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Modelling performance an air transport network operated by subsonic and supersonic aircraft

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ABSTRACT

This paper deals with modelling the performance of an air transport network operated by existing subsonic and the prospective supersonic commercial aircraft. Analytical models of indicators of the infrastructural, technical/technological, operational, economic, environmental, and social performance of the network relevant for the main actors/stakeholders involved are developed. The models are applied to the given long-haul air route network exclusively operated by subsonic and supersonic aircraft according to the specified "what-if" scenarios.

The results from application of the models indicate that supersonic flights powered by LH_2 (Liquid Hydrogen) could be more feasible than their subsonic counterparts powered by Jet A fuel, in terms of about three times higher technical productivity, 46% smaller size of the required fleet given the frequency of a single flight per day, 20% lower sum of the aircraft/airline operational, air passenger time, and considered external costs, up to two times higher overall social-economic feasibility, and 94% greater savings in contribution to global warming and climate change. These flights could be less feasible in terms of about 70-85% higher aircraft/airline operational costs, 70% and 19% higher fuel consumption and emissions of Green House Gases, respectively, and 6-13% higher noise compared to the specified acceptable levels.

Keywords: Air route network; performance; indicators; modelling; subsonic and supersonic aircraft/flights; social-economic feasibility

NOMENCLATURE

A	aircraft fuselage area (m^2)
A_w	reference wing area (m^2)
ADR	aircraft depreciation rate (%/year)
ATC	Air Traffic Control
b	aircraft wingspan (m)
BH	Block Hour
a_{cl}	longitudinal acceleration during climbing (m/s^2)
a_{de}	longitudinal deceleration during descending (m/s^2)
C	operating costs ($US/BH)
C_{D0}	coefficient of parasite drag at zero lift
C_F	fixed cost of flight ($US/flight)
C_o	variable (operating) cost of flight ($US/flight)
$\bar{c}$	average cost of flight ($US/p-km)
$\bar{c}_e$	average cost of GHG emissions ($US/p-km)
c_e	unit charge of GHG emissions ($US/kg, $US/ton)
$C_{d/cl}(\bar{v}_{cl})$	coefficient of aerodynamic drag during climbing at speed
$C_{d/de}(\bar{v}_{de})$	coefficient of aerodynamic drag during descending at speed
e_f	specific energy of fuel (J/kg)
e_m	emission rate of GHG (kg or ton of GHG/kg or ton of fuel)
$f(\Delta t)$	number of flights on route during time (Δt)
$f(UL)$	number of aircraft flights during useful line (UL)
FC	fuel consumption by flight (kg, ton)
ft	feet
g	gram
GDP	Gross Domestic Product generated by commercial air transportation ($US/year)
GWP	Global Warming Potential of GHG
H_{cl}	taking-off and climbing altitude (m, ft)
H_{cr}	cruising altitude (m, ft)
H_{de}	descending, final approach, and landing altitude (m, ft)
J	Joule
kg	kilogram
km	kilometre
kt	knot (nm/h)
l	effective length of aircraft (m);
L	litre
L_1, L_2	noise at distance (d_1) and (d_2) (dBA) $(d_1, d_2$ - nm, km)
LTO	Landing and Take-Off Cycle
m	meter
M	Mach number
MJ	Mega Joule

m_{cl}	initial aircraft mass during climbing phase (kg)
m_{cr}	initial aircraft mass at the beginning of cruising phase (kg)
m_{de}	initial aircraft mass during descending phase (kg)
N	Newton
nm	nautical mile
p	passenger
P	aircraft price including capital maintenance costs ($US/aircraft)
q	dynamic pressure (N/m^2)
R	route length (nm, km)
RV	aircraft residual value (%)
$\bar{r}_{GDP}$	average GDP generated by commercial air transportation ($US/p-km)
r_{fc}	cumulative rate of improvements in aircraft fuel efficiency
r	distance where thrust force (T) is applied (m, km, nm);
R/C	rate of climb of aircraft (ft/min)
R/D	rate of descent of aircraft (ft/min)
R_1, R_2	length of route in direction (1) and (2) (nm, km)
r_{cr}	length of cruising segment of the route (ft, m)
r_{cl}	length of climbing segment of the route (ft, m)
r_{de}	length of descending segment of the route (ft, m)
RPK	Revenue Passenger Kilometres
S	aircraft seating capacity (seats)
s	seconds
T	thrust force (N)
$t(R)$	average flight time on route (R) (h)
t_{LTO}	time of flight LTO cycle (min)
TP	technical productivity (10^3 seat-kt)
TW	transport work (10^3 p-km/route/day)
UL	aircraft useful life (years)
v	aircraft cruising speed (kt)
v_{cr}	average cruising speed (kt, km/h)
$\bar{v}_{cl}$	average climbing speed (m/s)
$\bar{v}_{de}$	average descending speed (m/s)
V_{RPK}	output of the commercial air transportation (RPK/year)
Vol	total volume (m^3)
α	value of passenger time ($US/p-h)
$\bar{\beta}_{cl}$	climbing angle (°)
$\bar{\beta}_{de}$	descending angle (°)
Λ	load factor of flight ($\Lambda \leq 1.0$)
η	propulsion efficiency of aircraft engines
η_{cl}	propulsion efficiency of aircraft engines during climbing
η_{cr}	propulsion efficiency of aircraft engines during cruising
η_{de}	propulsion efficiency of aircraft engines during descending

$n(\Delta\tau)$	required aircarft fleet for the network
$\overline{\rho}_{cl}$	average air density along climbing segment of the route (kg/m^3)
ρ_{cr}	air density at cruising altitude (kg/m^3)
$\overline{\rho}_{de}$	average air density along descending segment of the route (kg/m^3)
Θ	Mach angle (°)
$\tau(R)$	aircaft turnaround time on route *(R)* (h)
$\Delta\tau$	time of scheduling flights (day)
CO_2	Carbon Dioxide (kg/kg of fuel)
CO_{2e}	Carbon Dioxid Eqivalents
HC	Hydrocarbons (g/kg of fuel)
H_2O	Water Vapor (kg/kg of fuel)
LH_2	Liquid Hydrogen
NO_x	Nitrogen Oxides (g/kg of fuel)
PM	Particulate Matters (PM/kg of fuel)
SG	Specific Gravity (kg/L)
SE	Specific Energy (MJ/kg)
SO_x	Sulphur oxides (g/kg of fuel)

1.0 INTRODUCTION

Increasing of travel speed has been a human endeavour for a long time. In general, limitations on the time and monetary budget, maximizing travel distances during the shortest possible time and related costs have become the main driving forces in developing both inland HS (high speed) and air transport systems. A relatively simple calculation indicates that an increase in the operating speed generally brings marginal savings in the user/passenger travel time. These savings increase with increasing of the non-stop travel distance(s)[1]. Under such conditions, the possible implementation of commercial supersonic aircraft seems to be beneficial primarily in the case of long-haul flights. Currently, these flights are carried out by commercial subsonic aircraft. Additionally, combined with the aircraft seat capacity and the flight frequency, the supersonic speed could substantialyl increase the air route(s) technical productivity and thus bring obvious gains to the airlines. However, these gains in travel time and technical productivity remain questionable after being counterbalanced by the overall economics of these flights, including their operational costs and the environmental and social externalities. The earliest but retired Concorde and TY 144 supersonic aircraft did not achieve such an acceptable social-economic balance. Therefore, the question is whether the design and operational concepts of forthcoming supersonic aircraft, combined with an innovative consideration, could possibly indicate elements of their positive social-economic feasibility? This paper provides a framework for assessing this balance through modelling performance indicators of the given long-haul air route network, operated exclusively by supersonic aircraft or their subsonic counterparts, according to the given "what-if" scenarios.

Modelling implies development of the analytical models of particular indicators.

The considered performances are infrastructural, technical/technological, operational, economic, environmental, and social.

Infrastructural performance relates to airports as network nodes and the air routes connecting them as the network links. The airports and air routes can accommodate both subsonic and supersonic aircraft safely, efficiently, and effectively.

Technical/technological performance is considered as directly related to the design of both considered categories of aircraft. These are: length (m), wing aspect ratio, L/D (lift-to-drag) ratio, take-off weight (tons), number of engines (-), take-off speed (kt), cruising speed (Mach), landing speed (kt), range (nm; km), payload (seats), payload/weight ratio (-), fuel/weight ratio (-), and payload/fuel ratio (-)[13]. In the further consideration, these are exclusively used as the given case-based parameters.

Operational performance relates to the passenger demand and the supply of transport capacities serving this demand, under the given condition. In general, these are the flight frequency by the given aircraft type(s) and load factor, the air route travel time, the aircraft turnaround time, the route and network transport work and technical productivity, and the fleet size[13].

Economic performance is considered to be the flight(s) operating cost (as the basis for setting up airfares), the cost of passenger time while on board the flight(s), contribution to the regional/national gross domestic product (GDP), and the overall social-economic feasibility.

Environmental performance includes the aircraft/flight fuel consumption and related emissions of greenhouse gases (GHG), their costs, i.e. externalities, and the contribution of GHG emissions to global warming and climate change.

Social performance relates to the aircraft noise around airports and along the network routes (the latter by supersonic aircraft/flights), congestion/delays, safety, i.e. the risk of incidents/accidents, and their corresponding costs, i.e. externalities.

These performances are considered relevant for the particular actors/stakeholders involved. These can be the aerospace manufacturers, airlines, airports, users/air passengers, local communities, and aviation and non-aviation regulatory bodies and policy makers at the local, regional, national, or international level.

In addition to this introductory section, the paper consists of five other sections. Section 2 presents an overview of research and development of commercial subsonic and supersonic aircraft. Section 3 deals with development of the analytical models of performance indicators of the given long-haul air route network. This consists of airports as network nodes and air routes with the non-stop flights connecting them as the network links. Section 4 shows an application of the models of performance indicators to an existing long-haul air route network. This network consists of 25 longest air routes where the current subsonic flights are assumed to be completely replaced by their supersonic counterparts in the year 2050. The last Section (5) summarises some conclusions.

2.0 DEVELOPMENT OF COMMERCIAL SUBSONIC AND SUPERSONIC AIRCRAFT

2.1 Commercial subsonic aircraft

The commercial subsonic aircraft considered in the given context have been typically characterised by their range *(R)*, seat capacity *(S)*, and cruising speed *(v)*. By the year 2011, their range *(R)* had increased to about 7500nm, seat capacity *(S)* to 550 seats, and cruising speed *(v)* to about 500 kt (926km/h), i.e., $M = 0.87$ (*M* - Mach number)[2],[3],[4]. Such developments have generally influenced technical productivity *(TP)*[3], operational costs *(C)*[49,53], and fuel consumption *(FC)*[9,67-71] of the corresponding flights. Examples of their causal (regression) relationships are given by Eq. 1 (a, b, c) as follows:

- Technical productivity

$$TP(S, v) = -17.335 + 0.432 \cdot S + 0.071 \cdot v$$
$$t(-2.654)\ (15.000)\ (5.907)$$
$$R^2 = 0.983; F = 177.497; DW = 1.387; N = 11$$
$$(139 \leq R \leq 6800nm; 241 \leq S \leq 544) \qquad \dots (1a)$$

- Operating costs

$$C(R, S) = 177.479 + 0.301 \cdot R + 21.956 \cdot S$$
$$t(0.285)\ (1.764)\ (4.967)$$
$$R^2 = 0.712; F = 82.655; DW = 1.636; N = 70$$
$$(139 \leq R \leq 6800nm; 241 \leq S \leq 544) \qquad \dots (1b)$$

- Fuel consumption

$$FC(R, S) = (1 - r_{fc}) \cdot (-107.149 + 0.00953 \cdot R + 0.280 \cdot S)$$
$$t(-13.013)\ (12.707)\ (11.809)$$
$$R^2 = 0.957; F = 267.980; DW = 2.536; N = 27$$
$$(6000 \leq R \leq 13500km; 241 \leq S \leq 544) \qquad \dots (1c)$$

$TP(S, v)$	technical productivity (10^3 seat-kt)
$C(R, S)$	operating costs ($US/BH) (BH - block hour)
$FC(R,S)$	fuel consumption (tons/flight)
S	aircraft capacity (seats)
V	aircraft cruising speed (kt) (1kt = 1nm/h; nm-nautical mile)
R	route length (nm; km)
r_{fc}	cumulative rate of improvements in aircraft fuel efficiency by the year (2050 vs 2019/2020)

As can be seen, technical productivity *(TP)*, (operational) cost *(C)*, and fuel consumption *(FC)* have been strongly driven by the aircraft capacity *(S)*, cruising speed *(v)*, and range *(R)*. Aircraft capacity *(S)* has been the strongest driving force in all three relationships. In addition to fuel consumption *(FC)*, it has also indirectly been the strongest driver of the corresponding GHG emissions.

