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# On characteristic values for calculating factors of safety for dyke stability

D. VARKEY*, M. A. HICKS*, A. P. VAN DEN EIJNDEN* and P. J. VARDON*

Various simplified approaches are used to calculate the characteristic values of shear strength properties, which have then been used in deterministic stability analyses of a dyke cross-section. The calculated factors of safety are compared with the 5-percentile 'system response' of the dyke cross-section, calculated using the more exhaustive random finite-element method (RFEM), which is consistent with the requirements of Eurocode 7. The simplified methods accounting for variance reduction due to averaging of property values mostly give factors of safety within 10% of the RFEM solution, whereas the factor of safety based on the 5-percentile material properties is significantly over-conservative.

**KEYWORDS:** numerical modelling; slopes; statistical analysis



## NOTATION

| | |
|---|---|
| $a$ | factor accounting for the extent and quality of test results and levels of expertise |
| $b_i$ | coefficient of variable $X_i$ in the linearised performance function |
| COV | inherent coefficient of variation |
| $\mathrm{COV_m}$ | coefficient of variation due to measurement error |
| $\mathrm{COV_s}$ | coefficient of variation due to statistical error |
| $\mathrm{COV_t}$ | coefficient of variation due to transformation error |
| $\mathrm{COV_{total}}$ | total coefficient of variation |
| $c'$ | effective cohesion |
| $F$ | factor of safety |
| $G$ | system response function |
| $l_i$ | component of failure length in the $i$ direction |
| $X$ | variable |
| $X_{\mathrm{extr}}$ | expected extreme value of $X$ |
| $X_{\mathrm{k}}$ | characteristic value of $X$ |
| $X_{\mathrm{m}}$ | mean value of $X$ |
| $\Gamma^2$ | variance reduction factor |
| $\Gamma_i^2$ | $\Gamma^2$ in the $i$ direction; $\Gamma^2$ for $X_i$ |
| $\gamma$ | unit weight |
| $\eta$ | percentile of the underlying distribution corresponding to $X_{\mathrm{k}}$ |
| $\theta$ | scale of fluctuation |
| $\theta_i$ | $\theta$ in the $i$ direction |
| $\mu_i$ | mean of $X_i$ |
| $\sigma_i$ | standard deviation of $X_i$ |
| $\Phi$ | standard normal cumulative distribution function |
| $\phi'$ | effective friction angle |

## INTRODUCTION

Engineering practice often uses characteristic soil property values, which are meant to account for (among other things) the spatial nature of soil variability with respect to the extent of the failure mechanism, and partial factors – for example, as in Eurocode 7 (EC7) (CEN, 2004). Although EC7 gives only limited guidance on determining characteristic values, several simplified approaches have been proposed (Shen *et al*., 2019). However, a more rigorous approach is the random finite-element method (RFEM) (Fenton & Griffiths, 2008), which combines random field theory with the finite-element method within a Monte-Carlo framework (Griffiths *et al*., 2009; Hicks & Spencer, 2010). In particular, Hicks *et al*. (2019) used RFEM to account for spatial variability of soil properties in the reliability-based assessment and re-design of a dyke in the Netherlands. The assessment revealed that the factor of safety did not meet national safety requirements. However, it resulted in a 48% higher factor of safety compared to that obtained using a simple interpretation of EC7 based on 5-percentile property values, and thereby led to a less intrusive and more economic re-design.

This paper uses various simplified approaches to determine characteristic soil property values for the dyke cross-section analysed by Hicks *et al*. (2019). These values have then been used in deterministic finite-element slope stability analyses, and the resulting factors of safety ($F$) compared with $F = 0.98$, the 5-percentile system response previously computed using RFEM.

## CHARACTERISTIC VALUES AND DESIGN ACCORDING TO EC7

Section 2.4.5.2 of EC7 states that, if statistical methods are to be used in the derivation of characteristic values, clause (11) applies (Table 1). From this clause, it can be inferred that the characteristic value should be selected so as to give a minimum confidence level or reliability of 95% with respect to the system response, before application of partial factors. Hicks & Samy (2002), Hicks (2012) and Hicks & Nuttall (2012) investigated the implications of clause (11) and its footnote, and explained the relationship between them by relating the scale of fluctuation ($\theta$) – that is, the distance over which soil property values are significantly correlated, with the length of the potential failure surface. They also introduced an 'effective' property distribution that can be back-figured from the response of the system/structure, as simply illustrated in Fig. 1 for a single material property ($X$) represented by a normal distribution. The mean and standard deviation of this 'effective' property distribution are generally lower than those of the underlying

