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A Human-like Steering Model Sensitive to Uncertainty in the Environment

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Abstract—The interaction between a human driver and an automated driving system may improve when the automation is designed in such a way that it behaves in a human-like manner. This paper introduces a human-like steering model, in which the driver adapts to the risk due to uncertainty in the environment. Current steering models take a risk-neutral approach, while the fields of economics and sensorimotor control suggest that humans exhibit risk-sensitive behavior. The proposed model uses a risk-sensitive optimal feedback control structure to predict steering behavior. The paper studies the effect of the risk-sensitivity parameter and compares the prediction of the risk-neutral and risk-sensitive controllers in a simulated abstraction of two scenarios: (a) driving while being subjected to lateral wind gusts and (b) overtaking an unpredictably swerving car. The simulation results show that the risk-sensitive model adapts to the uncertainty in the environment. Experimental data will be needed to validate the predictions of our model.

I. INTRODUCTION

Automobile manufacturers and tech companies are investing heavily to make autonomous driving a reality. Trust appears to be an important construct that is predictive of whether consumers will want to ride an automated car, e.g., [1]. Hence, understanding how human trust in automated vehicles can be improved has become an important question. At the same time, we need to avoid designs that would lead to over-reliance on automated driving systems, as this is dangerous [2]. Studies show that by anthropomorphizing the technology with human-like characteristics, trust in automation can be improved [3][4]. Human centered automation which uses a model of the human operator as a template to automate tasks, could improve human-automation interaction [5]. The work in this paper takes inspiration from the above mentioned studies and attempts to capture human steering behavior for its use in making the control of (semi)automated vehicles anthropomorphic.

An important characteristic of human driver behavior is the adaptability to different driving situations. For example, there is ample evidence that drivers reduce speed in more complex traffic scenarios [6]. Additionally, most drivers will be familiar with the experience of driving towards the lane center in a filled parking lot, because of the unpredictability of the pedestrians' movements, a phenomenon which was recently quantified in a driving simulator study [7]. Similar is the experience of adopting a large safety margin to an erratically swerving car while overtaking it (Fig. 1). Such adaptations do not depend merely on the likelihood of the presence of the object but also on the statistical properties of its motion. In the

fields of robot motion planning this is known as *uncertainty in environment predictability* [8]. In this paper we aim to capture human steering behavior in the presence of such an uncertain environment.

The modeling of driver steering behavior has received much attention [9]. A large portion of the models in literature are deterministic, meaning that they do not include noise from any source [10][11][12]. Stochastic models incorporate noise that may arise due to an uncertain environment (e.g., bumpy road, wind gusts), sensors and/or actuators, and describe human driving behavior in the presence of variability. There are relatively few stochastic driver models and some of them incorporate additive noise in the predictions of a deterministic driver model [13][14]. These models do not study the effect of disturbances on human behavior, but study how uncertainty in predicting human behavior affects driver assistance systems. Other models incorporate the uncertainty into the parameters of the driver's internal model [15][16]. These stochastic models have provided important insights into the *spread* of driving behavior, but predicted that the average behavior does not change with the amount of uncertainty.

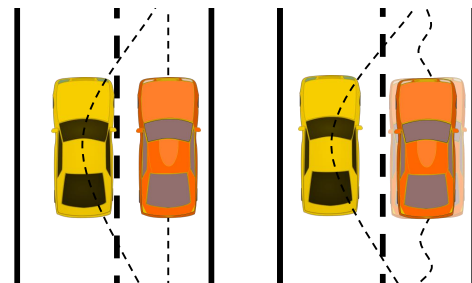


Fig. 1. **Illustration of the effect of environment uncertainty:** The expected difference in desired safety margin when overtaking a vehicle that drives steadily (left), compared to one that is erratically swerving (right).