Noise generated by commercial aircraft around airports, as one of the conditions for their certification, has been regulated at the local, national, and international level. Currently, all commercial aircraft meet the specified noise limits[5].

As far as safety is concerned, the accident rate of commercial aircraft has generally been decreasing over time; for example, from $0.55/10^6$ flights in 1998 to $0.03/10^6$ flights in 2017. Specifically, this rate with fatalities on the long-haul flights, carried out by the B777s and A330s aircraft between 1959 and 2016 was $0.20/10^6$ flights and $0.21/10^6$ flights, respectively. The most recent B787s, A350, and A380 have been without accidents with fatalities[6],[7].

2.2 Commercial supersonic aircraft

2.2.1 Past developments

Past development of commercial supersonic[1] aircraft had materialised in the commercialisation of the two aircraft – the French/British Aerospatiale/BAC Concorde and the Soviet Union's TY 144. The Concorde entered commercial service in 1976 and retired in 2003. The TY 144 entered the commercial service in 1977 and retired in 1978[8]–[11]. Both aircraft had a similar design, technical productivity, operating costs, fuel consumption, related GHG emissions, and noise. However, regarding the latest four above-mentioned features, they had not fulfilled expectations of the main actors/stakeholders. The airfares had been based on the high operating costs, mainly influenced by the high fuel consumption. This had been additionally compromised by constraining the overland operations at the supersonic speed (M > 1) aimed at mitigating and/or avoiding excessive noise from the sonic boom(s). Under such circumstances, these aircraft had been inferior to their slower subsonic counterparts, the B 707 and the B 747, regarding the operational costs and consequently airfares[12]–[14]. Additionally, the accidents (crashes) of the Concorde (25 Jul 2000, Paris) and the TY 144 (3 June 1973, Paris Air Show) raised some concerns about their safety, which consequently speeded up their retirement[15].

2.2.2 Past and current research and development

After the retirement of the Concorde and the TY144, research and development of supersonic aircraft continued. One of the earliest efforts dealt with identifying the relevant research topics regarding the design, economic efficiency, and safety of these aircraft. This was initially elaborated by improving the existing and developing some innovative research techniques[27]. This was followed by elaborating the concepts of commercial supersonic transport aircraft in terms of identifying new research opportunities regarding critical technologies and areas needing continuous development. These included the airframe design, control systems, engines, and materials, as well as the issue of reducing the sonic boom, fuel consumption and GHG emission, improving in-flight safety, and the certification requirements[28,29]. Further research summarized the developments of the concepts of supersonic aircraft over the past 30 years, from the engineering, economic, and safety/environmental/social perspective[30]. This was certification by research on the possible introduction of LH_2 as fuel for commercial air transportation. It elaborated the necessary conditions for smooth transition from conventional (Jet A) to new (LH_2) fuel, including the necessary modifications of the aircraft design. This resulted in the development of the concept of both subsonic and supersonic aircraft powered by LH_2[20],[31]. In particular, comprehensive research on developing the concept of a large supersonic aircraft, including the overall long-term aspects related to the high-speed transport, was carried out. For example, two EC (European Commission) projects, LAPCAT and ATLLAS, developed the methodology for aircraft design including optimal integration of their airframe, engines, and materials. Additionally, some dedicated experiments were carried out to evaluate the overall feasibility of the proposed design under different operating conditions[24],[25]. RAMJET or SCRAMJET engines were also specifically explored as

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[1]The current commercial aircraft operate at the subsonic speed of: $M < 1.0$. The other relevant speeds in the given context are: i) Transonic ($0.8 < M < 1.3$; 430 - 794 kt or 980 - 1470 km/h); and ii) Supersonic ($1.3 \leq M \leq 5.0$; 794-3308 kt or 1470 - 6126 km/h) (4).

propulsion systems for supersonic aircraft, including the challenges in their development, technical/technological and operational feasibility[32],[33].

As far as the most recent endeavours of the airspace industry are concerned, Boeing has announced the development of a hypersonic aircraft with the cruising speed of about $M = 5$. It is thought that it will be operational by the late 2030s[34]. Additionally, three U.S.-based start-up companies, Aerion, Spike, and Boom, are developing the new supersonic aircraft expected to be operational in the mid-2020s (see Table AI-1)[22],[23],[35]. The operational compatibility of the future supersonic aircraft has also been under scrutiny, with regard to the current and future ATC (air traffic control) system, and the corresponding flight operational rules and procedures. In particular, the prospective benefits and barriers to integrating these aircraft seamlessly in the U.S. NAS (National Airspace System) have been considered[12],[21],[39].

Several research efforts have recently focused on the assessment of the potential market for the supersonic flights. One deals with the development of the air transport market, including its long-haul segment where the supersonic aircraft would most likely operate[36]. The other deals directly with the estimation of the global market potential for supersonic transportation by evaluating worldwide data on the premium ticket sales[37].

One of the main concerns in developing the new concepts of supersonic aircraft has continued to be their economic efficiency. This has expected to be mainly influenced by the costs of substantial fuel consumption. The airfares based on such costs could make them eventually attractive primarily for business passengers with the presumably high value of their time. However, some research has indicated that if these aircraft were large and powered by LH_2 fuel charged at the reasonable prices, their costs and related airfares would be quite competitive to those of their current subsonic counterparts[24]. Since the environmental and social impacts of supersonic aircraft powered by Jet-A fuel (kerosene) have expected to be substantial, the corresponding GHG emissions and their noise have been also under scrutiny. Some research has reviewed the environmental issues and challenges of relevance to the design of supersonic business jets. Due to the inherent interrelation of the above-mentioned performance, a multidisciplinary design, analysis and optimisation have been considered as necessary for creating "low-boom" "low-drag" supersonic aircraft[38]. Additionally, a preliminary assessment of noise and GHG emissions by supersonic aircraft has indicated that their most likely design would not enable fulfilment of the current (2018/19) global standards for GHG emission of and local-airport noise. Particularly, according to the operating scenarios on the selected routes, their average fuel consumption and related GNG emissions (per passenger) would exceed those of their current subsonic counterparts several fold[18]. Also, the regulation of operations of the new supersonic aircraft, which is already underway, is primarily related to the sonic boom currently restricting their overland operation. One of these has been the U.S. FAA (Federal Aviation Administration) initiative for creating federal and international policies, regulations, and standards to certify the safe and efficient operation of civil supersonic aircraft[12],[19],[39].

The above-mentioned concepts of supersonic aircraft have been thought to carry out the long-haul flights with a rather positive balance between their effects and impacts. The effects have included travel speed, technical productivity, and economics. The impacts have included fuel consumption, GHG emissions, and noise. Some of these thoughts have also been systematically articulated as far-term (beyond the year 2035) research objectives of the two of the six strategic thrusts of the U.S. NASA strategic research program. The two relevant strategic thrusts have been "Innovation in Commercial Supersonic Aircraft" and "Transition to Alternative Propulsion and Energy"[16]. Consequently, the general requirements for design and operation of these aircraft have as follows[4],[8],[16]–[23]:

- Sufficient range for operating along the current and future long-haul, including the longest-haul non-stop routes;
- Flight costs and related air fares comparable to those of their subsonic counterparts and consequently attractive for both airlines and different categories of air passengers (low, medium, high income; business, leisure);
- Fuel consumption and related GHG emissions at the level at least neutral compared to their current subsonic counterparts; and
- Noise around airports and the sonic boom overland within the existing and forthcoming regulatory limits, the former similar to their subsonic counterparts.

The causal (regression) relationship between the technical productivity *(TP)*, cruising speed *(v)*, and capacity *(S)* of the past and current concepts of supersonic aircraft is estimated as follows (based on data in Table AI-1 (Appendix I)):

$$TP\,(S, v) = -308.004 + 0.959 \cdot S + 0.307 \cdot v$$
$$t\,(-10.575)\,(6.442)\,(10.467)$$
$$R^2 = 0.987; F = 296.077; DW = 1.553; N = 10 \quad (12 < S < 300; 860 < v < 2867\text{kt}) \quad \dots (2)$$

As can be seen, the aircraft capacity *(S)* and cruising speed *(v)* have very strongly influenced on technical productivity *(TP)*. Again, this influence is about three times greater thanks to the seat capacity (coefficient at the variable *(S)*) than thanks to the maximum speed (coefficient at the variable *(v)*). Additionally, the influence of speed on the technical productivity *(TP)* is for more than four times greater than in the case of subsonic aircraft (see Eq. 1a).

In regard to economics, some estimates indicate that the price of currently developing Boom Aircraft will be about $200 \cdot 10^6$ \$US and that of the previous EC LAPCAT Hydrogen Mach 5 Cruiser A2 $640 \cdot 10^6$ €. Based on the advertised prices on particular long-haul routes, the average operating cost of Boom Aircraft could be less than about 1.5 - 1.8 ¢/s-nm and that of the EC Hydrogen Mach 5 Cruiser A2 about 6.2 ¢/s-nm (s-nm – seat-nautical mile)[18],[22],[24],[25]. The fuel consumption, GHG emissions, and noise of these aircraft are expected to be in line with the above-mentioned requirements. In particular, the noise from the sonic boom is expected to be reduced to the level of about 70-80dB primarily through aircraft design and the increase in their cruising altitude, both of which could eventually allow for unrestricted overland operations[22].

3.0 MODELLING PERFORMANCE OF AN AIR ROUTE NETWORK

3.1 Objectives

The above-mentioned overview has indicated the existence of long-standing research efforts to develop the concepts of supersonic aircraft. However, those dealing with the systematic analysing, modelling, and comparing performance with those of the subsonic aircraft have been fragmentary or non-existent. This especially applies to the consideration of different operating scenarios including competition or eventually full replacement in the given (mainly long-haul) air route networks. Therefore, the objectives of this paper are to partially decrease this fragmentation and mainly increase interest in the topic within the academic community.
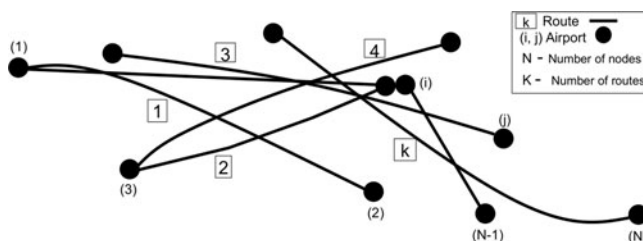
Figure 1. Simplified scheme of a long-haul air route network.

As such, the paper provides a framework for the systematic examination of the performance of a given long-haul air route network, exclusively operated by the prospectively forthcoming supersonic aircraft and their subsonic counterparts, according to the "what-if" scenario. This includes definition of the indicators of the above-mentioned network performance, development of their analytical models, and application of the models to the selected network case.

