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Transfer Matrix Method for Fundamental LP_{01} Core Mode Coupling in a Tilted FBG Sensor

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Abstract. Fibre Bragg gratings (FBGs) are obtained through a permanent and periodic refractive index modulation in the core of the single-mode optical fibre. For many years, they have been employed in telecommunication industry as a passive device for wavelength division multiplexing and dispersion compensation components, or in laser apparatus for laser fibre stabilization, Erbium amplifier gain flattening device and amplifier pump reflectors. In aerospace structures, FBGs are used as sensors for structural health monitoring of composite materials as they are able to perform measurements of several parameters inside the material in an elegant and low intrusiveness way. Based on the Bragg and optical fibre structure many kind of customizations can be applied on FBG sensors during the manufacturing process. Each of them gives to the FBG sensor different proprieties and sensing abilities. In this work, we address the numerical simulation of the reflected spectrum by a special FBG sensor called a tilted FBG (TFBG), in which the core refractive index modulation is performed in way to obtain a tilted Bragg super-structure. By considering the classic Coupled-Mode theory for weakly-guided waveguides, we solved the mode propagation equations with the Transfer Matrix method (TMM) obtaining the TFBG reflectivity for different tilt angles.

Introduction

Carbon fibre reinforced plastic (CFRP) is a class of composite materials which has gained a lot of interest over the last decades in aerospace industry. This kind of composites can save considerable weight of the aircraft which reduces CO_2 emission. Moreover, carbon fibres allow tailoring of mechanical properties due to inherent anisotropy of CFRP composites. In particular, contemporary CFRP composites are still susceptible to both mechanical and chemical aging and physical damages, especially impact damage [1, 2]. Furthermore, composite materials have distinct characteristics and their mechanical response to any possible type of external load (particularly, fatigue and failure modalities) is different and less known, compared to the traditional materials used in aeronautical structures such as aluminum, steel and titanium [2]. Therefore, it is of great importance to perform a constant control during operational life of such composite structures through structural health monitoring (SHM) techniques.

SHM of composite materials consists in the embedding or permanently integrating sensors into the structure. These embedded sensors provide detection of the material damages transferring data for processing and evaluation to an external data acquisition (DAQ) device. Fibre optics usage for SHM sensors is continuing to grow in recent years due to their inherent insensitivity to external electromagnetic fields and comparable mechanical properties and dimensions to carbon and optical fibres [3]. Recent research has particularly focused on fibre Bragg grating sensors (FBG), which are a type of point sensor and widely used in the field as they are able to detect the complex state of strain [4, 5]. FBG sensors consist of a local modulation of refractive index in the fibre core, which serves as a spectral filter for guided light [6]. The grating period and refractive index are sensitive to external strain, temperature and vibrations of any structure in which they are mounted to. However, ordinary FBG is not sensitive to surrounding refractive index change because light is guided mostly in fibre core. Tilted FBG can couple some portion of the light to the cladding modes drastically modifying the transmission spectrum [7]. Using this type of gratings one could in principle monitor temperature, strain and refractive index of the surrounding composite.

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Theory

There are several approaches for modeling of TFBG's spectral properties. A coupled-mode theory (CMT) has been applied for the case of tilted Bragg gratings and up to date it is the most intuitive and useful approach [8, 9]. Alternatively, antenna theory [10] and the volume current method [11] can be implemented. In CMT, a system of differential equations describes the amplitudes of the modes during propagation in the Bragg grating structure. The "weight" of the coupling between the modes is given by the coupling coefficient, which can be rigorously calculated by writing the energy exchange equations between the propagating modes in TFBG [12]. Usually, the case of a standard FBG coupling is allowed only between the modes propagating inside the core region as these modes can exchange energy between them. In this case, to good approximation, it is possible to obtain the reflectivity by considering just the coupling between the forward propagating fundamental core mode and the backward propagating core mode generated by reflection at the Bragg grating. However, in the TFBG case, the presence of the tilt angle allows the exchange of energy between the fundamental core mode and the cladding propagating modes, hence, multiple coupling coefficients (core-core modes and core-cladding modes) need be calculated.

Although a straightforward numerical calculation of coupled-mode differential equations offers precise results, this reveals to be time consuming and computationally inefficient. To overcome these issues, the TMM is often preferred to solve the CMM equations along the FBG length. It allows one to divide the FBG into multiple segments taking into account the grating chirp, apodization or phase shift [13]. The transfer matrices first links the electric fields of the modes at the ends of each segment, then each sub-matrix can be multiplied in sequence with the adjacent one to create a single matrix describing the whole FBG while considerably reducing calculation time [5].

