand accumulates in a backlog.

3-3 Complete Bond Graph Model of a Competitive Market

Since the full model is modular, by combining all subsystems as described in the previous Section a complete bond graph model of a competitive market can be constructed. This is shown in Figure 3-5, with the interpretations described in Figure 3-6.

The full model has a total of 6 states; 2 supply prices p_S, 2 market prices p_D, and 2 backlog levels q - all of which are assumed to be known at all times. Both suppliers each control 2 inputs. With u_S they can set their supply prices, and consequently have full control over their own quantity supplied. With u_D they can each set their market price, which in combination with the demand offsets $Q_{D,o}$ and the others' market price lead to the quantity demanded for their product. The demand offsets $Q_{D,o}$ are modeled as measured disturbances. More competitors could easily be added in a similar fashion due to the modularity. This is discussed in more detail in Section 6-1.

Model Assumption

- All states are known at any time to both suppliers and demanders.

The equations of motion are derived in Appendix C. The resulting state-space equations describing the model are as follows:

$$\dot{x} = Ax + Bu + Ed = \begin{bmatrix} \dot{p}_{S,1} \\ \dot{p}_{S,2} \\ \dot{p}_{D,1} \\ \dot{p}_{D,2} \\ \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{1}{C_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{C_2} \\ 0 & 0 & 0 & 0 & \frac{1}{C_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{C_2} \\ -\varepsilon_{S_1} & 0 & -\varepsilon_{D_{11}} & \varepsilon_{D_{12}} & 0 & 0 \\ 0 & -\varepsilon_{S_2} & \varepsilon_{D_{21}} & -\varepsilon_{D_{22}} & 0 & 0 \end{bmatrix} \begin{bmatrix} p_{S,1} \\ p_{S,2} \\ p_{D,1} \\ p_{D,2} \\ q_1 \\ q_2 \end{bmatrix} +$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{S,1} \\ u_{S,2} \\ u_{D,1} \\ u_{D,2} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} Q_{D_1,o} \\ Q_{D_2,o} \end{bmatrix} \tag{3-2}$$

3-4 Modeling Companies as Model-Predictive Controllers

Now that the complete time-domain market model has been constructed, competition needs to be added, which means that the suppliers will compete with each other over the demanders - the customers. They actively do so by changing - controlling - both their supply prices and their market prices. I choose to model this with model-predictive controllers.

Model-Predictive Control (MPC) solves a finite-horizon open-loop optimal control problem to calculate the optimal control action sequence based on the current states of the model. At the next time step, only the first control action is applied. Afterwards, the optimization starts again, based on the new initial states and the same finite time horizon, which means MPC is capable of *online optimization*. It is thus also known as *Receding Horizon Control*. It is also able to handle hard constraints on both inputs and states [65].

MPC can be used to optimize profits, by using an economic objective function, in the form of *Economic MPC* [66]. Since it uses the current states of the system in its optimization, MPC can include dynamic prices in its optimization and its objective function, in contrast to offline optimization.

Consequently, both the transient and the steady-state behaviour of a system are taken into account, similarly to how companies can slightly adjust their strategies throughout the year, despite their long-term planning. They make decisions based on imperfect information about their competitor, and can - to some extent - adapt later on when they receive the updated

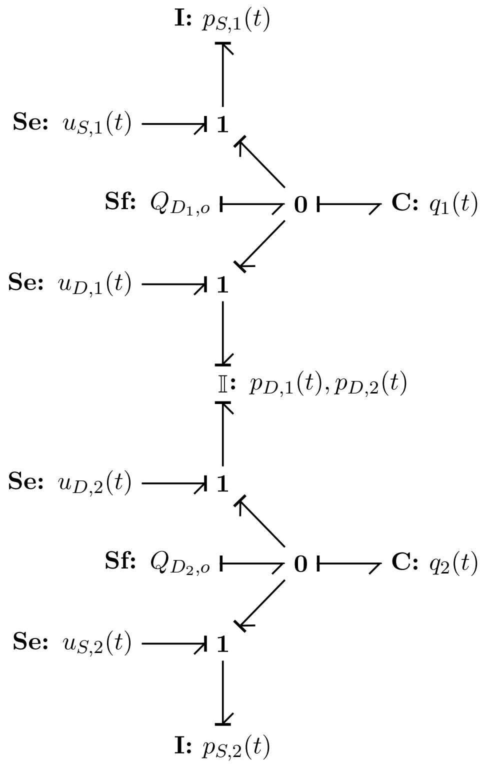


Figure 3-5: Full bond graph model of a competitive two-supplier market.

information. The receding horizon aspect is analogous to the time window they use for their strategy, e.g., 3 months or 5 years.

This Section first describes how MPC is currently being used to simulate various types of competition, both of which are relevant within a competitive market. Section 3-4-1 focuses on internal competition. Internal competition is present within a system when a controller is simultaneously optimizing several conflicting criteria. It is discussed how this relates to competition in a business sense and how common problems that arise are typically resolved. Section 3-4-2 then deals with external competition. This occurs on an industry level, when multiple individual controllers are present within a system, and they all compete with each other.

3-4-1 Internal competition: conflicting objective functions

When a model predictive controller has a conflicting objective function, it means that it is simultaneously optimizing multiple criteria that could be contradicting each other. A typical example of multi-objective MPC is the control of an autonomous vehicle. When the MPC objective function includes both the objectives "travel time" and "energy consumption", a conflict arises. When travel time becomes lower, energy consumption will typically increase [67]. Section 2-4-2 has also introduced an analytical competition model in which the potentially conflicting objectives "profit" and "market share" are simultaneously being optimized [14].

The latter resolves this problem by simply assigning fixed weights to each objective, making

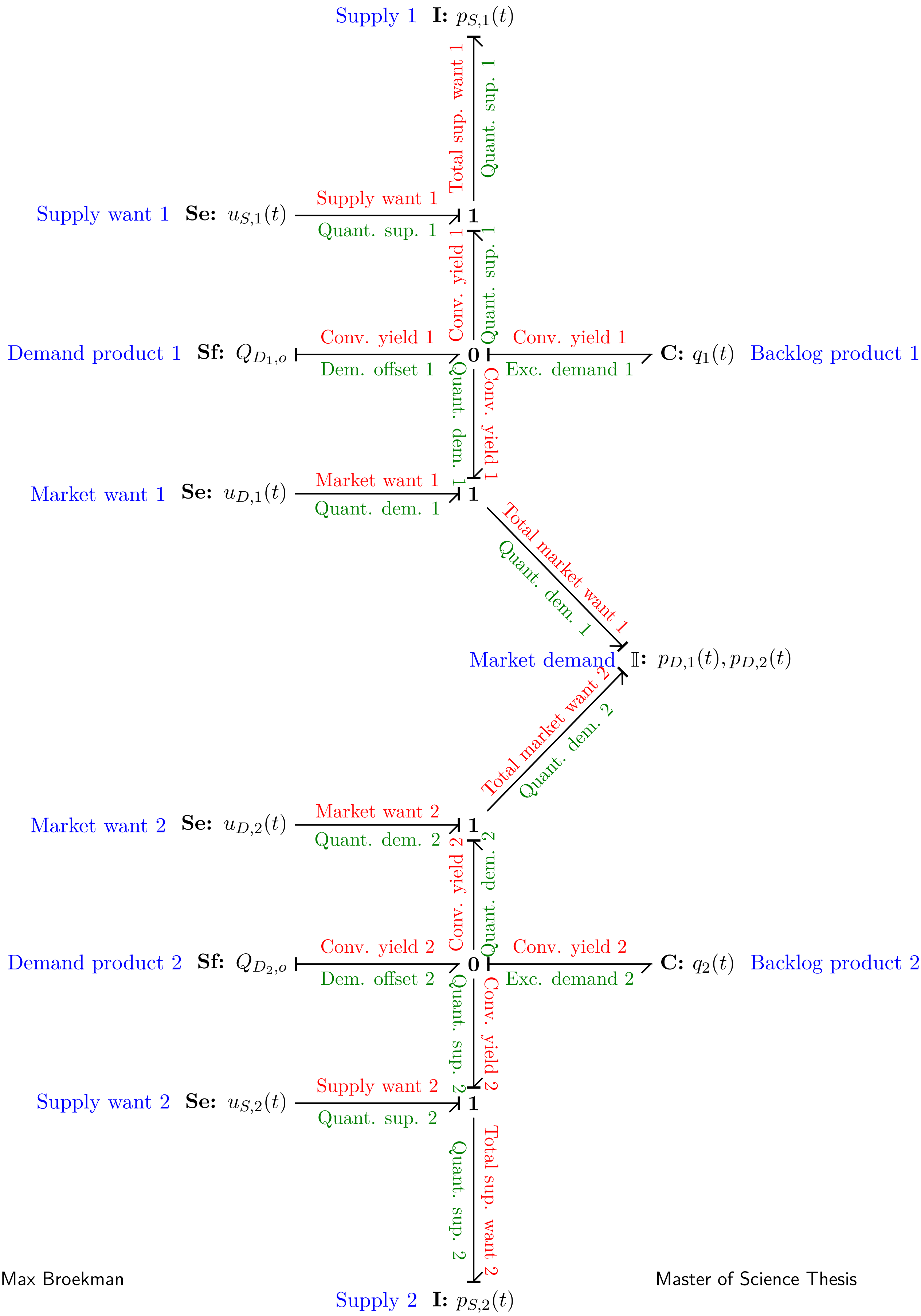


Figure 3-6: Full bond graph model economic interpretations.

the overall objective function a linear function of the two individual objectives. Although this methods has its limitations, it is very commonly used in MPC [68]. The scalarized objective function then takes the form:

$$\hat{J} = \Sigma_{i=1}^{k} w_i J_i \tag{3-3}$$

However, in the case of e.g., model-predictive control of an autonomous vehicle, such a simple approach would often not suffice. The objective function cannot always be static and should be able to change when the states of the system or its surroundings change, i.e., it should be vector-valued - and the objective function will typically be non-linear. This is similar to how companies make strategic decisions; they observe their rivals actions (e.g., changing their price - a state of the system) in real-time and readjust their strategy accordingly. This means they adjust their priorities, i.e., they assign different weights to their individual objectives.

Based on these different combinations of weights, different optimal outcomes arise. Jointly, these optimal outcomes form a *Pareto set*. Within this set, the outcomes are *non-dominated* by each other, which means that one outcome can be better than another outcome within the Pareto set with respect to one objective. However, there will not exist an outcome that is better than all other outcomes with respect to all objectives [69]. Put differently, another outcome within the Pareto set will be worse with respect to at least one objective.

Although MPC is typically used for reference tracking or the stabilization of a system [70], it can also be used to maximize profits. This gives rise to the field of *Economic MPC* [66]. From a company perspective, multi-objective (economic) MPC can then be used to besides profits, also optimize market share or revenues. This is further described in Section 6-1. In this thesis, I assume that companies (the controllers) are only interested in optimizing their profits. As described in 3-2-1, they do so by maximizing their supplier surplus.

Model Assumption

- A supplier's only objective is to maximize his profits.

3-4-2 External competition: rivaling controllers

Within large systems, multiple controllers can be present, leading to a multi-agent control structure. Typically, there will be a global objective or constraint, and the individual model predictive controllers all attempt to optimize some local and individual objective. In the case of e.g., a power network, individual controllers will aim to maximize their profits, while jointly ensuring that the overall system is stable [71]. Typically, *distributed control* is used to model these types of systems.

An analogous scenario arises in competitive markets. There is a large overall system - the industry, and individual companies attempt to maximize their own profits and outperform their rivals, without creating an extremely volatile market. In this case, market stability can be seen as a global constraint. In my model, this translates to a maximum change in supply prices and market prices per time interval, i.e., a maximum on the absolute input wants, controlled by the supplier.

Model Assumption

- Suppliers are limited in the price changes they can make.

Although the prices that rivals charge might be known, the different strategies of the competition (i.e., their objective functions or internal market models) are typically confidential. Various (distributed) MPC methods have been constructed to deal with these kinds of limited knowledge optimization problems [71]. Additionally, solutions have been suggested for when various agents, i.e., rivaling companies, jointly have to determine certain variables [72], when rivaling agents have to compete for the same resources [73], or when individual objective functions also depend on your rivals' control inputs [74].

As described before, suppliers are assumed to only care about maximizing their individual profits. Consequently, there is no joint objective in the competitive market model. Additionally, this thesis assumes that all states are known to both the suppliers and the demanders at all times. Therefore, the model that the model-predictive controller uses is assumed to be the same as the model of the system. Since the strategies of competitors are typically unknown, suppliers optimize their strategies based on constant competitor prices. Obviously, except in equilibrium scenarios, these prices will most likely not be constant, but model-predictive controllers can deal with this by simply revising their strategy at the next time step, based on current competitor market prices.

Model Assumptions

- The internal MPC model is the same as the model of the system.
- Suppliers assume that the market price of their competitors remain constant.

Consequently, at each new time step, both suppliers will choose an optimal input. Both inputs will then simultaneously be applied to the system, resulting in a new state, after which the process repeats itself. Since the suppliers' optimal input wants depend on both their own prices and the current market prices of their competitors, this leads to *interdependent optimization*, similar to non-cooperative games in game theory.

Competitive behaviour can then be observed as the result of companies continuously optimizing their surplus, taking into account the latest moves from their competition. Simultaneously, their competitors do the same thing, and the interaction that then arises between the companies shows the price dynamics of competition.

3-4-3 Discretization

Whereas the bond-graph model describes a continuous-time system, MPC is used in combination with a discrete-time model. Hence, the model needs to be discretized. For this, the *Forward Euler method* will be used, which numerically approximates the next state values as [75]:

$$w_{k+1} = w_k + \Delta t f(t_k, w_k) \tag{3-4}$$

wherein $f(t_k, w_k)$ represents the slope of the tangent in (t_k, w_k). The model state evolution can then similarly be described as:

$$x[k+1] = x[k] + T_s\Big[Ax[k] + Bu[k] + Ed[k]\Big] \tag{3-5}$$

For the complete model, this translates to:

$$x[k+1] = \begin{bmatrix} p_{S,1}[k+1] \\ p_{S,2}[k+1] \\ p_{D,1}[k+1] \\ p_{D,2}[k+1] \\ q_1[k+1] \\ q_2[k+1] \end{bmatrix} =$$

$$\begin{bmatrix} p_{S,1}[k] \\ p_{S,2}[k] \\ p_{D,1}[k] \\ p_{D,2}[k] \\ q_1[k] \\ q_2[k] \end{bmatrix} + T_s \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{1}{C_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{C_2} \\ 0 & 0 & 0 & 0 & \frac{1}{C_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{C_2} \\ -\varepsilon_{S_1} & 0 & -\varepsilon_{D_{11}} & \varepsilon_{D_{12}} & 0 & 0 \\ 0 & -\varepsilon_{S_2} & \varepsilon_{D_{21}} & -\varepsilon_{D_{22}} & 0 & 0 \end{bmatrix} \begin{bmatrix} p_{S,1}[k] \\ p_{S,2}[k] \\ p_{D,1}[k] \\ p_{D,2}[k] \\ q_1[k] \\ q_2[k] \end{bmatrix}$$

$$+T_s \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{S,1}[k] \\ u_{S,2}[k] \\ u_{D,1}[k] \\ u_{D,2}[k] \end{bmatrix} + T_s \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} Q_{D_1,o}[k] \\ Q_{D_2,o}[k] \end{bmatrix} \tag{3-6}$$

Besides the MPC requirement for a discrete-time model, the model now also holds a new economic interpretation. In contrast to a continuous-time model, inputs are constant during each sampling time period. Economically, this indicates that there exists an upper limit on how quickly companies can adjust their prices. They could e.g., easily do this once per year, but not once per second.

For simulation purposes only, T_s is further decreased by a factor of M - the *discretization parameter*, to ensure that the discrete-time model still accurately follows the continuous-time behaviour of the system. Consequently, the system inputs are then held constant for M times a T_s/M time period.

