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Adaptive Detection of Range Migrating Target in non-Gaussian Clutter

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Abstract—**Adaptive detection of fast moving targets by means of high range resolution radar is considered. It is assumed that a fast-moving target of interest has a few range cells migration during the coherent processing interval and the clutter power fluctuates rapidly along the range. Therefore, the target competes with the clutter responses in a few adjacent range cells, modeled by Compound-Gaussian process. The adaptive CFAR detector of range-migrating targets is designed and it is complemented by the algorithm for covariance matrix estimation from the reference data. The generalized approach for detection of range-extended migrating targets is provided. The performance of the proposed detectors is evaluated via numerical simulations, showing valuable improvement over the conventional techniques.**

I. INTRODUCTION

Modern wideband radars have enabled a sub-meter range resolution, thus providing additional possibilities for target detection and classification [1], [2]. However, the target detection in the high range resolution (HRR) mode has a few differences w.r.t. the detection in the low range resolution mode. Namely, clutter becomes non-Gaussian, targets become range extended and also, target range-walk (range migration) withing the coherent processing interval (CPI) becomes non-negligable.

The modern trend is to represent a non-Gaussian radar clutter by the compound noise models (compound-Gaussian (CG) process, Spherically Invariant Random Vectors (SIRV)) which allow separation of clutter spectrum characteristics from its PDF. The compound models accurately describe the scattering phenomena of clutter for short observation times [2]. Radar detection of a point target in CG and SIRV models has been extensively studied during the last decades, resulting in a number of handful constant false alarm rate (CFAR) detectors for point-like targets [2], [3]. These detectors exploit clutter covariance matrix (CM) estimated from the reference data, and therefore, referred to as adaptive detectors [2], [4].

The targets of interest (planes, cars etc.), observed by a high-resolution radar, are well modeled

as a set of independent point scatterers separated along the range [1]. The aforementioned detectors have been generalized for the case of range-extended (range-distributed) targets assuming either homogeneous, locally-homogeneous [5] or non-homogeneous [6] CG clutter along the target range extent.

Fast-moving target, observed in the HRR mode during relatively long CPI (say 50 - 100 ms) obey range migration phenomenon. This effect is well-studied for target feature extraction and it can be efficiently compensated via Keystone [7] or Radon [8] transforms. Ignoring of a range-walk results in a smeared target response in range and Doppler frequency (or velocity) [7]. Consequently, signal to clutter ratio (SCR) degrades, as well as the detection performance. Recently, some detectors of range-migrating targets in Gaussian [9] and locally Gaussian clutter [5] have been proposed. These detectors require knowledge of the clutter correlation properties (via CM) in two dimensions: in range and slow-time. So, detection of range-migrating targets requires clutter correlation in range to be considered. Thus, Dai et al. [5] showed that assumption of clutter independence between adjacent range cells leads to the non-CFAR performance of the adaptive detector for range-migrating targets.

The aim of this paper is to derive a CFAR detector of range migrating targets in slow-time and range correlated CG clutter and obtain an adaptive detector for range migrating target in CG clutter. To do this in Section II, the models of a target and clutter are provided and problem formulation is given. Next, in Section III the detector of a migrating point target in CG clutter with known speckle CM is developed and its generalization to the extended target case is provided. Further, in Section IV the algorithm for CM estimation and the adaptive detector are proposed. The performance of the proposed techniques is evaluated by numerical simulations in Section V. Conclusions are drawn in Section VI.

II. SIGNAL MODEL AND PROBLEM FORMULATION

A. Target model

Assume a wideband radar coherently transmits M wideband pulses. The signature of a point target, observed by the radar in a block of K adjacent range cells, can be expressed by $K \times M$ matrix $\mathbf{A}$ [7]:

$$\mathbf{A}_{k,m} = e^{j2\pi f_D T_r m} \cdot u_p\left(k - k_0 + \frac{v_0 T_r}{\delta_R} m\right), \quad (1)$$

where $m = 0 \ldots M-1$ is the pulse (slow-time) index, $k = 0 \ldots K-1$ is the range cell (fast-time) index, k_0 stands for the initial range cell of the target, moving with the constant radial velocity v_0, $f_D = 2v_0 f_c/c$ is target Doppler frequency at f_c. Transmitted signal occupies frequencies from f_c to $f_c + B$, where B is the bandwidth, T_r is the pulse repetition interval (PRI), $\delta_R = c/(2B)$ is the radar range resolution, and $u_p(x)$ denotes the impulse response of the transmitted waveform: $u_p(x) = \mathrm{sinc}\,(\pi x)$ is assumed hereinafter.

