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Analytical models to determine in-plane damage initiation and force capacity of masonry walls with openings

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Abstract

Masonry panels consisting of piers and spandrels in buildings are vulnerable to in-plane actions caused by seismicity and soil subsidence. Tectonic seismicity can be hazardous for the safety of masonry structures, whereas low-magnitude induced seismicity can be detrimental to their durability due to the accumulation of light damage. This is particularly true in the case of unreinforced masonry. Therefore, the development of models for the accurate prediction of both damage initiation and force capacity for masonry elements and structures is necessary.

In this paper a method based on analytical modelling for the prediction of the damage initiation mode and capacity of stand-alone masonry piers is presented, followed by the expansion of the model through a modular approach to masonry walls with asymmetric openings. The models account for all potential damage and failure modes for in-plane loaded walls.

The stand-alone piers model is applicable to all types of masonry construction. The wall with openings model can be applied as-is to simple buildings but can also be extended to more complex structures with

26 simple modifications. The model results are compared with numerous experimental cases and exhibit very
27 good accuracy.

28 **Keywords**

29 masonry – earthquake engineering – analytical modelling – limit analysis – in-plane loading

30 **Notation**

31	h	height
32	h_0	effective height
33	l	length
34	t	thickness
35	m	width of compressive stress fan at centre-height
36	b	width of compression strut
37	f_c	compressive strength of masonry
38	f_t	tensile strength of masonry
39	f_v	initial shear strength (cohesion)
40	μ	friction coefficient (tangent of friction angle)
41	σ	vertical stress
42	τ	shear stress
43	V	vertical force
44	H	horizontal force

45 **Highlights**

- 46 • Closed-form expressions predict the damage initiation mode and capacity of piers
- 47 • Analytical modelling predicts the in-plane shear capacity of masonry walls with openings
- 48 • The models are accurate against newly elaborated and existing experimental data

49 **Introduction**

50 **State of the art**

51 Masonry structures are vulnerable to seismic loading due to their low tensile and shear strength. While
52 out-of-plane effects can be severely detrimental to the safety of masonry structures, these are often offset
53 when adequate connections allow for the force distribution to the transversal walls via floor diaphragm
54 action. Even if such measures are taken, in-plane failure remains a problem to be dealt with.

55 Typically, four main failure modes may be clearly distinguished: a) rocking, b) sliding, c) biaxial failure
56 and d) compressive failure. These failure modes, listed in order of appearance under increasing levels of
57 applied vertical stress, define, in combination, a failure envelope for masonry piers under in-plane shear.

58 Rocking mode failure arises due to the very low tensile strength of masonry perpendicularly to the bed
59 joints, leading to a clear localisation of the bending crack. Models for the rocking capacity can be easily
60 derived through simple equilibrium in bending (Magenes and Calvi, 1997; Roca *et al.*, 2011). Other models
61 have been proposed in design codes (Ministerio delle Infrastrutture e dei Trasporti, 2009). Sliding due to
62 shear, typically localised in bed joints is often described using a Mohr-Coulomb failure criterion.
63 Expressions to determine the capacity in shear at the scale of structural member have been proposed in
64 the literature (Magenes and Calvi, 1997; Tomaževič, 2006) and used in design codes (CEN, 2005). Models
65 for diagonal failure are generally more complex due to the interaction of compression and tension in an
66 area of the pier that is not as clearly defined as in rocking or sliding. Several models for biaxial failure have
67 been proposed in the literature (Turnšek and Cacovic, 1971; Turnšek and Sheppard, 1980; Mann and
68 Muller, 1982), each with different considerations for the dimensions of the pier and the mechanical
69 properties of the masonry composite. The accuracy of biaxial failure models is strongly dependent on the
70 accuracy of the approach used for calculating the tensile strength of masonry, particularly in the horizontal
71 direction. Apart from resorting to computational modelling or simple empirical expressions, there does not
72 appear to be in the literature a demonstrably reliable analytical method for calculating the tensile strength
73 of masonry.

74 The formulation of models for the prediction of the force capacity of masonry walls with openings is
75 complicated by the frame action made possible by the spandrels, whose failure needs to be accounted for
76 (Beyer, 2012). Simple analytical models accounting for the interaction of failure modes of piers and
77 spandrels in walls with openings are currently lacking in the literature.

78 The available experimental inventory on masonry stand-alone piers subjected to in-plane shear under
79 vertical stress is extensive (Morandi *et al.*, 2018), and continuously updated (Messali *et al.*, 2020). It
80 includes masonry composites made of different materials, with widely different dimensions and aspect
81 ratios, different boundary conditions and in different bond types. Experimental tests on masonry walls with
82 windows or openings, accompanied by a characterization of the mechanical properties of the masonry
83 composite, are less frequent and feature a smaller variety of boundary conditions (Raijmaker and
84 Vermeltfoort, 1992; Foraboschi, 2009; Parisi, Augenti and Prota, 2014; Drougkas, Roca and Molins, 2019;
85 Korswagen *et al.*, 2019). The relatively small number of experimental tests on walls with openings
86 compared to those performed on single piers has resulted in very limited effort at developing simple
87 models for predicting the force capacity of these assemblages. The abundance of walls with openings in
88 actual practice indicates the potential usefulness of such models for quick capacity checks.

89 In addition to capacity calculation, the complications introduced by induced seismicity raise the issue
90 of correctly identifying the mode of damage initiation in masonry structures. Combined soil subsidence and
91 low-magnitude seismicity have been shown to impose mostly in-plane demands on masonry structures
92 (Terwel and Schipper, 2018; Van Staaldunin, Terwel and Rots, 2018; Drougkas *et al.*, 2020). These
93 demands are the source of light damage, linked to damage initiation rather than collapse.

94 Upper-bound approaches for the calculation of the capacity of masonry walls with openings have been
95 proposed in the literature (Vanin and Foraboschi, 2012). Similarly, computational efforts based on finite
96 element (Korswagen *et al.*, 2019; Drougkas *et al.*, 2020) and discrete element analysis (Sarhosis *et al.*, 2019)
97 are relatively abundant. However, a simple model with general applicability for the prediction of the force
98 capacity of masonry walls with openings, one based on the material properties of the masonry composite,
99 is still lacking. Such a model should allow a quick calculation of the capacity of a masonry structure, the

100 prediction of the critical failure mode and the evaluation of the influence of structural intervention on the
101 behaviour without resorting to complex finite element or macro-element modelling.

102 Furthermore, a simple model for the identification of the in-plane damage initiation mode of masonry
103 piers has not been yet proposed. The need of such a model arises from the increase in low-magnitude
104 induced seismicity near urban centres, which does not necessarily raise the risk of collapse but may be the
105 cause of light damage in masonry structures (Korswagen *et al.*, 2019). A-priori knowledge of the location
106 of damage initiation using simple approaches allows the application of targeted intervention at vulnerable
107 areas. Further, such a model can prove useful as a structural inspection tool, assisting in focusing damage
108 mapping efforts in existing masonry buildings on the areas where damage is expected to arise.

109 **Objectives**

110 The primary objective of the present paper is the presentation of a simple model for the calculation of
111 the in-plane shear capacity of masonry structures. In the context of the paper, the term masonry structure
112 refers to walls with door- or window-openings, in essence masonry portal frames with or without a base
113 spandrel. The model should account for frame action afforded by the spandrel, whose contribution is itself
114 limited by potential damage. Through a modular approach, this model is applicable to masonry elements
115 with multiple openings.

116 The secondary objective of the paper is the development of a simple model for the prediction of the
117 damage initiation mode in masonry piers subjected to in-plane shear. Essentially, this model should be able
118 to predict the failure mode that arises first in masonry piers under shear. Such a model can be used in
119 stand-alone piers or can be alternatively plugged-in to the proposed model for masonry structures.

120 The development of the masonry structure model is based on the assembly and evaluation of simple
121 models predicting the capacity of piers in well-defined failure modes. A new model for the biaxial failure of
122 masonry is here proposed which takes into account the effect of the masonry bonding pattern on the tensile
123 strength of masonry in a simple manner. These models are used to define a capacity envelope. The results
124 of the failure models are compared to numerous experimental results from the literature on stand-alone
125 piers.