In short, existing steering models do not account for human-like adaptation to uncertainty in environment predictability (Fig. 1). The field of economics has extensively studied the risk sensitive behavior exhibited by humans while making decisions about lotteries, investments, valuations of assets, etc [17][18]. In these studies subjects had to make discrete decisions. Using a reaching task, Nagengast et al. showed that human sensorimotor control, which is concerned with continuous tasks, is sensitive to risk as well [19]. They observed that, as the amount of uncertainty in the process was increased,

subjects adapted by increasing their effort. Medina et al. used a risk-sensitive controller to provide assistive feedback, that adapted to uncertainties in its predictions of the human operator's actions, in a teleoperation task [20].

In driving, there have been some applications of risk-sensitive controllers. Saito and Raksincharoensak implemented a risk-sensitive controller that decelerated the car in anticipation of an unexpected obstacle appearing suddenly from a poorly visible area [21]. Dunning et al. (2015) studied human response to risk in a simulated driving task and found that while driving on a road that was flanked by water (high danger) on one side and grass (low danger) on the other, lateral positions of the vehicle were biased towards the grass region [22]. That paper elegantly showed that humans are risk aware controllers but did not provide a steering model.

In conclusion, conventional steering models in the literature predict that average human steering behavior does not change with the amount of noise. This is either because these models do not consider the noise at all (deterministic models) or because they minimize the expected value of a cost function. According to the certainty equivalence principle, the optimal control solution obtained by minimizing the expected value of a quadratic cost of a linear system subjected to additive noise is the same as that obtained in the absence of noise. For example, the gains of a LQR (Linear Quadratic Regulator) are the same as those of a LQG (Linear Quadratic Gaussian) controller. However, research in the field of economics and sensorimotor control provides evidence for risk-sensitive human behavior.

Our goal in this paper is to use insights from economics and sensorimotor control to formulate a risk-sensitive steering model that incorporates the ability to adapt to uncertainty in the environment.

II. APPROACH

To incorporate the dependence of controller gains on variance of noise, in addition to the expected value, higher 'moments' (e.g., variance) of the cost function need to be included. Directly including these higher moments increases the non-linearity of the problem. Jacobson (1973) suggested an approach in which he minimized the expected value of the exponent of the cost function (1), and indirectly incorporated higher moments of the cost function [23].

$$J = \underset{\pi}{\operatorname{argmin}} \mathbf{E} \{ \exp[\sigma \mathcal{L}(\pi)] \} \quad (1)$$

- J = Cost function minimized by Jacobson
- \mathbf{E} is the expected value operator
- σ is a real valued parameter
- π is the control policy
- $\mathcal{L}(\pi)$ is quadratic cost function (random variable)

The Taylor expansion of the \log of cost J shows that this cost incorporates higher moments of the cost function (2). \mathcal{L}^* is the cost incurred by optimal policy.

$$\log[J] = \mathbf{E}[\mathcal{L}^*] + \frac{\sigma}{2} \operatorname{Var}[\mathcal{L}^*] + \dots \quad (2)$$

In this paper we will be using Jacobson's approach to design a human driver model that adapts its gains in response to the level of uncertainty.

It is important not to confuse *variance of cost* with *variance of trajectories*, as the risk-neutral controller does minimize the variance of its (input and output) trajectories by minimizing the expected value of the cost function as shown in (3), where $\operatorname{tr}()$ is the *trace* operator.

$$\begin{aligned} \mathbf{E}[\mathcal{L}] &= \mathbf{E} \left[\sum_{i=k}^{k+n} \mathbf{x}_i^T \mathbf{Q}_i \mathbf{x}_i + \mathbf{u}_i^T \mathbf{R}_i \mathbf{u}_i \right] \\ &= \underbrace{\operatorname{tr}(\mathbf{Q}_i \Sigma_x) + \mu_x^T \mathbf{Q}_i \mu_x}_{\text{variance and mean of states}} + \underbrace{\operatorname{tr}(\mathbf{R}_i \Sigma_u) + \mu_u^T \mathbf{Q}_i \mu_u}_{\text{variance and mean of input}} \end{aligned} \quad (3)$$

The risk-sensitive controller, in addition to minimizing the variance of its trajectories, takes into account the variance of cost itself (2).