Because of the migration effect, the target amplitude estimation and, therefore, its detection should be performed over the block of K adjacent range cells, called low range resolution segment (LRRS), which satisfies:

$$K \geq |v_{max}| M T_r / \delta_R + E_{max}. \quad (2)$$

Here v_{max} is the maximal target velocity and E_{max} is the maximum expected target extend in range cells. An extended target is modeled as a composition of point scatterers in the adjacent range cells within target physical dimensions. In the following we refer to the vectorized form of the target model by $\mathbf{a} = \mathrm{vec}(\mathbf{A}^T)$.

B. Clutter model

Since target detection should be performed in the LRRS, the generalization of CG clutter to multiple range cells, possibly correlated, is developed here. In order to consider possible clutter correlation in range the Dependent Interference Model (DIM) [10] has to be adapted, contrary to usually considered Independent Interference Model (IIM).

The CG model, being a product of two random variables, gives three ways to model spatial correlation: considering either the speckle component or texture to be correlated over the range, or both of them. To select one of these models, recall the result of [5], [9], where it was demonstrated that detection of a range-migrating target in Gaussian clutter requires estimation of clutter CM in two dimensions: range and slow-time. The compound-Gaussian model can be considered as the extension of the Gaussian model, which preserves the correlation properties of the former, and allows the power variation along the range. Therefore, for the CG clutter, we consider the speckle to be correlated over the range, while the texture to be independent from one
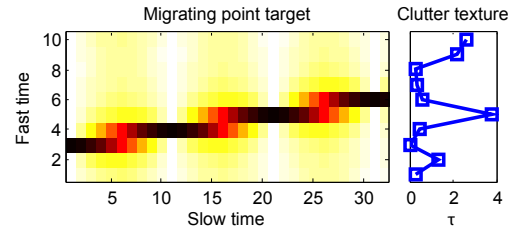

Figure 1. Range migrating point target in spiky clutter

range cell to another. The independence of the texture is imposed for model tractability.

The clutter response in the LRRS can be represented by $K \times M$ matrix $\mathbf{C}$, its vectorized counterpart is $KM \times 1$ vector $\mathbf{c} = \mathrm{vec}(\mathbf{C}^T)$, which is given element-wise by $\mathbf{c} = [c_0, c_1 \ldots c_{KM-1}]^T$. Hereinafter we also refer to the clutter in the k-th range cell by the sub-vector of length M: $\mathbf{c_k} = [c_{kM}, \ldots, c_{(k+1)M-1}]^T$, so $\mathbf{c} = [\mathbf{c_0}^T, \mathbf{c_1}^T, \ldots, \mathbf{c_{K-1}}^T]^T$. Similar definition of subvectors holds for other $KM \times 1$ vectors.

The clutter in each range cell is modeled as a CG random vector, given as [2]: $\mathbf{c_k} = \sigma_k \mathbf{g_k}$. The texture σ_k^2 is considered to be constant along slow-time and independent from one range cell to another. In a LRRS, the speckle component is a KM-dimensional zero mean complex Gaussian vector with known CM: $\mathbf{g} \sim \mathcal{CN}(\mathbf{0}_{KM}, \mathbf{Q})$. The covariance and cross-covariance matrices of clutter in the range cells are:

$$E\{\mathbf{c_i}\mathbf{c_j}^H\} = E\{\sigma_i\}E\{\sigma_j\}\mathbf{Q_{i,j}}, \quad (3)$$

where the expectation is taken over multiple realizations of the same process, but not over the range and $\mathbf{Q_{i,j}} = \mathbf{Q}_{iM\ldots(i+1)M-1, jM\ldots(j+1)M-1}$ defines $M \times M$ block of the speckle CM. The PDF of CG clutter in the LRRS, conditional on $\sigma_{\mathcal{K}}$, $\mathcal{K}: k = 0 \ldots K-1$, is:

$$p(\mathbf{c}|\sigma_{\mathcal{K}}) = \frac{\exp\left(-\sum_{i=0}^{K-1}\sum_{j=0}^{K-1}\frac{\mathbf{c}_i^H \mathbf{Q_{i,j}^{-1}} \mathbf{c}_j}{\sigma_i \sigma_j}\right)}{\pi^{KM}|\mathbf{Q}|\prod_{k=0}^{K-1}\sigma_k^{2M}}. \quad (4)$$

