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Semi-constant Spacing Policy for Leader-Predecessor-Follower Platoon Control via Delayed Measurements Synchronization ⋆

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Abstract: Constant spacing-based platooning systems cannot guarantee string stability if platoon members only use the preceding vehicle's information. To meet string stability specification, leader-predecessor-follower (LPF) platooning systems are proposed to incorporate the information of both the preceding vehicle and the platoon leader into the control loop. However, string stability of LPF platooning systems is very sensitive to communication and sensing delays. Even a delay of 5 milliseconds may render LPF platooning systems string-unstable. This paper focuses on a new approach to deal with communication and sensing delays in LPF platooning systems. A semi-constant spacing policy that synchronizes delayed measurements of system states obtained from different sources is proposed. This spacing policy aims at tracking the past information of the preceding vehicle to gurantee string stability. Moreover, the delay-synchronizing LPF platooning system puts the same requirements on controller parameters as the nominal LPF platooning system that is not affected by communication and sensing delays. Thus, control gains of the delay-synchronizing LPF platoon can be designed without considering delays.

Keywords: Communication delay, constant spacing policy, leader-predecessor-follower topology, platooning, individual vehicle stability, string stability, vehicle following

1. INTRODUCTION

Vehicle platooning has been attracting extensive attention in recent years for its potential for improving traffic throughput (Bian et al., 2019) and reducing fuel assumption (Gattami et al., 2011). To achieve vehicle platooning, initial efforts were taken to equip the subject vehicles with a controller to regulate a predefined constant inter-vehicle spacing, using only the predecessor's position as a reference (Swaroop and Hedrick, 1999). However, this kind of systems suffers from the problem of string instability (Swaroop and Hedrick, 1999; Seiler et al., 2015), i.e., amplification of oscillation from the predecessor to the follower, which may lead to stop-and-go traffic waves and even rear-end collisions. Furthermore, the string instability problem occurs regardless of the controller parameters, the control law, or the oscillation profile.

To achieve string stability, two kinds of measures are mainly adopted. The first one is to change the constant spacing (CS) policy to be velocity-dependent spacing policy (for instance, constant time gap policy), allowing inter-vehicle distance growing with the speed of the host vehicle (Naus et al., 2010; Ploeg et al., 2014; Wang, 2018; Ploeg et al., 2013; Monteil et al., 2018; Zhang et al., 2020). Improvement in string stability with velocity-dependent policy is realized at the expenses of compromising the traffic throughput benefits (Santhanakrishnan and Rajamani, 2003). Another way is to keep the CS policy and incorporate the platoon leader's information into the control loop under the leader-predecessor-follower (LPF) topology (Liu et al., 2001; Peters and Middleton, 2011; Bian et al., 2019). LPF based CS policy facilitates tighter platoon as the CS policy does not require larger spacing when platoon operates in higher speeds and it improves string stability at the expense of higher complexity of communication structure (Swaroop and Hedrick, 1999; Konduri et al., 2017).

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However, string stability performance of both the above-mentioned methods can be jeopardized by communication and sensing delays, intrinsic characteristic of any communication systems and onboard sensors (Liu et al., 2001; Ploeg et al., 2014). To design a string-stable platooning system considering communication and sensing delays, systems adopting the velocity-dependent spacing policy have to employ large inter-vehicle time gap (Ploeg et al., 2014), which compromises the benefit of vehicle platooning. For LPF based CS platooning systems, string instability is unavoidable even when the delays are small (Liu et al., 2001).

A few methods that deal with delay in platooning systems have been proposed, mainly resorting either to predictor-feedback (Molnár et al., 2017; Bekiaris-Liberis et al., 2017) or to delay-based spacing policy (Besselink and Johansson, 2015, 2017; Ge et al., 2017). Yet, these methods focus on the platooning systems with velocity-dependent spacing policy and using a predecessor-follower communication architecture. It is still a challenge to develop a method to deal with the delays for LPF platooning systems with constant spacing policy.

In this paper, we focus on the control design of delay-synchronizing LPF platooning system. Toward this end, a new semi-constant spacing (SCS) policy is proposed. The SCS policy modifies the CS policy such that the subject vehicle regulates its position with respect to the past position of the preceding vehicle. This modification results in the same requirements of both individual vehicle stability and string stability as the nominal delay-free case. Thus, controller gains of the delay-synchronizing LPF platoon can be designed without considering delays.

The remainder of the paper is organized as follows. Section 2 presents the vehicle platooning system dynamics and the basic assumptions on the operation of the system. Section 3 presents the control design and stability analysis of LPF platooning systems in the absence of communication/sensing delays. Section 4 shows the influence of delays on the platooning systems' string stability and reveals the mechanism behind the string instability. Section 5 describes the delay-synchronizing strategy and shows the individual vehicle stability and string stability of the delay-synchronizing LPF platoon. Section 6 verifies the theoretical analysis by simulation. Section 7 concludes the paper and remarks the future research.

2. VEHICLE PLATOONING SYSTEM DYNAMICS

A platoon with LPF communication topology is shown in Fig. 1. For a platoon of $m+1$ vehicles, the vehicles in the platoon are indexed from 0 to m with the one indexed with 0 being the platoon leader and the rest being followers. The sensing delay of the i-th vehicle is denoted as $\delta_{s,i}$; the communication delay between the i-th vehicle and its preceding vehicle is denoted as $\delta_{p,i}$; the communication delay between the i-th vehicle and the leader vehicle is denoted as $\delta_{l,i}$.

