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Ill-posedness in modelling 2D river morphodynamics

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Introduction

The set of equations used in modelling river morphodynamics needs to be (at least) well-posed to be representative of the real natural phenomenon. As we deal with a time dependent process the solution needs to be wave-like to be well-posed. In other words, the solution must have a domain of dependence and of influence. Otherwise, the future river state influences the present solution, which is physically unrealistic.

Based on an analysis of the system of equations to model one-dimensional river morphodynamics with unisize sediment and a Chezy-based friction term, Cordier (2011) concluded that the system is always well-posed. Stecca (2014) extended the analysis to a mixture of sediment with 2 size fractions and concluded that under degradational conditions the system may become ill-posed. This result supported the first analysis that found ill-posedness in mixed-size sediment morphodynamics conducted by RIbberink (1987) assuming a simpler model.

Here we extend these analyses by adding the effects of flow curvature which creates an intrinsically 3D flow referred to as secondary or spiral flow (Van Bendegom, 1947). In this study the flow is assumed bi-dimensional which implies that the secondary flow needs to be parameterized.

Model Description

The two-dimensional water flow is modelled using the Shallow Water Equations. The Exner (1920) equation accounts for the conservation of mass of bed sediment. The Hirano (1971) (or active layer) equation accounts for the conservation of mass per grain size fraction. The sediment in the topmost layer of the bed (the active layer) can be entrained and transported, and sediment can be deposited in the active layer. If the bed degrades, sediment from the substrate is transferred to the active layer and vice versa if it aggrades. The active layer has no vertical stratification, which implies that it is assumed to be fully mixed.

The parametrization of secondary flow is based on the advection and diffusion of the secondary flow intensity which is a measure of the magnitude of the velocity component normal to the depth-averaged velocity (Kalkwijk and Booij, 1986).

The system contains N+4 equations (being N the number of size fractions). The dependent variables are the flow depth h, the water discharge per unit width in x and y direction q_x and q_y, the secondary flow intensity I, the bed elevation η, and the volume of sediment of size fraction k per unit of bed area in the active layer M_{ak}. x, y and t are two space coordinates and a time coordinate. We refer the reader to Chavarrias and Ottevanger (2016) for a more detailed description of the system of equations.

Model Characterization

We characterize the system of equations obtaining its Monge cones (Courant and Hilbert, 1961). These cones in the x-y-t space represent the wave front of a perturbation. If the cones do not exist in the real domain the solution is not wave-like and the model is ill-posed.

We first consider the flow of water over a fixed bed (q_x=1 m^2/s, h=1 m, q_y=0). In Fig. 1a we plot the intersection of the cones with a plane at t=1 s. Observe the two cones, one with a circular section, and a degenerated second cone related to the advection of information related to vorticity (Vreugdenhil, 1989).

When considering mobile bed with unisize sediment, an additional star-shaped cone appears (De Vriend, 1987). This cone related to morphodynamic changes is smaller, as bed perturbations propagate slower than flow perturbations (Fig. 1c-e). The consideration of two sediment size fractions introduces a new cone into the system (Sieben, 1994, and Fig. 1f-h).

We consider a two sediment size fractions case which is known to be ill-posed assuming one-dimensional flow, i.e., degradation into a substrate finer than the active layer (see Chavarrias and Ottevanger (2016) for the specific values). In two dimensions one of the cones does not exist (Fig. 1i-k) implying that the model is ill-posed. Note that not for every direction the system is ill-posed. A new cone appears when we consider a third size fraction (Fig. 1l-n).

Eventually we consider the effect of a simplified secondary flow on a case with unisize sediment. We assume no diffusion and we neglect the possible effects of the source terms. Note that an additional cone appears which moves at the speed of the mean flow velocity (Fig. 1p). Its size is related to the secondary flow intensity. Moreover, the cone related to morphology (Fig. 1q) is turned with respect to the situation without secondary flow (Fig. 1e) due to the change in the sediment transport direction that secondary flow induces. Interestingly, the model is ill-posed. We have tested the model for decreasing values of secondary flow and it appears to be always ill-posed.

