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RESEARCH ARTICLE

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A risk-based fuzzy arithmetic model to determine safety integrity levels considering individual and societal risks

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Abstract

Risk-based techniques such as risk graph and Layer of Protection Analysis (LOPA) are used to determine the Safety Integrity Level (SIL) of safety instrumented functions to ensure that risk is reduced to a tolerable level. However, these techniques have some drawbacks. For instance, they need absolute and precise numbers to evaluate SIL parameters, which are rarely available or are highly uncertain. In addition, they are incapable of considering individual and societal risks simultaneously. Moreover, risk tolerance criteria are likely to be used incorrectly in the LOPA technique, and risk graph is difficult to calibrate. In the current paper, a novel comprehensive fuzzy arithmetic model has been developed to determine the required SILs in process industries. The fuzzy required Risk Reduction Factor (RRF) is calculated for both individual and societal risks. Fuzzy numbers are developed from crisp intervals, based on the expected interval of the fuzzy numbers. Expert fuzzy-scaled elicitation has been applied to obtain the SIL parameters. In the proposed model, the overall risk tolerance criterion and apportionment factor are defined as SIL parameters for both individual and societal risks to ensure that the applied risk criteria are compliant with the requirements of the system. In addition, an approach is introduced for determining the required SIL based on the fuzzy required RRF. The proposed methodology was demonstrated to alleviate the limitations, and thus, can be considered as a more precise alternative to the conventional methods.

KEYWORDS

fuzzy, individual risk, safety instrumented system, safety integrity level, societal risk

1 | INTRODUCTION

Despite existing mandatory safety rules and a wide range of methods for hazard identification and evaluation, industrial accidents still occur.[1–3] The lessons learned from past accidents have led to various types of protection layers to prevent accidents and/or mitigate their consequences.[4] Safety Instrumented Functions (SIFs) operated by Safety Instrumented Systems (SISs), such as high-level trip and high-pressure trip, are widely used in process industries.[5,6]

Applications of the most reliable SIFs are not always cost-effective. The required level of reliability to ensure that risk is reduced to a tolerable level is known as the Safety Integrity Level (SIL). The SIL is determined by calculating the required

TABLE 1 SIL rating in terms of RRF for demand mode.[8,9]

SIL rating	Range of RRF
NSR[a]	$< 10^0$
NSSR[b]	$[10^0, 10^1]$
SIL1	$[10^1, 10^2]$
SIL2	$[10^2, 10^3]$
SIL3	$[10^3, 10^4]$
SIL4	$[10^4, 10^5]$
NR[c]	$10^5 \leq$

[a]NSR refers to the case "no safety requirements."
[b]NSSR refers to the case "no special safety requirements."
[c]NR refers to the case "not recommended (a single SIF is not sufficient)."

Risk Reduction Factor (RRF) to fill the gap between the current risk and risk tolerance criterion (Equation 1).[7] The SIL rating and relevant ranges of the RRF are shown in Table 1 for a system working on-demand mode of operation.

$$RRF_{Req} = \frac{Current\ Risk}{Risk\ tolerance\ criterion} \tag{1}$$

The risk graph and the Layer of Protection Analysis (LOPA) techniques are the prevalent conventional techniques for determining the SIL based on risk.[6,10]

Risk graph is a simple method for determining the required SIL level.[7,11] This technique uses a number of risk parameters to describe the nature of a hazardous situation when the SIS fails or is not available. In risk graph technique, the risk is defined as a combination of four parameters: consequence, occupancy, unavoidability, and demand rate. The required SIL for a SIF is calculated according to the value of parameters and using the related path on the graph.

LOPA is a systematic technique to evaluate the effectiveness of Independent Protection Layers (IPLs) and to determine the required SIL level.[7,11,12] In LOPA, the frequency of an unwanted consequence is calculated by multiplying the Probabilities of Failure on Demand (PFDs) of the IPLs by the initiating event frequency. The total amount of risk reduction covered by the existing IPLs is determined, and then the necessity for more risk reduction is evaluated based on predetermined risk reference criterion. The SILs of the SIFs should be determined if risk reduction should be achieved via the implementation of SIFs.

Although being popular, these conventional techniques still have some major drawbacks.[13–18] First, incorrect use of risk tolerance criteria is likely when using these techniques, that is, using the risk tolerance criterion intended for an overall facility instead of the risk tolerance criterion for a single scenario. The risk criteria are most defined for an overall facility rather than for a single scenario. Comparing the risk of a single scenario to a risk tolerance criterion intended for an overall facility ignores the fact that other facility operations will also contribute to risk.[19] Thus, apportioning risk tolerance criteria to an appropriate level (e.g., individual scenarios) is needed in determining the required SIL.

Second, these conventional techniques do not consider uncertainty in SIL determination. For instance, crisp values for the risk tolerance criteria provided by various standards and guidelines are usually constant over time and do not consider organizational policies. In addition, the total number of scenarios for apportioning the risk tolerance criteria is subject to uncertainty. Furthermore, quantifying the unwanted consequences of an event by experts is subjective, if not practically impossible. In addition, due to data scarcity and experts' inadequate knowledge, an accurate estimation of the probabilities in the form of absolute and precise numbers is usually prone to high levels of uncertainty. Consequently, modeling the uncertainties becomes an integral part in SIL determination. A number of researchers have used uncertainty analysis to show the inconsistencies in experts' opinions.[20,21] Fuzzy logic is an effective way to deal with this type of uncertainty, which arises due to data scarcity and incomplete knowledge of experts.[22]

Finally, the conventional methods do not consider simultaneously the individual risk and societal risk. As important as every individual is, the reality is that the potential impact on a company increases for multiple fatality incidents compared to individual fatalities. The individual risk criterion may be met for a hazard scenario, but the societal risk criterion may not, or vice versa. For instance, when a large population of people is at risk, the societal risk for personnel may be high, but the individual risk may be low. On the other hand, when personnel is exposed to a large amount of hazardous material, the individual risk may be high, but the societal risk might be low.[19] Therefore, in SIL determination, both individual risk (i.e., provides a perspective on facility risk from an individual's point of view) and societal risk (i.e., provides a perspective on the risk to the company) should be addressed.

The lack of reliable and holistic models for determining the required SIL, which are also capable of considering all the above-mentioned limitations, is the primary motivation for this work. The main contribution of the paper is developing a comprehensive fuzzy arithmetic model to determine the required SIL while addressing both individual and societal risks in an uncertain environment. The specific novel features of the proposed model include (i) driving new fuzzy required RRF equations for both individual and societal risks, (ii) developing a new approach for apportioning individual and societal risk criteria to appropriate levels, and (iii) introducing an approach for determining the required SIL based on the fuzzy required RRF. Since the conventional techniques such as the risk graph technique and LOPA calculate required SIL by a fixed formula, this paper introduces a fuzzy arithmetic model instead of a fuzzy rule-based model to make more precise decisions with regard to the SIL determination. To improve the applicability of the model to a wide range of industries, fuzzy arithmetic equations are derived, which require only basic operations such as summation and subtraction. In order to employ the crisp intervals of SIL parameters in conventional methods, a numerical technique based on the expected interval of a fuzzy number is applied for transferring these intervals into trapezoidal membership functions. Furthermore, in the proposed technique, the overall risk tolerance criterion and apportionment factor are defined as SIL parameters for both individual and societal risks to ensure that the risk criteria are more compliant with the requirements of the system. An approach is introduced to determine the required SIL based on a novel fuzzy comparison of the calculated fuzzy RRF and fuzzy RRF of each SIL rating rather than defuzzification, which may mask valuable information and thus lead to inaccurate decisions. The proposed model is applied to a flammable liquid vessel to demonstrate its advantages over the conventional techniques.