The PDF clutter in the LRRS can be obtained as $p(\mathbf{c}) = E_{\sigma_{\mathcal{K}}}\{p(\mathbf{c}|\sigma_{\mathcal{K}})\}$, which requires multidimensional integration over PDFs of $\sigma_{\mathcal{K}}$ and has no explicit solution. The way to overcome this limitation is to consider each σ_k as an unknown deterministic parameter to be estimated in the GLRT. The same strategy was employed for non-migrating target detection to obtain a distribution-free test [3] and approaches the optimal test for moderate number of pulses in CPI ($M > 16$). Since the target range-walk is observed only for large M, hereafter σ_k^2 is tackled as unknown constant.

C. Problem formulation

The scenario under consideration is shown in Fig. 1. The detection problem of a point target (Fig. 1) can

be formulated as:

$$\mathbf{y_k} = \begin{cases} H_0: & \mathbf{c_k}, \\ H_1: & \alpha \mathbf{a_k} + \mathbf{c_k}, \end{cases} \qquad k = 0 \ldots K-1, \quad (5)$$

where α is a constant amplitude of a target in the LRRS under the hypothesis of its presence (H_1), $\mathbf{y_k}$, $\mathbf{c_k}$ and $\mathbf{a_k}$ are the sub-vectors, corresponding to the received data, the clutter and the target responses accordingly in the k-th range cell. The definition of the sub-vectors is identical to that of $\mathbf{c_k}$ given above. The generalization to the case of an extended target is straightforward.

III. Migrating target detection in range correlated CG clutter

A. Point target detector

As follows from the problem formulation and clutter model, the detection problem involves unknown parameters $\sigma_{\mathcal{K}}^2$ and α and attacked here with the generalized likelihood ratio test (GLRT): $\Lambda(\mathbf{y}) = \frac{\max_{\sigma_{\mathcal{K}}, \alpha} f^{(1)}(\mathbf{y}; \sigma_{\mathcal{K}}, \alpha)}{\max_{\sigma_{\mathcal{K}}} f^{(0)}(\mathbf{y}; \sigma_{\mathcal{K}})}$. Hereinafter, the superscript in braces stands for the hypothesis index $H_i, i \in \{0,1\}$. The likelihood function of the LRRS under H_1 is:

$$f^{(1)}(\mathbf{y}; \sigma_{\mathcal{K}}, \alpha) = \frac{\exp\left(-\sum_{i=0}^{K-1}\sum_{j=0}^{K-1} \frac{q_{i,j}^{(1)}}{\sigma_i \sigma_j}\right)}{\pi^{KM} |\mathbf{Q}| \prod_{k=0}^{K-1} \sigma_k^{2M}}. \quad (6)$$

and similarly $f^{(0)}(\mathbf{y}; \sigma_{\mathcal{K}}) = f^{(1)}(\mathbf{y}; \sigma_{\mathcal{K}}, \alpha)|_{\alpha=0}$. Herein $q_{i,j}^{(1)} = (\mathbf{y_i} - \alpha \mathbf{a_i})^H \mathbf{Q_{i,j}^{-1}}(\mathbf{y_j} - \alpha \mathbf{a_j})$ and $q_{i,j}^{(0)} = \mathbf{y_i^H Q_{i,j}^{-1} y_j}$, where $i, j \in \mathcal{K}$.

We start with the estimation of $\sigma_{\mathcal{K}}$ under both hypotheses by maximizing the logarithm of (6) w.r.t. each $\sigma_k, k \in \mathcal{K}$. Then the estimation of $\sigma_k^{(H)}$ is obtained as the positive solution of the quadratic equation:

$$\left(\sigma_k^{(H)}\right)^2 - \sigma_k^{(H)} \sum_{j=0, j\neq k}^{K-1} \frac{\Re\left(q_{k,j}^{(H)}\right)}{M\sigma_j^{(H)}} - \frac{q_{k,k}^{(H)}}{M} = 0, \quad (7)$$

for each $\sigma_k^{(H)}, k \in \mathcal{K}$. The equation for $\sigma_k^{(H)}$ depends on $\sigma_{j\in\mathcal{K}, j\neq k}^{(H)}$ and also on α under H_1: $q_{k,j}^{(1)} = q_{k,j}^{(1)}(\alpha)$.

