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Solving the Steady-State Power Flow Problem on Integrated Transmission-Distribution Networks: A Comparison of Numerical Methods

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***Abstract*—Steady-state power flow models are essential for daily operation of the electricity grid. The changing electrical environment requires a shift from separated power flow models to integrated transmission-distribution power flow models. Integrated models incorporate the coupling of the networks and the interaction that they have on each other, representing the power flow within this changing environment accurately. In this paper we conduct a comparison study on the numerical performance of methods that solve the integrated power flow problem. The methods of study can be divided into unified or splitting methods. In addition, the integrated networks can be modeled as homogeneous or as hybrid networks. Our study shows that the methods have several advantages and disadvantages, but that unified methods in combination with hybrid network models have the best numerical performance. Splitting methods running on hybrid network models have an advantage when full network data sharing between system operators is not allowed.**

***Index Terms*—power flow, numerical analysis, integrated systems, steady-state, transmission-distribution**

LIST OF ABBREVIATIONS (METHODS ONLY)

NR Newton-Raphson
- -P Power mismatch formulation
- -TCIM Three-phase current injection method

F3P Full three-phase method
IC Interconnected method
MSS Master-slave splitting method
- -homo (applied to) Homogeneous networks
- -hybrid (applied to) Hybrid networks
- -CAI Convergence alternating iterative (scheme)
- -MAI Multistep alternating iterative (scheme)

I. INTRODUCTION

The electricity grid is facing challenges due to the ongoing electrification and rise of renewable energy resources. The changing environment requires a more detailed analysis of the electricity network and of the interaction between transmission and distribution networks. Integrated transmission-distribution network models can be used to study this interaction.

It is not straightforward to integrate these separate domains. The distinct properties of transmission systems versus distribution systems resulted in the development of different system models. An important feature of transmission models is the assumption that the system is balanced, resulting in the simplification to a single-phase system. Distribution systems contain unbalanced loading and unbalanced operating conditions which require them to be modeled in three-phase.

Integrating both systems poses challenges for the solver and the connection method. Several methods have been proposed to solve the integrated system which can be divided into unified and splitting methods. In the unified approach, both systems are connected via a transformer and solved as one system. One way to solve unified systems is by modeling the transmission system in three-phase, which is called the full three-phase (F3P) approach. The other unified method, presented in [1], is the interconnected (IC) approach which respects the single-phase and three-phase notion of the transmission and distribution system respectively. The authors of [2] introduce the master-slave splitting method. This is an iterative approach in which the slave (distribution system) is solved, its solution is injected into the master (transmission system) and vice versa, until convergence is reached. Another way of simulating power flow in integrated networks is by using co-simulation techniques, such as the HELICS framework [3]. As we are interested in standalone simulations, we do not take co-simulation into account in our comparison.

In this paper we review the unified and splitting methods with a focus on their numerical performance, because large integrated network models need fast and robust solvers. We apply the methods on various integrated balanced-unbalanced test cases on which we solve the power flow problem. We compare the output of the proposed solution methods on convergence, CPU-time, and their sensitivity to the amount of PV-buses.

II. THE POWER FLOW PROBLEM

An electricity network model is represented as a graph consisting of buses $i = 1, ..., N$, representing generators, loads, and shunts and branches $e = 1, ..., M$, representing transformers and cables [4]. The steady-state power flow problem of a network is formulated as the determination of the

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voltages V_i at each bus such that its product with the complex conjugate of current I_i correspond with the specified complex power S_i. Voltages and currents are linked by the admittance Y, through Ohm's Law: $I = \mathbf{Y}V$. Power is generated in three phases: a, b, and c. Transmission systems are balanced systems which means that the three phases are equal in magnitude and phase-shift. Therefore, transmission systems are modeled only using phase a. Distribution systems need to be modeled in all three phases. The representation of Ohm's Law for all the buses $i = 1, .., N$ in a network, is the following:

$$I = \mathbf{Y}V \quad \leftrightarrow \quad \begin{bmatrix} I_1^p \\ \vdots \\ I_N^p \end{bmatrix} = \begin{bmatrix} Y_{11}^p & \dots & Y_{1N}^p \\ \vdots & \ddots & \vdots \\ Y_{N1}^p & \dots & Y_{NN}^p \end{bmatrix} \begin{bmatrix} V_1^p \\ \vdots \\ V_N^p \end{bmatrix}, \quad (1)$$

