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## Accepted Manuscript

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**Highlights**

- A semi-analytical model for heat flow with friction heat gain in BHE.
- The spectral analysis method is utilized.
- The model calculates temperature distribution in the BHE and the soil mass.

## A Spectral Model for Heat Transfer with Friction Heat Gain in Geothermal Borehole Heat Exchangers

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### Abstract

This paper introduces a semi-analytical model for the simulation of transient heat transfer with friction heat gain in a single U-tube geothermal borehole heat exchanger subjected to an arbitrary heat flux signal. The friction effect appears as a nonhomogeneous term in the governing equations, which constitutes a set of coupled partial differential equations describing heat flow in the three components of the borehole; pipe-in, pipe-out and grout. We utilize the spectral analysis for discretizing the time domain, and the eigenfunction expansion for discretizing the spatial domain to solve the governing initial and boundary value problem. The proposed model combines the exactness of the analytical methods with an important extent of generality in describing the geometry and boundary conditions of the numerical methods. The model is verified analytically against a simplified one-dimensional solution. A numerical example is given to illustrate the effect of friction on heat transfer in the borehole heat exchanger for different fluid velocities and viscosities. The analysis shows; for the geometry, materials fluid velocities and viscosities, typically utilized in shallow geothermal systems; the friction is not really significant. However, the main advantage of this work is on the solution technique that can be useful for many other applications, including fluid flow in narrow pipes, high fluid velocities, high fluid viscosities, and pipes made of composite materials and of complex geometry. Also, the method can be useful for solving other nonhomogeneous coupled partial differential equations.

**Keywords:** Friction in pipes, Borehole heat exchanger, BHE, GHP, GSHP, Spectral analysis, FFT.

### 1. Introduction

Friction heat gain due to fluid flow in a pipe arises from the energy loss, which might be the result of the viscous force generated at the contact area between the fluid and the inner surface of the pipe. Friction energy loss can be significant in many engineering applications dealing with fluid flow in pipes, and has been given a significant attention, especially to those related to mechanical engineering. For geothermal engineering, however, this effect has not been considered. Two factors have motivated us to explore the effect of heat gain due to friction in shallow geothermal systems. First, shallow geothermal systems make use of the relatively low temperatures in shallow ground depths to generate heat. In this technology, every degree Celsius counts and made useful. Second, the friction is a function of length, which is particular in geothermal systems. The fluid in most U-tube borehole heat exchangers travels 200 m at every cycle.

Geothermal engineering is a relatively new field of physical sciences dealing with mining heat from shallow and deep earth formations. The borehole heat exchanger (BHE) is an important technology in this field that makes use of the widely available geothermal energy in shallow layers for heating and cooling of buildings and other facilities. It works by circulating a fluid, mostly water with antifreeze solution, through a U-tube (or a co-axial) polyethylene pipe that is

inserted in a borehole. The borehole is filled with some grouted materials to fix the pipe and to ensure a good thermal interaction with the surrounding soil mass, (Fig. 1).

In shallow geothermal systems, in heating modes, gain of only few degrees centigrade from the ground is considered significant. This can make heat gain due to friction appealing and needs to be studied. Such a study might lead to improving the BHE technology, not merely the involved materials and the operation techniques. This constitutes the core subject of this work, which aims at studying the possible gain of heat from friction between the circulating fluid and the pipe.

Several computational models have been developed to simulate the thermal behavior of the BHE and the surrounding soil mass. These models vary from detailed numerical 3-D analysis to analytical solutions of simplified geometry and initial and boundary conditions. Due to the peculiarity of the involved geometry; which constitutes highly slender borehole heat exchangers embedded in a vast soil mass, and the presence of convection heat flow; the numerical models require extensive computational capacity and CPU time. Many numerical models have been introduced, such as those given in [1-3]. Nevertheless, none of these models considered the friction heat gain due to fluid flow in the borehole heat exchanger.

In contrast, the analytical models require smaller computational capacity and much less CPU time. Several analytical and semi-analytical models with different complexity have been introduced, including those given in [4-17]. As for the numerical models, none of these models considered friction heat gain.

There is a large number of researches done on head loss due to friction in fluid flow in pipes. References [18-22] are only few examples of research works in this field. However, it seems that there are only few computational models dealing explicitly with friction heat gain in pipes. In geothermal engineering, for instance, Ozudorgu et al [23] have included heat gain into their governing equations, but did not explicitly study the effect of friction on heat flow. Saeid et al [24] studied the effect of friction in heat flow in a low enthalpy deep wellbore, and found, for the studied flow rate and pipe roughness, insignificant gain of heat due to friction. In both works, the finite element method was utilized to solve the problem. No analytical solutions have been introduced for heat flow with friction, though, solutions of nonhomogeneous advective-diffusive transport equations, such as the one provided by Weigand [25] or van Genuchten and Alves [26], can be tailored and utilized to study friction heat gain. However, these solutions are designed for a one-dimensional object subjected to a mostly step force signal, and does not take into consideration the particular geometry of the pipe.

In this paper, a semi-analytical solution for transient heat flow with friction heat gain in a single U-tube borehole heat exchanger, subjected to an arbitrary heat flux signal, is introduced. We utilize the spectral analysis and the eigenfunction expansion to solve the problem. The friction effect appears as a nonhomogeneous term in the governing equations, which constitutes a set of coupled partial differential equations. We make use of the solution provided in [16,17] to solve the homogeneous part of the solution, and extend it to solve the particular part of the solution.

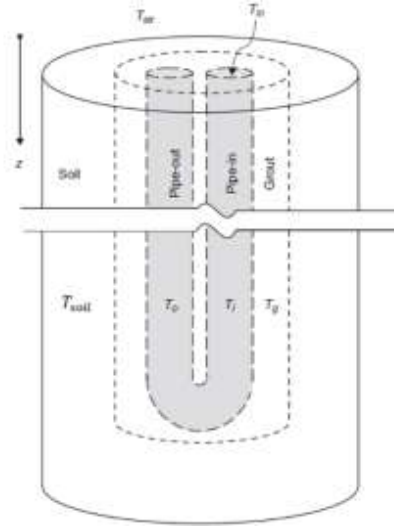


Figure 1: A schematic representation of the single U-tube borehole heat exchanger [27].

