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Identifying critical and vulnerable links: A new approach using the Fisher information matrix

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A R T I C L E I N F O

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A B S T R A C T

In traffic networks, some elements are more prone to suffer or to create disruptive situations, and the identification of these elements becomes a challenge due to the large number of possible threats. The following paper presents two new methodologies to identify and rank vulnerable and critical links of traffic networks. These methodologies use the Fisher Information Matrix, and the analysis of eigenvalues and eigenvectors, to systematically rank the links of a network. The identification is done by using traffic variables, such as the demand, the travel time, and the network's flow. For the ranking of the links, disruptions are considered in all the possible locations of the network, and the effects are systematically evaluated. In addition, the evaluation of traffic resilience is included in the process to validate the results. Finally, both methodologies are tested in a real network to infer on the validity of the results.

1. Introduction

Traffic networks are a key element in modern societies since daily activity revolves around them. However, these networks are constantly threatened by a wide range of perturbations, from natural to man-made hazards, and the analysis and evaluation of transport resilience is needed [1]. As a result, identification of critical locations in transport networks when affected by disruptive events has become an indispensable area of research.

In this paper, transport networks are evaluated as critical infrastructures and the ultimate goal of the presented methodologies is to strengthen and maintain the secure, functioning, and resilient performance of them as a system. With the aim of creating robust and resilient traffic networks against any type of perturbation, the present work analyses and discusses criticality and vulnerability within a traffic network. Identification of critical and vulnerable elements allows the improvement of their performance contributing for the improvement of the whole traffic network activity [2,3].

In the field of identification of critical and vulnerable elements of traffic networks, relevant work has been done previously [4–9], these previous works are discussed in Section 2. Most of the methodologies presented in the literature analyse specific scenarios, and often a large amount of data and measurements of several indexes or indicators are needed for their evaluation process [10–12]. Methodologies for ranking the elements of the network have been previously presented in the literature. However, when working under dynamic network conditions, the time needed to use the methodologies can be increased and a first filter to determine the group of links that will be analysed is recommended. The presented methodology allows to perform a full analysis in a systematic and separable way, and to identify links as either vulnerable or critical links. In addition, it is noted that sometimes, these two concepts, critical and vulnerable, have been used indistinctly [13]. This paper presents methodologies to address them separately. A definition for each of these concepts is presented in Section 2.

In order to achieve the presented objective, a definition of critical and vulnerable elements in a traffic network is introduced, highlighting the differences between them, and explaining the consequences of damaging each of these components. A novel methodology for the identification of the most critical and the most vulnerable elements is presented. This methodology will also rank every link of the network from the most to the least critical, and from the most to the least vulnerable. The capacity of ranking each of the links of the network provides relevant information not only about the weakest element but also regarding the whole behaviour the network.

The proposed methodology is based on the Fisher Information Matrix (FIM) and the analysis of its principal components, i.e., eigenvalues and eigenvectors. The FIM is a well-known methodology in areas such as, Statistics and Economy [14]. Nevertheless, to the best of the authors'

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knowledge, this is the first time that this methodology is presented in the area of traffic networks to identify the critical and vulnerable links when a network is affected by a perturbation. It is also noted that the presented methodologies could be adapted and used in other types of networks. If the networks are defined by links and nodes, and sensitivity vectors can be created, there is potential for the use of an adapted version of the presented methodology to define critical and vulnerable links.

In the field of transport networks, analysis of the principal components has been used in Graph Theory, specifically, in spectral graph theory [15]. In this case, the characteristic polynomial, eigenvalues, and eigenvectors of matrices associated with the graph, such as the Laplacian matrix, are analysed to determine the properties of the graph. Laplacian matrix of graphs has been widely used, and applied in physical and chemical theories to identify weak elements; see [16] for an extensive study of this matrix, and [17] for principal component analysis to evaluate vulnerability of hotspots. In the area of transport, graph theory has also been used for the analysis of vulnerable elements, see [18].

The analysis based on the Laplacian matrix only addresses the topological characteristics of the network. In contrast, the methods implemented in the presented paper to define the critical and vulnerable links of a traffic network, based on the FIM analysis, consider additional characteristics, such as the capacity of the link, the demand of the network and the possible routes between the Origin–Destination (OD) pairs, which are not incorporated in the Graph Theory.

In order to introduce and discuss this innovative approach, the present work is organized as follows; Section 2 analyses the definitions and the differences between critical and vulnerable links in traffic networks; Section 3.1 presents a detailed explanation of the FIM; Section 3.2 presents the new methodology to identify the vulnerable links, along with a simple example for better comprehension of it; in Section 3.3 the new methodology for the identification of critical links is introduced and applied to a simple network system and Section 4 presents a real network; finally, Section 5 draws the main conclusions of the work developed.

2. Critical vs vulnerable elements of traffic networks

The management of traffic networks under a perturbation requires a profound knowledge of the network in order to implement the most efficient measures to reduce the impact of these events. In [19], it is highlighted that one of the most powerful strategies to deal with these situations is to increase the knowledge of the effects that can cause disruptions to develop strategies to adapt the network to future perturbations. Identification of critical and vulnerable elements of a network as an essential adaptation strategy for traffic networks has large relevance. This paper focuses on the type of perturbations that can happen in a recurrent way in the daily operation of the network, and understand the effect of a perturbation as loss of capacity in the system components. However, it is noted that a resilient behaviour under recurrent perturbations can also help in the development of more robust networks not only under this type of frequent disruptions but can also create a greater ability to withstand system breakdowns, as explained in [20].

When a network is affected by a perturbation, the critical and vulnerable elements of the network demand particular attention. These elements will have a key role in the network performance. They can create disruptive situations. Even more, due to the connections existing among the elements of a traffic network, a perturbation of one link may result in partial or complete damage of the system.

Identification of an established definition in the literature for these two concepts, i.e., critical and vulnerable links, can be challenging. Different perspectives can be found among the experts. In 1994, vulnerability as a ‘susceptibility for rare, big risks’ was analysed by [21]. Later, in [22], the vulnerability of a link in a road transportation system was understood as ‘a susceptibility to incidents that can results in considerable reductions in road serviceability’, and in [23] vulnerability was defined as ‘sensitivity to threats and hazards’. In [24], this concept is analysed in terms of accessibility. In this work, the authors assume that networks are created to give access to different locations, and for that reason, the vulnerability of the elements is measured depending on the capacity of these to provide access in different scenarios. Other authors also consider accessibility as a main variable to define vulnerability, *e.g.*, [25,26].

In [13], the critical locations of a spatial network is studied, defining critical as a measure of connectivity, and using the topology of the network elements to identify them. In addition, the authors consider the critical locations as vulnerable too, which need to be protected against the possible perturbations and specifically included in the adaptation measures. A similar classification is used by [27], however, more focused on the link criticality.

While there are different approaches for the definition of vulnerable and critical links, it became crucial to differentiate between both of them.

According to [28], vulnerable is defined as ‘exposed to the possibility of being attacked or harmed, either physically or emotionally’, and critical as ‘having the potential to become disastrous; at a point of crisis’ or ‘having a decisive or crucial importance in the success, failure, or existence of something’. This characterization clearly remarks the difference between critical and vulnerable, where vulnerable is associated with exposure and critical with the key functionality in a system. A simple example to emphasize this is to think on the hands and the hearth of an individual, the hands will be vulnerable, highly exposed to harm, but the heart will be critical, a key element to the system . Consequently, this paper distinguishes both concepts as:

- The most vulnerable link of a traffic network will be the one suffering the major relative impact when damage happens at any point of the traffic network.
- The most critical link will be the one that creates the largest impact on the network when it is damaged.

In a transport network, it can happen that some links are both vulnerable and critical, but the authors believe that there is a necessity to distinguish between these types of elements of the system. Being able to identify and rank these two types of elements will make a difference in how decision-making schemes are implemented. The interdependences and behaviours of the links of the network will be better understood and stakeholders will be able to distinguish in more detail the consequences of a disruption. Finally, it is noted that vulnerable links are the links that are prone to failure, and therefore will be locally impacted with a higher frequency. That means, that increasing in travel time in these is highly frequent; even when damages in other parts of the network occur (i.e. prone to loss). Nonetheless, their relative role in the functionality of the whole network could be small.

2.1. Methodologies presented in the literature for the identification of vulnerable and critical links

Once both concepts are defined, a methodology to identify them in a traffic network is required.

Some authors have presented methodologies to find the vulnerable and critical links in a network. Four main approaches were identified in this regard:damaged-link assessment, topological evaluation, link removal analysis and data-driven methods.