The p represents the phase(s). In a transmission system, V_i^p,I_i^p, and Y_{ij}^p are as follows:

$$I_i^p = [I_i^a],\ V_i^p = [V_i^a],\ Y_{ij}^a = [Y_{ij}^a], \quad (2)$$

and in a distribution systems they are represented as:

$$I_i^p = \begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}_i,\ V_i^p = \begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix}_i, Y_{ij}^p = \begin{bmatrix} Y^{aa} & Y^{ab} & Y^{ac} \\ Y^{ba} & Y^{bb} & Y^{bc} \\ Y^{ca} & Y^{cb} & Y^{cc} \end{bmatrix}_{ij}. \quad (3)$$

The single-phase and three-phase relations between power S and voltage V of a bus i are respectively:

$$S_i^a = V_i^a \overline{I_i^a} = V_i^a \sum_{k=1}^N \overline{Y_{ik}^a V_k^a} \quad (4)$$

and:

$$S_i^p = V_i^p \overline{I_i^p} = V_i^p \sum_{k=1}^N \sum_{q=a,b,c} \overline{Y_{ik}^{pq} V_k^q}, \quad p \in \{a, b, c\}. \quad (5)$$

A. Newton-Raphson solution method

We solve the transmission and distribution systems using Newton-Raphson. Traditionally all networks were solved by the Newton-Raphson power mismatch formulation (NR-P) in which the power mismatch formulation $\Delta S_i = (S_{g,i} - S_{d,i}) - S(V_i) \approx 0$ is used to compute V_i [4]. The s stands for the specified power of the generator (g) and load (d) buses. The complex power S is split into an active (P) and reactive (Q) part and combined to form the power mismatch vector F:

$$F(x) = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} P_s - P(x) \\ Q_s - Q(x) \end{bmatrix}. \quad (6)$$

Using Newton-Raphson, $F(x)$ should converge to zero in the L_∞-norm. The x represents the state variables: $x_i = \begin{bmatrix} \delta_i & |V_i| \end{bmatrix}^T$ which form the voltage in the phasor notation $V_i = |V| \exp(\iota\delta)_i$ to compute $P_i(x_i) = Re(S(V_i))_i$ and $Q_i(x_i) = Im(S(V_i))$. We compute the state variables in an iterative process, where ν is the iteration counter:

$$\Delta x^\nu = -\mathbf{J^{-1}}(x^\nu) F(x^\nu), \quad (7)$$

$$x^{\nu+1} = x^\nu + \Delta x^\nu, \quad (8)$$

using the Jacobian $\mathbf{J}(x) = \begin{bmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial |V|} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial |V|} \end{bmatrix}$, until the norm of the power mismatch vector $|F|_\infty$ is lower than a certain tolerance value ε. We start with a flat profile as initial guess. Transmission systems are solved with NR-P. We use a version of Newton-Raphson using current mismatches to solve unbalanced distribution systems.

B. Newton-Raphson with current mismatches

For distribution systems it has been proved that NR with current mismatches has better convergence properties than with power mismatches [5]. Not only because of the unbalanced system, but also the high R/X ratios of distribution lines, the lower voltage level and the radial structure lead to different solution techniques [6]. This NR modification is called the Three-phase Current Injection Method (TCIM) [7]. Instead of applying the NR method to power mismatches, Ohm's Law is directly used, resulting in the current mismatch vector:

$$F(fx) = \begin{bmatrix} \Delta I^{Re,abc}(x) \\ \Delta I^{Im,abc}(x) \end{bmatrix} = \begin{bmatrix} I_s^{Re,abc} - I^{Re,abc}(x) \\ I_s^{Im,abc} - I^{Im,abc}(x) \end{bmatrix}. \quad (9)$$

The specified current I_s and computed current $I(x)$ are calculated using the injected complex power and Ohm's Law:

$$I_{s,i} = \left(\frac{\overline{S}}{V}\right)_i \quad \text{and} \quad I(x)_i = \mathbf{Y}V_i \quad (10)$$

The Jacobian is formed by taking the derivative of the real and imaginary current mismatch with respect to the real and imaginary voltage. We use the same tolerance value and initial guess as in NR-P.

