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An Analytical Model for Characterizing the Thrust Performance of a Low Pressure Micro-Resistojet

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ABSTRACT

There is a clear trend towards the developments of micro-propulsion system to enhance the capabilities of nano- and pico-satellites. A promising propulsion option to meet the strict requirements of these small satellites is the Low-Pressure Micro-Resistojet (LPM) which works under rarefied gas dynamic regime. To simplify the engineering design of this propulsion system an analytical model has been developed using the fundamental physical models. This analytical model is based on the Kinetic theory of gases and the Maxwell-Boltzmann distribution of molecular velocities to describe the macroscopic flow parameters such as mass flow rate, velocity and pressure, and then to estimate the thruster performance. The equations are well known, but they are applied in this case using a particular approach in order to describe the physics behind this micro-propulsion system. Comparisons between numerical simulations using the DSMC method and the results of the analytical model, as well as experimental results, have been carried out. The analytical model using an accurate estimation of the transmission coefficient compared to the numerical simulation presents a maximum difference of 3%.

Nomenclature

a Small slot cross-section dimension
A_e Thruster exit area
b Large slot cross-section dimension
C_p Specific heat of the propellant at constant pressure
d Microchannel diameter
E_0 Energy of inlet stream
E_e energy of outlet stream
f Maxwellian or equilibrium distribution
g_0 Earth gravitational acceleration at sea level [9.80665 m/s^2]
h_0 Enthalpy at the microchannel entrance
h_e Enthalpy at the microchannel exit
I_{sp} Specific Impulse
k Boltzmann constant [$1.3806488 \times 10^{-23}$ J/K]
Kn Knudsen number
l Microchannel length

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L_0 Characteristic dimension
m_a Molecule mass
$\dot{m}$ Mass flow rate
$\dot{m}_{fm}$ Mass flow rate in free-molecular limit ratio
n Number density
n_e Number density at the microchannel exit
P_a External ambient pressure
P_0 Plenum pressure
P_e Exhaust pressure or Pressure at channel/slot exit
$\dot{Q}$ Heat transferred to the gas
T Overall temperature
T_0 Plenum temperature
T_e Overall temperature at the microchannel exit
T_{int} Internal temperature
T_{tr} Translational kinetic temperature
T_w Microchannel wall temperature
u, v, w Velocity components in the x, y, z directions
u_e Velocity at the microchannel exit
α Transmission coefficient
γ Specific heat ratio
δ Microchannel length to diameter ratio
Δh_{pc} Enthalpy of the phase change
η Propulsion system efficiency
λ Mean free path
ϕ Slot length to small cross-section dimension ratio
ζ Total number of degree of freedom
$\wp_t$ Power transmitted to the phase change into the tank
$\wp_w$ Power transmitted by the heater chip to the flow
$\Im$ Thrust
$\Im_m$ Momentum thrust
$\Im_p$ Pressure thrust
ψ Quantity of mass, momentum or energy
$\dot{\psi}$ Flux ψ through a surface per unit area

1 Introduction

The standardization of nano- and pico-satellites has created a niche market which has been more and more explored in the last years. The CubeSat is the most popular standard for this class of satellites [1]. It uses commercial-off-the-shelf (COTS) products, widely available on the market for several subsystems and components. However, there is still a lack of commercially available options for the propulsion system of such satellites. Among the most promising micro-propulsion systems for this class of satellites, due to their reliability and simplicity, are pulsed-plasma thrusters (PPT), cold gas thrusters, and micro-resistojets [2–8].

Recent research has proven that micro-resistojets are especially promising for CubeSats and PocketQubes, mainly for performing maneuvers such as formation flying, station keeping of constellations and orbit transfer [2,9,10]. High integration capability, small volume, low mass, fast response, high thrust to mass ratio, high reliability, and easy integrability in a thrust array are the main advantages of the micro-resistojets. With a low power consumption such propulsion systems can achieve a thrust level in the range of 0.1-10 mN with a specific impulse in the range of 50-200 s [11].

Additionally, micro-resistojets have the advantage that they can use virtually any propellant, including ”green” ones, and still deliver a reasonable performance in terms of thrust and specific impulse. A recent publication selected and characterized nine ”green” propellants out of 95 different substances that can be used in micro-resistojets, namely Acetone, Ammonia, Butane, Cyclopropane, Ethanol, Isobutane, Methanol, Propene and Water [12]. These propellants can also be stored at ambient temperature and relatively low pressure (not higher than 10 bar) as a liquid and, in some cases such as water, also as a solid.

