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APPLIED MATHEMATICS
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"Quantum Zeno effect in qubits"

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Abstract

This thesis analyzes the occurence of the quantum Zeno effect in a qubit in different situations. A system with a particle with spin 1/2, which represents a qubit, and detector is considered. The detector is modeled by a coordinate q, which has a Gaussian distribution with dispersion σ. There is the free evolution of the qubit and the interaction with the detector. Moreover, there are calculated algebraic expressions for the probabilities of the qubit to be in one of its states at certain moments in time by considering the wave function and density matrix at that time and consequently tracing out the detector coordinate q. Furthermore, there is done an analysis using plots of the evolution of these probabilities in time for different situations.

In the situation with one continuous measurement and no free evolution, the qubit remains in its initial state, as expected.

In the situation where periods of only free evolution and only measurements alternate, the probabilities keep the same value during the measurement, so then the evolution of the system freezes, and the probabilities evolve in a sine form during the free evolution, in line with the expectation. To neglect free evolution during the measurement, $t_m \ll t_{ev}$ is assumed, so there are no series of fast subsequent measurements or a continuous measurement. In this case, the quantum Zeno effect does not occur.

Furthermore, we consider the situation where periods of only free evolution and periods with a measurement during free evolution alternate. Now the oscillations continue in time, due to the ongoing free evolution. The larger the influence of the interaction between the qubit and detector during the measurement and the smaller the influence of the magnetic field of the free evolution, the higher the equilibirium position of the oscillations of the probability for the qubit to remain in its initial state is in time. Moreover, the amplitude of these oscillations is smaller. However, due to the free evolution the qubit always has a probability to undergo a transition to its other state. The continuous measurement does not freeze the evolution of the system totally. The quantum Zeno effect does not occur.

In the situation where the dispersion σ of the detector coordinate q goes to 0, there is a perfect measurement. The larger σ, the larger the measurement error in the detector resulting in dissipation of the system. The oscillations of the probabilities damp in time.

Recommendations for further research include plotting the evolution of the probabilities for a longer period in time with another integration tool. Also, it would be interesting to work out the assumptions done in this analysis to more realistic conditions. Furthermore, another distribution of the detector coordinate q and other qubit states might be interesting to work out in follow-up research.

1 Introduction

The quantum Zeno effect is a phenomenon known from the quantum optics. The first general derivation of the effect was presented in 1974 [4] and the comparison with Zeno's paradox has been done in 1977 [7]. Since this moment, the effect has been observed a few times in different settings, like on ions in a two level quantum system [12], in an unstable quantum system [10] and as the modulation of the rate of quantum tunnelling in an ultra-cold lattice gas by the intensity of light used to image the atoms [9]. It is an interesting tool in the field of quantum information processing. There are lots of promising applications, like in error-correcting codes, entanglement production and state preparation. It is also used in commercial atomic magnetometers [6]. However, there is still a lot to be learned as well. This report will give theoretical insight in the quantum Zeno effect in qubits.

The purpose of this research is to see if the quantum Zeno effect appears in a qubit in a specific initial state. A system with a particle with spin 1/2, which represents a qubit, and detector with coordinate q and Gaussian initial state, is considered. There is the free evolution of the qubit, described by $H_0 = B\sigma_z$, with B the magnetic field, and the interaction with a detector, described by $H_{int} = g(t)q\sigma_x$ with $g(t)$ the interaction strength between the qubit and detector.

The evolution of the wave function of the system is determined by applying the Schrödinger equation. We consider two different situations. First, there has been taken into account a time interval of only free evolution alternated with a time interval of only measurement. Subsequently, we consider a time interval of only free evolution alternated with a time interval of measurement during free evolution. These two cases have first been analyzed by assuming the dispersion $\sigma \to 0$ and then without this assumption. Moreover, the density matrix has been determined and traced out over the detector coordinate. Consequently, the probabilities that the qubit remains in its initial state or undergoes a transition to its other state have been calculated for different settings of parameters. Finally, the quantum Zeno effect is analyzed.

Chapter 2 of this report discusses the theoretical background. This is followed by a description of the calculations and a discussion of the results in chapter 3. The conclusions drawn from these results as well as recommendations for comparable and follow-up research, follow in chapter 4.

2 Theoretical background

In order to better understand the quantum Zeno effect in qubits that is treated in this report, it is useful to have an overview of the theory behind this. This theory includes a description of the evolution of a system in time, the time ordering operator, qubits, the density matrix and trace, the Rabi frequency, the collapse of the wavefunction and something about the quantum Zeno effect.

2.1 System evolution

The evolution of a system in time is described by the time-dependent Schrödinger equation [5]

$$\mathrm{i}\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle. \tag{2.1.1}$$

with i the imaginary unit, $\hbar$ the reduced Planck constant, ψ the state vector of the quantum system, t the time and $\hat{H}$ the Hamiltonian operator. If the initial state is known, there are two ways to solve this differential equation.
Solving this differential equation directly gives

$$|\psi(t)\rangle = T\mathrm{exp}\Big(\int_0^t -\frac{\mathrm{i}\hat{H}}{\hbar}dt'\Big)|\psi(0)\rangle \tag{2.1.2}$$

with T the time ordering operator, discussed in the next subsection.

Working out the exponential gives the evolution of a system in time.
Another way [2] is to calculate the eigenstates ψ_1, ψ_2,..., ψ_n with corresponding eigenvalues λ_1, λ_2,..., λ_n of the Hamiltonian. The solution of 2.1.1 is of the form

$$|\psi(t)\rangle = c_1\mathrm{exp}(\lambda_1 t)\psi_1 + c_2\mathrm{exp}(\lambda_2 t)\psi_2 + ... + c_n\mathrm{exp}(\lambda_n t)\psi_n \tag{2.1.3}$$

with c_1, c_2, ... , c_n constants. These constants are calculated by filling in the state at different moments in time.
A physical realizable state has to be normalized, so ψ has to satisfy the following equation

$$\int_{-\infty}^{\infty}|\psi(x,t)|^2dx = 1 \tag{2.1.4}$$

where x is the position of the system at time t.

2.2 Time ordering operator

The time ordering operator T is defined as follows

$$T(A(t)B(t')) = \begin{cases} A(t)B(t'), & \text{if } t < t' \\ B(t')A(t) & \text{if } t' < t \end{cases}$$

with A and B time dependent operators [13].
Let C be another time dependent operator. T works a lot on integrals in exponentials in this article. Then it gives, approaching the integral by a sum,

$$T\mathrm{exp}\Big(\mathrm{i}\int_0^t C(t)dt\Big) \approx T\mathrm{exp}\Big(\mathrm{i}\sum_n C(t_n)\Big) = \mathrm{expi}C(t_1))\mathrm{exp}(iC(t_2))\mathrm{exp}(\mathrm{i}C(t_3))... \tag{2.2.1}$$

with $t_1 < t_2 < t_3 < ....$

2.3 Qubits

A classical bit has the two possible states 0 and 1. Two possible states for a qubit are $|0\rangle$ and $|1\rangle$, but it can also be in a superposition of these two states, namely

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \tag{2.3.1}$$

with $|\psi\rangle$ the state of the qubit and α and β complex numbers. The qubit states are normalized so the complex numbers satisfy the relation $|\alpha|^2 + |\beta|^2 = 1$. So in general a qubit's state is a unit vector in a two-dimensional complex vector space. When we measure a qubit, we get either the result $|0\rangle$ with probability $|\alpha|^2$ or the result $|1\rangle$ with probability $|\beta|^2$ [8].

2.4 Density matrix and trace

The density matrix ρ of a quantum system with a mixture of states $|\psi_i\rangle$ with respective probabilities p_i is given by [11]

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|. \tag{2.4.1}$$

All the postulates of quantum mechanics can be reformulated in terms of the density matrix. It is most commonly used for the description of quantum systems whose state is not known and the description of subsystems of a composite quantum system.
The diagonal elements ρ_{nn} of ρ give the probability of occupying a quantum state $|n\rangle$. The off-diagonal elements are complex and have a time dependent phase factor that describes the evolution of coherent superpositions.
Let ρ be a two dimensional density matrix. Since ρ is hermitian, we have $\rho_{12}* = \rho_{21}$.
The equation of motion for the density matrix follows from the definition of ρ and the time dependent Schrödinger equation. This results in the Von Neumann equation

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[H, \rho]. \tag{2.4.2}$$

The probability p_i to find the system with density matrix ρ in state ψ_i is given by the following equation

$$p_i = \langle\psi_i|\rho|\psi_i\rangle. \tag{2.4.3}$$

These states $|\psi_i\rangle$ are orthogonal.

Let $\rho = \begin{pmatrix} \rho_{ii} & \rho_{ij} \\ \rho_{ji} & \rho_{jj} \end{pmatrix}$.

Consider a qubit with two states; $|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$. With equation 2.4.3 follows that the probability that the qubit is in state $|0\rangle$ is given by

$$p\Big(\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}\Big) = \frac{\rho_{ii} + \rho_{ij} + \rho_{ji} + \rho_{jj}}{2} \tag{2.4.4}$$

and the probability that the qubit is in $|1\rangle$ is given by

$$p\Big(\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}\Big) = \frac{\rho_{ii} - \rho_{ij} - \rho_{ji} + \rho_{jj}}{2}. \tag{2.4.5}$$

The trace of a matrix plays an important role when working with the density matrix. It is defined as follows for matrix A,

$$tr(A) = \sum_i A_{ii}. \tag{2.4.6}$$

The expectation value of an observable M is easy to calculate using the trace, when the density matrix ρ is known. The following equation is valid

$$\langle M \rangle = tr(\rho M). \tag{2.4.7}$$

Another important equation which is valid for the density matrix is

$$tr(\rho) = 1. \tag{2.4.8}$$

Consider two physical systems A and B, whose state is described by a density matrix ρ^{AB}. The reduced density operator ρ^A for system A is defined by

$$\rho^A = tr_B(\rho^{AB}). \tag{2.4.9}$$

with tr_B the partial trace over system B. This is defined by

$$tr_B(\rho^{AB}) = tr_B(|a_1\rangle\langle a_2| \otimes |b_1\rangle\langle b_2|) = |a_1\rangle\langle a_2| tr(|b_1\rangle\langle b_2|) = |a_1\rangle\langle a_2|\langle b_2|b_1\rangle \tag{2.4.10}$$

with $|a_1\rangle$ and $|a_2\rangle$ in the state space of system A and $|b_1\rangle$ and $|b_2\rangle$ in the state space of system B. The $\otimes$ symbol denotes the tensor product of state spaces of component physical systems. The partial trace tr_B maps the density matrix ρ^{AB} on a composite space $\mathcal{H}^A \otimes \mathcal{H}^B$ onto the density matrix ρ^A on $\mathcal{H}^A$.

2.5 Rabi frequency

Consider a 2-level system undergoing a free evolution driven by a magnetic field. It will begin to oscillate between its two states. The continual change between this states is known as Rabi oscillation and the frequency that this occurs is called the Rabi frequency. An application of this frequency can be found in subsection 3.2.

2.6 Collapse of the wavefunction

When a wave function is initially in a superposition of several eigenstates, it reduces to a single eigenstate of the observable due to interaction with the external world. This is the collapse of the wave function. A quantum system evolves in time by continuous evolution and by collapse of the wave function. Observables represent classical variables. The observer measures the classical value of the observable. This is the eigenvalue of the eigenstate to which the wave function collapses. The wave function collapses to an eigenstate of the observable with a certain probability.

2.7 Quantum Zeno effect

The quantum Zeno effect is the inhibition of transitions between quantum states by frequent measurements [5]. So an unstable particle which is continuously observed will never decay. Consider a system starting in the excited state ψ_2. Take the ground state ψ_1. An obervation that the state has not decayed causes a collapse of the wavefunction to ψ_2. The probability that the state decays after this collapse grows quadratically in time for short enough time, so we can say $p_{2\to 1} = \alpha t^2$ with α a constant. If we observe the system at n regular time intervals, from $t = 0$ to $t = T$, the probability that the system is still in state ψ_2 after n measurements is $\left(1 - \alpha(\frac{T}{n})^2\right)^n \approx 1 - \frac{\alpha}{n}T^2$.

For $n \to \infty$ this probability becomes 1. This suggests that a continuously observed unstable system never decays at all. This is known as the quantum Zeno effect.
Misra and Sudarshan [7] were the first to call the effect by that name. In their article they gave a proof of their theorem. However, they assumed ideal measurements and continuous obervations and recommended to do more research on the collapse of the state vector and the outcomes of successive measurements.
Cook [3] proposed the following experiment on a trapped ion to demonstrate the quantum Zeno effect on an induced transition. Consider three energy levels of the ion. The level structure is shown in the following figure.

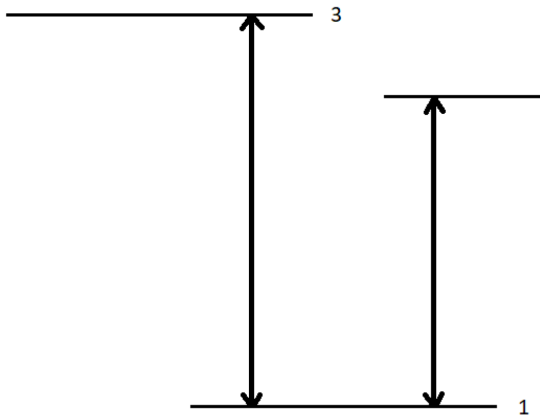


Figure 1. Energy-level diagram of the ion in Cook's proposed demonstration of the quantum Zeno effect.

Assume that spontaneous emission is negligible during the experiment. The state of the two level system formed by level 1 and 2 can be measured by use of level 3. This level has a strong optical transition to level 1 and can only decay to level 1. The state measurement is made by applying an optical pulse to the $1 \to 3$ transition. If the ion is in level 1 at the start of the pulse, it is promoted to level 3, from which it decays to level 1 by sending scattered photons. If the ion is in level 2, it scatters no photons.

The ion starts in level 1. Cook proposed to drive the $1 \to 2$ transition with an on-resonance π pulse (a square pulse of duration $T = \frac{\pi}{\Omega}$ with Ω the Rabi frequency) while simultaneously applying n short measurement pulses, each pulse applied at times $\tau = \frac{kT}{n}$. Each measurement pulse set the coherences ρ_{12} and ρ_{21} on 0, because the wave function was reduced. The emission of a photon constitutes a measurement, so the pulse length has to be long enough so that when the system is in level 1, the probability to emit at least one photon is high. After n measurements he found the probability that the ion starting in level 1 was in level 2 was

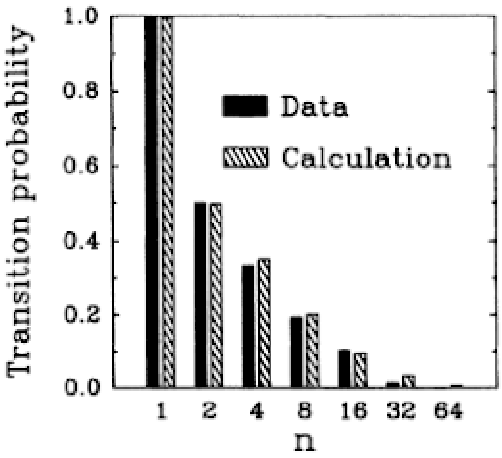
$$p_2(T) = \frac{1}{2}\Big(1 - \exp(-\frac{\pi^2}{2n})\Big). \tag{2.7.1}$$

This probability goes to 0 when $n \to \infty$. Then the quantum Zeno effect appears.
However, spontaneous emission can only be ignored for sufficiently short time intervals between measurements. Otherwise there would be no quantum Zeno effect.

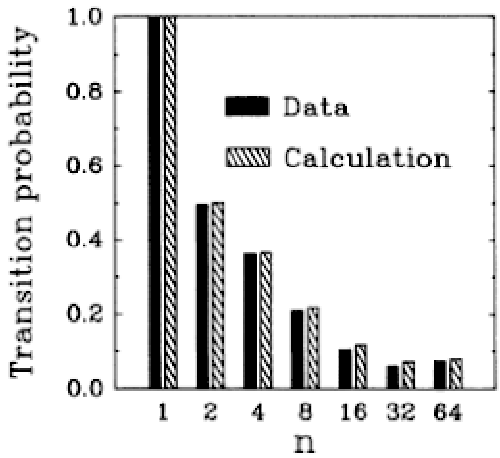
In 1989, Itano et al. [12] observed the quantum Zeno effect for a two-level atomic system. They did an experiment very similar to the proposed experiment of Cook. This was done with approximately 5000 9Be+ ions. These were cooled to below 250 mK. A resonant radiofrequent pulse was applied, which, if applied alone, would cause all the ions in the ground state to go to an excited state. After the pulse was applied, they measured whether the ions emitted photons or not. Those measurements suppressed the evolution of the system into the excited state. They found for the probability that the ion starting in level 1 was in level 2

$$p_2(T) = \frac{1}{2}\Big(1 - \cos^n(\frac{\pi}{n})\Big). \tag{2.7.2}$$

Their results are shown in figure 2a and 2b. Equation 2.7.1 was in agreement with the probabilities they found by measurements. The decrease of the transition probabilities with increasing n demonstrates the quantum Zeno effect.



(a) Graph of the experimental and calculated $1 \to 2$ transition probabilities as a function of the number of measurement pulses n found by Itano et al.



(b) Graph of the experimental and calculated $2 \to 1$ transition probabilities as a function of the number of measurement pulses n found by Itano et al.

Figure 2.

There are more experiments done where the quantum Zeno effect is observed.

In 2001, Mark G. Raizen et al. [10] observed the quantum Zeno effect for an unstable quantum system. This was done how it originally was proposed by Sudarshan and Misra. Cold sodium atoms were trapped in an accelerating optical lattice. For a large acceleration the atoms could escape the trapping potential via tunneling. The number of atoms remaining trapped during the initial period of non-exponential decay was measured repeatedly. Depending on the frequency of measurements, there was observed a decay, suppressed as compared to the unperturbed system.

In 2015, Mukund Vengalattore et al. [9] demonstrated a quantum Zeno effect as the modulation of the rate of quantum tunnelling in an ultra-cold lattice gas by the intensity of light used to image the atoms.

3 Results and discussion

3.1 Definition of variables and parameters

Consider a system, a particle with spin 1/2, and a detector with coordinate q, which we assume to be continuous, and momentum p. The particle with spin 1/2 represents a qubit. The system evolves under the Hamiltonian $H_0 = B\sigma_z$ [1], with B a magnetic field and σ_z a Pauli spin matrix, so there is the free evolution of spin. A measurement is done by measuring the x-component of the spin and the measurement reading on the detector is the shift of momentum p. The interaction Hamiltonian is given by $H_{int} = g(t)q\sigma_x$, with $g(t)$ the interaction strength between the system and detector and σ_x a Pauli spin matrix. The detector coordinate q is taken dimensionless, so $g(t)$ has the dimension of energy. The initial state of the detector is taken Gaussian, $|\phi(0)\rangle \propto \exp\left(-\frac{(q-q_0)^2}{2\sigma}\right)$, with q the detector coordinate, q_0 the mean of the detector coordinate q and σ the dispersion.

There follow subsections in which the system evolves under only H_0, H_{int} or a combination of these Hamiltonians. We define certain dimensionless parameters and variables in each subsection, in which we can express all the probabilities, wave functions and density matrices. These parameters and variables are defined as follows.

In subsection 3.2, the system evolves only under H_0. In this case, we define the dimensionless parameter $\alpha = \frac{Bt_{ev}}{\hbar}$, with t_{ev} the duration of one period of free evolution.

In subsection 3.3, the system evolves only under H_{int}. In this case, we define two variables, $q' = \frac{q}{\sqrt{2\sigma}}$ and $q'_0 = \frac{q_0}{\sqrt{2\sigma}}$. The parameter $\frac{1}{\sqrt{\sigma}}$ is dimensionless, so these variables are dimensionless.

In subsection 3.4, the system alternately evolves under H_0 during t_{ev} and H_{int} during t_m.
In the case of $\sigma \to 0$, we define two dimensionless parameters α and β and together with the dimensionless variables q and q_0, we can express all the calculated probabilities, wave functions and density matrices in these parameters and variables. Set $\alpha = \frac{Bt_{ev}}{\hbar}$ and $\beta = \frac{gt_m}{\hbar}$.
In the case of $\sigma \not\to 0$, we define two dimensionless parameters α and β together with two dimensionless variables q' and q'_0. We can express all the calculated probabilities, wave functions and density matrices in these parameters and variables. Set $\alpha = \frac{Bt_{ev}}{\hbar}$, $\beta = \frac{gt_m\sqrt{2\sigma}}{\hbar}$. and define two variables, $q' = \frac{q}{\sqrt{2\sigma}}$ and $q'_0 = \frac{q_0}{\sqrt{2\sigma}}$. The parameter $\frac{1}{\sqrt{\sigma}}$ is dimensionless, so these variables are dimensionless.

In subsection 3.5 there is always free evolution, also during measurements. The system alternately evolves under H_0 during t_{ev} and $H = H_0 + H_{int}$ during t_m.
In the case of $\sigma \to 0$, we define four dimensionless parameters, from which three dependent on the variable q, so α, $\beta(q)$, $\gamma(q)$ and $\delta(q)$. Together with the dimensionless variables q and q_0, we can express all the calculated probabilities, wave functions and density matrices in these parameters and variables. Set $\alpha = \frac{Bt_{ev}}{\hbar}$, $\beta(q) = \frac{t_m\sqrt{B^2+g^2q^2}}{\hbar}$, $\gamma(q) = \frac{B}{\sqrt{B^2+g^2q^2}}$ and $\delta(q) = \frac{gq}{\sqrt{B^2+g^2q^2}}$.
The following restrictions are valid for $\gamma(q)$ and $\delta(q)$: $\gamma(q)^2 + \delta(q)^2 = 1$ and $0 \leq \gamma(q) \leq 1$ and $0 \leq \delta(q) \leq 1$.
In the case of $\sigma \not\to 0$, we have again the dimensionless variables $q' = \frac{q}{\sqrt{2\sigma}}$ and $q'_0 = \frac{q_0}{\sqrt{2\sigma}}$. We define four dimensionless parameters, from which three dependent on the variable q', so α, $\beta(q')$, $\gamma(q')$ and $\delta(q')$. Set $\alpha = \frac{Bt_{ev}}{\hbar}$, $\beta(q') = \frac{t_m\sqrt{B^2+2\sigma g^2q'^2}}{\hbar}$, $\gamma(q') = \frac{B}{\sqrt{B^2+2\sigma g^2q'^2}}$ and $\delta(q') = \frac{gq'\sqrt{2\sigma}}{\sqrt{B^2+2\sigma g^2q'^2}}$.
The following restrictions are valid for $\gamma(q')$ and $\delta(q')$: $\gamma(q')^2 + \delta(q')^2 = 1$ and $0 \leq \gamma(q') \leq 1$ and $0 \leq \delta(q') \leq 1$.

The values of the parameters will be mentioned at every plot. All the plots in this thesis are made in Python [1].

[1] B includes here the Bohr magneton μ_B so B has the dimension of energy.

3.2 System evolution of free spin

In this section, we study a qubit which freely evolves under influence of a magnetic field. We will demonstrate how the probabilities of the qubit to be in one of its states develop in time.

Consider a system, a particle with spin 1/2, which represents a qubit, that can be in two states: $|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ and $|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$. The system evolves under the Hamiltonian $H_0 = B\sigma_z$, with B a magnetic field and σ_z a Pauli spin matrix, so there is the free evolution of spin. Take the initial state of the spin an eigenstate of the spin in the x-direction

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}, \tag{3.2.1}$$

so in qubit terms the initial state of the qubit is $|0\rangle$.

The free evolution of the qubit is calculated by solving Schrödingers equation 2.1.1. This gives

$$|\psi(t)\rangle = \exp\Big(-\frac{i}{\hbar}\int_0^t B\sigma_z dt'\Big)|\psi(0)\rangle. \tag{3.2.2}$$

Since B and σ_z do not depend on time, we set $\hat{\alpha} = -\frac{i}{\hbar}Bt$. The Taylor series of the exponential function give

$$\exp(\hat{\alpha}\sigma_z) = \sum_{n=0}^{\infty}\frac{(\hat{\alpha}\sigma_z)^n}{n!} = \sum_{n=0}^{\infty}\frac{(\hat{\alpha}\sigma_z)^{2n}}{(2n)!} + \sum_{n=0}^{\infty}\frac{(\hat{\alpha}\sigma_z)^{2n+1}}{(2n+1)!}. \tag{3.2.3}$$

Since $\sigma_z = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$, we notice that $\sigma_z^2 = I$. As a result, $\sigma_z^{2n} = I$ and $\sigma_z^{2n+1} = \sigma_z$. This gives

$$\exp(\hat{\alpha}\sigma_z) = I\sum_{n=0}^{\infty}\frac{\hat{\alpha}^{2n}}{(2n)!} + \sigma_z\sum_{n=0}^{\infty}\frac{\hat{\alpha}^{2n+1}}{(2n+1)!} \tag{3.2.4}$$

and with the Taylor series of the hyperbolic cosine and hyperbolic sine, this gives

$$\exp(\hat{\alpha}\sigma_z) = I\cosh(\hat{\alpha}) + \sigma_z\sinh(\hat{\alpha}) = \begin{pmatrix}\cosh(\hat{\alpha})+\sinh(\hat{\alpha}) & 0\\ 0 & \cosh(\hat{\alpha})-\sinh(\hat{\alpha})\end{pmatrix} = \begin{pmatrix}\exp(\hat{\alpha}) & 0\\ 0 & \exp(-\hat{\alpha})\end{pmatrix}. \tag{3.2.5}$$

With the initial state given as in equation 3.2.1 and substituting $\hat{\alpha} = -\frac{i}{\hbar}Bt$, the system evolves as follows in time,

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}\exp(-\frac{i}{\hbar}Bt)\\ \exp(\frac{i}{\hbar}Bt)\end{pmatrix}. \tag{3.2.6}$$

The density matrix of this system is given by

$$\rho = \frac{1}{2}\begin{pmatrix}1 & \exp(-2\frac{i}{\hbar}Bt)\\ \exp(2\frac{i}{\hbar}Bt) & 1\end{pmatrix} \tag{3.2.7}$$

using equation 2.4.1.

At $t = 0$ the qubit is in state $|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$. The probability that the qubit is in this state at time t is, using equation 2.4.4, given by

$$P_0(t) = \frac{1}{2} + \frac{1}{2}\cos(\frac{2}{\hbar}Bt) \tag{3.2.8}$$

and the probability that the qubit is in state $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$ is, using equation 2.4.5, given by

$$P_1(t) = \frac{1}{2} - \frac{1}{2}\cos(\frac{2}{\hbar}Bt). \tag{3.2.9}$$

The corresponding plot of P_0 and P_1 against time is shown in figure 3. The dimensionless parameter is $\alpha = \pi$. The Rabi frequency is given by $f = \frac{\hbar}{\pi}$.

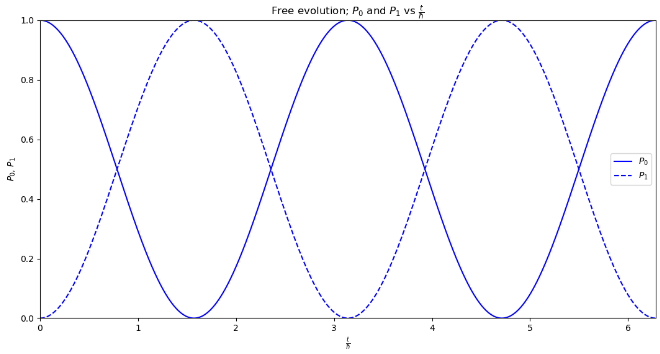


Figure 3. Graph of the probability of being in state 0 P_0 (continuous line) or in state 1 P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time.

3.3 Measurement of a system neglecting free evolution

In this section, we study a qubit, which only evolves under the influence of a detector. We will demonstrate how the probabilities of the qubit to be in one of its states develop in time.

Consider a system, a particle with spin 1/2, and a detector with coordinate q and momentum p. The particle with spin 1/2 represents the qubit, which can be in two states: $|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ and $|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$. Take the initial state of the spin an eigenstate of the spin in the x-direction

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}, \tag{3.3.1}$$

so in qubit terms the initial state of the qubit is $|0\rangle$. Take the initial state of the detector Gaussian

$$|\phi(0)\rangle \propto \exp\Big(-\frac{(q-q_0)^2}{2\sigma}\Big) \tag{3.3.2}$$

with q the detector coordinate which is assumed to be continuous, q_0 the mean of the detector coordinate q and σ the dispersion. The initial state is the product state $|\Phi(0)\rangle = |\psi(0)\rangle|\phi(0)\rangle$.

Neglect the free evolution of the system. One continuous measurement is done by measuring the x-component of the spin. So the system evolves only under the interaction Hamiltonian, given by $H_{int} = g(t)q\sigma_x$, with $g(t)$ the interaction strength between the system and the detector which is non-zero during a long period of time, q the detector coordinate and σ_x a Pauli spin matrix. When doing a measurement, the wave function will collapse to an eigenstate of the observable, in this case σ_x. These eigenstates are $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ and $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$.

The system evolves as follows in time, denoted by t,

$$|\Phi(t)\rangle = \exp\Big(-\frac{i}{\hbar}\int_0^t H_{int}dt'\Big)|\Phi(0)\rangle. \tag{3.3.3}$$

Assuming $g(t)$ to be independent in time, we get

$$|\Phi(t)\rangle = \exp(-\frac{i}{\hbar}gq\sigma_x t)|\Phi(0)\rangle. \tag{3.3.4}$$

Take $\hat{\beta} = -\frac{i}{\hbar}gqt$.

The Taylor series of the exponential function give

$$\exp(\hat{\beta}\sigma_x) = \sum_{n=0}^{\infty}\frac{(\hat{\beta}\sigma_x)^n}{n!} = \sum_{n=0}^{\infty}\frac{(\hat{\beta}\sigma_x)^{2n}}{(2n)!} + \sum_{n=0}^{\infty}\frac{(\hat{\beta}\sigma_x)^{2n+1}}{(2n+1)!}. \tag{3.3.5}$$

Since $\sigma_x = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$, we notice that $\sigma_x^2 = I$. As a result, $\sigma_x^{2n} = I$ and $\sigma_x^{2n+1} = \sigma_x$. This gives

$$\exp(\hat{\beta}\sigma_x) = I\sum_{n=0}^{\infty}\frac{\hat{\beta}^{2n}}{(2n)!} + \sigma_x\sum_{n=0}^{\infty}\frac{\hat{\beta}^{2n+1}}{(2n+1)!} \tag{3.3.6}$$

and with the Taylor series of the hyperbolic cosine and hyperbolic sine, this gives

$$\exp(\hat{\beta}\sigma_x) = I\cosh(\hat{\beta}) + \sigma_x\sinh(\hat{\beta}) = \begin{pmatrix}\cosh(\hat{\beta}) & \sinh(\hat{\beta})\\ \sinh(\hat{\beta}) & \cosh(\hat{\beta})\end{pmatrix}. \tag{3.3.7}$$

Substituting $\hat{\beta} = -\frac{i}{\hbar}gqt$ in equation 3.3.7, this results in

$$\exp(\hat{\beta}\sigma_x) = \begin{pmatrix}\cos(\frac{1}{\hbar}gqt) & -i\sin(\frac{1}{\hbar}gqt)\\ -i\sin(\frac{1}{\hbar}gqt) & \cos(\frac{1}{\hbar}gqt)\end{pmatrix}. \tag{3.3.8}$$

We find using equation 3.3.4 and the initial state, the following normalized state

$$|\Phi(t)\rangle = \frac{1}{\sqrt{2\sqrt{\pi\sigma}}}\exp\Big(-\frac{(q-q_0)^2}{2\sigma}\Big)\exp(-\frac{i}{\hbar}qgt)\begin{pmatrix}1\\1\end{pmatrix}. \tag{3.3.9}$$

The density matrix ρ is calculated by the following equation

$$\rho = |\Phi(t)\rangle_q\langle\Phi(t)|_{q^\dagger}. \tag{3.3.10}$$

Set $q = q^\dagger$, since q is a real observable coordinate. The density matrix is then given by

$$\rho = \frac{1}{2\sqrt{\pi\sigma}}\exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big)\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}. \tag{3.3.11}$$

Now the dectector coordinate q is traced out by taking the partial trace over q in order to get the density matrix only dependent on the qubit coordinates $i = |0\rangle$ and $j = |1\rangle$. This is given by equation 2.4.6 and this gives, the sum approached by an integral since q is a continuous variable,

$$\rho_{ii,ij,ji,jj} = tr_q\rho = \int_{-\infty}^{\infty} \frac{1}{2\sqrt{\pi\sigma}} \exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big) \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} dq. \tag{3.3.12}$$

Working out this integral for ρ_{ii} gives

$$\rho_{ii} = tr_q\rho = \int_{-\infty}^{\infty} \frac{1}{2\sqrt{\pi\sigma}} \exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big) dq = \left[\frac{\mathrm{erf}(\frac{q-q_0}{\sqrt{\sigma}})}{4}\right]_{-\infty}^{\infty} = \frac{1}{2}. \tag{3.3.13}$$

The same calculation is done for ρ_{ij}, ρ_{ji} and ρ_{jj}. At $t = 0$ the qubit is in state $|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. The probability that the qubit is in this state at time t is, using equation 2.4.4, given by

$$P_0(t) = 1 \tag{3.3.14}$$

and the probability that the qubit is in state $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ is, using equation 2.4.5, given by

$$P_1(t) = 0. \tag{3.3.15}$$

So the qubit will remain in its initial state and will not undergo a transition to another state. This is in line with the expectation, since the initial state of the qubit is $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and when doing an instant measurement of the observable σ_x on the qubit, it will remain with probability 1 in the eigenstate of this observable, so its initial state. The wavefunction will collapse to state $|0\rangle$. This is also in line with the quantum Zeno effect. Since we do a continuous measurement, the qubit will never undergo a transition to another state.