III. Review of Integration Methods

We connect a transmission network with a distribution network to form an integrated network. There are two types of integrated networks: a homogeneous network and a hybrid network. A homogeneous network consists of two integrated three-phase networks. A hybrid network consists of a single-phase transmission and a three-phase distribution network. We have selected several approaches from the literature to solve integrated electricity networks, which we divide into unified and splitting methods. Unified methods solve the integrated network as a whole using a transformer in between. The first unified method is called the full three-phase approach, which solves a homogeneous integrated network. The other unified approach is the interconnected method which solves an integrated hybrid network [1]. The splitting methods are called master-slave splitting (MSS) in the literature [2]. They can also be applied on homogeneous (MSS-homo) and hybrid (MSS-hybrid) networks. They solve homogeneous and hybrid networks by keeping these networks separated and using an iterative scheme on the boundary of the two domains, where convergence is based on the mismatch of voltage on this boundary [2]. Figure 1 gives an overview of the methods.

Any load bus of a transmission network can be the coupling bus of the integrated network. This bus is connected to the reference bus of the distribution system. In the unified methods, a transformer is placed between the two buses to connect the systems. The original transmission load bus changes to a zero-load bus and the original distribution reference bus changes to a load bus having the former load of the transmission system.

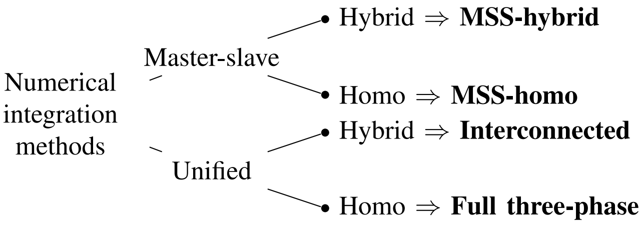

Fig. 1. Division of numerical methods to solve integrated systems.

A. The full three-phase approach

The first unified method that we explain is the F3P method. With this method, we model the transmission system as a three-phase system. Because we still assume that the system is balanced, we know that the load and generation are equal in all three phases:

$$\begin{bmatrix}S^a & S^b & S^c\end{bmatrix}_i^T = [1\ 1\ 1]^T \begin{bmatrix}S^a\end{bmatrix}_i \tag{11}$$

$$\begin{bmatrix}V^a & V^b & V^c\end{bmatrix}_i^T = [1\ a^2\ a]^T \begin{bmatrix}V^a\end{bmatrix}_i, \quad a = e^{\frac{2}{3}\pi\iota}, \tag{12}$$

$$Y_{ij}^{abc} = \begin{bmatrix} Y_{aa} & 0 & 0 \\ 0 & Y_{aa} & 0 \\ 0 & 0 & Y_{aa} \end{bmatrix}_{ij}. \tag{13}$$

The three-phase transmission system can now be connected to the distribution system, using the line method: The line method places a transformer between coupling bus k of the transmission system and coupling bus m of the distribution system, using impedance z_{mp} of the first distribution line between reference bus m and bus p.

B. The interconnected method

The interconnected (IC) method solves the system as a whole using a connecting transformer that connects the single-phase system to a three-phase system [1] using the line method via coupling buses k and m. The connecting transformer consists of admittance Y_{km}. We assume that bus k is perfectly balanced; only phase a needs to be represented in the single-phase side of the Y_{bus} matrix. We start with a three-phase relation between voltage and current using the Y_{bus} matrix, and transform these using the following transformation matrices:

$$T_1 = [1\ a^2\ a]^T,\ T_2 = \beta[1\ a\ a^2],\ a = e^{\frac{2}{3}\pi\iota},\ \beta = \frac{1}{3} \tag{14}$$

to define the following three-phase/single-phase relationships of the balanced bus k:

$$V_k^{abc} = T_1 V_k^a, \tag{15}$$

$$I_a^a = T_2 I_k^{abc}. \tag{16}$$

We apply (15) and (16) on bus k and m in (1):

$$I_k^a = T_2 I_k^{abc} = T_2 \mathbf{Y}_{kk}^{abc} T_1 V_k^a + T_2 \mathbf{Y}_{km}^{abc} V_m^{abc}, \tag{17}$$

$$I_m^{abc} = \mathbf{Y}_{mk}^{abc} T_1 V_k^a + \mathbf{Y}_{mm}^{abc} V_m^{abc}. \tag{18}$$

