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Revealing features and peculiarities of a nonlinear gradient elasticity model for seismic waves prediction

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Summary. The authors have previously introduced a novel gradient elasticity model for seismic wave predictions. The said model combines (i) the higher-order gradient terms that capture the influence of small-scale soil heterogeneity and/or microstructure and (ii) the nonlinear softening soil behaviour through the use of the hyperbolic soil model. The current study presents an in-depth analysis of the proposed model. The findings indicate that as nonlinearity increases, the bulk of the wave slows down, and its shape becomes more distorted in comparison to the response of the linear system. Furthermore, the wavenumber spectrum of the nonlinear-*elastic* response presents peaks at large wavenumbers. However, these are eliminated when a small amount of linear viscous damping is added indicating that they are not physically relevant. One model feature that does not disppear with the presence of damping is the formation of small-amplitude waves travelling in the opposite direction to the main wave. These findings shed light on the characteristics of the proposed nonlinear gradient elasticity model and its applicability for predicting the seismic site response.

Introduction and model

Predicting the seismic site response, which refers to the reaction of the top layers of soil to seismic waves, is crucial for designing structures in earthquake-prone regions. When seismic loads cause significant soil strains, considering the nonlinear behavior of the soil becomes essential for precise predictions. In Ref. [1], the authors have introduced a *nonlinear* gradient elasticity model for forecasting the seismic site response. This model incorporates the nonlinear constitutive behavior of the soil using a hyperbolic soil model, where the (secant) shear modulus G is dependent on the shear strain γ through a non-polynomial relation, as follows

$$G(\gamma) = \frac{G_0}{1 + \left(|\gamma|/\gamma_{\mathrm{ref}}\right)^{\beta}}, \tag{1}$$

where G_0 is the small-strain shear modulus and γ_{ref} and β are material constants.
Furthermore, the classical wave equation is extended to a nonlinear gradient elasticity model to capture the effects of small-scale heterogeneity or microstructure. This extension introduces higher-order gradient terms into the equation of motion, resulting in dispersive effects [2] that prevent the occurrence of unphysical jumps in the response. The equation of motion of the model reads [1]

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial z}\left(G(\gamma)\frac{\partial u}{\partial z} - B_1 L^2 \frac{\partial^2}{\partial z^2}\left(G(\gamma)\frac{\partial u}{\partial z}\right) + B_2 \frac{\rho L^2}{G_0}\frac{\partial^2}{\partial t^2}\left(G(\gamma)\frac{\partial u}{\partial z}\right)\right), \tag{2}$$

whereρ is the mass density of soil, B_1, B_2 are dimensionless constants, L is the characteristic length of the soil microstructure, z and t denote space and time, respectively, and u is the soil displacement. For infinitesimal strains, Eq. (2) reduces to the linear gradient elasticity model derived in [3].
The considered system has an infinite extent and is subject to initial conditions, which are prescribed in this study as trivial initial displacement and non-zero initial velocity with representing a spatial-derivative of the Gaussian pulse, and read

$$u(z, t=0) = 0, \qquad \frac{\partial u}{\partial t}(z, t=0) = A\frac{z}{\sigma^2} 2\sqrt{\frac{G_0}{\rho}}\,\mathrm{e}^{-\frac{z^2}{2\sigma^2}}, \tag{3}$$

where A and σ are the amplitude and standard deviation of the Gaussian pulse. The response of the system is determined by using a novel finite difference scheme (see Ref. [1]). The infinite extent of the system is ensured by enlarging the spatial domain such that the initial pulse cannot reach the boundaries in the finite simulation time.
The current study presents an in-depth analysis of the proposed model. More specifically, the following aspects are investigated: (i) the change in response for different levels of nonlinearity, which is varied through the amplitude A of the initial conditions; (ii) the evolution of the response wavenumber spectrum with time (this highlights the change in response wavenumber contribution for the continuously changing response pattern from the initial pulse to the final response); (iii) the emergence in the wavenumber spectrum of peaks at very large wavenumbers; and (iv) the formation of small-amplitude waves travelling in the opposite direction to the main wave. A thorough exploration of the characteristics of the proposed model can aid in its accurate utilization for predicting the seismic site response.

Results

The left panel in Fig. 1 shows that increasing nonlinearity changes the wave shape; the wave becomes broader at the base and sharper at its peak. Also, its velocity decreases with increasing nonlinearity, which is intuitive for a softening material. Due to the nonlinear behaviour, the wavenumber spectrum of the system's total energy presents considerable peaks at very large wavenumbers (right panel of Fig. 1 for a medium nonlinearity). While these peaks seem large, they contain a negligible amount of energy compared to the main peak. Furthermore, once a small amount of viscous damping is introduced into the system, these large-wavenumber peaks disappear, implying that they are not of physical relevance.

