

Prediction formula for the spectral wave period $T_{m-1,0}$ on mildly sloping shallow foreshores

Hofland, Bas; Chen, Xuexue; Altomare, Corrado; Oosterlo, Patrick

DOI

[10.1016/j.coastaleng.2017.02.005](https://doi.org/10.1016/j.coastaleng.2017.02.005)

Publication date

2017

Document Version

Accepted author manuscript

Published in

Coastal Engineering

Citation (APA)

Hofland, B., Chen, X., Altomare, C., & Oosterlo, P. (2017). Prediction formula for the spectral wave period $T_{m-1,0}$ on mildly sloping shallow foreshores. *Coastal Engineering*, 123(May), 21-28.
<https://doi.org/10.1016/j.coastaleng.2017.02.005>

Important note

To cite this publication, please use the final published version (if applicable).
Please check the document version above.

Copyright

Other than for strictly personal use, it is not permitted to download, forward or distribute the text or part of it, without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license such as Creative Commons.

Takedown policy

Please contact us and provide details if you believe this document breaches copyrights.
We will remove access to the work immediately and investigate your claim.

1 **Second Revision.** Accepted for publication in Coastal Engineering.

2 © 2017 Manuscript version made available under CC-BY-NC-ND 4.0 license

3 <https://creativecommons.org/licenses/by-nc-nd/4.0/>

4 Postprint of Coastal Engineering Volume 123, May 2017, Pages 21–28

5 Link to formal publication (Elsevier): <https://dx.doi.org/10.1016/j.coastaleng.2017.02.005>

6 Prediction formula for the spectral wave period

7 $T_{m-1,0}$ on mildly sloping shallow foreshores

8

9

10

Bas Hofland

11

Delft University of Technology, Delft, The Netherlands, b.hofland@tudelft.nl

12

Deltares, Delft, The Netherlands

13

Xuexue Chen

14

Delft University of Technology, Delft, The Netherlands, X.Chen-2@tudelft.nl

15

Corrado Altomare

16

Flanders Hydraulics Laboratory, Antwerp, Belgium, corrado.altomare@mow.vlaanderen.be

17

Ghent University, Ghent, Belgium

18

Patrick Oosterlo

19

Delft University of Technology, Delft, The Netherlands, P.Oosterlo@tudelft.nl

20

21

22 **Abstract**

23

24 During the last decades, the spectral wave period $T_{m-1,0}$ has become accepted as a characteristic wave period when
25 describing the hydraulic attack on coastal structures, especially over shallow foreshores. In this study, we derive an
26 empirical prediction formula for $T_{m-1,0}$ on shallow to extremely shallow foreshores with a mild slope. The
27 formula was determined based on flume tests and numerical calculations, mainly for straight linear foreshore
28 slopes. It is shown that the wave period increases drastically when the water depth decreases; up to eight times
29 the offshore value. The bed slope angle influences the wave period slightly. For short-crested wave fields, the
strong increase of $T_{m-1,0}$ starts closer to shore (at smaller water depths) than for long-crested wave fields.

30

31 **Keywords**

32

33 spectral wave period, $T_{m-1,0}$, shallow foreshore, very shallow foreshore, sea dike,

34

35 infragravity waves

36

37

38 **1. Introduction**

39

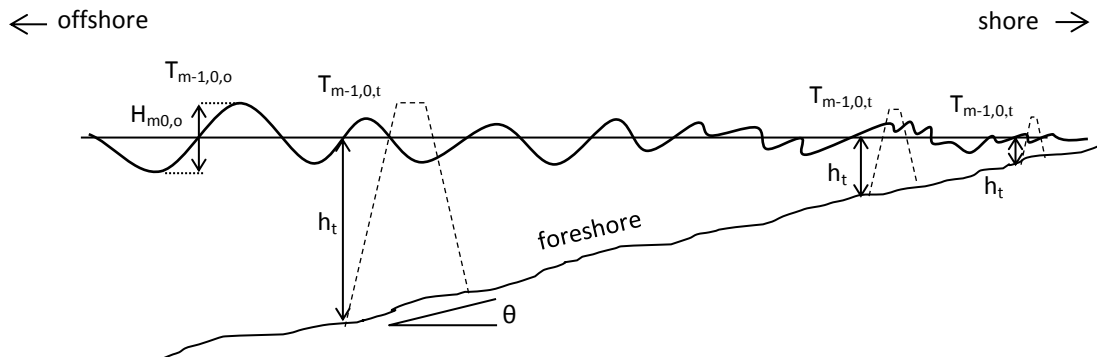
40 During the last decades, the spectral wave period $T_{m-1,0}$ has been accepted as a
41 characteristic period when describing the interaction between sea waves and coastal
42 structures. This wave period is used for describing many processes like wave run-up,
43 overtopping, reflection, and armour layer stability, especially when the structure has a
44 shallow foreshore. The period can

38 also be used in situations characterized by a multi-modal wave spectrum. It is sometimes
 39 called the wave energy period as it is the equivalent wave period needed to calculate the
 40 energy flux P for any irregular wave field in deep water: $P \propto H_{m0}^2 T_{m-1,0}$ (e.g. Battjes,
 41 1969), where H_{m0} is the spectral significant wave height.

42

43 In the present study, a foreshore is defined as the part of the seabed bathymetry seaward of the
 44 toe of a structure that has influence on waves. The configuration considered in this paper is
 45 shown in Figure 1. A shallow foreshore is typically characterized by depths smaller than about
 46 three to four times the significant wave height. A very shallow foreshore may be further
 47 defined where the wave height is reduced to about 50% of its offshore value. Such foreshores
 48 are considered to be mildly sloping when the slopes are gentler than 1:30, such that the waves
 49 are influenced by depth-effects over a certain distance. These definitions are discussed in
 50 more detail in Section 2.2.

51



52

53 Figure 1. The foreshore configuration that is treated in this study. Three possible locations for structures on the
 54 sea bed are indicated, as well as the conditions at the toes of these structures, $T_{m-1,0,t}$ and h_t . The
 55 offshore conditions are indicated by the offshore spectral wave period $T_{m-1,0,o}$ and the offshore wave
 56 height $H_{m0,o}$.

57

58 Hard-soft hybrid constructions, for example dike-in-dune constructions or large beach
 59 nourishments in front of sea walls such as those found in the Netherlands and Belgium, are
 60 characterized by large amounts of sand seaward of the hard structure. Therefore, once the
 61 sand has been eroded away in an extreme storm, the foreshore in front of the hard dike will be
 62 extremely shallow (in the order of one metre). In these situations, the magnitude of the wave
 63 period is not well known, although very large wave periods have been observed on shallow
 64 foreshores. Usually numerical or physical modelling is applied to predict the wave period, or
 65 the response of the structures directly (Van Gent, 1999a,b; Van Gent, 2004; Suzuki et al.,
 66 2014; Altomare et al., 2016). Design formulas for several types of response in these
 67 conditions do exist, however the near shore wave period is often required in these equations.

68

69 While the significant wave height H_{m0} at the local depth h over a shallow foreshore can be
70 predicted relatively well using a value for the breaker parameter H_{m0}/h (e.g. Goda,1975, CIRIA
71 et al., 2007), to the authors' knowledge no empirical formula exists for the prediction of the
72 wave period $T_{m-1,0,t}$. Therefore, an empirical formula to predict the wave period is proposed,
73 which is calibrated using various data sets that have been gathered in shallow and very
74 shallow foreshores in the Netherlands and Belgium.

