

## A probabilistic framework for offshore aquaculture suitability assessment using bivariate copulas

Santjer, R.; Mares-Nasarre, P.; Vilmin, L.; El Serafy, G. Y.H.; Morales-Nápoles, O.

**DOI**

[10.1016/j.aquaeng.2024.102479](https://doi.org/10.1016/j.aquaeng.2024.102479)

**Publication date**

2024

**Document Version**

Final published version

**Published in**

Aquacultural Engineering

**Citation (APA)**

Santjer, R., Mares-Nasarre, P., Vilmin, L., El Serafy, G. Y. H., & Morales-Nápoles, O. (2024). A probabilistic framework for offshore aquaculture suitability assessment using bivariate copulas. *Aquacultural Engineering*, 107, Article 102479. <https://doi.org/10.1016/j.aquaeng.2024.102479>

**Important note**

To cite this publication, please use the final published version (if applicable).  
Please check the document version above.

**Copyright**

Other than for strictly personal use, it is not permitted to download, forward or distribute the text or part of it, without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license such as Creative Commons.

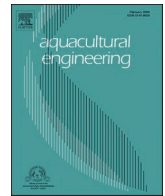
**Takedown policy**

Please contact us and provide details if you believe this document breaches copyrights.  
We will remove access to the work immediately and investigate your claim.



Contents lists available at ScienceDirect

# Aquacultural Engineering

journal homepage: [www.elsevier.com/locate/aque](http://www.elsevier.com/locate/aque)

## A probabilistic framework for offshore aquaculture suitability assessment using bivariate copulas

R. Santjer<sup>a,b,\*</sup>, P. Mares-Nasarre<sup>c</sup>, L. Vilmin<sup>a</sup>, G.Y.H. El Serafy<sup>a,b</sup>, O. Morales-Nápoles<sup>c</sup>

<sup>a</sup> Department of Data Science and Waterquality, Deltares, Boussinesqweg 1, Delft 2629 HV, the Netherlands

<sup>b</sup> Faculty of Electrical Engineering, Mathematics and Computer Science, Delft University of Technology, Mekelweg 4, Delft 2628 CD, the Netherlands

<sup>c</sup> Faculty of Civil Engineering and Geosciences, Delft University of Technology, Stevinweg 1, Delft 2628 CN, the Netherlands

### ARTICLE INFO

#### Keywords:

Probabilistic framework  
 Probabilistic dependence  
 Bivariate copula  
 Joint distribution  
 Offshore aquaculture  
 Suitability maps  
 Ecological variables

### ABSTRACT

Aquaculture at sea is gaining increasing importance, not only as a (local) food source but also due to its potential of being combined with other offshore activities such as wind parks. Nevertheless, experience of offshore aquaculture is limited. This study aims to provide a framework to evaluate offshore aquaculture suitability accounting for the probabilistic dependence between relevant variables. This framework is applied to obtain suitability maps of aquaculture for the North Sea for the blue mussel *Mytilus edulis* and the sugar kelp *Saccharina latissima*. For each of these species, three ecological variables are selected and the optimal growth and critical survival limits are defined. Here, suitability is defined as the probability of meeting these conditions. Data on the selected variables is extracted from a large-scale 3D hydrodynamic and ecological model of the northwest European Shelf, of which daily extremes are sampled. The probabilistic model is developed using bivariate copula models, which are fitted to each variable pair to describe their joint distribution function at each studied location. Empirical distribution functions are used to describe the univariate distribution function of each variable and location. Using Monte-Carlo simulations, the probability of meeting the optimal and critical limits is estimated and suitability maps accounting for the probabilistic dependence between the variables are generated. In addition, suitability maps disregarding the dependence are generated and compared to those accounting for the probabilistic dependence. It was found that considering the dependence between variables significantly improves the accuracy of the results for optimal and critical growth conditions for both species. The presented method allows to identify potential areas where blue mussel and sugar kelp cultivation is the most suitable. For instance, in this study, a north-south elongated area west of the German and Danish coast appears to be most suitable for blue mussels, while estuaries and rivers are found the most suitable for the sugar kelp.

### 1. Introduction

The increasing human world population (Tripathi et al., 2019) leads to an increased demand for sustainable food sources and green energy (Yong et al., 2022). Consequently, the production sector of aquaculture has experienced rapid global growth, continuing to expand despite the pandemic in recent years (Michler-Cieluch et al., 2009; FAO, 2022). Species such as blue mussels and especially sugar kelp have the potential to be used as food, source material for pharmaceutical industry, biomaterial or bioenergy sources such as biogas or bioethanol (Schiener, 2014; Kerrison et al., 2015; Fernand et al., 2017; Kammler et al., 2024), thus contributing to the solution to climate change (Yong et al., 2022). However, available space for aquaculture near-shore is rare. Thus,

species cultivation offshore becomes more relevant, even though the available space is limited and the competition at sea among various sectors pose another challenge to the novel sector of offshore aquaculture (Buck and Langan, 2017; Buck et al., 2018). As a result, aquaculture of certain species combined with wind energy production have emerged as a promising multi-use solution to address the above raised challenges as supported by the European Union (European-Commission, 2010, 2012). Moreover, there is evidence to suggest that offshore conditions can lead to increased growth and better product quality of some species (Buck, 2002; Brenner et al., 2012). However, while the offshore wind energy sector is experiencing a rapid development (Michler-Cieluch et al., 2009), offshore aquaculture, as well as the combination of several sectors, still needs to be further investigated and developed. Thus, the

\* Corresponding author at: Department of Data Science and Waterquality, Deltares, Boussinesqweg 1, Delft 2629 HV, the Netherlands  
 E-mail address: [r.santjer@tudelft.nl](mailto:r.santjer@tudelft.nl) (R. Santjer).

<https://doi.org/10.1016/j.aquaeng.2024.102479>

Received 14 February 2024; Received in revised form 16 October 2024; Accepted 20 October 2024

Available online 24 October 2024

0144-8609/© 2024 The Author(s). Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>).

presented study provides a framework to assess spatial suitability of offshore aquaculture from an environmental perspective.

Although the importance of co-use and multi-use has been recognised and emphasised by both the research community and policy makers (e.g. Buck et al., 2004; Wever et al., 2015), experience in the field of offshore aquaculture is limited and thus comes with several risks and uncertainties. First steps given by the scientific community point towards the technical and biological suitability of offshore cultivation of certain species (Buck et al., 2008; Maar et al., 2023). Technical feasibility was confirmed by Gagnon and Bergeron (2017), who analysed an exposed submerged mussel long-line structure, 4 km off the Canadian east coast. The measured forces on the structure were significantly smaller than the breaking strength of the lines and no damage such as mussel fall-off was observed. As defined by (Buck et al., 2024), “offshore” refers to a location with a distance of more than 3 nautical mile from the shore, while “exposed” relates to the physical oceanographic conditions. In an European project (UNITED Project, 2022), a pilot case was conducted to test the cultivation blue mussels and sugar kelp at an exposed offshore site. Regarding the biological feasibility, Buck (2002); Brenner et al. (2012) suggested that offshore conditions can lead to increased growth and better product quality of some species due to the good water quality and oxygen concentrations (Hopkins et al., 1996). Opposite conclusions were reached by Stechele et al. (2022), where less productivity was observed offshore than near-shore. However, it should be noted that these authors performed this research for a self-regulating cultivation technique, where wild larvae spat settles and cultivates at the structure. There are other factors in favour of offshore cultivation, such as limited inter-annual variability and lower risk of toxic algae blooms. Other benefits of the open ocean are sufficient water depths and the existence of infrastructure, especially when co-locating aquaculture with wind parks (Buck, 2002; Maar et al., 2023). This allows for shared vessels, operation and maintenance. Thus, offshore aquaculture as single-use, or in the context of multi-use, has a great potential leading to a growing need for tools to support the development of these novel sustainable offshore aquaculture practices (Bergström and Lindgarth, 2016).

In previous studies, different approaches have been used to explore the suitability of offshore aquaculture practices. Benassai et al. (2014) focused on the co-location of offshore wind and open-water aquaculture in Danish waters and the southern part of the North Sea. These authors developed a “sustainability” index which provided a first large-scale evaluation of the suitability of aquaculture in offshore wind park sites. This index was computed through a scoring model which considers physical limitations, such as water depth, wind velocity and water temperature, and biological parameters, such as chlorophyll-a concentrations.

Gimpel et al. (2015) developed a marine spatial planning (MSP) tool to evaluate spatial co-location scenarios of offshore aquaculture integrated in wind farms. This tool accounts for environmental, economic, inter-sectoral and social-cultural risks and opportunities and considers 13 species native in the German North Sea, resistant to offshore hydrodynamic conditions and economically interesting for the European market. Di-Tullio et al. (2017) also proposed a “sustainability index” to assess the co-location of offshore wind and open-water mussel cultivation using remote-sensing data for physical and biological variables, following the research done in Benassai et al. (2014). Geisler et al., 2018 conducted a feasibility study on various species at the FINO3 research platform in the German North Sea, located 80 km off the coast. In this feasibility study, a scoring model was applied, considering biological, technical and economical/political aspects for several different species. The aforementioned authors identified the blue mussel *Mytilus edulis* and sugar kelp *Saccharina latissima* as the most suitable species for cultivation at this location in the North Sea. Therefore, to the author’s knowledge, methods available in the literature to identify potential locations for offshore aquaculture are deterministic so they do not directly account for the natural variability (uncertainty) of the relevant variables

or the probabilistic dependence between them. This is, each variable is included in the analysis through a single statistic, but variables in nature typically present a stochastic behaviour. Moreover, the relationship between variables is not explicitly considered, although knowing information about one variable can provide insights about the distribution of another variable. For example, if the water temperature rises, it is likely that the concentration of dissolved oxygen will decrease but a deterministic estimation of one variable solely based on the other is typically not realistic. This indicates a probabilistic relationship between the two variables. In studies such as Ruiz-Velazco et al. (2013) or Santjer et al. (2023), explicitly accounting for the relationship between environmental variables was shown to play a relevant role in feasibility assessments. In Ruiz-Velazco et al. (2013), a linear relationship between variables was assumed to predict the shrimp production under commercial conditions, highlighting the importance of the interactions between relevant variables. Santjer et al. (2023) took the first steps to include the probabilistic dependence between variables and explored the applicability of different bivariate copulas to model the probabilistic dependence between environmental variables to assess the suitability of cultivating blue mussels and sugar kelp in a location in the German North Sea. Best fitting models were identified and a significant influence of the probabilistic dependence was reported. Moreover, bivariate copula models have been widely used in existing literature to model multivariate joint distribution of variables (e.g.: Leontaris et al., 2016; Ragno et al., 2023; Mares-Nasarre et al., 2024), being concluded that considering the dependence between the variables has a severe impact on the results. Thus, there are cases where the probabilistic dependence plays a key role and needs to be accounted for Ju et al. (2014). Another recent study made use of copula models, describing the joint dependence between two variables to predict sedimentation processes in aquaculture systems (López-Rebollar et al., 2024). Consequently, the present study aims to provide a probabilistic framework which accounts for the dependence between several ecological variables using bivariate copulas to assess the suitability of offshore aquaculture. This framework is applied in the south-eastern North Sea (see Fig. 1) to the cultivation of *Mytilus edulis*, hereon blue mussels, and *Saccharina latissima*, hereon sugar kelp. These two species have been identified as suitable using a deterministic scoring model by Geisler et al., 2018. This methodology

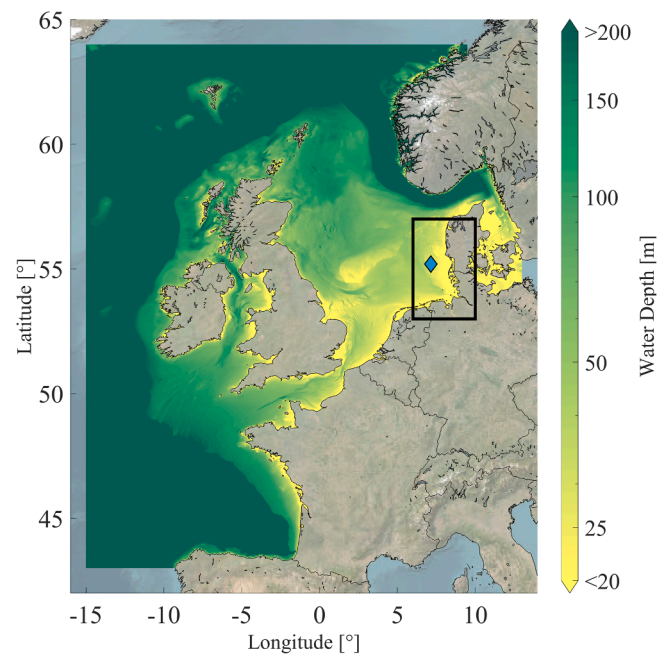


Fig. 1. Water depths for the European Shelf. The area of interest (AOI) of this study is indicated with a black frame, and the location of the FINO3 research platform is marked by a blue diamond.

allows to identify ecologically suitable locations for offshore cultivation and, thus, can potentially contribute to the development of multi-use approaches that could take advantage from offshore environments (Buck, 2002), the design of marine spatial plans and the planning of a more sustainable use of offshore environments.

The present paper is structured as follows. First, the three-dimensional hydrodynamic and ecological numerical model used to extract the observations of the environmental variables is described in Section 2.1. In order to assess the suitability of blue mussels and sugar kelp, three ecological variables per species are selected based on the existing literature in Section 2.2. Also, for each of the variables, the optimal growth and critical survival limits are defined. The methodology to build the probabilistic model proposed in this study is described in Section 2.4. For each observation point, daily extremes are extracted during the cultivation months of each species. The dependence for each pair of variables is first assessed through correlation analysis in Section 3.1 and, later, the best fitting bivariate copula model for each pair of variables is investigated across the AOI in Section 3.2). The probabilities of meeting the critical and optimal limits for each species are computed using the developed probabilistic models in Section 3.3. These results are compared to the probability calculations assuming independence between the variables to assess whether the dependence plays a key role

in the process. The strengths and limitation of the developed framework, its transferability to other species and areas and the obtained results are discussed in Section 4. Finally, conclusions are drawn in Section 5.

## 2. Materials and methods

In this section, the steps of the proposed probabilistic framework for performing suitability analysis in aquaculture are described, as well as the case studies used to showcase its application. The methodology is applied to two example species, namely blue mussels and sugar kelp, to assess their spatial suitability for offshore cultivation. A flowchart providing an overview of the method described below is displayed in Fig. 2.

