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Full Wavefield Modeling vs Marchenko Redatuming

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Wavefield reconstruction in the presence of a dipping layer: Full wavefield modeling vs Marchenko redatuming

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Summary

Wavefield reconstruction plays a crucial role in various seismic imaging and inversion techniques, including full waveform inversion (FWI), least-squares migration (LSM), and full wavefield migration (FWM). Conventionally, wavefield reconstruction has been achieved using model-based approaches. However, recent advancements in data-driven wavefield reconstruction algorithms have made them viable alternatives to traditional methods. Full wavefield modeling (FWMod) is one such model-based technique. Additionally, fully data-driven methods have emerged as promising alternatives. In this study, we specifically focus on Marchenko redatuming, a data-driven wavefield reconstruction method. To gain a better understanding of these methods, we conducted a comparative analysis of Marchenko redatuming and FWMod for models with horizontal boundaries, where both are expected to perform well, and also in a model where the datum level crosses a dipping boundary. In the later we see deficiencies in the Marchenko results, due to using approximate initial focusing function.

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Introduction

Wavefield reconstruction is at the heart of most seismic imaging and inversion algorithms, such as full waveform inversion (FWI), least-squares migration (LSM), and full wavefield migration (FWM). Traditionally, wavefield reconstruction is done with model-driven algorithms. However, recent developments in data-driven algorithms for wavefield reconstruction made them suitable alternatives to model-driven methods.

One model-driven wavefield reconstruction algorithm is full wavefield modeling (FWMod), as described by Berkhout (2014a). This algorithm is based on Rayleigh integral and reconstructs the wavefield depth by depth. FWMod is capable of modeling higher-order scattered events and multiple reflections. However, it has its shortcomings. Although some advances have been published (Li and Liu, 2021), this algorithm relies on phase shift operators based on a locally invariant model in practical implementations. This will result in an erroneous wavefield in the presence of vertical and lateral variations in the locality of the phase shift operator. On the other hand, the FWMod can be embedded within an inversion process called Joint Migration inversion Berkhout (2014b), where the reflectivity and velocity model are optimized to fit the data. Such an inversion approach brings a data-driven element into the process again.

On the other hand, fully data-driven methods have recently emerged. In this research, our focus is on Marchenko redatuming in particular. The Marchenko method is a promising data-driven wavefield reconstruction algorithm that only needs the reflection response at the acquisition surface and the travel time between focal depth and the acquisition surface (Wapenaar et al., 2014). With this information, this algorithm can reconstruct the wavefield while accounting for all orders of multiple reflections at any depth level. Nevertheless, it is known if the focal boundary passes through a discontinuity in the medium, it causes problems for Marchenko redatuming.

In this paper, we compare and analyze the wavefield of each algorithm. We aim to find a connection between these two reconstruction methods to see how they can help each other overcome their shortcomings. To do this, we compare their wavefields for two different media: 1) a horizontally layered medium and 2) a medium with a dipping layer that passes through the depth levels of FWMod and the focal boundary of the Marchenko method.

Theory

Full wavefield modelling

Full wavefield modeling is based on wavefield extrapolation using the Rayleigh integral, which in the frequency domain in the case of vertical extrapolation reads for a 2D medium (Berkhout, 2014a):

$$p(x',z_n,\omega) = -\int_{\mathscr{D}} \frac{\rho(x',z_n)}{\rho(x,z_m)} \frac{\partial G(x,z_m;x',z_n,\omega)}{\partial z} p(x,z_m,\omega)dx. \tag{1}$$

Here, ρ is the density, $G(x,z_m;x',z_n,\omega)$ is the Green's function between points (x,z_m) and (x',z_n). Equation 1 extrapolates wavefield p from depth level z_m to z_n. We can rewrite it in the form of a matrix-vector multiplication

$$\vec{P}(z_n) = \mathbf{W}(z_n,;z_m)\vec{P}(z_m), \tag{2}$$

where $\mathbf{W}$ is propagator matrix. In the wavenumber-frequency domain each column of $\mathbf{W}$ is

$$\tilde{w} = e^{-jk_z\Delta z}e^{jk_xx_i}, \tag{3}$$

where i is the index of the column. We can include scatterings and reflections in Equation 2 by defining a scattering term as

$$\delta\vec{S}(z_m) = \mathbf{R}^{\cup}(z_m)\vec{P}^{+}(z_m) + \mathbf{R}^{\cap}(z_m)\vec{P}^{-}(z_m), \tag{4}$$

where $\mathbf{R}^{\cup}$ is the reflectivity matrix from above and $\mathbf{R}^{\cap}$ is the reflectivity matrix from below. For computing the upgoing wavefield ($\vec{P}^{-}$) we use ($n = 0, 1, 2, ..., N-1$):

$$\vec{P}^{-}(z_n) = \sum_{m=n+1}^{N} \mathbf{W}(z_n, z_m)\delta\vec{S}(z_m). \tag{5}$$

For the downgoing wavefield ($\vec{P}^{+}$) we can use

$$\vec{P}^{+}(z_n) = \mathbf{W}(z_n, z_0)\vec{S}(z_0) + \sum_{m=0}^{n-1} \mathbf{W}(z_n, z_m)\delta\vec{S}(z_m). \tag{6}$$

Here $\vec{S}(z_0)$ is source vector at z_0. These equations are intertwined, and we need to solve them iteratively.