A. Mathematical formulation

We assume that the vehicle is traveling at a constant speed V while being subjected to an uncertain environment. Here we use the linear dynamic bicycle model, with steering angle u as the input and ε as a gaussian additive noise on the states.

$$\mathbf{x}_{k+1} = \mathbf{A}_k \mathbf{x}_k + \mathbf{B}_k u_k + \mathbf{G}_k \varepsilon_k \quad (4)$$

$$\mathbf{x}_k = \begin{bmatrix} e_k \\ \dot{e}_k \\ \psi_k \\ \dot{\psi}_k \\ 1 \end{bmatrix} \quad \mathbf{B}_k = \begin{bmatrix} 0 \\ a_{15} \Delta t \\ 0 \\ a_{25} \Delta t \\ 0 \end{bmatrix} \quad \mathbf{G}_k = \begin{bmatrix} \Delta t \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \varepsilon_k \sim N(0, s^2)$$

$$\mathbf{A}_k = \begin{bmatrix} 1 & \Delta t & 0 & 0 & 0 \\ 0 & 1 + a_{11} \Delta t & a_{12} \Delta t & a_{13} \Delta t & a_{14} \rho_k V \Delta t \\ 0 & 0 & 1 & \Delta t & 0 \\ 0 & a_{21} \Delta t & a_{22} \Delta t & 1 + a_{23} \Delta t & a_{24} \rho_k V \Delta t \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

$$\begin{aligned} a_{11} &= \frac{-2C_1 - 2C_2}{mV} & a_{12} &= \frac{2C_1 + 2C_2}{m} & a_{13} &= \frac{-2C_1 a + 2C_2 b}{mV} \\ a_{14} &= \frac{-2C_1 a + 2C_2 b}{mV} - V & a_{15} &= \frac{2C_1}{rm} & a_{21} &= \frac{-2C_1 a + 2C_2 b}{I_z V} \\ a_{22} &= \frac{2C_1 - 2C_2 b}{I_z} & a_{23} &= \frac{-2C_1 a^2 - 2C_2 b^2}{I_z V} & a_{25} &= \frac{2C_1 a}{r I_z} \\ a_{24} &= \frac{-2C_1 a^2 - 2C_2 b^2}{I_z} \end{aligned} \quad (6)$$

- CG is the center of gravity
- e is the lateral deviation of vehicle's CG from the lane center [m]
- ψ is the difference between vehicle heading and road heading ($\psi_{car} - \psi_{road}$) [rad]
- u is the steering angle [rad]
- ε is the additive *process* noise (e.g., wind gust) [m]

- s is the standard deviation of ε [m]
- C_1, C_2 are the front and rear tire lateral stiffness, respectively [N/rad]
- a, b are the distances of front and rear axles from vehicle's CG, respectively [m]
- m is the mass of the vehicle [kg]
- I_z is the moment of inertia of the vehicle about yaw axis [kgm²]
- V is the longitudinal vehicle speed [m/s]
- ρ is the road curvature ($\frac{1}{\text{road radius}}$), left turn \rightarrow positive, right turn \rightarrow negative, straight road 0 [m⁻¹]
- Δt is the time step [s]
- r is the steering ratio (steering angle : tire rotation) [-]

The steering model uses road preview up to a distance of d_p [m], in the form of road curvature ρ , from multiple points which are d_s [m] apart (Fig. 2).