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**Table 1.** Clause (11) extracted from Section 2.4.5.2 of EC7

| | |
|---|---|
| (11) | If statistical methods are used, the characteristic value should be derived such that the calculated probability of a worse value governing the occurrence of the limit state under consideration is not greater than 5%. NOTE: In this respect, a cautious estimate of the mean value is a selection of the mean value of the limited set of geotechnical parameter values, with a confidence level of 95%; where local failure is concerned, a cautious estimate of the low value is a 5% fractile. |

Source: CEN (2004)

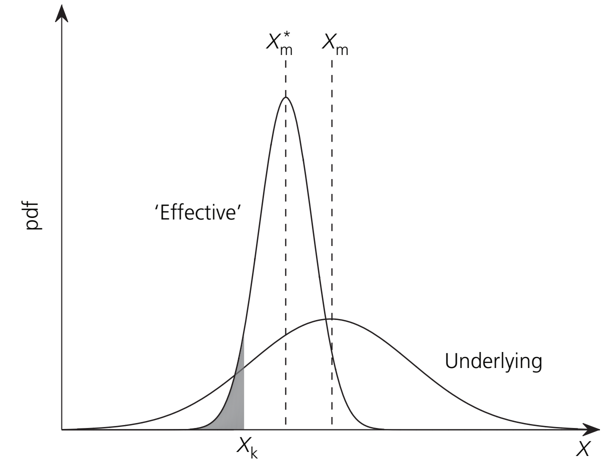


**Fig. 1.** Derivation of characteristic property value satisfying EC7 (source: based on Hicks (2012) and Hicks *et al*. (2019))

property distribution, due to the respective influences of weaker zones on the failure mechanism and averaging of soil properties along the failure surface. Consequently, the 5 percentile of the effective distribution, which represents the characteristic value ($X_k$) defined in clause (11), generally corresponds to a percentile ($\eta$) of the underlying distribution that is higher than 5%.

Unfortunately, the derivation of $X_k$ in Fig. 1 is not trivial, as demonstrated by Hicks & Samy (2002) and Hicks & Nuttall (2012), although some attempt has been made to approximate the process for simpler applications (Ching & Phoon, 2013). This is because it is a function of the underlying property distribution, the spatial correlation of properties and the problem being analysed. Moreover, the derivation becomes more complicated for multiple soil properties and multiple soil layers, because there are then many possible combinations of $X_k$ that can give the same reliability. One solution is to use a simplified approach for calculating the value of $X_k$, which can then, after application of partial factors, be used in deterministic analyses to obtain reliability-based values of *F*. For example, Dutch engineering practice calculates *F* for dykes by using the 5 percentile of either the underlying soil property distribution or a distribution which takes some account of spatial variability by using simplified variance reduction. Recently, Hicks *et al.* (2019) showed how RFEM can be used (with or without partial factors) to directly determine reliability-based factors of safety, without having to explicitly derive the characteristic values. Although this method is computationally intensive, it removes the need to determine $X_k$, is completely general, and automatically accounts for both variance reduction and the reduced mean due to weaker zones.

## ANALYSIS OF DYKE CROSS-SECTION

Figure 2 shows the idealised dyke cross-section analysed by Hicks *et al.* (2019), which is the same as that used previously by Kames (2015) in limit equilibrium slope stability analyses based on 5-percentile characteristic values. Table 2 lists, for each soil layer, the mean, 5 percentile and coefficient of variation (COV) of the shear strength parameters (cohesion $c'$ and tangent of friction angle $\phi'$), which were assumed to follow log-normal distributions, as well as the unit weight $\gamma$, which was assumed to be deterministic (Kames, 2015). Hicks *et al.* (2019) also assumed the vertical and horizontal scales of fluctuation to be $\theta_v = 0.5$ m and $\theta_h = 6.0$ m, respectively, based on CPT data from a similar site (de Gast *et al.*, 2017).

Hicks *et al.* (2019) used RFEM with the strength reduction method to compute the probability distribution of possible values of *F*, given the soil parameter statistics listed in Table 2. From that distribution, the 5-percentile response corresponded to $F = 0.98$ (before application of partial factors). They also demonstrated, by way of a simple approach, that the 5-percentile system response implied $X_k$ values corresponding to a single value of $\eta$ of 34%.