2.1.1. Damaged-link assessment

A model-based case study to identify the vulnerability of the road network is presented by [4]. The authors analyse different failures in the network, such as partial or complete closures of several links of the network, reductions of the free flow speed of all the links or only of the secondary roads, and different percentages of reduction or increment of traffic in the network. Once the damage is applied, link flow-based user equilibrium models are used to measure the effects, and identify the vulnerabilities. A case by case method is used to identify the vulnerable elements of the network, therefore, only the impacts analysed are included in the results, and a systematic method to analyse all the links is not attained.

Also using a case by case method, in [10], the critical locations of a network are analysed. The authors present a vulnerability index which measures the differences between the optimal performance of the network and the worst performance under a damaging scenario. To include the importance of a link in an OD pair, a bi-level method to identify the critical locations is presented by [29], using Game Theory.

In [5], it is introduced other vulnerability index considering the serviceability; being serviceability of a link defined as 'the possibility to use that link during a given period which then relates to the possibility of partial degradation of roads'. Another vulnerability index, developed by [6], assesses the link vulnerability through a two stages process; firstly, different attributes are chosen to characterize the vulnerability based on concepts such as the capacity of the link and the traffic flow, later on, these attributes are combined using fuzzy logic to obtain the link vulnerability index. In [30], vulnerability is evaluated by taking account of the path distance and passenger flow. The methodology is based on the complex network theory introducing a double-weighted analysis for the vulnerability analysis of urban metrorail transit systems.

Recently, in [9], critical links in a network are studied by a classification for decision-making that evaluates potential retrofits, before the impacting event that increase the functionality of the network. Criticality is studied in terms of replaceability.

2.1.2. Topological evaluation

Using another perspective, in [13], the critical location of a network are analysed using graph theory. The authors study the topology of the network, i.e. how the links are connected to each other, to be able to identify which are the problematic elements. Including aspects to identify critical locations, such as being the only element that connects two sub parts of the network (tunnels and bridges), elements included in a high number of routes (main roads), and highly dense parts of the network (city centre). Also, based on graph theory, a methodology for identifying critical nodes in urban public transportation networks is presented by [8].

2.1.3. Link-removal analysis

Another frequent approach in the literature review to identify critical links is removing the link from the network, and analysing the consequences. Based on [31] differentiation between vulnerable and critical, where vulnerable is associated to nodes and critical is evaluated in links, in [12], the authors develop an index to measure the criticality of links by evaluating the consequences of links closures in terms of travel time. In [11], it is also presented a method to identify the critical links. The authors describe the critical links as the ones creating a worse scenario when damaged, highlighting the importance of variables such as the demand of the links. An index to measure network robustness is introduced, analysing the effects in the travel time when each link is removed. In [32,33], the removal of links is used to locate the critical links, named in these papers as 'vital links', and in [7], link criticality of urban road network is evaluated by using the concept of macroscopic fundamental diagram (MFD), comparing the behaviour in normal condition and the behaviour when a link of a road network is disabled by a hazard.

2.1.4. Data-driven methods

Data driven approaches have been also used to detect critical points in the transport network, specially to identify those parts of the network that are likely to suffer high levels of congestion. Data-driven approaches use traffic flow prediction to detect those areas that can become congested and reduce the traffic performance in the system. Among the data-driven studies presented in the literature, some use historical datasets to identify those areas that are more likely to be affected, see [34,35]. Other present approaches that use traffic flow prediction techniques to identify critical areas. A methodology for the prediction of near future traffic condition is proposed by [36], estimating next time-step traffic volume on a single road segment. In [37], the authors use a Support Vector Regression based method to estimate next time-step traffic speed and volume, and in [38] a deep learning approach by using a Restricted Boltzmann Machine and a Recurrent Neural Network to predict the areas affected by congestion for all road segments in next time-step is proposed. Using a similar approach, in [39] an algorithm that predict traffic congestion and allows the identification of footprints of traffic congestion propagation in the near future is presented.

When predicting traffic flow conditions in a transport network, it is possible to work with short-term or long-term predictions. Short-term predictions tend to be more accurate, however they do not give the global perspective that can be achieved with long-term predictions.

Among the short-term traffic flow predictions methods, one that has been widely used in the literature for the identification of congestion problems is the analysis of time series, particularly those using Autoregressive Integrated Moving Average (ARIMA) models. ARIMA models are one of the most precise methods for the prediction of traffic flow [40], and have been used since the late 1970s to predict short-term traffic flows [41]. A disadvantage of this type of data driven approaches is the requirement of very large historical database if accurate models have to be developed. For that reason, some authors have tried to reduce the amount of data that are needed in these models. For example, a prediction scheme using Seasonal autoregressive integrated moving average (SARIMA) model for short-term prediction of traffic flow using only limited input data is presented by [42]. This approach for short-term prediction of traffic flow uses only previous 3 days of flow observations as input for predicting the next day flow values. It is also noted that long-term traffic flow predictions have interest for transport managers. The long-term view allows performing a more efficient traffic management, including a better design of new infrastructures or public transportation policies and/or the development of transport planning strategies. In [43], a clustering stage that can discover patterns within the traffic flow data registered by each road sensor is presented. This allows building prediction models for the discovered patterns. Also, based on long-term traffic flow predictions, and including the cascadian effects due to congestion, in [44] a risk-based interdependency analysis is proposed. This methodology can detect large-scale traffic congestions between interconnected junctions of the road network. The methodology presented in [44] uses a time-based dependency analysis for critical infrastructure dependency, based on one methodology presented in [45,46]. Similar approaches, based on risk-based method to analyse interdependencies and congestions have been used in other sectors such as in the aviation network, see [47] and in the maritime network, see [48].

The main drawback of long-term traffic flow predictions based on historical data lies on their potential misinterpretation of unexpected hazards that may have never been recorded, and suddenly appear on the network. Some examples of these events can be flash flooding, total or partial closures of roads due to for example maintenance works, among others. Short-term prediction model tend to be more accurate when forecasting traffic flow in the subsequent instants of an unexpected hazard, however when analysing hazards that can last several days other methodologies could be more precise. By nature, short-term models provide outputs based on sensors within a temporal window.

It is possible to infer that different methodologies try to address the presented challenge using distinct perspectives. Most of them are supported by strategy-specific assessments, analysing a group of probable damaged scenarios that can only partially address the objective of performing a comprehensive analysis.

Analysing the whole network may allow to extend the understanding of it. In [25], an approach to analyse the vulnerability of road networks under extreme weather events by studying extended affected areas instead of single link failures is presented. The authors analyse the studied area with uniformly shaped grids and size cells, where each of the cells represents the area affected by the hazard. In this case, the authors cover all the network, but a lot of resources are needed to complete the analysis, since the effect of each damaged area has to be analysed individually. To achieve a more comprehensive analysis, in [49], it is also presented a methodology to evaluate the ability of transport systems to maintain functionality when affected by a disruption, analysing the link criticality for single-link failures and multiple-link failures.

There is a demand for a systematic and holistic approach for local and global understanding of the network. Within this context, the proposed methodologies systematically analyse every link of the network and their interactions with the other links. All the links can be ranked from the most to the least vulnerable and from the most to the least critical with a full network analysis. By analysing the principal components of FIM, as detailed below, a substantial reduction of the efforts needed in the process of critical and vulnerable links identification is achieved. Moreover, with the presented approach, the perturbation suffered by a link does not necessarily need to result in its closure. Partial damages can be also analysed.

3. Systematic methodologies to identify critical and vulnerable links for full-traffic network resilience analysis

Graph theory and network analysis have several centrality measures that can help in the identification of traffic network vulnerabilities and criticalities. These centrality measures evaluate the importance of a node in a network, and are usually based on the topological characteristics of the network. Centrality measures can rank the nodes of a network by assigning a score to each of them. The most common centrality measures are degree centrality, closeness centrality, betweenness centrality and eigenvector centrality. Each of these centrality measures is described below:

- **Degree centrality** is defined as the number of links incident upon a node. In a directed graph, two degree matrices can be calculated, the in-degree matrix, defined by the number of links coming towards a node, being that the in degree of that node, and the out-degree matrix, defined by the number of links going out from a node.
- **Closeness centrality** measures the node average farness (inverse distance) to all other nodes. This means that those nodes with a high closeness value will have the shortest paths to the rest of the nodes of the network. The closeness centrality can be also defined as in-closeness, and out-closeness.
- **Eigenvector centrality** measures the influence of a node as a function of the connections of its neighbours. Eigenvector centrality is a natural progression of the degree centrality, that attempts to measure the level of connections in a network, including not only the links connecting the studied node but also the degree of connection of its neighbours.
- **Betweenness centrality** quantifies the number of times a node is used along the shortest path between two other nodes. In the context of analysing critical and vulnerable links in traffic network, it is common to use a betweenness centrality metric that analyse the links [50]. This link-betweenness centrality measures the number of times a link is used along the shortest paths.