From (17) and (18) we see that our new nodal admittance matrix becomes:

$$\mathbf{Y}_{km} = \begin{matrix} \\ 1 \\ 3 \end{matrix} \begin{matrix} \quad 1 \qquad\qquad 3 \\ \begin{bmatrix} T_2[\mathbf{Y}_{kk}^{abc}]T_1 & T_2[\mathbf{Y}_{km}^{abc}] \\ [\mathbf{Y}_{mk}^{abc}]T_1 & \mathbf{Y}_{mm}^{abc} \end{bmatrix} \end{matrix} \tag{19}$$

C. The master-slave splitting

The master-slave splitting separates the transmission (the master) and distribution (the slave) systems and is based on the idea that the connecting bus of the transmission system is the same as the connecting bus of the distribution system. This bus is called the boundary bus B. As with the unified methods, this bus can be any load bus of the transmission system and is the reference bus of the distribution system. In short, during one master-slave iteration the slave is solved, the power S_B at bus B is injected into the master, the master is solved, and the voltage V_B on bus B is injected into the slave. This process is repeated until convergence on the boundary is reached:

$$|V_B^{\nu+1} - V_B^{\nu}|_1 \leq \varepsilon \tag{20}$$

Algorithm III.1 shows how the iterative scheme of the MSS-method works. As the MSS-method solves the master and slave separately, it allows for using different algorithms per domain. We solve the slave with the advantageous NR-TCIM method and the master with the NR-P method, explained in sections II-A and II-B. The splitting methods have an advantage when it is not allowed to share full network information with other system operators. In these methods, only information of the boundary bus needs to be shared.

Algorithm III.1 General algorithmic approach of the Master-Slave splitting method

1: Set iteration counter $\nu = 0$. Initialize the voltage V_B^0 of the Slave.
2: Solve the slave system. Output: $S_B^{\nu+1}$.
3: Inject $S_B^{\nu+1}$ into the Master.
4: Solve the Master. Output: $V_B^{\nu+1}$.
5: Is $|V_B^{\nu+1} - V_B^{\nu}|_1 > \varepsilon$? Repeat step 2 till 5.

The MSS-method can be applied to homogeneous networks and to hybrid networks. The first one requiring a transformation of the entire master domain, the latter requiring a transformation of the boundary bus only.

1) The MSS-homogeneous method: The MSS-method applied to homogeneous networks requires a transformation of the single-phase transmission system to a three-phase system. The balanced transmission system is transformed in the same way as in the F3P-method. The voltage, power, and admittance of all the buses $i = 1, .., N$ are transformed to three-phase equivalents. This idea is summarized in equations (11) - (13).

2) The MSS-hybrid method: The MSS-method applied to hybrid systems keeps the transmission system in single-phase. Only a transformation of the boundary bus is then required.

As we first solve the slave, we receive the complex power S_B as three-phase output, which we have to transform to a single-phase quantity. Once we have solved the master, we have to transform the single-phase output of the voltage V_B of the master. The assumption that the boundary bus is balanced is required to form the relation between the single- and three-phase system. The balanced relation between three-phase and single-phase power and voltage is the following:

$$[S^a]_B = \beta[1\ 1\ 1]\begin{bmatrix}S^a & S^b & S^c\end{bmatrix}_B^T,\ \beta = \begin{cases}\frac{1}{3}, \text{ p.u. values}\\ 1, \text{ actual values}\end{cases} \quad (21)$$

$$\begin{bmatrix}V^a & V^b & V^c\end{bmatrix}_B^T = [1\ a^2\ a]^T\begin{bmatrix}V^a\end{bmatrix}_B, \quad a = e^{\frac{2}{3}\pi\iota}. \quad (22)$$