The corresponding plot for P_0 and P_1 against time is given in figure 4.

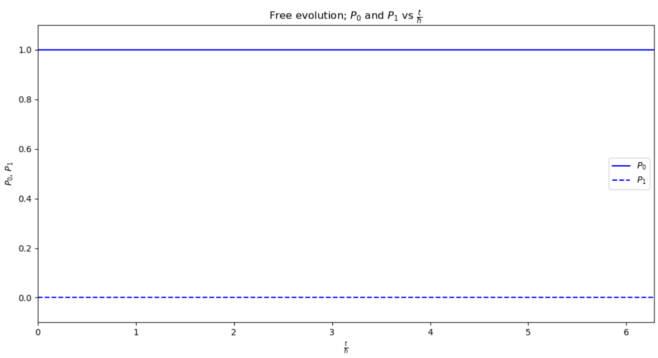


Figure 4. Graph of the probability of being in state 0 P_0 (continuous line) or state 1 P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time.

3.4 System evolution of a series of measurements alternated with free evolution of the spin

In this section, we study a qubit, which alternately evolves under influence of a magnetic field [2] and a detector [3]. We will demonstrate how the probabilities of the qubit to be in one of its states develop in time and see if the quantum Zeno effect can be observed.

[2]This represents the free evolution of the qubit.
[3]This represents a measurement done on the qubit.

Consider the same system as in subsection 3.3, so a particle with spin 1/2, and a detector with coordinate q and momentum p. The particle with spin 1/2 represents the qubit, which can be in two states: $|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ and $|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$. Now the system evolves also under the Hamiltonian $H_0 = B\sigma_z$, with B a magnetic field and σ_z a Pauli spin matrix, so there is the free evolution of spin. Take again the initial state of the spin an eigenstate of the spin in the x-direction

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}, \tag{3.4.1}$$

and the initial state of the detector Gaussian

$$|\phi(0)\rangle \propto \exp\Big(-\frac{(q-q_0)^2}{2\sigma}\Big) \tag{3.4.2}$$

with q the detector coordinate which is assumed to be continuous, q_0 the mean of the detector coordinate q and σ the dispersion. The initial state of the system is the product state $|\Phi(0)\rangle = |\psi(0)\rangle|\phi(0)\rangle$.

Consider a series of measurements. Each measurement is performed at time $t_k = \frac{kT}{n}$ during t_m under the Hamiltonian H_{int}. Assume T to be finite. The system evolves under H_0 during t_{ev} between the measurements, so then there is only free evolution. We take $t_m \ll t_{ev}$ so that we can neglect the Hamiltonian H_0 during the measurements. So during the free evolution we have $g(t) = 0$ and during the measurement we have $B = 0$. The periods of free evolution and measurement alternate. The wavefunction evolves during $t_{ev} + t_m$ as

$$|\Phi(t_{ev}+t_m)\rangle = \hat{T}\exp\Big(-\frac{i}{\hbar}\int_{t_{ev}}^{t_{ev}+t_m} H_{int}dt'\Big)\exp\Big(-\frac{i}{\hbar}\int_0^{t_{ev}} H_0 dt'\Big)|\Phi(0)\rangle \tag{3.4.3}$$

with $\hat{T}$ the time ordering operator.

3.4.1 Situation $\sigma \to 0$

In this subsection, the wavefunctions, density matrices and probabilities are expressed in the following dimensionless terms: $\frac{1}{\hbar}Bt_{ev}$, $\frac{1}{\hbar}gqt_m$ and $\frac{1}{\hbar}gq_0t_m$. These can be expressed as follows in the parameters $\alpha = \frac{Bt_{ev}}{\hbar}$ and $\beta = \frac{gt_m}{\hbar}$ and the dimensionless variables q and q_0. $\frac{1}{\hbar}Bt_{ev} = \alpha$, $\frac{1}{\hbar}gqt_m = \beta q$ and $\frac{1}{\hbar}gq_0t_m = \beta q_0$.

First, there is a period of length t_{ev} of free evolution of the system. We already derived in subsection 3.2 the following result

$$\exp\Big(-\frac{i}{\hbar}\int_0^{t_{ev}} H_0 dt\Big) = \exp\Big(-\frac{i}{\hbar}Bt_{ev}\sigma_z\Big) = \begin{pmatrix}\exp(-\frac{i}{\hbar}Bt_{ev}) & 0\\ 0 & \exp(\frac{i}{\hbar}Bt_{ev})\end{pmatrix}. \tag{3.4.4}$$

With the initial state given, we solve Schrödingers equation and derive that the system is in the following normalized state after time t_{ev},

$$|\Phi(t_{ev})\rangle = \frac{1}{\sqrt{2\sqrt{\pi\sigma}}}\exp\Big(-\frac{(q-q_0)^2}{2\sigma}\Big)\begin{pmatrix}\exp\Big(-\frac{i}{\hbar}Bt_{ev}\Big)\\ \exp\Big(\frac{i}{\hbar}Bt_{ev}\Big)\end{pmatrix}. \tag{3.4.5}$$

So this is the state after time t_{ev}. Now there will be done a measurement during t_m. The system evolves as follows under the interaction Hamiltonian,

$$|\Phi(t_{ev}+t_m)\rangle = \exp\Big(-\frac{i}{\hbar}\int_{t_{ev}}^{t_{ev}+t_m} H_{int}dt\Big)|\Phi(t_{ev})\rangle. \tag{3.4.6}$$

Assume $g(t)$ to be independent in time.[4] We find using equations 3.3.8, 3.4.5 and 3.4.6, the following normalized state at time $t_m + t_{ev}$,

$$|\Phi(t_{ev}+t_m)\rangle = \frac{1}{\sqrt{2\sqrt{\pi\sigma}}}\exp\Big(-\frac{(q-q_0)^2}{2\sigma}\Big)\begin{pmatrix}\cos\Big(\frac{1}{\hbar}gqt_m - \frac{1}{\hbar}Bt_{ev}\Big) - i\sin\Big(\frac{1}{\hbar}gqt_m + \frac{1}{\hbar}Bt_{ev}\Big)\\ \cos\Big(\frac{1}{\hbar}gqt_m + \frac{1}{\hbar}Bt_{ev}\Big) - i\sin\Big(\frac{1}{\hbar}gqt_m - \frac{1}{\hbar}Bt_{ev}\Big)\end{pmatrix}. \tag{3.4.7}$$

So the evolution of the wave function at discrete moments in time $k(t_{ev}+t_m)$ with k an integer is given by applying the matrix

$$A = \exp\Big(-\frac{i}{\hbar}\int_{t_{ev}}^{t_{ev}+t_m} H_{int}dt\Big)\exp\Big(-\frac{i}{\hbar}\int_0^{t_{ev}} H_0 dt\Big) = \begin{pmatrix}\exp(-\frac{i}{\hbar}Bt_{ev})\cos(\frac{1}{\hbar}gqt_m) & -i\exp(\frac{i}{\hbar}Bt_{ev})\sin(\frac{1}{\hbar}gqt_m)\\ -i\exp(-\frac{i}{\hbar}Bt_{ev})\sin(\frac{1}{\hbar}gqt_m) & \exp(\frac{i}{\hbar}Bt_{ev})\cos(\frac{1}{\hbar}gqt_m)\end{pmatrix} \tag{3.4.8}$$

a number of times on the initial wave function for each time period consisting of a period of free evolution followed by a period of a measurement. After each measurement, the probability that the qubit is in its initial state $|0\rangle$ and the probability that the qubit is in its other state $|1\rangle$ is calculated using the density matrix.

The density matrix ρ at time $t_{ev}+t_m$ is calculated by the following equation

$$\rho(t_{ev}+t_m) = |\Phi(t_{ev}+t_m)\rangle_q\langle\Phi(t_{ev}+t_m)|_{q^\dagger}. \tag{3.4.9}$$

Set $q = q^\dagger$. [5] The density matrix [6] is then given by

$$\rho(t_{ev}+t_m) = \frac{1}{2\sqrt{\pi\sigma}}\exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big)\begin{pmatrix}\tilde{\alpha}(t_{ev}+t_m) & \tilde{\beta}(t_{ev}+t_m)\\ \tilde{\gamma}(t_{ev}+t_m) & \tilde{\delta}(t_{ev}+t_m)\end{pmatrix} \tag{3.4.10}$$

with $\tilde{\alpha}(t_{ev}+t_m) = \cos^2\Big(\frac{1}{\hbar}gqt_m - \frac{1}{\hbar}Bt_{ev}\Big) + \sin^2\Big(\frac{1}{\hbar}gqt_m + \frac{1}{\hbar}Bt_{ev}\Big)$,
$\tilde{\beta}(t_{ev}+t_m) = \cos\Big(\frac{1}{\hbar}gqt_m + \frac{1}{\hbar}Bt_{ev}\Big)\cos\Big(\frac{1}{\hbar}gqt_m - \frac{1}{\hbar}Bt_{ev}\Big)$
$+\sin\Big(\frac{1}{\hbar}gqt_m + \frac{1}{\hbar}Bt_{ev}\Big)\sin\Big(\frac{1}{\hbar}gqt_m - \frac{1}{\hbar}Bt_{ev}\Big) + i\Big(\cos\Big(\frac{1}{\hbar}gqt_m - \frac{1}{\hbar}Bt_{ev}\Big)\sin\Big(\frac{1}{\hbar}gqt_m - \frac{1}{\hbar}Bt_{ev}\Big)$
$-\cos\Big(\frac{1}{\hbar}gqt_m + \frac{1}{\hbar}Bt_{ev}\Big)\sin\Big(\frac{1}{\hbar}gqt_m + \frac{1}{\hbar}Bt_{ev}\Big)\Big)$,
$\tilde{\gamma}(t_{ev}+t_m) = \cos\Big(\frac{1}{\hbar}gqt_m + \frac{1}{\hbar}Bt_{ev}\Big)\cos\Big(\frac{1}{\hbar}gqt_m - \frac{1}{\hbar}Bt_{ev}\Big)$
$+\sin\Big(\frac{1}{\hbar}gqt_m + \frac{1}{\hbar}Bt_{ev}\Big)\sin\Big(\frac{1}{\hbar}gqt_m - \frac{1}{\hbar}Bt_{ev}\Big) + i\Big(\cos\Big(\frac{1}{\hbar}gqt_m + \frac{1}{\hbar}Bt_{ev}\Big)\sin\Big(\frac{1}{\hbar}gqt_m + \frac{1}{\hbar}Bt_{ev}\Big)$
$-\cos\Big(\frac{1}{\hbar}gqt_m - \frac{1}{\hbar}Bt_{ev}\Big)\sin\Big(\frac{1}{\hbar}gqt_m - \frac{1}{\hbar}Bt_{ev}\Big)\Big)$
and $\tilde{\delta}(t_{ev}+t_m) = \cos^2\Big(\frac{1}{\hbar}gqt_m + \frac{1}{\hbar}Bt_{ev}\Big) + \sin^2\Big(\frac{1}{\hbar}gqt_m - \frac{1}{\hbar}Bt_{ev}\Big)$.

Now the dectector coordinate q is traced out by taking the partial trace over q in order to get the density matrix only dependent on the qubit coordinates $i = |0\rangle$ and $j = |1\rangle$. This is given by equation 2.4.6 and this gives, the sum approached by an integral since q is a continuous variable,

$$\rho_{ii,ij,ji,jj}(t_{ev}+t_m) = tr_q\rho(t_{ev}+t_m) = \int_{-\infty}^{\infty}\frac{1}{2\sqrt{\pi\sigma}}\exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big)\begin{pmatrix}\tilde{\alpha}(t_{ev}+t_m) & \tilde{\beta}(t_{ev}+t_m)\\ \tilde{\gamma}(t_{ev}+t_m) & \tilde{\delta}(t_{ev}+t_m)\end{pmatrix}dq. \tag{3.4.11}$$

[4] The evolution of the wave function at discrete moments in time for $g(t)$ time dependent is given in the appendix in subsection 5.3.1.

[5] This assumption is made at every moment in time and it will not be noticed anymore.

[6] Indeed $tr(\rho) = 1$ and $\rho_{12}* = \rho_{21}$. These conditions are valid for every density matrix in this report.

In this subsection, we consider the case where $\sigma \to 0$. Consequently, the exponential approaches the Dirac delta function around q_0, so $\exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big) \to \delta(q-q_0)$. This gives in equation 3.4.11

$$\rho_{ii,ij,ji,jj}(t_{ev}+t_m,q_0) \approx \frac{1}{2}\begin{pmatrix} \tilde{\alpha}(t_{ev}+t_m,q_0) & \tilde{\beta}(t_{ev}+t_m,q_0) \\ \tilde{\gamma}(t_{ev}+t_m,q_0) & \tilde{\delta}(t_{ev}+t_m,q_0) \end{pmatrix} \tag{3.4.12}$$

with $\tilde{\alpha}(t_{ev}+t_m,q_0) = \cos^2\Big(\frac{1}{\hbar}gq_0t_m - \frac{1}{\hbar}Bt_{ev}\Big) + \sin^2\Big(\frac{1}{\hbar}gq_0t_m + \frac{1}{\hbar}Bt_{ev}\Big)$,
$\tilde{\beta}(t_{ev}+t_m,q_0) = \cos\Big(\frac{1}{\hbar}gq_0t_m + \frac{1}{\hbar}Bt_{ev}\Big)\cos\Big(\frac{1}{\hbar}gq_0t_m - \frac{1}{\hbar}Bt_{ev}\Big)$
$+ \sin\Big(\frac{1}{\hbar}gq_0t_m + \frac{1}{\hbar}Bt_{ev}\Big)\sin\Big(\frac{1}{\hbar}gq_0t_m - \frac{1}{\hbar}Bt_{ev}\Big) + i\Big(\cos\Big(\frac{1}{\hbar}gq_0t_m - \frac{1}{\hbar}Bt_{ev}\Big)\sin\Big(\frac{1}{\hbar}gq_0t_m - \frac{1}{\hbar}Bt_{ev}\Big)$
$- \cos\Big(\frac{1}{\hbar}gq_0t_m + \frac{1}{\hbar}Bt_{ev}\Big)\sin\Big(\frac{1}{\hbar}gq_0t_m + \frac{1}{\hbar}Bt_{ev}\Big)\Big)$,
$\tilde{\gamma}(t_{ev}+t_m,q_0) = \cos\Big(\frac{1}{\hbar}gq_0t_m + \frac{1}{\hbar}Bt_{ev}\Big)\cos\Big(\frac{1}{\hbar}gq_0t_m - \frac{1}{\hbar}Bt_{ev}\Big)$
$+ \sin\Big(\frac{1}{\hbar}gq_0t_m + \frac{1}{\hbar}Bt_{ev}\Big)\sin\Big(\frac{1}{\hbar}gq_0t_m - \frac{1}{\hbar}Bt_{ev}\Big) + i\Big(\cos\Big(\frac{1}{\hbar}gq_0t_m + \frac{1}{\hbar}Bt_{ev}\Big)\sin\Big(\frac{1}{\hbar}gq_0t_m + \frac{1}{\hbar}Bt_{ev}\Big)$
$- \cos\Big(\frac{1}{\hbar}gq_0t_m - \frac{1}{\hbar}Bt_{ev}\Big)\sin\Big(\frac{1}{\hbar}gq_0t_m - \frac{1}{\hbar}Bt_{ev}\Big)\Big)$ and
$\tilde{\delta}(t_{ev}+t_m,q_0) = \cos^2\Big(\frac{1}{\hbar}gq_0t_m + \frac{1}{\hbar}Bt_{ev}\Big) + \sin^2\Big(\frac{1}{\hbar}gq_0t_m - \frac{1}{\hbar}Bt_{ev}\Big)$.

At $t=0$ the qubit is in state $|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$. The probability that the qubit is still in this state at $t_m + t_{ev}$ is, using equation 2.4.4, given by

$$P_0(t_m+t_{ev},q_0) = \frac{1}{2} + \frac{1}{2}\cos\big(\frac{2}{\hbar}Bt_{ev}\big) \tag{3.4.13}$$

and the probability that the qubit is in its other state $|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$ is, using equation 2.4.5, given by

$$P_1(t_m+t_{ev},q_0) = \frac{1}{2} - \frac{1}{2}\cos\big(\frac{2}{\hbar}Bt_{ev}\big) \tag{3.4.14}$$

As expected, $P_0(t_m+t_{ev},q_0) + P_1(t_m+t_{ev},q_0) = 1$. [7]

Now the second period, consisting of a period of free evolution and a measurement, takes place. The state after $t = 2t_{ev} + 2t_m$ is given by applying two times matrix A, calculated in equation 3.4.8, on the initial state. This gives

$$|\Phi(2t_{ev}+2t_m)\rangle = A^2|\Phi(0)\rangle = \frac{1}{\sqrt{2\sqrt{\pi\sigma}}}\exp\Big(-\frac{(q-q_0)^2}{2\sigma}\Big)\begin{pmatrix}\Phi_i\\ \Phi_j\end{pmatrix} \tag{3.4.15}$$

with $\Phi_i = \exp(-\frac{2i}{\hbar}Bt_{ev})\cos^2(\frac{1}{\hbar}gqt_m) - i(1+\exp(\frac{2i}{\hbar}Bt_{ev}))\sin(\frac{1}{\hbar}gqt_m)\cos(\frac{1}{\hbar}gqt_m) - \sin^2(\frac{1}{\hbar}gqt_m)$ and $\Phi_j = \exp(\frac{2i}{\hbar}Bt_{ev})\cos^2(\frac{1}{\hbar}gqt_m) - i(1+\exp(-\frac{2i}{\hbar}Bt_{ev}))\sin(\frac{1}{\hbar}gqt_m)\cos(\frac{1}{\hbar}gqt_m) - \sin^2(\frac{1}{\hbar}gqt_m)$.

The density matrix at $2t_{ev}+2t_m$ is given by

$$\rho(2t_{ev}+2t_m) = |\Phi(2t_{ev}+2t_m)\rangle\langle\Phi(2t_{ev}+2t_m)| = \frac{1}{2\sqrt{\pi\sigma}}\exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big)\begin{pmatrix}\tilde{\alpha}(2t_{ev}+2t_m) & \tilde{\beta}(2t_{ev}+2t_m)\\ \tilde{\gamma}(2t_{ev}+2t_m) & \tilde{\delta}(2t_{ev}+2t_m)\end{pmatrix} \tag{3.4.16}$$

[7] For every moment in time $P_0 + P_1 = 1$ is valid.

with $\tilde{\alpha}(2t_{ev}+2t_m) = \frac{1}{8}\exp(-4\frac{i}{\hbar}Bt_{ev})\Big(-2i\exp(8\frac{i}{\hbar}Bt_{ev})\sin(\frac{2}{\hbar}gqt_m)+2i\exp(2\frac{i}{\hbar}Bt_{ev})\sin(\frac{4}{\hbar}gqt_m)$
$-2i\exp(6\frac{i}{\hbar}Bt_{ev})\sin(\frac{4}{\hbar}gqt_m)-i\exp(8\frac{i}{\hbar}Bt_{ev})\sin(\frac{4}{\hbar}gqt_m)+8\ \exp(4\frac{i}{\hbar}Bt_{ev})+2i\sin(\frac{2}{\hbar}gqt_m)+i\sin(\frac{4}{\hbar}gqt_m)\Big)$,
$\tilde{\beta}(2t_{ev}+2t_m) = -\frac{1}{8}\exp(-4\frac{i}{\hbar}Bt_{ev})\Big(4\exp(4\frac{i}{\hbar}Bt_{ev})\cos(\frac{2}{\hbar}gqt_m)-2\exp(2\frac{i}{\hbar}Bt_{ev})\cos(\frac{4}{\hbar}gqt_m)+2\exp(6\frac{i}{\hbar}Bt_{ev})\cos(\frac{4}{\hbar}gqt_m)$
$+\exp(8\frac{i}{\hbar}Bt_{ev})\cos(\frac{4}{\hbar}gqt_m)+2\exp(2\frac{i}{\hbar}Bt_{ev})-4\exp(4\frac{i}{\hbar}Bt_{ev})-2\exp(6\frac{i}{\hbar}Bt_{ev})-\exp(8\frac{i}{\hbar}Bt_{ev})$
$-4\cos(\frac{2}{\hbar}gqt_m)-\cos(\frac{4}{\hbar}gqt_m)-3\Big)$,
$\tilde{\gamma}(2t_{ev}+2t_m) = \frac{1}{8}\exp(-4\frac{i}{\hbar}Bt_{ev})\Big(-4\exp(4\frac{i}{\hbar}Bt_{ev})\cos(\frac{2}{\hbar}gqt_m)+4\exp(8\frac{i}{\hbar}Bt_{ev})\cos(\frac{2}{\hbar}gqt_m)-2\exp(2\frac{i}{\hbar}Bt_{ev})\cos(\frac{4}{\hbar}gqt_m)$
$+2\exp(6\frac{i}{\hbar}Bt_{ev})\cos(\frac{4}{\hbar}gqt_m)+\exp(8\frac{i}{\hbar}Bt_{ev})\cos(\frac{4}{\hbar}gqt_m)+2\exp(2\frac{i}{\hbar}Bt_{ev})+4\exp(4\frac{i}{\hbar}Bt_{ev})-2\exp(6\frac{i}{\hbar}Bt_{ev})$
$+3\exp(8\frac{i}{\hbar}Bt_{ev})-\cos(\frac{4}{\hbar}gqt_m)+1\Big)$ and
$\tilde{\delta}(2t_{ev}+2t_m) = \frac{1}{8}\exp(-4\frac{i}{\hbar}Bt_{ev})\Big(2i\exp(8\frac{i}{\hbar}Bt_{ev})\sin(\frac{2}{\hbar}gqt_m)-2i\exp(2\frac{i}{\hbar}Bt_{ev})\sin(\frac{4}{\hbar}gqt_m)+2i\exp(6\frac{i}{\hbar}Bt_{ev})\sin(\frac{4}{\hbar}gqt_m)$
$+i\exp(8\frac{i}{\hbar}Bt_{ev})\sin(\frac{4}{\hbar}gqt_m)+8\exp(4\frac{i}{\hbar}Bt_{ev})-2i\sin(\frac{2}{\hbar}gqt_m)-i\sin(\frac{4}{\hbar}gqt_m)\Big)$.

Again we take the partial trace over the detector coordinate q in order to get the density matrix only dependent on the qubit coordinates $i = |0\rangle$ and $j = |1\rangle$

$$\rho_{ii,ij,ji,jj}(2t_{ev}+2t_m) = tr_q\rho(2t_{ev}+2t_m) = \int_{-\infty}^{\infty}\frac{1}{2\sqrt{\pi\sigma}}\exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big)\begin{pmatrix}\tilde{\alpha}(2t_{ev}+2t_m) & \tilde{\beta}(2t_{ev}+2t_m)\\ \tilde{\gamma}(2t_{ev}+2t_m) & \tilde{\delta}(2t_{ev}+2t_m)\end{pmatrix}dq. \tag{3.4.17}$$

In this case $\sigma\to 0$, so again $\exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big)\to\delta(q-q_0)$. This gives in equation 3.4.17

$$\rho_{ii,ij,ji,jj}(2t_{ev}+2t_m,q_0) \approx \frac{1}{2}\begin{pmatrix}\tilde{\alpha}(2t_{ev}+2t_m,q_0) & \tilde{\beta}(2t_{ev}+2t_m,q_0)\\ \tilde{\gamma}(2t_{ev}+2t_m,q_0) & \tilde{\delta}(2t_{ev}+2t_m,q_0)\end{pmatrix} \tag{3.4.18}$$

with $\tilde{\alpha}(2t_{ev}+2t_m,q_0) = \frac{1}{8}\exp(-4\frac{i}{\hbar}Bt_{ev})\Big(-2i\exp(8\frac{i}{\hbar}Bt_{ev})\sin(\frac{2}{\hbar}gq_0t_m)+2i\exp(2\frac{i}{\hbar}Bt_{ev})\sin(\frac{4}{\hbar}gq_0t_m)$
$-2i\exp(6\frac{i}{\hbar}Bt_{ev})\sin(\frac{4}{\hbar}gq_0t_m)-i\exp(8\frac{i}{\hbar}Bt_{ev})\sin(\frac{4}{\hbar}gq_0t_m)+8\exp(4\frac{i}{\hbar}Bt_{ev})+2i\sin(\frac{2}{\hbar}gq_0t_m)+i\sin(\frac{4}{\hbar}gq_0t_m)\Big)$,
$\tilde{\beta}(2t_{ev}+2t_m,q_0) = -\frac{1}{8}\exp(-4\frac{i}{\hbar}Bt_{ev})\Big(4\exp(4\frac{i}{\hbar}Bt_{ev})\cos(\frac{2}{\hbar}gq_0t_m)-2\exp(2\frac{i}{\hbar}Bt_{ev})\cos(\frac{4}{\hbar}gq_0t_m)$
$+2\exp(6\frac{i}{\hbar}Bt_{ev})\cos(\frac{4}{\hbar}gq_0t_m)+\exp(8\frac{i}{\hbar}Bt_{ev})\cos(\frac{4}{\hbar}gq_0t_m)+2\exp(2\frac{i}{\hbar}Bt_{ev})-4\exp(4\frac{i}{\hbar}Bt_{ev})-2\exp(6\frac{i}{\hbar}Bt_{ev})$
$-\exp(8\frac{i}{\hbar}Bt_{ev})-4\cos(\frac{2}{\hbar}gq_0t_m)-\cos(\frac{4}{\hbar}gq_0t_m)-3\Big)$,
$\tilde{\gamma}(2t_{ev}+2t_m,q_0) = \frac{1}{8}\exp(-4\frac{i}{\hbar}Bt_{ev})\Big(-4\exp(4\frac{i}{\hbar}Bt_{ev})\cos(\frac{2}{\hbar}gq_0t_m)+4\exp(8\frac{i}{\hbar}Bt_{ev})\cos(\frac{2}{\hbar}gq_0t_m)$
$-2\exp(2\frac{i}{\hbar}Bt_{ev})\cos(\frac{4}{\hbar}gq_0t_m)+2\exp(6\frac{i}{\hbar}Bt_{ev})\cos(\frac{4}{\hbar}gq_0t_m)+\exp(8\frac{i}{\hbar}Bt_{ev})\cos(\frac{4}{\hbar}gq_0t_m)+2\exp(2\frac{i}{\hbar}Bt_{ev})$
$+4\exp(4\frac{i}{\hbar}Bt_{ev})-2\exp(6\frac{i}{\hbar}Bt_{ev})+3\exp(8\frac{i}{\hbar}Bt_{ev})-\cos(\frac{4}{\hbar}gq_0t_m)+1\Big)$ and
$\tilde{\delta}(2t_{ev}+2t_m,q_0) = \frac{1}{8}\exp(-4\frac{i}{\hbar}Bt_{ev})\Big(2i\exp(8\frac{i}{\hbar}Bt_{ev})\sin(\frac{2}{\hbar}gq_0t_m)-2i\exp(2\frac{i}{\hbar}Bt_{ev})\sin(\frac{4}{\hbar}gq_0t_m)$
$+2i\exp(6\frac{i}{\hbar}Bt_{ev})\sin(\frac{4}{\hbar}gq_0t_m)+i\exp(8\frac{i}{\hbar}Bt_{ev})\sin(\frac{4}{\hbar}gq_0t_m)+8\exp(4\frac{i}{\hbar}Bt_{ev})-2i\sin(\frac{2}{\hbar}gq_0t_m)-i\sin(\frac{4}{\hbar}gq_0t_m)\Big)$.

The probability that the qubit is in state $|0\rangle$ after $2t_{ev}+2t_m$ is, using equation 2.4.4, given by

$$P_0(2t_m+2t_{ev},q_0) = \frac{3}{4}-\frac{1}{4}\cos(\frac{2}{\hbar}gq_0t_m)+\frac{1}{4}\cos(\frac{4}{\hbar}Bt_{ev})+\frac{1}{4}\cos(\frac{4}{\hbar}Bt_{ev})\cos(\frac{2}{\hbar}gq_0t_m) \tag{3.4.19}$$

and the probability that the qubit is in state $|1\rangle$ is, using equation 2.4.5, given by

$$P_1(2t_m+2t_{ev},q_0) = \frac{1}{4}+\frac{1}{4}\cos(\frac{2}{\hbar}gq_0t_m)-\frac{1}{4}\cos(\frac{4}{\hbar}Bt_{ev})-\frac{1}{4}\cos(\frac{4}{\hbar}Bt_{ev})\cos(\frac{2}{\hbar}gq_0t_m). \tag{3.4.20}$$

The calculations of the probabilities at $t = 3t_{ev} + 3t_m$ and at $t = 4t_{ev} + 4t_m$ have been done in the same way and are given in the appendix in subsection 5.1.1.

Unfortunately, there is not a pattern in these calculated probabilities. Therefore, the next probabilities are calculated numerically according to the same method as the algebraic expressions were determined. The wavefunction at time $t = kt_{ev} + kt_m$ with $k = 0, 1, 2, ..., n$ is given by $A^k|\Phi(0)\rangle$ with A given in equation 3.4.8 and $n < \infty$ an integer. Consequently, the density matrix is calculated for each k and then the detector coordinate q is traced out in order to get the density matrix only dependent on qubit coordinates. This is done in the limit of $\sigma \to 0$. From this resulting matrix, the probabilities are calculated that the qubit is in state $|0\rangle$ and in state $|1\rangle$. The first four numeric determined probabilities correspond with the values of the algebraic calculated probabilities. In this way, the probabilities are known on discrete moments in time $kt_{ev} + kt_m$ with $k = 0, 1, 2, ..., n$.

We also want to know what the probabilities of the qubit states are during each period of time evolution and during each period of measurement. Therefore, define matrix A_{ev} and A_m as follows.

$$A_{ev}(t) = \exp\big(-\frac{i}{\hbar}H_0 t\big) = \begin{pmatrix} \exp(-\frac{i}{\hbar}Bt) & 0 \\ 0 & \exp(\frac{i}{\hbar}Bt) \end{pmatrix} \tag{3.4.21}$$

and

$$A_m(t) = \exp\big(-\frac{i}{\hbar}H_{int} t\big) = \begin{pmatrix} \cos(\frac{1}{\hbar}gqt) & -i\sin(\frac{1}{\hbar}gqt) \\ -i\sin(\frac{1}{\hbar}gqt) & \cos(\frac{1}{\hbar}gqt) \end{pmatrix}. \tag{3.4.22}$$

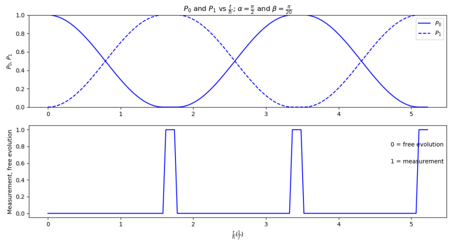
The time t is divided in small steps of length Δt. The wavefunction $A_{ev}^k|\Phi(0)\rangle$ with $k = 0, 1, 2, ..., n$ is calculated for each k during t_{ev}, with $t_{ev} = n\Delta t$ and $n < \infty$ an integer. Then a measurement is done during t_m and the wavefunction $A_m^l A_{ev}^n|\Phi(0)\rangle$ with $l = 0, 1, 2, ..., m$ is calculated for each l during t_m, with $t_m = m\Delta t$ and $m < \infty$ an integer. Then there is free evolution again and the wavefunction $A_{ev}^k A_m^m A_{ev}^n|\Phi(0)\rangle$ is calculated for each k during t_{ev}. During the subsequent measurement, the wavefunction $A_m^l A_{ev}^n A_m^m A_{ev}^n|\Phi(0)\rangle$ is calculated for each l during t_m, etc. For each time step, the density matrix is calculated using the wave function. The density matrix only dependent on qubit coordinates $|0\rangle$ and $|1\rangle$ is determined by taking the partial trace over the detector coordinate q in the limit of $\sigma \to 0$. From this matrix, the probabilities that the qubit is in state $|0\rangle$ and in state $|1\rangle$ are determined. As a result, the behaviour of the probabilities continuously in time is known by taking $\Delta t \to 0$.

Since in this case $\sigma \to 0$, the dispersion of the detector coordinate q is zero so the detector coordinate q has one value q_0. [8] There is a perfect measurement. When $q_0 = 0$, there is no interaction between the detector and the qubit. $H_{int} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ in this case. There is only free evolution so the probabilities will evolve in time as described in subsection 3.2.

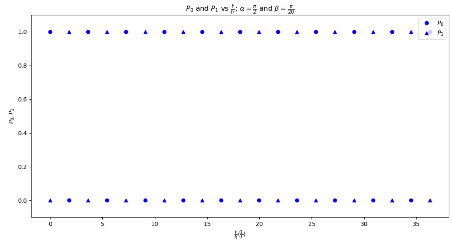
A result of the first three periods of free evolution alternated with a measurement and of the first 40 points in time at $k\frac{t_{ev}+t_m}{\hbar}$ with $k = 0, 1, ..., 40$ is seen in figure 5. The dimensionless parameters, defined in subsection 3.1 [9], are $\alpha = \frac{\pi}{2}$ and $\beta = \frac{\pi}{20}$, so in the limit of $t_m \ll t_{ev}$, and the dimensionless variable $q_0 = 1$, so there is interaction between the detector and the qubit.

[8] $\exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big) \to \delta(q - q_0)$, so this is mathematically correct.

[9] Remember $\alpha = \frac{Bt_{ev}}{\hbar}$ and $\beta = \frac{gt_m}{\hbar}$.