It is noted that the first three metrics provide information on connectivity such as link density in regions of the network which may of relevance to identify critical or vulnerable areas. Betweenness has been the mostly used metric in explicit analyses of criticality and vulnerability in traffic. These four centrality metrics recurrently focus on the topological characteristics of the network, and their analysis is commonly done under static conditions. Static analysis can be useful at an early stage analysis to know the basic properties of the network. It was highlighted in previous sections that there is a need to introduce dynamic aspects to this, and significant research have been done in this regard. Some recent papers proposed modified centrality measures implemented with dynamic traffic-related information. These dynamic versions of centrality measures allow the introduction of variables such as travel demand and travel times in the analysis, see [51–54], and also can incorporate real-time data as shown in [55].

When evaluating the network resilience, it is essential to understand the holistic behaviour of a network under disruptive scenarios. The previous approaches that incorporate dynamic aspects of the traffic behaviour in the centrality matrix do not evaluate the system under disruptions. Few authors have presented methodologies that can specifically evaluate the critical spots of the network based on the network resilience. In this context, it is shown by [50] that a modified betweenness centrality matrix with a stress testing approach could provide further inside into the analysis of resilience of a network, while taking into account demand levels and dynamic characteristics of traffic. In the same work, the authors emphasize the importance of modelling the network as a dynamic graph, whose link characteristics can change over time depending on traffic behaviour.

In the present work the FIM is used to tackle this need. FIM has been used before in different fields as a means to identify key aspects of engineering systems (e.g. critical locations in sensor placement). This approach has been used in [56], and it was also implemented in the area of bridges for the placement of sensors by [57]. In the area of damage identification of buildings, see [58], the FIM is used to define an optimal approach for the location of sensors. More recently, in [59], a method of multi-objective sensor locations optimization using the collected information by FIM is presented. In this study, the sensor locations are prioritized according to their ability to localize structural damage based on the eigenvector sensitivity method, and in the same area, a method based on the FIM matrix and the structural topology optimization concept to establish a methodology for the optimal placement of sensor for structural damage identification is presented by [60].

FIM has also been used in other areas of knowledge, for example, in [61], the sustainability of a regional system using FIM in the San Luis Basin, Colorado is evaluated. The use of FIM in this analysis provides a means of monitoring the variables of a system to characterize dynamic order, and, therefore, its regimes and regime shifts. In the area of ecological systems, FIM is used to track stability in multivariate systems, see [62], and in the same area, the use of FIM to create an environmental system index is demonstrated by [63]. This index can detect when the system is changing its configuration to a new dynamic regime and improve the decision-making process to promote more sustainable decisions.

These are just some examples of the application of FIM for categorization of variables, since different authors highlight in the literature that FIM can be applied to a broad range of systems. Thus, in [64], where phase transitions and relevant order parameters via statistical estimation theory using FIM are studied, it is highlighted the generality of FIM as a measure that can be applied to a broad range of systems, particularly those where the determination of order parameters is cumbersome. Another example of the multiple uses that FIM can have is [65], who discussed how FIM can collapses multiple variables into an index that can be monitored over time to assess changes in the dynamic behaviour of systems.

Therefore, it is clear that FIM can be applied to a broad range of problems and used in analysing patterns in data. FIM can be adapted to evaluate the importance of parameters in complex system behaviour, and one of the main advantages is the ability to incorporate several variables into an index that can be used to assess criticality and identify important trends in a system. In this case, the FIM allows encompassing in the analysis not only node-link influence, but also the whole dynamics of the network, including traffic-related variables, such as travel demand and travel time. This is of particular interest as traffic will not depend exclusively on node connections, i.e. topology, but also on routes available. The FIM provides a systematic approach to identify critical or vulnerable links. It evaluates a measure of sensitivity that is directly related to importance of a link's performance in a traffic network with current defined dynamics.

In addition, the advantages of FIM are also relevant when analysing unexpected disruptive scenarios that have not been previously recorded in the historical datasets. When using the FIM, a single stress test is needed at each link in order to define the whole matrix and to identify the critical and/or vulnerable links. This is an advantage because by construction the FIM sets the demand of the necessary stress tests required to provide insight on critical and vulnerable links and it is only parametric in respect to the stress level — i.e., the level of damage in the link.

3.1. Fisher Information Matrix

In statistics, the FIM allows a measure of the amount of information that an observable random variable x carries about an unknown parameter ϕ upon which the probability of x depends on.

For a N-variate multivariate normal distribution, $X \sim N(\boldsymbol{\mu}(\boldsymbol{\phi}), \boldsymbol{\Sigma}(\boldsymbol{\phi}))$, let the K-dimensional vector of parameters be $\boldsymbol{\phi} = [\phi_1, \phi_2, \dots, \phi_K]^T$, and the vector of random normal variables be $X = [X_1, X_2, \dots, X_N]^T$. Assuming that the mean vector is $\boldsymbol{\mu}(\boldsymbol{\phi}) = [\mu_1(\boldsymbol{\phi}), \mu_2(\boldsymbol{\phi}), \dots, \mu_N(\boldsymbol{\phi})]^T$, and let $\boldsymbol{\Sigma}_{NxN}(\boldsymbol{\phi})$ be the covariance matrix. Then, the (i,j) entry, for $1 \le i, j \le K$, of the FIM is defined as follows:

$$f_{i,j} = \frac{\partial \boldsymbol{\mu}^T}{\partial \phi_i} \boldsymbol{\Sigma}^{-1} \frac{\partial \boldsymbol{\mu}}{\partial \phi_j} + \frac{1}{2} tr \left(\boldsymbol{\Sigma}^{-1} \frac{\partial \boldsymbol{\Sigma}}{\partial \phi_i} \boldsymbol{\Sigma}^{-1} \frac{\partial \boldsymbol{\Sigma}}{\partial \phi_j} \right), \tag{1}$$

where $tr(\cdot)$ denotes the trace of a square matrix, and $(\cdot)^T$ denotes the transpose of a vector.

Assuming that $X = E[X]$, $\boldsymbol{\Sigma}(\boldsymbol{\phi}) = \boldsymbol{I}$, $\boldsymbol{I}$ being the identity matrix, Eq. (1) becomes

$$f_{i,j} = \frac{\partial \boldsymbol{\mu}^T}{\partial \phi_i} \frac{\partial \boldsymbol{\mu}}{\partial \phi_j}, \tag{2}$$

where the $f_{i,j}$ denotes the FIM element for the row i and the column j. Following the previous definition of the $f_{i,j}$ element, the FIM can be rewritten as a sum of matrices, as follows:

$$F = \sum_{n=1}^{N} \frac{\partial \mu_n}{\partial \boldsymbol{\phi}} \frac{\partial \mu_n^T}{\partial \boldsymbol{\phi}}, \tag{3}$$

and, since $X = E[X] = \boldsymbol{\mu}(\boldsymbol{\phi})$, Eq. (3) can be expressed as

$$F = \sum_{n=1}^{N} \frac{\partial X_n}{\partial \boldsymbol{\phi}} \frac{\partial X_n^T}{\partial \boldsymbol{\phi}}. \tag{4}$$

This approach to build the FIM analyses the variation of an expected value X, as a function of a variable ϕ. It is noted that this way of expressing the matrix is very relevant, since it allows the definition of the matrix by a sensitivity vector. Thus, the sensitivity vector is defined as $\theta_n = [\frac{\partial X_n}{\partial \phi_1}, \frac{\partial X_n}{\partial \phi_2}, \dots, \frac{\partial X_n}{\partial \phi_K}]$, and, the FIM is expressed as follows:

$$F = \sum_{n=1}^{N} \theta_n^T \theta_n. \tag{5}$$

If a traffic network is defined as a set of nodes connected by a set of links, for the identification of the vulnerable and critical links of the network, a sensitivity vector, θ_n, must be defined associated with each of the links of the network, for $1 \le n \le N$, with N being the total number of links of the network.

For each sensitivity vector, K components are defined, for $1 \le k \le K$, corresponding to each parameter analysed affecting the N links. In the approach proposed, the K components represent the influence of each link upon the other links of the network. Thus, $K = N$, and hereafter, N will be used to refer to the dimension of the $\boldsymbol{\phi}$ vector.

Based on this overall idea, two different methodologies are developed to identify the vulnerable and the critical links of a traffic network. Sensitivity vectors will be defined depending on whether a vulnerable or a critical analysis is being performed.

3.2. Identifying vulnerable links in traffic networks with FIM

When the aim is to identify the vulnerable links of a traffic network, the FIM is designed in the way that each component will carry the information of how vulnerable a link is when the network is suffering a perturbation.

