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On 1D morphodynamic network models

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INTRODUCTION

One-dimensional (1D) hydrodynamic network models for tidal predictions are common in engineering practice. 1D morphodynamic network models, however, have rarely been reported in the literature. This may be due to difficulties in dealing with two fundamental uncertainties in morphodynamic models: sediment transport and the distribution of sediment at bifurcations. The purpose of the present paper is to show that the morphodynamic behaviour of the model is extremely sensitive to certain parameters in the sediment transport formula and the nodal-point relation.

NODAL-POINT RELATIONS

In 1D models, the sediment distribution at a bifurcation has to be imposed by means of an internal boundary condition. Such a condition is called a *nodal-point relation*. The following nodal-point relation is recommended in [2].

$$\frac{S_1}{S_2} = \left[\frac{Q_1}{Q_2}\right]^k \left[\frac{B_2}{B_1}\right]^{1-k} \tag{1}$$

This is a relation between the specific sediment transport distribution and a power of the specific discharge distribution. S denotes sediment transport, Q denotes discharge and B denotes channel width. The indices denote the different channels and k is a positive exponent.

The distribution of sediment at a bifurcation depends on the local flow conditions and the geometry. It is very hard, to measure the distribution in nature. DELFT HYDRAULICS has measured the sediment distribution in a scale model of the non-tidal Rhine branches at Pannerden, the Netherlands [4]. Fig. 1 shows the sediment distribution against the discharge distribution. The data is fitted by a curve which relates sediment distribution to a power of the discharge distribution. The measurements support relation (1) as the best-fit curve is

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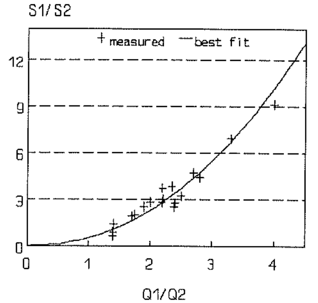
$$\frac{S_1}{S_2} = 0.5\left[\frac{Q_1}{Q_2}\right]^{2.2} \tag{2}$$

This agrees very well with the width ratio of the downstream branches at Pannerden, which is approximately equal to $B_1 : B_2 = 1 : 2$. More measurements are needed to decide whether Equation (1) has physical relevance.



SEDIMENT TRANSPORT FORMULA

The following equation encompasses many of the customary sediment-transport formula's

$$S = M\,(u - c)^n \tag{3}$$

In this equation, M is a constant and n is a positive exponent. The constant c is a threshold value which signifies the initiation of sediment transport. The parameters in formula (3) have to be determined by data fitting. The optimal choice of the parameters depends on the sediment transport formula. For instance, the threshold value c is zero if the Engelund-Hansen formula is used, whereas it is positive for the Meyer Peter-Müller formula.

The present paper studies how variations of c influence the morphodynamic behaviour of the model. A previous paper studied the influence of the exponent k [2]. The small parameter c has important consequences for morphodynamic behaviour.

DIFFERENTIAL EQUATION OF MORPHODYNAMIC EVOLUTION

The change of volume of a channel bed is determined by the residual sediment transport rates at the two ends of the channel. For a bifurcating channel this gives the following equation:

$$\begin{aligned} B_1 L_1 \frac{\mathrm{d}a_1}{\mathrm{d}t} &= \overline{S}_{1,\mathrm{in}} - \overline{S}_{1,\mathrm{out}} \\ B_2 L_2 \frac{\mathrm{d}a_2}{\mathrm{d}t} &= \overline{S}_{2,\mathrm{in}} - \overline{S}_{2,\mathrm{out}} \end{aligned} \tag{4}$$

where the bar denotes averaging over a tidal period and a denotes water depth. The channel boundary at the bifurcation is denoted by the subscript *in*.

S_{in} is determined by the nodal-point relation. The subscript *out* denotes the opposite boundary, where S_{out} is in equilibrium. Under the assumption that the bathymetry of the channel can be represented by the average depth, this is an ordinary differential equation.
Equation (4) can be written in the general form

$$\frac{\mathrm{d}a_1}{\mathrm{d}t} = f_1(a_1,a_2)$$
$$\frac{\mathrm{d}a_2}{\mathrm{d}t} = f_2(a_1,a_2) \qquad \textbf{(5)}$$

According to the qualitative theory of differential equations, the long-term behaviour of this system is determined by the equilibrium states, i.e., f_1 and f_2 are zero. The stability and the time-scales of the equilibrium follow from the eigenvalues of the Jacobian.