(a) Top graph: Graph of the probability of being in state $|0\rangle$ P_0 (continuous line) or state $|1\rangle$ P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = \frac{\pi}{2}$ and $\beta = \frac{\pi}{20}$. Bottom graph: Step function where 0 denotes a period of free evolution and 1 a period of a measurement as function of $\frac{t}{\hbar}$ with t the time.



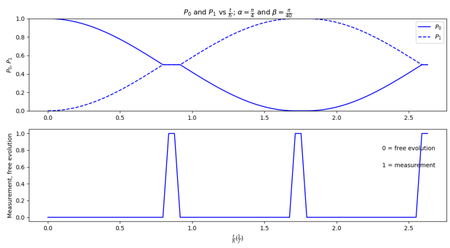
(b) Graph of the probability of being in state $|0\rangle$ P_0 (circles) or state $|1\rangle$ P_1 (triangles) as function of $\frac{t}{\hbar}$ with t the time at discrete points; $\alpha = \frac{\pi}{2}$ and $\beta = \frac{\pi}{20}$.

Figure 5.

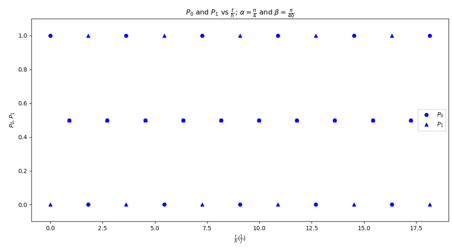
When there is free evolution, the system evolves under the Hamiltonian H_0 and as noticed in subsection 3.2, the probabilities of the qubit to be in state $|0\rangle$ or $|1\rangle$ will evolve in time according to a sine form between 0 and 1. When a measurement is done, the system evolves under H_{int}. The eigenvectors of the observable σ_x in H_{int} are $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ and $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$, so respectively $|0\rangle$ and $|1\rangle$. In this situation with parameters $\alpha = \frac{\pi}{2}$ and $\beta = \frac{\pi}{20}$, the measurement is done when the qubit is with a probability 1 in one of its states. When doing a measurement at that moment, the wave function will collapse to this state with probability 1 and the qubit will remain in this state. As we can see in figure 5b, where the probabilities in which state the qubit is are plotted after each measurement, the expected period of figure 5a will repeat like this in time. This is in line with the expectation.

A stronger measurement results in the same plot. The interaction between the qubit and detector becomes larger by increasing gq_0. H_{int} is directly proportional to gq_0. However, since there is only a measurement and no free evolution during the period of measurement, the evolution of the probabilities will freeze during this period, regardless of the strength of the measurement.

Now we take other parameter values for α and β, namely $\alpha = \frac{\pi}{4}$ and $\beta = \frac{\pi}{40}$, so still in the limit of $t_m \ll t_{ev}$. Again, $q_0 = 1$.



(a) Top graph: Graph of the probability of being in state $|0\rangle$ P_0 (continuous line) or state $|1\rangle$ P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = \frac{\pi}{4}$ and $\beta = \frac{\pi}{40}$. Bottom graph: Step function where 0 denotes a period of free evolution and 1 a period of a measurement as function of $\frac{t}{\hbar}$ with t the time.



(b) Graph of the probability of being in state $|0\rangle$ P_0 (circles) or state $|1\rangle$ P_1 (triangles) as function of $\frac{t}{\hbar}$ with t the time at discrete points; $\alpha = \frac{\pi}{4}$ and $\beta = \frac{\pi}{40}$.

Figure 6.

As can be concluded from figure 6b, the sine pattern from figure 6a will repeat. Now the measurement is done at moments in time when the qubit is with a probability of 1 or $\frac{1}{2}$ in one of its states. The wavefunction will collapse with a probability of 1 to the state where the qubit already is in, when the measurement is done at a moment when the qubit is with a probability of 1 in one of its states. When the measurement is done at the moment that the qubit is with a probability of

$\frac{1}{2}$ in one of its states, the wavefunction will collapse with a probability of $\frac{1}{2}$ to one of those states during the measurement. So during the measurement, these probabilities freeze. This is in line with our expectation.

Again, a stronger measurement will result in the same plot.

3.4.2 Situation $\sigma \not\to 0$

In this section, the wavefunctions, density matrices and probabilities are expressed in the following dimensionless terms: $\frac{1}{\hbar}Bt_{ev}$, $\frac{1}{\hbar}gqt_m$, $\frac{1}{\hbar}gq_0t_m$ and $(\frac{1}{\hbar}gt_m)^2\sigma$. These can be expressed as follows in the dimensionless parameters $\alpha = \frac{Bt_{ev}}{\hbar}$, $\beta = \frac{gt_m\sqrt{2\sigma}}{\hbar}$. and the dimensionless variables, $q' = \frac{q}{\sqrt{2\sigma}}$ and $q'_0 = \frac{q_0}{\sqrt{2\sigma}}$. $\frac{1}{\hbar}Bt_{ev} = \alpha$, $\frac{1}{\hbar}gqt_m = \beta q'$, $\frac{1}{\hbar}gq_0t_m = \beta q'_0$ and $(\frac{1}{\hbar}gt_m)^2\sigma = \frac{\beta^2}{2}$. The initial state of the detector is expressed as follows in the dimensionless variables, $|\phi(0)\rangle = \exp\big(-(q'-q'_0)^2\big)$.

The integral in the expression by calculating the partial trace over the detector coordinate q will now be calculated without the assumption $\sigma \to 0$. Consider this case at $t = t_{ev} + t_m$. The integral from equation 3.4.11 is calculated for each matrix element, resulting in the following equations for ρ_{ii}, ρ_{ij}, ρ_{ji} and ρ_{jj}. Equations 3.4.23 and 3.4.24 are worked out completely in appendix subsection 5.2.[10]

$$\begin{aligned}\rho_{ii}(t_{ev}+t_m) &= \int_{-\infty}^{\infty} \frac{1}{2\sqrt{\pi\sigma}} \exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big)\tilde{\alpha}(t_{ev}+t_m)dq \\ &= \frac{1}{2} + 2\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)\cos(\frac{1}{\hbar}gt_mq_0)\sin(\frac{1}{\hbar}gt_mq_0)\cos(\frac{1}{\hbar}Bt_{ev})\sin(\frac{1}{\hbar}Bt_{ev})\end{aligned} \tag{3.4.23}$$

$$\begin{aligned}\rho_{ij}(t_{ev}+t_m) &= \int_{-\infty}^{\infty} \frac{1}{2\sqrt{\pi\sigma}} \exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big)\tilde{\beta}(t_{ev}+t_m)dq \\ &= -\frac{\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)}{4}\Big(i\sin(2(\frac{1}{\hbar}Bt_{ev}+\frac{1}{\hbar}gt_mq_0)) + i\sin(2(\frac{1}{\hbar}Bt_{ev}-\frac{1}{\hbar}gt_mq_0)) \\ &\quad - 2\exp((\frac{1}{\hbar}gt_m)^2\sigma)\cos(\frac{2}{\hbar}Bt_{ev})\Big)\end{aligned} \tag{3.4.24}$$

$$\begin{aligned}\rho_{ji}(t_{ev}+t_m) &= \int_{-\infty}^{\infty} \frac{1}{2\sqrt{\pi\sigma}} \exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big)\tilde{\gamma}(t_{ev}+t_m)dq \\ &= \frac{\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)}{4}\Big(i\sin(2(\frac{1}{\hbar}Bt_{ev}+\frac{1}{\hbar}gt_mq_0)) + i\sin(2(\frac{1}{\hbar}Bt_{ev}-\frac{1}{\hbar}gt_mq_0)) \\ &\quad + 2\exp((\frac{1}{\hbar}gt_m)^2\sigma)\cos(\frac{2}{\hbar}Bt_{ev})\Big)\end{aligned} \tag{3.4.25}$$

$$\begin{aligned}\rho_{jj}(t_{ev}+t_m) &= \int_{-\infty}^{\infty} \frac{1}{2\sqrt{\pi\sigma}} \exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big)\tilde{\delta}(t_{ev}+t_m)dq \\ &= \frac{1}{2} - 2\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)\cos(\frac{1}{\hbar}gt_mq_0)\sin(\frac{1}{\hbar}gt_mq_0)\cos(\frac{1}{\hbar}Bt_{ev})\sin(\frac{1}{\hbar}Bt_{ev})\end{aligned} \tag{3.4.26}$$

So the result of equation 3.4.11 is $\rho_{ii,ij,ji,jj}(t_{ev}+t_m) = \begin{pmatrix}\rho_{ii}(t_{ev}+t_m) & \rho_{ij}(t_{ev}+t_m) \\ \rho_{ji}(t_{ev}+t_m) & \rho_{jj}(t_{ev}+t_m)\end{pmatrix}$.

The probability that the qubit is still in state $|0\rangle$ at $t_{ev}+t_m$ is, using equation 2.4.4, given by

[10]All the other integrals in this subsection are calculated using the same sort of steps.

$$P_0(t_m + t_{ev}) = \frac{1}{2} + \frac{1}{2}\cos(\frac{2}{\hbar}Bt_{ev}) \tag{3.4.27}$$

and the probability that the qubit is in state $|1\rangle$ is, using equation 2.4.5, given by

$$P_1(t_m + t_{ev}) = \frac{1}{2} - \frac{1}{2}\cos(\frac{2}{\hbar}Bt_{ev}). \tag{3.4.28}$$

Consider the general case of the partial trace at $t = 2t_{ev} + 2t_m$. The integral from equation 3.4.17 is calculated for each matrix element at this moment in time, resulting in the following equations

$$\begin{aligned}\rho_{ii}(2t_{ev} + 2t_m) = &\int_{-\infty}^{\infty} \frac{1}{2\sqrt{\pi\sigma}} \exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big)\tilde{\alpha}(2t_{ev} + 2t_m)dq = \\ &\frac{1}{2} + 2\exp(-4(\frac{1}{\hbar}gt_m)^2\sigma)\cos(\frac{1}{\hbar}gt_mq_0)\sin(\frac{1}{\hbar}gt_mq_0)\cos(\frac{1}{\hbar}Bt_{ev})\sin(\frac{1}{\hbar}Bt_{ev})\Big(2(2\cos^2(\frac{1}{\hbar}gt_mq_0) \\ &+ \exp(3(\frac{1}{\hbar}gt_m)^2\sigma) - 1)\cos^2(\frac{1}{\hbar}Bt_{ev}) - \exp(3(\frac{1}{\hbar}gt_m)^2\sigma)\Big)\end{aligned} \tag{3.4.29}$$

$$\begin{aligned}\rho_{ij}(2t_{ev} + 2t_m) = &\int_{-\infty}^{\infty} \frac{1}{2\sqrt{\pi\sigma}} \exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big)\tilde{\beta}(2t_{ev} + 2t_m)dq = \\ &\frac{1}{16}\Big(-i\exp(-4(\frac{1}{\hbar}gt_m)^2\sigma)\sin(\frac{4}{\hbar}Bt_{ev} + 4\frac{1}{\hbar}gt_mq_0) - 2i\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)\sin(\frac{4}{\hbar}Bt_{ev} + 2\frac{1}{\hbar}gt_mq_0) \\ &+ 2\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)\cos(\frac{4}{\hbar}Bt_{ev} + 2\frac{1}{\hbar}gt_mq_0) - 2i\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)\sin(\frac{4}{\hbar}Bt_{ev} - 2\frac{1}{\hbar}gt_mq_0) \\ &+ 2\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)\cos(\frac{4}{\hbar}Bt_{ev} - 2\frac{1}{\hbar}gt_mq_0) - 4\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)\cos(2\frac{1}{\hbar}gt_mq_0) \\ &- i\exp(-4(\frac{1}{\hbar}gt_m)^2\sigma)\sin(\frac{4}{\hbar}Bt_{ev} - 4\frac{1}{\hbar}gt_mq_0) - 2i\exp(-4(\frac{1}{\hbar}gt_m)^2\sigma)\sin(\frac{2}{\hbar}Bt_{ev} + 4\frac{1}{\hbar}gt_mq_0) \\ &- 2i\exp(-4(\frac{1}{\hbar}gt_m)^2\sigma)\sin(\frac{2}{\hbar}Bt_{ev} - 4\frac{1}{\hbar}gt_mq_0) - 2i\sin(\frac{4}{\hbar}Bt_{ev}) + 4\cos(\frac{4}{\hbar}Bt_{ev}) + 4i\sin(\frac{2}{\hbar}Bt_{ev}) + 4\Big)\end{aligned} \tag{3.4.30}$$

$$\begin{aligned}\rho_{ji}(2t_{ev} + 2t_m) = &\int_{-\infty}^{\infty} \frac{1}{2\sqrt{\pi\sigma}} \exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big)\tilde{\gamma}(2t_{ev} + 2t_m)dq = \\ &\frac{1}{16}\Big(i\exp(-4(\frac{1}{\hbar}gt_m)^2\sigma)\sin(\frac{4}{\hbar}Bt_{ev} + 4\frac{1}{\hbar}gt_mq_0) + 2i\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)\sin(\frac{4}{\hbar}Bt_{ev} + 2\frac{1}{\hbar}gt_mq_0) \\ &+ 2\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)\cos(\frac{4}{\hbar}Bt_{ev} + 2\frac{1}{\hbar}gt_mq_0) + 2i\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)\sin(\frac{4}{\hbar}Bt_{ev} - 2\frac{1}{\hbar}gt_mq_0) \\ &+ 2\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)\cos(\frac{4}{\hbar}Bt_{ev} - 2\frac{1}{\hbar}gt_mq_0) - 4\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)\cos(2\frac{1}{\hbar}gt_mq_0) \\ &+ i\exp(-4(\frac{1}{\hbar}gt_m)^2\sigma)\sin(\frac{4}{\hbar}Bt_{ev} - 4\frac{1}{\hbar}gt_mq_0) + 2i\exp(-4(\frac{1}{\hbar}gt_m)^2\sigma)\sin(\frac{2}{\hbar}Bt_{ev} + 4\frac{1}{\hbar}gt_mq_0) \\ &+ 2i\exp(-4(\frac{1}{\hbar}gt_m)^2\sigma)\sin(\frac{2}{\hbar}Bt_{ev} - 4\frac{1}{\hbar}gt_mq_0) + 2i\sin(\frac{4}{\hbar}Bt_{ev}) + 4\cos(\frac{4}{\hbar}Bt_{ev}) - 4i\sin(\frac{2}{\hbar}Bt_{ev}) + 4\Big)\end{aligned} \tag{3.4.31}$$

$$\rho_{jj}(2t_{ev}+2t_m) = \int_{-\infty}^{\infty} \frac{1}{2\sqrt{\pi\sigma}} \exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big)\tilde{\delta}(2t_{ev}+2t_m)dq =$$
$$\frac{1}{2} - 2\exp(-4(\frac{1}{\hbar}gt_m)^2\sigma)\cos(\frac{1}{\hbar}gt_mq_0)\sin(\frac{1}{\hbar}gt_mq_0)\cos(\frac{1}{\hbar}Bt_{ev})\sin(\frac{1}{\hbar}Bt_{ev})\Big(2(2\cos^2(\frac{1}{\hbar}gt_mq_0)$$
$$+\exp(3(\frac{1}{\hbar}gt_m)^2\sigma)-1)\cos^2(\frac{1}{\hbar}Bt_{ev}) - \exp(3(\frac{1}{\hbar}gt_m)^2\sigma)\Big) \tag{3.4.32}$$

So the result of equation 3.4.17 is $\rho_{ii,ij,ji,jj}(2t_{ev}+2t_m) = \begin{pmatrix} \rho_{ii}(2t_{ev}+2t_m) & \rho_{ij}(2t_{ev}+2t_m) \\ \rho_{ji}(2t_{ev}+2t_m) & \rho_{jj}(2t_{ev}+2t_m) \end{pmatrix}$.

The probability that the qubit is in state $|0\rangle$ after $2t_{ev}+2t_m$ is given by

$$\begin{aligned} P_0(2t_m+2t_{ev}) =& \frac{3}{4} + \frac{1}{4}\cos(\frac{4}{\hbar}Bt_{ev}) - \frac{1}{4}\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)\cos(2\frac{1}{\hbar}gt_mq_0) \\ &+ \frac{1}{4}\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)\cos(\frac{4}{\hbar}Bt_{ev})cos(\frac{2}{\hbar}gt_mq_0) \end{aligned} \tag{3.4.33}$$

and the probability that the qubit is in state $|1\rangle$ is given by

$$\begin{aligned} P_1(2t_m+2t_{ev}) =& \frac{1}{4} - \frac{1}{4}\cos(\frac{4}{\hbar}Bt_{ev}) + \frac{1}{4}\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)\cos(2\frac{1}{\hbar}gt_mq_0) \\ &- \frac{1}{4}\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)\cos(\frac{4}{\hbar}Bt_{ev})cos(\frac{2}{\hbar}gt_mq_0). \end{aligned} \tag{3.4.34}$$

The calculations of the probabilities at $t = 3t_{ev} + 3t_m$ are done in the same way and given in the appendix in subsection 5.1.2.

The general case for the probabilities at $t = 4t_{ev} + 4t_m$ was a really long expression, so those calculations are not included. Unfortunately, again there is not a pattern in these calculated probabilities. Therefore, the next steps are calculated numerically. This is done in the same way as described at the end of subsection 3.4.1, but now the partial trace is calculated without the simplification of $\sigma \to 0$. An integration tool is used in Python [1]. The first three numeric determined probabilities correspond with the values of the algebraic calculated probabilities.

We also calculated numerically the probabilities during the first three periods of time evolution and measurement. This is done in the same way as described at the end of subsection 3.4.1 with matrices A_{ev} and A_m, but now without the simplification of $\sigma \to 0$.

σ indicates the degree of spread in the detector coordinate q. $\sigma = 0$ indicates a perfect measurement. When $\sigma > 0$, there is a measurement error in the detector. The larger σ, the bigger the error and the harder it is for the detector to do a proper measurement and to tell correctly in which state the qubit is. This results in dissipation of the system. The oscillations in the probabilities will damp.

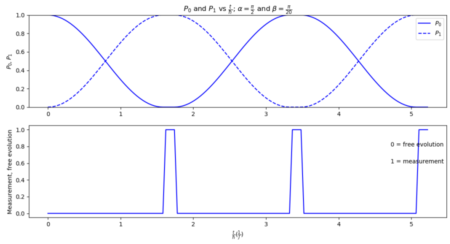
When $\sigma = 0$ and $q_0 = 0$, we notice that the algebraic calculated probabilities at $t_m + t_{ev}$ and $2t_m + 2t_{ev}$ correspond with the expressions of the probabilities when there is only evolution, as expected. This is harder to notice in the calculated probabilities at $3t_m + 3t_{ev}$, but the values of these probabilities are the same as the values during only free evolution at this moment.

When $\sigma > 0$ and $q_0 = 0$, there will be some interaction between the qubit and detector, since there are values of q which are unequal to 0. There is integrated from $-\infty$ to ∞ over q by the calculation of the partial trace over q so these values are taken along in this calculation. Since there is only a measurement and no free evolution during the period of measurement, the strength of the measurement does not matter and the probabilities will freeze during this period.

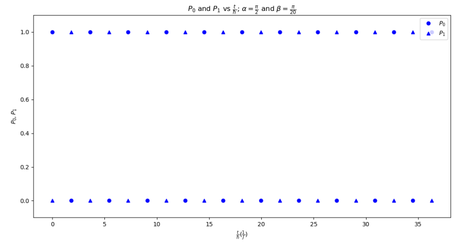
When we compare the algebraic expressions of the probabilities at $t_m + t_{ev}$ in the cases of $\sigma \to 0$ and $\sigma > 0$, we see no difference. When we compare the expressions at $2t_m + 2t_{ev}$, we notice that

all the terms dependent on $\beta q_0'$ and also a term dependent on α are multiplied with $\exp(\frac{-\beta^2}{2})$. The expressions of the probabilities at $3t_m + 3t_{ev}$ are harder to compare since they are quite different, but also at this moment in time we notice that all the terms dependent on $\beta q_0'$ and also terms dependent on α, which represent the free evolution, are multiplied with $\exp(\frac{-n\beta^2}{2})$ with n an integer. So probably after $t_m + t_{ev}$, there are exponentials dependent on β^2 in the algebraic expressions of the probabilities, which cause damping of the oscillations of the probabilities. The larger β, the larger σ, so the faster the oscillations will damp. The dissipation of the system will go faster.
So also in the algebraic expressions we see the damping of the oscillations of the probabilities, caused by σ.

A result of the first three periods of free evolution and measurement and the first 40 points in time at $k\frac{t_{ev}+t_m}{\hbar}$ with $k = 0, 1, ..., 40$ is given in the following figure. The dimensionless parameters from subsection 3.1 [11] are $\alpha = \frac{\pi}{2}$ and $\beta = \frac{\pi}{20}$ and the variable $q_0' = 1$.



(a) Top graph: Graph of the probability of being in state $|0\rangle$ P_0 (continuous line) or state $|1\rangle$ P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = \frac{\pi}{2}$ and $\beta = \frac{\pi}{20}$. Bottom graph: Step function where 0 denotes a period of free evolution and 1 a period of a measurement as function of $\frac{t}{\hbar}$ with t the time.

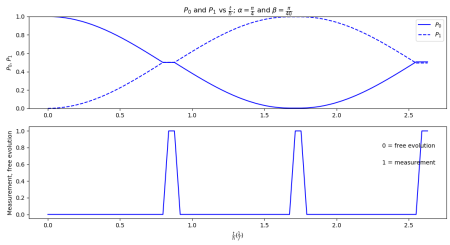


(b) Graph of the probability of being in state $|0\rangle$ P_0 (circles) or state $|1\rangle$ P_1 (triangles) as function of $\frac{t}{\hbar}$ with t the time at discrete points; $\alpha = \frac{\pi}{2}$ and $\beta = \frac{\pi}{20}$.

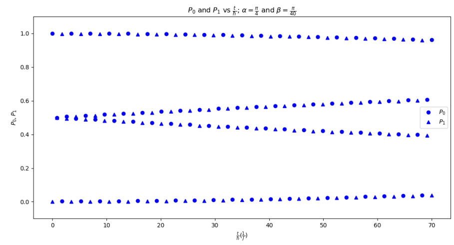
Figure 7.

These are the same plots as in figures 5a and 5b. So the simplification $\sigma \to 0$ did not make any difference for these parameter values. From equations 3.4.33 and 3.4.34 follows that for $\alpha = \frac{\pi}{2}$ the exponentials dependent on β^2 cancel to each other. This will probably happen to all the damping terms at every moment in time with this value of α.

Now the plots are made for the parameters $\alpha = \frac{\pi}{4}$ and $\beta = \frac{\pi}{40}$ and the variable $q_0' = 1$. Now the first 80 points in time at $k\frac{t_{ev}+t_m}{\hbar}$ are plotted instead of the first 40 points.



(a) Top graph: Graph of the probability of being in state $|0\rangle$ P_0 (continuous line) or state $|1\rangle$ P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = \frac{\pi}{4}$ and $\beta = \frac{\pi}{40}$. Bottom graph: Step function where 0 denotes a period of free evolution and 1 a period of a measurement as function of $\frac{t}{\hbar}$ with t the time.



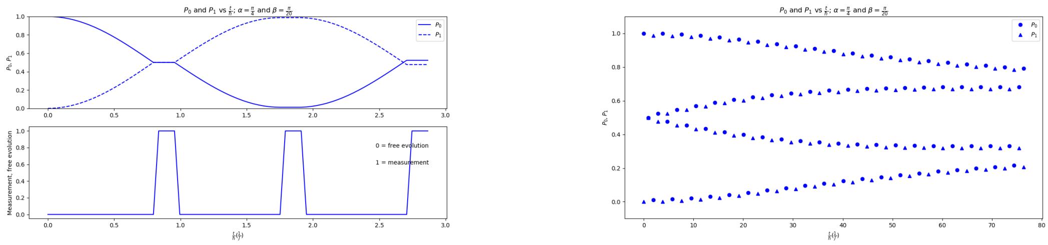
(b) Graph of the probability of being in state $|0\rangle$ P_0 (circles) or state $|1\rangle$ P_1 (triangles) as function of $\frac{t}{\hbar}$ with t the time at discrete points; $\alpha = \frac{\pi}{4}$ and $\beta = \frac{\pi}{40}$.

Figure 8.

[11] Remember $\alpha = \frac{Bt_{ev}}{\hbar}$, $\beta = \frac{gt_m\sqrt{2\sigma}}{\hbar}$ and $q_0' = \frac{q_0}{\sqrt{2\sigma}}$.

There are differences between the results when $\sigma \to 0$ and in this case when $\sigma \not\to 0$ at these parameter values. The probabilities now freeze at different values than 0, 0.5 or 1 at $nt_{ev} + nt_m$ with $n = 2, 3, ...$ in comparison to the case when $\sigma \to 0$ at these parameter values, shown in figure 6. As can be concluded from equations 3.4.27 and 3.4.28, there is no damping of the free evolution at $t_{ev} + t_m$. However, in the expressions of the probabilities at $2t_{ev} + 2t_m$ and $3t_{ev} + 3t_m$, the exponential damping terms are present and the free evolution is damped. This will continue in time, as we can see in figure 8b.

Now plots are made for a larger $g\sqrt{\sigma}$. So β increases while α and q_0' remain the same.



(a) Top graph: Graph of the probability of being in state $|0\rangle$ P_0 (continuous line) or state $|1\rangle$ P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = \frac{\pi}{4}$ and $\beta = \frac{\pi}{20}$. Bottom graph: Step function where 0 denotes a period of free evolution and 1 a period of a measurement as function of $\frac{t}{\hbar}$ with t the time.

(b) Graph of the probability of being in state $|0\rangle$ P_0 (circles) or state $|1\rangle$ P_1 (triangles) as function of $\frac{t}{\hbar}$ with t the time at discrete points; $\alpha = \frac{\pi}{4}$ and $\beta = \frac{\pi}{20}$.

Figure 9.

As can be seen by comparison with figures 6 and 8, the larger β so $g\sqrt{\sigma}$, the faster the oscillations caused by the free evolution will damp. Indeed, the larger σ, the larger the measurement error in the detector and the larger the dissipation of the system.

The quantum Zeno effect can occur when doing fast subsequent measurements or a continuous measurement. In this subsection $t_m \ll t_{ev}$ was assumed such that H_0 could be neglected during measurements. For fast subsequent measurements and a continuous measurement we want $t_{ev} \ll t_m$, so this is not the right situation for the quantum Zeno effect to occur. However, when only a measurement is done without evolution, the system freezes. The probabilities for the qubit to be in one of its states remain the same. Since there is no free evolution during the measurement, this is expected.

3.5 System evolution of a series of measurements during free evolution of the spin alternated with only free evolution of the spin

In this section, we study the same system as in subsection 3.4, but in this subsection there is during the measurement also free evolution. So periods of only free evolution and periods of a measurement, executed by the detector under the same circumstances as before, during free evolution alternate. Consequently, we will demonstrate how the probabilities of the qubit to be in one of its states develop in time and see if the quantum Zeno effect can be observed.

Consider the same system as in subsection 3.4, so a particle with spin 1/2, and a detector with coordinate q and momentum p. The particle represents the qubit with the states $|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ and $|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$. Take again the initial state of the spin an eigenstate of the spin in the x-direction

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}, \tag{3.5.1}$$

and the initial state of the detector Gaussian

$$|\phi(0)\rangle \propto \exp\Big(-\frac{(q-q_0)^2}{2\sigma}\Big) \tag{3.5.2}$$

with q the detector coordinate which is assumed to be continuous, q_0 the mean of the detector coordinate q and σ the dispersion. The initial state of the system is again the product state $|\Phi(0)\rangle = |\psi(0)\rangle|\phi(0)\rangle$.

Now there is during the period of measurement also free evolution, so the Hamiltonian under which the system evolves is in this case given by $\hat{H} = \hat{H}_0 + \hat{H}_{int}$. The wave function $|\psi(t)\rangle$ evolves in this case as follows in time

$$|\psi(t)\rangle = \exp\Big(-\frac{i}{\hbar}\int_0^t (B\sigma_z + g(t')q\sigma_x)dt'\Big)|\psi(0)\rangle. \tag{3.5.3}$$

We will write this exponential in matrix form and derive an expression for $|\psi(t)\rangle$.

Assume that $g(t)$ is time independent [12]. We have $\hat{H} = \begin{pmatrix} B & gq \\ gq & -B \end{pmatrix}$. Let $\hat{\alpha} = -\frac{i}{\hbar}t$. The Taylor series of the exponential function give

$$\exp(\hat{\alpha}\hat{H}) = \sum_{n=0}^{\infty}\frac{(\hat{\alpha}\hat{H})^n}{n!} = \sum_{n=0}^{\infty}\frac{(\hat{\alpha}\hat{H})^{2n}}{(2n)!} + \sum_{n=0}^{\infty}\frac{(\hat{\alpha}\hat{H})^{2n+1}}{(2n+1)!}. \tag{3.5.4}$$

We notice that $\hat{H}^2 = (B^2+g^2q^2)I$. From this follows $\hat{H}^{2n} = (B^2+g^2q^2)^n I$ and $\hat{H}^{2n+1} = (B^2+g^2q^2)^n\hat{H}$. This gives

$$\exp(\hat{\alpha}\hat{H}) = I\sum_{n=0}^{\infty}(B^2+g^2q^2)^n\frac{\hat{\alpha}^{2n}}{(2n)!} + \hat{H}\sum_{n=0}^{\infty}(B^2+g^2q^2)^n\frac{\hat{\alpha}^{2n+1}}{(2n+1)!} \tag{3.5.5}$$

and with the Taylor series of the hyperbolic cosine and sine, this gives

$$\exp(\hat{\alpha}\hat{H}) = I\cosh(\hat{\alpha}\sqrt{B^2+g^2q^2}) + \hat{H}\frac{\sinh(\hat{\alpha}\sqrt{B^2+g^2q^2})}{\sqrt{B^2+g^2q^2}}. \tag{3.5.6}$$

Let $\hat{\beta} = \frac{i}{\hbar}t\sqrt{B^2+g^2q^2}$. This results in the following matrix

$$\begin{pmatrix} \cosh(\hat{\beta}) + \frac{B}{\sqrt{B^2+g^2q^2}}\sinh(-\hat{\beta}) & \frac{gq}{\sqrt{B^2+g^2q^2}}\sinh(-\hat{\beta}) \\ \frac{gq}{\sqrt{B^2+g^2q^2}}\sinh(-\hat{\beta}) & \cosh(\hat{\beta}) - \frac{B}{\sqrt{B^2+g^2q^2}}\sinh(-\hat{\beta}) \end{pmatrix}. \tag{3.5.7}$$

With the initial state given as in equation 3.2.1 and using equation 3.5.3, the wavefunction evolves as follows in time,

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} \cos(\frac{1}{\hbar}t\sqrt{B^2+g^2q^2}) - i\frac{B+gq}{\sqrt{B^2+g^2q^2}}\sin(\frac{1}{\hbar}t\sqrt{B^2+g^2q^2}) \\ \cos(\frac{1}{\hbar}t\sqrt{B^2+g^2q^2}) - i\frac{-B+gq}{\sqrt{B^2+g^2q^2}}\sin(\frac{1}{\hbar}t\sqrt{B^2+g^2q^2}) \end{pmatrix}. \tag{3.5.8}$$

[12] The evolution of the wave function at discrete moments in time for $g(t)$ time dependent is given in the appendix in subsection 5.3.2.

Series of measurements

Consider a series of measurements. Each measurement is performed at time $t_k = \frac{kT}{n}$ during t_m under the Hamiltonian $H = H_{int} + H_0$, so now there is always free evolution of the system, also during the measurements. The system evolves under H_0 during t_{ev} between the measurements, so then there is only free evolution. Assume g to be time independent. The wavefunction evolves as follows

$$|\Phi(t_{ev} + t_m)\rangle = \hat{T} \exp\Big(- \frac{i}{\hbar} \int_{t_{ev}}^{t_{ev}+t_m} (H_{int} + H_0) dt' \Big) \exp\Big(- \frac{i}{\hbar} \int_0^{t_{ev}} H_0 dt' \Big) |\Phi(0)\rangle \qquad (3.5.9)$$

with $\hat{T}$ the time ordering operator.

3.5.1 Situation $\sigma \to 0$

In this section, the wavefunctions, density matrices and probabilities are expressed in the following dimensionless terms or combinations of these terms: $\frac{1}{\hbar} B t_{ev}$, $\frac{1}{\hbar} t_m \sqrt{B^2 + g^2 q^2}$, $\frac{1}{\hbar} t_m \sqrt{B^2 + g^2 q_0^2}$, $\frac{B}{\sqrt{B^2+g^2q^2}}$, $\frac{B}{\sqrt{B^2+g^2q_0^2}}$, $\frac{gq}{\sqrt{B^2+g^2q^2}}$ and $\frac{gq_0}{\sqrt{B^2+g^2q_0^2}}$. These can be expressed as follows in the dimesionless parameters α, $\beta(q)$, $\gamma(q)$ and $\delta(q)$ and the variables q and q_0, which are dimensionless: $\frac{1}{\hbar} B t_{ev} = \alpha$, $\frac{1}{\hbar} t_m \sqrt{B^2 + g^2 q^2} = \beta(q)$, $\frac{1}{\hbar} t_m \sqrt{B^2 + g^2 q_0^2} = \beta(q_0)$, $\frac{B}{\sqrt{B^2+g^2q^2}} = \gamma(q)$, $\frac{B}{\sqrt{B^2+g^2q_0^2}} = \gamma(q_0)$, $\frac{gq}{\sqrt{B^2+g^2q^2}} = \delta(q)$ and $\frac{gq_0}{\sqrt{B^2+g^2q_0^2}} = \delta(q_0)$.