3-4-4 Objective functions

Model-Predictive Control (MPC) uses an objective function, which determines what the optimal control sequence is going to be. This objective function is typically minimized. However, a supplier is interested in the maximization of its supplier surplus, as displayed in Figure 3-2a. Consequently, the suppliers' objective function can be represented as the negative of his surplus (the dependence on time step $[k]$ is left out):

$$J_1 = -\sum_{k=1}^{T} \big[\underbrace{Q_{D_1,o} p_{D_1} - \varepsilon_{D_{11}} p_{D,1}^2 + \varepsilon_{D_{12}} p_{D_1} p_{D_2}}_{\text{Revenue}} - \underbrace{\frac{1}{2} \varepsilon_{S,1} p_{S_1}^2}_{\text{Supply costs}} - \underbrace{\frac{1}{2} \frac{1}{C_1} q_1^2}_{\text{Backlog costs}} \big] \tag{3-7}$$

The first term represent the revenue (in \$/day) from selling its product on the market, and is defined as the product of the demand flow (#/day) and the market price (\$/#), i.e.,

$$\text{Revenue} = \big[\underbrace{Q_{D_1,o} - \varepsilon_{D_{11}} p_{D,1} + \varepsilon_{D_{12}} p_{D_2}}_{\text{Demand flow}} \big] \underbrace{p_{D_1}}_{\text{Market price}} \tag{3-8}$$

This is based on the demand offset for its product $Q_{D_1,o}$, and both market prices. The demand offset gets multiplied by the market price of its own product, which introduces a linear part to the otherwise quadratic equation. The revenue can be displayed graphically as the rectangular area to the bottom left of the market price on the demand curve in Figure 3-2b.

The second part of the objective function represents the supply costs, which depend on the suppliers' supply price. In Figure 3-2b, this is the triangular area below the supply curve, up to the supply price. Economically, it represents the suppliers' expenses associated with supplying its product to the market. Due to the upward-sloping supply curve, marginal costs increase linearly, resulting in quadratically increasing supply costs.

The last term of the objective function represents the backlog costs. When a supplier is producing more than he sells, he needs to store these goods (i.e., there is negative backlog). Storing a small quantity of goods is most likely cheap. However, the expenses associated with storage quickly increase when storage needs to be scaled up, due to the need to rent external storage space. Inversely, when there are more products being sold than there are produced and they cannot be retrieved from storage, a backlog builds up. Economically, this represents the fact that customers might have to wait for their already purchased order, which could result in reputational damages when this waiting time becomes too large. Hence, the costs associated with a backlog are again quadratic (and thus convex). Similarly to the supply costs, it represents a triangular area under the (backlog) curve, which explains the 1/2 term.

Additionally, a term is added to the objective function which represents a penalty on the inputs - the supplier wants - in the following form:

$$\frac{1}{2} u[k]^\top P u[k] \tag{3-9}$$

wherein P represents a symmetric penalty matrix. As described in Section 3-4-2, suppliers are limited in the price changes they can make, to preserve market stability. Section 3-4-5 will describe how this can be guaranteed with constraints. However, this can additionally be included in the objective function, in order to penalize larger deviations more severely. It can then be interpreted as a financial penalty for disturbing the market, such that the objective function still represents the supplier surplus.

Simultaneously, the second supplier (the competitor) solves a similar optimization problem, based on his individual supplier surplus, which is given by:

$$J_2 = -\sum_{k=1}^{T} [\underbrace{Q_{D_2,o}p_{D_2} - \varepsilon_{D_{22}}p_{D,2}^2 + \varepsilon_{D_{21}}p_{D_1}p_{D_2}}_{\text{Revenue}} - \underbrace{\frac{1}{2}\varepsilon_{S,2}p_{S_2}^2}_{\text{Supply costs}} - \underbrace{\frac{1}{2}\frac{1}{C_2}q_2^2}_{\text{Backlog costs}} - \underbrace{\frac{1}{2}u^\top Pu}_{\text{Price change penalty}}] \tag{3-10}$$

Since the profit for both suppliers is now expressed in terms of the system states, the price dynamics from the Economic-Engineering model are now also taken into account into the optimization that the suppliers solve.

3-4-5 Constraints

As described in the previous Section, each supplier aims to maximize their own supplier surplus. In order to do so, they solve an optimization problem. The states cannot just arbitrarily be changed to maximize the surplus, since the system is constrained to its own dynamics, i.e., the state-space equations. Additionally, several other constraints are imposed upon the system. Eq. (3-7) can consequently be adjusted as follows to represent the optimization problem for the first supplier (again the dependence on time step [k] is left out):

$$\min_{u_{S_1},u_{D_1}} -\sum_{k=1}^{T} [\underbrace{Q_{D_1,o}p_{D_1} - \varepsilon_{D_{11}}p_{D,1}^2 + \varepsilon_{D_{12}}p_{D_1}p_{D_2}}_{\text{Revenue}} - \underbrace{\frac{1}{2}\varepsilon_{S,1}p_{S_1}^2}_{\text{Supply costs}} - \underbrace{\frac{1}{2}\frac{1}{C_1}q_1^2}_{\text{Backlog costs}} - \underbrace{\frac{1}{2}u^\top Pu}_{\text{Price change penalty}}]$$

$$\begin{aligned} \text{subject to:} \quad & x[k+1] = Ax[k] + Bu[k] + Ed[k] \\ & u_{S_1,min} \leq u_{S,1} \leq u_{S_1,max} \\ & u_{D_1,min} \leq u_{D,1} \leq u_{D_1,max} \\ & q_{1,T,min} \leq q_1[T] \leq q_{1,T,max} \\ & x[0] = x_0 \end{aligned} \tag{3-11}$$

This objective function can be minimized by the supplier by changing his input wants u_{S_1} and u_{D_1}. These inputs are itself in the objective function, but they also affect the states (the prices), which are in the objective function as well.

To be able to solve this problem, Pontryagin's maximum principle can be used in discrete time, which forms a Hamiltonian by using a time-varying auxiliary multiplier $\mu[k]$ called the co-state variable [76] - similar to a Lagrange multiplier. This Hamiltonian is then composed of the objective function J and the state equations f in the following form:

$$H(x_k, u_k, \mu_k) = J(x_k, u_k) + \mu_k f(x_k, u_k) \tag{3-12}$$

The maximum principle then refers to the following two necessary conditions for optimality:

$$\frac{\partial H}{\partial u_k} = 0 \tag{3-13}$$

$$\mu_k - \mu_{k-1} = -\frac{\partial H}{\partial x_k} \tag{3-14}$$

As the state equations are constraints to the system, the co-state variable can then be interpreted as the marginal cost associated with disobeying these constraints. Hence, the state equations are soft constraints in this optimization problem.

Subsequently, both input wants are constrained. As mentioned before, suppliers are obligated to preserve a certain degree of market stability. Consequently, the higher the absolute values of these input constraints, the larger the potential price movements in the market.

Additionally, the higher the price change they make, the higher the financial penalty they incur through the $\frac{1}{2}u^\top Pu$ term. A high value of the elements on the diagonal of the matrix P can then provide additional stability to the system.

Additionally, a terminal region is added to improve performance [77]. This also acts in a stabilizing way, since it forces the suppliers to push their backlogs to a certain steady state deemed economically optimal at the end of their control horizon. This optimal state (region) indicates a preference for a specific long-term behaviour, and as such ensures that suppliers do not engage in cannibalistic behaviour to generate short-term results. A terminal region (instead of a terminal point) is used to take into account the unexpected behaviour of competitors. Since a supplier's backlog is also dependent on the market price of the competition (Section 3-3), which is unknown beforehand, a terminal point is not feasible and could lead to a less stable system.

Finally, the last constraint ensures that the system starts from a set of specific initial states. The objective function and constraints displayed here are unique for the first supplier, but although not displayed here, a similar version is used for the competition.

3-4-6 Tuning

Sampling time

For a decreasing sampling time T_s, rejection of unknown disturbances can to a certain extent improve [78]. This means suppliers can react to unexpected competitor moves more quickly. Economically, T_s can be seen as the smallest reaction time of a company, i.e., the time they need to adjust their strategy. However, a smaller value of T_s does come at the expense of computational power.

In this thesis, since the typical time span is in days (Section 4-2), T_s is chosen as 1/12 days, i.e., 2 hours. This has shown to be within 10% to 25% of the desired individual closed-loop response time.

Control and prediction horizon

The prediction horizon is chosen long enough so that competitive behaviour can be observed over a period of time, but eventually a steady-state scenario arises. Since the smallest reaction time is set to 2 hours, using a prediction horizon of 36 time steps allows one to look at the competitive behaviour taking place over a period of 3 days. This can be increased to observe e.g., quarterly behaviour, but at the expense of increased computational intensity. Additionally, uncertainty typically grows over time - it is very hard to determine what your competitor will do in 2 months time. Robust or stochastic Model-Predictive Control (MPC) could deal with this uncertainty, but this thesis adheres to a prediction horizon of 3 days.

The control horizon is chosen shorter than the prediction horizon, to reduce computational efforts; a control horizon of 5 time steps (10 hours) is used in this thesis. In isolation, a supplier could reach a steady-state scenario within this time span, and as such it is deemed sufficient for optimization purposes. Since the competitor will behave in unexpected ways and in any case only the first optimal input is used, the prediction horizon is larger, to show competitive behaviour over time.

3-4-7 Stability

As described in Section 3-4-2, suppliers assume that the market prices of their competitors stay constant while performing their optimization. Consequently, for the first supplier the term $\varepsilon_{D_{12}} p_{D_1} p_{D_2}$ in Eq. (3-7) becomes a function only of p_{D_1}.

The objective function is twice differentiable, such that the objective function's Hessian $\nabla^2 f(x)$ - the square matrix of second-order partial derivatives [79] - can be defined as:

$$\nabla^2 f(x)_{ij} = \frac{\partial^2 f(x)}{\partial x_i \partial x_j}, \; i,j = 1, ..., n \tag{3-15}$$

Due to the double differentiation, the linear terms drop out and only the quadratic terms remain. Since the parameter values that determine the corresponding constant are positive (Table 4-1), it follows that:

$$\nabla^2 f(x) \succeq 0 \tag{3-16}$$

i.e., the Hessian is semi-definite. Consequently, the objective function is convex, and it can easily be verified that the objective function Eq. (3-7), based on the parameter values in Table 4-1, has a global minimum (idem for supplier 2).

Although both suppliers individually solve a convex optimization problem wherein they assume the competition does not adjust their prices, in reality the market prices do change. As a result of the interactions between the suppliers, influenced by updated information on competitors' prices - i.e., competitive behaviour - the system as a whole could still exhibit unstable behaviour.

However, determining the joint stability of these two competing model-predictive controller is outside the scope of this thesis. The stability of the system including competitive interaction can be assessed through simulations, as is done in Sections 4-3 and 4-4. It is shown there that the system indeed stabilizes over time.

In order to improve the stability of the whole system, several constraints have been added to the optimization problem, as has been described in the previous Section. Bounds on the input wants limit suppliers in the size of the price adjustments they can make, and consequently limit the price movements in the market to preserve a certain degree of market stability. The terminal constraint guarantees that the backlog (or storage) levels return to a certain region - the terminal region - at the end of the prediction horizon. In Sections 4-3 and 4-4 it can be seen that these constraints are indeed satisfied.

3-5 Conclusions

The model developed in this Chapter solves the problem described in the previous Chapter, i.e., that current competition models do not include price dynamics. Price dynamics are built into a competitive market model using Economic-Engineering. In contrast to game-theoretic models, this makes it possible to observe the state developments of the system, instead of only calculating steady-state solutions. The next Chapter shows the results of simulating the model with Model-Predictive Control (MPC) in both the time and the frequency domain, which enables regulators and hedge funds to quantify the effects of competition on prices, demand flows, and valuations.

Chapter 4

Analyzing the Dynamics of Competition with Control Engineering Tools

4-1 Introduction

In Chapter 3 a model of a competitive market has been constructed, and Model-Predictive Control (MPC) systems have been designed to represent competitors. This Chapter shows the results of the model simulations. These simulations are based on illustrative parameter values, which are stated in Section 4-2.

Both a competitive and a cooperative scenario are simulated in the time domain in respectively Sections 4-3 and 4-4. The results are described and details are provided on the differences between the two scenarios in terms of market prices, demand flows and instantaneous supplier surpluses.

Sections 4-5 and 4-6 describe how these differences can be transformed in the frequency domain to calculate the cost of competition, i.e., the total value that has evaporated due to competition. Subsequently, Section 4-7 performs scenario analysis. This is done in both the time domain through assessing parametric uncertainty, and in the frequency domain with bode plots. Finally, Section 4-8 states the conclusion.

4-2 Model Parameters and Simulation Settings

For the simulation, it is assumed that both suppliers are different, i.e., the parameters describing e.g., their supply and demand curves or their backlogs are different. This is in line with the assumption in Section 3-2-2 that some degree of differentiation is present. The Model-Predictive Control (MPC) settings are similar. The YALMIP toolbox [80] in MATLAB is used to model the MPC, in combination with the Gurobi non-convex solver. This

Table 4-1: Simulation parameter values.

Variable	Interpretation	Value	Units
C_1	Storage parameter supplier 1	0.15	$(\#^2 \cdot \text{day})/\$$
C_2	Storage parameter supplier 2	0.25	$(\#^2 \cdot \text{day})/\$$
ε_{S_1}	Price elasticity of supply 1	10	$\#^2/(\$ \cdot \text{day})$
ε_{S_2}	Price elasticity of supply 2	6.7	$\#^2/(\$ \cdot \text{day})$
$\varepsilon_{D_{11}}$	Price elasticity of demand 1	10	$\#^2/(\$ \cdot \text{day})$
$\varepsilon_{D_{22}}$	Price elasticity of demand 2	6.7	$\#^2/(\$ \cdot \text{day})$
$\varepsilon_{D_{12}}$	Cross elasticity of demand	5	$\#^2/(\$ \cdot \text{day})$
$\varepsilon_{D_{21}}$	Cross elasticity of demand	5	$\#^2/(\$ \cdot \text{day})$
$Q_{D_1,o}$	Demand flow offset 1	3000	$\#/\text{day}$
$Q_{D_2,o}$	Demand flow offset 2	4000	$\#/\text{day}$
T_s	Sampling time	1/12	day
M	Discretization parameter	5	-
m	Control horizon	5	-
N	Prediction horizon	36	-

solver translates the optimization problem into a bilinear form and then solves it by applying spatial branching [81].

Model Assumption

- The suppliers are not identical.

Although model identification is one of the recommendations (Section 6-2), I do not perform it in this thesis. Hence, the parameters are illustrative and any numerical results are resting on these assumed parameters. Tables 4-1 and 4-2 display the parameter and constraint values used in this thesis.

4-3 Competition

In this Section, the time-domain results for a competitive scenario are shown. The model described in Section 3-3 is used. Within this model, two model-predictive controllers (see also Section 3-4) control the input wants. Competitive behaviour then arises as a result of both controllers (representing companies) continuously optimizing their surplus, while taking price dynamics and competitor moves into account.

4-3-1 Time domain simulation results

Inputs and prices

Simulating the model in a competitive scenario leads to the results in Figures 4-1 to 4-7. Figure 4-1 shows the *input wants* that both suppliers control, i.e., the *effort sources*. Both are limited to an absolute size of 500 $\$/(\# \cdot \text{day})$. For the initial 1.5 day time period, these

Table 4-2: Simulation constraints and initial states.

Variable	Value	Units
$p_{S,1}\|_{t_0}$	300	$\$/\#$
$p_{S,2}\|_{t_0}$	300	$\$/\#$
$p_{D,1}\|_{t_0}$	300	$\$/\#$
$p_{D,2}\|_{t_0}$	300	$\$/\#$
$q_1\|_{t_0}$	300	$\#$
$q_2\|_{t_0}$	300	$\#$
$\|u_{S,1}\|_{max}$	500	$\$/(\# \cdot \text{day})$
$\|u_{S,2}\|_{max}$	500	$\$/(\# \cdot \text{day})$
$\|u_{D,1}\|_{max}$	500	$\$/(\# \cdot \text{day})$
$\|u_{D,2}\|_{max}$	500	$\$/(\# \cdot \text{day})$
$\|q_{1,T}\|$	100	$\#$
$\|q_{2,T}\|$	100	$\#$
R_{pen}	0.1	$(\#^2 \cdot \text{day})/\$$

limits are reached, after which both controllers converge to a stable steady-state input. For both suppliers, their supply wants u_S and their market wants u_D eventually converge.

