Philosophical Anthropology and Geometric Design Methods in the Parthenon

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Abstract – This thesis aims to find a correlation between the geometric symbolism used in the design of the Greek Parthenon and the contemporary zeitgeist of philosophical anthropology. It will do so by creating a collective narrative combining the disciplines of architecture, mathematics and philosophy.

Our anthropological predisposition as humans to obtain knowledge has led us to be the only species on earth to create philosophical theories about our purpose on earth. Symbolism plays a defining role within architecture as a means to represent the philosophical anthropology zeitgeist of certain civilisations. The discipline of mathematics plays a leading role in the development of philosophical anthropology because it was a way of actualizing and physically displaying symbolism through geometry. Especially during the Mesopotamian, Egyptian and Greek civilisations, mathematics and philosophy have been closely related.

Through geometric analyses of the Parthenon, one of the most well-known and representative works of monumental Greek architecture, a strong relation to the philosopher and mathematician Pythagoras was found. The geometric techniques found in the analyses were examined in their philosophical context to relate them back to the philosophical anthropological zeitgeist of the Hellenic period in Greece. This thesis concludes that Pythagoras, and by extension the Pythagoreans, has had a strong influence on the zeitgeist of Greek philosophical anthropology and that this has become evident in the Greek built environment.

Key Words – Parthenon, Geometry, Philosophical anthropology, Architecture, Pythagoras, Pythagorean music scale, Golden section, Numerology.

Outline

I –INTRODUCTION

ARCHITECTURAL SYMBOLISM

When looking at the architecture of any civilisation, new or old, one can see that people have always used their built environment as a means to convey their own cultural values, were it consciously or unconsciously. From vernacular architecture to planned design or monumental architecture, architectural legacies continue to give insight to the functionality and social values of long gone civilizations and cultures and their political and religious development and organisation (Mazumdar, 1994). It is therefore only logical to conclude that archaeological findings of ancient civilisations' built environments provide answers to the way of thinking of these civilisations in terms of religion and general philosophies. In particular, their monumental buildings like temples provide insight into these specific parts of their lives.

Nowadays, ruins of temples are the embodiment of antiquity and its mysteries. For us, these ruins are like a bridge between the past and present, projected onto tangible entropized splendour. Back in their functional era, however, these temples functioned as a gateway between humans and their gods. These temples were places where people, for whatever reason, thought their prayers would be heard and communication between humans and the divine was facilitated. These monuments were human responses to the presence of the divine. Therefore, it is likely that these buildings were created with a great deal of symbolism to connect to whatever that specific civilisation thought to be divine (Rhodes, 1995).

This thesis aims to answer the question on how geometric symbolism is used in architecture and how this translates to the zeitgeist of philosophical anthropology within a particular civilisation. It will do so by preforming an analytical study of one of the more well-known temples of ancient architecture: the Parthenon. It will place this building in the historical narrative of Greek culture, mathematics and philosophy. The basis on which this thesis is built, is the mathematical and philosophical history from the earliest civilisations to the end of the Hellenistic period. The research question for this thesis reads as follows:

"How do the geometrical techniques used in the design of the Parthenon relate to the philosophical anthropology of the Greeks during the time period surrounding its construction?"

This thesis will aim to answer this question by clarifying firstly how philosophical anthropology of the Greeks fits into the historical narrative of mathematics and philosophy leading up to the Hellenistic period; Secondly, by finding the most influential figures in mathematics and philosophy during the construction of the Parthenon and lastly by analysing the leading theories on the geometrical design methods used on the Parthenon.

Philosophical Anthropology and Geometric Symbolism

Ever since humans have existed on earth, there has been a drive for the acquisition of knowledge. Anthropologist Yuval Noah Harari (2014) explains in his book *Sapiens* how the most fundamental thing that sets us humans apart from other animals is our exceptionally large brain. He explains how the growth of human brains has forced us to live in large groups, to form strong social ties, form languages, religions, civilisations and everything that is now considered self-evident in modern social societies. According to him, this is all caused by our anthropological predisposition to obtain knowledge.

Anthropology itself is engaged in the study of all aspect of human evolution. Combining both the practice of philosophy and anthropology, Schacht (1990) introduces the term 'philosophical anthropology' as a means for humans to explain, or reflect on, in a wide range of fields, human reality. Without being restricted to certain perspectives and methodologies specific to different disciplines, philosophical anthropology encompasses all knowledge produced by humans as a means to explain our existence in the universe. Even religion, a concept often subject to its own classification, can be seen as a form of philosophical anthropology. Moving forward, the term 'philosophical anthropology' will be used in this thesis to refer to the way of human thinking to pursue knowledge about the purpose of our species in the universe. Looking back to the origins of the human species, philosophical anthropology has been an continuous theme in the evolution of human beings and consequently the development of knowledge. This anthropological way of thinking has been evident in their legacy in the form of physical remainders for any civilisation's values. This is where archaeological architecture comes into play.

To understand how this architecture relates to philosophical anthropology, one has to be familiar with the different kinds of symbolism relating to architecture. Astakhova (2020) proposes several categories in which different kinds of symbolism can be classified. These categories include numerology, basic geometrical symbols and conceptual symbolism. Astakhova recognises two different classifications under this last term but they both relate to mythology, astronomy and philosophical anthropology in general. This is where architectural symbolism relates more to the topics in this thesis. In this type of symbolism, architecture is used to reflect the contemporary ideas of the universe through the use of, for example, solar orientation or symbolically recreating the model of the universe in a temple.

In general, when thinking about the history of mathematics, the development of geometry is considered to be one of the major contributors. As will be explained in the next chapter, geometry plays an important role in how mathematics was transformed over time from purely utilitarian into a scientific discipline in itself. Furthermore, geometry has played a defining role within the realms of mysticism and symbolism (Boyer, 1985).

The connections made by many philosophers and mathematicians around the time the Parthenon was constructed, between the political and social context of architecture and philosophical thought shows, suggests that architecture, monumental/religious architecture in particular, functions as a window for later generations to the philosophical anthropology practiced in that time (Hahn, 2017). This thesis will therefore connect the disciplines of architecture, philosophy and mathematics to create a collective narrative of architectural symbolism in ancient Greece.

METHODOLOGY

This thesis will start by illuminating the complex history of mathematics and how geometry has first greatly influenced its development and how later philosophy became intertwined. This chapter will try to explain how in ancient Greece, mathematics and philosophy could not be thought of as separate but rather as an intertwined science with different perspectives that support and stimulate each other. In this chapter, several civilisations and individuals will be brought up that play deciding roles in this historic narrative of mathematics and philosophy.

Following this foundational chapter, a case analysis will be preformed of one of the most monumental works of ancient Greek architecture: The Parthenon. As will be explained in this chapter, the Acropolis, and by extension the Parthenon, has played an important role in the history of Greek architecture and is one of the most well-known Greek temple. It is therefore chosen as the main case for this thesis. The chapter starts with a general history of the Acropolis, followed by a general physical description of the building. After that, a literature study will follow based on several different geometric analyses from different perspectives. With regard to the Parthenon, the theories on the design methods can be categorised into two main currents. Firstly the more well-known theory that the proportional ratios of the Parthenon were based on the golden section and secondly the theory that its design was based on Pythagoras' musical scale. During these analyses, the mathematical techniques and concepts found throughout will be summarised and placed within their philosophical context. Several different studies will be referenced in order to support these two main theories. Also, primary sources like that of Vitruvius' *On Architecture* and Plato's *Meno* will be cited.

Finally, in the conclusion these different mathematical concepts will be used to create a collective narrative on how the Parthenon, and by extension other Greek temples, were designed. The mathematical techniques found in the previous chapter will be connected to the foundational first chapter to explain how the utilisation of these mathematical concepts in architecture relate to the collective narrative of the Greek civilisation.

II – Mathematical and philosophical narrative

During the development of philosophy and mathematics from the earlies civilisations to the end of the Hellenic period, certain empires surrounding the Mediterranean area have played leading roles. If we look at history as a flowing continuum it becomes clear that certain periods and empires start to stand apart. These divisions are conveniently put in place to distinguish certain cultural levels and characteristics on which the following chapter will be based. It is important, however, for this chapter to consider that no division in time is an unbridged gap.