### 2.1. 3D hydrodynamic and water quality model

The environmental conditions in the area of interest, AOI, marked by a black rectangle in Fig. 1, are extracted from model outputs of the 3D Dutch Continental Shelf Model - Flexible Mesh (3D DCMSM-FM) (Zijl et al., 2023). Hydrodynamics and water quality processes are computed using the D-Flow Flexible Mesh (D-Flow FM) component from the Delft 3D Flexible Mesh Suite (Deltares, 2023a). Water quality processes are

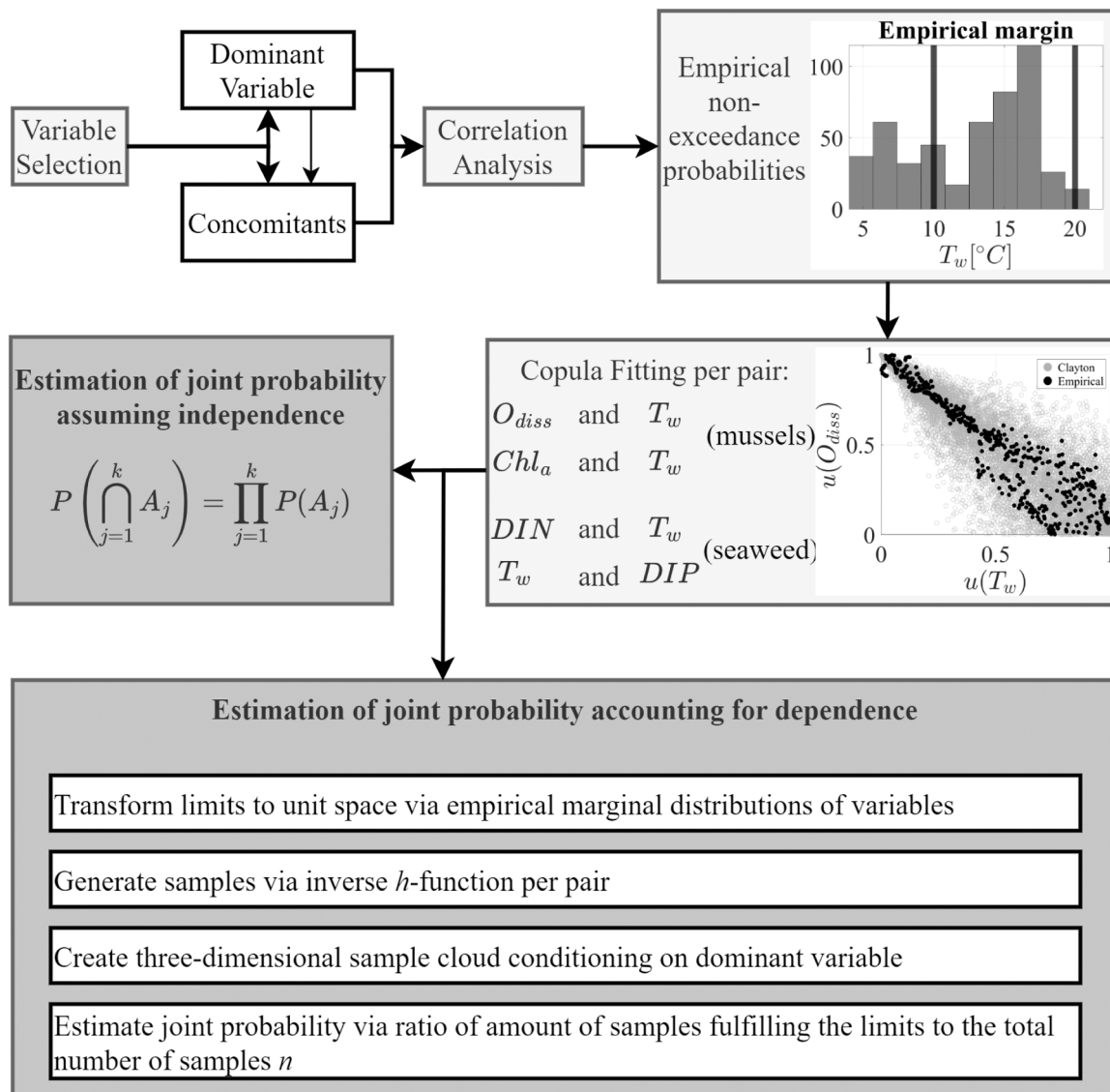


Fig. 2. Flowchart displaying the main procedures of the described methodology.



simulated with the D-Water Quality module (Deltares, 2023b), fully integrated within D-Flow FM.

In this study, the coupled hydrodynamic-water quality 3D DCSM-FM version described in (van Leeuwen et al., 2023) is used. The 3D DCSM-FM model domain covers the entire North-Western European Shelf from 15°W to 13°E and from 43°N to 64°N, including the North Sea (Fig. 1). The horizontal model grid is coarser near the offshore boundaries and in deep waters ( $\approx 4 \times 4$  nautical miles) and the resolution increases toward the shallower waters and in the Southern North Sea to  $0.5 \times 0.5$  nautical miles. The water column is represented using a z-sigma layer approach. The top 100 m (or less in shallower areas, such as at the AOI) are divided into a fixed number of 20 uniform layers (sigma-layers). Beneath 100 m-depth the water column is divided into z-layers, at fixed depths over the entire domain, with thicknesses increasing exponentially by a factor 1.19 towards the deeper layers. The hydrodynamic component of the model simulates water levels, currents and tides as well as temperature and salinity. The water quality component of the model simulates the most relevant processes in the cycling of major nutrients (nitrogen, phosphorus and silicate), of organic carbon and dissolved oxygen. Water quality processes parameterisation is based on Blauw et al. (2009).

The model was run for 2 years (2014 and 2015), using a 2-year spinup (2012–2013 conditions), allowing to account for some inter-annual variability. These years were selected, as these are the most recent years computed and validated by van Leeuwen et al. (2023). Model output is produced at an hourly timestep and a 4 nautical mile spatial resolution for the AOI (from 6°E to 10°E and 53.5°N to 57°N, see Fig. 1).

## 2.2. Variable selection and sampling

In this study, the proposed methodology is applied to assess the suitability of offshore cultivation of two species, blue mussel *Mytilus edulis* and sugar kelp *Saccharina latissima*, in the south-eastern North Sea. Here, their growth differences and variables affecting their growth are described for each species. Many factors can impact the growth of these species, such as hydrodynamic variables (waves and currents) and light availability, especially for exposed locations offshore. However, since there is already existing knowledge on the suitability of such offshore aquaculture based on hydrodynamic variables (e.g., Azevedo et al., 2019; Buck and Buchholz, 2005; Geisler et al., 2018), they are disregarded here. Instead, the focus is on ecological variables, as these are as important for assessing the suitability of sites (Schmidt et al., 2018). As salinity is not limiting for the respected species in the North Sea, it is not considered here either.

Three variables per species are considered to assess their cultivation suitability based on the existing literature (references in Table 1). The water temperature  $T_w$  [°C] is selected for both species. In addition, the

**Table 1**  
Overview of the selected three relevant variables and their limits for optimal growth and survival per species, based on literature.

Blue mussels			
Variable	Optimal	Critical	References
$T_w$ [°C]	10 – 20	$\leq 25$	Fly and Hilbish (2013); Kamermans et al. (2022)
$O_{diss}$ [g/m <sup>3</sup> ]	> 2	$\leq 2$	Belivermis et al. (2020); Tang and Riisgard (2018)
$Chl_a$ [mg/m <sup>3</sup> ]	3 – 10	> 0.5	Kamermans et al. (2022); Pascoe et al. (2009); Riisgard et al. (2011); Filgueira et al. (2009)
Sugar kelp			
Variable	Optimal	Critical	Reference
$T_w$ [°C]	5 – 15	$\leq 20$	Kerrison et al. (2015)
$DIN$ [g/m <sup>3</sup> ]	> 0.35	> 0.098	Jevne et al. (2020)
$DIP$ [g/m <sup>3</sup> ]	> 0.0484	> 0.0136	via Redfield Ratio Redfield (1934)

dissolved oxygen  $O_{diss}$  [g/m<sup>3</sup>] and chlorophyll-a concentrations  $Chl_a$  [mg/m<sup>3</sup>] are considered for the blue mussels. For the sugar kelp, the dissolved inorganic nitrogen  $DIN$  [g/m<sup>3</sup>] and the dissolved inorganic phosphorus concentration  $DIP$  [g/m<sup>3</sup>] are selected in addition to  $T_w$ . Further information about the variables is given below, while an overview of the selected variables can be found in Table 1. This Table also contains information about the optimal and critical limits per variable, which are further described below per species, and the references used to define the variables and their limits. Time series including the limits per variable for both species can be found in the Appendix A for the research platform FINO3 (see location of the blue diamond in Fig. 1, which is used as a reference location in this study.)

### 2.2.1. Blue Mussel, *Mytilus edulis*

In this study, the cultivation of the blue mussels *Mytilus edulis* is considered as growing at submerged long-lines at sea. Buck (2007) investigated such cultivation in the German Bight and concluded that these structures for mussel cultivation should be at least 6–7 m below mean surface level (MSL) to avoid high impacts of waves. The long-line was installed at a depth of 5 m, while the cultivation range was from 5 to 8 m. Another pilot project in the German North Sea (Strothotte et al., 2021) far offshore cultivated the species at depths of 7–11 m below MSL. Therefore, 9 m below MSL is selected in the current study and the selected ecological variables are extracted from the numerical model at that depth. Although the blue mussels are growing throughout the whole year, the main growth season is between March and October. Therefore, data from these months is selected for the analysis.

As already described, the selected variables are water temperature  $T_w$  [°C], dissolved oxygen  $O_{diss}$  [g/m<sup>3</sup>] and chlorophyll-a concentration  $Chl_a$  [mg/m<sup>3</sup>] based on existing literature (Bergström and Lindegarh, 2016; Tang and Riisgard, 2018; Kamermans et al., 2022; Li et al., 2022). Blue mussels are relatively resistant to extreme water temperatures (high and low), but optimal growth conditions exist for certain temperature ranges according to Fly and Hilbish (2013). The minimum dissolved oxygen concentration to enable growth of blue mussels should be above 2 g/m<sup>3</sup> (Tyler-Walters and Hiscock, 2008; Tang and Riisgard, 2018). This is similar to the concentrations of  $Chl_a$ , which should not fall below 0.5 mg/m<sup>3</sup> to ensure the survivability of the species. Thus, minima are selected to be most critical for both,  $O_{diss}$  and  $Chl_a$ .

### 2.2.2. Sugar kelp, *Saccharina latissima*

The second species considered in the current study is the sugar kelp *Saccharina latissima*. According to Peteiro and Freire (2011), the ropes for the sugar kelp are recommended to be at depths of 2–4 m below MSL. In the pilot project in the German North Sea (Strothotte et al., 2021), the sugar kelp is cultivated at depths of 0–4 m below MSL. Thus, here 2 m below MSL is selected. As the sugar kelp develops better in colder water temperatures, the main growth season is from September until May. Therefore, data from this period at a depth of 2 m is used in this study.

The growth and survival of the sugar kelp relies heavily on the water temperature  $T_w$ , as water temperatures above 20°C are lethal for the selected type of seaweed (Kerrison et al., 2015). The temperature ranges that lead to the highest growth rates are given in Table 1. Besides, two additional variables are inevitable for the growth of the sugar kelp according to Buck and Buchholz (2004): dissolved inorganic nitrogen,  $DIN$  (composed of both, ammonium, NH<sub>4</sub>, and nitrate, NO<sub>3</sub>), and dissolved inorganic phosphorus,  $DIP$  (phosphate, PO<sub>4</sub>), each in [g/m<sup>3</sup>]. Since too low concentrations for both,  $DIN$  and  $DIP$ , are critical for the growth and survival of the species, they are selected in this study.

### 2.2.3. Variable sampling

In this study, the daily maxima of the water temperatures  $T_w$  at the appropriate depth (9 m and 2 m below MSL for blue mussels and sugar kelp, respectively) are selected. The concomitants of 4 h before and after the maxima of  $T_w$  are determined for the other variables and their minimum values are selected.

Fig. 3 shows an example of the data used for the relevant variables  $T_w$ ,  $O_{diss}$  and  $Chl_a$  of the blue mussel case at the FINO3 location. The figure presents scatter plots per variable pair in their respective units, to allow for a comparison of pairwise relationships. The limits from Table 1 are represented by dashed lines. On the diagonal of Fig. 3, the empirical probabilistic distribution functions (pdf's) per variable are shown. Fig. F.1 in the Appendix F.1 shows the scatter data of the relevant variables  $T_w$ ,  $DIN$  and  $DIP$  for the sugar kelp.

### 2.3. Spearman rank correlation

Spearman's rank correlation coefficient (Spearman, 1987),  $r$ , is calculated between each pair of variables as a starting point of the dependence analysis to assess the strength of the dependence between variables and its significance (e.g. Hanea et al., 2006; Santjer et al., 2023; Mares-Nasarre et al., 2024).  $r$  computes Pearson's correlation coefficient (Pearson and Galton, 1895) between the ranks of each variable to assess monotonic relationships different to Pearson's which detects linear relationships. Thus, the Spearman's rank correlation coefficient  $r$  is used here to assess the degree of monotonic dependence between the random variables and is given by

$$r = \frac{Cov[R(X), R(Y)]}{\sigma_{R(X)}\sigma_{R(Y)}} \quad (1)$$

where  $Cov[R(X), R(Y)]$  is the covariance of the ranked variates of  $X$  and  $Y$ , such as  $T_w$  or  $O_{diss}$ , and  $\sigma_{R(X)}$  and  $\sigma_{R(Y)}$  are their standard deviations.  $r \in [-1, 1]$ , where  $r = 1$  and  $-1$  represent the perfect (monotonic) positive and negative correlation, respectively. In order to determine the significance of the observed correlations, the  $p$ -values are calculated. This significance test is done via the student-t distribution. Therefore, first the  $t$ -score is calculated (Zar, 1972):

$$t = r\sqrt{\frac{n-2}{1-r^2}} \quad (2)$$

where  $n$  are the degrees of freedom or sample size. Following, the  $p$ -value is defined as the combined area in both tails of the student-t distribution. Therefore, the  $p$ -value per tail can be determined via the cumulative distribution function of the student-t distribution evaluated at the  $t$ -score from Eq. (2) with  $(n-2)$  degrees of freedom. A given rank correlation is statistically significant if the  $p$ -value is below the significant level of  $\alpha = 0.05$  and thus, it is unlikely that the observed correlation is due to chance.

The goal of this analysis is to assess the strength of the dependence between the selected variables (see Section 2.2) and whether they are significant along the AOI. This can be used as a first assessment of whether incorporating dependence between the variables is significant for suitability analysis.

### 2.4. Copula modeling

Copulas are a popular approach to model dependence between random variables (e.g., Nelsen, 2006; Genest and Favre, 2007; Joe, 2015). The concept of copulas is based on Sklar's theorem (Sklar, 1959): any multivariate joint distribution can be described in terms of a set of univariate marginal distributions and a copula that models the dependence between the variables. Thus, copulas are multivariate distribution functions whose one-dimensional margins are uniform on the interval  $[0, 1]$ . For the bivariate case, as used in this study, copulas can be defined as follows:

Let  $H(x, y)$  for  $(x, y) \in \mathbb{R}^2$  be a joint distribution with univariate marginals  $F(x)$  and  $G(y)$ , then there exists a copula  $C$  in the unit square  $I^2 = ([0, 1] \times [0, 1])$ :

$$H(x, y) = C(F(x), G(y)) \quad (3)$$

Eq. (3) is satisfied for all  $(x, y) \in \mathbb{R}^2$ .

#### 2.4.1. Empirical non-exceedance probabilities

In order to later calculate probabilities of meeting the optimal and critical limits (see Table 1), the univariate uncertainty of each variable needs to be modelled per location. This is done using the univariate empirical distribution functions, as the limits from Table 1 are within the range of observations and thus no extrapolation is required. Table 2 shows the empirical non-exceedance probabilities corresponding to those limits at the location of the FINO3 research platform. Note that  $F_{T_w}(x)$  denotes here the empirical cumulative distribution function of the random variable  $T_w$  evaluated at  $x$ . Fig. 4 gives an example of the empirical distribution functions of the three variables  $T_w$ ,  $O_{diss}$  and  $Chl_a$  of the blue mussel case at the location of the FINO3 research platform, which are displayed together with the optimal limits indicated by vertical dashed lines. Also, the empirical copulas are presented. For an example of the case of the sugar kelp, see Fig. F.2 in Appendix F.1.

