A robust optimization approach to synchromodal container transportation

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by

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Abstract

This thesis addresses synchromodal planning at operational level from the perspective of a logistics service provider. The existing infrastructure and the transportation activities are studied and modeled as an optimization problem with simultaneous vehicle routing and container-to-mode assignment. A special characteristic of this problem is the uncertain data. In other words, it is assumed that the release times of the containers belong to an uncertainty interval, and no further statistical information is available. The problem is then classified according to an extensive framework previously developed within the project. An extensive body of literature is reviewed to identify current modeling approaches and their theoretical and practical limitations. This literature study shows that, although discrete time models have been intensively investigated, there are few studies which propose continuous modeling of time. The container routing problem is modeled as a mixed integer program with explicit time variables and lateness penalties. A robust formulation is then proposed to eliminate the uncertain parameters from the objective function and constraints. By solving the new model exactly, with the aid of an optimization solver, robust solutions are obtained corresponding to transportation plans which remain feasible for any realization of the release times within the pre-specified uncertainty interval. In order to introduce some flexibility in the transportation plan, the continuous time variables are modeled as affine functions of the uncertain parameters. The resulting two-stage decision model is tested for a small-sized instance in both situations, with high and low lateness penalties. The computational results show that the adjustable robust model yields on the one hand, route-dependent adjusted solutions for the case of penalized lateness, and on the other hand, a direct improvement of the objective function for the case of tolerated lateness. The results suggest that the adjustable robust optimization framework has sufficient potential to model the synchromodal container routing problem. This thesis concludes with addressing some of the limitations of the proposed model and indicating concrete approaches for countering them.

Preface

This thesis report is the final step towards obtaining my master's degree at Delft University of Technology. Carrying out this project has been an interesting and eventful journey, and I would like to address a few words of thanks to those who have helped me along the way.

First of all I would like to express my gratitude to my supervisors Etienne de Klerk (TU Delft), Frank Phillipson (TNO) and Alex Sangers (TNO) for providing the guidance, interest and knowledge that were so needed in my project, and for allowing me to pursue my ideas freely. Moreover, I would like to thank Tina Nane for her willingness to be part of my thesis committee.

Special thanks go to the entire Cybersecurity and Robustness department at TNO for providing a great work environment. I would like to thank Kishan and Lianne, my closest collaborators in the project, and the rest of the interns for the very happy and friendly atmosphere.

Now that my studies have come to an end, I realize that moving to Delft was probably one of the best decisions I ever made. Besides getting a good education, I had the chance of meeting so many special people to whom I greatly owe my happiness. I am grateful to my best friend Elena for the continuous support, to Piotr, Dennis, and the guys, for kindly adopting me in their group, to Beatrice and Blane, for the laughs and the home-cooked dinners. I have also been most fortunate to cross paths with Tom, to whom I would like to thank for all the love and encouragement.

Finally, my outmost gratitude goes to my parents and little brother, who have managed to support me in every way, despite the few thousands kilometers between us. This thesis is dedicated to them.

I. Chiscop
Delft, August 2018

Contents

Introduction

1.1. Synchromodality context

Freight transportation plays an essential role in supply chains by providing the efficient movement of feedstock, goods and finished products between producers and consumers. In the European Union (EU) particularly, freight transport accounts for almost 4.5% of the gross domestic product (GDP), whilst the shipping carries 90% of the EU's foreign trade [10]. However, freight transport also raises a number of issues such as pollutant emissions, noise and congestion, which are mainly due to the road transport. A few figures illustrate this assertion. In 2014 about 49% of the total freight transportation in EU countries was done via road, 11.7% via rail, 4.3% via inland waterways and 31.8% by sea[1] [29]. In terms of pollution, 72.9% of Greenhouse gas (GHG) emissions are due to road transport, 12.8% to maritime and 0.5% due to railways [2]. To address both the issues of congestion and polluting emissions, a modal shift has become desirable [48]. In order to explain this concept, we will briefly review the existing transport modes.

Nowadays freight transport is mostly carried out using containers of standardized dimensions. These can be loaded and unloaded, stacked, transported efficiently over long distances, and transferred from one mode of transport to another (container ships, rail transport flatcars, and semi-trailer trucks) without being opened. The handling system is completely mechanized such that all handling is done with cranes and special forklift trucks. All containers have their own identification number and are tracked using computerized systems. These aspects make containers a preferable choice for goods transportation. The transportation chain of such containers is partitioned in three different segments [44]: *pre-haul* (first mile for the pickup process at the customer's warehouse for instance), *long-haul* (transit of containers between different ports) and *end-haul* (last mile for the delivery process at the distribution center). In most cases, the origin or destination of containers is located in the hinterland and therefore the pre-haul and end-haul transportation is carried out by road. For the long-haul however, multiple transportation modes are available such as road, rail and waterways. In this scenario, we distinguish several types of transportation whose terminology is well-established in literature. We distinguish between unimodal transportation (transporting load by means of only one transportation mode) and multimodal transportation (using multiple modes). We further elaborate on different types of multimodal transportation. In *intermodal* freight transportation a load is transported from origin to destination in one transportation unit without handling the goods themselves when changing modes [44]. The three segment container transport chain previously described is an example of intermodal transport. *Co-modal* transportation as defined in [50], is the intelligent use of two or more modes of transport by a (group of) shipper(s) in a distribution system, either on their own or in combination, in order to obtain the best benefit from each mode, in terms of overall sustainability. Synchromodal freight transportation is the next step in terms of development, based on an efficient combination of intermodal and co-modal transportation. The Platform Synchromodality provides the following definition: "Synchromodality is the optimally flexible and sustainable deployment of different modes of transport in a network under

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[1]There is a certain amount of freight transport carried out by cargo aircrafts. However this is not relevant for the scope of this thesis.

the direction of a logistics service provider, so that the customer (shipper or forwarder) is offered an integrated solution for his (inland) transport." [3]. Synchromodality emphasizes the following aspects: the usage of various transport modes available in parallel to provide a flexible transport solution, the entrustment the logistics service provider with the choice of transportation mode and the possibility to switch in between transportation modes in real time [6].

Kind of transport Level of coordination shippers	Multimodal transport (general term)	
	Use of different modes in one transport from A to B	Use of different modes in a network
No operational logistics coordination between shippers: 1-to-1 link (chain) between user and provider of multimodal transport	**Intermodal transport**	**Co-modal transport**
Operational logistics coordination between shippers: many-to-many link (network) between users and providers of multimodal transport	**Synchromodal transport**	

Table 1.1: Intermodal, co-modal and synchromodal transport [50].

In view of the existing types of transportation, the modal shift previously mentioned refers to reducing the number of containers transported by road in the long-haul by dispatching them on barges or sea-vessels in a smart and efficient way based on the cooperation of shippers. In other words it is a transition from unimodal transport to either intermodal, co-modal or synchromodal transportation, depending on the resources and the cooperation of the agents in the transportation network. The necessity of this shift has also been recognized by some port authorities [48]. In [4], the Port of Rotterdam Authority presented their goal to reduce the total number of containers transported by truck between the terminals in Rotterdam and inland destinations in North-West Europe from 55% in 2010 to 35% by 2035. For this purpose, a synchromodal network of rail and inland waterway connecting The Netherlands, Belgium and Germany was initiated by a consortium led by the Europe Container Terminals (ECT) in Rotterdam [50]. The Extended Gate Services (EGS) network is based on the partnership between shipping lines and inland terminals [1]. The inland terminals of Amsterdam, Duisburg, Venlo, Moerdijk and Willebroek act as virtual extensions of the Rotterdam-based deep sea terminal, in such a way that containers are transshipped in minimal time from the deep sea terminal in Rotterdam to the inland terminals.

To support the continuous development, improvement and expansion of this network with advanced planning methodologies and predictive methods, the Netherlands Organization for Scientific Research has granted a five year project entitled *Complexity Methods for Predictive Synchromodality* (Comet-PS). From 2017 until 2022, this project aims to explore and exploit the benefits of synchromodal transportation by developing models, methods and tools based on predictive data and stochastic decision making to provide solutions to planning problems arising throughout the transportation network. A partner in this project is the Netherlands Organization for applied scientific research (TNO). TNO works in close cooperation with one of the transport service providers involved in this project and therefore, this thesis will focus on understanding and modeling their planning process and producing results that can eventually serve as a decision support tool for their planners.

1.2. Problem description
Synchromodal transportation can be studied from multiple perspectives. There are several agents acting in the transportation network, each with their own modes and terminals/warehouses but sharing the existing infrastructure. Although synchromodality entails collaboration between all these parties, this is not always the case. Therefore, it is necessary to understand how much information is actually available and shared, and what kind of optimization objectives are desired. The information within the network is available globally or locally. If the information is locally available, it means that only the agents themselves know, for example, where they are or what their status is at a certain time. If the information is global, this information is also known to the network operator, to all other agents or both. Furthermore, if all agents need to be individually optimized, the optimization objective is local. If the optimization objective is global, the best option for the entire network is the desired outcome.

Figure 1.1: Different models of a synchromodal network [33].

The logistics service provider (LSP) whose activity is serving as a case study in this thesis, is interested in reducing its own overall costs but has certain knowledge of the other agents in the network. This corresponds to a selfish approach to synchromodality as described in [33] and illustrated in Figure 1.1. Given these facts, the following question arises: how can the logistics service provider optimally plan his transportation activities in order to minimize the associated costs? By investigating the characteristics of the problem further, we can develop this question into a proper research inquiry. In the following subsection we give a description of the practical setting behind the activity of the LSP and identify the optimization problem in their planning process based on the information that was made available for us.

1.2.1. Practical setting
A logistics service provider is a company that uses its resources to offer and perform transportation services of goods from origin to destination. The company usually manages the goods being transported along the entire way and is responsible for storage and handling. In our case, the LSP has a few inland terminals and one warehouse. Moreover, it has a fleet of trucks and several kinds of chartered barges for transporting containers between the deep-sea or inland terminals and different customer warehouses in the hinterland. These barges may have different capacities. For instance, the larger ones may transport up to 156 TEU[2] in three layers.

The LSP receives transportation requests from customers on a daily basis. These requests consist of one or more standardized containers to be picked up at a terminal, and then transported either to another terminal or to the customer's warehouse. The transport between terminals is usually carried out by barge and, when this is not possible, by truck. The way in which these orders are handled within the LSP administration can be visualized in diagram 1.2.
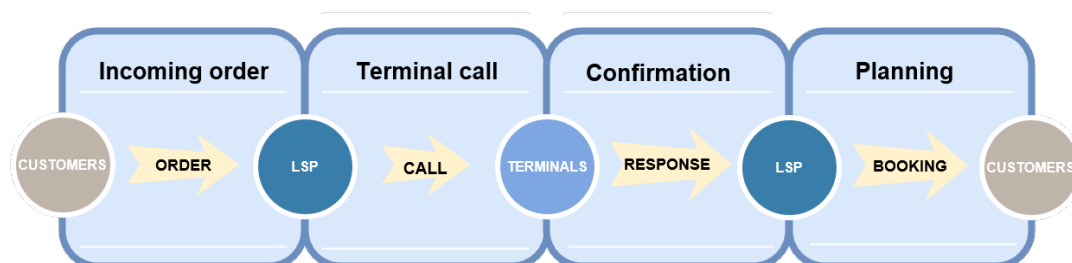

Figure 1.2: Administration of a transportation request at the LSP.

When a transportation order is received by the LSP the amount of information accompanying it may

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[2]Twenty feet equivalent unit.

vary. In general, the destination and due date namely, the latest time at which the containers should arrive at their destination, are always specified. Moreover, the terminal from where these containers should be picked up, the time at which they are available for pickup and the shipping company may be indicated. However, this is not always the case. If the pickup location is known, then the planners of the LSP will make a call towards that particular terminal in order to request a date and time-slot for the pickup. Depending on the working volume and the number of vessels to handle, the terminal may either confirm the proposed appointment, confirm the appointment on a different date, or not confirm an appointment at all. It is worth mentioning that the last two scenarios occur quite often in practice. Depending on the particular terminal, the time difference between the requested time and the confirmed time, otherwise known as the planning delay, can reach up to ten days. After a response has been received from the terminal, then the LSP planners need to evaluate the current positions and loads of the available barges and decide which one will execute the pickup and when, and inform the customer about this. This process is difficult and the resulting plan is often subjected to change due to the uncertain elements in the network. The planner aims to schedule the available barges in such a way that all containers are picked up on time, then timely delivered to their destinations with a minimum amount of costs. These costs emerge from the usage of transportation modes, stationing at the terminals before the actual handling of containers and the eventual failure of meeting the due dates at the customers.

Our goal is to make use of all the practical information available in order to formulate an optimization problem. Therefore, we need to further elaborate on what kind of elements are influencing the planning and what information is available to a planner at the moment that a decision must be made. To achieve this, we employ the framework for synchromodal problems developed in [33], in which the authors distinguish between *resources* and *demand elements*. Intuitively, the resource elements refer to the available transportation modes namely, barges or trucks, whilst the demand elements consist of freight containers[3]. The features of these elements may be:

- *controllable*: since we are discussing a decision problem, at least one element of the system must be in control. This can be for instance the allocation of demands to resources.

- *fixed*: a fixed element does not change within the scope of the problem.

- *dynamic*: a dynamic element might change over time or due to a change in the state of the system (e.g. the amount of containers changes the travel time of a barge), but this change is known or computable beforehand.

- *stochastic*: a stochastic element is not necessarily known beforehand. For instance it is not known when transportation orders will arrive, but the arrivals occur according to a Poisson process.

- *irrelevant*: It might occur that for certain problems not all elements are taken into consideration to model the system. Then these elements are irrelevant.

We will closely follow the classification in [33] to describe all the elements occurring in the planning process of the LSP. However, not all elements encountered in the practical setting are encompassed by this framework.

Resource elements

- *Resource type*: In this study, the LSP owns a fleet of barges of different known capacities and a uniform fleet of trucks. One may distinguish here between owned and subcontracted resources.

- *Resource features*: The resource capacities are fixed. The schedules of the barges and trucks are not fixed. Therefore the resource origin and resource destination are controllable elements. However, the resource departure time, resource travel time and resource arrival time are not controllable. This is a consequence of the delays which may occur either when receiving a confirmation from the terminal, or at the terminal itself, when the handling time takes longer than expected (this can happen due to a crane malfunction for instance). We will classify these elements outside the framework as *uncertain*, since there is no information available concerning their distribution. Finally, we also have a resource price. Here we can distinguish between the

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[3]In this thesis the demand elements will always correspond to one container.

price for employing a certain resource which is a fixed amount (per day for instance) and the price for handling services provided at the terminals. The latter depends on the load to be handled, which is an uncertain element at the beginning of the planning period.

- *Terminal Handling time*: This refers to time required to handle different types of modes at the terminal. It includes both the waiting time and the time allocated for loading/unloading containers. This is also an *uncertain* element since there exist incoming orders which do not specify the pickup time or locations. For instance, it may be the case that a barge is waiting at a terminal to pick up some containers which have not arrived there yet.

Demand elements

- *Demand type*: The LSP under study can transport containers of different sizes, of either 1 TEU or 2 TEU in load. Therefore, this element is fixed.

- *Demand-to-Resource allocation*: The assignment of containers to barges is essentially a decision that a planners have to make. Therefore, it is a controlled element.

- *Demand features*: The destination of a container, as well as its volume (in TEU) and due date at the customer to whom it belongs, are fixed elements. The demand origin (pick up terminal of a container) and its release date (moment in time at which it can be loaded on a barge) are uncertain elements. This uncertainty emerges from the missing data in the transportation order, as customers simply do not specify it.

- *Demand Penalty*: This term refers to costs that are incurred when the due date at the destination for a container is not met. Since these costs are in general customer-dependent, we can classify this element as dynamic.

The resource and demand elements described are the main input for creating a schedule for the barges and trucks. However, the planning process does not only rely on the information that is available, but also on the moment at which this information becomes available. At the beginning of the planning period, the planner knows the exact locations of all the barges and trucks in the fleet, their capacity, and has a list of orders with specified destinations and due dates to be picked up sometime in the next nine days. Moreover, at every moment in time, a planner has an estimation of the maximum and average delay of the deep sea terminals (based on historic data in the last thirty days). This is the initial amount of knowledge. As time progresses, more information becomes available. That is, pickup locations along with release times of containers are revealed, and terminals send confirmation for appointment times. Moreover, new transportation orders may come in, which are also required to be executed within the next 9 days. This information can become available at any time so the planner must create a schedule that can handle real-time switches.

Given this practical setting, one may formulate the decision making of the LSP planners as an optimization problem in which a routing of transportation modes and an assignment of the containers to modes must be provided under uncertain data in such a way that the total delay and costs are minimized.

1.2.2. Base instance
In order to be able to develop a mathematical model and later on explore solution methods, we consider the following simplified instance obtained by reducing the size of the real-life problem and introducing some assumptions. The network comprises of the following elements:

- *2 customers denoted C_1, C_2*: their physical location is known and it is accesible only by truck.

- *2 deep-sea terminals denoted T_1, T_2*: deep-sea vessel arrive here and unload the containers that belong to the two customers.

- *1 container terminal operated by the LSP denoted T*: barges leave from here and go to the deep-sea terminals to pick up containers.

- *1 hinterland terminal operated by the LSP denoted D*: it is the central terminal of the LSP, closest in distance to any customer.

We notice here that there is one main difference between the container terminal and the hinterland terminal of the LSP. The container terminal is the located in the port, nearby deep-sea terminals. On the other hand, the hinterland terminal is situated further away on the continent, in the proximity of customers. This is illustrated in Figure 1.3.

Figure 1.3: Geographical display of the network.

The LSP has the following resources:

- *3 barges*: all with capacity of 20 units. Two of the barges at the terminal T whilst the other one is situated at the central terminal D. There is a fixed cost per kilometer[4] traveled by a barge.

- *unlimited trucks*: all with capacity 1. There is a fixed cost per kilometer traveled by a truck.

Suppose we are given two transportation orders with the following specifications:

1. Customer C_1 asked the LSP to pickup 30 containers from T_1. The terminal has confirmed a time window for the pickup: $[10, 11]$[5]. These containers have an uncertain release time. They will be simultaneously released sometime in the interval $[10, 11]$. This order needs to arrive at the customer by time unit 20.

2. Customer C_2 has 10 containers to be picked up from terminal T_2. This terminal has also confirmed a time window for the pickup: $[15, 16]$. All 10 containers are already available. This order needs to arrive at the customer warehouse by time unit 20.

When developing this base model we have made several assumptions. We discuss them and their relation with the real practical setting below.