75

76 In this paper, first the existing knowledge on $T_{m-1,0}$ is treated in Section 2.1. Next, in Section
77 2.2, a classification of different types of shallow foreshore (shallow, very shallow, and
78 extremely shallow) is presented, which is important for understanding the generation
79 mechanism of low-frequency energy on such foreshores. In Section 2.3, relevant research
80 about low-frequency waves is introduced. In Section 3 the datasets and the numerical
81 calculations that are used to derive an empirical prediction formula for $T_{m-1,0}$ are described.
82 Subsequently, the derived prediction formula is presented in Section 4. This paper ends with a
83 discussion and conclusions in Sections 5 and 6.

84

85 **2. Literature review**

86 *2.1 Spectral wave period, $T_{m-1,0}$*

87 Many wave periods, such as the peak period T_p (defined as the frequency at the peak of the
88 wave spectrum), and the significant zero-crossing period $T_{1/3}$ (mean period of the highest third
89 of the waves) have been proven to be linked to many coastal processes for standard spectral
90 shapes and deep water conditions. However, for shallow foreshores, the spectral shape tends
91 to become flattened and/or double-peaked. Examples of wave spectrum shapes along different
92 locations on shallow foreshores are presented in Figure 2. Spectral shapes like the ones
93 presented in Figure 2 make most of these commonly used wave periods in deep water less
94 suitable to describe the coastal processes in shallow water. To weigh the contribution of
95 different parts of the spectrum to the relevant coastal process, several spectral periods are
96 applied, for example $T_{m0,1}$ or $T_{m-1,0}$. The spectral period $T_{m-1,0}$ is defined as:

97

$$T_{m-1,0} = \frac{m_{-1}}{m_0} \quad , \quad \text{with } m_n = \int_0^{\infty} S f^n df \quad , \quad (1)$$

98

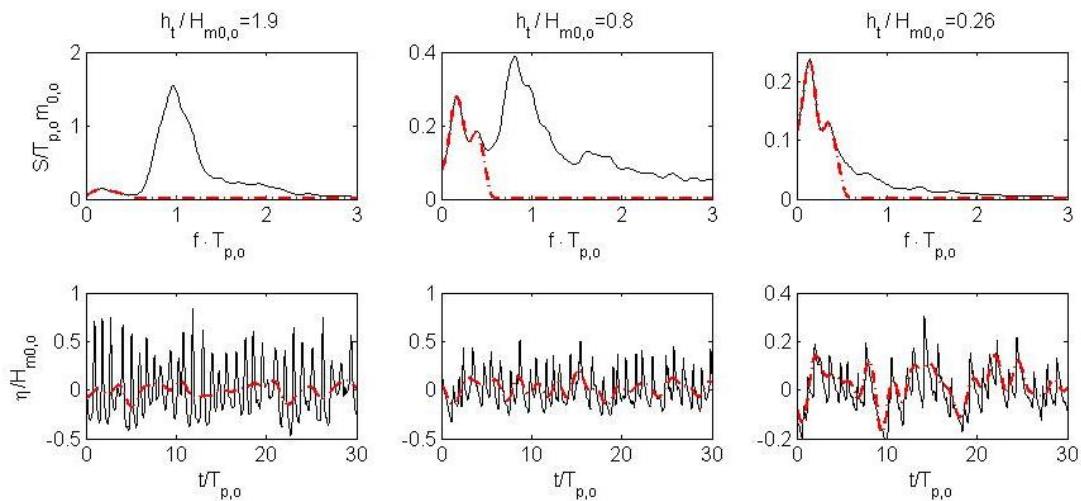
99 where f is frequency and S the spectral density of the water surface elevation. m_0 is the
100 variance of the water surface elevation. The mean energy wave period, $T_{m-1,0}$, gives more

101 weight to the lower frequencies, and therefore to the longer periods in the spectrum, than
 102 wave periods like T_p or $T_{1/3}$.

103

104 After Holterman (1998) made the first attempt to link wave run-up to several wave periods
 105 based on moments of the wave spectrum, the period $T_{m-1,0,t}$ was recommended by Van Gent
 106 (1999a, 2001) as the best suited wave period to describe wave run-up and overtopping process
 107 for single and double-peaked spectra. Various spectral-based wave periods have been
 108 correlated to wave run-up and $T_{m-1,0,t}$ has been found to have the highest correlation.
 109 Therefore, the overtopping discharge can be computed for a given $T_{m-1,0,t}$ and wave height,
 110 independent of the type of spectrum. Subsequent research discovered and validated the
 111 correlation of $T_{m-1,0,t}$ to a number of coastal processes, e.g., wave overtopping (Van Gent,
 112 1999a,b; Pozueta et al., 2005; Altomare et al., 2016), reflection (Dekker et al., 2007;
 113 Zanuttigh & Van der Meer, 2008), armour layer stability (Van Gent, 2004), and wave impacts
 114 (Chen et al., 2016). The use of $T_{m-1,0,t}$ is also incorporated in manuals such as the EurOtop
 115 manual (EurOtop, 2016) and the Rock Manual (CIRIA et al., 2007).

116



117

118

119

Figure 2. Example of measured wave spectra (top) and surface elevations η (bottom) for various water depths on a shallow foreshore, normalized by offshore wave parameters (subscript o) (data of Chen et al., 2016). The water depths are roughly indicated in Figure 1. Solid lines indicate the signals within the full frequency range, whereas the dash-dotted lines indicate the corresponding low-pass filtered signals (cut-off at $f_p/2$).

120

121

122

123

124 Presently, in engineering, the deep water ratio $T_{m-1,0,o} / T_{p,o} \approx 0.9$ for a single-peaked spectrum
 125 is often used to predict the wave period near the toe of a structure $T_{m-1,0,t}$ from a known
 126 offshore wave period $T_{p,o}$. Here the subscripts o and t represent the *offshore* and *toe* locations,
 127 respectively. Hence it is essentially stated that $T_{m-1,0,t} / T_{m-1,0,o} = 1$, independent of the location
 128 of the structure. The ratio of $T_{m-1,0,t} / T_{m-1,0,o}$ can actually reach values up to 8, as will be
 129 shown in Section 4. Therefore, the use of the ratio $T_{m-1,0,t} / T_{p,o}$ for the estimation of $T_{m-1,0,t}$ at

130 shallow foreshores is invalidated in this study. A prediction formula for $T_{m-1,0,t}$ over (very)
131 shallow foreshores is thus required.

132

133 *2.2 Foreshore*

134 Goda (2009) argued that there is no agreement about the terminology of the word foreshore in
135 many references. Manuals such as the Rock Manual (CIRIA et al., 2007) and Coastal
136 Engineering Manual (USACE, 2002) formally define a foreshore as the part of a beach
137 between a high and a low water level. However, in coastal structure research, the word
138 foreshore is defined differently. It implies the part of the seabed bathymetry seaward of the
139 toe of the structure that is characterized by depth-induced wave processes such as depth-
140 induced wave breaking. In the EurOtop manual (EurOtop, 2016), for example, the foreshore
141 is defined as the section in front of the dike/structure and it “can be horizontal or up to a
142 maximum slope of 1:10 [...] having a minimum length of one [fictitious deep water]
143 wavelength L_0 ”.

144

145 Because of wave breaking on a shallow foreshore, the wave height becomes depth limited.
146 Moreover, there is not one clear peak frequency visible anymore in the energy density
147 spectrum (e.g. Holterman, 1998, Van Gent, 2001). Also, using the Rayleigh distribution to
148 calculate the distribution of wave heights and wave run-up levels in deep water cannot be
149 applied anymore for shallow foreshores (e.g. Battjes and Groenendijk, 2000).

150

151 As certain formulae for e.g. wave overtopping or wave impact forces are intended to be used
152 for shallow or very shallow foreshores, because their validity depends on the type of wave
153 motion, it is required to characterize the shallowness of the foreshore explicitly. The
154 shallowness of the foreshores is best characterized by the water depth near the structure, h_t ,
155 normalized by the offshore wave height $H_{m0,o}$. In literature, some (approximate) limits can be
156 found with some interpretation for four classes of foreshore: deep, shallow, very shallow and
157 extremely shallow, see Table 1. The prediction formula for $T_{m-1,0,t}$ presented in this paper
158 includes all these classes of foreshores. The definitions of hydraulic and foreshore geometry
159 parameters are illustrated in Figure 1.