Let $b_k^{(H)} = -\left(\sum_{j=0, j\neq k}^{K-1} \left(M\sigma_j^{(H)}\right)^{-1} \Re(q_{k,j}^{(H)})\right)$ and $c_k^{(H)} = -q_{k,k}^{(H)}/M$, then each of K equations always (under any hypothesis $H_i, i \in \{0,1\}$, for any data set $q_{k,j}^{(H)}$ and any assumption on $\sigma_{j\in\mathcal{K}, j\neq k}^{(H)} > 0$) has two real roots, as $\left(b_k^{(H)}\right)^2 - 4c_k^{(H)} > 0, \forall k \in \mathcal{K}$. Moreover, from Vieta's formula, it follows that the roots of (7) satisfy $\sigma_k^{[1]}\sigma_k^{[2]} = c < 0$, so only one root is positive, which is the one of interest. Therefore, the solution of (7) is:

$$\hat{\sigma}_k^{(H)} = \frac{1}{2}\left(-b_k^{(H)} + \sqrt{\left(b_k^{(H)}\right)^2 - 4c_k^{(H)}}\right), \quad (8)$$

written as $\hat{\sigma}_k^{(H)} = g_k^{(H)}\left(\hat{\sigma}_{j\in\mathcal{K}, j\neq k}^{(H)}, \alpha\right)$ for notation simplicity. Then, under H_0 there exist the system of K equations for $\sigma_{\mathcal{K}}$ in the form (8). Similarly, under H_1, we have K equations (8) for σ_k, which depend on $K+1$ unknowns: $\sigma_{\mathcal{K}}$ and α. The last equation is:

$$\hat{\alpha} = \left(\mathbf{a}^H \left(\hat{\mathbf{M}}^{(1)}\right)^{-1} \mathbf{a}\right)^{-1} \mathbf{a}^H \left(\hat{\mathbf{M}}^{(1)}\right)^{-1} \mathbf{y}, \quad (9)$$

with $\hat{\mathbf{M}}^{(H)} = \mathbf{M}|_{\sigma_{\mathcal{K}} = \hat{\sigma}_{\mathcal{K}}^{(H)}}$ and the latter has the form:

$$\mathbf{M} = \begin{bmatrix} \sigma_0^2 \mathbf{Q_{0,0}} & \cdots & \sigma_0 \sigma_{K-1} \mathbf{Q_{0,K-1}} \\ \vdots & \ddots & \vdots \\ \sigma_{K-1}\sigma_0 \mathbf{Q_{K-1,0}} & \cdots & \sigma_{K-1}^2 \mathbf{Q_{K-1,K-1}} \end{bmatrix}. \quad (10)$$

Under both hypotheses, the systems are solved by the fixed point iteration for systems of equations.

Substitution of the estimates into the GLRT gives [11] the detector in the form:

$$\Lambda(\mathbf{y}) = \prod_{k=0}^{K-1} \left(\frac{\hat{\sigma}_k^{(0)}}{\hat{\sigma}_k^{(1)}}\right)^{2M} \underset{H_0}{\overset{H_1}{\gtrless}} T, \quad (11)$$

where T is the threshold to satisfy the appropriate probability of false alarm P_{FA}.

The test (11) is the generalization of the detector proposed for IIM clutter model in [12]. The latter can be obtained assuming $b_{\mathcal{K}}^{(H)} = \mathbf{0}_K$, which is the IIM of CG clutter.

B. Clutter map detector

Assume that high resolution clutter map is available to the radar processor. Within the framework described above, the clutter map provides the values of $\hat{\sigma}_{\mathcal{K}}$ for both hypothesizes. Substitution of $\hat{\sigma}_{\mathcal{K}}$ in (10) transforms the problem into classical detection of a target in Gaussian noise with known CM and treated with the standard matched filter detector [2]:

$$\frac{\left|\mathbf{a}^H \mathbf{M}^{-1} \mathbf{y}\right|^2}{\mathbf{a}^H \mathbf{M}^{-1} \mathbf{a}} \underset{H_0}{\overset{H_1}{\gtrless}} T'. \quad (12)$$

Hereinafter, the detector (12) is referred to as clairvoyant detector.