As indicated, one sensitivity vector is defined for each link of the network. To identify the vulnerability of the links as described in the previous section, a damage is introduced in each link by means of a reduction of the capacity of the link. This reduction of the capacity simulates the impact, which can be created due to different perturbations, such as climatological hazards, man-made events, road works or any other impact in the traffic network. Some examples of capacity reductions are presented as follows: (a) caused by weather events, such as floods, when a link is partially affected by a flood one or more lanes can be blocked, and its capacity will be reduced; (b) by inclement weather, it has been shown in the literature that heavy rain can reduce capacity up to 17% and heavy snow up to 27%, see [1]; or (c) by scheduled activities on the road, for example, road works or upgrade events, in this situation one or more lanes might need to be closed for the development of the works. Due to the multiple types of events that can cause a disruption in a link, the proposed methodology uses a reduction in the capacity to simulate the disruption created by any type of hazard, which facilitates the simulation of different hazards since most of the time disruptions are evaluated as a loss of capacity on the affected link.. After that, each sensitivity vector is associated with one link, and it will contain the information of the effect of the damage of that link on the rest of the links of the network.

The effect of this damage is measured in terms of cost by implementing a user equilibrium traffic assignment. The cost increment produced by the new conditions of the traffic network is evaluated by comparing the non-affected network with the disturbed situation. In this paper, the impact in the cost is measured in terms of travel time.

Therefore, to identify the vulnerable links, the sensitivity vector θ_n is defined as follows;

$$\theta_n = \left[\frac{\Delta t_1}{\Delta C_n}, \frac{\Delta t_2}{\Delta C_n}, \dots, \frac{\Delta t_n}{\Delta C_n}, \dots, \frac{\Delta t_N}{\Delta C_n} \right], \tag{6}$$

where ΔC_n is the variation of the capacity for each link, and Δt_i, with $i = 1 \dots N$, is the variation produced in the travel time of each link when the capacity of link n is reduced, which in practice is a backward finite difference.

The vulnerability information of the total traffic network will be gathered by building the FIM according to Eq. (5). Note that the term i,j of the FIM will have the following form,

$$F_{i,j} = \sum_{n=1}^{N} \frac{\Delta t_i}{\Delta C_n} \frac{\Delta t_j}{\Delta C_n}, \tag{7}$$

with the diagonal elements of the FIM being

$$F_{i,i} = \sum_{n=1}^{N} \left(\frac{\Delta t_i}{\Delta C_n} \right)^2. \tag{8}$$

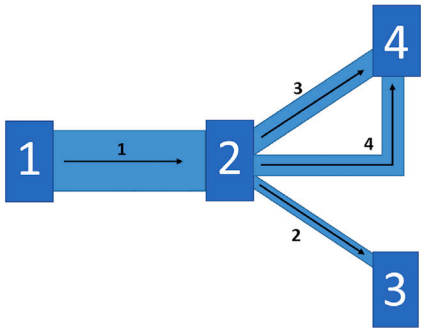

Fig. 1. Simple network.

Then, the eigenvector of the FIM will provide a ranking of the links containing the largest amount of information on the parameters, that is, the most vulnerable links of the network. The eigenvalues will provide the relative weight within the ranking, according to the following expression,

$$F\boldsymbol{v} = \lambda \boldsymbol{v}, \tag{9}$$

where $\boldsymbol{v}$ is the eigenvector associated with the eigenvalue λ. For the sake of clarification, if the obtained eigenvector is parallel to the i component, that is, with a value of 1 in the i position, the following relation should be fulfilled

$$F\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} \sum_{n=1}^{N} \frac{\Delta t_i}{\Delta C_n}\frac{\Delta t_1}{\Delta C_n} \\ \sum_{n=1}^{N} \frac{\Delta t_i}{\Delta C_n}\frac{\Delta t_2}{\Delta C_n} \\ \vdots \\ \sum_{n=1}^{N} \left(\frac{\Delta t_i}{\Delta C_n}\right)^2 \\ \vdots \\ \sum_{n=1}^{N} \frac{\Delta t_i}{\Delta C_n}\frac{\Delta t_N}{\Delta C_n} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \lambda \\ \vdots \\ 0 \end{bmatrix}, \tag{10}$$

that is,

$$\sum_{n=1}^{N} \left(\frac{\Delta t_i}{\Delta C_n}\right)^2 = \lambda \tag{11}$$

and the rest of components of the central vector in Eq. (10) are equal to zero. Thus, the eigenvalue λ will provide how the travel time of link i varies when the links of the traffic network suffer a reduction of their capacities.

3.2.1. Example for the identification of vulnerable links

A simple example is used to understand the advantages of the model in the identification of vulnerable links of a traffic network.

The traffic network in Fig. 1 is presented for the example. The network has 4 nodes and 4 links. Different thicknesses are used to represent the different capacities of the links. See Table 1 for a description of capacities and free flow travel times associated with each link. Link 1 has the largest capacity, 50 users, followed by links 3 and 4 with a capacity of 30 users both, and finally, link 2 with a capacity of 20 users. The routes analysed in this network are the ones shown in Table 2. The traffic assignment used for the creation of the FIM matrix for this and all the examples presented in the paper is a user traffic equilibrium together with the BPR function, using parameters of $\alpha = 0.15$, and $\beta = 4$.

This network is created to evidence the differences between a vulnerable link and a critical link because it can be extremely difficult to previously identify the most vulnerable or the most critical link of complex examples, and the results can be less expected. The same example will be used to identify the critical links of the network in the following section and both results can be compared.

This network represents a main link, link 1, between node 1 and 2, which has the largest capacity. This link is a key link in the network, giving access to the rest of the nodes, such as nodes 2, 3 and 4. Thus, link 1 will have a main role in the performance, and any damage in link 1 is expected to have a large impact on the network behaviour. For these reasons, link 1 is expected to be the most critical link of the network in this example.

Table 1
Characteristics of the example network.

Link	Capacity	Free flow travel time
1	50	1
2	20	1
3	30	1
4	30	1.2

Table 2
Example network. Routes selected.

Route Id	O-D	Links	Demand (users/ OD)
1	1–3	1–2	20
2	1–4	1–3	20
3	1–4	1–4	20

Table 3
Example network. Ranking of the vulnerable links.

Rank position	Eigenvalue	Vulnerability link rank
1	1.667	2
2	0.9976	1
3	0.0033	3–4

Link 2 appears in route 1, connecting node 1 and node 3, the users using route 1 need to use firstly the link 1, and afterwards the link 2 to reach their destination. Also, link 2 is the end of route 2, and does not give access to other nodes, for that reason, link 2 has a smaller capacity, and a smaller relevance in the network.

Despite being less critical, link 2 is the most vulnerable because it has the worst relation flow/capacity, and there is no other optional routes to relieve the situation. Then, any change in link 2 will have a large impact in route 2. However, the damages in link 2 will not affect the rest of the routes if the physical queues are not considered. It is noted the difference with the case of a damage in link 1, where not only one route but all them are expected to be impacted, including route 1, 2 and 3.

Therefore, link 2 will represent a vulnerable link in the network, since changes in other links of the network, for example a damage in link 1, will affect it; but a damage in the link 2 will create problems mainly in the same link, rather than in other links of the network.

Finally, route 2 and route 3 connect node 1 with node 4, in this OD pair, the users have two options to reach their destination, then, the redundancy of the network is larger and this will contribute to reduce the vulnerability of both links, 3 and 4. When one of the routes of 1–4 OD pair is damaged the users can choose the other one, and easily avoid the affected part of the network.

In conclusion, the most vulnerable link is expected to be link 2, and link 1 is expected to be the most critical in this example. Consequently, the methodology is applied in the example, and the results are shown in Table 3, also a visual representation of the vulnerability level of the links is shown in Fig. 2.

The eigenvalues obtained for the example are shown in Table 3, a ranking from the most to the least vulnerable link is presented too. As expected, the most vulnerable link is the link 2. The value of the eigenvalue is used to measure the difference between the levels of vulnerability of the links, when two links have similar eigenvalues means that their vulnerability level is similar too and vice versa. Then, link 1 is ranked with a medium level of vulnerability, and finally link 3 and 4, whose vulnerability level is significantly lower with an eigenvalue near to zero.

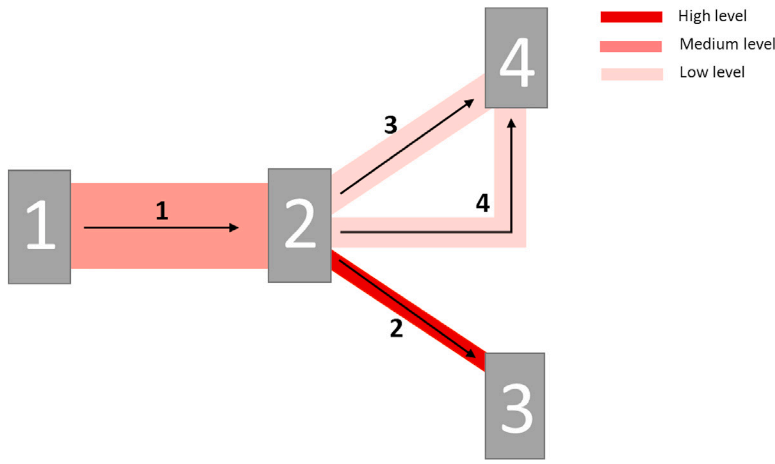

Fig. 2. Vulnerability level of the links, simple network.