### 5-Percentile design point

The 5-percentile design point is here defined as the most likely combination of parameters on the 'characteristic' surface (i.e. the 5-percentile system response surface, corresponding to $F = 0.98$). It was evaluated using the HLRF (Hasofer-Lind-Rackwitz-Fiessler) algorithm (Hasofer & Lind, 1974; Rackwitz & Fiessler, 1978), with the performance function $G = F - 0.98$ being evaluated by the finite-element method without accounting for spatial variability. Based on the location of the shear strain invariant contours observed in the previous RFEM analyses, six variables were considered in defining the 5-percentile design point – that is, two variables ($c'$ and tan $\phi'$) for soil

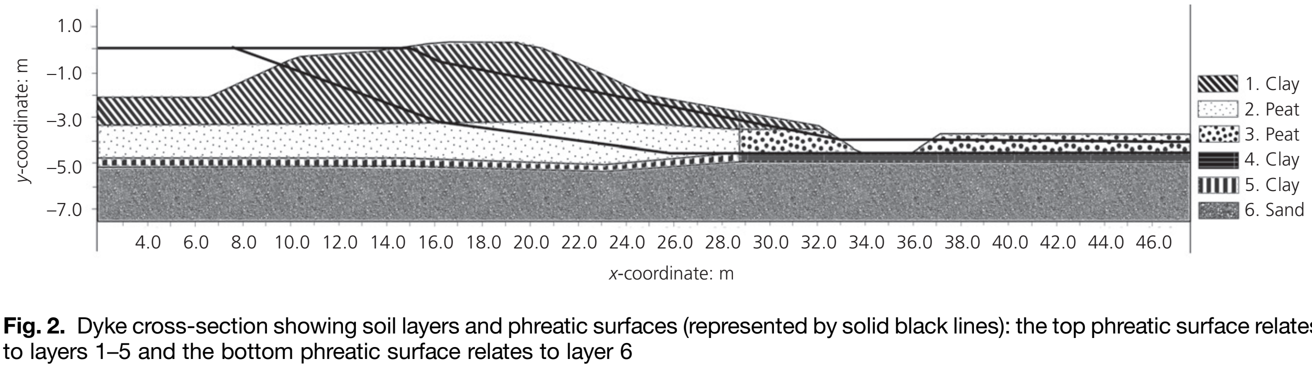


**Fig. 2.** Dyke cross-section showing soil layers and phreatic surfaces (represented by solid black lines): the top phreatic surface relates to layers 1–5 and the bottom phreatic surface relates to layer 6

**Table 2.** Unit weights and shear strength parameters for different layers of the dyke section

| Layer | $\gamma$: kN/m$^3$ | $c'$ | | | $\tan\phi'$ | | |
|---|---|---|---|---|---|---|---|
| | | Mean: kPa | 5-percentile value: kPa | COV | Mean | 5-percentile value | COV |
| 1 | 13.9* | 4.4 | 1.1 | 0.773 | 0.580 | 0.506 | 0.081 |
| 2 | 9.8 | 3.2 | 1.0 | 0.656 | 0.398 | 0.361 | 0.058 |
| 3 | 9.9 | 2.0 | 0.5 | 0.775 | 0.358 | 0.279 | 0.145 |
| 4 | 15.0 | 4.5 | 1.7 | 0.544 | 0.559 | 0.547 | 0.012 |
| 5 | 15.0 | 5.4 | 2.9 | 0.352 | 0.601 | 0.594 | 0.007 |
| 6 | 20.0 | 0.0 | 0.0 | 0.000 | 0.637 | 0.637 | 0.000 |

*$\gamma = 6.9$ kN/m$^3$ above phreatic surface.

**Table 3.** Most likely combination of characteristic soil property values corresponding to 5-percentile system response ($F = 0.98$) of the dyke section, the respective percentiles of the underlying distributions ($\eta$) and the sensitivity indices of the variables

| Layer | $c'$ | | | $\tan\phi'$ | | |
|---|---|---|---|---|---|---|
| | 5-percentile design point, $X_k$: kPa | $\eta$: % | Sensitivity index | 5-percentile design point, $X_k$ | $\eta$: % | Sensitivity index |
| 1 | 2.688 | 35.27 | 0.27 | 0.577 | 49.70 | 0.00 |
| 2 | 1.863 | 27.27 | 0.58 | 0.396 | 46.98 | 0.01 |
| 3 | 1.285 | 38.11 | 0.14 | 0.354 | 49.70 | 0.00 |

layers 1, 2 and 3. The parameters of layers 4, 5 and 6 were found to have negligible influence on $F$.