We transform the three-phase power output of the boundary bus $[S^{abc}]_B$ to $[S^a]_B$ using equation (22) and the single-phase voltage output $[V^a]_B$ to $[V^{abc}]_B$ using equation (21).
The MSS-methods do not require a transformation of the nodal admittance matrix. We transform the necessary boundary parameters directly. At every MSS-iteration, we make transformation (22) and (21) after step 2 and step 4 of algorithm III.1, respectively.

a) Two master-slave iterative schemes: The authors of [2] propose two iterative schemes to solve the integrated system, one of them is the Convergence Alternating Iterative (CAI) scheme in which the convergence tolerance of each subsystem is controlled. The other is the Multistep Alternating Iterative (MAI) scheme at which the number of sub-iterations is controlled. We try several number of sub-iterations to check the most optimal number of sub-iterations per test case. We compare both CAI and MAI-schemes on the MSS-methods.

IV. Results

We solve the integrated networks using the Matpower library[1] in which we create several integrated test cases. The focus of this paper is on the numerical performance of the methods: we are interested in the speed of the methods and its sensitivity to the amount of distributed generation. Therefore, we compare the output on CPU-time and number of iterations. Furthermore, we check the influence of the number of PV-buses in the distribution network on the number of iterations.

A. Test cases

We create integrated test networks from existing separate test cases. We use two small-size transmission and distribution test cases. We use one larger transmission test case and we created a larger distribution test case from the data of one small-size distribution test case. The larger transmission test-network should give us better insight into the difference of homogeneous and hybrid network models. We use the 9-bus, 118-bus, and 3120-bus Matpower balanced transmission test cases and 13-bus and 37-bus IEEE distribution test cases [9]. We create a larger distribution test case (245-buses) from the data of the 37-bus test case. The 13-bus test case is an unbalanced test case. We modify it to a 10-bus network by omitting the buses that are connected to the network via a regulator. We change the loading of the 37-bus network by shifting 20% of the loads of phase b equally to phase a and c, like the original authors [1] to create an unbalanced network. All the three-phase loads are connected in a grounded Wye-configuration. We also create three test cases that consist of multiple distribution networks (3, 5, and 10 respectively) connected to one transmission network. This results into the following test networks: T9-D13, T118-D37, T3120-D37, T9-3D13, T118-5D37, and T3120-10D245.
For the unified methods, we use NR-TCIM with $\varepsilon = 10^{-8}$ as convergence condition to solve the integrated problem. In the splitting methods, we solve the distribution system with NR-TCIM and set $\varepsilon_D = 10^{-8}$, the transmission system using NR-P and $\varepsilon_T = 10^{-8}$, and we define the tolerance value of the MSS-method also as $\varepsilon_{MSS} = 10^{-8}$.

B. Comparison of the integration methods

We model the integrated systems using the Matpower [8] library according to the theory described in sections II and III. We are interested in the numerical performance of the different integration methods. Therefore, we compare integration the methods on CPU-time and number of iterations. The results are described in Tab. I.

TABLE I
COMPARISON ON NUMBER OF ITERATIONS (FOR THE MSS-METHOD: BOTH THE NUMBER OF ITERATIONS OF THE METHOD ITSELF (I_{MSS}) AND THE NUMBER OF ITERATIONS PER SUB DOMAIN (I_T AND I_D)), AND CPU-TIME OF THE INTEGRATION METHODS, APPLIED ON FIVE TEST CASES. THE BOLD NUMBERS ARE THE LOWEST CPU-TIME.