126 Moving beyond the application of these models in stand-alone piers, the paper presents a method of
127 application to complex walls with openings, dealing with issues of force distribution and the development
128 of admissible failure modes depending on boundary conditions. This model is validated against case studies
129 from the literature, limiting the investigation to cases where a comprehensive determination of the
130 mechanical properties of the masonry composite is available.

131 The damage initiation model is developed along the lines of a proposed envelope, similarly to the model
132 for the capacity of piers. A comparison with the corresponding capacity envelope is provided.

133 **Analytical force capacity models for piers**

134 **Overview**

135 The dimensions of the pier are $l \times h \times t$ (length \times height \times thickness). For a given masonry
136 compressive strength f_c and a vertical applied stress σ (negative for compression), the length of the
137 compressed toe b_r , assuming a constant rectangular distribution of vertical stress, or b_t , assuming a
138 triangular distribution, is:

$$\begin{aligned} b_r &= -\frac{\sigma}{f_c} l \\ b_t &= -2\frac{\sigma}{f_c} l \end{aligned} \quad (1)$$

139 For a given set of geometric and material parameters of a stand-alone pier, the applied vertical force V
140 and the horizontal force capacity H are calculated as:

$$\begin{aligned} V &= l \cdot t \cdot \sigma \\ H &= l \cdot t \cdot \tau \end{aligned} \quad (2)$$

141 The shear stress capacity τ is calculated for each of the considered failure modes below. An envelope
142 curve of the capacity can be drawn by varying σ in the range $[0, f_c]$ and considering the minimum value of
143 τ obtained between the considered failure modes. The considered failure modes are illustrated in Figure 1.

144 The pier is always considered clamped at the base and may be in a cantilever or double-clamped
145 configuration when rotational restraint is provided. The boundary condition at the top determines the

146 effective height h_0 of the pier, with $h_0 = 1.0$ for a cantilever and $h_0 = 0.50$ for a double-clamped
147 configuration.

148 **Rocking mode capacity**

149 In a cantilever configuration, the vertical force V is applied at the centre of the top of the pier, while in
150 a clamped top configuration it is applied at a distance of $b_r/2$ from the edge. The compression strut extends
151 from the point of application of V to the centre of the compressed toe. Through equilibrium of forces and
152 moments, the capacity of a cantilever pier in rocking is:

$$\tau = -\sigma \left(\frac{l}{2} - \frac{b_r}{2} \right) / h \quad (3)$$

153 while for a clamped top the capacity is:

$$\tau = -\sigma (l - b_r) / h \quad (4)$$

154 In a more general formulation, the horizontal force capacity H can be expressed as the horizontal
155 component of a force acting between two points at a horizontal distance of l and a vertical distance of h
156 whose vertical component is equal to V :

$$H = V \frac{l}{h} \quad (5)$$

157 **Shear mode capacity**

158 For the shear capacity of the pier, the model proposed by Magenes and Calvi is used (Magenes and Calvi,
159 1997). In the notation of the present paper, the shear capacity is equal to:

$$\tau = \frac{f_v - \mu \cdot \sigma}{1 + h_0/l} \quad (6)$$

160 where f_v is the initial shear strength (cohesion) and μ is the friction coefficient (tangent of friction angle).
161 As noted in the cited work, these parameters are meant to be understood as globally representing the shear
162 characteristics of the masonry composite rather than that of the bed joints.

163 **Biaxial mode capacity**

164 A new approach based on principal stresses is proposed for calculating the capacity of the pier against
165 biaxial failure. For the interaction of tension and compression, a simple linear failure criterion in planar
166 stress is adopted:

$$f = \frac{\sigma_1}{f_t} - \frac{\sigma_2}{f_c} - 1 \quad (7)$$

167 where σ_1 is the maximum principal stress (tensile), σ_2 is the minimum principal stress (compressive) and
168 f_t is the tensile strength of the masonry composite. This failure criterion clearly describes the interaction
169 of tensile and compressive stresses in quasi-brittle materials with a shape approximating very closely a
170 linear Mohr-Coulomb criterion.

171 The compressive stress distribution in a cantilever pier is considered to assume a fan shape, extending
172 along the entire length l of the wall at the top and contracting to the width of the compressive strut b_t at
173 the base. A depiction of this fan shape is illustrated in Figure 2a. The width of the fan m at centre height,
174 where diagonal cracking typically originates, is:

$$m = \frac{l + b_t}{2} \quad (8)$$

175 In double-clamped piers, the stress fan assumes the shape shown in Figure 2b, with a laterally
176 expanding branch from top to mid-height and a contracting branch from mid-height to base. The angle θ_e
177 of the right external line of the fan with respect to the vertical is limited by the shear strength characteristics
178 of the masonry composite (Roca *et al.*, 2011). Considering that the vertical stress at the edge of the fan is
179 zero, the limit values for the tangent of this angle is:

$$\tan(\theta_e - \theta_c) \leq \mu \quad (9)$$

180 where θ_c is the angle of the line connecting the centres of the strut edges with respect to the vertical. In
181 this context, the friction coefficient of masonry does not coincide with the friction coefficient of the unit-
182 mortar interface. It is a parameter related to the masonry geometric bond and the resulting interlocking of
183 units, with a minimum value equal to the friction coefficient of the unit-mortar interface. As such, for

184 running bond masonry this coefficient is equal to $\mu = (l_u/2)/(h_u + h_m)$, for Flemish bond it is equal to $\mu =$
 185 $(3 l_u/4)/(h_u + h_m)$ and for English bond it is $\mu = (l_u/2)/(2h_u + 2h_m)$ with l_u , h_u and h_m being the length
 186 of the unit, height of the unit and height of the mortar bed joint respectively. Therefore, the importance of
 187 the masonry bonding pattern on both the shear and tensile strength of the masonry becomes apparent. The
 188 accuracy of this calculation of the friction coefficient is increased with the increase of the size of the
 189 masonry member, due to the clearer formation of diagonal cracks following the masonry bond. The
 190 maximum length for m is only limited by the length of the pier.

191 Based on these conditions, the width m of the fan at centre height of a double-clamped pier is:

$$m = b_t + \min \left[l - b_t, \frac{h}{2} \mu \right] \quad (10)$$

192 For the orthogonal stress state at the mid-height, it is assumed that the horizontal σ_x stress is zero and
 193 that the vertical stress σ_y is evenly distributed. Therefore, it follows that:

$$\begin{aligned} \sigma_x &= 0 \\ \sigma_y &= \sigma \frac{l}{m} \end{aligned} \quad (11)$$

194 According to Mohr's circle (Beer *et al.*, 2012), the average stress σ_m is:

$$\sigma_m = \frac{\sigma_x + \sigma_y}{2} \quad (12)$$

195 and, in combination with the adopted failure criterion according to eq. (7), the principal stresses are:

$$\begin{aligned} \sigma_1 &= \frac{2f_t \sigma_m + f_c f_t}{f_t + f_c} \\ \sigma_2 &= \frac{2f_c \sigma_m - f_c f_t}{f_t + f_c} \end{aligned} \quad (13)$$