The evolution of the wavefunction during t_{ev} is the same as described in equations 3.4.3, 3.4.4 and 3.4.5. The wavefunction after t_{ev} is given by

$$|\Phi(t_{ev})\rangle = \frac{1}{\sqrt{2\sqrt{\pi\sigma}}} \exp\Big(- \frac{(q-q_0)^2}{2\sigma} \Big) \begin{pmatrix} \exp\big(- \frac{i}{\hbar} B t_{ev} \big) \\ \exp\big(\frac{i}{\hbar} B t_{ev} \big) \end{pmatrix}. \qquad (3.5.10)$$

Now $|\Phi(t_{ev} + t_m)\rangle$ is given by

$$|\Phi(t_{ev} + t_m)\rangle = \exp\Big(- \frac{i}{\hbar} \int_{t_{ev}}^{t_{ev}+t_m} (H_{int}(t') + H_0(t')) dt' \Big) |\Phi(t_{ev})\rangle. \qquad (3.5.11)$$

Assume g to be time independent. The exponential is given in matrix form by equation 3.5.7 with t the measuring time t_m.

Applying this matrix to $|\Phi(t_{ev})\rangle$ gives

$$|\Phi(t_{ev}+t_m)\rangle = \frac{1}{\sqrt{2\sqrt{\pi\sigma}}} \exp\Big(-\frac{(q-q_0)^2}{2\sigma}\Big) \begin{pmatrix} \cos(\hat{\beta})\exp(-\hat{\alpha}) - i\frac{B}{\sqrt{B^2+g^2q^2}} \exp(-\hat{\alpha})\sin(\hat{\beta}) - i\frac{gq}{\sqrt{B^2+g^2q^2}} \exp(\hat{\alpha})\sin(\hat{\beta}) \\ \cos(\hat{\beta})\exp(\hat{\alpha}) + i\frac{B}{\sqrt{B^2+g^2q^2}} \exp(\hat{\alpha})\sin(\hat{\beta}) - i\frac{gq}{\sqrt{B^2+g^2q^2}} \exp(-\hat{\alpha})\sin(\hat{\beta}) \end{pmatrix} \qquad (3.5.12)$$

with $\hat{\alpha} = \frac{i}{\hbar} B t_{ev}$ and $\hat{\beta} = \frac{1}{\hbar} t_m \sqrt{B^2 + g^2 q^2}$.

Moreover, the evolution of the wave function at discrete moments in time $k(t_{ev} + t_m)$ with $k = 0, 1, 2, ..., n$ with $n < \infty$ an integer is given by applying the matrix

$$\begin{aligned} C &= \exp\Big(- \frac{i}{\hbar} \int_{t_{ev}}^{t_{ev}+t_m} (H_0 + H_{int}) dt \Big) \exp\Big(- \frac{i}{\hbar} \int_0^{t_{ev}} H_0 dt \Big) \\ &= \begin{pmatrix} \exp(-\hat{\alpha})\big(\cos(\hat{\beta}) - i\frac{B}{\sqrt{B^2+g^2q^2}} \sin(\hat{\beta})\big) & -i\frac{gq}{\sqrt{B^2+g^2q^2}} \exp(\hat{\alpha})\sin(\hat{\beta}) \\ -i\frac{gq}{\sqrt{B^2+g^2q^2}} \exp(-\hat{\alpha})\sin(\hat{\beta}) & \exp(\hat{\alpha})\big(\cos(\hat{\beta}) + i\frac{B}{\sqrt{B^2+g^2q^2}} \sin(\hat{\beta})\big) \end{pmatrix} \end{aligned} \qquad (3.5.13)$$

k times for each time period of time evolution and measurement on the initial wave function.

The density matrix ρ at time $t_{ev} + t_m$ is given by

$$\rho(t_{ev} + t_m) = \frac{1}{2\sqrt{\pi\sigma}} \exp\Big(- \frac{(q-q_0)^2}{\sigma}\Big) \begin{pmatrix} \tilde{\alpha}(t_{ev}+t_m) & \tilde{\beta}(t_{ev}+t_m) \\ \tilde{\gamma}(t_{ev}+t_m) & \tilde{\delta}(t_{ev}+t_m) \end{pmatrix} \tag{3.5.14}$$

with $\tilde{\alpha}(t_{ev}+t_m) = 1 + \frac{Bgq}{B^2+g^2q^2}\sin^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})(\exp(\frac{-2i}{\hbar}Bt_{ev}) + \exp(\frac{2i}{\hbar}Bt_{ev}))$
$+ \frac{gq}{\sqrt{B^2+g^2q^2}}\cos(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\sin(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\big(i\exp(\frac{-2i}{\hbar}Bt_{ev}) - i\exp(\frac{2i}{\hbar}Bt_{ev})\big)$,
$\tilde{\beta}(t_{ev}+t_m) = -\exp(-\frac{2i}{\hbar}Bt_{ev})\frac{B^2}{B^2+g^2q^2}\sin^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2}) +$
$\exp(\frac{2i}{\hbar}Bt_{ev})\frac{g^2q^2}{B^2+g^2q^2}\sin^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2}) + \exp(\frac{-2i}{\hbar}Bt_{ev})\cos^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})$
$- 2i\frac{B}{\sqrt{B^2+g^2q^2}}\exp(\frac{-2i}{\hbar}Bt_{ev})\cos(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\sin(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})$,
$\tilde{\gamma}(t_{ev}+t_m) = -\exp(\frac{2i}{\hbar}Bt_{ev})\frac{B^2}{B^2+g^2q^2}\sin^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2}) + \exp(-\frac{2i}{\hbar}Bt_{ev})\frac{g^2q^2}{B^2+g^2q^2}\sin^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})$
$+ \exp(\frac{2i}{\hbar}Bt_{ev})\cos^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})$
$+ 2i\frac{B}{\sqrt{B^2+g^2q^2}}\exp(\frac{2i}{\hbar}Bt_{ev})\cos(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\sin(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})$ and
$\tilde{\delta}(t_{ev}+t_m) = 1 - \frac{Bgq}{B^2+g^2q^2}\sin^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})(\exp(\frac{-2i}{\hbar}Bt_{ev}) + \exp(\frac{2i}{\hbar}Bt_{ev}))$
$+ \frac{gq}{\sqrt{B^2+g^2q^2}}\cos(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\sin(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\big(-i\exp(\frac{-2i}{\hbar}Bt_{ev}) + i\exp(\frac{2i}{\hbar}Bt_{ev})\big)$.

Consequently, the detector coordinate q is traced out again in the same way as in subsection 3.4.1 in order to get the density matrix only dependent on the qubit coordinates $i = |0\rangle$ and $j = |1\rangle$. Equation 3.4.11 gives in this situation where $\sigma \to 0$, so $\exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big) \to \delta(q-q_0)$,

$$\rho_{ii,ij,ji,jj}(t_{ev}+t_m, q_0) \approx \frac{1}{2}\begin{pmatrix} \tilde{\alpha}(t_{ev}+t_m,q_0) & \tilde{\beta}(t_{ev}+t_m,q_0) \\ \tilde{\gamma}(t_{ev}+t_m,q_0) & \tilde{\delta}(t_{ev}+t_m,q_0) \end{pmatrix} \tag{3.5.15}$$

with $\tilde{\alpha}(t_{ev}+t_m,q_0) = 1 + \frac{Bgq}{B^2+g^2q_0^2}\sin^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})(\exp(\frac{-2i}{\hbar}Bt_{ev}) + \exp(\frac{2i}{\hbar}Bt_{ev}))$
$+ \frac{gq_0}{\sqrt{B^2+g^2q_0^2}}\cos(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\sin(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\big(i\exp(\frac{-2i}{\hbar}Bt_{ev}) - i\exp(\frac{2i}{\hbar}Bt_{ev})\big)$,
$\tilde{\beta}(t_{ev}+t_m,q_0) = -\exp(-\frac{2i}{\hbar}Bt_{ev})\frac{B^2}{B^2+g^2q_0^2}\sin^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2}) +$
$\exp(\frac{2i}{\hbar}Bt_{ev})\frac{g^2q_0^2}{B^2+g^2q_0^2}\sin^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2}) + \exp(\frac{-2i}{\hbar}Bt_{ev})\cos^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})$
$- 2i\frac{B}{\sqrt{B^2+g^2q_0^2}}\exp(\frac{-2i}{\hbar}Bt_{ev})\cos(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\sin(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})$,
$\tilde{\gamma}(t_{ev}+t_m,q_0) = -\exp(\frac{2i}{\hbar}Bt_{ev})\frac{B^2}{B^2+g^2q_0^2}\sin^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2}) + \exp(-\frac{2i}{\hbar}Bt_{ev})\frac{g^2q_0^2}{B^2+g^2q_0^2}\sin^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})$
$+ \exp(\frac{2i}{\hbar}Bt_{ev})\cos^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})$
$+ 2i\frac{B}{\sqrt{B^2+g^2q_0^2}}\exp(\frac{2i}{\hbar}Bt_{ev})\cos(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\sin(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})$ and
$\tilde{\delta}(t_{ev}+t_m,q_0) = 1 - \frac{Bgq_0}{B^2+g^2q_0^2}\sin^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})(\exp(\frac{-2i}{\hbar}Bt_{ev}) + \exp(\frac{2i}{\hbar}Bt_{ev}))$
$+ \frac{gq_0}{\sqrt{B^2+g^2q_0^2}}\cos(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\sin(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\big(-i\exp(\frac{-2i}{\hbar}Bt_{ev}) + i\exp(\frac{2i}{\hbar}Bt_{ev})\big)$.

The probability that the qubit is in state $|0\rangle$ after $t_{ev}+t_m$ is, using equation 2.4.4, given by

$$\begin{aligned} P_0(t_m+t_{ev},q_0) =& \frac{1}{2} + \frac{1}{2}\sin^2\big(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2}\big)\cos(\frac{2}{\hbar}Bt_{ev})\frac{g^2q_0^2-B^2}{B^2+g^2q_0^2} \\ & - \frac{B}{\sqrt{B^2+g^2q_0^2}}\sin(\frac{2}{\hbar}Bt_{ev})\sin(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\cos(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2}) \\ & + \frac{1}{2}\cos(\frac{2}{\hbar}Bt_{ev})\cos^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2}) \end{aligned} \tag{3.5.16}$$

and the probability that the qubit is in state $|1\rangle$ is, using equation 2.4.5, given by

$$
\begin{aligned}
P_1(t_m + t_{ev}, q_0) =& \frac{1}{2} - \frac{1}{2}\sin^2\big(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2}\big)\cos(\frac{2}{\hbar}Bt_{ev})\frac{g^2q_0^2 - B^2}{B^2+g^2q_0^2} \\
&+ \frac{B}{\sqrt{B^2+g^2q_0^2}}\sin(\frac{2}{\hbar}Bt_{ev})\sin(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\cos(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2}) \\
&- \frac{1}{2}\cos(\frac{2}{\hbar}Bt_{ev})\cos^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2}).
\end{aligned}
\tag{3.5.17}
$$

The calculations of the probabilities at $t = 2t_{ev} + 2t_m$ are done in the same way and given in the appendix in subsection 5.1.3.

Unfortunately, there in not a pattern in these calculated probabilities. Therefore, the next probabilities are calculated numerically. This is done in the same way as described at the end of subsection 3.4.1., but now using matrix C, defined in equation 3.5.13, instead of A.

We also want to know what the probabilities of the qubit states are during each period of only free evolution and during each period of free evolution and measurement. Therefore, define matrix C_{ev} and C_{mev},

$$
C_{ev}(t) = \exp\big(-\frac{i}{\hbar}H_0t\big) = \begin{pmatrix} \exp(-\frac{i}{\hbar}Bt) & 0 \\ 0 & \exp(\frac{i}{\hbar}Bt) \end{pmatrix} \tag{3.5.18}
$$

and

$$
\begin{aligned}
C_{mev}(t) =& \exp\big(-\frac{i}{\hbar}(H_0 + H_{int})t\big) = \\
& \begin{pmatrix} \cos(\beta(q)) - i\gamma(q)\sin(\beta(q)) & -i\delta(q)\sin(\beta(q)) \\ -i\delta(q)\sin(\beta(q)) & \cos(\beta(q)) + i\gamma(q)\sin(\beta(q)) \end{pmatrix}
\end{aligned}
\tag{3.5.19}
$$

with $\beta(q) = \frac{1}{\hbar}t\sqrt{B^2+g^2q^2}$, $\gamma(q) = \frac{B}{\sqrt{B^2+g^2q^2}}$ and $\delta(q) = \frac{gq}{\sqrt{B^2+g^2q^2}}$.

The time t is divided in small steps of length Δt. The wavefunction $C_{ev}^k|\Phi(0)\rangle$ with $k = 0, 1, 2, ..., n$ is calculated for each k during t_{ev}, with $t_{ev} = n\Delta t$ and $n < \infty$ an integer. Then, with still the free evolution ongoing, a measurement is done during t_m and the wavefunction $C_{mev}^l C_{ev}^n|\Phi(0)\rangle$ with $l = 0, 1, 2, ..., m$ is calculated for each l during t_m, with $t_m = m\Delta t$ and $m < \infty$ an integer. Then there is free evolution again and the wavefunction $C_{ev}^k C_{mev}^m C_{ev}^n|\Phi(0)\rangle$ is calculated for each k during t_{ev}. During the subsequent measurement with ongoing evolution $C_{mev}^l C_{ev}^n C_{mev}^m C_{ev}^n|\Phi(0)\rangle$ is calculated for each l during t_m. For each time step Δt, the probabilities that the qubit is in state $|0\rangle$ and in state $|1\rangle$ are determined from the wavefunction using the same method as before. As a result, the behaviour of the probabilities continuously in time is known by taking $\Delta t \to 0$.

Since in this case, $\sigma \to 0$, the dispersion of the detector coordinate q is zero so the detector coordinate q has one value q_0. There is a perfect measurement. When $q_0 = 0$, this results in $H_{int} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ so then there is no interaction between the detector and qubit. As a result, there is only free evolution during all the periods since there is no measurement. The probabilities will evolve in time as described in subsection 3.2.

When $B = 0$ during the period of free evolution and measurement, there is no free evolution and only measurement during that period, so in that case we get the same result as shown in subsection 3.4. When $\gamma(q_0) = 0$ [13], we have $B = 0$ or $gq_0 \to \infty$ during all periods. If $B = 0$, there is no free evolution during all periods, so then there is only a measurement and we get the result as shown in subsection 3.3. If $gq_0 \to \infty$, the interaction between the qubit and detector has much more influence than the free evolution, resulting in plots as shown in subsection 3.4.

[13] Remember $\gamma(q) = \frac{B}{\sqrt{B^2+g^2q^2}}$.

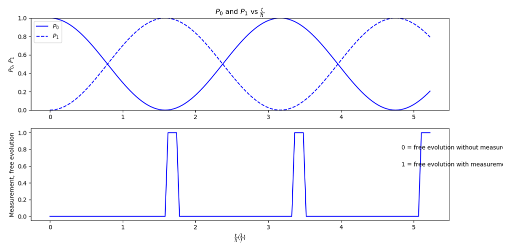
When $\delta(q_0) = 0$ [14], we have $gq_0 = 0$ or $B \to \infty$ during all periods. When $gq_0 = 0$, there is no measurement and only free evolution, resulting in the same results as in subsection 3.2. When $B \to \infty$ during all periods, the free evolution will have much more influence than the measurement, also resulting in the plot shown in subsection 3.2.

In this subsection, the strength of the measurement influences the resulting evolution of the probabilities in time during the period of measurement with now also free evolution. When increasing gq_0, so increasing $\beta(q_0)$ and $\delta(q_0)$, the interaction between the qubit and detector becomes larger, since H_{int} is directly proportional to gq_0. As a result of an increasing $\delta(q_0)$, $\gamma(q_0)$ will decrease because of the restriction on these parameters, $\gamma(q_0)^2 + \delta(q_0)^2 = 1$. The influence of the measurement will be bigger than the influence of the free evolution. This will result in an attempt to freeze the free evolution. The larger the influence of the measurement, the more the free evolution will be suppressed during the period of measurement and evolution.

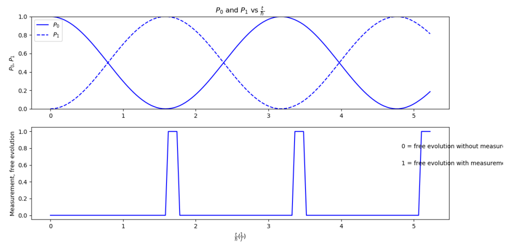
A result of the first three periods of free evolution and measurement alternated with only free evolution is shown in the following figure. We use the same value for the parameters α and $\beta(q_0)$ as in figure 5. The plots are made for different values of $\gamma(q_0)$ and $\delta(q_0)$ [15].

[14] Remember $\delta(q) = \frac{gq}{\sqrt{B^2+g^2q^2}}$.

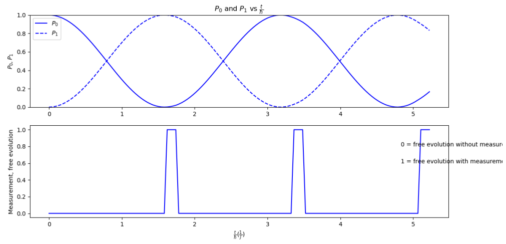
[15] $\alpha = \frac{1}{\hbar}Bt_{ev}$, $\beta(q) = \frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2}$, $\gamma(q) = \frac{B}{\sqrt{B^2+g^2q^2}}$, $\delta(q) = \frac{gq}{\sqrt{B^2+g^2q^2}}$.



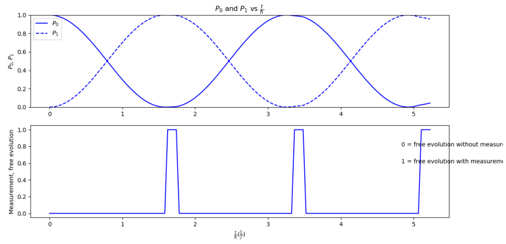
(a) Top graph: Graph of the probability of being in state $|0\rangle$ P_0 (continuous line) or state $|1\rangle$ P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = \frac{\pi}{2}$, $\beta(q_0) = \frac{\pi}{20}$, $\gamma(q_0) = 1$ and $\delta(q_0) = 0$. Bottom graph: Step function where 0 denotes a period of free evolution without a measurement and 1 a period of free evolution with a measurement as function of $\frac{t}{\hbar}$ with t the time.



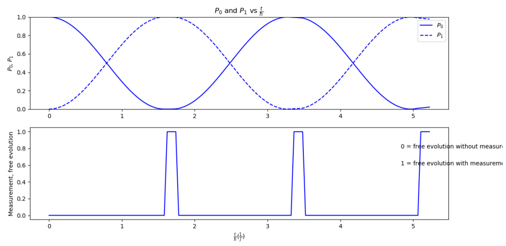
(b) Top graph: Graph of the probability of being in state $|0\rangle$ P_0 (continuous line) or state $|1\rangle$ P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = \frac{\pi}{2}$, $\beta(q_0) = \frac{\pi}{20}$, $\gamma(q_0) = \frac{3}{\sqrt{10}}$ and $\delta(q_0) = \frac{1}{\sqrt{10}}$. Bottom graph: Step function where 0 denotes a period of free evolution without a measurement and 1 a period of free evolution with a measurement as function of $\frac{t}{\hbar}$ with t the time.



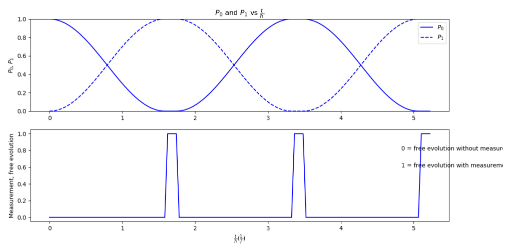
(c) Top graph: Graph of the probability of being in state $|0\rangle$ P_0 (continuous line) or state $|1\rangle$ P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = \frac{\pi}{2}$, $\beta(q_0) = \frac{\pi}{20}$, $\gamma(q_0) = \frac{2}{\sqrt{5}}$ and $\delta(q_0) = \frac{1}{\sqrt{5}}$. Bottom graph: Step function where 0 denotes a period of free evolution without a measurement and 1 a period of free evolution with a measurement as function of $\frac{t}{\hbar}$ with t the time.



(d) Top graph: Graph of the probability of being in state $|0\rangle$ P_0 (continuous line) or state $|1\rangle$ P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = \frac{\pi}{2}$, $\beta(q_0) = \frac{\pi}{20}$, $\gamma(q_0) = \frac{1}{\sqrt{5}}$ and $\delta(q_0) = \frac{2}{\sqrt{5}}$. Bottom graph: Step function where 0 denotes a period of free evolution without a measurement and 1 a period of free evolution with a measurement as function of $\frac{t}{\hbar}$ with t the time.



(e) Top graph: Graph of the probability of being in state $|0\rangle$ P_0 (continuous line) or state $|1\rangle$ P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = \frac{\pi}{2}$, $\beta(q_0) = \frac{\pi}{20}$, $\gamma(q_0) = \frac{1}{\sqrt{10}}$ and $\delta(q_0) = \frac{3}{\sqrt{10}}$. Bottom graph: Step function where 0 denotes a period of free evolution without a measurement and 1 a period of free evolution with a measurement as function of $\frac{t}{\hbar}$ with t the time.



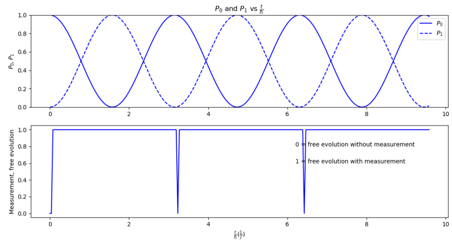
(f) Top graph: Graph of the probability of being in state $|0\rangle$ P_0 (continuous line) or state $|1\rangle$ P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = \frac{\pi}{2}$, $\beta(q_0) = \frac{\pi}{20}$, $\gamma(q_0) = 0$ and $\delta(q_0) = 1$.Bottom graph: Step function where 0 denotes a period of free evolution without a measurement and 1 a period of free evolution with a measurement as function of $\frac{t}{\hbar}$ with t the time.

Figure 10.

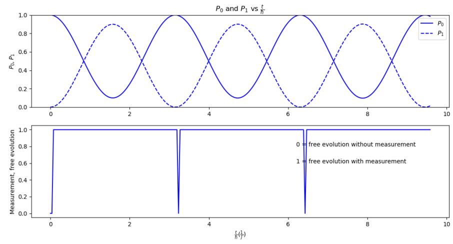
Six cases for $\gamma(q_0)$ and $\delta(q_0)$ are plotted. When $\gamma(q_0) = 0$ and $\delta(q_0) = 1$, $gq_0 \to \infty$ so the interaction between the qubit and detector has much more influence than the free evolution. The probabilities freeze during the period of free evolution and measurement, as expected. When $\gamma(q_0) = 1$ and $\delta(q_0) = 0$, $gq_0 = 0$ so there is only free evolution the whole time. The probabilities keep oscillating in a sine form, as if there is no measurement, as expected. Furthermore, the bigger $\delta(q_0)$ so the smaller $\gamma(q_0)$, the more the free evolution is suppressed during the periods of free evolution and measurement. When $\delta(q_0)$ increases, gq_0 increases so the interaction between the qubit and detector increases, since H_{int} is directly proportional to gq_0. The resulting plots are in line with the expectation.

These graphs will continue like this in time, oscillating between 0 and 1. The quantum Zeno effect does not occur. In this case there are no series of fast subsequent measurements or a continuous measurement.

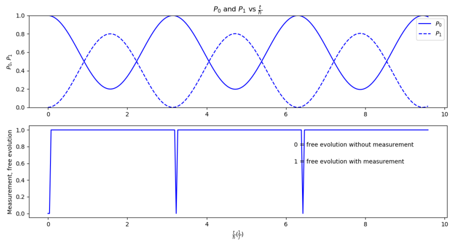
Another result of the first three periods of free evolution and measurement is shown in the following figure. We now do not make the assumption of $t_m \ll t_{ev}$, since H_0 is not neglected during measurements, so we can look to the case where $t_{ev} \ll t_m$. The dimensionless parameters are given by $\alpha = \frac{\pi}{80}$ and $\beta(q_0) = \pi$. The plots are made for different values of $\gamma(q_0)$ and $\delta(q_0)$.



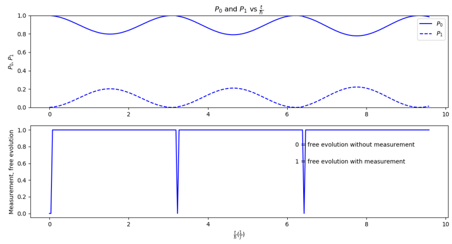
(a) Top graph: Graph of the probability of being in state $|0\rangle$ P_0 (continuous line) or state $|1\rangle$ P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = \frac{\pi}{80}$, $\beta(q_0) = \pi$, $\gamma(q_0) = 1$ and $\delta(q_0) = 0$. Bottom graph: Step function where 0 denotes a period of free evolution without a measurement and 1 a period of free evolution with a measurement as function of $\frac{t}{\hbar}$ with t the time.



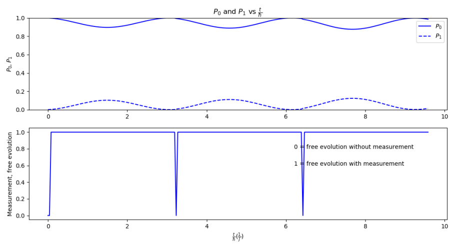
(b) Top graph: Graph of the probability of being in state $|0\rangle$ P_0 (continuous line) or state $|1\rangle$ P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = \frac{\pi}{80}$, $\beta(q_0) = \pi$, $\gamma(q_0) = \frac{3}{\sqrt{10}}$ and $\delta(q_0) = \frac{1}{\sqrt{10}}$. Bottom graph: Step function where 0 denotes a period of free evolution without a measurement and 1 a period of free evolution with a measurement as function of $\frac{t}{\hbar}$ with t the time.



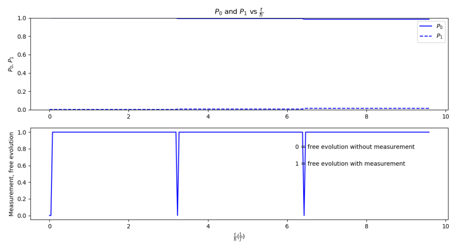
(c) Top graph: Graph of the probability of being in state $|0\rangle$ P_0 (continuous line) or state $|1\rangle$ P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = \frac{\pi}{80}$, $\beta(q_0) = \pi$, $\gamma(q_0) = \frac{2}{\sqrt{5}}$ and $\delta(q_0) = \frac{1}{\sqrt{5}}$. Bottom graph: Step function where 0 denotes a period of free evolution without a measurement and 1 a period of free evolution with a measurement as function of $\frac{t}{\hbar}$ with t the time.



(d) Top graph: Graph of the probability of being in state $|0\rangle$ P_0 (continuous line) or state $|1\rangle$ P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = \frac{\pi}{80}$, $\beta(q_0) = \pi$, $\gamma(q_0) = \frac{1}{\sqrt{5}}$ and $\delta(q_0) = \frac{2}{\sqrt{5}}$. Bottom graph: Step function where 0 denotes a period of free evolution without a measurement and 1 a period of free evolution with a measurement as function of $\frac{t}{\hbar}$ with t the time.



(e) Top graph: Graph of the probability of being in state $|0\rangle$ P_0 (continuous line) or state $|1\rangle$ P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = \frac{\pi}{80}$, $\beta(q_0) = \pi$, $\gamma(q_0) = \frac{1}{\sqrt{10}}$ and $\delta(q_0) = \frac{3}{\sqrt{10}}$. Bottom graph: Step function where 0 denotes a period of free evolution without a measurement and 1 a period of free evolution with a measurement as function of $\frac{t}{\hbar}$ with t the time.



(f) Top graph: Graph of the probability of being in state $|0\rangle$ P_0 (continuous line) or state $|1\rangle$ P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = \frac{\pi}{80}$, $\beta(q_0) = \pi$, $\gamma(q_0) = 0$ and $\delta(q_0) = 1$. Bottom graph: Step function where 0 denotes a period of free evolution without a measurement and 1 a period of free evolution with a measurement as function of $\frac{t}{\hbar}$ with t the time.

Figure 11.

There is now a long period of free evolution with measurement, alternated with a very short period of free evolution without measurement. Again six cases for $\gamma(q_0)$ and $\delta(q_0)$ are plotted. When $\gamma(q_0) = 0$ and $\delta(q_0) = 1$, $gq_0 \to \infty$, so the interaction between the qubit and detector has much more influence than the free evolution during the period of measurement and free evolution. The probabilities freeze during this period. In the very short period of free evolution, the probabilities evolve according to a sine form. When $\gamma(q_0) = 1$ and $\delta(q_0) = 0$, $gq_0 = 0$ so there is only free evolution the whole time. The probabilities keep oscillating in a sine form between 0 and 1, as if there is no measurement, as expected. Furthermore, the bigger $\delta(q_0)$ so the smaller $\gamma(q_0)$, the more the free evolution is suppressed during the period of free evolution and measurement, as can be seen in the plots. The amplitude of the oscillations due to free evolution is smaller by a bigger $\delta(q_0)$. There is also a short period with only free evolution, in which the system evolves only under H_0, resulting in a part of a sine oscillation.

In the following figure the first 160 points at $k\frac{t_{ev}+t_m}{\hbar}$ with $k = 0, 1, ..., 160$ are plotted. As can be concluded from this plot, the same cycle will start after a number of alternating periods.

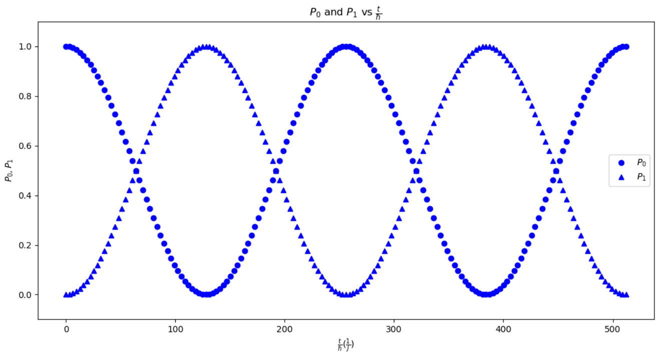


Figure 12. Graph of the probability of being in state $|0\rangle$ P_0 (circles) or state $|1\rangle$ P_1 (triangles) as function of $\frac{t}{\hbar}$ with t the time at discrete points; $\alpha = \frac{\pi}{80}$, $\beta(q_0) = \pi$, $\gamma(q_0) = 1$ and $\delta(q_0) = 0$.

In this case there are series of fast subsequent long measurements. The stronger the measurement, the longer the probability for the qubit to remain in its initial state stays high in time. However, the measurement does not freeze the evolution of the system totally. The quantum Zeno effect does not occur.

3.5.2 Situation $\sigma \not\to 0$

In this section, the integrals which appear when taking the partial trace to trace out the detector coordinate q were too difficult to calculate algebraically, so there are no algebraic expressions for the probabilities at a certain moment in time. Since σ is unequal to 0, we define the following dimensionless variables [16] $q' = \frac{q}{\sqrt{2\sigma}}$ and $q'_0 = \frac{q_0}{\sqrt{2\sigma}}$. Furthermore, we define four dimensionless parameters, from which three dependent on the variable q', so α, $\beta(q')$, $\gamma(q')$ and $\delta(q')$. Set $\alpha = \frac{Bt_{ev}}{\hbar}$, $\beta(q') = \frac{t_m\sqrt{B^2+2\sigma g^2 q'^2}}{\hbar}$, $\gamma(q') = \frac{B}{\sqrt{B^2+2\sigma g^2 q'^2}}$ and $\delta(q') = \frac{gq'\sqrt{2\sigma}}{\sqrt{B^2+2\sigma g^2 q'^2}}$. The following restrictions are valid for $\gamma(q')$ and $\delta(q')$: $\gamma(q')^2 + \delta(q')^2 = 1$ and $0 \leq \gamma(q') \leq 1$ and $0 \leq \delta(q') \leq 1$. Probably, based on the former algebraic expressions, the wavefunctions, density matrices and probabilities will be expressed in the following dimensionless terms or combinations of these terms: $\frac{1}{\hbar}Bt_{ev}$, $\frac{1}{\hbar}t_m\sqrt{B^2 + 2\sigma g^2 q'^2}$, $\frac{1}{\hbar}t_m\sqrt{B^2 + 2\sigma g^2 {q'_0}^2}$, $\frac{B}{\sqrt{B^2+2\sigma g^2 q'^2}}$, $\frac{B}{\sqrt{B^2+2\sigma g^2 {q'_0}^2}}$, $\frac{\sqrt{2\sigma}gq'}{\sqrt{B^2+2\sigma g^2 q'^2}}$, $\frac{\sqrt{2\sigma}gq'_0}{\sqrt{B^2+2\sigma g^2 {q'_0}^2}}$ and $(\frac{1}{\hbar}t_m)^2(B^2 + 2\sigma g^2 q'^2)$. These can be expressed as follows in the dimensionless parameters α, $\beta(q')$, $\gamma(q')$ and $\delta(q')$ and the variables q' and q'_0, which are dimensionless:
$\frac{1}{\hbar}Bt_{ev} = \alpha$, $\frac{1}{\hbar}t_m\sqrt{B^2 + 2\sigma g^2 q'^2} = \beta(q')$, $\frac{1}{\hbar}t_m\sqrt{B^2 + 2\sigma g^2 {q'_0}^2} = \beta(q'_0)$, $\frac{B}{\sqrt{B^2+2\sigma g^2 q'^2}} = \gamma(q')$,

[16] These variables are based on the initial state of the detector.

