# Client Self-Defense against Model Poisoning in Federated Learning

Master Thesis Chaoyi Zhu



# Client Self-Defense against Model Poisoning in Federated Learning

by

## Chaoyi Zhu

to obtain the degree of Master of Science at the Delft University of Technology, to be defended publicly on Wednesday June 14th, 2023 at 10:00 AM.

Student number: 5500524 Thesis committee: Dr. Lydia Chen, TU Delft, supervisor Dr. Zekeriya Erkin, TU Delft Dr. Zaid Al-Ars, TU Delft Dr. Stefanie Roos, TU Delft

An electronic version of this thesis is available at http://repository.tudelft.nl/.



### Preface

In recent years, the rise of federated learning has revolutionized the field of distributed machine learning. By allowing participants to collaboratively train models while keeping their data decentralized, federated learning has opened up new possibilities for privacy-preserving and efficient machine learning in various domains. However, this groundbreaking approach also brings about new challenges and vulnerabilities.

One of the major concerns in federated learning is the susceptibility to poisoning attacks. Since participants can manipulate their data and models locally without centralized oversight, there is a lack of assurance regarding the integrity of the training process. While there have been server-side defenses proposed to mitigate these attacks by modifying or rejecting local updates uploaded by clients, they are not foolproof. In particular, bursty adversarial patterns with a high variance in the number of malicious clients can circumvent the existing defenses, leaving the system vulnerable to compromise.

In this master thesis, we propose a novel client-side defense mechanism, called LeadFL, which addresses the limitations of existing server-side defenses and aims to thwart backdoor and targeted attacks in federated learning. The core idea behind LeadFL is the introduction of a unique regularization term in local model training, effectively nullifying the Hessian matrix of local gradients. Through extensive research and analysis, we provide convergence analysis of LeadFL and demonstrate its robustness guarantee in terms of a certified radius, which quantifies the distances between benign and poisoned models.

Our empirical evaluation reveals the effectiveness of LeadFL in mitigating bursty adversarial patterns for both IID and non-IID data distributions. Notably, LeadFL consistently reduces the backdoor accuracy from more than 75% using state-of-the-art defenses to less than 10%, all while maintaining minimal impact on the accuracy of the main task.

In the process of conducting the research for this thesis, the findings have been distilled and presented in the form of a research paper titled *LeadFL*: *Client Self-Defense against Model Poisoning in Federated Learning*. It is noteworthy to mention that this paper has been accepted by 40<sup>th</sup> International Conference on Machine Learning (ICML 2023), an esteemed conference in the field of machine learning.

Profound gratitude is extended to Prof. Lydia Chen, whose mentorship, expertise, and support have been integral to this research, enriching this work and contributing significantly to my academic growth. Additional thanks are given to Prof. Stefanie Roos for her valuable input and academic rigour, which have greatly enhanced the quality of this research. Equally important, heartfelt thanks are extended to family and friends who have provided constant encouragement, patience, and selfless support.

Chaoyi Zhu Delft, June 2023

## Contents

Pr	eface			i
1	Rese	earch Pa	aper	1
2	<b>Bacl</b> 2.1 2.2 2.3	<b>cgroun</b> Fedeau Poisor Defens 2.3.1 2.3.2 2.3.3 2.3.4	d rted learning	12 12 13 14 15 17 19 20
3	Lead 3.1 3.2 3.3	<b>dFL An</b> Assum Conve Robus	alysis ptions and Definitions	<b>22</b> 22 23 24
4	<b>Add</b> 4.1 4.2	itional Comp Additi 4.2.1 4.2.2 4.2.3 4.2.3 4.2.4 4.2.5	Experiments         arison Between FL-WBC and LDP         onal Results         Results of different client selections         Results of the larger scale setting         Results of CMA and CTMA under 9-pixel backdoor attack         Results of single-pixel backdoor attack         Results of single image targeted attack	27 28 28 29 30 30 30
5	Con	clusion	l	37

# Research Paper

In this chapter, we present our core motivations, theories, and results, adhering to the structure of a research paper. Notably, this research paper has been accepted at  $40^{th}$  International Conference on Machine Learning (ICML 2023). The subsequent chapters are structured as follows: Chapter 2 delves into an extensive background on federated learning, offering an in-depth exploration of poisoning attacks and their corresponding defense strategies. In Chapter 3, we provide detailed proof of convergence and robustness for our algorithm introduced in the research paper. Lastly, in Chapter 4, we showcase additional experiments that further corroborate the effectiveness of our proposed method.

#### LeadFL: Client Self-Defense against Model Poisoning in Federated Learning

#### Abstract

Federated Learning is highly susceptible to backdoor and targeted attacks as participants can manipulate their data and models locally without any oversight on whether they follow the correct process. There are a number of server-side defenses that mitigate the attacks by modifying or rejecting local updates submitted by clients. However, we find that bursty adversarial patterns with a high variance in the number of malicious clients can circumvent the existing defenses. We propose a client-self defense, LeadFL, that is combined with existing server-side defenses to thwart backdoor and targeted attacks. The core idea of LeadFL is a novel regularization term in local model training such that the Hessian matrix of local gradients is nullified. We provide the convergence analysis of LeadFL and its robustness guarantee in terms of certified radius. Our empirical evaluation shows that LeadFL is able to mitigate bursty adversarial patterns for both iid and non-iid data distributions. It frequently reduces the backdoor accuracy from more than 75% for state-of-the-art defenses to less than 10% while its impact on the main task accuracy is always less than for other client-side defenses.

#### 1. Introduction

Federated Learning (FL) realizes collaborative learning without the need to share possibly sensitive raw data. Clients submit intermediate local models to a server, the federator, who aggregates these models. In order to achieve models of high accuracy, high-quality local models and effective aggregation algorithms are needed. Adversarial clients can reduce the accuracy, either overall or on specific tasks, by manipulating their local data and the submitted model. For instance, malicious clients can launch backdoor attacks, which mislead the model to make inaccurate inferences on images with certain triggers.

The attack severity is closely related to the number of malicious clients that are chosen over time. Federated learning proceeds in rounds. Usually, in each round, a certain number of clients are selected from a large pool. If the selection is random and by the server, the number of malicious clients chosen varies greatly even if the overall fraction of malicious clients in the pool stays constant. Figure 1 displays an example for 5 selected clients over 5 rounds. As a consequence, in some rounds, the fraction, malicious clients make up the majority of the clients, which allows them to launch a strong attack.



Figure 1: Bursty adversarial patterns with the number of malicious clients chosen varying greatly between rounds

Defense mechanisms (Blanchard et al., 2017; Fung et al., 2020; Muñoz-González et al., 2019; Xia et al., 2019; Mhamdi et al., 2018; Ozdayi et al., 2021; Panda et al., 2022; Yin et al., 2018; Nguyen et al., 2022; Rieger et al., 2022; Gupta et al., 2022; Xu et al., 2022) have been designed to mitigate the attacks. The majority of these attacks are serverside, meaning the federator assigns updates that appear to be malicious a low weight during aggregation or completely excludes them from the aggregation. These defenses have been shown to be effective against sophisticated attacks when the number of malicious selected clients is constant and low (Nguyen et al., 2022; Panda et al., 2022; Rieger et al., 2022). In addition to demonstrating the empirical effectiveness, theoretical frameworks, such as certified radius on models (Panda et al., 2022) and inference samples (Xie et al., 2021) provide theoretical guarantees of the defense effectiveness.

In contrast, client-side defenses (Sun et al., 2021) have the client modify the training process. The most notable clientside defense is FL-WBC (Sun et al., 2021), which can deal with bursty attack patterns. The authors find that a strong bursty attack in one round has a lingering effect on the model and the duration and severity of that effect depends on the sparsity of the Hessian matrix of gradients: the higher the sparsity, the longer the attack effect. FL-WBC perturbs the Hessian matrix of gradients by adding random noise into clients' local models to reduce sparsity. Such uncalibrated random noise unfortunately leads to the degradation in the global model accuracy. Moreover, there is no theoretical guarantee that FL-WBC is robust against backdoor attacks.

In this paper, we design LeadFL, a client-side defense that enhances server-side defenses to deal with bursty adversarial patterns while not affecting global model accuracy significantly. The core of LeadFL is an optimization framework that optimally perturbs the Hessian matrix of local gradients and local models using a regularisation term such that their Hessian matrix is close to the identity matrix.We verify the effectiveness of the proposed regularized Hessian optimization by deriving the convergence analysis and certified radius analysis, which quantifies the distances between benign and poisoned models. Specifically, we make the following technical contributions:

- We design LeadFL, a novel client-side defense based on Hessian matrix optimization, to mitigate the impact of bursty adversarial patterns for backdoor and targeted attacks.
- We derive the convergence analysis and certified radius analysis, proving LeadFL that is effective.
- We empirically combine LeadFL with different serverside defenses and find that the combination can effectively defend against strong attacks while other combinations of server-side and client-side defenses fail.
   We reduce the backdoor accuracy by up to 65% and achieve a lower impact on main task accuracy than other combined defenses.

#### 2. Background and Prior Art

We first introduce necessary concepts and then analyze the state-of-the-art defenses against model poisoning.

#### 2.1. Model Poisoning Attacks in Federated Learning

**Federated Learning** In Federated Learning (FL) (Konečný et al., 2016), K clients indexed by  $k = \{1 \cdots K\}$  are selected from a total of N clients at global round t to train a learning model by using the local data to minimize the loss function  $\mathcal{L}(\theta^k)$  with model weights of  $\theta^k$ ). Specifically, at its local round of i, the client k uses stochastic gradient descent to update weights as follows:

$$\boldsymbol{\theta}_{t,i+1}^{k} \leftarrow \boldsymbol{\theta}_{t,i}^{k} - \eta_{t,i} \nabla \mathcal{L}\left(\boldsymbol{\theta}_{t,i}^{k}\right)$$

where  $\eta_{t,i}$  is the learning rate and each local round is computed on a mini-batch of data samples uniformly chosen from client *k*'s local data set.

Periodically, namely, at every global round t, the federator

selects a subset of clients and updates the global model weights. The most common aggregation method is FedAvg (McMahan et al., 2017), which averages the selected local models with weights proportional to their sample sizes.

**Poisoning Attacks** Malicious clients may join the training process. We assume them to have the similar computational capability as benign clients and they cannot access the weights or data of other clients. Their objectives are to reduce the model accuracy on certain tasks, termed targeted attacks (Chen et al., 2017; Bhagoji et al., 2019), or to mislead the global model to make wrong inferences on data sets with certain triggers, termed backdoor attacks (Xie et al., 2019; Bagdasaryan & Shmatikov, 2021), without degrading the overall model accuracy. To obtain such a poisoned model, malicious clients train their local models on malicious data to minimize the malicious loss functions  $\mathcal{L}_M$  as follows:

$$\boldsymbol{\theta}_{t,i+1}^{k} \leftarrow \boldsymbol{\theta}_{t,i}^{k} - \eta_{t,i} \left[ \pi \nabla \mathcal{L} \left( \boldsymbol{\theta}_{t,i}^{k} \right) + (1-\pi) \nabla \mathcal{L}_{M} \left( \boldsymbol{\theta}_{t,i}^{k} \right) \right]$$

Note that data samples in the malicious data set are assumed have the same distribution as the benign training data. The only difference is that for targeted attacks, the labels are altered to belong to a certain target class whereas for backdoor attacks, data samples with certain patterns are inserted into the dataset.

Model poisoning attacks are typically stealthy and difficult to detect, as the malicious dataset is usually small and does not affect the accuracy of the global model(Fung et al., 2020; Steinhardt et al., 2017; Tolpegin et al., 2020; Bagdasaryan & Shmatikov, 2021).

#### 2.2. Prior Art on Defenses

**Server-side defenses** To defend against the adversarial parties, the federator may employ (i) robust aggregation by computing the median (Yin et al., 2018) of all or subset of client updates, e.g., Trimmed-mean (Yin et al., 2018), or (ii) filtering by removing outliers in the set of updates based on pair-wise distance, e.g., MultiKrum (Blanchard et al., 2017). Bulyan (Mhamdi et al., 2018) combines both approaches by first filtering the outliners using MultiKrum and then applying robust aggregation using Trimmed Mean. These defenses are designed for general adversarial attacks where the number of malicious clients is strictly less than the benign clients.

**Defenses for poisonous model attacks**. Recognizing the increasing threat from poisoning attacks, the prior art designs attack-specific defenses by bounding the norms of updates or adding noise. SparseFed (Panda et al., 2022) mitigates model poisoning attacks in FL by only updating the most relevant weights of the aggregated models. Deep-Sight (Rieger et al., 2022) mitigates backdoor attacks in FL through clustering the last layer of deep models to fil-



(a) Number of malicious clients selected out of 5 selected clients per round.

(b) Backdoor accuracy of 3 server-side defenses and a client-side defense (FL-WBC).

Figure 2: The lingering impact of bursty backdoor attack on federated learning for FashionMNIST.

ter outliers. CRFL (Xie et al., 2021) exploits clipping and smoothing methods to provide certified robustness against backdoor attacks.

Aforementioned defenses mainly take place at the federator, under an implicit assumption that the number of malicious clients selected in each global round is lower than the number of benign clients. To the best of our knowledge, FL-WBC (Sun et al., 2021) and Local Differential Privacy (LDP) (Naseri et al., 2022) are the only client-side defenses against model poisoning attacks in federated learning. In LDP, benign clients add noise to updates before sending updates to the server.

#### 3. Hessian Matrix

We first demonstrate the long-term impact of bursty adversarial patterns on state-of-the-art defenses that we already discussed in Section 2.2. Details about the framework used can be found in Section 5. Using a total of 100 clients with 25 of them being malicious, we selected 5 clients per round. Fig. 2a shows the number of malicious clients selected. The learning task is image classification on FashionMNIST and the malicious clients execute a 9-pixel attack (Bagdasaryan & Shmatikov, 2021). As displayed in Figure 2b, none of the defenses can defend against the attack, i.e., the final backdoor accuracy is around 90%, though Bulyan and Multikrum are able to filter out the malicious updates occasionally. This example highlights the ineffectiveness of existing defenses against bursty adversarial patterns. While the attack only directly affects some rounds, the effect lingers.

Attack Effect and Hessian Matrix The effect of attacks taking place at round t,  $\delta_t$ , can be formalized (Sun et al., 2021) as follows  $\delta_t \triangleq \theta_t - \theta_t^M$  where  $\theta_t$  represents the global model weights at round t without the presence of malicious updates and  $\theta_t^M$  is the model weights from the malicious clients.

Based on (Sun et al., 2021), the estimated attack effect,  $\hat{\delta}_t$ ,

can be written as the function of Hessian matrix

$$\hat{\delta}_t = \frac{N}{K} \left[ \sum_{k \in \mathbb{S}_t} p^k \prod_{i=0}^{I-1} \left( \boldsymbol{I} - \eta_t \boldsymbol{H}_{t,i}^k \right) \right] \hat{\delta}_{t-1}, \qquad (1)$$

where  $p^k$  is aggregation weight for client k,  $H_{t,i}^k \triangleq \nabla^2 \mathcal{L}(\theta_{t,i}^k)$  is the Hessian matrix at local iteration *i* of global round *t* and *I* is the identify matrix.