Discussion and Future Research

The bi-dimensionality of the system of equations introduces an additional aspect in the study of well-posedness. While it is clear that the absence of a cone in one single direction implies that the model is mathematically ill-posed, it is not clear what are the implications of a model being ill-posed only in certain directions when numerically solving it.

The fact that the inclusion of a parameterized secondary flow induces the unisize model to be ill-posed may indicate that the neglected mechanisms (diffusion and source term) play an important role in the model. The diffusive term dampens small wavelengths (Gray and Ancey, 2011) which are the most critical ones for the ill-posedness of a system (Joseph and Saut, 1990). Yet, due to the shape of the advection-diffusion equation which models the secondary flow intensity, diffusion does not regularize the model (at least if the source term is neglected).

The source term depends on the radius of curvature which depends on the streamwise gradient of the flow velocity in transverse direction. Preliminary tests of the effect of this gradient show that it does not regularize the model if diffusion is not included. The role of diffusion needs to be studied in this case.

Eventually, the study of the role of diffusion would also be useful to assess the role of other diffusive mechanisms as the effects of bed-slope on sediment transport.

Conclusions

We have conducted a preliminary study of the well-posedness of a 2D model for predicting mixed-sediment river morphodynamics including the effect of secondary flow. We show that an additional cone carrying information through the domain appears for every size fraction that we include. Ill-posedness becomes a property depending on the direction. The consequences of this property need to be further assessed. A simple treatment of secondary flow seems to switch the mathematical character of the unisize model which makes it ill-posed. Further research is required to assess the effect of the secondary flow terms that have been neglected.

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References

Van Bendegom, L. (1947), Eenige beschouwingen over riviermorphologie en rivierverbetering, De Ingenieur, 59, 1-11 (in Dutch).

Chavarrias V. and Ottevanger, W. (2016), Mathematical analysis of the well-posedness of the Hirano active layer concept in 2D models, Tech. Rep. 1230044-000-ZWS-0035, Deltares, Delft, The Netherlands.

Cordier, S., M. Le, and T. M. de Luna (2011), Bedload transport in shallow water models: Why splitting (may) fail, how hyperbolicity (can) help, Adv. Water Resour., 34 (8), 980-989.

Exner, F. M. (1920), Zur physic der dünen. Klasse, 129(2a) 929-952. Akad. Wiss. Wien Math. Naturwiss. (in German).

Gray, J. M. N. T. & Ancey, C. (2011) Multi-component particle-size segregation in shallow granular avalanches J. Fluid Mech., Cambridge Univ. Press, 678, 535-588.

Hirano, M. (1971), River bed degradation with armouring. Trans. Jpn. Soc. Civ. Eng. 3(2), 194-195.

Joseph, D. and Saut, J. (1990) Short-wave instabilities and ill-posed initial-value problems Theor. Comput. Fluid Mech., Springer-Verlag, 1, 191 – 227

Kalkwijk, J. P., and Booij, R. (1986), Adaptation of secondary flow in nearly-horizontal flow, J. Hydr. Res., 24(1) 19-37.

Ribberink J. S. (1987), Mathematical modelling of one-dimensional morphological changes in rivers with non-uniform sediment. Ph.D. thesis, Delft University of Technology.

Sieben, A. (1994), Notes on the mathematical modelling of alluvial mountain rivers with graded sediment, Tech. Rep. 94-3, Delft University of Technology.

Stecca, G., Siviglia, A., and Blom, A. (2014) Mathematical analysis of the Saint-Venant-Hirano model for mixed-sediment morphodynamics Water Resour. Res., 50, 7563-7589.

Vreugdenhil, C. B., (1989) *Computational hydraulics, an introduction*, Springer Verlag, Berlin, 182pp.

De Vriend, H. (1987), Analysis of horizontally two-dimensional morphological evolutions in shallow water, J. Geophys. Res., 92, 3877-3893.