The Low Pressure Micro-Resistojet (LPM) is an extension of a similar design previously developed and tested in [13]. This propulsion system concept is divided into three main parts: tank, feed system, and thruster. The tank stores the propellant in solid or liquid state, and a heater is used to sublimate/evaporate the propellant. The feed system is basically composed of a valve which receives the opening or closing command allowing the passage of the propellant vapour. The thruster is composed of a plenum and a heater chip where the propellant gas is expelled to the outer space. The heater chip is usually made of silica wafer and presents a grid of straight channels or slots which heat up the propellant increasing its velocity [11]. This propulsion system has the main characteristic to work under low pressure up to 600 Pa meaning it is under rarefied flow regime.

Rarefied gas dynamics is important in different applications such as vacuum technology and space dynamics [14,15], but it is becoming even more popular in the microfluidics field. The Knudsen number (Kn) is the dimensionless number which defines the degree of gas rarefaction. It depends on the average distance travelled by the molecules between collisions, known as the mean-free path (λ), and a characteristic dimension of the flow (L_0), and is expressed as $Kn = \lambda/L_0$. Different rarefied gas dynamics regimes are defined, depending on the value of Kn: continuum flow regime with slip flow ($0.1 > Kn > 0.01$), transitional flow regime ($10 > Kn > 0.1$) and free-molecular flow regime ($Kn > 10$) [16].

From space propulsion theory, thrust $\Im$ and specific Impulse I_{sp} are the two parameters used to estimate the thruster performance. The theoretical thrust is well known to be the sum of momentum thrust $\Im_m$ and pressure thrust $\Im_p$ as

$$\Im = \Im_m + \Im_p = \dot{m}u_e + (P_e - P_a)A_e \tag{1}$$

where $\dot{m}$ is the mass flow rate, u_e is the exhaust velocity, P_e is the exhaust pressure, P_a the external ambient pressure. The specific impulse is the total impulse delivered per unit weight of consumed propellant expressed as

$$I_{sp} = \frac{\Im}{\dot{m}g_0} \tag{2}$$

where g_o is the Earth gravitational acceleration at sea level. In order to estimate the thruster performance, researchers proposed a simplified model that they used just as an approximation, besides of the relevant experiment results [13]. In their model, they mainly neglect the pressure thrust and overestimate the momentum thrust.

In this paper, an analytical model to estimate the thruster performance is presented in order to facilitate the engineering design and parametric study of this propulsion concept. In this model the pressure and momentum thrust are considered, improving the model presented in [13]. Additionally, the efficiency of the system is estimated taking into account the necessary power to sublimate/evaporate the propellant in the tank and heat up the expelled propellant gas. A validation of the analytical model is performed by comparing its results to different numerical simulations and some available experimental results. The current analytical model presents a much better accuracy than the model presented in [13] when compared to the numerical and experimental results.

2 Operation principle

The architecture of the LPM concept is presented in Figure 1. The propellant is stored in the tank in solid or liquid state. Due to the working principle of the LPM, it is important to keep the tank pressure as low as possible, otherwise a pressure reducer or a proportional valve would be necessary to reduce the vapour pressure downstream the tank making the system more complex. An open/close valve is placed between the tank and the plenum in order to control the mass flow rate. An important part of the system is the heater chip that is a flat plate with a grid of microchannels (or micro-slots) in which the particles are accelerated and expelled into space. The gas flow goes from the continuum flow regime inside the tank to the free molecular regime in outer space.

Basically, the propellant is evaporated or sublimated inside the tank. The propellant vapour goes through the feeding system (tubing and valve) and reaches the plenum. In the plenum the nominal pressure is kept between 50 and 300 Pa. The propellant molecules go towards the microchannels where they collide with the high temperature wall increasing their velocity generating higher thrust and specific impulse. The wall temperature is heated by electric power attached to the heater chip, similar to any conventional micro-resistojet.