The prices - shown in Figure 4-2 - can be seen to initially heavily fluctuate, but also eventually stabilize after approximately 2 days. As expected, for both suppliers their market price exceeds their supply price, i.e., they both have a positive *gross profit margin*, as described in Section 3-2-2. In this case, supplier 1 eventually generates a profit margin of approximately 58%, whereas supplier 2 stabilizes at a slightly higher 60% gross profit margin.

Supply and demand

Figure 4-3 and Figure 4-4 display the quantities supplied and demanded for both suppliers. Both flows of goods fluctuate heavily at first, and then gradually stabilize at around 2.5 days. Consequently, the excesses demand - shown in Figure 4-5 - converge to zero. Additionally, it can be seen that their is some out-of-phase behaviour between the two competitors, like in the excesses demand in Figure 4-5. That arises because companies optimize partly based on their competitors' current market prices. However, the competitor most likely does not hold these prices constant, hence there is some overshoot.

To illustrate this, consider what happens around 0.5 days. Company 2 has a little excess supply (Figure 4-5), and reduces their supply price (Figure 4-2) to reduce production of additional goods, i.e., lower their supply to again match the demand. However, at the same time, the first company is just making their products more expensive, which benefits their competitor, who now sees an increased demand for their (substitute) product (Figure 4-4). The result is a big overshoot, and thus an excess demand for the second product.

Backlog levels change rapidly, and occasionally turn into storage, i.e., negative backlog. When the excesses demand converge to zero, the backlog levels stabilize. After roughly 1.5 days, both backlog levels stay within their absolute terminal bounds of 100 units, indicated with the red dashed lines.

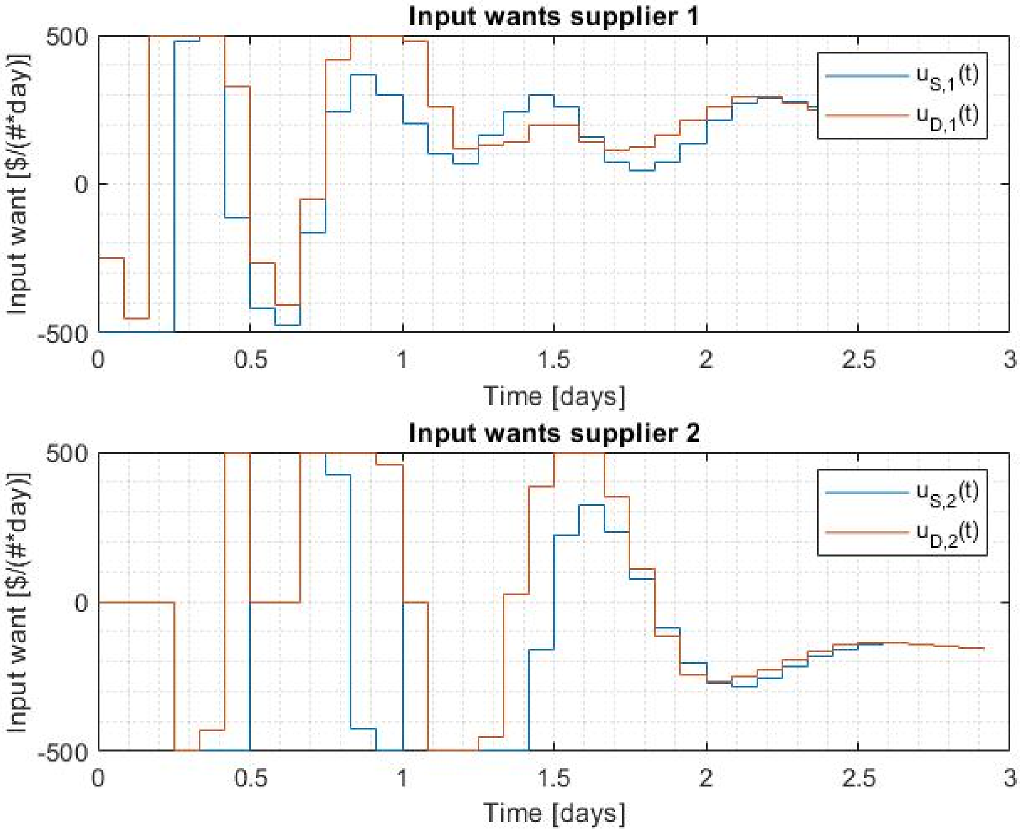


Figure 4-1: Supply input wants ($u_{S,1}(t)$ & $u_{S,2}(t)$, in blue) and market input wants ($u_{D,1}(t)$ & $u_{D,2}(t)$, in red) reach their limits, but stabilize after 2.5 days.

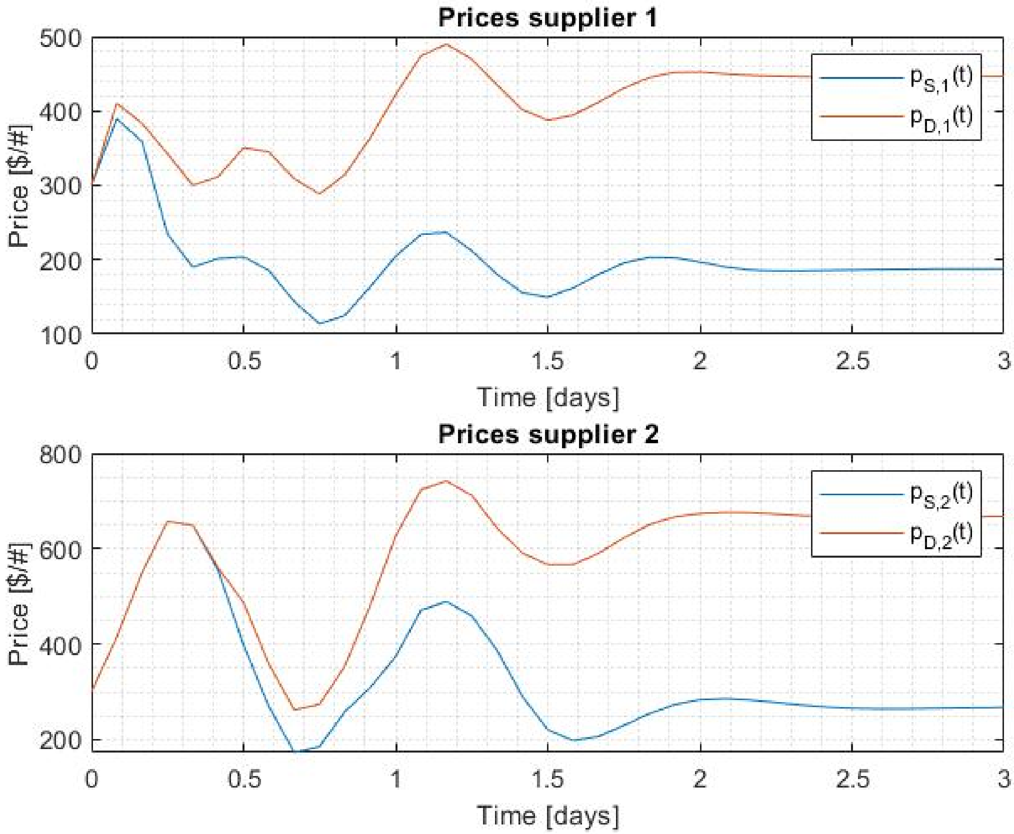


Figure 4-2: Supply prices ($p_{S,1}(t)$ & $p_{S,2}(t)$, in blue) and market prices ($p_{D,1}(t)$ & $p_{D,2}(t)$, in red). Stabilization occurs after 2 days. Market prices are respectively 2.38 and 2.5 times as high as supply prices, indicating 58% and 60% gross profit margins.

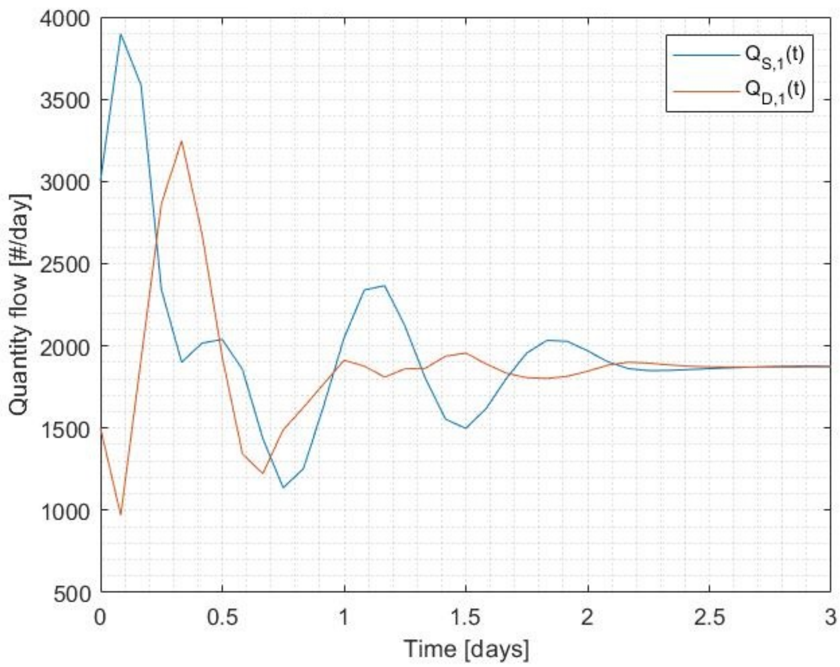


Figure 4-3: Supply flow (blue) and demand flow (red) supplier 1. Stabilization after 2.5 days around 1,875 #/day.

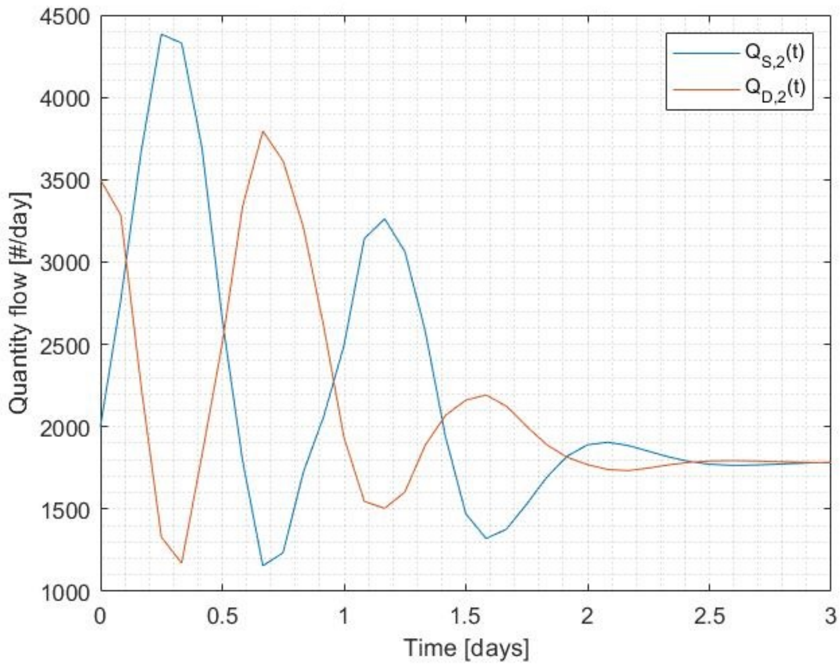


Figure 4-4: Supply flow (blue) and demand flow (red) supplier 2. Stabilization after 2.5 days around 1,765 #/day.

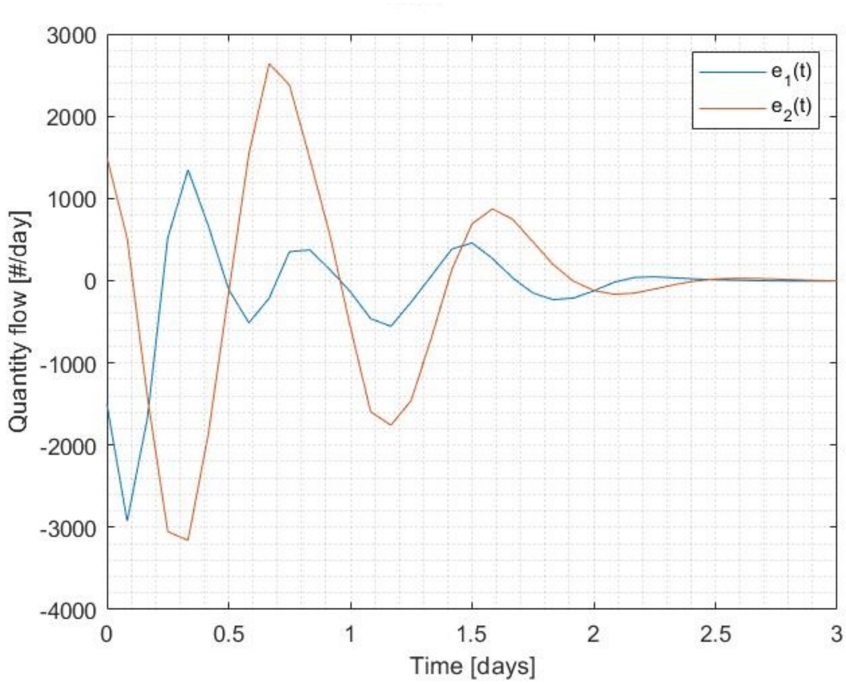


Figure 4-5: Excesses demand for both suppliers fluctuate heavily - especially for supplier 2 (red) - but go to zero after 2.5 days.

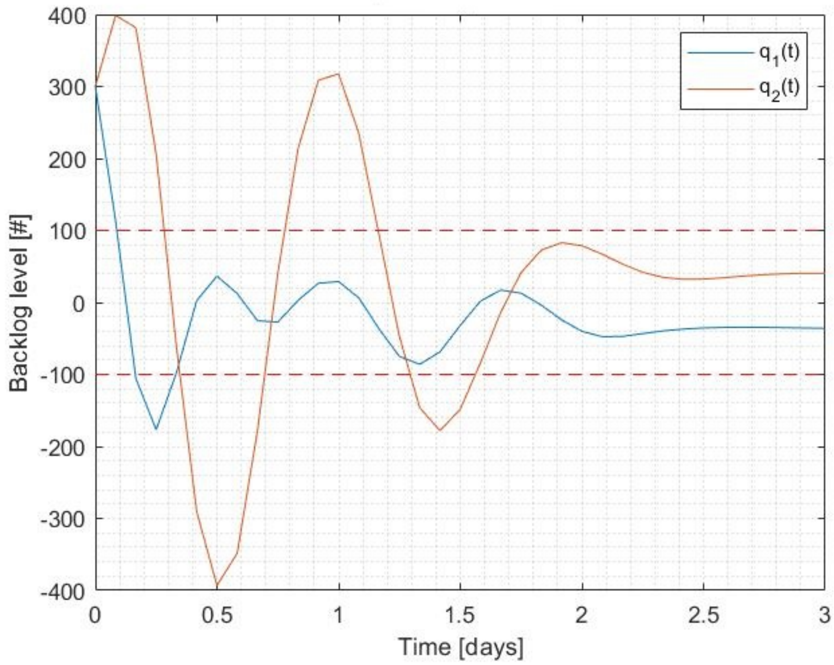


Figure 4-6: Backlog levels stabilize and stay within their terminal bounds after 1.5 days.

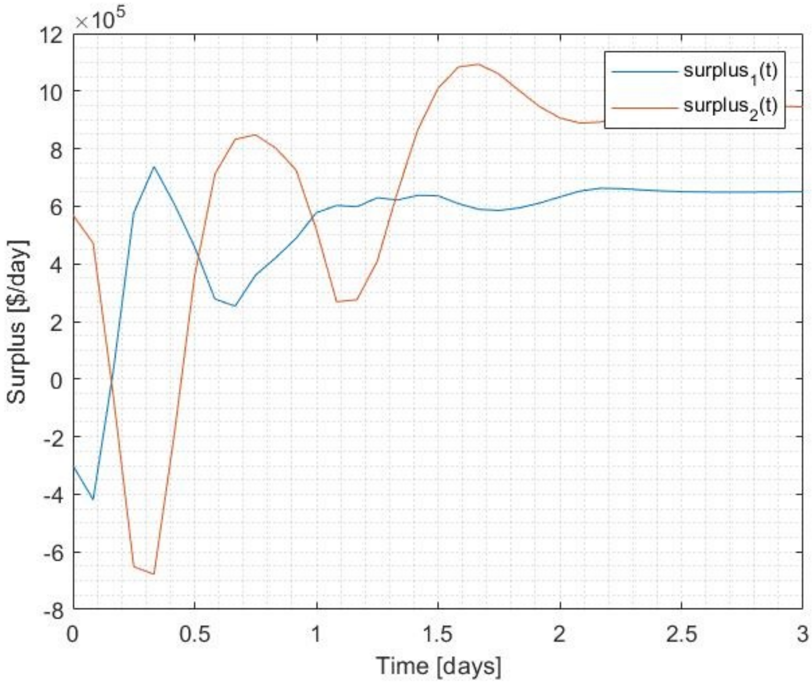


Figure 4-7: Instantaneous surpluses for both suppliers. Supplier 2 (red) eventually generates a higher instantaneous surplus after incurring large initial losses. Stabilization occurs after 2 days.