The Hessian matrix  $\boldsymbol{H}_{t,i}^k$  is observed to be highly sparse during the training process, for both benign and malicious clients. Therefore, the weights of  $\hat{\delta}_{t-1}$  in Eq. 1 are close to  $\sum_{k \in \mathbb{S}_t} p^k \prod_{i=0}^{I-1} (\boldsymbol{I})$ . As a consequence,  $\delta_t$  causes notable changes and due to the relation between  $\hat{\delta}_{t-1}$  and  $\hat{\delta}_t$ , the effect lingers.

**Insight** To mitigate the effect of poisoned weights, benign clients can perturb the Hessian matrix such that that coefficient of  $\delta_{t-1}$  is minimized, i.e.,  $\prod_{i=0}^{I-1} (I)$ . As noted earlier, the Hessian matrix here is sparse, FL-WBC (Sun et al., 2021) proposes to add random noise to the benign clients' model weights such that their Hessian matrix is no longer sparse and the impact of  $\delta_{t-1}$  is thus reduced. However, as the noise is randomly added, the coefficient may not necessarily be reduced, unfortunately enhance the attacking effects, and further degrade the model accuracy, shown by extensive experiments in Appendix C. We are thus motivated to find alternatives to perturb the Hessian Matrix more effectively to reduce coefficients without degrading the model's accuracy.

#### 4. LeadFL

In this section, we describe LeadFL, a novel client-side defense and can be agilely combined with any existing serverside defense.

#### 4.1. Algorithm Design

The core idea of LeadFL is to mitigate the attack effect by minimizing the coefficient term  $(I - \eta_t H_{t,i}^k)$  in Equation 1. Essentially, we aim to add perturbation to the Hessian matrix such that this coefficient term vanishes. We show that this is equivalent to adding the same amount of perturbation in model updates  $\theta_{t,i+1}^k$ , which motivates our proposed novel regularization term. We first summarize the proposed regularized model update protocol before conducting an analysis:

$$\widetilde{\boldsymbol{\theta}}_{t,i+1}^{k} \leftarrow \boldsymbol{\theta}_{t,i}^{k} - \eta_{t} \nabla \mathcal{L}\left(\boldsymbol{\theta}_{t,i}^{k}\right)$$
(2)

$$\boldsymbol{\theta}_{t,i+1}^{k} \leftarrow \widetilde{\boldsymbol{\theta}}_{t,i+1}^{k} - \eta_{t} \alpha \operatorname{clip}\left(\nabla\left(\boldsymbol{I} - \eta_{t} \widetilde{\boldsymbol{H}}_{t,i}^{k}\right), q\right) \quad (3)$$

where  $\widetilde{\theta}_{t,i+1}^k$  is the intermediate model weights in local iteration t + 1 of client k.  $\widetilde{H}_{t,i}$  is the estimation of the Hessian matrix of the local model before adding the regularization term in this local iteration,  $\alpha$  is a hyper-parameter to control the magnitude of the regularization term, and clip is the operation of bounding the regularization term to a threshold *q* to ensure convergence.

**Hessian Matrix Estimation** As the Hessian matrix is the second-order derivative of the loss function, we resort to the estimation of the Hessian matrix proposed in (LeCun et al., 1989). We only focus on the diagonal terms due to the intractability of estimating non-diagonal terms. Specifically, the diagonal elements in  $H_{t,i}^k$  can be estimated from the change of the gradient between local iteration *i* and *i* + 1:  $\nabla \mathcal{L} \left( \theta_{t,i+1}^k \right) - \nabla \mathcal{L} \left( \theta_{t,i}^k \right)$ . In this term, the change of the gradient can be approximated by the change of the model parameters during the local iterations, i.e.,

$$\hat{\boldsymbol{H}}_{t,i}^{k} = \left(\Delta \boldsymbol{\theta}_{t,i+1}^{k} - \Delta \boldsymbol{\theta}_{t,i}^{k}\right) / \eta_{t}.$$

where  $\Delta \boldsymbol{\theta}_{t,i+1}^k = \boldsymbol{\theta}_{t,i+1}^k - \boldsymbol{\theta}_{t,i+1}^k$  and  $\Delta \boldsymbol{\theta}_{t,i}^k = \boldsymbol{\theta}_{t,i}^k - \boldsymbol{\theta}_{t,i}^k$ .

The estimation of the Hessian Matrix before adding the regularization term in Equation 3 can be rewritten as a function of model parameter changes.

$$\widetilde{\boldsymbol{H}}_{t,i}^{k} = \left(\widetilde{\boldsymbol{\theta}}_{t,i+1}^{k} - \boldsymbol{\theta}_{t,i}^{k} - \Delta \boldsymbol{\theta}_{t,i}^{k}\right) / \eta_{t}$$
(4)

Adding Perturbation Our objective now is to perturb the estimated Hessian matrix such that the coefficient term  $(I - \eta_t H_{t,i}^k)$  is minimized, i.e.,

$$\hat{H}_{t,i}^k \leftarrow \operatorname{argmin}_{\widetilde{H}_{t,i}^k} \left( I - \eta_t \widetilde{H}_{t,i}^k \right)$$

Combining it with Equation 4, optimizing  $\widetilde{H}$  is then equivalent to

$$\boldsymbol{\theta}_{t,i+1}^{k} \leftarrow \operatorname{argmin}_{\widetilde{\boldsymbol{\theta}}_{t,i+1}^{k}} \left( \boldsymbol{I} - \left( \widetilde{\boldsymbol{\theta}}_{t,i+1}^{k} - \boldsymbol{\theta}_{t,i}^{k} - \Delta \boldsymbol{\theta}_{t,i}^{k} \right) \right)$$
(5)

**Gradient Clipping** To ensure that the model can converge after the regularization term is added, the gradients are clipped with the threshold q during the local training. The clipping function is defined as:

$$\operatorname{clip}\left(\nabla\left(\mathbf{I} - \eta_{t,i}\widetilde{\mathbf{H}}_{t,i}^{k}\right), q\right)_{r,c} = \begin{cases} \nabla\left(\mathbf{I} - \eta_{t,i}\widetilde{\mathbf{H}}_{t,i}^{k}\right)_{r,c}, \left|\nabla\left(\mathbf{I} - \eta_{t,i}\widetilde{\mathbf{H}}_{t,i}^{k}\right)_{r,c}\right| \leq q \\ q, \left|\nabla\left(\mathbf{I} - \eta_{t,i}\widetilde{\mathbf{H}}_{t,i}^{k}\right)_{r,c}\right| > q \end{cases}$$

where r and c are the indexes of rows and columns of the Matrix.

Algorithm 1 LeadFL and robust aggregation **Input:** number of global rounds T, local learning rate  $\eta_t$ , regularization rate  $\alpha$ , clipping bound q, # of clients selected in a round Kfor communication round  $t = 0, 1, \dots, T - 1$  do Server randomly chooses K clients **parallel** on clients  $k = 1, 2, \cdots, K$  do Update model weights as global weights from the last round  $\theta_t^k \leftarrow \theta_t$ ; for local iteration  $i = 0, 1, \dots$  do Compute gradients and update weights Compute gradients and applied to the regularization  $\widetilde{\theta}_{t,i+1}^k \leftarrow \theta_{t,i}^k - \eta_t \nabla \mathcal{L} \left( \theta_{t,i}^k \right);$ Estimate Hessian matrix  $\widetilde{H}_{t,i}^k = \left( \widetilde{\theta}_{t,i+1}^k - \theta_{t,i}^k - \Delta \theta_{t,i}^k \right) / \eta_t$ Compute and Clip gradients of the regularization term  $\mathbf{R}_{t,i}^{k} = \operatorname{clip}\left(\nabla\left(\mathbf{I} - \eta_{t}\widetilde{\mathbf{H}}_{t,i}^{k}\right), q\right);$ Update weights  $\boldsymbol{\theta}_{t,i+1}^{k} \leftarrow \widetilde{\boldsymbol{\theta}}_{t,i+1}^{k} - \eta_t \alpha \boldsymbol{R}_{t,i}^{k}$ ; end for Compute updates  $u_t^k = \theta_t^k - \theta_t$ ; end parallel Aggregate updates using server-side defense:  $\boldsymbol{u}_t = \text{Aggregation}\left(\{\boldsymbol{u}_t^k\}_{k=1}^K\right)$ Update  $\boldsymbol{\theta}_{t+1} \leftarrow \boldsymbol{\theta}_t + \boldsymbol{u}_t$ end for Output:  $\{\boldsymbol{\theta}^t\}_{t=0}^{T-1}$ 

Algorithm To compute the model updates as shown in Equations 2 and 3, we adopt a two-step backpropagation process. We first allow the losses to backpropagate and then estimate the diagonal values of the Hessian matrix. Our second step is to use the estimated Hessian Matrix to compute the proposed regularization term and to allow the regularization loss to backpropagate. We summarize the key steps of LeadFL in Algorithm 1 and includes option of combining it with a server-side defense.

#### 4.2. Convergence Analysis

In this part, we show that LeadFL converges under the same assumptions as other methods when there are no malicious clients attacking the FL system. We summarize these common assumptions in Appendix A.1.

In our defense, we add a new backpropagation process to perturb the Hessian matrix as shown in Equation 3. This extra backpropagation can be seen as a modification of gradients  $\nabla \mathcal{L}$ :

$$\nabla \mathcal{L}'\left(\boldsymbol{\theta}_{t,i}^{k}\right) = \nabla \mathcal{L}\left(\boldsymbol{\theta}_{t,i}^{k}\right) + \operatorname{clip}\left(\nabla\left(\boldsymbol{I} - \eta_{t}\widetilde{\boldsymbol{H}}_{t,i}^{k}\right), q\right)$$
(6)

Based on Assumption 3.1.1 to 3.1.5, we can derive the

convergence guarantee of our defense on FedAvg as follows.

**Theorem 4.1** (Convergence Guarantee). Let Assumptions A.1 to A.5 hold and  $l, \mu, \sigma_k, G, K, N, \Gamma, \mathcal{L}^*$  be as defined therein and in Definition A.6. Choose  $\kappa = \frac{l}{\mu}$ ,  $\gamma = \max\{8\kappa, E\}$  and the learning rate  $\eta_t = \frac{2}{\mu(\gamma+t)}$ . Then we have the following bound for LeadFL:

$$\mathbb{E}\left[\mathcal{L}\left(\boldsymbol{\theta}_{T}\right)\right] - \mathcal{L}^{*} \leq \frac{\kappa}{\gamma + T - 1} \left(\frac{2(B + C)}{\mu} + \frac{\mu\gamma}{2} \mathbb{E}\left\|\boldsymbol{\theta}_{0} - \boldsymbol{\theta}^{*}\right\|^{2}\right)$$

where

$$C = \frac{N - K}{N - 1} \frac{4}{K} E^2 (d^2 q^2 + G^2)$$
$$B = \sum_{k=1}^{N} p_k^2 (d^2 q^2 + \sigma_k^2) + 6l\Gamma + 8(E - 1)^2 (d^2 q^2 + G^2)$$

The proof is shown in Chapter 3.

#### 4.3. Robustness Analysis

In this subsection, we use the certified radius framework proposed by (Panda et al., 2022) to analyze the robustness of LeadFL. We consider two types of threat models: periodic poisoned model submissions and bursty poisoned model submissions. Due to the space limitation, we provide the definitions and assumptions in Chapter 3.

The certified radius is the upper bound on the distance between a poisoned model and a benign model. From (Xie et al., 2021), minimizing the certified radius improves robustness because close models are likely to have the same predictions. Based on the aforementioned assumptions and definitions, the certified radius for general protocols is proposed by (Panda et al., 2022).

**Theorem 4.2.** Let f be a c-coordinatewise-Lipschitz protocol on a dataset D. Then  $R(\rho) = \Lambda(T)(1 + dc)^{\Lambda(T)}\rho$ is a certified radius for f, where  $\Lambda(t)$  is the cumulative learning rate  $\Lambda(t) = \sum_{t=0}^{T-1} \eta_t$ , d is the dimension of model parameters.

#### 4.3.1. SCENARIO I

Scenario I assumes a simplified model of bursty adversarial patterns, namely the most extreme pattern where periodically, a large number of clients is malicious and there are no malicious clients in the other rounds. Concretely, malicious clients submit poisoned updates in global round  $T_A$ . Afterwards, there are no malicious updates submitted between round  $T_A$  and round T - 1.

**Theorem 4.3** (Certified Radius in Scenario I). Let Assumptions A.9 hold and  $T_A$ , c be as defined therein. We assume

that LeadFL with FedAv aggregation is used. The certified radius satisfies:

$$R(\rho) = \left(\frac{N}{K}\right)^{T-T_A} \left| \prod_{t=T_A}^{T} \left[ \sum_{k \in S_t} p^k \prod_{i=0}^{I-1} \left(I - \eta_t H_{t,i}^k\right) \right] \right|$$
$$\cdot \sum_{t=0}^{T_A - 1} \eta_t (1 + dc)^{\sum_{t=0}^{T_A - 1} \eta_t} \rho$$

The proof is in Chapter 3. As LeadFL aims to minimize  $(I - \eta_t H_{t,i}^k)$  in the local training as by Equation 6. Hence, LeadFL achieves a low certified radius.

#### 4.3.2. SCENARIO II

Here, we consider a more general threat model, the number of malicious clients varies between rounds with resulting bursty adversarial patterns. Concretely, we assume that the clients are selected randomly. We furthermore assume the presence of a server-side defense that filters out updates.

The probability of a server-side defense filtering out all malicious updates is correlated to the number of malicious clients selected in a communication round. For an attack atk, we use  $g_{atk}(\cdot)$  to represent the above correlation. The probability of a server-side defense filtering out all malicious updates in global round t can be presented as  $\phi_{atk}^t = g_{atk}(K_m^t)$ , where  $K_m^t$  is the number of malicious clients selected in round t. We then can derive the certified radius of LeadFL combined with any given server-side defense under bursty adversarial patterns as:

**Theorem 4.4** (Certified Radius in Scenario II). Let Assumption A.9 hold. The certified radius of the threat model is

$$R(\rho) = (1+dc)^{\sum_{t \in \Phi_T} \eta_t} \rho \cdot \left( \left| \prod_{t \in \Gamma_T} \left[ \frac{N}{|S_t^*|} \sum_{k \in S_t^*} p^k \prod_{i=0}^{I-1} \left( I - \eta_t H_{t,i}^k \right) \right] \right| + |\Phi_T| \sum_{t \in \Phi_T} \eta_t \right)$$

where  $\Phi_T$  is the set of communication rounds that serverside defenses cannot filter out all malicious updates.  $\Gamma_T$  is the set of communication rounds that server-side defenses filter out all malicious updates.  $S_t^*$  is a set of clients whose updates are not filtered out by the server-side defense in round t.  $|\Phi_T|$  and  $|S_t^*|$  are the cardinality of the set  $\Phi_T$  and  $S_t^*$ , where  $\mathbb{E}[|\Phi_T|] = \sum_{t=0}^{T-1} g_{atk}(K_m^t)$ .