2 | LITERATURE REVIEW

Risk graphs and LOPA techniques are the most frequent techniques in determining the required SIL in a wide range of industries. For example, Ahn et al.[23] employed these conventional techniques for a molten carbonate fuel cell stack, and Yang et al.[24] utilized them for a shale gas station.

Due to the importance of SIL determination in safety and reliability engineering, more and more researchers put effort into improving these conventional techniques.

Several alternative models have been developed for determining the required SIL by modifying SIL parameters. Blackmore[25] proposed to modify the risk graph by defining different classifications and ranges for the SIL parameters. Likewise, Baybutt[26] introduced an improved risk graph with four SIL parameters, including "initiating cause frequency," "enabling events/conditions and other modifiers," "safeguards failure probability," and "consequences of the hazardous event or scenario." Piesik et al.[10] developed a risk graph by considering "security" as a new SIL parameter.

Baybut[14] suggested an approach within the framework of LOPA to determine the required SIL while addressing the incorrect use of risk tolerance criteria. In another work, they[17] developed a procedure for SIL determination using LOPA. The procedure uses a risk model to calculate the risk of an overall facility to be compared with the overall risk tolerance criteria considering both the individual risk and the societal risks. Recently, Cheraghi and Taghipour[70] developed a mathematical optimization model to determine the SILs for SIFs of a facility. Their model considers both individual and societal risk perspectives and applies the risk tolerance criteria intended for an overall facility, rather than the risk tolerance criteria for a single scenario.

Fuzzy logic has been effectively used to improve the performance and credibility of the techniques for the determination of required SIL. Ormos and Ajtonyi[27] proposed a fuzzy model for determining the required SIL. Their model was based on the fuzzy inference system, the hazardous event severity matrix, and the risk graph. Similarly, Simon et al.[28] applied a fuzzy inference system for developing a fuzzy version of the risk graph. In their model, SIL parameters are assessed by aggregating experts' opinions. Nait-Said et al.[29] also proposed a modified risk graph model to determine the required SIL using a fuzzy inference system. Their model uses fuzzy scales to evaluate the SIL parameters, which are derived from corresponding crisp partitions. Raeesivand and Kasaeyan[30] developed a similar fuzzy risk graph based on a fuzzy inference system and experts' opinions. All the aforementioned fuzzy risk graph techniques based on the fuzzy inference system consider four input variables (consequence, occupancy, unavoidability, and demand rate) and only one output variable (required SIL).

Similarly, Qorbali et al.[31] improved the risk graph method, originally introduced by Baybutt,[26] using a fuzzy approach. Chang et al.[5] applied the Minimum SIL Table,[32] the risk graph and LOPA techniques to determine the required SIL. They combined these techniques with the fuzzy logic and Monte Carlo simulation, and discussed its application to offshore

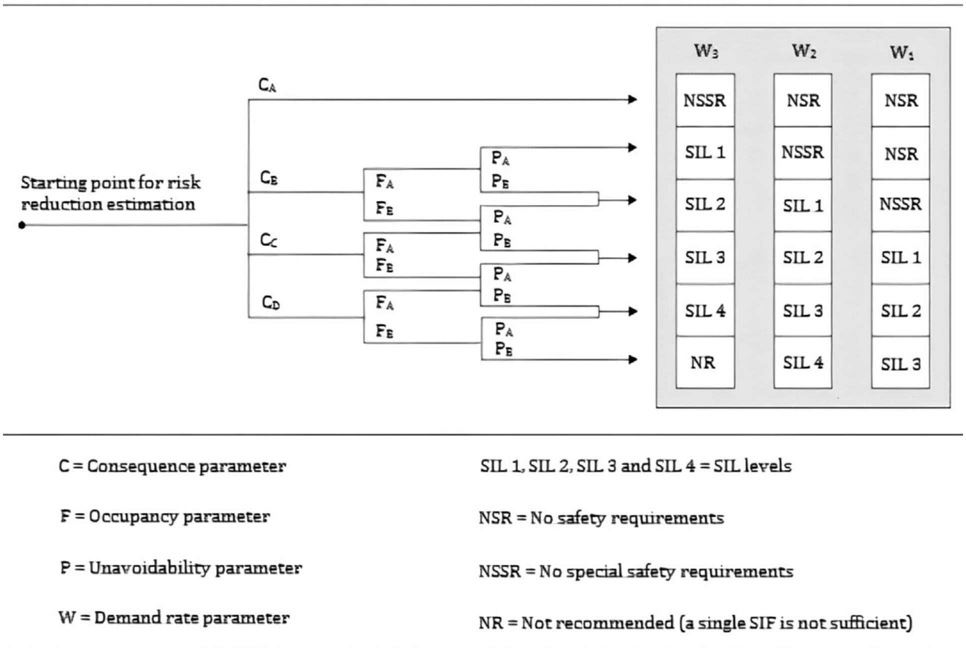


FIGURE 1 A typical risk graph.[7]

industry. Markowski and Mannan[33] developed a fuzzy LOPA approach for risk evaluation of major hazards related to the transportation of flammable substances in long pipelines. They used a fuzzy inference system with two inputs (frequency and severity) and one output (risk) for calculating risk level. In their study, the severity is derived from another fuzzy inference system, and the frequency is calculated using fuzzy multiplication. Similarly, Khalil et al.[34] proposed a cascaded fuzzy-LOPA model to determine the required SIL for applications in the natural gas industry. Their model is constructed based on two fuzzy inference systems. The first fuzzy inference system calculates the severity of each scenario where safety and economical aspects are considered as input variables, and severity is considered as the output variable. In the second fuzzy inference system, severity and frequency are combined together as inputs for determining the required SIL. Ouazraoui et al.[35] used fuzzy multiplication to calculate frequency in the LOPA technique. The frequency was then compared to a tolerable risk using possibility and necessity measures. They calculated the required risk reduction by solving a possibilistic decision-making problem. Hong et al.[36] developed a fuzzy inference system and a probabilistic hybrid approach to quantify the uncertainty in the frequency of an initiating event and the PFD of an independent protection layer.

Reviewing the abovementioned works reveals that, as opposed to the conventional techniques, fuzzy models have significantly improved handling of uncertainties and imprecision in SIL determination. However, the previous fuzzy models generally fail to simultaneously consider individual and societal risks. In addition, the issue of using the risk tolerance criteria is not sufficiently addressed in these fuzzy models. This paper aims to address these issues by developing a novel comprehensive fuzzy arithmetic model.

3 | METHODOLOGY

3.1 | Conventional methods

Risk graph and LOPA techniques are briefly discussed in the following subsections.

3.1.1 | Risk graph technique

The risk graph determines the required SIL by taking into account four parameters: consequence (C), occupancy (F), unavoidability (P), and demand rate (W).[7,11] A typical risk graph is shown in Figure 1, and its parameters and their values are explained in Table 2.