3 Analytical model

The thruster part of the propulsion system that is composed by plenum and microchannel (or micro-slot) is designed to work in a transitional flow regime defined by a Knudsen number Kn between 0.1 and 10. The flow behaviour inside the microchannel plays the main role in the thruster performance. In order to estimate this behaviour the kinetic theory of Maxwell is used. According to Equation 1 the main unknown parameters to define the thrust are mass flow rate, exhaust velocity and exhaust pressure. Based on that, this section presents the analytical equations to define them. Additionally, it presents the analytical equations to estimate the needed electric power and the efficiency of the propulsion system.

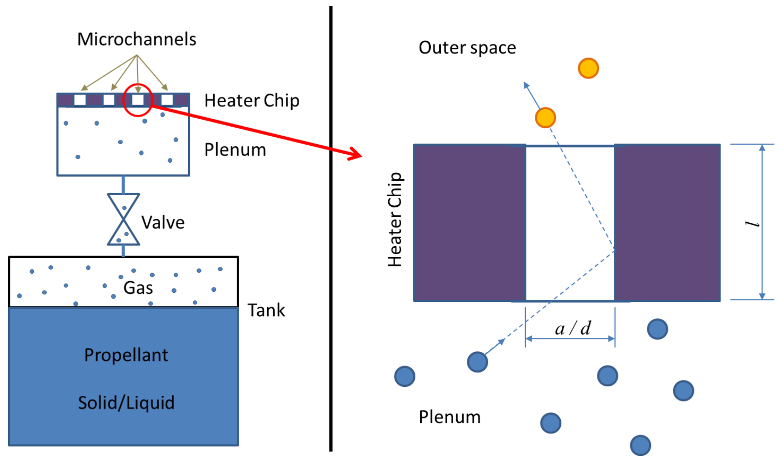

Fig. 1. Schematic of the LPM and its working principle.

Assuming that there is thermodynamic equilibrium inside the plenum and given the degree of rarefaction, the Maxwellian or equilibrium distribution f for the thermal velocity of the molecules in thermodynamic equilibrium is given in [17]:

$$f(u,v,w) = \left(\frac{m_a}{2\pi kT}\right)^{3/2} e^{-\frac{m_a(u^2+v^2+w^2)}{2kT}}. \tag{3}$$

where m_a is the molecule mass, k is the Boltzmann's constant, T is the thermodynamic temperature and u, v and w is the velocity of the molecule for a cartesian axis x, y and z respectively [17]. Additionally, the flux $\dot{\psi}$ of mass, momentum or energy of a quantity ψ through a surface per unit area in the axis x direction, which is the flow direction towards the microchannels, is defined in [13]

$$\dot{\psi} = n \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{0}^{\infty} u \psi f(u,v,w) du dv dw \tag{4}$$

where, n is the molecular number density.

Mass flow rate The actual mass flow rate $\dot{m}$ that goes through the channel depends to the geometry of the microchannel which is discussed below and the mass flow rate in the free-molecular limit ($\dot{m}_{fm}$) which is defined by setting $\psi = m_a$ in the Equation 4 and multiplying by the microchannel exit area A_e. Then we have

$$\dot{m}_{fm} = n\sqrt{\frac{km_aT}{2\pi}}A_e. \tag{5}$$

As said before the geometry of the expansion microchannel defines the actual mass flow rate through the microchannel. This relation is described by the transmission coefficient (α), which is expressed in the form of

$$\alpha = \frac{\dot{m}}{\dot{m}_{fm}}. \tag{6}$$

Then, combining Equation 5 and 6 with the ideal gas equation ($n = P/kT$), the actual mass flow rate can be estimated as [13]:

$$\dot{m} = \alpha P_0 \sqrt{\frac{m_a}{2\pi kT_0}} A_e \tag{7}$$

where P is replaced with P_0 which is the plenum pressure and T is replaced with T_0 which is the plenum temperature. Several empirical equations have been proposed to evaluate the transmission coefficient depending on the geometry [14], for instance:

Short Uniform Circular Cross Section (channels):

$$\alpha = 1 + \delta^2 - \delta\sqrt{\delta^2+1} - \frac{\left[(2-\delta^2)\sqrt{\delta^2+1} + \delta^3 - 2\right]^2}{4.5\delta\sqrt{\delta^2+1} - 4.5ln(\delta + \sqrt{\delta^2+1})} \tag{8}$$

where δ is the channel length to diameter ratio l/d. It is valid for a $\delta < 50$. By way of example, a cylindrical channel with an aspect ratio of 5 has a transmission coefficient of 0.19.
Short Uniform Rectangular Cross Section (slots):

$$\alpha = 0.5(1 + \sqrt{1+\phi^2} - \phi) - \frac{1.5\left[\phi - ln(\phi + \sqrt{1+\phi^2})\right]^2}{\phi^3 + 3\phi^2 + 4 - (\phi^2+4)\sqrt{1+\phi^2}} \tag{9}$$

where, if a and b are the cross-sectional dimensions with $b \geq a$, and l is the length in the direction of the gas flow, ϕ is the slot length to small cross-sectional dimension ratio l/a. This expression is valid for $b \gg a$ and $b \gg l$. As an example, in a slot with an aspect ratio of 5, the transmission coefficient is 0.36.

Exhaust velocity The exhaust velocity u_e is the average gas velocity at microchannel exit and depends on the translational kinetic temperature. It can be modeled by setting $\psi = u$ in the Equation 4 and divide by the total number flux through the surface as

$$u_e = \frac{n\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty} u^2 f(u)dudvdw}{n\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty} uf(u)dudvdw}. \tag{10}$$

Solving the Equation 10 we have

$$u_e = \sqrt{\frac{\pi k T_{Tr}}{2m_a}}. \tag{11}$$

Exhaust pressure The exhaust pressure P_e at the expansion microchannel exit is another important parameter to define the thruster performance. In order to model P_e we can rewrite the mass flow rate according to the microchannel exit parameters as

$$\dot{m} = m_a n_e u_e A_e \tag{12}$$

where n_e is the molecular number density at microchannel exit. According to Bird [18], the scalar pressure is related to the translational kinetic temperature meaning that the pressure can be written equivalently to the perfect gas equation of state. Based on it and replacing n_e in Equation 12 we have

$$\dot{m} = m_a \frac{P_e}{kT_{tr}} u_e A_e. \tag{13}$$

Now, replacing the equations 7 and 11 in the equation 13, and rearranging we have

$$P_e = \frac{\alpha P_0}{\pi}\sqrt{\frac{T_{tr}}{T_0}}. \tag{14}$$

Translational kinetic temperature The translational kinetic temperature is the unknown parameter to define the main parameters related to the thruster performance. During a gas expansion into a very low pressure environment there is a gas temperature drop [19–21]. Using the conservation of energy and the equipartition principle we can analyse the energy throughout the microchannel. According to the first law of thermodynamics for an open system considering the microchannel as the control volume (Figure 2), we can write the equation in the steady state as

$$\dot{Q} = E_e - E_0 = \dot{m}\left(h_e + \frac{u_e^2}{2}\right) - \dot{m}h_0 \tag{15}$$

where $\dot{Q}$ is the heat transferred to the gas, h_e is the enthalpy at the microchannel exit, h_0 the enthalpy at the microchannel entrance. For an ideal gas, the variation of enthalpy between microchannel entrance and the exit can be replaced by

$$h_e - h_0 = C_p\left(T_e - T_0\right) \tag{16}$$

where C_p is the specific heat of the propellant at constant pressure and T_e is the overall temperature at the microchannel exit. The heat transferred $\dot{Q}$ to the gas can be estimated by

$$\dot{Q} = \dot{m}C_p\left(T_w - T_0\right). \tag{17}$$

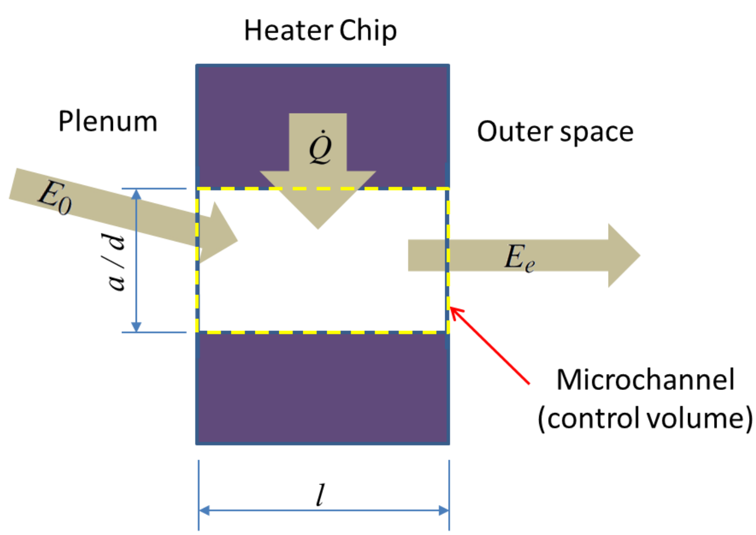

Fig. 2. Balance of the energy inside of the microchannel where E_0 is the energy which gets in the control volume, E_e is the energy which gets out the control volume and $\dot{Q}$ is the heat transferred to the gas inside the control volume.