Instantaneous surplus

The instantaneous surplus can be calculated as the negative of the objective function. It is shown for both suppliers in Figure 4-7. Again, some sort of out-of-phase behaviour seems to be present, but after about 2 days it stabilizes. It can be seen that supplier 2 generates a higher profit, which is mostly caused by the higher demand offset, i.e., $Q_{D_2,o} > Q_{D_1,o}$, as shown in Table 4-1. However, it can also be seen that the second supplier incurred large losses at first (around 0.3 days). This happened because he was massively producing, but the low market price of his competitor (Figure 4-2) led to a scenario with very little demand for the supplier's own product (Figure 4-4).

4-4 Cooperation

In the previous Section, a competitive scenario has been described. In some cases however, it could be possible - and even beneficial - for companies to work together. This could happen e.g., when two companies merge, or when a large hedge fund buys two competitors. In this cooperative scenario, one controller (one company) would be able to control all supplier inputs.

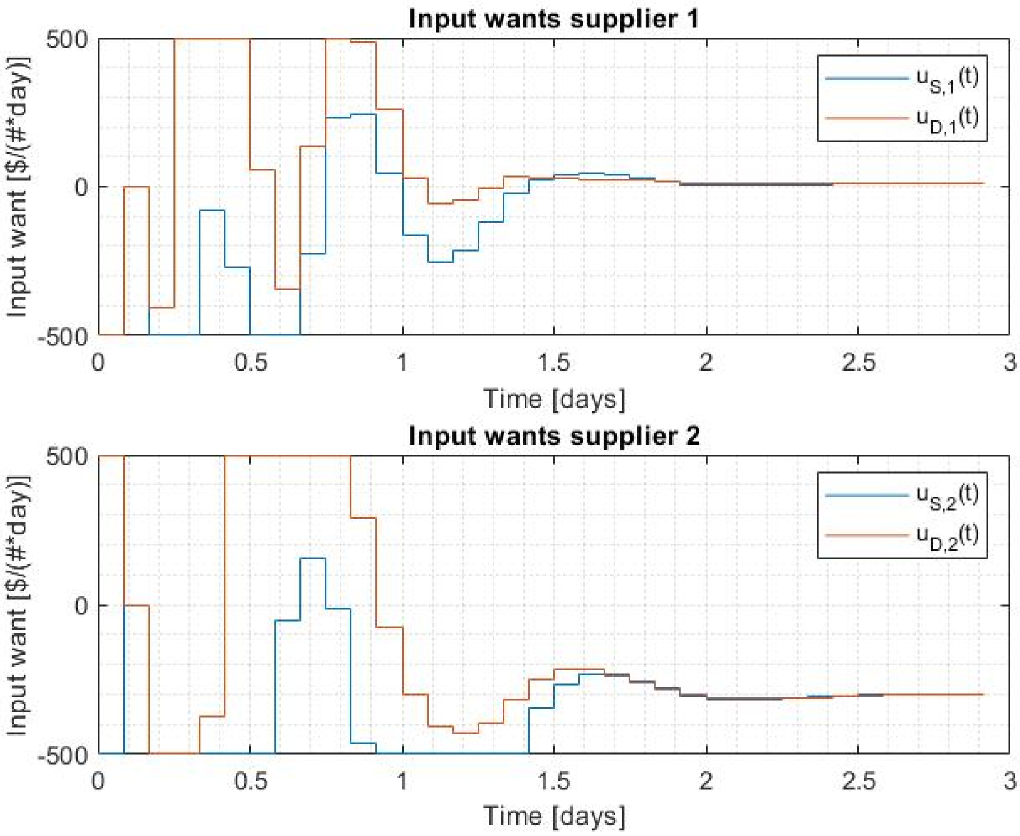


Figure 4-8: Supply input wants ($u_{S,1}(t)$ & $u_{S,2}(t)$, in blue) and market input wants ($u_{D,1}(t)$ & $u_{D,2}(t)$, in red) again reach their limits, but stabilize after 2 days - quicker than in the competitive scenario.

4-4-1 Time domain simulation results

Inputs and prices

The results of the model simulations in a cooperative scenario are shown in Figures 4-8 to 4-14. Figure 4-8 shows that again the input wants reach their limits. However, this happens for a shorter period of time compared to the competition scenario, and stabilization takes place half a day quicker. Whereas in the competitive scenario both pairs of input wants still were around 40% of their maximum value in steady-state, under cooperation the second company applies more negative input wants, while the input wants of company one become close to zero.

Figure 4-9 shows the price dynamics under cooperation and compares them to the competitive scenario. In both cases, supply prices are higher under competition and market prices are lower. This confirms that as described in Section 2-5, consumers are better off under competition. In the cooperative scenario, gross profit margin have increased for both companies, to respectively 75% and 70%. Additionally, price fluctuations have decreased with cooperation, and stabilization happens sooner.

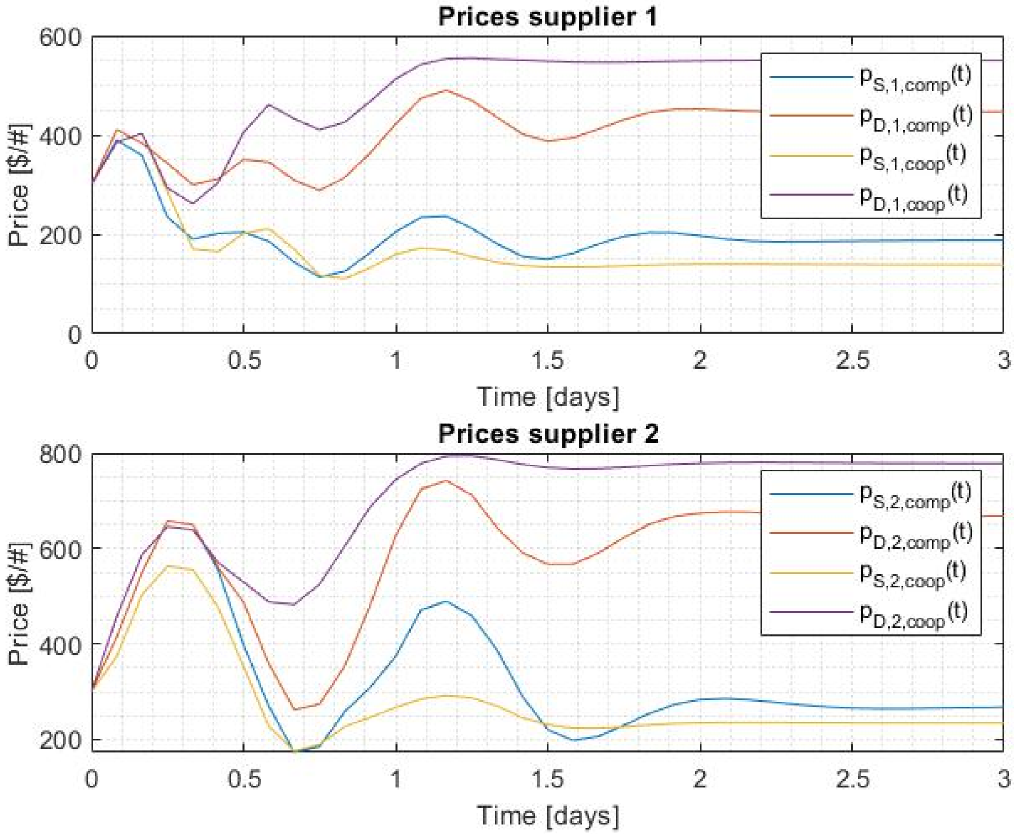


Figure 4-9: Supply prices (in blue for competition and in yellow for cooperation) and market prices (in red for competition and in magenta for cooperation). Stabilization occurs after 2 days. Supply prices have fallen under cooperation, whereas market prices have risen. Market prices are respectively 4 and 3.33 times as high as supply prices, indicating 75% and 70% gross profit margins - respectively 17 and 10 percentage points higher compared to the competitive scenario.

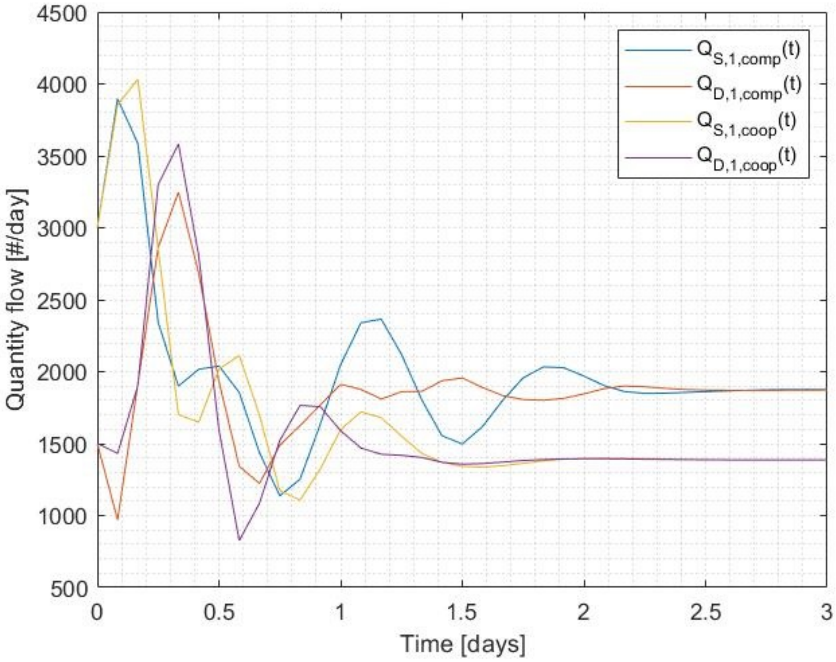


Figure 4-10: Supply flow (yellow) and demand flow (magenta) under cooperation for supplier 1. Supply flow and demand flow under competition is displayed for comparison in respectively blue and red. Stabilization occurs a day earlier (after 1.5 days) at 1,385 #/day under cooperation, 26% lower compared to competition.

Supply and demand

As a direct result of the lower supply prices and the higher market prices, quantity flows for both products go down when the companies work together. This is shown in Figures 4-10 and 4-11. Again, fluctuations have decreased and stabilization occurs almost a day sooner. Similar effects can be seen in the excesses demand (Figure 4-12) and the backlog levels (Figure 4-13). Out-of-phase behaviour still occurs, but to a lesser extent.

Instantaneous surplus

The instantaneous surplus in Figure 4-14 takes on a roughly similar shape compared to the competition scenario. The second company has an initial period of losses (although this time the losses are smaller), but eventually the supplier becomes more profitable than before. Stabilization happens earlier, and company one stabilizes at roughly the same surplus level. Section 4-6 describes how the cost of competition can be derived from these surplus figures.

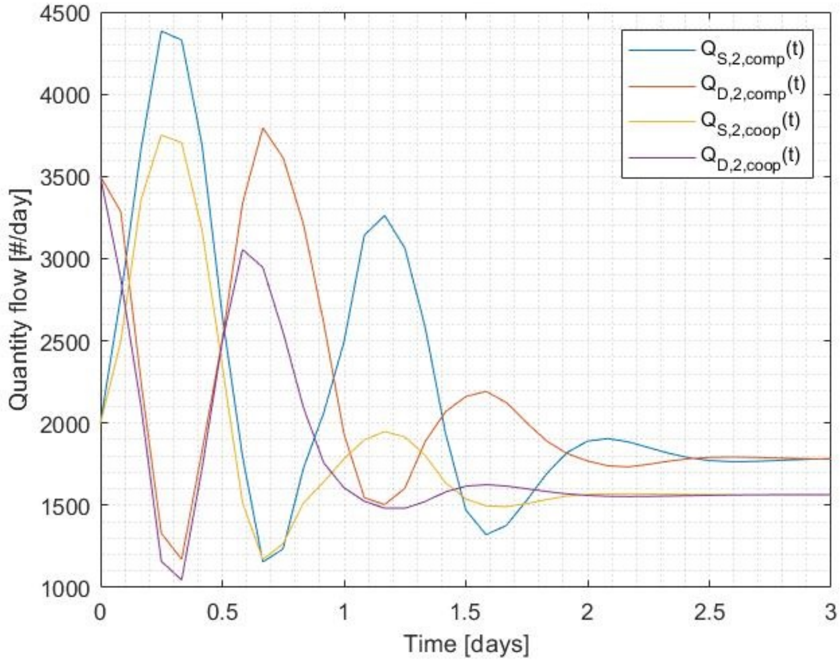


Figure 4-11: Supply flow (yellow) and demand flow (magenta) under cooperation for supplier 2. Supply flow and demand flow under competition is displayed for comparison in respectively blue and red. Stabilization occurs half a day earlier (after 2 days) at 1,565 #/day under cooperation, 11% lower compared to competition.

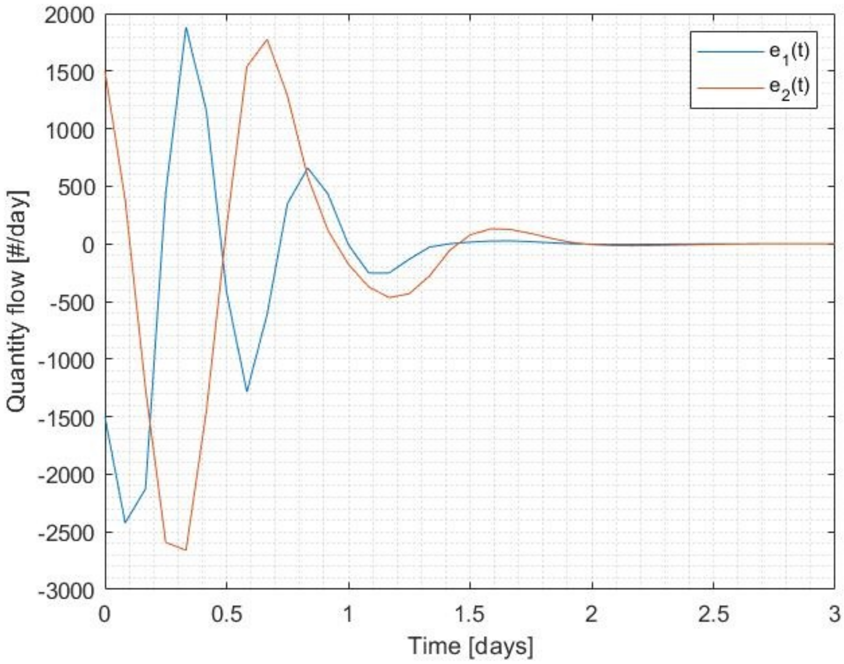


Figure 4-12: Excesses demand for both suppliers go to zero and fluctuate less compared to competition. Stabilization occurs a day sooner after 1.5 days.