TABLE 2 Descriptions of the risk graph's parameters extracted from IEC 61511[7] and UKOOA.[37]

Risk parameter	Classification	Range	Comments
Consequence (C)			
Number of fatalities	C_A	Minor injury	The vulnerability is determined by the nature of the hazard being protected against. The following factors can be used:
C = NP × V	C_B	[0.01, 0.1] probable fatalities per event	V = 0.01 (Small release of flammable or toxic material)
NP = Number of people at risk	C_C	[0.1, 1] probable fatalities per event	V = 0.1 (Large release of flammable or toxic material)
V = Vulnerability	C_D	> 1 probable fatalities per event	V = 0.5 (As above but also a high probability of catching fire or highly toxic material)
			V = 1 (Rupture or explosion)
Occupancy (F)			
Occupancy is calculated by determining the proportional length of time the area exposed to the hazard, is occupied during a normal working period.	F_A	Rare to more frequent exposure in the hazardous zone. occupancy < 0.1	
	F_B	Frequent to permanent exposure in the hazardous zone. occupancy ≥ 0.1	
Unavoidability (P)			
Probability of unavoiding the hazardous event, if the protection system fails to operate.	P_A	Adopted if all conditions in column 4 are satisfied. < 0.1 probability hazard cannot be avoided	P_A should only be selected if all the following are true:
	P_B	Adopted if not all the conditions are satisfied. ≥ 0.1 probability hazard cannot be avoided	- Facilities are provided to alert the operator that the SIS has failed.
			- Independent facilities are provided to shut down such that the hazard can be avoided or which enable all persons to escape to a safe area.
			- The time between the operator being alerted and a hazardous event occurring exceeds 1 h, or is definitely sufficient for the necessary actions.
Demand rate (W)			
The number of times per year that the hazardous event would occur in absence of SIF under consideration	W_1	< 0.1D event per year	For demand rates higher than 10D per year, higher integrity is needed.
	W_2	[0.1D, D] event per year	D is a calibration factor. (UKOOA suggests D ≃ 0.33)
	W_3	[D, 10D] event per year	

Since the risk of a path in Figure 1 is $R = C \times F \times P \times W$[38] and using Equation (1), the risk tolerance criteria for the worst-case, the geometric-mean-case, and the best-case are respectively 3.3×10^{-4}, 1.0×10^{-6}, and 3.3×10^{-9} fatalities per year. The worst-case is obtained when all the risk parameters are at the maximum of their ranges. The geometric-mean-case is achieved when all the risk parameters are at the geometric mean of their ranges. The best-case is obtained when all the risk parameters are at the minimum of their ranges.

3.1.2 | LOPA technique

LOPA calculates the required RRF by comparing the current risk and a predetermined risk tolerance criterion. Since the current risk and relevant risk tolerance criterion are both defined based on the same consequence, using Equation (1), the required RRF for determining the required SIL can be calculated as.[7,11,12]

$$RRF_{Req} = \frac{f_i^C}{TR^C} \tag{2}$$

where f_i^C is the frequency of consequence C for initiating event i, and TR^C is the maximum tolerable risk frequency of the consequence C.

f_i^C is calculated as.[39]

$$f_i^C = f_i^I \times \prod_{l=1}^{L} PEC_{il} \times \prod_{j=1}^{J} PFD_{ij} \times \prod_{k=1}^{K} PCM_k \tag{3}$$

where f_i^I is the frequency of initiating event i, PEC_{il} is the probability of the lth enabling condition pertinent to initiating event i, PFD_{ij} is the probability of failure on demand of the jth IPL that protects against the consequence C for initiating event i, and PCM_k is the probability of the kth conditional modifier applicable to consequence C. Enabling condition is defined as a condition that makes the beginning of a scenario possible (e.g., time-at-risk). The conditional modifier is a probabilistic condition (e.g., probability of ignition) in a scenario when risk criteria endpoints are expressed in impact terms (e.g., fatalities) instead of in primary loss event terms (e.g., release).

The frequency of an initiating event and the probabilities in Equation (3) can be obtained from databases[12,39–42] as well as experts' opinions and vendor data.

3.2 | Proposed model

The overall overview of the proposed methodology is depicted in Figure 2. In the first step, the fuzzy arithmetic models for obtaining fuzzy required RRF, considering both individual risk and societal risk are developed. Next, the appropriate membership functions are constructed from crisp intervals based on the expected interval of a fuzzy number. Then, the input parameters' values are gathered from the expert team and Chief Executive Officer (CEO) in the form of fuzzy linguistic variables. After aggregating the expert opinions, the fuzzy required RRFs for both individual and societal risks are determined. To select the appropriate level of SIL for SIF for both individual and societal risks, the calculated fuzzy required RRF should be compared with all fuzzy RRF of SIL ratings (i.e., NSR, NSSR, SIL1, SIL2, SIL3, SIL4, and NR). At the end, the final required SIL for the SIF is selected based on the higher SIL for both individual risk and societal risks.

3.2.1 | Developing a fuzzy arithmetic model for the required RRF

3.2.2 | Individual risk approach

Using Equation (3), the required RRF based on the individual risk can be calculated as:

$$RRF_{Req}^{Individual} = f_i^I \times \prod_{l=1}^{L} PEC_{il} \times \prod_{j=1}^{J} PFD_{ij} \times \prod_{k=1}^{K-1} PCM_k \times V \times (IRC_1)^{-1} \tag{4}$$

where V is the vulnerability (see Table 2), and IRC_1 is the individual risk tolerance criterion for a single scenario. Note that V is a conditional modifier parameter.