Now, replacing Equations 16 and 17 into Equation 15 we have

$$C_p\left(T_w - T_e\right) = \frac{u_e^2}{2}. \tag{18}$$

The specific heat of the propellant at constant pressure C_p for an ideal gas depends on the total number of degree of freedom ζ as follows

$$C_p = \frac{(\zeta+2)}{2}\frac{k}{m_a}. \tag{19}$$

For monatomic molecules the translational kinetic temperature T_{tr} is equal to the overall temperature T. However, for diatomic and poly-atomic molecules the overall temperature T is divided into translational kinetic temperature T_{tr} and internal temperature T_{int}. Looking at the overall temperature at the microchannel exit T_e, we can assume that the internal gas temperature T_{int} at the microchannel exit is equal to the wall temperature T_w. Then, we can write

$$T_e = \frac{(3T_{tr} + (\zeta-3)T_{int})}{\zeta} = \frac{(3T_{tr} + (\zeta-3)T_w)}{\zeta}. \tag{20}$$

Additionally, the total number of degree of freedom ζ also defines the well known specific heat ratio γ as

$$\gamma = \frac{\zeta+2}{\zeta}. \tag{21}$$

Replacing the Equations 11, 19, 20 and 21 into the Equation 18 we have the translational kinetic temperature T_{tr} in terms of the wall temperature T_w and specific heat ratio γ as

$$T_{tr} = \left(\frac{6\gamma}{\pi+6\gamma}\right)T_w. \tag{22}$$

Thruster Performance Replacing Equations 7, 11, 14 and 22 into Equation 1 and assuming P_a equal to zero in space, we have trust as

$$\Im = \alpha P_0 A_e \frac{(\pi+2)}{2\pi}\sqrt{\frac{T_w}{T_0}\left(\frac{6\gamma}{\pi+6\gamma}\right)}. \tag{23}$$

Now, replacing Equations 7 and 23 into the Equation 2, we have specific impulse as

$$I_{sp} = \frac{A_e}{g_0}\frac{(\pi+2)}{\sqrt{2\pi}}\sqrt{\frac{kT_w}{m_a}\left(\frac{6\gamma}{\pi+6\gamma}\right)}. \tag{24}$$

Thruster Power The thruster power $\wp_w$ is the power necessary to heat up the propellant inside of the microchannel. It can be estimated by using

$$\wp_w = \dot{Q} \tag{25}$$

where $\dot{Q}$ is the heat transferred to the propellant, Equation 17.

Tank Power As previously described the tank has to be designed such that the propellant is predominantly in a liquid or solid state. Basically, the phase change occurs inside the tank and only gas is expected to pass through the feeding system. We can therefore assume that the thermodynamic equilibrium inside the tank is most of the time in a saturation region during stand by. However, when the thruster is working the tank is not in equilibrium anymore. In this case, the phase change is an endothermic process, meaning that to keep a certain sublimation/evaporation rate (or mass flow rate) it is necessary to increase the enthalpy by giving power to the system. In other words, a heater device shall be used in the tank in order to offset the power lost due to the mass flow rate exiting the tank. The required power in the tank $\wp_t$ is defined by

$$\wp_t = \dot{m}\Delta h_{pc} \tag{26}$$

where Δh_{pc} is the enthalpy required by the phase change.