This example is extremely simple, and the performance of the network can be easily understood without any methodology. However, when the networks grow in number of links, nodes and routes, a systematic methodology becomes indispensable, since the calculation and the effort to identify the vulnerable and critical links increase significantly.

3.3. Identifying critical links in a traffic network with FIM

In this section a methodology to identify the critical links of the network is presented, based on the same principles than the previous methodology. For the identification of the critical links of a traffic networks, the FIM is built as described in Eq. (5), and the sensitivity vector, θ_n, is defined as follows,

$$\theta_n = \left[\frac{\Delta t_n}{\Delta C_1}\Delta T_1, \frac{\Delta t_n}{\Delta C_2}\Delta T_2, \ldots, \frac{\Delta t_n}{\Delta C_n}\Delta T_n, \ldots, \frac{\Delta t_n}{\Delta C_N}\Delta T_N\right], \tag{12}$$

where Δt_n is the variation of the travel time of link n when the capacity of link i with $i = 1 \ldots N$, ΔC_i, is reduced, and ΔT_i is the variation of the total travel times of the network when the link i is damaged.

The variation of the total travel times of the network, ΔT_i, is introduced in the methodology because the global performance of the network needs to be considered in order to identify the critical links. In this way, it is possible to incorporate the effects of the global behaviour of the network when the link analysed is being damaged.

Note that the term i, j of the FIM will have the following form,

$$F_{i,j} = \sum_{n=1}^{N} \frac{\Delta t_n}{\Delta C_i}\frac{\Delta t_n}{\Delta C_j}\Delta T_i \Delta T_j, \tag{13}$$

with the diagonal elements of the FIM being

$$F_{i,i} = \sum_{n=1}^{N} \left(\frac{\Delta t_n}{\Delta C_i}\Delta T_i\right)^2. \tag{14}$$

Then, the eigenvector of the FIM defined in that manner will provide a ranking of the links causing the largest cumulative sensitivity, or in other words, the most critical links of the network, and the eigenvalues will provide the relative weight within the ranking

For instance, if the obtained eigenvector is parallel to the i component, that is, with a value of 1 in the i position, the following relation should be fulfilled

$$F\begin{bmatrix}0\\0\\\vdots\\1\\\vdots\\0\end{bmatrix} = \begin{bmatrix}\sum_{n=1}^{N}\frac{\Delta t_n}{\Delta C_i}\frac{\Delta t_n}{\Delta C_1}\Delta T_i\Delta T_1\\ \sum_{n=1}^{N}\frac{\Delta t_n}{\Delta C_i}\frac{\Delta t_n}{\Delta C_2}\Delta T_i\Delta T_2\\ \vdots\\ \sum_{n=1}^{N}\left(\frac{\Delta t_n}{\Delta C_i}\Delta T_i\right)^2\\ \vdots\\ \sum_{n=1}^{N}\frac{\Delta t_n}{\Delta C_I}\frac{\Delta t_n}{\Delta C_N}\Delta T_i\Delta T_N\end{bmatrix} = \begin{bmatrix}0\\0\\\vdots\\\lambda\\\vdots\\0\end{bmatrix}, \tag{15}$$

Table 4
Example network. Ranking of the critical links and resilience of the traffic network when the corresponding link is affected by a reduction of 50% of its capacity.

Rank position	Eigenvalue	Critical link rank	Resilience (%)
1	0.4872	1	92.99
2	0.0046	2	97.08
3	0	3–4	99.99

that is,

$$\sum_{n=1}^{N} \left(\frac{\Delta t_n}{\Delta C_i}\Delta T_i\right)^2 = \lambda \tag{16}$$

and the rest of components of the central vector in Eq. (15) are equal to zero. Therefore, the eigenvalue λ will inform on the influence of a modification of the capacity of link i over the whole traffic network, scaled by the total cost that such change causes upon the overall performance.

3.3.1. Example for the identification of critical links

The same network that was used in the previous section to identify the vulnerable links is used in this section to identify the critical links. See Fig. 1 for a description of the network, and Tables 1 and 2 for a description of the routes, the capacities, and the demand of the network.

As explained before, link 1 is expected to be the most critical element, because the rest of the network has a big dependency on this link and any damage in link 1 will affect the whole network in a large amount than any damage in the rest of the links.

The results of the methodology are shown, in Table 4, and as expected, the most critical link of the network is link 1(see Fig. 3) .

To validate the results, the resilience of the network is evaluated. Perturbations in different areas will be introduced to corroborate the results, because when a critical link is damaged, the resilience will be more reduced than when a less critical link is damaged. Therefore, four scenarios are considered, damaging only one link at each scenario. For this case, the damage introduced is a reduction of the capacity to a half of its initial value.

The methodology used to evaluate the resilience is the one presented by [66]. The proposed approach measures the resilience of the network as the capacity of the network to absorb a perturbation. The model analyses the progressive response of the traffic network to a time-varying impact using a Dynamic Restricted Equilibrium Assignment. The resilience value is assessed as the normalized area over the exhaustion curve takes into account important aspects, such as the cost increment, the user stress level, and the system impedance to alter its

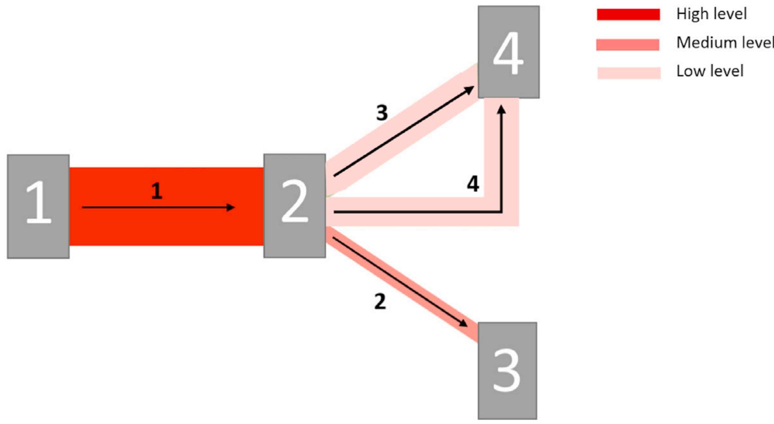

Fig. 3. Critical level of the links, example network.

previous state. These parameters are obtained during the evaluation of the new restricted traffic assignment that includes an impedance to simulate the actual capacity of adaptation to the changes. This helps to simulate the lack of knowledge of the new situation and the lack on information of the behaviour of other users.

The resilience approach used in this analysis considers not only the effect on the travel over-cost due to the perturbation, but the effect on the users by means of the network stress level. This assesses concepts such as network redundancy, adaptability and local vulnerability. It is noted that the proposed formulation reflects the important interdependence existing between the weather impact and the traffic conditions.

In this approach, the cost level is calculated with the data obtained from the dynamic equilibrium-restricted assignment model, as described in Eqs. (17) and (18), and the links travel cost is evaluated with the BPR function.

$$C_T(t) = \sum_{a\in A} c_a(t), \tag{17}$$

$$\tau_k(t) = (C_T(t) - C_0)/(C_{th} - C_0), \tag{18}$$

where $\tau_k(t)$ is the normalized cost level, $C_T(t)$ is the actual total cost for a time t of the traffic network, and c_a the cost for each link a. C_0 is the initial total cost ($t = 0$), and C_{th} is a cost threshold. The cost threshold, C_{th}, is determined by the system breakdown point.

The stress level is calculated as describe in Eq. (19)

$$\sigma_k(t) = max_{pq\in D}((1/\alpha)(\sum_{r\in R_{pq}} |\rho_r(t) - 1|)/(\eta_{pq})), \tag{19}$$

where n_{pq} is the number of routes with OD pair pq, α is the system impedance, and ρ_r is the limit of the system impedance for each route, r.

Finally, the exhaustion level is a combination of the cost and the stress level, and it is calculated as explained in Eq. (20).

$$\psi_k(t) = (1/2)(1 + \sigma_k(t))(\tau_k(t))^b, b >= 1, \tag{20}$$

where b is a coefficient to penalize the cost level when it is larger than the cost threshold.

After that the resilience is calculated as the normalized area over the exhaustion curve, as indicated in Eq. (21).

$$\chi_k^p = 100(\int_{t_{p0}}^{t_{p1}} 1 - \psi_k(t)dt)/(t_{p1} - t_{p0}), \tag{21}$$

where t_{p0} and t_{p1} denote the initial and the final time of the perturbation event respectively.