#### 2.4.2. Characteristics of parametric copula models applied in this study

In this study, five commonly used parametric bivariate copula models, with different characteristics are considered to capture the possible asymmetric behaviour that the joint distributions of each pair may exhibit. Copula models can be used to describe these asymmetries, such as tail dependence. Upper tail dependence occurs when the correlation between the high values of the two modelled random variables is greater than that between lower values. Lower tail dependence occurs when the opposite behavior is observed. For more information see Appendix B.

The copula models considered in this study are described below. Fig. 5 shows samples generated from the five different bivariate copulas considered in this study with the same rank correlation of  $r = 0.8$ , namely (a) Gaussian, (b) Frank, (c) Gumbel, (d) Clayton, and two t-copulas with different degrees of freedom  $\nu$  in (e) and (f). Their bivariate cumulative distribution functions (cdf's) are given in Appendix C.

The density of the Gaussian copula is ellipse-shaped and thus symmetric (see Fig. 5 (a)). The distribution of the t-copula is also elliptical but has a star-like shape and stronger dependence in the tails. Different to the other considered copula models in this study, the bivariate t-copula has two copula parameters. The first parameter is the same as for the Gaussian copula, the correlation coefficient  $\rho$ , while the second parameter is the shape parameter  $\nu$ , describing the degree of freedom. High degrees of freedom make the t-copula converge towards the Gaussian copula (see Fig. 5 (f) for  $\nu = 10$ ). The density of the Frank copula is also symmetric and has similar characteristics to the Gaussian copula, with the difference that it has greater dependence in both tails than in the centre. The Gaussian, Frank and t-copula are symmetrical and thus are not characterised by any tail dependence (Joe, 2015). Different to that, the Gumbel copula has an upper tail dependence in the top right quadrant, as the correlation is higher in this quadrant of the bivariate distribution, while the Clayton copula has a lower tail dependence in the bottom left quadrant (see Fig. 5 (c) and (d)). Because of the different tail dependencies of these two copula models, rotation might be needed during the copula fitting process. Therefore, the quadrant with the highest correlation of the variable pair is determined and the Gumbel and Clayton copula are rotated clockwise by  $90^\circ$ ,  $180^\circ$  or  $270^\circ$ , if necessary, such that their tail dependence is in the quadrant with the strongest correlation.

#### 2.4.3. Goodness-of-fit measures for copula selection

Goodness-of-fit (GOF) measures are needed to assess and compare the performance of different parametric copula models. Two GOF measures are applied here. The first one is the Cramér-von Mises (CvM) statistic  $S_n$ , as described by Genest et al. (2009). It determines the sum of squared differences between the empirical joint cdf and the joint cdf of the parametric copula model.  $S_n$  for a sample length of  $n$  is determined as:

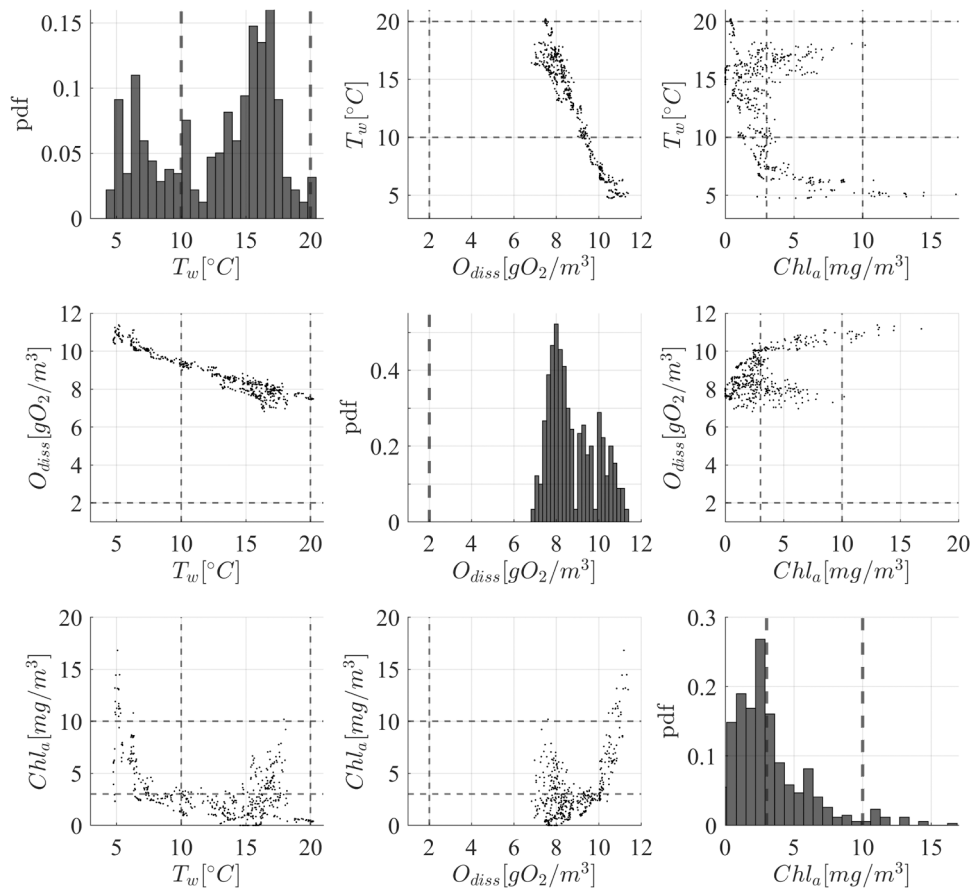


Fig. 3. Overview of sampled data for blue mussels at the location of the FINO3 research platform together with the optimal limits from Table 1, while on the diagonal the empirical probabilistic distribution functions (pdfs) are shown.

Table 2

Overview of the selected three relevant variables and the univariate probability of meeting the limits (optimal and critical, see Table 1), based on their empirical marginal distributions, here for an example location at the FINO3 research platform.

Blue mussels		
Variable	Optimal	Critical
$T_w$	$F_{T_w}(10) = 0.298$ $F_{T_w}(20) = 0.989$	$F_{T_w}(25) = 1$
$O_{diss}$	$1 - F_{O_{diss}}(2) = 1$	$1 - F_{O_{diss}}(2) = 1$
$Chl_a$	$F_{Chl_a}(3) = 0.601$ $F_{Chl_a}(10) = 0.959$	$1 - F_{Chl_a}(0.5) = 0.918$
Sugar kelp		
Variable	Optimal	Critical
$T_w$	$F_{T_w}(5) = 0.084$ $F_{T_w}(15) = 0.802$	$F_{T_w}(20) = 1$
$DIN$	$1 - F_{DIN}(0.35) = 0$	$1 - F_{DIN}(0.098) = 0.671$
$DIP$	$1 - F_{DIP}(0.0484) = 0$	$1 - F_{DIP}(0.0136) = 0.562$

$$S_n(\mathbf{u}) = n \sum_{|n|} \{C_n(\mathbf{u}) - C_{\hat{\theta}_n}(\mathbf{u})\}^2, \mathbf{u} \in [0, 1]^2 \quad (4)$$

Here,  $C_n(\mathbf{u}) = \frac{1}{n} \sum_{i=1}^n 1(U_i \leq \mathbf{u})$  is the empirical copula and  $C_{\hat{\theta}_n}(\mathbf{u})$  is the parametric copula with estimated parameter  $\hat{\theta}_n$  from the sample. The lower  $S_n(\mathbf{u})$ , the better the GOF. For the calculation of the CvM statistics, the BANSHEE toolbox version 1.3 was used (see Paprotny et al. (2020) and Mendoza-Lugo and Morales-Nápoles (2023) for the MATLAB implementation and Koot et al. (2023) for the Python implementation).

The second measure to assess the GOF of the copula models are the semi-correlations, as described by Joe (2015). For this method, the

uniform margins are transformed to standard normal  $N(0, 1)$  margins through the inverse of the standard normal univariate Gaussian distribution. This allows to better evaluate asymmetries such as tail dependence. Then, the correlation coefficients per quadrant are calculated. For positive correlated samples, the upper and lower semi-correlation coefficients are defined as:

$$\begin{aligned} \hat{\rho}_N^+ &= \text{Cor}[Z_1, Z_2 | Z_1 > 0, Z_2 > 0], \\ \hat{\rho}_N^- &= \text{Cor}[Z_1, Z_2 | Z_1 < 0, Z_2 < 0]. \end{aligned} \quad (5)$$

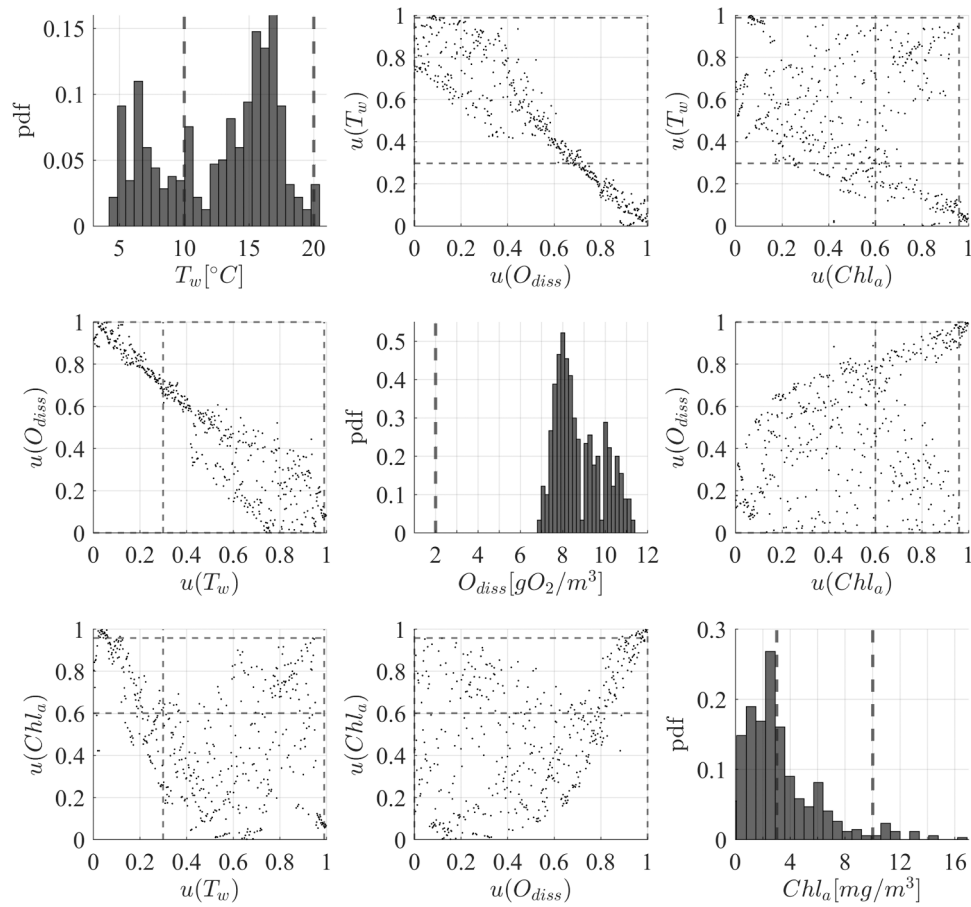
The smaller the difference of the semi-correlation coefficients between the empirical and parametric copula, the better the fit. If the GOF results of the two described methods do not align, the results of  $S_n$  are followed.

### 2.5. Joint multivariate probability calculation

Once the parametric copulas for each pair of variables and locations are selected and fitted, the joint probabilities of meeting optimal growth and critical survival limits are computed for the respective growth period of the species. These probabilities are computed for each location with and without considering dependence between the variables to assess the role of dependence. Assuming independence, the joint probability can be calculated as follows:

$$P\left(\bigcap_{j=1}^k A_j\right) = \prod_{j=1}^k P(A_j) \quad (6)$$

where  $P(A_j)$  is the probability of variable  $A_j$  meeting the optimal (or critical) condition. These probabilities are determined using the empirical cdf's of the variable data sets (see Section 2.4). When



**Fig. 4.** Overview of uniform observations of the sampled data for blue mussels at the location of the FINO3 research platform together with the empirical non-exceedance probabilities of the optimal limits (see Table 2), while on the diagonal the univariate empirical probabilistic distribution functions are shown together with the optimal limits from Table 1.

considering dependence between variables, the fitted bivariate copulas are used as in higher dimensions multivariate copula modelling becomes more difficult and loses part of its flexibility. Here, a pair copula construction or a one-tree vine-copula (Joe, 2015) is applied similar to previous studies (e.g. Mares-Nasarre et al., 2024), as shown in Fig. 6. The one-tree vine-copula consists of three nodes which represent the three relevant variables per species and two edges ( $C_{ij}$ ), which are the bivariate copulas. Note that the variables  $X_1$  and  $X_3$  are then conditionally independent given  $X_2$ .

The model is built using the best fitting bivariate copulas for the pairs  $T_w$  and  $O_{diss}$ , and  $T_w$  and  $Chl_a$  for the blue mussels and bivariate copulas for the pairs  $T_w$  and  $DIN$ , and  $T_w$  and  $DIP$  for the sugar kelp. This is, the conditioning variable for both species (variable  $X_2$  in Fig. 6) is set to water temperature  $T_w$ , which is the dominant variable.  $T_w$  is easy to measure and influences the other selected variables, called conditioned variables.

The joint probability of meeting certain conditions per species and location is calculated here through numerical simulation as follows:

1. The limits defined in Table 1 are transformed to unit space via the empirical marginal distributions of the variables (see Table 2).
2. For each of the three variables per species, random samples with size  $n = 10,000$  in  $[0,1]$  are drawn from a uniform distribution, denoted as  $u_1$ ,  $u_2$  and  $u_3$ .  $u_1$  represents random samples of  $T_w$  per species, while  $u_2$  represents the conditional probabilities  $P(O_{diss}|T_w)$  for blue mussels and  $P(DIN|T_w)$  for sugar kelp. Consequently,  $u_3$  represents the conditional probabilities  $P(Chl_a|T_w)$  for blue mussels and  $P(DIP|T_w)$  for sugar kelp, respectively.

3. A set of  $n$  samples per variable pair ( $u_1$  and  $u_2$ , and  $u_1$  and  $u_3$ , respectively) is generated for each species using Monte-Carlo simulations. This is done via the cdf and the inverse  $h$ -function (Aas et al., 2009) for the best fitting copula models. This results in sample clouds per variable pair conditioning on  $T_w$ . More information about the inverse  $h$ -function can be found in Appendix D.
4. In order to determine the joint probability of these variables satisfying the optimal or critical growth requirements, the samples fulfilling these conditions are counted (see an example in Fig. 7, where the green samples fulfil the requirements). The ratio of this amount of samples to the total number of samples  $n$  provides an estimate of the probability accounting for the dependence between the pairs.

### 3. Results

#### 3.1. Correlation analysis of the data

A correlation analysis is performed via the Spearman rank correlation coefficient as described in Section 2.3. The ranges of these rank correlation coefficients for the offshore AOI (see Fig. 1) are shown in Table 3.

As shown in Table 3, the correlations are strong for the offshore AOI, mostly linked with  $p$ -values below the significance level (here  $\alpha = 0.05$ ) being then significant correlations (see Section 2.3). For the variable pairs of the sugar kelp, correlations are occasionally found not to be significant. This indicates that the dependence between the variables might be relevant and, thus, needs to be considered in the suitability analysis. The results in the following sections will mainly focus on the blue mussel due to the stronger dependence between the variables.