196 The radius of Mohr's circle R is:

$$R = \frac{\sigma_1 - \sigma_2}{2} \quad (14)$$

197

198 and the resulting shear stress τ_m is:

$$\tau_m = \sqrt{R^2 - \sigma_m^2} \quad (15)$$

199 This shear stress τ_m acts along the length m of the fan at the evaluated position. Therefore, the
 200 equivalent stress τ along the length l of the pier is:

$$\tau = \tau_m \frac{m}{l} \quad (16)$$

201 The determination of the uniaxial horizontal tensile strength of masonry f_t is a complicated issue. It is
 202 a function of the tensile strength $f_{t,u}$ of the units, the tensile strength $f_{t,m}$ of the mortar, the tensile strength
 203 $f_{t,i}$ of the unit-mortar interface and the shear strength $f_{v,b}$ of the bed joints. Further, it is strongly affected
 204 by the masonry bonding pattern, which governs the length at which shear stresses develop. While for
 205 masonry in regular bond pattern the vertical tensile strength may be taken as the tensile strength of the
 206 unit-mortar interface, the staggered arrangement of the units in, for example, running bond, complicates
 207 the failure mechanism. A simple model for the horizontal tensile strength of masonry is therefore
 208 introduced. It is based on the identification of three failure modes for the masonry composite in horizontal
 209 tension: a) tensile failure of the upper head joint unit-mortar interface together with shearing of the bed
 210 joint along the length of half a unit and tensile failure of the lower head joint unit-mortar interface, b) tensile
 211 failure of the upper head joint unit-mortar interface together with tensile failure of the bed joint and tensile
 212 failure of the lower unit, c) tensile failure of the upper unit together with tensile failure of the bed joint and
 213 tensile failure of the lower unit. These modes are illustrated in Figure 3 and are expressed analytically as:

$$\begin{aligned} f_{t,a} &= \frac{f_{t,i} \frac{h_u}{2} + f_{v,b} l_o + f_{t,i} \frac{h_u}{2}}{h_u + h_m} \\ f_{t,b} &= \frac{f_{t,i} \frac{h_u}{2} + f_{t,m} h_m + f_{t,u} \frac{h_u}{2}}{h_u + h_m} \\ f_{t,c} &= \frac{f_{t,u} \frac{h_u}{2} + f_{t,m} h_m + f_{t,u} \frac{h_u}{2}}{h_u + h_m} \\ f_t &= \min[f_{t,a}, f_{t,b}, f_{t,c}] \end{aligned} \quad (17)$$

214 where l_o is the overlap length between the beds of the units which contributes to the shear mechanism.
215 This length can be easily determined for the most common masonry bonds. For running bond it is equal to
216 $l_u/2$, for Flemish and English bond it is equal to $l_u/4$ and in stack bond it is equal to 0. In addition to regular
217 masonry with mortared joints, eq. (17) can account for dry masonry through the contribution of $f_{v,b}$ and
218 for masonry with unfilled head joints by considering $f_{t,i} = 0$.

219 **Compression mode capacity**

220 The capacity of the pier in compression is calculated through a simple superposition of the normal
221 stresses at the base of the pier due to the applied vertical stress σ and the bending moment caused by τ
222 applied at the top of the pier. Limiting the minimum stress to the compressive strength $-f_c$, the shear
223 capacity is equal to:

$$\tau = \frac{(l - b_r)}{6h} f_c \quad (18)$$

224 **Model results and validation**

225 All failure models yield non-negative results for $\sigma \in [0, f_c]$ and produce a capacity envelope as
226 qualitatively shown in Figure 4 for a cantilever pier, defined by the minimum value among the models for
227 a given value of σ . In the case of a clamped pier, the τ envelope is altered only in the region of low vertical
228 stress σ , as both rocking and shear capacity increase. This results in an increase in the range of biaxial
229 failure towards the range of lower vertical stress σ . This shift is critical given that most masonry piers, due
230 to their large dimensions, function at a relatively low level of average vertical stress from self-weight and
231 service loads in buildings.

232 The results of the model combination are tested against the dataset of experimental results assembled
233 by Morandi et al (Morandi *et al.*, 2018). The dataset includes 188 experimental results of masonry piers
234 subjected to in-plane shear under vertical stress. Material properties are included in the dataset. However,
235 this data is not always fully reported. In the absence of a reported tensile strength f_t this was calculated
236 according to eq. (17). A conservative value of 0.100 N/mm^2 was assumed for $f_{t,i}$ in masonry with mortared
237 head joints, and the tensile strength of the mortar and units was taken as 10% of their respective

238 compressive strengths (Drougkas, Roca and Molins, 2015). Rather than assigning nominal values, the cases
239 where f_v or μ were not reported were disregarded. This filtering resulted in 36 cases with reported f_t and
240 27 cases with no reported f_t to be considered for analysis, for a total of 63 cases, that is 33% of all reported
241 cases in the cited dataset.

242 The results of the comparison are plotted in Figure 5. When relying on the reported f_t (Figure 5a) the
243 obtained coefficient of determination R^2 is 0.955 and the mean percentage error MPE is -9.55% , indicating
244 excellent global agreement between the experimental data and analysis results and a tendency of the model
245 to underestimate the capacity. The proposed envelope rarely overestimates the capacity of the piers by
246 more than 15%. The accuracy of the model is noticeably increased when not relying on the reported f_t
247 (Figure 5b) but rather by relying only on the f_t as calculated using eq. (17). The obtained R^2 is slightly
248 increased to 0.962 and the MPE is increased to -5.42% , indicating an enhancement of the model's accuracy,
249 especially in the cases with higher capacity. This improvement validates the accuracy of the proposed
250 model for the tensile strength of masonry and the calculated biaxial failure envelope. Due to the accuracy
251 of the obtained results, the envelope described by these failure models is considered appropriate for
252 application in the analysis of more complex wall structures.

253 **Analytical damage initiation models for piers**

254 **Overview**

255 The proposed damage initiation model for stand-alone piers functions similarly to the capacity model.
256 However, instead of calculating the peak shear force for a specific failure type, it calculates the shear force
257 activating a specific failure type. It may, therefore, be used for identifying the sequence of damage mode
258 initiation and propagation in stand-alone piers loaded in-plane.

259 Under the assumption that the pier is uncracked before damage initiation, the normal stresses can be
260 easily computed through superposition of the stresses due to σ and τ applied at the top of the pier. Similarly,
261 the distribution of shear stress along the length of the pier assumes a parabolic shape, with the maximum
262 shear stress being 1.5 times the average (Timoshenko, 1940).

263 **Rocking mode initiation**

264 Damage initiation in rocking occurs under the following conditions: a) a constant vertical stress
265 distribution at the top and a triangular vertical stress distribution at the base are assumed, b) for the
266 maximum stress at the least compressed toe: $\sigma_{\max} = f_{t,i}$, c) for the minimum stress at the compressed toe:
267 $\sigma_{\min} \geq -f_c$.

268 According to moment and force equilibrium, and based on the above conditions, the resulting value for
269 the minimum stress is:

$$\sigma_{\min} = 2\sigma - f_{t,i} \quad (19)$$

270 while the damage initiation shear stress is:

$$\tau = (f_{t,i} - \sigma) \frac{l}{6h} \quad (20)$$

271 **Shear mode initiation**

272 The conditions for shear mode initiation are: a) a trapezoidal vertical stress distribution is assumed at
273 the base, b) for the maximum stress at the least compressed toe: $\sigma_{\max} \leq f_{t,i}$, c) for the minimum stress at
274 the compressed toe: $\sigma_{\min} \geq -f_c$, d) the maximum shear stress due to the trapezoidal distribution needs to
275 reach the shear strength. Therefore, $\tau = (f_v - \mu \cdot \sigma_{\max})/1.5$.

276 Based on these assumptions and applying moment equilibrium, the values for the minimum and
277 maximum stress are:

$$\sigma_{\max} = \frac{f_v \cdot h - (2 \cdot \mu \cdot h + l)\sigma}{h \cdot \mu + l} \quad (21)$$

$$\sigma_{\min} = -\frac{l \cdot \sigma + 4 \cdot f_v \cdot h}{4 \cdot h \cdot \mu + l}$$

279 while the value for the damage initiation shear stress is:

$$\tau = 2 \frac{f_v - \sigma \cdot \mu}{12 \cdot h \cdot \mu + 3 \cdot l} l \quad (22)$$

280 **Biaxial mode initiation**

281 The biaxial mode initiation stress is calculated similarly to the capacity according to eq. (8) through eq.
282 (16). Due to the assumption that no other damage initiation mode has arisen, the pier remains uncracked
283 and the stress fan is vertical, the horizontal force being resisted by friction. Therefore, the width of the fan
284 m is equal to the length of the pier l . Due to the parabolic shear stress distribution along the length of the
285 pier, the damage initiation shear stress is equal to:

$$\tau = \frac{\tau_m}{1.5} \quad (23)$$

286 **Model results and validation**

287 The three mode initiation models can be combined to produce a damage initiation envelope. This
288 envelope is additionally delimited by the compressive failure model as defined in eq. (18). The brittleness
289 of the compressive failure mode results in the coincidence of damage initiation and force capacity. Plotting
290 the damage initiation envelope for a masonry pier results in a typical curve shown in Figure 6, where a
291 comparison with the capacity envelope is shown. The damage initiation envelope is always below the
292 capacity envelope.