$\frac{B}{\sqrt{B^2+2\sigma g^2 {q'_0}^2}} = \gamma(q'_0)$, $\frac{\sqrt{2\sigma} g q'}{\sqrt{B^2+2\sigma g^2 {q'}^2}} = \delta(q')$, $\frac{\sqrt{2\sigma} g q'_0}{\sqrt{B^2+2\sigma g^2 {q'_0}^2}} = \delta(q'_0)$ and $(\frac{1}{\hbar} t_m)^2 (B^2 + 2\sigma g^2 {q'}^2) = \beta(q')^2$.

In this section we study again alternating periods of only free evolution and of a measurement during free evolution, but now without the simplification of $\sigma \to 0$. The integrals which appear when taking the partial trace to trace out the detector coordinate q were too difficult to calculate algebraically. Therefore, first of all the probabilities at $k\frac{t_{ev}+t_m}{\hbar}$ with $k = 0, 1, 2, 3, ...n$ were determined numerically with the integrator tool in Python [1]. This is done in the same way as described at the end of subsection 3.5.1, but now the partial trace is calculated without the simplification of $\sigma \to 0$.

We also calculated numerically the probabilities during the first three periods of only free evolution alternated with the first three periods of a measurement during free evolution to know the behaviour of the probabilities continuously during these periods. This is done in the same way as described at the end of subsection 3.5.1, so by using C_{ev} and C_{mev}, but now without the simplification of $\sigma \to 0$.

Since this is the most complete case, without the simplification of $\sigma \to 0$ and with also free evolution during the measurement, we will discuss this case extensively.

First of all, the case that no measurement is done is analyzed. This is the case when $\delta(q'_0) = 0$ so $gq'_0 = 0$, so the system only evolves under H_0. There is only free evolution, resulting in the same results found in subsection 3.2. The dimensionless parameters are given as follows, $\alpha = \pi$, $\beta(q'_0) = 0$, $\gamma(q'_0) = 1$ and $\delta(q'_0) = 0$. The first three periods of free evolution are seen in figure 13.

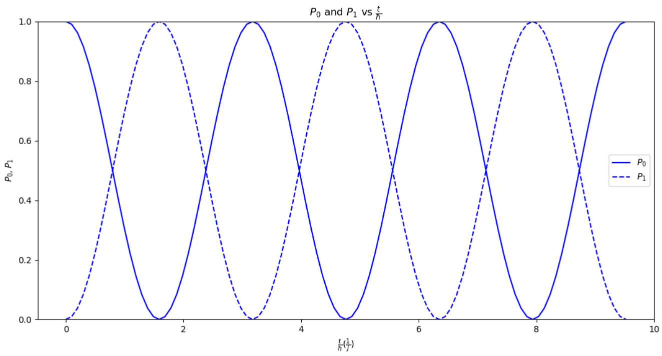


Figure 13. Graph of the probability of being in state $|0\rangle$ P_0 (continuous line) or state $|1\rangle$ P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = \pi$, $\beta(q'_0) = 0$, $\gamma(q'_0) = 1$ and $\delta(q'_0) = 0$.

This will continue like this in time, in line with the expectation. The Rabi frequency is in this case $f = \frac{\hbar}{\pi}$.

Now the situation is analyzed when $B = 0$ at every moment in time, so $\alpha = 0$ and $\gamma(q'_0) = 0$. In this case there is only a measurement without free evolution. This results in the following plot.

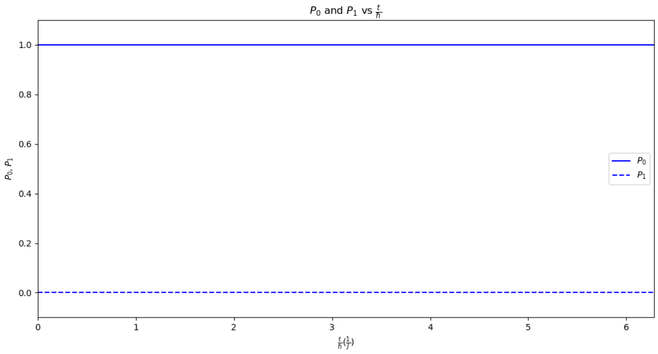
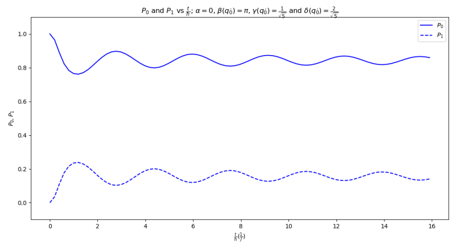


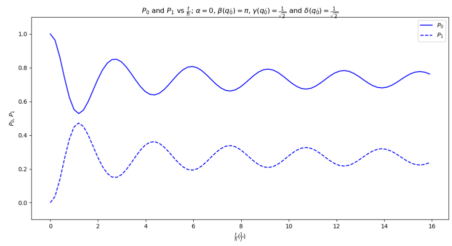
Figure 14. Graph of the probability of being in state $|0\rangle$ P_0 (continuous line) or state $|1\rangle$ P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = 0$, $\beta(q_0') = \pi$, $\gamma(q_0') = 0$ and $\delta(q_0') = 1$.

This is in line with our expectation. There is one continuous measurement, so the qubit remains in its initial state, since this is an eigenstate of H_{int} under which the system evolves during the measurement.

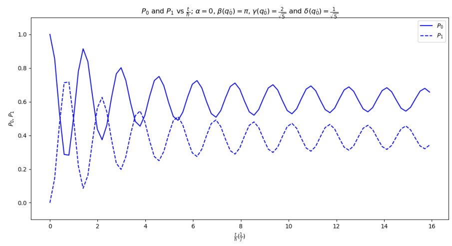
Now the situation is analyzed in which $t_{ev} = 0$, so there is one ongoing measurement during free evolution. This is done for several values of $\gamma(q_0')$ and $\delta(q_0')$ [17] under their restriction. The resulting plots are given below.



(a) Graph of the probability of being in state $|0\rangle$ P_0 (continuous line) or state $|1\rangle$ P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = 0$, $\beta(q_0') = \pi$, $\gamma(q_0') = \frac{1}{\sqrt{5}}$ and $\delta(q_0') = \frac{2}{\sqrt{5}}$.



(b) Graph of the probability of being in state $|0\rangle$ P_0 (continuous line) or state $|1\rangle$ P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = 0$, $\beta(q_0') = \pi$, $\gamma(q_0') = \frac{1}{\sqrt{2}}$ and $\delta(q_0') = \frac{1}{\sqrt{2}}$.



(c) Graph of the probability of being in state $|0\rangle$ P_0 (continuous line) or state $|1\rangle$ P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = 0$, $\beta(q_0') = \pi$, $\gamma(q_0') = \frac{2}{\sqrt{5}}$ and $\delta(q_0') = \frac{1}{\sqrt{5}}$.

Figure 15.

When $\gamma(q_0') = 1$ and $\delta(q_0') = 0$, there is only free evolution and no measurement. This was analyzed

[17]Remember $\gamma(q_0') = \frac{B}{\sqrt{B^2+2\sigma g^2 {q_0'}^2}}$ and $\delta(q_0') = \frac{\sqrt{2\sigma} g q_0'}{\sqrt{B^2+2\sigma g^2 {q_0'}^2}}$.

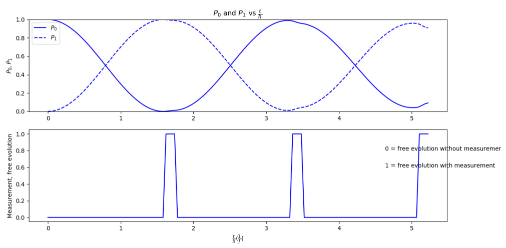
in figure 13. When $\gamma(q'_0) = 0$ and $\delta(q'_0) = 1$, there is one continuous measurement and no free evolution, what was analyzed in figure 14.

Furthermore, plots are made for intermediary situations. Then we notice that the larger $\delta(q'_0)$, so the smaller $\gamma(q'_0)$, the smaller the amplitude of the oscillations and the higher the equilibrium position of P_0, so the lower the equilibirium position of P_1. When doing a measurement during free evolution, the wave function will evolve in time according to equation 3.5.9. This is a linear combination of eigenvectors from the total Hamiltonian H with time dependent exponentials. The qubit states are eigenvectors from H_{int}. As a result, when doing a measurement during free evolution with a higher $\delta(q'_0)$, so a stronger interaction between the qubit and detector and a bigger part of H_{int} in H, the probability for the qubit to be in its initial state $|0\rangle$ will be in a higher range, and the probability for the qubit to be in state $|1\rangle$ will be in a lower range. Furthermore, when the part of H_{int} in H is bigger, the amplitude of the oscillations will be smaller since the free evolution will have less influence.

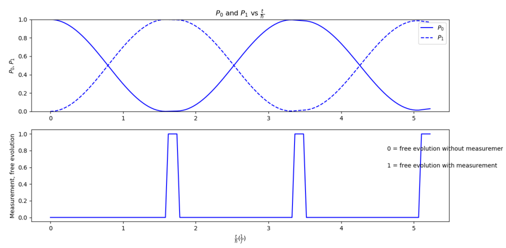
Moreover, the oscillations damp. For larger $\delta(q'_0)$, so larger $gq'_0\sqrt{\sigma}$ this damping goes faster. There are probably terms dependent on σ in the expressions of the probabilities, like in the situation of subsection 3.4.2, which cause this damping. This is in contrast to the case where $\sigma \to 0$. Then the probabilities did not damp and there were no damping terms in the algebraic expressions. σ indicates a measurement error in the detector. The larger σ, the bigger the error and the harder it is for the detector to tell in which state the qubit is, resulting in dissipation of the system. As a result, the oscillations of the probabilities will damp.

The quantum Zeno effect states that a continuously observed system never decays at all. The continuous measurement freezes the free evolution. When there is no free evolution, the system freezes, as expected. However, when there is free evolution present, the wavefunction will decay to a certain eigenstate of the Hamiltonian $H = H_0 + H_{int}$, which is not a qubit state. Therefore, due to the free evolution, the probabilities for the qubit to be in one of its states will keep oscillating, but in a smaller range due to the continuous measurement. The value around the probabilities oscillate is dependent on the strength of the magnetic field and the strength of the interaction between the qubit and detector. The stronger the interaction between the qubit and detector and the weaker the magnetic field, the smaller the probability is that the qubit undergoes a transition to its other state. The quantum Zeno effect does not occur since the free evolution does not freeze totally.

Now we look to the situation where $t_m \ll t_{ev}$. We take the same value for α and $\beta(q'_0)$ [18] as in figure 10. The values of the dimensionless parameters $\gamma(q'_0)$ and $\delta(q'_0)$ [19] are changed. This results in the following plots.



(a) Top graph: Graph of the probability of being in state $|0\rangle$ P_0 (continuous line) or state $|1\rangle$ P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = \frac{\pi}{2}$, $\beta(q'_0) = \frac{\pi}{20}$, $\gamma(q'_0) = \frac{1}{\sqrt{17}}$ and $\delta(q'_0) = \frac{4}{\sqrt{17}}$. Bottom graph: Step function where 0 denotes a period of free evolution without a measurement and 1 a period of free evolution with a measurement as function of $\frac{t}{\hbar}$ with t the time.



(b) Top graph: Graph of the probability of being in state $|0\rangle$ P_0 (continuous line) or state $|1\rangle$ P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = \frac{\pi}{2}$, $\beta(q'_0) = \frac{\pi}{20}$, $\gamma(q'_0) = \frac{1}{\sqrt{101}}$ and $\delta(q'_0) = \frac{10}{\sqrt{101}}$. Bottom graph: Step function where 0 denotes a period of free evolution without a measurement and 1 a period of free evolution with a measurement as function of $\frac{t}{\hbar}$ with t the time.

Figure 16.

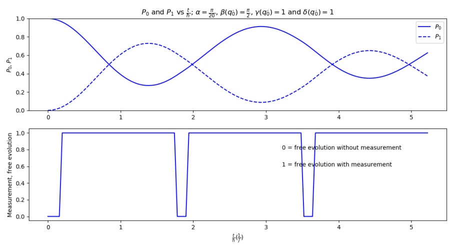
[18] Remember $\alpha = \frac{Bt_{ev}}{\hbar}$ and $\beta(q'_0) = \frac{1}{\hbar} t_m \sqrt{B^2 + 2\sigma g^2 {q'_0}^2}$.

[19] Remember $\gamma(q'_0) = \frac{B}{\sqrt{B^2+2\sigma g^2 {q'_0}^2}}$ and $\delta(q'_0) = \frac{\sqrt{2\sigma} g q'_0}{\sqrt{B^2+2\sigma g^2 {q'_0}^2}}$.

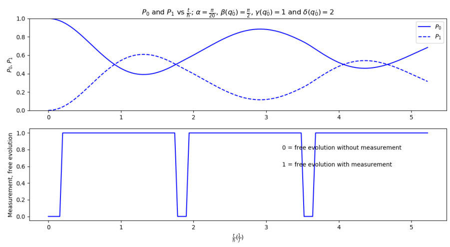
In figure 16a, we set $q_0' = 1$ ($q_0 = 4$ and $\sigma = 8$) and in figure 16b $q_0' = \frac{2}{\sqrt{6}}$ ($q_0 = 2$ and $\sigma = 3$). As can be seen, the higher σ, the stronger the damping of the oscillations, so in this case the dissipation of the system is indeed stronger. The higher $\delta(q_0')$, so gq_0', the stronger the interaction between the qubit and detector, so the more influence the measurement has during the period of the measurement during free evolution. This can be seen in figure 16b, since there the probabilities freeze more than in figure 16a during this period. The difference in this case with figure 10 is that there is now dissipation of the system, caused by terms dependent on σ.

In this case there is no quantum Zeno effect, since $t_m \ll t_{ev}$. There is no series of fast subsequent measurements or a continuous measurement.

Now we will look to the case where $t_{ev} \ll t_m$. We consider fast subsequent measurements while there is always free evolution. First, the parameters are taken as follows, $\alpha = \frac{\pi}{20}$ and $\beta(q_0') = \frac{\pi}{2}$. $\gamma(q_0')$ and $\delta(q_0')$ are changed under their restriction. This gives the following result for the first three periods.



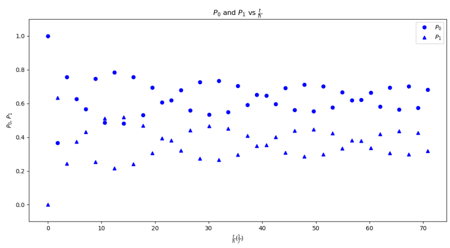
(a) Top graph: Graph of the probability of being in state $|0\rangle$ P_0 (continuous line) or state $|1\rangle$ P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = \frac{\pi}{20}$, $\beta(q_0') = \frac{\pi}{2}$, $\gamma(q_0') = \frac{1}{\sqrt{2}}$ and $\delta(q_0') = \frac{1}{\sqrt{2}}$. Bottom graph: Step function where 0 denotes a period of free evolution without a measurement and 1 a period of free evolution with a measurement as function of $\frac{t}{\hbar}$ with t the time.



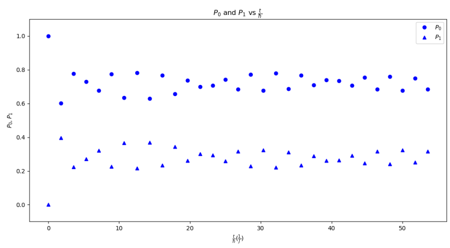
(b) Top graph: Graph of the probability of being in state $|0\rangle$ P_0 (continuous line) or state $|1\rangle$ P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = \frac{\pi}{20}$, $\beta(q_0') = \frac{\pi}{2}$, $\gamma(q_0') = \frac{1}{\sqrt{5}}$ and $\delta(q_0') = \frac{2}{\sqrt{5}}$. Bottom graph: Step function where 0 denotes a period of free evolution without a measurement and 1 a period of free evolution with a measurement as function of $\frac{t}{\hbar}$ with t the time.

Figure 17.

Here are the results in this same sitution for the first 40 points in time at $k\frac{t_{ev}+t_m}{\hbar}$ with $k = 0, 1, ..., 40$.



(a) Graph of the probability of being in state $|0\rangle$ P_0 (circles) or state $|1\rangle$ P_1 (triangles) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = \frac{\pi}{20}$, $\beta(q_0') = \frac{\pi}{2}$, $\gamma(q_0') = \frac{1}{\sqrt{2}}$ and $\delta(q_0') = \frac{1}{\sqrt{2}}$.



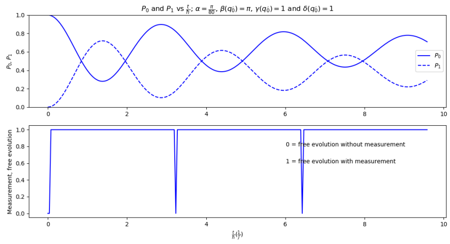
(b) Graph of the probability of being in state $|0\rangle$ P_0 (circles) or state $|1\rangle$ P_1 (triangles) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = \frac{\pi}{20}$, $\beta(q_0') = \frac{\pi}{2}$, $\gamma(q_0') = \frac{1}{\sqrt{5}}$ and $\delta(q_0') = \frac{2}{\sqrt{5}}$.

Figure 18.

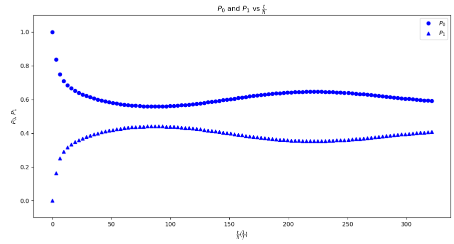
From these plots can be concluded like before that the stronger the interaction between the detector and qubit and the smaller the influence of the free evolution, the higher the equilibrium position of P_0 and the smaller the amplitude of the oscillations. The initial state of the qubit is $|0\rangle$ and this is an eigenstate of H_{int}. The first measurement takes place when the qubit has a big probability to be in this state, so the probability will stay high to be in this state, so P_0 will oscillate around

a higher value than P_1.
Moreover, the oscillations of the probabilities damp, due to damping terms dependent on σ. Since the periods of measurements are longer than the periods with only free evolution, the oscillations damp out quite fast. From figure 18 can be concluded that the oscillations go further in time and damp more. The discrete points are also in oscillating damping sine wave forms, since the points are plotted every $\frac{t_{ev}+t_m}{\hbar}$. The free evolution determines the period of the continuous sine waves, so $\frac{t_m}{\hbar}$ causes a shift, resulting in these sine patterns in the plots of these discrete points in time.

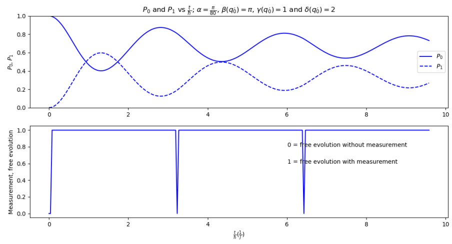
Finally, we take the same parameters as in figure 11. This gives the following results.



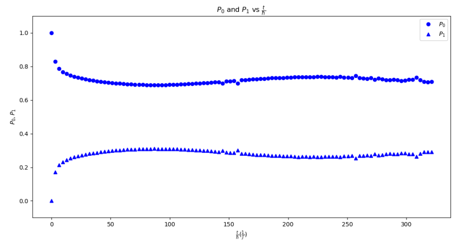
(a) Top graph: Graph of the probability of being in state $|0\rangle$ P_0 (continuous line) or state $|1\rangle$ P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = \frac{\pi}{80}$, $\beta(q_0') = \pi$, $\gamma(q_0') = \frac{1}{\sqrt{2}}$ and $\delta(q_0') = \frac{1}{\sqrt{2}}$. Bottom graph: Step function where 0 denotes a period of free evolution without a measurement and 1 a period of free evolution with a measurement as function of $\frac{t}{\hbar}$ with t the time.



(b) Graph of the probability of being in state $|0\rangle$ P_0 (circles) or state $|1\rangle$ P_1 (triangles) as function of $\frac{t}{\hbar}$ with t the time at discrete points; $\alpha = \frac{\pi}{80}$, $\beta(q_0') = \pi$, $\gamma(q_0') = \frac{1}{\sqrt{2}}$ and $\delta(q_0') = \frac{1}{\sqrt{2}}$.



(c) Top graph: Graph of the probability of being in state $|0\rangle$ P_0 (continuous line) or state $|1\rangle$ P_1 (dashed line) as function of $\frac{t}{\hbar}$ with t the time; $\alpha = \frac{\pi}{80}$, $\beta(q_0') = \pi$, $\gamma(q_0') = \frac{1}{\sqrt{5}}$ and $\delta(q_0') = \frac{2}{\sqrt{5}}$. Bottom graph: Step function where 0 denotes a period of free evolution without a measurement and 1 a period of free evolution with a measurement as function of $\frac{t}{\hbar}$ with t the time.



(d) Graph of the probability of being in state $|0\rangle$ P_0 (circles) or state $|1\rangle$ P_1 (triangles) as function of $\frac{t}{\hbar}$ with t the time at discrete points; $\alpha = \frac{\pi}{80}$, $\beta(q_0') = \pi$, $\gamma(q_0') = \frac{1}{\sqrt{5}}$ and $\delta(q_0') = \frac{2}{\sqrt{5}}$.

Figure 19.

The cases where $\gamma(q_0') = 0$ or $\delta(q_0') = 0$ are the same as in the cases analyzed before and not shown again.
In this case there are series of long fast subsequent measurements with ongoing evolution. The probabilities damp, probably caused by a damping term in the expression of the probabilities dependent on σ. Again when $\delta(q_0')$ is increased and $\gamma(q_0')$ decreased, the amplitude of the oscillations is smaller and the equilibrium position of P_0 is higher.

We can conclude from the first 100 discrete points plotted on $k\frac{t_{ev}+t_m}{\hbar}$ with $k = 0, 1, ..., 100$ that the oscillations will keep going in time. There is a vague sine form in these plots. This is caused by the $\frac{t_m}{\hbar}$ term in $k\frac{t_{ev}+t_m}{\hbar}$, since $\frac{t_{ev}}{\hbar}$ determines the period of the sine waves. There is now a longer period of measurement and free evolution and a shorter period of only free evolution which alternate than in the case analyzed before. The biggest difference between figure 18a and 19b, and 18b and 19d is that there are bigger oscillations in figure 18a and 18b. In figure 19b and 19d there is noticed a really small oscillation. Moreover, the bigger $\delta(q_0')$, the smaller this oscillation.

There is plotted a value of P_0 and P_1 every $\frac{t_{ev}+t_m}{\hbar}$, so after each period of only free evolution of duration t_{ev} and of a measurement during free evolution of duration t_m. In the case of figure 19, the period of measurement and free evolution is longer and the period of only free evolution shorter, so the measurement is present during a longer period in time. This results in smaller oscillation of the probabilities plotted every $\frac{t_{ev}+t_m}{\hbar}$, since the measurement suppresses the free evolution and consequently also the oscillations. Furthermore, the larger $\delta(q_0')$ so the stronger the measurement, the smaller the amplitudes of the oscillations will be, since the free evolution is suppressed more by the stronger measurement.
In figure 19 a few points deviate from the pattern. This is probably caused by an error in the integrator used in Python [1].

In these cases we analyzed $t_{ev} \ll t_m$, so a series of fast subsequent long measurement were executed. The measurement does not freeze the free evolution of the system totally, so the quantum Zeno effect does not occur.

4 Conclusions and recommendations

In this thesis, the quantum Zeno effect is analyzed in a qubit in different situations. For nearly every situation, we determined algebraic expressions of the probabilities of the qubit to be in one of its states at certain moments in time. This is done by calculating the wave function and density matrix at that time and consequently tracing out the detector coordinate. Furthermore, we did an analysis using plots of the evolution of these probabilities for each situation.

Conclusions

In the situation with only free evolution, the system evolves only under the Hamiltonian H_0 and formulas are derived for the evolution of the probabilities of the qubit states in time. These oscillate in time in a sine form in line with our expectation.

In the situation with one continuous measurement, the system evolves only under the Hamiltonian H_{int} and the qubit remains in its initial state. Since there is no free evolution, this is expected.

Moreover, we considered the situation where periods of only free evolution and only measurement alternated. To neglect the free evolution during the measurement, $t_m \ll t_{ev}$ was assumed. First, we made a simplification where the dispersion of the detector coordinate $\sigma \to 0$ and then the situation was analyzed without this simplification. In both situations, the probabilities had the same value during the measurement and evolved in a sine form during the free evolution. The difference between both situations was that the oscillations damped in the situation where $\sigma \not\to 0$, caused by exponential terms dependent on σ in the algebraic expressions of these probabilities. The larger σ, the larger the measurement error in the detector, so the faster the oscillations damp, since the dissipation of the system is larger. In the situation where $\sigma \to 0$, the oscillations did not damp.
In this situation $t_m \ll t_{ev}$, so there are no series of fast subsequent measurements or a continuous measurement. The quantum Zeno effect did not occur.

Finally, we considered the situation where periods of only free evolution and periods with a measurement during free evolution alternated. Again, we first made a simplification of $\sigma \to 0$ and then the situation was analyzed without this simplification. Now there was also free evolution during the measurement, so the oscillations of the probabilities continued during this period. The larger the influence of the interaction between the qubit and detector and the smaller the influence of the magnetic field, the higher the equilibrium position of the probability for the qubit to remain in its initial state and the smaller the oscillations of the probabilities. Again, in the case where $\sigma \to 0$ the probabilities did not damp, in contrast to the case where $\sigma \not\to 0$. The larger σ, the faster the oscillations damp. This is probably again caused by exponential terms dependent on σ which occur in this case and cause dissipation of the system.
When there is free evolution during the measurement, the probabilities keep oscillating. The measurement does not freeze the evolution of the system totally, so the quantum Zeno effect does not occur. However, the oscillations due to free evolution are smaller and the equilibrium position is higher, when there is a strong continuous measurement.

Recommendations

In this report several assumptions are made. One of the recommendations for follow-up research would be to work out this assumptions for more realistic conditions.

First of all, the interaction strength between the qubit and detector g is taken time indepedent in this report and depends on the precise measurement setup. It would be interesting to see what happens when g changes in time. Furthermore, we assumed the initial state of the detector to be Gaussian. It might be interesting to take another initial state instead of a Gaussian. Moreover, the dispersion σ was not related to spin. It would be interesting to see what happens when this parameter becomes related to spin.

In this report, the alternating time intervals of in one case free evolution and measurement and in the other case free evolution without measurement and free evolution with measurement had alternating the same length. It might be interesting to change the length of these intervals, so for example first have a long period of free evolution without measurement, than have a short period of free evolution with measurement, then a short period of free evolution without measurement, etc.

It would be interesting to look to other qubit states and another initial state of the qubit. Now eigenstates of H_{int} were taken. It could be interesting to look for example to $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

It might be interesting to see what happens when the free evolution is restricted, for example to add an extra term in this Hamiltonian, H_0. Furthermore, there could be done research to the situation of no perfect decay of the wave function.

It would sometimes be interesting to look how the probabilities continuously evolve over a longer period in time. However, the plots were made in Python 3.6.5. When attempting to calculate the evolution of the probabilities further in time, the maximum number of subdivisions of the integrator had been achieved. As a result, the error in these calculated probabilities became too big. Perhaps an other integrator could be used or there could be done a numeric approach on these integrals.

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5 Appendix

5.1 Algebraic expressions of probabilities further in time

We also determined algebraic expressions of the probabilities of the qubit states at moments further in time than given in section 3.4 and 3.5.
In subsection 3.4, where the system alternately evolves under H_0 and H_{int}, there are also expressions of the probabilities at $t = 3t_{ev} + 3t_m$ and at $t = 4t_{ev} + 4t_m$ for the case of $\sigma \to 0$ and at $t = 3t_{ev} + 3t_m$ for the case of $\sigma \not\to 0$.
In subsection 3.5, where the system alternately evolves under H_0 and $H = H_0 + H_{int}$, there are also expressions of the probabilities at $t = 2t_{ev} + 2t_m$ for the case of $\sigma \to 0$.
These calculations are given in the following subsections 5.1.1, 5.1.2. and 5.1.3.

5.1.1 Series of measurements alternated with free evolution of the spin; $\sigma \to 0$

The calculations of the algebraic expressions of the probabilities at $t = 3t_{ev}+3t_m$ and $t = 4t_{ev}+4t_m$ are given below.

The state of the qubit after $t = 3t_{ev}+3t_m$ is given by applying three times matrix A from equation 3.4.8 on the initial state. This gives

$$|\Phi(3t_{ev} + 3t_m)\rangle = A^3|\Phi(0)\rangle = \frac{1}{\sqrt{2\sqrt{\pi\sigma}}} \exp\Big(-\frac{(q-q_0)^2}{2\sigma}\Big)\begin{pmatrix}\Phi_i\\ \Phi_j\end{pmatrix} \tag{5.1.1}$$

with $\Phi_i = \exp(-\frac{3i}{\hbar}Bt_{ev})\cos^3(\frac{1}{\hbar}gqt_m) - 2\exp(-\frac{i}{\hbar}Bt_{ev})\sin^2(\frac{1}{\hbar}gqt_m)\cos(\frac{1}{\hbar}gqt_m)$
$- \exp(\frac{i}{\hbar}Bt_{ev})\sin^2(\frac{1}{\hbar}gqt_m)\cos(\frac{1}{\hbar}gqt_m) - i\exp(-\frac{i}{\hbar}Bt_{ev})\sin(\frac{1}{\hbar}gqt_m)\cos^2(\frac{1}{\hbar}gqt_m)$
$-i\exp(\frac{i}{\hbar}Bt_{ev})\sin(\frac{1}{\hbar}gqt_m)\cos^2(\frac{1}{\hbar}gqt_m) - i\exp(\frac{3i}{\hbar}Bt_{ev})\sin(\frac{1}{\hbar}gqt_m)\cos^2(\frac{1}{\hbar}gqt_m) + i\exp(\frac{i}{\hbar}Bt_{ev})\sin^3(\frac{1}{\hbar}gqt_m)$
and $\Phi_j = \exp(\frac{3i}{\hbar}Bt_{ev})\cos^3(\frac{1}{\hbar}gqt_m) - 2\exp(\frac{i}{\hbar}Bt_{ev})\sin^2(\frac{1}{\hbar}gqt_m)\cos(\frac{1}{\hbar}gqt_m)$
$- \exp(-\frac{i}{\hbar}Bt_{ev})\sin^2(\frac{1}{\hbar}gqt_m)\cos(\frac{1}{\hbar}gqt_m) - i\exp(-\frac{i}{\hbar}Bt_{ev})\sin(\frac{1}{\hbar}gqt_m)\cos^2(\frac{1}{\hbar}gqt_m)$
$-i\exp(\frac{i}{\hbar}Bt_{ev})\sin(\frac{1}{\hbar}gqt_m)\cos^2(\frac{1}{\hbar}gqt_m) - i\exp(-\frac{3i}{\hbar}Bt_{ev})\sin(\frac{1}{\hbar}gqt_m)\cos^2(\frac{1}{\hbar}gqt_m) + i\exp(-\frac{i}{\hbar}Bt_{ev})\sin^3(\frac{1}{\hbar}gqt_m)$.