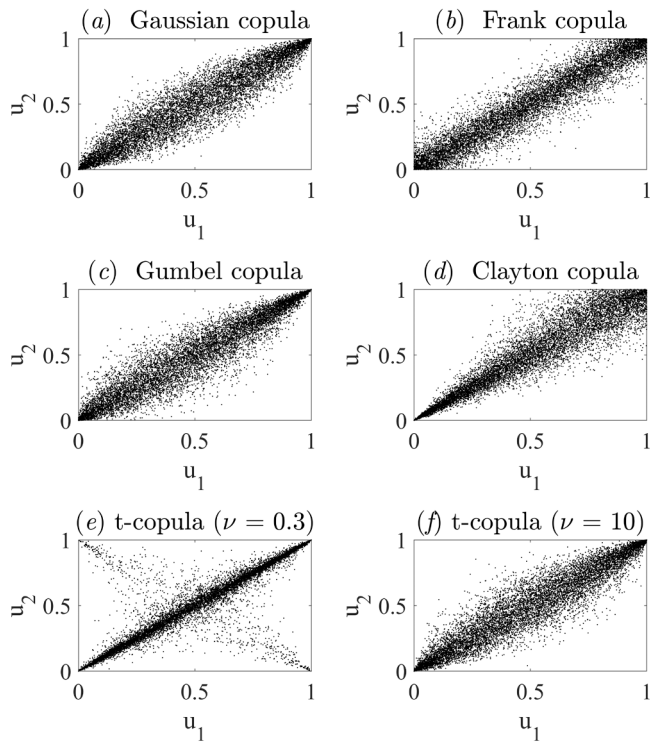


Fig. 5. Samples generated for random variables with rank correlation of  $r = 0.8$  for (a) Gaussian, (b) Frank, (c) Gumbel, (d) Clayton and (e) and (f) t-copula with  $\nu = 0.3$  and  $\nu = 10$  respectively, each for  $n = 10,000$  samples.

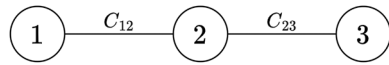


Fig. 6. One-tree vine with three nodes and two edges.

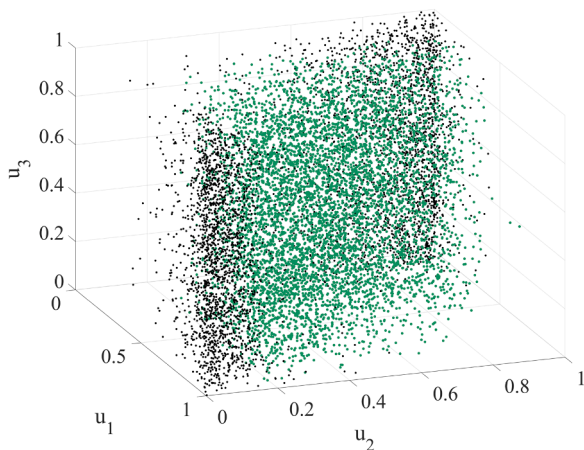


Fig. 7. Three-dimensional sample cloud for an arbitrary example, with  $u_1$  being the conditioning variable, where the samples in green are meeting the required conditions.

Partial results for the sugar kelp will be presented; the complete results can be found in the Appendix F3.

For further results on the spatial variability of the rank correlation coefficients and corresponding  $p$ -values, the reader is referred to the Appendix E.1 (Fig. E.1 and Fig. E.2) for the blue mussels and to the Appendix F.2 (Fig. F.3 and Fig. F.4) for the sugar kelp.

Table 3

Correlation ranges in the offshore part of the south-eastern North Sea.

Blue mussel	
$T_w - O_{diss}$	-0.95 to -0.9
$T_w - Chl_a$	-0.68 to -0.2
Sugar kelp	
$T_w - DIN$	-0.6 to 0
$T_w - DIP$	-0.7 to 0

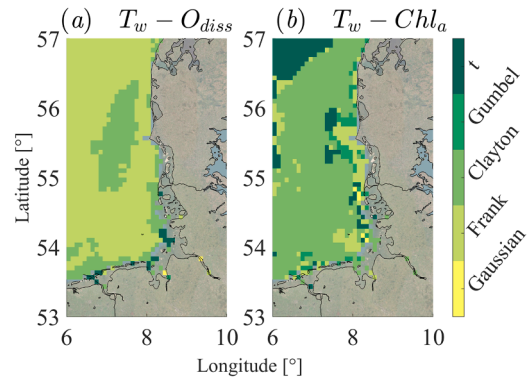


Fig. 8. Best fitting copulas across the AOI for both variable pairs for the blue mussels: (a)  $T_w$  and  $O_{diss}$  and (b)  $T_w$  and  $Chl_a$ .

### 3.2. Copula selection and fitting

As introduced in Fig. 4 copulas are used here to model the dependence between the pairs of studied variables. Best copula model is selected by fitting a selection of copula families (Gaussian, Frank, Gumbel, Clayton and t-) and comparing them using two GOF techniques: (1) the CvM statistic, and (2) the semi-correlations, as exposed in Fig. 4<sup>1</sup>.

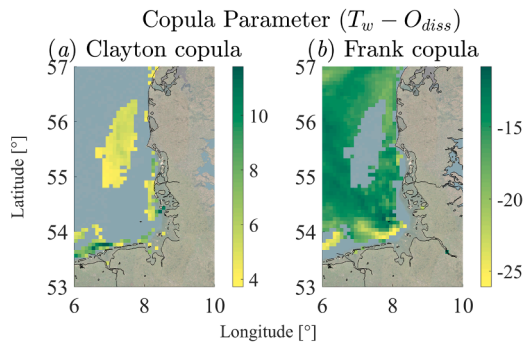
The spatial variability of the best fitting copula model for the variable pairs  $T_w$  and  $O_{diss}$ , and  $T_w$  and  $Chl_a$  is displayed in Fig. 8 for the blue mussel case, where each copula is represented by a different colour. For the sugar kelp, the results for the pairs  $T_w$  and  $DIN$ , and  $T_w$  and  $DIP$  are shown in the Appendix F.3 (Fig. F.5 (a) and Fig. F.5 (b)).

In the case of the blue mussels, the Frank copula is the best fit for the variable pair  $T_w$  and  $O_{diss}$  across most of the investigated area, as displayed in Fig. 8 (a), with a parameter that ranges from -24 to -12 (see Fig. 9 (b)). The Clayton copula is the optimal fit for the central region of the North Sea and its parameter varies between 4 and 11, as shown in Fig. 9 (a). The main difference in the results for these two areas is the correlation in the upper left tail quadrant: if the correlation in this quadrant for the pair of  $T_w$  and  $O_{diss}$  is below -0.9, the Clayton copula provides the best fit. When correlation goes above -0.9, the symmetrical Frank copula becomes the best fit. Note that  $T_w$  and  $O_{diss}$  are negatively correlated so it is required to rotate the Frank and Clayton copula. As this variable pair is characterised by a strong correlation in the upper right quadrant, the rotation angle for the Clayton copula is consistently 270° for the entire area. The rotation angle of the Clayton copula for the pair of  $T_w$  and  $O_{diss}$  can be found in Fig. E.3 (c) in the Appendix E.2. Different to the Clayton copula, the Frank copula is symmetrical and is thus rotated by 90° for negative correlations.

Regarding the pair  $T_w$  and  $Chl_a$ , the Clayton copula provides the best fit (see Fig. 8 (b)) with the parameter increasing gradually from approximately 0 in the south-east to 1.3 in the north-west of the area, similar to the correlation pattern (see Fig. E.1 (b) in the Appendix E.1).

<sup>1</sup> Because of the extensive volume of results obtained from these two fitting methods, the findings are not presented in this study but can be made available upon request.



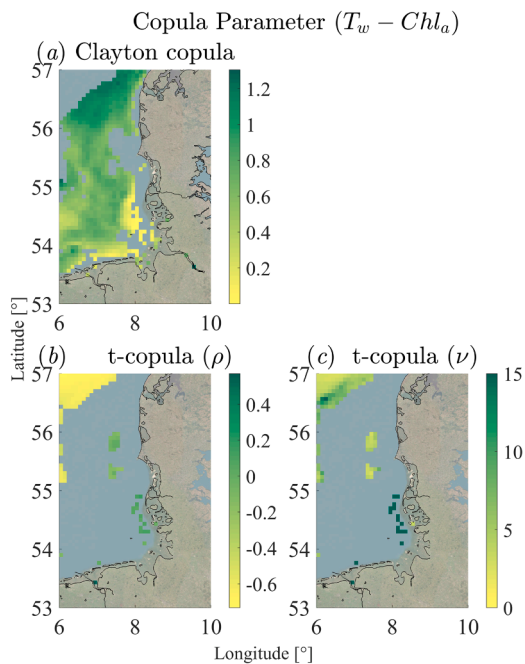


**Fig. 9.** The spatial copula parameter for the Clayton copula (a) and the Frank copula (b) for the variable pair of  $T_w$  and  $O_{diss}$  for the blue mussel.

Again, the variables are negatively correlated, so rotation of the copulas is required. Different to the pair of  $T_w$  and  $O_{diss}$ , the degree of rotation angle for the Clayton copula is less uniform, although  $270^\circ$  is still the most prominent (see Fig. E.3 (d) in the Appendix E.2).

The t-copula is the best fitting copula model for the pair  $T_w$  and  $Chl_a$  in the northwestern part, where the correlation is the strongest. The bivariate t-copula presents two parameters:  $\rho$  (correlation coefficient) and  $\nu$  (degrees of freedom);  $\rho$  varies between  $-0.6$  and  $-0.7$ , while the parameter  $\nu$  ranges from 2 to 10 (see Fig. 10 (b) and (c)). However, note that for a  $\nu > 10$ , the t-copula approximates the Gaussian copula. Finally, near the coastline, where the correlation is not found significant, the fitting of copula models is difficult and spread between the different considered models.

Regarding the sugar kelp, the fitting of copula models for the variable pair  $T_w$  with  $DIN$  does not present one dominant copula model across the entire studied area (see Fig. F.5 (a) in the Appendix F.3) due to the low observed correlations. In regions with a stronger correlation near the coastline and further offshore, the Frank or Gaussian copula models appear to be the most suitable fit. Thus, no significant tail dependence is observed between the variables. The variability of the parameters for these two fitted copulas is shown in Fig. F.6 in the Appendix F.4. For a great part of the studied area offshore, the t-copula is predominant.



**Fig. 10.** The spatial copula parameter for (a) the Clayton copula, (b)  $\rho$  and (c)  $\nu$  of the t-copula for the variable pair of  $T_w$  and  $Chl_a$  for the blue mussel.

However, the copula parameter  $\nu$  reaches values up to  $10^7$ , approximating a Gaussian copula.

Different to the previous variable pair, the fitting of copula models is less varied across the area for the variable pair  $T_w$  with  $DIP$  (as depicted in Fig. F.5 (b) in the Appendix F.3). In regions with correlations between  $-0.4$ – $0$ , the t-copula is the best fit with very high  $\nu$  values (see Fig. F.7 in the Appendix F.4). In regions where the correlation is stronger (near the coast and further offshore), the Gumbel and Frank copulas are identified as the best fit. For the Gumbel copula, the parameter varies between 1 and 2.2, while for the Frank copula ranges between  $-8$  and  $-2$ . Note that the samples for the Gumbel copula are rotated by  $270^\circ$  (see Fig. F.8 (b) in the Appendix F.4) and for the symmetrical Frank copula by  $90^\circ$ , as the correlation of this variable pair of  $T_w$  and  $DIP$  is negative.

In summary, the fitting of copula families in the blue mussel case within the offshore AOI is rather uniform. In the nearshore area, however, where the nearshore processes take a relevant role, the correlation between variables is lower leading to less clear patterns. Similarly, for the sugar kelp, where correlations are in general lower, the fitting and selection of the best copula model becomes more challenging.

### 3.3. Probability calculation and suitability maps

After selecting the copula models to fit the dependence between the variables, the probabilities of meeting the optimal and critical thresholds for the three variables during the main growth season of each species is calculated as described in Section 2.5. These probabilities are also calculated under the independence assumption to investigate whether incorporating the dependence between variables improves the estimations.

First, the probabilities of meeting the critical limits (see Table 1) are investigated. For the blue mussels, these probabilities are for most of the area close to 1, regardless of whether dependence is considered or not (see Fig. E.5 (a) and (b) in the Appendix E.3). For the sugar kelp, it can be seen that the probability of meeting the critical limits is high close to the coast but decreases with increasing distance to the shore, as shown in Fig. F.10 in the Appendix F.5. In Fig. F.10 (c), the differences between the probabilities computed accounting for the dependence and disregarding it are displayed, reaching a maximum of about 0.09. Close to the coast, the differences are around 0, while offshore, the differences in probabilities are negative for the area with tail dependence (Gumbel model) between  $DIP$  and  $T_w$  and positive where symmetrical copula models (i.e., Frank and t-copulas) are the best fit. Note that these differences become more remarkable for the areas with low probabilities of meeting the critical thresholds (northwestern part of the AOI). When computing the difference relative to the actual probability considering dependence, the difference can be up to 40 %.

With regard to the optimal growth conditions for the blue mussels, the highest probabilities of meeting them are observed near the coast, exceeding 0.4, as displayed in Fig. 11 (a). As the distance from the shore increases, the probability gradually decreases, reaching approximately 0.2 for most of the offshore area. In an elongated area running from south to north along the German and Danish coast, a similar probability to that near the shore is observed. This pattern is caused by the differences in the univariate probabilities; the probability of meeting the optimal conditions is relatively consistent and high for  $T_w$  and  $O_{diss}$ , while the probability of  $Chl_a$  meeting the optimal conditions is more variable. It is highest close to the coast and within the aforementioned elongated shape, ranging from 0.6 to 0.8. In the rest of the area, the probability is relatively low, ranging from 0.2 to 0.4. This leads to  $Chl_a$  being the most limiting factor, as already pointed out by Geisler et al., 2018.

Regarding the difference between considering or disregarding the dependence between the variables, a maximum absolute difference of about 0.1 is found, as shown in see Fig. 11 (b). The darker area in Fig. 11 (b) indicates a positive difference between the dependent and independent approach, thus ignoring the dependence between variables

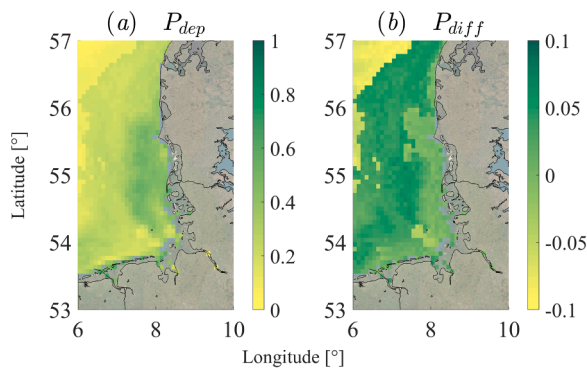


Fig. 11. Probabilities of meeting the optimal conditions for the suitability of blue mussel *Mytilus edulis*: (a) probabilities computed using the dependence models, and (b) (absolute) differences between the probabilities computed accounting for the dependence and assuming independence between the variables.

would underestimate the joint probability. In contrast to that, the lighter area northwest indicates a negative difference, such that assuming independence might overestimate the joint probability for these areas.