Table 3 shows the most likely combination of characteristic values, as well as the corresponding $\eta$ values and the sensitivity indices of the variables. The results imply that $F$ is less sensitive to $\tan\phi'$ for all layers, with the characteristic values of $\tan\phi'$ corresponding to $\eta$ values approaching 50%. Conversely, $F$ is most sensitive to $c'$ for the underlying peat layer (layer 2); the characteristic value of $c'$ for this layer corresponds to $\eta = 27.27\%$.

*Characteristic values for the dyke section computed using various analytical equations*

The approaches described above and in Hicks *et al.* (2019) to back-calculate the characteristic values require a reliability-based $F$ from a fully stochastic analysis – for example, using RFEM. However, several simpler (albeit more approximate) solutions exist. Hence, characteristic values for $c'$ and $\tan\phi'$ for layers 1 to 3 of the dyke section have been calculated using the methods reviewed below, and, using the computed $X_k$ values for these layers and mean values ($X_m$) for the other (not influential) layers, deterministic slope stability assessments have been carried out using finite elements with the strength reduction method.

*Schneider (1997) equation.* It was proposed by Schneider (1997) that:

$$X_k = X_m \times (1 - \mathrm{COV} \times 0.5) \quad (1)$$

The resulting characteristic values, value of $\eta$ and value of $F$ are listed in Table 4. This shows that the $X_k$ values are mostly underestimated relative to the 5-percentile design point values, especially for $\tan\phi'$, resulting in a slightly conservative value of $F$ (i.e. relative to $F = 0.98$).

**Table 4.** Characteristic soil property values for the dyke section computed using equation (1) (Schneider, 1997), value of $\eta$ and resulting value of $F$

| Layer | $c'$ | | $\tan\phi'$ | | $F$ |
|---|---|---|---|---|---|
| | $X_k$: kPa | $\eta$: % | $X_k$ | $\eta$: % | |
| 1 | 2.472 | 30.85 | 0.555 | 30.85 | 0.96 |
| 2 | 1.984 | 30.85 | 0.386 | 30.85 | |
| 3 | 1.122 | 30.85 | 0.329 | 30.85 | |

*Schneider & Schneider (2012) equation.* Equation (1) was extended by Schneider & Schneider (2012) to include variance reduction ($\Gamma^2$) (Vanmarcke, 1977) due to averaging of soil property values along the failure surface. The derivation was based on the total coefficient of variation $\mathrm{COV}_{\mathrm{total}}$ (Phoon & Kulhawy, 1999):

$$\mathrm{COV}_{\mathrm{total}} = \sqrt{\Gamma^2 \times \mathrm{COV}^2 + \mathrm{COV}_{\mathrm{m}}^2 + \mathrm{COV}_{\mathrm{t}}^2 + \mathrm{COV}_{\mathrm{s}}^2} \quad (2)$$

Assuming that the COVs due to measurement (m), transformation (t) and statistical (s) errors are negligible, so that $\mathrm{COV}_{\mathrm{total}} \approx \mathrm{COV} \times \Gamma$, Schneider & Schneider (2012) proposed the following equations for $X_k$. When $X$ is modelled as a normal distribution

$$X_k = X_m \times (1 - \mathrm{COV} \times \Gamma \times 1.645) \quad (3)$$

whereas for a log-normal distribution of $X$

$$X_k = X_m \times \left(0.192^{\sqrt{\ln(1+(\mathrm{COV}\times\Gamma)^2)}} / \sqrt{1 + (\mathrm{COV} \times \Gamma)^2}\right) \quad (4)$$

where $\Gamma^2 = \Gamma_x^2 \times \Gamma_y^2 \times \Gamma_z^2$ and $\Gamma_i^2$ is the variance reduction due to the averaging of property values over the failure length $l_i$ in the direction $i$, given by

$$\Gamma_i^2 = \left(\frac{\theta_i}{l_i} \times \left(1 - \frac{\theta_i}{3 \times l_i}\right)\right); \; \theta_i < l_i$$
$$\Gamma_i^2 = \left(1 - \frac{l_i}{3 \times \theta_i}\right); \; \theta_i \geq l_i \quad (5)$$

Equations (3) and (4) imply that $X_k$ is the 5 percentile of a distribution with a COV that is reduced relative to the underlying distribution. Although this aspect is similar to the concept of 'effective' property distribution described in section 'Characteristic values and design according to EC7', equations (3) and (4) do not consider the reduction in the mean of the distribution arising from the influence of weak zones. Moreover, they require the estimation of $\Gamma_i^2$ and thereby $l_i$, which may not be straightforward.