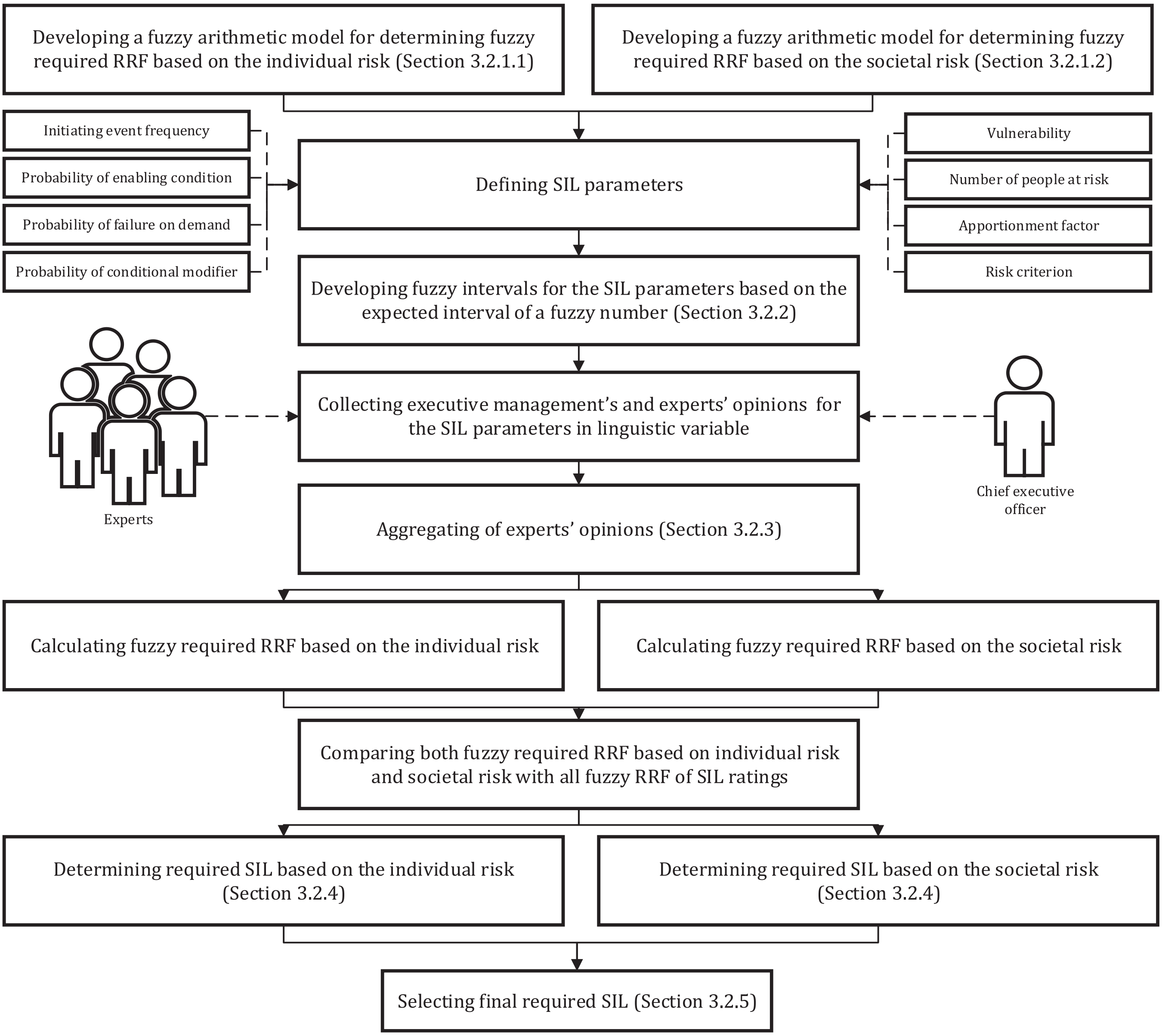


FIGURE 2 Flowchart of the proposed methodology to determine the required SIL.

Also IRC_1 can be calculated as:

$$IRC_1 = \frac{IRC}{AI} \tag{5}$$

where IRC is the overall individual risk tolerance criterion, and AI is the apportionment factor for individual risk tolerance criterion (AI can be estimated considering the number of scenarios that could potentially cause a fatality).

If IRC_1 in Equation (4) is replaced with, Equation (5), Equation (4) can be rewritten as:

$$RRF_{Req}^{Individual} = f_i^I \times \prod_{l=1}^{L} PEC_{il} \times \prod_{j=1}^{J} PFD_{ij} \times \prod_{k=1}^{K-1} PCM_k \times V \times (IRC)^{-1} \times AI \tag{6}$$

Taking the logarithm of both sides of Equation (6):

$$log\ RRF_{Req}^{Individual} = log\ f_i^I + log \prod_{l=1}^{L} PEC_{il} + log \prod_{j=1}^{J} PFD_{ij} + log \prod_{k=1}^{K-1} PCM_k + log\ V + log\ (IRC)^{-1} + log\ AI \quad (7)$$

$$log\ RRF_{Req}^{Individual} = log\ f_i^I + logPEC_{i1} + \cdots + logPEC_{iL} + \log PFD_{i1} + \cdots + log\ PFD_{iJ} + \log PCM_1 + \cdots + \log PCM_{K-1} + log\ V - \log IRC + log\ AI \quad (8)$$

Considering $log\ RRF_{Req}^{Individual}$, $\log f_i^I$, $logPEC_{il}$, $\log \text{PFD}_{\text{ij}}$, $\log PCM_k$,$log\ V$, $\log IRC$, and $\log AI$ as fuzzy membership functions, Equation (8) can be written as:

$$log\ \widetilde{RRF_{Req}^{Individual}} = \widetilde{log\ f_i^I} \oplus log\widetilde{PEC}_{i1} \oplus ... \oplus log\widetilde{PEC}_{iL} \oplus \log \widetilde{PFD}_{i1} \oplus ... \oplus \log \widetilde{PFD}_{iJ} \oplus \log \widetilde{PCM}_1 \oplus ... \oplus \log \widetilde{PCM}_{K-1} \oplus \widetilde{\log V} \ominus \widetilde{\log IRC} \oplus \widetilde{\log AI} \quad (9)$$

where fuzzy summation, subtraction and multiplication are denoted by $\oplus$, $\ominus$, and $\otimes$, respectively.

3.2.3 | Societal risk approach

The required RRF based on the societal risk can be calculated as:

$$log\ RRF_{Req}^{Societal} = f_i^I \times \prod_{l=1}^{L} PEC_{il} \times \prod_{j=1}^{J} PFD_{ij} \times \prod_{k=1}^{K-1} PCM_k \times V \times NP \times (SRC_1)^{-1} \quad (10)$$

where NP is the number of people at risk (see Table 2), and SRC_1 is the societal risk tolerance criterion for a single scenario. SRC_1 can be calculated as:

$$SRC_1 = \frac{SRC}{AS} \quad (11)$$

where SRC is the overall societal risk tolerance criterion, and AS is the apportionment factor for the societal risk tolerance criterion.

Therefore, Equation (10) can be written as:

$$log\ \widetilde{RRF_{Req}^{Societal}} = \widetilde{log\ f_i^I} \oplus log\widetilde{PEC}_{i1} \oplus ... \oplus log\widetilde{PEC}_{iL} \oplus \log \widetilde{PFD}_{i1} \oplus ... \oplus \log \widetilde{PFD}_{iJ} \oplus \log \widetilde{PCM}_1 \oplus ... \oplus \log \widetilde{PCM}_{K-1} \oplus \widetilde{\log V} \oplus \widetilde{\log NP} \ominus \widetilde{\log SRC} \oplus \widetilde{\log AS} \quad (12)$$

Equations (9) and (12) are developed for calculating the fuzzy required RRF for individual and societal risks, respectively.

3.2.4 | Development of fuzzy intervals for the parameters of the required SIL

Experts are generally more comfortable with linguistic variables rather than numerical judgments, when asked to determine an uncertain quantity (e.g., the initiating event frequency, the PFD of an IPL, and other SIL parameters).[43] Building appropriate membership functions which correspond to linguistic variables is a key step before subsequent operations.[44] There are various forms of a fuzzy membership function that can be used to represent uncertainty, while triangular and trapezoidal functions are widely used.[45–47] The selection of the form of a membership function depends on the characteristics of input and output variables, and is generally chosen according to expert experience. Although in the majority of reliability and safety analysis cases, there are no significant differences in the outputs of different types of fuzzy number,[48,49] the trapezoidal fuzzy membership function has been applied in this paper due to its simplicity and ability to model crisp intervals without dependency on experts' intuitions. After describing a trapezoidal fuzzy number, a numerical technique for developing a trapezoidal fuzzy number from a crisp interval is elaborated.