For the sugar kelp, the probabilities of meeting the optimal growth conditions across the entire area are very low regardless considering or not the dependence between the studied variables (see Fig. F.9 (a) and (b) in the Appendix F.5). While the probability of meeting the optimal conditions for  $T_w$  all along the domain and  $DIN$  close to the coast is relative high,  $DIP$  marginally fails to meet the specified conditions. The probability of  $DIP$  meeting the optimal concentrations is close to 0 for the entire area, except in estuaries and rivers, where the probability reaches a maximum of more than 0.6. This could be primarily explained by the nutrient influxes from coastal regions for both,  $DIN$  and  $DIP$ . The results of the present study align with the findings from previous studies. For instance, Gimpel et al. (2015) found that the offshore cultivation of blue mussels is in general feasible for the AOI, while for sugar kelp, regions with higher water temperatures and nutrient rich waters due to river inflows are mostly suitable.

In summary, the critical limits for the blue mussel species are generally met differently to those for the sugar kelp, especially further offshore. For the optimal growth conditions, the overall suitability across the entire area is higher for the blue mussels compared to the sugar kelp, as  $DIP$  marginally fails to meet the optimal limits. Besides, the strengths of the correlation between variables has a direct impact on whether the dependence makes a difference in the computed calculations and, thus, needs to be accounted for. If the correlation is not significant, the simplified independent probability calculation may be sufficient. However, accounting for the dependence when asymmetries such as tail dependence are present might be needed when dealing with extremes, namely minimum and maximum observations of the variables. Also, in cases where low probabilities are computed (e.g. the optimal requirements for the blue mussel species in Fig. 11 or critical conditions for the sugar kelp in Fig. F.10 in the Appendix F.5), the differences between accounting for the dependence or disregarding it can be up to values between 30 % and 40 %. This highlights the importance of considering the dependence between variables in suitability studied.

#### 4. Discussion

In this study, a probabilistic framework is proposed to assess the suitability of offshore aquaculture accounting for the dependence between relevant ecological variables. This methodology is applied as a case study to two species, the blue mussel *Mytilus edulis* and the sugar kelp *Saccharina latissima*. The growth offshore in the North Sea is investigated by computing the probability of occurrence of the optimal and critical conditions of these species. Such conditions are

parameterised using three variables per species during their growth season: temperature, dissolved oxygen and chlorophyll-a concentrations for the blue mussels, and temperature, dissolved inorganic nitrogen and phosphorus concentrations for the sugar kelp. Thresholds describing critical conditions for the survival of the species and optimal conditions for their growth are defined based on the existing literature. Empirical distribution functions are applied here to model the univariate uncertainty of each variable (see Section 2.4.1), while bivariate copula models are recommended in Section 2.5 to account for the probabilistic dependence between variables. Empirical distribution functions were used here since the optimal and critical limits are generally for most of the variables within the observed ranges and there is no need to extrapolate out of them. However, if those thresholds would be changed towards more extreme probabilities, parametric univariate margins would be needed.

The probabilities to meet the critical survival and optimal growth limits for each species were computed both accounting for the dependence using bivariate copulas and disregarding it. The results show that this may lead to significant differences in the computed probabilities and, thus, in the conclusions of the suitability study. Here, five parametric families of bivariate copulas were considered, namely Gaussian, Frank, Gumbel, Clayton, and t-copula. Such parametric copulas were selected as they are commonly used and they cover different shapes of symmetric dependence, as well as the asymmetry of tail dependence. However, no other asymmetries were considered within the considered models. This is, for a positive dependence, asymmetries between the upper right and lower left quadrant are considered, but not between the upper left and lower right quadrant. Thus, further extensions of this work may focus on analyzing the role of such asymmetries in the computation of the probabilities (Jaeger and Morales-Nápoles, 2017).

Regarding the results, high probabilities of meeting the optimal or critical requirements for the growth of the blue mussels are obtained. On the contrary, the probabilities of meeting optimal conditions for the sugar kelp are low due to the variables marginally failing to meet the specified optimal growth conditions. This is, the computed probabilities are sensitive to the defined thresholds. However, slightly different limits are provided for some of the studied variables depending on the source and there is no general consensus about the relevance of some ecological variables for the species to grow (Bergström and Lindegarh, 2016; Kamermans et al., 2022). Moreover, there is limited literature to define limits for  $DIN$  and  $DIP$  for sugar kelp. In this study, the conditions for optimal and critical  $DIN$  concentrations are taken from Jevne et al. (2020), where the growth of small sporophytes under controlled conditions in tanks for 20 days of experimental time was investigated. These experimental results were also used to determine the conditions for  $DIP$  via the molar Redfield ratio of 16:1 ( $molN/molP$ ) (Redfield, 1934). Therefore, field observations would be desirable to validate such limits, as well as further research on defining the critical and optimal threshold for growth. However, the goal of the applications here is to showcase the proposed probabilistic methodology and, thus, the computed probabilities can be recalculated in light of new literature.

Possible extensions of this study could include hydrodynamic factors like waves and currents that can be crucial for species (Kerrison et al., 2015) and the safety of the installed structures for their growth. Also, interactions between species, such as integrated multi-trophic aquaculture (IMTA) or influences of large-scale cultivation systems on e.g. the water quality can be included in extensions of this study. If the incorporation of further variables leads to higher dimensional models, copula-based Bayesian Networks (e.g. Hanea et al., 2006) or Vine Copula models (Joe, 2015) might be applied. Moreover, long-term effects such as the increase in water temperature or potential changes on the nutrients for the species due to climate change can be investigated and, if found significant, modelled using non-stationary margins (Ragno et al., 2019). Finally, future studies could aim to account for the relationship between consecutive extremes as, for example, the sugar kelp *Saccharina latissima* can tolerate short-term exposure to high  $DIP$

concentrations without negative effects for a few days (Lubsch and Timmermans, 2019). This is not investigated here, as this study analyses daily extremes independently, rather than in sequence.

The proposed method is highly transferable to other species, such as fish or oysters, to define cultivation suitability through probabilities by accounting for the probabilistic dependence structure among variables. Beyond aquaculture, renaturation or species reintroduction, not just in the marine context, are a possible field of application.

## 5. Conclusion

The offshore aquaculture sector is still in early stages. Some methodologies exist for spatial planning, but they do not account for the dependence between the relevant growth variables of species. In this study, a method accounting for this dependence between variables is proposed and its results are compared to the results when disregarding it (independent case). To model the joint distributions of the studied variables, bivariate copulas are applied. Using the developed models, the spatial suitability for the cultivation of two species, specifically blue mussels *Mytilus edulis* and sugar kelp *Saccharina latissima*, is assessed in the south-eastern North Sea. This is done by determining the probabilities of meeting certain conditions for three ecological variables per species. The results show that there is a significant difference in the probabilities when incorporating the dependence compared to assuming probabilistic independence. This is especially remarkable for areas with low probabilities (less suitable areas for cultivation) as not accounting for dependence can underestimate the suitability by up to 40 %. However, if the correlation between the variables is low, the independent approach may be sufficient, such as in most of the studied area for the sugar kelp. Besides, the results of this study indicate that both species' cultivation is generally suitable in the defined area with regard to their

survival conditions. For the optimal growth conditions, an offshore north-south elongated area along the German and Danish coast seems to be most suitable for blue mussel cultivation next to near-coast areas. Different to that, optimal conditions for the sugar kelp are only met in rivers and their estuaries.

## CRedit authorship contribution statement

**R. Santjer:** Writing – original draft, Writing – review & editing, Methodology, Formal analysis, Conceptualisation. **P. Mares-Nassarre:** Writing – review & editing, Conceptualisation. **L. Vilmin:** Writing – review & editing. **G.Y.H. El Serafy:** Funding acquisition. **O. Morales-Nápoles:** Writing – review & editing, Methodology, Conceptualisation.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

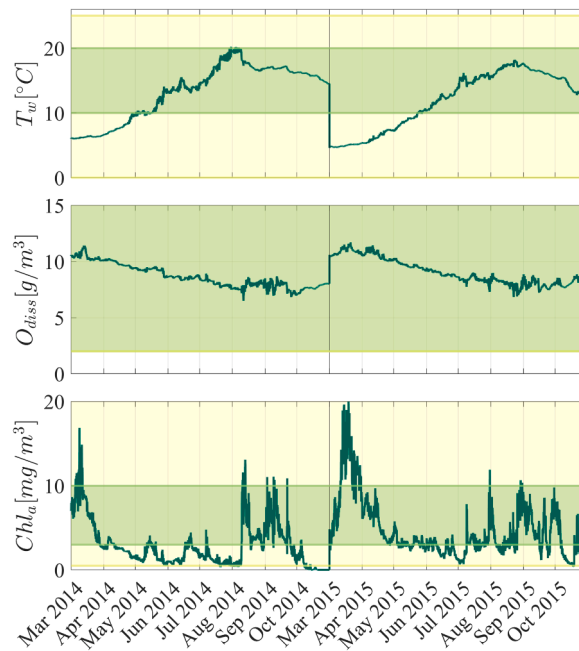
## Data Availability

Data will be made available on request.

## Acknowledgements

The first, third and fourth authors were financially supported by the European Union's Horizon 2020 Research and Innovation Programme under Grant Agreement no 862915. Special thanks to Eva Strothotte for her biological expertise and discussions.

## Appendix A. Timeseries of the studied variables



**Fig. A.1.** Time series of the relevant variables for the blue mussels, where the green background marks the optimal growth conditions and the yellow parts mark the critical survival conditions.

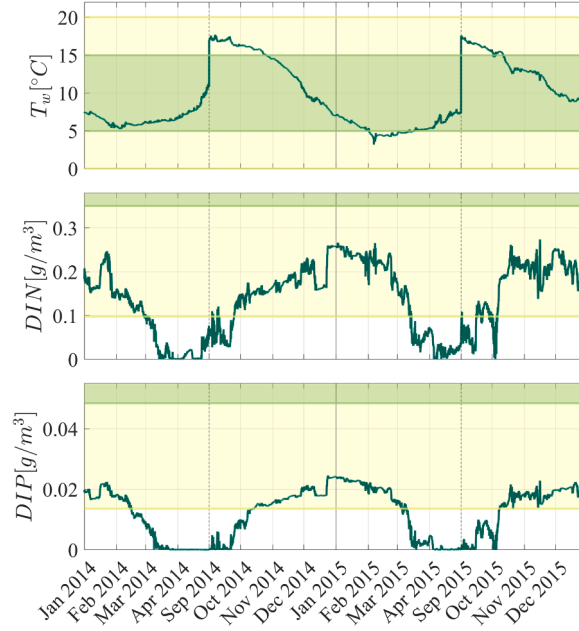


Fig. A.2. Time series of the relevant variables for the sugar kelp, where the green background marks the optimal growth conditions and the yellow parts mark the critical survival conditions.

### B. Tail dependence

Here, the tail dependencies are described.

The upper tail dependence parameter  $\lambda_U$  can be described as follows:

$$\lambda_U = \lim_{u \rightarrow 1} P[X > F_X^{-1}(u) | Y > G_Y^{-1}(u)], \tag{B.1}$$

$$\lambda_U \in [0, 1]$$

While the lower tail dependence parameter  $\lambda_L$  is determined by:

$$\lambda_L = \lim_{u \rightarrow 0} P[X \leq F_X^{-1}(u) | Y \leq G_Y^{-1}(u)], \tag{B.2}$$

$$\lambda_L \in [0, 1]$$

Where X and Y are continuous random variables with distribution functions |F| and |G|. If  $\lambda_U > 0$  then X and Y are upper tail dependent while if  $\lambda_L > 0$ , the random variables are lower tail dependent (Nelsen, 2006).

### C. Cumulative distribution functions of bivariate parametric copulas

Below, the bivariate cumulative distribution function (cdf) of the respected copula families are given (Czado, 2019; Nelsen, 2006; Joe, 2015).

The cdf for the bivariate Gaussian copula is given by

$$C(u_1, u_2; \rho_{12}) = \Phi_2(\Phi^{-1}(u_1), \Phi^{-1}(u_2); \rho_{12}) \tag{C.1}$$

where  $\Phi_2$  is the joint cdf of a bivariate normal distribution,  $\Phi^{-1}$  is the inverse cdf of a univariate standard normal distribution and  $\rho_{12}$  being the correlation coefficient.

The cdf for the bivariate t-copula is given by

$$C(u_1, u_2, \nu_{12}, \rho_{12}) = \int_0^{u_1} \int_0^{u_2} \frac{t(T_{\nu_{12}}^{-1}(v_1), T_{\nu_{12}}^{-1}(v_2); \nu_{12}, \rho_{12})}{t_{\nu_{12}}(T_{\nu_{12}}^{-1}(v_1)) t_{\nu_{12}}(T_{\nu_{12}}^{-1}(v_2))} dv_1 dv_2 \tag{C.2}$$

where  $T_{\nu_{12}}^{-1}(\cdot)$  is the inverse of the cdf of a univariate t-distribution with  $\nu_{12}$  degrees of freedom,  $t_{\nu_{12}}(\cdot)$  is the pdf of a univariate t-distribution with  $\nu_{12}$  degrees of freedom and  $t(\cdot, \cdot, \nu_{12}, \rho_{12})$  is the bivariate pdf of the t-distribution.

The cdf for the bivariate Frank copula is given by

$$C(u_1, u_2, \delta_{12}) = -\frac{1}{\delta_{12}} \ln \left( \frac{1 - e^{-\delta_{12}} - (1 - e^{-\delta_{12}u_1})(1 - e^{-\delta_{12}u_2})}{1 - e^{-\delta_{12}}} \right) \tag{C.3}$$

where  $-\infty < \delta_{12} < \infty$  is the copula parameter, describing the dependence, while  $\delta_{12} \rightarrow 0^+$  means independence.

The cdf for the bivariate Clayton copula is given by

$$C(u_1, u_2, \delta) = (u_1^{-\delta} + u_2^{-\delta} - 1)^{\frac{1}{\delta}} \tag{C.4}$$

where  $0 < \delta < \infty$  is the copula parameter, describing the dependence, while independence occurs with  $\delta \rightarrow 0$ .

The cdf for the bivariate Gumbel copula is given by

$$C(u_1, u_2, \delta) = \exp \left[ - \{ (-\ln u_1)^\delta + (-\ln u_2)^\delta \}^{\frac{1}{\delta}} \right] \tag{C.5}$$

where  $\delta > = 1$  is parameter of dependence, while independence is described with  $\delta = 1$ .

**D. (Inverse) conditional distribution functions**

Conditional distribution functions associated with a bivariate copula are used in this study. These are usually referred to as *h*-functions by (Aas et al., 2009). For a bivariate copula  $C_{XY}$ , the *h*-function is defined as:

$$h(x, y, \theta_{xy}) = F(x|y) = \frac{\partial C_{XY}(x, y, \theta_{xy})}{\partial y} \tag{D.1}$$

with *X* being the conditioned variable, *Y* being the conditioning variable, and  $\theta_{xy}$  being the set of parameters for the copula of the joint distribution function of *x* and *y*. It represents the cdf when *X* and *Y* are uniform.

Its inverse with respect to the first variable *X*,  $h^{-1}(X, Y, \theta_{xy})$  is equivalent to the inverse of the conditional distribution function. For the copulas used here (Gaussian, Gumbel, Clayton, Frank, and t-copula), the *h*-function is given by an explicit analytical expression that can be analytically inverted for all pair-copulas except for Gumbel, which requires numerical inversion (Aas et al., 2009; Czado, 2019). Below, the inverse conditional distribution functions (inverse *h*-functions) of the respected copula families are given, as well as the *h*-function for the Gumbel copula (Aas et al., 2009; Joe, 2015).