3.2 Assumptions

The models of performance indicators of the given air route network are based on the following assumptions reflecting the "what-if" operating scenarios:

- The air route network has a point-to-point spatial configuration consisting of airports as the O-D (origin-destination) nodes of aircraft/flights and their passengers, the long-haul air routes[2] as the network links as shown in Fig. 1;

- The characteristics of both subsonic and supersonic aircraft/flights are considered in modelling the particular indicators;

- The supersonic aircraft are fully operational including sufficiently (long) range for flying non-stop on all routes of the network;

- The airlines operate either a fleet of subsonic aircraft powered by Jet A or a fleet of supersonic aircraft powered by LH_2 (Liquid Hydrogen) fuel; both fleets are homogenous, i.e. n consist of the same aircraft types;

- The profiles of subsonic flights continue to be as at present both around the airports (following the SIDs (standard instrument departure) routes and STARs (standard terminal arrival route(s)), and en-route (4D RNAV trajectories). The profiles of supersonic flights need to be standardised. This implies that the ATC (air traffic control) would assign the aircraft dedicated three-dimensional (airspace) corridors mainly separated from the subsonic traffic. These would enable: i) taking-off from the origin airport and proceeding along the dedicated SIDs through the terminal area of the origin airport; ii) leaving the terminal area, climbing up to the cruising altitude while accelerating to the supersonic cruising speed; iii) cruising with the constant supersonic speed on the constant cruising altitude; iv) ending cruising and descending from the cruising altitude, while decelerating to the entry speed of the terminal area of the destination airport; v) entering the terminal area and proceeding along the dedicated STARs, again separated from the

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[2]The long-haul non-stop routes are considered to be those with the length from 6000 to 8000 nm (nm - nautical mile; 1nm = 1.852 km).
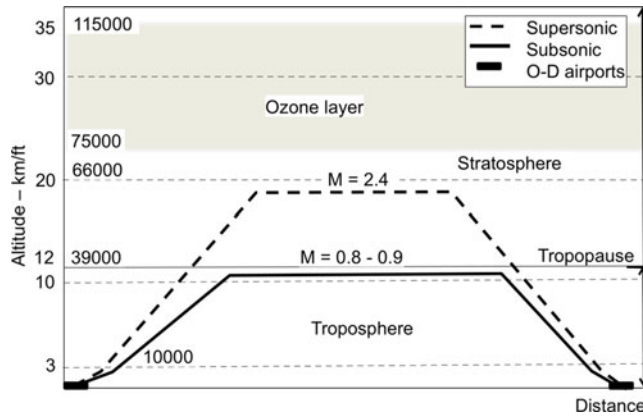
Figure 2. Simplified scheme of the vertical profile of subsonic and supersonic flight(s).

subsonic arriving traffic; and vi) entering the final approach path and landing. During take-off and landing, these aircraft are considered as in the heavy (or super heavy) wake-vortex category. Ideally these aircraft could share the SIDs and STARs of their subsonic counterparts within the corresponding noise constraints[12],[39]; a simplified scheme of the vertical profile of both categories of flights is shown in Fig. 2.

- A single airline or several airlines and/or their alliances operate in the network. Their market relationships, such as collaboration or competition, are not considered. The number of scheduled flights and their load factors on each network route are the same; this implies equality/uniformity of the O-D passenger demand accommodated by these flights under the given conditions.

- The direct fuel consumption and related GHG emissions of both categories of aircraft/flights are considered only.

- The GHG from burning particular fuels (Jet A and LH_2) are assumed to impact the environment independently, i.e. without interrelating with each other[40].

- The airport airside infrastructure (runways, taxiways, and apron/gate complex) is assumed to be suitable for accommodating supersonic aircraft safely, effectively, and efficiently; in general some modifications of the apron/gate parking stands and provision of LH_2 fuel delivery would be needed.

3.3 The models of performance indicators

The analytical models of performance indicators for the air route network shown in Fig. 1 are developed for its representative (average) route. As such, they can be applied to each network route and estimated in both absolute and relative terms. The corresponding values for the entire network can be obtained by adding up these estimated values for all routes. The models of indicators of infrastructural and technical/technological performance are assumed to be implicitly given and therefore not modelled.

3.3.1 Indicators of operational performance

The operational performance indicators are passenger demand, flight frequency on a route and network aircraft turnaround time on a route, transport work, technical productivity, and the size of the aircraft fleet.

Passenger demand and flight frequency on a route: The passenger demand on a network route *(k)* in the single direction, served by either category of aircraft during time *(Δτ)*, is assumed to be $(Q_k(\Delta\tau))$[37]. The flight frequency by either aircraft category to accommodate this demand can be estimated as follows[13]:

$$f_k (\Delta\tau) = Q_k (\Delta\tau) / (\lambda_k \cdot S_k) \qquad \ldots (3a)$$

k route in the given network *(k = 1,2,..., K)*;

$\Delta\tau$ time interval in which the flights by either aircraft category are scheduled on route *(k)* (h, day, year);

λ_k, S_k average aircraft load factor and capacity, respectively, of a flight on route *(k)* carried out by either aircraft category $(\lambda_k \leq 1.0)$ (-; seats/dep);

K number of routes in the network.

Aircraft turnaround time on a route: The turnaround time of an aircraft of either category on route *(k)* is expressed as follows:

$$\tau_k (R_k) = t_{0/k/1} + t_k (R_{k/1}) + t_{0/k/2} + t_k (DR_{k/2}) \qquad \ldots (3b)$$

$R_{k/1}, R_{k/2}$ length of route *(k)* in direction (1) and (2), respectively (nm, km);

$t_k(R_{k/1}), t_k(R_{k/2})$ flight time by aircraft category *(i)* in the direction (1) and (2), respectively, on route *(k)*, (h);

$t_{0/k/1}, t_{0/k/2}$ handling time of an aircraft of either category at the apron/gate complex of the end airports, before operating on route *(k)* in direction (1) and (2), respectively, (h).

The time *(tk (Rk/1))* or *(tk (Rk/2))* in Eq.3b is approximated as follows:

$$t_k (R_{k/.}) = t_{k/cl/.} + t_{k/cr/.} + t_{k/de/.} + 1/2 \cdot t_{k/LTO} \qquad \ldots (3c)$$

$t_{k/cl/.}$ climbing time of a flight of either category operating on route *(k)* in the given direction (min);

$t_{k/cr/.}$ cruising time of a flight of either category operating on route *(k)* in the given direction (min);

$t_{k/de/.}$ descending time of a flight of either category operating on route *(k)* in the given direction (min);

$t_{k/LTO}$ LTO (Landing and Take-Off)[3] cycle of a flight of either category before and after operating on route *(k)* (min) *(i = 1, 2)*.

The other symbols are analogous to those in the previous equations.

The detailed analytical models for estimating the flight time components in Eq. 3c are given in Appendix II.

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[3]This includes the taxing-out and take-off time at the origin and the landing and taxing-in time at the destination airport[41].

Transport work, technical productivity, and size of the aircraft fleet: The transport work of the flight carried out on route *(k)* of the network by either category of aircraft during time *(Δτ)* is estimated as follows:

$$TW_k(\Delta\tau) = f_k(\Delta\tau) \cdot R_k \cdot \lambda_k \cdot S_k \qquad \ldots(3d)$$

where all symbols are analogous to those in the previous equations.

The technical productivity of the flight carried out on route *(k)* of the network by either category of aircraft during time *(Δτ)* is estimated as follows:

$$TP_k(\Delta\tau) = f_k(\Delta\tau) \cdot \lambda_k \cdot S_k \cdot \overline{v}_k(R_k) \qquad \ldots(3e)$$

$\overline{v}_k(R_k)$ average flight speed carried out by either aircraft category on route *(R_k)* in both directions (1) and (2) (km/h or kt).

From Eq. 3c, the average speed $\overline{v}_k(R_k)$ in Eq. 3e is expressed as follows:

$$\overline{v}_k(R_k) = R_k / \tau_k(R_k) \qquad \ldots(3f)$$

From Eq. 3a and 3b, the required fleet of either aircraft category to serve the network under given conditions is expressed as follows:

$$n(\Delta\tau) = \sum_{k=1}^{K} f_k(\Delta\tau) \cdot \tau_k(R_k) \qquad \ldots(3g)$$

The other symbols are analogous to those in the previous equations.

3.3.2 Indicators of economic performance

The indicators of economic performance are flight cost, cost of passenger time, and contribution to GDP (gross domestic product).

Flight cost: The total cost of a single flight carried out by either category of aircraft on route *(k)* of the network can be estimated as follows:

$$C_{k/T} = C_{k/F} + C_{k/o} \qquad \ldots(4a)$$

where

$C_{k/F}$ fixed cost of a flight carried out on route *(k)* ($US/flight);
$C_{k/o}$ is the variable, i.e., operating cost of a flight carried out on route *(k)* ($US/flight).

The fixed cost *(C_{k/F})* in Eq. 4a can be estimated as follows[42]:

$$C_{k/F} = \left(\frac{P \cdot ADR}{\sum_{k=1}^{K} f_k(UL)} \right) = \left[\frac{P \cdot (100 - RV/UL)}{\sum_{k=1}^{K} f_k(UL)} \right] \qquad \ldots(4b)$$

P aircraft price including the capital maintenance costs during its useful life ($US/aircraft);

ADR aircraft annual depreciation rate (%);

RV aircraft residual value (%);

UL aircraft useful life (years);

$f_k(UL)$ number of flights carried out on network route (k) during the aircraft's useful life.

The other symbols are analogous to those in Eq. 3a.

The operating cost $(C_{k/o})$ in Eq. 4a includes the cost of fuel, crew, maintenance, insurance, fees (airport, ATC), and others (www.PlaneStats.com). For the flights carried out by the subsonic aircraft this cost is usually estimated by the empirical data. For the flights carried out by the supersonic aircraft, this cost can be approximated by using the corresponding available data in combination with an analogy with the cost of their subsonic counterparts[24]. From Eq. 4a, the average cost is expressed as follows:

$$\overline{C}_k = \frac{C_{k/T}}{R_k \cdot \lambda_k \cdot S_k} = \frac{C_{k/F} + C_{k/o}}{R_k \cdot \lambda_k \cdot S_k} \qquad \dots (4c)$$

$\overline{c}_k$ average cost of a flight carried out on route (k) of the network ($US/p-km).

The other symbols are analogous to those in the previous equations.

As mentioned above, in combination with the route length, the average cost $(\overline{c}_k)$ in Eq. 4c can be considered as the basis for setting up the airfares.

Cost of passenger time: Using supersonic instead of subsonic flights is expected to bring savings in the passenger time and related costs. These average savings by a flight of either category carried out on route (k) can be estimated as follows:

$$\overline{c}_{k/sv} = \left[\alpha_k \cdot \left(t_{k/1}^* - t_{k/2}^*\right)\right] / (R_k \cdot \lambda_k \cdot S_k) \qquad \dots (4d)$$

$t_{k/1}^* t_{k/2}^*$ flight time on route (k) by subsonic $(i = 1)$ and supersonic $(i = 2)$ aircraft, respectively, (h) $(t_{k/1}^* > t_{k/2}^*)$;

α_k average value of time of a user/passenger travelling on route (k) ($US/h-p).

The other symbols are analogous to those in the previous equations.

Contribution to GDP: The average contribution of the subsonic or supersonic flight(s) carried out in the network to the GDP is estimated as follows:

$$\overline{r}_{GDP} = (GDP/V_{RPK}) \qquad \dots (4e)$$

$\overline{r}_{GDP}$ average GDP generated by commercial air transportation in the given region ($US/p-km);

GDP total GDP generated by commercial air transportation in the given region during the specified period of time ($US/year);

V_{RPK} output of commercial air transportation in the given region during the specified period of time (RPK/year) (RPK - revenue passenger kilometre).