To calculate the variance reduction for the dyke section, a deterministic analysis based on mean soil properties was used to provide a representative failure mechanism (Fig. 3(a)). The length of the failure surface was calculated based on the curve fitted through the failure points in Fig. 3(b). The estimated lengths of the horizontal and vertical components of the surface passing through each soil layer are given in Table 5, along with the respective values of $\Gamma$ (equation (5)), the $X_k$ values (equation (4)), and resulting value of *F*. The characteristic values and thereby $\eta$ values are greatly underestimated for layer 3, resulting in a conservative estimate of *F*. Although it is unsurprising that a failure length smaller than $\theta$, as in layer 3, would result in $X_k$ tending towards the 5 percentile (as has been computed by equation (4)), the higher $\eta$ values of the 5-percentile design point for layer 3 (Table 3) are due to the lower relative influence of layer 3 on the failure mechanism.

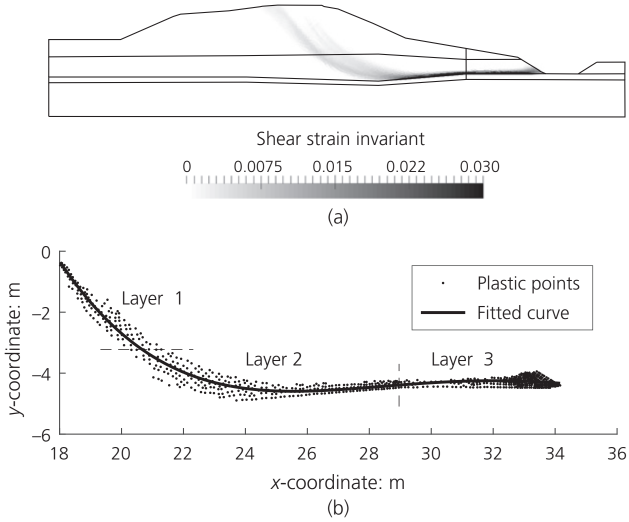


**Fig. 3.** Deterministic analysis of the dyke section based on mean soil property values: (a) shear strain invariant contours at slope failure; (b) failure surface fitted through plastic points in order to calculate variance reduction using equation (5)

*Equation proposed by CEN.* An evolution committee of CEN, the European Committee for Standardisation, which plans to publish a revised version of EC7, has proposed (Orr, 2017):

$$X_k = X_m - a \times (X_m - X_{extr}) \times \sqrt{\theta_v / l_v} \tag{6}$$

where $X_{extr}$ is the expected extreme value, which Orr (2017) proposed to be at a distance of 3 standard deviations from the mean, $l_v$ is the vertical component of the failure length and $a$ is a factor accounting for the extent and quality of field and laboratory investigations and levels of expertise (with lower values of $a$ corresponding to high-quality tests and reliable results).

Based on the values of $a$ suggested by Orr (2017), the characteristic soil property values computed using equation (6) and resulting values of *F* are listed in Tables 6(a)–6(c). Note that, in using equation (6), an upper limit for $\theta_v/l_v$ of 1.0 has been implemented in order to avoid the possibility of $X_k < X_{extr}$. The table shows that the $X_k$ values for tan $\phi'$ are greatly underestimated (even though, as indicated by the 5-percentile design point, the dyke section is less sensitive to tan $\phi'$). Conversely, the $X_k$ values for $c'$ for