Egyptian and Mesopotamian background

Before the first theoretical forms of mathematics were developed, it had its humble beginnings in pre-history when in early civilisations, as is generally agreed, it consisted of no more than the distinction between one and many. This soon transformed into a primitive or primal numeral basis. To communicate anything related to numbers, some form of communication was needed, probably non-verbal at first. One could easily use ones fingers to indicate a small amount of objects (Boyer, 1985). Anatomical characteristics like these form the foundation of the first numeral basis and counting methods.[1] It is a common theory that geometry originated in Egypt, a civilisation which will be discussed later in this chapter. However, the earliest human civilisations show proof that these humans did have a aptitude for noticing spatial relations which paved the way for geometry in a purely utilitarian function (Harari, 2014).

A few of the first ever documented examples of the use of early mathematics were found in Egypt. The most extensive of Egyptian papyri, commonly referred to as the Ahmes Papyrus, shows the use of how the Egyptians primarily used mathematics in a practical context (Boyer, 1985). With the annual overflowing of the Nile, the Egyptians had need for so called 'rope stretchers'. Surveyors who, after the annual flooding of the Nile, kept measuring the lands over to correctly return it to its owner. They did so by means of knotted ropes to construct right angles and other basic geometrical forms (Paulson, 2005). This spatial practice of geometry was one of the examples of how Egyptians used mathematics in a purely utilitarian sense. The geometry in the Ahmes Papyrus was therefore also purely utilitarian. This functional use of geometry does, however, not mean that the Egyptians were only aware of a very simple, primitive geometry. The Ahmes Papyrus shows that the Egyptians struggled with certain problems in higher geometry and even shows attempts at creating the first ever mathematical proof (Boyer, 1985). One of these geometrical problems was to find the area of a circle by using the area of a square and even came close to approximating the value of *pi* to 3 1/6th. These higher geometrical problems later became important for the design of monumental architecture by later civilisations as will become evident in the remainder of this thesis. However, since the use of mathematics by the Egyptians was mainly utilitarian, they did not carry their work further since there was no need for that back then.

Simultaneously with the Egyptian empire, the Mesopotamian valley was home to another great dynasty by the Sumerians. Instead of papyrus, the Sumerians/Babylonians[2] used clay tablets to document their lives. Their contributions to algebraic mathematics are astounding, though, they led historians to believe there might have been more to their advances in mathematics than merely utilitarian purposes (Boyer, 1985). Since Egyptian mathematics was mostly (if not completely) functional, it would have been easy to believe Mesopotamian mathematics would be no different. However, practical use for their advanced mathematics are difficult to imagine. This led historians to believe that there might be more to it than purely practical applications. Much like their algebraic mathematics, the Mesopotamians contributed a great deal to geometric mathematics. Their geometry was even more sophisticated than that of the Egyptians and even though still mainly utilitarian, their higher forms of geometry suggested to the origins of geometry as a branch of theoretical mathematics.

[1] It is only by coincidence that today's decimal system is the way it is because of anatomical characteristic of humans, as Aristotle noted (Boyer, 1985). Would we have evolved from a intelligent species with, say, seven fingers in total, today's decimal system would most likely be based on the number seven.
[2] The term used in the remainder of this thesis to refer to this civilisation will be Mesopotamian. This is because, even though Babylon is known to be the greatest city of their empire, the Sumerians were referred to as Mesopotamians well before this city was ever erected (Boyer, 1985).

It is important to note, however, that even though the Mesopotamian mathematics was potentially more theoretical than that of the Egyptians', the main purpose of mathematics was still utilitarian. This is proven by the fact that both civilisations' mathematics lack a clear distinction between an approximation and an exact value[3]. The first 'official' theoretical purpose of mathematics was introduced during the rise of a new intellectual civilisation while simultaneously introducing a new era: The Hellenic era.

Thales and Pythagoras

In understanding the connection between geometry and architecture and how it developed in early antiquity, mathematics' frontrunner Thales proofs to be a figure of importance. As a man of whom generally not a lot is known, Thales produced the first ever mathematical/geometric proofs. He is known to have travelled to both Egypt and Mesopotamia from which he gathered lots of mathematical knowledge (Boyer, 1985). More relevant to this thesis however is his contemporary Pythagoras. This mathematician and philosopher also travelled to Egypt and Mesopotamia having not only obtained mathematical knowledge but also knowledge of religion and philosophy. In general, Pythagoras is known as a more mystic scholar. Although not much is known about the man in person, lots of the information of him and his teachings originated at his school and the contributions of the Pythagoreans (Keestra, 2006).[4] The Pythagoreans introduced a new chapter in the history of mathematics in which it had a much more theoretical function. The Pythagoreans were the first to merge mathematics with philosophy and comprehensive references to this group will be made throughout the remainder of this thesis.

This is where the historic narrative of mathematics merges with philosophy. The Pythagoreans used mathematics to explain the universe around them. Their famous saying 'all is number' represents their belief that everything around them was made up of integer numbers and that every natural phenomenon could be brought back to simple numbers.

Particularly in the field of geometry, the Pythagoreans contributions shaped much of what was considered theoretical geometry back then. Their contributions existed of, amongst other concepts, the Pythagorean pentagram, which produced incommensurable ratios occurring in what later came to be known as the golden section; Mystic numbers, which ascribed various meanings to certain numbers[5]; proportional and arithmetic cosmology (Boyer, 1985). The latter was used primarily by the Pythagoreans as a basis for their unification of all aspects of the universe. The Pythagoreans believed that mathematical knowledge provided insights to the eternal and unchanging structure of reality. Many of these concepts can be recognised in monumental architecture from around that time because the Pythagorean cult was extremely active and popular during the time that the Parthenon was built and therefore had a lot of influence in Greek culture.

Evolution into Philosophy

Simultaneously with the marriage between mathematics and philosophy by Pythagoras, the epicentre of mathematical advancement moves to Greece; in particular to the Greek philosophers. The three forefathers of modern philosophy are known as Socrates, Plato and Aristotle. Socrates is generally known to be the 'father of western philosophy'[6]. He was the first philosopher to pursue abstract intellectual concepts or ethics and morality as opposed to the 'physical sciences' this predecessors pursued (Mark, 2023). He, however, did not share his pupil's aptitude towards mathematics and is therefore for not necessarily relevant to this thesis. His pupil Plato has lots of similarities with Pythagoras. He, too, incorporated a lot of mysticism in his works. It is even generally agreed that Plato was part of a later branch of the Pythagoreans, which would explain the many

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[3] The distinction between an approximation and an exact value is very important within the discipline of modern mathematics because it determines whether it should be regarded as proper mathematical theory or layman practice.

[4] The Pythagorean school was often thought of as a sort of cult due to their mystic nature, strict code of conduct, vegetarian lifestyle and general philosophy of metempsychosis.

[5] Numerology had been used by many early civilisations and would be used on many occasions by future cultures. The latter specifically in religions.

[6] Albeit not from his own sources. Most of what is known of this philosopher comes from his pupil Plato, who produced numerous dialogues which featured Socrates.

similarities between him and Pythagoras. Though Plato is mostly known for his contributions towards philosophy, he cannot be neglected as a figurehead in mathematical history. He, however, does not pursue mathematical knowledge for mathematics' sake but rather uses mathematics as a means to support his philosophical theories (Heath, 1963). Like his predecessor Pythagoras, his theories of numerology and arithmetic led historians to believe that he saw something almost divine to numbers[7]. This is only confirmed by his general philosophical theories on anthropology which have something mystic to them (Gottlieb et al., 2001). Geometry in particular was something Plato used to glorify. In his dialogue of Timaeus, he portrayed God as a tradesman using shapes and ratios to construct the universe. For one of his theories, the duality of reality, he uses geometry as a means to prove it. For him, everything around us consisted of two realities (Heath, 1963). The first reality was the physical world that we live in and the second reality a higher immaterial world construed with ideal forms (Bursill-Hall, 2002). To him, geometry was a clear way to prove the existence of his higher ideal world: a window to a higher dimension as it were. The geometry used in monumental architecture of that time would therefore also function as a window to a higher dimension. He explained his view towards the function of geometry by saying that "Geometry is concerned, not with material things, but with mathematical points, lines, triangles, etc, as objects of pure thought. A diagram in geometry is only an illustration; the triangle which we draw is an imperfect representation of the real triangle of which we think. Constructions, then, or the processes of adding, squaring, and the like, are not of the essence of geometry, but are actually antagonistic to it." (Boyer, 1985)

Aristotle, on the other hand, had a much more rational attitude towards mathematics. Where Plato utilised mathematics for his philosophies, Aristotle's rational ideology contributed to the definitions and hypotheses of mathematics (Boyer, 1985). This rational mindset is indirectly what led Euclid to write his Elements, a mathematical textbook on all elementary mathematics. This book utilizes a rational and analytical framework to produce the first ever geometric proves, elaborated on the works of Thales, which shaped the world of modern mathematics (Boyer, 1985). This book is known up to today as the 'bible' of mathematics and is even the most purchased book in history, second only to the actual bible (News Editor, 2018).