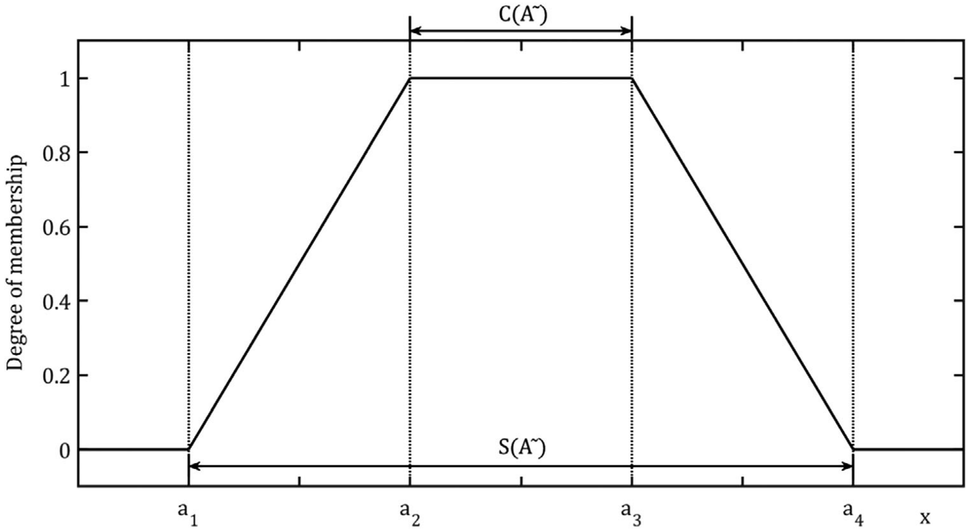


FIGURE 3 Schematic diagram of a trapezoidal fuzzy number with its features.[52]

The membership function $\mu_{\tilde{A}}$ of a trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ can be defined as.[50,51]

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ 1, & a_2 \le x \le a_3 \\ \dfrac{a_4 - x}{a_4 - a_3}, & a_3 \le x \le a_4 \\ 0, & x > a_4 \end{cases} \tag{13}$$

or can be written as[69]:

$$\mu_{\tilde{A}}(x) = \max\left(\min\left(\frac{x - a_1}{a_2 - a_1}, 1, \frac{a_4 - x}{a_4 - a_3}\right), 0\right) \tag{14}$$

where a_1, a_2, a_3, and a_4 are the four characteristic values of a trapezoidal fuzzy number.

The core and support of a trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ can be presented as $C(\tilde{A}) = [a_2\ a_3]$ and $S(\tilde{A}) = [a_1\ a_4]$, respectively. If $a_2 = a_3$, then $\tilde{A}$ is called a triangular fuzzy number, and if $a_1 = a_2 = a_3 = a_4$, then $\tilde{A}$ is called a crisp number or a fuzzy singleton.[47,52] Also, $\tilde{A}$ is called a positive trapezoidal fuzzy number if $a_1 \ge 0\ (\mu_{\tilde{A}}(x) = 0,\ \forall x < 0)$. $\tilde{A}$ is called a negative trapezoidal fuzzy number if $a_4 \le 0\ (\mu_{\tilde{A}}(x) = 0,\ \forall x > 0)$.[53,54]

A typical trapezoidal fuzzy number and its features are depicted in Figure 3.

Considering the two trapezoidal fuzzy numbers $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$, and a scalar (crisp number) K, the operational laws are defined as.[53,54]

$$\tilde{A} \oplus \tilde{B} = (a_1, a_2, a_3, a_4) \oplus (b_1, b_2, b_3, b_4) = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4) \tag{15}$$

$$\tilde{A} \ominus \tilde{B} = (a_1, a_2, a_3, a_4) \ominus (b_1, b_2, b_3, b_4) = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1) \tag{16}$$

$$\tilde{A} \otimes \tilde{B} = (a_1, a_2, a_3, a_4) \otimes (b_1, b_2, b_3, b_4) = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4);\quad \tilde{A} \text{ and } B \text{ are both positive trapezoidal fuzz numbers} \tag{17}$$

$$\tilde{A} \otimes \tilde{B} = (a_1, a_2, a_3, a_4) \otimes (b_1, b_2, b_3, b_4) = (a_1 b_4, a_2 b_3, a_3 b_2, a_4 b_1);\quad \tilde{A} \text{ isanegativetrapezoidalfuzzynumber, and isapositivetrapezoidalfuzzynumber} \tag{18}$$

$$\tilde{A} \otimes \tilde{B} = (a_1, a_2, a_3, a_4) \otimes (b_1, b_2, b_3, b_4) = (a_4b_4, a_3b_3, a_2b_2, a_1b_1); \quad \tilde{A} \text{ and } \tilde{B} \text{ are both negative trapezoidal fuzzy numbers} \tag{19}$$

$$-\left(\tilde{A}\right) = (-a_4, -a_3, -a_2, -a_1) \tag{20}$$

$$K\tilde{A} = (Ka_1, Ka_2, Ka_3, Ka_4)\ ;\ K > 0,\ K \in R \tag{21}$$

$$K\tilde{A} = (Ka_4, Ka_3, Ka_2, Ka_1)\ ;\ K < 0,\ K \in R \tag{22}$$

Developing a fuzzy interval (or a fuzzy number) that corresponds to a linguistic variable from a crisp interval can be referred to as the converse problem of calculating the expected interval of a fuzzy number. In this way, the boundaries of a crisp interval are considered as the expected interval of a desired fuzzy number.[29,55,69]

Some research has been conducted to introduce the expected value of a fuzzy number.[56,57] The expected interval $E(\tilde{A})$ of the fuzzy number $\tilde{A}$ is a closed interval bounded by the expectations calculated from its upper and lower distribution functions (Equation 23). As such, the expected value ($EV(\tilde{A})$) of the fuzzy number $\tilde{A}$ is defined as the center of the expected interval (Equation 24).[56,57]

$$E\left(\tilde{A}\right) = \left[E_*\left(\tilde{A}\right)\ E^*\left(\tilde{A}\right)\right] \tag{23}$$

$$EV\left(\tilde{A}\right) = \frac{E_*\left(\tilde{A}\right) + E^*\left(\tilde{A}\right)}{2} \tag{24}$$

where $E_*(\tilde{A})$ and $E^*(\tilde{A})$ are respectively called lower and upper expected values of the fuzzy number $\tilde{A}$. If $\tilde{A}$ is a trapezoidal fuzzy number, they can be calculated as.[56,57]

$$E^*\left(\tilde{A}\right) = \int_{-\infty}^{+\infty} x dF_*(x) = a_3 + \int_{a_3}^{+\infty} \mu_{\tilde{A}}(x)\ dx = a_3 + \int_{a_3}^{a_4} \mu_{\tilde{A}}(x)\ dx \tag{25}$$

$$E_*\left(\tilde{A}\right) = \int_{-\infty}^{+\infty} x dF^*(x) = a_2 - \int_{-\infty}^{a_2} \mu_{\tilde{A}}(x)\ dx = a_2 - \int_{a_1}^{a_2} \mu_{\tilde{A}}(x)\ dx \tag{26}$$

where $F_*(x)$ is the lower distribution function of $\tilde{A}$, and $F^*(x)$ is the upper distribution function of $\tilde{A}$. They can be calculated as:

$$F_*(x) = \begin{cases} 0, & x < a_3 \\ 1 - \mu_{\tilde{A}}(x), & x \geq a_3 \end{cases} \tag{27}$$

$$F^*(x) = \begin{cases} \mu_{\tilde{A}}(x), & x < a_2 \\ 1, & x \geq a_2 \end{cases} \tag{28}$$