Efficiency Finally, the propulsion system efficiency η can be estimated using the following relationship between thrust, specific impulse and consumed power:

$$\eta = \frac{\Im I_{sp} g_o}{2\left(\wp_t + \wp_w\right)}. \tag{27}$$

4 Results and Discussions

Table 1 compares the numerical results presented in [22], which presents an uncertainty not bigger than 2.6 %, to the results obtained with the analytical method presented in the previous section. The results shown in the Table 1 are related to Nitrogen propellant flowing in a single microchannel with an aspect ratio of 5 (length of 500 μm and diameter of 100 μm). The plenum temperature is 300 K, the plenum pressure varies in the range from 50 Pa to 300 Pa, and the wall temperature varies in the range from 300 to 900 K. The main source of uncertainty of the analytical model is the empirical Equation 8 to define the transmission coefficient, since it does not take into account the thermal effect and it is based only on the geometry of the channel. This difference in the evaluation of the transmission coefficient becomes higher at higher wall temperature and higher plenum pressure, as consequence, it affects the thrust calculated by the analytical model. Using the transmission coefficient from the numerical simulation into the analytical model the thrust error does not exceed 3 % when comparing to the numerical results.

Table 1. Comparison between the numerical results presented in [22] and the results of the analytical model, for Nitrogen flowing in a single microchannel with aspect ratio of 5 and plenum temperature of 300 K.

P_0 [Pa] / T_w [K]	α_{num}	α_{ana}	α error	$\Im_{num}$ [nN]	$\Im_{ana}$ [nN]	$\Im$ error
50 / 300	0.190	0.190	0.0%	63.6	63.0	-0.9%
50 / 573	0.185	0.190	2.7%	85.0	87.1	2.5%
50 / 700	0.183	0.190	3.7%	92.8	96.3	3.7%
50 / 900	0.182	0.190	4.4%	104.2	109.2	4.8%
150 / 300	0.195	0.190	-2.4%	196.7	189.1	-3.9%
150 / 573	0.180	0.190	5.6%	249.3	261.3	4.8%
150 / 700	0.177	0.190	7.0%	271.3	288.8	6.5%
150 / 900	0.174	0.190	8.7%	301.3	327.5	8.7%
300 / 300	0.198	0.190	-4.2%	402.3	378.2	-6.0%
300 / 573	0.177	0.190	6.9%	493.4	522.6	5.9%
300 / 700	0.173	0.190	9.1%	531.9	577.7	8.6%
300 / 900	0.169	0.190	11.2%	587.9	659.1	12.1%

Figure 3 shows the comparison among the numerical results presented in [22], the analytical model proposed in [13] and the current analytical model for a single microchannel using Nitrogen with aspect ratio of 5 and plenum temperature of 300 K. The analytical model proposed in [13] presents an error of about 30 % when compared to the numerical results. Similarly,

Figure 4 shows the comparison among the numerical results presented in [23], the analytical model proposed in [13] and the current analytical model for a single microchannel using Water with aspect ratio of 5 and plenum temperature of 300 K. The analytical model proposed in [13] presents an error between 20 and 30 % when compared to the numerical results while the current analytical model presents an error up to 10 %.

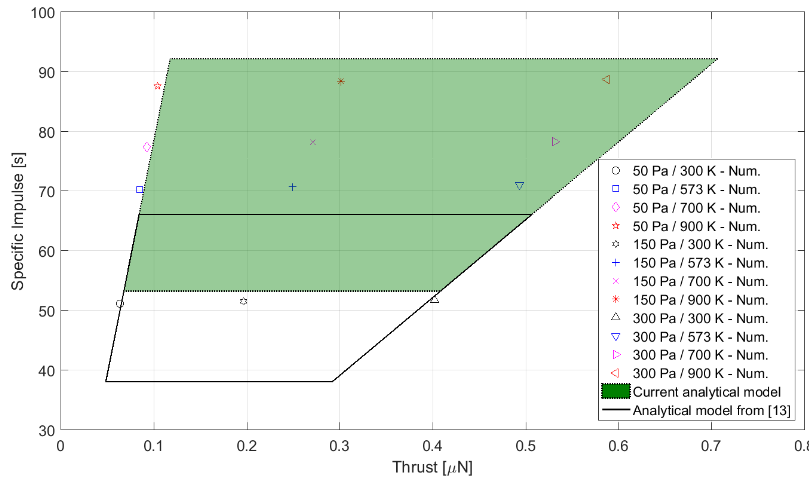

Fig. 3. Thrust versus Specific Impulse for different values of the plenum pressure ranging from 50 to 300 Pa and wall temperature ranging from 300 to 900 K. Comparison between the numerical results presented in [22], the analytical model proposed in [13] and the current analytical model for a single microchannel using Nitrogen with aspect ratio of 5 and plenum temperature of 300 K.