Marchenko redatuming

Marchenko redatuming is an innovative data-driven technique that can recover the Green's function generated by a source at the surface and recorded by a virtual receiver just above the target area's surface, including all orders of multiple-scattered events.

The following coupled Marchenko-type representations are solved iteratively to retrieve the Green's functions at the redatuming level (Wapenaar et al., 2014):

$$G^{-}_{Mar}(\mathbf{x}_v, \mathbf{x}_r, \omega) = \int_{\mathscr{D}_{acq}} R(\mathbf{x}_r, \mathbf{x}_s, \omega) f_1^{+}(\mathbf{x}_s, \mathbf{x}_v, \omega)\, d\mathbf{x}_s - f_1^{-}(\mathbf{x}_r, \mathbf{x}_v, \omega), \tag{7}$$

and

$$G^{+}_{Mar}(\mathbf{x}_v, \mathbf{x}_r, \omega) = -\int_{\mathscr{D}_{acq}} R(\mathbf{x}_r, \mathbf{x}_s, \omega) f_1^{-}(\mathbf{x}_s, \mathbf{x}_v, \omega)^{*}\, d\mathbf{x}_s + f_1^{+}(\mathbf{x}_r, \mathbf{x}_v, \omega)^{*}. \tag{8}$$

In these equations, $\mathscr{D}_{acq}$ represents the acquisition surface where $\mathbf{x}_s$ and $\mathbf{x}_r$ are situated. G^{-}_{Mar} and G^{+}_{Mar} denote the up-going and down-going components of the Marchenko redatumed Green's function, respectively. Additionally, $f_1^{-}(\mathbf{x}_s, \mathbf{x}_v, \omega)$ and $f_1^{+}(\mathbf{x}_s, \mathbf{x}_v, \omega)$ denote the up-going and down-going parts of the focusing function, respectively, with the subscript "v" denoting a virtual point situated on the redatuming level denoted by $\mathscr{D}_{tar}$. Furthermore, $R(\mathbf{x}_r, \mathbf{x}_s, \omega)$ refers to the dipole response of the medium at the acquisition surface.

Examples

We defined two models to analyze the effects of a dipping reflector passing through a boundary that acts as the focusing boundary for Marchenko redatuming and a depth level for FWMod. The first model consists of two horizontal reflectors and the second one has two dipping ones. Both models have a constant velocity of $1500 m/s$ and variable density (Fig. 1). To compute the FWMod wavefields, we use a $10m$ spatial sampling for both model grids and source/receiver sampling. However, to compute the reflection response and the direct arrival for the Marchenko method, we use a finite difference algorithm with spatial sampling of $2.5m$ for model grids and $10m$ for source/receiver spacing. In both cases, we use a time sampling of $4ms$.

Results

Figure 2 shows shot gathers with upgoing wavefields of the model with horizontal layers with (virtual) receivers at a depth of $150m$ for the middle source, and Figure 3 shows the same for the model with a dipping layer with (virtual) receivers at a depth of $100m$ for three different source locations. For the second model, the reconstruction boundary passes through the first reflector. Figure 3 a and d show the location below the reflector with a source at $-500m$, Figure 3 b and e show the location at the reflector with a source at $0m$, and Figure 3 c and f show the location above the reflector with a source at $500m$. Moreover, Figure 4 compares traces of a few columns of Figure 3.

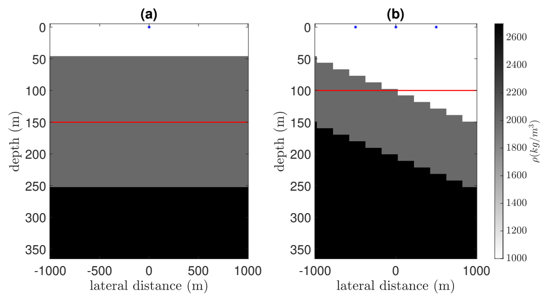

Figure 1: Density models: a) Horizontally layered medium, and b) dipping layered medium. Red lines show the reconstruction boundary. Blue stars show the source locations used for analysis.

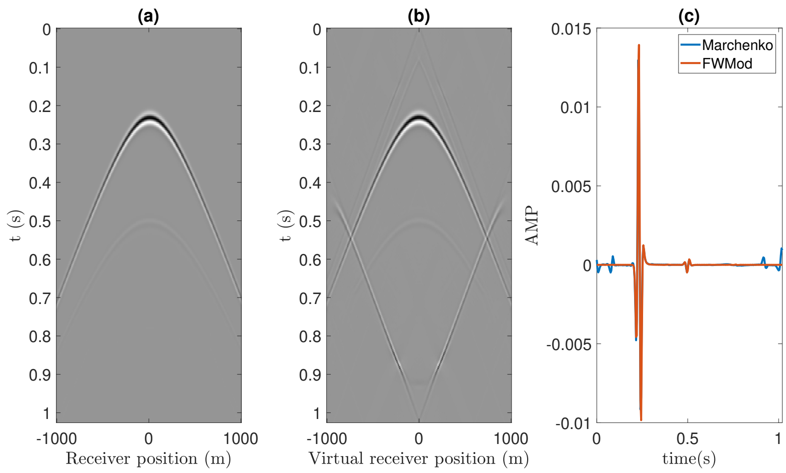

Figure 2: Middle source shot gather with upgoing wavefields for horizontally layered medium with (virtual) receivers at a depth of $150m$. In these panels, the second reflector and one multiple reflection are visible. a) FWMod, and b) Marchenko redatuming c) Comparison of the middle trace ($0m$).