## 2. Governing equations

Heat flow with friction heat gain in a single U-tube borehole heat exchanger, consisting of pipe-in, denoted as  $i$ ; pipe-out, denoted as  $o$ ; and grout, denoted as  $g$ , can be described as

### Pipe-in

$$\rho c_r \frac{\partial T_i}{\partial t} \Delta V_i - \lambda_r \frac{\partial^2 T_i}{\partial z^2} \Delta V_i + \rho c_r u \frac{\partial T_i}{\partial z} \Delta V_i = b_{ig} (T_i - T_g) \Delta S_{ig} + \Delta Q_f \quad (1)$$

### Pipe-out

$$\rho c_r \frac{\partial T_o}{\partial t} \Delta V_o - \lambda_r \frac{\partial^2 T_o}{\partial z^2} \Delta V_o - \rho c_r u \frac{\partial T_o}{\partial z} \Delta V_o = b_{og} (T_o - T_g) \Delta S_{og} + \Delta Q_f \quad (2)$$

### Grout

$$\rho_g c_g \frac{\partial T_g}{\partial t} \Delta V_g - \lambda_g \frac{\partial^2 T_g}{\partial z^2} \Delta V_g = b_{ig} (T_g - T_i) \Delta S_{ig} + b_{og} (T_g - T_o) \Delta S_{og} \quad (3)$$

in which the subscripts  $r$  and  $g$  represent the circulating fluid (refrigerant) and the grout, respectively;  $T_i, T_o$  and  $T_g$  ( $K$ ) are the cross-sectional average temperatures in pipe-in, pipe-out and grout, respectively;  $\lambda_r$  and  $\lambda_g$  ( $W/mK$ ) are the thermal conductivity of the circulating fluid and grout, respectively;  $u$  ( $m/s$ ) is the circulating fluid cross-sectional average velocity;  $b_{ig}$  ( $K$ ) is the reciprocal of the thermal resistance between pipe-in and grout;  $b_{og}$

is the reciprocal of the thermal resistance between pipe-out and grout; and  $\rho c_r$  ( $J/m^3K$ ) is the volume heat capacity, with  $c_r$  ( $J/kgK$ ) the specific heat,  $\rho$  ( $kg/m^3$ ) the mass density.

$\Delta V_i, \Delta V_o$  and  $\Delta V_g$  are the partial volume of pipe-in, pipe-out and the grout respectively, and  $\Delta S_{ig}, \Delta S_{og}$  are the partial surface areas at the contact between pipe-in and grout, and pipe-out and grout, respectively.  $\Delta Q_f$  is the change in heat flux due to friction between the circulating fluid and the pipe internal wall, derived below. For clarity of notation, in what follows, the subscript  $r$  will not be included.

## 2.1 Initial and boundary conditions

For a single U-tube borehole heat exchangers, the initial and boundary conditions are typically:

$$\begin{aligned} T_i(z, 0) &= T_o(z, 0) = T_g(z, 0) = T_{st}(z) \\ T_i(0, t) &= T_{in}(t) \\ T_i(L, t) &= T_o(L, t) \end{aligned} \quad (4)$$

$$-\lambda_g \frac{\partial T_g(z, t)}{\partial z} A_g - b_{ig} (T_g - T_i) \Delta S_{ig} - b_{og} (T_g - T_o) \Delta S_{og} = b_{gs} (T_g - T_{soil}) \Delta S_{gs} \quad (5)$$

where  $T_{st}$  is the steady state soil temperature before operating the geothermal system;  $T_{in}$  is the fluid temperature at the inlet of pipe-in ( $z = 0$ ), coming from the heat pump;  $T_{soil}$  is the soil temperature immediately surrounding the BHE;  $b_{ig}$  is the reciprocal of the thermal resistance between the grout and the soil;  $A_g$  is the partial surface area at the contact between the grout and the soil;  $b_{og}$  is the cross-sectional area of the grout; and  $L$  is the length of the BHE. At the bottom of the BHE, ( $z = L$ ) the fluid temperature in pipe-in is equal to that in pipe-out, neglecting the elbow part because it is too small compared to the length.

Eqs. (1)-(3) and (5) state that, as physically occurring, the coupling between the BHE components, and between them and the soil formation occurs via the grout, which works as an intermediate medium that transfers heat from one component to another. Unlike the commonly utilized delta-circuit formulation [5], heat flow in the grout is explicitly formulated.

## 2.2 Friction heat gain term, $\Delta Q_f$

When a fluid moves in a pipe, it encounters frictional resistance due to the roughness of the inner surface of the pipe wall. This causes a loss of energy as a heat, which is equivalent to the loss of power consumed to overcome the viscous force at the contact surface between the fluid and the pipe.

Head loss in fluid flow in a pipe due to friction is commonly described using the Darcy–Weisbach equation, as

$$h_{loss} = f_D \frac{Lu^2}{2gd_i} \quad (6)$$

where  $L$  is the length of the pipe,  $d_i$  is its inner diameter,  $u$  is the average velocity of the fluid,  $g$  is the gravity and  $f_D$  is the Darcy friction factor, a dimensionless quantity. Several formulations describing  $f_D$  are available in literature. Here, we utilize the Colebrook equation [28] for the turbulent flow. For laminar and turbulent flow,  $f_D$  is described as

$$f_D = \begin{cases} \frac{64}{\text{Re}} & \text{Re} < 2000 \\ \frac{1}{\sqrt{f_D}} = -2.0 \log \left( \frac{e}{3.7d_i} + \frac{2.51}{\text{Re} \sqrt{f_D}} \right) & \text{Re} > 2000 \end{cases} \quad (7)$$

where  $e$  (m) is the tubing surface roughness, and  $\text{Re}$  is the Reynolds number, defined as

$$\text{Re} = \frac{\rho u d_i}{\mu} \quad (8)$$

where  $\mu$  (Pa.s) is the dynamic viscosity.

The fluid pressure associated with the head loss is expressed as

$$\Delta P = \rho g h_{loss} = \frac{f_D \rho u^2 L}{2d_i} \quad (9)$$

In fluid mechanics, the power loss is given by

$$\text{Power} = \Delta P V \quad (10)$$

where  $V(m^3/s)$  is the volumetric fluid rate.

As the heat gain due to friction is equivalent to the power loss, substituting Eq.(9) into Eq. (10), the heat gain can be described as

$$\Delta Q_f = \frac{f_D \rho u^3}{8} \Delta S \quad (11)$$

where  $\Delta S$  is the inner surface area of the pipe at depth  $z$ .