In this example, the TFGB is made in single mode fibre with a core diameter of 8.2 um and a cladding diameter of 125 um without coating. We consider weakly tilted FBGs with a tilt angle in the range $\theta < 20^o$. In this configuration light is coupled from fibre fundamental LP_{01} mode to a backward propagating Bragg mode and also backward propagating cladding modes. The coupled mode equations for core modes are described as follows [12]:

$$\frac{dR_{01}}{dz} = if_{01-01}R_{01} + ig^{+}_{01-01}S_{01}exp(-2i\delta_{01-01}z), \tag{1}$$

$$\frac{dS_{01}}{dz} = -iR_{01-01}g^{-}_{01-01}exp(2i\delta_{01-01}z) - iS_{01}f_{01-01}. \tag{2}$$

with

$$\delta_{01-01} = \beta_{01} - K_g cos(\theta) \tag{3}$$

where R_{01} and S_{01} are the amplitudes of forward and backward core mode propagating waves, $\beta_{01} = 2\pi n_{eff}/\lambda$, θ is the grating tilt angle, z is a propagation distance, $K_g = \pi/\Lambda$, δ_{01-01} is a demodulation parameter, Λ the grating period. The coupling coefficients f_{01-01} and $g^{\pm}_{01-01}$ for forward and backward propagation core modes are calculated with the following equation:

$$g^{\pm}_{01-01} = \frac{\omega}{4}\varepsilon_0 n_1^2 n(z) v \int_0^{2\pi} d\varphi \int_0^{a_1} rdr\; exp(\mp 2iK_{gB} rcos(\varphi)sin(\theta)) \times (E_r^{01}E_r^{01*} + E_\varphi^{01}E_\varphi^{01*}) \tag{4}$$

$$f_{01-01} = \frac{\omega}{2}\varepsilon_0 n_1^2 n(z) \int_0^{2\pi} d\varphi \int_0^{a_1} rdr\; (E_r^{01}E_r^{01*} + E_\varphi^{01}E_\varphi^{01*}) \tag{5}$$

where E_r^{01}, E_φ^{01}, E_r^{01*} and E_φ^{01*} are electric field of the fundamental core mode radial and azimuthal components and their conjugates, ω is angular frequency of light, a_1 and n_1 are the core radius and refractive index respectively, ε_0 is the vacuum permittivity, v is a visibility, and $n(z)$ is refractive index modulation along the Bragg grating.

A numerical method such as Runge-Kutta can be applied for solving this system of differential equations [14]. However, it is time consuming and therefore piece-wise approach is preferred to straightforward numerical calculation. In the transfer matrix method (TMM) the grating is divided to multiple segments. Each one of them is described by a 2x2 matrix:

$$\boldsymbol{F_i} = \begin{bmatrix} cosh(\gamma_B\Delta z) - i\frac{\hat{\sigma}}{\gamma_B}sinh(\gamma_B\Delta z) & -i\frac{\kappa}{\gamma_B}sinh(\gamma_B\Delta z) \\ i\frac{\kappa}{\gamma_B}sinh(\gamma_B\Delta z) & cosh(\gamma_B\Delta z) + i\frac{\hat{\sigma}}{\gamma_B}sinh(\gamma_B\Delta z) \end{bmatrix} \tag{6}$$

with

$$\gamma_B = \sqrt{\kappa^2 - \hat{\sigma}^2} \tag{7}$$

$$\hat{\sigma} = \delta + \sigma - \frac{d\phi}{2dz} \tag{8}$$

Here κ is an "ac" coupling coefficient, σ is the "dc" (period-averaged) coupling coefficient, δ is the detuning wave number, dz is a length of each segment and ϕ is the phase modulation of the grating, which is equal to 0 for uniform gratings. In this paper we ignore the effects of the dc induced-index change in order to directly compare the Bragg peak shape for different tilt angles. Electric field of forward and backward propagating modes can be calculated after passing through each segment as following:

$$\begin{bmatrix} R_i \\ S_i \end{bmatrix} = \boldsymbol{F_i} \begin{bmatrix} R_{i-1} \\ S_{i-1} \end{bmatrix} \tag{9}$$

After calculating transfer matrices for each segment it is straightforward to calculate the global transfer matrix describing the whole grating:

$$\boldsymbol{F} = \boldsymbol{F_i} * \boldsymbol{F_{i-1}} * ... * \boldsymbol{F_1} \tag{10}$$

Thus, considerable computation time is required only for calculation of transfer matrices, and it is much faster to calculate the electric field of coupled modes after the interaction with the FBG.