For a better understanding of the resilience evaluation, the author is referred to [67,68] and to [69,70] for the sensitivity analysis of the parameters involved.

The results obtained are showed in the last column of Table 4. As expected, when the most critical link of the network is damaged, the largest impact is obtained in the value of resilience, in this case the resilience is reduce to 92.99%. The impact in the value of resilience is reduced as less critical links are damaged, confirming that perturbations in the most critical links of the network will create the worse performance of the whole network.

4. Case study: Cuenca network

Finally, the methodology presented is implemented in a real case. The selected network is the city of Cuenca in Spain, see Fig. 4 for a description of the network. The Cuenca network has 232 nodes, 672 links, and 207 routes, considering a total demand of 20,700 users. In this example, the free flow travel time of the links has been calculated assuming that the velocity is the same for all the links, therefore it will depend on the length of the link, i.e., proportional to the distance between nodes. The link capacity is 400 users for the links of the city centre (links from 1 to 300), and 800 users for the external links of the network.

For a better understanding of the results in this complex case, the Cuenca network has been represented to visualize the number of routes that cross each link. In Fig. 5, the thickness of the links has been modified depending on the quantity of routes passing across each link. Then, the thicker the link is, the higher the number of routes passing through the link. It is highlighted that when analysing the vulnerable and critical links of a network, the number of users using the link will be a determining variable.

Using the methodologies previously described, the vulnerable and the critical links of the network are obtained and ranked from the most vulnerable and critical to the least. Table 5 shows the results for the 14 most vulnerable links, and Table 6, the results for the 14 most critical links.

With the aim of validating the rank of the critical links, the resilience of the network is evaluated damaging the identified link, in the same way that in the previous example. The values obtained for resilience are shown in column 3 of Table 6. The damage introduced to simulate a perturbation is a reduction of the capacity of the selected link. The disruption lasts 8 days, during the first 4 days the capacity is reduced, reaching the lower level in the 4th day, with a 35% of the initial capacity. In the last 4 days, the value of the capacity of the link gradually increases until the 100% of the initial capacity.

Then, the value of resilience of the first row, 70.92, is obtaining damaging only the most critical link, 72, in the second row, the second link of the ranking is the only one being damaged, and in the same way for the following values. Significant differences in the values of resilience are found, which exhibits the relevance of the methodology. In the case that the perturbation is affecting the most critical link, link

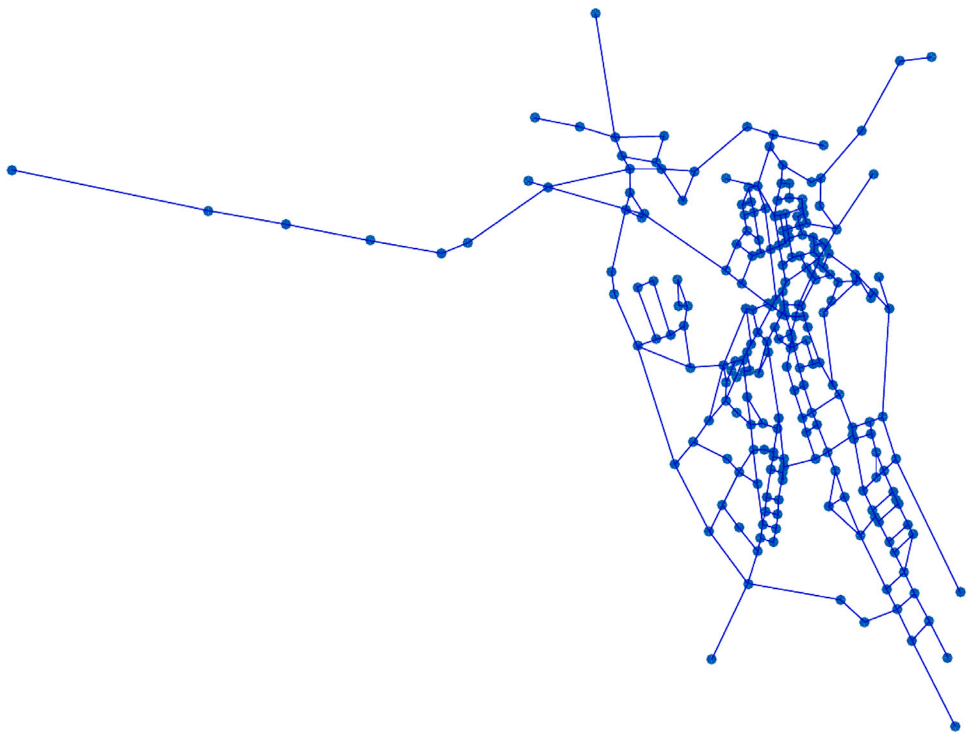

Fig. 4. Network represented by nodes and links. City: Cuenca, Spain.

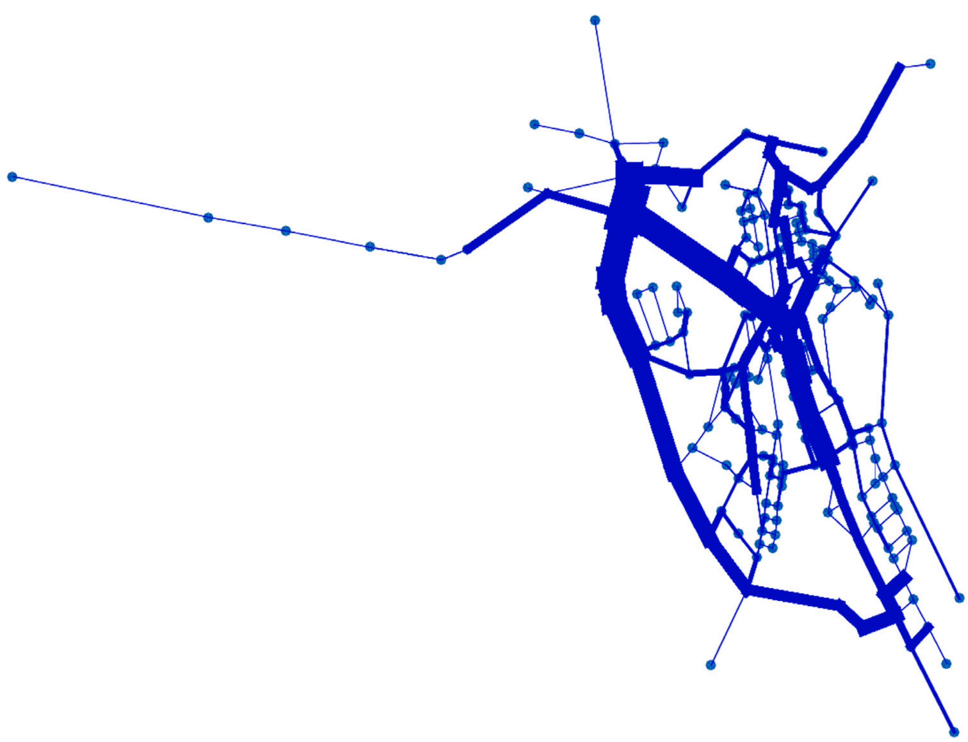

Fig. 5. Cuenca Network, Spain. The thickness represents the number of routes crossing each link.

72, the resilience of the network is reduced almost 30%, what it is a considerable impact caused by only one link damaged. However, if the same perturbation is affecting the following link in the route, link 349, ranked in 7th position, the reduction of resilience is around 5%.

When resilience is calculated, its value is expected to increase as less critical links are affected. Observing the results in Table 6, the expected tendency is obtained. However, slight variations can be found in links ranked as 7th, 10th and 12th, where decimal reductions in the resilience are noted with respect to link 6th, 9th, and 11th respectively. These small variations occur for links with similar values of their eigenvalues, i.e., values of criticality very similar, as in the case of the links 6th and 7th, 9th and 10th, and 11th and 12th.