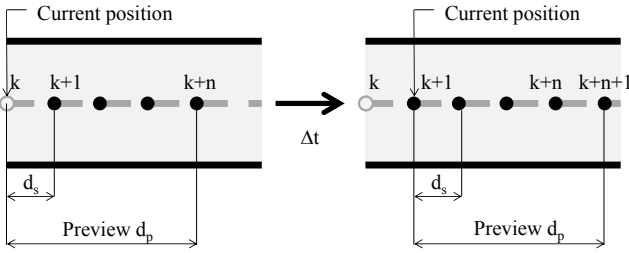


Fig. 2. Black solid lines indicate the lane boundaries, and the grey dashed line marks the lane center. (Left) Road curvature information ($\rho_k \rightarrow \rho_{k+n}$) is previewed by the model for n upcoming time steps ($k \rightarrow k+n$), and used to calculate the optimal control policies $\pi(k \rightarrow k+n)$. Here $d_s = V\Delta t$ and $d_p = Vn\Delta t$. (Right) After 1 time step, in which only the control policy for $\pi(k)$ was implemented, the calculations are repeated to obtain the optimal policy $\pi(k+1 \rightarrow k+n+1)$.

Algorithm 1 Implementation of the risk-sensitive controller

Input: $\mathbf{A}_k, \mathbf{B}_k, \mathbf{G}_k, \mathbf{Q}_k, \mathbf{R}_k, \mathbf{P}_k, \sigma$

Output: \mathbf{K}_k (implement only \mathbf{K}_1)

Initialization : $\mathbf{W}_n = \mathbf{Q}_n$

for $k = n-1$ to 1 do

$$\tilde{\mathbf{W}}_{k+1} = \mathbf{W}_{k+1} + \sigma \mathbf{W}_{k+1} \mathbf{G}_k (\mathbf{P}_k - \sigma \mathbf{G}_k^T \mathbf{W}_{k+1} \mathbf{G}_k)^{-1} \mathbf{G}_k^T \mathbf{W}_{k+1}$$

$$\mathbf{W}_k = \mathbf{Q}_k + \mathbf{A}_k^T \left[\tilde{\mathbf{W}}_{k+1} - \mathbf{W}_{k+1} \mathbf{B}_k (\mathbf{R}_k + \mathbf{B}_k^T \tilde{\mathbf{W}}_{k+1} \mathbf{B}_k)^{-1} \right.$$

$$\left. \mathbf{B}_k^T \tilde{\mathbf{W}}_{k+1} \right] \mathbf{A}_k$$

$$\mathbf{K}_k = (\mathbf{R}_k + \mathbf{B}_k^T \tilde{\mathbf{W}}_{k+1} \mathbf{B}_k)^{-1} \mathbf{B}_k^T \tilde{\mathbf{W}}_{k+1} \mathbf{A}_k$$

end for

The optimal control policy is calculated by minimizing the cost in (1), where \mathcal{L} is quadratic in nature and given by (7). \mathbf{Q}_k and \mathbf{R}_k are positive semi-definite and positive definite matrices, respectively.

$$\mathcal{L} = \sum_{i=k}^{k+n} \mathbf{x}_i^T \mathbf{Q}_i \mathbf{x}_i + \mathbf{u}_i^T \mathbf{R}_i \mathbf{u}_i \quad (7)$$

σ is the real valued risk-sensitivity parameter. As can be seen from (2), $\sigma = 0$ ignores the effect of variance of cost