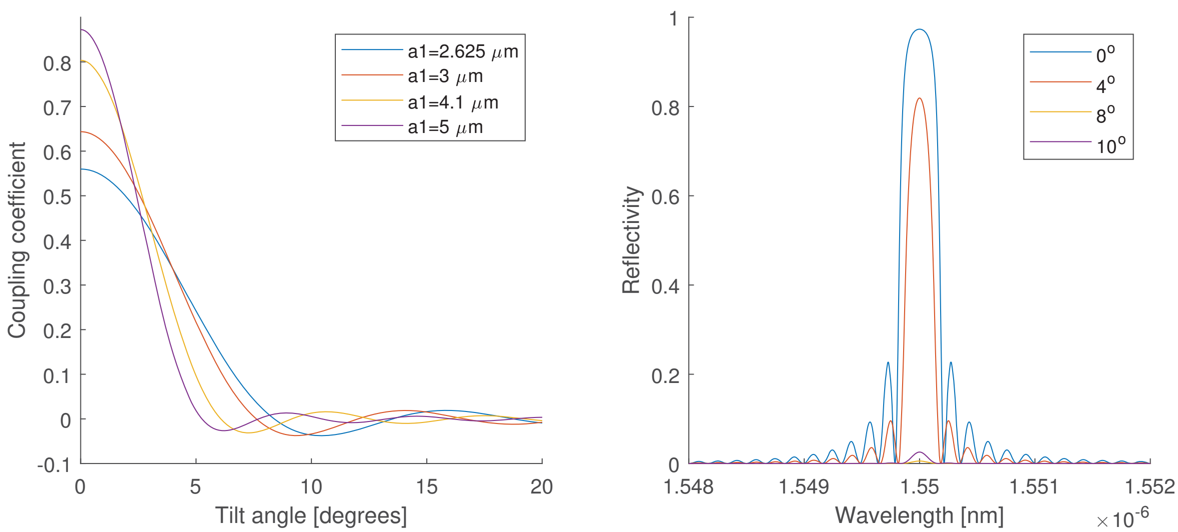

FIGURE 1. Dependence of the coupling coefficient on grating tilt angle for different core radii (left), reflection spectrum of TFBGs with different tilt angle for core radius $a_1 = 2.625\mu m$ (right).

Results

For numerical analysis of CMT with TMM, a model of optical fibre was evaluated with the following parameters: cladding refractive index $n_2 = 1.44$, core and cladding refractive index difference $\Delta = 0.0055$, surrounding refractive index $n_3 = 1$, core radius $a_1 = 2.625\ \mu m$, cladding radius $a_2 = 62.5\ \mu m$, $v = 1$, $\sigma = 10^{-4}$. Figure 1 (left) shows dependence of coupling coefficient g^{+}_{01-01} on tilt angle of TFBG for the fibre core radii $a1 = 2.625, 3, 4.1, 5\ \mu m$. Most of these radii correspond to commercially available optical fibre. The coupling coefficient has maximum value for a non-tilted FBG and is larger for fibres with greater core radius. Furthermore, it rapidly declines with a minimum value around 10 degrees for $a_1 = 2.625\ \mu m$. The first minimum is observed at a smaller tilt angle for a larger core radius and for $a_1 = 5\ m$ the dip is already observed at 6 degrees. Afterwards, the coupling coefficient starts oscillating around zero with decaying amplitude.

Reflection of the TFBG Bragg peak approaches zero when the coupling coefficient equals zero and can be calculated as $R = |F_{21}/F_{11}|^2$ [8]. Figure 1 (right) shows the normalized reflection spectra of TFBG with core radius $a1 = 2.625\ \mu m$ with different tilt angles. The reflection amplitude of the Bragg peak follows the absolute value of the coupling coefficient. One can also observe side lobes on the both sides of the Bragg peak. These peaks are caused by the uniform reflective index modulation profile [8]. A proper apodization of the grating can suppress these side lobes, but it will affect also the coupling coefficient for different tilt angles as follows from equation 5. A full width at half maximum (FWHM) of non-tilted FBG is around 0.2 nm and can be varied with exposure parameters during the manufacturing. However, for sensing applications it is important to be able to precisely track the position of the Bragg peak. So the amplitude of the Bragg peak for a tilt angle $\theta > 5^o$ can be obtained by increasing the refractive index modulation or the TFBG length. Larger tilt angles of TFBGs allows one to increase a range of refractive index and/or strain measurements, because at $\theta > 5^o$ the effective refractive index of some cladding modes has low enough value [7]. In future works the coupling coefficient of fundamental core mode to cladding modes with low azimuthal number will be evaluated using TMM. It can allow one to study reflection spectrum of TFBG sensor at different stages of composite material curing and also during an aircraft operation.

Conclusions

In this paper the TMM was used to simulate the reflectivity of different TFBG sensors in which the Bragg gratings are imposed with several tilt angles. The numerical method proposed above allows the determination of the reflected or transmitted spectrum of the fundamental forward-backward coupling core mode. Other simulations can be performed by considering changes of the optical fibre parameters, apodization profile, and/or the depth and phase of the core refractive index modulation. With this technique is possible to provide the spectral shape of the Bragg peak of the TFBG sensor before it is manufactured, hence it allows the determination of the optimum parameters for the manufacturing. However, this is not the unique purpose as the same technique can be applied to measure the spectrum when the TFBG is loaded by non-uniform external perturbations. The future work concerns the application of this method to simulate the coupling between the forward propagating core mode and the backward propagating cladding modes.

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