### 3. Spectral analysis of BHE heat equations

Applying the Fourier transform to Eqs.(1)-(3), gives

$$i\omega \rho c_r \hat{T}_i \Delta V_i - \lambda_r \frac{d^2 \hat{T}_i}{dz^2} \Delta V_i + \rho c_r u \frac{d \hat{T}_i}{dz} \Delta V_i = b_{ig} (\hat{T}_i - \hat{T}_g) \Delta S_{ig} + \Delta \hat{Q}_f \quad (12)$$

$$i\omega \rho c_r \hat{T}_o \Delta V_o - \lambda_r \frac{d^2 \hat{T}_o}{dz^2} \Delta V_o - \rho c_r u \frac{d \hat{T}_o}{dz} \Delta V_o = b_{og} (\hat{T}_o - \hat{T}_g) \Delta S_{og} + \Delta \hat{Q}_f \quad (13)$$

$$i\omega \rho_g c_g \hat{T}_g \Delta V_g - \lambda_g \frac{d^2 \hat{T}_g}{dz^2} \Delta V_g = b_{ig} (\hat{T}_g - \hat{T}_i) \Delta S_{ig} + b_{og} (\hat{T}_g - \hat{T}_o) \Delta S_{og} \quad (14)$$

in which the transformed quantity is defined as  $T \leftrightarrow \hat{T}$ . These equations are ordinary differential equations, two of which are nonhomogeneous. Solution of these nonhomogeneous equations is conducted by solving separately the homogenous part and the particular part, and then summed together algebraically.

#### 3.1 Homogeneous solution

The homogeneous solution of Eqs.(12)-(14) is given in details in Al-Khoury [16,17].

#### 3.2 Particular solution

As for the homogeneous solution, the particular solution of Eqs. (12) and (13) can be represented by an exponential complex function of the form [29]:

$$\hat{T}_{pi} = C_i e^{-ikz}, \quad \hat{T}_{po} = C_o e^{ikz}, \quad \hat{T}_{pg} = C_g e^{-ikz} \quad (15)$$

where  $\hat{T}_{pi}$ ,  $\hat{T}_{po}$  and  $\hat{T}_{pg}$  are the particular temperature frequency response of pipe-in, pipe-out and grout respectively.

Also, as for the homogeneous solution, the BHE system can be divided into two sub-systems: pipe-in – grout and pipe-out – grout.



### Pipe-in – grout

The particular solution of pipe in-grout equations, Eq. (12), can be expressed as

$$\begin{aligned}\hat{T}_{pi} &= C_{i1}e^{-ik_1z} + C_{i2}e^{-ik_2z} \\ \hat{T}_{pgi} &= C_{gi1}e^{-ik_1z} + C_{gi2}e^{-ik_2z}\end{aligned}\quad (16)$$

where  $\hat{T}_{pi}$  and  $\hat{T}_{pgi}$  are the particular temperature frequency response of pipe-in and grout respectively.  $C_{i1} \dots C_{gi2}$  are integration constants that need to be determined.  $\hat{T}_{pi}$  and  $\hat{T}_{pgi}$  are coupled via Eq.(14), as

$$i\omega\rho_g c_g \hat{T}_{pgi} \Delta V_g - \lambda_g \frac{d^2 \hat{T}_{pgi}}{dz^2} \Delta dV_g - b_{ig} (\hat{T}_{pgi} - \hat{T}_{pi}) \Delta S_{ig} = 0 \quad (17)$$

which is the corresponding particular heat equation of the grout in contact with pipe-in only. Substituting Eq.(16) into Eq.(17) gives

$$\begin{aligned}C_{gi1} &= \alpha_{gi1} C_{i1} \\ C_{gi2} &= \alpha_{gi2} C_{i2}\end{aligned}\quad (18)$$

where

$$\begin{aligned}\alpha_{gi1} &= \frac{-b_{ig} \Delta S_{ig}}{i\omega\rho_g c_g \Delta V_g + k_1^2 \lambda_g \Delta V_g - b_{ig} \Delta S_{ig}} \\ \alpha_{gi2} &= \frac{-b_{ig} \Delta S_{ig}}{i\omega\rho_g c_g \Delta V_g + k_2^2 \lambda_g \Delta V_g - b_{ig} \Delta S_{ig}}\end{aligned}\quad (19)$$

Substituting Eq.(16) into Eq.(12), and with some mathematical arrangements, gives

$$\begin{aligned}((i\omega\rho c \Delta V_i + \lambda k_1^2 \Delta V_i - ik_1 \rho c u \Delta V_i - b_{ig} \Delta S_{ig}) C_{i1} + b_{ig} \Delta S_{ig} C_{gi1}) e^{-ik_1z} + \\ ((i\omega\rho c \Delta V_i + \lambda k_2^2 \Delta V_i - ik_2 \rho c u \Delta V_i - b_{ig} \Delta S_{ig}) C_{i2} + b_{ig} \Delta S_{ig} C_{gi2}) e^{-ik_2z} = \Delta \hat{Q}_f\end{aligned}\quad (20)$$

Substituting Eq.(18) into Eq.(20) yields

$$\begin{aligned}((i\omega\rho c \Delta V_i + \lambda k_1^2 \Delta V_i - ik_1 \rho c u \Delta V_i - b_{ig} \Delta S_{ig}) + b_{ig} \Delta S_{ig} \alpha_{gi1}) C_{i1} e^{-ik_1z} + \\ ((i\omega\rho c \Delta V_i + \lambda k_2^2 \Delta V_i - ik_2 \rho c u \Delta V_i - b_{ig} \Delta S_{ig}) + b_{ig} \Delta S_{ig} \alpha_{gi2}) C_{i2} e^{-ik_2z} = \Delta \hat{Q}_f\end{aligned}\quad (21)$$

At  $z = 0$ , the heat gain due to friction is zero. Thus, the first equation of Eq.(16) gives

$$C_{i1} + C_{i2} = 0 \quad (22)$$

Applying Eq.(22) to Eq.(21) yields

$$\begin{aligned}C_{i1} &= \frac{\Delta \hat{Q}_f}{\alpha_{i1} - \alpha_{i2}} \\ C_{i2} &= \frac{\Delta \hat{Q}_f}{\alpha_{i2} - \alpha_{i1}}\end{aligned}\quad (23)$$

where

$$\begin{aligned}\alpha_{i1} &= (i\omega\rho c\Delta V_i + \lambda k_1^2\Delta V_i - ik_1\rho c u\Delta V_i - b_{ig}\Delta S_{ig} + b_{ig}\Delta S_{ig}\alpha_{gi1})e^{-ik_1z} \\ \alpha_{i2} &= (i\omega\rho c\Delta V_i + \lambda k_2^2\Delta V_i - ik_2\rho c u\Delta V_i - b_{ig}\Delta S_{ig} + b_{ig}\Delta S_{ig}\alpha_{gi2})e^{-ik_2z}\end{aligned}\quad (24)$$

