Interactive animation on Directional derivatives & Level curves

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ABSTRACT

We present our short series of interactive animations on directional derivatives and level curves. This visualisation was developed for students of first-year mathematics courses at the Delft University of Technology. The interactive animation is an animated video that can be interacted with while it is paused or playing. The user can change, for example, the function that is plotted, drag points of interest or change the angle of the camera to explore the scene.

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1 INTRODUCTION

Our poster is a website that provides both information on the technical side of our solution and the opportunity to interact with the animation. The interactive poster can be accessed via this link. What follows in this document is an explanation of the animation and what the viewer can experience when trying it themselves.

2 RESULTS

We strongly recommend exploring the full interactive experience over the still images in this document. The short series can be explored via the following links:

- Part 1: Level curves
- Part 2: Directional derivatives
- Part 3: Directional derivatives & Level curves
- Part 4: Gradient & Level curves

Below are screenshots from each part of the interactive animation series and an explanation of the visualisation. When exploring the interactive animation, a narrator provides this explanation.

2.1 Part 1: Level curves

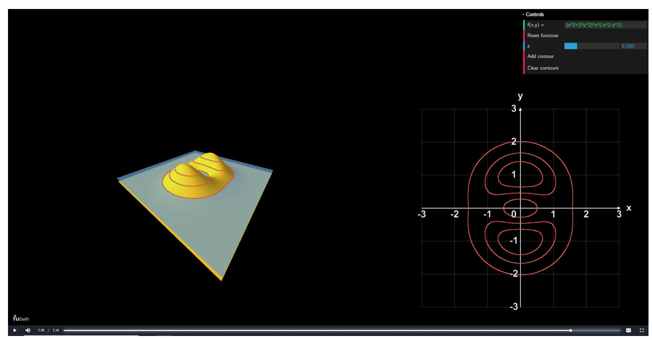

Figure 1: Screenshot from part 1 (Level Curves); a 3D and 2D plot showing how level curves are obtained. In the right top, adjustable parameters are shown.

Figure 1 provides a screenshot from part 1 of the series, on level curves. We present a view with two panels. A 3D plot on the left, and a 2D plot on the right. On the bottom a classic video player bar is located with controls to play or pause. The video player bar also allows skipping through the timeline of the animation. At the top right is the controls menu with parameters that can be adjusted. Parameters in the controls menu can be adjusted either by filling in values directly, or by dragging the corresponding objects in the scene. In the 3D scene, the user can also control the camera by rotating and zooming.

In the 3D view, we see the graph of a function of two variables and the intersection of the graph with a horizontal plane. The two-variable function can be changed in the controls menu, allowing for quite some variation in the animations and wide applicability.

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The curve that results from the intersection of the function and the horizontal plane can be projected back to the xy-plane. This gives us a level curve, as plotted in 2D on the right. The active level curve can be changed by the user by dragging the plane or adjusting the planes height in the controls menu. In the animation, we add level curves for several equally spaced heights. Users can add extra level curves at the location of the plane via the controls menu.

2.2 Part 2: Directional Derivatives

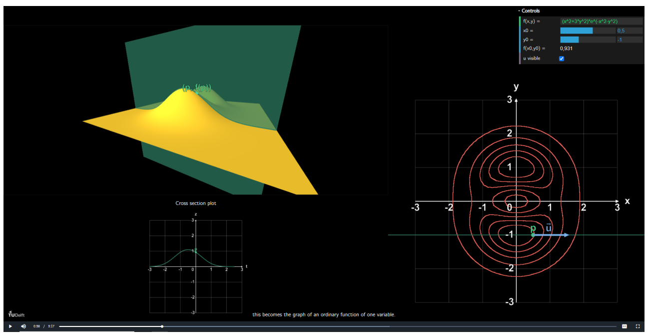

Figure 2: Screenshot from part 2 (Directional Derivatives); a 3D and two 2D plots illustrate the concept of the directional derivative.

Figure 2 is a screenshot from part 2 of the series, on directional derivatives. The view from part one has been expanded with an additional 2D plot.

In the 3D view, the animation adds a point P in the xy-plane, and the corresponding point (P,f(P)) on the 2D contour plot. The x and y coordinates for P can be adjusted in the controls menu.

The animation then adds a line through P in the direction of u in the contour plot and a corresponding vertical plane above this line in 3D. The visibility of u in the contour plot can be toggled on and off in the controls menu.

The intersection of the plane and the graph can be regarded as the graph of an ordinary 1-variable function, which is plotted in the newly added 2D plot on the left bottom. The animation explains that the directional derivative at p in the direction of u is the derivative of this new 1-dimensional function.

2.3 Part 3: Directional Derivatives & Level Curves

Part 3 of the series explains the relationship between level curves and directional derivatives, as shown in Figure 3. The view is expanded with a third 2D plot on the bottom left.

In the contour plot, a rotation arc is added for the orientation of the vector u and the corresponding vertical plane. The amount of rotation can be controlled with the theta value in the controls menu by the user.

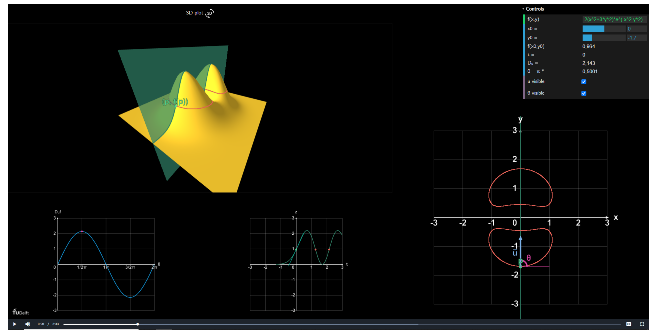

Figure 3: Screenshot from part 3 (directional derivatives & level curves); a 3D and three 2D plots explaining the relationship between level curves and directional derivatives.

The animation shows that varying theta yields different directional derivatives. Once the rotation goes full circle, the directional derivative repeats itself. The graph of the directional derivative is a sinusoidal curve, plotted in the newly added 2D graph on the far left bottom. The directional derivative is 0 if u is tangent to the level curve through P. There are exactly two directions tangent to the level curve, both opposite each other. The plots show this intuitively.

2.4 Part 4: Gradient & Directional Derivatives

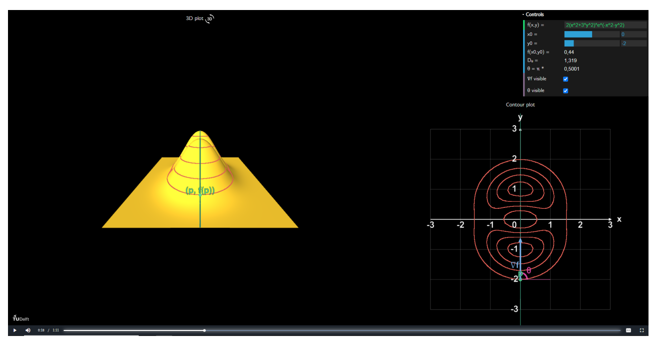

Figure 4: Screenshot from part 4 (directional derivatives & gradients); a 3D and three 2D plots explaining the relationship between level curves and gradients.

Figure 4, a screenshot from part 4 of the series, explains the relationship between directional derivatives and gradients. Only the 3D plot and the 2D contour plot are shown, together with the controls menu, in which visibility of several items can be toggled.

The animation draws the gradient in the picture of level curves and shows the gradient is always perpendicular to the level curves. This can be explored by changing the x and y coordinates of p in the controls menu. The animation shows that length of the gradient is inversely proportional to the distance between level curves.