- The planning period starts at midnight or otherwise interpreted, at time step 0 and covers one full day, until time step 24 respectively.

- We assume fixed time windows at the deep-sea terminals. In practice we saw that a terminal can either answer an appointment call or not. In this scenario, we assume that we have confirmed appointment calls at the beginning of the planning period.

- If a barge arrives either too early or too late at a deep-sea terminal, it can be handled right away. So we assume that there is no waiting time involved.

- We assume that there is no handling time.

- Once it has been loaded, a barge may leave the deep-sea terminal right away.

- At any point in time, there are trucks available at every terminal, which can transport the released containers to other locations.

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[4]We will elaborate on transportation costs of barges and trucks later in the thesis.
[5]We will take a time unit as being one hour. Therefore, regard this interval as the time between 10:00 and 11:00.

- There is a waterway connecting the terminals. The customers' warehouses can only be reached by truck.

- The travel times in between any two locations of the barges and trucks are known.

Given this simple instance, we are interested in minimizing the overall costs and the total delay at the customers. In order to maintain a uniform objective, we can associate costs with the delay in such a way that the final objective will represents the costs overall. This simple instance will serve as a starting point in developing a mathematical model that determines an assignment of containers to transport modes, and also a specific routing of the containers. Whilst this base model is not of any practical relevance, it will serve as a basic tool to understand, and later on, to incorporate more complex features of the transport network.

After analyzing the base instance, we understand that our choice for modeling approaches is somewhat restricted by the lack of probabilistic knowledge. In this case, we will study the container routing problem from a robust perspective. In other words, since we cannot employ stochastic models, we will look at robust optimization techniques.

1.3. Research question

In view of the instance example introduced in the previous section, we can formulate the following research question:

How can we simultaneously provide a container-to-mode assignment and a routing of modalities under uncertain data and with the objective of minimizing the total costs?

We can provide an answer by first tackling these sub-questions:

1. How can we model the simple instance described in Section 1.2.2 in order to encompass all the assumptions?

2. What solution methods can be used to obtain a schedule and container assignment for every modality?

3. What can be said about the quality and practical relevance of our solution?

4. Does the chosen approach successfully incorporate elements of synchromodality?

1.4. Report structure

This report has been organized in the following way. Chapter 1 gives an introduction into synchromodal container transport and presents a simplified instance of the general problem under study. The relevant literature concerning synchromodal and intermodal transport planning at different levels is reviewed in Chapter 2. Chapter 3 gives an overview of the most common modeling approaches for container transport encountered in literature. A deterministic problem formulation is presented in the fourth chapter, followed by the robust approach in Chapter 5. The results of our computational study are summarized and discussed in Chapter 6. The final part of this report, namely Chapter 7, is dedicated to the conclusions and some recommendations for related future work.

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Literature review

In this chapter we review some of the existing literature on synchromodal problems in order to present the current state of the research progress in this field. As discussed in the previous chapter, synchromodality is a relatively new concept which aims to enhance the efficiency of intermodal and unimodal transport networks. Therefore, developing methods for a synchromodal planning relies heavily on advancing and refining the existing approaches to well-studied intermodal and unimodal problems. Due to this fact, we will include in our summary not only literature which relates to synchromodality directly, but also papers that tackle freight transportation problems in a more general perspective, without focusing on the real-time switching or cooperation between the agents present in the transportation network.

Synchromodal transportation problems can be classified according to the time span of the decisions which must be made. Crainic and Laporte [28] describe three levels of decision problems: strategic (long term), tactical (medium term) and operational (short term) decisions. *Strategic* planning at a company level refers to decision taken by the highest level of management, involving a large capital investment over a long period of time. Examples of strategic decisions include the design of the physical network, the location of main facilities (terminals, rail yards etc.) and resource acquisition (fleet of barges and trucks). *Tactical* planning problems refer to the rational and efficient allocation and use of the existing resources over a medium term horizon. Tactical decisions concern aspects such as the general operating rules for each terminal, the work allocation among terminals and the choice of route and service to operate. Finally, *operational* planning is performed by local management and concerns activities which are about to take place. The most important operational decisions relate to scheduling the transport and maintenance services, routing and dispatching of vehicles and allocating resources (freight) to transport modes. These three types of decisions can be visualized in Figure 2.1. We will use this distinction to further structure our literature survey.

2.1. A look towards synchromodality

Tavasszy, Behdani and Konings [47] give a first outlook upon synchromodality. They provide a detailed description of the trends in intermodal European transportation from the beginning of containerized barge transport on the river Rhyne in the 1960's to present. To keep up with the growing trend of freight demand, the number of barge and rail terminals in ports is increasing. The authors regard this as the main cause for fragmented container flows and transport inefficiency, as a barge needs to load freight at multiple terminals. In this scenario, they highlight the necessity for an integrated view in the planning and management of different modalities. This refers to the combination of transport services on different modalities to provide a customized service to a shipper with a particular type of product to transport and a specific set of logistics requirements. Integrated service planning along with the subsequent real-time switching between modalities are the two elements of synchromodality discussed in detail in this paper.

Van Riessen, Negenborn and Dekker [49] provide an overview of relevant topics and research opportunities in synchromodal container transport. This overview is however limited in the sense that the
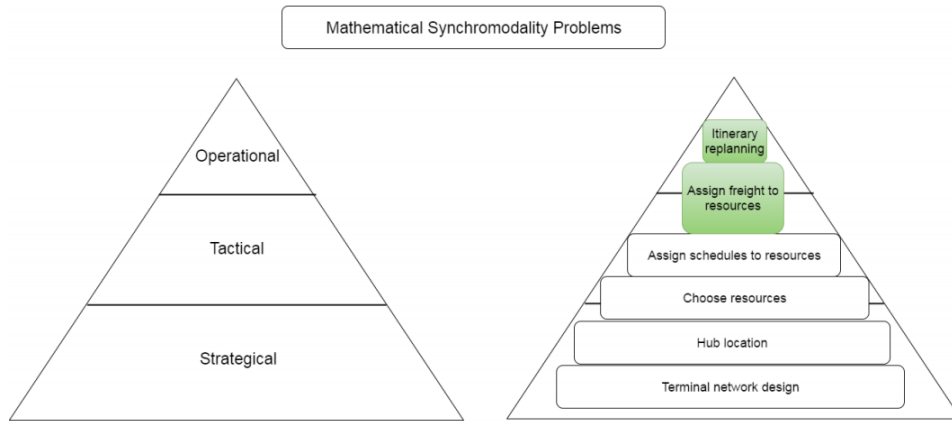
Figure 2.1: Mathematical synchromodal problems, from [33]. The problems listed with a green background are to be studied in this thesis.

authors consider synchromodal problems arising in the case of the hinterland network of European Gateway Services (EGS), a subsidiary of the Rotterdam container terminal operator ECT. In relation to this case, they identify three elements required for enabling synchromodal planning: integrated network planning (create a plan based on a combination of services on different modalities is created by a network operator), real-time network planning (create a plan that can be updated in real-time, as soon as new information becomes available) and planning flexibility (persuade customers to allow flexible planning of their transportation orders). The latter elements is somewhat ambiguous and requires further clarification. Planning flexibility mainly refers to customers fully entrusting the network operator with the decision on a transportation plan by specifying when and where their freight needs to arrive, but not the modality by which it should travel. This is also sometimes referred to as an *a-modal booking*. The authors briefly describe potential approaches to be explored for each of the three proposed research topics.

Pfoser, Treiblmaier and Schauer [38] provide a list of critical success factors of synchromodality based on a literature review. They conclude that synchromodal transport processes can be enabled by satisfying the following prerequisites: mutual collaboration and trust of the agents in the network, sophisticated dynamic planning, high quality ICT/ITS technology for information exchange between stakeholders, smartly designed physical infrastructure, comprehensive legal and political framework, awareness and mental shift of customers and fair pricing of synchromodal transport services. Based on four of these factors, a comparative analysis was performed to assess how far the implementation of synchromodality is in the ports of Rotterdam, Antwerp and Hamburg [18]. One of the main conclusions here is that the EGS network in Rotterdam is limited in its synchromodal services in the sense that it is affiliated to the ECT terminal. The collaboration with other terminals is still missing, hence synchromodality in Rotterdam is realized within a delimited geographic area and within only one organizational entity namely, EGS.

It has been already stressed that sharing and exchanging information between stakeholders is a an essential aspect of synchromodal planning. However, this is difficult to achieve due to a lack of mutual trust. Other factors which affect the coordination of the stakeholders such as the unequal distribution of the costs and benefits of coordination are discussed in [31]. Even when stakeholders are willing to share their information there is a still a question of what, when and how it should be transmitted. Singh and van Sinderen [43] have developed a data format which can represent real-time information on weather, location, traffic, water levels and disasters in a concise way. This is particularly useful for representing disruptions, to which an automated planning system can react in real-time. However, developing this format is an on-going research project and has yet to be implemented in practice.

Synchromodal problems often describe complex activities that take place within a network and the amount of information relevant to the behavior of that network may vary. Therefore it is useful to de-

scribe these problems in a comprehensive and consistent way. In [33] a framework is proposed to both classify the problems and ease the search for solution methods. We have already made use of this framework in Chapter 1, to describe the elements in our problem.

This is a brief overview of the most relevant papers in which synchromodality is the main topic. A more extended literature review, also including a categorization of papers according to pre-requisites, activities and effects of synchromodality can be found in [42]. For a more general literature review on multimodal freight transport planning, concerning both planning problems and solution methods, the reader is referred to [44].

2.2. Intermodal transportation

As mentioned earlier in this chapter, synchromodal problems can be seen as a further extension or enhancement of the intermodal problems which do not incorporate cooperation between stakeholders, real-time switching and a-modal booking. Therefore, it is worthwhile noting relevant papers covering intermodal transportation.

The article by Bektas and Crainic [26], gives an overview of an intermodal network together with a detailed description of its main agents. They discuss the perspective of shippers, who generate demand for transportation. Although intermodal services are composed of both the combination of transportation modes and the transfer facilities, a shipper regards this as one integrated service and has the same expectations in terms of speed, reliability and availability as from an unimodal service. The authors explain that this represents a major challenge for the carriers, who need to provide cost-effective and potentially customized services to the shippers. Some of planning issues that carriers need to solve are discussed here namely, the design of the physical infrastructure (at the strategic level) and the allocation of resources and routing of vehicles (at the operational level). Container terminals are also discussed as a transfer service provider in the network.

Crainic and Kim [27] address several problems arising in intermodal transportation and classify them according to the strategic, tactical and operational level of planning. Port dimensioning is discussed as an important strategic problem. Within tactical planning, they investigate system and service network design for carriers and seaport container terminals. The operational planning issues are focused on repositioning of the empty containers. The authors review several models for these problems, incorporating the particular features one at a time.

2.2.1. Strategic planning problems

Strategic planning problems concern decisions over long time horizons. In the context of intermodal transportation, these refer to infrastructure design and placement of terminals, warehouses or yards. In practice, intermodal transport networks function as a *consolidation* system in order to maximize the utilization of modalities [44]. This means that instead of shipping every cargo directly from origin to destination, low volume cargo is placed in a handling center where it is bundled into larger freight flows and only then further transported to its destination. Such a handling center is called a *hub* and routing cargo through a hub enables the usage of higher capacity modalities. Deciding on where to locate such hubs is the main strategic planning problem studied in literature. A detailed review of hub locations problems is given in [30]. The authors classify hub network design problems according to the number of hub nodes, the capacity of the hubs, the cost of locating the hub nodes, the allocation of a non-hub node to hub-nodes and the cost of connecting non-hub nodes to hub nodes. Mathematical models are presented for each considered problem and an overview of exact and heuristic methods is provided a the end of their paper.

As expected, we could find no literature on strategical planning problems which also incorporates elements of synchromodality. Synchromodal transport requires adjusting the planning as soon as new information becomes available whilst strategic planning involves long term decisions whose result is not subjected to change. Therefore, it is understandable that analyzing strategical problems in synchromodal transportation is practically the same as in intermodal transportation. However, strategic planning problems remain outside the scope of this thesis and will not be further mentioned in this

work.

2.2.2. Tactical planning problems

As mentioned in [44], tactical planning problems aim to optimally utilize the existing infrastructure by selecting the services to be operated and the corresponding transportation modes, allocating the necessary capacities and planning their itineraries and frequencies. The main decisions to be made at the tactical level can be further categorized as in [28] and are as follows:

1. Service selection: choosing the services together with the routes on which they will be operated.

2. Traffic distribution: routing of freight (commodities) through the network. Here it must be decided which services are used, which terminals are passed through and the specific operations to be carried out on the freight are.

3. Terminal policies: establishing the policy that regulates the operations executed at a terminal.

4. Empty balancing: repositioning empty vehicles.

5. Crew and motive power scheduling: assigning crew and vehicles to the planning.

These decision problems are often referred to as *Service Network Design*. Crainic [25] gives a thorough review of these problems, their mathematical formulations and solution methods. In his paper we see that service network design problems are often modeled as *Fixed-Cost Capacitated Multicommodity Network Design Problems* on space time graphs. These can be quite difficult to solve due to the large number of decision variables, which increases rapidly as instances become larger and the considered time horizon is extended. This fact is recognized by Wieberneit [51] in her more recent review on service network design problems. She presents a review of solution methods and suggests some heuristic alternatives. In [53], scheduled service network design problems with stochastic freight demand are considered.

Synchromodality aspects have also been incorporated in the study of tactical planning problems. In [11], the authors propose a service design problem with stochastic freight demand and allow for rerouting. They develop a two-stage stochastic program and introduce a set of integer decision variables for vehicle rerouting in the second optimization stage. Moreover, the paper by Behdani et al. [12] presents a mathematical model for a synchromodal service schedule. The model includes both the frequency and capacity of different transportation modes, determines an optimal sequence and timing for the services of each transport mode but also an assignment of containers to services.

2.2.3. Operational planning problems

Operational planning problems concern short-time decisions that need to be made by local management to schedule services and maintenance activities, allocate freight to resources and provide route for these resources. Operational planning problems are very similar to tactical problems in terms of the decisions to be made. However, Crainic and Laporte [28] highlight the main difference between these two levels of planning: "while in the tactical planning one is concerned with the *where* and *how* issues (selecting services of given types and traffic routes between spatial locations), here one is interested above all in *when* issues: when to start a given service, when the vehicle arrives at destinations or an intermediary terminal, when is the traffic delivered, etc."

As already mentioned, freight routing is one of the main planning decisions at the operational level and it refers to assigning optimal routes for the resources to move commodities from their origins to the respective destinations through the existing infrastructure network. The review by Sun et al. [46] provides an overview of optimization models and solution methods for freight routing planning problems in multi-modal transportation networks. The authors classify the problems according the their objective function, amount of commodities, commodity integrity[1], network capacity and transportation service

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[1]This term is used to determine whether a commodity can be split into multiple loads or not. For example, a batch of containers with the same origin and destinations may be regarded as one commodity, but each individual container might be allowed to take a different transportation route. We would call this a splittable commodity.

pattern. They consider studies with both deterministic and stochastic parameters in the model formulations and finally present a successful genetic algorithm for the multi-modal transportation freight routing problem. The earlier review of Caris et al. [20] has a broader vision on models for decision support in intermodal transport, including topics such as policy support terminal design, drayage operations and ICT innovations along freight routing. Bontekoning et al. [17] review a few papers dealing with intermodal rail-road freight routing from either the perspective of a shipper (methods providing an optimal route for a specific shipment) or of a network operator (methods which seek optimal routing for minimizing the costs of the entire network).

In a multi-modal transportation network, a single commodity corresponds to a specific origin-destination pair [46]. Models which optimize the routing of a single commodity have a limited and rather selfish perspective in the sense that they minimize costs for a specific customer. Xiong and Wang [54] focus on the best route selection for a shipment through the network of a multimodal carrier. The problem is modeled as a mixed integer linear program incorporating time windows at both terminal and customer locations. The authors opt for a Taguchi-based [5] multi-objective genetic algorithm which minimizes the total transportation cost and the total transportation time. Although such a model provides optimal transportation service for the commodity in cause, in a real-life network spanning a large geographic region, there are many commodities to be transported. The paper of Chang [21] tackles this aspect and formulates the intermodal routing problem as a multi-objective multimodal multicommodity flow problem with time windows and concave costs. The model assumes fixed transportation mode schedules, splittable commodities and calculation of transportation cost based on the effect of economies of scale (EOS). The latter refers to the fact that the cost of a transportation mode depends on the weight being transported by that mode. Chang divides the original problem into a set of smaller sub-problems using relaxation and decomposition techniques. Since constraint relaxation induces infeasible solutions to some of the sub-problems, a reoptimization algorithm is applied to re-assure feasibility. An advantage of this method is that it can efficiently solve large-scale instances as well, as shown by the author through an example network of 112 nodes and 407 directed arcs.

The intermodal freight routing problem can also be formulated as a weighted constrained shortest path model as in the paper by Cho et al.[22]. They suggest a dynamic programming algorithm to minimize the transportation cost and time. Caramia and Guerriero [19] model freight routing as a capacitated vehicle routing problem with time windows. The authors consider a multi-objective function in which the travel time, the operative cost and a transportation mean sharing index have to be simultaneously optimized. A particular feature of this model is that it allows mode switching only at certain nodes in the network. They propose a heuristic algorithm based on local search, which is implemented in cascade first at the tactical phase (assigning modes to transportation paths) and then at operational phase (routing freight demands on the candidate transportation paths from the previous step).

In all the papers reviewed so far, a multiple objective was used as this is suitable to represent both the perspective of shippers who wish to minimize costs and ensure sufficient demand at their warehouses, and that of carriers who aim for an efficient routing and resource utilization. There are two approaches for multiple objective optimization treatment: weighted sum methods [19, 21, 22] and Pareto frontier methods [54]. The master thesis of Ortega del Vecchyo [37] models the assignment of freight containers to transportation modes as a multicommodity flow problem on a space-time network and studies different objectives to be optimized in the network: cost, linear-anti-flexibility, mean robustness and customer satisfaction. These attributes are quantified mathematically in a novel way and then a lexicographic method is used to obtain Pareto optimal solutions. The author points out that Pareto solutions are particularly useful in the context of developing decision support models as they provide a planner multiple solutions to choose from.