### Pipe-out – grout

Pipe-out is a continuation of pipe-in at  $z=L$ , and as the friction is a function of the length travelled by the fluid, the particular solution of pipe-out can be expressed as

$$\begin{aligned}\hat{T}_{po} &= C_{o1}e^{-ik_1(2L-z)} + C_{o2}e^{-ik_2(2L-z)} \\ \hat{T}_{pgo} &= C_{go1}e^{-ik_1(2L-z)} + C_{go2}e^{-ik_2(2L-z)}\end{aligned}\quad (25)$$

where  $\hat{T}_{po}$  and  $\hat{T}_{pgo}$  are the particular temperature frequency response of pipe-out and grout respectively.  $C_{o1}...$  are integration constants that need to be determined.  $\hat{T}_{po}$  and  $\hat{T}_{pgo}$  are coupled via Eq.(14), as

$$i\omega\rho_g c_g \hat{T}_{pgo} \Delta V_g - \lambda_g \frac{d^2 \hat{T}_{pgo}}{dz^2} \Delta V_g - b_{og} (\hat{T}_{pgo} - \hat{T}_{po}) \Delta S_{og} = 0 \quad (26)$$

which is the corresponding particular heat equation of the grout in contact with pipe-out only.

Substituting Eq.(25) into Eq.(26) gives

$$\begin{aligned}C_{go1} &= \alpha_{go1} C_{o1} \\ C_{go2} &= \alpha_{go2} C_{o2}\end{aligned}\quad (27)$$

where

$$\begin{aligned}\alpha_{go1} &= \frac{-b_{og}\Delta S_{og}}{i\omega\rho_g c_g \Delta V_g + k_1^2 \lambda_g \Delta V_g - b_{og}\Delta S_{og}} \\ \alpha_{go2} &= \frac{-b_{og}\Delta S_{og}}{i\omega\rho_g c_g \Delta V_g + k_2^2 \lambda_g \Delta V_g - b_{og}\Delta S_{og}}\end{aligned}\quad (28)$$

Similar to pipe-in, substituting Eq.(25) and (27) into Eq.(13), yields

$$\begin{aligned}C_{o1} &= \frac{\Delta \hat{Q}_f}{\alpha_{o1} - \alpha_{o2}} \\ C_{o2} &= \frac{\Delta \hat{Q}_f}{\alpha_{o2} - \alpha_{o1}}\end{aligned}\quad (29)$$

where

$$\begin{aligned}\alpha_{o1} &= (i\omega\rho c\Delta V_o + \lambda k_1^2\Delta V_o - ik_1\rho c u\Delta V_o - b_{og}\Delta S_{og} + b_{og}\Delta S_{og}\alpha_{go1})e^{-ik_1(2L-z)} \\ \alpha_{o2} &= (i\omega\rho c\Delta V_o + \lambda k_2^2\Delta V_o - ik_2\rho c u\Delta V_o - b_{og}\Delta S_{og} + b_{og}\Delta S_{og}\alpha_{go2})e^{-ik_2(2L-z)}\end{aligned}\quad (30)$$

### Grout

The particular solution of the grout is considered as an average value of the particular solutions of  $\hat{T}_{pgi}$ , Eq. (16), and  $\hat{T}_{pgo}$ , Eq. (25), represented as

$$\hat{T}_{pg} = \frac{1}{2}(\hat{T}_{pgi} + \hat{T}_{pgo}) \quad (31)$$

### 3.3 General solution of BHE heat equations

The general solution of the single U-tube BHE heat equations can be obtained by summing over the homogeneous and particular solutions for all involved eigenfunctions and frequencies, as

#### Pipe-in

$$T_i(z,t) = \sum_n \left( A_i e^{-ik_1 z} + B_i e^{-ik_2 z} + C_{i1} e^{-ik_1 z} + C_{i2} e^{-ik_2 z} \right) e^{i\omega_n t} \quad (32)$$

#### Pipe-out

$$T_o(z,t) = \sum_n \left( A_o e^{ik_1 z} + B_o e^{ik_2 z} + C_{o1} e^{-ik_1(2L-z)} + C_{o2} e^{-ik_2(2L-z)} \right) e^{i\omega_n t} \quad (33)$$

#### Grout

$$T_g(z,t) = \frac{1}{2} \sum_n \left[ \begin{aligned} & \left( A_{ig} + A_{og} + C_{gi1} \right) e^{-ik_1 z} + \left( B_{ig} + B_{og} + C_{gi2} \right) e^{-ik_2 z} \\ & + C_{go1} e^{-ik_1(2L-z)} + C_{go2} e^{-ik_2(2L-z)} \end{aligned} \right] e^{i\omega_n t} \quad (34)$$

where  $A_{ig}$ ,  $A_{og}$ ,  $B_{ig}$ , and  $B_{og}$  are the homogeneous solution integration constants; defined in [16], and  $C_{gi1}$ ,  $C_{gi2}$ ,  $C_{go1}$ ,  $C_{go2}$  are defined in Eqs.(18) and (27) .

## 4. Model Verification

Exact solution describing heat flow with friction heat gain in a single U-tube BHE does not exist. Accordingly, verification of the model accuracy is done by comparing its computational results with those obtained from an analytical solution of a simplified case. The van Genuchten and Alves [26] solution of a one-dimensional advective-dispersive solute transport equation including a nonhomogeneous term is utilized for this purpose.

van Genuchten and Alves solved the following one-dimension partial differential equation

$$R \frac{\partial c}{\partial t} - D \frac{\partial^2 c}{\partial z^2} + F \frac{\partial c}{\partial z} + \mu c - \gamma = 0 \quad (35)$$

with the following initial and boundary conditions:

$$\begin{aligned} c(z,0) &= A(z) = \frac{\gamma}{\mu} + \left( C_{\text{int}} - \frac{\gamma}{\mu} \right) e^{\frac{(F-\bar{u})}{2D} z} \\ c(0,t) &= \begin{cases} C_{\text{in}} & 0 < t < t_o \\ 0 & t > t_o \end{cases} \\ \frac{\partial c}{\partial z}(\infty,t) &= 0 \end{aligned} \quad (36)$$

where  $R, D, F, \mu$  and  $\gamma$  are constants, and  $\sqrt{\frac{Rz - \bar{u}t}{2(DRt)^{\frac{1}{2}}}}$ . The initial value,  $C_{in}$  in Eq.(36), is determined by solving the steady state condition of Eq. (35). In this way, the nonhomogeneous term will be included in the initial condition, and there is no need to solve for the particular solution. This is possible because  $\gamma$  is independent of time.