[7] A direct link can be detected between the Pythagoreans and Plato. Plato's numerology (or the 'Platonic number') can be traces back to the Pythagoreans which most likely have adapted it from Babylonian numerology.

III – Parthenon

One of the most well-known sites of ancient Greek architecture is the Acropolis in Athens. With its history tracing back to well into the 13th century BC, the Acropolis gradually became the symbol of Greek architecture and religion (UNESCO World Heritage Centre, n.d.). The Acropolis as we know it today shows the height of the architectural archaic- through the classical period of Greece. However, its history began way earlier. Though sources of the beginning of the archaic period, when the Acropolis is thought to have originated, are scarce, scholars generally agree on the fact that the settlement of the site began with the construction of a cyclopean fortification (Glowacki, 1998)(figure 1). Typical for the Mycenaean period, this type of defensive architecture was a form of masonry with massive boulders, parts of which still remain at the current site of the Acropolis.

Figure 1 - Remaining Cyclopean fortification at the Acropolis (Athens city guide, 2019)

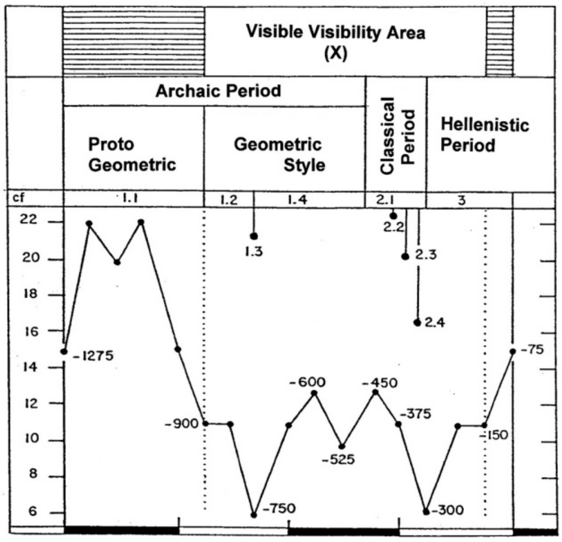

Figure 2 - Greek era's categorisation (Kuilman, 2013)

The site was thought to have been filled with temples, none of which are still standing today but evidence of which are still noticeable in the bedrock of the mountain. Excavations of the site have led scholars to believe that the Acropolis had been used throughout the archaic period as a cult site (Rhodes, 1995). More and more bronze and ceramic evidence has been found dating back to the beginning of the Mycenaean period to the middle of the eighth century which suggests ritualistic activity on the mountain.

From the beginning of the sixth century and onward, the Acropolis started to develop more intensely due to the changing political and religious climate. More temples were built to honour different entities of the same goddess Athena. Furthermore, the Greco-Persian wars (492-449) influenced the architectural development of the Acropolis even more so.

Under the reign of Darius, the Persian king ruling from 522 to 486 BC, Persia attempted to conquer Europe in 514 BC (Encyclopaedia Britannica, 1998). However, two commanders of his army, Histiaeus of Miletus and his son-in-law Aristagoras of Miletus, opted to stage an uprising and sought the help of several Greek city-states. The Athenians were one of the few to back the commanders and thereby became actively involved in the Greco-Persian wars. Throughout the wars, the Acropolis functioned as a stronghold against the Persians. By that time, the Acropolis was already filled with several religious temples, including the older version of the Parthenon. This older version is estimated to be destroyed in September 480 BC when the Acropolis was destroyed under the command of Persian king Xerxes I, son of Darius. (Huot, 1998). After this victory by the Persian army, however, they were soon thereafter defeated at the naval battle at Salamis, after which the Greco-Persian war slowly withered away over the course of 13 years.

After the destruction of the Acropolis in 480, the site was left a ruin for the next 33 years. Though the site did remain a religious sanctuary, the architectural interventions were limited to the renovations of the walls and levelling of the site through terracing (Kousser, 2009). After this period, however, the Athenians decided to rebuild the Acropolis, making it the Acropolis we know today.

DESCRIPTION OF THE PARTHENON

The new Parthenon was built on the exact location of the older version, recycling many of its unburnt materials. As a symbol of the victory of Greece over Persia, the ornamentations on the Parthenon depicted scenes of the Greco-Persian wars within the contexts of Greek religion. Myths taken from Greek religion were used to display the history of the Greco-Persian wars as a battle between right and wrong[8] (figure 3).

Figure 3 - Mesotope ornamentations depicting a symbolic war scene (Kousser, 2009)

The reconstruction of the Parthenon began right after the decision was made to rebuild the Acropolis after 33 years. Construction is thought to have started around 447 BC, taking approximately 15 years resulting in its completion on 432 BC under the supervision of architects Ictinus and Callicrates. Built in the Doric order, the Parthenon consists of a double colonnade of 8 Doric columns (as opposed to the standard 6 for classical Doric temples) in frond elevation and 17 in side elevation, standing on a three step crepidoma, carrying an entablature. The column colonnade encloses the interior of the temple that consists of two enclosed chambers: the Cella and the 'Sekos' (Encyclopedia

[8] These ornamentations, however, did not display a heroic narrative of the Greek victory over the Persians. They displayed the struggles and even sometimes the defeat of the Greece civilisations against the Persians. The centaur in this particular ornamentation symbolises the barbaric Persians as victorious over the civilised Greek man.

Britannica, 2023). A lot has been written about the geometric proportions of the Parthenon. Rightly so because the architecture in this building displays countless feats of geometric symbolism.

Today, the Parthenon is seen as one of the most representative works of Greek architecture, which symbolises a culture that is seen as the bedrock on which modern civilisation is built. During the fifth century however, this symbolism had an even greater significance because it stood for the Greek triumph over the Persians and the pinnacle of the Greek civilisation. Traces of the contemporary zeitgeist of the Greeks can therefore be found throughout its design. It symbolises the nationalistic mindset of the Greeks after their victory over the Persians, its construction shows the cutting-edge technology that was available at the time due to the many mathematical advances made by the Greeks and the religious function of the temple shows the philosophical anthropology of the Greek civilisation at the time. Because of this, the Parthenon is the perfect example of how the general philosophical anthropology was embedded in architecture.

POPULAR MISCONCEPTIONS

Before a geometric analysis of the Parthenon is made, a couple popular misconceptions and definitions need to be set straight. Throughout history, numerous theories have been proposed about the architectural design of the Parthenon. The most familiar would have to be the application of the golden spiral on the elevation of the temple as seen in the figure below.

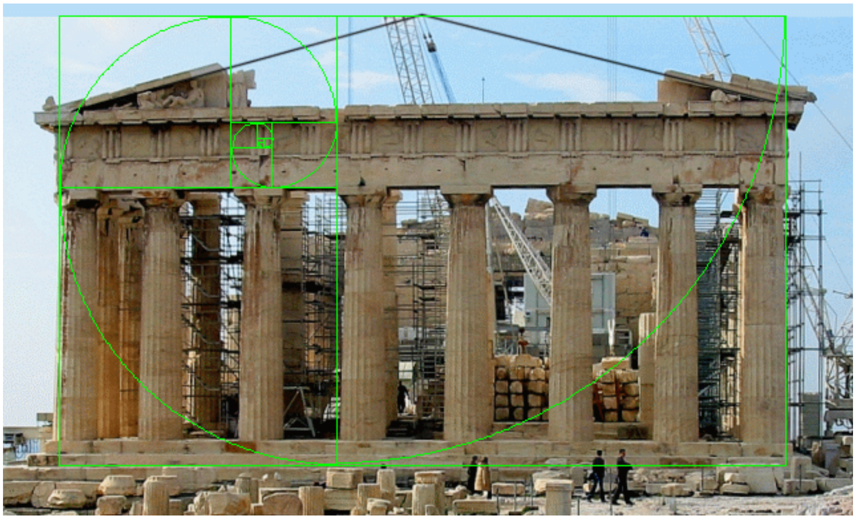

Figure 4 - Golden spiral projected across the front facade of the Parthenon (Usvat, n.d.)