A large amount of literature in multimodal transportation is focused on deterministic model formulations. In other words, parameters such as travel time, transshipping time at terminals and demand are usually assumed to be fixed. However, freight demands normally exhibit high uncertainty over time and travel times are often influenced by weather conditions or traffic congestion. Moreover, the time required for terminal operations is dependent on the resource distribution in the network. Therefore, uncertainties are common phenomena in a multimodal transportation network, thus contributing to the

complexity of freight routing problems. Uncertain parameters are often modeled as random variables which may have different realizations. Kooiman et. al [35] study the problem of assigning containers having stochastic release dates to barges with fixed schedules. The release dates are assumed to have a known uniform distribution which is dependent on their fixed due date. The authors present propose rule based decision making algorithms and a simulation approach. They conclude that for large instances, the simulation outperforms the rule based methods. The master thesis of Huizing [32] also explores the container-to-mode assignment problem assuming fixed schedules for the barges. He formulates a multicommodity flow problem in which the travel times of the modes are assumed to be normally distributed. Two classical methods are used for removing the uncertainty from the proposed stochastic linear program namely, replacing the random variables first by their expected values and then by pessimistic estimates of their values [36]. The results obtained were satisfactory as the solutions were within roughly 5% of the optimum value.

Sumalee et al. [45] study a multimodal transport network assignment with stochastic demand and supply in the context of urban public transportation. Although their model might not be suitable for freight transport, the authors describe mathematically the relationship between passengers' waiting times at stations and weather conditions. This dependence might potentially be used for modeling terminal waiting times. Moreover, the travel demands are assumed to follow independent Poisson distributions. The authors suggest that Lognormal and Normal distributions might be equally or more suitable choices.

Stochastic freight routing or more specifically, stochastic container-to-mode assignment, has already been studied in synchromodal context. Zhang and Pel [56] develop a model consisting of four components: a demand generator, a super-network processor, a schedule-based flow assignment module and a system performance evaluator. Transport demand is generated for a 24-hour period by random sampling from the annual transport demand (which is known). The super-network is used to represent the entire available infrastructure and schedules of transportation modes. A container-to-mode assignment is obtained by repeatedly solving a cheapest route problem. Rivera and Mes [40] study the problems of selecting services and transfers in a synchromodal network over a multi-period horizon. The synchromodality factor is introduced here by the assumption that every transportation order can be re-routed at any moment. Moreover, new transportation requests can enter the system during the planning horizon. This means that new information might become available to the planner at all times. It is assumed that the planner has probabilistic knowledge about the arrival of new transportation requests. The authors propose a Markov Decision Process model to represent his scenario and minimize the costs for the entire planning horizon using an approximate dynamic programming approach.

2.3. Contribution made by this work

As already mentioned, uncertainties are widely spread in multimodal transportation networks and highly influence the performance of operational activities. The vast majority of literature in synchromodal planning deals with deterministic scenarios whilst the few studies which address uncertain parameters, always assume that they follow a known distribution. Difficulties arise, however, when there is no knowledge of the distribution function. It may often be the case that real data is either faulty, insufficient or simply not available and then one cannot infer the distribution. In the field of vehicle routing, few studies present a robust optimization approach, modeling travel times as uncertain quantities [7, 8]. This thesis attempts to generate fresh insight into synchromodal planning under uncertainties by applying a robust optimization framework to a time-continuous freight routing model. To the best of our knowledge, this work is the first to address routing with uncertain release times of containers. The findings should make an important contribution to the field of robust intermodal transport by providing proof-of-concept that adjustable robust optimization can successfully model problem with simultaneous container assignment and vehicle routing, and provide solutions which remain feasible despite the uncertain release times.

3

Overview of existing models

This chapter provides an overview of the most commonly used formulations and methods for multi-modal freight routing at operational level. One approach seems to be particularly popular namely, multicommodity flow problems. We will present our base instance in these different formulations in order to argue how suitable they are for further encompassing more complex (uncertain) aspects.

3.1. Discrete-time multicommodity flow problems

The studies which model multimodal freight routing as multicommodity flow problems follow either a discrete [32] or a continuous time step approach [21]. Each of these approaches comes with its own advantages and drawbacks. Whilst a continuous time model enables straightforward modeling of time windows, waiting and service time, lateness etc., a discrete approach might yield a simpler and easier to solve model since the succession of activities in time is already embodied in the network structure. We illustrate and discuss both approaches using the instance presented in section 1.2.2. Some simplifications or additional assumptions will be made whenever the chosen formulation cannot encompass the complete scenario described by our base instance.

3.1.1. The minimum cost multicommodity flow problem on a space-time network

When the transportation modes follow a fixed schedule and all the parameters in the network are fixed, the freight routing problem can be modeled as a minimum cost multicommodity flow problem on a space-time graph. We provide the definition as given in [37]:

Definition 1. We call a graph $G = (V, A)$ a space-time network (or space-time graph) if its node set V is of the form $S \times \{1, 2, ..., T\}$, where S represents a set of distinct locations, $T \in \mathbb{Z}^+$ represents the amount of time steps, and every arc $((a, p), (b, q)) \in A$ satisfies $p < q$. We refer to the node (a, p) as location a at time p, and to T as the time horizon of G.

We can represent the available infrastructure and the fixed routes of the barges by directed arcs which traverse nodes describing physical locations at a particular point in time. In order to be able to further model our instance as a space-time network we allow the following simplifications:

1. Assume that there is no waiting nor handling time at a terminal, namely a barge can be loaded and leave right away.

2. Assume that all containers will become available at fixed points in time.

3. Suppose that the barges will follow a fixed schedule. That is, for each barge, there is a sequence of terminals to be visited at particular times.

4. Assume that the appointment times at customer are fixed and must always be met.

We incorporate these simplifications into our instance and model it as a space-time graph, as it can be seen in Figure 3.1. We further elaborate on the cost structure of the network. Since barges are
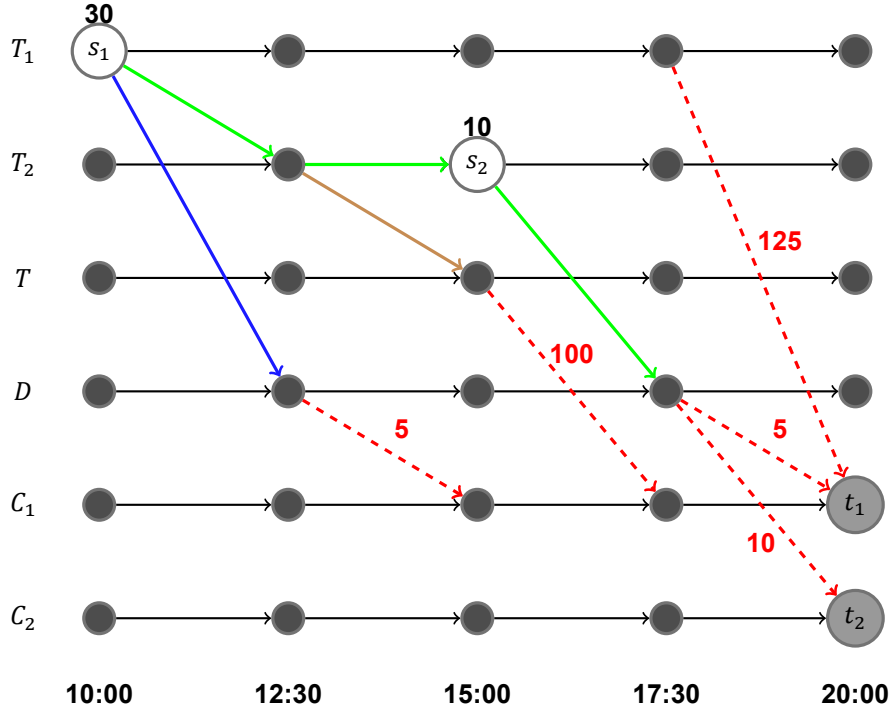
Figure 3.1: Simplified base instance on a space-time network. The sources s_1 and s_2 mark the release times and locations of the two transportation requests of 30 and 10 containers, respectively. The sinks t_1 and t_2 denote the due time and destination of these containers. The interpretation of the directed arcs in the network is as follows: black arc denote waiting arcs at a location with infinite capacity and cost zero. Similarly, color-filled arcs denote the routes of the three barges. Each of these arcs has capacity 20 and cost zero. Finally, the red dashed arcs mark the 'trucking' arcs. They have infinite capacity and fixed costs written in red.

assumed to have fixed schedules in this case, that will be carried out regardless of the incoming transportation requests, it is naturally to assume that the cost for sending one container by any barge is zero. As for the trucks, the situation is slightly different. A truck may only transport one container at a time, therefore we can assume that the cost for transporting one container by truck is equal to the cost of using that truck to travel the respective distance. The derivation of this cost is shown in Appendix A.

Due to the fact that we have removed all uncertainties from our base instance, the only goal in this simplified model is to minimize the costs associated with transporting containers by truck. This can be acheived by solving the *non-negative integral minimum cost multicommodity flow problem* (MCMCF) on the proposed space-time network.
The MCMCF problem involves simultaneously shipping multiple commodities through a single network such that the total flow is subjected to arc capacity constraints and has minimum cost. In a general setting consider some directed graph $D = (V, A)$ and a set of commodities K. We define the following decision variables and parameters:

1. Variables $x_{i,j}^k$ representing the amount of flow of commodity k that is being transported over the arc (i, j);

2. Parameters $c_{i,j}$ denoting the capacity of an arc (i, j), or simply said the maximal amount of flow that can be transported over this arc;

3. Labels s_k for the source node of commodity k and labels t_k for the unique destination node of commodity k;

4. Parameters $f_{i,j}$ denoting the cost of sending one unit of flow of any commodity over the arc (i, j);

5. Parameters d_k describing the demand of flow of commodity k that need to be transported through the network.

The MCMCF problem can then be described by the following integer linear program:

$$\min \quad \sum_{(i,j)\in A} \sum_{k\in K} f_{i,j} x_{i,j}^k$$

$$s.t. \quad \sum_{k\in K} x_{i,j}^k \leq c_{i,j} \qquad \forall (i,j) \in A \tag{3.1}$$

$$\sum_{(s_k,j)\in A} x_{s_k,j}^k = d_k \qquad \forall k \in K \tag{3.2}$$

$$\sum_{(i,t_k)\in A} x_{i,t_k}^k = d_k \qquad \forall k \in K \tag{3.3}$$

$$\sum_{(i,v)\in A} x_{i,v}^k = \sum_{(v,j)\in A} x_{v,j}^k \quad (\forall k \in K)(\forall v \in V \backslash \{s_k, t_k\}) \tag{3.4}$$

$$x_{i,j}^k \in \mathbb{N}_0 \qquad (\forall (i,j) \in A)(\forall k \in K) \tag{3.5}$$

Constraints (3.1) assert that the total flow of commodities on any arc cannot exceed the arc capacity. Equality (3.2) states that d_k units of flow of commodity k must leave the source. Analogously, equality (3.3) assert that the total demand of commodity k will reach its prescribed destination. Constraints (3.4) ensure flow conservation at every node in the network. Finally, the last set of constraints require that any amount of flow of any commodity to be transported must be a positive integer. These constraints emerge from the fact that we identify one unit of flow with one container. One can now understand that the optimal container-to-mode assignment is an instance of the MCMCF problem and its corresponding integer linear program can be solved with the aid of a solver. The solution thus obtained for the simplified base instance is shown in Figure 3.2.
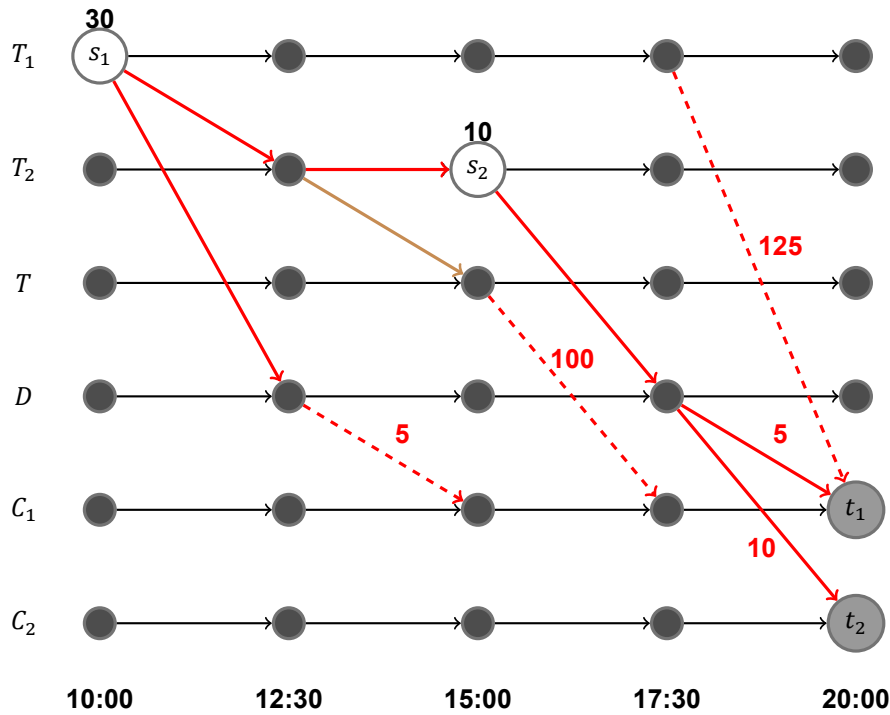
Figure 3.2: Solution of the simplified base instance on the space-time network. The red-filled arcs give the container assignment to modes. Wee see that 20 containers travel from terminal T_1 to the central terminal D and are then delivered by truck to the customer C_1. This operation has cost 100. The other 10 containers belonging to commodity 1 travel by another barge to terminal T_2, wait at this location while the other 10 containers are loaded, and then arrive at the central terminal D at time 17:30. The final delivery is again done by truck for both commodities resulting in a cost of 150. Therefore, this container assignment gives a total minimum cost of 250.

We have illustrated how the freight routing problem with fixed mode schedules and deterministic characteristics can be modeled as a MCMCF problem and solved exactly by means of an ILP solver. This approach can be extended to incorporate stochastic elements and we will demonstrate this in the following subsection.

3.1.2. The minimum cost multicommodity flow problem with stochastic elements

The previous arc-based formulation of the MCMCF problem on a space-time network may also include uncertainty in the transportation data in the form of stochastic parameters. As a simple example, we consider an incoming order of containers at a deep-sea terminal, that needs to be picked up by one of the barges of the logistics service provider. The exact time at which these containers will become available for pick-up is not known. However, the planner has some probabilistic knowledge of these release times. Therefore, one needs to address the problem of assigning containers to barges which follow fixed schedules, given that the release times of these containers follow known distributions. We illustrate this scenario in Figure 3.3, with the aid of a slightly adjusted version of the instance shown in Figure 3.1. Since the source node of commodity 1 is essentially modeled as a random variable with
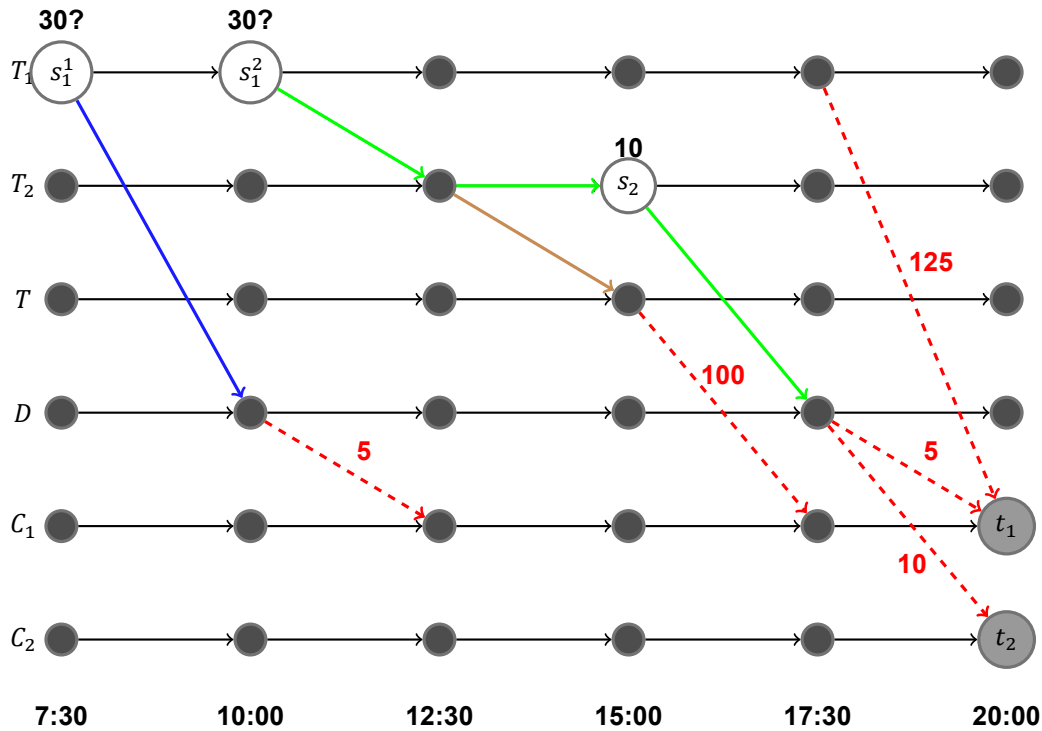
Figure 3.3: Adjusted base instance on a space-time network with stochastic release times. The thirty containers of commodity 1 can be released either at 7:30 or at 10:30. In other words, the source s_1 is found at the node s_1^1 with probability 1/3 and at node s_1^2 with probability 2/3. The ten containers of commodity 2 have a fixed release time at source node s_2. The meaning of color-filled, dashed arcs and black waiting arcs is kept as before.

a known distribution, we can no longer make use of an ILP solver to obtain a solution. Instead, it is desirable to find a way of reducing or removing the uncertainty in the release time of commodity 1. The classical methods of addressing uncertainty in linear programs include replacing the random variables by their expected values, replacing the random variables by pessimistic estimates of their values or reformulating the problem as a multi-stage stochastic program. These methods were already known and commonly used by early 1960s. For a more detailed description and a discussion on how these methods relate to each other, we refer to the work of Madansky [36]. These first two methods were implemented under the name of *Expected Future Iteration* and *Partially Pessimistic Future Iteration* for the MCMCF flow problem with stochastic travel times in the thesis by Huizing [32]. In Figure 3.4 we illustrate an application of the Expected Future Iteration. In this particular case, since the source node is a non-numeric parameter, the method reduces to replacing s_1 by the realization with the highest probability namely, s_1^2. Then the problem reduces to the deterministic MCMCF problem and can be solved with an ILP solver.