The most critical links are represented in Fig. 6. Due to the difficulty of showing the results in a real network with a large number of links,

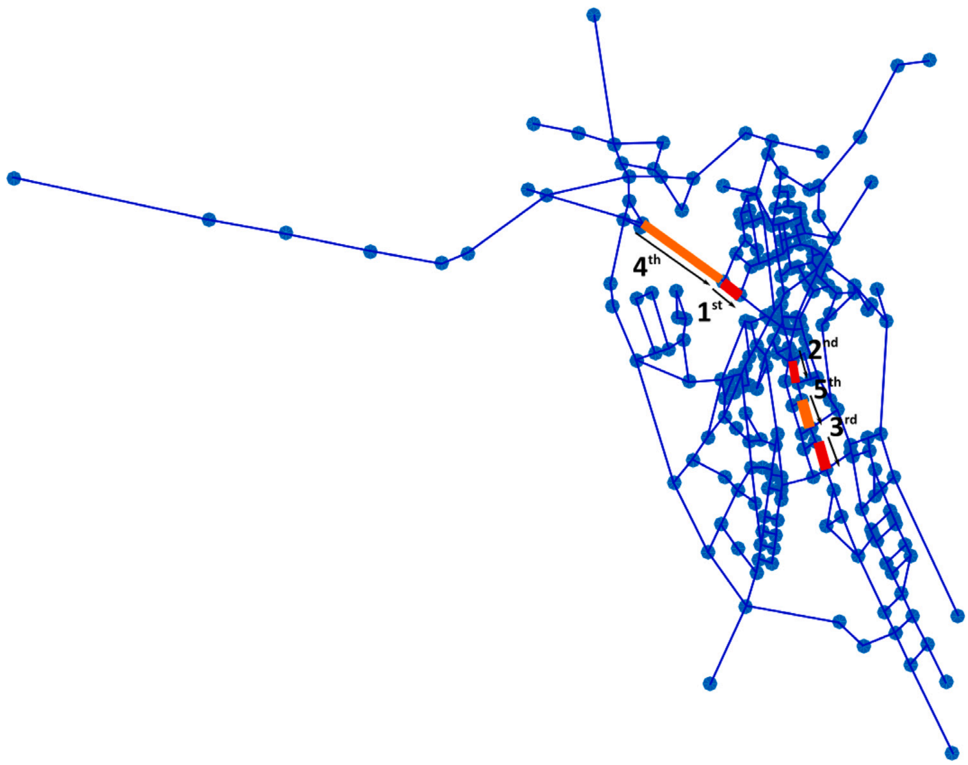

Fig. 6. Cuenca Network, Spain. Critical links that reduce the resilience of the network more than a 5%.

Table 5
Cuenca Network, Spain. Ranking of the 14 most vulnerable links.

Rank position	Normalized eigenvalue	Vulnerable link
1	1.0000	72
2	0.4461	221
3	0.3040	265
4	0.0879	229
5	0.0364	228
6	0.0359	4
7	0.0315	258
8	0.0306	438
9	0.0282	5
10	0.0221	157
11	0.0110	349
12	0.0074	622
13	0.0041	348
14	0.0028	2

Table 6
Cuenca Network, Spain. Ranking of 14 most critical links.

Rank position	Normalized eigenvalue	Critical link	Resilience (%)
1	1.0000	72	70.92
2	0.1716	221	82.19
3	0.0624	265	85.68
4	0.0040	438	91.82
5	0.0039	229	93.29
6	0.0007	228	95.63
7	0.0006	349	95.55
8	0.0006	258	96.32
9	0.0004	622	96.50
10	0.0004	4	96.08
11	0.0002	157	97.06
12	0.0001	5	96.61
13	0.0001	348	97.45
14	0.0000	619	98.45

only the most critical ones are shown in the network. Fig. 6 represents the critical links that create a reduction in the resilience of the network of more than a 5%, those ones that reduce the resilience more than a 10%, critical rank positions 1 to 3, are shown in red colour, and those ones that reduce the resilience between 5% and 10%, rank positions from 4 to 5, are shown in orange in the figure. The rest of the links will have an impact smaller, reducing the resilience less than a 5%.

As expected, the most critical links of the network are located in the areas where a high number of routes are supplied. In this network, two main corridors can be identified, the first one is the circular surrounding the city, and the other one through the city, see Fig. 5, therefore special attention is needed for the latter, since all the critical links are situated there.

Comparing the rankings for vulnerable and critical links, it is noted that the links on the 3 first positions of the ranking, are the same for both cases, showing that in real networks both link types can have the same location. Main links of the network, as the ones obtained in the results of this case, can be the critical ones, creating the largest impact when damaged. Also main links can be identified as the most vulnerable, suffering a large impact when other parts of the network are damaged. In such a case, investment in reducing the vulnerability of those links will result in the largest improvement of the resilience of the system. Nevertheless, the difference between critical and vulnerable links is fundamental. For example, the link 438 is ranked in the 4th position in the critical ranking, creating a reduction of around 9% of the total resilience when damaged. However, it is in the 8th position in the vulnerability ranking.

Sometimes, it will be necessary to focus on the critical links, for example when a hazard is forecasted and a collapse of the network needs to be avoided, in this case, the criticality of the links needs to be reduced, for example, by providing redundancy to the system (e.g., with alternative routes). Other occasions, the identification of the most vulnerable links becomes more relevant, for example, to avoid the loss of accessibility of some nodes.

4.1. Comparison with other centrality metrics

In this section a comparison is presented. The proposed methodology is compared with other centrality metrics, using in and out degree

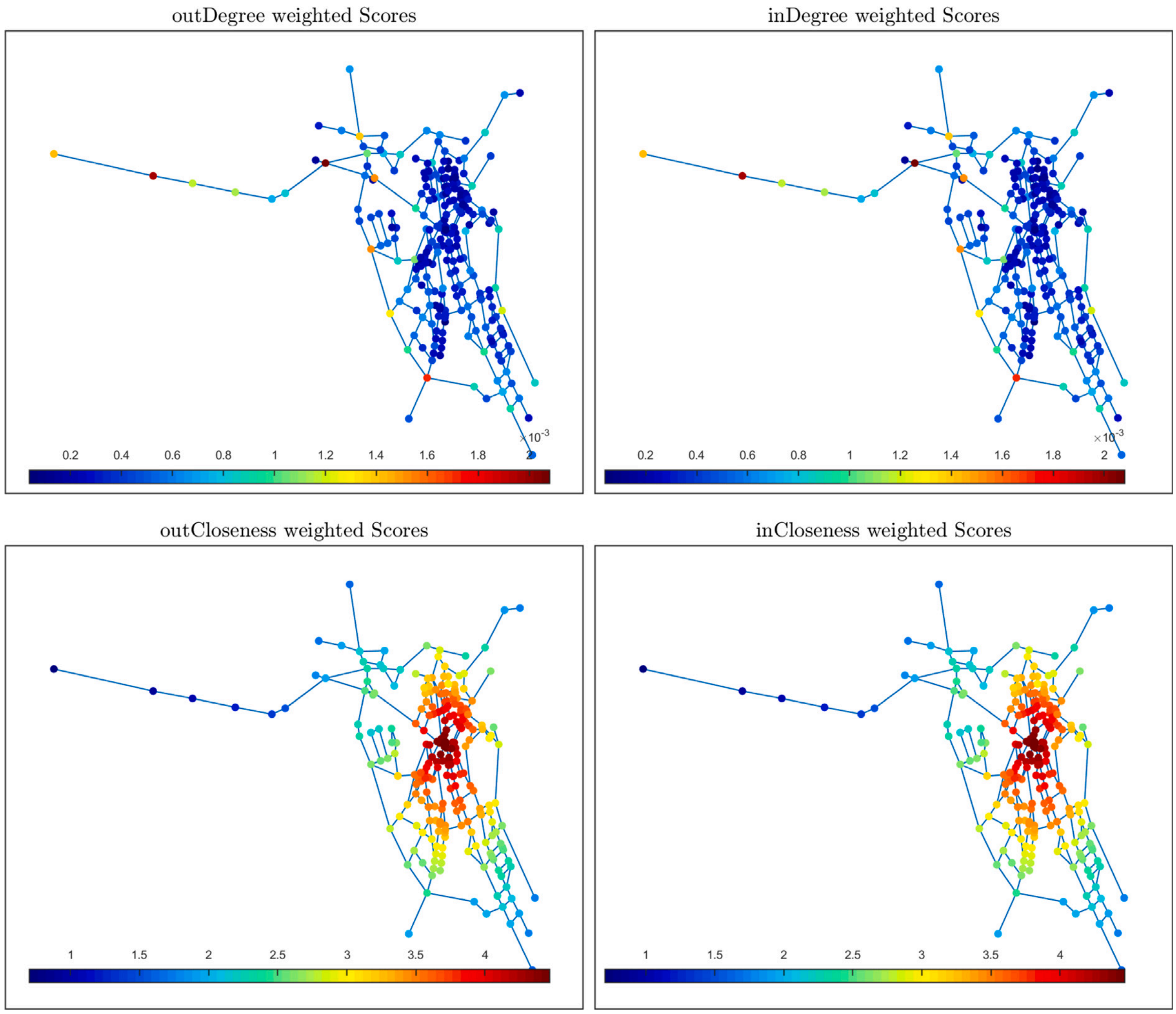

Fig. 7. Cuenca Network evaluated by centrality metrics, including in-degree and out-degree centrality and in-degree and out-degree closeness.

centrality, in and out closeness centrality and betweenness centrality. The eigenvector centrality is not used in this comparison, as [71] noted that although some extensions have been proposed for the use of eigenvector centrality in directed graphs, the most commonly used version of eigenvector centrality requires the graph to be undirected. The results when compared to alternative metrics shows that using a FIM provides further insights on the importance of different links in the network.