The solution of this problem is

$$c(z,t) = A(z) + \frac{1}{2}(C_{in} - C_{int}) \left( \begin{array}{l} e^{\frac{(F-\bar{u})z}{2D}} \operatorname{erfc} \left[ \frac{Rz - \bar{u}t}{2(DRt)^{\frac{1}{2}}} \right] \\ + e^{\frac{(F+\bar{u})z}{2D}} \operatorname{erfc} \left[ \frac{Rz + \bar{u}t}{2(DRt)^{\frac{1}{2}}} \right] \end{array} \right) \quad 0 < t \leq t_o \quad (37)$$

and

$$c(z,t) = A(z) + \frac{1}{2}(C_{in} - C_{int}) \left( \begin{array}{l} e^{\frac{(F-\bar{u})z}{2D}} \operatorname{erfc} \left[ \frac{Rz - \bar{u}t}{2(DRt)^{\frac{1}{2}}} \right] + e^{\frac{(F+\bar{u})z}{2D}} \operatorname{erfc} \left[ \frac{Rz + \bar{u}t}{2(DRt)^{\frac{1}{2}}} \right] \end{array} \right) - \frac{1}{2}C_{in} \left( \begin{array}{l} e^{\frac{(F-\bar{u})z}{2D}} \operatorname{erfc} \left[ \frac{Rz - \bar{u}(t-t_o)}{2(DR(t-t_o))^{\frac{1}{2}}} \right] + e^{\frac{(F+\bar{u})z}{2D}} \operatorname{erfc} \left[ \frac{Rz + \bar{u}(t-t_o)}{2(DR(t-t_o))^{\frac{1}{2}}} \right] \end{array} \right) \quad t > t_o \quad (38)$$

To compare with the proposed spectral model, the van Genuchten and Alves parameters need to be adjusted to match the physical parameters of the model. Comparing Eq.(1) to Eq.(35), these parameters are adjusted such that:

$$\begin{aligned} R &= \rho c \Delta V_i \\ D &= \lambda_r \Delta V_i \\ F &= \rho c_r u \Delta V_i \\ \mu &= -b_{ig} \Delta S_{ig} \\ \gamma &= \Delta Q_f - \mu T_g \\ C_{int} &= T_{st} \\ C_{in} &= T_{in} \end{aligned} \quad (39)$$

We utilized the two models to solve heat flow with heat gain due to friction in an insulated heat pipe. The geometry and material parameters are as the following:

Pipe length	= 100m
Pipe radius, $r_i$	= 0.016 m
Fluid $\rho c$	= 4.1298E6 J/m <sup>3</sup> K
Fluid $\lambda$	= 0.56 W/m K
Fluid velocity, $u$	= 1 and 20 m/s

The initial steady state temperature, and the temperature at the pipe inlet are:

$$\begin{aligned}
T_{st} \quad t=0, z=10^\circ\text{C} \\
T_{in} \quad t, z=0 = \begin{cases} 40^\circ\text{C} & 0 < t \leq 1000\text{s} \\ 0^\circ\text{C} & 1000 < t < \infty\text{s} \end{cases} \quad (40)
\end{aligned}$$

In the spectral model,  $T_{in}$  is equal to  $T_{st} + \Delta T_{in}$ , where, in this case,  $\Delta T_{in} = 30^\circ\text{C}$ . The coefficient of the thermal interaction between the pipe and the surrounding material (grout in the spectral model),  $b_{ig}$ , was made relatively small ( $0.1 \text{ W/m}^2\text{K}$ ) to insure insulation. The input temperature time histories of  $T_{in}$  and  $T_{st}$  were transformed to the frequency domain using the forward FFT. 4096 samples, with a sample rate of 1s, were used, giving a time window of 4096s.

The calculation results of the temperature at  $z = 100\text{m}$ , as calculated by the van Genuchten and Alves solution and the spectral model, are shown in Fig.3. Fig. 4a shows the temperature distributions along the pipe after 50s with fluid average velocity equals to 1m/s, and Fig. 4b shows the temperature distributions along the pipe after 500s for both velocities. Apparently, the two results are nearly identical for both fluid flow average velocities and along the depth of the pipe, though the van Genuchten and Alves solution exhibited some oscillation in the high velocity case.

Physically, Fig. 4 shows that with a relatively small fluid flow velocity, the temperature does not change along the pipe, while it increases by more than  $2^\circ\text{C}$  for the high velocity case. As  $b_{ig}$  is relatively small, the pipe is effectively insulated, and this increase in temperature from the top to the bottom is merely due to friction.

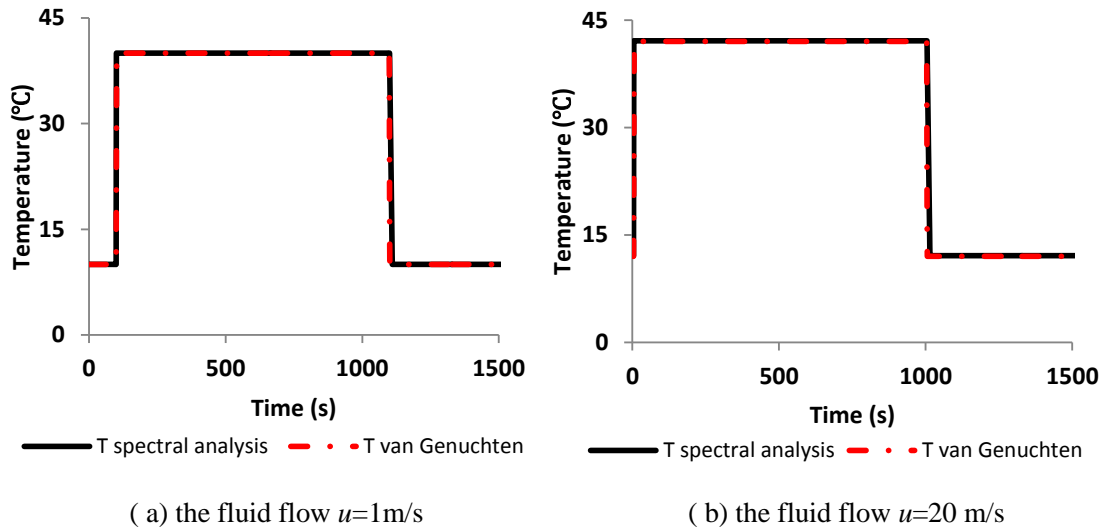


Figure 3: Spectral model vs. van Genuchten and Alves solution with time at  $z=100\text{m}$