 The solution given in Figure 3.4 highlights the impact of uncertainty on the overall transportation cost. Had containers of commodity 1 been released earlier, at source node s_1^1, we would have obtained the same as in Figure 3.2 with cost 250. On the other hand, assuming that the release will be at the later time step s_1^2 yields a schedule of cost 2400. This striking difference in the total cost indicates that one
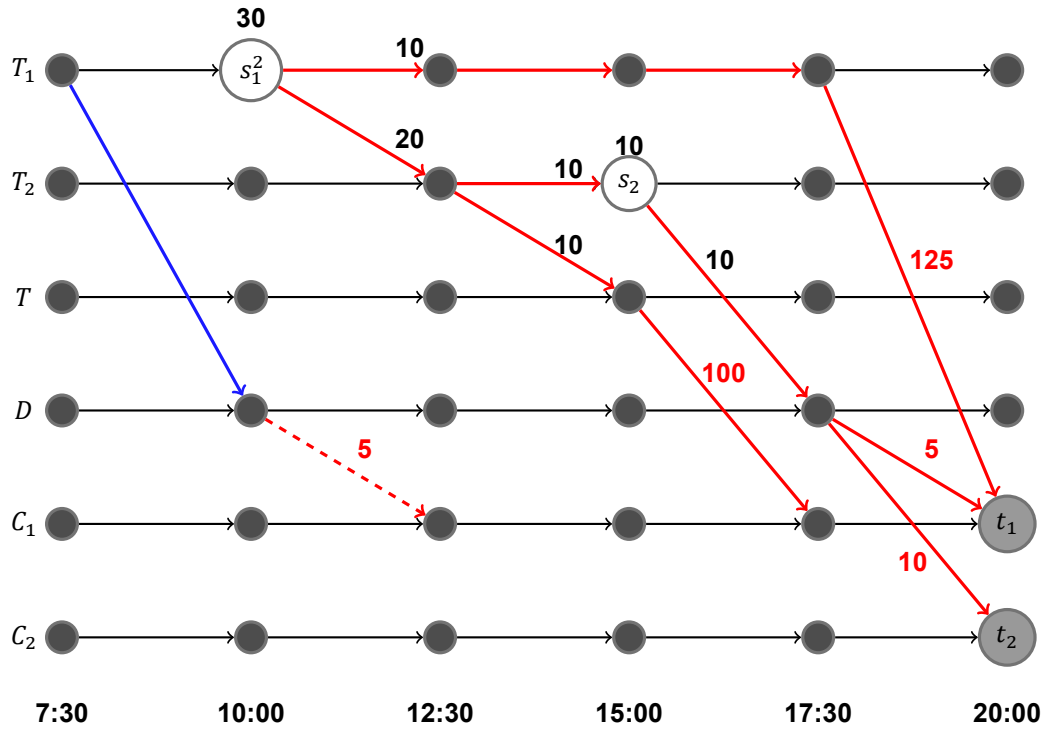
Figure 3.4: Solution of the simplified base instance on the space-time network with stochastic release times for commodity 1. The red-filled arcs give the container assignment to modes. The amount of commodity flow is written in black above the assigned arcs to indicate where transshipment takes place. Since we assume that commodity 1 is going to be released at the node with highest probability namely s_1^2, it becomes impossible to make use of barge 1. Therefore, the optimal solution is now to transport 20 containers of commodity 1 by barge 2 and truck the remaining 10. At terminal T_2, 10 containers of commodity 1 will be taken over by barge 3 whilst the rest remain on barge 2. The ten containers of commodity 2 will also travel by barge 2 and be trucked for the final destination from depot D. This assignment gives a total cost of 2400.

should handle uncertainties carefully. Consider this situation in a practical setting, at the actual decision level. If the logistics service provider assumes that the containers of commodity 1 will be released later simply based on the probabilistic knowledge, he or she may cancel the appointment at terminal T_1 or load the barge with other incoming container orders. However, if in reality the containers will actually be released earlier, this will result in a compromised appointment, which can affect the LSP's reliability, and in a drastic increase of the costs. Therefore, replacing random variables by the realization of highest probability might not always give a reasonable transportation plan.

So far, we looked at the problem of assigning containers to barges with fixed transportation schedules in two different scenarios: with fixed transportation elements and with stochastic release times of the commodities. In the MCMCF formulation, both problems can be solved exactly with an ILP solver, provided that one replaces the random variables by certain estimated values. Although this formulation is quite intuitive, and can be extended to account for flexible schedule of the transportation modes [32], it has two main drawbacks. The first and most obvious disadvantage of using a discrete time MCMCF formulation is the scaling of the model. In other words, if one wishes to represent arrival and departure times of barges accurately, considering that moving a container by crane from platform to barge require approximately two and a half minutes, then the time step to be considered should be of one or a few minutes. Since a transportation plan should be made for at least one day, this would result in a very large number of integer decision variables which is raises computational difficulties for the ILP solver. Secondly, all three methods commonly used to handle uncertainty in the data input of the problem rely on probabilistic information of this data. That is, one needs to have certain insight into the probability distribution of the uncertain parameters in order to be able to work with them. However, in practice this information might simply not be available. All in all, it appears that the MCMCF formulation on a space-time network is not sufficiently flexible to incorporate all the complex elements in the transportation network and requires a too high level of insight into the realizations of the uncertain parameters. In

the next subsection, we look into time-continuous models in order to allow for a better representation of the time parameters in the transportation network.

3.2. Multicommodity flow problems with continuous time variables

We have already established that the treatment of time is an important aspect in container routing problems. As the discrete-time models presented in the previous section were shown to be too limited to exploit in the context of a transportation network with uncertainties, we will now focus on finding time-continuous approaches, which are expected to be more suitable for modeling our base instance.

3.2.1. Multicommodity scheduled service network design

In her doctoral thesis, Sharypova [41] studies the problem of scheduled service network design for container freight transportation along inland waterways. She proposes a continuous-time mixed integer linear programming model which takes as input the transportation network, the available fleet of vehicles, the demand and supply of containers, the sailing time of vehicles, and the structure of costs, and provides a minimum cost container distribution plan together with a vehicle and container routing schedule. This service design formulation is intended for usage at a tactical level but can be re-interpreted to also serve short-term planning at operational level. We will present a slightly adjusted version of the model as given in [41], with the exception that some of the additional costs for vehicle utilization and handling activities at terminals have been removed.

Let $G = (V, A)$ denote a directed graph describing the transportation network. The vertex set V corresponds to the set of physical locations in our base instance namely $\{T_1, T_2, T, D, C_1, C_2\}$, whilst and arc (i, j) in the set A marks a direct[1] trip between the physical locations i and j. We denote by M the set of vehicles and by K the set of commodities. It is important to mention that a commodity is defined as a set of containers which have the same origin, destination, release time at the origin and due time at destination. We let D_k represent the number of containers in commodity k that need to be transported from the node of its origin the its destination. Furthermore, consider the following input parameters:

- $c_{i,j}^m$: unit transportation cost paid to transport one container by vehicle $m \in M$;

- h_i: unit handling cost at location $i \in V$;

- s_i: service time at node $i \in V$;

- d_i^k: demand of commodity $k \in K$ at node $i \in V$;

- $(o(k), d(k))$: origin-destination node pair of commodity $k \in K$;

- r_k: release time of commodity $k \in K$ at its origin;

- due_k: due time of commodity $k \in K$ at its final destination;

- $t_{i,j}^m$: traveling time of vehicle $m \in M$ from node $i \in V$ to node $j \in V$;

- c^m: maximum capacity of vehicle $m \in M$.

In particular, we define the demand of commodity k at node i in the following way:

$$d_i^k = \begin{cases} D_k & \text{if } i = o(k) \\ -D_k & \text{if } i = d(k) \\ 0 & \text{otherwise} \end{cases}$$

All the parameters described above will have a known fixed value except for the release times r_k. We assume that for every commodity $k \in K$, its release time lies in a bounded interval, namely $r_k \in [l_k, u_k]$ where the values of the bounds l_k and u_k are known. The set $V_m \subset V$ represents the subset of locations which are accessible by vehicle m.

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[1]We refer to a trip of a vehicle with no intermediate stops as a direct trip.

The following decision variables are introduced:

- $x_{i,j}^{k,m}$: integer variable describing number of containers of commodity $k \in K$ transported from location $i \in V$ to location $j \in V$ by vehicle $m \in M$;

- $y_{i,j}^m$: binary variable which takes value 1 if vehicle $m \in M$ travels from location $i \in V$ to location $j \in V$ and 0 otherwise;

- z^m: binary variable which takes value 1 is vehicle $m \in M$ is used in the transportation plan and 0 otherwise;

- A_i^m: continuous variable describing the arrival time of vehicle $m \in M$ at location $i \in V$;

- D_i^m: continuous variable describing the departure time of vehicle $m \in M$ from location $i \in V$;

- $q_i^{k,m,l}$: continuous variable describing the amount of containers of commodity $k \in K$ moved from vehicle $m \in M$ to vehicle $l \in M$ at location $i \in V_m \cap V_l$;

- $\theta_i^{m,l}$: binary variable which takes value 1 if a transshipment occurs between vehicle $m \in M$ and vehicle $l \in M$ at location $i \in V_m \cap V_l$;

- $\tau_i^{k,m}$: binary variable which takes value 1 if any container of commodity $k \in K$ is loaded on vehicle $m \in M$ at node $i \in \{o(k), d(k)\}$.

We further denote the set of outgoing arcs from node $i \in V$ by $V^+(i)$, and the set of ingoing arcs by $V^-(i)$. Given these parameters and decision variables, the container routing problem can be formulated as the following mixed integer program:

$$\min \quad \sum_{k \in K} \sum_{m \in M} \sum_{(i,j) \in A} c_{i,j}^k x_{i,j}^{k,m}$$

$$s.t. \quad \sum_{(j) \in V^+(i)} y_{i,j}^m - \sum_{j \in V^+(i)} y_{j,i}^m = 0 \qquad \forall m \in M, \forall i \in V_m \qquad (3.6)$$

$$\sum_{m \in M} \sum_{j \in V^+(i)} x_{i,j}^{k,m} - \sum_{m \in M} \sum_{j \in V^-(i)} x_{j,i}^{k,m} = d_i^k \qquad \forall i \in V, \ \forall k \in K \qquad (3.7)$$

$$\sum_{k \in K} x_{i,j}^{k,m} \leq c^m y_{i,j}^m \qquad \forall m \in M, \forall (i,j) \in A \qquad (3.8)$$

$$\sum_{k \in K} q_i^{k,m,l} > 0 \iff \theta_i^{m,l} = 1 \qquad \forall m,l \in K, \forall i \in V_m \cap V_l \qquad (3.9)$$

$$\sum_{j \in V^-(i)} x_{j,i}^{k,m} = \sum_{l \in K} q_i^{k,m,l} \qquad \forall m \in M, \forall k \in K, \forall i \in V_m \setminus \{d(k)\} \quad (3.10)$$

$$\sum_{j \in V^+(i)} x_{i,j}^{k,m} = \sum_{l \in K} q_i^{k,l,m} \qquad \forall m \in M, \forall k \in K, \forall i \in V_m \setminus \{o(k)\} \quad (3.11)$$

$$\sum_{l \in K} q_i^{k,m,l} = 0 \qquad \forall m \in M, \forall k \in K, \forall i \in \{o(k), d(k)\} \quad (3.12)$$

$$\sum_{j \in V^+(i)} x_{i,j}^{k,m} > 0 \iff \tau_i^{k,m} = 1 \qquad \forall m \in M, \forall k \in K, \forall i \in \{o(k)\} \qquad (3.13)$$

$$\sum_{j \in V^-(i)} x_{j,i}^{k,m} > 0 \iff \tau_i^{k,m} = 1 \qquad \forall m \in M, \forall k \in K, \forall i \in \{d(k)\} \qquad (3.14)$$

$$\theta_i^{m,l} = 1 \Rightarrow D_i^l - A_i^m - s_i \geq 0 \qquad \forall m,l \in M, \forall i \in V_m \cap V_l \qquad (3.15)$$

$$y_{i,j}^m \Rightarrow D_i^m + t_{i,j}^m - A_j^m \leq 0 \qquad \forall m \in M, \forall (i,j) \in A \qquad (3.16)$$

$$D_i^m \geq A_i^m + s_i \qquad \forall m \in M, \forall i \in V_m \qquad (3.17)$$

$$D_i^m \geq r_k \tau_i^{k,m} \qquad \forall k \in K, \forall i \in \{o(k)\}, \forall m \in M \qquad (3.18)$$

$$\tau_i^{k,m} \Rightarrow A_i^m \leq due_k \qquad \forall k \in K, \forall i \in \{d(k)\}, \forall m \in M \qquad (3.19)$$

$$\sum_{j \in V^+(i)} y_{i,j}^m \leq z^m \qquad \forall m \in M, \forall i \in V_m \qquad (3.20)$$

$$x_{i,j}^{k,m} \in \mathbb{N} \qquad \forall m \in M, \forall (i,j) \in A, \forall k \in K \qquad (3.21)$$

$$q_i^{k,m,l} \in \mathbb{N} \qquad \forall m,l \in M, \forall i \in V_m \cap V_l, \forall k \in K \qquad (3.22)$$

$$A_i^m, D_i^m \geq 0 \qquad \forall m \in M, \forall i \in V_m \qquad (3.23)$$

$$y_{i,j}^m \in \{0,1\} \qquad \forall m \in M, \forall (i,j) \in A \qquad (3.24)$$

$$\theta_i^{m,l} \in \{0,1\} \qquad \forall m,l \in M, \forall i \in V_m \cap V_l \qquad (3.25)$$

$$\tau_i^{k,m} \in \{0,1\} \qquad \forall k \in K, \forall m \in M, \forall i \in \{o(k), d(k)\} \quad (3.26)$$

$$z^m \in \{0,1\} \qquad \forall m \in M \qquad (3.27)$$

In order to describe the practical meaning of the (in)equalities above in a more efficient way, we will categorize them.

Objective function: The sum of transportation costs for all the containers is to be minimized.

Flow conservation constraints: Constraints (3.6) assert that a vehicle which is arriving at a location in the network, should also leave that location afterwards. Constraints (3.7) ensure that the demand of commodities is satisfied at every node in the network.

Vehicle capacity: Constraints (3.8) indicate that the amount of flow transported by a vehicle on any arc should not exceed the maximum capacity of the vehicle.

Transshipment conditions: Constraints (3.9) assert that there is transshipment from vehicle m to vehicle l if only only if there exist some containers to be moved from vehicle m to vehicle l. These constraints can be written in the following linearized form:

$$\sum_{k \in K} q_i^{k,m,l} \geq M_1(1 - \theta_i^{m,l}) + \epsilon \qquad \forall m, l \in K, \forall i \in V_m \cap V_l \qquad (3.28)$$

$$\sum_{k \in K} q_i^{k,m,l} \leq M_1 \theta_i^{m,l} \qquad \forall m, l \in K, \forall i \in V_m \cap V_l \qquad (3.29)$$

with M_1 being a large positive integer and ϵ a small positive constant. Typically, one may consider $M_1 = 1000$ and $\epsilon = 10^{-4}$. The equality in (3.10) describe what happens to containers of a certain commodity once they arrive at a location (which logically should not be their final destination since their trip ends here). That is, all containers of the same commodity k brought by vehicle m to location i should either be moved to other vehicles ($m \neq l$) or remain on board of vehicle m ($m = l$). Similarly, constraints (3.11) guarantee that upon their departure from node i, all containers of the same commodity are either transshipped to another vehicle, or continue their journey on the same vehicle m which originally brought them to i. Constraints (3.12) enforce that there is no transshipment possible at the origin and destination node of any commodity.

Commodity flow at origin and destination: Constraints (3.13) describe the fact that a container of commodity k can be on board of vehicle m at its origin node $o(k)$ if and only if the amount of containers which have left this node on board of vehicle m is positive. Analogously, constraints (3.14) states that if there is a positive flow of commodity k arriving at their destination on vehicle m, then this commodity is on board of vehicle m at node $d(k)$. We can linearize these conditions by the following inequalities:

$$\sum_{j \in V^+(i)} x_{i,j}^{k,m} \geq M_2(\tau_i^{k,m} - 1) + \epsilon \qquad \forall m \in M, \forall k \in K, \forall i \in \{o(k)\} \qquad (3.30)$$

$$\sum_{j \in V^+(i)} x_{i,j}^{k,m} \leq M_2 \tau_i^{k,m} \qquad \forall m \in M, \forall k \in K, \forall i \in \{o(k)\} \qquad (3.31)$$

$$\sum_{j \in V^-(i)} x_{j,i}^{k,m} \geq M_2(\tau_i^{k,m} - 1) + \epsilon \qquad \forall m \in M, \forall k \in K, \forall i \in \{d(k)\} \qquad (3.32)$$

$$\sum_{j \in V^-(i)} x_{j,i}^{k,m} \leq M_2 \tau_i^{k,m} \qquad \forall m \in M, \forall k \in K, \forall i \in \{d(k)\}, \qquad (3.33)$$

where M_2 and ϵ have the same meaning as before.

Vehicle synchronization for transshipment: Constraints (3.15) indicate that if there is a movement of containers from vehicle m to vehicle l at node i, then vehicle l may only depart with the loaded commodity if this has been already brought to node i by vehicle m. The linear version of this constraints is as follows:

$$D_i^l - A_i^m - s_i \geq M_3(\theta_i^{m,l} - 1) \qquad \forall m, l \in M, \forall i \in V_m \cap V_l. \qquad (3.34)$$

Validity of time variables: Constraints (3.16) impose that a vehicle m traveling from node i to node j should arrive at node j after it has departed from node i. Moreover, the time between these two events is at least as long as the time required by vehicle m to travel from i to j. Constraints (3.17) assert that a vehicle can depart from a location only after it has finished being loaded or unloaded there. Constraints (3.16) have the following linear equivalent:

$$D_i^m + t_{i,j}^m - A_j^m \leq M_4(1 - y_{i,j}^m) \qquad\qquad \forall m \in M, \forall(i,j) \in A. \qquad (3.35)$$

Time window constraints: Constraints (3.18) ensure that a vehicle can depart with containers on board only if they have become available for pick-up. Constraints (3.19) guarantee that all containers from a certain commodity arrive at the destination before the due date of that commodity and can be re-written into the following linear inequality:

$$A_i^m \leq due_k + M_5(1 - \tau_i^{k,m}) \qquad\qquad \forall k \in K, \forall i \in \{d(k)\}, \forall m \in M. \qquad (3.36)$$

Vehicle usage: Constraints (3.20) indicate that if a vehicle k is used at some point in the transport plan, then it should be served (load or unload containers) only once at every location node.