Overall social-economic feasibility: The overall social-economic feasibility of the subsonic or supersonic flight(s) carried out on route (k) of the network is estimated as the

difference between its total cost and contribution to GDP. From Eq. 4 (c, d, e) and 5 d (see below) this equals:

$$\Delta c_k = \bar{r}_{GDP} - \left(\bar{c}_k + \bar{c}_{k/sv} + \bar{c}_{k/e} \right) \qquad \ldots (4f)$$

$\bar{c}_{k/e}$ average cost of GHG emissions of a flight carried out on route *(k)* of the network ($US/p-km).

The other symbols are analogous to those in the previous equations.

If *(Δc_k)* is positive, the flight on the route *(k)* is overall social-economically feasible, and vice versa.

3.3.3 Indicators of environmental performance

The indicators of environmental performance are fuel consumption, emissions of GHG, and contribution to the global warming and climate change.

Fuel consumption: The fuel consumption of subsonic flights in the network is estimated by using the available empirical data. That of the supersonic flights is estimated by the analytical models considering the mechanical forces acting on the aircraft during the particular phases of flight—climb, cruise, and descend—and the corresponding energy consumption. The summed quantity is then increased for the factor including the fuel consumed during the LTO cycle(s). In general, in each of the above-mentioned flight phases the fuel consumption is estimated as follows:

$$FC = \left[(1/\eta) \cdot T \cdot r \right] / e_f \qquad \ldots (5a)$$

T thrust force during the given phase of flight (N) (N - Newton);

r distance along which the thrust force *(T)* is applied (m, km, nm);

e_f specific energy of the fuel used (J/kg) (J - Joule);

η propulsion efficiency of the aircraft engines during the given phase of flight ($\eta_{to} < 1.0$).

After expanding Eq. 5a in Appendix III for the particular flight phases - climbing *(cl)*, cruising *(cr)*, descending *(de)*, and *(LTO)* cycle, the total fuel consumption of a flight carried out by supersonic aircraft on route *(k)* of the network can be estimated as follows:

$$FC_k = FC_{k/cl} + FC_{k/cr} + FC_{k/de} + (1/2) \cdot FC_{k/LTO} \qquad \ldots (5b)$$

GHG emissions: GHG emissions by subsonic aircraft can be estimated by using available empirical data. Based on Eq. 5b, the GHG emissions by both subsonic and supersonic flight are estimated as follows:

$$EM_k = FC_k \cdot \sum_{m=1}^{M} e_m \qquad \ldots (5c)$$

FC_k fuel consumption by a flight carried out on route *(k)* by either category of aircraft (kg, ton);

e_m emission rate of the *(m)*-th GHG from burning the given type of fuel (kg or ton of GHG/kg or ton of fuel);

M number of different GHG emitted by burning the given type of fuel.

If internalised, the total and average cost of GHG emissions of a flight carried out on route *(k)* by either category of aircraft can be estimated as follows:

$$C_{k/e} = FC_k \cdot \sum_{m=1}^{M} c_{e/m} \cdot e_m \quad \text{and} \quad \overline{c}_{k/e} = \frac{C_{k/e}}{R_k \cdot \lambda_k \cdot S_k} \qquad \ldots (5d)$$

$c_{e/m}$ unit charge of the *(m)*-th GHG ($US/kg, $US/ton).

The other symbols are analogous to those in the previous equations.

Contribution to global warming and climate change: The GHG emitted by subsonic or supersonic flight powered by either type of fuel (Jet A or LH_2) contribute to global warming and climate change. Each GHG has its GWP (global warming potential) estimated for the future long-term period (for example, 100 years ahead)[43],[44]. Based on Eq. 5c, the GWP of any subsonic or supersonic flight carried out on route *(k)* can be estimated as follows (tons of GHG/flight):

$$GWP_k = EM_k \cdot \sum_{m=1}^{M} GWP_m \qquad \ldots (5e)$$

GWP_m Global Warming Potential of the *(m)*-th GHG (-).

The other symbols are analogous to those in the previous equations.

The relative savings in GWP by carrying out the supersonic *(i = 2 - LH_2 fuel)* instead of the subsonic *(i = 1 - Jet A fuel)* flight(s) on route *(k)* of the network are estimated as follows:

$$\Delta GWP(k) = \left(1 - \frac{GWP(k, 2)}{GWP(k, 1)} \right) \cdot 100 \qquad \ldots (5f)$$

All symbols are analogous to those in the previous equations.

3.3.4 Indicators of social performance

The indicators of social performance are noise, congestion and delays, and safety.

Noise: The noise produced by the subsonic aircraft around airports has been permanently regulated and used as the criteria for their (noise) certification[5]. The noise by the forthcoming supersonic aircraft, in addition to that around airports, has been and will continue to be the subject to specific regulation along the (overland) segments of air routes due to the sonic boom[12],[39]. This noise by a supersonic flight passing above an observer on the ground can be estimated as follows[45]:

$$L_2 = L_1 - 20 \cdot log\, d_1/d_2 \qquad \ldots (6a)$$

L_1, L_2 noise at the reference distance *(d_1)* and at the distance *(d_2)* of an observer on the ground from the noise source, i.e. the flying over aircraft, respectively (dBA) *(d_1, d_2 - nm, km)*.

The distance (d_1) is very close to the noise source, i.e. the flying over aircraft. The distance (d_2) is estimated as follows[11]:

$$d_2 = H/\sin\theta \quad \text{and} \quad \theta = \sin^{-1}(1/M) \qquad \dots (6b)$$

H altitude of supersonic flight (m, ft);
θ Mach angle (°);
M Mach number ($M > 1$).

At present, the costs of noise as an externality by supersonic aircraft are quite uncertain and are therefore not elaborated in the given context.

Congestion, delays and safety: Both categories of flights are carried out under the equivalent operational conditions at all airports of the network. As mentioned above, they are assumed to be "ultimately" free from the substantial congestion and delays. The same applies to their safety, i.e. the risk of and actual occurrence of the air traffic incidents/accidents. Therefore, the corresponding indicators of this performance and related costs/externalities are not elaborated.

4.0 AN APPLICATION OF MODELS OF PERFORMANCE INDICATORS

4.1 Inputs

The indicators of particular performance of a given air route network are estimated by two categories of data: real-life input data on subsonic flights and hypothetical input data on "what-if" scenario-based supersonic flights. In both cases adjustments are made to reflect operations of the network and flights in the year 2050. This is assumed to be the year that supersonic flights will be launched. The input data is also categorised in regard to the particular performance.

4.1.1 Infrastructural performance

The indicators of infrastructural performance are represented by the characteristics of the existing air route network shown in Fig. 3 and given in Table 1.

The same network is assumed to be operated exclusively either by the above-mentioned subsonic aircraft or their supersonic counterparts in the year 2050.

4.1.2 Technical/technological performance

The fleet of subsonic aircraft contains the average aircraft type based on Eq. 1 (a, b, c). The simplified layout of considered supersonic aircraft is shown in Fig. 4 (the EC's LAPCAT Hydrogen Mach 5 A2 concept)[24],[25],[32].

Additionally, the design-related characteristics of an average subsonic and supersonic aircraft belonging to the corresponding fleets are given in Table 2. On the one hand, these characteristics can be considered as inputs; on the other, they can represent indicators of technical/technological performance of the network and flights.

4.1.3 Operational performance

The inputs for estimating the indicators of operational performance of the network are synthesised from the relevant empirical data (subsonic flights) and the hypothetical "what-if" operational scenario-based data (supersonic flights) given in Table 1 and 2.

Figure 3. Simplified geographical scheme of the air route network consisting of 25 world's longest routes in the given example (period: the year 2018) [65].
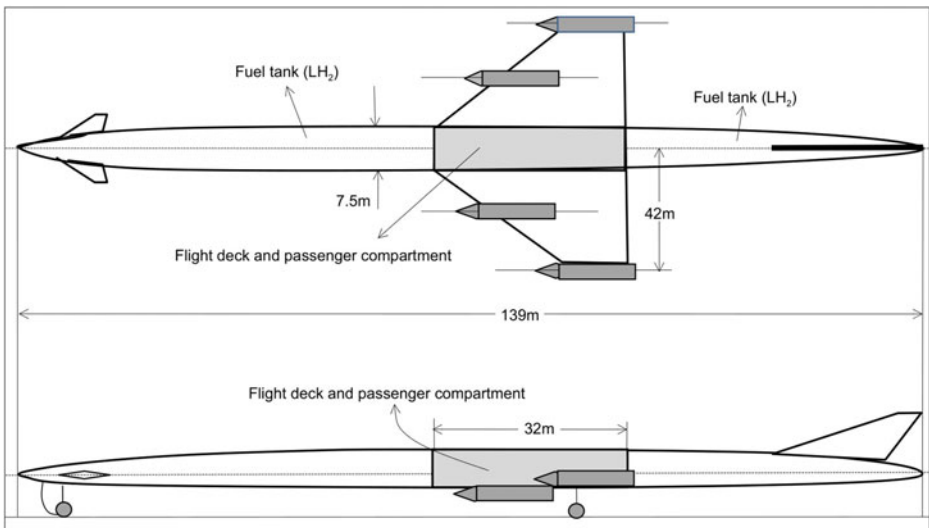

Figure 4. Simplified layout of supersonic aircraft in the given example (LAPCAT Hydrogen Mach 5 A2 concept) [24], [25].

4.1.4 Economic performance

Flight cost: The inputs for estimating the total operating cost of subsonic flight(s) are derived from Eq. 1b. The inputs for estimating the total operating cost of supersonic flight(s) operated at the speed $M = 2.4$ are derived as follows.

The price of a supersonic aircraft including the capital maintenance cost during the life-cycle is assumed to be: $P = 450 \cdot 10^6$ \$US. This is similar to that of the A380 aircraft[46],[47]. The aircraft residual value at the end of useful life of: $UL = 20$ years is assumed to be: $RV = 10\%$, which gives ADR (Annual Depreciation Rate): $= 4.5\%$/year[42]. The inputs for estimating the variable cost component are as follows: the fuel cost - 1.00 and 0.85 \$US/kg of

Table 1

Characteristics of the existing long-haul air route network and subsonic non-stop flights in the given example (Fig. 2) (Period: the year 2018) [65])

No.	Route	Route Length R (nm)	Flight Time $t(R)$ (h:min)	Aircraft Type
1	Auckland - Doha	7853	17:17	Boeing 777-200LR
2	Perth - London Heathrow	7837	17:20	Boeing 787-9
3	Auckland - Dubai	7672	16:34	Airbus A380-800
4	Los Angeles - Singapore	7620	17:21	Boeing 787-9
5	Houston - Sydney	7460	17:07	Boeing 787-9
6	Dallas/Ft. Worth - Sydney	7445	16:47	Airbus A380-800
7	San Francisco - Singapore	7333	16:57	Airbus A350-900
8	San Francisco - Singapore	7338	16:46	Boeing 787-9
9	Johannesburg - Atlanta	7327	16:04	Boeing 777-200LR
10	Abu Dhabi - Los Angeles	7285	16:08	Boeing 777-300ER
11	Dubai - Los Angeles	7241	15:32	Airbus A380-800
12	Jeddah - Los Angeles	7235	16:11	Boeing 777-300
13	Doha - Los Angeles	7212	15:55	Boeing 777-200LR
14	Manila - Toronto	7139	16:27	Boeing 777-300
15	Dubai - Houston	6622	15:48	Boeing 777-300ER
16	Dallas/Ft. Worth - Hong Kong	7057	16:08	Boeing 777-300ER
17	Dubai - San Francisco	7023	15:05	Airbus A380-800
18	Hong Kong - New York JFK	6992	15:18	Boeing 777-300ER
19	Hong Kong - Newark EWR	6994	15:18	Airbus A350-900
20	Hong Kong - Newark EWR	6994	15:21	Boeing 777-200ER
21	Doha - Houston	6523	15:54	Boeing 777-200LR
22	Dubai - Dallas/Ft. Worth	6983	15:26	Boeing 777-300ER
23	Shanghai - Mexico City	6973	16:55	Boeing 787-8
24	Guangzhou - New York JFK	6962	15:25	Boeing 777-300ER
25	Johannesburg - New York JFK	6936	15:33	Airbus A340-600
	Average:	**6532**	**16:00**	

LH_2[48]; the crew cost - 2000 \$US/h. The fixed, fuel, and crew costs are assumed to account for 70% of the total operating cost[49]. The inputs for both subsonic and supersonic flights are adjusted to the prospective conditions in the year 2050.