Note that the value  $|\Phi_T|$  depends entirely on the server-side defense. In the absence of a server-side defense, the certified radius is hence large, so we need the server-side defense to lower it.

LeadFL: Client Self-Defense against Model Poisoning in Federated Learning

Table 1: Comparison of defenses under 9-pixel pattern backdoor attack on IID and non-IID FashionMNIST dataset.

Distribution		IID								Non-IID								
Server-side Defense	SparseFed		Mu	ti-Krum		Bulyan			SparseFed	Multi-Krum				Bulyan				
Client-side Defense	None	None	LDP	FL-WBC	Ours	None	LDP	FL-WBC	Ours	None	None	LDP	FL-WBC	Ours	None	LDP	FL-WBC	Ours
MA	85.9	89.3	87.0	87.2	87.9	89.2	86.0	86.0	86.3	84.9	85.6	76.7	77.2	79.1	77.4	73.4	71.7	74.0
BA Avg	97.9	82.6	76.0	77.5	32.9	78.8	74.1	70.6	21.6	99.8	88.7	80.4	74.0	39.5	92.5	71.9	73.7	32.3
BA Final	99.9	93.2	79.6	80.6	0.0	90.6	62.2	86.5	0.3	99.9	93.3	86.7	70.3	1.2	88.6	94.7	69.0	2.0

Table 2: Comparison of defenses under 9-pixel pattern backdoor attack on IID and non-IID CIFAR10 dataset.

Distribution		IID								Non-IID								
Server-side Defense	SparseFed		Mu	lti-Krum		Bulyan			SparseFed	Multi-Krum				Bulyan				
Client-side Defense	None	None	LDP	FL-WBC	Ours	None	LDP	FL-WBC	Ours	None	None	LDP	FL-WBC	Ours	None	LDP	FL-WBC	Ours
MA	50.9	76.3	48.0	43.3	56.9	76.2	41.5	42.2	54.8	55.3	70.7	43.2	42.9	55.3	61.7	36.7	36.2	51.4
BA Avg	95.8	77.5	53.1	56.9	35.6	79.1	46.7	51.3	43.9	45.2	85.8	55.4	54.4	45.2	87.5	48.8	48.1	46.8
BA Final	98.5	80.5	43.8	40.5	25.6	87.0	23.4	35.5	21.4	34.4	96.2	52.4	35.4	34.4	95.2	29.8	47.7	27.3

#### 5. Evaluation

In this section, we demonstrate the effectiveness of LeadFL for multiple server-side defenses. We consider heterogeneous data distributions and compare against state-of-the-art client-side defense mechanisms. Furthermore, our ablation study confirms that a combination of server-side and clientside defenses succeeds in mitigating attacks that are highly effective in the presence of either of the two.

We perform all experiments using PyTorch's deep learning framework (Paszke et al., 2019) in combination with the FLTK Testbed <sup>1</sup>. We reimplemented FL-WBC, LDP, and the targeted attacks based on the source code of FL-WBC <sup>2</sup> to compare them with our defense. Additionally, we reimplement SparseFed and backdoor attacks based on the source code provided by (Panda et al., 2022) <sup>3</sup> and (Bagdasaryan & Shmatikov, 2021) <sup>4</sup>, respectively. Our code can be found at https://github.com/CarlosChu-c/LeadFL.

#### 5.1. Evaluation Metrics

Our goal is to achieve high accuracy for the main task but mitigate the backdoor. Thus, we primarily focus on the following three metrics:

- Main Task Accuracy (MA): The main task accuracy is the fraction of correctly classified samples of the model on test data without the trigger. As other works, we consider the maximum accuracy achieved during training.
- **Backdoor Accuracy (BA)**: The backdoor accuracy qualifies how successful the attacker is in integrating a backdoor into the model. We measure backdoor accuracy as the percentage of samples with the trigger that are classified as intended by the attacker. We found

<sup>1</sup>https://github.com/JMGaljaard/ fltk-testbed that the backdoor accuracy does not converge during our experiments, hence we consider both the average and the final backdoor accuracy. The final backdoor accuracy is the one of the model that is later used but it does not give a full picture due to the high variance in backdoor accuracy over rounds, which is why we also include the average backdoor accuracy.

 Mitigation rounds: Our attacks do not have the same strength in every round due to the fact that the number of malicious clients selected varies between rounds. When a lot of malicious clients are involved, the backdoor accuracy spikes and then decreases again. After a strong attack that achieves a temporary backdoor accuracy of more than 50%, we define the mitigation rounds as the number of communication rounds until the backdoor accuracy drops below 50%.

#### 5.2. Evaluation Setup

In each simulation run, we have a set of clients. During each round, the server selects clients. The clients train and apply the client-side defense during training. Afterwards, the server aggregates the local updates submitted by the clients, applying the server-side defense during aggregation.

Client Selection and Rounds There are 100 clients in total, of which 25 are malicious. There are 80 global rounds and 10 local rounds. The server selects 10 clients per global round. For most experiments, the selection is random but consistent over experiments, i.e., for two experiments, the clients selected in round t are the same to enable comparison between the different settings. Figure 2a displays the number of malicious clients per round. In order to ensure that our results are not an artifact of this one specific client selection, we present results for other selections in the Chapter 4.

In previous work, periodic attacks alternating between a large number of malicious selected clients and no malicious selected clients have been evaluated. In addition to random selecting, we hence also use a selection corresponding to

<sup>&</sup>lt;sup>2</sup>https://github.com/jeremy313/FL-WBC
<sup>3</sup>https://github.com/sparsefed/sparsefed
<sup>4</sup>https://github.com/ebagdasa/backdoors101

LeadFL: Client Self-Defense against Model Poisoning in Federated Learning



Table 3: Comparison of defenses under 9-pixel pattern backdoor attack on IID CIFAR100 dataset.

Figure 3: Comparison of 9-pixel back door attack on FashionMNIST and CIFAR10 when the attack is periodic.

such a periodic attack: For every 10 global rounds, 6 of 10 clients are malicious in the first two rounds while the remaining 8 rounds only have honest clients.

Backdoor Accuracy

**Datasets and Model Architecture** We conduct experiments on FashionMNIST, CIFAR10 and CIFAR100, which are both benchmark tasks in image classification. For Fashion-MNIST, each of the 100 clients receives 600 images out of 60.000. For CIFAR10 and CIFAR100, each client gets 500 out of 50.000.

In the IID setting, samples are uniformly distributed to clients. In the non-IID setting, we deploy the limited label strategy (McMahan et al., 2017) that is also used for the evaluation of FL-WBC in FashionMNIST and CIFAR10: Of the 10 classes in each of the two datasets, each client is assigned 5 random classes. They are then assigned an equal number of randomly selected samples from each of their classes. The clients' datasets are selected independently.

We adopt the same model architectures as FL-WBC (Sun et al., 2021) on FahionMNIST and CIFAR10. On FashionMNIST, we employ two convolutional layers and one fully-connected layer. Our CIFAR10 model consists of two convolutional layers and three fully-connected layers. And on CIFAR100, we employ ResNet9 (He et al., 2016), which is a more complicated model. The detail of the model architecture and hyperparameters can be found in Chapter 3.

Attacks and Defenses For attacks, we evaluate both stateof-the-art backdoor and targeted attacks. In terms of backdoor attacks, we use the 9-pixel pattern backdoor attacks and the single-pixel backdoor attacks from (Bagdasaryan & Shmatikov, 2021). As a targeted attack, we evaluate the single-image targeted attacks from (Bhagoji et al., 2019): All malicious clients add one incorrectly classified image to their otherwise clean dataset; it is the same image for all clients. We use the settings that achieved the best results in the original papers.

Here, we use Multi-Krum (Blanchard et al., 2017) and Bulyan (Mhamdi et al., 2018) as server-side defenses. We also compare SparseFed (Panda et al., 2022), one of the stateof-the-art defenses against poisoning attacks in FL. Moreover, we considered CMA (Yin et al., 2018) and CTMA (Yin et al., 2018) but they had very little effect in comparison to the other defenses, so we only include the corresponding results in the Chapter 4.Note that our protocol can enhance any other server-side defense as well.

For client-side defenses, we choose FL-WBC and LDP as the baseline. For these two defenses, we apply Laplace noise with mean = 0 and std = 0.2 as in the original papers. For our defense, we set the clipping norm q = 0.2. For the regularization term, we use hyperparameter tuning to choose  $\alpha = 0.4$  for FashionMNIST,  $\alpha = 0.25$  for CIFAR10, and  $\alpha = 0.15$  for CIFAR100.

#### 5.3. Results

Table 1, 2 and 3 show the results for the 9-pixel backdoor attack. In our threat model, SparseFed presents limited effectiveness in defending against poisoning attacks, achieving higher Backdoor accuracy than Multi-Krum and Bulyan across all three datasets. And it can be seen that our defense achieves the highest main task accuracy and lowest backdoor accuracy. In comparison to the case without a client-side defense, the main task accuracy is reduced by less than 10% whereas the final backdoor accuracy is 0 or close to 0 for FashionMNIST. For CIFAR10, the main task accuracy of



Figure 4: Comparison of 9-pixel pattern backdoor accuracy on FashionMNIST and CIFAR10. The server-side defense here is Multi-Krum. Black hollow circles indicate that the system is attacked very strongly in that round.



Figure 5: Various defenses' tradeoffs between main task accuracy and mitigation rounds on FashionMNIST and CIFAR10.

our defense is between 50% and 60% and the final backdoor accuracy is between 20% and 35%. The average backdoor accuracy for our defense is higher than the final accuracy but always below 50% and lower than the backdoor accuracy of the state-of-the-art defenses. For CIFAR100, the main task accuracy is about 30% and the final backdoor accuracy is only between 3% and 7%. Indeed, without a client-side defense, the final and average backdoor accuracy is always above 75% in FashionMNIST and CIFAR10, and 55% in CIFAR100, meaning that the server-side defense on its own is ineffective. The other client-side defenses are considerably less effective than ours: For FashionMNIST, they have a final and average backdoor accuracy of above 69%, a stark contrast to our defense, especially for the final backdoor accuracy. For CIFAR10, the difference is less pronounced, with backdoor accuracies that are only about 10% higher than for our defense. However, the main task accuracy of the other defenses falls below 50% for CIFAR10. For CI-FAR100, although the main task accuracy of our defense is only about 2% higher than other client-side defenses, the average backdoor accuracy and final backdoor accuracy of our defense is about 20% lower than other defenses.

While our defense is hence an improvement over existing defenses, there are notable differences between settings. Non-IID distributions of data reduce the main task accuracy and increase the backdoor accuracy for all defenses. The result is unsurprising: The more uniform benign clients are, the easier it is to detect malicious clients whose model updates differ. However, if client data and hence models already differ between benign clients, it becomes more difficult to identify and mitigate malicious behavior.

In order to analyze how the backdoor accuracy is affected by the number of attackers, we consider the backdoor accuracy over the duration of the experiment. Figure 4 displays the backdoor accuracy. We can see that whenever the number of malicious clients exceeds the number of benign clients, i.e., if there are at least 6 malicious clients selected in a round, the backdoor is successfully embedded into the model, as shown by a high backdoor accuracy of close to 100%. In subsequent rounds with a lower amount of malicious clients, the backdoor accuracy decreases. Our defense exhibits a faster decrease in backdoor accuracy than the other defenses, which results in the lower final and average backdoor accuracy seen above. The same pattern is observed for both datasets and levels of data heterogeneity, although the speed of recovery is faster for iid data distributions.

We compare this behavior for random client selection with the periodic attack described in Section 5.2. We observe the same pattern, displayed in Figure 3, as when selecting clients randomly, only that it is now periodic. For the periodic setting, we derive the number of mitigation rounds, as defined in Section 5.1. As the delay between two attacks is always the same and the attacks are of the same severity, the periodic setting enables use to compare recovery in a fair manner. We can then analyze whether there is a trade-off between mitigation rounds, i.e., strength of the defense, and main task accuracy. Concretely, for each experiment and each attack, we compute the number of mitigation rounds. If the backdoor accuracy does not recover during the 8 rounds between two attacks, we use  $\geq 8$  for the number of mitigation rounds. For a defense d, we then compute  $MA_{r,d}$ , the average main task accuracy over all experiments for d that have r mitigation rounds<sup>5</sup>. Figure 5 shows the results. Our defense achieves a better trade-off between main task accuracy and recovery, i.e., for the same number of mitigation rounds, it has a higher main task accuracy. An exception is the case  $\geq 8$  with no client-side defense having a higher main task accuracy, which makes sense as if our defense does not lead to recovery, not applying a defense is the better option. However, usually our defense successfully mitigates the attack and if it does so, it has a higher main task accuracy than other defenses.

All the presented results are for the 9-pixel attack. The results for the 1-pixel attack and the single-image targeted attack are similar (see Chapter 4).

#### 6. Conclusion

To defend against model poisoning attacks with bursty adversarial patterns, we propose a novel client-side self defense, LeadFL, which perturbs the local model updates by adding a novel regularization term based on the Hessian matrix of the gradients. Thanks to the optimized regularization, LeadFL effectively thwarts backdoor and targeted attacks with a low degradation of the main task accuracy, proven theoretically and empirically. Evaluated on FashionMNIST, CIFAR10 and CIFAR100, LeadFL combined with a serverside defense can reduce the backdoor accuracy by up to 65 % in comparison only using a server-side defense.

#### References

- Bagdasaryan, E. and Shmatikov, V. Blind backdoors in deep learning models. In 30th USENIX Security Symposium (USENIX Security 21), pp. 1505–1521, 2021.
- Bhagoji, A. N., Chakraborty, S., Mittal, P., and Calo, S. Analyzing federated learning through an adversarial lens. In Chaudhuri, K. and Salakhutdinov, R. (eds.), *Proceedings of the 36th International Conference on Machine Learning*, volume 97 of *Proceedings of Machine Learning Research*, pp. 634–643. PMLR, 09–15 Jun 2019.
- Blanchard, P., El Mhamdi, E. M., Guerraoui, R., and Stainer, J. Machine learning with adversaries: Byzantine tolerant gradient descent. In *Advances in Neural Information Processing Systems*, volume 30. Curran Associates, Inc., 2017.