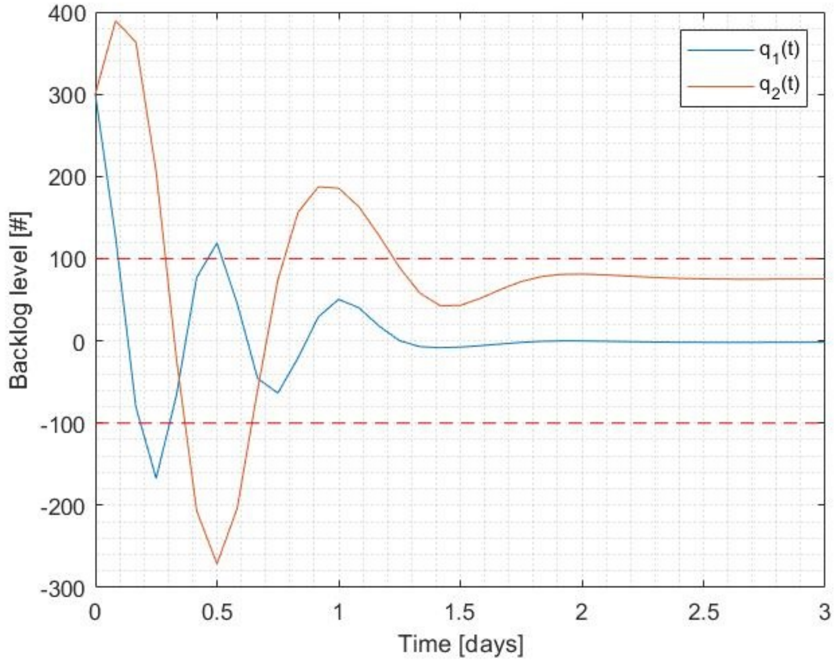


Figure 4-13: Backlog levels again stabilize and stay within their terminal bounds after 1.5 days. Compared to competition backlog fluctuations have slightly decreased under cooperation.

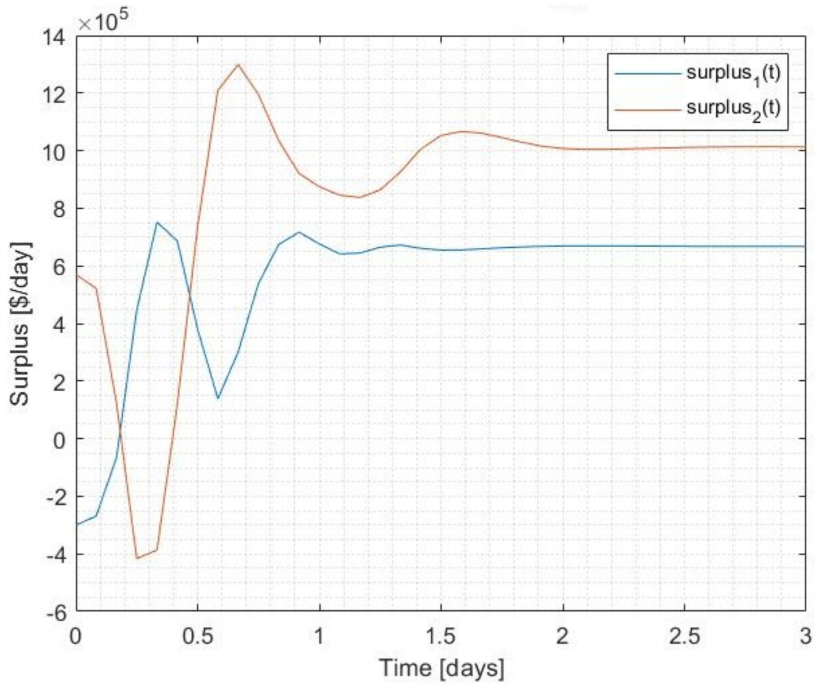


Figure 4-14: Instantaneous surpluses for both suppliers. Supplier 2 (in red) eventually generates a higher instantaneous surplus after incurring large initial losses. The initial losses are lower than under competition. Both suppliers, but especially supplier 2, generate more instantaneous surplus under cooperation.

4-5 Determining the Net Present Value (NPV) with the Laplace Transform

So far, all simulations have been in the time domain, resulting in time-domain variables and signals, such as the prices and the surpluses. Surpluses, or profits, can be used to calculate the value of (part of) a company. One of the most common methods for valuation is the Net Present Value (NPV) method [82].

The NPV method applies *discounting* to all (expected) future cash flows, in order to assign a value to a company. This value depends on the *discount rate* r as follows:

$$\text{NPV} = \sum_{t=0}^{T} \frac{CF_n}{(1+r)^n} \tag{4-1}$$

This reflects that money now (assuming a positive interest rate) is worth more than money at a later time. The discount rate reflects the cost of borrowing money or the return expected by investors, based on the risk involved in the project or company to which valuation is applied. Assuming that cash flows are constant and continue to infinity, the value of this so-called *perpetuity* can be calculated by using the properties of a geometric series. Assuming zero cash flows at $t = 0$, Eq. (4-1) can be rewritten as:

$$\text{NPV} = \frac{CF}{(1+r)} + \frac{CF}{(1+r)^2} + \cdots + \frac{CF}{(1+r)^T} \tag{4-2}$$

Subsequently, multiplying or dividing by $(1+r)$ and subtracting one from the other eventually leads to the perpetuity formula:

$$\text{NPV} = \frac{CF}{r} \tag{4-3}$$

Within Economic Engineering, the value of a business is calculated in the frequency domain. This is done by using the Laplace transform - for which, in contrast to the typical NPV calculations, standard solutions exist - to calculate the NPV from a series of cash flows [83] as follows:

$$\text{NPV}(s) = \mathcal{L}\{C(t)\} = \int_0^\infty C(t)e^{-st}dt \tag{4-4}$$

$C(t)$ represents a cash flow signal in the time domain, and s is expressed as:

$$s = \sigma + i\omega \tag{4-5}$$

wherein σ is the real component of s and therefore represents the discount rate. ω is the complex and periodic element and describes the frequency of business cycles or a form of volatility.

Since the system is discretized, as described in Section 3-4-3, the Z-transform can be used instead of the Laplace transform, leading to:

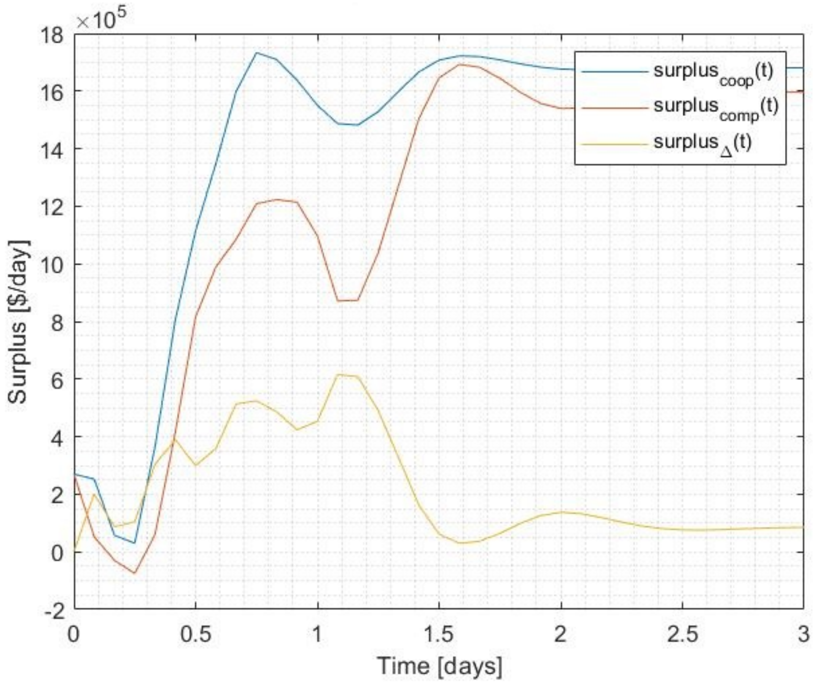


Figure 4-15: Total instantaneous surpluses. Under cooperation (in blue) it has increased by roughly 5% compared to competition (in red). The difference is shown in yellow.

$$\mathrm{NPV}[z] = \mathcal{Z}\{C[k]\} = \sum_{k=0}^{\infty} C[k] z^{-n} \tag{4-6}$$

in which case it takes on a form similar to Eq. (4-1).

4-6 Determining the Cost of Competition

Figure 4-15 displays the summed surpluses in a competitive and a cooperative scenario, together with their difference. It can be seen that with cooperation, the initial period of losses (jointly) has disappeared. In steady-state, the total surplus has also increased. The difference is plotted in yellow, and shown separately in Figure 4-16. It is highest in the beginning, reaching values upward of 600,000 $/day, and eventually stabilizes around roughly 80,000 $/day.

This difference can be interpreted as the *cost of competition* in the time domain. Put differently, by not cooperating, companies jointly miss out on over 80,000 $ of profits per day.

Using the formulas in the previous Section in combination with the time-domain instantaneous surplus signals, it now becomes possible to valuate the individual companies and this cost of competition. Table 4-3 shows the valuations of both suppliers in the competitive and cooperative scenario, based on a discount rate of 5% with no business cycles ($\omega = 0$).

The total value has increased in the cooperative scenario, meaning that the cost of competition, i.e., the evaporated value due to competition, can now be quantified. It can be calculated

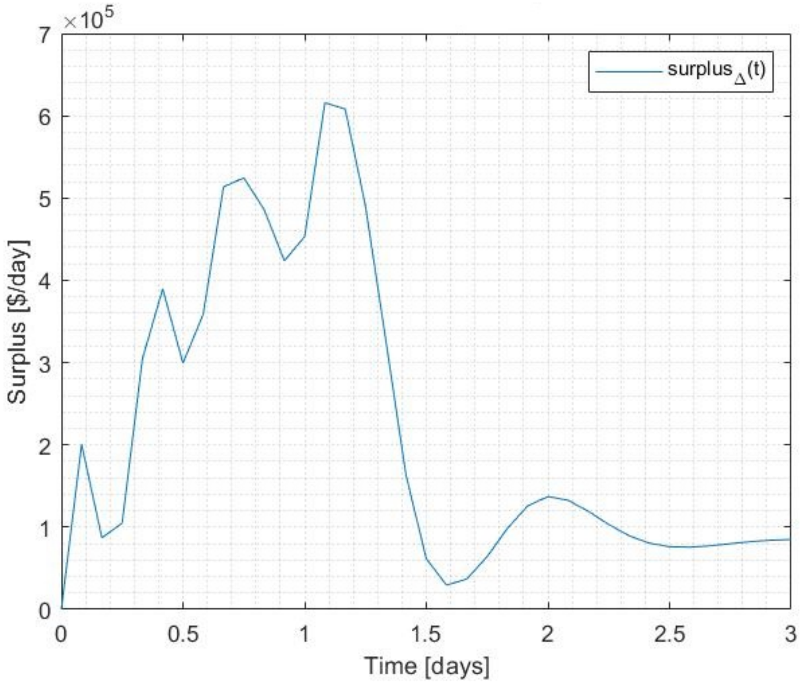


Figure 4-16: Cost of competition in the time domain. It stabilizes around 80,000 $/day.

Table 4-3: Company valuations - both individual and total - under competition (left) and cooperation (right). Both individual valuations have increased under cooperation - especially the valuation of company 2 - and the total has increased by 5.3%: the cost of competition.

Scenario	Competition	Cooperation
Company 1	4,749M $	4,871M $
Company 2	6,900M $	7,398M $
Total	11,649M $	12,269M $

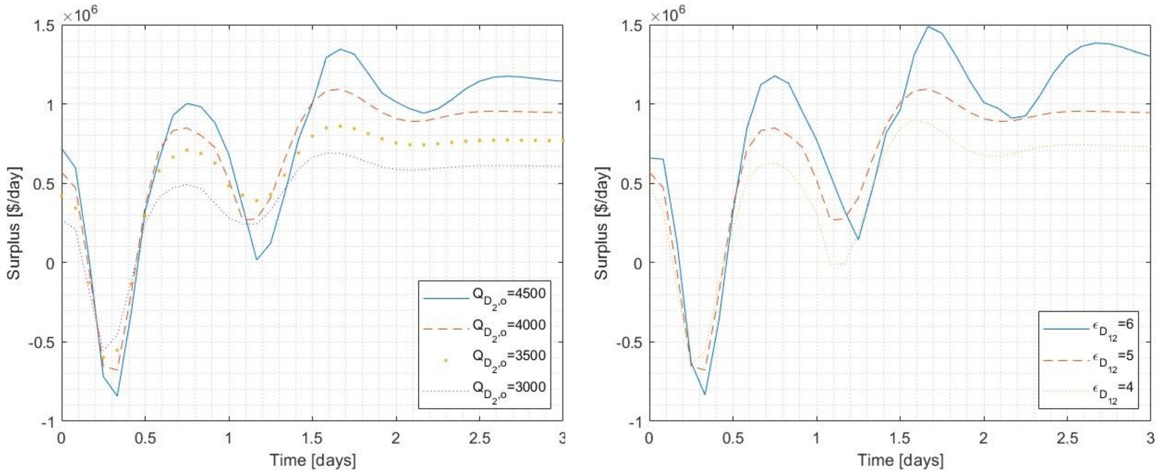
by taking the difference between the two totals, leading to a (negative) valuation of 621M $. It should be noted however, that this number is very sensitive to changes in the discount rate, as is shown in Table 4-4.

These findings confirm the classical economic findings described before, which state that with more competition, profits go down. In this case, the evaporated value due to competition is thus 621M $.

Put differently, in this case (based on the total valuations), hedge funds or M&A departments can make a 5.3% Return on Investment (ROI) by acquiring both companies and merging them.

Table 4-4: Cost of competition at different discount rates. The cost of competition value is very sensitive to changes in the discount rate - doubling the discount rate will approximately halve the cost of competition value.

Discount rate	Cost of competition value
2.5%	1,242M \$
5%	621M \$
10%	310M \$



(a) Changing $Q_{D_2,o}$

(b) Changing $\varepsilon_{D_{12}}$

Figure 4-17: Effect of changing parameter values. Increasing $Q_{D_2,o}$ or $\varepsilon_{D_{12}}$ will increase the instantaneous surplus - and thereby also the NPV - of the second supplier.

4-7 Scenario Analysis in the Time and Frequency Domain

4-7-1 Parametric uncertainty

As described before, the parameters used in this thesis are illustrative and not based on system identification. The simulations are highly dependent on these parameters, and changing them can thus significantly affect the results, as is shown in this Section. Specifically, the effects of changing parameters on the surplus of an individual supplier in the competition scenario is shown.

Figure 4-17a shows the changes in surplus for the second supplier for different values of $Q_{D_2,o}$. It can be seen that increasing it leads to higher steady-state values. Due to the properties of the perpetuity formula Eq. (4-3), NPV scales almost linearly with the surplus, and as such increasing $Q_{D_2,o}$ has a positive effect on the second supplier's NPV. The fluctuations are however also increasing, meaning the initial losses are also larger. The first supplier's steady-state values have increased as well, although to a lesser extent.

Figure 4-17a shows that increasing the cross elasticity of demand $\varepsilon_{D_{12}}$ has a similar effect. It increases both the steady-state surplus and the fluctuations, but also the first supplier's surplus now increases significantly.

On a final note, the system is simulated using the parameters described in Table 4-1. Although it is stable now, e.g., significantly increasing $\varepsilon_{D_{12}}$ could render the system unstable. Consequently, additional stability analysis and system identification are required, as is mentioned in Section 6.

4-7-2 Bode plots for assessing the effect of demand cycles on market prices

Bode plots are a popular tool within control engineering to visualize the frequency response of a dynamic system. A bode plot shows how a sinusoidal input on the form $u = \sin(\omega t)$ propagates through a system, i.e., how it affects the output $y = y_0 \sin(\omega t + \phi)$. It uses a transfer function of the following form to do this:

$$H(s) = \frac{P(s)}{Q(s)} \tag{4-7}$$

in which $Q(s)$ represents the Laplace transform of an input and $P(s)$ represents the Laplace transform of an output. With $s = j\omega$, the magnitude or gain is defined as $y_0 = |H(j\omega)|$ and the phase shift as the argument $\phi = \angle H(j\omega)$ [84]. It can be calculated from the state-space equations as:

$$H(s) = C(sI - A)^{-1}B + D \tag{4-8}$$

Within Economic Engineering, it has also been used to perform scenario analysis in the frequency domain [85, 86]. In this Section, it is used to show how inputs from the measured disturbance $Q_{D_{1,o}}$ propagate to the output states $p_{D,1}$ and $p_{D,2}$. Put differently, it shows the *liquidity* between an input and an output signal.

In this thesis, the measured demand disturbances are assumed to be constant. Consequently, the bode plots in this Section show the effect on market prices of inaccurately estimating or ignoring demand cycles in the disturbance.