Cost of passenger time: The inputs for estimating the prospective savings in the cost of passenger time if using supersonic instead subsonic flights are represented by the average value of passenger time of: $\alpha_k = 74$ \$US/h-p. Based on 50% medium and 50% high income passengers, both performing 50% business and 50% leisure trips. This value is assumed to be also relevant in the year 2050 (h - hour; p - passenger)[50],[51].

Contribution to GDP: The inputs for estimating the average contribution of commercial air transportation to GDP are obtained from the long-term annual forecasts for global commercial air passenger transportation and its contribution to GDP[52]. For the year 2050, $GDP_{2050} = 5.4 \cdot 10^{12}$ \$US/year and $V_{RPK/2050} = 14.1 \cdot 10^{12}$ RPK/year, which gives: $\bar{r}_{GDP/2050} = (GDP/V_{RPK})_{2050} = 5.59 \cdot 10^{12}$ \$US/$14.1 \cdot 10^{12}$ RPK $= 0.3965$ \$US/p-km.

Table 2
Some design characteristics of subsonic and supersonic aircraft in the given example [24], [28], [32]

Design Characteristics	Aircraft Category	
	Subsonic[a] $M < 1.0$	Supersonic[b] $M = 2.4$
Length (m)	69.00	139
Wing span (m)	64.32	42
Wing aspect ratio	9.19	1.87
L/D (lift-to-drag) ratio (max)	17.77	8.8 [c]
Take-off weight (tons)	289	400
Number of engines (-)	2	4
Take-off speed (kt)	160	185
Cruising speed (Mach)	0.87	2.4
Landing speed (kt)	145	175
Capacity (seats)	300	300
Load factor (-)	0.70	0.70

[a] Averages of the aircraft types operating on 25 longest routes in the given example[62], [63], [64];
[b] Based on the EC Hydrogen Mach 5 Cruiser A2 [13], [24], [25];
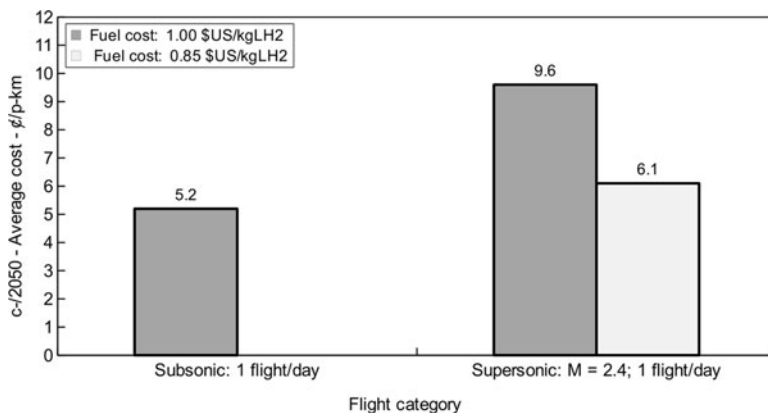[c] Based on $(L/D)_{max} = 4 \cdot (1 + 3/M)$ (M is Mach number) [72].

Figure 5. Indicators of economic performance: The average operating cost of particular categories of flights carried out on an average route of the network in the given example (period: the year 2050).

4.1.5 Environmental performance

Fuel consumption: The regression equation in Eq. 1c and the inputs in Table 2 are used for estimating the fuel consumption of subsonic flight(s). The inputs in Table 2, 3, and 4 are used in the corresponding models (Appendix III) to estimate the fuel consumption of the supersonic flight(s).

Table 3
Characteristics of supersonic flight(s) in the given example

Phase of Flight	Cruising Speed $M = 2.4$
Climb	
Altitude (H) $(10^3$ feet)	0-3-10-65
Climb angle (β_{cl}) $(^0)$	9.64
Flying speed (v) (kt)[a]	185-250-1376
Average rate of climb (R/C) (ft/min)[b]	2806
Average speed $(\bar{v}_{cl})$ (m/s)	306
Avg. longitudinal acceleration $(a_{cl}{}^+)$ (m/s^2)	0.468
Distance (r_{cl}) (km, nm)	289.7; 156.4
Time (t_{cl}) (min)[c]	$1.2 + 22.09 = 23.29$
Propulsion efficiency (η_{cl}) (-)	0.444
Coeff. of the aerodynamic drag: $C_{d/cl}$ $(\bar{v}_{cl})$ (-)[d]	0.3
Average air density $(\bar{\rho}_{cl})$ (kg/m^3)	0.4135
Cruise	
Altitude (H) $(10^3$ feet)	65
Air density (ρ_{cr}) (kg/m^3)[e]	0.08891
Coeff. of parasite drag at zero lift (C_{D0})[f]	0.006 or 0.001523
Flying speed (v) (m/s)	729
Effective aircraft length (l) (m)[g]	139
Wingspan (b) (m)	$2 \cdot 42$
Radius of the aircraft fuselage (r) (m)[g]	3.75
Total volume (Vol) (m^3)[g]	6138
Dynamic pressure (q) (N/m^2)	24189
Distance (r_{cr}) (km; nm)	11426/6169
Time (t_{cr}) (h)	8.303
Propulsion efficiency (η_{cr}) (-)[g]	0.444
Descend	
Altitude (H) $(10^3$ feet)	65-10-3-0
Descend angle (β_{de}) $(^0)$	9.23
Flying speed (v) (kt)[a]	1376-250-175
Average rate of descend (R/D) (ft/min)[b]	2619
Average speed $(\bar{v}_{de})$ (m/s)	309
Avg. longitudinal deceleration $(a_{de}{}^-)$ (m/s^2)	0.435
Distance (r_{de}) (km, nm)	382.4; 206.5
Time (t_{de}) (min)[c]	$23.67 + 1.34 = 25.01$
Propulsion efficiency (η_{de})(-)	0.444
Coeff. of the aerodynamic drag: $C_{d/de}$ $(\bar{v}_{de})$ (-)[d]	0.3
Average air density $(\bar{\rho}_{de})$ (kg/m^3)	0.4135

[a]Take-off speed is 185 kt; the landing speed is 175 kt; the speed at FL10 is 250 kt (FL10 = 10000ft);

[b] the rate of climb: R/C (h) = 9000 - 0.1308 · H; the rate of descent: R/D(H) = - 8000 + 0.1154 · H (based on Concorde and TY 144) (H is the altitude (10^3 feet));

[c] the climb/descend time: $t\,(H_1, H_2) = (1/b) \cdot \{\ln\,[a\,(H_1) - b \cdot H_2] - \ln\,[a\,(H_1) - b \cdot H_1)]\}$ (H_1, H_2 is the initial and the end altitude, respectively) [(8)];

[d] attack angle is $= 4^0$;

[e] At FL 60-65 (10^3 ft);

f [(28)];

g [(24)]; [(25)].

Table 4

Characteristics of aircraft fuels, emissions of GHG, costs/externalities, and GWP (Global Warming Potential) in the given example [66], [67]

Characteristic	Fuel Type/Value		
	Jet A	*LH$_2$*	*GWP^a*
SG - Specific Gravity (kg/L)	0.804	0.071	
SE – Specific Energy (MJ/kg)	43.2	120	
CO_2 (kg/kg of fuel)	3.162	0.0	1
H_2O (kg/kg of fuel)	1.230	3.128	0.1
NO_x (g/kg of fuel)^a	9.0-18.0	0.05-0.25	265
SO_x (g/kg of fuel)	0.084	0.00	-
PM (PM/kg of fuel)^b	3.0-6.0·10^{15}	0.00	-
HC (g/kg of fuel)	18.0	0.00	-
Emission rate (kgCO$_{2e}$/kg of fuel)^c	4.42358	3.12815	
Cumulative GWP ^{d}	6.86250	0.35255	
Avearge cost of GHG emissions ($US/tonCO$_{2e}$)			
• CO$_{2e}$ ^{e}	212	-	
• NO$_x$ ^{f}	-	86.4	

^{a}Direct combustion;
^{b} engine power is 45–100%; PM - Particulate Matters;
^{c} carbon dioxide equivalent;
^{d} global warming potential (values for 100-year time horizon) [43], [44];
^{e} high impact [48], [73];
^{f} high Impact [74].

GHG emissions, contribution to global warming and climate change, and costs/externalities: The inputs for estimating GHG emissions of both subsonic and supersonic flights relate to the characteristics of Jet A and LH$_2$ fuel, their contribution to global warming and climate change and the related costs as externalities is given in Table 4. The related cost of emissions of particular GHG is adjusted for the year 2050.

4.1.6 Social performance

The cruising altitude ranging as: $H = 36 - 60 \cdot 10^3$ ft and speed of $M = 2.4$ are used as the "what-if" scenario-based inputs for estimating the level of noise produced by supersonic flight(s) as experienced by an observer on the ground.

4.2 Analysis of the results

Based on the above-mentioned inputs, the performance indicators are estimated for the average (representative) route of the network where subsonic or supersonic flights are exclusively carried out. These estimates, however, do not compromise in any way the relevance of findings and the related conclusions referring to the entire network. If needed, the corresponding inputs for estimating the performance indicators of each individual route can be estimated in order to obtain the corresponding totals for the entire network.

Table 5

Indicators of operational performance of an average route of the network in the given example: Transport work, productivity, and required fleet of particular aircraft categories (period: the year 2050)

Inputs/Aircraft category	Subsonic $M < 1.0$	Supersonic $M = 2.4$
$(S \cdot \lambda)$ - Number of passengers onboard [a]	210	210
$t(R)$ - Average flight time (h) [b]	16.00	8.68
$\bar{v}$ - Avearge aircaft speed (kt; km/h) [c]	408; 756	1164; 2155
$\tau(R)$ - Turnaround time along the route (h) [d]	$2 + 2 \cdot 16 = 34$	$4 + 2 \cdot 8.68 = 21.36$
TW - Transport work (10^3 p-km/route/day)	2540	2540
TP - Productivity (10^3 p-km/h/day)	158.760	452.55
$F(\cdot)$ - Number of flights per route/year [e]	329	329
$n(\Delta \tau)$ - Required fleet for the network [f]	35	24

[a] Based on the aircraft capacity of S = 300 seats/dep and load factor λ = 0.7;
[b] includes the standard LTO cycle of t_{LTO} = 34.38min (Masiol and Harrison, 2014) (h - hour);
[c] based on the average route length of R = 6532 nm (12097km) (Table 1) and the average flight duration t(R);
[d] based on the aircraft handling time at the origin and the destination airport of τ_0 = 1 h;
[e] based on the aircaft availability of U = 90%/year and the daily frequency of f(△ τ) = 1dep/day;
[f] based on the flight frequency of f(△ τ) = 1dep/day.

4.2.1 Infrastructural and technical/technological performance

As mentioned above, the indicators of infrastructural and technical/technological performance are not particularly elaborated. They are assumed as given in the inputs for estimating those of the other performances.

4.2.2 Operational performance

The inputs in Tables 1, 2, and AI-1 (Appendix I) are used for estimation of the indicators of operational performance as given in Table 5.

As can be seen, the flight time by the supersonic aircraft operating at a speed of $M = 2.4$ would be almost two times shorter than that of their subsonic counterparts. Consequently, thanks to the shorter route turnaround time, the required fleet of supersonic aircraft would be lower by about 46%. The transports work on an average route and consequently in the network would be equal for both categories of flights. This is mainly due to the equal flight frequencies, seat capacity, load factor, and the average route length. However, thanks to the higher cruising speed, the technical productivity of supersonic flights would be about 2.9 times greater than that of their subsonic counterparts.