All the vehicles in the platoon are assumed to share the same dynamic model as follows:

$$\dddot{p}(t) = \ddot{v}(t) = \dot{a}(t) = \frac{u(t) - a(t)}{\tau}, 0 \leq i \leq m \qquad (1)$$

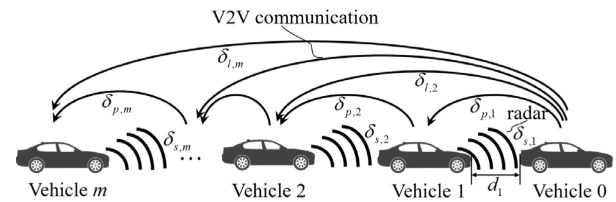


Fig. 1. LPF platooning system.

where p, v, and a are the rear-bumper position, speed and acceleration respectively. Variable u is the control, which can be interpreted as the desired acceleration. τ is the time constant of vehicle driveline dynamics, which is assumed to be identical for all the platoon members.

The following assumptions are made regarding the operation of the system:

Assumption 1. The preceding vehicle's speed and position are obtained by the subject vehicle's onboard sensors with delay $\delta_{s,i}$; the preceding vehicle's acceleration is obtained by vehicle-to-vehicle (V2V) communication with delay $\delta_{p,i}$; all the information (position, speed, and acceleration) about the lead vehicle are obtained by V2V communication delayed by $\delta_{l,i}$. All signals from the platoon leader and preceding vehicle are transmitted together with appropriate timestamps.

Assumption 2. Communication delays are time-varying in nature but bounded: $\delta_{p,i}$ is bounded in $[\underline{\delta_{p,i}}, \overline{\delta_{p,i}}]$; $\delta_{l,i}$ is bounded in $[\underline{\delta_{l,i}}, \overline{\delta_{l,i}}]$; the difference between $\delta_{l,i}$ and $\delta_{l,i-1}$ is bounded in $[\underline{\Delta\delta_{l,i}}, \overline{\Delta\delta_{l,i}}]$; sensor delay $\delta_{s,i}$ is deterministic and constant.

Assumption 3. The vehicles in the platoon have different sensor delays, communication delays, and desired distances, which represents heterogeneity.

In view of the performance of the state-of-the-art onboard sensors and communication technology, the aforementioned assumptions are not restrictive.

3. NOMINAL LPF PLATOONING SYSTEM WITHOUT COMMUNICATION AND SENSING DELAYS

This section focuses on the control design and stability analysis of LPF platooning systems without communication/sensing delays, which sets a nominal controller as benchmark. We briefly revisit the spacing policy, control law, as well as string stability analysis as proposed in previous works, e.g., Swaroop and Hedrick (1999); Liu et al. (2001). The LPF platooning system in this paper regulates on the control of the platoon followers and the platoon leader's control is out of scope.

3.1 Nominal Control Law

Constant spacing policy is proposed in previous works, e.g., Swaroop and Hedrick (1999); Liu et al. (2001):

$$d_{r,p,i}(t) = L_i, \qquad (2)$$

where i is the index of the subject vehicle; d_r is the desired spacing; L_i is the constant, vehicle-specific spacing. The desired spacing with respect to the lead vehicle indicated by this policy is

$$d_{r,l,i}(t) = \sum_{j=1}^{i} L_j. \tag{3}$$

The spacing policy leads to spacing error with respect to the preceding vehicle and the lead vehicle, respectively, as:

$$e_{p,i}(t) = d_{r,p,i}(t) - d_{p,i}(t) = p_i(t) - p_{i-1}(t) + L_i, \tag{4}$$

$$e_{l,i}(t) = d_{r,l,i}(t) - d_{l,i}(t) = p_i(t) - p_l(t) + \sum_{j=1}^{i} L_j, \tag{5}$$

where e is the spacing error; d is the actual spacing; p_i, p_{i-1}, and p_l are the rear-bumper positions of the subject vehicle, the preceding vehicle, and the lead vehicle, increasing with the driving direction.

The following control law was proposed in Liu et al. (2001):