- Chen, X., Liu, C., Li, B., Lu, K., and Song, D. Targeted backdoor attacks on deep learning systems using data poisoning. *arXiv preprint arXiv:1712.05526*, 2017.
- Fung, C., Yoon, C. J., and Beschastnikh, I. The limitations of federated learning in sybil settings. In 23rd International Symposium on Research in Attacks, Intrusions and Defenses (RAID 2020), pp. 301–316, 2020.
- Gupta, A., Luo, T., Ngo, M. V., and Das, S. K. Long-short history of gradients is all you need: Detecting malicious and unreliable clients in federated learning. In *European Symposium on Research in Computer Security*, pp. 445– 465. Springer, 2022.
- He, K., Zhang, X., Ren, S., and Sun, J. Deep residual learning for image recognition. In *Proceedings of the IEEE* conference on computer vision and pattern recognition, pp. 770–778, 2016.
- Konečný, J., McMahan, H. B., Yu, F. X., Richtarik, P., Suresh, A. T., and Bacon, D. Federated learning: Strategies for improving communication efficiency. In *NIPS Workshop on Private Multi-Party Machine Learning*, 2016.
- LeCun, Y., Denker, J., and Solla, S. Optimal brain damage. *Advances in neural information processing systems*, 2, 1989.
- Li, X., Huang, K., Yang, W., Wang, S., and Zhang, Z. On the convergence of fedavg on non-iid data. In *International Conference on Learning Representations*, 2019.
- McMahan, B., Moore, E., Ramage, D., Hampson, S., and y Arcas, B. A. Communication-efficient learning of deep networks from decentralized data. In *Artificial intelligence and statistics*, pp. 1273–1282. PMLR, 2017.
- Mhamdi, E. M. E., Guerraoui, R., and Rouault, S. The hidden vulnerability of distributed learning in byzantium. In *Proceedings of the 35th International Conference on Machine Learning*, pp. 3521–3530. PMLR, 2018. ISSN: 2640-3498.
- Muñoz-González, L., Co, K. T., and Lupu, E. C. Byzantinerobust federated machine learning through adaptive model averaging, 2019.
- Naseri, M., Hayes, J., and De Cristofaro, E. Local and central differential privacy for robustness and privacy in federated learning. In *Proceedings 2022 Network and Distributed System Security Symposium*. Internet Society, 2022. ISBN 978-1-891562-74-7. doi: 10.14722/ndss. 2022.23054.

<sup>&</sup>lt;sup>5</sup>We round it up the nearest integer.

- Nguyen, T. D., Rieger, P., Chen, H., Yalame, H., Möllering, H., Fereidooni, H., Marchal, S., Miettinen, M., Mirhoseini, A., Zeitouni, S., et al. {FLAME}: Taming backdoors in federated learning. In *31st USENIX Security Symposium (USENIX Security 22)*, pp. 1415–1432, 2022.
- Ozdayi, M. S., Kantarcioglu, M., and Gel, Y. R. Defending against backdoors in federated learning with robust learning rate. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 35, pp. 9268–9276, 2021.
- Panda, A., Mahloujifar, S., Bhagoji, A. N., Chakraborty, S., and Mittal, P. Sparsefed: Mitigating model poisoning attacks in federated learning with sparsification. In *International Conference on Artificial Intelligence and Statistics*, pp. 7587–7624. PMLR, 2022.
- Paszke, A., Gross, S., Massa, F., Lerer, A., Bradbury, J., Chanan, G., Killeen, T., Lin, Z., Gimelshein, N., Antiga, L., Desmaison, A., Kopf, A., Yang, E., DeVito, Z., Raison, M., Tejani, A., Chilamkurthy, S., Steiner, B., Fang, L., Bai, J., and Chintala, S. Pytorch: An imperative style, high-performance deep learning library. In Advances in Neural Information Processing Systems 32, pp. 8024– 8035. Curran Associates, Inc., 2019.
- Rieger, P., Nguyen, T. D., Miettinen, M., and Sadeghi, A.-R. DeepSight: Mitigating backdoor attacks in federated learning through deep model inspection. In *Proceedings* 2022 Network and Distributed System Security Symposium. Internet Society, 2022. ISBN 978-1-891562-74-7. doi: 10.14722/ndss.2022.23156.
- Steinhardt, J., Koh, P. W. W., and Liang, P. S. Certified defenses for data poisoning attacks. Advances in neural information processing systems, 30, 2017.
- Sun, J., Li, A., DiValentin, L., Hassanzadeh, A., Chen, Y., and Li, H. Fl-wbc: Enhancing robustness against model poisoning attacks in federated learning from a client perspective. *Advances in Neural Information Processing Systems*, 34, 2021.
- Tolpegin, V., Truex, S., Gursoy, M. E., and Liu, L. Data poisoning attacks against federated learning systems. In *European Symposium on Research in Computer Security*, pp. 480–501. Springer, 2020.
- Xia, Q., Tao, Z., Hao, Z., and Li, Q. Faba: An algorithm for fast aggregation against byzantine attacks in distributed neural networks. In *Proceedings of the Twenty-Eighth International Joint Conference on Artificial Intelligence, IJCAI-19*, pp. 4824–4830. International Joint Conferences on Artificial Intelligence Organization, 7 2019. doi: 10.24963/ijcai.2019/670.

- Xie, C., Huang, K., Chen, P.-Y., and Li, B. Dba: Distributed backdoor attacks against federated learning. In *International Conference on Learning Representations*, 2019.
- Xie, C., Chen, M., Chen, P.-Y., and Li, B. CRFL: Certifiably robust federated learning against backdoor attacks. In *Proceedings of the 38th International Conference on Machine Learning*, pp. 11372–11382. PMLR, 2021. ISSN: 2640-3498.
- Xu, J., Huang, S.-L., Song, L., and Lan, T. Byzantinerobust federated learning through collaborative malicious gradient filtering. In 2022 IEEE 42nd International Conference on Distributed Computing Systems (ICDCS), pp. 1223–1235. IEEE, 2022.
- Yin, D., Chen, Y., Kannan, R., and Bartlett, P. Byzantinerobust distributed learning: Towards optimal statistical rates. In *Proceedings of the 35th International Conference on Machine Learning*, pp. 5650–5659. PMLR, 2018. ISSN: 2640-3498.

# 2

### Background

The rapid development of computational devices and the big data revolution have led to the creation and accessibility of vast amounts of data. However, the centralization of such extensive data collections for model training on a single cloud server can be prohibitively expensive and time-consuming (Li et al., 2021). Moreover, the dispersion of private data across local clients leads to the emergence of isolated data islands (Kairouz et al., 2021). In this context, federated learning has emerged as a promising distributed machine learning framework that finds application in numerous domains (Konečnỳ et al., 2016). In this paradigm, data remains distributed across a multitude of clients, and the model training process is decentralized. At the commencement of each training round, the server disseminates the model to all clients. Subsequently, each client locally trains the model utilizing its own data, then returns

Despite the advantages offered by federated learning in terms of enhancing machine learning training speed and safeguarding data privacy, recent research highlights that this framework is not immune to poisoning attacks. These attacks can be orchestrated by a portion of the local clients who alter their model updates with malicious intent, such as to compromise the global model's performance or to cause the model to behave unpredictably with specific inputs (Bagdasaryan et al., 2020; Wang et al., 2020). There are two primary types of such poisoning attacks. The first, known as *Untargeted poisoning attacks*, is designed to make the model behave inappropriately on specific inputs subtly controlled by the attackers.

This chapter will delve into the background of federated learning, poisoning attacks, and the defenses devised to counter these threats.

#### 2.1. Fedearted learning

In the federated learning paradigm, each client retains a local model, while a central server maintains a global model for the purpose of model aggregation, as depicted in Figure 2.1. This process bears resemblance to centralized training, involving multiple iterative rounds. At the beginning of each round, the server disseminates the global model to the clients, facilitating them in updating their individual models. The clients then employ their local datasets to train their models and subsequently transmit the

updates, which could be gradients or weights, back to the central server. The server aggregates these local models using algorithms such as FedAvg.

To elaborate, at the start of the  $t^{th}$  round, the server distributes the weight of the aggregated model, denoted as  $\theta_t^{global}$ , to each client  $(k_1, k_2, ..., k_m)$ . Each client then uses its local dataset to train and update its local model as follows:

$$\theta_{i,t}^{\text{global}} \xrightarrow[trained \text{ on local dataset}]{} \theta_{i,t+1}^{\text{local}}$$
(2.1)

Here, *i* denotes the  $i_{th}$  client. Upon completion of local training, each client uploads its trained model to the server. For aggregating models across n clients, the FedAvg algorithm computes the new global model as a weighted average of the submitted local models, as follows:

$$\theta_{i,t+1}^{\text{global}} = \sum_{i=1}^{m} \frac{n_i}{\sum_{s=1}^{m} n_s} \theta_{i,t+1}^{\text{local}}$$
(2.2)

In this equation,  $n_i$  represents the size of the local dataset of client  $k_i$ . Thus, a client's influence in model aggregation is proportional to the size of its local dataset.

Similarly, clients can also submit only model updates for modifying the global model. If the model updates of the  $i_{th}$  client in the  $t_{th}$  round are denoted as  $\Delta_{i,t} = \theta_{i,t+1}^{\text{local}} - \theta_{i,t}^{\text{global}}$ , then FedAvg using the model gradient can be formulated as:

$$\theta_{i,t+1}^{\text{global}} = \theta_{i,t}^{\text{global}} + \sum_{i=1}^{m} \frac{n_i}{\sum_{s=1}^{m} n_s} \Delta_{i,t}$$
(2.3)

#### 2.2. Poisoning Attacks in Federated Learning

Federated learning systems are susceptible to a variety of threats, such as poisoning attacks, inference attacks, and gradient inversion attacks. Among these, poisoning attacks stand out as the most severe. The adversaries in poisoning attacks are clients, which makes them quite different from gradient inversion and model inversion attacks, where the server acts as the adversary. However, typically in real-world scenarios, the server is assumed to be reliable and trusted. Poisoning attacks are also more threatening compared to inference attacks. This is due to the fact that the malicious parties in a poisoning attack have the ability to directly alter the model updates of clients to fulfill their harmful intentions. On the other hand, inference attacks merely allow the adversary to infer private data from other clients without reducing performance, under the assumption that the server is honest-but-curious. In this subsection, we will delve into the details of model poisoning attacks within the context of federated learning.

#### Categorized by attackers' objective

Based on the attacker's objective, poisoning attacks can be split into two groups: untargeted and targeted poisoning attacks. Untargeted poisoning attacks are designed to undermine the performance of the global model, while targeted poisoning attacks are used by assailants aiming to subtly alter a model in response to specific inputs under their control, such as a unique user or a set of inputs. Among targeted attacks,



**Figure 2.1:** An overview of the federated learning system. The Global Model, updated after each global round, is maintained on the server side. The Local Model is trained on each client's Local Data. Following every global round, the server aggregates the individual local models to create a new, updated Global Model.

backdoor attacks are the most covert and hazardous. These attacks trigger the model to behave anomalously when presented with specific cues, like particular pixel patterns in images or specific text words.

#### Categorized by attack techniques

As depicted in Figure 2.2, poisoning attacks can be classified into data poisoning attacks (Xie et al., 2019; Sun et al., 2019; Wang et al., 2020; Bagdasaryan et al., 2020) and model poisoning attacks (Blanchard et al., 2017; Yin et al., 2018; Xie et al., 2020; Baruch et al., 2019; Fang et al., 2020; Shejwalkar & Houmansadr, 2021), based on the attack method. In a data poisoning attack, an attack modifies or distorts the distribution of a local client's training data, which subsequently poisons the local and, after aggregation, the global model. For instance, in a label-flipping attack, the training data label is flipped to diverge the model, and in a backdoor attack, particular patterns are added to the training data that lead the model to misclassify data samples. Model poisoning attacks involve manipulating model updates before aggregation. This poisoned model update can affect the global model by changing the convergence direction or decelerating convergence speed. For instance, in a sign-flipping attack, the malicious clients change the sign of the model updates to poison the global model.

#### 2.3. Defenses against Poisoning Attacks

Defenses against poisoning attacks in federated learning can be categorized into four classes according to their approaches: 1) filtering-based defenses: 2.3.1, 2) DP-derived defenses: 2.3.2, 3) optimizer-based defenses: 2.3.3, and 4) ensembling-based defenses:



**Figure 2.2:** A comparison of data poisoning attacks and model poisoning attacks: Data poisoning attacks corrupt users' local training data, while model poisoning attacks directly manipulate the model updates.

#### 2.3.4.

In filtering-based defenses, the server tries to discard malicious updates or anomalous coordinates of updates, leveraging all updates submitted to the server over single or multiple rounds. Differential privacy-derived defenses involve the server or clients employing clipping or noise addition techniques, derived from differential privacy, to enhance the robustness of the federated learning system against poisoning attacks. Optimizer-based defenses involve modifying the optimizer, like SGD, used in the optimization process to mitigate the impact of poisoning attacks. Ensembling-based defenses involve choosing diverse sets of clients to avoid the influence of attackers. These defenses can also be classified into server-side and client-side based on their implementation location. And some defenses utilize multiple strategies to safeguard against poisoning attacks.

#### 2.3.1. Filtering-based

In these defense mechanisms, the server identifies and eliminates malicious or unreliable updates, which are determined based on the updates from one or multiple rounds. Consequently, all the defenses in this category are implemented on the server side, meaning the server, not the benign clients, conducts the defenses. There are several types of filtering-based defenses, including distance-based, statistics-based, clustering-based, and spectral anomaly detection defenses. Each of these categories will be further explored in the sections that follow.

#### **Distance-based Methods**

Distance-based methods involve the server identifying malicious updates by comparing the distances between updates to defend against poisoning attacks in distributed learning. Krum (Blanchard et al., 2017) is a seminal method in this category. It chooses an update  $\Delta_i$  from the  $i_{th}$  client that is nearest to its n - m - 2 neighboring updates in the  $l_2$  norm. The downside of Krum is that the convergence speed is slow, which leads to the proposal of the Multi-Krum method (Blanchard et al., 2017). Multi-Krum selects *k* gradients in an iterative Krum way. It selects a gradient using Krum from the remaining set, adds it to a selection set, and removes it from the remaining set. FABA (Xia et al., 2019) eliminates malicious updates based on their difference from the average update, reducing computational complexity. AFA (Muñoz-González et al., 2019) detects malicious updates based on the cosine-similarities with averaged updates, enabling the number of attacks to remain unknown. FoolsGold (Fung et al., 2020) concentrates on Sybil attacks, maintaining a history of updates from each client in each round. It computes the pairwise similarity between aggregated historical updates to estimate client similarity more accurately. In comparison, Krum, MultiKrum, and FABA all assume the number of attackers is below a certain threshold. However, in AFA, the number of attackers can remain unknown.

#### **Statistics-Based Methods**

The defenses in this category attempt to filter out the poisoning updates based on statistical information such as median, trimmed mean, or magnitude of updates (Yin et al., 2018). Some of these defenses filter updates element-wise rather than clientwise. Coordinate-wise median focuses on the statistical performance of the robust aggregation methods, while Coordinate-wise trimmed mean is similar, but removes the top and bottom  $\beta$  values and computes the averages of the rest of the values as its aggregate of this dimension (Yin et al., 2018). Bulyan combines distance-based and statistic-based defense, focusing on the problem that a single gradient dimension with a very large value can bypass Krum and Multi-Krum and diverge the model (Mhamdi et al., 2018). The defense mechanism proposed in (Ozdayi et al., 2021) identifies malicious updates using sign information and adjusts the learning rate of each parameter to steer the model away from malicious influences. This is achieved by a voting system based on the sign of updates, with a learning rate negation if updates in a certain direction are insufficient. On the other hand, (Panda et al., 2022) updates only the top-k highest magnitude elements of gradients, detecting malicious coordinates by their magnitude. This method also includes a robustness verification framework and aims to minimize the certified radius, the maximum drift of a poisoned model from a benign one.