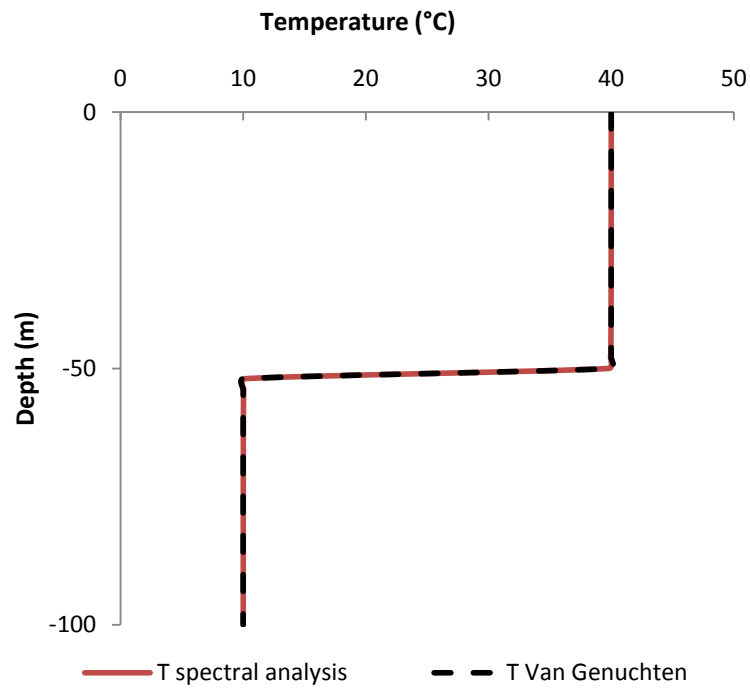


Figure 4a: Spectral model vs. van Genuchten and Alves solution along the pipe at time=50s with fluid average velocity  $u=1\text{m/s}$ .

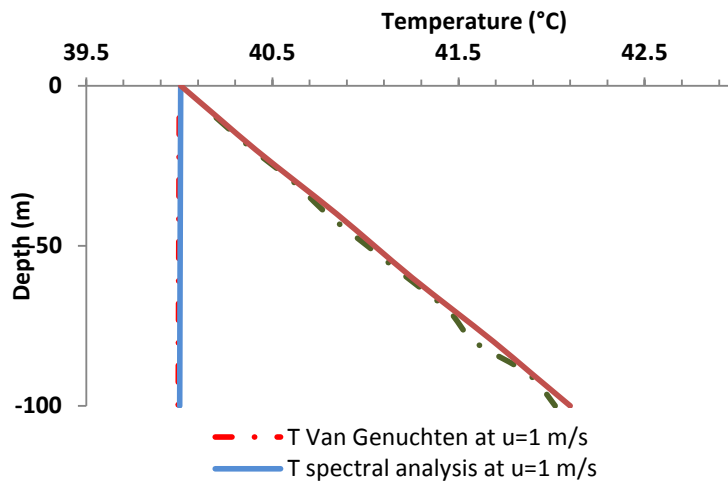


Figure 4b: Spectral model vs. van Genuchten and Alves solution along the pipe at time=500s

## 5. Numerical Examples

As discussed earlier, the proposed spectral model is capable of calculating the temperature distribution in all BHE components and in the surrounding soil mass (not shown in this paper) for short and long terms. Here, we introduce numerical examples illustrating its computational

capabilities for analyzing an in-time varying signal for a relatively long term. The material and geometrical properties are given in Table 1.

The initial temperature in the soil and the borehole is assumed 10°C. The air temperature is also set to 10°C (see[17]). The fluid temperature at the inlet is assumed to vary between *on* and *off*, as

$$T_{in} = \begin{cases} 20 & t < 30 \text{ day} \\ 10 & 30 \text{ day} \leq t < 45 \text{ day} \\ 18 & 45 \text{ day} \leq t < 75 \text{ day} \\ 10 & 75 \text{ day} \leq t < 90 \text{ day} \\ 16 & 90 \text{ day} \leq t < 120 \text{ day} \\ 10 & 120 \text{ day} \leq t \end{cases} \quad (41)$$

where it can be seen that the BHE has a 15 days off after every 30 days of operation.

Frequency discretization of  $T_{in}$ ,  $Q_f$  and  $T_{air}$  signals was conducted using the forward FFT with 16,384 ( $2^{14}$ ) samples and a sample rate of 1 hour, giving a time window of approximately 22 months. Spatial discretization of the soil mass was conducted using 100 Bessel function roots. It is worth mentioning that, as the friction term is a function of fluid velocity and its effect vanishes by stopping the system operation, it must be discretized using FFT. Its time distribution is equivalent to the  $T_{in}$  signal, but its magnitude, for any specific  $z$ , is determined from Eq. (11).

Table 1: Material and geometrical parameters

<i>Parameter</i>	<i>Value</i>
<b>Borehole:</b>	
Borehole length	100 m
Borehole diameter	0.127 m
Pipe inner diameter	0.032 m
Pipe wall thickness	0.0029 m
Pipe roughness	3 E-6
Pipe thermal conductivity	0.42 W/(m.K)
<b>Grout:</b>	
Grout density	1420 kg/m <sup>3</sup>
Grout thermal conductivity	0.6 W/(m.K)
Grout specific thermal capacity	1197 J/(kg.K)
<b>Soil:</b>	
Soil density	1680 kg/m <sup>3</sup>
Soil thermal conductivity	2.15 W/(m.K)
Soil specific thermal capacity	400 J/(kg.K)

The thermal coefficients  $b_{ig}$ ,  $b_{og}$  and  $b_{gs}$  are determined based on Al-Khoury [17, 27] thermal resistance formulation.

The effects of fluid velocities and viscosities are studied hereafter.