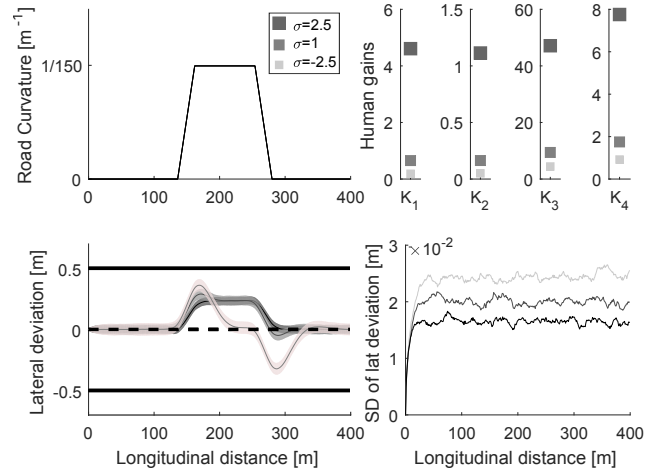


Fig. 3. **Effect of risk-sensitivity (σ):** Three different levels of σ are used to show the risk-averse ($\sigma = 2.5$ and 1) and risk-taking ($\sigma = -2.5$) behavior. *Top-left:* Input to the model is road curvature. *Bottom-left:* Lateral deviation from lane center is shown on a road with effective width ($\text{width}_{road} - \text{width}_{car}$) of 1m using, solid lines (average steering behavior) and bands around them (± 1 standard deviation). *Top-right:* $K_1, K_2, K_3,$ and K_4 are the gains of the controller (corresponding to $e_k, \dot{e}_k, \psi_k, \dot{\psi}_k$, the 4 states of the system (5)). *Bottom-right:* Standard deviation of lateral deviation from lane center is highest for the risk-taking driver, while it decreases as the driver becomes more risk-averse.

and hence makes the controller risk-neutral. Only non-zero values of σ can be chosen for a risk-sensitive controller. Fig. 3 shows the simulation results when three different σ values (-2.5, 1, and 2.5) were used (while keeping all other parameters unchanged; Table I). The proposed steering model drove on a road whose curvature is given in the top-left plot of Fig. 3. As the value of σ increases from -2.5 to 2.5, the driver becomes more risk-averse and the controller gains increase. This indicates that the driver puts in more effort and the trajectories become 'tighter', while exhibiting lower variance in the trajectory (bottom-right plot). In essence, the driver considers the variability to be detrimental to his/her performance and hence puts in more effort to reduce risk. On the other hand, the driver represented by $\sigma = -2.5$ exhibits a risk-taking behavior by lowering his/her gains, resulting in a 'loose' trajectory. This driver considers the variability to be working in his/her favor and hence reduces his/her gains and relaxes. We propose that this σ parameter can represent a driver specific behavior that accounts for an individual's risk versus reward trade-off. We can classify controllers based on the value of σ in the following way:

- 1) $\sigma = 0$: Risk-neutral
- 2) $\sigma \neq 0$: Risk-sensitive
 - a) $\sigma > 0$: Risk-averse
 - b) $\sigma < 0$: Risk-taking

III. SIMULATION RESULTS

In this section the predictions of the risk-neutral and risk-sensitive controllers are compared by simulating abstractions of two scenarios, namely: (a) driving while being subjected to

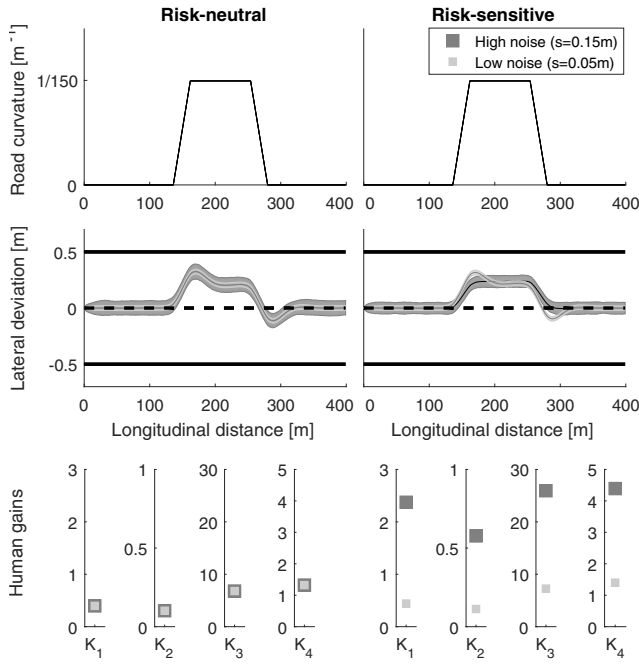


Fig. 4. **Effect of level of noise (wind gust):** The predictions from a risk-neutral ($\sigma = 0$) and a risk-sensitive ($\sigma = 1$) controller are shown in column 1 and 2, respectively. *Row 1:* Input to the model is road curvature and is the same for both controllers. *Row 2:* In the plots for lateral deviation from lane center, the solid lines indicate the average steering behavior, while the bands around them represent the (± 1) standard deviation. *Row 3:* K_1, K_2, K_3 , and K_4 are the gains of the controller (corresponding to $e_k, \dot{e}_k, \psi_k, \dot{\psi}_k$, the 4 states of the system (5)).

lateral wind gusts, and (b) overtaking an unexpectedly swerving vehicle. The simulation parameter values are provided in Table I.