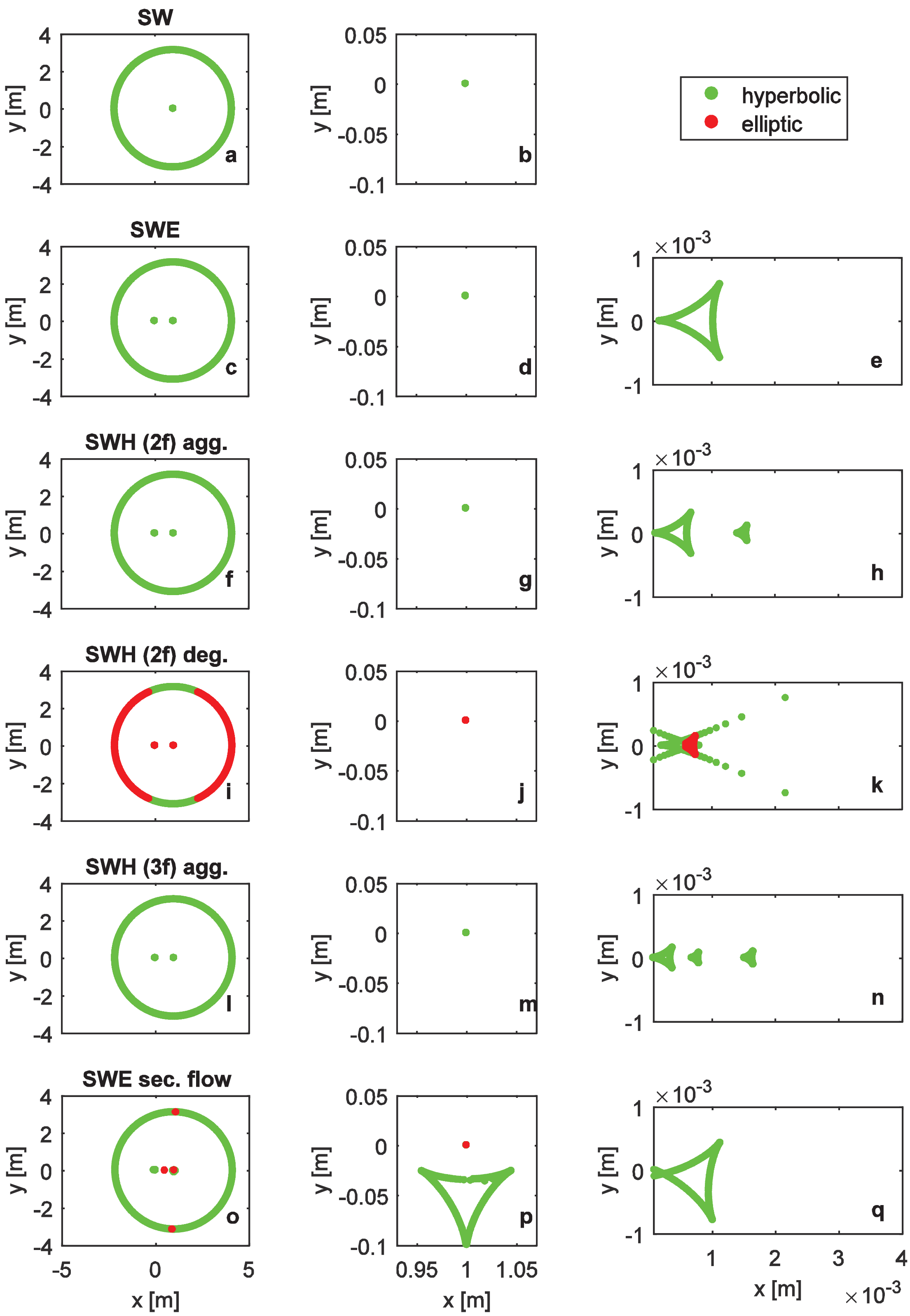

Figure 1. Intersection of the Monge cones at t=1 for: (a-b) the Shallow Water Equations (SW), (c-e) SW coupled to the Exner equation (SWE), (f-h) SW coupled to the Hirano equation (SWH) for 2 size fractions in aggradational conditions, (i-k) SWH for 2 size fractions in degradational conditions, (l-n) SWH for 3 size fractions in aggradational conditions, and (o-q) SWE considering secondary flow.