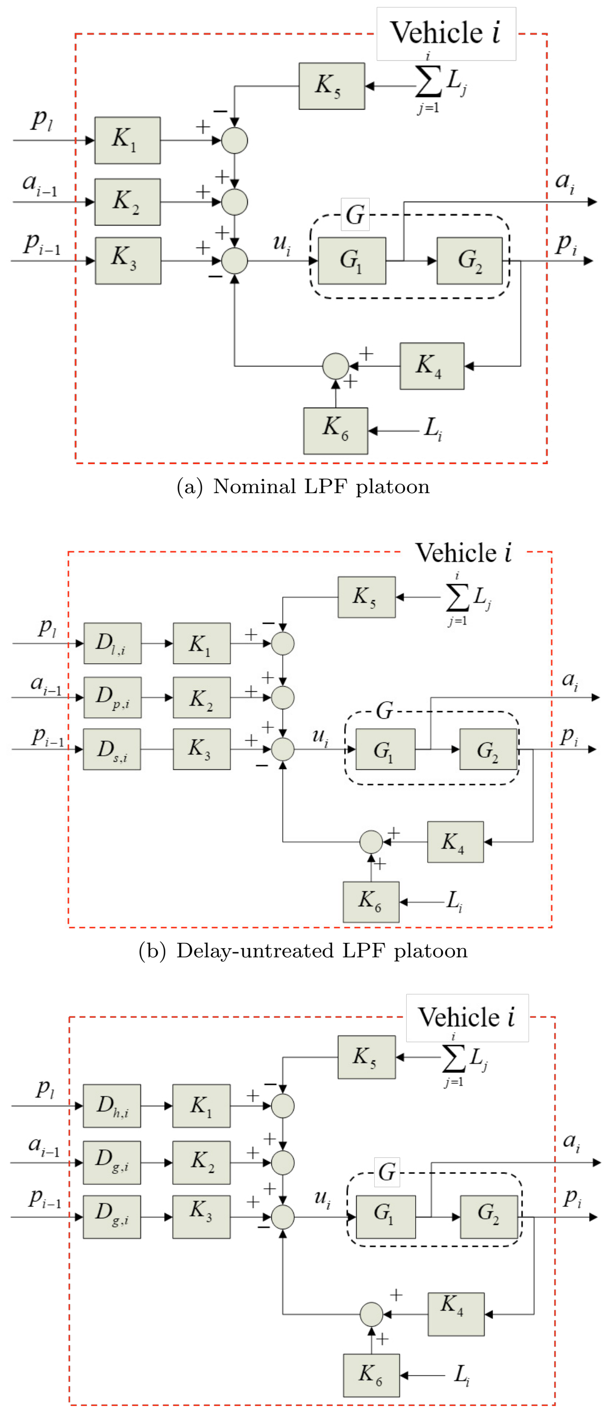
$$\begin{aligned} u_i(t) &= \frac{1}{1+q_3}\Big[a_{i-1}(t) + q_3 a_l(t) - (q_1+\lambda)\dot{e}_{p,i}(t) - q_1\lambda e_{p,i}(t) \\ &\quad - (q_4+\lambda q_3)\dot{e}_{l,i}(t) - \lambda q_4 e_{l,i}(t)\Big] \\ &= \frac{1}{1+q_3}\Big[a_{i-1}(t) + q_3 a_l(t) - (q_1+\lambda)(v_i(t) - v_{i-1}(t)) \\ &\quad - q_1\lambda(p_i(t) - p_{i-1}(t) + L_i) - (q_4+\lambda q_3)(v_i(t) - v_l(t)) \\ &\quad - \lambda q_4\Big(p_i(t) - p_l(t) + \sum_{j=1}^{i} L_j\Big)\Big], \end{aligned} \tag{6}$$

where u is the control; q_1, q_3, q_4, and λ are controller parameters; v_l and a_l are, respectively, the speed and acceleration of the lead vehicle. This control law incorporates the acceleration, speed difference, and spacing errors to both the lead vehicle and the preceding vehicle. The control law is designed such that the subject vehicle tracks the position of both the preceding vehicle (with desired distance L_i) and the lead vehicle (with desired distance $\sum_{j=1}^{i} L_j$).

The block diagram of the nominal LPF platooning system is shown in Fig. 2(a), in which K_1, K_2, K_3, and K_4 are respectively the transfer functions associated with the lead vehicle's information, the preceding vehicle's acceleration, the preceding vehicle's position, and the subject vehicle's position, and read as $K_1 = \frac{q_3 s^2 + (q_4+q_3\lambda)s + q_4\lambda}{1+q_3}$, $K_2 = \frac{1}{1+q_3}$, $K_3 = \frac{(q_1+\lambda)s + q_1\lambda}{1+q_3}$, $K_4 = \frac{(q_1+\lambda+q_4+q_3\lambda)s + (q_1\lambda+q_4\lambda)}{1+q_3}$ (Liu et al., 2001). K_5 and K_6 are the transfer functions associated with the constant spacing and read as $K_5 = \frac{\lambda q_4}{1+q_3}$ and $K_6 = \frac{\lambda q_1}{1+q_3}$. Transfer functions G_1 and G_2 together represent vehicle dynamics, and read as $G_1 = \frac{1}{1+\tau s}$, $G_2 = \frac{1}{s^2}$ (Öncü et al., 2014; Ploeg et al., 2014).

3.2 Stability Analysis of the Nominal Controller

The criteria for both individual vehicle stability and string stability are given in this section. As no delay is involved in the statements of the following lemmas, their proofs can be found in literature, see, e.g., Liu et al. (2001); Öncü et al. (2014); Ploeg et al. (2014). Yet, as we adopt similar proof strategies for our control design (enabled by its delay-synchronizing structure), we also detail the respective proofs for the delay-free case below.



(a) Nominal LPF platoon

(b) Delay-untreated LPF platoon

(c) Delay-synchronizing LPF platoon

Fig. 2. Block diagrams of different system/control setups.

Lemma 1. (Individual vehicle stability criterion) The LPF platooning system is stable if $(q_1+q_4)\lambda > 0$, $1+q_3 > 0$, and $\lambda(1+q_3) > (\lambda\tau - 1)(q_1+q_4)$.

Proof. The block diagram indicates the following relation in the frequency domain (note that the Laplace transform is carried out ignoring the constant spacing L_i, as this does not affect the analysis)

$$\left(K_4(s) + \frac{1}{G_1(s)G_2(s)}\right)P_i(s) = (K_2(s)s^2 + K_3(s))P_{i-1}(s) + K_1(s) \times P_l(s). \tag{7}$$

Rearranging the above equation gives

$$A(s)P_i(s) = B(s)P_{i-1}(s) + C(s)P_l(s), \tag{8}$$

with

$$\begin{aligned} A(s) =&(1+q_3)s^2(1+\tau s) + (q_1+\lambda+q_4+q_3\lambda)s \\ &+ (q_1\lambda + q_4\lambda), \\ B(s) =&s^2 + (q_1+\lambda)s + q_1\lambda, \\ C(s) =&q_3 s^2 + (q_4+q_3\lambda)s + q_4\lambda. \end{aligned} \tag{9}$$

