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# Improving convergence of quasi dynamic assignment models

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## 1 Introduction

For decades congestion levels around the world are rising. To properly incorporate the effects of congestion into strategic transport models, a shift from static capacity restrained towards capacity constrained and dynamic traffic assignment models has occurred. In this paper we focus on quasi dynamic assignment models (more specific: static-capacity and storage constrained models by the definitions in Bliemer et al (2015)). These models explicitly capture the flow metering and spillback effects of congestion, but assume stationary demand during a single time period (e.g. a whole peak hour) and are therefore more scalable and mathematically tractable, both important properties for strategic transport models.

Although computational capabilities of current hardware allow for large scale application of such models, the incorporation of capacity constraints causes route cost functions to be much more sensitive and to be inseparable over space (the latter occurs when routes share bottleneck nodes). Furthermore, the incorporation of storage constraints further increases inseparability (which occurs when queues spill back onto upstream links) and causes cost functions to become implicit. As such quasi dynamic models do not fully contain the favorable mathematical properties that are exploited in many algorithms to solve their capacity restrained counterparts and in fact do not necessarily comply with the requirements for existence and/or uniqueness of the user equilibrium (theorems 1.4 and 1.8 in Nagurney (1993)).

Although in reality these unfavorable properties exist, a substantial body of research suggests that their (spatial) occurrence is limited and as such “...have minimal practical temporal and spatial consequences...” (Peeta and Ziliaskopoulos (2001)). However, several large scale applications using the quasi dynamic assignment model STAQ (first described in Brederode et al (2010)) have shown that especially the addition of storage constraints causes poor or non-convergence in real world applications. Further investigations in this paper will show that also the capacity constraints on their own can cause serious convergence issues.

Contributions in this paper are (i) to give an overview of methods in literature and logical extensions to those methods that could improve convergence of quasi dynamic assignment models, (ii) to reveal and illustrate mechanisms that cause the convergence issues using examples on theoretical networks and (iii) to investigate to what extent enhancements to existing algorithms can be used to (partly) get around the convergence issues encountered. Ultimately, this research should lead to a method that generically solves quasi dynamic assignment models.

## 2 Methodology

The scope of the research is narrowed down by assuming that (i) a path based model is used and that (ii) travelers have perception errors on route travel times, leading to the stochastic user equilibrium (SUE). We choose to use the multinomial logit (MNL) model to calculate route choice probabilities, such that route demand  $f_p$  is defined by:

$$f_p = \exp(-\mu_{od}c_p) / \sum_{p' \in P_{od}} \exp(-\mu_{od}c_{p'}) D_{od}, \quad (1)$$

where  $c_p$  is the route cost on route  $p$ ,  $\mu_{od}$  is the scale parameter describing the degree of travelers' perception errors on route travel times (where perfect knowledge is assumed when  $\mu_{od}$  approaches infinity) and  $D_{od}$  is the travel demand for OD pair  $od$ . Here (and in most real world applications)  $\mu_{od}$

a global scale parameter  $\mu$  is normalized over ODpairs by  $\mu_{od} = \mu / \min_{p \in P_{od}} c_p^0$ , where  $c_p^0$  is the free flow cost on route  $p$ . This normalization ensures that the relative effect of perception errors is the same on all ODpairs (regardless of their average route travel time). As measure of convergence we use the gap function derived in Bliemer et al (2013) that will reach zero upon convergence when using MNL:

$$G = \frac{\sum_{(o,d)} \sum_{p \in P_{od}} f_p (c_p + \mu_{od}^{-1} \ln f_p - \psi_{od})}{\sum_{(o,d)} D_{od} \psi_{od}}, \quad (2)$$