160

161 **Offshore** is defined here as $h_t/H_{m0,o} > 4$, as that is the water depth at which no depth-induced
162 wave breaking occurs according to the Battjes and Groenendijk (2000) equation. Other
163 references (Holterman, 1998; TAW, 2002) give a similar limit of 3 to 4.

164

165 **Shallow** is defined here as $1 < h_t/H_{m0,o} < 4$. This is the depth where the water depth starts to
166 influence the wave breaking. The wave spectrum observed here is still similar to that offshore

167 (here JONSWAP) with a clear single peak, but some minor (higher and lower) second-order
 168 effects are visible, see the left panels of Figure 2 where the typical wave signal on a shallow
 169 foreshore is depicted.

170

171 **Very shallow** is defined as $0.3 < h_t/H_{m0,o} < 1$. This is the water depth where the wave height is
 172 reduced to 50% to 60% of the offshore wave height by depth-induced wave breaking as
 173 defined by e.g. Holterman (1998), TAW (2002), and EurOtop (2016). As the breaker
 174 parameter ($H_{m0,t}/h_t$) on a mildly sloping foreshore is also somewhere between 0.5 to 0.6, that
 175 gives a definition of the shallow foreshore of $h_t/H_{m0,o} < 1$. Van Gent (1999a) presented data in
 176 the very shallow range, where the flattening of the spectra becomes apparent. In the middle
 177 panels of Figure 2 the typical wave signal on a very shallow foreshore is depicted. The
 178 majority of the offshore spectrum has been dissipated, and a large amount of low-frequency
 179 energy has emerged.

180

181 **Extremely shallow** is defined in the present paper as $h_t/H_{m0,o} < 0.3$, or more shallow than
 182 studied by Van Gent (1999a). In the right panels of Figure 2 the typical wave signal on an
 183 extremely shallow foreshore is depicted. Nearly most of the high frequency part of the
 184 spectrum has been dissipated, and the low-frequency energy is dominant. Altomare et al.
 185 (2016) and Chen et al. (2016) present data in this range.

186

Deep (Holterman, 1998; Battjes & Groenendijk, 2000; TAW, 2002)	$\frac{h_t}{H_{m0,o}} > 4$
Shallow (Holterman, 1998; TAW, 2002)	$1 < \frac{h_t}{H_{m0,o}} < 4$
Very Shallow (Van Gent, 1999)	$0.3 < \frac{h_t}{H_{m0,o}} < 1$
Extremely Shallow (Altomare et al., 2016; Chen et al., 2016)*	$\frac{h_t}{H_{m0,o}} < 0.3$

187 Table 1. Consistent classification of foreshore depths, based on the water depth at toe of structure h_t , normalized
 188 by the offshore wave height $H_{m0,o}$.

189 *) here different classifications are used.

190

191 Also other parameters are used to classify the shallowness of a foreshore, such as the
 192 steepness of the wave field (Altomare et al., 2016; EurOtop, 2016), or the surf-similarity
 193 parameter ζ (EurOtop, 2016). However, using these parameters, that include the local wave
 194 period, to classify foreshores is not convenient in the present research, as the aim is to obtain

195 a prediction of this local wave period. Moreover, non-breaking swells on deep foreshores
196 would then formally also classify deeper foreshores as shallow foreshore, whereas shallow
197 foreshore in the present context implies the presence of heavy wave breaking. When the surf-
198 similarity parameter based on the structure slope is used to define a shallow foreshore, steep
199 structure slopes would imply the presence of a shallow foreshore. This has no physical
200 relevance.

201

202 *2.3 Infragravity wave research*

203 Munk (1949) and Tucker (1950) were the first to relate the presence of low-frequency or
204 infragravity waves in the shoaling and surf zones to the group structure of the incident short
205 waves. Infragravity waves are long waves of periods typically with an order of 100 s in
206 prototype (Van Dongeren et al, 2007). Two generation mechanisms of this kind of waves have
207 been identified: the shoaling of the low-frequency long waves, and the time-varying
208 breakpoint mechanism (or surf-beat). Both these mechanisms are associated with the
209 modulation of the wave height on the scale of the wave group. In the first mechanism, the
210 variation in radiation stress at the time scale of the incident wave group forces bound
211 infragravity waves (Longuet-Higgins and Stewart, 1962). These bound waves are in anti-
212 phase with the forcing wave groups. Alternatively, the time varying location of the breakpoint,
213 due to the group structure of the incident short waves, results in the generation of free low-
214 frequency waves (Symonds et al., 1982). The type of generation mechanism seems to be
215 dependent on the slope of the foreshore and steepness of the incoming (low-frequency) waves
216 (Battjes et al., 2004, Van Dongeren et al., 2007). The first mechanism (shoaling of bound long
217 waves) is believed to be dominant on a mild slope, where the mild slope is characterized by a
218 low value of surf-similarity-like parameter, β_b :

219

$$\beta_b = \frac{\theta}{\omega} \sqrt{\frac{g}{h_b}}, \quad (2)$$

220

221 where θ is the bed slope, ω is the angular frequency of the long waves, h_b is the mean breaker
222 depth of the primary short waves, and g the gravitational acceleration. In the mild slope
223 regime ($\beta_b < 0.3$), the low-frequency waves are shown to be breaking, yielding a low
224 reflection of the long waves (Van Gent, 2001; Van Dongeren et al., 2007).

225

226 The generation mechanisms of the low-frequency wave energy may be different for short-
227 crested waves. Short-crested waves exhibit less long-wave energy generation on beaches (e.g.

228 Herbers et al., 1994) and less wave overtopping occurs at structures with very shallow
229 foreshores when short-crested seas are applied (Suzuki et al., 2014).

230

231 In conclusion, there is much research on the origin on the low-frequency energy on (very)
232 shallow foreshores. For the mildly sloping foreshores treated here, the generation of low-
233 frequency energy appears to have a different generation mechanism than that for steep
234 beaches. Moreover, the value of the wave period $T_{m-1,0}$ is influenced by the presence of low-
235 frequency waves. This wave period is shown to be important for many wave-structure
236 interaction processes, and can be used to assess the response of coastal structures with
237 shallow foreshores. However, no empirical prediction tool is available to predict this wave
238 period. This paper aims to provide such an engineering tool.

239

240 **3. Data sets**

241 In order to derive a prediction formula for $T_{m-1,0,b}$, several data sets of physical model studies
242 have been selected, and some additional numerical calculations have been performed. An
243 overview is given in Table 2. In the first part of this section the general set-up for all these
244 studies is described, followed by the specifics of the separate studies.

245

246 In all studies, the bottom was horizontal from the wave maker to the toe of the foreshore,
247 representing deep water (deeper than $4H_{m0,o}$). For all tests (except Deltares, 2011; and
248 FHR13_168), the foreshore was characterized by an initial linear slope followed by a
249 horizontal part, as shown in Figure 3. Furthermore, instead of having the sea dike, a
250 horizontal platform was inserted just after the foreshore. Damping material (e.g. gravel or
251 foam) was located after the platform to reduce the possible reflection of (long) waves as much
252 as possible.