A. Risk-neutral versus risk-sensitive (Effect of wind gusts)

In this scenario (Fig. 4) each controller drove on a curved road while the vehicle was subjected to two levels of lateral wind gusts:

- Low noise with $s = 0.05m$
- High noise with $s = 0.15m$

where s is the standard deviation of the effect of wind gust on the lateral deviation of the vehicle. As seen from Fig. 4, the risk-neutral ($\sigma = 0$) controller changes, neither its trajectory (lateral deviation) nor its gains. On the other hand, the risk-sensitive controller adapts to the different levels of noise. In this simulation we considered a risk-averse ($\sigma = 1$) type of risk-sensitive controller. This risk-sensitive controller increases its gains for the higher noise condition and also follows a 'tighter' trajectory while negotiating the curve. Hence, the risk-sensitive model predicts that humans will adapt their behavior by increasing their gains, in response to higher noise levels. Experimental evidence for such increase in gains for a risk-averse behavior has also been found in point-to-point reaching tasks in sensorimotor tasks [19].

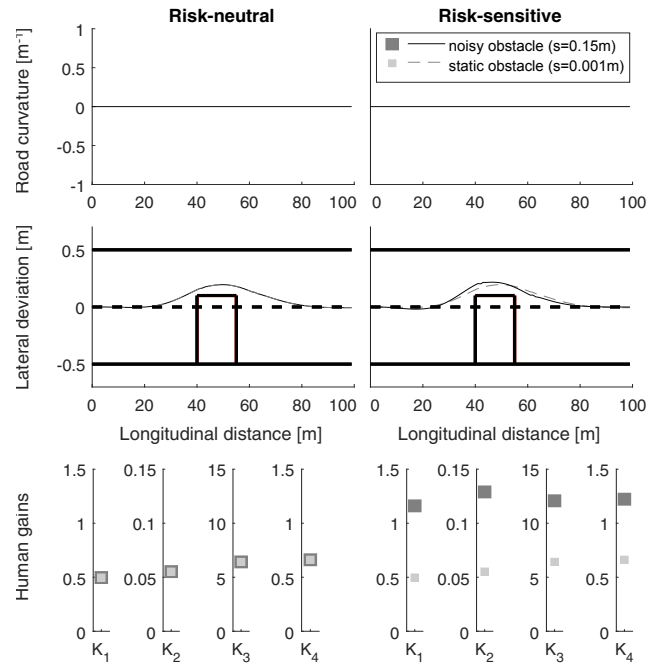


Fig. 5. **Effect of level of noise (noisy obstacle):** The predictions from a risk-neutral ($\sigma = 0$) and a risk-sensitive ($\sigma = 1$) controller are shown in column 1 and 2, respectively. *Row 1:* Both controllers are driving on a straight road ($\rho = 0$). *Row 2:* In the risk-neutral case (*left*) trajectories for noisy and static obstacles are identical and hence overlap. The risk-sensitive controller (*right*) leaves a larger space for the noisy obstacle, as compared to the static obstacle. *Row 3:* K_1, K_2, K_3 , and K_4 are the gains of the controller (corresponding to $e_k, \dot{e}_k, \psi_k, \dot{\psi}_k$, the 4 states of the system (5)).

B. Risk-neutral versus risk-sensitive (noisy obstacle)

This scenario (Fig. 5) simulates an abstraction of the example given in Fig. 1, where the two controllers drive on a straight road while encountering 2 different obstacles. Both the obstacles are stationary in their longitudinal position, but the uncertainty in their lateral position (s) varies:

- Static obstacle with $s = 0.001m$
- Noisy obstacle with $s = 0.15m$

In this case as well, the risk-sensitive controller simulated is risk-averse ($\sigma = 1$). This risk-sensitive controller increases its gains and leaves the noisy obstacle more space as compared to the static obstacle, while overtaking it. In comparison, the risk-neutral controller does not treat the two obstacles differently.