C. Extended target detector

The targets of interest, observed in HRR radar mode (with a meter or sub-meter range resolution), become extended in range. Because of the target migration and also the clutter correlation in range, the key assumption of [6] on data independence in adjacent range cells is obviously not valid. Therefore, for the detection of range-extended targets with the range-walk, the theory of subspace detectors [2] is adapted here. Assume the target vectorized signal $\mathbf{s}$ lies in known subspace of dimension R: $\mathbf{\Psi} = [\mathbf{a}(0), \ldots, \mathbf{a}(R-1)]$, so $\mathbf{s} = \mathbf{\Psi}\boldsymbol{\alpha}$,

where $\boldsymbol{\alpha} = [\alpha_0, \ldots, \alpha_{R-1}]^T$. Since the target extension in range is considered, the subspace vectors correspond to the target steering vectors at different range cells k_0, but with the same velocity v_0 in (1). The reflection from the moving parts of the target, such as wheels, blades etc, is neglected.

Denote by $\mathbf{s}$ the signature of the extended target in the LRRS (instead of $\alpha \mathbf{a}$ for a point target). The quadratic form under H_1 then becomes $q_{i,j}^{(1)} = (\mathbf{y_i} - \mathbf{s_i})^H \mathbf{Q_{i,j}^{-1}} (\mathbf{y_j} - \mathbf{s_j})$, where $\mathbf{s_i}$ is the sub-vector of the target signal in the i-th range cell. Then, the estimation of $\hat{\sigma}_{\mathcal{K}}^{(H)}$ has the form of (7).

The estimation of $\hat{\boldsymbol{\alpha}}$ can be found by maximizing the likelihood function w.r.t. each element of $\boldsymbol{\alpha}$:

$$\hat{\alpha}_r = \frac{\mathbf{a}^H(r) \left(\hat{\mathbf{M}}^{(1)}\right)^{-1} \left(\mathbf{y} - \sum_{j=0, j \neq r}^{R-1} \hat{\alpha}_j \mathbf{a}(j)\right)}{\mathbf{a}^H(r) \left(\hat{\mathbf{M}}^{(1)}\right)^{-1} \mathbf{a}(r)}. \tag{13}$$

Detection of a target spread over R range cells requires solving numerically K equations for $\hat{\sigma}_{\mathcal{K}}^{(1)}$ and R equations for $\hat{\alpha}_r$ together. The detection rule for the range-extended target can be shown in the form (11) with the appropriate definition of $q_{i,j}^{(1)}$.

Note, that in general, the signal subspace is not known in advance. So, to make such a detector applicable, some assumption on the target extent should be made based on the prior knowledge of the scene or extracted from the data using some model order selection techniques.

IV. Covariance Matrix Estimation and Adaptive Detector

In the previous section, we assumed known speckle CM $\mathbf{Q}$ in slow-time/range, which is generally not the case in a real application. In this section the approach to estimate clutter CM in a LRRS from the reference data is proposed.

In Gaussian clutter, the sample covariance matrix (SCM) is known to be the maximum likelihood estimation [2]. In CG clutter, the ML estimation of CM is defined as the solution of the transcendental equation [10] depending on the PDF of texture. The practical approach is the approximate ML (AML) estimator [10], [4], which considers clutter texture in each range cell as an unknown deterministic parameter.

Assume $L > KM$ independent and target free reference LRRSs with CG clutter having homogeneous speckle component are available. The received data in the l-th reference cell $\mathbf{z}(l) = [z_0(l), z_1(l), \ldots, z_{KM-1}(l)]^T$ can be arranged by the range cells as: $\mathbf{z}(l) = [\mathbf{z_0}^T(l), \mathbf{z_1}^T(l), \ldots, \mathbf{z_{K-1}}^T(l)]^T$. The received data in the l-th reference LRRS is: $\mathbf{z}(l) = \mathbf{W}(l)\mathbf{g}(l)$, where texture is accounted via

$\mathbf{W}(l) = \text{diag}\left(\sigma_0(l), \ldots, \sigma_{K-1}(l)\right) \otimes \mathbf{I}_M$ and $\mathbf{g}(l) \sim \mathcal{CN}(\mathbf{0}_{KM}, \mathbf{Q})$ is the speckle. The clutter in the reference LRRS l is conditionally Gaussian $\mathbf{z}(l)|_{\mathbf{W}(l)} \sim \mathcal{CN}(\mathbf{0}_{KM}, \mathbf{M}(l))$, with:

$$\mathbf{M}(l) = E\{\mathbf{z}(l)\mathbf{z}^H(l)\}$$
$$= \mathbf{W}(l)E\{\mathbf{g}(l)\mathbf{g}^H(l)\}\mathbf{W}(l) = \mathbf{W}(l)\mathbf{Q}\mathbf{W}(l) \tag{14}$$

Given the structure of clutter CM in the l-th LRRS, we adopt the two-step maximization procedure of [10] to derive the range slow-time speckle CM estimator.