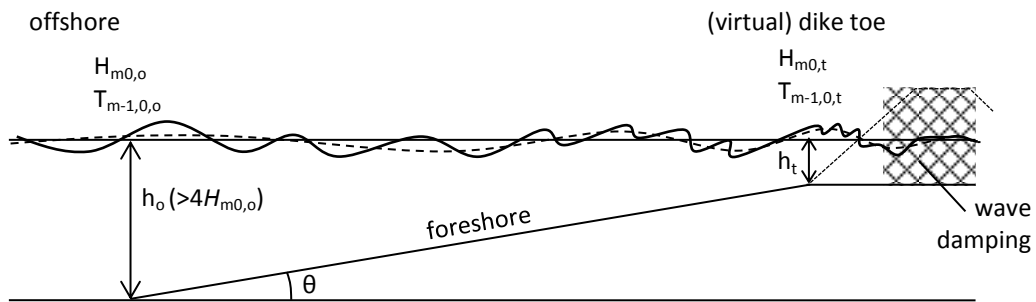
253

254 The waves (offshore and at the structure) were measured at the horizontal sections. Classical
255 reflection analysis methods (e.g. Mansard and Funke, 1980) are not suitable in shallow water
256 conditions because non-linear effects dominate in cases with very shallow foreshores (Van
257 Gent, 1999a) and the presence of long waves. Instead, the measurements of wave height and
258 period at the dike toe (h_t) have been conducted using a single wave gauge at the location of
259 the (virtual) dike toe without the presence of the reflecting structure.

260

261 The wave conditions for all test series consisted of irregular waves with values for the
262 offshore wave steepness, $s_{m-1,0,o} = H_{m0,o} / \frac{g}{2\pi} T_{m-1,0,o}^2$, ranging from around 0.01 to around 0.05.

263 Typically standard JONSWAP spectra were applied.



265

266

267

Figure 3. Typical setup for the model studies used. The dashed line indicates the (bound) long wave. h_o indicates the offshore water depth, and θ indicates the foreshore slope.

268

269 **Van Gent (1999a)** measured the wave parameters for foreshore slopes of 1:100 and 1:250 in
 270 the Scheldt Flume of Deltares (1 m \times 1.2 m \times 55 m). JONSWAP and double-peaked spectra
 271 were applied. The entire (smoothed) measured spectral range was utilized to determine the
 272 values of $T_{m-1,0,t}$. The waves were generated with Active Reflection Compensation and 2nd
 273 order wave generation.

274

275 **Chen et al. (2016)** measured the wave conditions in a wide flume (4 m \times 1.4 m \times 70 m) at
 276 Flanders Hydraulics Research (FHR). The foreshore extended over the entire width and was
 277 split in four sections around the top horizontal part. Passive wave absorption was present in
 278 the outer two sections, and at the two sections in the middle of the flume a dike section was
 279 present. The possible build-up of low (or high) frequency energy was investigated using
 280 wavelet analysis, and was absent. Only the energy corresponding to the first seiching mode
 281 was slightly increased. Hence, the entire spectrum was used to determine $T_{m-1,0,t}$, except for
 282 the frequency band corresponding to a slight seiching oscillation (over the small frequency
 283 resolution $\Delta f = 0.01$ Hz) that was removed. First order wave generation was used.

284

285 **Altomare et al. (2016)** describe three more experimental campaigns that have been carried
 286 out in the same wide flume at FHR between 2012 and 2015 (datasets: 13-116, 00-025, 13-
 287 168), having as main objectives the characterization of wave overtopping and loading on
 288 coastal defences with shallow to extremely shallow foreshores. The foreshore slopes were
 289 smooth. Passive reflection compensation, wave generation, and data processing were done in
 290 a similar fashion as Chen et al. (2016). For tests 13-168 the setup differed somewhat. It was
 291 characterized by a 1:50 (upper) foreshore slope with a length of 21 m. A 1:15 transition slope
 292 of 5 m long was constructed between the wave maker and the start of the foreshore to obtain a

293 sufficient depth at the wave maker location. Offshore wave heights of 2.4 to 7 cm were
294 applied. In test series 13-168, 2nd order wave generation was used.

295

296 **XBeach.** The numerical model XBeach was used to model a similar setup as applied in the
297 tests. XBeach is a nearshore numerical model used to assess the coastal response during storm
298 conditions, and has extensively been calibrated and validated (Roelvink et al., 2009; Smit et
299 al., 2010; www.xbeach.org). In the currently applied non-hydrostatic mode it solves the non-
300 linear shallow water equations, including a non-hydrostatic pressure correction, based on the
301 approach of Stelling and Zijlema (2003). The wave breaking behaviour is improved by
302 disabling this non-hydrostatic pressure term when the water level gradient exceeds a certain
303 steepness. After this, the bore-like dissipation term in the momentum-conserving shallow
304 water equations takes over (Smit et al., 2010).

305

306 XBeach calculations were performed for the cases of Van Gent (1999a) with a JONSWAP
307 spectrum, a 1:100 slope, and wave steepness $s_{m-1,0,o} = 0.043$, as well as for additional
308 shallower cases that were not tested. For these tests the value of $k_p h_o$ ranged from 0.63 to 1.18,
309 where k_p is the wave number based on the peak period. The short wave celerity of wave
310 components with $kh < 3$ is within 3%. The calculations are well below this limit, and with
311 decreasing water depth the accuracy increases. To get well-converged statistics of the long
312 bound waves, 5000 waves were used in the calculations. Furthermore, both long-crested (1D
313 calculations) and short-crested (2D calculations) waves were used for all conditions. For the
314 short-crested waves, a directional spreading with a standard deviation of $\sigma = 25^\circ$ was applied.
315 The main wave angle was normal to the coast line. The numerical flume was 45 m long (513
316 grid cells) for the 1D cases. The 2D calculations used the same length and a width of 40 m
317 and 101 grid cells. For the bed friction, a friction coefficient of $c_f = 0.002$ (concrete bed) was
318 used. At both the generation and the downstream side of the domain a weakly reflective
319 boundary condition was used. For the 2D cases, periodic boundary conditions were applied at
320 the lateral boundaries, to prevent edge effects. In the post-processing, the entire (smoothed)
321 spectral range was used to determine the value of $T_{m-1,0}$.

322

323 **Deltares (2011)** obtained measurements of $T_{m-1,0,t}$ in a commercial project where an irregular
324 natural shallow foreshore was applied. The foreshore slope was 1:10 up to $h / H_{m0,o} \approx 2.7$,
325 followed by a horizontal part of about 3 m, and an irregular sloping part with a mean slope of
326 about 1:200 to the toe of a 1:1.5 rubble mound slope. The foreshore was not constant over the
327 width, and waves were travelling onto the slope under a 30° obliquity. Two test conditions
328 were repeated with long- and short-crested waves, while all other conditions were identical.

Source	slope 1:cot θ	spectrum	lab.	$h_t / H_{m0,0}$	dir. spreading	# tests	order of wave generation
Van Gent, 1999	1:100	JONSWAP	Deltares	0.34-2.52	no	12	2 nd
Van Gent, 1999	1:250	JONSWAP	Deltares	0.34-2.52	no	12	2 nd
Van Gent, 1999	1:100	Double peaked	Deltares	0.67-2.52	no	30	2 nd
Van Gent, 1999	1:250	Double peaked	Deltares	0.67-2.52	no	12	2 nd
Chen et al., 2016	1:35	JONSWAP	Flanders Hydraulics	0.15-0.83	no	49	1 st
Altomare et al., 2016 FHR13_116	1:35	JONSWAP	Flanders Hydraulics	-0.05-0.20	no	45	1 st
Altomare et al., 2016 FHR00_025	1:35	JONSWAP	Flanders Hydraulics	-0.14-0.25	no	21	1 st
XBeach calculations	1:100	JONSWAP	numerical, XBeach	0.08-2.52	no	9	2 nd
XBeach calculations	1:100	JONSWAP	numerical, XBeach	0.08-2.52	$\sigma=25^\circ$	9	2 nd
Altomare et al., 2016 FHR13_168	1:15, 1:50	JONSWAP	Flanders Hydraulics	0.00-0.86	no	28	2 nd
Deltares, 2011 (in-house data)	irreg.	JONSWAP	Deltares	1.34-1.37	$\sigma=5^\circ$	2	1 st
Deltares, 2011 (in-house data)	irreg.	JONSWAP	Deltares	1.34-1.37	$\sigma=22^\circ$	2	1 st