Solution space: Finally, constraints (3.21)-(3.27) define the range of every decision variable and thus, the entire solution space.

Once the non-linear constraints are replaced with their equivalent linear versions in (3.28)-(3.36), the model proposed by Sharypova becomes a mixed integer linear program which can be solved with the aid of a commercial solver when all the input parameters are given fixed values. For example, consider the simplified instance in Figure 3.5.

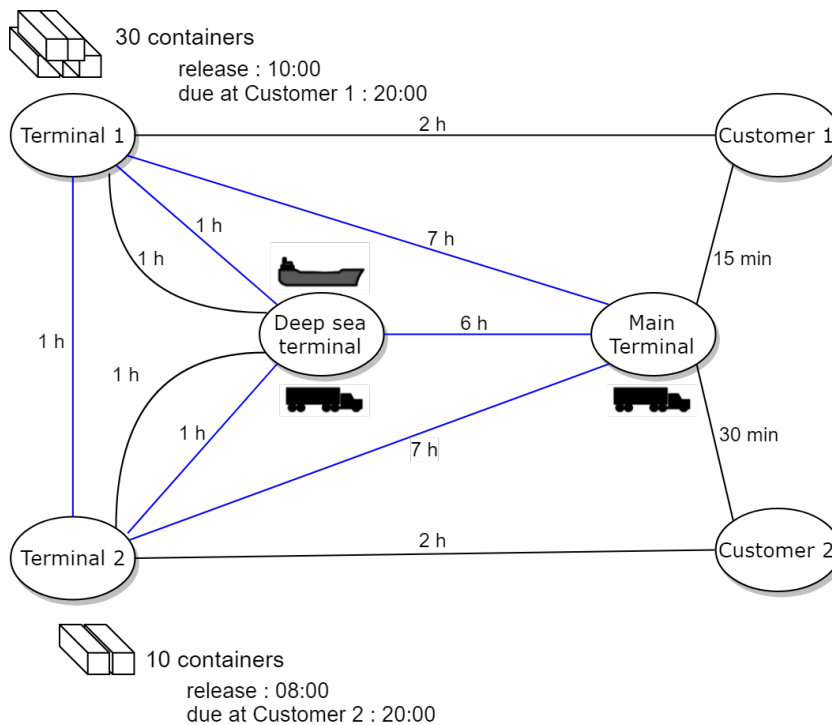Figure 3.5: Simple example of a transportation network with flexible vehicle schedules. The arcs colored in blue may only be traversed by barges, whilst those colored in black can only be traversed by trucks. The travel time of the vehicle on in between the given pair of locations is written on the corresponding edge. The goal is to provide a minimum cost transportation plan in which the two orders are delivered on time.

In this example, there are two transportation requests that must be carried out from their origin to their destination before their due time. There are three barges of capacity 20 situated at the Deep sea terminal, along with a sufficiently large number of trucks with capacity 1 available at both the Main Terminal and the Deep sea terminal. We assume that all vehicles are available for utilization well before the first containers are released and that at every location there is a uniform service time of one hour. There are no costs for traveling by barge. However, every truck-arc has a cost associated which is defined as follows: the two arcs from the Main Terminal to the Customers have costs equal to 5. Each of the rest of the truck arcs has a cost of 10. Solving the mixed integer linear program described earlier with this particular set of parameters yields an optimal schedule of total cost equal to 200. The assignment of containers is as follows: twenty containers from Terminal 1 are carried by one barge to the Main Terminal and further trucked to Customer 1. Another barge will handle the ten remaining containers and bring them to the Main Terminal, from which they will be trucked. Finally, a third barge will transport the ten containers released at Terminal 2, bringing them to the Main Terminal. Again, the last leg of the trip to the final customer destination is done by trucks. As a result of this transportation plan, all thirty containers of the first order arrive at their destination at 18:15, whilst the rest of ten containers corresponding to the second order, reach their destination at 18:30.

We observe that in the optimal solution three barges must be used to meet the strict deadlines of the transportation orders. Had these deadlines been scheduled at a later moment, one could have carried the total load of forty containers with only two barges. Although in our base instance this aspect goes unnoticed since we have not defined arc costs for the barges, using one vehicle less can yield a big cost reduction in practice. Moreover, in the logistics reality, it may often occur that the penalty inquired for being late with servicing a customer is not nearly as high as the cost incurred for utilizing more vehicles. Therefore, it is important to allow for flexibility in meeting the appointment times at the destinations of shipments and to model overall costs appropriately, seeking to reflect as much as possible their impact in a real transportation network.

Although the model proposed by Sharypova includes fixed deadlines at customers and does not allow for multiple visits of a vehicle at a location, it incorporates flexible schedules for the transportation modes and time variables which record the start and end time of all the transport activities in the network. Aspects such as service time at a location, waiting time, and actual travel time are very easy to model with this approach. Moreover, within the larger scope of dealing with uncertainty in the transport parameters, there is a great body of literature related to solution methods for mixed integer linear programs with uncertain data. Concerning scalability, this mixed integer program can solve exactly instances with up to fifty commodities. All in all, it appears that this model is a suitable choice for describing our base instance, possibly extending it and solving it to obtain a transportation plan.

3.2.2. Multicommodity bulk shipping

When studying modeling approaches for freight transportation, it is worthwhile to also consider liquid bulk logistics. This refers to the problem of routing a fleet of vessels engaged in the pick-up and delivery of different liquid bulk products, which is often encountered by oil companies. Liquids are considered bulk cargo, and can be described as commodities that cannot be handled as individual pieces. Although in terms of capacity, handling of commodities and overall objective, these problems are very different from freight routing, some aspects of the transport network still remain very similar, and as a consequence, the models are often built on the same kind of networks and impose the same constraints (flow conservation, timely arrival at customer locations etc.). In essence, liquid bulk routing problems can give new insights into modeling techniques that might counter some assumptions in the freight routing formulations. In this regard, we recall that the model of Sharypova did not allow for multiple visits of a vehicle at a location. However, this assumption is not realistic and is discussed by Al-Khayyal and Hwang [9]. They propose a model for finding a minimum cost routing in a network for a fleet of vessels carrying liquid products. The highlight of their work is the actual design of the network namely, a node (i, m) in the network, where i denotes the location and m marks the arrival number of that location within the planning horizon. An example is shown in Figure 3.6.
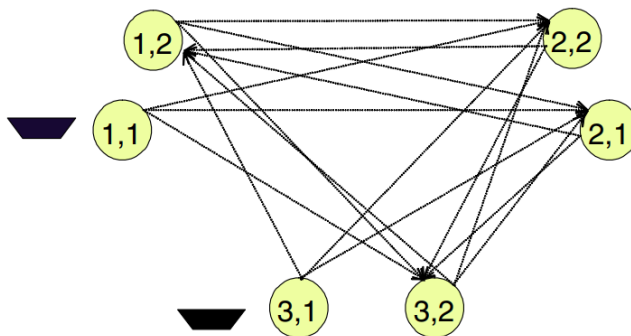
Figure 3.6: Example of a network model with two ships, three locations and two possible arrivals at every location from [9].

Given the network structure exemplified above, the authors define the following binary variables to characterize the movement of vehicles:

$$
x^k_{i,n,j,p} = \begin{cases} 1 & \text{if ship } k \text{ has a route segment that includes location } i \text{ as the } n\text{th arrival} \\ & \text{followed immediately by a visit to location } j \text{ as the } p\text{th arrival,} \\ 0 & \text{otherwise} \end{cases}
$$

The clear advantage of using these decision variables is that vehicle may visit a location multiple times. However, there is a striking drawback of this formulation. Since it is not known beforehand how many visits will be made to each location during a planning horizon, it is necessary to create enough nodes (i, m), to allow as many visits as needed for an optimal transportation plan. This implies that one should have some intuition for assigning reasonable values to the number of visits m. In the case of large instances, this number is difficult to estimate and setting it to a high amount yields a model in which the amount of binary variables becomes exponentially large. Although this approach may give a computationally heavy model, it may still serve as an alternative solution depending on the length of the planning horizon and the available information about the transportation network. For the interested reader, a similar idea of duplicating the locations is explored by Rivera and Mes [39].

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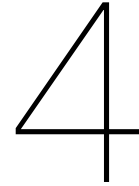
Deterministic problem formulation

In this chapter we present a mathematical model for the freight routing problem described as our base instance. We will use Sharypova's model [41] as the basis of our research and further develop it to incorporate all aspects which are of interest in the context of uncertain parameters in the transportation network. In Section 4.1 we describe all the modifications brought to the original mixed integer linear program. In Section 4.2 we elaborate on a further extension of the model that can be used to incorporate multiple trips of a vehicle to a certain location.

4.1. Deterministic model

Sharypova's model [41] serves as a starting point in our problem formulation. This model provides a transportation schedule of minimum cost which meets the strict delivery deadlines at the destinations of commodities. Since our goal is to investigate the impact of uncertainty on the transportation plan, it is reasonable to allow for more flexibility in the network namely, replace the strict deadlines of commodities by the so-called *soft due times*. This implies that a commodity may arrive later than its due date at its destination, in which case a penalty cost is incurred. To model this aspect, we introduce *lateness* decision variables L_i^m, describing how late vehicle m is at location i. Clearly, these variables are only defined when location i is a destination node for some commodity. Moreover, in our base instance described in section 1.2.2, we assumed that a barge arriving at a deep-sea terminal must wait a certain amount of time before it starts being loaded or unloaded. Thus, we will incorporate a terminal specific parameter w_i, representing the amount of time that a barge has to wait at terminal i. Finally, the components of the objective function must be addressed.

Whilst in Chapter 3 we discussed an objective function related to vehicle utilization, transportation and handling costs, in our case the focus will be on time related components. Generally speaking, we are interested in minimizing the utilization of trucks. However, in practice it is often the case that trucks and trails are rented by the hour and motivated by this, we will aim for minimizing the trucking hours. A trucking hour is one hour in which a truck has been utilized for transportation purposes. It is important to remark here that the amount of trucks used overall or the time traveled by a truck without being loaded are not important quantities in this setting. Moreover, we complete our multi-objective function by incorporating the total lateness recorded in the planning. This term quantifies by how much time container arrivals differ from their specified due date. Weights are associated with each of the two objectives for scaling purposes. These weights allow us to prioritize one objective over the other one, as in practice one might often find that arriving an hour late at a location might be preferable to renting a truck for another hour. A thorough discussion on these weights will follow in Chapter 6.

In the previous chapter, we have already mentioned all the other parameters and decision variables, but for the sake of completeness, we display them again here, including the newly introduced lateness variables.

Sets

$$V = \text{set of locations}$$
$$A = \text{set of traveling arcs between locations}$$
$$M = \text{set of vehicles}$$
$$K = \text{set of commodities}$$
$$V_m = \text{set of locations that can be accessed with vehicle } m$$

Decision variables

$x_{i,j}^{k,m} = $ number of containers of commodity $k \in K$ transported from location $i \in V$ to location $j \in V$ by vehicle $m \in M$

$$y_{i,j}^m = \begin{cases} 1 & \text{if vehicle } m \in M \text{ travels from location } i \in V \text{ to location } j \in V \\ 0 & \text{otherwise} \end{cases}$$

$$z^m = \begin{cases} 1 & \text{if vehicle } m \in M \text{ is used in the transportation plan} \\ 0 & \text{otherwise} \end{cases}$$

$$\theta_i^{m,l} = \begin{cases} 1 & \text{if a transshipment occurs between vehicle } m \in M \text{ and vehicle } l \in M \text{ at location} \\ & i \in V_m \cap V_l \\ 0 & \text{otherwise} \end{cases}$$

$$\tau_i^{k,m} = \begin{cases} 1 & \text{if any container of commodity } k \in K \text{ is loaded on vehicle } m \in M \text{ at node} \\ & i \in \{o(k), d(k)\} \\ 0 & \text{otherwise} \end{cases}$$

$A_i^m = $ arrival time of vehicle $m \in M$ at location $i \in V$

$D_i^m = $ departure time of vehicle $m \in M$ from location $i \in V$

$L_i^m = $ lateness/arrival delay of vehicle $m \in M$ at location $i \in \{d(k)|k \in K\}$

$q_i^{k,m,l} = $ amount of containers of commodity $k \in K$ moved from vehicle $m \in M$ to vehicle $l \in M$ at location $i \in V_m \cap V_l$

Parameters

$s_i = $ service time at node $i \in V$

$d_i^k = $ demand of commodity $k \in K$ at node $i \in V$

$w_i = $ waiting time at terminal location $i \in V$

$r_k = $ release time of commodity $k \in K$ at its origin

$due_k = $ due time of commodity $k \in K$ at its final destination

$t_{i,j}^m = $ traveling time of vehicle $m \in M$ from node $i \in V$ to node $j \in V$

$c^m = $ maximum capacity of vehicle $m \in M$

$(o(k), d(k)) = $ origin-destination node pair of commodity $k \in K$

$\omega_{1,2} = $ weights of the objective functions

Given the previous decision variables and parameters, the model for container assignment and vehicle routing is:

$$\min \quad \omega_1 \sum_{k \in K} \sum_{m \in M} \sum_{(i,j) \in A} t_{i,j}^m x_{i,j}^{k,m} + \omega_2 \sum_{m \in M} \sum_{i \in V} L_i^m$$

$s.t.$

$$\sum_{(j) \in V^+(i)} y_{i,j}^m - \sum_{j \in V^+(i)} y_{j,i}^m = 0 \qquad \forall m \in M, \forall i \in V_m \tag{4.1}$$

$$\sum_{m \in M} \sum_{j \in V^+(i)} x_{i,j}^{k,m} - \sum_{m \in M} \sum_{j \in V^-(i)} x_{j,i}^{k,m} = d_i^k \quad \forall i \in V, \ \forall k \in K \tag{4.2}$$

$$\sum_{k \in K} x_{i,j}^{k,m} \leq c^m y_{i,j}^m \qquad \forall m \in M, \forall (i,j) \in A \tag{4.3}$$

$$\sum_{k \in K} q_i^{k,m,l} > 0 \ \Leftrightarrow \ \theta_i^{m,l} = 1 \qquad \forall m, l \in K, \forall i \in V_m \cap V_l \tag{4.4}$$

$$\sum_{j \in V^-(i)} x_{j,i}^{k,m} = \sum_{l \in K} q_i^{k,m,l} \qquad \forall m \in M, \forall k \in K, \forall i \in V_m \setminus \{d(k)\} \tag{4.5}$$

$$\sum_{j \in V^+(i)} x_{i,j}^{k,m} = \sum_{l \in K} q_i^{k,l,m} \qquad \forall m \in M, \forall k \in K, \forall i \in V_m \setminus \{o(k)\} \tag{4.6}$$

$$\sum_{l \in K} q_i^{k,m,l} = 0 \qquad \forall m \in M, \forall k \in K, \forall i \in \{o(k), d(k)\} \tag{4.7}$$

$$\sum_{j \in V^+(i)} x_{i,j}^{k,m} > 0 \ \Leftrightarrow \ \tau_i^{k,m} = 1 \qquad \forall m \in M, \forall k \in K, \forall i \in \{o(k)\} \tag{4.8}$$

$$\sum_{j \in V^-(i)} x_{j,i}^{k,m} > 0 \ \Leftrightarrow \ \tau_i^{k,m} = 1 \qquad \forall m \in M, \forall k \in K, \forall i \in \{d(k)\} \tag{4.9}$$

$$\theta_i^{m,l} = 1 \Rightarrow D_i^l - A_i^m - s_i \geq 0 \qquad \forall m, l \in M, \forall i \in V_m \cap V_l \tag{4.10}$$

$$y_{i,j}^m = 1 \Rightarrow D_i^m + t_{i,j}^m - A_j^m \leq 0 \qquad \forall m \in M, \forall (i,j) \in A \tag{4.11}$$

$$D_i^m \geq A_i^m + s_i \qquad \forall m \in M, \forall i \in V_m \tag{4.12}$$

$$D_i^m \geq r_k \tau_i^{k,m} \qquad \forall k \in K, \forall i \in \{o(k)\}, \forall m \in M \tag{4.13}$$

$$\tau_i^{k,m} = 1 \Rightarrow L_i^m \geq A_i^m - due_k \qquad \forall k \in K, \forall i \in \{d(k)\}, \forall m \in M \tag{4.14}$$

$$\sum_{j \in V^+(i)} y_{i,j}^m \leq z^m \qquad \forall m \in M, \forall i \in V_m \tag{4.15}$$

$$x_{i,j}^{k,m} \in \mathbb{N}_0 \qquad \forall m \in M, \forall (i,j) \in A, \forall k \in K \tag{4.16}$$

$$q_i^{k,m,l} \in \mathbb{N}_0 \qquad \forall m, l \in M, \forall i \in V_m \cap V_l, \forall k \in K \tag{4.17}$$

$$A_i^m, D_i^m, L_i^m \geq 0 \qquad \forall m \in M, \forall i \in V_m \tag{4.18}$$

$$y_{i,j}^m \in \{0,1\} \qquad \forall m \in M, \forall (i,j) \in A \tag{4.19}$$

$$\theta_i^{m,l} \in \{0,1\} \qquad \forall m, l \in M, \forall i \in V_m \cap V_l \tag{4.20}$$

$$\tau_i^{k,m} \in \{0,1\} \qquad \forall k \in K, \forall m \in M, \forall i \in \{o(k), d(k)\} \tag{4.21}$$

$$z^m \in \{0,1\} \qquad \forall m \in M \tag{4.22}$$

We recall that the objective is the weighted sum of trucking hours and total lateness. Constraints (4.2) ensure flow conservation at a location, while constraints (4.3) account for the demand requirement at the origin and destination of every commodity. Constraints (4.4-4.8) regulate the occurrence of transshipment of containers from one vehicle to another depending on their current location. The inequalities in (4.9)-(4.10) assure that every commodity leaves its origin and arrives at its destination by means of some vehicle. The following four sets of inequalities (4.11)-(4.14) validate the time-related variables. Constraints (4.15) are of particular importance, as they establish the definition of lateness variables. Inequalities (4.16) provide the relation between used travel routes and the number of vehicles. Finally, the remaining constraints define the range of each decision variable.

The mixed integer program presented above describes a transportation problem which can be viewed as a complex extension of the capacitated vehicle routing problem with time windows (abbreviated as CVRP-TW). Since VRP is known to be NP-hard, we understand that there is no polynomial-time algorithm to solve the freight routing problem. Therefore, we expect that solving this problem even for small data instances with state-of-the-art optimization solvers might require a considerable computational effort.