$P(s)$ in the above equation is the Laplace transform of $p(t)$. Routh-Hurwitz stability criterion shows that the sufficient conditions to guarantee the individual vehicle stability of the system are $(q_1+q_4)\lambda > 0$, $1+q_3 > 0$, and $\lambda(1+q_3) > (\lambda\tau - 1)(q_1+q_4)$, which completes the proof of Lemma 1. ■

Lemma 2. (String stability criterion) The LPF platooning system is string stable if

$$\sup_{\omega>0}\left|\frac{\frac{1}{1+q_3}\left[s^2+(\lambda+q_1)s+\lambda q_1\right]}{s^2(1+\tau s)+\frac{\lambda(1+q_3)+q_1+q_4}{1+q_3}s+\frac{\lambda(q_1+q_4)}{1+q_3}}\right| \le 1,\ s=j\omega$$

Proof. Substituting the vehicle index i in equation (8) with $i-1$ gives

$$A(s)P_{i-1}(s) = B(s)P_{i-2}(s) + C(s)P_l(s). \tag{10}$$

Subtracting equation (10) from (8) then leads to

$$A(s)\left[P_i(s) - P_{i-1}(s)\right] = B(s)\left[P_{i-1}(s) - P_{i-2}(s)\right]. \tag{11}$$

which gives the transfer function between e_i and e_{i-1} as:

$$\Gamma^{\text{error}}_{i,i-1}(s) = \frac{P_i(s)-P_{i-1}(s)}{P_{i-1}(s)-P_{i-2}(s)} = \frac{B(s)}{A(s)}. \tag{12}$$

To guarantee that the spacing error does not amplify, the following should hold

$$\sup_{\omega>0}\left|\Gamma^{\text{error}}_{i,i-1}(j\omega)\right| \le 1. \tag{13}$$

This is exactly the condition given by Lemma 2. ■

4. DESTABILIZATION EFFECTS OF COMMUNICATION AND SENSING DELAYS

The influence of communication and sensing delays on LPF platooning systems is revealed in this section. The mechanism behind the sting instability is also discussed. To ease the following discussions, communication delays are assumed to be constant in this section.

As indicated by Assumption 1, the position and speed of the preceding vehicle are delayed by $\delta_{s,i}$, the acceleration of the preceding vehicle is delayed by $\delta_{p,i}$, and all the information associated with the leader vehicle are delayed by $\delta_{l,i}$. The block diagram of the delay-untreated LPF platooning systems affected by delays is shown in Fig. 2(b). The three delay blocks in the block diagram, respectively, read as $D_{s,i} = e^{-\delta_{s,i}s}$, $D_{l,i} = e^{-\delta_{l,i}s}$, $D_{p,i} = e^{-\delta_{p,i}s}$.

When affected by communication/sensing delays, the transfer functions have the following recursive relation:

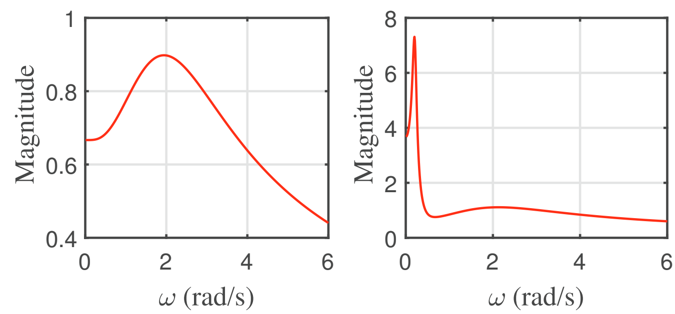
$$A(s)P_i(s) = \begin{cases} B_1(s)P_{i-1}(s)e^{-\delta_{s,i}s} + B_2(s)P_{i-1}(s)e^{-\delta_{p,i}s} \\ +C(s)P_l(s)e^{-\delta_{l,i}s}, \quad\quad if\ i>2 \\ B_1(s)P_{i-1}(s)e^{-\delta_{s,i}s} + B_2(s)P_{i-1}(s)e^{-\delta_{p,i}s} \\ +C_1(s)P_l(s)e^{-\delta_{s,i}s} + C_2(s)P_l(s)e^{-\delta_{l,i}s}, \\ \quad\quad if\ i=2 \end{cases} \tag{14}$$

with

$$\begin{aligned} B_1(s) &= (q_1+\lambda)s + q_1\lambda, \\ B_2(s) &= s^2, \\ C_1(s) &= (q_4+q_3\lambda)s + q_4\lambda, \\ C_2(s) &= q_3 s^2. \end{aligned}$$

4.1 String Instability of the Delay-untreated Controller

A comparison of the error transfer function of LPF platooning system with and without delays is shown in Fig. 3, which clearly indicates the string instability of LPF platooning systems affected by communication/sensing delays.



(a) Bode plot of $\Gamma^{\text{error}}_{2,1}$ without delays (b) Bode plot of $\Gamma^{\text{error}}_{2,1}$ with delays

Fig. 3. Error transfer function for $\lambda = 1.0$, $q_1 = 0.8$, $q_3 = 0.5$, $q_4 = 0.4$, $\tau = 0.25$ s, $\delta_{s,i} = 0.02$ s, $\delta_{p,i} = 0.1$ s, $\delta_{l,i} = 0.1 \times i$ s.

The string instability of delay-untreated LPF platoons originates from two conflicting control objectives, which is detailed as follows.