#### **Clustering Methods**

Clustering is used in all defenses in this category to filter out outliers before the aggregation process. Compared to other methods used to detect anomalies, clustering can still be used when outliers are only slightly less than benign values, which fits the threat model of poisoning attacks. FLAME uses pairwise cosine distances to measure the angular differences between all model updates and applies the HDBSCAN clustering algorithm (Nguyen et al., 2022). DeepSight consists of three steps in anomaly detection: classification, clustering, and identifying poisoned clusters (Rieger et al., 2022). SignGuard involves Norm-based Thresholding and Sign-based Clustering to filter out gradients with the Norm larger or smaller than a threshold (Xu et al., 2022). MUD-HoG focuses on the history of gradients of each client and filters out all the updates from malicious and unreliable clients instead of filtering out the updates that are predicted as malicious and unreliable (Gupta et al., 2022).

#### **Spectral Anomaly Detection Methods**

Spectral anomaly detection has been used in several papers to defend against data poisoning attacks in centralized learning settings. The method in a certain work utilizes an encoder and decoder structure to approximate low-dimensional embeddings (Li et al., 2020). Once trained on benign updates, the encoder-decoder model can identify backdoored updates with much higher generation errors. VAE (Variation Auto-encoder) is used as the encoder-decoder in this work (Li et al., 2020).

#### 2.3.2. DP-derived

Recent advancements in federal learning algorithms have laid out the theoretical basis for differential privacy (DP). The categories of differential privacy algorithms are 1) Local DP, wherein every client introduces noise or limits the updates or gradients during local training, and 2) Central DP (CDP), where the server enforces DP during aggregation. In these algorithms, a clipping operation limits each client's update magnitude, assuring strict privacy guarantees for the subsequent perturbation step. This step involves introducing noise to the model's update for the safeguarding of client privacy. It has been demonstrated that the clipping and noise introduction phases effectively combat poisoning attacks, particularly those that are targeted. This section presents defense measures comprising algorithms derived from different privacy forms.

#### **Differential Privacy in Federated Learning**

The commonly used ( $\epsilon$ ,  $\delta$ )-differential privacy is defined as follows. An algorithm  $\mathcal{M}$  is ( $\epsilon$ ,  $\delta$ )-differentially private if

$$P(\mathcal{M}(\mathcal{D}) \in \mathcal{S}) \le e^{\epsilon} P\left(\mathcal{M}\left(\mathcal{D}'\right) \in \mathcal{S}\right) + \delta \tag{2.4}$$

where  $\mathcal{D}$  and  $\mathcal{D}'$  represent neighboring datasets, and  $\mathcal{S}$  is an arbitrary subset of  $\mathcal{M}$ 's outputs.

#### Clipping

This section presents defenses that include clipping operations. It introduces papers that enhance the clipping function.

(Karimireddy et al., b) applies iterative centered clipping, robust against gradient noise distribution. The centered clipping method follows:

$$v_{t+1} = v_t + \frac{1}{n} \sum_{i=1}^n (\Delta_i - v_t) \min\left(1, \frac{L_t}{\|\Delta_i - v_t\|}\right)$$
(2.5)

where  $v_t$  represents the aggregated update in the *t*-th round,  $\Delta_i$  is the *i*-th client's update in the *t*-th round, and  $L_t$  is the *t*-th round's clipping threshold. The paper's proof demonstrates that the centered clipping method yields fewer errors than preceding clipping methods.

(Xie et al., 2021) provides a thorough proof of the proposed defense's certified robustness. The certification elucidates the robustness of the proposed defense's relationship with different federal learning parameters like the number of attackers, training iterations, and poisoning rate. The clipping method's clipping bound is a hyper-parameter that can be adjusted to manage the robustness-accuracy trade-off.

The authors of (Nguyen et al., 2022) discovered that benign model updates' L2norms diminish during training. Therefore, the clipping bound should adapt to the decrease. Their FLAME algorithm employs the L2-norms of updates' median in a round as the clipping bound. If more than 50% of clients are benign, this method assures the clipping bound always falls within the L2 norms between benign and global models.

(Naseri et al., 2022) proposes local differential privacy and contrasts it with central differential privacy. The clipping and noises phase is executed following each client's local round in local difference privacy.

#### Noising

The citation (Xie et al., 2021) introduces isotropic Gaussian noise  $\epsilon_t \sim \mathcal{N}(0, \sigma_t^2 \mathbf{I})$  directly to the aggregated global model after clipping it.  $\sigma_t$  here is a hyper-parameter linked to certain robustness.

(Nguyen et al., 2022) applies an adaptive noising method to adjust the Gaussian noise's variance added to the aggregated model updates. According to prior research, a model can classify backdoored examples better the more noise introduced to a model during training. However, more noise causes more substantial performance degradation. Hence, this defense attempts to strike a balance between defending against attacks and maintaining the model's accuracy in the main task. The defense adds Gaussian noise based on the median distance between each client's model and the global model. The Gaussian noise is  $\epsilon_t \sim \mathcal{N}(0, \sigma^2 \mathbf{I})$ , where  $\sigma = \lambda \cdot S_t$ .  $S_t$  is the previous round's median distances between each client's model. And  $\lambda$  is a hyper-parameter derived from differential privacy. Consequently, if the model changes substantially in this round, the amount of noise will increase, effectively eliminating the backdoor attack effect from the aggregated model.

The paper (Naseri et al., 2022) tested Local Differential Privacy (LDP) and Central Differential Privacy (CDP). The central differential privacy aligns closely with the paper (Xie et al., 2021). In local differential privacy, Gaussian noise  $\epsilon_t \sim N(0, \sigma_t^2 \mathbf{I})$  is added to the local model in each local training epoch. According to the paper, like CDP, LDP can mitigate backdoor attacks and doesn't necessitate trusting the server.

FL-WBC (Sun et al., 2021) aims to perturb the Hessian matrices' kernel of benign clients to mitigate the attack effect while maintaining the benign task's performance. The authors first introduce a quantitative estimator called AEP (Attack Effect on Parameter), which includes a quantitative estimation of the attack's impact on the target model's parameters. The authors demonstrate that AEPs are hidden in local models' Hessian matrices' kernel. Since Hessian matrices are high-dimensional, the authors estimate the diagonal elements by the gradient change. The perturbation is then executed by adding noise to the weight elements, where the change of gradients is not significant. In essence, the proposed method is a client-side defense, which adds adaptive noise to the model during local training based on an approximation of Hessian matrices.

#### Smoothing

Here, smoothing refers to randomized parameter smoothing applied to the final global model during the inference (testing) stage. CRFL (Xie et al., 2021) enhances the randomized smoothing proposed in prior work by creating a new smoothed classifier

 $h_s$ . The prediction results for  $h_s$  are a majority vote over predictions for h, whose parameters are drawn from a probability distribution  $\mu$ . The final prediction result is denoted as  $h_s$  (w;  $x_{\text{test}}$ ) and c is a class that belongs to  $\mathcal{Y}$ . The smoothing process is shown below:

$$h_{s}(w; x_{\text{test}}) = \arg \max_{c \in \mathcal{Y}} H_{s}^{c}(w; x_{\text{test}})$$
  
where  $H_{s}^{c}(w; x_{\text{test}}) = \mathbb{P}_{W \sim \mu(w)}[h(W; x_{\text{test}}) = c]$ 

$$(2.6)$$

The authors use a Gaussian distribution as  $\mu$  in the implementation:  $\mu(w) = N(w, \sigma_T^2 \mathbf{I})$ . This approach has the downside of difficulty achieving a continuous distribution on a deep learning model, consisting of a multitude of parameters. In practice, this defense employs Monte Carlo estimation to acquire an estimate of the final result. That is, the final global model is perturbed by generating Gaussian random noise *M* times to generate *M* different models. Finally, these M different models' voting results are counted, and the final predicted outcome is the voting winner. This method's effect on robustness can be theoretically certified.

The DP-derived defense uses clipping, noise, and smoothing methods to diminish the model's performance while enhancing the FL system's robustness. Hence, this defense type mainly focuses on target and backdoor attacks. The effect of these attacks will be mitigated along with the decrease in global accuracy. Several works in this category aim to strike a better balance between utility and robustness by adaptively clipping updates or adding adaptive noise updates. And there are two works (Naseri et al., 2022) and (Sun et al., 2021) that attempt to perform DP from the client's perspective. According to (Naseri et al., 2022), Local Differential Privacy (LDP) and Central Differential Privacy (CDP) are more effective than merely clipping updates by norm in defending against backdoor attacks. However, local differential privacy can better protect the system when the server is untrustworthy. Additionally, client-side defenses are much harder to exploit by known attackers, so attackers cannot easily devise adaptive attacks. Furthermore, in light of the numerous works that have been published on the proof of the DP algorithm, paper A extends this work by implementing a certified defense based on DP. Although it may not perform as well as other defenses, this work's probability makes it very impressive.

#### 2.3.3. Optimizer-based

Strategies under this category aim to tweak optimizers such as SGD, a common tool during training, to counteract poisoning assaults.

The study by (Karimireddy et al., b) introduces a technique known as worker momentum to address time-coupled attacks, where the impact of an attack can accumulate over multiple rounds, even if the attack directly affects only one round. The worker momentum method involves averaging the stochastic gradients of the workers over past gradients, which reduces the variance of benign workers and makes time-coupled Byzantine perturbations more detectable. This is achieved by using a momentum controller, which adjusts the momentum of each client's update based on the gradient of the loss function and the previous momentum. The method then aggregates the momentum and applies it to update the model. As a result, this method can effectively defend against most poisoning attacks. The same method is also employed in a subsequent work by the same author (Karimireddy et al., a).

In preceding robust federated aggregation methodologies, the variance to norm ratio of non-Byzantine gradients remained under a specified limit. According to (El-Mhamdi et al.), the concept of momentum from centralized SGD is adapted for federated learning, with the difference being its application on the client side rather than on the server. As all gradients from each client already contain momentum, all momentum calculations are conducted on the workers, thereby eliminating additional computational costs. The application of momentum allows model parameters to navigate the loss function with a certain level of inertia, aggregating both the actual gradient and the stochastic noise. Commonly, cumulative errors exhibit a moderate growth rate, with previous errors partly offsetting future ones. However, if consecutive gradients maintain a sufficiently low solid angle, prior real gradients don't counteract future real gradients, which can result in the norm of gradients growing faster than its variance, thereby dampening the impact of the attack.

#### 2.3.4. Ensemble-based

Ensemble-based defenses vary from previous methods by incorporating multiple models or updates for a more resilient system.

(Cao et al.) presented an ensemble federated learning method, at its time, the only certified approach. This strategy includes selecting k clients from a total of n, yielding

 $\binom{n}{k}$  client sets and equivalently, global models. During inference,  $p_i$  represents the

proportion of global models predicting data samples as label *i*. The ultimate prediction is the label with the highest  $p_i$ . This ensemble process enhances the model's robustness against model poisoning attacks through voting. A certified security level is also defined to measure the quantifiable level of a data example. If the malicious client count doesn't exceed this level, the global model can reliably predict the same label as the clean model. This is achieved by basing predictions on majority votes from the global models. Considering most models learn from regular clients due to *k* clients' subsampling, the accuracy of majority voting can be certified.

However, established robust aggregation techniques, such as anomaly detection, might fail in non-I.I.D settings due to the heterogeneous nature of model updates. (Karimireddy et al., a) counters this challenge with an ensemble-based aggregation method, known as bucketing. It groups clients' updates in each round into buckets, averages the updates within each bucket, and applies robust aggregation methods, such as Krum, on these averaged updates. This reduces variance in the updates and makes the ensemble update more homogeneous, thus improving the defense performance by increasing the success rate of robust aggregation methods, especially in non-I.I.D settings.

The first ensemble-based defense (Cao et al.) relies on dividing clients into groups and using majority voting to confirm the aggregated model's accuracy, but this method proves time-intensive. The experiment saw the training of 435 global models from various client subsets, resulting in a significant accuracy drop in non-I.I.D. settings. However, the certified accuracy remains comparable in non-I.I.D and I.I.D environments when just 1% of clients are malicious. But in an I.I.D environment, this rises to 6.1%. The second defense strategy (Karimireddy et al., a) pairs well with other aggregation techniques such as Krum, Multi-Krum, and Bulyan, making it potentially more practical and beneficial.

# LeadFL Analysis

This chapter delves deeply into the convergence and robustness analysis of LeadFL. The theorems related to convergence and robustness that were outlined in the research paper will be thoroughly proven in the following sections.

#### **3.1. Assumptions and Definitions**

This section enumerates the assumptions adopted from prior works (Li et al., 2019; Sun et al., 2021; Panda et al., 2022).

Assumption 3.1.1 (Smoothness).  $\mathcal{L}$  is  $\ell$ -smooth if  $\forall x, y \in \mathbb{R}^d$ 

$$\mathcal{L}(\boldsymbol{x}) - \mathcal{L}(\boldsymbol{y}) + (\boldsymbol{x} - \boldsymbol{y})^T \mathcal{L}(\boldsymbol{x}) \le \frac{\ell}{2} \|\boldsymbol{x} - \boldsymbol{y}\|_2^2$$

**Assumption 3.1.2** (Convex).  $\mathcal{L}$  is  $\mu$ -strongly convex if  $\forall x, y \in \mathbb{R}^d$ ,

$$\mathcal{L}(\mathbf{x}) - \mathcal{L}(\mathbf{y}) + (\mathbf{x} - \mathbf{y})^T \nabla \mathcal{L}(\mathbf{y}) \ge \frac{\mu}{2} \|\mathbf{x} - \mathbf{y}\|_2^2$$

**Assumption 3.1.3** (Bound of Variance). Let  $\xi_t^k$  be sampled from the *k*-th device's local data uniformly at random. The variance of stochastic gradients in each device is bounded:

$$\mathbb{E}\left\|\nabla\mathcal{L}^{k}\left(\boldsymbol{\theta}_{t}^{k},\boldsymbol{\xi}_{t}^{k}\right)-\nabla\mathcal{L}^{k}\left(\boldsymbol{\theta}_{t}^{k}\right)\right\|^{2}\leq\sigma_{k}^{2}\text{ for }k=1,\cdots,N.$$

Assumption 3.1.4 (Bound of Norm). The expected squared norm of stochastic gradients is uniformly bounded, i.e.,  $\mathbb{E} \left\| \nabla \mathcal{L}^k \left( \boldsymbol{\theta}_{t,i}^k, \xi_{t,i}^k \right) \right\|^2 \le G^2$  for all k = 1, ..., N, i = 0, ..., I-1 and t = 0, ..., T-1

In our FL system, *K* clients are randomly selected among *N* clients each round. Then we adapt the following assumption from (Li et al., 2019).

**Assumption 3.1.5** (Selection of Clients). Assume  $S_t$  contains a subset of K indices uniformly sampled from [N] without replacement. Assume the data is balanced in the sense that  $p_1 = \cdots = p_N = \frac{1}{N}$ . The aggregation step of FedAvg performs  $\theta_t \leftarrow \frac{N}{K} \sum_{k \in S_t} p_k \theta_t^k$ .