The inverse conditional distribution function (hinv) for the Gaussian copula is given by

$$h_{12}^{-1}(u_1, u_2, \rho_{12}) = \Phi \left\{ \Phi^{-1}(u_1) \sqrt{1 - \rho_{12}^2} + \rho_{12} \Phi^{-1}(u_2) \right\} \tag{D.2}$$

where  $\rho_{12}$  is the parameter of the copula,  $\Phi$  is the cdf of the standard normal distribution and  $\Phi^{-1}$  its inverse.

The inverse conditional distribution function (hinv) for the t-copula is given by

$$h_{12}^{-1}(u_1, u_2, \rho_{12}, \nu_{12}) = t_{\nu_{12}} \times \left\{ t_{\nu_{12}+1}^{-1}(u_1) \sqrt{\frac{(\nu_{12} + (t_{\nu_{12}}^{-1}(u_2))^2)(1 - \rho_{12}^2)}{\nu_{12} + 1}} + \rho_{12} t_{\nu_{12}}^{-1}(u_2) \right\} \tag{D.3}$$

where  $\rho_{12}$  is the parameter of the copula,  $\nu$  the degree of freedom, and  $t_{\nu_{12}}$  the standard cdf of the t-distribution and  $t_{\nu_{12}}^{-1}$  its inverse function.

The inverse conditional distribution function (hinv) for the Frank copula is given by

$$h_{12}^{-1}(u_1, u_2, \delta_{12}) = -\log \left\{ 1 - \frac{1 - e^{-\delta_{12}}}{(u_1^{-1} - 1)e^{-\delta_{12}u_2} + 1} \right\} / \delta_{12} \tag{D.4}$$

where  $\delta_{12}$  is the parameter of the copula.

The inverse conditional distribution function (hinv) for the Clayton copula is given by

$$h_{12}^{-1}(u_1, u_2, \delta_{12}) = \left\{ (u_1 u_2^{\delta_{12}+1})^{-\frac{\delta_{12}}{\delta_{12}+1}} + 1 - u_2^{-\delta_{12}} \right\}^{-1/\delta_{12}} \tag{D.5}$$

where  $0 < \delta_{12} < \infty$  is the parameter of the copula, controlling the dependence.

The conditional distribution function for the Gumbel copula is given by

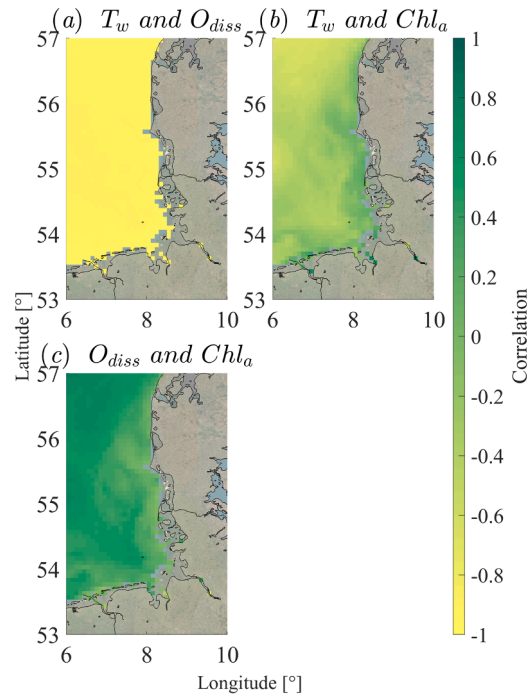


$$h(u_1, u_2, \delta_{12}) = C_{12}(u_1, u_2) \frac{1}{u_2} (-\log u_2)^{\delta_{12}-1} \times \{ (-\log u_1)^{\delta_{12}} + (-\log u_2)^{\delta_{12}} \}^{1/\delta_{12}-1} \tag{D.6}$$

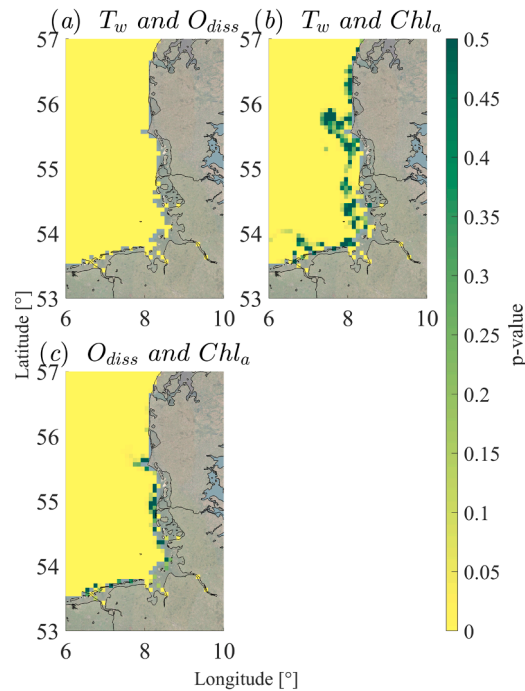
where  $C_{12}(u_1, u_2)$  is the copula, as defined in Equation (C.5) and  $\delta_{12} \geq 1$  is the parameter of the copula, controlling the dependence. Note that it is necessary to obtain the inverse of the h-function numerically.

**E. Additional results for the blue mussel case**

*E.1. Correlation analysis*



**Fig. E.1.** Correlations between the selected three variable pairs for the blue mussel case: (a)  $T_w$  and  $O_{diss}$ , (b)  $T_w$  and  $Chl_a$  and (c)  $O_{diss}$  and  $Chl_a$ .



**Fig. E.2.**  $p$ -values for the correlations between the selected three variable pairs for the blue mussel case: (a)  $T_w$  and  $O_{diss}$ , (b)  $T_w$  and  $Chl_a$  and (c)  $O_{diss}$  and  $Chl_a$ .

E.2. Angle of rotation for Gumbel and Clayton copula

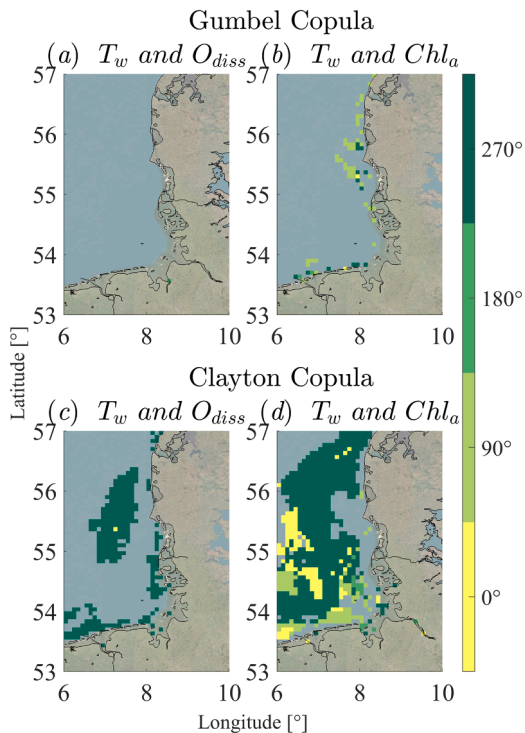


Fig. E.3. Rotation of the Gumbel Copula in (a) and (b) and rotation of the Clayton copula in (c) and (d) for the variable pairs of  $T_w$  and  $O_{diss}$ , and  $T_w$  and  $Chl_a$ , respectively.

E.3. Probability calculation

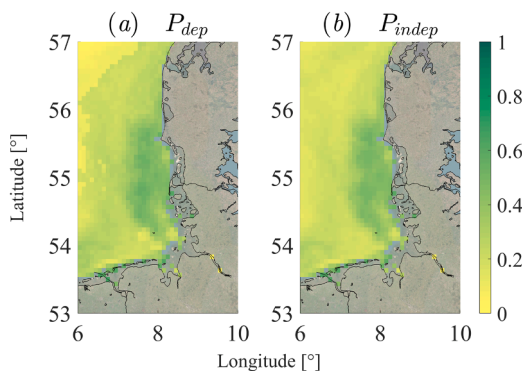
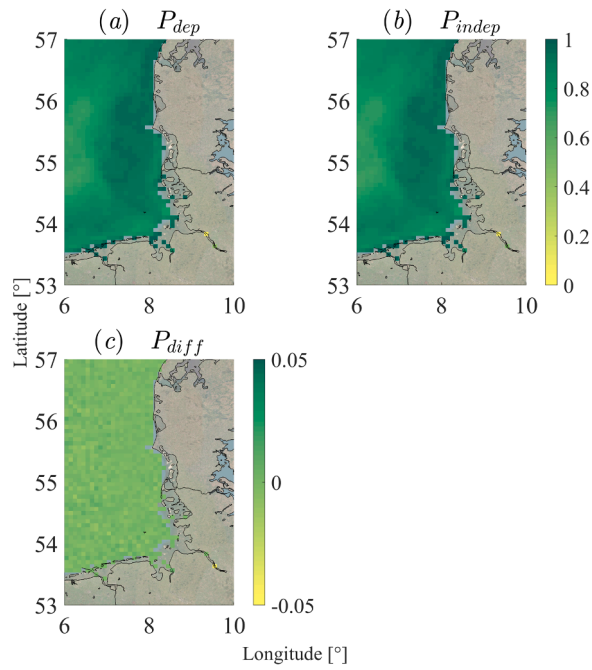


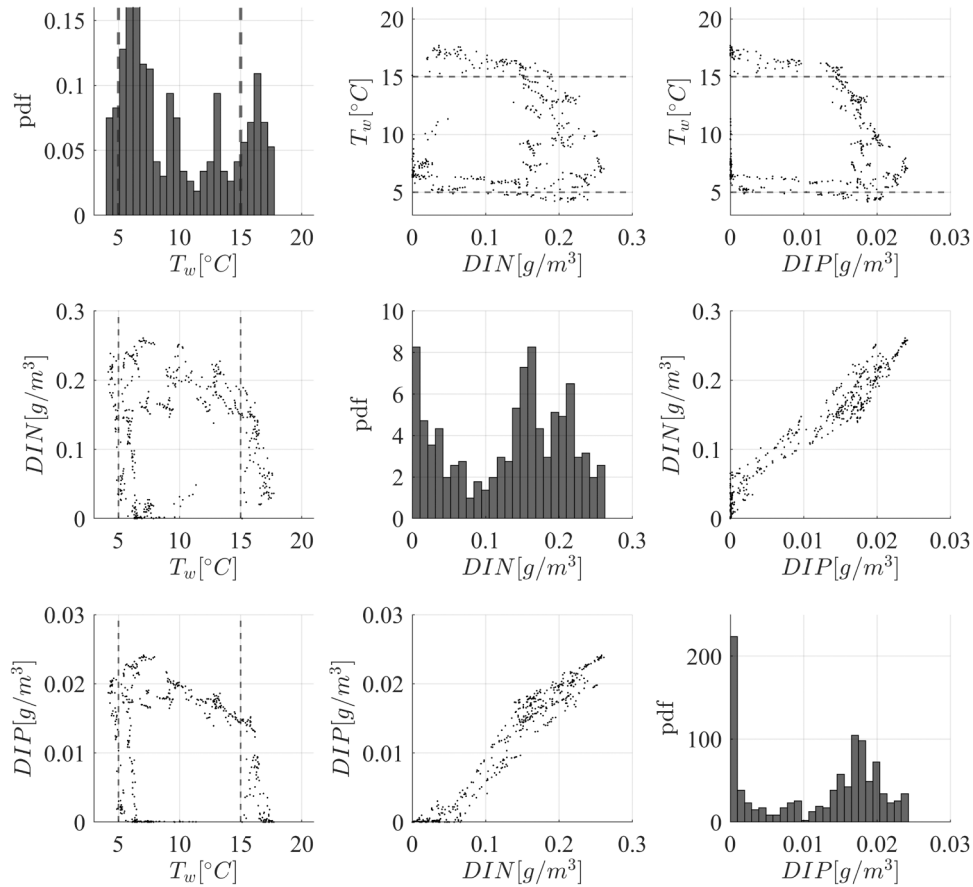
Fig. E.4. The spatial probabilities with (a) accounting for dependence ( $P_{dep}$ ) and (b) without ( $P_{indep}$ ), describing the suitability of blue mussel *Mytilus edulis* for the optimal growth conditions.



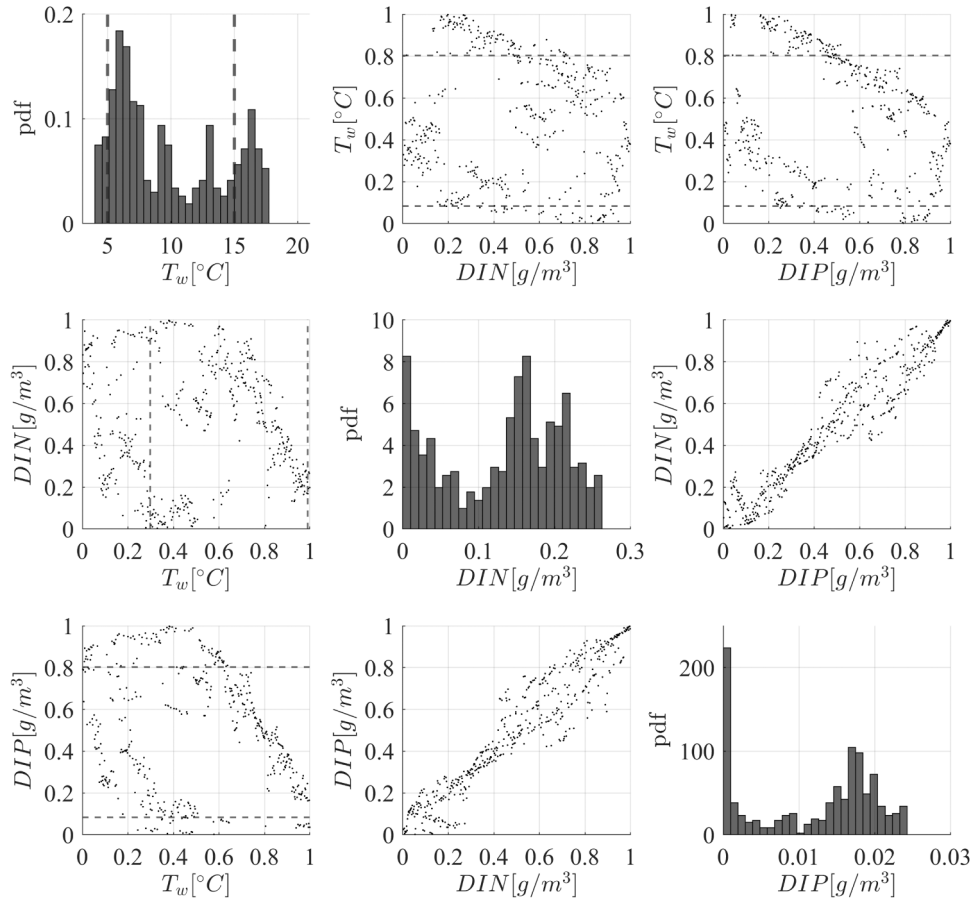
**Fig. E.5.** The spatial probabilities with (a) considering ( $P_{dep}$ ), (b) without considering dependence ( $P_{indep}$ ), describing the suitability of blue mussel *Mytilus edulis* for the critical growth conditions; while in (c), the differences between the two approaches is displayed.