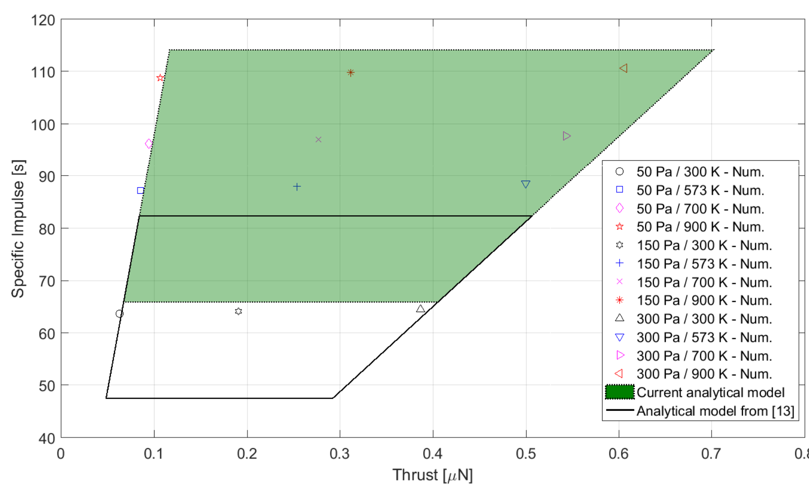

Fig. 4. Thrust versus Specific Impulse for different values of the plenum pressure ranging from 50 to 300 Pa and wall temperature ranging from 300 to 900 K. Comparison between the numerical results presented in [23], the analytical model proposed in [13] and the current analytical model for a single microchannel using Water with aspect ratio of 5 and plenum temperature of 300 K.

A numerical analysis carried out in [24] which studies the behaviour of the flow inside a slot is used to compare to the current analytical model in Table 2. A single slot with aspect ratio of 5 (length of 500 μm, short slot dimension of 100 μm and large slot dimension of 5.375 mm) under plenum temperature of 300 K using Nitrogen as propellant is the analysed geometry. A plenum pressure varying from 50 to 200 Pa and wall temperature varying from 300 to 900 K were considered. In this case the thrust differences are considered acceptable since the uncertainty in the simulations is reported to be around 4.4 %. Additionally, there is also the uncertainty from the empirical Equation 9 that does not consider the thermal effect. The same results compared to the model proposed in [13] present an error between 35 and 45 %.

Table 2. Comparison between the numerical results presented by [24] and the analytical model for a single slot using Nitrogen with aspect ratio of 5 and plenum temperature of 300 K.

P_0 [Pa] / T_w [K]	$\Im_{num}$ [μ N]	$\Im_{ana}$ [μ N]	$\Im$ error
50 / 300	6.8	6.3	-7.9%
50 / 600	9.2	8.9	-3.4%
50 / 900	10.9	10.9	-0.2%
100 / 300	13.6	12.6	-7.9%
100 / 600	18.1	17.8	-1.7%
100 / 900	21.4	21.7	1.4%
150 / 300	20.4	18.9	-7.9%
150 / 600	26.8	26.6	-0.8%
150 / 900	31.9	32.6	2.1%
200 / 300	27.3	25.2	-8.3%
200 / 600	36.0	35.5	-1.4%
200 / 900	42.6	43.5	2.1%

The analytical model is also compared to the experiment performed in [13]. The heater chip was made out of silicon wafer with thickness of 500 μm presenting 44 slots with dimension of 100 μm wide and 5.375 mm long meaning aspect ratio of 5. The experiment was performed using a nano-Newton thrust stand in a large vacuum chamber that was capable to keep the background pressure below 10^{-2} Pa. They provide the fit equations for the thrust as a function of T_w as well as the specific impulse as a function of T_w, both for a constant mass flow rate of 50 sccm. Different propellants were used such as Nitrogen (N_2), Argon (Ar) and Carbon dioxide (CO_2). Figure 5 shows the thrust comparison among experiment and analytical results presented in [13] and the current analytical model. In an analogous way, Figure 6 shows the specific impulse comparison. The differences between experimental and the current analytical results can be considered acceptable, taking into account the uncertainties in the test measurements and the not completely clear description of the experiment setup and procedures.