4.2. Additional remarks

The mixed integer program described in Section 4.1 has many binary and integer variables which makes it difficult to solve. Therefore, it is important to ensure that the solution space is as restricted as possible. In order to do so, we include the following *strong forcing* constraints:

$$x_{i,j}^{k,m} \leq \min\{D_k, C^m\} y_{i,j}^m \quad \forall (i,j) \in A, \forall k \in K, \forall m \in M,$$

where D^k is the demand of containers of commodity k, to be transported from their origin location to their destination. These constraints can be derived as flow cover inequalities and have been shown to be effective in improving the LP-relaxation of multi-commodity network design problems [23]. Therefore, they are added to the mixed integer program presented in the previous section.

A final remark concerns the modeling of trucks. Since a truck in general only has the capacity to transport one or two containers, it was preferable to not model them individually, as the size of the instance would have been too large. Instead, a number of trucks with very large capacity (set to 3000 in our instances) was enabled at every location. This number was set equal to the number of commodities to ensure that there is enough transport capacity for timely deliveries.

5

Robust problem formulation

In Chapter 4 we have presented a deterministic model for the freight routing problem which gives optimal solutions if the input data is assumed to be fully correct. However, in practice, this is almost never the case, as perturbations in data occur due to estimation, prediction or implementation errors. This sort of uncertainty may drastically affect the quality of the solution and it is not considered in deterministic optimization. Nevertheless, it can be handled by stochastic optimization (SO) and robust optimization (RO). Stochastic programming is a commonly used method which optimizes the problem by making use of the parameters' expected value. This approach generates a number of scenarios that represent the possible realizations of the stochastic parameters, assigns a probability to each of these scenarios and finally, creates a model optimizing over all scenarios. Stochastic programming cannot be used when detailed statistical information is missing or when the number of scenarios becomes too large, making the problem intractable. In [32], it was shown that already for a small instance the freight routing problem with fixed vehicle routes and stochastic travel times, the scenario tree becomes prohibitively large. Although we will not investigate stochastic programming further, we refer the interested reader to [16].The robust optimization framework on the other hand, is based on obtaining solutions which remain feasible for any realization of the parameters within a pre-defined uncertainty set. For this reason, we will explore in this chapter how can the adjustable robust optimization be used in order to deal with the uncertainty in the release times of the containers. In Section 5.1 we present the robust optimization paradigm and explain how to formulate and solve the robust counterpart. The robust mathematical formulation of the freight routing problem is given in Section 5.2.

5.1. Robust optimization paradigm

Robust optimization is an increasingly popular methodology to model mathematical optimization programs with uncertain data. Instead of assuming a known probability distribution, the uncertain data is presumed to reside in a user-specified set of realizations, called the uncertainty set. We consider a general formulation of an uncertain linear optimization problem:

$$\min_{x}\{c^T x : Ax \le b\}, \tag{P_0}$$

where $c \in \mathbb{R}^n$, $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$. Suppose that the matrix A is uncertain and it belongs to a bounded uncertainty set $\mathcal{U}_A \subset \mathbb{R}^{m \times n}$. In a similar fashion we assume that right hand side vector b belongs to uncertainty set $\mathcal{U}_b \subset \mathbb{R}^m$, whilst the objective coefficients c reside in the uncertainty set $\mathcal{U}_c \subset \mathbb{R}^n$. The sets $\mathcal{U}_A$, $\mathcal{U}_b$ and $\mathcal{U}_c$ specify all possible realizations of the uncertain data and are collectively referred to as the uncertainty set $\mathcal{U}$. The robust optimization paradigm as described by Ben-Tal et al. [14] relies on the following assumptions:

A.1 All decision variables $x \in \mathbb{R}^n$ represent here-and-now decisions: they should be assigned specific numerical values as a result of solving the problem before the actual data 'reveals itself'.

A.2 The decision maker is fully responsible for consequences of the decisions to be made when, and only when, the actual data is within the prespecified uncertainty set $\mathcal{U}$.

A.3 All the constraints of the uncertain problem in case are 'hard': we cannot tolerate violations of constraints when the data is in the uncertainty set $\mathcal{U}$.

These assumptions indicate what are the relevant feasible solutions of the linear uncertain problem P_0. The first assumption **A.1** asserts that the solution vector should have fixed values or otherwise said, it should not contain any components to which there has not been assigned a numerical value. By assumptions **A.2** and **A.3** this solution vector should satisfy all the constraints, regardless of the realization of the data in the uncertainty set $\mathcal{U}$. Such a solution is called *robust feasible* [14]. Thus we understand that robust optimization is concerned with finding robust feasible solutions for problems with a predefined uncertainty set.

5.1.1. The robust counterpart

We observe that the linear uncertain problem P_0 exhibits uncertainty in all parameters. In fact, one can show that this problem can be reformulated in such a way that only the matrix A will contain uncertain entries. Firstly, the uncertainty in the objective function can be removed by introducing an additional continuous decision variable $t \in \mathbb{R}$. Problem P_0 is then equivalent to:

$$\min_{x,t}\{t : c^T x - t \leq 0 \quad \forall c \in \mathcal{U}_c, \quad Ax \leq b \quad \forall A \in \mathcal{U}_A, \forall b \in \mathcal{U}_b\}.$$

Secondly, the uncertain components of vector b can be transferred to the matrix A in the following way: vector b is added as a column of A and value $x_{n+1} = -1$ is added as an extra component to the vector x. Then the problem P_0 can be written as:

$$\min_{x,t}\{t : c^T x - t \leq 0 \quad \forall c \in \mathcal{U}_c, \quad Ax \leq 0 \quad \forall A \in \mathcal{U}_A \cup \mathcal{U}_b\},$$

Given these two reformulations, we conclude that it is always safe to assume that uncertain quantities occur only in the matrix of coefficients. This being said, we can finally give a most general form of the uncertain linear problem as:

$$\min_{x}\{c^T x : Ax \leq b \quad \forall A \in \mathcal{U}\} \tag{P}$$

The robust reformulation of problem P is referred to as the *robust counterpart* (RC) problem [14] and we will present it as given in [55]. We assume that the coefficient matrix $A(\zeta)$ is an affine[1] function of the uncertain parameter ζ:

$$\min_{x}\{c^T x : A(\zeta)x \leq b \quad \forall \zeta \in \mathcal{Z}\}, \tag{RC}$$

where $\mathcal{Z} \subset \mathbb{R}^p$ denotes the user defined uncertainty set. Recall that a solution x is robust feasible if the constraints $A(\zeta)x \leq b$ are satisfied for every value of $\zeta \in \mathcal{Z}$. As discussed in [14], the robust counterpart of an uncertain linear optimization problem with a certain objective is a 'constraint-wise' construction. In other words, the original ith row constraint $(Ax)_i \leq b_i \Leftrightarrow a_i^T x \leq b_i$, (with a_i being the ith row in A) from the nominal problem is replaced by $a_i^T x \leq b_i \quad \forall [a_i; b_i] \in \mathcal{U}_i$, where $\mathcal{U}_i$ is the projection of $\mathcal{U}$ on the space of data of ith constraint: $\mathcal{U}_i = \{[a_i; b_i] : [A, b] \in \mathcal{U}\}$. Therefore, we can address the uncertainty by a single constraint. For instance, we extract one constraint from the robust counterpart problem RC modeled as an affine expression in terms of ζ:

$$(a_i + P\zeta)^T x \leq b_i \quad \forall \zeta \in \mathcal{Z}, \tag{5.1}$$

where $a_i \in \mathbb{R}^n$ is interpreted as the nominal value of the data, $P \in \mathbb{R}^{n \times p}$ and $b_i \in \mathbb{R}$. The idea behind this process is to reformulate the robust counter part constraint-wise in such a way that it becomes computationally tractable. The expression in (5.1) has infinitely many constraints due to the *for all* ($\forall$) quantifier and it is thus intractable in general. In [55] the authors provide a compact overview of the steps to be followed in order to remove this quantifier. We will closely follow their approach. Consider a polyhedral uncertainty set defined as:

$$\mathcal{Z} = \{\zeta : D\zeta + q \geq 0\}, \tag{5.2}$$

where $D \in \mathbb{R}^{m \times p}$, $\zeta \in \mathbb{R}^p$ and $q \in \mathbb{R}^n$.

---

[1] A function $f : A \to B$ is affine if and only if the mapping $x \mapsto f(x) - f(0)$ is linear.

In a worst case reformulation, when the realization of the uncertain data yields the largest objective value, one can re-write the nominal problem P as:

$$a_i^T x + \max_{\zeta}\{(P^T x)^T \zeta : \quad D\zeta + q \geq 0\} \leq b_i. \tag{5.3}$$

By strong duality, the inner maximization problem in the expression above can be replaced by its dual. Therefore, expression (5.3) is equivalent to:

$$a_i^T x + \min_{w}\{q^T w : \quad D^T w = -P^T x, \quad w \geq 0\} \leq b_i. \tag{5.4}$$

We see that in order to satisfy inequality (5.4), it suffices to find at least one w. Hence, the final formulation of the RC is given by:

$$\exists w : \quad a_i^T x + q^T x \leq b_i, \quad D^T w = -P^T x, \quad w \geq 0, \tag{5.5}$$

which is an LP feasibility problem.

From everything that we have done so far, we conclude that solving the robust counterpart of a general linear optimization problem with continuous variables and a polyhedral uncertainty set reduces to finding a feasible solution to the linear problem described in (5.5). Therefore, the robust counterpart of an uncertain linear program (LP) with a polyhedral uncertainty set is in fact a computationally tractable LP. Moreover, this property also holds for the so-called box uncertainty set of the form: $\mathcal{Z} = \{\zeta : \|\zeta\|_\infty \leq 1\}$, since the robust counter part in this case is simply given by $a_i^T x + \|P^T x\|_1 \leq b_i$ [55]. For a thorough mathematical discussion on tractability properties of the robust counterpart for various uncertainty sets, the reader is referred to the book of Ben-Tal et al. [14].

5.1.2. Adjustable robust optimization

The robust optimization formulation given in Section 5.1.1 is static in the sense that the numerical values of all decision variables must be determined before the uncertain quantities reveal their true value. For this reason, the solutions obtained by solving the robust counterpart are indeed robust feasible but sometimes very conservative: they are only optimal for the worst case realizations of the uncertain data. With this static approach it may often be the case that the objective function of the solution becomes unnecessarily high given the actual data realizations attained in practice. This concept is also known as the *price of robustness*, described by Bertsimas and Sim [15] as the trade-off between the optimal solution and robustness. In order to achieve a reasonable price of robustness, the adjustable robust optimization framework has been proposed [13]. In this framework, assumption **A.1** from the robust optimization paradigm is relaxed, meaning that we allow for some *wait-and-see* decision variables. In other words, some decision variables can be adjusted at a later point in time according to the realization of the data. Most commonly, these adjustable decisions are modeled as functions of the uncertain data. In view of this, the adjustable robust counterpart (ARC) can be formulated as:

$$\min_{x, y(\cdot)}\{c^T x : A(\zeta)x + By(\zeta) \leq b\} \quad \forall \zeta \in \mathcal{Z}, \tag{ARC}$$

where $x \in \mathbb{R}^n$ represents a first-stage here-and-now decision vector that is made before $\zeta \in \mathbb{R}^p$ is realized, $y \in \mathbb{R}^k$ denotes the second-stage wait-and-see decision vector that can be computed according to the realization of ζ, and $B \in \mathbb{R}^{n \times k}$ is a given coefficient matrix. For the scope of this thesis it is sufficient to assume that the matrix B does not contain any uncertain elements. In general, it is difficult to optimize over functions, so a commonly used approach is to express the the adjustable decision variables as affine functions of the uncertain data namely:

$$y(\zeta) = y_0 + Q\zeta. \tag{5.6}$$

In the expression above, $y_0 \in \mathbb{R}^k$ and $Q \in \mathbb{R}^{k \times p}$ are here-and-now decisions to be optimized by the model in the first stage. Substituting the expression for y given in equation (5.6) into the ARC we obtain the affinely adjustable robust counterpart (AARC):

$$\min_{x, y_0, Q}\{c^T x : A(\zeta)x + By_0 + BQ\zeta \leq b \quad \forall \zeta \in \mathcal{Z}\}. \tag{AARC}$$

Since the AARC is linear in both the decision variables and the uncertain parameter, it can be solved by following the same reformulation steps as in the previous section. Therefore, the AARC has the same tractability as the original robust counter part, regardless of the uncertainty set chosen. Two important remarks are required here. First of all, the AARC might contain many more decision variables than the RC due to the size of matrix Q. Secondly, although the AARC will likely require more computational effort, the solution thus obtained will be at least as good as the one given by solving the RC.

Up to this point, we have presented both the static and the affinely adjustable robust counterpart problems and showed that in the case of linear programs with polyhedral or box uncertainty, both formulations are tractable. The fact that we can provide adjustable robust feasible solutions, makes the robust optimization approach extremely appealing for further applying it to our freight routing problem. Nevertheless, as shown in Chapter 4, the model developed includes many binary and integer variables for which the mathematical treatment in sections 5.1.1 and 5.1.2 is not directly applicable. Therefore, we will further discuss how robust optimization techniques can be used in the context of mixed integer programs.

5.1.3. Robust optimization for mixed integer programs

A mixed integer program is a mathematical program which contains both real valued decision variables and variables restricted to take integer values. It is well known that determining whether a feasible solution of a given mixed integer program with rational coefficients exists is in the class of NP-complete problems [24]. As such, we expect that a robust counterpart of a mixed integer linear program is also intractable. Consider the general form of a mixed integer program:

$$\min_{x,y}\{c^T x + d^T y : \quad Ax + Gy \leq p, \quad x \in \mathbb{Z}^n, y \in \mathbb{R}^k\}, \tag{MIP}$$

where $c \in \mathbb{Q}^n$ and $d \in \mathbb{Q}^k$ are given cost vectors, $A \in \mathbb{Q}^{l \times n}$ and $G \in \mathbb{Q}^{l \times k}$ are coefficients matrices and $p \in \mathbb{Q}^l$. We assume that the matrix A is the only element affected by uncertainty. This assumption is motivated by our deterministic model in Chapter 4, in which the uncertain release time is multiplied with a binary variable in constraints (4.14). Thus we consider a model of the form:

$$\min_{x \in \mathbb{Z}^n, y \in \mathbb{R}^k}\{c^T x + d^T y : \quad A(\zeta)x + Gy \leq p, \quad \forall \zeta \in \mathcal{Z}\}. \tag{RC-MIP}$$

Since uncertainty was showed to appear constraint-wise in a general linear program, we can once again model uncertainty affected constraints by an affine transformation of the uncertainty term $\zeta \in \mathcal{Z}$ namely, every element of A can be written as a summation between a linear combination of the components of vector ζ and a constant:

$$A(\zeta)^T = \begin{bmatrix} a_1(\zeta) & a_2(\zeta) & \dots & a_n(\zeta) \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & \dots & a_n \end{bmatrix} + \begin{bmatrix} P_1\zeta & P_2\zeta & \dots & P_n\zeta \end{bmatrix}. \tag{5.7}$$

The robust counterpart then contains constraints of the form:

$$\min_{x \in \mathbb{Z}^n, y \in \mathbb{R}^k}\{c^T x + d^T y : \quad (a_i + P\zeta)^T x + g_i^T y \leq p_i, \quad \forall i \in 1, \dots, l \quad \forall \zeta \in \mathcal{Z}\}, \tag{5.8}$$

where $a_i \in \mathbb{Q}^n$ is the nominal value, $P \in \mathbb{R}^{n \times p}$ is the matrix with vectors $P_1, P_2, \dots, P_n \in \mathbb{R}^p$ as columns, g_i^T is a vector corresponding to the ith row of matrix G and p_i is the ith entry of vector p. Just as in the case of a general linear program, we now wish to bring the RC-MIP problem into a reasonable form, removing the 'for all' ($\forall$) operator. The uncertainty set to be considered is the simple box uncertainty:

$$\mathcal{Z} = \{\|\zeta\|_\infty \leq 1\}. \tag{5.9}$$

This kind of uncertainty set is the most intuitive for the freight routing problem, since the release time of a container is assumed to belong to a certain bounded interval of time. Using the worst-case values of the uncertain parameter ζ, the robust counter part RC-MIP is re-formulated as:

$$\min_{x \in \mathbb{Z}^n, y \in \mathbb{R}^k}\{c^T x + d^T y : \quad a_i^T x + \|P^T x\|_1 + g_i^T y \leq p_i \quad \forall i \in 1, \dots, l\}. \tag{5.10}$$

We observe that expression (5.9) is a convex optimization problem that can be re-written as a linear mixed integer problem by introducing auxiliary decision variables. The next step from here is to adjust the continuous variables which in our freight routing model correspond to decision related to time. We assume that they can be written as affine functions of the uncertainty as in expression (5.6). By doing so, we obtain the following adjustable robust counterpart:

$$\min_{x \in \mathbb{Z}^n, y_0 \in \mathbb{R}^k, Q \in \mathbb{R}^{k \times p}} \{c^T x + d^T(y_0 + Q\zeta) : \quad A(\zeta)x + G(y_0 + Q\zeta) \leq p, \quad \forall \zeta \in \mathcal{Z}\}. \qquad (5.11)$$

In the case of box uncertainty, this can be formulated as a convex problem very similar to (5.9):

$$\min_{x \in \mathbb{Z}^n, y_0 \in \mathbb{R}^k, Q \in \mathbb{R}^{k \times p}} \{c^T x + d^T(y_0 + Q\zeta) : \quad a_i^T x + \|P^T x + Q^T g_i\|_1 + g_i^T y_0 \leq p_i\}. \qquad (5.12)$$

The final form of this mixed integer problem without uncertainty removed from the objective and constraints is:

$$\min_{x \in \mathbb{Z}^n, y_0 \in \mathbb{R}^k, Q \in \mathbb{R}^{k \times p}, t \in \mathbb{R}} \{t : c^T x + d^T y_0 + \|Q^T d\|_1 - t \leq 0, \quad a_i^T x + \|P^T x + Q^T g_i\|_1 + g_i^T y_0 \leq p_i \quad \forall i \in 1, \ldots, l\}.$$
$$\text{(ARC-MIP)}$$

Finding a solution to the ARC-MIP reduces to solving a mixed integer program bigger in size than the original MIP. Nevertheless, it provides a suitable modeling framework for the freight routing problem and a way to find static and adjustable robust feasible solutions. Since the robust optimization approach has been discussed for both a general linear program and the mixed integer case, we are now ready to present a robust model for the freight routing problem.