### 5.1. Fluid velocity effect

To study the effect of velocity, two fluid velocities are assumed: 0.5 and 5 m/s. The thermal parameter for the circulating fluid is shown in Table 2.

Table 2: the circulating fluid thermal parameters

<i>Parameter</i>	<i>Value</i>
Fluid density	1000 kg/m <sup>3</sup>
Fluid thermal conductivity	0.56 W/(mK)
Fluid specific thermal capacity	4186 J/(kg.K)
Fluid dynamic viscosity	0.001 Pa.s
Fluid velocities	0.5 and 5 m/s

Fig. 5a shows the temperature variations with no friction versus time for fluid velocity 0.5 m/s at  $z = 0$ . Fig. 5b shows the temperature distributions along the BHE. Analysis with friction heat gain (not shown in the figure) reveals that, for this fluid velocity, the difference is negligible.

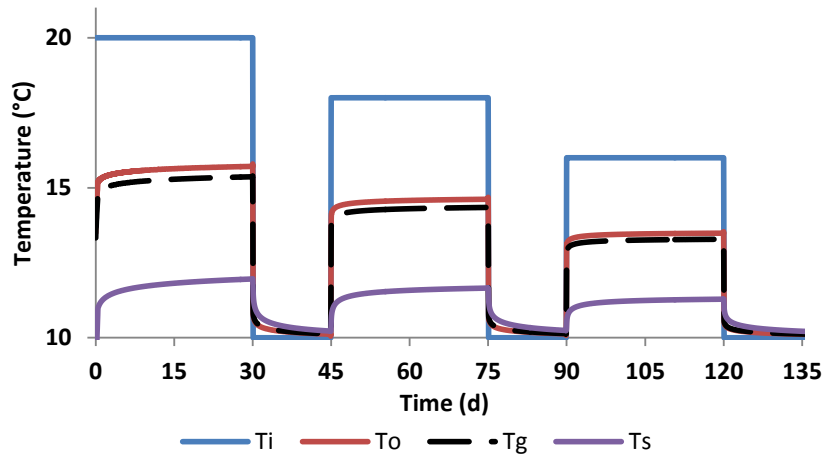


Figure 5a: Temperature variations for BHE components and soil vs. time for  $u=0.5$  m/s with  $b_{ig} = b_{og} = 126.53$  and  $b_{gs} = 27.52$  W/m<sup>2</sup>K



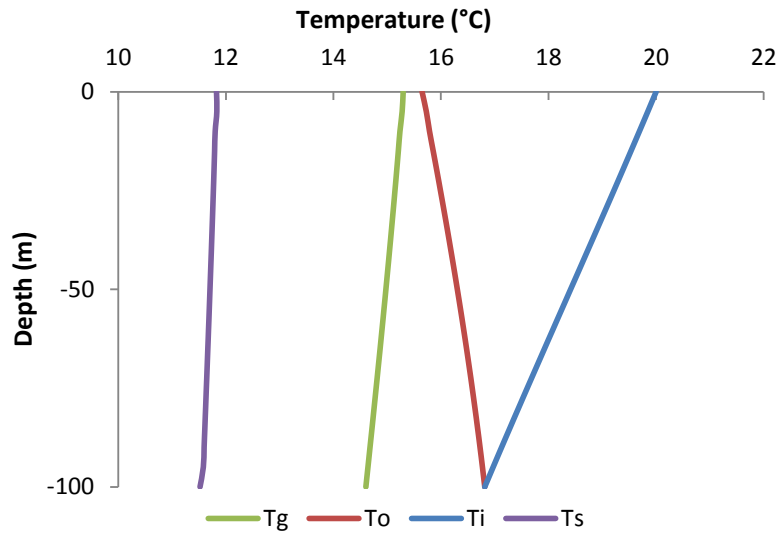
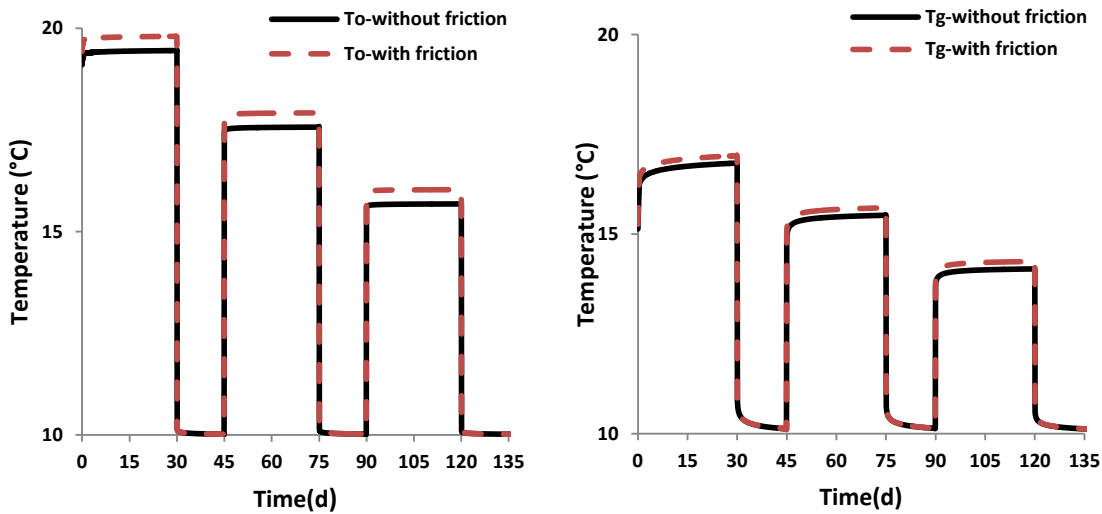


Figure 5b: Temperature distributions for BHE components and soil along the z-axis, for  $u=0.5$  m/s after 20 days with  $b_{ig} = b_{og} = 126.53$  and

Fig. 6 shows the temperature variations with and without friction versus time for pipe-out ( $T_o$ ) and grout ( $T_g$ ), for a fluid velocity equals to 5 m/s. The figure reveals that the temperature in pipe-out increased by approximately  $0.4$  °C and in the grout increased by approximately  $0.2$  °C. Apparently, the friction effect is higher for this flow rate.



(a) Fluid temperature at the outlet

(b) Grout temperature at the surface

Figure 6: Temperature variations in pipe-out and grout vs. time with and without friction for  $u=5$  m/s with  $b_{ig} = b_{og} = 132.26$  and  $b_{gs} = 27.4$  W/m<sup>2</sup>K.