4.2.3 Economic performance

Flight cost

The total operating cost of a subsonic flight is derived from Eq. 1b and the adjustments reflecting the expected conditions in the year 2050, as follows: $C(R, S)_{2050} = [(a_0 + a_1 \cdot R + a_2 \cdot S)_{2019} \cdot t(R)] [(1 - p_{fc}) + p_{fc} \cdot (1 + i_r)^{30} \cdot (1 - r_{fc})] = [(177.479 + 0.301 \cdot 6532 + 21.965 \cdot 300) \cdot 16] \cdot [(1-0.4) + 0.4 \cdot (1 + 0.012)^{30} \cdot (1 - 0.4)] = 131761$ \$US/flight. The average cost per flight is equal to: $\bar{c}_{-/2050} = C(R, S)_{2050} / (R \cdot \lambda \cdot S) = 131761/(6532 \cdot 1.852 \cdot 0.7 \cdot$

Table 6
Indicators of economic performance of an average route of the network in the given example: Cost of passenger time and its potential savings at particular categories of flights (period: the year 2050)

Inputs	Flight Category	
	Subsonic $M < 1.0$	**Supersonic** $M = 2.4$
$t(R)$ -Average flight time (h)	16.00	8.681
α - Value of passenger time ($US/p-h) [a]	74	74
Cost of time per flight ($10^3$$US/flight) [b]	248.640	134.903
Total costs of time in the network ($10^3$$US/day) [c]	6216	3372.56
Total savings in the cost of time ($10^3$$US/day) [c]	0	2843.50
$\bar{c}_{sv/2050}$- Average savings in the cost of time ($US/p-km)	0	0.0448

[a] *Based on 50% medium- and 50% high-income passngers on board and their 50% business and 50% leisure trips* [50], [51];
[b] *based on the route length of $R = 12097$ km (6532nm), the aircraft seat capacity of $S = 300$seats/dep, and the load factor of $\lambda = 0.7$;*
[c] *based on 25 flights/day in the network.*

300) $= 0.052$ $US/p-km ($p_{fc} = 0.4$ is the share of fuel cost in the total aircraft operating cost in Eq. 1b[49],[53]); $r_{fc} = 0.4$ is the cumulative rate of improvement in aircraft fuel efficiency in the year 2050 vs 2019/2020[54]; $i_r = 1.2\%$ is the annual rate of increasing prices of crude oil, which proportionally influences an increase in the price of Jet-A fuel during the period 2018/19-2050[55].

The total cost of a flight carried out by supersonic aircraft is estimated as follows:

The fixed cost of the fleet of 24 aircraft ($M = 2.4$) in Table 5, carrying out 329 flights/year on each of 25 routes of the network in Table 1, is estimated as follows: $C_{F/2050}$ $= [24 \cdot (450 \cdot 10^6) \cdot 0.045]/(329 \cdot 25) = 59088$ ($US/flight)[24], the crew cost: $c_{cw/2050} = 2000$ $US/h $\cdot$ 8.681 h $= 17362$ $US/flight[56], and the fuel cost: $c_{fc/2050} = 93516$ kgLH$_2$/flight $\cdot$ 1$US/kg LH$_2 = 93516$ $US/flight (see also below)[48]. If the above-mentioned (three) cost components are assumed to account for about 70% of the total flight operating cost, the corresponding total cost will be: $C_{T/2050} = (C_{F/2050} + c_{cw/2050} + c_{fc/2050})/0.7 = (59088 + 17362 + 93516)/0.7 = 242809$ $US/flight. The average cost of single flight carried out on the route: $R = 6532$ nm (12097 km) (by the aircraft of: $S = 300$ seats and load factor: $\Lambda = 0.70$ is equal to: $\bar{c}_{-/2050} = C_{T/2050}/(R \cdot \lambda \cdot S) = 242809/(6532 \cdot 1.852 \cdot 0.7 \cdot 300) = 0.096$ $US/p-km. If the fuel cost is 0.85 $US/kgLH$_2$[48], the corresponding average cost of the supersonic flight would be: $\bar{c}_{-/2050} = 155939/(6532 \cdot 1.852 \cdot 0.7 \cdot 300) = 0.061$ $US/p-km. Figure 5 shows these average cost per flight.

As can be seen, at the frequency of 1 flight/day, the average operating cost of the supersonic flight, depending on the fuel cost, would be between 18% (0.85 $US/kgLH$_2$) and 85% (1 $US/kgLH$_2$) higher than that of the subsonic flight. This example indicates that the supersonic flights would generally be economically inferior to their subsonic counterparts under the given conditions.

Cost of passenger time: The cost of passenger time and the potential savings in this cost by using the supersonic instead of the subsonic flight(s) are given in Table 6.
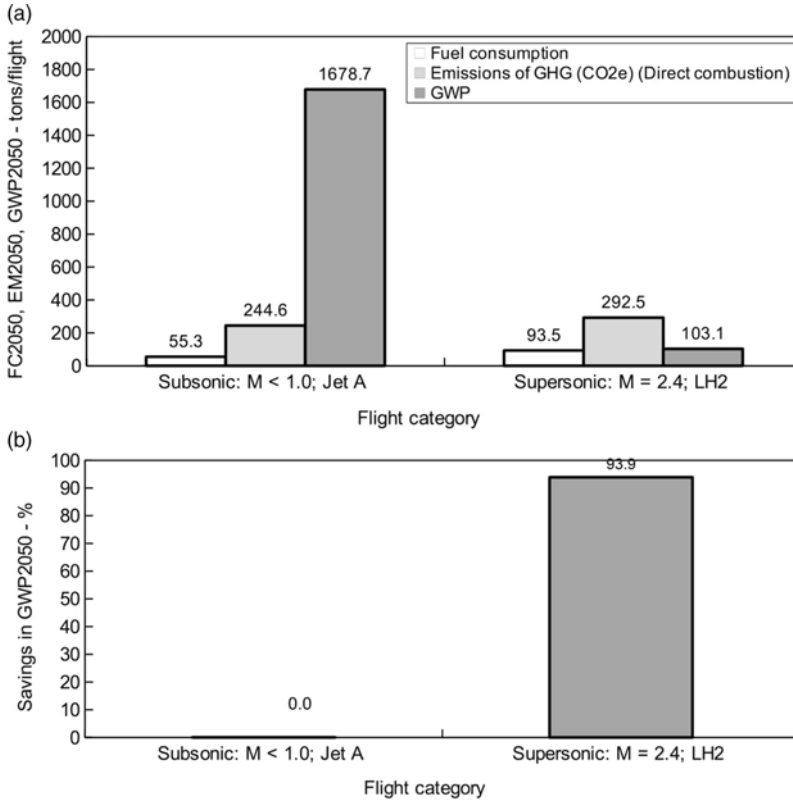
Figure 6. Indicators of environmental performance of particular categories of flights carried out on an average route of the network in the given example (period: the year 2050) a) fuel consumption, emissions of GHG, and GWP; b) savings in the contribution to GWP.

As can be seen the cost of passenger time would be about 50% lower for the supersonic flights. The corresponding savings in this cost would be about 4.5 ¢/p-km.

4.2.4 Environmental performance

The fuel consumption of subsonic flights in the given example is derived from Eq. 1c, while respecting the prospective improvements in the aircraft fuel efficiency of $r_{fc} \approx 0.4$ by the year 2050[57]. Consequently, the fuel consumption of a flight carried out by an aircraft with a seat capacity of $S = 300$ seats along the route $R = 12097$km (6532nm) would be $FC_{2050}(R) = 55.3$ tons/flight (Jet A fuel). Under the same conditions, by applying the models in Appendix III to the inputs from Table 4, the average fuel consumption of the supersonic flight is estimated as: $FC_{2050}(R) = 1.02 \ (FC_{cl} + FC_{cr} + FC_{de}) = 1.02 \cdot (9245 + 76089 + 6339) = 1.02 \cdot 91687 \approx 93516$kg/flight (LH$_2$). In this case the factor 1.02 is applied to include the fuel consumed during the LTO cycle. The inputs in Table 4 and the above-estimated fuel consumption are used for estimating GHG emissions and their absolute and relative contribution to global warming and climate change as shown in Fig. 6 (a, b).

Figure 6a shows that the average fuel consumption by the supersonic flight would be about 70% higher than that of the subsonic flight. The corresponding GHG emissions would be also higher by about 19%. At the same time, the GWP of the subsonic flight would be about 16
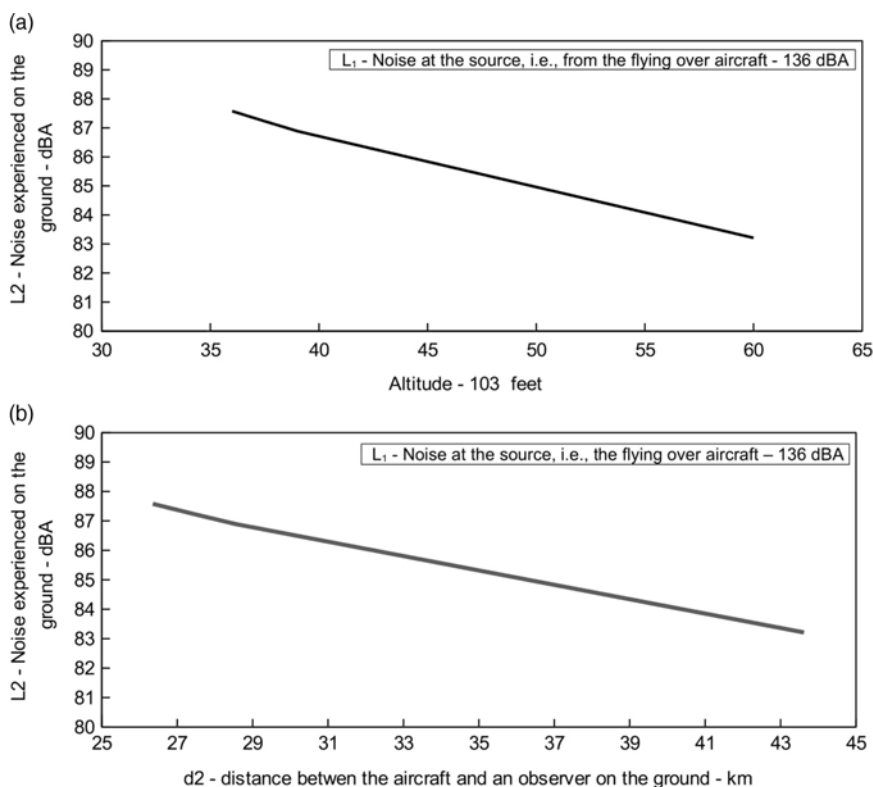
Figure 7. Indicators of social performance: Relationship between the noise levels generated by supersonic flight ($M = 2.4$) passing above an observer on the ground in the given example (period: the year 2050). a) Noise vs cruising altitude; b) noise vs distance from an observer on the ground.

times higher than that of its supersonic counterpart (CO_2 and H_2O dominate in Jet-A and H_2O in LH_2 fuel). Figure 6b shows that despite the higher fuel consumption and related GHG emissions, the supersonic flight(s) could substantially contribute to savings (about 94%) in the overall GWP and consequently global warming and climate change, both compared to their subsonic counterparts.

4.2.5 Social performance

The noise generated by the supersonic flight(s) operating on an average route of the network is shown in Fig. 7 (a, b).