At the first step of maximization, we assume the texture estimates $\hat{\sigma}_{\mathcal{K}}(l)$ are available $\forall l \in \mathcal{L}$, where $\mathcal{L} : l = 0, \ldots, L-1$. Then, from (14) the speckle CM MLE from L reference LRRSs is given:

$$\hat{\mathbf{Q}} = \frac{1}{L} \sum_{l=0}^{L-1} \hat{\mathbf{W}}^{-1}(l)\mathbf{z}(l)\mathbf{z}^H(l)\hat{\mathbf{W}}^{-1}(l), \tag{15}$$

where $\hat{\mathbf{W}}(l) = \mathbf{W}(l)|_{\sigma_{\mathcal{K}}(l) = \hat{\sigma}_{\mathcal{K}}(l)}$.

At the second step, we suppose speckle CM estimation $\hat{\mathbf{Q}}$ is available and derive the MLE of $\sigma_k(l), \forall k \in \mathcal{K}, \forall l \in \mathcal{L}$. The estimation of $\sigma_k(l), \forall k \in \mathcal{K}$ in each LRRS l corresponds to (8) under H_0 and obtained iteratively via:

$$\hat{\sigma}_k(l) = g_k^{(0)}\left(\hat{\sigma}_{j \in \mathcal{K}, j \neq k}(l), \hat{\mathbf{Q}}\right). \tag{16}$$

So the algorithm for the range slow-time speckle CM estimation involves two nested loops. The inner loop updates the texture estimation in all the reference LRRSs by means of (16); and the outer loop updates the speckle CM by means of (15).

A few comments are in order regarding the estimator (15), (16). First, the algorithm should be initialized with some estimation of CM $\mathbf{Q}$. With no prior information, the initialization is made with the SCM:

$$\hat{\mathbf{Q}}_{\text{SCM}} = \frac{1}{L} \sum_{l=0}^{L-1} \mathbf{z}(l)\mathbf{z}^H(l). \tag{17}$$

Second, the number of iterations in the outer loop $I_{\mathbf{Q}}$ can be limited to a few [10], since the further improvement in CM estimation has a minor impact. Third, the iterative procedure in the inner loop might be initialized with $\hat{\sigma}_{\mathcal{K}}(\mathcal{L})$ obtained at the previous step of the outer loop and then estimated in a few iterations I_σ. The latter relies on the fact that $\hat{\mathbf{Q}}$ does not vary significantly from one iteration to another. Note, that each loop can be stopped when the corresponding convergence criteria, e.g. $C(i) = \frac{||\hat{\mathbf{W}}_{i+1}^{(H)} - \hat{\mathbf{W}}_i^{(H)}||_2}{||\hat{\mathbf{W}}_i^{(h)}||_2} \leq \epsilon$ for outer loop is satisfied. Forth, the iterative estimation (15) is normalized by $\text{Tr}(\hat{\mathbf{Q}})$ at each iteration for the identification reasons [10], [4].

The adaptive detector has the form of the detector defined above, namely (11), where the known matrix $\mathbf{Q}$ is substituted with its estimation from the reference dataset $\hat{\mathbf{Q}}$.
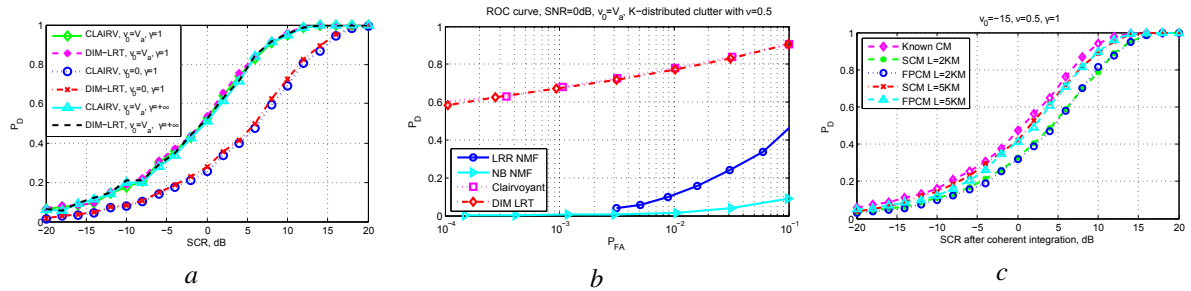
Figure 2. *a* - Detection probability of range migrating target in CG clutter with: $v_0 = 0$ m/s and $v_0 = 15$ m/s; *b* - ROC curves of migrating target detector in CG clutter; *c* - Detection probability of the adaptive detector for the range migrating target in CG clutter with $v_0 = 15$