EQUILIBRIUM STATES AND THEIR STABILITY

To analyze Equation (4) mathematically, it is simplified by assuming that the flow is steady, i.e., the tide is considered only at one point of time. The validity of the conclusions for steady flow have been verified by numerical simulations for tidal flow [1]. For steady flow, it is relatively easy to relate flow velocity, and hence sediment transport, to the water depth. So, the functions f_1 and f_2 in Equation (5) are relatively easy to determine.
The long-term behaviour of Equation (4) is determined by its equilibrium states and their stability. Consider the case in which the branches are symmetric: all geometrical and morphological parameters, except the water depth, are identical. The width of the channels is half the width of the main channel, before bifurcation. There are three obvious equilibrium states: branch 1 and branch 2 have depth equal to the main channel (state *A*); branch 2 is closed (state *B*); branch 1 is closed (state *C*). These three equilibrium states, in fact, can be derived from a one-channel equilibrium.
The stability of state *A* has been analyzed in [3] (for $c=0$), where it is concluded that *A* is stable if, and only if, k is greater than $n/3$. Extension of this analysis shows that the stability of state *B* and *C* depends on the threshold value c. If c is larger than zero, these states are always stable. If c is equal to zero (power-law formula), then *B* and *C* are stable if, and only if, k is less than $n/3$.

The long-term behaviour of Equation (4) can be represented graphically by a phase diagram, which shows the evolution of a_1 against a_2.

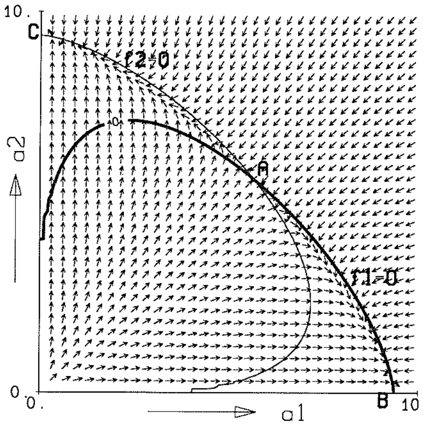


Fig. 2 Phase diagram for $k = 1$

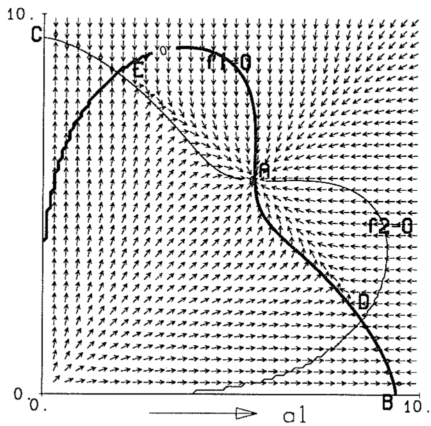


Fig. 3 Phase diagram for $k = 10$

The phase diagram can be computed numerically by plotting the vector $(f_1(a_1,a_2),f_2(a_1,a_2))$ at grid points (a_1,a_2). The magnitude of the vector represents the speed of the evolution and the direction represents the local direction of the evolution. Phase diagrams have been drawn in Figs. 2 and 3. Because of large variations of the speed, the vectors in Figs. 2 and 3 have been normalized. The equilibrium states are the intersections of the isolines $f_1=0$ and $f_2=0$. The isoline $f_1=0$ includes the a_2 axis and the isoline $f_2=0$ includes the a_1 axis.
The parameters which have been used for these phase diagrams are given in the table below. The channels are symmetric. The sediment transport formula is the Meyer Peter-Müller formula.

length	width	Chézy	D_{mean}	Q	S
2400 m	130 m	45 $m^{0.5}/s$	1.45 mm	1600 m^3/s	1.27 m^3/s