**Definition 3.1.6** (Loss of clients). Denote  $\mathcal{L}^*$  and  $\mathcal{L}^*_k$  as the minimum value of  $\mathcal{L}$  and  $\mathcal{L}_k$ , where  $\mathcal{L}$  is the loss of a model trained on the combination of datasets from all the clients and  $\mathcal{L}_k$  is the loss of a model trained on the dataset of client k. We can set  $\Gamma = \mathcal{L}^* - \sum_{k=1}^N p_k \mathcal{L}^*_k$  which can quantify the degree of noniid. If the data are iid, then  $\Gamma$  goes to zero as the number of samples grows. If the data are noniid, then  $\Gamma$  is nonzero, and its magnitude reflects the heterogeneity of the data distribution.

**Definition 3.1.7** (Poisoning Attack). For a protocol  $f = (\mathcal{G}, \mathcal{A}, \eta)$  we define the set of poisoned protocols  $F(\rho)$  to be all protocols  $f^* = (\mathcal{G}^*, \mathcal{A}, \eta)$  that are exactly the same as f except that the gradient oracle  $\mathcal{G}^*$  is a  $\rho$ -corrupted version of  $\mathcal{G}$ . That is, for any round t and any model  $\theta_t$  and any dataset D we have we have  $\mathcal{G}^*(\theta_t, D) = \mathcal{G}(\theta_t, D) + \epsilon$  for some  $\epsilon$  with  $\|\epsilon\|_1 \leq \rho$ 

**Definition 3.1.8** (Certified Radius). Let *f* be a protocol and  $f^* \in F(\rho)$  be a poisoned version of the same protocol. Let  $\theta_T$ ,  $\theta_T^*$  be the benign and poisoned final outputs of the above protocols. We call *R* a certified radius for *f* if  $\forall f^* \in F(\rho)$ ;  $R(\rho) \ge |\theta_T - \theta_T^*|_1$ 

**Assumption 3.1.9** (Coordinate-wise Lipschitz). The protocol  $f(\mathcal{G}, \mathcal{A}, \eta)$  is c-coordinatewise Lipschitz if for any round  $t \in [T]$ , models  $\theta_t, \theta_t^* \in \mathcal{M}$ , and a dataset D we have that the outputs of the gradient oracle on any coordinate cannot drift too much farther apart. Specifically, for any coordinate index  $i \in [d]$ 

$$\left|\mathcal{G}\left(\theta_{t}^{*}, D\right)[i] - \mathcal{G}\left(\theta_{t}, D\right)[i]\right| \leq c \cdot \left|\theta_{t}^{*} - \theta_{t}\right|_{1}$$

#### **3.2. Convergence Analysis**

#### Proof of 3.2.1 (Theorem 4.1 in the research paper)

**Theorem 3.2.1** (Convergence Guarantee). Let Assumptions 3.1.1 to 3.1.5 hold and  $l, \mu, \sigma_k, G, K, N, \Gamma, \mathcal{L}^*$  be as defined therein and in Definition 3.1.6. Choose  $\kappa = \frac{l}{\mu}$ ,  $\gamma = \max\{8\kappa, E\}$  and the learning rate  $\eta_t = \frac{2}{\mu(\gamma+t)}$ . Then we have the following bound for LeadFL:

$$\mathbb{E}\left[\mathcal{L}\left(\boldsymbol{\theta}_{T}\right)\right] - \mathcal{L}^{*} \leq \frac{\kappa}{\gamma + T - 1} \left(\frac{2(B + C)}{\mu} + \frac{\mu\gamma}{2} \mathbb{E}\left\|\boldsymbol{\theta}_{0} - \boldsymbol{\theta}^{*}\right\|^{2}\right)$$

where

$$C = \frac{N - K}{N - 1} \frac{4}{K} E^2 (d^2 q^2 + G^2)$$
$$B = \sum_{k=1}^{N} p_k^2 (d^2 q^2 + \sigma_k^2) + 6l\Gamma + 8(E - 1)^2 (d^2 q^2 + G^2)$$

*Proof*: The expected distance between the gradients before and after regularization can be bounded.

$$\mathbb{E} \left\| \nabla \mathcal{L}'_{k} \left( \boldsymbol{\theta}_{t,i}^{k}, \boldsymbol{\xi}_{t,i}^{k} \right) - \nabla \mathcal{L}_{k} \left( \boldsymbol{\theta}_{t,i}^{k}, \boldsymbol{\xi}_{t,i}^{k} \right) \right\|_{2}^{2}$$
  
=
$$\mathbb{E} \left\| \operatorname{clip} \left( \nabla \left( \mathbf{I} - \eta_{t} \hat{\mathbf{H}}_{t,i}^{k} \right), q \right) \right\|_{2}^{2}$$
  
$$\leq \mathbb{E} \| q \|_{2}^{2} = d^{2}q^{2}$$
  
(3.1)

Using the bounds above and Assumption 3.1.3, we can derive new bounds for the variance of modified gradients  $\mathbb{E} \left\| \nabla \mathcal{L}'_k \left( \boldsymbol{\theta}^k_{t,i}, \boldsymbol{\xi}^k_{t,i} \right) - \nabla \mathcal{L}_k \left( \boldsymbol{\theta}^k_{t,i} \right) \right\|^2$ 

$$\begin{split} & \mathbb{E} \left\| \nabla \mathcal{L}_{k}^{\prime} \left( \boldsymbol{\theta}_{t,i}^{k}, \boldsymbol{\xi}_{t,i}^{k} \right) - \nabla \mathcal{L}_{k} \left( \boldsymbol{\theta}_{t,i}^{k} \right) \right\|^{2} \\ & \leq \mathbb{E} \left\| \nabla \mathcal{L}_{k}^{\prime} \left( \boldsymbol{\theta}_{t,i}^{k}, \boldsymbol{\xi}_{t,i}^{k} \right) - \nabla \mathcal{L}_{k} \left( \boldsymbol{\theta}_{t,i}^{k}, \boldsymbol{\xi}_{t,i}^{k} \right) \right\|^{2} \\ & + \mathbb{E} \left\| \nabla \mathcal{L}_{k} \left( \boldsymbol{\theta}_{t,i}^{k}, \boldsymbol{\xi}_{t,i}^{k} \right) - \nabla \mathcal{L}_{k} \left( \boldsymbol{\theta}_{t,i}^{k} \right) \right\|^{2} \\ & \leq d^{2}q^{2} + \sigma_{k}^{2}, \end{split}$$

where we use the triangle inequality.

Similarly, we can also derive bounds the expected squared norm of modified gradients using Assumption 3.1.4.

$$\begin{split} & \mathbb{E} \left\| \nabla \mathcal{L}'_{k} \left( \boldsymbol{\theta}_{t,i}^{k}, \boldsymbol{\xi}_{t,i}^{k} \right) \right\|^{2} \\ \leq & \mathbb{E} \left\| \nabla \mathcal{L}'_{k} \left( \boldsymbol{\theta}_{t,i}^{k}, \boldsymbol{\xi}_{t,i}^{k} \right) - \nabla \mathcal{L}_{k} \left( \boldsymbol{\theta}_{t,i}^{k}, \boldsymbol{\xi}_{t,i}^{k} \right) \right\|^{2} \\ & + \mathbb{E} \left\| \nabla \mathcal{L}_{k} \left( \boldsymbol{\theta}_{t,i}^{k}, \boldsymbol{\xi}_{t,i}^{k} \right) \right\|^{2} \\ \leq & d^{2}q^{2} + G^{2}, \end{split}$$

Applying the bounds for the variance and the expected squared norm of modified gradients after applying LeadFL, we can derive our convergence guarantee from Theorem 3 in (Li et al., 2019) by replacing these bounds.

#### **3.3. Robustness Analysis**

#### Proof of Theorem 3.3.1 (Theorem 4.3 in the research paper)

**Theorem 3.3.1** (Certified Radius in Scenario I). Let Assumptions 3.1.9 hold and  $T_A$ , *c* be as defined therein. We assume that LeadFL with FedAv aggregation is used. The certified radius satisfies:

$$R(\rho) = \left(\frac{N}{K}\right)^{T-T_A} \left| \prod_{t=T_A}^{T} \left| \sum_{k \in S_t} p^k \prod_{i=0}^{l-1} \left( I - \eta_t H_{t,i}^k \right) \right| \right|$$
$$\cdot \sum_{t=0}^{T_A - 1} \eta_t (1 + dc)^{\sum_{t=0}^{T_A - 1} \eta_t} \rho$$

*Proof.* Equation (1) in the research paper can be rewritten as follows:

$$\boldsymbol{\theta}_{t} - \boldsymbol{\theta}_{t}^{*} = \frac{N}{K} \left[ \sum_{k \in S_{t}} p^{k} \prod_{i=0}^{I-1} \left( I - \eta_{t} H_{t,i}^{k} \right) \right] \left( \boldsymbol{\theta}_{t-1} - \boldsymbol{\theta}_{t-1}^{*} \right)$$
(3.2)

This equation can be used iteratively to get:

$$\boldsymbol{\theta}_{T} - \boldsymbol{\theta}_{T}^{*} = \left(\frac{N}{K}\right)^{T-T_{A}} \prod_{t=T_{A}}^{T} \left[\sum_{k \in S_{t}} p^{k} \prod_{i=0}^{I-1} \left(I - \eta_{t} H_{t,i}^{k}\right)\right]$$

$$\left(\boldsymbol{\theta}_{T_{A}} - \boldsymbol{\theta}_{T_{A}}^{*}\right)$$
(3.3)

Apply the Theorem 4.2 in the research paper, we can get:

$$\left|\boldsymbol{\theta}_{T_{A}} - \boldsymbol{\theta}_{T_{A}}^{*}\right| \leq \cdot \sum_{t=0}^{T_{A}-1} \eta_{t} (1+dc)^{\sum_{t=0}^{T_{A}-1} \eta_{t}} \rho$$
(3.4)

Combine Equations 3.3 and 3.4, the certified radius can be derived:

$$\left|\boldsymbol{\theta}_{T}-\boldsymbol{\theta}_{T}^{*}\right| \leq \left(\frac{N}{K}\right)^{T-T_{A}} \left|\prod_{t=T_{A}}^{T} \left[\sum_{k\in S_{t}} p^{k} \prod_{i=0}^{I-1} \left(I-\eta_{t} H_{t,i}^{k}\right)\right]\right|$$

$$\cdot \sum_{t=0}^{T_{A}-1} \eta_{t} (1+dc)^{\sum_{t=0}^{T_{A}-1}} \rho$$
(3.5)

#### **Proof of Theorem 3.3.2(Theorem 4.4 in the research paper)**

**Theorem 3.3.2** (Certified Radius in Scenario II). *Let Assumption 3.1.9 hold. The certified radius of the threat model is* 

$$R(\rho) = (1+dc)^{\sum_{t\in\Phi_T}\eta_t} \rho \cdot \left( \left| \prod_{t\in\Gamma_T} \left[ \frac{N}{|S_t^*|} \sum_{k\in S_t^*} p^k \prod_{i=0}^{I-1} \left( I - \eta_t H_{t,i}^k \right) \right] \right| + |\Phi_T| \sum_{t\in\Phi_T} \eta_t \right)$$

where  $\Phi_T$  is the set of communication rounds that server-side defenses cannot filter out all malicious updates.  $\Gamma_T$  is the set of communication rounds that server-side defenses filter out all malicious updates.  $S_t^*$  is a set of clients whose updates are not filtered out by the server-side defense in round t.  $|\Phi_T|$  and  $|S_t^*|$  are the cardinality of the set  $\Phi_T$  and  $S_t^*$ , where  $\mathbb{E}[|\Phi_T|] = \sum_{t=0}^{T-1} g_{atk}(K_m^t)$ .

*Proof.* Denote  $f^* = (\mathcal{G}^*, \mathcal{A}, \eta) \in f(\rho)$  as an arbitrary  $\rho$ -poisoned version of f in Definition 3.1.7. And let  $u_1, \ldots, u_T$  and  $u_1^*, \ldots, u_T^*$  be the model updates that the benign oracle  $\mathcal{G}$  would produce on models  $\theta_0, \ldots, \theta_{T-1}$  and  $\theta_0^*, \ldots, \theta_{T-1}^*$ , respectively. We also define  $\hat{u}_1, \ldots, \hat{u}_T$  to be the output of the adversarial gradient oracle  $\mathcal{G}^*$  on models  $\theta_0, \ldots, \theta_{T-1}$ . By the definition of  $\rho$ -poisoning in Definition 3.1.7, we have  $|\hat{u}_t - u_t^*|_1 \leq \rho$ .

Based on the definition of model updates, we use the triangle inequality to get the following inequality between  $|\theta_t - \theta_t^*|$  and  $|\theta_{t-1} - \theta_{t-1}^*|$  when the system is attacked in round t - 1

$$\left|\boldsymbol{\theta}_{t}-\boldsymbol{\theta}_{t}^{*}\right|=\left|\boldsymbol{\theta}_{t-1}-\eta_{t}\boldsymbol{u}_{t}-\boldsymbol{\theta}_{t-1}^{*}+\eta_{t}\hat{\boldsymbol{u}}_{t}\right|\leq\left|\boldsymbol{\theta}_{t-1}-\boldsymbol{\theta}_{t-1}^{*}\right|+\eta_{t}\left|\boldsymbol{u}_{t}-\hat{\boldsymbol{u}}_{t}\right|$$
(3.6)

Using the triangle inequality again, we can get

$$|u_t - \hat{u}_t| = |u_t - u_t^* + u_t^* - \hat{u}_t| \le |u_t - u_t^*| + |u_t^* - \hat{u}_t|$$
(3.7)

According to Definition 3.1.7 and coordinate-wise Lipshitz in Assumption 3.1.9:

$$|u_t - \hat{u}_t| \le |u_t - u_t^*| + |u_t^* - \hat{u}_t| = dc |\theta_{t-1} - \theta_{t-1}^*| + \rho$$
(3.8)

By plugging the above equation into Equation 3.6, we get

$$\left|\boldsymbol{\theta}_{t}-\boldsymbol{\theta}_{t}^{*}\right| \leq \left|\boldsymbol{\theta}_{t-1}-\boldsymbol{\theta}_{t-1}^{*}\right| + \eta_{t} \left(dc\left|\boldsymbol{\theta}_{t-1}-\boldsymbol{\theta}_{t-1}^{*}\right| + \rho\right) = \left(1+dc\eta_{t}\right)\left|\boldsymbol{\theta}_{t-1}-\boldsymbol{\theta}_{t-1}^{*}\right| + \rho\eta_{t} \quad (3.9)$$

According to Bernoulli's inequality, we have

$$\left|\boldsymbol{\theta}_{t} - \boldsymbol{\theta}_{t}^{*}\right| \leq (1 + dc)^{\eta_{t}} \left|\boldsymbol{\theta}_{t-1} - \boldsymbol{\theta}_{t-1}^{*}\right| + \rho \eta_{t}$$
(3.10)

Now we get the inequality between  $|\theta_t - \theta_t^*|$  and  $|\theta_{t-1} - \theta_{t-1}^*|$  when the system is attacked in round t - 1.