329 Table 2. Data sets used in this study.

330

331

4. Results

332

The data of Van Gent (1999a) and Chen et al. (2016) are plotted in the left graph of Figure 4,

333

with the relative depth on the horizontal axis and the ratio of nearshore to offshore spectral

334

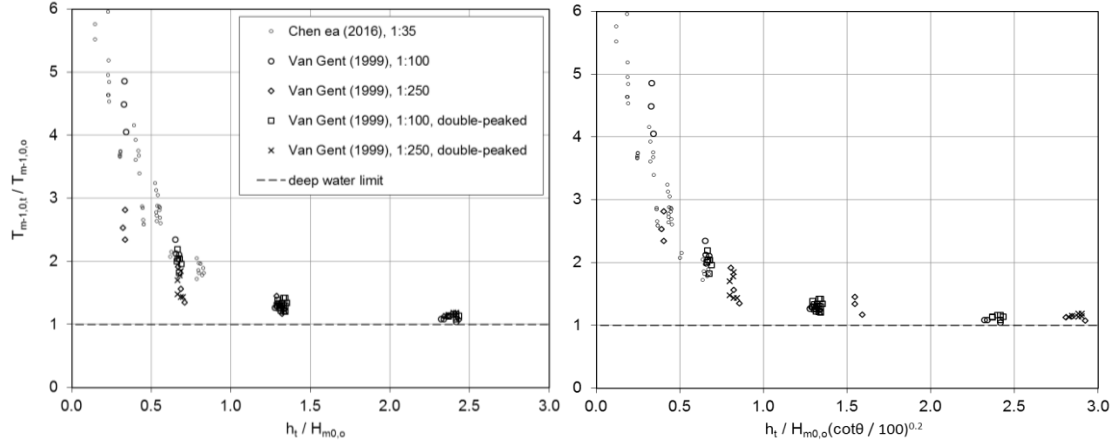
wave period on the vertical axis. These data sets represent tests with a wide range in foreshore

335

slopes over a comparable range of dimensionless depths. It can be seen that the wave periods

336

increase with decreasing relative depth h_t/H_{m0} on the foreshore, but much scatter is present.



337

338

339

Figure 4. Measured evolution of wave period $T_{m-1,0,t}$ as function of relative water depth for selected flume tests with (right) and without (left) slope correction.

340

341

Next, a parameter is introduced in which, besides the relative depth, also the foreshore slope θ

342

is incorporated as follows:

343

$$\tilde{h} = \frac{h_t}{H_{m0,0}} \left(\frac{\cot \theta}{100} \right)^{0.2}. \quad (3)$$

344

345

Here θ is the slope angle of the foreshore. The exponent on the slope term is determined

346

empirically, by minimizing the scatter. The inclusion of the slope seems to yield a slightly

347

better data collapse, as shown in the right graph of Figure 4. The R^2 -value (coefficient of

348

determination) of the best fit (with a shape as presented later) was respectively 0.91 and 0.94

349

for these data, without and with the slope influence in the dimensionless foreshore depth

350

formulation in eq. (3). Since the slope has an effect on the wave transformation processes

351

according to eq. (2), this influence is credible. So, despite the limited improvement of the fit

352

using this influence, it is maintained. According to eq. (2), a kind of surf-similarity parameter

353

based on the foreshore slope like $\tan \theta / \sqrt{s_{m-1,0,0}}$ could be expected to be better related to the

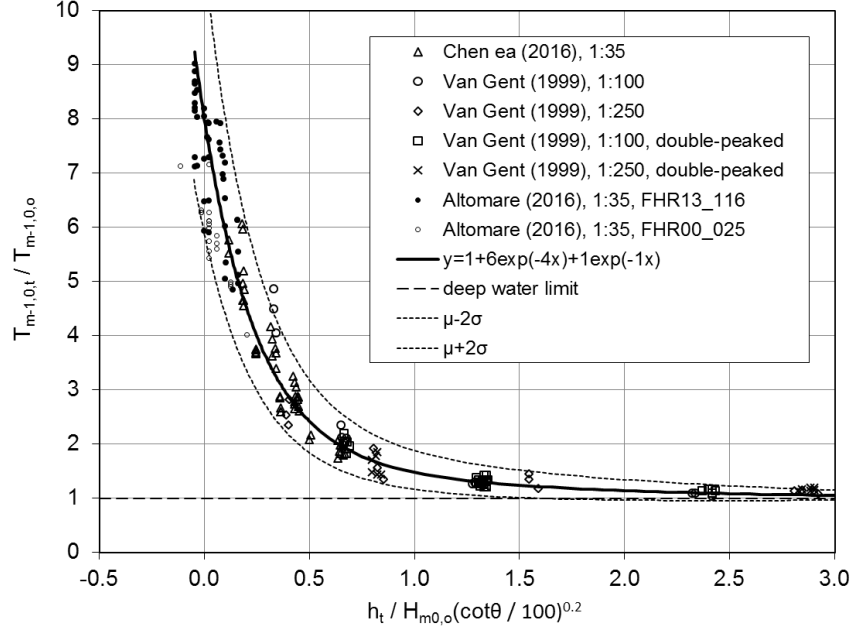
354

evolution of the low-frequency energy, and hence to the spectral wave period. However, the

355

data collapse only deteriorated when using this parameter.

356



357

358 Figure 5. Data of the (increase in) measured wave period $T_{m-1,0}$ of long-crested waves on a straight mildly
 359 sloping foreshore, as a function of relative depth with slope correction. The solid line is the fit through
 360 the data given in eq. (4). The dashed lines indicate the $\pm 2\sigma$ (root-mean-square variation) error bands.

361

362 In Figure 5 all measurement data obtained with a straight foreshore are presented. The data
 363 collapse rather well. It can be seen that for shallow foreshores, the wave period $T_{m-1,0,t}$
 364 increases slightly with decreasing depth, up to values of about 1.5 the offshore value. For very
 365 shallow foreshores $T_{m-1,0,t}$ increases quicker with depth, up to values of about 3.5 times the
 366 offshore value. For extremely shallow foreshores the increase in wave period is even more, up
 367 to values of about 8 times the offshore value at the water line (start of the swash zone).

368

369 The fit that is presented in Figure 5 for the increase of the spectral wave period in the test data
 370 is given by:

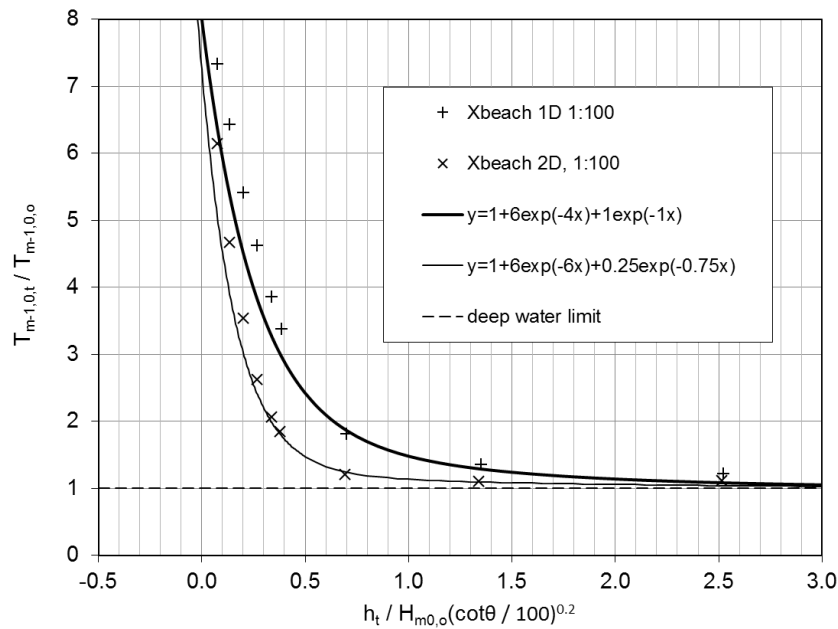
371

$$\frac{T_{m-1,0,t}}{T_{m-1,0,o}} - 1 = 6 \exp(-4 \tilde{h}) + \exp(-\tilde{h}), \quad (4)$$