5.2. Robust model

In the robust model the release times of the commodities are uncertain. We recall that every container has a predefined earliest and latest pick-up time from its terminal of origin, and the moment at which it is actually released from the terminal and available for loading on the vehicle is contained in this time window. In mathematical terms we have:

$$r_k \in [e_k, l_k] \quad \forall k \in K,$$

where e_k and l_k mark the earliest and the latest pickup time, respectively. In practice, these two quantities are made available in advance by the terminal where the pickup should occur. The release is known to take place sometime between these two moments. This can be modeled as follows:

$$r_k = \frac{1}{2}e_k(1 - \zeta_k) + \frac{1}{2}l_k(1 + \zeta_k) \quad \forall k \in K.$$

where $\zeta_k \in [-1, 1]$ is the actual uncertain parameter based on which the release r_k can be computed. Therefore, just as in Section 5.1.3, the uncertainty set is the simple boxed uncertainty given by:

$$\mathcal{Z} = \{\zeta \in \mathbb{R}^k : \zeta_k \in [-1, 1]\}.$$

Based on this uncertainty set, we introduce an adjustable robust model which contains two stages of decisions: the first stage variables that must be determined before the value of the uncertain parameter becomes known, and second stage decision variables which can change their value according to the realization of the parameters. In our robust model, the first stage variables $x_{i,j}^{k,m}, y_{i,j}^m, z^m, \theta_i^{m,l}, \tau_i^{k,m}$ and $q_i^{k,m,l}$ concern the routing, the sequence of terminal visits, the assignment and transshipment of containers. The second stage decisions are the continuous variable D_i^m, A_i^m and L_i^m which account for the explicit departure and arrival times and are modeled as adjustable variables. The idea of adjusting time variables to the uncertain parameters originates from Agra et al. [8], who give a robust formulation for a maritime inventory routing problem with uncertain vessel sailing times. Therefore, we define $D_i^m(\zeta), A_i^m(\zeta)$ and $L_i^m(\zeta)$ as the arrival time, departure time and lateness, respectively, when scenario ζ (a vector containing release times of all commodities) has been revealed.

The first stage solution must ensure that, for each possible realization of the release times in the uncertainty set, the containers are transported from their origin to their destination without missing any of their planned transshipment on the way. In other words, these decisions should result in a robust plan that can be carried out regardless of delayed releases of containers. In the original deterministic model, all time-related constraints (4.10)-(4.14) become:

$$\theta_i^{m,l} = 1 \Rightarrow D_i^l(\zeta) - A_i^m(\zeta) - s_i \geq 0 \qquad \forall m,l \in M, \forall i \in V_m \cap V_l, \forall \zeta \in \mathcal{Z} \qquad (5.13)$$

$$y_{i,j}^m = 1 \Rightarrow D_i^m(\zeta) + t_{i,j}^m - A_j^m(\zeta) \leq 0 \qquad \forall m \in M, \forall (i,j) \in A, \forall \zeta \in \mathcal{Z} \qquad (5.14)$$

$$D_i^m(\zeta) \geq A_i^m(\zeta) + s_i \qquad \forall m \in M, \forall i \in V_m, \forall \zeta \in \mathcal{Z} \qquad (5.15)$$

$$D_i^m(\zeta) \geq r_k \tau_i^{k,m} \qquad \forall k \in K, \forall i \in \{o(k)\}, \forall m \in M, \forall \zeta \in \mathcal{Z} \qquad (5.16)$$

$$\tau_i^{k,m} = 1 \Rightarrow L_i^m(\zeta) \geq A_i^m(\zeta) - due_k \qquad \forall k \in K, \forall i \in \{d(k)\}, \forall m \in M, \forall \zeta \in \mathcal{Z}. \qquad (5.17)$$

As we have already discussed in the previous section, a common approach to handle adjustable variables is to use affine decision rules. In this case, we can write the arrival and departure times as affine functions of the uncertain release times:

$$D_i^m(\zeta) = D_{i,0}^m + \sum_{k \in K} D_{i,k}^m \zeta_k \quad \forall i \in V, \forall m \in M \qquad (5.18)$$

$$A_i^m(\zeta) = A_{i,0}^m + \sum_{k \in K} A_{i,k}^m \zeta_k \quad \forall i \in V, \forall m \in M \qquad (5.19)$$

$$L_i^m(\zeta) = L_{i,0}^m + \sum_{k \in K} L_{i,k}^m \zeta_k \quad \forall i \in \{d(k) : k \in K\}, \forall m \in M \qquad (5.20)$$

The newly introduced variables $D_{i,0}^m \geq 0$, $D_{i,k}^m \in \mathbb{R}$ and so on must be determined in the first stage, together with the routing, assignment and transshipment decisions. We are interested in robust feasible solutions that satisfy constraints (5.12)-(5.16) for any realization of the release time vector $\zeta \in \mathcal{Z}$. Such a solution must also satisfy the following re-formulated constraints:

$$\theta_i^{m,l} = 1 \Rightarrow D_{i,0}^l + \sum_{k \in K} D_{i,k}^l \zeta_k \geq s_i + A_{i,0}^m + \sum_{k \in K} A_{i,k}^m \zeta_k \qquad \forall m,l \in M, \forall i \in V_m \cap V_l, \forall \zeta \in Z \quad (5.21)$$

$$y_{i,j}^m = 1 \Rightarrow D_{i,0}^m + \sum_{k \in K} D_{i,k}^m \zeta_k + t_{i,j}^m \leq A_{j,0}^m + \sum_{k \in K} A_{j,k}^m \zeta_k \qquad \forall m \in M, \forall (i,j) \in A, \forall \zeta \in Z \quad (5.22)$$

$$D_{i,0}^m + \sum_{k \in K} D_{i,k}^m \zeta_k \geq A_{i,0}^m + \sum_{k \in K} A_{i,k}^m \zeta_k + s_i \qquad \forall m \in M, \forall i \in V_m, \forall \zeta \in Z \quad (5.23)$$

$$D_{i,0}^m + \sum_{k \in K} D_{i,k}^m \zeta_k \geq \left(\frac{1}{2} e_k (1 - \zeta_k) + \frac{1}{2} l_k (1 + \zeta_k)\right) \tau_i^{k,m} \qquad \forall k \in K, \forall i \in \{o(k)\}, \forall m \in M, \forall \zeta \in Z \quad (5.24)$$

$$\tau_i^{k,m} = 1 \Rightarrow L_{i,0}^m + \sum_{k \in K} L_{i,k}^m \zeta_k \geq A_{i,0}^m + \sum_{k \in K} A_{i,k}^m \zeta_k - due_k \quad \forall k \in K, \forall i \in \{d(k)\}, \forall m \in M, \forall \zeta \in Z. \quad (5.25)$$

As shown earlier in Section 5.2, the uncertainty ζ_k can be removed from the constraints by assuming a worst case realization of the data. For example, constraints (5.20) can be written as follows:

$$\theta_i^{m,l} = 1 \Rightarrow D_{i,0}^l - s_i - A_{i,0}^m \geq \sum_{k \in K} (A_{i,k}^m - D_{i,k}^l) \zeta_k \quad \forall m,l \in M, \forall i \in V_m \cap V_l, \forall \zeta \in Z.$$

Since this inequality should hold for any realization of ζ_k, we impose the following constraint:

$$\theta_i^{m,l} = 1 \Rightarrow D_{i,0}^l - s_i - A_{i,0}^m \geq \sum_{k \in K} |A_{i,k}^m - D_{i,k}^l| \quad \forall m,l \in M, \forall i \in V_m \cap V_l.$$

Moreover, we note that in the constraints above there is no uncertain parameter anymore and all the decision variables are to be determined in the first stage. Moreover, the absolute value can be removed

from the expression by introducing an additional decision variable $\alpha_{m,l}^{i,k}$:

$$\theta_i^{m,l} = 1 \Rightarrow D_{i,0}^l - s_i - A_{i,0}^m \geq \sum_{k \in K} \alpha_{m,l}^{i,k} \qquad \forall m, l \in M, \forall i \in V_m \cap V_l$$

$$-\alpha_{m,l}^{i,k} \leq A_{i,k}^m - D_{i,k}^l \leq \alpha_{m,l}^{i,k} \qquad \forall k \in K, \forall m, l \in M, \forall i \in V_m \cup V_l.$$

One can reformulate constraints (5.21)-(5.24) in a similar fashion and obtain the following inequalities:

$$y_{i,j}^m = 1 \Rightarrow D_{i,0}^m + t_{i,j}^m - A_{j,0}^m + \sum_{k \in K} \beta_{k,m}^{i,j} \leq 0 \qquad \forall m \in M, \forall (i,j) \in A \qquad (5.26)$$

$$-\beta_{k,m}^{i,j} \leq D_{i,k}^m - A_{j,k}^m \leq \beta_{k,m}^{i,j} \qquad \forall (i,j) \in A, \forall k \in K, \forall m \in M$$

$$D_{i,0}^m - A_{i,0}^m - s_i \geq \sum_{k \in K} \gamma_{k,m}^i \qquad \forall m \in M, \forall i \in V_m \qquad (5.27)$$

$$-\gamma_{k,m}^i \leq A_{i,k}^m - D_{i,k}^m \leq \gamma_{k,m}^i \qquad \forall i \in V_m, \forall k \in K, \forall m \in M$$

$$D_{i,0}^m \geq \delta_{k_0,m} + \sum_{k \neq k_0} \epsilon_{i,k}^m \qquad \forall k_0 \in K, \forall i \in \{o(k)\}, \forall m \in M \qquad (5.28)$$

$$-\delta_{k_0,m}^i \leq \frac{1}{2}(l_{k_0} - e_{k_0})\tau_i^{k,m} - D_{i,k_0}^m \leq \delta_{k_0,m}^i \qquad \forall k_0 \in K, \forall i \in \{o(k)\}, \forall m \in M$$

$$-\epsilon_{i,k}^m \leq D_{i,k}^m \leq \epsilon_{i,k}^m \qquad \forall k \in K, \forall i \in \{o(k)\}, \forall m \in M$$

$$\tau_i^{k,m} = 1 \Rightarrow L_{i,0}^m + due_k - A_{i,0}^m \geq \sum_{k \in K} \eta_{k,m}^i \qquad \forall k \in K, \forall i \in \{d(k)\}, \forall m \in M \qquad (5.29)$$

$$-\eta_{k,m}^i \leq A_{i,k}^m - L_{i,k}^m \leq \eta_{k,m}^i \qquad \forall k \in K, \forall i \in \{d(k)\}, \forall m \in M$$

We observe that in the constraints above there is no uncertain parameter ζ_k present anymore and all the decision variables are to be determined in the first stage. The adjustable robust counterpart of the deterministic model presented in Chapter 4 is thus composed from constraints (4.1)-(4.9), (4.15)-(4.22) and (5.25)-(5.28). The objective of the mixed integer program is modeled according to the method described in the beginning of Section 5.1.1, meaning that the following expression is added to finalize the model:

$$\max \quad t \quad \text{where} \quad \omega_1 \sum_{k \in K} \sum_{m \in M} \sum_{(i,j) \in A} t_{i,j}^m x_{i,j}^{k,m} + \omega_2 \sum_{m \in M} \sum_{i \in V} \left(L_{i,0}^m + \sum_{k \in K} \mu_{i,k}^m \right) \leq t \qquad (5.30)$$

$$-\mu_{i,k}^m \leq L_{i,k}^m \leq \mu_{i,k}^m, \quad \forall k \in K, \forall i \in \{d(k)\}, \forall m \in M$$

Solving the robust model will determine all the routing, assignment and transshipment variables. The value obtained for the objective value corresponds to the worst case realization of the data. Nevertheless, using the adjustable time variables in the second stage, when the data is revealed, we can improve the value of the objective without re-solving the model. That is due to the fact that in the second stage the lateness term in the objective can still be adjusted and reduced when the realization of the data is favorable.

6

Computational results

In this chapter, we report on the solutions found for the deterministic and robust formulations of the freight routing problem and compare them to past approaches. In Section 6.1. it is explained how the test instances were generated. Section 6.2 contains the results for the deterministic model, whilst the findings for the robust approach are shown in Section 6.3. All results are interpreted and discussed in Section 6.4.

6.1. Instance generation

In order to test the models that were given in chapters 4 and 5, we generated multiple instances. These were inspired from the work of Kishan Kalicharan [34], a project colleague from TNO who has designed a transport network of eight terminal locations based on Google Maps data. Since some of these locations represented clustered terminals, the original instances were modified to include only nodes which correspond to actual physical locations in real life. For comparison purposes, the number of locations was kept the same. The barge travel times on waterways were assumed to be fixed and their values were approximated using online tools which compute sea distances based on the speed of the vessel. In our transport infrastructure, we assume that some of the locations are terminals, where containers can be transshiped, and some of them are customers, serving as end-locations for the containers. There is also direct connection between every pait of locations in our model. Furthermore, we assume that the service time is the same at every location. There is a set of commodities (bookings with one or more containers) that need to be transported from the terminals to the customer locations. As in [34], the demand value of each commodity is randomly chosen in the interval $[0, 125]$.

Barges and trucks are available for container transport. The capacity of barges is assumed to be of 100 containers. These barges always start at a particular terminal which in real-life interpretation in a hub-location. We assume that there is an infinite amount of trucks of large capacity available at every location. To ensure that all containers can be transported, the total capacity of all vehicles is always larger than the total demand of all commodities [41]. Finally, the due dates and release times of the containers are chosen in such a way that the difference between them is strictly larger than the time required by barge to travel on the direct connection arc from the origin of the containers to the their destination.

To assess the computational difficulty of our models we create instances with 8 nodes, 6 and 12 barges, and 5, 10, 20 and 30 commodities. In total, we generate 10 instances which are tested for three different objective functions by varying the values of the weights ω_1 and ω_2. We denote a problem instance by km, where k the number of commodities and m the number of available barges. Both the deterministic and the robust model were implemented in AIMMS Developer version 4.53, a mathematical optimization modeling tool, and solved with CPLEX optimization solver (Version 12.8, 32-bit). Numerical experiments were carried out on a DELL Latitude E7240 laptop with an Intel(R) Core(TM) i5-4310U CPU 2.00 GHz 2.60 GHz processor and 8 GB RAM memory. This laptop is operational on a 64-bit operating system.

6.2. Results of deterministic model

In the freight routing problem we are interested in providing an assignment of containers to vehicles whilst minimizing the total number of trucking hours and overall lateness. In order to get an idea of how the allowed lateness affects the solution time of the freight routing problem, the deterministic model in Chapter 4 was tested for three different objective functions. These were obtained by varying the weights w_1 and w_2. Since we have no knowledge of the real costs of trucking activities in practice, we shall gradually increase the weight ω_2 of lateness and keep the first weight $\omega_1 = 1$. The following three objective functions are considered:

- Objective 1: $\sum_{k \in K} \sum_{m \in M} \sum_{(i,j) \in A} t^m_{i,j} x^{k,m}_{i,j} + \sum_{m \in M} \sum_{i \in V} L^m_i$

- Objective 2: $\sum_{k \in K} \sum_{m \in M} \sum_{(i,j) \in A} t^m_{i,j} x^{k,m}_{i,j} + 0.1 \cdot \sum_{m \in M} \sum_{i \in V} L^m_i$

- Objective 3: $\sum_{k \in K} \sum_{m \in M} \sum_{(i,j) \in A} t^m_{i,j} x^{k,m}_{i,j} + 1000 \cdot \sum_{m \in M} \sum_{i \in V} L^m_i$

These values give us a reasonable way to assess which objective yields a solution fast enough. Since these weights are chosen in a way that highly penalizes lateness, the results of our model should be comparable to those obtained when completely removing lateness variables. Table 6.1 shows the solution and computation time for each instance and each of the three objective functions that were chosen. The results obtained for the original model with no lateness allowed are given in Table 6.2.

Data instance	Obj. 1	Gap (%)	Runtime (s)	Obj. 2	Gap (%)	Runtime (s)	Obj. 3	Gap (%)	Runtime (s)
k5m6	100	0	1.94	100	0	1.94	100	0	1.8
k10m6	149.5	0	22.56	149.5	0	22.92	149.5	0	72.53
k20m6	235.5	0	263	235.5	0	245.36	235.5	0	565.52
k30m6	na	na	3600*	na	na	3600*	na	na	3600*
k5m12	100	0	2.06	100	0	3.52	100	0	2.22
k10m12	149.5	0	19.23	149.5	0	24.02	149.5	0	18.9
k20m12	235.5	0	303.47	235.5	0	380.89	235.5	0	178.36
k30m12	na	na	3600*	na	na	3600*	na	na	3600*

Table 6.1: Objective value, solution time (CPU seconds) and gaps between the lower and upper bounds for the freight routing model with lateness allowed.
* an upper bound of 3600s was set on the running time of the solver.

What immediately stands out from the results above is the computational difficulty of the deterministic

Data instance	Objective	Gap (%)	Runtime (s)
k5m6	100	0	4.84
k10m6	149.5	0	114.08
k20m6	na	na	3600*
k30m6	na	na	3600*
k5m12	100	0	30.38
k10m12	149.5	0	21.66
k20m12	235.5	0	319.8
k30m12	na	na	3600*

Table 6.2: Objective value, solution time (CPU seconds) and gaps between the lower and upper bounds for the freight routing model with no lateness allowed.
* an upper bound of 3600s was set on the running time of the solver.

models for freight routing as for instances with thirty commodities k30m6 and k30m12 the solver could not find a feasible solution within one hour for any of the models considered. However, we see that the model incorporating lateness performs better than the original version in terms of the computational time required and the solution found. For example, instance k20m6 can be solved to optimality for all the three objective functions considered in the case of allowed lateness but not for the original model. This might be due to the fact that the model including lateness is always feasible, and therefore, it is

easier for the solver to find an initial feasible solution than in the situation of hard due dates for the commodities.

In general, we note that the computational time significantly increases for all cases considered when the number of commodities increases. On the other hand, the number of vehicles does not seem to drastically influence the computational time of the instances that we have tested since there are no compelling differences between instances with six or twelve barges. In particular, for the instance with twenty commodities the solver found an optimal solution three times faster when the number of available barges was doubled. This result confirms our expectation, as a larger fleet of barges offers more routing possibilities and requires less transshipments of containers to trucks.