IV. DISCUSSION

To improve trust in automation, it may be necessary to impart automated systems with anthropomorphic features. The human-like adaptation examined in this paper is the risk-sensitivity of steering behavior to uncertainty in the environment. The difference in predictions of the risk-neutral and risk-sensitive controllers can be clearly seen in the simulation studies presented in section III. In the risk-sensitive case, the difference in the trajectories (of low and high noise conditions) is dependent on the values of the parameters, especially σ . For

TABLE I
PARAMETER VALUES

Parameters	$C_1=C_2$	$a=b$	m	I_z	r	d_p	d_s	R	V	q	s	σ
Units	N/rad	m	kg	kgm ²	-	m	m	-	m/s	-	m	-
Effect of σ (Fig. 3)	30000	1.4	1600	3136	15	40	-	-	20	0.2	0.1	2.5,1,-2.5
Wind gusts (Fig. 4)									20	0.2	0.05,0.15	1
Obstacles (Fig. 5)									8.33	0.3	0.001, 0.15	1
Effect of Q (Fig. 6)									20	0.2, 0.4	0.1	1
Effect of ρ (Fig. 7)									20	0.2	0.1	1

State cost matrix: $Q(1,1) = q$ and $Q(rest) = 0$. Uncertainty: s is the standard deviation of the lateral position. Time step: $\Delta t = d_s/V$. Road curvature ρ is provided in figures corresponding to the condition.

example, in the noisy obstacle scenario (subsection III-B) the maximum lateral distance between the trajectories for a static obstacle ($s = 1mm$) and a noisy obstacle ($s = 15cm$) is $\approx 5cm$, which is quite small.

This simulation was performed with $\sigma = 1$, which was chosen as a common parameter value in the two scenarios (subsections III-A and III-B), for ease of comparison. From the simulations presented in Fig. 3 it can be inferred that a larger value of σ will result in larger differences in the trajectories of low and high noise conditions. A realistic value of σ that represents a human driver should be estimated using experimental data.

σ acts as a weighting factor between the expected value of the cost and the variance of the cost. One possible way to estimate the value of σ for an individual driver would be to ask him/her to drive in conditions with different levels of expected cost, and different levels of variance in cost. Such an empirical study will possibly give us an estimate of the risk-sensitivity of drivers.

Both the simulations presented in section III are abstractions of real-life scenarios, which are based on a number of assumptions. Firstly, it is assumed that the driver can perfectly estimate the uncertainty in the environment and that this estimation by-passes the visual and cognitive processes. Secondly, the wind gusts are assumed to affect the vehicle only in its lateral deviation, without affecting its longitudinal motion while traveling on a curved road. The wind gusts are represented using an additive gaussian noise with zero mean, and hence do not have a preferred direction.

This study is limited by the fact that the simulations were performed using a linear vehicle dynamics model at a constant longitudinal speed. Also, the model adapts to risk arising only from *uncertainty* in the environment and not from the *type* of environment. For example, the controller perceives a wooden box with a variability of 15cm in its lateral position equivalent to a pedestrian of the same dimensions and uncertainty.

The proposed steering model could be applied in automated cars in order to adopt 'human-like' trajectories while overtaking. A lack of such adaptive behavior was a concern for some Tesla users [24]. Another possible application could be to convey the level of risk to the driver, using haptic shared control [25]. The field of robotics has made significant advances in the area of path planning in the presence of an uncertain environment. But the uniqueness of our work is its

aim of mimicking human-like steering behavior. Whether the proposed model holds true, needs to be tested empirically.