Figure 7a shows that the noise produced by a supersonic flight passing above an observer on the ground would decrease with the increase of the cruising altitude. Figure 7b shows that increasing the distance between the overflying aircraft and an observer on the ground, due to an increase of the cruising altitude, would contribute to the decreasing of the experienced noise of an observer on the ground. The levels of noise generally between 83 and 88 dBA do not reflect barely audible explosion (physical phenomenon) and have been the subject of undesirable psychological reactions. As such, these noise levels appear to be about 6-13% above U.S. NASA's suggested tolerable levels from the sonic boom, set at about 78 dB[39]. Additionally, the size of the area covered by the noise from the Mach cone needs to be taken
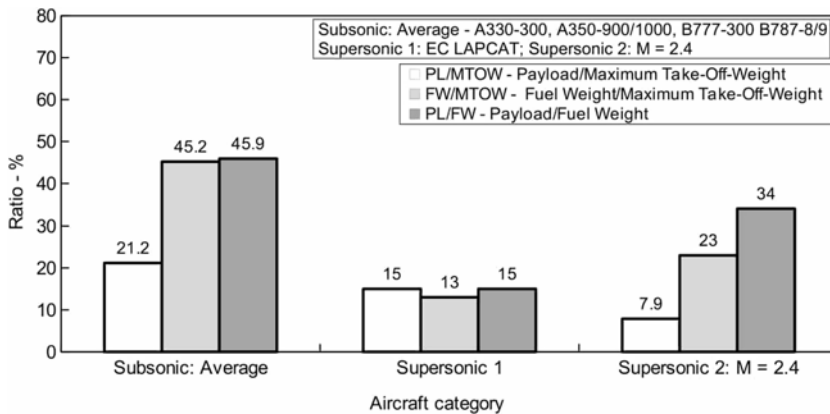
Figure 8. Some (design-related) derived indicators of technical/technological performance of subsonic and ssssupersonic aircraft in the given example (Period: the year 2050) [24], [28], [32], [62], [63].

into account. Both these are and will certainly be used as inputs in considering the possible noise constraints on operations of the supersonic aircraft[12],[39],[58].

4.2.6 Some derived indicators of performance

Aircraft design: The maximum take-off weight, payload including only the passengers and their baggage, and the fuel consumption allow for the estimation of some the design-related derived indicators of the technical/technological performance of both subsonic and supersonic aircraft. These are expressed by the ratios such as: PL/MTOW (Payload/Maximum Take-Off-Weight), FW/MTOW (Fuel Weight/Maximum Take-Off-Weight), and PL/FW (Payload/Fuel Weight) as shown in Fig. 8.

As can be seen, the particular ratios would be quite different for subsonic and considered supersonic aircraft. For example, the ratio PL/MTOW is about 21% for subsonic and 8% at supersonic aircraft. (Full payload for supersonic aircraft: 31500kg; 1 passenger + baggage = 105kg[59]). The ratio FW/MTOW is about 45% for subsonic and 23% for supersonic aircraft, the latter also influenced by the fuel type. Finally, the rate PL/FW is about 46% for subsonic and 34% for supersonic aircraft, the latter again influenced by the fuel type.

Economics and environment: The derived indicators of economic and environmental performance of an average route and the entire network are considered through the relationship between the flight operating cost, the cost of GHG emissions, i.e. externalities, and the savings in the cost of passenger time, as shown in Fig. 9 (a, b).

Figure 9 shows the difference between the average total cost and its components of the subsonic and supersonic flight on an average route of the network. As can be seen, the average total cost would be lower for the supersonic than for the subsonic flight, by about 20%. This would be achieved thanks to its lower externalities and higher potential savings in passenger time despite the higher operational costs. Consequently, this example indicates that internalising all costs of the particular actors/stakeholders involved in both the demand and the supply side of the given air route network could eventually make supersonic flights economically feasible under the given conditions.

The overall social-economic feasibility: The relationship between the average total monetary contribution to the GDP and the average total cost of both subsonic and supersonic flight(s) carried out on an average route of the network is shown in Fig. 10.
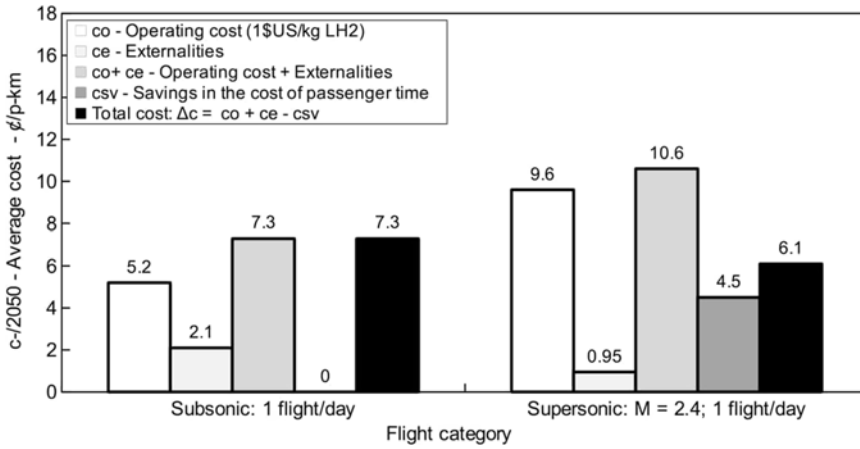
Figure 9. Indicators of the economic performance: The average costs of particular categories of flights carried out on an average route of the network in the given example (period: the year 2050).
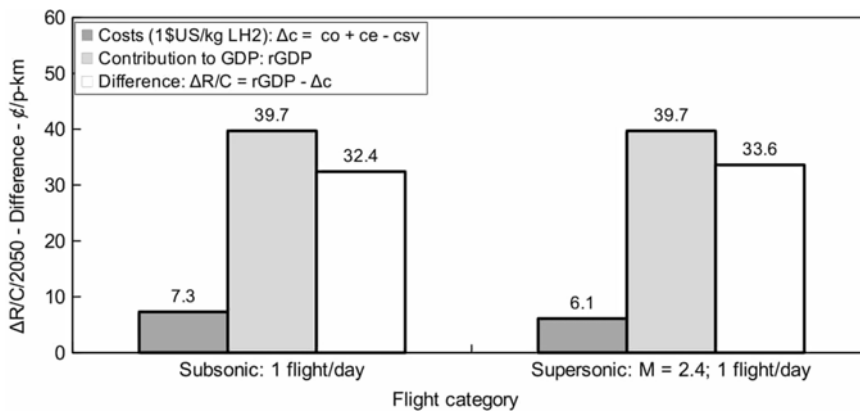

Figure 10. Indicators of social-economic performance: The average contribution to GDP and the average total costs of particular categories of flights carried out on an average route of the network in the given example (period: the year 2050).

As can be seen, in the cases of both subsonic and supersonic flights the average contribution to GDP would be overall higher than the average total cost thus making their difference generally net positive. This difference would be about 5% greater for supersonic flights compared to subsonic flights. The above figures indicate that supersonic flights could eventually be overall social-economically feasible but only under the considered circumstances.

The above-mentioned results enable synthesising some qualitative pros and cons of supersonic flights, relevant for the particular actors/stakeholders involved, which are summarised as follows:

These pros and cons indicate that the full implementation of the future supersonic commercial flights is and will remain a challenge for all above-mentioned main actors/stakeholders involved.

ACTOR/STAKEHOLDER	PROS	CONS
Aircraft manufacturers	• Potential profitability of manufacturing the aircraft	• Complexity in delivering the economically, environmentally, and socially feasible aircraft design • Inherent risk of failure to deliver the expected/required design
Airlines	• Shorter flight time and higher technical productivity enabling more flights if justified by air passenger demand • Smaller fleet size • Lower costs of GHG emissions (if internalised)	• Inherent uncertainty in deployment • Higher operating cost requiring higher airfares, which could compromise expected air passenger demand
Airports	• Do not require substantial re-design (modification) of the airside and landside area	• Provision of the supply of the new (LH_2) fuel • Noise as the subject for further regulation
Users/air passengers	• Substantial savings in time cost	• Higher potentially deterring airfares based on higher operational cost and internalised externalities (GHG emissions)
Local communities	• Availability of supersonic air transport services at the nearby airports	• Additional costs of insulating or relocating houses due to the increased noise if these would not be regulated similarly to current subsonic aircraft/flights
Policy makers/society	• Contribution to reducing global warming and climate change • Overall social-economic/ feasibility	• Higher fuel consumption and related GHG emissions, particularly the impact from the LH_2 generated NO_x on O_3 (Ozone) layer • Noise around airports and from the sonic boom, as the subject of further regulation • Uncertainty and inherent complexity of implementation • Complexity in internalizing and then considering all direct and indirect impacts/costs and effects/revenues in assessing the overall social/economic feasibility/benefits

6 CONCLUSIONS

This paper has modelled the performance of a given long-haul air transport network operated by existing subsonic and prospectively forthcoming supersonic commercial aircraft. Infrastructural, technical/technological, operational, economic, environmental, and social performance has been considered. Analytical models of the performance indicators have been developed.

The indicator models have been applied in estimating the performance of an existing air route network, consisting of 25 long-haul routes operated exclusively by commercial subsonic aircraft (currently) and supersonic (prospectively by the year 2050) aircraft.

The input for the estimation of the performance indicators of the network and its average route was empirical data for subsonic aircraft/flights, available data from the corresponding design concepts of supersonic aircraft/flights, and the elements of the specified "what-if" operational scenarios The subsonic aircraft are assumed to be powered by Jet-A fuel and the supersonic aircraft by LH_2 (Liquid Hydrogen) fuel.

The results have indicated that:

- The supersonic and subsonic flights carried out on the same routes with the same frequency, seat capacity, and load factor would perform the same transport work. The supersonic flights would have about 2.9 time higher technical productivity thanks to the much higher cruising speed. The required fleet size of supersonic aircraft would be about 46% smaller than that of the subsonic ones, thanks to the higher cruising speed and the consequently shorter turnaround time on the route(s).

- The operating cost of supersonic flights, depending on the fuel cost, would be about 18-85% higher than that of the subsonic flights. This has confirmed concerns about their future economic feasibility from the airline perspective. However, these flights would provide the substantial savings in passenger time costs, of up to 46%.

- The fuel consumption and related GHG emissions of the supersonic flights would be about 70% and 19%, respectively, higher than those of the subsonic flights. This would primarily be due to much higher fuel consumption of LH_2 fuel, mainly caused by much higher operating speeds despite its much higher energy content, compared to Jet-A fuel. However, despite being higher, the costs GHG emissions have shown to be much lower thanks to the prospective charges/externalities. Combined with their GWP (global warming potential), GHG emissions by supersonic flights would contribute to savings in global warming and climate change of up to about 94%. The supersonic aircraft would generally not substantially compromise land use as an environmental externality. The possible impact of the logistics of supplying LH_2 fuel in this context remains to be further considered.

- The noise levels produced by supersonic aircraft at airports and during cruising at higher altitudes would be about 6-13% above certain prescribed tolerable level(s). Therefore, this (noise) externality will continue to be an important subject in the further dealings with commercialization of supersonic aircraft/flights.

- The selected derived design-related indicators of the technical/technological performance of supersonic aircraft, such as the payload/weight, fuel/weight, and payload/fuel ratio would be about 13%, 22%, and 12%, respectively, lower than that of subsonic aircraft. This indicates inherent specificities and challenges in their future design.

- If all the above-mentioned costs were fully internalised as externalities, supersonic flights could be about 20% more beneficial than their subsonic counterparts. This would be the

case only if the noise burden and the risk of air traffic incidents/accidents (i.e. safety) externalities are not taking fully into account.

Furthermore, estimation of the particular performance indicators for the network operated by supersonic aircraft was carried out by using secondary sources and available data predominantly from the U.S. airline industry. Some of them have been based on analogy with those related to subsonic aircraft/flights carried out in the network according to equivalent scenarios. Therefore, in addition to this initial step, future research could be directed towards:

- Refining the existing and developing additional performance indicators, including their more refined estimation by using more global and reliable data;
- Developing more realistic future scenarios of operating air route network(s) implying estimation of passenger demand, respecting the eventual airline/flight category competition; and
- Obtaining more refined estimates of the contribution of particular flight categories to the GDP and particularly to the environmental and social costs/externalities.