Figures 4-18 and 4-19 show the two bode plots. It should be noted that these are the cleaned-up versions, i.e., terms like $7e^{-15}$ that are caused by numerical computation errors are thus dropped. The two Bode plots (assuming illustrative parameters) are then based on the following transfer functions:

$$H_1(s) = \frac{6.667s^4 + 355.6s^2}{s^6 + 186.7s^4 + 6444s^2} \tag{4-9}$$

$$H_2(s) = \frac{133.3s^2}{s^6 + 186.7s^4 + 6444s^2} \tag{4-10}$$

The poles and zeros can be deducted from these, which are shown in Tables 4-5 and 4-6. Since the transfer functions both have the same denominator, they have the same (6) poles. The complex pole pairs at $\pm 6.7604i$ and $\pm 11.8742i$ cause the two sharp peaks in both Bode plots, and a 180° phase shift. Pole-zero cancellation occurs for both transfer functions with

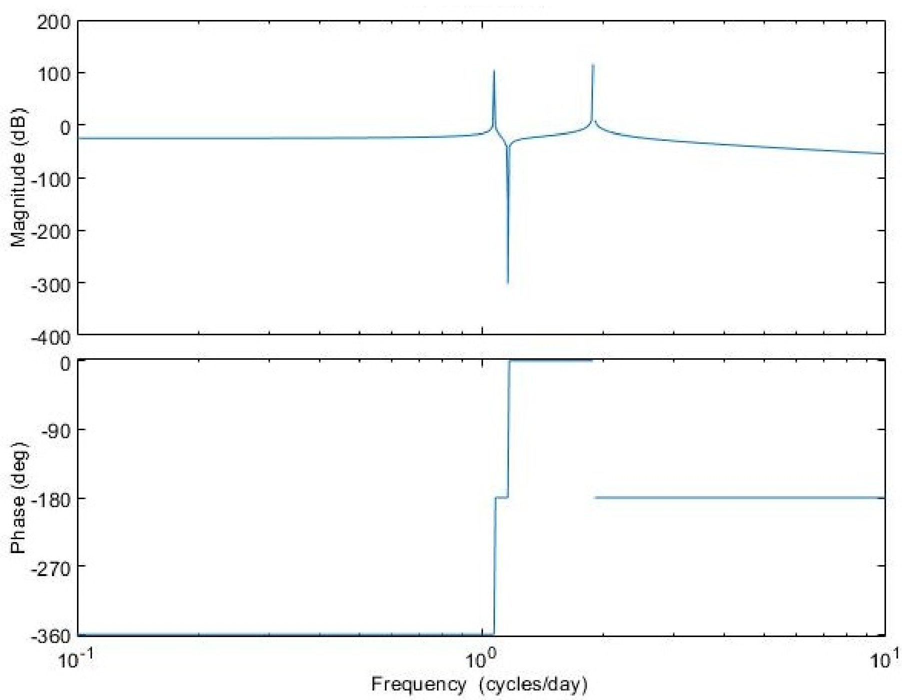


Figure 4-18: Bode plot: demand offset input $Q_{D_1,o}$ to market price 1 output $p_{D,1}$. Overlooking demand cycles will impact $p_{D,1}$ most for cycle frequencies of 1.08 or 1.89 cycles per day.

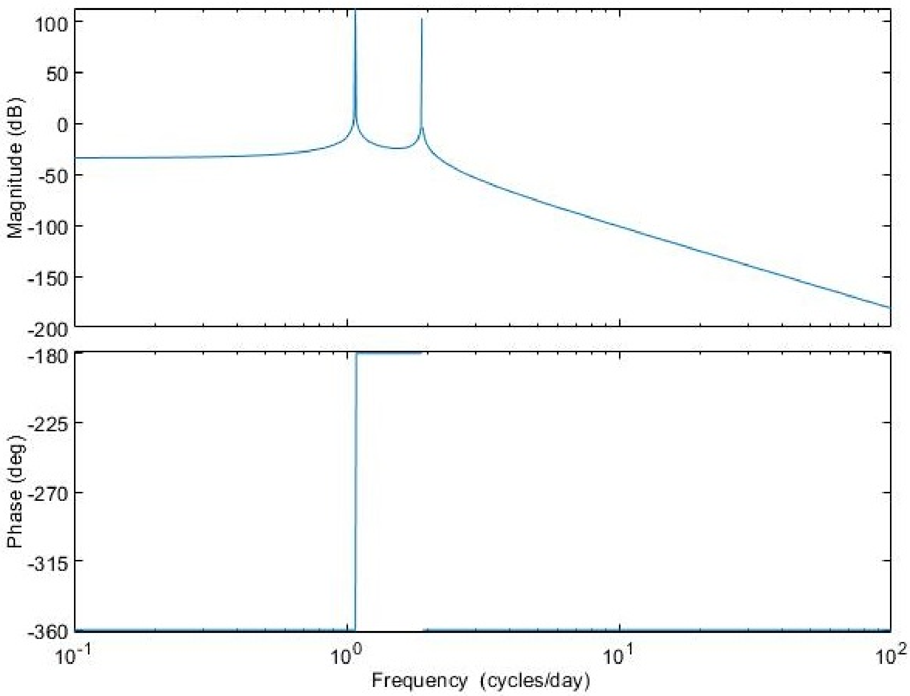


Figure 4-19: Bode plot: demand offset input $Q_{D_1,o}$ to market price 2 output $p_{D,2}$. Overlooking demand cycles will impact $p_{D,2}$ most for cycle frequencies of 1.08 or 1.89 cycles per day.

Table 4-5: Poles and zeros $H_1(s)$ - demand offset input $Q_{D_1,o}$ to market price 1 output $p_{D,1}$.

Poles (6)	Zeros (4)
0 (2x)	0 (2x)
±11.8742i	±7.3032i
±6.7604i	

Table 4-6: Poles and zeros $H_2(s)$ - demand offset input $Q_{D_1,o}$ to market price 2 output $p_{D,2}$.

Poles (6)	Zeros (2)
0 (2x)	0 (2x)
±11.8742i	
±6.7604i	

the double poles and zeros at the origin, and finally the complex zero pair at $\pm 7.3032i$ causes the sharp drop in $|H_1(j\omega)|$.

Consequently, overlooking demand cycles is most consequential to market prices at the peaks where the cycle frequencies are approximately 1.08 and 1.89 cycles per day. Additionally, there is a roll-off for higher frequencies in the second Bode plot, due to the additional complex pole pair. Note that the frequency for the Bode plots is in cycles per day and not in radians per day, hence there is a factor 2π difference between the cycle lengths and the complex parts of the poles and zeros. Although Bode plot are a useful tool, system identification is still required to draw conclusions based on actual identified parameters, as is recommended in Section 6-2.

4-8 Conclusions

Running simulations as is done in this Chapter allows regulators and hedge funds to quantify the effects of competition. In the time domain, regulators are able to model how much market prices will drop under competition and what the effect of that drop on demand flows will be. Hedge funds and Mergers & Acquisitions (M&A) departments can use the frequency-domain Laplace transform to calculate how much value can be added by merging two companies.

In contrast to existing competition models, the simulations are able to show the price dynamics of the system. Additionally, the resulting decrease in both profits and prices - which means consumers are better off - in competitive scenarios is in line with classical economic findings.

Chapter 5

Conclusions

In this thesis, Economic-Engineering theory is applied to model the dynamic aspects of competition with control engineering tools. Currently existing competition models are either not quantifiable or do not include price dynamics. This thesis shows that by modeling a competitive market with bond graphs, the price dynamics of competition can be added to the model. The inclusion of price dynamics into the competition model developed in this thesis sets it apart from existing models and makes it possible for regulators and hedge funds to dynamically simulate competitive behaviour.

In this thesis, it is demonstrated how a two-supplier competitive scenario can be simulated. The model can be extended to more suppliers by expanding the bond graph with additional subsystems, although this would increase the complexity of the model.

The model parameters need to be estimated through system identification in order to model real-world scenarios. In contrast to existing data-driven models, the model parameters in this thesis have an economic interpretation. This will assist in the identification effort, since the economic interpretations of the parameters clarify what data exactly is needed (e.g., price elasticities, storage costs) to identify the parameter values.

This thesis additionally shows how Economic Model-Predictive Control (MPC) is a useful tool to model company decision making, assuming a company's only objective is profit maximization. In a scenario where profit maximization is not the only objective of a company - e.g., when a company wants to concurrently maximize both its market share and its profits, multi-objective Model-Predictive Control (MPC) can be included in the model to simulate company decision making under internally conflicting objectives. This thesis has only considered simultaneous moves; a Stackelberg model can additionally be developed to simulate company decision making with sequential moves. This would make it possible to model the presence of first-moving dominant companies in a competitive market.

With the model developed in this thesis, regulators can make informed decisions on e.g., blocking mergers and acquisitions or putting additional regulations into place, since they can use the model to simulate how market prices and demand flows would change as a result of an increase or decrease in competition. On the other hand, hedge funds and Mergers &

Acquisitions (M&A) departments can use this model to quantify the effects of competition on company valuations. With this model, they are able to simulate how a merger between two companies would affect the price dynamics in a competitive market and they can subsequently quantify by how much the combined valuation of the two companies would increase after a merger.

Chapter 6

Recommendations

Various aspects of the model could still be further improved or adjusted, since this thesis has only made a first effort of using Economic Engineering to model competitive behaviour. Section 6-1 presents an overview of several concepts that could be included in a future version of the model. Section 6-2 subsequently elaborates on how system identification can then be used to simulate real-life scenarios.

6-1 Model Extensions

6-1-1 Adding more companies

As mentioned in Section 3-3, the model is modular, meaning that additional suppliers can be added in a straightforward manner. Each additional competitor would bring an extra 3 states, 2 inputs, and 1 measured disturbance to the model. The dimensions of the mass tensor - which is used to model price-demand relationships - would increase by 1.

This modular way of modeling performs well in oligopoly scenarios, i.e., with only a few companies present in the system. However, when the number of companies grows, the order of the model and the computational complexity of the optimization grow as well. In such a situation, one supplier subsystem can be used to model the rest of the market, i.e., several smaller competitors grouped together.

6-1-2 Advertising

In the model used in this thesis, suppliers can change the market price of their product through their market input wants $u_D(t)$. The price of a product is an element of what marketers refer to as the *marketing mix*. This comprises everything a company can do to convince (potential) customers to buy their product or service, and can be described with the "4 Ps" of marketing: price, product, place, and promotion [87].

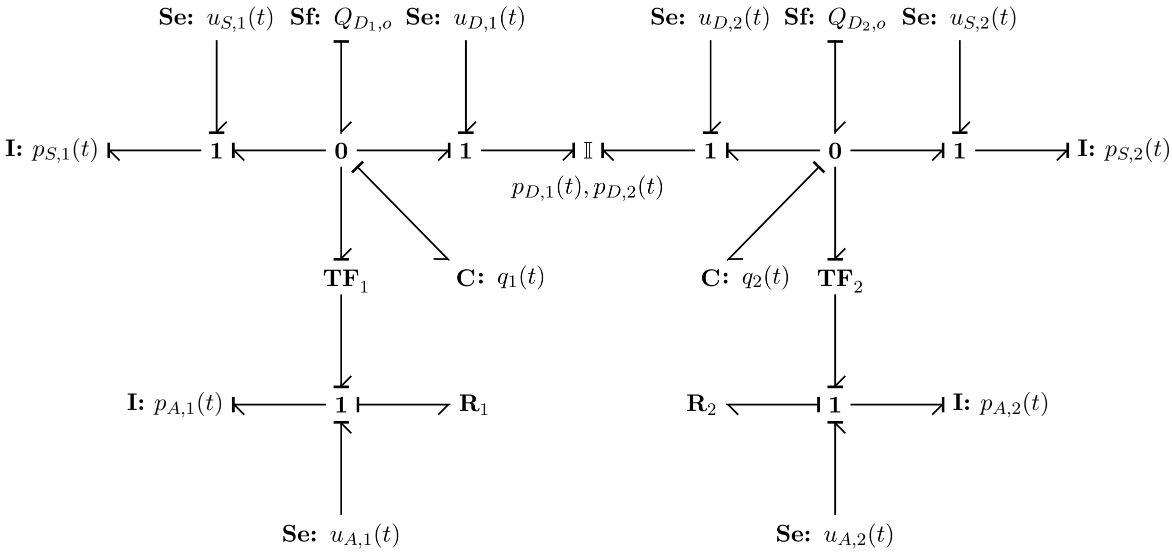


Figure 6-1: Bond graph model extension: advertisements as a separate product (with price $p_A(t)$) affecting the demand curve through a transformer.

Besides the "price" element, "product" is obviously also included in the model. It is captured by the (cross) elasticity parameters and the multi-port I-element. Since this thesis does not study the distribution or transport of products, the "place" element is ignored.

This leaves "promotion" as the obvious choice to extend the model with. Promotion is related to how e.g., advertising can affect the perceived quality of a product, and thereby either shift or rotate the demand curves. The exact effect however is specific to e.g., the product, the industry or the type of good under consideration [88]. To add the effect of advertising to the model, advertisements can be modeled as a separate product (with its own price $p_A(t)$) in which suppliers can invest. Through a *transformer* it affects the rest of the system, including the price dynamics and the demand flows. This could take on the form shown in Figure 6-1.

6-1-3 Modeling uncertainty

In the current model, the suppliers have knowledge of all the states of the system. They do not know how the competition is going to act based on those states and their profit maximization optimal control problem. This is currently the only uncertainty for a supplier, which is not very realistic.

Additionally, (unmeasured) noise could be added to the measured disturbances (the demand offsets) or to the inputs. Economically, this could represent e.g., inaccurate estimates of the current demand on the supplier side. To simulate this, Monte Carlo simulations can be used to run a variety of scenarios under uncertainty, leading to a wide range of possible outcomes. Stochastic or robust Model-Predictive Control (MPC) can also be used to incorporate these uncertainties directly into the optimal control problems of the suppliers [89].

Additionally, the discounting of future cash flows could be taken into account in the objective functions, leading to weighted objective functions, as described in Section 3-4-1. Since further out cash flows are more prone to estimation errors and other uncertainties, they can be given less weight compared to profits in the nearby future.

6-1-4 Revenue or market share optimization

As described in Section 3-4-1, this thesis assumes that suppliers' only objective is to maximize their profits. However, in reality companies might care about additional objectives - such as market share or revenues - as well. Multi-objective MPC can be used to optimize for several objectives at once, by assigning a weight to each of them.

Alternatively, one or more suppliers could initially optimize for market share or revenues to push their competition out of the market. They might even be willing to operate at a significant loss for a certain period of time. When their competition has accrued a certain level of losses and is assumed to be bankrupt, a company could switch to optimizing its profits again, which are likely to be higher in this newly created monopoly scenario. A hybrid control algorithm can be used to switch objectives based on the competitor's current situation.

6-1-5 Sequential moves

The model developed in this thesis is based on the game-theoretic Bertrand competition model. In the basic model, both suppliers simultaneously make their decisions. In game theory (as described in Section 2-4-3), the Stackelberg model extension assigns a dominant status to one of the companies, allowing them to make the first move. The other company can then observe that move and react to it. In this way, sequential moves can be modeled, and the first company (typically one with a large market share) will benefit from a significant first-mover advantage. By adjusting the optimal control algorithms, this can be included in the model described in this thesis as well.

6-2 System Identification

The parameters used in this thesis are purely illustrative, and enable performing the simulations shown in Chapter 4. In order to actually use the model on a real-life scenario, (grey box) *system identification* will need to be performed. However, this is outside the scope of this thesis. In order to estimate the parameters of the model in an accurate way, data on Mergers & Acquisitions (M&A) will be required. This data is confidential in a lot of cases, making system identification more difficult. Once identified, a real-life case study can be done and additional stability analysis can also be performed on the model, including or excluding aforementioned model extensions. In the model described in this thesis, e.g., a different sampling time T_s can have a significant impact of the stability of the system, which could then be studied in more depth.

Appendix A

Using Economic-Engineering Analogies to Model Supply and Demand

This Section first describes how Economic Engineering can be used to build price dynamics into an economic model. This is also done in Chapter 3, where a dynamic model of a competitive market is developed. First, Economic Engineering is described in Section A-1. Its analogs are introduced and a basic example illustrates how these analogs can be used to model a simple economic system with two different modeling methods.

Section A-2 subsequently shows that the mechanism of supply and demand can be modeled as a closed-loop system, by introducing a model that has been developed within the Economic Engineering group. This is also used to further illustrate the modeling methods described in Section A-1. More information on the background of Economic Engineering can be found in Appendix B-1.