293 The range of normalised vertical stresses for which damage initiates through pure rocking is greater
294 than the range where rocking determines the capacity. This is true for the shear mode as well. Conversely,
295 the range of biaxial mode initiation is limited compared to the capacity envelope. Due to the usually low
296 level of global vertical stress under which masonry piers typically operate in buildings (Heyman, 1966), it
297 is expected that the majority of piers will feature rocking or shearing damage initiation, followed by
298 rocking, shearing or, less commonly, biaxial failure.

299 The damage initiation model is validated against a series of experiments carried out on piers at Delft
300 University of Technology (Esposito and Ravenshorst, 2017; Korswagen *et al.*, 2017), coupled with extensive
301 material characterisation (Jafari and Esposito, 2016, 2017). The geometric and material parameters are
302 reported in Table 1. The experiments include two different sets of materials, different masonry bonds,
303 different boundary conditions and varying vertical pre-compression levels.

304 The damage initiation and final failure mode was reported in three of the cases (TUD_COMP_20,
305 TUD_COMP_21, TUD_COMP_22), while for one of the cases (TUD_COMP_47/48) the crack pattern was
306 objectively registered using digital image correlation (DIC). Systematic documentation and objective
307 interpretation of damage initiation in experimental reports is often problematic without the use of DIC or
308 other optical methods for crack tracking. Damage initiation is typically reported in terms of visible diagonal
309 cracking, which cannot arise without prior initiation of some degree of rocking damage. Localised toe
310 crushing may also be reported, but this phenomenon is associated with practically all damage initiation
311 and capacity models and is, therefore, not indicative of the overall failure mode by itself. Nevertheless, even
312 damage reported in simple terms can assist in interpreting damage initiation modes in masonry piers.

313 Overall, the model exhibits very good accuracy in both capacity calculation and in predicting the
314 damage initiation and failure mode. In cases TUD_COMP_21, TUD_COMP_22 and TUD_COMP_47/48 the
315 model was able to predict the shift from a damage initiation mode based on rocking/sliding to a failure
316 mode based on diagonal cracking. The number of suitable experimental cases suitable for validation of the
317 proposed model, which need to include comprehensive material characterisation and unambiguous
318 reporting of the damage initiation force and mode, is currently small, especially compared to the number
319 of cases suitable for validation of the capacity model. Further experimental investigation focusing on
320 damage initiation is thus motivated.

321 **Strut & fan model for walls with openings**

322 **General model description**

323 In the context of the proposed approach, modelling of masonry walls with openings under in-plane
324 loads requires: 1) the discretisation of the frame into individual components, 2) the distribution of forces
325 and stresses in these components, 3) the identification of potential failure modes according to the
326 arrangement of the components and the boundary conditions.

327 The discretisation of a masonry wall with a single opening is shown in Figure 7, along with the notation
328 used hereafter for dimensions and loads. The wall consists of 8 components arranged in a regular 3×3
329 grid. Three components for the spandrel (S_1 , S_2 and S_3), two components for the piers (P_1 and P_3) and three

330 components for the base (B_1 , B_2 and B_3) are considered. The piers can have different lengths, thus allowing
331 the analysis of asymmetric structures. Each component can be assigned its own thickness t and set of
332 material properties. Additionally, the vertical load at the top of each pier and of the spandrel can be
333 different. A height of $h_1 = 0$ reduces the model to a portal frame, while all the other dimensions can only
334 be greater than 0. The horizontal loading direction is towards the positive of the x axis. Vertical
335 compression is applied towards the negative of the y axis.

336 In the modular approach proposed, the wall W is composed of a pair of sub-systems: L (left) and R
337 (right), connected with a central spandrel. Each sub-system consists of a single base, pier and spandrel. The
338 capacity of the wall is dependent on the capacity of the individual sub-systems and the effect of their
339 interaction through spandrel action.

340 **Boundary conditions and spandrel function**

341 As in the case of stand-alone piers, the wall is considered clamped at the base. For the boundary
342 condition at the top, the wall may be in a) cantilever, b) clamped or c) clamped with vertical translational
343 restraint configuration.

344 The boundary conditions and construction details at the top of the wall affect the function of the
345 spandrel in providing frame action. In particular, for a cantilever configuration, two cases are distinguished:
346 a) a “weak” connection with the piers, due to the absence of structural elements above the spandrel, and b)
347 a “strong” connection with the piers, provided by steel or reinforced concrete capping beams or a strong
348 lintel. In the former, the S_2 spandrel component responds to horizontal loading by “rocking” between the
349 two piers: a hinge is formed at the top right corner of S_1 and another at the left bottom corner of S_3 . In the
350 latter case, the spandrel elements respond jointly. For the clamped and clamped with vertical restraint
351 configurations, it is always considered that the spandrel provides a “strong” connection. The two types of
352 spandrel function are illustrated in Figure 8.

353 The combined effect of boundary conditions, spandrel action and assumption on load transfer from the
354 top to the spandrel blocks during flexure (Beyer, 2012) control the static determinacy of the wall system,
355 which may be treated as a portal frame with internal hinges. The “weak” spandrel provides two internal

356 hinges, while the “strong” spandrel provides one. Therefore, the static indeterminacy of the wall in the first
357 case is 1 while for the second case it is 2. The added rotational restraint from a double-clamped condition
358 adds an additional degree of indeterminacy.

359 **Modelling assumptions**

360 The stress distribution in the components is represented through a system of compressive struts and
361 fans. The compressive struts develop between two formed plastic hinges. Fans develop between two
362 continuous lines of applied vertical displacement or between one such line and a plastic hinge.

363 Concerning the distribution of the compressive stresses, it is assumed that the vertical stress σ_1 is
364 distributed to pier P_1 , while both vertical stresses σ_2 and σ_3 are borne by pier P_3 , due to the loading
365 direction (Roca, 2006). The transfer of vertical load from the spandrel S_2 above the opening constitutes the
366 frame action of the wall.

367 Plastic hinges are formed due to yielding in compression and have a width of b calculated as per the
368 pier model through eq. (1).

369 **Sub-system failure shapes**

370 Eight arrangements of plastic hinges are possible for a sub-system, illustrated in Figure 9. Stress fans
371 are depicted in light blue, with the direction of the stress flow indicated by arrows. Compressive struts are
372 indicated in deeper blue colour. The expansion of the stress fan between hinges as expressed in eq. (10)
373 and illustrated in Figure 2 is not shown for clarity of the illustrations. The plastic hinges are formed at the
374 edges of the struts or at the convergence locus between a laterally contracting and an expanding stress fan.
375 The locations of the plastic hinges coincide with the points of contact between blocks, i.e. the points where
376 the piers meet the base or the spandrel, where stresses due to in-plane shear tend to concentrate. The
377 resulting mechanisms are representative of those found in experimental practice and used to interpret the
378 failure mode of walls with openings (Vanin and Foraboschi, 2012).

379 The disposition of the struts and fans determines where the failure checks are performed. This point is
380 illustrated by commenting on the difference between shapes 1, 2 and 3. In shape 1, the spandrel, pier and

381 base are checked individually. In shape 2 all three components are checked as one. In shape 3 the base is
382 checked individually while the spandrel and pier are checked as one component.