The corresponding density matrix is given by

$$\rho(3t_{ev}+3t_m) = |\Phi(3t_{ev}+3t_m)\rangle\langle\Phi(3t_{ev}+3t_m)| = \frac{1}{2\sqrt{\pi\sigma}}\exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big)\begin{pmatrix}\tilde\alpha(3t_{ev}+3t_m) & \tilde\beta(3t_{ev}+3t_m)\\ \tilde\gamma(3t_{ev}+3t_m) & \tilde\delta(3t_{ev}+3t_m)\end{pmatrix} \tag{5.1.2}$$

with $\tilde\alpha(3t_{ev}+3t_m) = \cos^6(\frac{1}{\hbar}gqt_m) + \sin^6(\frac{1}{\hbar}gqt_m) + \sin(\frac{1}{\hbar}gqt_m)\cos^5(\frac{1}{\hbar}gqt_m)(i\exp(-\frac{6i}{\hbar}Bt_{ev})$
$-i\exp(\frac{6i}{\hbar}Bt_{ev}) + i\exp(-\frac{4i}{\hbar}Bt_{ev}) - i\exp(\frac{4i}{\hbar}Bt_{ev}) + i\exp(-\frac{2i}{\hbar}Bt_{ev}) - i\exp(\frac{2i}{\hbar}Bt_{ev})) + 3\sin^2(\frac{1}{\hbar}gqt_m)\cos^4(\frac{1}{\hbar}gqt_m)$
$+\sin^3(\frac{1}{\hbar}gqt_m)\cos^3(\frac{1}{\hbar}gqt_m)(-3i\exp(-\frac{4i}{\hbar}Bt_{ev}) + 3i\exp(\frac{4i}{\hbar}Bt_{ev}) - 2i\exp(-\frac{2i}{\hbar}Bt_{ev}) + 2i\exp(\frac{2i}{\hbar}Bt_{ev}))$
$+ 3\sin^4(\frac{1}{\hbar}gqt_m)\cos^2(\frac{1}{\hbar}gqt_m) + \sin^5(\frac{1}{\hbar}gqt_m)\cos(\frac{1}{\hbar}gqt_m)(2i\exp(-\frac{2i}{\hbar}Bt_{ev}) - 2i\exp(\frac{2i}{\hbar}Bt_{ev}))$,

$\tilde\beta(3t_{ev}+3t_m) = \exp(2i\frac{1}{\hbar}Bt_{ev})\sin^6(\frac{1}{\hbar}gqt_m) + \exp(-6i\frac{1}{\hbar}Bt_{ev})\cos^6(\frac{1}{\hbar}gqt_m) + \sin^2(\frac{1}{\hbar}gqt_m)\cos^4(\frac{1}{\hbar}gqt_m)(\exp(6i\frac{1}{\hbar}Bt_{ev})$
$+2\exp(4i\frac{1}{\hbar}Bt_{ev}) + 3\exp(\frac{2i}{\hbar}Bt_{ev}) + 2 - \exp(-\frac{2i}{\hbar}Bt_{ev}) - 4\exp(-4i\frac{1}{\hbar}Bt_{ev})) + \sin^4(\frac{1}{\hbar}gqt_m)\cos^2(\frac{1}{\hbar}gqt_m)(4\exp(-\frac{2i}{\hbar}Bt_{ev})$
$+ 2 - \exp(2i\frac{1}{\hbar}Bt_{ev}) - 2\exp(4i\frac{1}{\hbar}Bt_{ev}))$,

$\tilde\gamma(3t_{ev}+3t_m) = \exp(-2i\frac{1}{\hbar}Bt_{ev})\sin^6(\frac{1}{\hbar}gqt_m) + \exp(6i\frac{1}{\hbar}Bt_{ev})\cos^6(\frac{1}{\hbar}gqt_m) + \sin^2(\frac{1}{\hbar}gqt_m)\cos^4(\frac{1}{\hbar}gqt_m)(\exp(-6i\frac{1}{\hbar}Bt_{ev})$
$+2\exp(-4i\frac{1}{\hbar}Bt_{ev}) + 3\exp(-\frac{2i}{\hbar}Bt_{ev}) + 2 - \exp(\frac{2i}{\hbar}Bt_{ev}) - 4\exp(4i\frac{1}{\hbar}Bt_{ev})) + \sin^4(\frac{1}{\hbar}gqt_m)\cos^2(\frac{1}{\hbar}gqt_m)(4\exp(\frac{2i}{\hbar}Bt_{ev})$
$+ 2 - \exp(-2i\frac{1}{\hbar}Bt_{ev}) - 2\exp(-4i\frac{1}{\hbar}Bt_{ev}))$ and

$\tilde\delta(3t_{ev}+3t_m) = \cos^6(\frac{1}{\hbar}gqt_m) + \sin^6(\frac{1}{\hbar}gqt_m) + \sin(\frac{1}{\hbar}gqt_m)\cos^5(\frac{1}{\hbar}gqt_m)(i\exp(\frac{6i}{\hbar}Bt_{ev})$
$-i\exp(-\frac{6i}{\hbar}Bt_{ev}) + i\exp(\frac{4i}{\hbar}Bt_{ev}) - i\exp(-\frac{4i}{\hbar}Bt_{ev}) + i\exp(\frac{2i}{\hbar}Bt_{ev}) - i\exp(-\frac{2i}{\hbar}Bt_{ev})) + 3\sin^2(\frac{1}{\hbar}gqt_m)\cos^4(\frac{1}{\hbar}gqt_m)$
$+ \sin^3(\frac{1}{\hbar}gqt_m)\cos^3(\frac{1}{\hbar}gqt_m)(3i\exp(-\frac{4i}{\hbar}Bt_{ev}) - 3i\exp(\frac{4i}{\hbar}Bt_{ev}) + 2i\exp(-\frac{2i}{\hbar}Bt_{ev}) - 2i\exp(\frac{2i}{\hbar}Bt_{ev}))$
$+ 3\sin^4(\frac{1}{\hbar}gqt_m)\cos^2(\frac{1}{\hbar}gqt_m) + \sin^5(\frac{1}{\hbar}gqt_m)\cos(\frac{1}{\hbar}gqt_m)(2i\exp(\frac{2i}{\hbar}Bt_{ev}) - 2i\exp(-\frac{2i}{\hbar}Bt_{ev}))$.

Again we take the partial trace over the detector coordinate q and this gives, the sum approached by an integral, the density matrix only dependent on the qubit coordinates $i = |0\rangle$ and $j = |1\rangle$

$$\rho_{ii,ij,ji,jj}(3t_{ev}+3t_m) = tr_q\rho(3t_{ev}+3t_m) = \int_{-\infty}^{\infty} \frac{1}{2\sqrt{\pi\sigma}} \exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big) \begin{pmatrix} \tilde{\alpha}(3t_{ev}+3t_m) & \tilde{\beta}(3t_{ev}+3t_m) \\ \tilde{\gamma}(3t_{ev}+3t_m) & \tilde{\delta}(3t_{ev}+3t_m) \end{pmatrix} dq. \tag{5.1.3}$$

Again $\sigma \to 0$, so this gives in equation 5.1.3

$$\rho_{ii,ij,ji,jj}(3t_{ev}+3t_m, q_0) \approx \frac{1}{2} \begin{pmatrix} \tilde{\alpha}(3t_{ev}+3t_m, q_0) & \tilde{\beta}(3t_{ev}+3t_m, q_0) \\ \tilde{\gamma}(3t_{ev}+3t_m, q_0) & \tilde{\delta}(3t_{ev}+3t_m, q_0) \end{pmatrix} \tag{5.1.4}$$

with $\tilde{\alpha}(3t_{ev}+3t_m, q_0) = \cos^6(\frac{1}{\hbar}gq_0t_m) + \sin^6(\frac{1}{\hbar}gq_0t_m) + \sin(\frac{1}{\hbar}gq_0t_m)\cos^5(\frac{1}{\hbar}gq_0t_m)(i\exp(-\frac{6i}{\hbar}Bt_{ev})$ $-i\exp(\frac{6i}{\hbar}Bt_{ev}) + i\exp(-\frac{4i}{\hbar}Bt_{ev}) - i\exp(\frac{4i}{\hbar}Bt_{ev}) + i\exp(-\frac{2i}{\hbar}Bt_{ev}) - i\exp(\frac{2i}{\hbar}Bt_{ev})) + 3\sin^2(\frac{1}{\hbar}gq_0t_m)\cos^4(\frac{1}{\hbar}gq_0t_m)$ $+\sin^3(\frac{1}{\hbar}gq_0t_m)\cos^3(\frac{1}{\hbar}gq_0t_m)(-3i\exp(-\frac{4i}{\hbar}Bt_{ev}) + 3i\exp(\frac{4i}{\hbar}Bt_{ev}) - 2i\exp(-\frac{2i}{\hbar}Bt_{ev}) + 2i\exp(\frac{2i}{\hbar}Bt_{ev}))$ $+ 3\sin^4(\frac{1}{\hbar}gq_0t_m)\cos^2(\frac{1}{\hbar}gq_0t_m) + \sin^5(\frac{1}{\hbar}gq_0t_m)\cos(\frac{1}{\hbar}gq_0t_m)(2i\exp(-\frac{2i}{\hbar}Bt_{ev}) - 2i\exp(\frac{2i}{\hbar}Bt_{ev}))$,

$\tilde{\beta}(3t_{ev}+3t_m, q_0) = \exp(2i\frac{1}{\hbar}Bt_{ev})\sin^6(\frac{1}{\hbar}gq_0t_m) + \exp(-6i\frac{1}{\hbar}Bt_{ev})\cos^6(\frac{1}{\hbar}gq_0t_m) + \sin^2(\frac{1}{\hbar}gq_0t_m)\cos^4(\frac{1}{\hbar}gq_0t_m)$ $(\exp(6i\frac{1}{\hbar}Bt_{ev}) + 2\exp(4i\frac{1}{\hbar}Bt_{ev}) + 3\exp(\frac{2i}{\hbar}Bt_{ev}) + 2 - \exp(-\frac{2i}{\hbar}Bt_{ev}) - 4\exp(-4i\frac{1}{\hbar}Bt_{ev}))$ $+ \sin^4(\frac{1}{\hbar}gq_0t_m)\cos^2(\frac{1}{\hbar}gq_0t_m)(4\exp(-\frac{2i}{\hbar}Bt_{ev}) + 2 - \exp(2i\frac{1}{\hbar}Bt_{ev}) - 2\exp(4i\frac{1}{\hbar}Bt_{ev}))$,

$\tilde{\gamma}(3t_{ev}+3t_m, q_0) = \exp(-2i\frac{1}{\hbar}Bt_{ev})\sin^6(\frac{1}{\hbar}gq_0t_m) + \exp(6i\frac{1}{\hbar}Bt_{ev})\cos^6(\frac{1}{\hbar}gq_0t_m) + \sin^2(\frac{1}{\hbar}gq_0t_m)\cos^4(\frac{1}{\hbar}gq_0t_m)$ $(\exp(-6i\frac{1}{\hbar}Bt_{ev}) + 2\exp(-4i\frac{1}{\hbar}Bt_{ev}) + 3\exp(-\frac{2i}{\hbar}Bt_{ev}) + 2 - \exp(\frac{2i}{\hbar}Bt_{ev}) - 4\exp(4i\frac{1}{\hbar}Bt_{ev}))$ $+ \sin^4(\frac{1}{\hbar}gq_0t_m)\cos^2(\frac{1}{\hbar}gq_0t_m)(4\exp(\frac{2i}{\hbar}Bt_{ev})$ $+ 2 - \exp(-2i\frac{1}{\hbar}Bt_{ev}) - 2\exp(-4i\frac{1}{\hbar}Bt_{ev}))$ and

$\tilde{\delta}(3t_{ev}+3t_m, q_0) = \cos^6(\frac{1}{\hbar}gq_0t_m) + \sin^6(\frac{1}{\hbar}gq_0t_m) + \sin(\frac{1}{\hbar}gq_0t_m)\cos^5(\frac{1}{\hbar}gq_0t_m)(i\exp(\frac{6i}{\hbar}Bt_{ev})$ $-i\exp(-\frac{6i}{\hbar}Bt_{ev}) + i\exp(\frac{4i}{\hbar}Bt_{ev}) - i\exp(-\frac{4i}{\hbar}Bt_{ev}) + i\exp(\frac{2i}{\hbar}Bt_{ev}) - i\exp(-\frac{2i}{\hbar}Bt_{ev})) + 3\sin^2(\frac{1}{\hbar}gq_0t_m)\cos^4(\frac{1}{\hbar}gq_0t_m)$ $+\sin^3(\frac{1}{\hbar}gq_0t_m)\cos^3(\frac{1}{\hbar}gq_0t_m)(3i\exp(-\frac{4i}{\hbar}Bt_{ev}) - 3i\exp(\frac{4i}{\hbar}Bt_{ev}) + 2i\exp(-\frac{2i}{\hbar}Bt_{ev}) - 2i\exp(\frac{2i}{\hbar}Bt_{ev}))$ $+3\sin^4(\frac{1}{\hbar}gq_0t_m)\cos^2(\frac{1}{\hbar}gq_0t_m) + \sin^5(\frac{1}{\hbar}gq_0t_m)\cos(\frac{1}{\hbar}gq_0t_m)(2i\exp(\frac{2i}{\hbar}Bt_{ev}) - 2i\exp(-\frac{2i}{\hbar}Bt_{ev}))$.

The probability that the qubit is in state $|0\rangle$ after $3t_{ev}+3t_m$ is given by

$$\begin{aligned} P_0(3t_m+3t_{ev}, q_0) =& \frac{1}{2} + \frac{1}{2}\cos(\frac{2}{\hbar}Bt_{ev})\sin^6(\frac{1}{\hbar}gq_0t_m) + \frac{1}{2}\cos(\frac{6}{\hbar}Bt_{ev})\cos^6(\frac{1}{\hbar}gq_0t_m) \\ &+ \sin^2(\frac{1}{\hbar}gq_0t_m)\cos^4(\frac{1}{\hbar}gq_0t_m)(1 + \frac{1}{2}\cos(\frac{6}{\hbar}Bt_{ev}) + \cos(\frac{2}{\hbar}Bt_{ev}) - \cos(\frac{4}{\hbar}Bt_{ev})) \\ &+ \sin^4(\frac{1}{\hbar}gq_0t_m)\cos^2(\frac{1}{\hbar}gq_0t_m)(1 + \frac{3}{2}\cos(\frac{2}{\hbar}Bt_{ev}) - \cos(\frac{4}{\hbar}Bt_{ev})) \end{aligned} \tag{5.1.5}$$

and the probability that the qubit is in state $|1\rangle$ is given by

$$\begin{aligned} P_1(3t_m+3t_{ev}, q_0) =& \frac{1}{2} - \frac{1}{2}\cos(\frac{2}{\hbar}Bt_{ev})\sin^6(\frac{1}{\hbar}gq_0t_m) - \frac{1}{2}\cos(\frac{6}{\hbar}Bt_{ev})\cos^6(\frac{1}{\hbar}gq_0t_m) \\ &- \sin^2(\frac{1}{\hbar}gq_0t_m)\cos^4(\frac{1}{\hbar}gq_0t_m)(1 + \frac{1}{2}\cos(\frac{6}{\hbar}Bt_{ev}) + \cos(\frac{2}{\hbar}Bt_{ev}) - \cos(\frac{4}{\hbar}Bt_{ev})) \\ &- \sin^4(\frac{1}{\hbar}gq_0t_m)\cos^2(\frac{1}{\hbar}gq_0t_m)(1 + \frac{3}{2}\cos(\frac{2}{\hbar}Bt_{ev}) - \cos(\frac{4}{\hbar}Bt_{ev})). \end{aligned} \tag{5.1.6}$$

The state after $t = 4t_{ev} + 4t_m$ is given by applying four times matrix A from equation 3.4.8 on the initial state. This gives

$$|\Phi(4t_{ev}+4t_m)\rangle = A^4|\Phi(0)\rangle = \frac{1}{\sqrt{2\sqrt{\pi\sigma}}}\exp\Big(-\frac{(q-q_0)^2}{2\sigma}\Big)\begin{pmatrix}\Phi_i\\ \Phi_j\end{pmatrix} \qquad (5.1.7)$$

with $\Phi_i = \exp(-\frac{4i}{\hbar}Bt_{ev})\cos^4(\frac{1}{\hbar}gqt_m) + \sin^4(\frac{1}{\hbar}gqt_m) - 2\sin^2(\frac{1}{\hbar}gqt_m)\cos^2(\frac{1}{\hbar}gqt_m)$
$-3\exp(-\frac{2i}{\hbar}Bt_{ev})\sin^2(\frac{1}{\hbar}gqt_m)\cos^2(\frac{1}{\hbar}gqt_m) - \exp(\frac{2i}{\hbar}Bt_{ev})\sin^2(\frac{1}{\hbar}gqt_m)\cos^2(\frac{1}{\hbar}gqt_m) + 2i\sin^3(\frac{1}{\hbar}gqt_m)\cos(\frac{1}{\hbar}gqt_m)$
$+2i\exp(\frac{2i}{\hbar}Bt_{ev})\sin^3(\frac{1}{\hbar}gqt_m)\cos(\frac{1}{\hbar}gqt_m) - i\sin(\frac{1}{\hbar}gqt_m)\cos^3(\frac{1}{\hbar}gqt_m)(1+\exp(-\frac{2i}{\hbar}Bt_{ev})+\exp(\frac{2i}{\hbar}Bt_{ev})$
$+\exp(\frac{4i}{\hbar}Bt_{ev}))$
and $\Phi_j = \exp(\frac{4i}{\hbar}Bt_{ev})\cos^4(\frac{1}{\hbar}gqt_m) + \sin^4(\frac{1}{\hbar}gqt_m) - 2\sin^2(\frac{1}{\hbar}gqt_m)\cos^2(\frac{1}{\hbar}gqt_m)$
$-3\exp(\frac{2i}{\hbar}Bt_{ev})\sin^2(\frac{1}{\hbar}gqt_m)\cos^2(\frac{1}{\hbar}gqt_m) - \exp(-\frac{2i}{\hbar}Bt_{ev})\sin^2(\frac{1}{\hbar}gqt_m)\cos^2(\frac{1}{\hbar}gqt_m) + 2i\sin^3(\frac{1}{\hbar}gqt_m)\cos(\frac{1}{\hbar}gqt_m)$
$+2i\exp(-\frac{2i}{\hbar}Bt_{ev})\sin^3(\frac{1}{\hbar}gqt_m)\cos(\frac{1}{\hbar}gqt_m) - i\sin(\frac{1}{\hbar}gqt_m)\cos^3(\frac{1}{\hbar}gqt_m)(1+\exp(-\frac{2i}{\hbar}Bt_{ev})+\exp(\frac{2i}{\hbar}Bt_{ev})$
$+\exp(-\frac{4i}{\hbar}Bt_{ev}))$.

The corresponding density matrix is given by

$$\rho(4t_{ev}+4t_m) = |\Phi(4t_{ev}+4t_m)\rangle\langle\Phi(4t_{ev}+4t_m)| = \frac{1}{2\sqrt{\pi\sigma}}\exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big)\begin{pmatrix}\tilde{\alpha}(4t_{ev}+4t_m) & \tilde{\beta}(4t_{ev}+4t_m)\\ \tilde{\gamma}(4t_{ev}+4t_m) & \tilde{\delta}(4t_{ev}+4t_m)\end{pmatrix} \qquad (5.1.8)$$

with

$\tilde{\alpha}(4t_{ev}+4t_m) = \cos^8(\frac{1}{\hbar}gqt_m) + \sin^8(\frac{1}{\hbar}gqt_m) + \sin(\frac{1}{\hbar}gqt_m)\cos^7(\frac{1}{\hbar}gqt_m)(-i\exp(\frac{2i}{\hbar}Bt_{ev}) + i\exp(-\frac{2i}{\hbar}Bt_{ev})$
$-i\exp(\frac{4i}{\hbar}Bt_{ev}) + i\exp(-\frac{4i}{\hbar}Bt_{ev}) - i\exp(\frac{6i}{\hbar}Bt_{ev}) + i\exp(-\frac{6i}{\hbar}Bt_{ev}) - i\exp(\frac{8i}{\hbar}Bt_{ev}) + i\exp(-\frac{8i}{\hbar}Bt_{ev}))$
$+4\sin^2(\frac{1}{\hbar}gqt_m)\cos^6(\frac{1}{\hbar}gqt_m) + \sin^3(\frac{1}{\hbar}gqt_m)\cos^5(\frac{1}{\hbar}gqt_m)(6i\exp(\frac{4i}{\hbar}Bt_{ev}) - 6i\exp(\frac{-4i}{\hbar}Bt_{ev}) + 3i\exp(\frac{2i}{\hbar}Bt_{ev})$
$-3i\exp(-\frac{2i}{\hbar}Bt_{ev}) + 5i\exp(\frac{6i}{\hbar}Bt_{ev}) - 5i\exp(-\frac{6i}{\hbar}Bt_{ev})) + 6\sin^4(\frac{1}{\hbar}gqt_m)\cos^4(\frac{1}{\hbar}gqt_m)$
$+\sin^5(\frac{1}{\hbar}gqt_m)\cos^3(\frac{1}{\hbar}gqt_m)(-8i\exp(\frac{2i}{\hbar}Bt_{ev}) + 8i\exp(\frac{-2i}{\hbar}Bt_{ev}) - 7i\exp(\frac{4i}{\hbar}Bt_{ev}) + 7i\exp(-\frac{4i}{\hbar}Bt_{ev}))$
$+4\sin^6(\frac{1}{\hbar}gqt_m)\cos^2(\frac{1}{\hbar}gqt_m) + \sin^7(\frac{1}{\hbar}gqt_m)\cos(\frac{1}{\hbar}gqt_m)(2i\exp(\frac{2i}{\hbar}Bt_{ev}) - 2i\exp(-\frac{2i}{\hbar}Bt_{ev}))$,

$\tilde{\beta}(4t_{ev}+4t_m) = \cos^8(\frac{1}{\hbar}gqt_m)\exp(-\frac{8i}{\hbar}Bt_{ev}) + \sin^8(\frac{1}{\hbar}gqt_m) + \sin^2(\frac{1}{\hbar}gqt_m)\cos^6(\frac{1}{\hbar}gqt_m)(\exp(\frac{8i}{\hbar}Bt_{ev})$
$+2\exp(\frac{6i}{\hbar}Bt_{ev}) + 3\exp(\frac{4i}{\hbar}Bt_{ev}) + 4\exp(\frac{2i}{\hbar}Bt_{ev}) + 3 - 3\exp(\frac{-4i}{\hbar}Bt_{ev}) - 6\exp(\frac{-6i}{\hbar}Bt_{ev}))$
$+\sin^4(\frac{1}{\hbar}gqt_m)\cos^4(\frac{1}{\hbar}gqt_m)(-4\exp(\frac{6i}{\hbar}Bt_{ev}) - 7\exp(\frac{4i}{\hbar}Bt_{ev}) - 4\exp(\frac{2i}{\hbar}Bt_{ev}) + 2 + 8\exp(\frac{-2i}{\hbar}Bt_{ev})$
$+11\exp(\frac{-4i}{\hbar}Bt_{ev})) + \sin^6(\frac{1}{\hbar}gqt_m)\cos^2(\frac{1}{\hbar}gqt_m)(4\exp(\frac{4i}{\hbar}Bt_{ev}) + 6\exp(\frac{2i}{\hbar}Bt_{ev}) - 6\exp(-\frac{2i}{\hbar}Bt_{ev}))$,

$\tilde{\gamma}(4t_{ev}+4t_m) = \cos^8(\frac{1}{\hbar}gqt_m)\exp(\frac{8i}{\hbar}Bt_{ev}) + \sin^8(\frac{1}{\hbar}gqt_m) + \sin^2(\frac{1}{\hbar}gqt_m)\cos^6(\frac{1}{\hbar}gqt_m)(\exp(-\frac{8i}{\hbar}Bt_{ev})$
$+2\exp(-\frac{6i}{\hbar}Bt_{ev}) + 3\exp(-\frac{4i}{\hbar}Bt_{ev}) + 4\exp(-\frac{2i}{\hbar}Bt_{ev}) + 3 - 3\exp(\frac{4i}{\hbar}Bt_{ev}) - 6\exp(\frac{6i}{\hbar}Bt_{ev}))$
$+\sin^4(\frac{1}{\hbar}gqt_m)\cos^4(\frac{1}{\hbar}gqt_m)(-4\exp(-\frac{6i}{\hbar}Bt_{ev}) - 7\exp(-\frac{4i}{\hbar}Bt_{ev}) - 4\exp(-\frac{2i}{\hbar}Bt_{ev}) + 2 + 8\exp(\frac{2i}{\hbar}Bt_{ev})$
$+11\exp(\frac{4i}{\hbar}Bt_{ev})) + \sin^6(\frac{1}{\hbar}gqt_m)\cos^2(\frac{1}{\hbar}gqt_m)(4\exp(-\frac{4i}{\hbar}Bt_{ev}) + 6\exp(-\frac{2i}{\hbar}Bt_{ev}) - 6\exp(\frac{2i}{\hbar}Bt_{ev}))$

and

$\tilde{\delta}(4t_{ev}+4t_m) = \cos^8(\frac{1}{\hbar}gqt_m) + \sin^8(\frac{1}{\hbar}gqt_m) + \sin(\frac{1}{\hbar}gqt_m)\cos^7(\frac{1}{\hbar}gqt_m)(-i\exp(-\frac{2i}{\hbar}Bt_{ev}) + i\exp(\frac{2i}{\hbar}Bt_{ev})$
$-i\exp(-\frac{4i}{\hbar}Bt_{ev}) + i\exp(\frac{4i}{\hbar}Bt_{ev}) - i\exp(-\frac{6i}{\hbar}Bt_{ev}) + i\exp(\frac{6i}{\hbar}Bt_{ev}) - i\exp(-\frac{8i}{\hbar}Bt_{ev}) + i\exp(\frac{8i}{\hbar}Bt_{ev}))$
$+4\sin^2(\frac{1}{\hbar}gqt_m)\cos^6(\frac{1}{\hbar}gqt_m) + \sin^3(\frac{1}{\hbar}gqt_m)\cos^5(\frac{1}{\hbar}gqt_m)(6i\exp(-\frac{4i}{\hbar}Bt_{ev}) - 6i\exp(\frac{4i}{\hbar}Bt_{ev}) + 3i\exp(-\frac{2i}{\hbar}Bt_{ev})$
$-3i\exp(\frac{2i}{\hbar}Bt_{ev}) + 5i\exp(-\frac{6i}{\hbar}Bt_{ev}) - 5i\exp(\frac{6i}{\hbar}Bt_{ev})) + 6\sin^4(\frac{1}{\hbar}gqt_m)\cos^4(\frac{1}{\hbar}gqt_m)$
$+\sin^5(\frac{1}{\hbar}gqt_m)\cos^3(\frac{1}{\hbar}gqt_m)(8i\exp(\frac{2i}{\hbar}Bt_{ev}) - 8i\exp(\frac{-2i}{\hbar}Bt_{ev}) + 7i\exp(\frac{4i}{\hbar}Bt_{ev}) - 7i\exp(-\frac{4i}{\hbar}Bt_{ev}))$
$+4\sin^6(\frac{1}{\hbar}gqt_m)\cos^2(\frac{1}{\hbar}gqt_m) + \sin^7(\frac{1}{\hbar}gqt_m)\cos(\frac{1}{\hbar}gqt_m)(2i\exp(-\frac{2i}{\hbar}Bt_{ev}) + 2i\exp(-\frac{2i}{\hbar}Bt_{ev}))$.

Again we take the partial trace over the detector coordinate q and this gives, the sum approached by an integral, the density matrix only dependent on the qubit coordinates $i = |0\rangle$ and $j = |1\rangle$

$$\rho_{ii,ij,ji,jj}(4t_{ev}+4t_m) = tr_q\rho(4t_{ev}+4t_m) = \int_{-\infty}^{\infty}\frac{1}{2\sqrt{\pi\sigma}}\exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big)\begin{pmatrix}\tilde{\alpha}(4t_{ev}+4t_m) & \tilde{\beta}(4t_{ev}+4t_m)\\ \tilde{\gamma}(4t_{ev}+4t_m) & \tilde{\delta}(4t_{ev}+4t_m)\end{pmatrix}dq. \qquad (5.1.9)$$

Again $\sigma \to 0$, so this gives in equation 5.1.9

$$\rho_{ii,ij,ji,jj}(4t_{ev} + 4t_m, q_0) \approx \frac{1}{2}\begin{pmatrix}\tilde{\alpha}(4t_{ev} + 4t_m, q_0) & \tilde{\beta}(4t_{ev} + 4t_m, q_0)\\ \tilde{\gamma}(4t_{ev} + 4t_m, q_0) & \tilde{\delta}(4t_{ev} + 4t_m, q_0)\end{pmatrix} \tag{5.1.10}$$

with

$$\begin{aligned}
&\tilde{\alpha}(4t_{ev} + 4t_m, q_0) = \cos^8(\tfrac{1}{\hbar}gq_0t_m) + \sin^8(\tfrac{1}{\hbar}gq_0t_m)\\
&+ \sin(\tfrac{1}{\hbar}gq_0t_m)\cos^7(\tfrac{1}{\hbar}gq_0t_m)(-i\exp(\tfrac{2i}{\hbar}Bt_{ev}) + i\exp(-\tfrac{2i}{\hbar}Bt_{ev}) - i\exp(\tfrac{4i}{\hbar}Bt_{ev}) + i\exp(-\tfrac{4i}{\hbar}Bt_{ev})\\
&- i\exp(\tfrac{6i}{\hbar}Bt_{ev}) + i\exp(-\tfrac{6i}{\hbar}Bt_{ev}) - i\exp(\tfrac{8i}{\hbar}Bt_{ev}) + i\exp(-\tfrac{8i}{\hbar}Bt_{ev}))\\
&+ 4\sin^2(\tfrac{1}{\hbar}gq_0t_m)\cos^6(\tfrac{1}{\hbar}gq_0t_m) + \sin^3(\tfrac{1}{\hbar}gq_0t_m)\cos^5(\tfrac{1}{\hbar}gq_0t_m)(6i\exp(\tfrac{4i}{\hbar}Bt_{ev}) - 6i\exp(\tfrac{-4i}{\hbar}Bt_{ev}) +\\
&3i\exp(\tfrac{2i}{\hbar}Bt_{ev})\\
&- 3i\exp(-\tfrac{2i}{\hbar}Bt_{ev}) + 5i\exp(\tfrac{6i}{\hbar}Bt_{ev}) - 5i\exp(-\tfrac{6i}{\hbar}Bt_{ev})) + 6\sin^4(\tfrac{1}{\hbar}gq_0t_m)\cos^4(\tfrac{1}{\hbar}gq_0t_m)\\
&+\sin^5(\tfrac{1}{\hbar}gq_0t_m)\cos^3(\tfrac{1}{\hbar}gq_0t_m)(-8i\exp(\tfrac{2i}{\hbar}Bt_{ev})+8i\exp(\tfrac{-2i}{\hbar}Bt_{ev})-7i\exp(\tfrac{4i}{\hbar}Bt_{ev})+7i\exp(-\tfrac{4i}{\hbar}Bt_{ev}))\\
&+4\sin^6(\tfrac{1}{\hbar}gq_0t_m)\cos^2(\tfrac{1}{\hbar}gq_0t_m)+\sin^7(\tfrac{1}{\hbar}gq_0t_m)\cos(\tfrac{1}{\hbar}gq_0t_m)(2i\exp(\tfrac{2i}{\hbar}Bt_{ev})-2i\exp(-\tfrac{2i}{\hbar}Bt_{ev})),
\end{aligned}$$

$$\begin{aligned}
&\tilde{\beta}(4t_{ev}+4t_m, q_0) = \cos^8(\tfrac{1}{\hbar}gq_0t_m)\exp(-\tfrac{8i}{\hbar}Bt_{ev})+\sin^8(\tfrac{1}{\hbar}gq_0t_m)+\sin^2(\tfrac{1}{\hbar}gq_0t_m)\cos^6(\tfrac{1}{\hbar}gq_0t_m)(\exp(\tfrac{8i}{\hbar}Bt_{ev})\\
&+ 2\exp(\tfrac{6i}{\hbar}Bt_{ev}) + 3\exp(\tfrac{4i}{\hbar}Bt_{ev}) + 4\exp(\tfrac{2i}{\hbar}Bt_{ev}) + 3 - 3\exp(\tfrac{-4i}{\hbar}Bt_{ev}) - 6\exp(\tfrac{-6i}{\hbar}Bt_{ev}))\\
&+\sin^4(\tfrac{1}{\hbar}gq_0t_m)\cos^4(\tfrac{1}{\hbar}gq_0t_m)(-4\exp(\tfrac{6i}{\hbar}Bt_{ev}) - 7\exp(\tfrac{4i}{\hbar}Bt_{ev}) - 4\exp(\tfrac{2i}{\hbar}Bt_{ev}) + 2 + 8\exp(\tfrac{-2i}{\hbar}Bt_{ev})\\
&+11\exp(\tfrac{-4i}{\hbar}Bt_{ev}))+\sin^6(\tfrac{1}{\hbar}gq_0t_m)\cos^2(\tfrac{1}{\hbar}gq_0t_m)(4\exp(\tfrac{4i}{\hbar}Bt_{ev})+6\exp(\tfrac{2i}{\hbar}Bt_{ev})-6\exp(-\tfrac{2i}{\hbar}Bt_{ev})),
\end{aligned}$$

$$\begin{aligned}
&\tilde{\gamma}(4t_{ev}+4t_m, q_0) = \cos^8(\tfrac{1}{\hbar}gq_0t_m)\exp(\tfrac{8i}{\hbar}Bt_{ev})+\sin^8(\tfrac{1}{\hbar}gq_0t_m)+\sin^2(\tfrac{1}{\hbar}gq_0t_m)\cos^6(\tfrac{1}{\hbar}gq_0t_m)(\exp(-\tfrac{8i}{\hbar}Bt_{ev})\\
&+ 2\exp(-\tfrac{6i}{\hbar}Bt_{ev}) + 3\exp(-\tfrac{4i}{\hbar}Bt_{ev}) + 4\exp(-\tfrac{2i}{\hbar}Bt_{ev}) + 3 - 3\exp(\tfrac{4i}{\hbar}Bt_{ev}) - 6\exp(\tfrac{6i}{\hbar}Bt_{ev}))\\
&+\sin^4(\tfrac{1}{\hbar}gq_0t_m)\cos^4(\tfrac{1}{\hbar}gq_0t_m)(-4\exp(-\tfrac{6i}{\hbar}Bt_{ev})-7\exp(-\tfrac{4i}{\hbar}Bt_{ev})-4\exp(-\tfrac{2i}{\hbar}Bt_{ev})+2+8\exp(\tfrac{2i}{\hbar}Bt_{ev})\\
&+11\exp(\tfrac{4i}{\hbar}Bt_{ev}))+\sin^6(\tfrac{1}{\hbar}gq_0t_m)\cos^2(\tfrac{1}{\hbar}gq_0t_m)(4\exp(-\tfrac{4i}{\hbar}Bt_{ev})+6\exp(-\tfrac{2i}{\hbar}Bt_{ev})-6\exp(\tfrac{2i}{\hbar}Bt_{ev}))
\end{aligned}$$

and

$$\begin{aligned}
&\tilde{\delta}(4t_{ev} + 4t_m, q_0) = \cos^8(\tfrac{1}{\hbar}gq_0t_m) + \sin^8(\tfrac{1}{\hbar}gq_0t_m)\\
&+ \sin(\tfrac{1}{\hbar}gq_0t_m)\cos^7(\tfrac{1}{\hbar}gq_0t_m)(-i\exp(-\tfrac{2i}{\hbar}Bt_{ev}) + i\exp(\tfrac{2i}{\hbar}Bt_{ev}) - i\exp(-\tfrac{4i}{\hbar}Bt_{ev}) + i\exp(\tfrac{4i}{\hbar}Bt_{ev})\\
&- i\exp(-\tfrac{6i}{\hbar}Bt_{ev}) + i\exp(\tfrac{6i}{\hbar}Bt_{ev}) - i\exp(-\tfrac{8i}{\hbar}Bt_{ev}) + i\exp(\tfrac{8i}{\hbar}Bt_{ev}))\\
&+ 4\sin^2(\tfrac{1}{\hbar}gq_0t_m)\cos^6(\tfrac{1}{\hbar}gq_0t_m) + \sin^3(\tfrac{1}{\hbar}gq_0t_m)\cos^5(\tfrac{1}{\hbar}gq_0t_m)(6i\exp(-\tfrac{4i}{\hbar}Bt_{ev}) - 6i\exp(\tfrac{4i}{\hbar}Bt_{ev})\\
&+3i\exp(-\tfrac{2i}{\hbar}Bt_{ev})-3i\exp(\tfrac{2i}{\hbar}Bt_{ev})+5i\exp(-\tfrac{6i}{\hbar}Bt_{ev})-5i\exp(\tfrac{6i}{\hbar}Bt_{ev}))+6\sin^4(\tfrac{1}{\hbar}gq_0t_m)\cos^4(\tfrac{1}{\hbar}gq_0t_m)\\
&+\sin^5(\tfrac{1}{\hbar}gq_0t_m)\cos^3(\tfrac{1}{\hbar}gq_0t_m)(8i\exp(\tfrac{2i}{\hbar}Bt_{ev})-8i\exp(\tfrac{-2i}{\hbar}Bt_{ev})+7i\exp(\tfrac{4i}{\hbar}Bt_{ev})-7i\exp(-\tfrac{4i}{\hbar}Bt_{ev}))\\
&+4\sin^6(\tfrac{1}{\hbar}gq_0t_m)\cos^2(\tfrac{1}{\hbar}gq_0t_m)+\sin^7(\tfrac{1}{\hbar}gq_0t_m)\cos(\tfrac{1}{\hbar}gq_0t_m)(2i\exp(-\tfrac{2i}{\hbar}Bt_{ev})+2i\exp(-\tfrac{2i}{\hbar}Bt_{ev})).
\end{aligned}$$

The probability that the qubit is in the state $|0\rangle$ at $4t_{ev} + 4t_m$ is given by

$$\begin{aligned}
P_0(4t_m + 4t_{ev}, q_0) =& \frac{1}{2} + \frac{1}{2}\sin^8(\frac{1}{\hbar}gq_0t_m) + \frac{1}{2}\cos(\frac{8}{\hbar}Bt_{ev})\cos^8(\frac{1}{\hbar}gq_0t_m)\\
&+ \sin^2(\frac{1}{\hbar}gq_0t_m)\cos^6(\frac{1}{\hbar}gq_0t_m)(\frac{3}{2} + \frac{1}{2}\cos(\frac{8}{\hbar}Bt_{ev}) - 2\cos(\frac{6}{\hbar}Bt_{ev}) + 2\cos(\frac{2}{\hbar}Bt_{ev}))\\
&+ \sin^4(\frac{1}{\hbar}gq_0t_m)\cos^4(\frac{1}{\hbar}gq_0t_m)(1 + 2\cos(\frac{2}{\hbar}Bt_{ev}) + 2\cos(\frac{4}{\hbar}Bt_{ev}) - 2\cos(\frac{6}{\hbar}Bt_{ev}))\\
&+ 2\cos(\frac{4}{\hbar}Bt_{ev})\sin^6(\frac{1}{\hbar}gq_0t_m)\cos^2(\frac{1}{\hbar}gq_0t_m)
\end{aligned} \tag{5.1.11}$$

and the probability that the qubit is in the state $|1\rangle$ is given by

$$
\begin{aligned}
P_1(4t_m+4t_{ev},q_0) =& \frac{1}{2}-\frac{1}{2}\sin^8(\frac{1}{\hbar}gq_0t_m)-\frac{1}{2}\cos(\frac{8}{\hbar}Bt_{ev})\cos^8(\frac{1}{\hbar}gq_0t_m) \\
&-\sin^2(\frac{1}{\hbar}gq_0t_m)\cos^6(\frac{1}{\hbar}gq_0t_m)(\frac{3}{2}+\frac{1}{2}\cos(\frac{8}{\hbar}Bt_{ev})-2\cos(\frac{6}{\hbar}Bt_{ev})+2\cos(\frac{2}{\hbar}Bt_{ev})) \\
&-\sin^4(\frac{1}{\hbar}gq_0t_m)\cos^4(\frac{1}{\hbar}gq_0t_m)(1+2\cos(\frac{2}{\hbar}Bt_{ev})+2\cos(\frac{4}{\hbar}Bt_{ev})-2\cos(\frac{6}{\hbar}Bt_{ev})) \\
&-2\cos(\frac{4}{\hbar}Bt_{ev})\sin^6(\frac{1}{\hbar}gq_0t_m)\cos^2(\frac{1}{\hbar}gq_0t_m).
\end{aligned}
\tag{5.1.12}
$$

5.1.2 Series of measurements alternated with free evolution of the spin; $\sigma \not\to 0$

The calculations of the algebraic expressions of the probabilities at $t = 3t_{ev} + 3t_m$ are given below.