The control law (6) gives the desired spacing between the subject vehicle and the leader vehicle as

$$[p_l(t-\delta_{l,i}) - p_i(t)]_{\text{desired}} = \sum_{j=1}^{i} L_j. \tag{15}$$

The argument of p_l is $t-\delta_{l,i}$ rather than t, which is originated from the communication delay and implies the actual control objective is to track the lead vehicle's past position. The equation above is equivalent to:

$$[p_l(t) - p_i(t)]_{\text{desired}} = \sum_{j=1}^{i} L_j + \int_{t-\delta_{l,i}}^{t} v_l(t')dt'. \tag{16}$$

On the other hand, for each vehicle, the desired spacing with respect to its preceding vehicle is

$$[p_{i-1}(t-\delta_{s,i}) - p_i(t)]_{\text{desired}} = L_i, \tag{17}$$

which gives the desired spacing between the subject vehicle and the lead vehicle as:

$$[p_l(t) - p_i(t)]_{\text{desired}} = \sum_{j=1}^{i} L_j + \sum_{j=0}^{i-1}\int_{t-\delta_{s,j+1}}^{t} v_j(t')dt'. \tag{18}$$

The difference between equation (16) and (18) shows that tracking the preceding vehicle's position is actually conflicting with tracking the lead vehicle's position. When there is no communication/sensing delays, the conflict vanishes. Yet, when delays are taken into account in the control design, the string instability emerges.

4.2 Fixing String Instability

As discussed in the previous section, it is the conflict between tracking the preceding vehicle's position and tracking the lead vehicle's position that causes string instability. To fix the string instability caused by communication and sensing delays, the key point is to harmonize the two tracking objectives. To this end, the prevailing constant spacing policy needs to be adapted. The mechanism behind the proposed SCS policy, presented in the next section, relies on synchronization of the delays of sensing and communication measurements, which leads to elimination of the conflict.

5. DELAY-SYNCHRONIZING LPF PLATOONING SYSTEM

This section details the delay-synchronizing strategy for LPF platooning systems including presentation of control design and stability analysis.

5.1 Control Design

A semi-constant spacing policy is proposed as:

$$\tilde{d}_{p,r,i}(t) = L_i + \int_{t-g_i}^{t} v_{i-1}(t')dt', \tag{19}$$

where

$$g_i \geq \max(\delta_{s,i}, \overline{\delta_{p,i}}, \overline{\Delta\delta_{l,i}}). \tag{20}$$

The tilde symbols hereinafter are used to distinguish the delay-synchronizing LPF platoon variables with the nominal counterparts. g_i is a design parameter. It is interpreted as memory time window and is formulated to guarantee that the information of the preceding vehicle g_i seconds ago is available to the subject vehicle at the current time as long as inequality (20) is respected, as well as to synchronize the delays of communicated and sensing measurements.

Note that g_i is not determined by $\delta_{l,i}$, but by $\Delta\delta_{l,i}$ (partially), i.e. the difference between $\delta_{l,i}$ and $\delta_{l,i-1}$. Therefore, the delay-synchronizing LPF system puts no requirement on the realization of the leader's information (it can be either single hop or multiple hops) and there is no need to employ larger spacing for the vehicles at the tail of the platoon than those at the head. Furthermore, communication/sensing delays are normally small and the contribution of integral part of the desired spacing policy is marginal when compared with the constant part, thus the proposed spacing policy is quite alike the constant spacing (that is why the formulated spacing policy is called semi-constant spacing policy).

Under this formulation, the spacing error with respect to the preceding vehicle is

$$\begin{aligned}\tilde{e}_{p,i}(t) &= p_i(t) - \left[p_{i-1}(t) - \int_{t-g_i}^{t} v_{i-1}(t')dt'\right] + L_i \\ &= p_i(t) - p_{i-1}(t-g_i) + L_i.\end{aligned} \tag{21}$$

Similarly, the desired spacing with respect to the lead vehicle is defined as

$$\tilde{d}_{l,r,i}(t) = \sum_{j=1}^{i} L_j + \int_{t-h_i}^{t} v_l(t')dt', \tag{22}$$

with

$$h_i = \sum_{j=1}^{i} g_i, \tag{23}$$

which leads the spacing error with respect to the lead vehicle to be

$$\tilde{e}_{l,i}(t) = p_i(t) - p_l(t-h_i) + \sum_{j=1}^{i} L_j. \tag{24}$$

Correspondingly, control law is then formulated as in the similar way as Eq. (6):

$$\begin{aligned}\tilde{u}_i(t) =& \frac{1}{1+q_3}\Big[a_{i-1}(t-g_i) + q_3 a_l(t-h_i) \\ &- (q_1+\lambda)(v_i(t) - v_{i-1}(t-g_i)) \\ &- q_1\lambda(p_i(t) - p_{i-1}(t-g_i) + L_i) \\ &- (q_4+\lambda q_3)(v_i(t) - v_l(t-h_i)) \\ &- \lambda q_4\Big(p_i(t) - p_l(t-h_i) + \sum_{j=1}^{i} L_j\Big)\Big].\end{aligned} \tag{25}$$

The difference between the adapted control law (25) and the original one (6) lies in the time arguments. The time arguments associated with the preceding vehicles information are all $t - g_i$ and those associated with the lead vehicle are $t - h_i$. The two objectives indicated by the proposed control law are: 1) tracking preceding vehicle's past state g_i seconds ago; 2) tracking the lead vehicle's past state h_i seconds ago. As shown by equation (23), the delays between the consecutive vehicles' time arguments (g_i) add up to the delay between the subject vehicle and the lead vehicle (h_i), thus the delays are synchronized and the two aforementioned objectives are consistent. Moreover, the information required in equation (25) is all available as:

$$g_i \geq \delta_{s,i}, \tag{26}$$

$$g_i \geq \overline{\delta_{p,i}} \geq \delta_{p,i}, \tag{27}$$

and

$$\begin{aligned}h_i = \sum_{j=1}^{i} g_i &\geq \sum_{j=2}^{i} \overline{\Delta\delta_{l,j}} + \delta_{l,1} \\ &\geq \sum_{j=2}^{i} (\delta_{l,j} - \delta_{l,j-1}) + \delta_{l,1} = \delta_{l,i},\end{aligned} \tag{28}$$