V. CONCLUSION

- Steering models in the literature take a risk-neutral approach. That is, existing models do not adapt to the level of uncertainty in the environment.
- In this study we propose the approach of minimizing both, the expected value, and the variance of the cost. This results in a risk-sensitive steering model that adapts to the level of uncertainty in the environment.
- The risk sensitivity parameter (σ) may be used to individualize the risk-perception of different drivers.
- The risk-sensitive optimal feedback controller proposed in this paper adapts itself to the uncertainty in the environment, as demonstrated for two scenarios : (a) driving while being subjected to lateral wind gusts and (b) overtaking an unpredictably swerving vehicle.

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APPENDIX

Two parameters (state cost matrix in Fig. 6 and road curvature in Fig. 7) which have the same effect in risk-neutral and risk-sensitive controllers are shown.

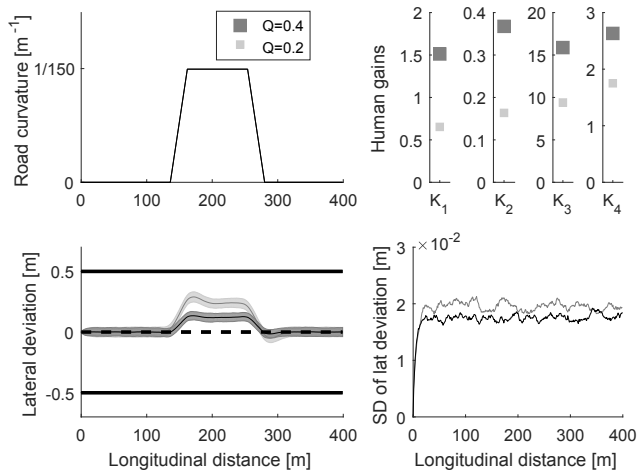


Fig. 6. **Effect of state cost matrix (Q):** As the value of $Q(1,1) = q$ increases, the lateral deviations from the lane center are punished more. *Top-left:* Road curvature as a function of longitudinal distance covered is the input to the model. *Bottom-left:* For a higher value of q the 'curve-cutting' is reduced. *Top-right:* K_1, K_2, K_3 , and K_4 are the gains of the controller corresponding to the states $e_k, \dot{e}_k, \psi_k, \dot{\psi}_k$ (5). As the state cost increases, the gains increase. *Bottom-right:* The standard deviation of lateral deviation is lowered as the value of q increases.

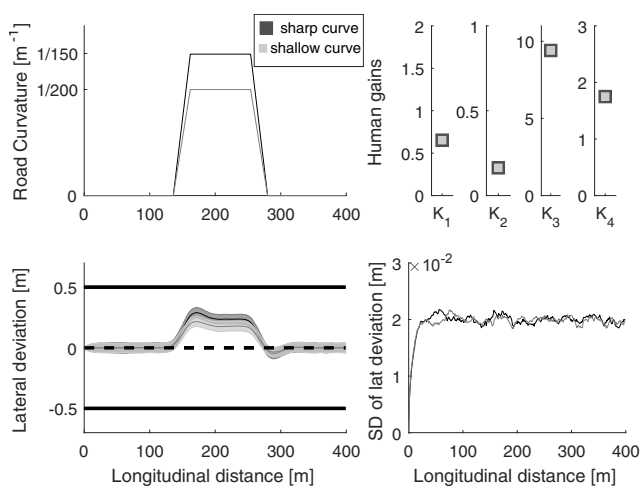


Fig. 7. **Effect of road curvature (ρ):** *Top-left:* Two different road curvatures are used as input to the model. As the curvature increases, the curve becomes sharper. *Bottom-left:* Sharper the curve, more is the curve cutting behavior. *Top-right:* K_1, K_2, K_3 , and K_4 are the gains of the controller corresponding to the states $e_k, \dot{e}_k, \psi_k, \dot{\psi}_k$. Gains remain unchanged for the two curvature, predicting that the human will not increase or decrease his/her effort due to a change in road curvature. *Bottom-right:* The standard deviation of lateral deviation also remains the same for both the curves.

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