Since we introduced server-side defense, we rewrite Equation 3.2

$$\boldsymbol{\theta}_{t} - \boldsymbol{\theta}_{t}^{*} = \frac{N}{\left|\boldsymbol{S}_{t}^{*}\right|} \left[ \sum_{k \in \boldsymbol{S}_{t}^{*}} \boldsymbol{p}^{k} \prod_{i=0}^{I-1} \left( \boldsymbol{I} - \eta_{t} \boldsymbol{H}_{t,i}^{k} \right) \right] \left(\boldsymbol{\theta}_{t-1} - \boldsymbol{\theta}_{t-1}^{*}\right)$$
(3.11)

Then we get the following relationship between  $|\theta_t - \theta_t^*|$  and  $|\theta_{t-1} - \theta_{t-1}^*|$  when server-side defense filters out all malicious updates in round t - 1.

$$\left|\boldsymbol{\theta}_{t}-\boldsymbol{\theta}_{t}^{*}\right| \leq \frac{N}{\left|\boldsymbol{S}_{t}^{*}\right|} \left|\sum_{\boldsymbol{k}\in\boldsymbol{S}_{t}^{*}} \boldsymbol{p}^{\boldsymbol{k}} \prod_{i=0}^{I-1} \left(\boldsymbol{I}-\eta_{t}\boldsymbol{H}_{t,i}^{\boldsymbol{k}}\right)\right| \left|\boldsymbol{\theta}_{t-1}-\boldsymbol{\theta}_{t-1}^{*}\right|$$
(3.12)

Finally, we can use Equation 3.10 and 3.12 to prove the Theorem by induction hypothesis

$$R(\rho) = \left|\boldsymbol{\theta}_T - \boldsymbol{\theta}_T^*\right| = (1 + dc)^{\sum_{\boldsymbol{\theta} \in \Phi_T} \eta_t} \rho\left(\left|\prod_{t \in \Gamma_T} \left[\frac{N}{|S_t^*|} \sum_{k \in S_t^*} p^k \prod_{i=0}^{I-1} \left(I - \eta_t H_{t,i}^k\right)\right]\right| + \left|\Phi_T\right| \sum_{t \in \Phi_T} \eta_t\right)$$
(3.13)

# 4

## Additional Experiments

In this chapter, we present supplementary experiments that are not included in the main part of our research paper. For all datasets, we choose the learning rate  $\eta = 0.01$  and batch size Ba = 32 for all clients. The model architectures for two datasets are shown in Table 4.1.

#### 4.1. Comparison Between FL-WBC and LDP



Figure 4.1: Comparison between FL-WBC and LDP on different datasets. The black circles represent the communication round that malicious clients conduct the attack.

The only difference between FL-WBC and LDP (Local Differential Privacy) is that FL-WBC adds noise to only the smaller elements in Hessian Matrix by estimating the matrix, whereas LDP includes noise for all elements. Therefore, LDP can also be used to perturb the null space of the Hessian Matrix. We, therefore, believe that a detailed comparison between the two is necessary.

The experiment compares the FL-WBC given std of Laplace noise s = 0.4 with LDP s = 0.4 on both FashionMNIST and CIFAR10 datasets with IID settings under single-image targeted attack. The threat model is the same as the paper (Sun et al.,

FashionMNIST	CIFAR10
5×5 Conv2d 1-16	3×3 Conv2d 3-32
5×5 Conv2d 16-32	3×3 Conv2d 32-32
FC-10	2×2 MaxPool
	3×3 Conv2d 32-64
	3×3 Conv2d 64-64
	2×2 MaxPool
	3×3 Conv2d 64-128
	3×3 Conv2d 128-128
	2×2 MaxPool
	FC-128
	FC-10

Table 4.1: Model Architectures for FashionMNIST and CIFAR10 dataset

2021). The results in Figure 4.1 show a slight difference between FL-WBC and LDP in all settings. The FashionMNIST-IID dataset shows almost no difference between the two defenses approach. Both FL-WBC and LDP successfully defend the attack and maintain almost the same benign accuracy in the first 100 communication rounds. With the CIFAR10-IID setup, the FL-WBC and LDP successfully defend the attack for the first 80 communication rounds. However, both defenses lead to a loss of model accuracy. The benign accuracy of FL-WBC and LDP have the same distribution, and both results are below 50%. In other words, there is no significant difference between the results of FL-WBC and LDP in this experiment.

#### 4.2. Additional Results

#### 4.2.1. Results of different client selections

In the research paper, only one client selection result is shown. In order to ensure that our results are not an artifact of this one specific client selection, we present results for another selection result shown in Figure 4.2. We can observe that, the backdoor accuracy is still very high at the round when the extreme attack is conducted. Our defense still exhibits a faster decrease in backdoor accuracy than the other defenses.

Distribution		IID								
Server-side Defense		Multi-Krum				Bulyan				
Client-side Defense	None	LDP	FL-WBC	Ours	None	LDP	FL-WBC	Ours		
MA	86.7	84.1	83.7	84.4	85.2	83.9	84.0	84.2		
BA Avg	77.2	73.5	67.9	52.8	82.8	76.1	72.9	45.7		
BA Final	71.0	65.1	62.3	35.7	79.9	53.2	46.9	26.9		

**Table 4.2:** Comparison of defenses under 9-pixel pattern backdoor attack on IID FashionMNIST dataset.The number of clients is 1000



**Figure 4.2:** Results of different random client selections. The backdoor attack here is the 9-pixel-pattern backdoor attack. The server-side defense here is Multi-Krum. Black hollow circles indicate that the system is attacked very strongly in that round.



**Figure 4.3:** Comparison of backdoor accuracy on FashionMNIST and CIFAR10 with both IID and non-IID settings. The backdoor attack here is single pixel backdoor attack. The server-side defense here is Multi-Krum. Black hollow circles indicate that the system is attacked very strongly in that round.

#### 4.2.2. Results of the larger scale setting

In the research paper, the number of clients in our system is 100. In this subsection, we present the results of experiments with an increased number of clients, totaling 1000, while maintaining 25% of malicious clients. The dataset is evenly distributed among all clients, and each round involves the selection of 100 clients, with other settings remaining consistent with Table 1 in the research paper. As shown in Table 4.2, our method still achieves the highest MA and lowest BA compared to other client-side defenses in the larger-scale experiments.



**Figure 4.4:** Comparison of backdoor accuracy on FashionMNIST and CIFAR10 with both IID and non-IID settings. The attack here is single image targeted attack. The server-side defense here is Multi-Krum. Black hollow circles indicate that the system is attacked very strongly in that round.

#### 4.2.3. Results of CMA and CTMA under 9-pixel backdoor attack

In the research paper, we only show the results of MultiKurm and Bulyan server-side defenses. Table 4.3 and 4.4 contain the results of CMA and CTMA.

#### 4.2.4. Results of single-pixel backdoor attack

Table 4.5, 4.6 and Figure 4.3 show the performance of defenses under single-pixel backdoor attacks.

#### 4.2.5. Results of single image targeted attack

We also measure **Malicious Confidence (MC)**: In (Bhagoji et al., 2019), the authors present a single-image attack where a malicious client inserts exactly one image with the wrong label in their dataset. The malicious confidence is the confidence of the global model in their classification of the malicious image. We consider both average and final confidence. Note that this metric is only relevant for single-image attacks. Table 4.7, 4.8 and Figure 4.4 show the performance of defenses under single image targeted attacks.

Distribution	Server-side	Client-side	МЛА	BA	BA
Distribution	Defense	Defense		Avg	Final
		None	89.8	98.4	100.0
	Nono	LDP	88.7	90.7	98.8
	INONE	FL-WBC	88.1	90.2	99.5
		Ours	89.0	95.0	95.7
		None	90.0	93.8	100.0
	CMA	LDP	87.1	95.8	99.3
	CIVIA	FL-WBC	87.2	96.7	99.2
		Ours	87.6	96.7	99.6
		None	89.7	96.6	100.0
		LDP	88.4	97.8	99.9
		FL-WBC	90.0	98.9	99.6
		Ours	87.5	91.9	96.8
		None	89.3	82.6	93.2
	Multi-Krum	LDP	87.0	76.0	79.6
		FL-WBC	87.2	77.5	80.6
		Ours	87.9	32.9	0.0
		None	89.2	78.8	90.6
	Bulyon	LDP	86.0	74.1	62.2
	Dulyan	FL-WBC	86.0	70.6	86.5
		Ours	86.3	21.6	0.3
		None	87.6	99.7	100.0
	None	LDP	82.2	94.9	99.4
		FL-WBC	84.0	94.7	96.6
		Ours	84.9	97.5	97.1
		None	85.6	98.9	100.0
	CMA	LDP	78.5	97.6	99.9
		FL-WBC	78.5	97.6	99.9
		Ours	80.2	98.2	92.5
		None	85.3	99.2	100.0
Non-IID		LDP	81.9	99.6	99.9
		FL-WBC	82.4	99.3	99.9
		Ours	80.8	95.1	67.3
		None	85.6	88.7	93.3
	Multikrum	LDP	76.7	80.4	86.7
		FL-WBC	77.2	74.0	70.3
		Ours	79.1	39.5	1.2
		None	77.4	92.5	88.6
	Bulyan	LDP	73.4	71.9	94.7
	Duryan	FL-WBC	71.7	73.7	69.0
		Ours	74.0	32.3	2.0

 Table 4.3: Comparison of defenses under 9-pixel pattern backdoor attack on FashionMNIST dataset with both IID and non-IID settings.

Distribution	Server-side	Client-side	МА	BA	BA
Distribution	Defense	Defense		Avg	Final
		None	71.3	93.5	99.6
	Nono	LDP	55.1	77.0	79.7
	INOTIC	FL-WBC	56.2	77.1	89.5
		Ours	60.4	70.2	58.7
		None	74.1	93.3	99.2
	CMA	LDP	12.8	67.6	70.6
	CIVIA	FL-WBC	12.6	62.3	69.9
		Ours	64.6	82.3	79.0
		None	75.8	95.2	99.7
		LDP	56.6	95.1	96.4
	CIMA	FL-WBC	56.4	94.8	97.1
		Ours	61.3	79.4	49.4
		None	76.3	77.5	80.5
	Multi Krum	LDP	48.0	53.1	43.8
		FL-WBC	43.3	56.9	40.5
		Ours	35.6	25.6	
		None	76.2	79.1	87.0
	Bulyon	LDP	41.5	46.7	23.4
	Dulyan	FL-WBC	42.2	51.3	35.5
		Ours	54.8	43.9	21.4
		None	73.7	97.0	100.0
	None	LDP	50.8	86.3	92.5
	INOTIC	FL-WBC	52.6	83.2	89.6
		Ours	60.5	76.3	67.7
		None	69.6	97.3	99.9
	CMA	LDP	13.5	58.4	69.2
		FL-WBC	13.2	63.9	69.7
		Ours	60.3	87.5	90.1
		None	73.0	98.3	100.0
Non-IID		LDP	54.2	97.5	99.1
		FL-WBC	51.1	97.2	99.8
		Ours	56.9	90.1	84.5
		None	70.7	85.8	96.2
	Multi-Krum	LDP	43.2	55.4	52.4
		FL-WBC	42.9	54.4	35.4
		Ours	55.3	45.2	34.4
		None	61.7	87.5	95.2
	Bulwan	LDP	36.7	48.8	29.8
	Buryan	FL-WBC	36.2	48.1	47.7
		Ours	51.4	46.8	27.3

**Table 4.4:** Comparison of defenses under 9-pixel pattern backdoor attack on CIFAR10 dataset with bothIID and non-IID settings.

Distribution	Server-side	Client-side	МА	BA	BA
Distribution	Defense	Defense	WIA	Avg	Final
		None	90.1	96.2	99.7
	Nono	LDP	88.0	88.0	95.1
	INOTIC	FL-WBC	87.8	85.2	97.7
		Ours	88.1	91.2	95.7
		None	89.9	89.1	99.8
	CMA	LDP	87.0	96.0	97.9
	CMA	FL-WBC	87.0	96.7	99.3
		Ours	87.7	91.2	98.0
		None	89.8	92.0	99.7
		LDP	88.2	96.3	99.6
	CIMA	FL-WBC	87.6	96.0	99.2
		Ours	88.2	84.4	92.1
		None	89.4	28.1	39.7
	Multi Vaum	LDP	87.0	70.7	90.7
	wiuiu-Kruin	FL-WBC	71.4	76.8	
		Ours	87.5	70.3	43.4
		None	89.1	72.2	89.7
	Destroom	LDP	85.9	68.8	79.4
	bulyan	FL-WBC	85.3	72.4	78.3
		Ours	86.7	67.6	85.8
		None	88.0	98.8	99.9
	None	LDP	83.4	90.9	97.8
		FL-WBC	82.2	91.0	98.1
		Ours	82.8	93.9	97.0
		None	85.7	95.4	99.9
	CMA	LDP	76.4	96.3	99.2
	CMA	FL-WBC	79.3	96.6	99.6
		Ours	79.5	93.4	93.7
		None	85.5	97.4	99.9
Non IID		LDP	81.0	98.2	99.7
	CIMA	FL-WBC	81.7	98.6	99.9
		Ours	81.5	94.5	92.8
		None	86.5	79.2	85.1
	Multi Vaure	LDP	80.5	72.7	64.5
	wiuiu-Kruin	FL-WBC	78.2	73.8	82.6
		Ours	81.7	54.4	87.9
		None	85.2	83.9	85.9
	Bulyon	LDP	73.7	66.9	85.2
	Buiyan	FL-WBC	70.1	72.5	71.4
		Ours	75.0	62.6	46.2

 Table 4.5: Comparison of defenses under single pixel backdoor attack on FashionMNIST dataset with both IID and non-IID settings.

Distribution	Server-side	Client-side	МА	BA	BA
Distribution	Defense	Defense	IVIA	Avg	Final
		None	71.3	93.5	99.6
	Nama	LDP	55.1	77.0	79.7
	INONE	FL-WBC	56.2	77.1	89.5
		Ours	62.1	70.2	58.7
		None	74.1	93.3	99.2
	CMA	LDP	12.8	67.6	70.6
	CMA	FL-WBC	12.6	62.3	69.9
		Ours	64.6	82.3	79.0
		None	75.8	95.2	99.7
		LDP	56.6	95.1	96.4
	CIMA	FL-WBC	56.4	94.8	97.1
		Ours	61.3	79.4	49.4
		None	76.3	77.5	80.5
	Multi Vrum	LDP	48.0	53.1	43.8
	wiuiu-Kruiii	FL-WBC	43.3	40.5	
		Ours	56.9	35.6	25.6
		None	76.2	79.1	87.0
	Bulyon	LDP	41.5	46.7	23.4
	Dulyan	FL-WBC	42.2	51.3	35.5
		Ours	55.8	43.9	26.4
		None	74.5	92.3	99.8
	None	LDP	52.7	73.0	81.2
		FL-WBC	51.4	70.2	85.0
		Ours	62.3	62.8	55.5
		None	67.8	89.5	98.4
	CMA	LDP	13.1	67.5	69.5
	CMA	FL-WBC	13.7	66.3	68.5
		Ours	59.2	82.7	74.1
		None	73.6	92.6	99.8
Non IID		LDP	54.8	93.6	94.2
	CIMA	FL-WBC	48.7	94.0	95.2
		Ours	58.1	77.7	78.7
		None	71.0	68.7	78.5
	Multi Vuun	LDP	40.7	34.8	19.3
		FL-WBC	39.6	35.9	20.0
		Ours	51.5	26.6	31.9
		None	62.7	69.8	78.3
	Bultran	LDP	33.6	25.4	16.6
	Buiyan	FL-WBC	37.5	22.1	15.5
		Ours	49.1	17.6	10.3

**Table 4.6:** Comparison of defenses under single pixel backdoor attack on CIFAR10 dataset with bothIID and non-IID settings.