Therefore, $E^*(\tilde{A})$, $E_*(\tilde{A})$, $E(\tilde{A})$, and $EV(\tilde{A})$ of a trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ can be calculated as:

$$E^*\left(\tilde{A}\right) = a_3 + \int_{a_3}^{a_4} \frac{a_4 - x}{a_4 - a_3}\, dx = a_3 + \frac{a_4}{a_4 - a_3} \int_{a_3}^{a_4} dx - \frac{1}{a_4 - a_3} \int_{a_3}^{a_4} x\, dx$$
$$= a_3 + \frac{a_4}{a_4 - a_3}(a_4 - a_3) - \frac{1}{a_4 - a_3}\left(\frac{a_4^2 - a_3^2}{2}\right) = \frac{a_3 + a_4}{2} \tag{29}$$

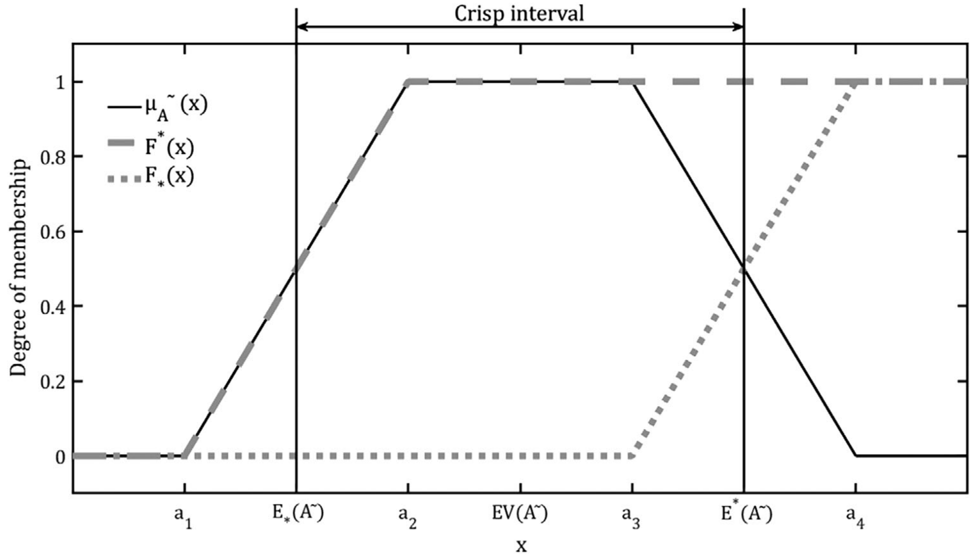


FIGURE 4 A trapezoidal fuzzy number and its related crisp interval.

$$E_* \left(\tilde{A}\right) = a_2 - \int_{a_1}^{a_2} \frac{x - a_1}{a_2 - a_1}\, dx = a_2 - \frac{1}{a_2 - a_1} \int_{a_1}^{a_2} x\, dx + \frac{a_1}{a_2 - a_1} \int_{a_1}^{a_2} dx$$

$$= a_2 - \frac{1}{a_2 - a_1} \left(\frac{a_2^2 - a_1^2}{2} \right) + \frac{a_1}{a_2 - a_1} \left(a_2 - a_1\right) = \frac{a_1 + a_2}{2} \tag{30}$$

$$E\left(\tilde{A}\right) = \left[\frac{a_1 + a_2}{2}\ \frac{a_3 + a_4}{2}\right] \tag{31}$$

$$EV\left(\tilde{A}\right) = \frac{a_1 + a_2 + a_3 + a_4}{4} \tag{32}$$

Figure 4 illustrates a trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ along with $F_*(x)$, $F^*(x)$, $E^*(\tilde{A})$, $E_*(\tilde{A})$, $E(\tilde{A})$, $EV(\tilde{A})$ and the related crisp interval.

As described above, $E_*(\tilde{A})$ of the trapezoidal fuzzy number $\tilde{A}$ is given by the lower bound of the related crisp interval, and $E^*(\tilde{A})$ of the trapezoidal fuzzy number $\tilde{A}$ is given by the upper bound of the related crisp interval. Then, $EV(\tilde{A})$ is calculated using Equation (24). Subsequently, the centers of the intervals $[E_*(\tilde{A})\ EV(\tilde{A})]$ and $[EV(\tilde{A})\ E^*(\tilde{A})]$ are defined as a_2 and a_3, respectively (Equations (33) and (34)).[29,69] Accordingly, a_1 and a_4 can be calculated using Equations (35) and (36), respectively.

$$a_2 = \frac{E_*\left(\tilde{A}\right) + EV\left(\tilde{A}\right)}{2} \tag{33}$$

$$a_3 = \frac{EV\left(\tilde{A}\right) + E^*\left(\tilde{A}\right)}{2} \tag{34}$$

$$a_1 = 2E_*\left(\tilde{A}\right) - a_2 = 2E_*\left(\tilde{A}\right) - \left(\frac{E_*\left(\tilde{A}\right) + EV\left(\tilde{A}\right)}{2}\right) = \frac{3E_*\left(\tilde{A}\right) - EV\left(\tilde{A}\right)}{2} \tag{35}$$

$$a_4 = 2E^*\left(\tilde{A}\right) - a_3 = 2E^*\left(\tilde{A}\right) - \left(\frac{EV\left(\tilde{A}\right) + E^*\left(\tilde{A}\right)}{2}\right) = \frac{3E^*\left(\tilde{A}\right) - EV\left(\tilde{A}\right)}{2} \tag{36}$$

In order to eliminate the risk of introducing a hole in the input domain and to allow a smooth mapping of the system, neighboring fuzzy numbers should overlap.[58,59] The sum of the membership degrees ($\mu_{\tilde{A}}(x)$) of neighboring fuzzy numbers for each point in the overlap should be less than or equal to 1.0.[60] So, the membership degree of the intersection point of neighboring fuzzy numbers should be less than or equal to 0.5, that is, $\mu_{\tilde{A}}(intersection\ point) \leq 0.5$.[61] Several researchers have proposed a value of 0.5 in modeling safety risk.[58,62] It should be noted that developing a fuzzy interval from a crisp interval may result in a linguistic label with meaningless values, or the sum of the membership degrees of neighboring fuzzy numbers for a point in the overlap may be greater than 1.0. Therefore, developed fuzzy numbers need to be modified by changing their supports ($S(\tilde{A}) = [a_1\ a_4]$). Table 3 presents the development of fuzzy numbers from crisp intervals for RRF of SIL ratings and the SIL parameters. In addition, Figure 5 shows these generated fuzzy numbers.

3.2.5 | Expert's opinions aggregating

After collecting experts' opinions for a SIL parameter, they are needed to be aggregated into a single opinion. There are various aggregation methods (e.g., minimum, maximum, and arithmetic mean) for this purpose. The most prevalent aggregation method is the weighted average method which can be defined as.[63,64]

$$\tilde{A}_n = \frac{\sum_{m=1}^{M} W_m \tilde{P}_{nm}}{\sum_{m=1}^{M} W_m} \; ; n = 1, 2, 3, \ldots, N; \; m = 1, 2, 3, \ldots, M \tag{37}$$

where $\tilde{A}_n$ is the aggregated fuzzy number for the SIL parameter n, W_m is the weight of expert m (calculated using Equation (38)), $\tilde{P}_{nm}$ is the corresponding fuzzy number of SIL parameter n given by expert m, and N and M are the number of SIL parameters and experts.

$$W_m = \frac{\sum_{r=1}^{R} ws_{rm}}{\sum_{m=1}^{M} \sum_{r=1}^{R} ws_{rm}} \; ; r = 1, 2, 3, \ldots, R; \; m = 1, 2, 3, \ldots, M \tag{38}$$

where ws_{rm} is the weighting score of attribute r for expert m (obtained from Table 4), and R is the number of attributes.