This general theory of how the Parthenon was proportioned based on this image of the golden spiral has been disproven various times by various historians, mathematicians and archaeologists over time (Hammer, 2016). The measurements on which this theory was based seem to be taken not nearly precisely enough (Kappraff, 2002). Also, the term *Golden spiral* was not used until the renaissance (Azeez, 2021). However, it is important to create a distinction between this image of the *Golden spiral* and the concept of the *golden ratio*. Even though the golden spiral is based on the proportions of the golden ratio, this shape does not encompass all of the mathematical properties of the original ratio since the golden ratio is far more abstract and can be applied to so much more than just geometric shapes like this spiral. Using the golden spiral to justify unfounded theories which state that the Parthenon's architects utilized this shape is therefore unjust. The fact that we now know that the golden spiral was not used, does not mean that the *ratio* was not used (!).

Though the concepts of the *golden spiral* had not existed before the renaissance, evidence of the use of the ratio 1:1.618.. can be found all throughout the Archaic, Classical and Hellenistic period and even before. Euclid defined this ratio in his *Elements* as "a whole that is to the larger part as the larger is to the smaller" (Boyer, 1985) (figure 5).

$$(a+b) / a = a/b = \Phi$$

Figure 5 - Golden section as defined by Euclid (own work)

But even before this theoretical book on mathematics, the ratio known now as *phi* was in use during the Egyptian and Mesopotamian empires (Boyer, 1985). Evidence of the use of this ratio can be found throughout the entire history of mathematics and philosophy as laid out in the first chapter of this thesis. The golden ratio appears in geometrical forms such as the pentagram, which was extensively studied by Pythagoreans for its mystical properties; Plato mentioned the golden ratio while trying to explain the universe; Even more rational thinkers like Aristotle have referred to the golden ratio (Dmytruk, 2017).

THE MODULAR DESIGN METHOD

Roman architect Vitruvius wrote the first ever preserved theoretical works on architecture (Morgan, 1960). He thought Greek temples to be of such importance to architecture that he felt the need to devote an entire section of his books to them. In this section he states that, in order to achieve perfect symmetry, one has to take one base measurement as a standard. “Proportion is a correspondence among the measures of the members of an entire work, and of the whole to a certain part selected as standard. From this result the principles of symmetry.” (Morgan, 1960). Throughout his books he refers back to this symmetry. He uses it while explaining the fundamentals of architecture: Order, arrangement, eurythmy, property, economy and symmetry; and he keeps referring back to it to support his claim on how certain buildings should have an aesthetic appeal. In his third book in particular, he describes how temples should be symmetrical due to the use of proportions. These proportions ought to be determined from a single module measurement. In the third chapter of his fourth book he describes how Doric temples should be proportioned my means of this base module.

> “Let the front of a Doric temple, at the place where the columns are put up, be divided, if it is to be tetrastyle, into twenty-seven parts; if hexastyle, into forty-two. One of these parts will be the module (in Greek ἐμβάτς); and this module once fixed, all the parts of the work are adjusted by means of calculations based upon it.” (Morgan 1960).

He then goes on to explain in detail how this now determined module is used to proportion the rest of the temple. Especially temples built after the second quarter of the fifth century seem to be more consistent in following certain design processes since before, architects primarily designed their temples using general rules of thumb instead of a predetermined design process. Classical architecture historian Mark Wilson Jones (2001) suggests that, even though many researchers use Vitruvius’ source as described above as a foundation for their arguments on modular temple design, these modules were more than just an abstract unit of measurement. He suggested that these were based on physical attributes. This argument is supported by the research of Anne Bulckens (Kappraff, 2002) in which she brought the Parthenon module, deducted from the ‘theoretical triglyph’ as per Vitruvius’ writings, back to the measurements of a Greek foot length of 343.04 mm. It is important to note, however, that the Parthenon is neither in tetrastyle nor hexastyle, as per Vitruvius’ classifications of Doric temples. Instead of the common six column front façade, the Parthenon has eight. Following Bulckens research, the front of the temple, or the stylobate, is divided into 36 modules. Jones (2001) argues that the triglyph, the only element in Doric designs that physically measure the width of one module, is the main determiner in the measurements of the entire temple. This differs from Bulckens’ theories that the measurements of the triglyph was based on the module length. Instead he argues that the module length was based on the earlier determined triglyph length. Nevertheless, whether the triglyph width or the module width came first, it is generally agreed upon that once the dimensions of this module unit were determined, the rest of the temples measurements were based on this.

Golden Section vs. Pythagorean Music

Once this modular unit was determined, (and by extension the horizontal dimensions of the front façade), the length of the temple was to be determined. Again, the various hypotheses put forth by researchers seem to fall under two main theories. Architectural history professor Rocco Leonardis constructed a design method for Greek temples in the pre-Socratic phase of Greek temples[9] based on the geometric analyses of Greek temples like the temple of Athena at Pompeii, the temple of Concord at Akragas and the Parthenon (Leonardis, 2016). This method is primarily based on what he argues to be the first step in designing a Greek temple: To determine the measurements of the stylobate and crepidoma based on the before mentioned module unit. He does this by utilizing two mathematical techniques. The first being the doubling of the square (or Plato's *Meno*) and the second being the utilization of the proportional ratio of the golden section[10].

Leonardis describes the process as follows: Using the predetermined width of the stylobate, a square is drawn. The first mathematical technique is then used to create a square with an area one fourth that of the first square. The second mathematical technique, the golden section, is then used to create a third square which is used to determine the width of the crepidoma. The length of the temple would then be determined by again halving a square, this time the one with the width of the crepidoma, placing it in the middle of the crepidoma square and applying the golden section technique. The length of the Parthenon is hereby determined by taking this extra length created by the golden section technique and adding a second crepidoma square at the end. Leonardis visualised this process in several diagrams of different temples. All of these temples have different width to length ratios but he is able to justify these differences by a slight alteration in the last step of determining the length of the crepidoma.

[9] He divided architectural Greece in three phases: Frist the pre-Socratic phase, which emphasized philosophical and religious applications in architecture; Secondly the Hellenistic Alexandria, which was influenced by Euclid's contributions to geometry and thirdly the late Hellenistic period. The arena of geometers Archimedes and Appolonius. (Leonardis, 2016)
[10] Note that the terms Plato's *Meno* and the golden section were not in use during the construction of the Parthenon but were only defined at a later date.

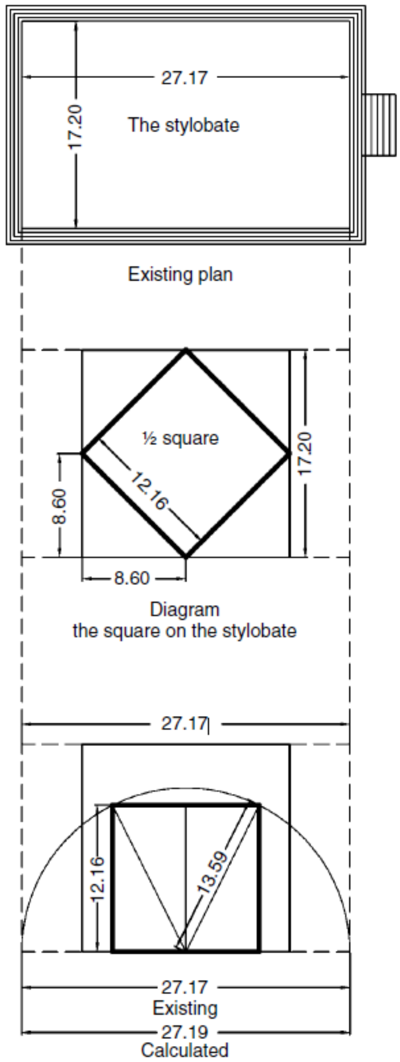

Figure 6 - Golden section method applied to the temple at Pompeii (Leonardis, 2016)