V. SIMULATION RESULTS

In this section, the performance of the proposed algorithms is assessed by numerical simulations. The parameters of the radar are fixed to: $f_c = 10$ GHz, $B = 1$ GHz ($\delta_R = 0.15$ m), $T_r = 1$ ms, $M = 32$. We set the maximum expected velocity of a target to: $|v_0| \leq v_{max} = v_a = c/(2f_cT_r) = 15 m/s$; for a point target detection we set $K = 5$ to satisfy (2).

The texture $\sigma_{\mathcal{K}}^2$ follows Gamma distribution, so the clutter follows the K-distribution, a special case of CG; the shape and scale parameters are $\nu = 0.5$ and $\mu = 1$. The known speckle CM has the structure $\mathbf{Q} = \mathbf{R} \otimes \mathbf{S}$, so the speckle correlation in slow-time is defined by $M \times M$ matrix $\mathbf{S}$ and in range by $K \times K$ matrix $\mathbf{R}$. For P_{FA} and P_D assessment, we run 10^6 and 10^3 Monte-Carlo trials accordingly.

A. Known speckle covariance matrix

1) Detection performance: The analysis of the detection performance as a function of target SCR = $\frac{|\alpha|^2}{E\{\sigma^2\}}\mathbf{a}^H\mathbf{Q}^{-1}\mathbf{a}$ is shown in Fig. 2, *a* for target velocities $v_0 = 0$ m/s and $v_0 = v_a = 15$ m/s and different clutter spatial correlation: $\mathbf{R}_{k,j} = e^{-\gamma|k-j|}$ with $\gamma = 1$ and $\gamma \to +\infty$; $\mathbf{S} = \mathbf{I}_M$, $P_{FA} = 10^{-5}$. Comparison in Fig. 2, *a* includes the proposed detector (refereed as DIM-LRT), and the clairvoyant detector (12). The loss of the proposed detector in comparison to the clairvoyant one is about 1 dB in each scenario. The analysis shows that target detection performance does not depend on clutter spatial correlation, but depends on the target velocity. Thus, the detection gain for the target with velocity $v_0 = 15$ m/s, which has a range-walk of about 3 range cells, with respect to the stationary obeys about 7 dB gain for the given clutter parameters. This phenomenon can be well explained by the diversity of clutter, obtained by coherent integration of the target response in a few range cells. The faster the target, the more it migrates, the less probability to miss the target due to clutter spike in one range cell, so the higher the probability of detection. This behavior is akin to detection of range-extended targets in CG clutter, where the detection performance depends on

the target spread (see e.g. [6]). The observed diversity gain is not linear and saturates as the number of the range cells increases; we have observed that the major improvement is obtained by the first 3 range cells migration, and fully saturates for the range-walk over 5 range cells.

2) ROC curves analysis: For analisys here a point target with SCR = 0 dB, $v_0 = 15 m/s$ and in clutter with known speckle CM are considered. Fig. 2, *b* shows the ROC curves for four detectors, namely: LRR NMF - Normalized matched filter applied to a LRRS, which consider a locally Gaussian clutter model; NB NMF - Normalized matched filter applied per range cell, assuming no target migration occur by ignoring the migration term in (1); the proposed detector (DIM-LRT) and the clairvoyant detector (12). The results demonstrate the advantages of applying CFAR detector for range migrating targets. Note that NB NMF suffers from incorrect target model, while for the LRR NMF is degraded due to incorrect clutter model, leading to non-CFAR behavior of the latter.