3.2.6 | Determining the required SIL based on the fuzzy required RRF

$SIL_{Req}^{Individual}$, the required SIL based on the individual risk, and $SIL_{Req}^{Societal}$, the required SIL based on the societal risk, can be calculated as:

$$SIL_{Req}^{Individual} = \min\left(SILn | \log \widetilde{RRF}_{SILn} \geq log \widetilde{RRF}_{Req}^{Individual}\right) \tag{39}$$

$$SIL_{Req}^{Societal} = \min\left(SILn | \log \widetilde{RRF}_{SILn} \geq log \widetilde{RRF}_{Req}^{Societal}\right) \tag{40}$$

where $SIL\ n$ is SIL rating (see Table 1); $\log \widetilde{RRF}_{SIL\ n}$ is the membership functions for RRF of $SIL\ n$ in log scale (see Table 3 and part (k) of Figure 5); $log\ \widetilde{RRF}_{Req}^{Individual}$ is the membership function of the required RRF based on the individual risk in log scale (calculated from Equation (9)), and $log\ \widetilde{RRF}_{Req}^{Societal}$ is the membership function of the required RRF based on the societal risk in log scale (calculated from Equation (12)).

In other words, if $\log \widetilde{RRF}_{SIL\ n} < log\ \widetilde{RRF}_{Req}^{Individual}$, the risk reduction from SIF is not enough for meeting the individual risk tolerance criterion. In addition, if $\log \widetilde{RRF}_{SIL\ n} = log\ \widetilde{RRF}_{Req}^{Individual}$,the selection of $SIL\ n$ is enough as the required level, and it is exact for meeting the individual risk tolerance criterion. If $\log \widetilde{RRF}_{SIL\ n} > log\ \widetilde{RRF}_{Req}^{Individual}$, the risk reduction from SIF is higher than the required risk reduction to meet the individual risk tolerance criterion. Therefore, for individual risk, the minimum SIL level can be selected as the required level when $\log \widetilde{RRF}_{SIL\ n} \geq log\ \widetilde{RRF}_{Req}^{Individual}$. Such strategy for selecting the required SIL in individual risk is applied for societal risk. So, for societal risk, the minimum SIL level that can satisfy $\log \widetilde{RRF}_{SIL\ n} \geq log\ \widetilde{RRF}_{Req}^{Societal}$ is selected as the required level.

To compare two membership functions $\tilde{A}$ and $\tilde{B}$, the membership function $\tilde{A} \ominus \tilde{B}$ can be used. For example, if the area for $x > 0$, that is, A_1, is higher than the area for $x < 0$, that is, A_2, we can say that $\tilde{A} > \tilde{B}$ (Figure 6).

TABLE 3 Development of fuzzy numbers from crisp intervals for RRF of SIL rating and the SIL parameters.

Linguistic variable	Symbol	Crisp interval: Linear scale	Crisp interval: Log scale	$E = [E_* E^*]$	EV	Trapezoidal fuzzy number (a_1, a_2, a_3, a_4)	Modified trapezoidal fuzzy number
Initiating event frequency (f^I)		**[3.162×10^{-6} 3.162]**	**[−5.5 0.5]**				
Negligible	NE	<3.162×10^{-5}	←4.5	[−5.5 −4.5]	−5	(−5.75 −5.25 −4.75 −4.25)	(−5.5 −5.5 −4.75 −4.25)
Approximately negligible	AN	[3.162×10^{-5} 3.162×10^{-4}]	[−4.5 −3.5]	[−4.5 −3.5]	−4	(−4.75 −4.25 −3.75 −3.25)	–
Absolutely low	AL	[3.162×10^{-4} 3.162×10^{-3}]	[−3.5 −2.5]	[−3.5 −2.5]	−3	(−3.75 −3.25 −2.75 −2.25)	–
Very low	VL	[3.162×10^{-3} 3.162×10^{-2}]	[−2.5 −1.5]	[−2.5 −1.5]	−2	(−2.75 −2.25 −1.75 −1.25)	–
Low	LO	[3.162×10^{-2} 3.162×10^{-1}]	[−1.5 −0.5]	[−1.5 −0.5]	−1	(−1.75 −1.25 −0.75 −0.25)	–
Relatively high	RH	[3.162×10^{-1} 3.162]	[−0.5 0.5]	[−0.5 0.5]	0	(−0.75 −0.25 0.25 0.75)	(−0.75 −0.25 0.5 0.5)
Probability of enabling condition (PEC)		**[10^{-2} 1]**	**[−2 0]**				
Not likely	NL	<10^{-1}	←1	[−2 −1]	−1.5	(−2.25 −1.75 −1.25 −0.75)	(−2 −2 −1.25 −0.75)
Possible	PO	≥10^{-1}	≥−1	[−1 0]	−0.5	(−1.25 −0.75 −0.25 0.25)	(−1.25 −0.75 0 0)
Probability of failure on demand (PFD)		**[3.162×10^{-4} 1]**	**[−3.5 0]**				
Not excepted	NX	<3.162×10^{-3}	←2.5	[−3.5 −2.5]	−3	(−3.75 −3.25 −2.75 −2.25)	(−3.5 −3.5 −2.75 −2.25)
Unlikely	UL	[3.162×10^{-3} 3.162×10^{-2}]	[−2.5 −1.5]	[−2.5 −1.5]	−2	(−2.75 −2.25 −1.75 −1.25)	–
Likely	LI	[3.162×10^{-2} 3.162×10^{-1}]	[−1.5 −0.5]	[−1.5 −0.5]	−1	(−1.75 −1.25 −0.75 −0.25)	(−1.75 −1.25 −0.75 −0.375)
Excepted	EX	> 3.162×10^{-1}	>−0.5	[−0.5 0]	−0.25	(−0.625 −0.375 −0.125 0.125)	(−0.75 −0.375 0 0)
Probability of conditional modifier (PCM)		**[10^{-2} 1]**	**[−2 0]**				
Not likely	NL	<10^{-1}	←1	[−2 −1]	−1.5	(−2.25 −1.75 −1.25 −0.75)	(−2 −2 −1.25 −0.75)
Possible	PO	≥10^{-1}	≥−1	[−1 0]	−0.5	(−1.25 −0.75 −0.25 0.25)	(−1.25 −0.75 0 0)

(Continues)

TABLE 3 (Continued)