**Table 5.** Characteristic soil property values for the dyke section computed using equation (4) (Schneider & Schneider, 2012), $\eta$ values and resulting value of *F*

| Layer | $l_h$: m | $l_v$: m | $\Gamma$ | $c'$ | | tan $\phi'$ | | *F* |
|---|---|---|---|---|---|---|---|---|
| | | | | $X_k$: kPa | $\eta$: % | $X_k$ | $\eta$: % | |
| 1 | 3.1 | 2.7 | 0.380 | 2.627 | 34.04 | 0.551 | 27.66 | 0.89 |
| 2 | 8.3 | 1.0 | 0.478 | 1.842 | 26.63 | 0.380 | 22.16 | |
| 3 | 5.1 | 0.0 | 0.845 | 0.624 | 8.77 | 0.290 | 8.43 | |

**Table 6.** Characteristic soil property values for the dyke section computed using equation (6) (Orr, 2017), $\eta$ values and resulting value of *F*, for different values of *a*: (a) $a = 0.5$; (b) $a = 0.75$; (c) $a = 1.0$

| Layer | $l_v$: m | $c'$ | | | tan $\phi'$ | | | *F* |
|---|---|---|---|---|---|---|---|---|
| | | $X_{extr}$: kPa | $X_k$: kPa | $\eta$: % | $X_{extr}$ | $X_k$ | $\eta$: % | |
| (a) | | | | | | | | |
| 1 | 2.7 | 0.447 | 3.548 | 51.10 | 0.454 | 0.558 | 28.95 | 1.04 |
| 2 | 1.0 | 0.445 | 2.226 | 37.92 | 0.334 | 0.375 | 16.30 | |
| 3 | 0.0 | 0.202 | 1.101 | 29.89 | 0.230 | 0.294 | 9.76 | |
| (b) | | | | | | | | |
| 1 | 2.7 | 0.447 | 3.122 | 43.67 | 0.454 | 0.539 | 19.40 | 0.89 |
| 2 | 1.0 | 0.445 | 1.739 | 23.56 | 0.334 | 0.364 | 6.54 | |
| 3 | 0.0 | 0.202 | 0.651 | 9.81 | 0.230 | 0.262 | 1.81 | |
| (c) | | | | | | | | |
| 1 | 2.7 | 0.447 | 2.696 | 35.43 | 0.454 | 0.525 | 11.90 | 0.69 |
| 2 | 1.0 | 0.445 | 1.252 | 10.21 | 0.334 | 0.353 | 1.99 | |
| 3 | 0.0 | 0.202 | 0.202 | 0.13 | 0.230 | 0.230 | 0.13 | |

layer 1 are overestimated, due to a relatively smaller value of $\theta_v/l_v$ leading to greater spatial averaging. Table 6 shows that the $X_k$ values are very sensitive to the value of $a$ and $F$ varies from moderately unconservative to extremely conservative, depending on $a$.

*Effective random dimensions-quantile value method.* A method to approximate the 5 percentile of the system response function ($G$) directly, through the reformulation of the characteristic values based on the concept of number of effective random dimensions (ERD) in a quantile value method (QVM), was recently proposed by Ching *et al.* (2020). The method relies on the linearisation of $G$ around the parameter means:

$$b_i = G(\mu_1, \ldots, \mu_i + 0.5 \times \sigma_i, \ldots, \mu_n) - G(\mu_1, \ldots, \mu_i - 0.5 \times \sigma_i, \ldots, \mu_n) \tag{7}$$

where $b_i$ is the coefficient of variable $X_i$ in the linearised $G$ and $\mu_i$ and $\sigma_i$ are the mean and standard deviation of $X_i$.

For uncorrelated variables, ERD is then calculated as

$$\mathrm{ERD} = \frac{(|b_1| + |b_2| + \cdots + |b_n|)^2}{\sum_i b_i^2} \tag{8}$$

**Table 7.** Characteristic soil property values for the dyke section computed using ERD-QVM-Γ, $\eta$ values and resulting value of $F$

| Layer | $c'$ | | | $\tan\phi'$ | | | $F$ |
|---|---|---|---|---|---|---|---|
| | $b$ | $X_k$: kPa | $\eta$: % | $b$ | $X_k$ | $\eta$: % | |
| 1 | 0.064 | 2.728 | 36.1 | 0.002 | 0.562 | 36.1 | 0.96 |
| 2 | 0.136 | 2.046 | 32.7 | 0.010 | 0.387 | 32.7 | |
| 3 | 0.097 | 0.918 | 21.4 | 0.005 | 0.316 | 21.4 | |