B. Adaptive detector of a point target

We analyze the detection performance of the adaptive detector in two scenarios: first, we assume that the reference data contains only the speckle clutter component, so the ML CM estimation is obtained with the SCM from this reference data. Second, we assume the reference data follows the CG model, and use the proposed in Section IV estimator referred as FPCM. We consider $L = 2KM$ and $L = 5KM$ and run 10^6 Monte-Carlo trials to set the threshold satisfying $P_{FA} = 10^{-4}$ in each case. For the FPCM estimation we used $I_\sigma = 5$ and $I_Q = 20$. Simulations results for the target moving with velocity $v_0 = 15$ m/s in range correlated clutter with $\gamma = 1$ are shown in Fig. 2, *c*. The detection loss for the case of $L = 2KM$ is about 3.5 dB and 4 dB for SCM of the speckle and the proposed estimation of the CM accordingly, and about 0.9 and 1.5 dB for the case of the $L = 5KM$. Both cases agree well with the theoretical performance degradation of the adaptive detectors [2].
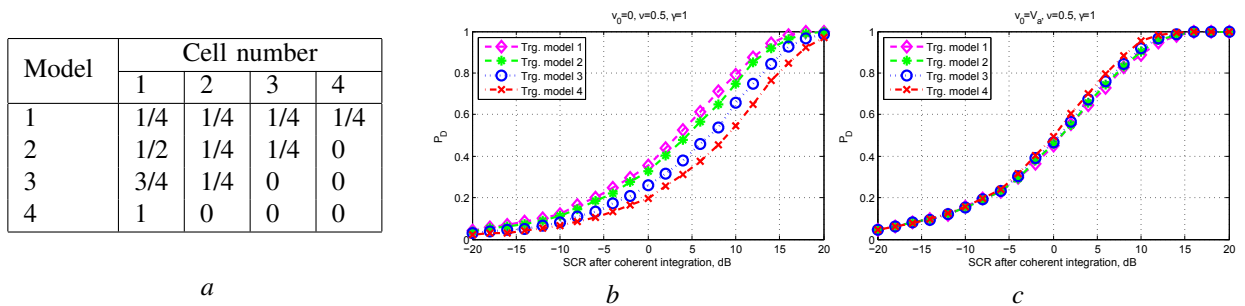
Model	Cell number			
	1	2	3	4
1	1/4	1/4	1/4	1/4
2	1/2	1/4	1/4	0
3	3/4	1/4	0	0
4	1	0	0	0

a

b

c

Figure 3. *a* - Extended target model with $E_{max} = 4$; *b, c* - Detection performance for the extended target: *a* - Stationary target: $v_0 = 0$ m/s; *b* - Migrating target: $v_0 = 15$ m/s.

C. Extended target detector

Herein the performance of the proposed detector for range-extended migrating targets is analyzed. A target of interest with known extent $E_{max} = 4$ is modeled with different spatial distribution [6], given in Figure 3, *a*. For this scenario we set $K = 8$ to satisfy (2) for $|v_0| \leq v_a$. Since the analytical expression of the P_{FA} is not available, the threshold for $P_{FA} = 10^{-4}$ with $E_{max} = 4$ was estimated numerically. Range correlated clutter with: $\gamma = 1$ and $\rho \to +\infty$ is considered. Herein we define $\text{SCR} = \frac{\mathcal{E}}{E\{\sigma^2\}} \mathbf{a}^H \mathbf{Q}^{-1} \mathbf{a}$, where $\mathcal{E} = \sum_{i=r}^{4} w_r |\alpha_r|^2$ and w_r are the coefficients from Figure 3, *a*.

The detection performance is evaluated for the velocities of the target $v_0 = 0$ and $v_0 = v_a = 15$ m/s. The results in Fig. 3, *b* show that non-coherent averaging along target extent improves detection performance, similarly to the results in [6]. The detection performance of a migrating extended target, on the other hand, is almost independent of the target extent and always better than that of a stationary one, as shown in Fig. 3, *c*. That is due to the fact that averaging over the target and over clutter provides the same gain - due to diversity. Thus, the detection performance of a migrating target depends only on its SCR, but not on the shape of its response.

VI. CONCLUSION

The problem of range migrating target detection in non-homogeneous clutter has been considered. In order to solve this problem we have introduced the model of range-correlated compound-Gaussian clutter in a block of range cells, which provides a model of spiky clutter with correlation in both range and slow-time. We used this model to derive an adaptive detector in two steps. First, we assumed that clutter speckle CM in range and slow-time is known, and derived the detector. Second, we substituted the known CM with its estimation to obtain the adaptive detector. We provided an algorithm for the range/slow-time CM estimation from the reference CG clutter. It is demonstrated that

considering target range-walk and its range extent along non-Gaussian clutter provides a novel way to exploit clutter diversity. The achieved diversity gain improves detection performance of the fast targets and can be used together with the integration over target extent against clutter fluctuation (akin burst-to-burst integration in conventional radars).

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