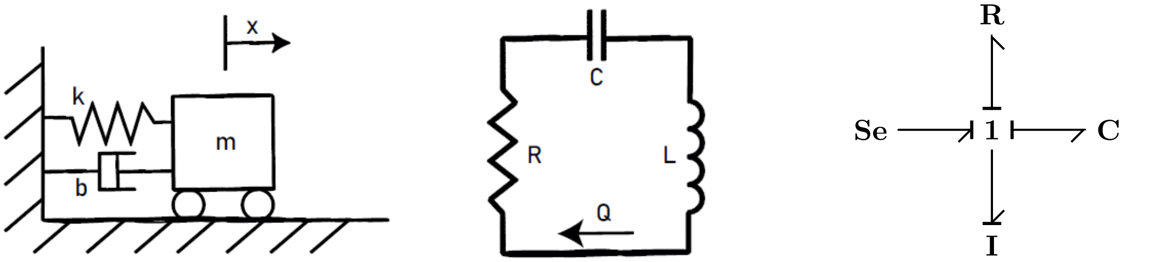
A-1 Economic Engineering Modeling Methods

The Economic-Engineering modeling framework has been developed by Mendel [90], within the Economic Engineering research group at the Delft Center for Systems and Control (DCSC). It is an experimental approach, largely based on the idea of formulating analogs, based on physical engineering domains, for economic variables. Consequently, economic systems and mechanisms can be modeled, analyzed and controlled similarly to e.g., mechanical or electrical systems, since the economic analogs describe systems that behave similarly to their physical counterparts. Moreover, it gives control engineers a more intuitive interpretation of economic variables and systems.

In contrast to most economic research, this allows one to include price dynamics and causality in an economic model. Similar attempts to model economic systems with engineering analogies have been made, but the chosen analogies differ, which result in fundamental differences between the various approaches. These attempts include the works of Franksen [91], Brewer

Table A-1: Physical and economic domain analogs.

General	Mechanics		Electronics		Economics	
	Variable	Unit	Variable	Unit	Variable	Unit
Effort (e)	Force (F)	N	Voltage (U)	V	Cost ($\dot{p}$)	\$/#·yr
Flow (f)	Velocity (v)	m/s	Current (I)	A	Flow ($\dot{q}$)	#/yr
Momentum (p)	Momentum (p)	Ns	Flux linkage (λ)	Vs	Price (p)	\$/#
Displacement (q)	Displacement (x)	m	Charge (Q)	C	Assets (q)	#
Power (P)	$F(t)v(t)$	Nm/s	$U(t)I(t)$	VA	Growth (G)	\$/yr^2
Energy (E)	$\int Fdx \mid \int vdp$	Nm	$\int edQ \mid \int Id\lambda$	J	Cash flow (CF)	\$/yr



(a) Mass-spring-damper system **(b)** Resistor-capacitor-inductor circuit **(c)** Bond graph of basic systems

Figure A-1: Analogous basic physical systems [3].

[92, 93, 94], and Machado [95]. Due to the fundamental differences, other attempts are hereafter not discussed in detail. Solely the work of Mendel is used in this study. Several analogs in the physical and economic domains are shown in Table A-1.

A commonly used method within engineering for modeling dynamical systems is bond graph modeling. This method has also repeatedly been used within the Economic Engineering research group, in particular due to its domain neutrality [96, 85, 97]. Bond graphs give a graphical and intuitive representation of a dynamical system, and can be used to model different domains (e.g., mechanical, electrical) and their interconnections. As such, it is a convenient and widely-used tool for Economic Engineering, which uses analogs of these different physical domains to describe economic systems. Once the bond graph model has been constructed, equations of motion can be derived, and the system - regardless of its physical domain, can be represented in state-space form. This familiar state-space representation then allows for controls to be applied to the system [98, 99].

To illustrate this, consider that basic systems like the mechanical mass-spring-damper system in Figure A-1a and the electrical resistor-capacitor-inductor circuit in Figure A-1b will have analogous state-space representations. Figure A-1c shows the corresponding bond graph model, which describes both systems. When using domain-neutral variables, these two models can be described by the following state-space model:

$$x = \begin{bmatrix} q \\ p \end{bmatrix} \tag{A-1}$$

$$\dot{x} = Ax + Bu = \begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{I} \\ -\frac{1}{C} & -\frac{R}{I} \end{bmatrix} \begin{bmatrix} q \\ p \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} S_e(t) \tag{A-2}$$

In Eq. (A-1), the state variable q refers to either the *displacement* or the *charge*, and p represents the *momentum* or the *flux linkage*.

By modeling economic systems similarly to mechanical or electrical systems, *price dynamics* are automatically included in the model. In the economic domain - as can be seen in Table A-1, q represents a number of assets or goods, and p is the price. The price dynamics then describe how the momentum (or flux linkage) changes over time.

The C-element in Figure A-1c (a spring or capacitor), accumulates a flow and acts as a store of assets q in the economic domain. It subsequently returns an effort (a force or voltage), which corresponds to the storage costs. The I-element (a mass or inductor) accumulates an effort and stores the other state variable - p. It then returns a flow (a velocity or a current), which is analogous to a quantity demanded or supplied. It can thus be used to model the law of supply and the law of demand. The R-element in Figure A-1c (a damper or a resistor) induces costs - like transport costs - that depend on the flow (of goods). Different R-elements can also be used to model consumption. The effort source S_e (an external force or a voltage source) adds *consumer want* to the system. Finally, the 1-junction (a shared velocity or electrical series connection) represents *market clearing*, i.e., no excess goods are left on the market, which means that the supply equals the demand. More information on bond graph modeling and its relation to Economic Engineering can be found in Appendix B-2.

The next Section will provide a more elaborate example of a closed-loop economic system, which is modeled both with a bond graph and with a block diagram, which has the advantage of being more graphically intuitive than a bond graph. For the development of the new time-domain competition model, which is proposed in Chapter 3, only bond graphs will be used to add price dynamics to the existing game theory competition models, because of the advantages described before.

A-2 Supply and Demand as a Control System

Within the Economic Engineering group, the supply and demand mechanism has been modeled as a closed-loop dynamical system [4]. The corresponding block diagram representation can be seen in Figure A-2. It is similar to a 1-supplier Bertrand competition model, i.e., the Bertrand competition model applied to a monopoly scenario, as described in Sections 2-2-1 and 2-4-3.

The error going into the controller is the difference between the reference signal - the quantity demanded $Q_D(t)$, and the quantity supplied $Q_S(t)$ - the output. It thus represents the *excess demand*. Since the reference signal is not a part of the process, the relationship between the price $p(t)$ and the quantity demanded $Q_D(t)$ can be nonlinear. The control input resulting from the error signal can then be interpreted as the Economic-Engineering analog of a force, i.e., a rate of change in price. This control input will be positive (negative) for a positive (negative) error signal, leading to an increase (decrease) in price $p(t)$, which will lower (increase) the quantity demanded and increase (lower) the quantity supplied, thereby

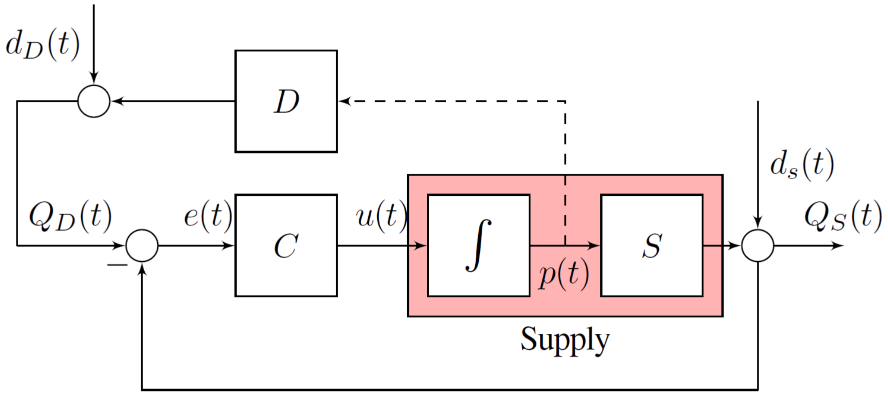


Figure A-2: Closed-loop supply and demand model in block diagram form [4].

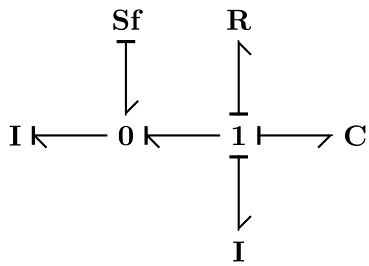


Figure A-3: Basic supply and demand bond graph model.

pushing the error signal towards zero. The variables $d_s(t)$ and $d_D(t)$ are respectively output and reference disturbances and can be used to model supply and demand shocks.

Hutters and Mendel [4] have subsequently also shown how a PID-controller can be interpreted economically within this model. The proportional element represents a direct *adjustment* of the rate of change in price. The integral part accumulates the excess demand into a *backlog*, which is a measure of scarcity. Finally, the derivative part represents *speculation* and is based on the rate of change in the error signal, i.e., the excess demand.

Figure A-3 shows how this system can be modeled as a bond graph. The modulated flow source $S_f(t)$ then represents the quantity demanded (the reference signal) and the I-element returns the quantity supplied. Based on the difference between the two, the R-, C-, and anti-causal I-element - respectively representing the proportional, integral, and derivative elements of a PID-controller - return an effort (a rate of change in price) which pushes the error to zero.

Appendix B

Economic Engineering Background

B-1 Economic Engineering Analogs

The analogs used within Economic Engineering have been introduced in Section A-1. This Section will provide the background of these analogs and the relationships between them.

Effort and flow

The effort e and flow f are known as *power variables*, since their product equals the instantaneous power flowing in a part of the system:

$$P(t) = e(t)f(t) \tag{B-1}$$

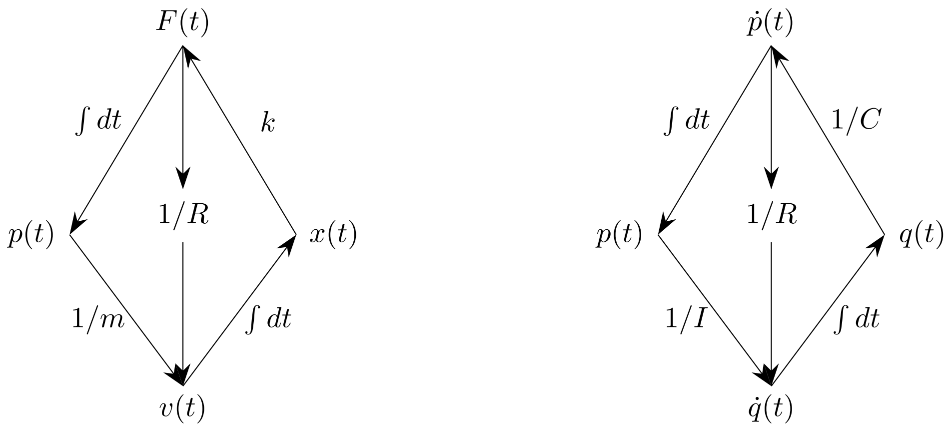
In the economic domain, the effort and the flow represent a *cost* or a *benefit*, and a *flow of goods* respectively. Thus, their product has units of *growth* [\$/yr^2].

Momentum and displacement

The momentum p and displacement q are also known as *energy variables* and represent respectively a *price* or an *asset position* in the economic domain. They are related to the effort and flow as follows:

$$p(t) \equiv \int^t e(t)dt = p_0 + \int_{t_0}^t e(t)dt \tag{B-2}$$

$$q(t) \equiv \int^t f(t)dt = q_0 + \int_{t_0}^t f(t)dt \tag{B-3}$$



(a) Mechanical domain **(b)** Economic domain

Figure B-1: Tetrahedrons of state.

in which p_0 and q_0 are respectively the initial momentum and the initial displacement at time t_0. The energy - or *cash flow* - is calculated as the time integral of power:

$$E(t) \equiv \int^t P(t)dt = \int^t e(t)f(t)dt \tag{B-4}$$

or alternatively as:

$$E(t) = \int^t e(t)dq(t) = \int^t f(t)dp(t) \tag{B-5}$$

Tetrahedron of state

The relationships between the power variables and the energy variables can be seen in the so called *tetrahedron of state*, which is shown in Figure B-1 for the mechanical and economic domains. Additional parameters represent resistance, compliance and inertia, which is discussed in Appendix B-2-2.

By using a system's energy and power (energy flow), its behavior can be modeled. A graphically intuitive method for modeling a system's behavior is *bond graph modeling*, which is described in Appendix B-2.

B-2 Bond Graphs

Bond graphs model the flow of energy within a system, i.e., the power, by using so called *bonds*. Several standard elements are used for connecting these bonds. The inputs and outputs of the system are respectively called *sources* and *sinks*. These are referred to as *active elements*. A system can contain three different kinds of *passive elements*. These elements can model resistance, compliance or inertia. All of these active and passive elements are *single-port elements*. *Two-port elements* consist of two types, *transformers* and *gyrators*, and they can

(a) Flow source **(b)** Effort source **(c)** Flow sink **(d)** Effort sink

Figure B-2: Bond graph sources and sinks.

convert energy, while conserving power. Finally, *0-junctions* and *1-junctions* can have multiple ports and divide the power across them. All these elements are discussed below and their relation to common components of different physical systems is described.

B-2-1 Sources and sinks

Sources are single-port and active elements that put energy into the system. Since there are two power variables, there also exist two different types of sources; *flow sources* and *effort sources*. A flow source, denoted by S_f, puts energy into the system with a specific flow value. The corresponding value of the effort depends on the connected system. For an ideal flow source, this value is independent of the system. In the mechanical domain, a body with a constant velocity (e.g., a car), can be modeled with a flow source. A current source is the analogy in the electrical domain. Analogously, in the economic domain a flow source represents the production of goods, which are introduced into the system.
An effort source is denoted by S_e. In contrast to a flow source, this element puts in energy with a specific effort value. The corresponding flow value then depends on the connected system, and in the case of an ideal effort source, the set effort value is independent of the system. An external force applied to the system or a voltage source are both examples of effort sources. In the economic domain, an effort source can represent an added benefit or cost.
Finally, *flow sinks* and *effort sinks* take energy out of the system, respectively in the form of a set flow or effort. The different types of sources and sinks can be seen in Figure B-2.
In bond graphs, *half-arrows* define the positive direction of the energy flow, which can be assigned arbitrarily. For sources, this direction is typically away from the element, while the half-arrow is pointing towards the element in the case of sinks. Causality strokes indicate the causality between the flow and the effort, i.e., whether an element with an incoming effort generates a delayed flow or vice versa. The flow is defined on the side of the causality stroke, while the effort is defined on the other side. The flow source and flow sink thus have the element on the side of the causality stroke, while the effort source and effort sink elements are opposite the causality stroke.

B-2-2 Passive elements

Three different types of passive elements are used in bond graph modeling. Similar to sources and sinks, these are also single-port elements. Resistance is represented by the R-element, and can be modeled in two different ways. This is shown in Figure B-3a and Figure B-3b. Both elements dissipate energy. Consequently, the half-arrows are always pointing in the direction of the element. Dampers and resistors are typical examples in the mechanical and electrical domain. In the economic domain, resistance can be used to model consumption or expenses. For both R-elements a static relationship between the incoming or outgoing flow or effort exists, but the two differ in their causality. Assuming linearity, this relationship for the R-element displayed in Figure B-3a is of the form:

$$e = \Phi_R(f) = Rf \tag{B-6}$$

Consequently, the rate at which the energy of the system is being dissipated, or the power, is given by:

$$P = Rf^2 \tag{B-7}$$

This is how a damper or a shock absorber can be modeled, which converts kinetic energy into heat. The linear relationship between effort and flow of the R-element in Figure B-3b is of the form:

$$f = \Phi_R^{-1}(e) = \frac{1}{R}e \tag{B-8}$$

In the electrical domain, this is equivalent to Ohm's law, with can be described mathematically as $I = \frac{V}{R}$. The value of the R-element represents the proportionality of the current to the voltage. The power dissipation in this type of R-element is then given by:

$$P = Rf^2 = \frac{1}{R}e^2 \tag{B-9}$$

The C-element models compliance or capacitance. It is shown in Figure B-3c. It acts as a store of potential energy, and the half-arrow is directed into the element. Capacitors and springs are typical examples within the electrical and mechanical domain, and within the economic domain it represents an inventory, i.e., a storage of assets. C-elements have a static relationship between its effort and displacement. Assuming linearity, this relationship is of the form:

$$e = \Phi_C(q) = \frac{1}{C}\int f\ dt = \frac{1}{C}q \tag{B-10}$$

For a spring, $k = \frac{1}{C}$, wherein k represents the spring constant or its stiffness. Combined with Eq. (B-10), this leads to $e = kq$, such that k indicates the proportionality between the spring extension and the force, formalized by Hooke's law.