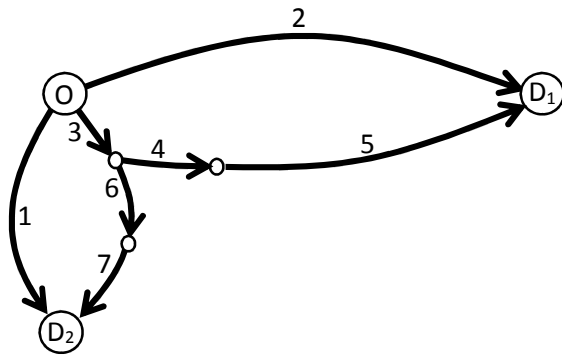
where  $\psi_{od} = \min_{p \in P_{od}} [c_p + \mu_{od}^{-1} \ln f_p]$  represents the minimum stochastic path cost. Note that by omitting the summation over OD pairs in both numerator and denominator, the gap value for a single OD can be obtained, useful when investigating which ODpairs cause convergence issues.

In order to enforce and speed up convergence, stochastic route based traffic assignment models typically average route demands over iterations  $i$  using some ‘averaging scheme’ that applies iteration specific step sizes  $\alpha_i$ . To ensure that the averaging scheme itself does not cause divergence, the conditions  $\sum \alpha_i = \infty$  and  $\sum \alpha_i^2 < \infty$  must hold (Blum (1954)) meaning that the step sizes must decrease in every iteration. The simplest averaging scheme tested is the method of successive averages (MSA) that uses  $\alpha_i = 1/i$ . A well-known problem of MSA is that convergence tends to slow down as the number of iterations increases. Tested variations on MSA that give more emphasis on later iterations using pre-determined step sizes include raising  $i$  to some power  $\lambda < 1$ , to reset  $i$  every other  $n$  iterations while maintaining current route costs (MSA-reset), to use constant step sizes or to use  $\alpha_i = i^d / \sum_{1..i} i^d$  (MwSA, Liu et al (2007)) where  $d$  is a constant. More intelligent averaging schemes tested use information of previous iteration(s) to determine  $\alpha_i$  are the Self Regulating Average (SRA, Liu et al (2007)) and SRA with dynamic step size (Taale and Pel (2015)).

## 2.1 Test Cases

For all test cases (an updated version of) the quasi dynamic assignment model STAQ (Brederode et al (2010)) was used. STAQ uses a static model with capacity constraints to determine flow metering effects and combines this with the dynamic link transmission model (LTM) to determine average spillback effects. Both submodels use the node model described in Tampere et al (2011). Its low computational cost allows for quickly running a lot of convergence schemes. Because the model basically is a static version of the LTM, results described in the remainder of this paper are likely to be also applicable on fully dynamic path based assignment models with stochastic route choice.

To investigate the mechanisms that cause convergence issues a generic test network displayed in Figure 1 is constructed. In this network the effects on convergence of adding capacity and storage constraints can be isolated because the two aspects of interest (sensitivity and extent of inseparability of the implicit cost functions, see section 1) can be controlled by adjusting link lengths. Assuming a demand of 2000 veh on both OD pairs and free flow link speeds and capacities defined on the right side of Figure 1, the upstream nodes of links 5 and 7 form potential bottleneck locations. For both ODpairs two routes exists: a constrained route using a potential bottleneck node and an unconstrained (direct) route.



Link #	Length (l) [km]	Free speed (v) [km/h]	Capacity (C) [veh/h]
1	5	100	2000
2	10	100	2000
3	$l_3$	120	4000
4	$l_4$	120	2000
5	$10-l_3-l_4$	120	1000
6	$l_6$	120	2000
7	$5-l_3-l_6$	120	1500

Figure 1: generic network geometry (left) and link attributes (right)

The lengths of links 3, 4 and 6 are variable, while the definitions of link lengths 5 and 7 ensure that in all specific networks derived from this generic network, the constrained routes have lower free flow travel time than the constrained route, meaning that in all specific networks, both bottlenecks are activated under SUE conditions. Three specific networks are considered:

- The Independent network, in which  $l_3=0$ ,  $l_4>0$  and  $l_6>0$ . In this network all routes are independent
- The Dependent network, in which  $l_3>0$ ,  $l_4=0$  and  $l_6=0$ . In this network link 3 is shared by two routes.
- The Spillback network, in which  $l_3>0$ ,  $l_4>0$  and  $l_6=0$ . In this network  $l_4$  is chosen such that spillback from link 4 onto its upstream link (3) occurs under SUE conditions.