The values for Q and S denote the transports in the main channel, before bifurcation.
Figs. 2 and 3 demonstrate the sensitivity of the model to the value of k, as has been analyzed in [2] for the case $c=0$. The only difference between the figures is that the nodal-point relation has exponent $k=1$ in Fig. 2 and exponent $k=10$ in Fig. 3. The influence of the theshold velocity c is demonstrated by Fig. 3, as follows. It has been shown in [2] that for $c=0$ only three equilibrium states are present independent of c. These are the states A,

B, C. For small values of k, the phase diagram is of the type of Fig. 2. For large values of k, A turns into a stable state and B, C are unstable. As Fig. 3 demonstrates, there are two extra unstable states D and E in the case that c is greater than zero. The states B and C now remain stable for large values of k. For small values of c, the states D and E are close to B and C. For large values of c, they are close to A. For c equal to zero, i.e., a power-law transport formula, B coincides with D and C coincides with E.
In summary, the exponents k and n mainly influence the state A, whereas the threshold value c influences the states B, C, D, E.

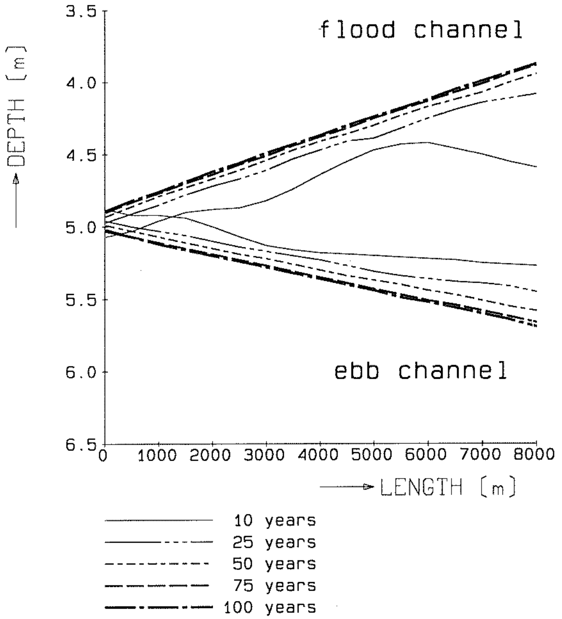
APPLICATIONS

The results can be readily applied for morphodynamic modelling of non-tidal rivers, for which the differential equation (4) is relatively easy to compute. For tidal rivers and estuaries, the differential equation is more involved as sediment transport has to be averaged over a period of the tide. In this case, it is not as easy to determine the phase diagram. The morphodynamic behaviour of bifurcating tidal channels is the same as the behaviour of non-tidal channels.
Fig. 4 shows the evolution of an ebb-channel flood-channel configuration in an estuary. The channels have common boundaries at both sides. The boundary at $x=0$ represents the sea-boundary; the boundary at the end of the channel is connected to a large storage area. The sediment transport formula is the Engelund-Hansen formula, for which $c=0$ and $n=5$. The nodal-point relation has exponent $k=3$. In this case, the states B and C are unstable, so the system settles down in state A, independent of the initial condition.
The channels have opposite geometry. The width of the flood channel increases linearly with its length. The ebb channel is the equal to the flood channel, only in the opposite direction.
The tide is a flood-dominated M_2-M_4 tide, with amplitude equal to 0.5 m for M_2 and 0.1 m for M_4. The main parameters of the flood channel are: length of 8000 m, a Chézy value of 50 $m^{0.5}/s$ and channel width increasing from 45 m to 55 m. The initial water depth of the channel is 5 m. The parameters are the same for the ebb channel, only here width decreases from 55 to 45 m.
The evolution of this configuration is shown in Fig. 4. Clearly, it settles down in equilibrium, with a relatively shallow flood channel and a relatively deep ebb channel.

CONCLUSIONS

Theoretical considerations and numerical simulations show that the behaviour of 1D morphodynamic models is extremely sensitive to the parameters k, n

and c in the nodal-point relation and the sediment transport formula. The analysis presented in this paper is a useful tool to determine the behaviour of network morphodynamic models. In a previous paper, this analysis has been used to study the parameters k and n. In the present paper, the analysis has been extended to study the influence of the threshold value c.



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