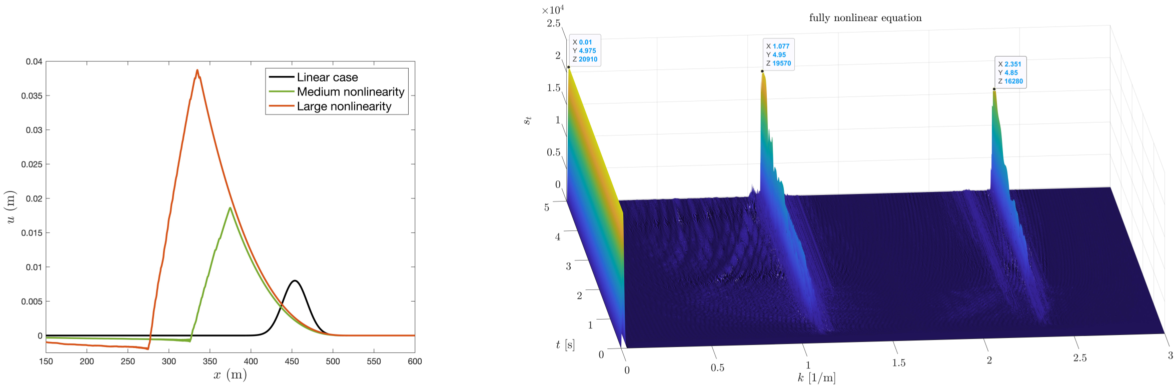

Figure 1: Left panel: Snapshot of displacement fields for three levels of induced nonlinearity (obtained by adjusting the amplitude A of the initial pulse). Right panel: The total spectral energy density versus time for medium nonlinearity.

The nonlinear system also leads to the formation of small-amplitude waves that propagate in the opposite direction to the main wave, as shown in Fig. 2 (clearly visible in the left panel). These small-amplitude waves appear to be excited only after a finite amount of time (around $t = 1$ s in the left panel while around $t = 2$ s in the right one). Unlike the large-wavenumber peaks, the backward propagating waves are also present in the system with little viscous damping, albeit less pronounced. This feature is a characteristic of the system's nonlinearity since it is not seen in the linear model (with or without higher-order terms).

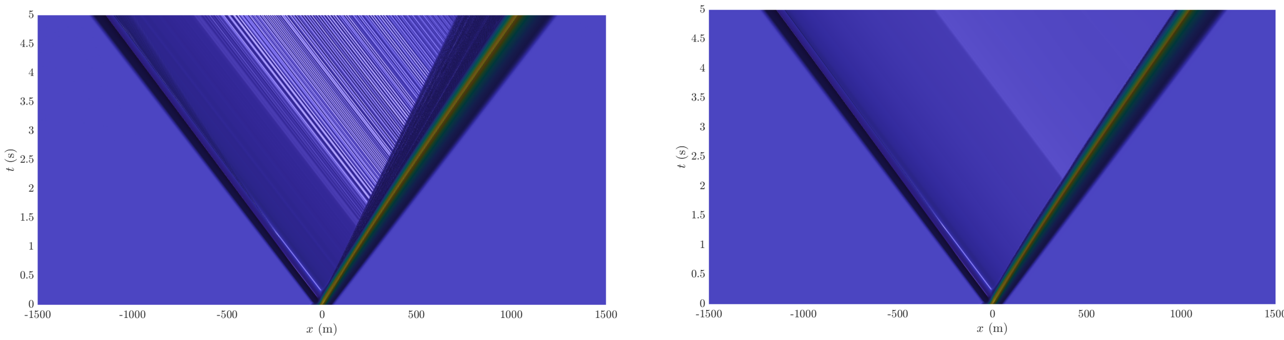

Figure 2: Top view of the displacement field in space and time for an elastic system (left panel) and one with a small amount of material viscous damping (right panel).

Conclusions

The current study presents an in-depth analysis of a previously proposed gradient elasticity model for seismic wave predictions. The model in question integrates (i) higher-order gradient terms to account for the impact of small-scale soil heterogeneity and/or microstructure, and (ii) nonlinear softening soil behavior utilizing the hyperbolic soil model. The results show that the propagation speed of the wave decreases with increasing nonlinearity, and its shape becomes more distorted. Additionally, the wavenumber spectrum of the nonlinear-elastic response exhibits significant peaks at high wavenumbers. However, these peaks disappear when a small amount of linear viscous damping is introduced, indicating their lack of physical relevance. One notable feature that persists despite damping is the emergence of small-amplitude waves traveling opposite to the main wave. This appears to be a consequence of the model's nonlinear characteristics, as it is absent in its linear gradient elasticity counterpart. A comprehensive investigation into the attributes of the proposed model can determine its applicability in predicting the seismic site response.

References

[1] Dostal L., Hollm M., Metrikine A.V., Tsouvalas A., van Dalen K.N. (2022) Localized stationary seismic waves predicted using a nonlinear gradient elasticity model. *Nonlinear Dyn* **107**:1107-1125.

[2] Metrikine A. (2006) On causality of the gradient elasticity models. *J. Sound Vib.* **297**:727-742.

[3] Metrikine, A.V., Askes, H. (2002) One-dimensional dynamically consistent gradient elasticity models derived from a discrete microstructure: part 1: generic formulation. *Eur. J. Mech. ASolids* **21**:555–572.