To implement the control law (25), the choice of g_i should be made such that the signals involved are available. One straightforward way is to use the nearest samples before and after the time instant $t - \delta_{p,i}$ to interpolate the preceding vehicle's information. A similar practice can be applied to obtain the lead vehicle's information. This is made possible thanks to the assumption that timestamp is transmitted together with the preceding vehicle acceleration. The block diagram of this control law is shown in Fig. 2(c), in which $D_{g,i} = e^{-g_i s}$, $D_{h,i} = e^{-h_i s}$.

5.2 Stability Analysis

Lemma 3. (Individual vehicle stability) The delay synchronizing LPF platooning system is stable if $(q_1 + q_4)\lambda > 0$, $1 + q_3 > 0$, and $\lambda(1 + q_3) > (\lambda\tau - 1)(q_1 + q_4)$.

The individual vehicle stability proof is similar with that of the nominal LPF platooning systems without delays and thus is omitted. ■

Lemma 4. (String stability) The delay-synchronizing LPF platooning system is string stable if

$$\sup_{\omega>0}\left|\frac{\frac{1}{1+q_3}\left[s^2+(\lambda+q_1)s+\lambda q_1\right]}{s^2(1+\tau s)+\frac{\lambda(1+q_3)+q_1+q_4}{1+q_3}s+\frac{\lambda(q_1+q_4)}{1+q_3}}\right| \le 1,\ s=j\omega.$$

Proof. The block diagram indicates that

$$\left(K_4(s)+\frac{1}{G_1(s)G_2(s)}\right)P_i(s)=(K_2(s)s^2+K_3(s))P_{i-1}(s)e^{-g_i s} + K_1(s)\times P_l(s)e^{-h_i s}. \tag{29}$$

Rearranging the above equation then gives

$$A(s)P_i(s)=B(s)P_{i-1}(s)e^{-g_i s}+C(s)P_l(s)e^{-h_i s}. \tag{30}$$

Substituting vehicle index i with $i-1$ in the above equation and multiplying by $e^{-g_i s}$, $A(s)P_{i-1}(s)e^{-g_i s}$ can be derived as:

$$A(s)P_{i-1}(s)e^{-g_i s}=B(s)P_{i-2}(s)e^{-(g_i+g_{i-1})s} + C(s)P_l(s)e^{-h_i s}. \tag{31}$$

Subtracting equation (31) from (30) gives that

$$A(s)[P_i(s)-P_{i-1}(s)e^{-g_i s}] = B(s)[P_{i-1}(s)-P_{i-2}(s)e^{-g_{i-1}s}]e^{-g_i s} \tag{32}$$

Thus,

$$\tilde{\Gamma}^{\text{error}}_{i,i-1}(s)=\frac{P_i(s)-P_{i-1}(s)e^{-g_i s}}{P_{i-1}(s)-P_{i-2}(s)e^{-g_{i-1}s}}=\frac{B(s)}{A(s)}e^{-g_i s}. \tag{33}$$

Hence, the sufficient condition for string stability is:

$$\sup_{\omega>0}\left|\tilde{\Gamma}^{\text{error}}_{i,i-1}(j\omega)\right|=\sup_{\omega>0}\left|\frac{B(j\omega)}{A(j\omega)}e^{-g_i j\omega}\right|\le\sup_{\omega>0}\left|\frac{B(j\omega)}{A(j\omega)}\right|\le 1. \tag{34}$$

Substituting $A(s)$ and $B(s)$ into the above equation gives the relation stated in Lemma 4. ■

Notice that the delay-synchronizing LPF platoon has the same individual vehicle stability conditions and string stability conditions as the nominal LPF platoon without delays. This greatly eases the parameter tuning task. Parameters of delay-synchronizing LPF platoon can be predetermined without considering communication delay. The only added work for designing a delay-synchronizing LPF platooning system is to determine g_i and h_i according to equation (20) and (23). The speed/acceleration error transfer functions of the proposed delay-synchronizing method cannot be guaranteed to not exceed unity. Nevertheless, this can be avoided by carefully design the longitudinal following controller of the leader, which is beyond the scope of this paper.

6. VERIFICATION

In this section, simulations are carried out to verify the effectiveness of the proposed delay-synchronizing LPF platoon strategy (with SCS policy and control law (25)) against the nominal LPF platoon (with CS policy, control law (6) and zero communication/sensing delays) and the delay-untreated LPF platoon (with CS policy, control law (6) and untreated communication/sensing delays).