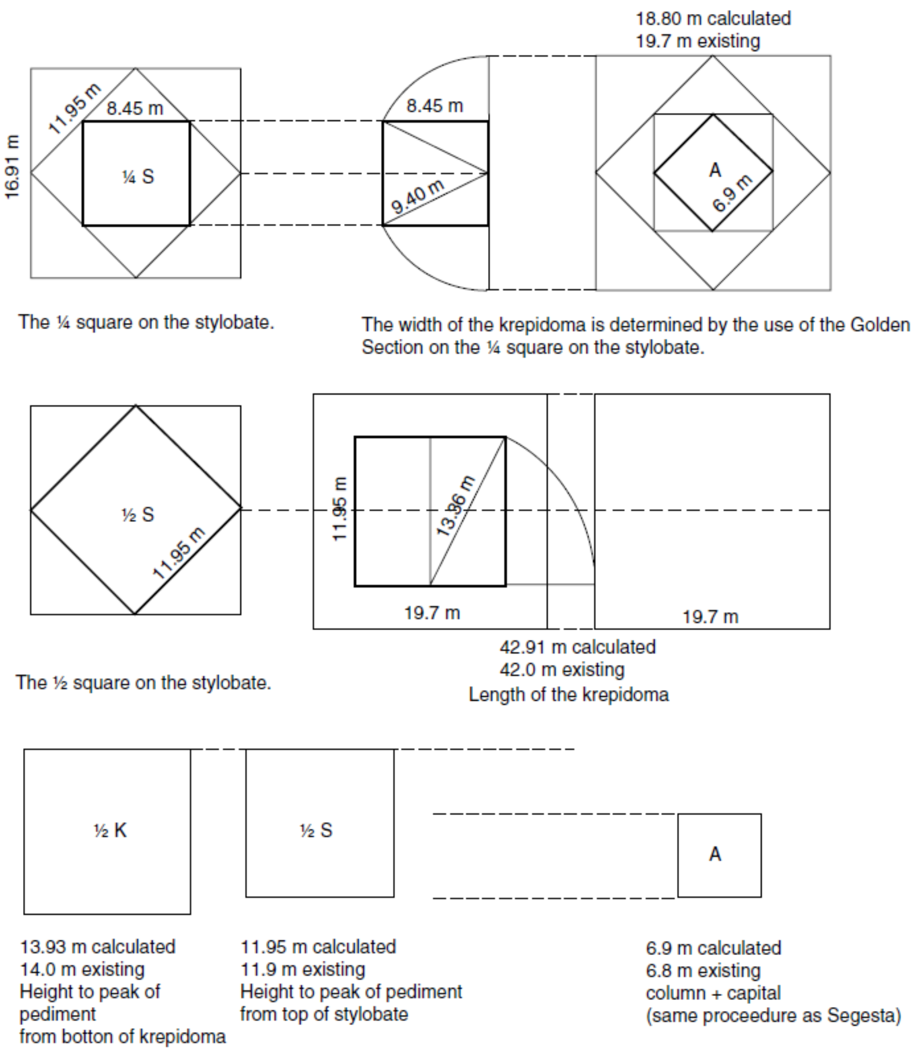

Figure 7 - Golden sectio method applied to the temple at Concord (Leonardis, 2016)

He backs up his argument by stating that for his analysis of the temple at Pompeii (figure 6), the measurements from his diagram match the actual measurements of the temple within two centimetres. Considering that the actual length of the crepidoma of this temple is 27.17 metres, this comes down to a margin of error of 0.07% which is extremely narrow and thus making his argument very credible. Leonardis argues that this method could have been applied, not only to the temple at Pompeii, but also to many other temples built during this time. When comparing the temple of Concord and the unfinished temple at Segesta, the same fundamental methods could be applied to come to the same actual measurements of these temples. The margin of error for these temples is approximately 1,0%. Furthermore, the unfinished temple at Segesta functions as proof of the design process of the Greek temples built during the fifth century BC. Because of the unfinished state of the building, it can be determined that the crepidoma and the stylobate were determined first since these were among the only parts of the building that were actually built.

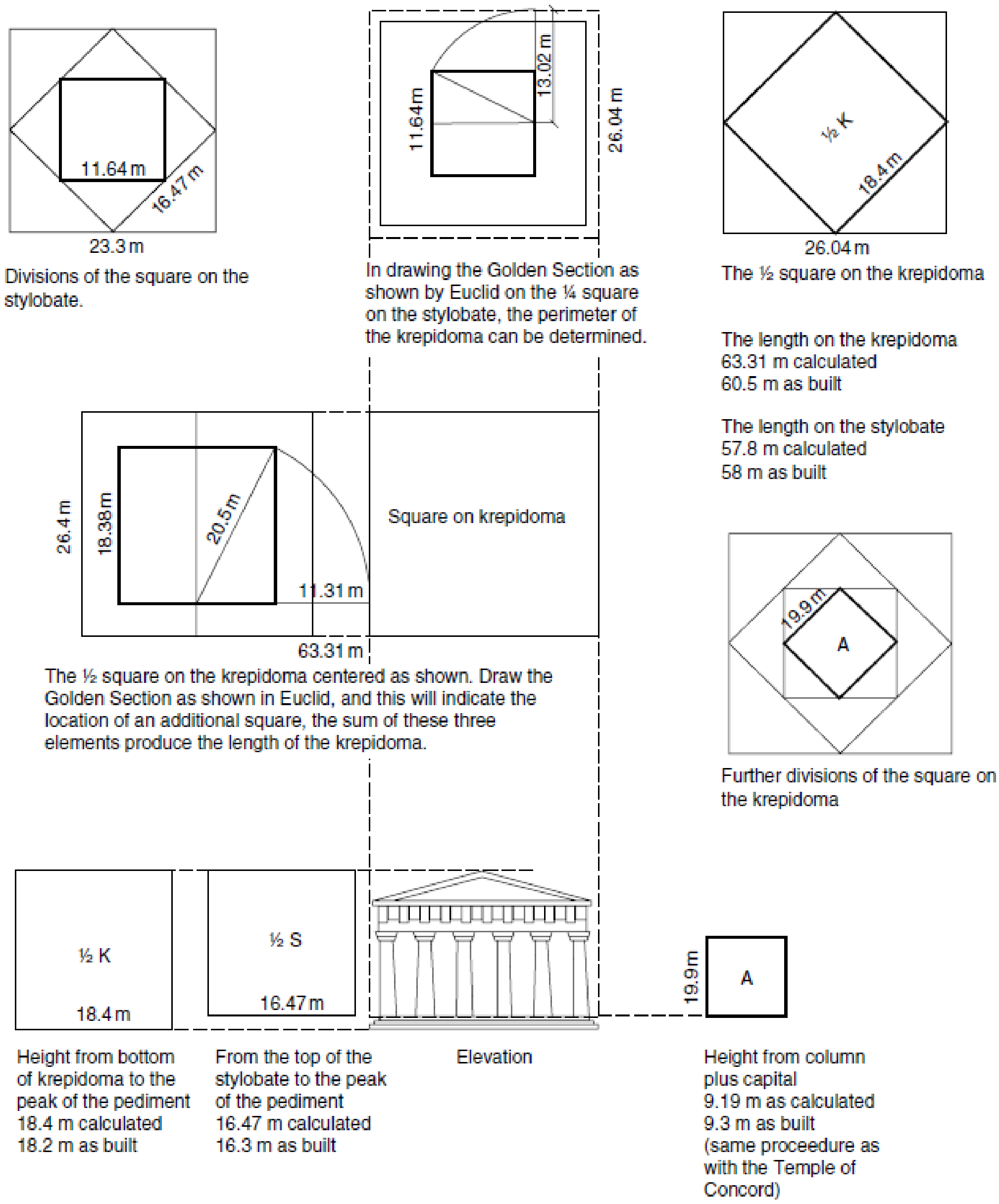

Figure 8 - Golden section method applied to the unfinished temple at Segesta (Leonardis, 2016)

He then goes on to show how this method is applied to the Parthenon (figure 9). The reason he suggests the use of this method in the Parthenon is because this pattern can be recognised in so many temples from that time period, the similarities cannot be ignored (Leonardis, 2016).