These all would contribute to the more detailed and reliable evaluation of the overall social-economic feasibility of the possible further development and implementation of commercial supersonic aircraft/flights by the year 2050.

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6 APPENDICES

Appendix I. Inputs for estimating the relationship between the technical productivity of supersonic aircraft and the influencing factors (Eq. 2)

Table AI-1
Speed, seat capacity, and technical productivity of the past and future concepts of supersonic aircraft

Aircraft type	Capacity S (Seats)	Maximum Cruising Speed v (kt)	Technical Productivity TP (10^3s-kt)
Concorde (Past)	128	1170	149.760
TY 144 (Past)	140	1233	172.620
Small [a]	15	1032	15.480
Medium [b]	200	1262	252.400
Large [c]	300	1376	412.800
LAPCAT Hydrogen Mach 5 Cruiser A2 [d]	300	2867	860.100
Boom Aircraft [e]	55	1262	69.410

Table AI-1
Continue

Aircraft type	Capacity S (Seats)	Maximum Cruising Speed v (kt)	Technical Productivity TP (10^3 s-kt)
NASA N + 2 [f]	70	1032	72.241
Aerion AS2	12	860	10.320
Spike S-512 [g]	18	917	16.512

[a] *Supersonic Business Jet;*
[b] *Overland Supersonic Commercial Transport* [16];
[c] *High-Speed Civil Transport* [28];
d [24]; e [22]; f [75]; g [23], [76] *(s- seat; kt- knot)*

Appendix II. Models for estimating the time of supersonic flight on a given route

The total flight time:

$$t_k \left(R_k \right) = t_{k/cl/.} + t_{k/cr/.} + t_{k/de/.} + t_{k/LTO} \qquad \ldots \text{(AII-1)}$$

Climbing time and distance:

- Time:

$$t_{k/cl/.} (H_{k/cr} - H_{k/cl}) = \frac{(H_{k/cr} - H_{k/cl})}{R/C\,(H)} = (1/b) \left\{ \begin{array}{l} \ln \left[a \left(H_{k/cl} \right) - b \cdot (H_{k/cr} \right] - \\ ln \left[a \left(H_{k/cl} \right) - b \cdot H_{k/cl} \right] \end{array} \right\} \qquad \ldots \text{(AII-2)}$$

The aircraft rate of climb R/C (H) is expressed as: $R/C\,(H) = a - b \cdot H$, i.e. as approximately linearly decreasing with altitude ((H) is the altitude - 10^3 feet) [8].

- Distance:

$$r_{k/cl/.} = (H_{k/cr} - H_{k/cl}) \big/ \sin \beta_{k/cl} \qquad \ldots \text{(AII-3)}$$

Descending time and distance:

- Time:

$$t_{k/de/.} (H_{k/cr} - H_{k/de}) = \frac{(H_{k/cr} - H_{k/de})}{R/D\,(H)} = (1/b) \left\{ \begin{array}{l} \ln \left[a \left(H_{k/cr} \right) - b \cdot (H_{k/de} \right] - \\ ln \left[a \left(H_{k/de} \right) - b \cdot H_{k/de} \right] \end{array} \right\} \qquad \ldots \text{(AII-4)}$$

The aircraft rate of descent $R/D(H)$ is expressed as: $R/D\,(H) = -a + b \cdot H$, i.e., as approximately linearly increasing with the decrease of altitude ((H) is the altitude - 10^3 feet)) [8].

- Distance:

$$r_{k/de/.} = \left(H_{k/cr} - H_{k/de} \right) / \sin \beta_{k/de} \qquad \ldots \text{(AII-5)}$$

Cruising time and distance:

$$t_{k/cr/.} = \left(R_k - r_{k/cl/.} - r_{k/de/.}\right)/v_{k/cr} \qquad \ldots (AII\text{-}6)$$

The LTO (Landing and Taking-Off) cycle

The LTO cycle $(t_{k/i/LTO})$ includes the time of taxing out, take-off, landing, and taxing in time. It can be standardised for the supersonic aircraft similarly as for their subsonic counterparts[60].

R_k	length of route (k) between the origin and destination airport of a given flight (nm, km)
$r_{k/cl/.}$	segment of route (k) enabling the aircraft of either category to climb to cruising altitude and accelerate from the take-off to the cruising speed (nm, km)
$r_{k/de/.}$	segment of route (k) enabling the aircraft of either category to descend from the cruising altitude and decelerate from the cruising to the final approach speed (nm, km)
$H_{k/cl}$, $H_{k/cr}$	take-off and cruising altitude, respectively, on route (k) operated by the aircraft of either category (ft, m)
$R/C\ (H_{k/cr} - H_{k/cl})$	rate of climb of the aircraft of either category from the take-off to the cruising altitude $(H_{k/cl})$ and $(H_{k/cr})$, respectively, along route (k) (ft/min);
$H_{k/cr}$, $H_{k/de}$	cruise and the final approach and landing altitude, respectively, on route (k) operated by the aircraft of either category (ft)
$R/D\ (H_{k/cr} - H_{k/de})$	rate of descent of the aircraft of either category from the cruising to the final approach altitude $(H_{k/cr})$ and $(H_{k/de})$, along route (k) (ft/min);
$v_{k/cr}$	cruising speed of an aircraft of either category on route (k) (kt, km/h)
$\beta_{k/cl}$, $\beta_{k/de}$	climb and descent angle, respectively, of the corresponding segments of the route (k) operated by the aircraft of either category (°)

Appendix III: Models for estimating fuel consumption of supersonic flight(s) on a given route

Four forces generally act on an aircraft in flight: thrust, aerodynamic drag, lift, and weight. They are always in balance, enabling safe flight. The lift force always balances the aircraft weight and the thrust force balances the aircraft aerodynamic drag and partially the component of its weight during the taking-off, climbing, descending, and landing phase. The thrust force during the climbing, cruising, and descending phase of flight is relevant for estimating the energy consumption in the given context [28], [61].

A-III: Climbing

- Thrust (N)

$$T_{cl} = m_{cl} \cdot a_{cl} + 0.5 \cdot C_{d/cl}\left(\overline{v}_{cl}\right) \cdot \overline{\rho}_{cl} \cdot \overline{v}_{cl}^2 \cdot A + m_{cl} \cdot g \cdot \sin \overline{\beta}_{cl} \qquad \ldots (AIII\text{-}1)$$

$$m_{cl} = m_{to} - FC_{to} \qquad \ldots \text{(AIII-2)}$$

$$\bar{v}_{cl} = (v_{cr} - v_{to})\,/2 \qquad \ldots \text{(AIII-3)}$$

$$m_{cl}^* = m_{to} - FC_{to} - 1/2 \cdot FC_{cl/0} \qquad \ldots \text{(AIII-4)}$$

- Energy (J)

$$E_{cl} = T_{cl} \cdot r_{cl} \qquad \ldots \text{(AIII-5)}$$

- Fuel consumption (kg)

$$FC_{cl} = (1/\eta_{cl}) \cdot \left(E_{cl}/e_f\right) \qquad \ldots \text{(AIII-6)}$$

A	aircraft fuselage area (m^2)
m_{cl}	initial aircraft mass during the climbing phase (kg)
a_{cl}^+	average longitudinal acceleration during the climbing phase (m/s^2)
$C_{d/cl}\,(\bar{v}_{cl})$	coefficient of aerodynamic drag during the climbing phase at average speed $(\bar{v}_{cl})$ (-)
$\bar{v}_{cl}$	average aircraft speed during the climbing phase (m/s)
r_{cl}	length of the climbing segment of the route (m)
e_f	specific energy of the given fuel used (J/kg) (J – Joule)
$\bar{\rho}_{cl}$	average air density along the climbing segment of the route (kg/m^3)
$\bar{\beta}_{cl}$	climbing angle (°)
η_{cl}	propulsion efficiency of the aircraft engines during the climbing phase

B-III: Cruising

- Thrust (N)

$$T_{cr} = q \cdot A_w \cdot C_{D0} + \frac{(m_{cr} \cdot g)^2}{q \cdot \pi \cdot b^2} + \frac{128 \cdot q \cdot Vol^2}{\pi \cdot l^4} + \frac{(M^2 - 1) \cdot (m_{cr} \cdot g)^2}{q \cdot \pi \cdot l^2} \qquad \ldots \text{(BIII-1)}$$

$$m_{cr} = m_{to} - FC_{to} - FC_{cl} \qquad \ldots \text{(BIII-2)}$$

$$Vol \approx \pi \cdot r^2 \cdot l \qquad \ldots \text{(BIII-3)}$$

- Energy (J)

$$E_{cr} = T_{cr} \cdot r_{cr} \text{ and } r_{cr} = R_k - r_{cl} \cos \beta_{cl} - r_{de} \cos \beta_{de} \qquad \ldots \text{(BIII-4)}$$

- Fuel consumption (kg)

$$FC_{cr} = (1/\eta_{cr}) \cdot \left(E_{cr}/e_f\right) \qquad \ldots \text{(BIII-5)}$$

q	dynamic pressure (N/m^2) (N - Newton)
A_w	reference wing area (m^2)

C_{D0}	coefficient of parasite drag at zero lift (i.e. the skin drag and all other drag except induced drag)
l	effective length of the aircraft (m)
b	aircraft wingspan (m)
m_{cr}	initial aircraft mass at the beginning of the cruising segment of the route (kg)
M	Mach number (-)
r	radius of the aircraft fuselage (m)
Vol	total volume (m^3)
R_k	length of route *(k)* in the given network (ft, m)
r_{cr}	length of the cruising segment of the route (ft, m)
r_{cl}, r_{de}	length of climbing and descending segment of the route, respectively (ft, m)
e_f	specific energy of the given fuel used (J/kg) (J - Joule)
ρ_{cr}	air density at cruising altitude (kg/m^3)
η_{cr}	propulsion efficiency of the aircraft engines during the cruising phase of flight

C-III: Descending

- Thrust (N)

$$T_{de} = m_{de} \cdot a_{de} + 0.5 \cdot C_{d/de}\,(\overline{v}_{de}) \cdot \overline{\rho}_{de} \cdot \overline{v}_{de}^2 \cdot A - m_{de} \cdot g \cdot \sin \overline{\beta}_{de} \qquad \ldots \text{(CIII-1)}$$

$$m_{de} = m_{to} - FC_{to} - FC_{cl} - FC_{cr} \qquad \ldots \text{(CIII-2)}$$

$$\overline{v}_{de} = (v_{cr} - v_{ld})\,/2 \qquad \ldots \text{(CIII-3)}$$

$$m_{de}^* = m_{to} - FC_{to} - FC_{cl} - FC_{cr} - 1/2 \cdot FC_{de} \qquad \ldots \text{(CIII-4)}$$

- Energy (J)

$$E_{de} = \left(\frac{1}{\eta_{de}}\right) \cdot T_{de} \cdot r_{de} \qquad \ldots \text{(CIII-5)}$$

- Fuel consumption (kg)

$$FC_{de} = (1/\eta_{de}) \cdot \left(E_{de}/e_f\right) \qquad \ldots \text{(CIII-6)}$$

m_{de}	initial aircraft mass during the descending phase (kg)
a_{de}^-	average aircraft longitudinal deceleration during the descending phase (m/s^2)
$C_{d/de}\,(\overline{v}_{de})$	coefficient of aerodynamic drag during the descending phase at the average speed ($\overline{v}_{de}$) (-)
$\overline{v}_{de}$	average aircraft speed during the descending phase (m/s)
e_f	specific energy of the given fuel used (J/kg) (J - Joule)
r_{de}	length of the descending segment of the route (m)
$\overline{\rho}_{de}$	average air density along the descending segment of the route (kg/m^3)
$\overline{\beta}_{de}$	descending angle (°)
η_{de}	is the propulsion efficiency of the aircraft engines during the descending phase