383 Individual failure checks are executed according to the model for stand-alone piers: a) all components
384 are checked against biaxial failure according to eq. (16), b) all components are checked in compression
385 according to eq. (18), c) piers are checked in shear according to eq. (6), c) rocking failure is checked
386 according to eq. (5) by calculating the horizontal force component between plastic hinges or, in the absence
387 of a second plastic hinge, by assuming a resultant force at the centre of a fan extending towards the
388 direction of loading (positive x direction).

389 The failure checks in sub-system L are straightforward due to the sub-system only bearing the vertical
390 and horizontal forces applied on S_1 . Sub-system B bears the vertical and horizontal forces applied on both
391 S_2 and S_3 . For a “weak” spandrel, V_2 is transferred to the lower right corner of S_3 , while for a “strong”
392 spandrel it is applied at the centre of S_2 , providing an increased lever-arm and increased rocking capacity.
393 In the case of a “strong” spandrel, an additional biaxial strength check is performed for S_2 , considering a
394 stress fan from the top of S_2 to the top of P_2 , where it assumes a width as defined in Figure 9.

395 Based on these calculations, the capacity $\tau_{A,i}$ and $\tau_{B,i}$ of each sub-system L or R for the failure shapes
396 $i \in [1,8]$ is calculated.

397 **Combination of sub-system failure shapes**

398 The sub-system failure shapes are combined in pairs. Each pair defines a potential failure mode and
399 total capacity for the wall. These capacity sums can be expressed as:

$$C(i,j) = (\tau_{L,i} + \tau_{R,j}), \quad i, j \in [1,8] \quad (24)$$

400 The capacity of the wall τ_W is defined as the minimum element in C . However, due to their interaction
401 in the wall and due to boundary conditions, not all sub-system failure shapes are allowed in the complete
402 wall structure. The boundary conditions and geometry of the wall affect the stress distribution and
403 potential failure modes as follows: a) cantilever walls with “weak” spandrels require the formation of 2
404 hinges, b) cantilever walls with “strong” spandrels require 3 hinges, c) hinges cannot form at the top of

405 cantilever walls with “strong” spandrels for maintaining continuity of the applied vertical stress, d) double-
406 clamped walls require 4 hinges, e) double-clamped walls with vertical restraint require 4 hinges, but the
407 bending failure mode is inactivated, f) the central part B_2 of the base restricts the rotation of components
408 B_1 and B_3 (Caliò, Marletta and Pantò, 2012), therefore, no plastic hinge can form at the base of B_1 or B_3 ,
409 unless a sufficient gap is provided between the base components.

410 Based on the above conditions, and assuming that no gaps are provided between the base components,
411 the allowable failure shape combinations are: a) $C(3,3)$, $C(3,7)$, $C(3,8)$, $C(7,3)$, $C(7,7)$, $C(7,8)$, $C(8,7)$,
412 and $C(8,8)$ for a cantilever wall with a “weak” spandrel, b) $C(1,3)$, $C(3,1)$, $C(1,7)$ and $C(7,1)$ for a cantilever
413 wall with a “strong” spandrel, c) $C(1,1)$, $C(1,6)$ and $C(6,1)$ for the double-clamped, with or without vertical
414 restraint.

415 **Model results and validation**

416 The proposed model for the masonry wall capacity is validated against experimental cases from the
417 literature (Raijmaker and Vermeltoort, 1992; Foraboschi, 2009; Lobato Paz, 2009; Vanin and Foraboschi,
418 2012; Parisi, Augenti and Prota, 2014; Esposito and Ravenshorst, 2017; Korswagen *et al.*, 2017). Among
419 the findings in the literature, the list of cases used was confined to those in which material parameters were
420 reported. It includes both walls with window openings and portal frames, i.e. where $h_1 = 0$. Due to the
421 small number of such available campaigns, nominal shear characteristics were assumed where they were
422 missing in order to not overly limit the application cases (Van der Pluijm, 1992). The tensile strength as
423 calculated according to eq. (17) and the friction angle of the masonry as calculated in subsection 0 are also
424 reported. Concerning boundary conditions, walls were tested in cantilever with “strong” spandrel (C) and
425 double-clamped with vertical restraint (V) configuration. The vertical load was only applied on the piers
426 in a few instances. All parameters used and results obtained are presented in Table 2

427 The case studies involve different types of loading regimes. Three cases involve monotonic loading
428 (Raijmaker and Vermeltoort, 1992; Lobato Paz, 2009; Parisi, Augenti and Prota, 2014), three cases involve
429 loading-unloading cycles in one direction (Foraboschi, 2009; Vanin and Foraboschi, 2012; Korswagen *et*
430 *al.*, 2017) and one case involves cyclic loading in two directions (Esposito and Ravenshorst, 2017). Since

431 the proposed model does not account for degradation due to repeated or cyclic loading, the modelling
432 approach is not altered to accommodate this fact. Simulation of cyclic response in the context of this model
433 would involve the degradation of the shear strength in the bed joints, the opening of the head joints in
434 tension, which would reduce the horizontal tensile strength of masonry, and the adjustment of the strut
435 disposition due to diagonal cracking.

436

437 The difference in wall capacity due to uneven piers, accompanied by a shift in failure mode, is captured
438 in the simulation of the experiments by Esposito & Ravenshorst (Esposito and Ravenshorst, 2017). The
439 quarter-scale experiments by Lobato (Lobato Paz, 2009) and full-scale experiments by Foraboschi & Vanin
440 (Foraboschi, 2009; Vanin and Foraboschi, 2012) illustrate the shift in capacity due to an increasing vertical
441 load, an increase indicative of the global friction angle of the masonry. The accuracy of the model in
442 simulating large piers connected by a spandrel, namely portal frames without the base, is shown in the
443 simulation of the experiments by Parisi et al (Parisi, Augenti and Prota, 2014). Finally, the model captures
444 the significant effect of boundary conditions on the response, as illustrated in the high capacity obtained in
445 the experiments by Raijmaker, which was vertically restrained, resulting in a force capacity nearly double
446 that of a double-clamped model (Raijmaker and Vermeltoort, 1992).

447 Overall, the model exhibits good accuracy, with no marked tendency to under- or overestimate the force
448 capacity. A slight divergence from the linear trend obtained in the experiments by Lobato is obtained,
449 pointing towards a possible discrepancy between the actual friction angle of masonry and the value
450 obtained in the model. The capacity obtained by Parisi et al is well approximated, potentially due to the
451 simple failure mode registered in the piers. The differences between the analysis results and the
452 experimental results in the related experimental cases by Foraboschi and Vanin & Foraboschi are not easy
453 to explain. Despite the repeated nature of the loading in the experiments, the model underestimates the
454 force capacity nearly throughout. It is possible that this systematic error is due to the unit-mortar interface
455 tensile strength or shear strength being higher than assumed. This is also potentially true in the
456 experiments by Korswagen and Esposito & Ravenshorst. The large overestimation obtained in one of the

457 experiments by Foraboschi (under 0.09 N/mm^2 vertical stress), due to the very low level of applied vertical
458 stress, can only be accounted for in the model by a change in the spandrel action. A “weak” spandrel
459 assumption results in a calculated peak force of 11.7 kN versus the experimental value of 18.2 kN. It is
460 possible, therefore, that the actual conditions during the experiment were an intermediate between a
461 “strong” and a “weak” spandrel.

462 A comparison of the experimentally obtained failure modes, where these were reported, with the
463 numerically calculated results is presented in Table 3. Overall, there is very good agreement between the
464 experimental and numerical results, especially regarding the prediction of the failure mode of the left sub-
465 system. This sub-system appears to be susceptible to bending failure due to the lower overall vertical load
466 borne by it, validating the assumption of the transfer of vertical load to the right sub-system by the
467 spandrel. Regarding the failure of the right sub-system, the model is able to predict the failure mode for
468 nearly all cases. Reports on the experimental results through photos and text descriptions reveal a more
469 mixed failure mode, with cracking being shown in both the spandrel and the bases, such as in the
470 experiments by Vanin & Foraboschi. In this sense the model is capable of predicting the failure mode at
471 least in part. Finally, regarding the discrepancy in the prediction for the Esposito and Ravenshorst case,
472 where the sub-system failed at the base, with the spandrel failing in subsequent loading cycles, the model
473 predicts a capacity of the spandrel roughly 1 kN or 6% higher than for the base. The discrepancy is therefore
474 considered minor as a very slight change in material properties would produce the recorded failure type.