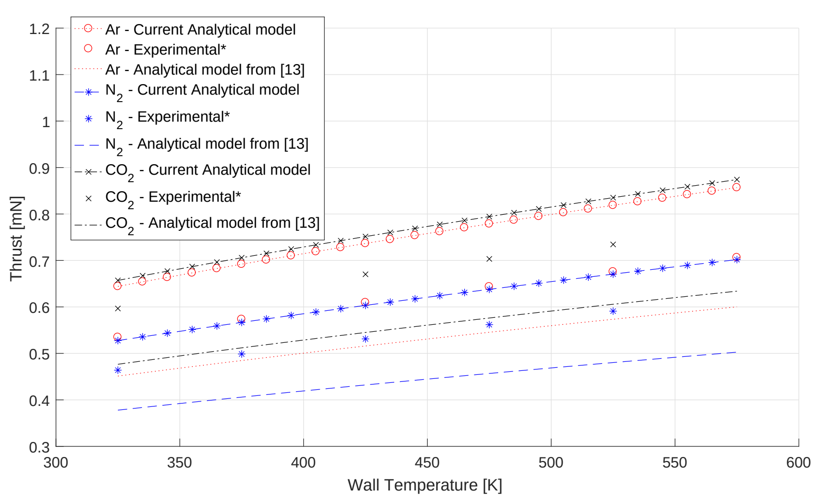

Fig. 5. Thrust as a function of T_w for different propellants and constant mass flow rate. *The experimental results were plotted from the fit equations presented in [13].

This analytical model is recommended to be used in order to estimate the LPM performance during the initial engineering design phase, and is particularly useful to perform parametric analysis to define how different design parameters influence

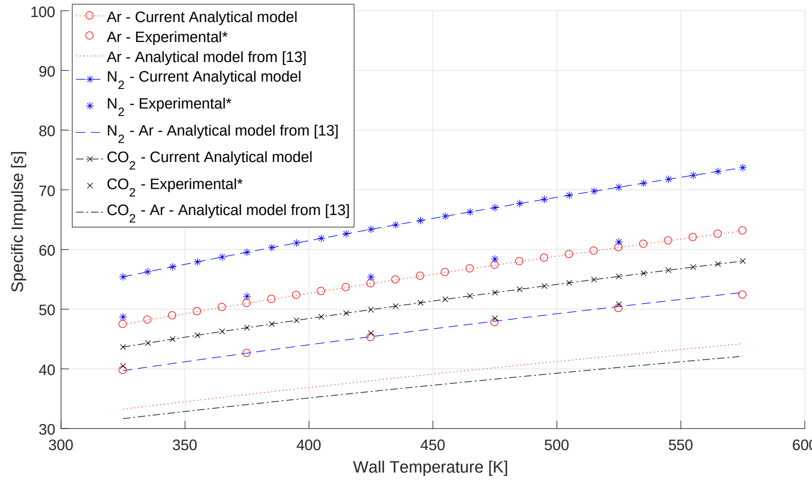

Fig. 6. Specific impulse as a function of T_w for different propellants and constant mass flow rate. *The experimental results were plotted from the fit equations presented in [13].

the thruster performance. However, it presents a limitation when the microchannel is not straight as in this case the flow characteristics can not be reproduced by this simplified analytical model. For instance, a divergent microchannel is typically characterized by supersonic flow at the microchannel exit, differently to the straight microchannel where the flow remains sonic, and by inefficient heat transfer since it depends on the particle-surface collisions, which are reduced in a divergent microchannel [22]. Besides the straight microchannel, there is another case where the analytical model can still be used as a good approximation: the optimal case where the first part of the microchannel is divergent and the second part is straight as presented in [22]. It is due to the fact that the exit flow is sonic as well.

5 Conclusion

In this paper an analytical model to estimate the performance of Low-Pressure Micro-Resistojet was presented. This analytical model simplifies the engineering design of the propulsion system, allowing faster decision-making and more detailed parametric analysis. The proposed analytical model presents a much better accuracy when compared to the analytical model proposed in [13]. The main uncertainty in the proposed model has been identified to be related to the transmission coefficient equation, an empirical equation that does not take into account the heat effect. Using a more accurate estimation of the transmission coefficient obtained from numerical analysis, the difference between the performance predictions obtained by numerical DSMC results and the results given by the analytical model becomes lower than 3%.

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