Finally, inertia can be modeled with an I-element, as shown in Figure B-3d. The I-element stores kinetic energy and similar to the other passive elements, the positive direction of energy flow is into the element. An inductance or a mass can be modeled with an I-element, which has a static relationship between its flow and momentum. Within economic engineering, this element is used to model market mechanisms; incoming costs are accumulated into a price, which determines e.g., the quantity demanded. It also models the law of supply, i.e., at a higher price, the quantity supplied will be higher as well. With assumed linearity, this relationship takes the following form:

$$f = \Phi_I(p) = \frac{1}{I}\int e\ dt = \frac{1}{I}p \tag{B-11}$$

(a) Resistance 1 **(b)** Resistance 2 **(c)** Compliance **(d)** Inertia

Figure B-3: Passive bond graph elements.

For the example of a mass, with $m = I$, Eq. (B-11) can then be rewritten as $f = \frac{1}{m}p$, such that $\frac{1}{m}$ indicates how the momentum of the mass relates to its velocity.

B-2-3 Two-port elements

Transformers and gyrators are two-port elements. They can convert the energy they receive in one port to the other, while conserving power. Both a transformer and a gyrator can be used to connect different physical or economic domains in one model. The different types of transformers and gyrators can be seen in Figure B-4. Since power is conserved (for ideal transformers or gyrators), both obey the energy balance:

$$e_1 f_1 = e_2 f_2 \tag{B-12}$$

A transformer applies a relationship between the two efforts, and between the two flows. Typical examples of transformers are electrical transformers in the electrical domain or levers in the mechanical domain. There are two possible causality assignments, as shown in Figure B-4a and Figure B-4b. The reason for this is that when an effort or a flow in set as an input, respectively the other effort or flow has to be an output. For the first transformer, the relationship is of the following form:

$$e_1 = re_2, \qquad f_2 = rf_1 \tag{B-13}$$

in which r represent a parameter known as the *transformer modulus*. Rearranging and multiplying the incoming and outgoing power variables shows that Eq. (B-12) is satisfied. Consequently, the relationship for the second transformer, as in Figure B-4b, can be defined as:

$$f_1 = \frac{f_2}{r}, \qquad e_2 = \frac{e_1}{r} \tag{B-14}$$

Whereas a transformer applies a relationship between the same type of power variables, a gyrator does the opposite. It thus applies a relationship between the incoming effort and the outgoing flow, and between the incoming flow and the outgoing effort. An example is a DC motor, in which an input effort (voltage) is converted to an output flow (angular velocity). Similarly to transformers, two versions exist, due to two different causality assignments. The gyrator shown in Figure B-4c applies the following relationship:

$$e_1 = gf_2, \qquad e_2 = gf_1 \tag{B-15}$$

(a) Transformer 1 **(b)** Transformer 2 **(c)** Gyrator 1 **(d)** Gyrator 2

Figure B-4: Bond graph transformers and gyrators.

(a) 0-junction **(b)** 1-junction

Figure B-5: Bond graph junctions.

in which g represents a parameter known as the *gyrator modulus*. Rearranging and multiplying the incoming and outgoing power variables again shows that Eq. (B-12) is satisfied. Subsequently, the relationship for the second gyrator, as in Figure B-4d, can be defined as:

$$f_1 = \frac{e_2}{g}, \qquad f_2 = \frac{e_1}{g} \tag{B-16}$$

B-2-4 Junctions

In contrast to the elements mentioned before, 0-junctions and 1-junctions can have more than two ports, and they can divide the power across them. They can be used to interconnect multiple subsystems and they represent the law of conservation of energy in bond graph models. For a 3-port junction, this implies:

$$e_1 f_1 + e_2 f_2 + e_3 f_3 = 0 \tag{B-17}$$

A 0-junction, or *common effort junction*, can be seen in Figure B-5a. The efforts on all the connected bonds are equal, and the flows sum to zero. In the electrical domain, this resembles a node with a shared voltage across all its components, and in the mechanical domain this can be thought of as a connection between system components where all components experience an equal force. In the economic domain, it represents the *conservation of money*. For this 0-junction, this can be described as:

$$e_1 = e_2 = e_3 \tag{B-18}$$

$$f_1 + f_2 + f_3 = 0 \tag{B-19}$$

Since efforts are equal in a 0-junction, only one bond can cause this effort. Consequently, for a n-port 0-junction, only 1 causality stroke is adjacent to the element.
Figure B-5b displays a 1-junction, or *common flow junction*. The flows on all the connected bonds are equal, and the efforts sum to zero. This is similar to a series connection in an electrical circuit or a shared velocity in mechanics. In the economic domain, it represents

market clearing, i.e., the supply equals the demand and no excess goods are left. The 1-junction in Figure B-5b can be described as:

$$f_1 = f_2 = f_3 \tag{B-20}$$

$$e_1 + e_2 + e_3 = 0 \tag{B-21}$$

Since flows are equal in a 1-junction, only one bond can cause this flow. Thus, for a n-port 1-junction, $n - 1$ causality strokes are adjacent to the element.

Appendix C

Model Derivations

The complete bond graph model has been introduced in Chapter 3. In this Chapter, the equations of motion will be derived.

C-1 Equations of Motion Derivation

$$f_1 = f_2 = f_3 \tag{C-1}$$

$$e_2 = e_1 + e_3 \tag{C-2}$$

$$e_1 = u_{S,1} \tag{C-3}$$

$$f_2 = \frac{1}{I_{S,1}} \int e_2 \; dt = \varepsilon_{S_1} p_{S,1} \tag{C-4}$$

$$e_3 = e_4 = e_5 = e_6 \tag{C-5}$$

$$f_4 = f_3 + f_5 + f_6 \tag{C-6}$$

$$f_4 = Q_{D_1,o} \tag{C-7}$$

$$e_5 = \frac{1}{C_1} \int f_5 \; dt = \frac{1}{C_1} q_1 \tag{C-8}$$

$$f_6 = f_7 = f_8 \tag{C-9}$$

$$e_8 = e_6 + e_7 \tag{C-10}$$

$$e_7 = u_{D,1} \tag{C-11}$$

$$f_8 = \frac{1}{\mathbb{I}_{1,1}} \int e_8 \; dt - \frac{1}{\mathbb{I}_{1,2}} \int e_9 \; dt = \frac{1}{\mathbb{I}_{1,1}} p_{D,1} - \frac{1}{\mathbb{I}_{1,2}} p_{D,2} = \varepsilon_{D_{11}} p_{D,1} - \varepsilon_{D_{12}} p_{D,2} \tag{C-12}$$

$$f_9 = -\frac{1}{\mathbb{I}_{2,1}} \int e_8 \; dt + \frac{1}{\mathbb{I}_{2,2}} \int e_9 \; dt = -\frac{1}{\mathbb{I}_{2,1}} p_{D,1} + \frac{1}{\mathbb{I}_{2,2}} p_{D,2} = -\varepsilon_{D_{21}} p_{D,1} + \varepsilon_{D_{22}} p_{D,2} \tag{C-13}$$

$$f_9 = f_{10} = f_{11} \tag{C-14}$$

$$e_9 = e_{10} + e_{11} \tag{C-15}$$

$$e_{10} = u_{D,2} \tag{C-16}$$

$$e_{11} = e_{12} = e_{13} = e_{14} \tag{C-17}$$

$$f_{12} = f_{11} + f_{13} + f_{14} \tag{C-18}$$

$$f_{12} = Q_{D_2,o} \tag{C-19}$$

$$e_{13} = \frac{1}{C_2} \int f_{13} \; dt = \frac{1}{C_2} q_2 \tag{C-20}$$

$$f_{14} = f_{15} = f_{16} \tag{C-21}$$

$$e_{16} = e_{14} + e_{15} \tag{C-22}$$

$$e_{15} = u_{S,2} \tag{C-23}$$

$$f_{16} = \frac{1}{I_{S,2}} \int e_{16} \; dt = \varepsilon_{S_2} p_{S,2} \tag{C-24}$$

C-2 State-space Representation

Combining Eq. (C-2), Eq. (C-3), Eq. (C-4), Eq. (C-5) and Eq. (C-8) leads to:

$$\dot{p}_{S,1} = e_2 = e_1 + e_3 = e_1 + e_5 = \frac{1}{C_1} q_1 + u_{S,1} \tag{C-25}$$

Combining Eq. (C-17), Eq. (C-20), Eq. (C-22), Eq. (C-23) and Eq. (C-24) leads to:

$$\dot{p}_{S,2} = e_{16} = e_{14} + e_{15} = e_{13} + e_{15} = \frac{1}{C_2} q_2 + u_{S,2} \tag{C-26}$$

Combining Eq. (C-5), Eq. (C-8), Eq. (C-10), Eq. (C-11) and Eq. (C-12) leads to:

$$\dot{p}_{D,1} = e_8 = e_6 + e_7 = e_5 + e_7 = \frac{1}{C_1} q_1 + u_{D,1} \tag{C-27}$$

Combining Eq. (C-13), Eq. (C-15), Eq. (C-16), Eq. (C-17) and Eq. (C-20) leads to:

$$\dot{p}_{D,2} = e_9 = e_{10} + e_{11} = e_{10} + e_{13} = \frac{1}{C_2} q_2 + u_{D,2} \tag{C-28}$$

Combining Eq. (C-1), Eq. (C-4), Eq. (C-6), Eq. (C-7), Eq. (C-8), Eq. (C-9) and Eq. (C-12) leads to:

$$\dot{q}_1 = f_5 = f_4 - f_3 - f_6 = f_4 - f_2 - f_8 = -\varepsilon_{S_1} p_{S,1} - \varepsilon_{D_{11}} p_{D,1} + \varepsilon_{D_{12}} p_{D,2} + Q_{D_1,o} \tag{C-29}$$

Combining Eq. (C-13), Eq. (C-14), Eq. (C-18), Eq. (C-19), Eq. (C-20), Eq. (C-21) and Eq. (C-24) leads to:

$$\dot{q}_2 = f_{13} = f_{12} - f_{11} - f_{14} = f_{12} - f_9 - f_{16} = -\varepsilon_{S_2} p_{S,2} + \varepsilon_{D_{21}} p_{D,1} - \varepsilon_{D_{22}} p_{D,2} + Q_{D_2,o} \tag{C-30}$$

Rearranging leads to:

$$\dot{x} = Ax + Bu + Ed = \begin{bmatrix} \dot{p}_{S,1} \\ \dot{p}_{S,2} \\ \dot{p}_{D,1} \\ \dot{p}_{D,2} \\ \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{1}{C_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{C_2} \\ 0 & 0 & 0 & 0 & \frac{1}{C_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{C_2} \\ -\varepsilon_{S_1} & 0 & -\varepsilon_{D_{11}} & \varepsilon_{D_{12}} & 0 & 0 \\ 0 & -\varepsilon_{S_2} & \varepsilon_{D_{21}} & -\varepsilon_{D_{22}} & 0 & 0 \end{bmatrix} \begin{bmatrix} p_{S,1} \\ p_{S,2} \\ p_{D,1} \\ p_{D,2} \\ q_1 \\ q_2 \end{bmatrix} +$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{S,1} \\ u_{S,2} \\ u_{D,1} \\ u_{D,2} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} Q_{D_1,o} \\ Q_{D_2,o} \end{bmatrix} \tag{C-31}$$

Appendix D

MATLAB Code

```
yalmip('clear')
clear all

tic

% Data
C1 = 0.15;
C2 = 0.25;
Is1 = 0.1;
Is2 = 0.15;
Id11 = 0.1;
Id22 = 0.15;
Id12 = 0.2;
Id21 = Id12;
Qdo1 = 3000;
Qdo2 = 4000;

% Definition of the LTI system
A = [0 0 0 0 1/C1 0; 0 0 0 0 0 1/C2; 0 0 0 0 1/C1 0; 0 0 0 0 0 1/C2; -1/
    Is1 0 -1/Id11 1/Id12 0 0; 0 -1/Is2 1/Id21 -1/Id22 0 0];
B = [1 0 0 0;0 1 0 0; 0 0 1 0;0 0 0 1; 0 0 0 0; 0 0 0 0];
E = [0;0;0;0;Qdo1;Qdo2];
x0 = [300;300;300;300;300;300];
Ts = 1/12;
M = 5;

% Definition of system dimensions
nx = 6;
nu = 2;
N = 5;
timespan = 36;

% Definition of quadratic cost functions --> max vs. min, so "-" added
    everywhere
```

```
Q1 = [0.5/Is1 0 0 0 0 0; 0 0 0 0 0 0; 0 0 1/Id11 -1/Id12 0 0; 0 0 0 0 0 0; 0 0 0 0 0.5/C1 0; 0 0 0 0 0 0];
P1 = [0 0 -Qdo1 0 0 0];
Q2 = [0 0 0 0 0 0; 0 0.5/Is2 0 0 0 0; 0 0 0 0 0 0; 0 0 -1/Id21 1/Id22 0 0; 0 0 0 0 0 0; 0 0 0 0 0 0.5/C2];
P2 = [0 0 0 -Qdo2 0 0];
Rpen = 0.1;
R = [Rpen 0; 0 Rpen]; % penalize input wants

% Variables
u1 = sdpvar(repmat(nu,1,N),repmat(1,1,N)); % N matrices size nu x 1
u2 = sdpvar(repmat(nu,1,N),repmat(1,1,N));
x = sdpvar(repmat(nx,1,N+1),repmat(1,1,N+1));

% Constraints and objective functions
constraint_terminal = 1; % on
terminal_level = 100;
max_input = 500;
constraints1 = [];
objective1 = 0;
constraints2 = [];
objective2 = 0;
for k = 1:N
 objective1 = objective1 + x{k}'*Q1*x{k} + P1*x{k} + u1{k}'*R*u1{k};
 constraints1 = [constraints1, x{k+1} == fDT(x{k},[u1{k}(1);0;u1{k}(2);0],Ts,M,A,B,E)];
 constraints1 = [constraints1, -max_input <= u1{k}<= max_input];
    if k == N && constraint_terminal == 1
     constraints1 = [constraints1, -terminal_level <= x{k+1}(5)<= terminal_level];
    end
end
for k = 1:N
 objective2 = objective2 + x{k}'*Q2*x{k} + P2*x{k} + u2{k}'*R*u2{k};
 constraints2 = [constraints2, x{k+1} == fDT(x{k},[0;u2{k}(1);0;u2{k}(2)],Ts,M,A,B,E)];
 constraints2 = [constraints2, -max_input <= u2{k}<= max_input];
    if k == N && constraint_terminal == 1
     constraints2 = [constraints2, -terminal_level <= x{k+1}(6)<= terminal_level];
    end
end

ops = [sdpsettings('solver','gurobi','gurobi.NonConvex',2)];
controller1 = optimizer(constraints1,objective1,ops,x{1},[u1{:}]);
controller2 = optimizer(constraints2,objective2,ops,x{1},[u2{:}]);

x = x0;
implementedU = [];
states = [x];
for i = 1:timespan
  U1 = controller1{x};
  U2 = controller2{x};
```

```
  x = fDT(x,[U1(1,1);U2(1,1);U1(2,1);U2(2,1)],Ts,M,A,B,E);
  implementedU = [implementedU [U1(1,1);U2(1,1);U1(2,1);U2(2,1)]];
  states = [states x];
end

toc

% Functions
function dxdt = fCT(x,u,A,B,E)
    dxdt = A*x + B*u + E;
end

function xk1 = fDT(xk,uk,Ts,M,A,B,E)
delta = Ts/M;
xk1 = xk;
    for j=1:M
        xk1 = xk1 + delta*fCT(xk1,uk,A,B,E);
    end
end
```

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Glossary

List of Acronyms

DCSC	Delft Center for Systems and Control
M&A	Mergers & Acquisitions
MPC	Model-Predictive Control
NPV	Net Present Value
ROI	Return on Investment