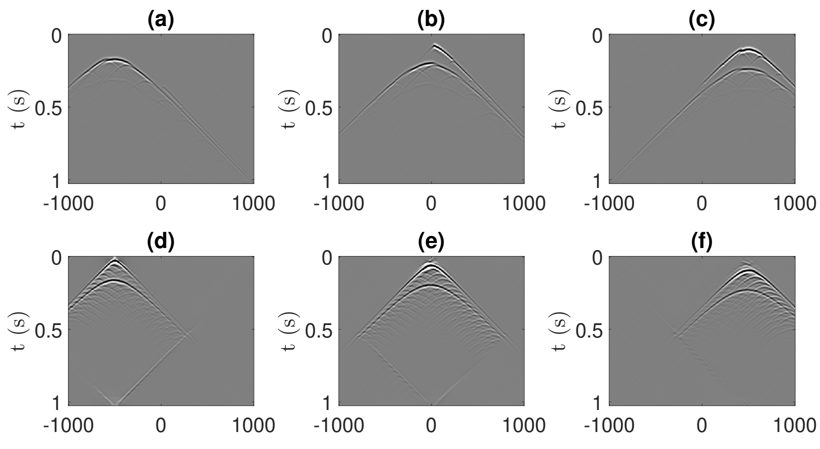

Figure 3: Shot gather with upgoing wavefields for dipping layered medium with (virtual) receivers at a depth of $100m$. a) FWMod with a source at $-500m$, b) FWMod with a source at $0m$, c) FWMod with a source at $500m$, d) Marchenko redatuming with a source at $-500m$, e) Marchenko redatuming with a source at $0m$, and f) Marchenko redatuming with a source at $500m$.

Analyzing the upgoing wavefields for the horizontally layered medium (figure 2) shows that reconstructed wavefields by Marchenko redatuming and FWMod methods are similar despite some aperture effects. We can observe the reflected event from the lower boundary and the first-order internal multiple.

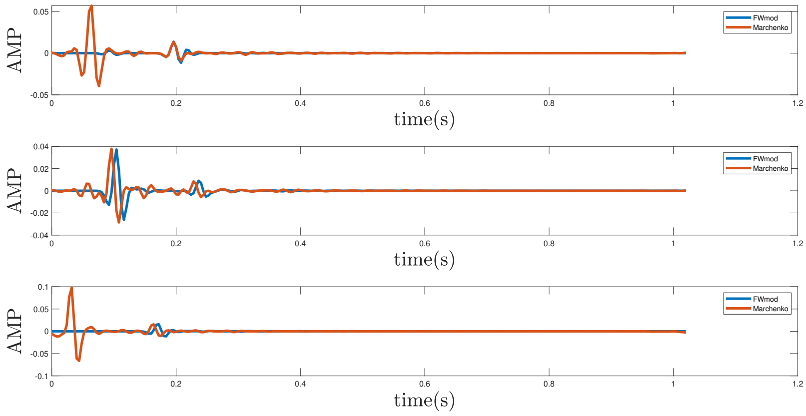

Figure 4: Middle traces from Figure 3. a) FWMod and Marchenko with a source at 0m, b) FWMod and Marchenko with a source at 500m, c) FWMod and Marchenko with a source at $-500m$.

However, for the dipping medium, we observe a large difference. In this case, the FWMod (figures 3 a, b, and c) reconstructs the upgoing wavefield according to the model. Since part of the recording surface is above the dipping layer, it is logical to see part of the reflected event from the first reflector in Figures 2a, b, and c. Meanwhile, Marchenko redatuming considers the first layer a boundary that goes completely below the focusing depth. This error is due to incorrect transmission effects incorporated into the initial downgoing wavefield for the Marchenko redatuming. Since we can not create a smooth dipping layer with our current model-building method, we also see the effects of the staircases of the model in the wavefields. Due to different model grid sizes, these diffraction patterns differ between Marchenko and FWMod wavefields. In addition, the difference in the model grid sizes shows itself as a time shift in the traces recorded above (figure 4b) and above (figure 4c) the dipping reflector. Thus, the observed differences can all be understood, but the main issue is the incorrect first event in the Marchenko method.

Conclusions

Although it is hardly reported, Marchenko redatuming's problem with focusing boundaries that pass through a reflector is a well-known one. In this paper, we compared and analyzed the results of Marchenko redatuming and FWMod to better understand this problem. As the next step, we want to use FWMod to help Marchenko redatuming overcoming this problem.

Acknowledgements

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