6.1 Experiment Design

An LPF platoon with 22 vehicles (indexed from 0 to 21) is simulated. The vehicle indexed with 0 is the leader and the rest are followers. Simulation settings are as follows:

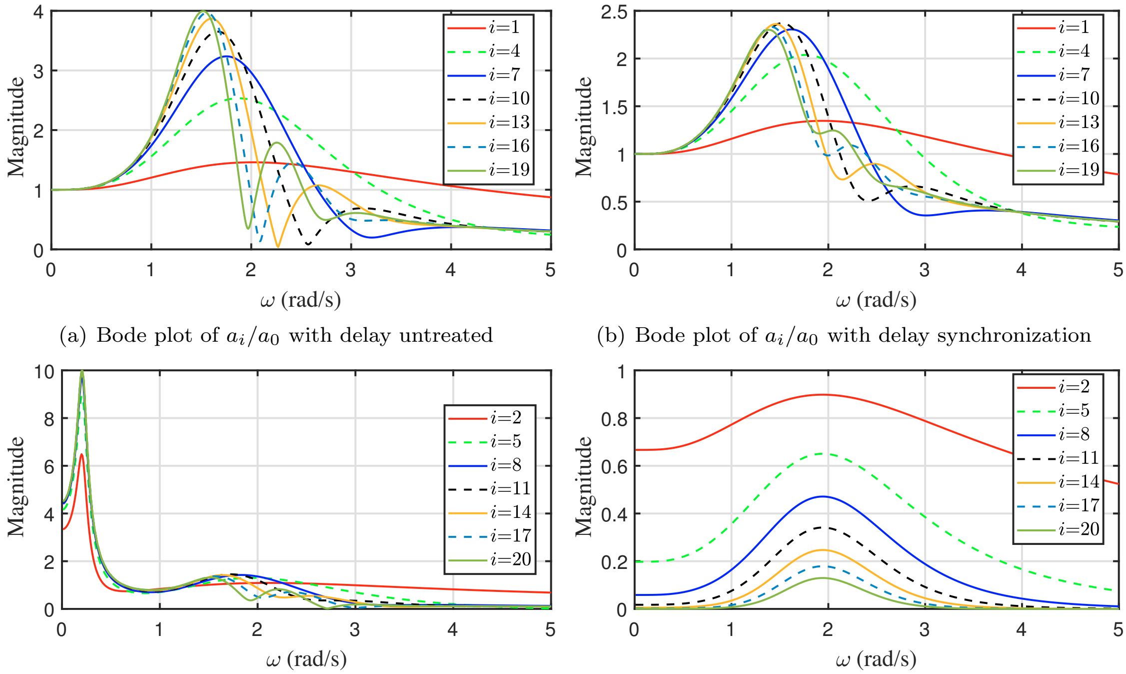
- Simulation parameters: simulation horizon 100 s; communication frequency 10 Hz; sensing frequency 10 Hz; control frequency 10 Hz.
- Controller parameters: $\lambda = 1.0$, $q_1 = 0.8$, $q_3 = 0.5$, $q_4 = 0.4$; Desired constant spacing: $L_i = 10$ m $(i \ge 1)$;
- Vehicle dynamics parameters: estimated time constants are all 0.25 s for all vehicles; real time constants are uniformly distributed in [0.20, 0.30] seconds;
- Delay parameters: sensor delay is 0.02 seconds, communication delays with the preceding vehicle are uniformly distributed in [0.08, 0.10] seconds, communication delays with the lead vehicle are uniformly distributed in $[0.08\times i,\ 0.1\times i]$ seconds;
- Memory time window: $g_i = 0.1$ s $(i \ge 1)$.
- Initial state: simulations start with equilibrium state; the lead vehicle decelerates from 30 m/s to 5 m/s with a constant desired deceleration (i.e., u_0) -1 m/s^2.

The requirement on homogeneous dynamics shown by Eq. (1) is relaxed in the simulation setting. The real time constants for the platoon members are assumed to be different from the estimated ones. This is to show the robustness of the proposed delay-synchronizing method against vehicle dynamics heterogeneity.

6.2 Simulation Results

The proposed delay-synchronizing strategy helps to enhance the string stability of the platoon, thus reduce traffic oscillation. This can be seen from both the Bode plots, which are obtained by adopting the average communication delays, and the motion profiles of the LPF platoon members. Under the same experiment setting, the acceleration and spacing error transfer functions of the delay-synchronizing LPF platooning system are much smaller than that of the delay-untreated system, as shown in Fig. 4. Moreover, the spacing error transfer function of the delay-synchronizing LPF platooning system converges toward zero (showing string stability) as the errors propagate to the platoon tail, while that of the delay-untreated platoon diverges (showing string instability). Fig. 5 indicates that the delay-synchronizing LPF operates similarly with the nominal LPF. While the oscillations of errors $e_{p,i}$ and $e_{l,i}$ of delay-synchronizing platoon gradually decrease as they are propagating to the tail, those of the delay-untreated platoon continuously increase. Furthermore, due to the variations in communication delay, the delay-untreated platoon exhibits significant oscillations in both the state and control even when the lead vehicle runs at steady state.