475 **Sensitivity study**

476 The influence of a number of geometric and material parameters on the predicted force capacity is
477 investigated through a sensitivity study. The Esposito & Ravenshorst case study is elected for this task due
478 to the extensive material characterisation campaign that accompanies it. The parameters include the
479 compressive strength of masonry f_c , the tensile strength of the unit-mortar interface $f_{t,i}$ and the initial
480 shear strength of the unit-mortar interface f_v . Additionally, the influence of the length of units l_u , the height
481 of units h_u and the thickness of the mortar joints t_m is similarly investigated. The results are presented in
482 Figure 10. For ease of presentation in a single graph, the parameters have been normalised by division with

483 their reference values as listed in Table 2. The normalised parameters are presented using a hat operator,
484 meaning that the normalised compressive strength of masonry is shown as \hat{f}_c and the force capacity is
485 shown as \hat{H}_{mod} .

486 The force capacity is not particularly sensitive to the compressive strength of masonry \hat{f}_c . This is
487 consistent with the rocking of the piers being governed primarily by the geometry of the structure.
488 Conversely, the tensile strength of the unit-mortar interface $\hat{f}_{t,i}$ plays a more important role, being directly
489 involved in the calculation of the tensile strength of the masonry. Similarly, the shear strength \hat{f}_v exerts the
490 greatest influence on the peak force due to it being involved in both the horizontal tensile strength and
491 shear strength of the masonry.

492 The geometric properties of the units and mortar appear to play a very significant role in the force
493 capacity. Firstly, it is found that increasing the length of units \hat{l}_u increases the force capacity due to an
494 increase in the horizontal tensile strength of masonry. Secondly, the thickness of the mortar joints \hat{t}_m does
495 not strongly affect the force capacity, although it is noted that an increase in the mortar joint in reality
496 would lead to a slight decrease in the compressive strength of masonry. Finally, increasing the height of the
497 units \hat{h}_u leads to a reduction of the force capacity due to a decrease in the horizontal tensile strength of
498 masonry.

499 In addition to the study of numerical parameters, the bonding pattern and boundary conditions are
500 included in the study. These results are presented in Table 4, the reference case being the one with an
501 English bond pattern and a cantilever with “strong” spandrel boundary condition.

502 While the stack bond results in a marginal reduction of the calculated force capacity, switching to a
503 Flemish or running bond leads to a roughly 23% increase. In all three cases (stack, Flemish, running bond)
504 the failure of the wall was due to bending of the left sub-system and biaxial failure of the spandrel in the
505 right sub-system. In the first case the change in tensile strength of masonry was not enough for making a
506 substantial change in the result. However, in the latter two cases, the increase in the tensile strength was
507 enough to make a difference in the capacity of the spandrel. The boundary conditions, as expected, have a
508 strong effect on the calculated force capacity. Double clamped and vertically restrained conditions result

509 in an increase of 14.3% and 45.5% of the force capacity respectively. Conversely, the “weak” spandrel
510 cantilever results in a 15.4% decrease in the force capacity, illustrating the effect of a rigid top beam or
511 lintel.

512 The sensitivity study illustrates the delicate interaction of all parameters involved in the prediction of
513 the in-plane shear force capacity of masonry walls. The dimensions of the units, the bonding pattern,
514 boundary conditions and properties of the unit-mortar interface all play a significant role in the load
515 bearing mechanism and should, therefore, always be the subject of careful investigation.

516 **Conclusions**

517 An analytical model for the prediction of the in-plane capacity of masonry piers and walls with openings
518 is presented. The model considers all major geometric and material parameters, including the bond type,
519 for the calculation of the capacity. Additionally, an analytical model is proposed for the prediction of the
520 damage initiation mode in masonry piers under in-plane shear. Apart from geometric and material
521 properties, no further numerical parameters or major empirical assumptions are needed for analysis.

522 The model accounts for all potential failure modes normally encountered in masonry walls subjected
523 to a combination of in-plane vertical and horizontal loading. Unequal vertical loading, asymmetric piers
524 and local variations in material properties can be easily introduced in the analysis.

525 The basis of the model is validated against numerous standalone pier experimental tests, while the
526 model for walls with openings is similarly validated against several case studies with different material
527 properties, dimensions, bonding patterns and boundary conditions.

528 The model provides a very efficient and accurate method for the capacity assessment of simple
529 structures subjected to in-plane shear loading under vertical stress. The damage initiation model provides
530 a simple means of highlighting weaknesses in masonry piers, thus allowing efficient intervention design
531 for the strengthening of masonry structures against damage initiation. The advantages of the model include
532 the calculation of the damage initiation and capacity forces with simple analytical expressions and no
533 computational resources nor reliance on empirical simplifications. This facilitates the quick completion of

534 sensitivity studies, which, given the large number of material and geometric parameters involved in the
535 problem, have been demonstrated to be crucial in understanding the behaviour of masonry walls with
536 openings. Further, the resulting failure mechanisms can be unambiguously evaluated by the user, whether
537 the model is used for research or in engineering practice. Disadvantages of the model include the lack of
538 capabilities for generating force-displacement data, a reliance on numerous material parameters and an
539 inability to account for cyclic loading or load reversal. It is noted that the first disadvantage is shared with
540 all methods based on limit-analysis, the second is shared with most other computational approaches
541 currently available and the third can be addressed in a future effort involving more detailed constitutive
542 modelling.

543 The proposed model presents opportunities for future work pertaining to the simulation of structural
544 reinforcement, such as in the form of embedded bars. The contribution of horizontal bars can be introduced
545 in the tensile strength for the biaxial failure check. Vertical bars can increase the rocking mode capacity
546 when anchored at the base of cantilever walls, or at the base and top of double-clamped piers. Finally,
547 diagonal bars can restore or increase the cohesion in damaged zones.

548 **Data availability statement**

549 All data, models, or code that support the findings of this study are available from the corresponding
550 author upon reasonable request.

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645 **Figure 3** Illustration of potential horizontal tensile failure modes for masonry. Planes of
646 failure for each mode indicated in dashed lines.

647 **Figure 4** Capacity envelope for piers, derived from the four considered failure modes.

648 **Figure 5** Results of pier capacity model against dataset of experimental data (Morandi *et al.*,
649 2018): a) relying on the reported f_t , b) relying only on calculated f_t . Dashed lines indicate 15%
650 difference, solid line marks equality.

651 **Figure 6** Damage initiation model envelope for piers, derived from the four considered
652 damage initiation modes in dotted lines. Capacity envelope in solid lines.

653 **Figure 7** Discretisation of masonry wall with central opening into components.

654 **Figure 8** a) “Strong” and b) “weak” spandrel function under horizontal loading.

655 **Figure 9 Potential failure shapes for base-pier-spandrel sub-system. Disposition of stress fans**
656 **and compressive struts.**

657 **Figure 10 Sensitivity study: effect of a) material properties and b) geometric parameters on**
658 **force capacity.**

Table 1 Experimental case studies for pier model validation: geometric and material parameters. Force capacity prediction error in parentheses.