The integral from equation 5.1.3 is calculated for each matrix element at this moment in time, with as result the following equations.

$$
\begin{aligned}
\rho_{ii}(3t_{ev}+3t_m) =& \int_{-\infty}^{\infty}\frac{1}{2\sqrt{\pi\sigma}}\exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big)\tilde{\alpha}(3t_{ev}+3t_m)dq = \\
&\frac{1}{2}\Big(\exp(-9(\frac{1}{\hbar}gt_m)^2\sigma)\big(((64\cos^5(\frac{1}{\hbar}gt_mq_0)+\cos^3(\frac{1}{\hbar}gt_mq_0)(64\exp(5(\frac{1}{\hbar}gt_m)^2\sigma)-64) \\
&+(20\exp(8(\frac{1}{\hbar}gt_m)^2\sigma)-32\exp(5(\frac{1}{\hbar}gt_m)^2\sigma)+12)\cos(\frac{1}{\hbar}gt_mq_0))\sin(\frac{1}{\hbar}gt_mq_0)\cos^5(\frac{1}{\hbar}Bt_{ev}) \\
&+((24\exp(5(\frac{1}{\hbar}gt_m)^2\sigma)-24\exp(8(\frac{1}{\hbar}gt_m)^2\sigma))\cos(\frac{1}{\hbar}gt_mq_0) \\
&-48\exp(5(\frac{1}{\hbar}gt_m)^2\sigma)\cos^3(\frac{1}{\hbar}gt_mq_0))\sin(\frac{1}{\hbar}gt_mq_0)\cos^3(\frac{1}{\hbar}Bt_{ev}) \\
&+8\exp(8(\frac{1}{\hbar}gt_m)^2\sigma)\cos(\frac{1}{\hbar}gt_mq_0)\sin(\frac{1}{\hbar}gt_mq_0)\cos(\frac{1}{\hbar}Bt_{ev})\big)\sin(\frac{1}{\hbar}Bt_{ev})+\exp(9(\frac{1}{\hbar}gt_m)^2\sigma)\big)\Big)
\end{aligned}
\tag{5.1.13}
$$

$$
\begin{aligned}
\rho_{ij}(3t_{ev}+3t_m) = &\int_{-\infty}^{\infty} \frac{1}{2\sqrt{\pi\sigma}} \exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big)\tilde{\beta}(3t_{ev}+3t_m)dq = \\
&-\frac{1}{2}\Big(\exp(-9(\frac{1}{\hbar}gt_m)^2\sigma)\Big(\big((64i\cos^6(\frac{1}{\hbar}gt_mq_0)+\cos^4(\frac{1}{\hbar}gt_mq_0)(64i\exp(5(\frac{1}{\hbar}gt_m)^2\sigma)-96i) \\
&+(28i\exp(8(\frac{1}{\hbar}gt_m)^2\sigma)-64i\exp(5(\frac{1}{\hbar}gt_m)^2\sigma)+36i)\cos^2(\frac{1}{\hbar}gt_mq_0) \\
&+8i\exp(9(\frac{1}{\hbar}gt_m)^2\sigma)-14i\exp(8(\frac{1}{\hbar}gt_m)^2\sigma)+8i\exp(5(\frac{1}{\hbar}gt_m)^2\sigma)-2i)\cos^5(\frac{1}{\hbar}Bt_{ev}) \\
&+(-48i\exp(5(\frac{1}{\hbar}gt_m)^2\sigma)\cos^4(\frac{1}{\hbar}gt_mq_0)+(48i\exp(5(\frac{1}{\hbar}gt_m)^2\sigma)-32i\exp(8(\frac{1}{\hbar}gt_m)^2\sigma))\cos^2(\frac{1}{\hbar}gt_mq_0) \\
&-10i\exp(9(\frac{1}{\hbar}gt_m)^2\sigma)+16i\exp(8(\frac{1}{\hbar}gt_m)^2\sigma)-6i\exp(5(\frac{1}{\hbar}gt_m)^2\sigma))\cos^3(\frac{1}{\hbar}Bt_{ev}) \\
&+(8i\exp(8(\frac{1}{\hbar}gt_m)^2\sigma)\cos^2(\frac{1}{\hbar}gt_mq_0)+2i\exp(9(\frac{1}{\hbar}gt_m)^2\sigma) \\
&-4i\exp(8(\frac{1}{\hbar}gt_m)^2\sigma))\cos(\frac{1}{\hbar}Bt_{ev})\big)\sin(\frac{1}{\hbar}Bt_{ev}) \\
&+(-32\exp(5(\frac{1}{\hbar}gt_m)^2\sigma)\cos^4(\frac{1}{\hbar}gt_mq_0)+(32\exp(5(\frac{1}{\hbar}gt_m)^2\sigma)-32\exp(8(\frac{1}{\hbar}gt_m)^2\sigma))\cos^2(\frac{1}{\hbar}gt_mq_0) \\
&-12\exp(9(\frac{1}{\hbar}gt_m)^2\sigma)+16\exp(8(\frac{1}{\hbar}gt_m)^2\sigma)-4\exp(5(\frac{1}{\hbar}gt_m)^2\sigma))\cos^6(\frac{1}{\hbar}Bt_{ev}) \\
&+(32\exp(5(\frac{1}{\hbar}gt_m)^2\sigma)\cos^4(\frac{1}{\hbar}gt_mq_0)+(48\exp(8(\frac{1}{\hbar}gt_m)^2\sigma)-32\exp(5(\frac{1}{\hbar}gt_m)^2\sigma))\cos^2(\frac{1}{\hbar}gt_mq_0) \\
&+20\exp(9(\frac{1}{\hbar}gt_m)^2\sigma)-24\exp(8(\frac{1}{\hbar}gt_m)^2\sigma)+4\exp(5(\frac{1}{\hbar}gt_m)^2\sigma))\cos^4(\frac{1}{\hbar}Bt_{ev}) \\
&(-16\exp(8(\frac{1}{\hbar}gt_m)^2\sigma)\cos^2(\frac{1}{\hbar}gt_mq_0)-10\exp(9(\frac{1}{\hbar}gt_m)^2\sigma)+8\exp(8(\frac{1}{\hbar}gt_m)^2\sigma))\cos^2(\frac{1}{\hbar}Bt_{ev}) \\
&+\exp(9(\frac{1}{\hbar}gt_m)^2\sigma)\Big)\Big)
\end{aligned}
\tag{5.1.14}
$$

$$
\begin{aligned}
\rho_{ji}(3t_{ev}+3t_m) = &\int_{-\infty}^{\infty} \frac{1}{2\sqrt{\pi\sigma}} \exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big)\tilde{\gamma}(3t_{ev}+3t_m)dq = \\
&\frac{1}{2}\Big(\exp(-9(\frac{1}{\hbar}gt_m)^2\sigma)\Big(\Big((64i\cos^6(\frac{1}{\hbar}gt_mq_0) + \cos^4(\frac{1}{\hbar}gt_mq_0)(64i\exp(5(\frac{1}{\hbar}gt_m)^2\sigma) - 96i) \\
&+ (28i\exp(8(\frac{1}{\hbar}gt_m)^2\sigma) - 64i\exp(5(\frac{1}{\hbar}gt_m)^2\sigma) + 36i)\cos^2(\frac{1}{\hbar}gt_mq_0) \\
&+ 8i\exp(9(\frac{1}{\hbar}gt_m)^2\sigma) - 14i\exp(8(\frac{1}{\hbar}gt_m)^2\sigma) + 8i\exp(5(\frac{1}{\hbar}gt_m)^2\sigma) - 2i)\cos^5(\frac{1}{\hbar}Bt_{ev}) \\
&+ (-48i\exp(5(\frac{1}{\hbar}gt_m)^2\sigma)\cos^4(\frac{1}{\hbar}gt_mq_0) + (48i\exp(5(\frac{1}{\hbar}gt_m)^2\sigma) - 32i\exp(8(\frac{1}{\hbar}gt_m)^2\sigma))\cos^2(\frac{1}{\hbar}gt_mq_0) \\
&- 10i\exp(9(\frac{1}{\hbar}gt_m)^2\sigma) + 16i\exp(8(\frac{1}{\hbar}gt_m)^2\sigma) - 6i\exp(5(\frac{1}{\hbar}gt_m)^2\sigma))\cos^3(\frac{1}{\hbar}Bt_{ev}) \\
&+ (8i\exp(8(\frac{1}{\hbar}gt_m)^2\sigma)\cos^2(\frac{1}{\hbar}gt_mq_0) + 2i\exp(9(\frac{1}{\hbar}gt_m)^2\sigma) \\
&- 4i\exp(8(\frac{1}{\hbar}gt_m)^2\sigma))\cos(\frac{1}{\hbar}Bt_{ev})\Big)\sin(\frac{1}{\hbar}Bt_{ev}) \\
&+ (32\exp(5(\frac{1}{\hbar}gt_m)^2\sigma)\cos^4(\frac{1}{\hbar}gt_mq_0) + (-32\exp(5(\frac{1}{\hbar}gt_m)^2\sigma) + 32\exp(8(\frac{1}{\hbar}gt_m)^2\sigma))\cos^2(\frac{1}{\hbar}gt_mq_0) \\
&+ 12\exp(9(\frac{1}{\hbar}gt_m)^2\sigma) - 16\exp(8(\frac{1}{\hbar}gt_m)^2\sigma) + 4\exp(5(\frac{1}{\hbar}gt_m)^2\sigma))\cos^6(\frac{1}{\hbar}Bt_{ev}) \\
&+ (-32\exp(5(\frac{1}{\hbar}gt_m)^2\sigma)\cos^4(\frac{1}{\hbar}gt_mq_0) + (-48\exp(8(\frac{1}{\hbar}gt_m)^2\sigma) + 32\exp(5(\frac{1}{\hbar}gt_m)^2\sigma))\cos^2(\frac{1}{\hbar}gt_mq_0) \\
&- 20\exp(9(\frac{1}{\hbar}gt_m)^2\sigma) + 24\exp(8(\frac{1}{\hbar}gt_m)^2\sigma) - 4\exp(5(\frac{1}{\hbar}gt_m)^2\sigma))\cos^4(\frac{1}{\hbar}Bt_{ev}) \\
&(16\exp(8(\frac{1}{\hbar}gt_m)^2\sigma)\cos^2(\frac{1}{\hbar}gt_mq_0) + 10\exp(9(\frac{1}{\hbar}gt_m)^2\sigma) - 8\exp(8(\frac{1}{\hbar}gt_m)^2\sigma))\cos^2(\frac{1}{\hbar}Bt_{ev}) \\
&- \exp(9(\frac{1}{\hbar}gt_m)^2\sigma))\Big)
\end{aligned}
\tag{5.1.15}
$$

$$
\begin{aligned}
\rho_{jj}(3t_{ev}+3t_m) = &\int_{-\infty}^{\infty} \frac{1}{2\sqrt{\pi\sigma}} \exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big)\tilde{\delta}(3t_{ev}+3t_m)dq = \\
&-\frac{1}{2}\Big(\exp(-9(\frac{1}{\hbar}gt_m)^2\sigma)\Big(\Big((64\cos^5(\frac{1}{\hbar}gt_mq_0) + \cos^3(\frac{1}{\hbar}gt_mq_0)(64\exp(5(\frac{1}{\hbar}gt_m)^2\sigma) - 64) \\
&+ (20\exp(8(\frac{1}{\hbar}gt_m)^2\sigma) - 32\exp(5(\frac{1}{\hbar}gt_m)^2\sigma) + 12)\cos(\frac{1}{\hbar}gt_mq_0))\sin(\frac{1}{\hbar}gt_mq_0)\cos^5(\frac{1}{\hbar}Bt_{ev}) \\
&+ ((24\exp(5(\frac{1}{\hbar}gt_m)^2\sigma) - 24\exp(8(\frac{1}{\hbar}gt_m)^2\sigma))\cos(\frac{1}{\hbar}gt_mq_0) \\
&- 48\exp(5(\frac{1}{\hbar}gt_m)^2\sigma)\cos^3(\frac{1}{\hbar}gt_mq_0))\sin(\frac{1}{\hbar}gt_mq_0)\cos^3(\frac{1}{\hbar}Bt_{ev}) \\
&+ 8\exp(8(\frac{1}{\hbar}gt_m)^2\sigma)\cos(\frac{1}{\hbar}gt_mq_0)\sin(\frac{1}{\hbar}gt_mq_0)\cos(\frac{1}{\hbar}Bt_{ev})\Big)\sin(\frac{1}{\hbar}Bt_{ev}) - \exp(9(\frac{1}{\hbar}gt_m)^2\sigma))\Big)
\end{aligned}
\tag{5.1.16}
$$

So the result of equation 5.1.3 is $\rho_{ii,ij,ji,jj}(3t_{ev}+3t_m) = \begin{pmatrix} \rho_{ii}(3t_{ev}+3t_m) & \rho_{ij}(3t_{ev}+3t_m) \\ \rho_{ji}(3t_{ev}+3t_m) & \rho_{jj}(3t_{ev}+3t_m) \end{pmatrix}$.

The probability that the qubit is in state $|0\rangle$ after $3t_{ev}+3t_m$ is, using equation 2.4.4, given by

$$
\begin{aligned}
P_0(3t_m+3t_{ev}) =&(16\exp(-4(\frac{1}{\hbar}gt_m)^2\sigma)\cos^4(\frac{1}{\hbar}gt_mq_0)+(16\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)-16\exp(-4(\frac{1}{\hbar}gt_m)^2\sigma))\cos^2(\frac{1}{\hbar}gt_mq_0)\\
&+6-8\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)+2\exp(-4(\frac{1}{\hbar}gt_m)^2\sigma))\cos^6(\frac{1}{\hbar}Bt_{ev})\\
&+(-16\exp(-4(\frac{1}{\hbar}gt_m)^2\sigma)\cos^4(\frac{1}{\hbar}gt_mq_0)+(-24\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)\\
&+16\exp(-4(\frac{1}{\hbar}gt_m)^2\sigma))\cos^2(\frac{1}{\hbar}gt_mq_0)\\
&-10+12\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)-2\exp(-4(\frac{1}{\hbar}gt_m)^2\sigma))\cos^4(\frac{1}{\hbar}Bt_{ev})\\
&+(8\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)\cos^2(\frac{1}{\hbar}gt_mq_0)+5-4\exp(-(\frac{1}{\hbar}gt_m)^2\sigma))\cos^2(\frac{1}{\hbar}Bt_{ev})
\end{aligned}
\tag{5.1.17}
$$

and the probability that the qubit is in state $|1\rangle$ is, using equation 2.4.5, given by

$$
\begin{aligned}
P_1(3t_m+3t_{ev}) =&1-(16\exp(-4(\frac{1}{\hbar}gt_m)^2\sigma)\cos^4(\frac{1}{\hbar}gt_mq_0)\\
&+(16\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)-16\exp(-4(\frac{1}{\hbar}gt_m)^2\sigma))\cos^2(\frac{1}{\hbar}gt_mq_0)\\
&+6-8\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)+2\exp(-4(\frac{1}{\hbar}gt_m)^2\sigma))\cos^6(\frac{1}{\hbar}Bt_{ev})\\
&-(-16\exp(-4(\frac{1}{\hbar}gt_m)^2\sigma)\cos^4(\frac{1}{\hbar}gt_mq_0)+(-24\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)\\
&+16\exp(-4(\frac{1}{\hbar}gt_m)^2\sigma))\cos^2(\frac{1}{\hbar}gt_mq_0)\\
&-10+12\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)-2\exp(-4(\frac{1}{\hbar}gt_m)^2\sigma))\cos^4(\frac{1}{\hbar}Bt_{ev})\\
&-(8\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)\cos^2(\frac{1}{\hbar}gt_mq_0)+5-4\exp(-(\frac{1}{\hbar}gt_m)^2\sigma))\cos^2(\frac{1}{\hbar}Bt_{ev}).
\end{aligned}
\tag{5.1.18}
$$

5.1.3 Series of measurements during free evolution alternated with only free evolution of the spin; $\sigma \to 0$

The calculations of the algebraic expressions of the probabilities at $t = 2t_{ev} + 2t_m$ are given below.

The state of the qubit after $t = 2t_{ev} + 2t_m$ is given by applying two times matrix C from equation 3.5.13 on the initial state. This gives

$$
|\Phi(2t_{ev}+2t_m)\rangle = C^2|\Phi(0)\rangle = \frac{1}{\sqrt{2\sqrt{\pi\sigma}}}\exp\Big(-\frac{(q-q_0)^2}{2\sigma}\Big)\begin{pmatrix}\Phi_i\\ \Phi_j\end{pmatrix} \tag{5.1.19}
$$

with $\Phi_i = \exp(-\frac{2i}{\hbar}Bt_{ev})\cos^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})+\sin(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\cos(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})(-i\frac{gq}{\sqrt{B^2+g^2q^2}}$
$-i\frac{gq}{\sqrt{B^2+g^2q^2}}\exp(\frac{2i}{\hbar}Bt_{ev})-2i\frac{gq}{\sqrt{B^2+g^2q^2}}\exp(-\frac{2i}{\hbar}Bt_{ev}))$
$+\sin^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})(-\frac{Bgq}{B^2+g^2q^2}-\frac{g^2q^2}{B^2+g^2q^2}+\frac{Bgq}{B^2+g^2q^2}\exp(\frac{2i}{\hbar}Bt_{ev})-\frac{B^2}{B^2+g^2q^2}\exp(-\frac{2i}{\hbar}Bt_{ev}))$
and $\Phi_j = \exp(\frac{2i}{\hbar}Bt_{ev})\cos^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})+\sin(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\cos(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})(-i\frac{gq}{\sqrt{B^2+g^2q^2}}$
$-i\frac{gq}{\sqrt{B^2+g^2q^2}}\exp(-\frac{2i}{\hbar}Bt_{ev})+2i\frac{gq}{\sqrt{B^2+g^2q^2}}\exp(\frac{2i}{\hbar}Bt_{ev}))$
$+\sin^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})(\frac{Bgq}{B^2+g^2q^2}-\frac{g^2q^2}{B^2+g^2q^2}-\frac{Bgq}{B^2+g^2q^2}\exp(\frac{2i}{\hbar}Bt_{ev})-\frac{B^2}{B^2+g^2q^2}\exp(-\frac{2i}{\hbar}Bt_{ev}))$.

The density matrix is given by

$$\rho(2t_{ev}+2t_m) = |\Phi(2t_{ev}+2t_m)\rangle\langle\Phi(2t_{ev}+2t_m)| = \frac{1}{2\sqrt{\pi\sigma}} \exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big) \begin{pmatrix} \tilde{\alpha}(2t_{ev}+2t_m) & \tilde{\beta}(2t_{ev}+2t_m) \\ \tilde{\gamma}(2t_{ev}+2t_m) & \tilde{\delta}(2t_{ev}+2t_m) \end{pmatrix} \tag{5.1.20}$$

with $\tilde{\alpha}(2t_{ev}+2t_m) = \cos^4(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2}) + \sin^4(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\big(\frac{B^4}{(B^2+g^2q^2)^2}$
$+ \frac{g^4q^4}{(B^2+g^2q^2)^2} + 2\frac{B^2g^2q^2}{(B^2+g^2q^2)^2} + 2\frac{Bg^3q^3}{(B^2+g^2q^2)^2} - \frac{Bg^3q^3}{(B^2+g^2q^2)^2}\exp(-\frac{4i}{\hbar}Bt_{ev}) - \frac{Bg^3q^3}{(B^2+g^2q^2)^2}\exp(\frac{4i}{\hbar}Bt_{ev})\big)$
$+ \sin^3(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\cos(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\big(\frac{B^2gq}{(B^2+g^2q^2)\sqrt{B^2+g^2q^2}}(i\exp(\frac{2i}{\hbar}Bt_{ev}) - i\exp(-\frac{2i}{\hbar}Bt_{ev})$
$- 3i\exp(-\frac{4i}{\hbar}Bt_{ev}) + 3i\exp(\frac{4i}{\hbar}Bt_{ev})) + \frac{g^3q^3}{(B^2+g^2q^2)\sqrt{B^2+g^2q^2}}(i\exp(\frac{2i}{\hbar}Bt_{ev}) - i\exp(-\frac{2i}{\hbar}Bt_{ev}))\big)$
$+ \sin^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\cos^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\big(2\frac{B^2}{B^2+g^2q^2} + 2\frac{g^2q^2}{B^2+g^2q^2}$
$+ \frac{Bgq}{B^2+g^2q^2}(\exp(\frac{2i}{\hbar}Bt_{ev}) + \exp(-\frac{2i}{\hbar}Bt_{ev}) + 3\exp(\frac{4i}{\hbar}Bt_{ev}) + 3\exp(-\frac{4i}{\hbar}Bt_{ev})\big)$
$+\sin(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\cos^3(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})(-i\frac{gq}{\sqrt{B^2+g^2q^2}}\exp(\frac{2i}{\hbar}Bt_{ev}) + i\frac{gq}{\sqrt{B^2+g^2q^2}}\exp(-\frac{2i}{\hbar}Bt_{ev})$
$- i\frac{gq}{\sqrt{B^2+g^2q^2}}\exp(\frac{4i}{\hbar}Bt_{ev}) + i\frac{gq}{\sqrt{B^2+g^2q^2}}\exp(\frac{-4i}{\hbar}Bt_{ev}))$,
$\tilde{\beta}(2t_{ev}+2t_m) = \cos^4(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\exp(-\frac{4i}{\hbar}Bt_{ev})$
$+ \sin^3(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\cos(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\big(4i\frac{B^3}{(B^2+g^2q^2)\sqrt{B^2+g^2q^2}}\exp(\frac{-4i}{\hbar}Bt_{ev})$
$+ \frac{Bg^2q^2}{(B^2+g^2q^2)\sqrt{B^2+g^2q^2}}(-2i + 4i\exp(\frac{-2i}{\hbar}Bt_{ev}) + 2i\exp(\frac{4i}{\hbar}Bt_{ev}))\big)$
$+ \sin^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\cos^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\big(-6\frac{B^2}{B^2+g^2q^2}\exp(\frac{-4i}{\hbar}Bt_{ev})$
$+ \frac{g^2q^2}{B^2+g^2q^2}(1 + \exp(\frac{4i}{\hbar}Bt_{ev}) - 2\exp(\frac{-2i}{\hbar}Bt_{ev}) + 2\exp(\frac{2i}{\hbar}Bt_{ev}))\big)$
$- 4i\frac{B}{\sqrt{B^2+g^2q^2}}\exp(-\frac{4i}{\hbar}Bt_{ev})\sin(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\cos^3(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})$,

$\tilde{\gamma}(2t_{ev}+2t_m) = \cos^4(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\exp(\frac{4i}{\hbar}Bt_{ev})$
$+ \sin^3(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\cos(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\big(-4i\frac{B^3}{(B^2+g^2q^2)\sqrt{B^2+g^2q^2}}\exp(\frac{4i}{\hbar}Bt_{ev})$
$+ \frac{Bg^2q^2}{(B^2+g^2q^2)\sqrt{B^2+g^2q^2}}(2i - 4i\exp(\frac{2i}{\hbar}Bt_{ev}) - 2i\exp(-\frac{4i}{\hbar}Bt_{ev}))\big)$
$+ \sin^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\cos^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\big(-6\frac{B^2}{B^2+g^2q^2}\exp(\frac{4i}{\hbar}Bt_{ev})$
$+ \frac{g^2q^2}{B^2+g^2q^2}(1 + \exp(-\frac{4i}{\hbar}Bt_{ev}) - 2\exp(\frac{2i}{\hbar}Bt_{ev}) + 2\exp(-\frac{2i}{\hbar}Bt_{ev}))\big)$
$+ 4i\frac{B}{\sqrt{B^2+g^2q^2}}\exp(\frac{4i}{\hbar}Bt_{ev})\sin(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\cos^3(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})$

and

$\tilde{\delta}(2t_{ev}+2t_m) = \cos^4(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2}) + \sin^4(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\big(\frac{B^4}{(B^2+g^2q^2)^2}$
$+ \frac{g^4q^4}{(B^2+g^2q^2)^2} + 2\frac{B^2g^2q^2}{(B^2+g^2q^2)^2} + 2\frac{Bg^3q^3}{(B^2+g^2q^2)^2} + \frac{Bg^3q^3}{(B^2+g^2q^2)^2}\exp(-\frac{4i}{\hbar}Bt_{ev}) + \frac{Bg^3q^3}{(B^2+g^2q^2)^2}\exp(\frac{4i}{\hbar}Bt_{ev})\big)$
$+\sin^3(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\cos(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\big(\frac{B^2gq}{(B^2+g^2q^2)\sqrt{B^2+g^2q^2}}(-i\exp(\frac{2i}{\hbar}Bt_{ev}) + i\exp(-\frac{2i}{\hbar}Bt_{ev})$
$3i\exp(-\frac{4i}{\hbar}Bt_{ev}) - 3i\exp(\frac{4i}{\hbar}Bt_{ev})) + \frac{g^3q^3}{(B^2+g^2q^2)\sqrt{B^2+g^2q^2}}(i\exp(-\frac{2i}{\hbar}Bt_{ev}) - i\exp(\frac{2i}{\hbar}Bt_{ev}))\big)$
$+ \sin^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\cos^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\big(2\frac{B^2}{B^2+g^2q^2} + 2\frac{g^2q^2}{B^2+g^2q^2}$
$+ \frac{Bgq}{B^2+g^2q^2}(-\exp(\frac{2i}{\hbar}Bt_{ev}) - \exp(-\frac{2i}{\hbar}Bt_{ev}) - 3\exp(\frac{4i}{\hbar}Bt_{ev}) - 3\exp(-\frac{4i}{\hbar}Bt_{ev})\big)$
$+ \sin(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})\cos^3(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q^2})(i\frac{gq}{\sqrt{B^2+g^2q^2}}\exp(\frac{2i}{\hbar}Bt_{ev}) - i\frac{gq}{\sqrt{B^2+g^2q^2}}\exp(-\frac{2i}{\hbar}Bt_{ev})$
$+ i\frac{gq}{\sqrt{B^2+g^2q^2}}\exp(\frac{4i}{\hbar}Bt_{ev}) - i\frac{gq}{\sqrt{B^2+g^2q^2}}\exp(\frac{-4i}{\hbar}Bt_{ev}))$.