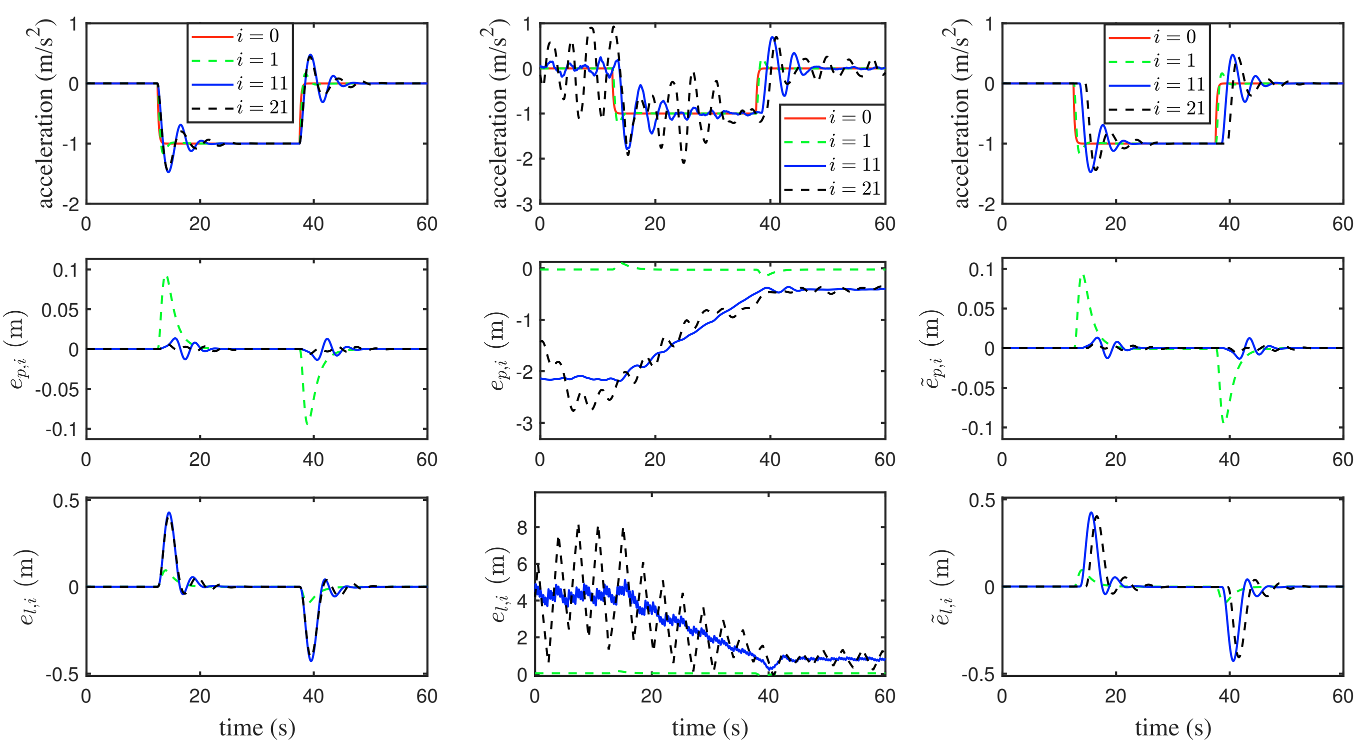
The simulation results clearly show the conflict between tracking the preceding vehicle's position and the lead vehicle's position in the delay-untreated LPF platooning system. Fig. 5 shows that the steady-state spacing errors of the delay-untreated LPF platoon are not always zero. Yet, the conflict between the two objectives is not present in the delay-synchronizing system.



(a) Bode plot of a_i/a_0 with delay untreated (b) Bode plot of a_i/a_0 with delay synchronization

(c) Bode plot of $\Gamma_{i,1}^{\text{error}}$ with delay untreated (d) Bode plot of $\tilde{\Gamma}_{i,1}^{\text{error}}$ with delay synchronization

Fig. 4. Bode plot of the respective transfer functions with $\lambda = 1.0$, $q_1 = 0.8$, $q_3 = 0.5$, $q_4 = 0.4$, $\tau \sim [0.2,\ 0.3]$ s, $\delta_{s,i} = 0.02$ s, $\delta_{p,i} = 0.09$ s, $\delta_{l,i} = 0.09 \times i$ s, $g_i = 0.1$ s.



(a) Simulation results of the nominal LPF platoon (b) Simulation results of the delay-untreated LPF platoon (c) Simulation results of the delay-synchronizing LPF platoon

Fig. 5. Simulation results: delay-compensated LPF platooning system is string stable, while the delay-untreated LPF platooning system is not. $\lambda = 1.0$, $q_1 = 0.8$, $q_3 = 0.5$, $q_4 = 0.4$, $\tau \sim [0.2,\ 0.3]$ s, $\delta_{s,i} = 0.02$ s, $\delta_{p,i} \sim [0.08,\ 0.10]$ s, $\delta_{l,i} \sim [0.08 \times i,\ 0.10 \times i]$ s, $g_i = 0.1$ s.

Moreover, the LPF platoon under the proposed strategy shows good robustness against vehicle dynamics heterogeneity. Although in the experiments, the actual time constants (which are uniformly distributed in [0.2, 0.3] seconds) do not perfectly match the estimated ones (which are 0.25 seconds), both the individual vehicle stability and string stability are not significantly affected.

Overall, the simulation results are in accordance with the theoretical analysis and prove the effectiveness of the proposed delay-synchronizing spacing policy.

7. CONCLUSIONS

String instability in LPF platooning systems is caused by the conflict between tracking the preceding vehicle's position and tracking the lead vehicle's position under heterogeneous delays of different information sources. To eliminate the conflict, the semi-constant spacing policy is proposed. By employing this novel spacing policy, string stability of LPF platooning systems can be guaranteed, as opposed to LPF platooning systems with untreated communication & sensing delays. The parameters of the delay-synchronizing LPF control law can be predetermined without considering delays, which greatly eases the control design. Moreover, the proposed strategy also shows robustness against parameter uncertainty of the vehicle driveline dynamics in simulation.

Apart from the constant spacing policy, state-of-the-art LPF platooning systems also adopt other spacing policies (for instance, constant time gap policy) to regulate the inter-vehicle distances. Handling the communication/sensing delays of these systems is seen as the next research step. Moreover, in this paper, the vehicles in the platoon are restricted to have the same dynamics. Control design of heterogeneous LPF platoon strategies will also be a topic of future research.

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