372

373 Two exponential terms are required to fit the data well both for the shallow and the extremely
 374 shallow foreshores. It can be seen that for extremely shallow conditions, the first term at the
 375 right hand side is dominant, and for shallow conditions the second term. When the equation is
 376 used for shallow foreshores ($\tilde{h} > 1$), only the second term on the right-hand side of eq. (4)
 377 can be used. The root-mean-square variation (σ) of the measurements compared to the fit (μ)
 378 varies linearly from σ/μ 0.18 at $\tilde{h} = 0$, to $\sigma/\mu = 0$ at $\tilde{h} = 4$. In Figure 5, the $\pm 2\sigma$ lines are

379 drawn. Eq. (4) gives slightly higher values than the best least-squares fit for all measurements.
 380 However, the numerical computations gave slightly larger values for $T_{m-1,0,t}$. In Figure 6, the
 381 1D XBeach results are shown. It can be seen that the XBeach computation results follow the
 382 line of eq. (4) as well. From the data collapse of the different sources, it seems that the
 383 spectral wave period can be predicted fairly well for long-crested waves using the
 384 normalizations that were used. The water level at the toe that is reported, is the still water
 385 level before the tests, so without wave set-up. Therefore, negative water levels are given in
 386 Figure 5.
 387



388
 389 Figure 6. Numerical calculations of evolution of wave period $T_{m-1,0,t}$ as function of relative water depth for long-
 390 crested (XBeach 1D) and short-crested (XBeach 2D) waves.

391

392 4.1 Influence of directional spreading

393 All measurement data discussed until now were obtained from flume tests, i.e. long-crested
 394 wave conditions. However, for short-crested seas the generation mechanisms of the low-
 395 frequency wave energy may be different (see Section 2.3). Therefore additional 2D
 396 computations have been performed with XBeach. A snapshot of a 2D XBeach calculation
 397 with short-crested waves is given in Figure 7. The computational XBeach results with short-
 398 crested waves have been included in Figure 6. It can be seen that the increase of $T_{m-1,0,t}$ is less
 399 than that for the short-crested waves, and occurs much closer to shore than that for the long-
 400 crested waves. The equation for the fit given in Figure 6 for the short-crested waves has a
 401 similar shape as eq. (4) and is given by:

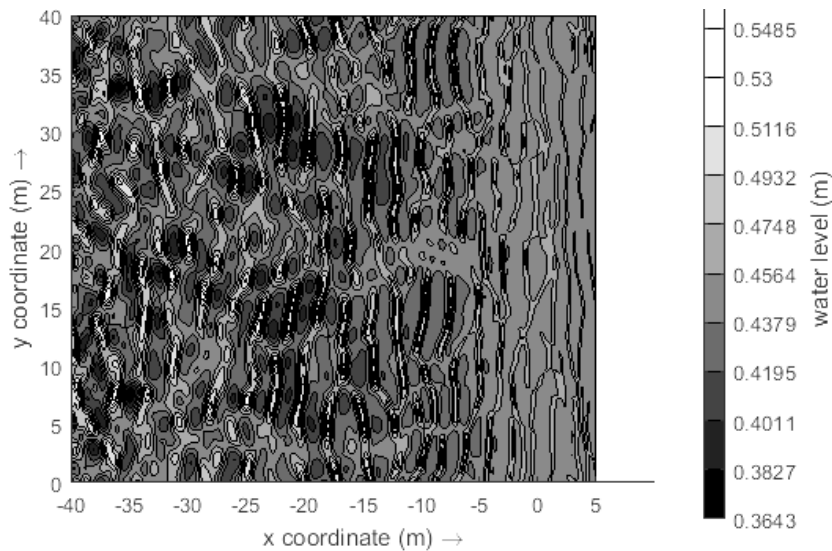
402

$$\frac{T_{m-1,0,t}}{T_{m-1,0,0}} - 1 = 6 \exp(-6\tilde{h}) + 0.25 \exp(-0.75\tilde{h}) , \quad (5)$$

403

404 Some existing data of a commercial project at Deltares (2011) is given in Figure 8 (squares
 405 and circles). Otherwise identical tests were done with short- and long-crested waves on a
 406 shallow foreshore. The results are plotted together with the fits of eqs. (4) and (5). For these
 407 few measurements on a shallow foreshore, the wave period $T_{m-1,0,t}$ agrees rather well with eqs.
 408 (4) and (5).

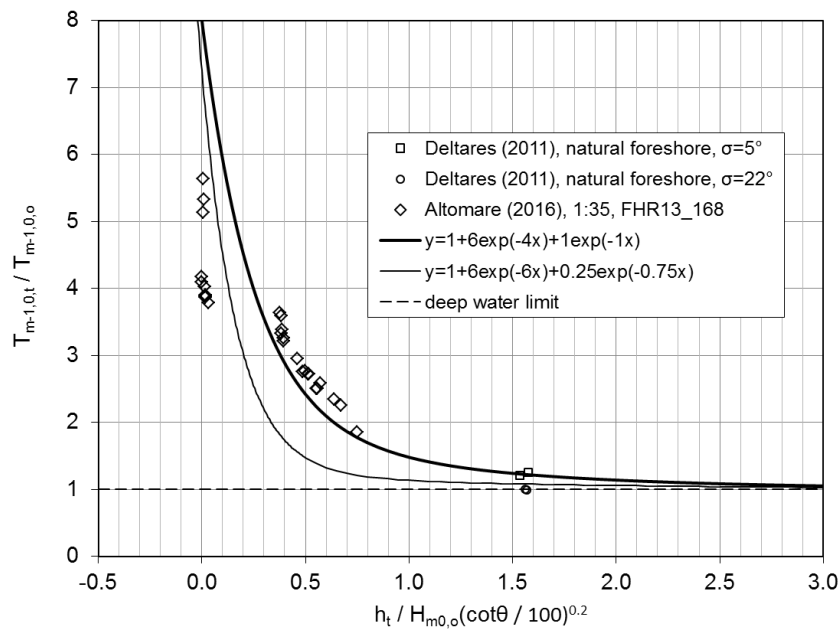
409



410

411 Figure 7. Snapshot of short-crested wave field over a shallow foreshore computed by XBeach (model scale).

412



413

414 Figure 8. Evolution of wave period $T_{m-1,0}$ as function of relative water depth compared to non-straight foreshores.

415 4.2 Influence of non-straight foreshore

416 Some scarce data was obtained with non-straight foreshores. This data is compared to the fits
417 for the straight foreshore in Figure 8.

418

419 The data of Deltares (2011) that is presented in Figure 8 was obtained for an irregular natural
420 foreshore. The few measurements represent shallow water conditions. Despite the irregular
421 nature of the foreshore, the resulting wave period, which has a limited influence of this
422 shallowness, is still comparable to the formula.

423

424 In the foreshore of test series FHR 13_168 (Altomare et. al. (2015) a change in foreshore
425 slope was situated at depths of 7 to 10 times the offshore wave height. The results for the very
426 shallow foreshore conditions are close to the general trend of eq. (4). However, the results for
427 the extremely shallow foreshore conditions are much lower than eq. (4). It is not clear
428 whether this change in foreshore slope (at a rather deep level) or another influence has altered
429 the wave period evolution for these tests with the lower water levels. This aspect is further
430 discussed in the next section.