	F3P		*MSS-homo-CAI*				*MSS-homo-MAI*			
	its	*CPU*	I_{MSS}	I_T	I_D	*CPU*	I_{MSS}	I_T	I_D	*CPU*
test case	#	*sec*	#	#	#	*sec*	#	#	#	*sec*
T9-D13 (7)	4	**0.105**	4	4	4	0.299	6	2	2	0.283
T118-D37 (118)	5	0.125	3	6	4	0.343	3	4	3	0.303
T3120-D37 (3003)	5	0.707	3	6	4	2.23	3	4	4	1.94
T9-3D13 (7-9)	5	**0.118**	5	4	4	0.418	5	3	3	0.421
T118-5D37 (114-118)	5	0.144	3	6	4	0.580	3	4	4	0.576
T3120-10D245 (3003-3012)	5	1.59	7	6	4	5.99	7	4	4	5.09
	IC		*MSS-hybrid-CAI*				*MSS-hybrid-MAI*			
	its	*CPU*	I_{MSS}	I_T	I_D	*CPU*	I_{MSS}	I_T	I_D	*CPU*
test case	#	*sec*	#	#	#	*sec*	#	#	#	*sec*
T9-D13 (7)	4	0.110	4	4	4	0.295	6	2	2	0.307
T118-D37 (118)	4	**0.114**	3	6	3	0.329	3	4	3	0.284
T3120-D37 (3003)	5	**0.203**	3	6	4	0.620	6	2	2	0.492
T9-3D13 (7-9)	5	0.134	4	4	4	0.412	4	3	3	0.408
T118-5D37 (114-118)	4	**0.135**	3	6	4	0.557	3	4	4	0.549
T3120-10D245 (3003-3012)	5	**0.484**	6	6	4	3.16	6	4	4	2.76

This table shows us that the unified methods are faster than the splitting methods and that hybrid network models produce faster results than homogeneous networks. This is inline with the expectations. Splitting methods iterate between the two networks until convergence on the boundary has been reached. Therefore, it needs to solve the separate transmission and distribution multiple times which will increase the total CPU-time of the MSS-method. Homogeneous network contain a three-phase representation of the transmission network and thus needs to process a larger Jacobian matrix: If we consider a transmission system with N buses, then the Jacobian matrix increases to a size of $6N$ x $6N$ instead of $2N$ x $2N$ at transmission side. The larger test case shows that this difference become significant, especially when the number of

[1]Matpower is a package of free, open-source Matlab-language M-files for solving steady-state power system simulation and optimization problems [8]

transmission buses is much larger than the number of distribution buses. This table also shows that multiple distribution networks do not influence the amount of iterations of all the methods. The CPU-time of the MAI and CAI schemes are very similar. Although the MAI-schemes are in general faster than the CAI-schemes, they have a disadvantage because the required number of sub-iterations have to be pre-determined to make sure the system converges.

C. *Effect of amount of PV-buses*

The distribution domains of the integrated networks contained only one PV-bus. We are interested in whether the number of PV buses increases the number of iterations. We perform this research on the T9-D13 test case and T118-D37 test case. We compare the increase in iterations of the integrated systems with the separated D13 and D37 networks. The results are summarized in Tab. II.

TABLE II
THE NUMBER OF ITERATIONS TO SOLVE THE INTEGRATED SYSTEM WHEN THE AMOUNT OF PV BUSES IS INCREASED. THE OUTER LEFT COLUMN REPRESENTS THE NUMBER OF PV BUSES IN THE NETWORK. THE MSS METHODS ONLY STATE THE NUMBER OF MSS ITERATIONS. THE - MEANS THAT THE METHOD DID NOT CONVERGE. THE $*$ MEANS THAT THE PV BUSES HAD AN INFLUENCE ON THE TRANSMISSION NETWORK.

	D-net		*F3P*		*MSS-homo-CAI*		*MSS-homo-MAI*	
PV-buses	D13	D37	T9-D13	T118-D37	T9-D13	T118-D37	T9-D13	T118-D37
1	2	2	4*	5	12*	23	12*	23
2	2	4	4*	5	15*	-	15*	-
3	2	4	4*	-	-	-	-	-
	D-net		*IC*		*MSS-hyb-CAI*		*MSS-hyb-MAI*	
1	2	2	4*	4	12*	23	12*	23
2	2	4	4*	5	15*	-	15*	-
3	2	4	4*	-	-	-	-	-

We see that the unified methods are less sensitive to the increase of distributed generation. This can be due to the representation of the coupling bus as a balanced reference bus in the splitting methods, which may not be accurate. We see in the unified methods that adding one or two PV-buses have no effect on the total number of iterations, but that the unified methods do not converge when three or more PV-buses are added.