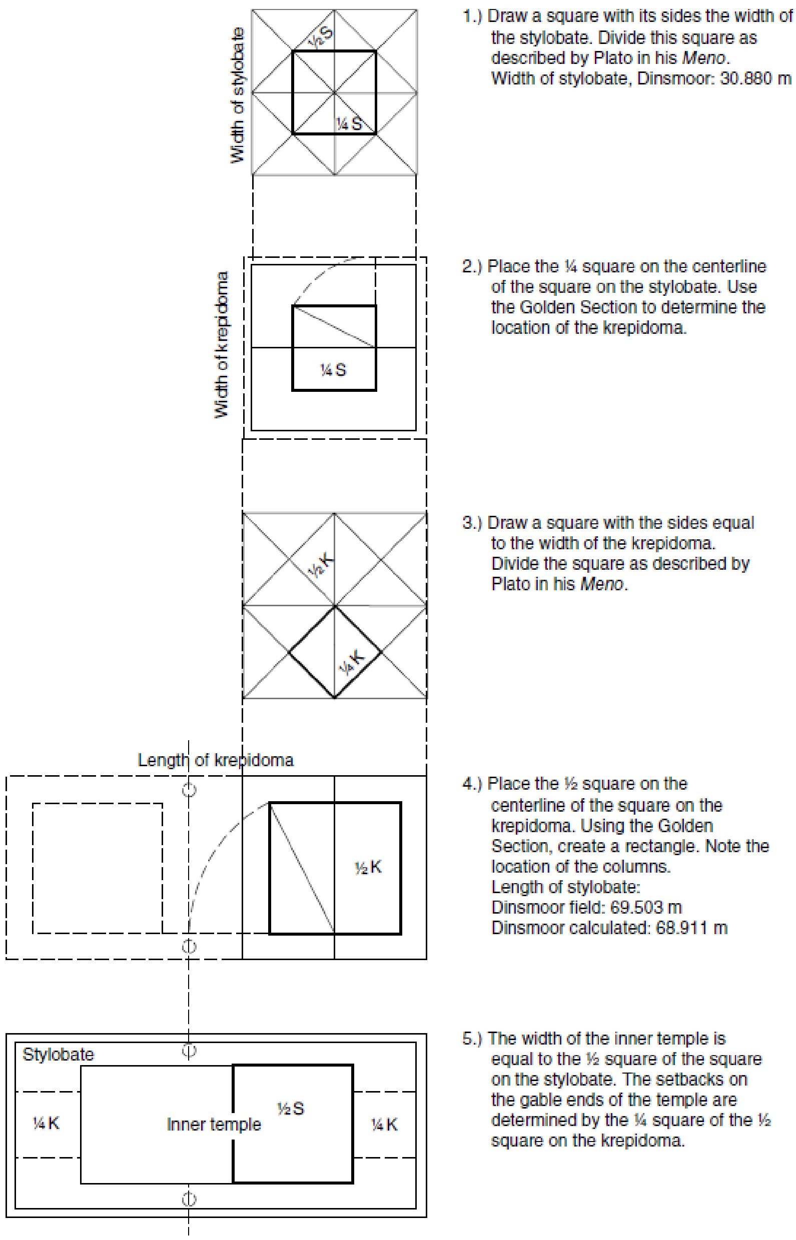

Figure 9 - Golden section method applied to the Parthenon (Leonardis, 2016)

Leonardis then goes on to explain how these same mathematical techniques are used to determine the height of the temple, placement of columns and detailing. He also addresses the fact that the use of the golden section might also have influenced the detailing of the columns. Namely, the golden section would be used to create a pentagram (which was an important form in Pythagoras' philosophy) which later would be used to divide a circle into five or ten parts. The correlation between the golden section and the pentagram will be elaborated later in this chapter. This technique would be used to mark off 20 equal width flutes in the Doric columns.

Doubling/Halving the Square

The mathematical techniques used here proof the importance of irrational numbers in Greek art. Firstly with the technique of doubling or halving the square for its use of the irrational number $\sqrt{2}$. This technique has been used throughout the history of mathematics. Documentation of this technique was found in Mesopotamian civilisations, Vitruvius referred to it when describing areas and Plato used in his dialogue he called *Meno* (Hoerber, 1960). The method for doubling a square was simply to take the diagonal of that square and using it as the width for the second square, the area of which would be double that of the original square. When turned around, ergo halving the square, the method that was primarily used in Leonardis' analysis, it relates to the Pythagorean music scale. His theories in music involve several ratios including his 'Pythagorean octave' which had a ratio of 2:1 (Hubbard, 2021). This particular concept will be elaborated on later in this chapter.

The 'Golden Section'

Secondly, the use of what Leonardis called the 'golden section' supports the argument for the influence of irrational numbers in Greek architecture. The term 'golden section' is ambiguous because of the ongoing debate on whether the actual golden section was used in the design of the Parthenon. But as mentioned before, the ratio occurring in this famous shape had been used far before the image was constructed[11]. Throughout the history of geometry, the term 'golden section' has been used to describe the use of this ratio and not necessarily this particular shape/section. This term will therefore be used in this thesis to refer to the ratio instead of the actual image (figure10).

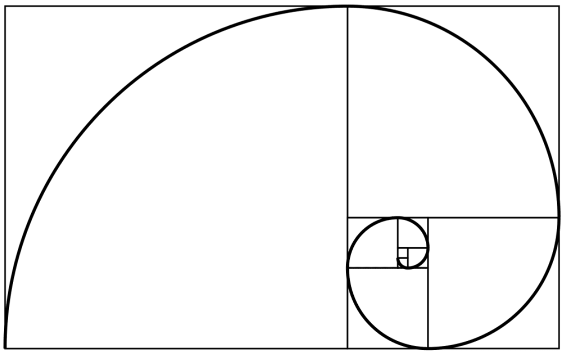

Figure 10 - Golden spiral

The discovery of irrational number traces back to the Pythagoreans. The original thought of the Pythagoreans was that everything existed from numbers and that everything could be separated in integers (De Bruin, 2004). We now know that this is not true for the before mentioned irrational numbers like $\sqrt{2}$ and *Phi*. This was discovered by the Pythagorean Hippasus. He was fascinated by the cult's official symbol, the pentagram, and its symmetry. The pentagram was thought to represent the concept of metempsychosis, or reincarnation, due to the fact that this symbol could be infinitely reproduced within itself (Fossa, 2006)(figure 11).

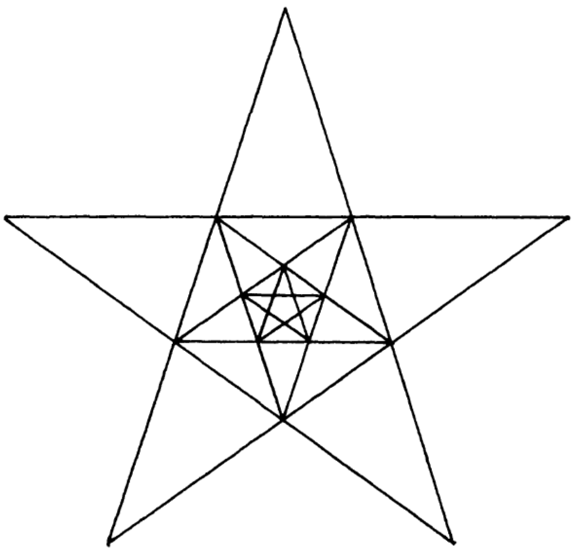

Figure 11 - Infinitely inscribed pentagrams (Choike 1980)

[11] The actual term of the 'golden section' first appeared in the renaissance. (Leonardis, 2016)

Using two of the mathematical theories by the Pythagoreans, which ironically were thought to prove their original philosophy, Hippasus concluded that there was not one integer that could explain the respective dimensions of the inscribed pentagrams to their former, thereby discovering the existence of incommensurability[12]. It is thought that this discovery led to the death of Hippasus himself since his fellow Pythagoreans found this discovery to be so unsettling that he was drowned as a consequence. His discovery was seen as highly sacrilegious because it negated the fact that everything could be explained by integers, something the Pythagoreans built their entire philosophy on. However, because of the incommensurable properties of the pentagram, and the golden section by extension, it became a symbol of higher geometric mathematics and philosophy (Lundy, 1998). This is why it was used by many Greek artists (and by extension architects) because of their search of perfect proportions (Choike, 1980).

The way the golden section was utilised according to Leonardis (2016) was by taking a square, taking the diagonal of one halve and using a compass to extend the square by 0.618 in relation to the width of the original square (figure 12). This would result in a width to length ratio of 1:1.618, ergo the golden ratio.

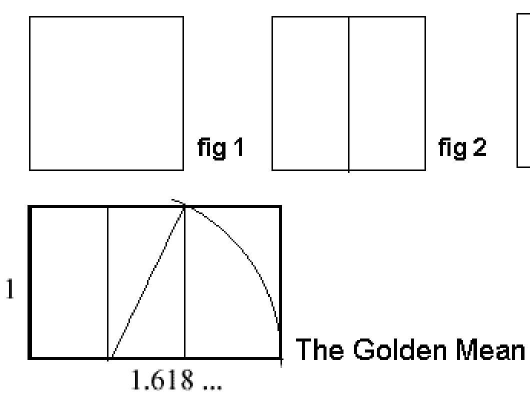

Figure 12 - Geometric method to construct the golden section (Smith, 2015)

Pythagorean Music Scale

PhD Anne Bulckens proposes a different method to proportioning the plan ratio of the Parthenon. Bulckens also uses the before mentioned base measurement to start. She found that this base measurement, the theoretical triglyph of 857,6 mm, contained both her own variation of the Parthenon foot measurement[13] of 343,04 and the dactyls, a common unit of measurement in the Parthenon. Bulckens suggests the influence of Pythagoras and calls the Parthenon “one of the finest examples of Pythagorean theory at work” (Kappraff, 2002). As opposed to the theory that the golden section plays a dominant role in the proportions of the Parthenon, she found that these proportions more resemble the musical scale of Pythagoras and states that the inspiration for the design of the Parthenon could be drawn from music.