431

432 5. Discussion

433 First we discuss whether the application linear wave generation influences the results. The
434 tests of Chen et al. (2016) and most of Altomare et al. (2016; FHR13_116, FHR00_025) were
435 made using linear wave generation. Using this kind of wave generation might increase the
436 value of $T_{m-1,0,t}$, as spurious low-frequency waves are created. However, the tests with linear
437 wave generation do not seem to have a different trend than those with 2nd order wave
438 generation. Only the results from dataset 13_168 with 2nd order wave generation (Altomare et
439 al., 2016), show much lower values of $T_{m-1,0,t} / T_{m-1,0,o}$ for extremely shallow water at the toe,
440 see Figure 8, while the results did align for the very shallow foreshore cases. However, in
441 these tests also the foreshore was not straight. In the XBeach computations, which do have 2nd
442 order wave generation, the wave periods increase to even somewhat larger values than were
443 measured for extremely shallow foreshore depths. So from these results it seems that the 1st
444 order wave generation does not have a large influence.

445

446 For more complicated cross sections, such as bar systems, eq. (5) has not yet been validated
447 by the few data point presented in Section 4. However, as the bed slope has a relatively small
448 influence in the parameter defined in eq. (3), it could be expected that the main trend might
449 hold for somewhat more complex geometries. The few data points that were given in Section
450 4.2 do seem to corroborate this. De Bakker et al. (2016) also observed that the influence of a
451 concave or convex foreshore on the low-frequency wave energy evolution was much less than

452 that of the (average) slope. Furthermore, for oblique wave attack, refraction will influence the
453 waves.

454

455 The degree of mildness of a foreshore slope can be estimated using a steepness parameter like
456 β_b in eq. (2). As eq. (2) was developed for regular bound waves (bichromatic primary waves),
457 it is assumed that the mean breaker depth h_b is equal to $2H_{m0,o}$, and that the angular frequency
458 of the bound long waves is $\omega = 2\pi/5T_{m-1,0,o}$. Using these assumptions, a rough estimate of the
459 steepness parameter β_b for the present tests was 0.02 to 0.35, with values lower than 0.3 for
460 more than 90% of the tests. In Section 2.3, it was discussed that for $\beta_b < 0.3$ the mild-slope
461 type of long-wave generation according to Longuet-Higgins and Stewart (1962) occurs.
462 Hence, we can conclude that the present equation is valid for mild slopes. It is not known
463 whether the development of the mean wave energy period $T_{m-1,0}$ will be the same for steeper
464 slopes than presently studied (1:35). In terms of a newly defined slope parameter, the limiting
465 slope for the present equation is obtained by rewriting $\beta_b < 0.35$ including the previously
466 mentioned assumptions for long wave period and breaker depth, which yields as range of
467 validity:

468

$$\theta T_{m-1,0,o} \sqrt{\frac{g}{H_{m0,o}}} < 0.62 . \quad (6)$$

469

470 Most tests were done with JONSWAP spectra that are characterized by a relatively narrow
471 peak. Other spectral shapes than JONSWAP were only included for $h_t / H_{m0,o} > 0.67$ (Van Gent,
472 1999a), but for this region there was a good data collapse. Hence, it seems that the wave
473 period $T_{m-1,0,t}$ is not very dependent on the offshore spectrum type. These different spectral
474 shapes were double-peaked spectra, that considered of two superimposed JONSWAP spectra
475 with the same peakedness. So, strictly speaking, the comparable results for the single-peaked
476 and double-peaked spectra might be due to the fact that each peak leads to the same type of
477 low-frequency wave generation without much interaction between the peaks. Hence, spectra
478 with separate broader peaks may still yield somewhat different wave periods $T_{m-1,0,t}$.

479

480 6. Conclusions

481 The spectral mean wave energy period $T_{m-1,0}$ has become accepted as a characteristic period
482 when describing the hydraulic attack on coastal structures. A prediction formula has been
483 derived for the wave period $T_{m-1,0}$ on shallow to extremely shallow foreshores with a mild
484 slope. A shallow foreshore is defined here as a bathymetry seaward of a structure that is
485 deeper than $h_t/H_{m0,o} = 4$, a very shallow foreshore as $h_t/H_{m0,o} < 1$, and an extremely shallow

486 foreshore as $h_t/H_{m0,o} < 0.3$. A mild slope of the foreshore is defined here as
487 $\theta T_{m-1,0,o}\sqrt{g/H_{m0,o}} < 0.62$ (see Figure 1 for the nomenclature). The prediction formula for
488 $T_{m-1,0}$ was determined based on tests and calculations for straight linear foreshore bed slopes
489 and perpendicular wave attack. The wave period $T_{m-1,0}$ increases drastically when the water
490 depth decreases, up to about 8 times the offshore value for extremely shallow foreshores. This
491 increase of $T_{m-1,0}$ with decreasing depth was somewhat less for milder slopes. For short-
492 crested wave fields the strong increase of the wave period $T_{m-1,0}$ starts closer to shore (at
493 smaller water depths) than for long-crested wave fields. For some cases with double-peaked
494 offshore spectra and irregular foreshores the increase of the wave period $T_{m-1,0}$ with
495 decreasing depth follows the same trend as for long-crested waves. However, it is
496 recommended to determine and/or extend the range of validity of the formulations for
497 different degrees of short-crestedness, spectral peak width, average foreshore slope, and
498 foreshore slope irregularities. A good prediction of the wave period $T_{m-1,0,t}$ will improve the
499 capability to make (conceptual) designs for coastal structures on shallow to extremely shallow
500 foreshores.

501

502 **Acknowledgements**

503 We thank the anonymous reviewers for their constructive comments, and dr. Marion Tissier
504 and dr. James Salmon for proof-reading the manuscript. The data of Chen et al. (2016) that
505 are used in this paper were obtained in research project Hydraulic impact of overtopping
506 waves on a multi-functional dike, project number 12176 (1.1.1), funded by technology
507 foundation STW.

508

509 **List of symbols**

510	f	: frequency	[s ⁻¹]
511	g	: gravitational acceleration	[ms ⁻²]
512	h	: water depth	[m]
513	h_b	: a mean breaking depth	[m]
514	H_{m0}	: spectral significant wave height, = $4\sqrt{m_0}$	[m]
515	k_p	: wave number based on the peak period, = $2\pi/(gT_p^2/2\pi)$	[m ⁻¹]
516	L_o	: fictitious offshore wave length, = $g/2\pi T_{m-1,0,o}^2$	[m]
517	m_n	: n th order moment of surface elevation	[m ² /s ⁿ]
518	P	: wave energy flux	[Wm ⁻¹]
519	$s_{m-1,0,o}$: offshore wave steepness, = $H_{m0,o} / \frac{g}{2\pi} T_{m-1,0,o}^2$	[-]
520	S	: the spectral density of the water surface elevation	[m ² s]
521	t	: time	[s]
522	$T_{1/3}$: significant wave period, mean period of the highest third of the	
523		waves in a record	[s]
524	T_m	: mean wave period	[s]
525	$T_{m-1,0}$: spectral mean wave energy period, = m_{-1}/m_0	[s]
526	T_p	: peak wave period	[s]
527	β_b	: kind of surf-similarity parameter for bound long waves	[-]
528	η	: the surface elevation	[m]
529	θ	: foreshore slope	[rad]
530	μ	: mean value	
531	σ	: standard deviation	
532	ω	: angular frequency (of bound long waves)	[s ⁻¹]
533			
534	o	: subscript indicating an offshore location	
535	t	: subscript indicating a location at the toe of a structure	
536			