V. CONCLUSION

We performed a comparison study on different methods to solve integrated networks. In this study, we were most interested in the numerical performance of the methods, because the changing electrical environment requires more detailed network models and a model of the interaction between the networks. Larger, integrated networks require fast robust power flow methods to react quickly on events happening on the grids. We divided the available integration methods into unified and splitting methods, and the modeling of the integrated networks into homogeneous and hybrid networks. The splitting methods were further divided into convergence schemes: the CAI- and MAI-scheme. In total, six methods were part of our comparison study. We compared these methods on CPU-time, number of iterations, and sensitivity to the amount of distributed generation.

Based on this comparison study, the interconnected method scores the best on these three objectives. Although the full three-phase method had better performance in the small-size test cases, the significant increase in CPU-time of large networks makes this method less advantageous. The splitting methods are slower than the unified methods, but are still of interest when system operators are not allowed to share full network information. The MSS-hybrid-CAI method is then most favorable: Hybrid methods are faster than homogeneous methods and we do not have to predefine the number of inner sub iterations, as we have to in the MAI methods.

We can conclude that the IC-method is most favorable when full network information can be shared and the MSS-hybrid-CAI method otherwise. In future work, we continue with these methods where we will extend the analysis to real test cases, including networks up to millions of buses. To solve these very large systems in reasonable amount of time, we have to adapt the methods, using Newton-Krylov methods and preconditioning techniques [10], for a parallel or GPU environment. Then, the MSS-hybrid-CAI method gets the advantage when multiple distribution networks are connected to one transmission network: These separate distribution networks can be solved in parallel. In coming works we will discuss the necessary adaptations and review these two methods once again based on the obtained results.

REFERENCES

[1] G. N. Taranto and J. M. Marinho, "A Hybrid Three-Phase Single-Phase Power Flow Formulation," *IEEE Transactions on Power Systems*, vol. 23, no. 3, pp. 1063–1070, 2008.

[2] H. B. Sun and B. M. Zhang, "Global state estimation for whole transmission and distribution networks," *Electric Power Systems Research*, vol. 74, pp. 187–195, 2005.

[3] B. Palmintier, D. Krishnamurthy, P. Top, S. Smith, J. Daily, and J. Fuller, "Design of the helics high-performance transmission-distribution-communication-market co-simulation framework," in *2017 Workshop on Modeling and Simulation of Cyber-Physical Energy Systems (MSCPES)*, pp. 1–6, April 2017.

[4] P. Schavemaker and L. van der Sluis, "Energy Management Systems," in *Electrical Power System Essentials*, ch. 6, Sussex, United Kingdom: John Wiley & Sons, Inc., 2008.

[5] B. Sereeter, K. Vuik, and C. Witteveen, "Newton power flow methods for unbalanced three-phase distribution networks," *Energies*, vol. 10, no. 10, p. 1658, 2017.

[6] J. A. Martinez and J. Mahseredjian, "Load Flow Calculations in Distribution Systems with Distributed Resources. A Review," *2011 IEEE Power and Energy Society General Meeting*, pp. 1–8, 2011.

[7] P. A. N. Garcia, J. L. R. Pereira, S. Carneiro, and V. M. Da Costa, "Three-phase power flow calculations using the current injection method," *IEEE Transactions on Power Systems*, vol. 15, no. 2, pp. 508–514, 2000.

[8] R. D. Zimmerman, C. E. Murillo-Sánchez, and R. J. Thomas, "MATPOWER: Steady-state operations, planning, and analysis tools for power systems research and education," *IEEE Transactions on Power Systems*, vol. 26, no. 1, pp. 12–19, 2011.

[9] K. P. Schneider, B. A. Mather, B. C. Pal, C. W. Ten, G. J. Shirek, H. Zhu, J. C. Fuller, J. L. Pereira, L. F. Ochoa, L. R. De Araujo, R. C. Dugan, S. Matthias, S. Paudyal, T. E. McDermott, and W. Kersting, "Analytic Considerations and Design Basis for the IEEE Distribution Test Feeders," *IEEE Transactions on Power Systems*, vol. 33, no. 3, pp. 3181–3188, 2018.

[10] R. Idema, G. Papaefthymiou, D. Lahaye, C. Vuik, and L. van der Sluis, "Towards faster solution of large power flow problems," *IEEE Transactions on Power Systems*, vol. 28, pp. 4918–4925, Nov 2013.