Regarding the two model formulations, with and without allowing for lateness, we see that the results obtained are the same. This suggests that despite the penalties, the overall lateness obtained if all containers were transported by barge on the main leg of the trip is still much larger than the cost resulting from trucking everything. This is fully due to the choice of values for the parameter ω_2. One could indeed assign lower numerical values to this weight to obtain solutions with late arrivals of commodities. However, since lateness is used mostly for computational reasons here, we will not look into those situations. In terms of the objective function used to generate the results in Table 6.1, optimizing the problem for Objective 3 is the most computationally expensive at least in the case of the first three instances. However, when the number of available barges is increased to twelve, the running time of the solver for Objective 3 is much lower than for Objectives 1 and 2. One possible explanation for this is the fact that the weight $\omega_2 = 1000$ adds a large contribution to the cost solution and thus the solver begins by finding a very expensive feasible solution and then reduces it by re-assigning the commodities over the available barges. If more barges are enabled, then more capacity is available for re-assigning and transporting containers by water instead.

Overall, the results in this section provide an important insight into the computational difficulty required by the deterministic model and help us set an expectation on the numerical effort for the robust model. The largest instances that we could solve namely, k20m6 and k20m12 that have been used are comparable to the transportation activity of a real logistics service provider.

6.3. Results of robust model

In this section we focus on solving the robust model explained in Section 5.2, in which the release times of the commodities belong to a predefined uncertainty set. As we already know, the robust mixed integer linear program will determine the routing of vehicles and assignment and transshipment of containers in the first stage, leaving the time variables to be determined in the second stage, when the uncertain release times have been revealed. It is expected that the robust solution is more conservative and thus, of higher cost, than the original deterministic solution. Our goal is to investigate the difference between these solutions and assess whether the 'price of robustness' is acceptable given the size of the instance, the level of uncertainty and the practical implications. Moreover, we would like to know what is the influence of the adjustable variables on the solution and objective function when lateness has a high and low penalty.

6.3.1. High lateness penalties

For our numerical test we will only consider an instance of manageable size, namely instance k5m12 with Objective 3. This objective is chosen because it has the highest lateness penalty and it has recorded the fastest computational time for the deterministic case, a fact which can be noticed in the last column of Table 6.1. Larger instances have not been considered due to two main reasons. Firstly, the solver would require a very large amount of time to solve them. Secondly, the simple k5m12 instance is already sufficiently diverse to allow us to study different features of the solution. In view of comparison purposes, we assume that all containers have an uncertain release date in an interval of fixed length. We consider six possible interval lengths of 2, 4, 6, 8, 10 and 12 hours, which encompass scenarios ranging from small to extremely large delays. For clarification, an uncertainty interval of two hours for instance, suggests that the release of a container can occur one hour before or after its nominal release value.

The solutions obtained by solving the robust model with different sizes for the interval uncertainty are given in Table 6.3. Some remarks are in place concerning the last two columns of this table. When using robust optimization, it is important to assess what is the 'price of robustness' namely, what is the additional cost to be paid when immunizing the solution with respect to the uncertain parameters. In order to do that, we have also considered the situation when the release times are already available at the beginning of the planning and solved the deterministic problem for two different realizations: the best case, in which every container is released at the earliest opportunity ($\zeta = -1$) and an 'average' case ($\zeta = 0$), when the release times occurs at the midpoint of the uncertainty interval. Then we calculated by how much the robust cost increases from the deterministic solution for both cases, and displayed those values in Table 6.3. As we expected, the cost of the robust solution increases as the the size of the interval of the release time is enlarged. Moreover, we observe that there is a certain amount of delay that the planning can handle. Namely, for instances with a release delay within three hours, the solution attained is identical to the one obtained by solving deterministic model with nominal release time values. Regarding the 'price of robustness', we see that for a data realization at the midpoint of the uncertainty set (not a favorable situation), the difference between the robust solution and the best deterministic solution is at most 22.73% (corresponding to a $\pm$6h margin for delay). The objective value

Data instance k5m12	Robust solution	Gap(%)	Runtime(s)	Increase best case	Increase 'average' case
2h interval	100	0	198.63	0 %	0%
4h interval	100	0	1854.22	0%	0%
6h interval	100	0	379.25	0%	0%
8h interval	1415	8.13	36000*	1315%	8.84%
10h interval	1465	11.26	36000*	1365%	12.69%
12h interval	1595.5	18.52	36000*	1495.5%	22.73%

Table 6.3: Objective value of the robust model, gap between the current solution and the best lower bound found so far, computation time time (CPU seconds), percentage increase from the deterministic objective for $\zeta = -1$, and percentage increase from the deterministic objective for $\zeta = 0$. * an upper bound of 36000s was set to the execution time of the solver.

for the robust solution alone is not sufficiently insightful to assess how the transportation changes when the uncertainty interval increases. To give a measure of this, we include the number of commodities that are transported only by truck and the utilization of barges in Table 6.4. It is apparent from this table that when commodities are released with significant delay (larger than three hours), it becomes impossible to transport them by barge. However, the uncertainty intervals that we considered were still not sufficiently large to enforce a transportation plan with no barge being utilized.

Data instance k5m12	Number of commodities fully trucked	Number of barges used
2h interval	0	6
4h interval	0	5
6h interval	0	5
8h interval	1	2
10h interval	4	1
12h interval	3	2

Table 6.4: Transportation characteristics of the robust planning.

The numbers of adjusted time variables for every uncertainty interval are highlighted in Table 6.4. From here, we can immediately notice that the affine rules that were proposed indeed induce the adjusting of arrival and departure times. However, it is striking that for uncertainty intervals of fours and eight hours, no adjustment occurs. Another important observation is that in the case when adjustment occurs, it does not affect all time variables which were assigned a numerical value in the solution.

Data instance k5m12	Number of adjusted time variables	Number of adjusted arrivals	Number of adjusted departures
2h interval	7	3	4
4h interval	0	0	0
6h interval	3	1	2
8h interval	0	0	0
10h interval	0	0	0
12h interval	10	5	5

Table 6.5: Statistics concerning the number of time variables that are adjusted by the affine rules. A random realization $\zeta \neq 0$ was used for checking whether the time variables have adjusted or not.

6.3.2. Low lateness penalties

When the lateness penalties are assigned sufficiently small numerical values, the adjustable variables can directly influence the value of the objective function. In order to illustrate this idea we have repeated the test from the previous section using Objective 1 (having a lateness penalty of 0.1). However, due to the high computational effort required by the solver, we were not able to record results in a reasonable amount of time as to include them in this report. Nevertheless, we will demonstrate the benefits of adjusting time variables using a simple example which still incorporates all the transportation elements that were shown by the other generated instances. Consider the transportation network in Figure 6.1. We assume that there are two containers to be released at Location 1 in the interval [8,10]. One needs to arrive at Location 2 by time unit 14 whilst the other has the same due time but it is destined for Location 3. It is assumed that there are sufficient barges and trucks to carry out these transport requests.
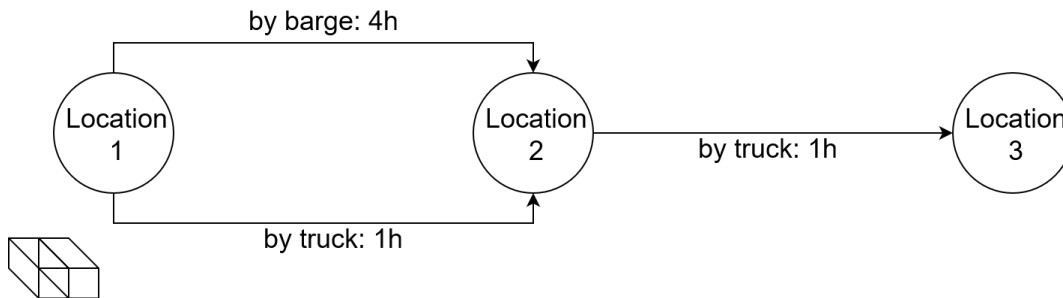

Figure 6.1: Example of a transportation network with three nodes and two commodities.

For this particular instance, the worst case solution given by the robust model in Chapter 5 with Objective 1 has cost 1.2. This corresponds to the situation when these containers are released at time unit 10. Both containers will be taken by barge to Location 2, and from there one will be trucked to its final destination, where it records a lateness of two hours. However, if the release occurs at time unit 8, then the adjusted cost will be of only 1 cost unit, since there is no lateness recorded at Location 3 anymore.

This result also shows that unlike in the results shown in Section 6.3.1, in this case it is possible to adjust the time variables along the routes which include a transshipment. This is due to the fact that we have included in the model constraints of the form

$$A_{i,k}^m \geq \lambda, \quad \forall i \in V, k \in K, m \in M, \tag{6.1}$$

where λ is some small scalar (chosen to be 1 in this example). These ensure in this case that the affine coefficients of the adjustable arrival times will be non-zero. We see that by including these constraints in the model, we can guarantee the adjustment of all time variables.

This example demonstrates that the affinely adjustable robust optimization framework can be used to obtain improved solutions for the freight routing problem with a high tolerance for lateness. However, the choice for the lateness parameter as well as for the scalar λ is instance-specific. Therefore,

at this stage of the research, it is difficult to make assertions about how the robust model in Chapter 5 can be used for any general instance.

6.4. Discussion

In this section, we discuss the most important findings from the numerical experiments and where necessary, provide more insight into the results obtained by closely inspecting the solutions.

The deterministic formulation of the freight routing can be successfully solved exactly with the branch-and-bound method for instances as large in size as those including twelve vehicles and twenty commodities. These instances are comparable to what is encountered in practice. However, the relatively large computational time require by the solver is likely due to two main factors namely, the large amount of (binary) decision variables in the model as well the many symmetries of the problem. Given the results in Table 6.1, we see that allowing for lateness with high penalties yields a model which can be solved faster than the original version with hard deadlines.

First of all, the computational results of the simple instance showed that we can obtain robust feasible solutions for the container freight routing problem by solving exactly the robust counterpart. These solutions correspond to transportation plans that can be carried whenever the release time of a container falls withing a prespecified interval. An increase in the size of this uncertainty interval induces higher solution costs, since containers which have a short delivery span will not be transported by barge. This results is fully confirmed by the data in Table 6.2. However, we see that the price to be paid for the robustness of solutions is quite high. As an example we consider the k5m12 case with an 8h interval. When we assume that the release times of the containers can deviate from their nominal value by four hours, and they in fact are released on the earliest time possible, the transportation plan obtained is 1315% more expensive than the plan that could have been achieved if all data was known beforehand. If we assume a less favorable realization, in which half of the commodities are released at their nominal value, and the other half at the latest time possible, the increase is only of 8.84%. Given the lack of information on the real-life situation, therefore difficult to assess if the price of robustness is acceptable when modeling highly uncertain releases for the containers. However, we can state that in a practical instance in which one can infer from historic data that parameters often attain 'bad realizations', the robust transport plan can be employed in exchange for a reasonable cost increase.

The price of robustness can also be regarded from a slightly different perspective. We consider a situation in which given some uncertainty intervals, one makes a deterministic plan assuming a certain nominal value for the releases. If the actual realization of the parameters is worse than the nominal values, then the deterministic solution is likely to be infeasible. This enforces re-planning of the current vehicles and container assignment. Although there are many ways in which one can re-route, the newly obtained transportation solution might have a higher cost than the robust solution that could have prevailed over the delays.

We have applied the affinely adjustable robust optimization framework in order to allow the arrival and departure times of the vehicle to change according to the realization of the container release and induce some degree of flexibility in the planning. Whilst the objective value of a solution remained unchanged, due to very high lateness penalties, some vehicles might be able to arrive or depart earlier at certain locations. Concerning the actual adjustment of variables, it was at first surprising to notice the low proportion of time variables are affected by the changes in data realization. However, at a closer inspection of the container routes given by the solutions we were able to find a possible explanation for this. We have found that adjustment is only effective for arrival and departure times on a particular kind of route. In other words, for direct routes, on which a commodity is shipped from its origin location to their destination by means of a single vehicle, adjustment takes place. Otherwise, if a switch of vehicle, transshipment or additional commodity pickup occurs on the way, then only arrivals and departures that can be adjusted are those at the beginning location of the route. When inspecting the solution, we have found that the instances k5m12 with 4h, 8h and 10h intervals, in which no adjustment took place, indeed included no direct routes. Moreover, adjustment seemed to be particularly successful for the case with the highest uncertainty interval. Essentially, since all commodities

are directly trucked from origin and destination in this case, all the arrivals and departures are adjusted.

It is difficult to further explain why adjustment only affects direct routes, but it might be related to the fact that the vehicle synchronization and transshipment constraints in the robust model force the affine coefficients of the adjustable variables to take the value zero. This suggests that on non-direct routes it is difficult to ensure adjustment with respect to all the decisions made on that route: transshipment, vehicle switch, loading or unloading of commodities. Another factor which may influence adjustment is the symmetry of solutions and the fact that the same objective can be achieved by many different routes. For instance, there are solutions in which a barge is assigned for every commodity, resulting in significantly less transshipments, which have the same objective as a solution with a smaller barge utilization. Nevertheless, we were able to produce an example in which the robust model with an additional sets of constraints gives a fully adjusted solution, which is cheaper than the worst-case scenario, if the data assumes a favorable realization. Therefore, we have shown that the adjustment of variables can result in a direct improvement of the objective function. For a generalization of this result, a sensitivity analysis of the instance parameters on the lateness term in the objective is required.

7

Conclusion

This chapter provides complete answers to the research questions addressed in this study, and summarizes the recommendations for future work concerning robust techniques applied to synchromodal transport.

7.1. Conclusions

This study set out to answer the following research question:

How can we simultaneously provide a container-to-mode assignment and a routing of modalities under uncertain data and with the objective of minimizing the total costs?

In order to answer this question, we proposed four sub-questions, which will be answered below.

1. *How can we model the simple instance described in Section 1.2.2 in order to encompass all the assumptions?*
In this thesis we have formulated a robust model for the freight routing problem with uncertain release times of the containers. As described in Chapter 5, the model minimizes the total costs due to trucking activities and lateness penalties whereby all the commodities are transported from their origin location to their destination. To allow for more flexibility in the resulting transport plan, the arrival times, departure times and lateness variables have been modeled as affine adjustable functions of the uncertain release times.

2. *What solution methods can be used to obtain a schedule and container assignment for every modality?*
To solve both the deterministic model and the robust counterpart, we have used an exact solver which implements the branch-and-bound method. Multiple instances were tested for the deterministic model, whilst the robust counterpart was only tested for an instance of manageable size.

3. *What can be said about the quality and practical relevance of our solution?*
All the solutions available for the deterministic model were optimal. This means that the freight routing problem with no uncertain elements can be optimally solved in a reasonable amount of time for instances encompassing at at most twelve barges and twenty commodities. For the robust model implemented for the simple instance with twelve barges and five commodities the results were at most within 20% of the best found lower bound. Although not optimal, these solutions have an immediate interpretation as decisions in the robust transportation plan. They dictate a container assignment and vehicle routing obtained in the first stage, followed by an associated time schedule in the second stage, after uncertain data is revealed.
The practical application of the robust model in this thesis is restricted in the sense that the model is built on the assumption that all uncertain release times are revealed simultaneously in the second stage. This fact however, is not always true in practical settings. Moreover, the sizes of the instances

in real life might be larger than the ones considered here. Finally, due to lack of information on real-world data, we cannot asses if the 'price of robustness' is an accurate measure for the additional costs inquired for a robust transportation plan.

4. *Does the chosen approach successfully incorporate elements of synchromodality?*
The robust optimization approach that we have chosen currently incorporates two characteristics of synchromodal transport namely, the ability of the planner to choose the appropriate transport means for delivering the containers, and the capacity of the transport plan to handle uncertain release times. There is some flexibility in the planning induced by the adjustment of time variables. This adjustment was found to be route dependent, and for a particular example, we were able to counter it by introducing additional constraints into the robust model. However, given the lack of a general result we are reticent in classifying it as a successful description of a synchromodal aspect.

7.2. Recommendations

This research has thrown up many questions and topics in need of further investigation. In this section, we discuss further research directions that we deem to be potentially fruitful.

First of all, the results for both the deterministic and robust model can be improved in both quality and in the size of the instances that can actually be solved. Several MIP based heuristics can be used here to explore the branch-and-bound tree more effectively. Examples of such heuristics include simple local branching, and variable neighborhood branching. We expect that the effectiveness of such approximation algorithms highly depends on the choice of binary variables for branching.

Secondly, one of our main result was that the time variables only get adjusted for direct routes in the robust solution. This can possibly be countered by a different description of the uncertainty set (for instance a polyhedral uncertainty set), or by decisions rules more developed than the affine dependence on uncertainty. Modifying these however is likely to results in a problem formulation more complex than a mixed integer program, which is inherently more difficult to solve.

Finally, we found that robust optimization can successfully model transportation problems with uncertain parameter data. However, the two-stage approach is too restrictive to fully encompass synchromodal elements such real-time vehicle switching. Therefore, we believe that a multi-stage approach, in which decisions are made on the short-term and adjusted continuously from one stage to another, has a great potential in describing synchromodal problems. Nevertheless, a multi-stage approach would also require significant theoretical developments since one would then be interested in adjusting binary and integer variables, along with the continuous ones.

A
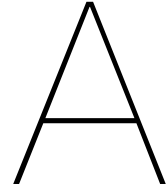
Distance matrix and cost derivation for base instance

The six locations described in the instance introduced in section 1.2.2 represent actual physical locations in The Netherlands. Therefore we were able to obtain the road distance between them using a Google Maps API. Information about the waterway distance was not available, therefore we have used the road distance in between the terminals since we expect that the road and waterway are sufficiently close in terms of the total length of the journey. Moreover, we only consider distances which are relevant for the available arcs in the space-time network. For example, since we do not allow traveling in between the two customers we leave the $C_1 - C_2$ distance value out.

Locations	T_1	T_2	T	D	C_1	C_2
T_1	-	-	35	220	250	-
T_2	-	-	35	230	-	-
T	35	220	-	200	200	210
D	220	230	200	-	10	20
C_1	250	-	200	10	-	-
C_2	-	-	210	20	-	-

Table A.1: Distance (by road) in km between every relevant pair of locations.

We consider the transportation costs to be given by the fuel cost per km for every mode. Wiegmans and Konings [52] state that the fuel cost of a small vessel (barge) with capacity of 90 TEU is approximately 7.5 €/km whilst the fuel cost of a truck is 0.44 €/km. We approximate this value by 0.5 €/km to ensure integral costs. Since the barges are assumed to have fixed schedules, we do not need to take into account their corresponding transportation costs. Instead, we are trying to minimize the transportation costs arising from trucking containers. Therefore, in the space time network we assign a cost of zero for transporting a container by barge. For the arcs traversed by trucks we assign the value obtained by multiplying the fuel cost with the traveled distance on that arc.

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