The influence of inseparable route costs due to capacity constraints (routes sharing a bottleneck) can be examined by comparing assignment results on the dependent with the independent network, whereas the influence of spatial inseparability of the cost function due to storage capacity constraints (spillback on upstream links) can be examined by comparing assignment results on the spillback network with the dependent network. It is expected that increased inseparability worsens convergence and thus that the independent network will converge best, the dependent network will converge worse and the dependent network with spillback will converge worst.

The influence of sensitivity of the implicit cost functions has been examined by scaling all link lengths, thereby controlling the ratio between free flow travel time and travel time in congestion, while maintaining the same solution under free flow conditions. The extent to which results are transferable into real size networks has already partly been examined by application on a network of the city of Den Bosch (the Netherlands) consisting of 150 centroids, 8500 links, 7000 nodes and 25000 routes. Because this is still work in progress and due to space constraints, both analyses will not be discussed here.

### 3 Preliminary results and conclusions (existing methods)

All averaging schemes mentioned in section 2 were implemented and tested using recommended parameter values provided in literature. When ranked by the number of iterations needed to achieve true SUE conditions (gap value to machine precision) SRA proved to be the best method on all networks. Results are displayed on the left hand side of figure 2, MSA results were added as a reference. Considering the left part of figure 2, the gap values show oscillations in the first 9-22 iterations until a value around  $1E-02$  is reached. Further investigation showed that these oscillations are formed due to iterations in which the averaging scheme overshoots, causing the bottleneck on (one of) the constrained routes to deactivate, and thus become inconsistent with the state under SUE conditions. Only after this 'unstable phase' the correct state is maintained and convergence accelerates and smoothens. Because MSA takes less iterations to stabilize into the correct state, it outperforms SRA until about iteration 24-30, after which SRA clearly takes over. Apparently, SRA suffers from using the information from iterations in the unstable phase, causing the step sizes to be decreased too much. After reaching the stable state, SRA maintains relatively large step sizes whereas MSA's continue to decline slowing down convergence.

Comparing the different networks, both MSA and SRA runs show unexpected results: using MSA the dependent network shows better convergence than the independent network, whereas using SRA, the dependent network with spillback shows better convergence than the dependent network. Additional runs in which demand was increased to ensure that bottlenecks are never deactivated (and thus no unstable phase occurs) did show expected results and much better convergence for all networks and averaging schemes, suggesting that the extent to which the unstable phase occurs determines the speed of convergence in later iterations.

#### 4 Preliminary Results and conclusions (enhanced methods)

Based on test results (partly) discussed in section 3 two enhancements to SRA are proposed and tested. Analysis of gap values per OD showed that the least converging ODpair contributes the most to poor gap values. Therefore we propose a new averaging scheme called SRA-ODspecific, which determines OD pair specific step sizes, giving SRA a higher degree of freedom during step size optimization. Furthermore, realizing that the normalization of the scale parameter based on free flow instead of congested cost yields too large scale parameters and thus too high sensitivity of the route choice model on congested ODpairs, we propose to normalize the scale parameter in each iteration based on the current maximum route cost for the considered ODpair. We choose to use the maximum (not the weighted average) route cost because this will stabilize the most sensitive ODpairs the most, since differences between minimum and maximum route costs are the largest on those ODpairs. After the instable phase, differences between minimum and maximum route costs will decline and the ‘overcompensation’ of the normalized scale parameter automatically diminishes. Additional test runs confirmed that using the maximum route cost clearly outperforms using the (weighed) averaged or minimum route cost for normalization of the scale parameter.