## 5.2. Fluid viscosity effect

To study the effect of viscosity, two solutions with different viscosities are assumed: 30% propylene glycol solution, and a solution with a 0.5 Pa.s viscosity.

The thermal parameter for the 30% propylene glycol solution is shown in Table 3.

Table 3: 30% propylene glycol thermal parameters

<i>Parameter</i>	<i>Value</i>
30% propylene glycol: at 15 °C	
Fluid density	1031 kg/m <sup>3</sup>
Fluid thermal conductivity	0.426 W/(mK)
Fluid specific thermal capacity	3834 J/(kg.K)
Fluid dynamic viscosity	0.00369 Pa.s
Fluid velocity	0.5 m/s

Fig. 7 shows the fluid temperature at the outlet of pipe-out ( $T_o$ ), for both: with friction and without friction. Apparently, the viscosity of this solution has no effect on the friction.

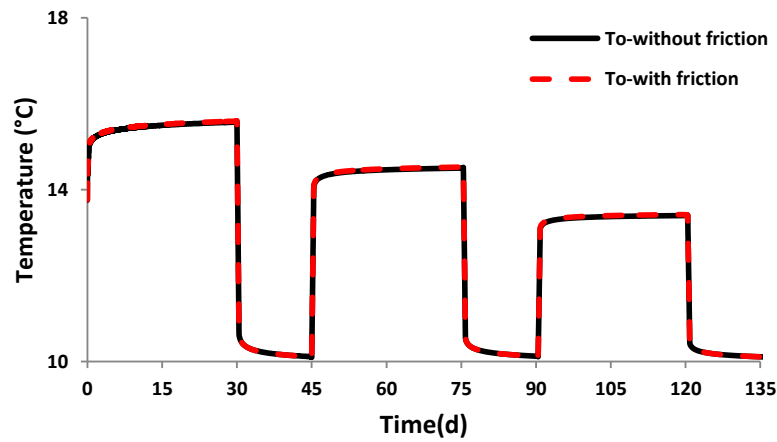


Figure 7: Outlet temperature of BHE with 30% propylene glycol solution

Suppose, for the sake of argument, we use a solution with 0.5 Pa.s viscosity, with thermal parameters given in Table 4. All other geometrical and thermal parameters are similar to the previous case.

Table 4: A solution with high viscosity

<i>Parameter</i>	<i>Value</i>
Fluid density	1000. kg/m <sup>3</sup>
Fluid thermal conductivity	0.56 W/(mK)
Fluid specific thermal capacity	4186 J/(kg.K)
Fluid dynamic viscosity	0.5 Pa.s
Fluid velocity	0.5 m/s

Fig. 8 shows the temperature variations with and without friction versus time for pipe-out ( $T_o$ ) and grout ( $T_g$ ), The figure reveals that the temperature in pipe-out increased by approximately 1 °C and in the grout increased by approximately 0.5 °C . Apparently, the friction effect is higher for highly viscous fluids.

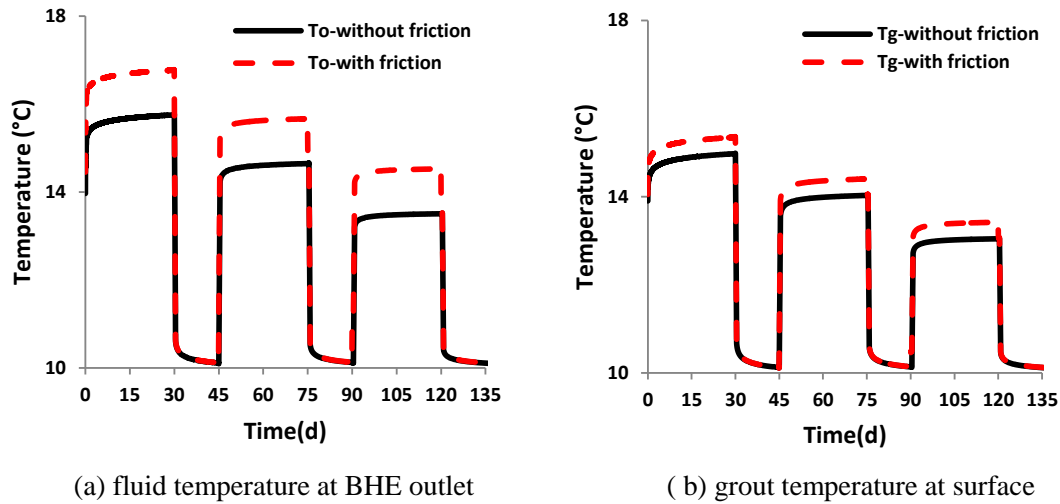


Figure 8: Temperature variations in pipe-out and grout vs. time with and without friction, for  $u=0.5\text{m/s}$ ,  $\mu = 0.5\text{ pa. s}$ ,  $b_{ig} = b_{og} = 128.89$  and

## 6. Conclusions

A semi-analytical model for the simulation of transient heat transfer with friction heat gain in a single U-tube geothermal borehole heat exchanger subjected to an arbitrary force signal has been derived and tested. The friction effect appears as a nonhomogeneous term in the governing equations, which constitutes a set of coupled partial differential equations describing heat flow in the three components of the borehole; pipe-in, pipe-out and grout. The spectral analysis is utilized to discretize the time domain; and the eigenfunction expansion is utilized to discretize the spatial domain. The model is verified analytically against a simplified one-dimensional transport equation given by van Genuchten and Alves. A numerical example is given to illustrate the effect of friction on heat transfer for different fluid velocities, and viscosities. The analysis shows that; for the geometry, materials, fluid velocities and viscosities, typically utilized for shallow geothermal systems; the friction is not really significant. However, the main advantage of this work is on the solution technique that can be useful for many other applications, including fluid flow in narrow pipes, high fluid velocities, high fluid viscosities, and pipes with composite

materials. Also, the method can be useful for solving other nonhomogeneous coupled partial differential equations.

The proposed model combines the exactness of the analytical methods with a great extent of generality in describing the geometry and boundary conditions of the numerical methods. The CPU time for calculating temperature distributions in all involved shallow geothermal system components; using 16,384 FFT samples, for the time domain, and 100 Fourier-Bessel series samples, for the spatial domain; is in the order of 1 second in a normal Intel PC. As the solution is highly accurate and computationally efficient, it can be suitable for inverse problems.

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