Bulckens recognises a recurring ratio within the temple of 3:2, an important ratio used in Pythagoras’ musical theory. Together with the more obvious ratio of 9:4 (the ratio of the crepidoma in plan and elevation, figure 13), she constructs a general ratio 9:6::6:4 which can be reconstructed into the ratio of 3:2. She recognises this ratio in the temples column height including the entablature in relation to the width of the stylobate and the length of the cella in relation to the stylobate length. All these lengths are based on the measurements of the dactyl mentioned before. Bulckens goes on to relate the proportion of the Parthenon to Pythagoras’ numerology. She states that the measurements of

[12] There is another theory that suggests that some Pythagoreans did in fact already know about the existence of incommensurability but that this knowledge was of such significant religious value that only the higher ranks or a certain branch of the Pythagoreans were allowed to have this knowledge. The fact that Hippasus made this information public is suggested to be the reason for the controversy around the subject and by extension his own demise.

[13] The common Greek foot length was around 294 mm.

the temple are based on sexagesimal and 10 base numeral systems which are derived from Mesopotamian and Egyptian numeral systems, which influenced Pythagoras' numerology. This will be elaborated later in this chapter. With her analysis of the measurements and ratios of the Parthenon, she accomplished a margin of error of 0.1% which makes her research incredibly precise and credible.

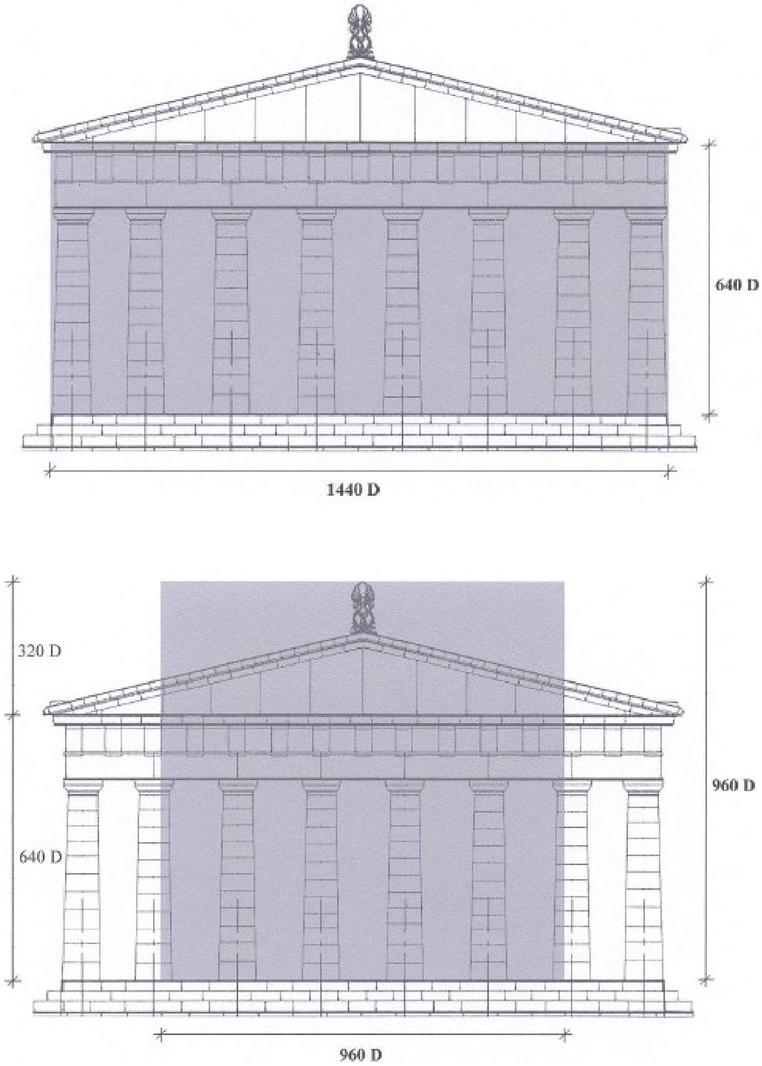

Figure 13 - Proportions of the front facade of the Parthenon by Anne Bulckens (Kappraff, 2002)

Though music might seem to have little in common with architecture, when the underlying foundations of both architecture and music are considered, it becomes evident that they both are founded on mathematics. Especially from the perspective of Pythagoras or the Pythagoreans and their philosophy of numbers and geometry, it becomes more believable that musical ratio theories can be used to influence the ratios used in architectural design (Jencks, 2021). As mentioned before, Bulckens found that the proportions of the Parthenon existed from the recurring 9:6::6:4 proportions and that these proportions were based on Pythagoras' musical ratio 3:2 . In his musical theory, the ratio 3:2 refers to what is known in the musical world as a 'perfect fifth'.

The theory behind the Pythagorean musical scale was based on their ideology that 'all is number'. The Pythagoreans found that they could make different harmonious notes with the use of strings of different lengths. They discovered that, in order to create a harmoniously coherent sequence of notes, the lengths of the strings had to have relative ratios based on small integers (Dudley, 1998). For example, in order to produce two notes, one of which is exactly one octave higher than the other, the length of the string had to be exactly half that of the other, resulting in a ratio of 2:1 (figure 14). (This is how the Pythagorean music scale relates to the halving-of-the-square technique of Leonardis.) Besides this division of octaves into the ratio 2:1, the Pythagorean music scale is closely related to the ratio 3:2. This is known as the 'perfect fifth', (four notes higher than the original) which is considered to be the most pure because this ‘cord’ was the most easy to tune based on hearing alone[14]. Therefore, this scale was considered to be the most important in this musical theory (Pitkänen, 2014).

Note	Ratio	Decimal
E	9/4	2.250
D	2	2.000
C	16/9	1.778
A	3/2	1.500
G	4/3	1.333
E	9/8	1.125
D	1	1.000
C	8/9	0.889

Figure 14 - The Pythagorean music scale with its respective ratios (*Pythagorean Scales*, n.d.)

As mentioned before, the techniques put forth by Leonardis also relate to the Pythagorean musical scale. This feeds into the theory of Bulckens that the overall design of the Parthenon was based on this musical scale which further strengthens her position.

Though Bulckens explicitly rejects the role of the golden ratio in the Parthenon and Leonardis based his entire argument on this, these two methods seem to produce roughly the same results. When applying the golden section method of Leonardis, by taking the crepidoma width as a unit of 1, the crepidoma length comes to roughly a unit of 2.29 (figure 15). This ratio of 1:1.29 can be multiplied by four to produce the ratio of 4:9.16 which results in a margin of error of 1.8%.

[14] This musical theory, like most of the Greeks theories and philosophies, is most likely based on earlier Mesopotamian or Egyptian theories on art and mathematics.

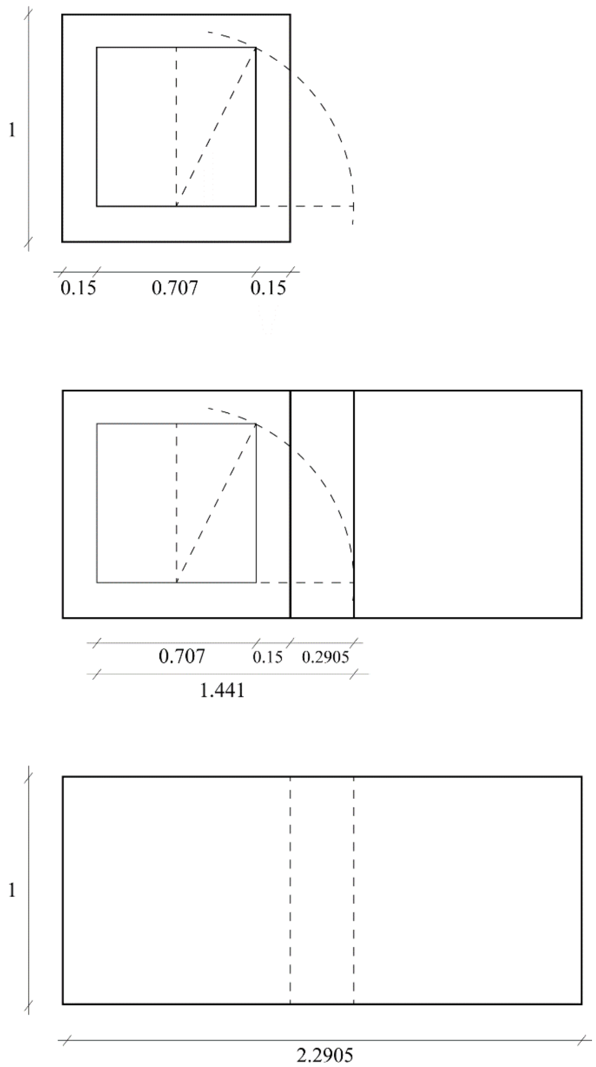

Figure 15 - Proportional comparison between Golden Section method and Pythagorean musicl scale method (Own work)

Bulckens' measurements might be more precise for the exact measurements of the Parthenon, but Leonardis' methods apply beyond the proportions of just the Parthenon. Therefore, both methods ought to be taken into consideration.