Parameter	Symbol	Unit	Case study			
Specimen name	-	-	TUD_COMP_20	TUD_COMP_21	TUD_COMP_22	TUD_COMP_47/48
Reference	-	-	(Esposito and Ravenshorst, 2017)	(Esposito and Ravenshorst, 2017)	(Esposito and Ravenshorst, 2017)	(Esposito and Ravenshorst, 2017)
Pier length	l	mm	1100	3070	3070	3070
Pier height	h	mm	2778	2710	2710	2710
Pier thickness	t	mm	102	100	210	100
Unit length	l_u	mm	214	210	210	210
Unit height	h_u	mm	72	50	50	50
Mortar bed joint height	h_m	mm	10	10	10	10
Unit compressive strength	f_{cu}	N/mm ²	13.26	28.30	28.30	28.30
Mortar compressive strength	f_{cm}	N/mm ²	7.57	3.81	3.81	3.81
Masonry compressive strength	f_c	N/mm ²	6.35	14.02	10.67	11.35
Unit-mortar interface tensile strength	f_{ti}	N/mm ²	0.12	0.15	0.15	0.09
Initial shear strength	f_v	N/mm ²	0.13	0.12	0.12	0.14
Masonry bond	-	-	Running	Running	English	Running
Vertical stress	$-\sigma$	N/mm ²	0.63	0.36	0.36	0.46
Boundary condition	-	-	Cantilever	Double-clamped	Cantilever	Cantilever
Damage initiation - experimental	-	-	Rocking	Shear/rocking	Rocking	Rocking
Failure mode - experimental	-	-	Rocking	Biaxial	Rocking	Biaxial
Shear force capacity - experimental	H_{exp}	kN	15.1	98.1	117.2	112.5
Damage initiation - model	-	-	Rocking	Rocking	Rocking	Rocking
Failure mode - model	-	-	Rocking	Biaxial	Rocking	Biaxial
Shear force capacity - model	H_{mod}	kN	13.5 (-10.6%)	100.6 (2.5%)	120.2 (2.6%)	106.2 (-5.6%)

Table 2 Comparison of wall with opening capacity model with experimental results from the literature. Predicted force error in parentheses.

Ref.	h_1	h_2	h_3	l_1	l_2	l_3	t	f_c	f_v	μ	f_t	$-\sigma$	Boundary conditions	Masonry bond	H_{exp}^+	H_{mod}^+	H_{exp}^-	H_{mod}^-
–	mm	mm	mm	mm	mm	mm	mm	N/mm ²	N/mm ²	–	N/mm ²	N/mm ²	–	–	kN	kN	kN	kN
(Lobato Paz, 2009)	90	90	90	150	75	112.5	35	18.9	0.46	2.42	1.70	0.645	<i>C</i>	Running	4.9	5.7 (16.3%)	-	-
												1.132			7.7	9.6 (24.7%)	-	-
												1.858			13.1	12.5 (-4.6%)	-	-
												2.540			13.2	14.5 (9.8%)	-	-
												3.236			15.9	16.3 (2.5%)	-	-
												4.036			17.7	17.9 (1.1%)	-	-
(Parisi, Augenti and Prota, 2014)	0	2300	1000	1700	1700	1700	310	3.73	0.15	1.36	0.26	0.373 ^b	<i>C</i>	Running	184	171.5 (-6.8%)	-	-
(Vanin and Foraboschi, 2012)	325	1170	845	930	880	930	240	1.21	0.15 ^a	2.36	0.16	0.300 ^b	<i>C</i>	Flemish	63	51.7 (-17.9%)	-	-
												0.179 ^b			48	37.8 (-21.3%)	-	-
												0.090 ^b			18	19.4 (7.7%)	-	-
(Foraboschi, 2009)	380	1210	1180	1025	1070	1025	250	1.21	0.15 ^a	2.64	0.16	0.270 ^b	<i>C</i>	Flemish	64.2	60.4 (-5.9%)	-	-
												0.179 ^b			59.8	43.2 (-27.8%)	-	-
												0.090 ^b			18.2	24.4 (34.1%)	-	-
(Korswagen et al., 2017)	530	1510	650	870	780	1420	100	11.35	0.13	1.75	0.37	0.120	<i>C</i>	Running	22.2	19.3 (-13.1%)	-	-
(Esposito and Ravenshorst, 2017)	540	1680	490	870	1000	1200	210	10.67	0.2	0.88	0.32	0.340	<i>C</i>	English	85.4	84.2 (-1.4%)	94.1	90.4 (-3.9%)
(Raijmaker and Vermeltfoort, 1992)	350	350	350	430	210	325	100	10.5	0.35	1.75	0.66	0.300	<i>V</i>	Running	41.5	45.6 (9.9%)	-	-

^a assumed value

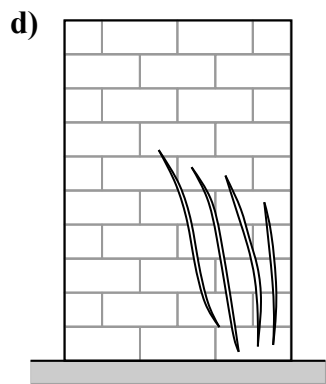
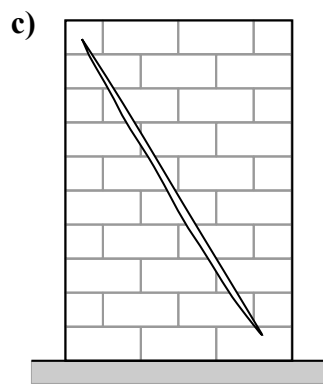
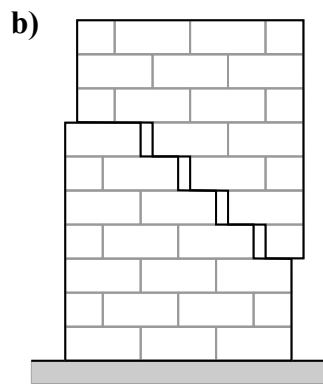
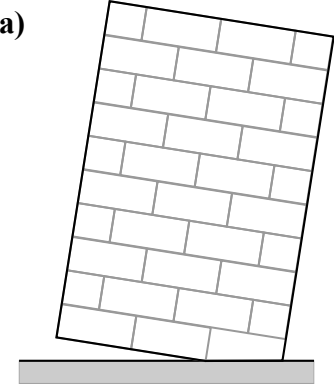
^b vertical load applied over pillars only

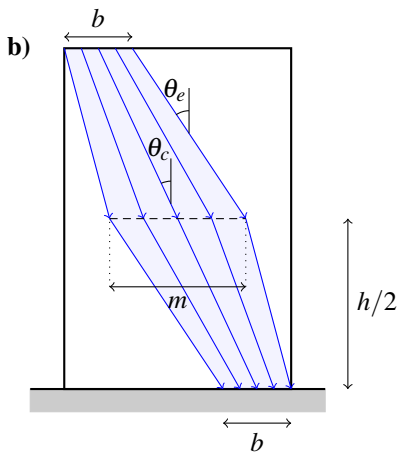
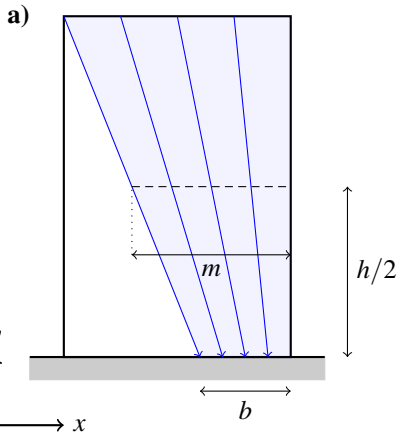
Table 3 Comparison of experimentally obtained and numerically derived failure modes for walls with openings.