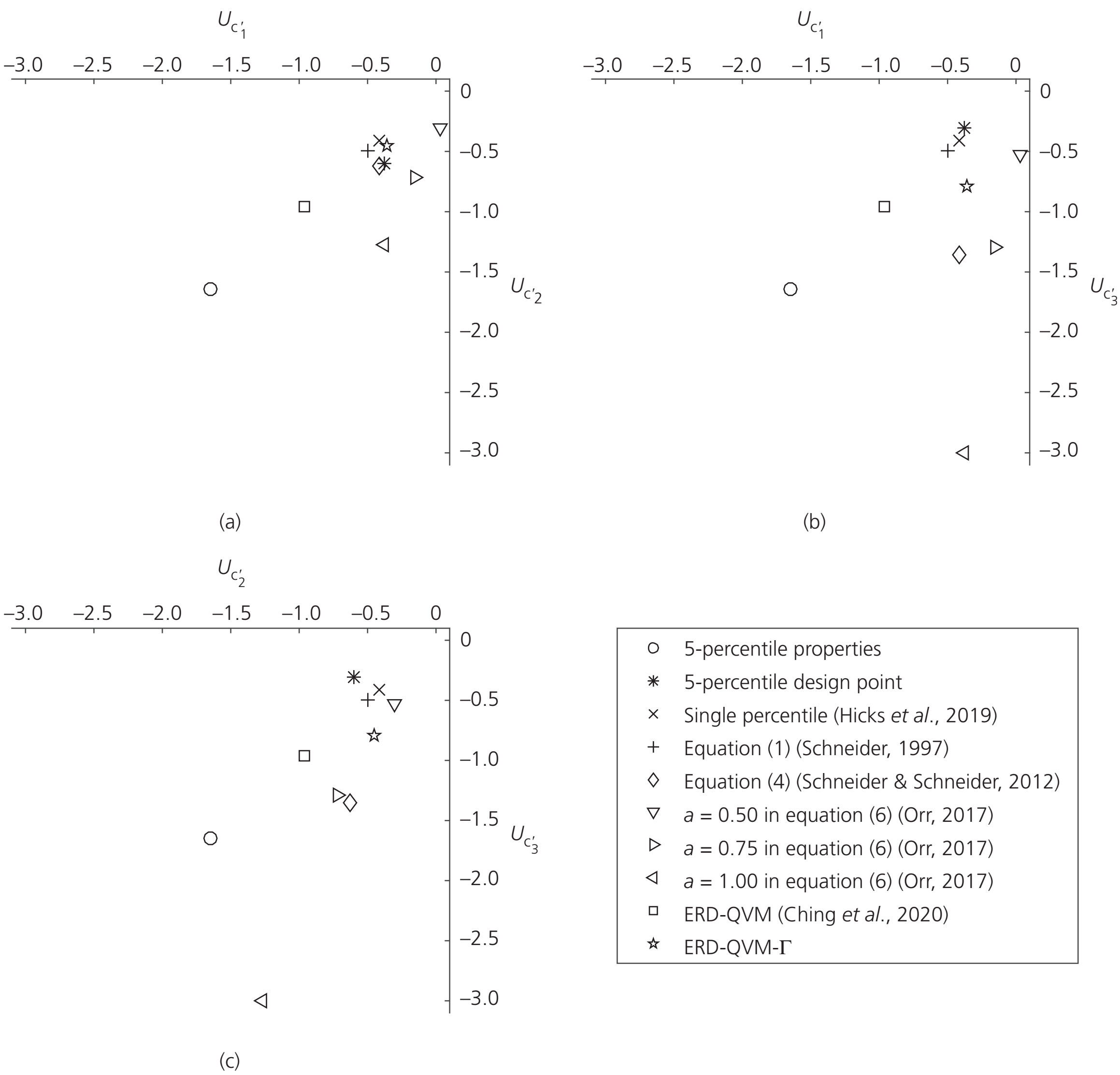


**Fig. 4.** Characteristic values in standard normal space of $c_1'$, $c_2'$ and $c_3'$ for layers 1, 2 and 3, respectively, computed using various methods: (a) layers 1 and 2; (b) layers 1 and 3; (c) layers 2 and 3