Distribution	Server-side	Client-side	МА	MC	MC		
Distribution	Defense	Defense		Avg	Final		
		None	89.9	94.6	98.9		
	None	LDP	87.4	79.7	95.8		
	INONE	FL-WBC	88.3	82.7	75.5		
		Ours	88.2	82.0	95.8		
		None	89.3	98.1	99.9		
	CMA	LDP	86.1	88.6	77.9		
	CMA	FL-WBC	86.7	86.1	83.6		
		Ours	87.6	93.6	91.0		
		None	89.0	94.2	99.4		
		LDP	87.3	95.2	93.9		
	CINA	FL-WBC	87.4	95.2	98.8		
		Ours	87.5	89.4	99.9		
		None	89.1	89.5	99.5		
	Multi Vrum	LDP	86.5	57.9	88.7		
		FL-WBC	86.4	86.4 58.1 86.8 44.9			
		Ours	86.8	44.9	7.4		
		None	89.1	92.0	100.0		
	Bulton	LDP	85.0	34.0	0.1		
	Duiyan	FL-WBC	85.5	86.2	84.1		
		Ours	86.7	33.9	14.0		
		None	87.7	97.2	99.6		
	None	LDP	83.3	86.1	99.2		
		FL-WBC	83.7	92.9	95.7		
		Ours	83.9	78.9	96.7		
		None	87.1	94.6	99.9		
	CMA	LDP	81.2	87.5	92.6		
	CIVIA	FL-WBC	79.4	81.8	94.5		
		Ours	82.2	81.5	83.1		
		None	86.7	95.4	99.8		
Non-IID		LDP	84.8	93.3	87.1		
		FL-WBC	84.8	92.9	93.7		
		Ours	84.4	78.4	84.0		
		None	86.3	87.6	99.6		
	Multi Vrum	LDP	81.6	55.2	92.1		
		FL-WBC	80.6	59.4	77.0		
		Ours	84.5	46.1	5.7		
		None	76.8	86.8	99.9		
	Bultzon	LDP	71.8	54.2	77.1		
	Buiyan	FL-WBC	73.9	83.5	99.8		
		Ours	77.2	39.8	0.0		

 Table 4.7: Comparison of defenses under single image targeted attack on FashionMNIST dataset with both IID and non-IID settings.

Distribution	Server-side	Client-side	МЛА	MC	MC
Distribution	Defense	Defense	IVIA	Avg	Final
		None	70.5	98.9	99.9
	None	LDP	51.1	97.2	97.2
	INONE	FL-WBC	50.8	95.5	100.0
		Ours	60.4	89.1	100.0
		None	71.0	99.8	99.9
	CMA	LDP	13.9	96.2	100.0
	CMA	FL-WBC	13.9	99.6	100.0
		Ours	60.6	97.0	100.0
		None	68.4	99.8	99.9
		LDP	47.0	98.9	100.0
	CIMA	FL-WBC	47.4	98.4	100.0
		Ours	60.8	96.1	99.9
		None	75.4	94.7	100.0
	Multi Vaum	LDP	46.3	86.6	99.9
	wiuiu-Kruin	FL-WBC	46.7	82.5	99.9
		Ours	56.6	47.7	2.4
		None	73.6	84.1	93.8
	Dultron	LDP	41.6	49.0	0.0
	Dulyan	FL-WBC	41.9	6.5	0.0
		Ours	53.6	1.7	0.0
		None	72.6	99.0	100.0
	None	LDP	48.6	83.3	100.0
		FL-WBC	50.6	86.0	97.7
		Ours	60.3	90.2	100.0
		None	65.9	99.6	99.6
	CMA	LDP	13.0	98.0	100.0
	CMA	FL-WBC	15.1	96.0	100.0
		Ours	61.5	98.4	100.0
		None	70.5	98.9	99.9
Non IID		LDP	47.3	97.3	99.1
	CIMA	FL-WBC	47.8	96.9	100.0
		Ours	55.0	95.5	100.0
		None	70.5	92.4	100.0
	Multi Vrum	LDP	43.8	60.6	100.0
		FL-WBC	43.3	65.0	100.0
		Ours	51.7	55.3	0.0
		None	68.1	84.7	100.0
	Bultzen	LDP	39.8	63.9	100.0
	Buiyan	FL-WBC	38.4	62.9	100.0
		Ours	49.1	25.1	3.5

 Table 4.8: Comparison of defenses under single image targeted attack on FashionMNIST dataset with both IID and non-IID settings.

# 5 Conclusion

In conclusion, this research tackles the challenges posed by bursty adversarial patterns in federated learning, specifically focusing on client-side defenses. Previous studies have explored client-side defenses, such as FL-WBC (Sun et al., 2021), which addresses bursty attack patterns. However, it has been observed that FL-WBC's use of uncalibrated random noise to perturb the Hessian matrix of gradients can lead to a degradation in global model accuracy, and its robustness against backdoor attacks lacks theoretical guarantees.

To address these limitations, we have introduced LeadFL, a novel client-side defense mechanism designed to enhance server-side defenses while mitigating the impact of bursty adversarial patterns without significantly affecting global model accuracy. The core of LeadFL lies in its optimization framework, which perturbs the Hessian matrix of local gradients and local models using a regularization term to maintain the proximity of the Hessian matrix to the identity matrix.

In support of the effectiveness of our proposed regularized Hessian optimization, we have derived convergence analysis and certified radius analysis, quantifying the distances between benign and poisoned models. These analyses provide theoretical evidence of LeadFL's efficacy in combating bursty adversarial patterns.

Furthermore, we have conducted empirical evaluations by combining LeadFL with different server-side defenses. The results demonstrate that our proposed combination effectively defends against strong attacks, outperforming other combinations of server-side and client-side defenses. Specifically, we have achieved a reduction of up to 65% in backdoor accuracy while maintaining a lower impact on the accuracy of the main task compared to previous defense approaches.

However, it is worth noting that LeadFL, despite its effectiveness, does introduce a slight influence on overall accuracy. Although this influence is relatively minor compared to other defense mechanisms, even small perturbations in accuracy can be significant in certain critical applications. We are committed to further enhancing our algorithm to minimize the impact on the accuracy of the main task. Our future research will focus on improving the performance of LeadFL to ensure even better preservation of the main task accuracy.

## Bibliography

- Bagdasaryan, E., Veit, A., Hua, Y., Estrin, D., and Shmatikov, V. How to backdoor federated learning. In *International Conference on Artificial Intelligence and Statistics*, pp. 2938–2948. PMLR, 2020.
- Baruch, G., Baruch, M., and Goldberg, Y. A little is enough: Circumventing defenses for distributed learning. *Advances in Neural Information Processing Systems*, 32, 2019.
- Bhagoji, A. N., Chakraborty, S., Mittal, P., and Calo, S. Analyzing federated learning through an adversarial lens. In Chaudhuri, K. and Salakhutdinov, R. (eds.), *Proceedings of the 36th International Conference on Machine Learning*, volume 97 of *Proceedings* of Machine Learning Research, pp. 634–643. PMLR, 09–15 Jun 2019.
- Blanchard, P., El Mhamdi, E. M., Guerraoui, R., and Stainer, J. Machine learning with adversaries: Byzantine tolerant gradient descent. In *Advances in Neural Information Processing Systems*, volume 30. Curran Associates, Inc., 2017.
- Cao, X., Jia, J., and Gong, N. Z. Provably secure federated learning against malicious clients. 35(8):6885–6893. ISSN 2374-3468, 2159-5399. doi: 10.1609/aaai.v35i8.16849.
- El-Mhamdi, E.-M., Guerraoui, R., and Rouault, S. Distributed momentum for byzantineresilient stochastic gradient descent. pp. 37.
- Fang, M., Cao, X., Jia, J., and Gong, N. Local model poisoning attacks to {Byzantine-Robust} federated learning. In 29th USENIX Security Symposium (USENIX Security 20), pp. 1605–1622, 2020.
- Fung, C., Yoon, C. J., and Beschastnikh, I. The limitations of federated learning in sybil settings. In 23rd International Symposium on Research in Attacks, Intrusions and Defenses (RAID 2020), pp. 301–316, 2020.
- Gupta, A., Luo, T., Ngo, M. V., and Das, S. K. Long-short history of gradients is all you need: Detecting malicious and unreliable clients in federated learning. In *European Symposium on Research in Computer Security*, pp. 445–465. Springer, 2022.
- Kairouz, P., McMahan, H. B., Avent, B., Bellet, A., Bennis, M., Bhagoji, A. N., Bonawitz, K., Charles, Z., Cormode, G., Cummings, R., et al. Advances and open problems in federated learning. *Foundations and Trends® in Machine Learning*, 14(1–2):1–210, 2021.
- Karimireddy, S. P., He, L., and Jaggi, M. BYZANTINE-ROBUST LEARNING ON HETEROGE- NEOUS DATASETS VIA BUCKETING. pp. 13, a.
- Karimireddy, S. P., He, L., and Jaggi, M. Learning from history for byzantine robust optimization. In *Proceedings of the 38th International Conference on Machine Learning*, pp. 5311–5319. PMLR, b. ISSN: 2640-3498.

- Konečný, J., McMahan, H. B., Yu, F. X., Richtárik, P., Suresh, A. T., and Bacon, D. Federated learning: Strategies for improving communication efficiency. *arXiv preprint arXiv:1610.05492*, 2016.
- Li, Q., Wen, Z., Wu, Z., Hu, S., Wang, N., Li, Y., Liu, X., and He, B. A survey on federated learning systems: vision, hype and reality for data privacy and protection. *IEEE Transactions on Knowledge and Data Engineering*, 2021.
- Li, S., Cheng, Y., Wang, W., Liu, Y., and Chen, T. Learning to detect malicious clients for robust federated learning. *arXiv preprint arXiv:2002.00211*, 2020.
- Li, X., Huang, K., Yang, W., Wang, S., and Zhang, Z. On the convergence of fedavg on non-iid data. In *International Conference on Learning Representations*, 2019.
- Mhamdi, E. M. E., Guerraoui, R., and Rouault, S. The hidden vulnerability of distributed learning in byzantium. In *Proceedings of the 35th International Conference on Machine Learning*, pp. 3521–3530. PMLR, 2018. ISSN: 2640-3498.
- Muñoz-González, L., Co, K. T., and Lupu, E. C. Byzantine-robust federated machine learning through adaptive model averaging, 2019.
- Naseri, M., Hayes, J., and De Cristofaro, E. Local and central differential privacy for robustness and privacy in federated learning. In *Proceedings* 2022 Network and Distributed System Security Symposium. Internet Society, 2022. ISBN 978-1-891562-74-7. doi: 10.14722/ndss.2022.23054.
- Nguyen, T. D., Rieger, P., Chen, H., Yalame, H., Möllering, H., Fereidooni, H., Marchal, S., Miettinen, M., Mirhoseini, A., Zeitouni, S., et al. {FLAME}: Taming backdoors in federated learning. In 31st USENIX Security Symposium (USENIX Security 22), pp. 1415–1432, 2022.
- Ozdayi, M. S., Kantarcioglu, M., and Gel, Y. R. Defending against backdoors in federated learning with robust learning rate. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 35, pp. 9268–9276, 2021.
- Panda, A., Mahloujifar, S., Bhagoji, A. N., Chakraborty, S., and Mittal, P. Sparsefed: Mitigating model poisoning attacks in federated learning with sparsification. In *International Conference on Artificial Intelligence and Statistics*, pp. 7587–7624. PMLR, 2022.
- Rieger, P., Nguyen, T. D., Miettinen, M., and Sadeghi, A.-R. DeepSight: Mitigating backdoor attacks in federated learning through deep model inspection. In *Proceedings* 2022 Network and Distributed System Security Symposium. Internet Society, 2022. ISBN 978-1-891562-74-7. doi: 10.14722/ndss.2022.23156.
- Shejwalkar, V. and Houmansadr, A. Manipulating the byzantine: Optimizing model poisoning attacks and defenses for federated learning. In *NDSS*, 2021.
- Sun, J., Li, A., DiValentin, L., Hassanzadeh, A., Chen, Y., and Li, H. Fl-wbc: Enhancing robustness against model poisoning attacks in federated learning from a client perspective. *Advances in Neural Information Processing Systems*, 34, 2021.

- Sun, Z., Kairouz, P., Suresh, A. T., and McMahan, H. B. Can you really backdoor federated learning? *arXiv preprint arXiv:1911.07963*, 2019.
- Wang, H., Sreenivasan, K., Rajput, S., Vishwakarma, H., Agarwal, S., Sohn, J.-y., Lee, K., and Papailiopoulos, D. Attack of the tails: Yes, you really can backdoor federated learning. *Advances in Neural Information Processing Systems*, 33:16070–16084, 2020.
- Xia, Q., Tao, Z., Hao, Z., and Li, Q. Faba: An algorithm for fast aggregation against byzantine attacks in distributed neural networks. In *Proceedings of the Twenty-Eighth International Joint Conference on Artificial Intelligence, IJCAI-19*, pp. 4824–4830. International Joint Conferences on Artificial Intelligence Organization, 7 2019. doi: 10.24963/ijcai.2019/670.
- Xie, C., Huang, K., Chen, P.-Y., and Li, B. Dba: Distributed backdoor attacks against federated learning. In *International Conference on Learning Representations*, 2019.
- Xie, C., Koyejo, O., and Gupta, I. Fall of empires: Breaking byzantine-tolerant sgd by inner product manipulation. In *Uncertainty in Artificial Intelligence*, pp. 261–270. PMLR, 2020.
- Xie, C., Chen, M., Chen, P.-Y., and Li, B. CRFL: Certifiably robust federated learning against backdoor attacks. In *Proceedings of the 38th International Conference on Machine Learning*, pp. 11372–11382. PMLR, 2021. ISSN: 2640-3498.
- Xu, J., Huang, S.-L., Song, L., and Lan, T. Byzantine-robust federated learning through collaborative malicious gradient filtering. In 2022 *IEEE 42nd International Conference on Distributed Computing Systems (ICDCS)*, pp. 1223–1235. IEEE, 2022.
- Yin, D., Chen, Y., Kannan, R., and Bartlett, P. Byzantine-robust distributed learning: Towards optimal statistical rates. In *Proceedings of the 35th International Conference on Machine Learning*, pp. 5650–5659. PMLR, 2018. ISSN: 2640-3498.