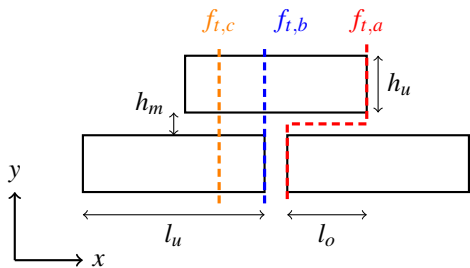
Reference	$-\sigma$ N/mm ²	Load direction	Experimental results		Numerical results	
			Left sub-system	Right sub-system	Left sub-system	Right sub-system
(Lobato Paz, 2009)	0.645	+	Bending	Bending	Bending	Bending
	1.132	+	-	-	Bending	Bending
	1.858	+	Bending/spandrel	Base/Pier	Bending	Base
	2.540	+	-	-	Bending	Base
	3.236	+	-	-	Bending	Pier
	4.036	+	Bending/spandrel	Pier/Base	Bending	Pier
(Parisi, Augenti and Prota, 2014)	0.373	+	Bending	Spandrel	Bending	Spandrel
(Vanin and Foraboschi, 2012)	0.300	+	Bending	Spandrel/base	Bending	Base
	0.179	+	Bending	Spandrel/base	Bending	Base
	0.090	+	Bending	Spandrel/bending	Bending	Bending
(Foraboschi, 2009)	0.270	+	-	-	Bending	Bending
	0.179	+	-	-	Bending	Bending
	0.090	+	-	-	Bending	Bending
(Korswagen <i>et al.</i> , 2017)	0.120	+	Bending	Bending/base	Bending	Bending
(Esposito and Ravenshorst, 2017)	0.340	+	Bending	Base	Bending	Spandrel
	0.340	-	Bending	Base	Bending	Spandrel
(Raijmaker and Vermeltoort, 1992)	0.300	+	Spandrel	Base	Spandrel	Base

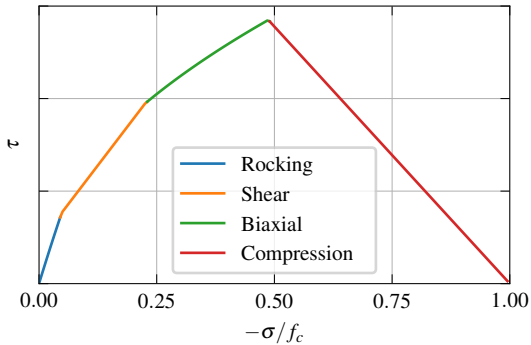
Table 4 Sensitivity study: effect of masonry bonding pattern and boundary conditions on force capacity.

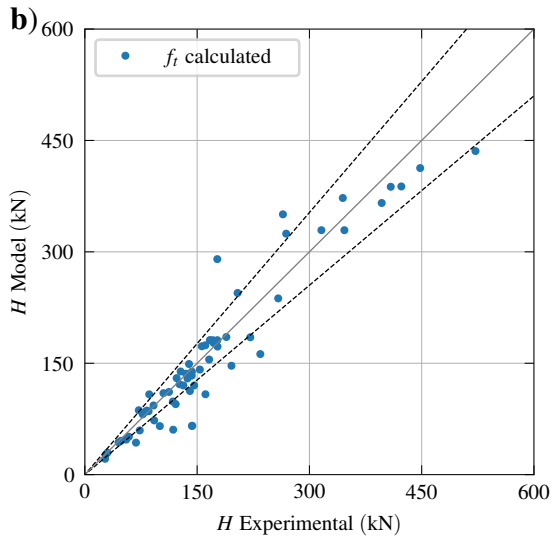
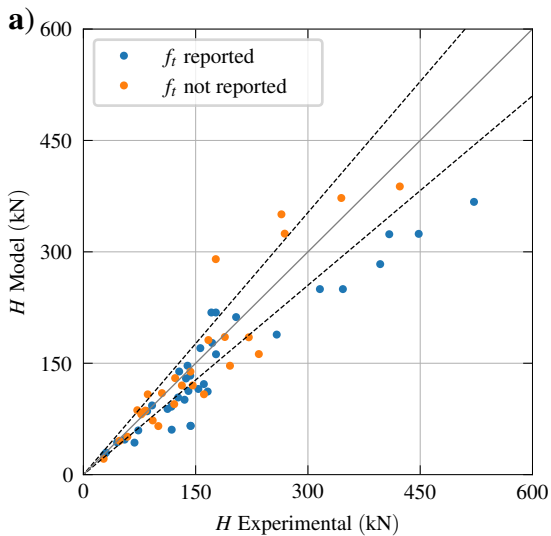
Parameter	Variation	\hat{H}_{mod}
Masonry Bond	Stack	0.964
	English	1.000
	Flemish	1.232
	Running	1.240
Boundary conditions	Cantilever with "weak" spandrel	0.846
	Cantilever with "strong" spandrel	1.000
	Double-clamped	1.143
	Vertically restrained	1.455

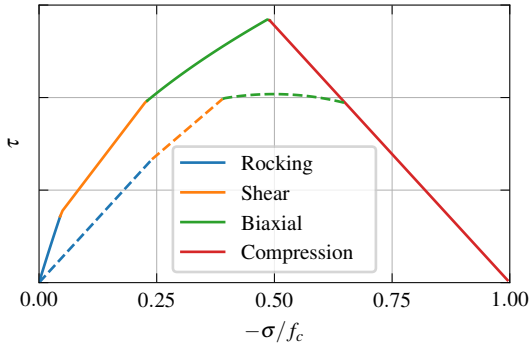


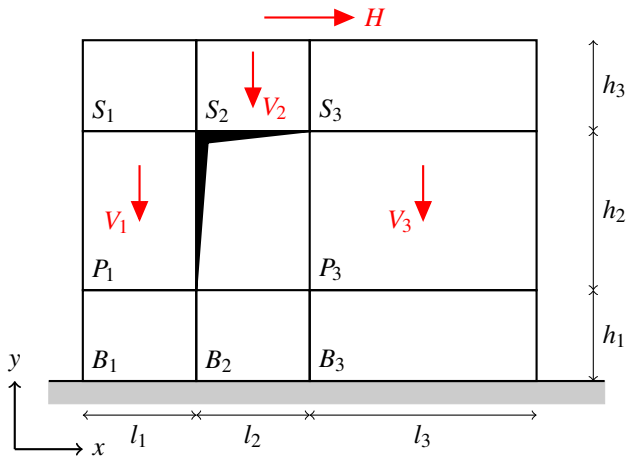




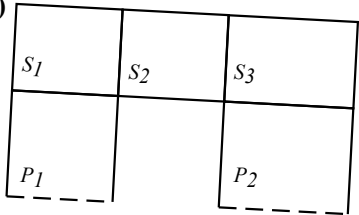




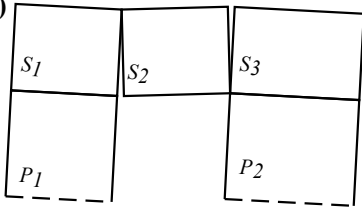




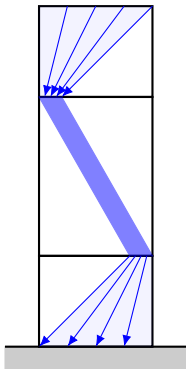
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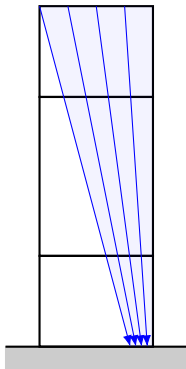
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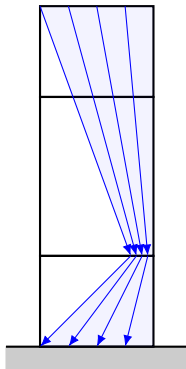
Shape 1



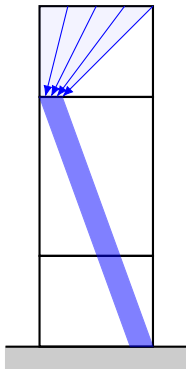
Shape 2



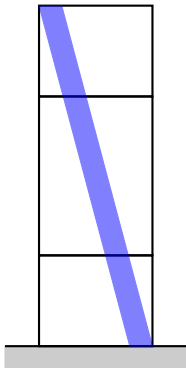
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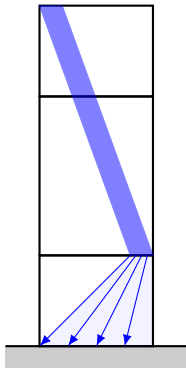
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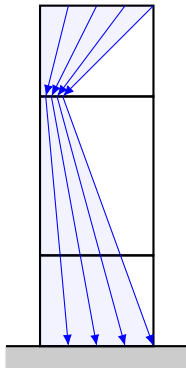
Shape 5



Shape 6



Shape 7



Shape 8