Fig. 7 presents the results for both in and out degree metrics, and in and out closeness (weighted by the link distance). Because most of the links serve nodes in and out directions, the locations of the scores are similar for both in and out directions. Both metrics have limited capability to be informative in supporting an explicit identification of critical or vulnerable links. As discussed previously, these fail to captivate the dynamic behaviour of the network but still carry important information on different regions of the network.

Fig. 8 shows the results of link-betweenness centrality in the network. Compared with the previous metrics, betweenness provides more information, since not only topology measures such as the number of links in and out a node are evaluated but also the current routes that users have available. Betweenness can be used to reduce the number of links that are analysed in large networks and then performing the critical and vulnerable link identification in a smaller sub-set of the network. However, in-line with the previous metrics it fails to comprise the dynamic behaviour necessary to identify the most critical and vulnerable links in a context of resilience analysis, when the performance of the network under unexpected disruptions needs to be evaluated.

Table 7
Cuenca Network. Ranking of links by betweenness centrality metric.

Rank	Link-betweenness	Link
1	0.1809	622
2	0.1508	349
–	0.1508	352
4	0.1407	619
5	0.1357	348
6	0.1307	438
7	0.1256	72
8	0.1206	557
9	0.1156	221
–	0.1156	459

In Table 7, a link rank based on the link betweenness centrality metric is presented. The top five links in the betweenness ranking are 622, 349, 352, 619, and 348, and even though four of them also appear in the criticality rank their order is significantly different. For example, link 622 occupies the first position in the betweenness ranking, but it is ranked in the 9th place in the criticality ranking. It is noted that when link 622 is affected by a disruption the overall value of resilience decreases to 96.50. This is reasonably small when compared with the

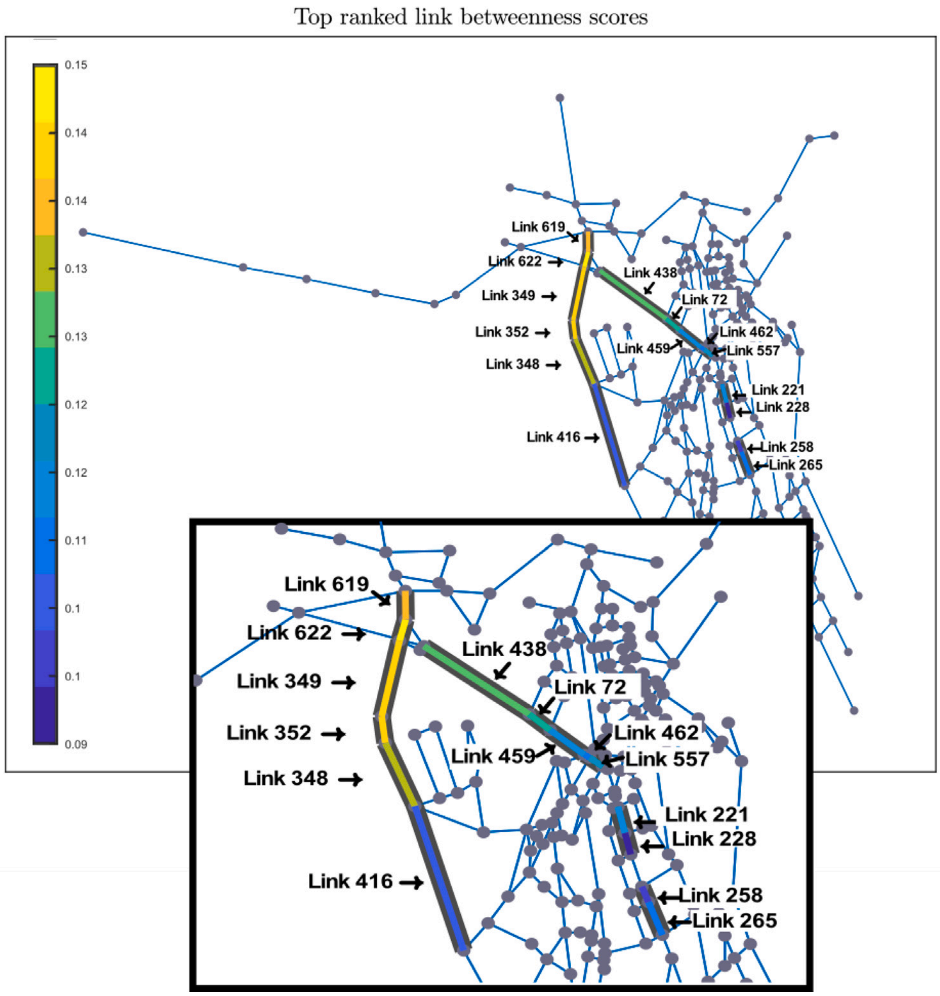

Fig. 8. Cuenca network evaluated by link betweenness centrality metric.

top links in the criticality rank, which reduce resilience up to 70.92. It is also noted that the link ranked in first place in the criticality ranking is link 72, and in the rank created by the betweenness metric it only appears in the 7th place. This highlights the fact that perturbation-based methodologies are required when analysing the critical and vulnerable links under disrupted scenarios. As highlighted in [50], different constructions of static metrics may allow comprehending criticality and vulnerability of links, however, some aspects of a network operation are only understood using full dynamic considerations.

It is noted that other transformations of the betweenness centrality may provide further accuracy in regard to the identification of the links that have the largest influence on the resilience of the network, such as augmented measures that use traffic flow, *e.g.*, [72,73]. Nonetheless, a particular advantage of the FIM in this regard is that it does not demand any prior information on the network (*e.g.*, topology or traffic flows), being only performance-based.

5. Conclusions

This paper presents two methodologies, one to identify and rank the vulnerable links of a traffic network, and the other one to identify and rank the critical links of a network. Both methodologies use the Fisher Information Matrix for the identification of the links, and the FIM is defined in a particular way for each methodology.

The paper shows how using the eigenvector associated to each eigenvalue of the matrix, it is possible to identify and rank the vulnerable and the critical links of the network. Then, the value of each eigenvalue determines the position in the rank, being the largest the one that identifies the most vulnerable and the most critical, and going down in the rank as the value of the eigenvalue decreases. Therefore, in just one analysis, all the links of the network are analysed.

This work distinguishes between vulnerable and critical links. In this way, specific strategies can be implemented depending on the desired objectives.

The importance of the critical links is highlighted when the resilience of the network is analysed. Results were in agreement with a resilience assessment based on a stress-based test, and the methodology implemented was able to systematically provide a measure of vulnerability and criticality in traffic networks. This novel strategy proposed can also inform decision-making as a means to increase the resilience of a network by improving the conditions of the most critical links.

The presented methodologies analyse the traffic performance, and in the same way, the disruptions affecting the traffic. For that reason, this methodology uses traffic-related variables such as the flow, the capacity, the travel time, and the demand. The impacts are evaluated in terms of travel time, but this can be easily convertible to economic cost.

The complete closure or removal of the link is not mandatory, a partial impact in the link with the desired intensity can be incorporated. Moreover, the assessment of vulnerability and criticality with the approach proposed demands limited *a priori* knowledge on the network. All links are ranked in a global analysis and relatively with each other without needing to navigate inputs to the system.

It is noted that the results obtained can be further improved by introducing more viewpoints. For example, characteristics such as crucial locations, proximity to natural elements such as rivers, sea, and mountains, and critical infrastructures such as bridges and tunnels, can be specifically incorporated in the results, since these elements already include a certain degree of vulnerability by nature. In addition, it would be of interest to perform further research in how the proposed methodology can inform decision making to the analysis of cascading effects in transport networks. Identifying a critical link in a certain perturbation is expected to be informative for this type of analysis. When critical links are stressed, or closer to failure, they will be more likely to create further disruptions and propagate possible damages across the network. Even more, and to be able to produce information valuable for risk assessments based on this methodology, more external factors can be weighted on the operational conditions of the networks, that then will feed the FIM methodology. In this way, the FIM analysis can be also relevant to increase the intrinsic resilience of the network, which is very important in the face of increasingly deep uncertainties.

To conclude, the present work emphasizes the interest and flexibility of the FIM matrix as a systematic methodology for global traffic network analysis (that includes interactions) and local assessment (through vulnerability), and future works should seek to improve the insights provided. As highlighted thorough this work, the FIM is a powerful tool to find cumbersome relationships and order. Hence, future works should provide further comparison of its application to traffic networks. These should also infer on the resolution of the matrix. An example of such can be found in competing ranks. It was seen that the methodology proposed correlated quite well with the resilience assessment when the difference in criticality (and impact in the system's resilience) between links surpasses a threshold value (of over 1% in value of resilience), however, in the cases of links that have similar values of criticality and carry similar impacts in the resilience of the system, the ranking provided was not so efficient and further investigation is required in this regard.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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