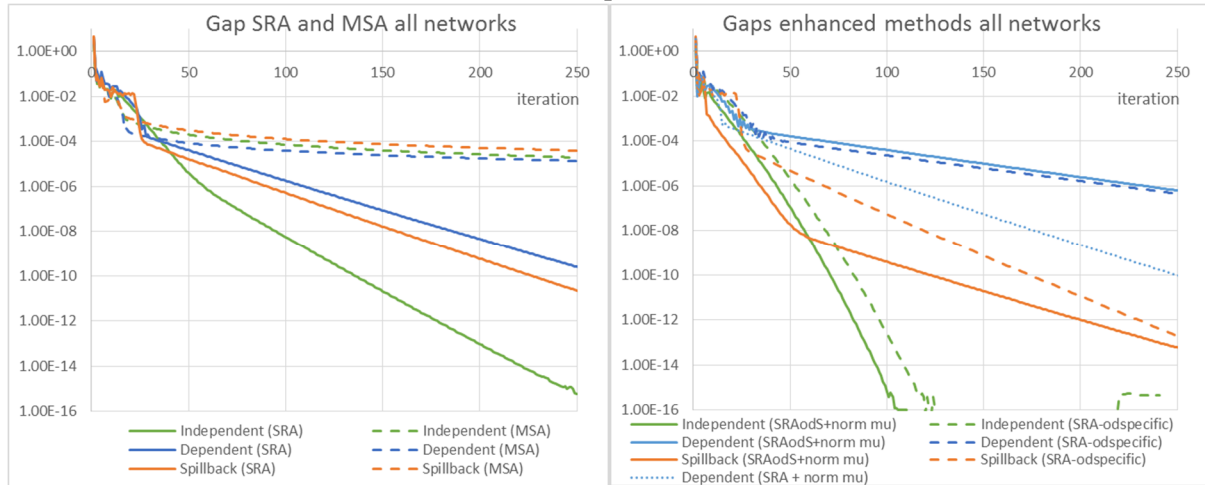


Figure 2: Convergence on test networks (left: MSA vs SRA, right: enhanced methods)

Results of the enhanced methods are displayed on the right hand side of figure 2. Compared to SRA, SRA-odspecific accelerates convergence on the independent network after iteration 50, reaching machine precision in just over 120 iterations. This big improvement makes sense when realizing that on the independent network, routecost is separable over both ODpairs, which is exactly what SRA-ODspecific implicitly assumes. On the other networks, routecost is inseparable over ODpairs, which explains why SRA outperforms SRA-odspecific on these networks. Normalization of  $\mu_{od}$  in each iteration in combination with SRA-ODspecific shortens the stabilization phase on the Independent and Spillback networks, thereby greatly improving convergence. On the Dependent network, the negative effect of SRA-odspecific apparently outweighs any potential positive effect of  $\mu_{od}$  normalization, indicated by the slightly worse performance compared to SRA-odspecific. Therefore, for this network a run with normal SRA and normalized  $\mu_{od}$  was run (dotted blue line in right hand side graph) which shows that also for this network, normalizing  $\mu_{od}$  can indeed improve results (compare to SRA).

## 5 Overall conclusions and further research directions

In this abstract methods in literature and logical extensions to those methods that could improve convergence of quasi dynamic assignment models were investigated. Two important mechanisms causing convergence issues were identified (existence of an ‘instable phase’ and spatial inseparability of route cost functions) using examples on theoretical networks and two enhancements to existing methods were proposed and successfully demonstrated.

Based on this research, novel methods that will be tested in the full study include SRA applied to ODpairs clustered by usage of bottlenecks, two hybrid forms of MSA and SRA, diagonalization and/or dampening of spillback effects, capping routecost for routes where the ratio between time in congestion and total travel time is too high and discarding diverging iterations instead of only lowering their step size (as SRA does).

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