537 **References**

- 538 Altomare, C., T. Suzuki, X. Chen, T. Verwaest, A. Kortenhuis. (2016) *Wave overtopping of sea dikes*
539 *with very shallow foreshores*. Coastal Engineering, Volume 116, October 2016, pp. 236-257,
540 ISSN 0378-3839.
- 541 Battjes, J.A. (1969) *Discussion of paper by J.N. Svašek: Statistical evaluation of wave conditions in a*
542 *deltaic area*. Proc. Symp. on Research on Wave Action. Vol. 1, paper 1. Waterloopkundig
543 Laboratorium (presently Deltares), Delft, the Netherlands.
- 544 Battjes, J.A. and Groenendijk, H.W. (2000). *Wave height distributions on shallow foreshores*. Coastal
545 Engineering 40(3), pp 161–182.
- 546 Battjes, J.A., H.J. Bakkenes, T.T. Janssen, and A.R. van Dongeren (2004) *Shoaling of subharmonic*
547 *gravity waves*. Journal of Geophysical Research 109, C02009, doi:10.1029/2003JC001863.
- 548 Chen, X., B. Hofland, W.S.J. Uijttewaai. (2016) *Maximum overtopping forces on a dike-mounted wall*
549 *with a shallow foreshore*. Coastal Engineering 116, October 2016, pp. 89–102.
- 550 CIRIA, CUR, CETMEF. (2007) *The Rock Manual. The use of rock in hydraulic engineering (2nd edition)*.
551 C683, CIRIA, London
- 552 De Bakker, A. T. M., M. F. S. Tissier, and B. G. Ruessink (2016) *Beach steepness effects on nonlinear*
553 *infragravity-wave interactions: A numerical study*. J. Geophys. Res. Oceans, 121, 554–570,
554 doi:10.1002/2015JC011268.
- 555 Dekker, J., S. Caires, M.R.A. van Gent (2007) *Reflection of non-standard wave spectra by sloping*
556 *structures*. Proc. Coastal Structures 2007.
- 557 Deltares (2011). Technical report no. 1204569-000-HYE-0012. Deltares, October 2011.
- 558 EurOtop (2016) Manual on wave overtopping of sea defences and related structures. An overtopping
559 manual largely based on European research, but for worldwide application. Authors: Van der
560 Meer, J.W. (ed.), Allsop, N.W.H., Bruce, T., De Rouck, J., Kortenhuis, A., Pullen, T.,
561 Schüttrumpf, H., Troch, P. and Zanuttigh, B. Pre-release of 2nd edition, October 2016
562 www.overtopping-manual.com.
- 563 Goda, Y. (1975) Irregular wave deformation in the surf zone. Coastal Engineering in Japan
564 18, 13–26 (JSCE)
- 565 Goda, Y. (2009) *Derivation of unified wave overtopping formulas for seawalls with smooth,*
566 *impermeable surfaces based on selected CLASH datasets*. Coastal Engineering 56(4), pp.
567 385–399.
- 568 Herbers, T. H. C., S. Elgar, and R. T. Guza (1994) *Infragravity-frequency (0.005-0.05 Hz) motions on*
569 *the shelf. Part I: Forced waves*. J. Phys. Oceanogr., 24, 917–927.
- 570 Holterman, S. R. (1998) *Wave runup on dikes with a shallow foreshore. (In Dutch) Golfoploop op*
571 *dijken met ondiep voorland*. MSc thesis, TU Delft, Delft. <http://repository.tudelft.nl>

572 Longuet-Higgins, M. S., and R. W. Stewart (1962) *Radiation stress and mass transport in gravity*
573 *waves, with application to 'surf beats'*. J. Fluid Mech., 13, 481–504.

574 Mansard and Funke (1980). *The measurement of incident and reflected spectra using a least squares*
575 *method*. Proceedings 17th International Coastal Engineering Conference, ICCE, pp. 154–172.

576 Munk, W. H. (1949), Surf beats, Eos Trans AGU, 30, 849– 854.

577 Pozueta, B., M.R.A. van Gent, H. Van den Boogaard, J.R. Medina (2005) *Neural network modelling of*
578 *wave overtopping at coastal structures*. Coastal Engineering 2004: proc. 29th int. conference
579 Lisbon, Portugal, 19-24 September 2004, Vol.4 ; p. 4275-4287

580 Roelvink, D., A. Reniers, A. van Dongeren, J. Van Thiel de Vries, R. McCall, J. Lescinski (2009)
581 *Modeling storm impacts on beaches, dunes and barrier islands*. Coastal Engineering 56,
582 1133–1152.

583 Smit, P., Stelling, G., Roelvink, J., Van Thiel de Vries, J., McCall, R., Van Dongeren, A., ... & Jacobs,
584 R. (2010). *XBeach: Nonhydrostatic model: Validation, verification and model description*.
585 Tech. rept. Delft University of Technology, Delft, the Netherlands.

586 Stelling, G. and M. Zijlema (2003). *An accurate and efficient finite-difference algorithm for non-*
587 *hydrostatic free-surface flow with application to wave propagation*. International Journal for
588 Numerical Methods in Fluids 43(1): 1-23.

589 Suzuki, T., C. Altomare, T. Verwaest, K. Trouw and M. Zijlema (2014) *Two-dimensional wave*
590 *overtopping calculation over a dike in shallow foreshore by SWASH*. Proceedings ICCE
591 2014.

592 Symonds, G., D. A. Huntley, and A. J. Bowen (1982) *Two dimensional surfbeat: Long wave*
593 *generation by a time varying breakpoint*. J. Geophys. Res., 87, 492– 498.

594 TAW (2002) *Technical Report Wave Run-up and Wave Overtopping at Dikes*. Delft, the Netherlands.
595 Report of the Technische Adviescommissie voor de Waterkeringen.

596 Tucker, M. J. (1950), *Surf beats: Sea waves of 1 to 5 min. period*. Proc. R. Soc. London, Ser. A, 202,
597 565– 573.

598 U.S. Army Corps of Engineers (2002) *Coastal Engineering Manual (CEM)*. Engineer Manual 1110-2-
599 1100, U.S. Army Corps of Engineers, Washington, D.C. (6 volumes).

600 Van Dongeren, A., J. Battjes, T. Janssen, J. van Noorloos, K. Steenhauer, G. Steenbergen, and A.
601 Reniers (2007) *Shoaling and shoreline dissipation of low-frequency waves*. Journal of
602 Geophysical Research, Vol. 112.

603 Van Gent, M.R.A. (1999a) *Physical model investigations on coastal structures with shallow*
604 *foreshores*. Delft Hydraulics (Deltares) Report H3608, January 1999, Delft.

605 Van Gent, M.R.A. (1999b), *Wave run-up and wave overtopping for double peaked wave energy*
606 *spectra*. Delft Hydraulics (Deltares) Report H3351-2, January 1999, Delft.

- 607 Van Gent, M.R.A. (2001), *Wave run-up on dikes with shallow foreshores*. ASCE, Journal of
608 Waterway, Port, Coast. & Ocean Engineering., Vol.127, No. 5.
- 609 Van Gent, M.R.A., Smale, A.J. and Kuiper, C. (2004). *Stability of rock slopes with shallow foreshores*.
610 In: J.A. Melby (ed), Proc 4th int coastal structures conf, Portland, OR, 26–30 Aug 2003.
611 ASCE, Reston, VA.
- 612 Zanuttigh, B., and J.W. van der Meer (2008) *Wave reflection from coastal structures in design*
613 *conditions*. Coastal Engineering Volume 55, Issue 10, October 2008, Pages 771-779
- 614 Zijlema, M., Stelling, G.S. and Smit P. (2011) SWASH: An operational public domain code for
615 simulating wave fields and rapidly varied flows in coastal waters. Coastal Engineering 58.
616 pp. 992-1012.