**F. Additional results for the sugar kelp**

*F.1. Example of the sampled data*

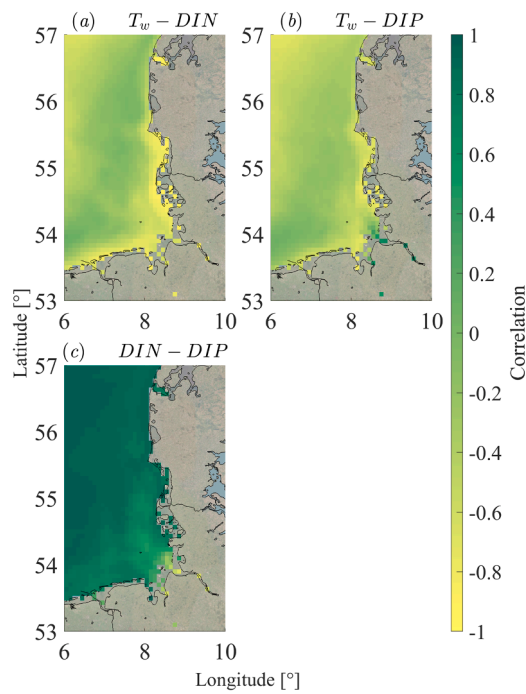


**Fig. F.1.** Overview of sampled data for sugar kelp at the location of the FINO3 research platform together with the optimal limits from [Table 1](#), while on the diagonal the empirical probabilistic distribution functions (pdfs) are shown.



**Fig. F.2.** Overview of uniform observations of the sampled data for sugar kelp at the location of the FINO3 research platform together with the empirical non-exceedance probabilities of the optimal limits (see Table 2), while on the diagonal the univariate empirical probabilistic distribution functions are shown together with the optimal limits from Table 1.

*F.2. Correlation analysis*



**Fig. F.3.** Correlations between the selected three variable pairs for the sugar kelp: (a)  $T_w$  and  $DIN$ , (b)  $T_w$  and  $DIP$  and (c)  $DIN$  and  $DIP$ .

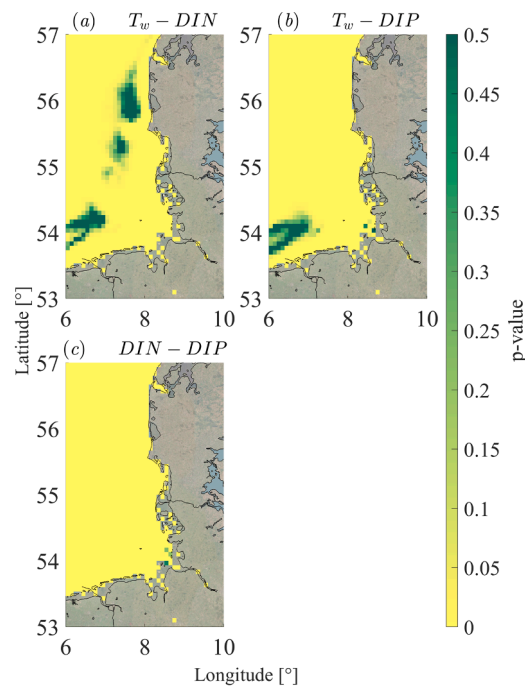


Fig. F.4.  $p$ -values for the correlations between the selected three variable pairs for the sugar kelp: (a)  $T_w$  and  $DIN$ , (b)  $T_w$  and  $DIP$  and (c)  $DIN$  and  $DIP$ .

F.3. Copula fitting results

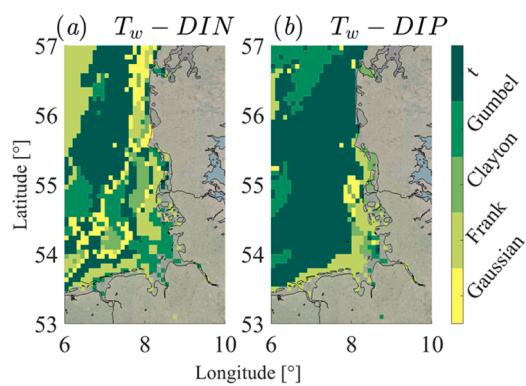


Fig. F.5. Best fitting copulas across the AOI for both variable pairs for the sugar kelp: in (a)  $T_w$  and  $DIN$  and in (b)  $T_w$  and  $DIP$ .



F.4. Copula parameter and angle of rotation for Gumbel and Clayton copula

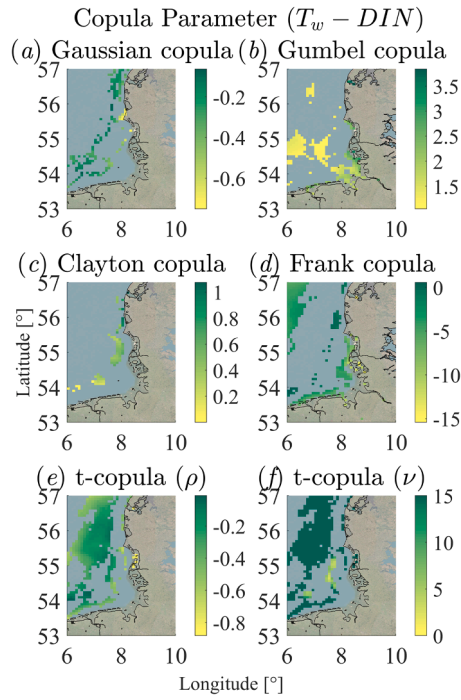


Fig. F.6. The spatial copula parameter for the respected copula families for the variable pair of  $T_w$  and  $DIN$  for the sugar kelp.

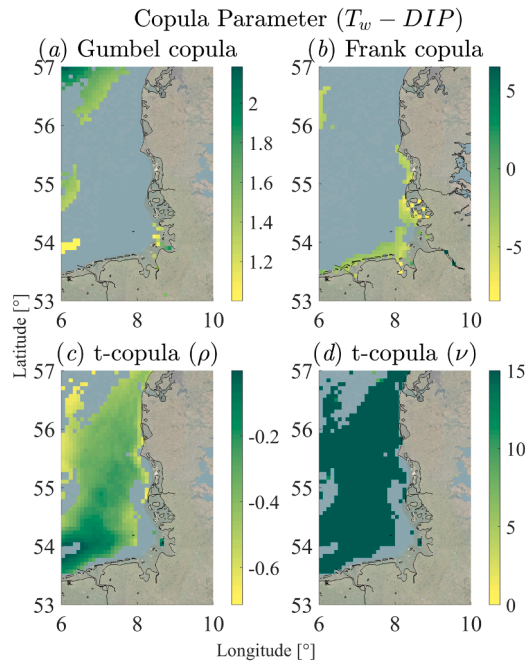


Fig. F.7. The spatial copula parameter for the Gumbel, Frank and t-copula for the variable pair of  $T_w$  and  $DIP$  for the sugar kelp.

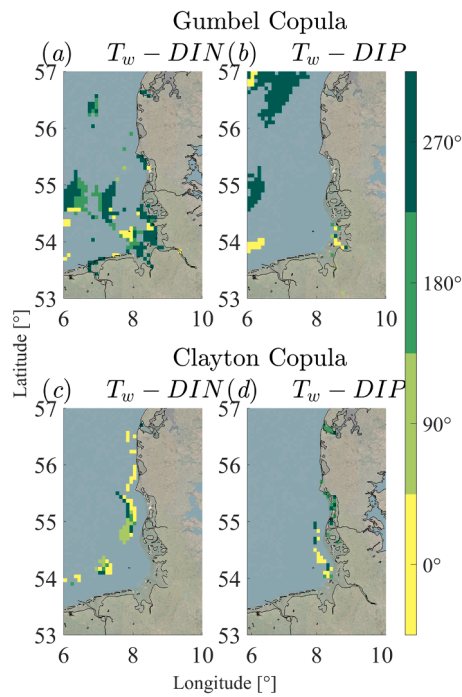


Fig. F.8. Rotation of the Gumbel and Clayton Copula for the sugar kelp for the variable pairs of  $T_w$  and  $DIN$  and  $T_w$  and  $DIP$ .

F.5. Probability calculation

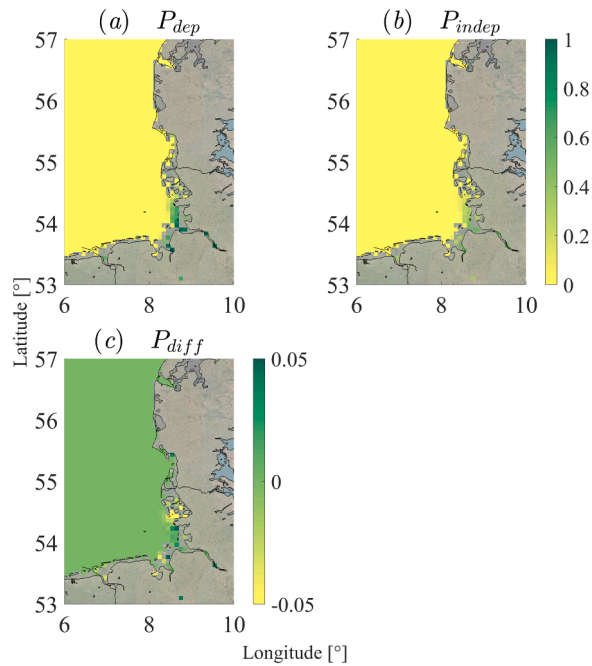
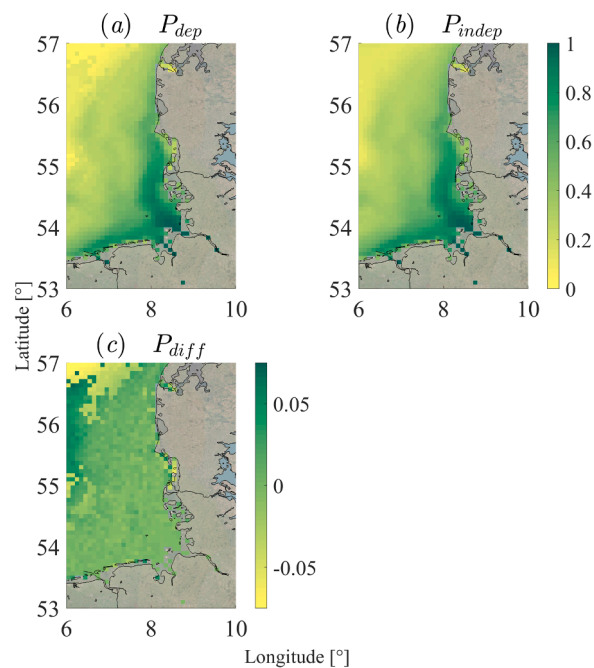


Fig. F.9. The spatial probabilities with (a) considering dependence ( $P_{dep}$ ), (b) without considering dependence ( $P_{indep}$ ), describing the suitability of the sugar kelp *Saccharina latissima* for the optimal growth conditions and (c) the difference between the two approaches.



**Fig. F.10.** The spatial probabilities with (a) considering dependence ( $P_{dep}$ ), (b) without considering dependence ( $P_{indep}$ ), describing the suitability of sugar kelp *Saccharina latissima* for the critical growth conditions and (c) the differences between the two approaches.

## References

- Aas, K., Czado, C., Frigessi, A., Bakken, H., 2009. Pair-copula constructions of multiple dependence. *Insur.: Math. Econ.* 44, 182–198. <https://doi.org/10.1016/j.insmatheco.2007.02.001>.
- Azevedo, I.C., Duarte, P.M., Marinho, G.S., Neumann, F., Sousa-Pinto, I., 2019. Growth of *saccharina latissima* (laminariales, phaeophyceae) cultivated offshore under exposed conditions. *Phycologia* 58, 504–515. <https://doi.org/10.1080/00318884.2019.1625610>.
- Belivermiş, M., Swarzenski, P.W., Oberhänsli, F., Melvin, S.D., Metian, M., 2020. Effects of variable deoxygenation on trace element bioaccumulation and resulting metabolome profiles in the blue mussel (*Mytilus edulis*). *Chemosphere* 250. <https://doi.org/10.1016/j.chemosphere.2020.126314>.
- Benassai, G., Mariani, P., Stenberg, C., Christoffersen, M., 2014. A sustainability index of potential co-location of offshore wind farms and open water aquaculture. *Ocean Coast. Manag.* 95, 213–218. <https://doi.org/10.1016/j.ocecoaman.2014.04.007>.
- Bergström, P., Lindegarth, M., 2016. Environmental influence on mussel (*Mytilus edulis*) growth - a quantile regression approach. *Estuar., Coast. Shelf Sci.* 171, 123–132. <https://doi.org/10.1016/j.ecss.2016.01.040>.
- Blaauw, A.N., Los, H.F.J., Bokhorst, M., Erftemeijer, P.L.A., 2009. Gem: a generic ecological model for estuaries and coastal waters. In: *Hydrobiologia*, 618, pp. 175–198.
- Brenner, M., Buchholz, C., Heemken, O., Buck, B.H., Koehler, A., 2012. Health and growth performance of the blue mussel (*Mytilus edulis* L.) from two hanging cultivation sites in the german bight: a nearshore-offshore comparison. *Aquac. Int.* 20, 751–778. <https://doi.org/10.1007/s10499-012-9501-0>.
- Buck, B.H., 2002. Open ocean aquaculture and offshore windparks: Eine Machbarkeitsstudie über die multifunktionale Nutzung von Offshore-Windparks und Offshore-Marikultur im Raum Nordsee. *Rep. Polar Mar. Res.* 254.
- Buck, B.H., 2007. Experimental trials on the feasibility of offshore seed production of the mussel *Mytilus edulis* in the german bight: Installation, technical requirements and environmental conditions. *Helgol. Mar. Res.* 61, 87–101. <https://doi.org/10.1007/s10152-006-0056-1>.
- Buck, B.H., Bjelland, H.V., Bockus, A., Chambers, M., Costa-Pierce, B.A., Dewhurst, T., Ferreira, J.G., Fore, H.M., Fredriksson, D.W., Goseberg, N., Holmyard, J., Isbert, W., Krause, G., Markus, T., Papandroulakis, N., Sclodnick, T., Silkes, B., Strand, A., Troell, M., Wiecezorek, D., van den Burg, S.W.K., Heasman, K.G., 2024. Resolving the term “offshore aquaculture” by decoupling “exposed” and “distance from the coast. *Front. Aquac.* 3, 1428056. <https://doi.org/10.3389/faqc.2024.1428056>.
- Buck, B.H., Buchholz, C.M., 2004. The offshore-ring: A new system design for the open ocean aquaculture of macroalgae. *J. Appl. Phycol.* 16, 355–368. <https://doi.org/10.1023/B:JAPH.0000047947.96231.ea>.
- Buck, B.H., Buchholz, C.M., 2005. Response of offshore cultivated *Laminaria saccharina* to hydrodynamic forcing in the North Sea. *Aquaculture* 250, 674–691. <https://doi.org/10.1016/j.aquaculture.2005.04.062>.
- Buck, B.H., Krause, G., Rosenthal, H., 2004. Extensive open ocean aquaculture development within wind farms in Germany: the prospect of offshore co-management and legal constraints. *Ocean Coast. Manag.* 47, 95–122. <https://doi.org/10.1016/j.ocecoaman.2004.04.002>.
- Buck, B.H., Krause, G., Michler-Cieluch, T., Brenner, M., Buchholz, C.M., Busch, J.A., Fisch, R., Geisen, M., Zielinski, O., 2008. Meeting the quest for spatial efficiency: progress and prospects of extensive aquaculture within offshore wind farms. *Helgol. Mar. Res.* 62, 269–281. <https://doi.org/10.1007/s10152-008-0115-x>.
- Buck, B.H., Langan, R., 2017. Aquac. Perspect. Multi-Use Sites Open Ocean. doi:10.1007/978-3-319-51159-7.
- Buck, B.H., Troell, M.F., Krause, G., Angel, D.L., Grote, B., Chopin, T., 2018 State of the art and challenges for offshore integrated multi-trophic aquaculture (IMTA). doi: 10.3389/fmars.2018.00165.
- Czado, C., 2019. Analyzing Dependent Data with Vine Copulas - A Practical Guide With R. vol. 222 ed. 10.1007/978-3-030-13785-4.
- Deltares, 2023a. D-FLOW Flexible Mesh. User Manual, Deltares.
- Deltares, 2023b. D-Water Quality Processes Library Description. Technical Reference Manual, Deltares.
- Di-Tullio, G.R., Mariani, P., Benassai, G., Luccio, D.D., Grieco, L., 2017. Sustainable use of marine resources through offshore wind and mussel farm co-location. *Ecol. Model.* 367, 34–41. <https://doi.org/10.1016/j.ecolmodel.2017.10.012>.
- European-Commission, 2010. A strategy for smart, sustainable and inclusive growth.
- European-Commission, 2012. Blue growth opportunities for marine and maritime sustainable growth.
- FAO, 2022. The state of world fisheries and aquaculture 2022. towards blue transformation doi:10.4060/cc0461en.
- Fernand, F., Israel, A., Skjermo, J., Wichard, T., Timmermans, K.R., Golberg, A., 2017. Offshore macroalgae biomass for bioenergy production: Environmental aspects, technological achievements and challenges. *Renew. Sustain. Energy Rev.* 75, 35–45. <https://doi.org/10.1016/j.rser.2016.10.046>.
- Filgueira, R., Fernández-Reiriz, M., Labarta, U., 2009. Clearance rate of the mussel *Mytilus galloprovincialis* in response to extreme chlorophyll ranges. *Cienc. Mar.* 35, 405–417.
- Fly, E.K., Hilbish, T.J., 2013. Physiological energetics and biogeographic range limits of three congeneric mussel species. *Oecologia* 172, 35–46. <https://doi.org/10.1007/s00442-012-2486-6>.
- Gagnon, M., Bergeron, P., 2017. Observations of the loading and motion of a submerged mussel longline at an open ocean site. *Aquac. Eng.* 78, 114–129. <https://doi.org/10.1016/j.aquaeng.2017.05.004>.
- Geisler, R., Schulz, C., Michl, S., Strothotte, E., 2018. Feasibility study: Offshore-Aquakultur Am Standort Der Forschungsplattform FINO3.
- Genest, C., Favre, A.C., 2007. Everything you always wanted to know about copula modeling but were afraid to ask. *J. Hydrol. Eng.* 347–368. <https://doi.org/10.1061/ASCE1084-0699200712:4347>.
- Genest, C., Rémillard, B., Beaudoin, D., 2009. Goodness-of-fit tests for copulas: a review and a power study. *Insur.: Math. Econ.* 44, 199–213. <https://doi.org/10.1016/j.insmatheco.2007.10.005>.
- Gimpel, A., Stelzenmüller, V., Grote, B., Buck, B.H., Floeter, J., Núñez-Riboni, I., Pogoda, B., Temming, A., 2015. A gis modelling framework to evaluate marine spatial planning scenarios: co-location of offshore wind farms and aquaculture in the