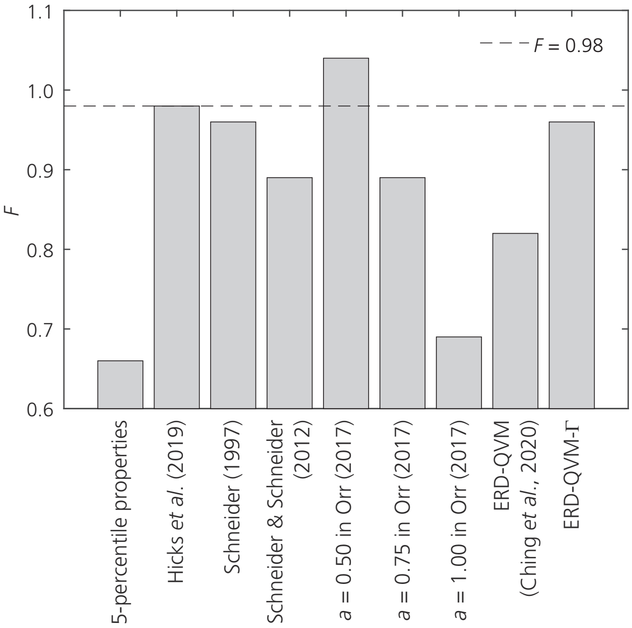


**Fig. 5.** Comparison of factors of safety obtained by the various methods with $F = 0.98$ (corresponding to the 5-percentile system response based on RFEM)

The required $\eta$ that achieves the target exceedance probability of 5% is then

$$\eta = \Phi\left(\frac{\Phi^{-1}(0.05)}{\sqrt{\mathrm{ERD}}}\right) \times 100\% \tag{9}$$

where $\Phi$ is the standard normal cumulative distribution function.

Applying this method to the six variables gives $\mathrm{ERD} = 2.93$, $\eta = 17\%$ and thereby $F = 0.82$. Due to the need for linearisation against all variables, the method does not allow the direct inclusion of spatial variability.

*Effective random dimensions-quantile value method-Γ (proposed in this study).* By combining the ERD-QVM with the method in section 'Schneider & Schneider (2012) equation' to account for spatial variability, equations (7) and (9) can be modified to:

$$\begin{aligned} b_i = {} & G(\mu_1, \ldots, \mu_i + 0.5 \times \sigma_i \times \Gamma_i, \ldots, \mu_n) \\ & - G(\mu_1, \ldots, \mu_i - 0.5 \times \sigma_i \times \Gamma_i, \ldots, \mu_n) \end{aligned} \tag{10}$$

$$\eta_i = \Phi\left(\frac{\Phi^{-1}(0.05)}{\sqrt{\mathrm{ERD}}} \times \Gamma_i\right) \times 100\% \tag{11}$$

where $\Gamma_i^2$ is the variance reduction for $X_i$.

Applying this method to the six variables and using $\Gamma_i$ from Table 5 gives $\mathrm{ERD} = 3.08$, and thereby the $\eta$ values listed in Table 7 and $F = 0.96$.

### *Comparison of methods*

Figure 4 illustrates, in standard normal space, the characteristic values of $c'$ computed for layers 1 to 3 using the different methods. The values corresponding to the RFEM-based simple approach in Hicks *et al.* (2019) and the 5-percentile design point lie on the characteristic surface of points resulting in $F = 0.98$. Figure 5 shows that the value computed using $a = 0.50$ in equation (6) lies on the unconservative side ($F > 0.98$) of the characteristic surface, whereas the values computed using other simplified methods are on the conservative side ($F < 0.98$). Although there are other variables that define the characteristic surface (i.e. tan $\phi'$ for layers 1 to 3), these have not been illustrated in Fig. 4 for reasons of clarity.

## CONCLUSIONS

Figure 5 compares the factors of safety obtained by the finite-element method using the characteristic soil properties obtained by the various simplified methods, and compares them with $F = 0.98$ obtained using RFEM (corresponding to the 5-percentile system response). Aside from the over-conservative values of $F$ computed using 5-percentile property values, ERD-QVM and equation (6) when based on unreliable data, all other methods give values of $F$ within 10% of the benchmark solution (both conservative and unconservative). In this study, Schneider (1997) equation and ERD-QVM-Γ give the best approximations, although which method is the best will be problem-dependent. The more rigorous approach reported by Hicks *et al.* (2019) is computationally intensive; however, it by-passes the need to explicitly determine characteristic values, is completely general and can lead to economy of design, so it may be prudent to use such an approach in larger projects.

## ACKNOWLEDGEMENTS

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