Again we trace out the detector coordinate q in the limit of $\sigma \to 0$ in order to get the density matrix only dependent on the qubit coordinates $i = |0\rangle$ and $j = |1\rangle$. Equation 3.4.17 gives

$$\rho_{ii,ij,ji,jj}(2t_{ev}+2t_m, q_0) \approx \frac{1}{2}\begin{pmatrix} \tilde{\alpha}(2t_{ev}+2t_m, q_0) & \tilde{\beta}(2t_{ev}+2t_m, q_0) \\ \tilde{\gamma}(2t_{ev}+2t_m, q_0) & \tilde{\delta}(2t_{ev}+2t_m, q_0) \end{pmatrix} \tag{5.1.21}$$

with $\tilde{\alpha}(2t_{ev}+2t_m,q_0) = \cos^4(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2}) + \sin^4(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\big(\frac{B^4}{(B^2+g^2q_0^2)^2}$
$+ \frac{g^4q_0^4}{(B^2+g^2q_0^2)^2} + 2\frac{B^2g^2q_0^2}{(B^2+g^2q_0^2)^2} + 2\frac{Bg^3q_0^3}{(B^2+g^2q_0^2)^2} - \frac{Bg^3q_0^3}{(B^2+g^2q_0^2)^2}\exp(-\frac{4i}{\hbar}Bt_{ev}) - \frac{Bg^3q_0^3}{(B^2+g^2q_0^2)^2}\exp(\frac{4i}{\hbar}Bt_{ev})\big)$
$+ \sin^3(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\cos(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\big(\frac{B^2gq_0}{(B^2+g^2q_0^2)\sqrt{B^2+g^2q_0^2}}(i\exp(\frac{2i}{\hbar}Bt_{ev}) - i\exp(-\frac{2i}{\hbar}Bt_{ev})$
$- 3i\exp(-\frac{4i}{\hbar}Bt_{ev}) + 3i\exp(\frac{4i}{\hbar}Bt_{ev})) + \frac{g^3q_0^3}{(B^2+g^2q_0^2)\sqrt{B^2+g^2q_0^2}}(i\exp(\frac{2i}{\hbar}Bt_{ev}) - i\exp(-\frac{2i}{\hbar}Bt_{ev}))\big)$
$+ \sin^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\cos^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\big(2\frac{B^2}{B^2+g^2q_0^2} + 2\frac{g^2q_0^2}{B^2+g^2q_0^2}$
$+ \frac{Bgq_0}{B^2+g^2q_0^2}(\exp(\frac{2i}{\hbar}Bt_{ev}) + \exp(-\frac{2i}{\hbar}Bt_{ev}) + 3\exp(\frac{4i}{\hbar}Bt_{ev}) + 3\exp(-\frac{4i}{\hbar}Bt_{ev})\big)$
$+\sin(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\cos^3(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})(-i\frac{gq}{\sqrt{B^2+g^2q_0^2}}\exp(\frac{2i}{\hbar}Bt_{ev}) + i\frac{gq_0}{\sqrt{B^2+g^2q_0^2}}\exp(-\frac{2i}{\hbar}Bt_{ev})$
$- i\frac{gq}{\sqrt{B^2+g^2q_0^2}}\exp(\frac{4i}{\hbar}Bt_{ev}) + i\frac{gq}{\sqrt{B^2+g^2q_0^2}}\exp(\frac{-4i}{\hbar}Bt_{ev})),$
$\tilde{\beta}(2t_{ev}+2t_m,q_0) = \cos^4(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\exp(-\frac{4i}{\hbar}Bt_{ev})$
$+ \sin^3(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\cos(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\big(4i\frac{B^3}{(B^2+g^2q_0^2)\sqrt{B^2+g^2q_0^2}}\exp(\frac{-4i}{\hbar}Bt_{ev})$
$+ \frac{Bg^2q^2}{(B^2+g^2q_0^2)\sqrt{B^2+g^2q_0^2}}(-2i + 4i\exp(\frac{-2i}{\hbar}Bt_{ev}) + 2i\exp(\frac{4i}{\hbar}Bt_{ev}))\big)$
$+ \sin^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\cos^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\big(-6\frac{B^2}{B^2+g^2q_0^2}\exp(\frac{-4i}{\hbar}Bt_{ev})$
$+ \frac{g^2q_0^2}{B^2+g^2q^2}(1 + \exp(\frac{4i}{\hbar}Bt_{ev}) - 2\exp(\frac{-2i}{\hbar}Bt_{ev}) + 2\exp(\frac{2i}{\hbar}Bt_{ev}))\big)$
$- 4i\frac{B}{\sqrt{B^2+g^2q_0^2}}\exp(-\frac{4i}{\hbar}Bt_{ev})\sin(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\cos^3(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2}),$

$\tilde{\gamma}(2t_{ev}+2t_mq_0) = \cos^4(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\exp(\frac{4i}{\hbar}Bt_{ev})$
$+ \sin^3(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\cos(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\big(-4i\frac{B^3}{(B^2+g^2q_0^2)\sqrt{B^2+g^2q_0^2}}\exp(\frac{4i}{\hbar}Bt_{ev})$
$+ \frac{Bg^2q_0^2}{(B^2+g^2q_0^2)\sqrt{B^2+g^2q_0^2}}(2i - 4i\exp(\frac{2i}{\hbar}Bt_{ev}) - 2i\exp(-\frac{4i}{\hbar}Bt_{ev}))\big)$
$+ \sin^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\cos^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\big(-6\frac{B^2}{B^2+g^2q_0^2}\exp(\frac{4i}{\hbar}Bt_{ev})$
$+ \frac{g^2q_0^2}{B^2+g^2q_0^2}(1 + \exp(-\frac{4i}{\hbar}Bt_{ev}) - 2\exp(\frac{2i}{\hbar}Bt_{ev}) + 2\exp(-\frac{2i}{\hbar}Bt_{ev}))\big)$
$+ 4i\frac{B}{\sqrt{B^2+g^2q_0^2}}\exp(\frac{4i}{\hbar}Bt_{ev})\sin(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\cos^3(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})$

and

$\tilde{\delta}(2t_{ev}+2t_m,q_0) = \cos^4(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2}) + \sin^4(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\big(\frac{B^4}{(B^2+g^2q_0^2)^2}$
$+ \frac{g^4q^4}{(B^2+g^2q_0^2)^2} + 2\frac{B^2g^2q_0^2}{(B^2+g^2q_0^2)^2} + 2\frac{Bg^3q_0^3}{(B^2+g^2q_0^2)^2} + \frac{Bg^3q_0^3}{(B^2+g^2q_0^2)^2}\exp(-\frac{4i}{\hbar}Bt_{ev}) + \frac{Bg^3q_0^3}{(B^2+g^2q_0^2)^2}\exp(\frac{4i}{\hbar}Bt_{ev})\big)$
$+\sin^3(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\cos(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\big(\frac{B^2gq_0}{(B^2+g^2q_0^2)\sqrt{B^2+g^2q_0^2}}(-i\exp(\frac{2i}{\hbar}Bt_{ev}) + i\exp(-\frac{2i}{\hbar}Bt_{ev})$
$3i\exp(-\frac{4i}{\hbar}Bt_{ev}) - 3i\exp(\frac{4i}{\hbar}Bt_{ev})) + \frac{g^3q_0^3}{(B^2+g^2q_0^2)\sqrt{B^2+g^2q_0^2}}(i\exp(-\frac{2i}{\hbar}Bt_{ev}) - i\exp(\frac{2i}{\hbar}Bt_{ev}))\big)$
$+ \sin^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\cos^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\big(2\frac{B^2}{B^2+g^2q_0^2} + 2\frac{g^2q_0^2}{B^2+g^2q_0^2}$
$+ \frac{Bgq_0}{B^2+g^2q_0^2}(-\exp(\frac{2i}{\hbar}Bt_{ev}) - \exp(-\frac{2i}{\hbar}Bt_{ev}) - 3\exp(\frac{4i}{\hbar}Bt_{ev}) - 3\exp(-\frac{4i}{\hbar}Bt_{ev})\big)$
$+ \sin(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\cos^3(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})(i\frac{gq_0}{\sqrt{B^2+g^2q_0^2}}\exp(\frac{2i}{\hbar}Bt_{ev}) - i\frac{gq_0}{\sqrt{B^2+g^2q_0^2}}\exp(-\frac{2i}{\hbar}Bt_{ev})$
$+ i\frac{gq_0}{\sqrt{B^2+g^2q_0^2}}\exp(\frac{4i}{\hbar}Bt_{ev}) - i\frac{gq_0}{\sqrt{B^2+g^2q_0^2}}\exp(\frac{-4i}{\hbar}Bt_{ev})).$

The probability that the qubit is in the state $|0\rangle$ after $2t_{ev}+2t_m$ is, using equation 2.4.4, given by

$$
\begin{aligned}
P_0(2t_m + 2t_{ev}, q_0) =& \frac{1}{2} + \frac{1}{2}\cos^4(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\cos(\frac{4}{\hbar}Bt_{ev}) \\
&+ \sin^3(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\cos(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\big(2\frac{B^3}{(B^2+g^2q_0^2)\sqrt{B^2+g^2q_0^2}}\sin(\frac{4}{\hbar}Bt_{ev}) \\
&+ 2\frac{Bg^2q_0^2}{(B^2+g^2q_0^2)\sqrt{B^2+g^2q_0^2}}\sin(\frac{2}{\hbar}Bt_{ev}) - \frac{Bg^2q_0^2}{(B^2+g^2q_0^2)\sqrt{B^2+g^2q_0^2}}\sin(\frac{4}{\hbar}Bt_{ev})\big) \\
&+ \sin^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\cos^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\big(-3\frac{B^2}{B^2+g^2q_0^2}\cos(\frac{4}{\hbar}Bt_{ev}) \\
&+ \frac{1}{2}\frac{g^2q_0^2}{B^2+g^2q^2}\cos(\frac{4}{\hbar}Bt_{ev}) + \frac{1}{2}\frac{g^2q_0^2}{B^2+g^2q_0^2}\big) \\
&- 2\frac{B}{\sqrt{B^2+g^2q_0^2}}\sin(\frac{4}{\hbar}Bt_{ev})\sin(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\cos^3(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})
\end{aligned} \tag{5.1.22}
$$

and the probability that the qubit is, using equation 2.4.5, in its other state $|1\rangle$ is given by

$$
\begin{aligned}
P_1(2t_m + 2t_{ev}, q_0) =& \frac{1}{2} - \frac{1}{2}\cos^4(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\cos(\frac{4}{\hbar}Bt_{ev}) \\
&- \sin^3(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\cos(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\big(2\frac{B^3}{(B^2+g^2q_0^2)\sqrt{B^2+g^2q_0^2}}\sin(\frac{4}{\hbar}Bt_{ev}) \\
&+ 2\frac{Bg^2q_0^2}{(B^2+g^2q_0^2)\sqrt{B^2+g^2q_0^2}}\sin(\frac{2}{\hbar}Bt_{ev}) - \frac{Bg^2q_0^2}{(B^2+g^2q_0^2)\sqrt{B^2+g^2q_0^2}}\sin(\frac{4}{\hbar}Bt_{ev})\big) \\
&- \sin^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\cos^2(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\big(-3\frac{B^2}{B^2+g^2q_0^2}\cos(\frac{4}{\hbar}Bt_{ev}) \\
&+ \frac{1}{2}\frac{g^2q_0^2}{B^2+g^2q_0^2}\cos(\frac{4}{\hbar}Bt_{ev}) + \frac{1}{2}\frac{g^2q_0^2}{B^2+g^2q_0^2}\big) \\
&+ 2\frac{B}{\sqrt{B^2+g^2q_0^2}}\sin(\frac{4}{\hbar}Bt_{ev})\sin(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2})\cos^3(\frac{1}{\hbar}t_m\sqrt{B^2+g^2q_0^2}).
\end{aligned} \tag{5.1.23}
$$

5.2 Partial trace integral calculations

The integrals for the partial trace when $\sigma \not\to 0$ are all the same sort of integrals. The same steps are used for each integral. The integrals in equations 3.4.23 and 3.4.24 are worked out in detail.

Equation 3.4.23

$$
\begin{aligned}
\rho_{ii}(t_{ev}+t_m) &= \int_{-\infty}^{\infty} \frac{1}{2\sqrt{\pi\sigma}} \exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big)\tilde{\alpha}(t_{ev}+t_m)dq \\
&= \int_{-\infty}^{\infty} \frac{1}{2\sqrt{\pi\sigma}} \exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big)\Big(\cos^2\Big(\frac{1}{\hbar}gqt_m - \frac{1}{\hbar}Bt_{ev}\Big) + \sin^2\Big(\frac{1}{\hbar}gqt_m + \frac{1}{\hbar}Bt_{ev}\Big)\Big)dq
\end{aligned} \tag{5.2.1}
$$

Let $x = q$, $a = q_0$, $b = \sigma$, $c = \frac{1}{\hbar}gt_m$ and $d = \frac{1}{\hbar}Bt_{ev}$.

This gives

$$
\begin{aligned}
\rho_{ii}(t_{ev}+t_m) &= \int_{-\infty}^{\infty} \frac{1}{2\sqrt{\pi b}} \exp\Big(-\frac{(x-a)^2}{b}\Big)\cos^2\Big((cx-d)+\sin^2(cx+d)\Big)dx \\
&= \int_{-\infty}^{\infty} \frac{1}{2\sqrt{\pi b}} \exp\Big(-\frac{(x-a)^2}{b}\Big)\cos^2\Big(cx-d\Big)dx + \int_{-\infty}^{\infty} \frac{1}{2\sqrt{\pi b}} \exp\Big(-\frac{(x-a)^2}{b}\Big)\sin^2\Big(cx+d\Big)dx
\end{aligned} \tag{5.2.2}
$$

First we will solve

$$
\begin{aligned}
&\int_{-\infty}^{\infty} \exp\Big(-\frac{(x-a)^2}{b}\Big)\sin^2(cx+d)dx \\
&= \int_{-\infty}^{\infty} -\frac{(\exp(i(cx+d))-\exp(-i(cx+d))^2\exp\Big(-\frac{(x-a)^2}{b}\Big)}{4}dx \\
&= \int_{-\infty}^{\infty} -\frac{\exp\Big(-\frac{(x-a)^2}{b}+2(2icx+2id)-2icx-2id\Big)}{4} - \frac{\exp\Big(-\frac{(x-a)^2}{b}-2icx-2id\Big)}{4} + \frac{\exp\Big(-\frac{(x-a)^2}{b}\Big)}{2}dx \\
&= -\frac{\exp(2id)}{4}\int_{-\infty}^{\infty}\exp\Big(2icx-\frac{(x-a)^2}{b}\Big)dx \\
&\quad -\frac{\exp(-2id)}{4}\int_{-\infty}^{\infty}\exp\Big(-2icx-\frac{(x-a)^2}{b}\Big)dx + \frac{1}{2}\int_{-\infty}^{\infty}\exp\Big(-\frac{(x-a)^2}{b}\Big)dx
\end{aligned} \tag{5.2.3}
$$

We are going to solve this in parts. Set $u = \frac{bx+b(-ibc-a)}{b^{\frac{3}{2}}}$ so $dx = \sqrt{b}du$.

$$
\begin{aligned}
&\int_{-\infty}^{\infty} \exp\Big(2icx-\frac{(x-a)^2}{b}\Big)dx = \int_{-\infty}^{\infty}\exp\left(-\Big(\frac{x}{\sqrt{b}}-\frac{\sqrt{b}(2ic+\frac{2a}{b})}{2}\Big)^2+\frac{b(2ic+\frac{2a}{b})^2}{4}-\frac{a^2}{b}\right)dx \\
&= \int_{-\infty}^{\infty}\sqrt{b}\exp\left(-u^2+b(ic+\frac{a}{b})^2-\frac{a^2}{b}\right)dx = \frac{\sqrt{\pi}\sqrt{b}\exp\left(b(ic+\frac{a}{b})^2-\frac{a^2}{b}\right)}{2}\int_{-\infty}^{\infty}\frac{2\exp(-u^2)}{\sqrt{\pi}}du \\
&= \left[\frac{\sqrt{\pi}\sqrt{b}\exp\left(b(ic+\frac{a}{b})^2-\frac{a^2}{b}\right)}{2}\operatorname{erf}(u)\right]_{-\infty}^{\infty} = \sqrt{\pi}\sqrt{b}\exp\left(b(ic+\frac{a}{b})^2-\frac{a^2}{b}\right)
\end{aligned} \tag{5.2.4}
$$

Now set $u = \frac{bx+b(ibc-a)}{b^{\frac{3}{2}}}$ so $dx = \sqrt{b}du$.

$$
\begin{aligned}
&\int_{-\infty}^{\infty} \exp\Big(-2icx-\frac{(x-a)^2}{b}\Big)dx = \int_{-\infty}^{\infty}\exp\left(-\Big(\frac{x}{\sqrt{b}}-\frac{\sqrt{b}(-2ic+\frac{2a}{b})}{2}\Big)^2+\frac{b(-2ic+\frac{2a}{b})^2}{4}-\frac{a^2}{b}\right)dx \\
&= \int_{-\infty}^{\infty}\sqrt{b}\exp\left(-u^2+b(ic-\frac{a}{b})^2-\frac{a^2}{b}\right)dx = \frac{\sqrt{\pi}\sqrt{b}\exp\left(b(ic-\frac{a}{b})^2-\frac{a^2}{b}\right)}{2}\int_{-\infty}^{\infty}\frac{2\exp(-u^2)}{\sqrt{\pi}}du \\
&= \left[\frac{\sqrt{\pi}\sqrt{b}\exp\left(b(ic-\frac{a}{b})^2-\frac{a^2}{b}\right)}{2}\operatorname{erf}(u)\right]_{-\infty}^{\infty} = \sqrt{\pi}\sqrt{b}\exp\left(b(ic-\frac{a}{b})^2-\frac{a^2}{b}\right)
\end{aligned} \tag{5.2.5}
$$

Now set $u = \frac{x-a}{\sqrt{b}}$ so $dx = \sqrt{b}du$.

$$\int_{-\infty}^{\infty} \exp\Big(-\frac{(x-a)^2}{b}\Big)dx = \frac{\sqrt{\pi b}}{2}\int_{-\infty}^{\infty}\frac{2\exp(-u^2)}{\sqrt{\pi}}du = \sqrt{\pi b} \tag{5.2.6}$$

Now we are going to solve the other part.

$$\begin{aligned}
&\int_{-\infty}^{\infty} \exp\Big(-\frac{(x-a)^2}{b}\Big)\cos^2(cx-d)dx\\
=&\int_{-\infty}^{\infty} \frac{(\exp(i(cx-d)) + \exp(-i(cx-d))^2\exp\Big(-\frac{(x-a)^2}{b}\Big)}{4}dx\\
=&\int_{-\infty}^{\infty} -\frac{\exp\Big(-\frac{(x-a)^2}{b}+2icx-2id\Big)}{4} - \frac{\exp\Big(-\frac{(x-a)^2}{b}-2icx+2id\Big)}{4} + \frac{\exp\Big(-\frac{(x-a)^2}{b}\Big)}{2}dx\\
=&\frac{\exp(-2id)}{4}\int_{-\infty}^{\infty}\exp\Big(2icx-\frac{(x-a)^2}{b}\Big)dx\\
&+\frac{\exp(2id)}{4}\int_{-\infty}^{\infty}\exp\Big(-2icx-\frac{(x-a)^2}{b}\Big)dx + \frac{1}{2}\int_{-\infty}^{\infty}\exp\Big(-\frac{(x-a)^2}{b}\Big)dx
\end{aligned} \tag{5.2.7}$$

These integrals are solved in equations 5.2.4, 5.2.5 and 5.2.6. Taking everything together gives

$$\begin{aligned}
\rho_{ii}(t_{ev}+t_m) =&\frac{1}{2} + 2\exp(-bc^2)\cos(ac)\sin(ac)\cos(d)\sin(d)\\
=&\frac{1}{2} + 2\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)\cos(\frac{1}{\hbar}gt_mq_0)\sin(\frac{1}{\hbar}gt_mq_0)\cos(\frac{1}{\hbar}Bt_{ev})\sin(\frac{1}{\hbar}Bt_{ev})
\end{aligned}. \tag{5.2.8}$$

Equation 3.4.23

$$\begin{aligned}
\rho_{ij}(t_{ev}+t_m) =& \int_{-\infty}^{\infty}\frac{1}{2\sqrt{\pi\sigma}}\exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big)\tilde{\beta}(t_{ev}+t_m)dq\\
=&\int_{-\infty}^{\infty}\frac{1}{2\sqrt{\pi\sigma}}\exp\Big(-\frac{(q-q_0)^2}{\sigma}\Big)\Bigg(\cos\Big(\frac{1}{\hbar}gqt_m+\frac{1}{\hbar}Bt_{ev}\Big)\cos\Big(\frac{1}{\hbar}gqt_m-\frac{1}{\hbar}Bt_{ev}\Big)\\
&+\sin\Big(\frac{1}{\hbar}gqt_m+\frac{1}{\hbar}Bt_{ev}\Big)\sin\Big(\frac{1}{\hbar}gqt_m-\frac{1}{\hbar}Bt_{ev}\Big) + i\Big(\cos\Big(\frac{1}{\hbar}gqt_m-\frac{1}{\hbar}Bt_{ev}\Big)\sin\Big(\frac{1}{\hbar}gqt_m-\frac{1}{\hbar}Bt_{ev}\Big)\\
&-\cos\Big(\frac{1}{\hbar}gqt_m+\frac{1}{\hbar}Bt_{ev}\Big)\sin\Big(\frac{1}{\hbar}gqt_m+\frac{1}{\hbar}Bt_{ev}\Big)\Big)\Bigg)dq
\end{aligned} \tag{5.2.9}$$

Let $x = q$, $a = q_0$, $b = \sigma$, $c = \frac{1}{\hbar}gt_m$ and $d = \frac{1}{\hbar}Bt_{ev}$.
This gives

$$\begin{aligned}
\rho_{ij}(t_{ev}+t_m) = \int_{-\infty}^{\infty}\frac{1}{2\sqrt{\pi b}}\exp\Big(-\frac{(x-a)^2}{b}\Big)\Bigg(&\cos(cx+d)\cos(cx-d)+\sin(cx+d)\sin(cx-d)\\
&+i\Big(\cos(cx-d)\sin(cx-d)-\cos(cx+d)\sin(cx+d)\Big)\Bigg)dx
\end{aligned} \tag{5.2.10}$$

We will solve it in parts. First we will use partial integration:

$$
\begin{aligned}
&\int_{-\infty}^{\infty}\exp\Big(-\frac{(x-a)^2}{b}\Big)\sin(cx+d)\sin(cx-d)dx\\
=&\Bigg[\frac{\sqrt{\pi b}\mathrm{erf}(\frac{x-a}{\sqrt{b}})\sin(cx-d)\sin(cx+d)}{2}\Bigg]_{-\infty}^{\infty}\\
&-\int_{-\infty}^{\infty}\mathrm{erf}(\frac{x-a}{\sqrt{b}})\big(\cos(cx-d)\sin(cx+d)+\sin(cx-d)\cos(cx+d)\big)dx
\end{aligned}
\tag{5.2.11}
$$

$$
\begin{aligned}
&\int_{-\infty}^{\infty}\exp\Big(-\frac{(x-a)^2}{b}\Big)\cos(cx+d)\cos(cx-d)dx\\
=&\Bigg[\frac{\sqrt{\pi b}\mathrm{erf}(\frac{x-a}{\sqrt{b}})\cos(cx-d)\cos(cx+d)}{2}\Bigg]_{-\infty}^{\infty}\\
&+\int_{-\infty}^{\infty}\mathrm{erf}(\frac{x-a}{\sqrt{b}})\big(\cos(cx-d)\sin(cx+d)+\sin(cx-d)\cos(cx+d)\big)dx
\end{aligned}
\tag{5.2.12}
$$

So

$$
\begin{aligned}
&\int_{-\infty}^{\infty}\exp\Big(-\frac{(x-a)^2}{b}\Big)\big(\sin(cx+d)\sin(cx-d)+\cos(cx+d)\cos(cx-d)\big)dx\\
=&\Bigg[\frac{\sqrt{\pi b}\mathrm{erf}(\frac{x-a}{\sqrt{b}})\cos(cx-d)\cos(cx+d)}{2}+\frac{\sqrt{\pi b}\mathrm{erf}(\frac{x-a}{\sqrt{b}})\sin(cx-d)\sin(cx+d)}{2}\Bigg]_{-\infty}^{\infty}
\end{aligned}
\tag{5.2.13}
$$

Now we solve another part. First we write everything in exponentials and use linearity.

$$
\begin{aligned}
&\int_{-\infty}^{\infty}\exp\Big(-\frac{(x-a)^2}{b}\Big)\sin(cx+d)\cos(cx+d)dx\\
=&\frac{-i\exp(2id)}{4}\int_{-\infty}^{\infty}\exp\Big(2icx-\frac{(x-a)^2}{b}\Big)dx+\frac{i\exp(-2id)}{4}\int_{-\infty}^{\infty}\exp\Big(-2icx-\frac{(x-a)^2}{b}\Big)dx
\end{aligned}
\tag{5.2.14}
$$

We already solved these equations in 5.2.4 and 5.2.5.

In the same way we solve the following integral

$$
\begin{aligned}
&\int_{-\infty}^{\infty}\exp\Big(-\frac{(x-a)^2}{b}\Big)\sin(cx-d)\cos(cx-d)dx\\
=&\frac{-i\exp(-2id)}{4}\int_{-\infty}^{\infty}\exp\Big(2icx-\frac{(x-a)^2}{b}\Big)dx+\frac{i\exp(2id)}{4}\int_{-\infty}^{\infty}\exp\Big(-2icx-\frac{(x-a)^2}{b}\Big)dx
\end{aligned}
\tag{5.2.15}
$$

Taking everything together gives

$$
\begin{aligned}
\rho_{ij}(t_{ev}+t_m)=&-\frac{\exp(-bc^2)}{4}\Big(i\sin(2(cx+d))-i\sin(2(cx-d))-2\exp(bc^2)\cos(2d)\Big)\\
=&-\frac{\exp(-(\frac{1}{\hbar}gt_m)^2\sigma)}{4}\Big(i\sin(2(\frac{1}{\hbar}Bt_{ev}+\frac{1}{\hbar}gt_mq_0))+i\sin(2(\frac{1}{\hbar}Bt_{ev}-\frac{1}{\hbar}gt_mq_0))\\
&-2\exp((\frac{1}{\hbar}gt_m)^2\sigma)\cos(\frac{2}{\hbar}Bt_{ev})\Big)
\end{aligned}
\tag{5.2.16}
$$

5.3 Interaction strength g(t)

In this report we made the assumption that $g(t)$ was time independent. We also worked out the situation in which this is not the case. First for the situation where series of measurements were alternated with free evolution of the spin (subsection 5.3.1) and consequently for the situation where series of measurements during free evolution were alternated with only free evolution of the spin (subsection 5.3.2).

5.3.1 Series of measurements alternated with free evolution of the spin

Assume $g(t)$ is time dependent. This gives

$$|\Phi(t_{ev}+t_m)\rangle = \exp\Big(-\frac{i}{\hbar}q\sigma_x\int_{t_{ev}}^{t_{ev}+t_m} g(t)dt\Big)|\Phi(t_{ev})\rangle. \tag{5.3.1}$$

Take $\beta(t_{ev},t_m) = -\frac{i}{\hbar}q\int_{t_{ev}}^{t_{ev}+t_m} g(t)dt$.

The Taylor series of the exponential function give

$$\exp(\beta(t_{ev},t_m)\sigma_x) = \sum_{n=0}^{\infty}\frac{(\beta(t_{ev},t_m)\sigma_x)^n}{n!} = \sum_{n=0}^{\infty}\frac{(\beta(t_{ev},t_m)\sigma_x)^{2n}}{(2n)!} + \sum_{n=0}^{\infty}\frac{(\beta(t_{ev},t_m)\sigma_x)^{2n+1}}{(2n+1)!}. \tag{5.3.2}$$

and equivalent to equation 3.3.6, this gives

$$\exp(\beta(t_{ev},t_m)\sigma_x) = I\sum_{n=0}^{\infty}\frac{\beta(t_{ev},t_m)^{2n}}{(2n)!} + \sigma_x\sum_{n=0}^{\infty}\frac{\beta(t_{ev},t_m)^{2n+1}}{(2n+1)!}. \tag{5.3.3}$$

With the Taylor series of the hyperbolic cosine and sine, this exponential results in

$$\exp(\beta(t_{ev},t_m)\sigma_x) = I\cosh(\beta(t_{ev},t_m)) + \sigma_x\sinh(\beta(t_{ev},t_m)) = \begin{pmatrix}\cosh(\beta(t_{ev},t_m)) & \sinh(\beta(t_{ev},t_m))\\ \sinh(\beta(t_{ev},t_m)) & \cosh(\beta(t_{ev},t_m))\end{pmatrix}. \tag{5.3.4}$$

Substituting $\beta(t_{ev},t_m) = -\frac{i}{\hbar}q\int_{t_{ev}}^{t_{ev}+t_m} g(t)dt$ in 5.3.4, we find using equations 3.4.5 and 5.3.1,

$$|\Phi(t_{ev}+t_m)\rangle = \frac{1}{\sqrt{2\sqrt{\pi\sigma}}}\exp\Big(-\frac{(q-q_0)^2}{2\sigma}\Big)\begin{pmatrix}\cos\Big(\frac{1}{\hbar}q\int_{t_{ev}}^{t_{ev}+t_m} g(t)dt - \frac{1}{\hbar}Bt_{ev}\Big) - i\sin\Big(\frac{1}{\hbar}q\int_{t_{ev}}^{t_{ev}+t_m} g(t)dt + \frac{1}{\hbar}Bt_{ev}\Big)\\ \cos\Big(\frac{1}{\hbar}q\int_{t_{ev}}^{t_{ev}+t_m} g(t)dt + \frac{1}{\hbar}Bt_{ev}\Big) - i\sin\Big(\frac{1}{\hbar}q\int_{t_{ev}}^{t_{ev}+t_m} g(t)dt - \frac{1}{\hbar}Bt_{ev}\Big)\end{pmatrix}. \tag{5.3.5}$$

So in fact the evolution of the wave function at discrete moments in time $k\frac{t_{ev}+t_m}{\hbar}$ is now given by applying the following matrix k times on the initial state for each time period consisting of a period of free evolution followed by a measurement.

$$\begin{aligned}A_2 &= \exp\Big(-\frac{i}{\hbar}\int_{t_{ev}}^{t_{ev}+t_m} H_{int}(t)dt\Big)\exp\Big(-\frac{i}{\hbar}\int_0^{t_{ev}} H_0 dt\Big)\\ &= \begin{pmatrix}\exp(-\frac{i}{\hbar}Bt_{ev})\cos(\frac{1}{\hbar}q\int_{t_{ev}}^{t_{ev}+t_m} g(t)dt) & -i\exp(\frac{i}{\hbar}Bt_{ev})\sin(\frac{1}{\hbar}q\int_{t_{ev}}^{t_{ev}+t_m} g(t)dt)\\ -i\exp(-\frac{i}{\hbar}Bt_{ev})\sin(\frac{1}{\hbar}q\int_{t_{ev}}^{t_{ev}+t_m} g(t)dt) & \exp(\frac{i}{\hbar}Bt_{ev})\cos(\frac{1}{\hbar}q\int_{t_{ev}}^{t_{ev}+t_m} g(t)dt)\end{pmatrix}\end{aligned} \tag{5.3.6}$$

5.3.2 Series of measurements during free evolution alternated with only free evolution of the spin

Assume that $g(t)$ is time dependent. We have $\hat{H}(t) = \begin{pmatrix} B & g(t)q \\ g(t)q & -B \end{pmatrix}$. The evolution of the wavefunction is given by equation 3.5.3 and we can write this as

$$|\psi(t)\rangle = \exp\Big(-\frac{i}{\hbar}\big(B\sigma_z t + q\sigma_x \int_0^t g(t'))dt'\Big)|\psi(0)\rangle \tag{5.3.7}$$

or

$$|\psi(t)\rangle = \exp\Big(-\frac{i}{\hbar}\begin{pmatrix} Bt & q\int_0^t g(t')dt' \\ q\int_0^t g(t')dt' & -Bt \end{pmatrix}\Big)|\psi(0)\rangle. \tag{5.3.8}$$

Take X $= \begin{pmatrix} Bt & q\int_0^t g(t')dt' \\ q\int_0^t g(t')dt' & -Bt \end{pmatrix}$ and set $\hat{\alpha} = -\frac{i}{\hbar}$. The Taylor series of the exponential function give

$$\exp(\hat{\alpha}X) = \sum_{n=0}^{\infty}\frac{(\hat{\alpha}X)^n}{n!} = \sum_{n=0}^{\infty}\frac{(\hat{\alpha}X)^{2n}}{(2n)!} + \sum_{n=0}^{\infty}\frac{(\hat{\alpha}X)^{2n+1}}{(2n+1)!}. \tag{5.3.9}$$

We notice that $X^2 = (B^2t^2+q^2(\int_0^t g(t')dt')^2)I$. From this follows $X^{2n} = (B^2t^2+q^2(\int_0^t g(t')dt')^2)^n I$ and $X^{2n+1} = (B^2t^2+q^2(\int_0^t g(t')dt')^2)^n X$. This gives

$$\exp(\hat{\alpha}X) = I\sum_{n=0}^{\infty}\Big(B^2t^2+q^2\big(\int_0^t g(t')dt'\big)^2\Big)^n\frac{\hat{\alpha}^{2n}}{(2n)!} + X\sum_{n=0}^{\infty}\Big(B^2t^2+q^2\big(\int_0^t g(t')dt'\big)^2\Big)^n\frac{\hat{\alpha}^{2n+1}}{(2n+1)!} \tag{5.3.10}$$

and with the Taylor series of the hyperbolic cosine and sine, and substituting $\hat{\alpha} = -\frac{i}{\hbar}$, this gives

$$\exp(-\frac{i}{\hbar}X) = I\cosh\Big(-\frac{i}{\hbar}\sqrt{B^2t^2+q^2\big(\int_0^t g(t')dt'\big)^2}\Big) + X\frac{\sinh\Big(-\frac{i}{\hbar}\sqrt{B^2t^2+q^2\big(\int_0^t g(t')dt'\big)^2}\Big)}{\sqrt{B^2t^2+q^2\big(\int_0^t g(t')dt'\big)^2}}. \tag{5.3.11}$$

We find the following matrix with $\hat{\gamma} = \frac{i}{\hbar}\sqrt{B^2t^2+q^2\big(\int_0^t g(t')dt'\big)^2}$

$$\begin{pmatrix} \cosh(\hat{\gamma}) + \frac{Bt}{\sqrt{B^2t^2+q^2\big(\int_0^t g(t')dt'\big)^2}}\sinh(-\hat{\gamma}) & \frac{q\int_0^t g(t')dt'}{\sqrt{B^2t^2+q^2\big(\int_0^t g(t')dt'\big)^2}}\sinh(-\hat{\gamma}) \\ \frac{q\int_0^t g(t')dt'}{\sqrt{B^2t^2+q^2\big(\int_0^t g(t')dt'\big)^2}}\sinh(-\hat{\gamma}) & \cosh(\hat{\gamma}) - \frac{Bt}{\sqrt{B^2t^2+q^2\big(\int_0^t g(t')dt'\big)^2}}\sinh(-\hat{\gamma}) \end{pmatrix}. \tag{5.3.12}$$

With the initial state given as in equation 3.2.1 and using equation 5.3.8, the wavefunction evolves as follows in time,

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \cos\left(\frac{1}{\hbar}\sqrt{B^2t^2 + q^2\left(\int_0^t g(t')dt'\right)^2}\right) - i\frac{Bt+q\int_0^t g(t')dt'}{\sqrt{B^2t^2+q^2\left(\int_0^t g(t')dt'\right)^2}} \sin\left(\frac{1}{\hbar}\sqrt{B^2t^2 + q^2\left(\int_0^t g(t')dt'\right)^2}\right) \\ \cos\left(\frac{1}{\hbar}\sqrt{B^2t^2 + q^2\left(\int_0^t g(t')dt'\right)^2}\right) - i\frac{-Bt+q\int_0^t g(t')dt'}{\sqrt{B^2t^2+q^2\left(\int_0^t g(t')dt'\right)^2}} \sin\left(\frac{1}{\hbar}\sqrt{B^2t^2 + q^2\left(\int_0^t g(t')dt'\right)^2}\right) \end{pmatrix}. \tag{5.3.13}$$