- German EEZ. Mar. Policy 55, 102–115. <https://doi.org/10.1016/j.marpol.2015.01.012>.
- Hanea, A., Kurowiczka, D., Cooke, R., 2006. Hybrid method for quantifying and analysing bayesian belief nets. Qual. Reliab. Eng. Int. 22, 709–729. <https://doi.org/10.1002/qre.808>.
- Hopkins, D.D., Goldburg, R.J., Martson, A., 1996. An environmental critique of government regulations and policies for open ocean aquaculture. Ocean Coast. Law J. 2.
- Jaeger, W.S., Morales-Nápoles, O., 2017. A vine-copula model for time series of significant wave heights and mean zero-crossing periods in the North Sea. ASCE-ASME J. Risk Uncertain. Eng. Syst., Part A: Civ. Eng. 3, 04017014.
- Jevn, L.S., Forbord, S., Olsen, Y., 2020. The effect of nutrient availability and light conditions on the growth and intracellular nitrogen components of land-based cultivated *Saccharina latissima* (phaeophyta). Front. Mar. Sci. 7. <https://doi.org/10.3389/fmars.2020.557460>.
- Joe, H., 2015. Dependence Modeling with Copulas. In: Monographs on Statistics and Applied Probability, 134. CRC Press, Boca Raton. <https://doi.org/10.1201/b17116>.
- Ju, Y.R., Chen, W.Y., Liao, C.M., 2014. Model-based risk assessment for milkfish and tilapia exposed to arsenic in a traditional polyculture system with seasonal variations. Aquac. Eng. 62, 1–8. <https://doi.org/10.1016/j.aquaeng.2014.07.001>.
- Kamermaers, P., Saurel, C., Boudry, P., Kamermaers, P., 2022. Interacting climate change effects on mussels (*Mytilus edulis* and *m. galloprovincialis*) and oysters (*Crassostrea gigas* and *Ostrea edulis*): Experiments for bivalve individual growth models. Aquat. Living Resour. 35. <https://doi.org/10.1051/alr/2022001>.
- Kammler, S., Romero, A.M., Burkhardt, C., Baruth, L., Antranikian, G., Liese, A., Kaltschmitt, M., 2024. Macroalgae valorization for the production of polymers, chemicals, and energy. Biomass Bioenergy 183, 107105. <https://doi.org/10.1016/j.biombioe.2024.107105>.
- Kerrison, P.D., Stanley, M.S., Edwards, M.D., Black, K.D., Hughes, A.D., 2015. The cultivation of European kelp for bioenergy: Site and species selection. doi:10.1016/j.biombioe.2015.04.035.
- Koot, P., Mendoza-Lugo, M.A., Paprotny, D., Morales-Nápoles, O., Ragno, E., Worm, D.T., 2023. Pybanshee version (1.0): A python implementation of the Matlab toolbox BANSHEE for non-parametric Bayesian Networks with updated features. SoftwareX 21. <https://doi.org/10.1016/j.softx.2022.101279>.
- Leontaris, G., Morales-Nápoles, O., Wolfert, A.R., 2016. Probabilistic scheduling of offshore operations using copula based environmental time series - an application for cable installation management for offshore wind farms. Ocean Eng. 125, 328–341. <https://doi.org/10.1016/j.oceaneng.2016.08.029>.
- Li, Q., Zhang, F., Sun, S., 2022. The survival and responses of blue mussel *Mytilus edulis* to 16-day sustained hypoxia stress. Mar. Environ. Res. 176. <https://doi.org/10.1016/j.marenvres.2022.105601>.
- López-Rebollar, B.M., Arévalo-Mejía, R., Díaz-Delgado, C., Latif, S., Ouarda, T.B., 2024. Settling velocity and effective density analysis for aquaculture floc particles: an approach through bivariate parametric copula. Aquac. Eng. 107, 102459. <https://doi.org/10.1016/j.aquaeng.2024.102459>.
- Lubsch, A., Timmermans, K.R., 2019. Uptake kinetics and storage capacity of dissolved inorganic phosphorus and corresponding dissolved inorganic nitrate uptake in *Saccharina latissima* and *Laminaria digitata* (phaeophyceae). J. Phycol. 55, 637–650. <https://doi.org/10.1111/jpy.12844>.
- Maar, M., Holbach, A., Boderskov, T., Thomsen, M., Buck, B.H., Kotta, J., Bruhn, A., 2023. Multi-use of offshore wind farms with low-trophic aquaculture can help achieve global sustainability goals. Commun. Earth Environ. 4, 447. <https://doi.org/10.1038/s43247-023-01116-6>.
- Mares-Nasarre, P., van Gent, M.R., Morales-Nápoles, O., 2024. A copula-based model to describe the uncertainty of overtopping variables on mound breakwaters. Coast. Eng. 189, 104483. <https://doi.org/10.1016/j.coastaleng.2024.104483>.
- Mendoza-Lugo, M.A., Morales-Nápoles, O., 2023. Version 1.3-BANSHEE - A Matlab toolbox for non-parametric Bayesian Networks. SoftwareX 23. <https://doi.org/10.1016/j.softx.2023.101479>.
- Michler-Cieluch, T., Krause, G., Buck, B.H., 2009. Reflections on integrating operation and maintenance activities of offshore wind farms and mariculture. Ocean Coast. Manag. 52, 57–68. <https://doi.org/10.1016/j.ocecoaman.2008.09.008>.
- Nelsen, R., 2006. An Introduction to Copulas. Second edition ed., Springer Series in Statistics.
- Paprotny, D., Morales-Nápoles, O., Worm, D., Ragno, E., 2020. BANSHEE - A Matlab toolbox for non-parametric Bayesian Networks. SoftwareX 12. <https://doi.org/10.1016/j.softx.2020.100588>.
- Pascoe, P.L., Parry, H.E., Hawkins, A.J., 2009. Observations on the measurement and interpretation of clearance rate variations in suspension-feeding bivalve shellfish. Aquat. Biol. 6, 181–190. <https://doi.org/10.3354/ab00123>.
- Pearson, K., Galton, F., 1895. VII. note on regression and inheritance in the case of two parents. Proc. R. Soc. Lond. 58, 240–242. <https://doi.org/10.1098/rsp1895.0041>.
- Peteiro, C., Freire, O., 2011. Offshore cultivation methods affects blade features of the edible seaweed *Saccharina latissima* in a bay of Galicia, northwest Spain. Russ. J. Mar. Biol. 37, 319–323. <https://doi.org/10.1134/S1063074011040110>.
- Ragno, E., AghaKouchak, A., Cheng, L., Sadegh, M., 2019. A generalized framework for process-informed nonstationary extreme value analysis. Adv. Water Resour. 130, 270–282. <https://doi.org/10.1016/j.advwatres.2019.06.007>.
- Ragno, E., Antonini, A., Pasquali, D., 2023. Investigating extreme sea level components and their interactions in the Adriatic and Tyrrhenian Seas. Weather Clim. Extrem. 41, 100590. <https://doi.org/10.1016/j.wace.2023.100590>.
- Redfield, A., 1934. On the proportions of organic derivatives in sea water and their relation to the composition of plankton. James Johnstone Meml. 176, 177–192.
- Riisgård, H.U., Egede, P.P., Saavedra, I.B., 2011. Feeding behaviour of the mussel, *Mytilus edulis*: New observations, with a minireview of current knowledge. J. Mar. Biol. 2011, 1–13. <https://doi.org/10.1155/2011/312459>.
- Ruiz-Velazco, J.M., Estrada-Pérez, M., Hernández-Llamas, A., Nieto-Navarro, J.T., Peña-Messina, E., 2013. Stock model and multivariate analysis for prediction of semi-intensive production of shrimp *Litopenaeus vannamei* as a function of water quality and management variables: A stochastic approach. Aquac. Eng. 56, 34–41. <https://doi.org/10.1016/j.aquaeng.2013.04.003>.
- Santjer, R., Mares-Nasarre, P., El-Serafy, G., Morales-Nápoles, O., 2023. A case study of ecological suitability of mussel and seaweed cultivation using bivariate copula functions. Proc. 33rd Eur. Saf. Reliab. Conf. 1877–1884. [https://doi.org/10.3850/978-981-18-8071-1\\_P172-cd](https://doi.org/10.3850/978-981-18-8071-1_P172-cd).
- Schiener, P., 2014. Bioethanol Production from Macroalgae (Doctoral thesis). University of Edinburgh, Edinburgh, UK.
- Schmidt, W., Raymond, D., Parish, D., Ashton, I.G., Miller, P.I., Campos, C.J., Shutler, J. D., 2018. Design and operation of a low-cost and compact autonomous buoy system for use in coastal aquaculture and water quality monitoring. Aquac. Eng. 80, 28–36. <https://doi.org/10.1016/j.aquaeng.2017.12.002>. (<https://www.sciencedirect.com/science/article/pii/S0144860917302133>).
- Sklar, A., 1959. Fonctions de répartition à n dimensions et leurs marges 8.
- Spearman, C., 1987. The proof and measurement of association between two things. Am. J. Psychol. 100, 441–471. <https://doi.org/10.2307/1422689>.
- Stechele, B., van der Zande, D., Alvera-Azcárate, A., Delbare, D., Lacroix, G., Nevejan, N., 2022. Biological site suitability for exposed self-regulating cultivation of blue mussel (*Mytilus edulis*): A Belgian case study. Aquac. Eng. 98. <https://doi.org/10.1016/j.aquaeng.2022.102264>.
- Strothotte, E., Jaeger, M., Pforth, J., De-Clercq, A., Stechele, B., Nevejan, N., Knoop, J., Kerkhove, T., Petit, S., Vandercammen, D., Pilgrim, L., Pribadi, A., Fernandez, G.V., Lataire, E., Groenendaal, B., Drigkopoulou, I., Vlacheas, P., Demestichas, P., Tzanettis, I., Foteinos, V., Trichias, K., Brouwers, E., Sørensen, H., Triest, J., 2021. Deliverable 7.2: Blueprint for the offshore site operation, with the addition of: "European UNITED project, Grant Agreement no. 862915"
- Tang, B., Riisgård, H.U., 2018. Relationship between oxygen concentration, respiration and filtration rate in blue mussel *Mytilus edulis*. J. Oceanol. Limnol. 36, 395–404. <https://doi.org/10.1007/s00343-018-6244-4>.
- Tyler-Walters, H., Hiscock, K., 2008. *Mytilus edulis* - Common Mussel. doi:10.17031/marlinsp.1421.1.
- Tyler-Walters, H., Hiscock, K., 2008. *Mytilus edulis* - Common Mussel. doi:10.17031/marlinsp.1421.1.
- UNITED Project, 2022. Multi-use offshore platforms demonstrators for boosting cost-effective and eco-friendly production in sustainable marine activities. (<https://www.h2020united.eu/publications>). accessed: 2024-09-23.
- van Leeuwen, S.M., Lenhart, H.J., Prins, T.C., Blauw, A., Desmit, X., Fernand, L., Friedland, R., Foteinos, V., Lacroix, G., van der Linden, A., Lefebvre, A., van der Molen, J., Plus, M., Baroni, I.R., Silva, T., Stegert, C., Troost, T.A., Vilmin, L.M., 2023. Deriving pre-eutrophic conditions from an ensemble model approach for the north-west European Seas. Front. Mar. Sci. 10.
- Wever, L., Krause, G., Buck, B.H., 2015. Lessons from stakeholder dialogues on marine aquaculture in offshore wind farms: Perceived potentials, constraints and research gaps. Mar. Policy 51, 251–259. <https://doi.org/10.1016/j.marpol.2014.08.015>.
- Yong, W.T.L., Thien, V.Y., Rupert, R., Rodrigues, K.F., 2022. Seaweed: A Potential Climate Change Solution. doi:10.1016/j.rser.2022.112222.
- Zar, J.H., 1972. Significance testing of the spearman rank correlation coefficient. J. Am. Stat. Assoc. 67, 578–580. <https://doi.org/10.1080/01621459.1972.10481251>.
- Zijl, F., Zijlker, T., Laan, S., Groenenboom, J., 2023. 3D DCSSM FM: A sixth-generation model for the NW European Shelf, with the addition of: "Technical Report, Deltares.