Numerology

Numerology, or number mysticism, is the discipline that ascribes meaning to numbers beyond their mathematical and utilitarian function (Dudley, 1998). It is generally agreed that this discipline started with Pythagoras but evidence of number mysticism can be found preceding the Greek civilisation (Fanthorpe et al., 2013). Pythagorean numerology as we know it today is primarily derived from his travels to Mesopotamia and Egypt. The Egyptian philosophy on numbers is recognisable in the Pythagorean teaching "All is number" since this ancient civilisation believed that numbers were the drivers for every energetic current in the universe (Francini, 2009).

With regard to the Parthenon, there are numerous cases in which numerology applies to its measurements. The dactyl measurements of the metopes and triglyph (or module) are respectively 60 and 40, which are important numbers in the sexagesimal numeral system; the area of the stylobate is six times 777,600 dactyls, which on its own contains the number 7 (as a dedication to the goddess Athena), the sexagesimal system and the base 10 system (Kappraff, 2002).

The importance of the sexagesimal numeral system is found in Greek astronomy (McEvilley, 2003). This numeral system is based on the amount of days in a year, and consequently the amount of hours in a day (or in half a day) (Neugebauer, 1971). As suggested in the introduction, astronomy was a big part of early Greek philosophical anthropology since theories on the workings of the universe were a way for the Greeks to explain their existence.

Though numerology as a discipline is generally thought of to be mainly mystic in nature, the mathematical reasoning behind the sexagesimal numerical system can be extremely rational and scientific, even to modern standards. The geometric theories and arithmetic's that are used by these scholars to try to construct models of the cosmos are extremely extensive and, at the time, some of the most cutting-edge, ground-breaking and innovative mathematical techniques and theories (Neugebauer, 1971). Since the sexagesimal numeral system was based on- and primarily used with astronomy, it represented (and still does) the mathematical advances made by the Greeks and its importance within the philosophical anthropological way of thinking. The use of this numeral system in monumental architecture is therefore evidence to the anthropological way of thinking at that time, especially relating to astronomy and the mysticism related to this discipline.

Next to the sexagesimal numeral system there is the 10 based numeral system. This relates even more to the Pythagorean teachings since the their numeral philosophy was mainly based on this 10 based system. The tetractys is a symbol of 10 dots, organized in a triangular shape to represent the first four integers (figure 16). Besides the pentagram from before, this tetractys is another important symbol for the Pythagoreans because it further supports their argument that integer numbers make up everything. In this case, the first four integers (1, 2, 3 and 4) together make the perfect number 10. This number was seen as perfect because it symbolised unity as a result of multiplication. Furthermore, the integers from which this symbol is constructed relates to the four elements of space in the Pythagorean philosophy: 1 being a single point, 2 a line, 3 a triangle and four a space. 10 was therefore a symbol for all spaces imaginable (Encyclopaedia Britannica, 2005). As opposed to the more rational reasoning behind the sexagesimal system, the 10 based system represents, again, the more mystic nature of the Pythagoreans. The fact that this numeral system is prominently used in the design of the Parthenon once more suggests the potent influence of Pythagorean philosophy.

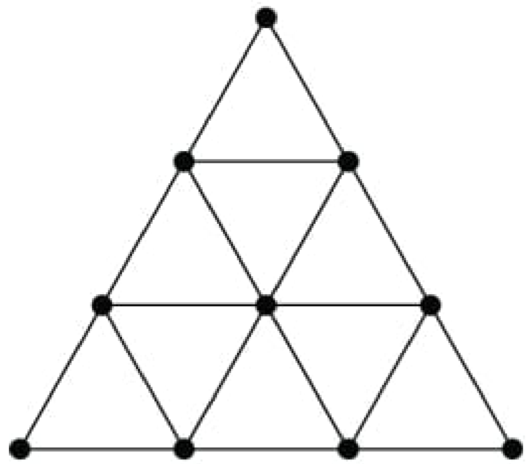

Figure 16 - The Pythagorean tetractys (Shaw & Shaw, 2014)

IV – CONCLUSION

After the analysis of the Parthenon from the viewpoint of geometric design, this thesis shows how the design of the Parthenon has been deeply influenced by the philosophical anthropology zeitgeist of the time, which in turn has been greatly influenced by Pythagoras and the Pythagoreans. The aim of this thesis was to determine the relation between the geometrical techniques used in the design of the Parthenon and the philosophical anthropology of the Greeks at that time.

The first chapter of this thesis provided a short summary on the historical narrative of mathematics and philosophy. It showed how these two branches of knowledge were deeply intertwined, particularly at the start of the Hellenic era. Pythagoras and the Pythagoreans were one of the first to merge philosophy with mathematics. This was true, especially, for the geometric branch of mathematics. Geometry started to become an important aspect of the philosophical anthropology of the Greeks. Pythagoras was a highly mystic philosopher and he passed this on to his followers. Their saying 'all is number' suggests a strong connection to philosophical anthropology and even with the discovery of incommensurability, their philosophy was highly accredited and had a lot of influence on the philosophical way of thinking in Greece at the time (De Bruin, 2004). This is also partly because they incorporated geometry in their philosophical theories, making it tangible and thus fit for physical representations like architecture.

The analyses of the Parthenon brought forth several different geometrical and mathematical concepts that were used in its design. The golden section, doubling the square, musical scales, numerology, pentagrams; all of these seem to have a common denominator. They all seem to connect with the teachings of Pythagoras. There is no denying that Pythagoras, and by extension the Pythagoreans, had a lot of influence on Greek history. Maybe not necessarily in the day-to-day life of the average Greek person but definitely in the sort of philosophical anthropology that was common during that time. During this period, deeply infected with Pythagorean philosophy, the buildings that were built leave traces of this zeitgeist. As an architect, one does not simply design in a purely utilitarian way. Even today, architecture does not only revolve around practicality. Aesthetics and symbolism have always been the cornerstones of design. Especially when speaking of monumental architecture in a society that is known for its philosophers and of which a big part of its everyday culture revolves around religion and philosophical anthropology, symbolism is an important aspect of architecture.

It is important to consider the fact that Pythagoras was deeply influenced by the Mesopotamian and Egyptian civilisations. Therefore, Pythagoras should be seen as a physical historical figure through which the influence of these former civilisations are portrayed. It is only logical to assume that with the gradual transfer of all the mathematical knowledge, the knowledge of philosophy and numerology were also transferred. So these would not have been introduced in Greek civilisation by one man only based on his travels. However, the documentation of Pythagoras' travels, however scarce, does provide more tangible evidence than this, which is purely speculation. The question to what specific topics Pythagoras picked up from these civilisations might be an interesting topic for further research.

As mentioned in the introduction, no division in time is an unbridged gap. Historians like to differentiate between the Egyptian, Mesopotamian, Greek and Roman civilisations as separate chapters in the historical narrative for convenience. It would be more realistic to see this narrative as several currents of simultaneous development, merging together and separating into different disciplines over time as human knowledge expanded. This thesis focusses on what appears to be a single point in this complex narrative of architecture, philosophy and mathematics. It could be interesting to see how the correlation between these disciplines developed as the Greek civilisation flowed into the architecture of the Roman Empire. Pythagoras' pupil Plato has had a significant influence on the development of philosophy and so does his more rational pupil Aristotle. A potential topic for further research could be to see how architecture changed during and after the lifetimes of these scholars and determine whether the more rational mindset of philosophy was gradually applied to architecture.

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