Linguistic variable	Symbol	Crisp interval: Linear scale	Crisp interval: Log scale	E = [E_* E^*]	EV	Trapezoidal fuzzy number (a_1, a_2, a_3, a_4)	Modified trapezoidal fuzzy number
Vulnerability (V)		**[3.162×10^{-3} 1]**	**[−2.5 0]**				
Low	LO	<3.162×10^{-2}	←1.5	[−2.5 −1.5]	−2	(−2.75 −2.25 −1.75 −1.25)	(−2.5 −2.5 −1.75 −1.25)
Medium	ME	[3.162×10^{-2} 3.162×10^{-1}]	[−1.5 −0.5]	[−1.5 −0.5]	−1	(−1.75 −1.25 −0.75 −0.25)	(−1.75 −1.25 −0.75 −0.406)
High	HI	[3.162×10^{-1} 7.5×10^{-1}]	[−0.5 −0.125]	[−0.5 −0.125]	−0.3125	(−0.594 −0.406 −0.219 −0.031)	(−0.75 −0.406 −0.219 −0.094)
Very high	VH	>7.5×10^{-1}	>−0.125	[−0.125 0]	−0.0625	(−0.156 −0.094 −0.031 0.031)	(−0.219 −0.094 0 0)
Number of people at risk (NP)		**[1 10^4]**	**[0 4]**				
Very low	VL	<2	<0.3	[0 0.3]	0.15	(−0.075 0.075 0.225 0.375)	(0 0 0.225 0.4)
Low	LO	[2 5]	[0.3 0.7]	[0.3 0.7]	0.5	(0.2 0.4 0.6 0.8)	(0.225 0.4 0.6 0.775)
Medium	ME	[5 10]	[0.7 1]	[0.7 1]	0.85	(0.625 0.775 0.925 1.075)	(0.6 0.775 0.925 1.25)
High	HI	[10 10^2]	[1 2]	[1 2]	1.5	(0.75 1.25 1.75 2.25)	(0.925 1.25 1.75 2.25)
Very high	VH	[10^2 10^3]	[2 3]	[2 3]	2.5	(1.75 2.25 2.75 3.25)	–
Absolutely high	AH	>10^3	>3	[3 4]	3.5	(2.75 3.25 3.75 4.25)	(2.75 3.25 4 4)
Apportionment factor for the individual risk criterion (AI)		**[1 10^3]**	**[0 3]**				
Low	LO	<10	<1	[0 1]	0.5	(−0.25 0.25 0.75 1.25)	(0 0 0.75 1.25)
Medium	ME	[10 10^2]	[1 2]	[1 2]	1.5	(0.75 1.25 1.75 2.25)	–
High	HI	>10^2	>2	[2 3]	2.5	(1.75 2.25 2.75 3.25)	(1.75 2.25 3 3)
Apportionment factor for the societal risk criterion (AS)		**[1 10^3]**	**[0 3]**				
Low	LO	<10	<1	[0 1]	0.5	(−0.25 0.25 0.75 1.25)	(0 0 0.75 1.25)
Medium	ME	[10 10^2]	[1 2]	[1 2]	1.5	(0.75 1.25 1.75 2.25)	–
High	HI	>10^2	>2	[2 3]	2.5	(1.75 2.25 2.75 3.25)	(1.75 2.25 3 3)
Individual risk criterion (IRC)		**[3.162×10^{-7} 1]**	**[−6.5 0]**				
Very low	VL	<3.162×10^{-6}	←5.5	[−6.5 −5.5]	−6	(−6.75 −6.25 −5.75 −5.25)	(−6.5 −6.5 −5.75 −5.25)
Low	LO	[3.162×10^{-6} 3.162×10^{-5}]	[−5.5 −4.5]	[−5.5 −4.5]	−5	(−5.75 −5.25 −4.75 −4.25)	–

(Continues)

TABLE 3 (Continued)

Linguistic variable	Symbol	Crisp interval: Linear scale	Crisp interval: Log scale	$E = [E_* E^*]$	EV	Trapezoidal fuzzy number (a_1, a_2, a_3, a_4)	Modified trapezoidal fuzzy number
Medium low	ML	$[3.162\times10^{-5}$ $3.162\times10^{-4}]$	[−4.5 −3.5]	[−4.5 −3.5]	−4	(−4.75 −4.25 −3.75 −3.25)	–
Fair	FA	$[3.162\times10^{-4}$ $3.162\times10^{-3}]$	[−3.5 −2.5]	[−3.5 −2.5]	−3	(−3.75 −3.25 −2.75 −2.25)	–
Medium high	MH	$[3.162\times10^{-3}$ $3.162\times10^{-2}]$	[−2.5 −1.5]	[−2.5 −1.5]	−2	(−2.75 −2.25 −1.75 −1.25)	–
High	HI	$[3.162\times10^{-2}$ $3.162\times10^{-1}]$	[−1.5 −0.5]	[−1.5 −0.5]	−1	(−1.75 −1.25 −0.75 −0.25)	(−1.75 −1.25 −0.75 −0.375)
Very high	VH	$>3.162\times10^{-1}$	>−0.5	[−0.5 0]	−0.25	(−0.625 −0.375 −0.125 0.125)	(−0.75 −0.375 0 0)
Societal risk criterion (SRC)		**$[3.162\times10^{-7}$ 1]**	**[−6.5 0]**				
Very low	VL	$<3.162\times10^{-6}$	←5.5	[−6.5 −5.5]	−6	(−6.75 −6.25 −5.75 −5.25)	(−6.5 −6.5 −5.75 −5.25)
Low	LO	$[3.162\times10^{-6}$ $3.162\times10^{-5}]$	[−5.5 −4.5]	[−5.5 −4.5]	−5	(−5.75 −5.25 −4.75 −4.25)	–
Medium low	ML	$[3.162\times10^{-5}$ $3.162\times10^{-4}]$	[−4.5 −3.5]	[−4.5 −3.5]	−4	(−4.75 −4.25 −3.75 −3.25)	–
Fair	FA	$[3.162\times10^{-4}$ $3.162\times10^{-3}]$	[−3.5 −2.5]	[−3.5 −2.5]	−3	(−3.75 −3.25 −2.75 −2.25)	–
Medium high	MH	$[3.162\times10^{-3}$ $3.162\times10^{-2}]$	[−2.5 −1.5]	[−2.5 −1.5]	−2	(−2.75 −2.25 −1.75 −1.25)	–
High	HI	$[3.162\times10^{-2}$ $3.162\times10^{-1}]$	[−1.5 −0.5]	[−1.5 −0.5]	−1	(−1.75 −1.25 −0.75 −0.25)	(−1.75 −1.25 −0.75 −0.375)
Very high	VH	$>3.162\times10^{-1}$	>−0.5	[−0.5 0]	−0.25	(−0.625 −0.375 −0.125 0.125)	(−0.75 −0.375 0 0)
Risk Reduction Factor (RRF) of SIL rating		**$[10^{-1} 10^{6}]$**	**[−1 6]**				
No safety requirements	NSR	<1	<0	[−1 0]	−0.5	(−1.25 −0.75 −0.25 0.25)	(−1 −1 −0.25 0.25)
No special safety requirements	NSSR	[1 10]	[0 1]	[0 1]	0.5	(−0.25 0.25 0.75 1.25)	–
SIL1	SIL1	$[10\ 10^{2}]$	[1 2]	[1 2]	1.5	(0.75 1.25 1.75 2.25)	–
SIL2	SIL2	$[10^{2}\ 10^{3}]$	[2 3]	[2 3]	2.5	(1.75 2.25 2.75 3.25)	–
SIL3	SIL3	$[10^{3}\ 10^{4}]$	[3 4]	[3 4]	3.5	(2.75 3.25 3.75 4.25)	–
SIL4	SIL4	$[10^{4}\ 10^{5}]$	[4 5]	[4 5]	4.5	(3.75 4.25 4.75 5.25)	–
Not recommended (a single SIF is not sufficient)	NR	$>10^{5}$	>5	[5 6]	5.5	(4.75 5.25 5.75 6.25)	(4.75 5.25 6 6)

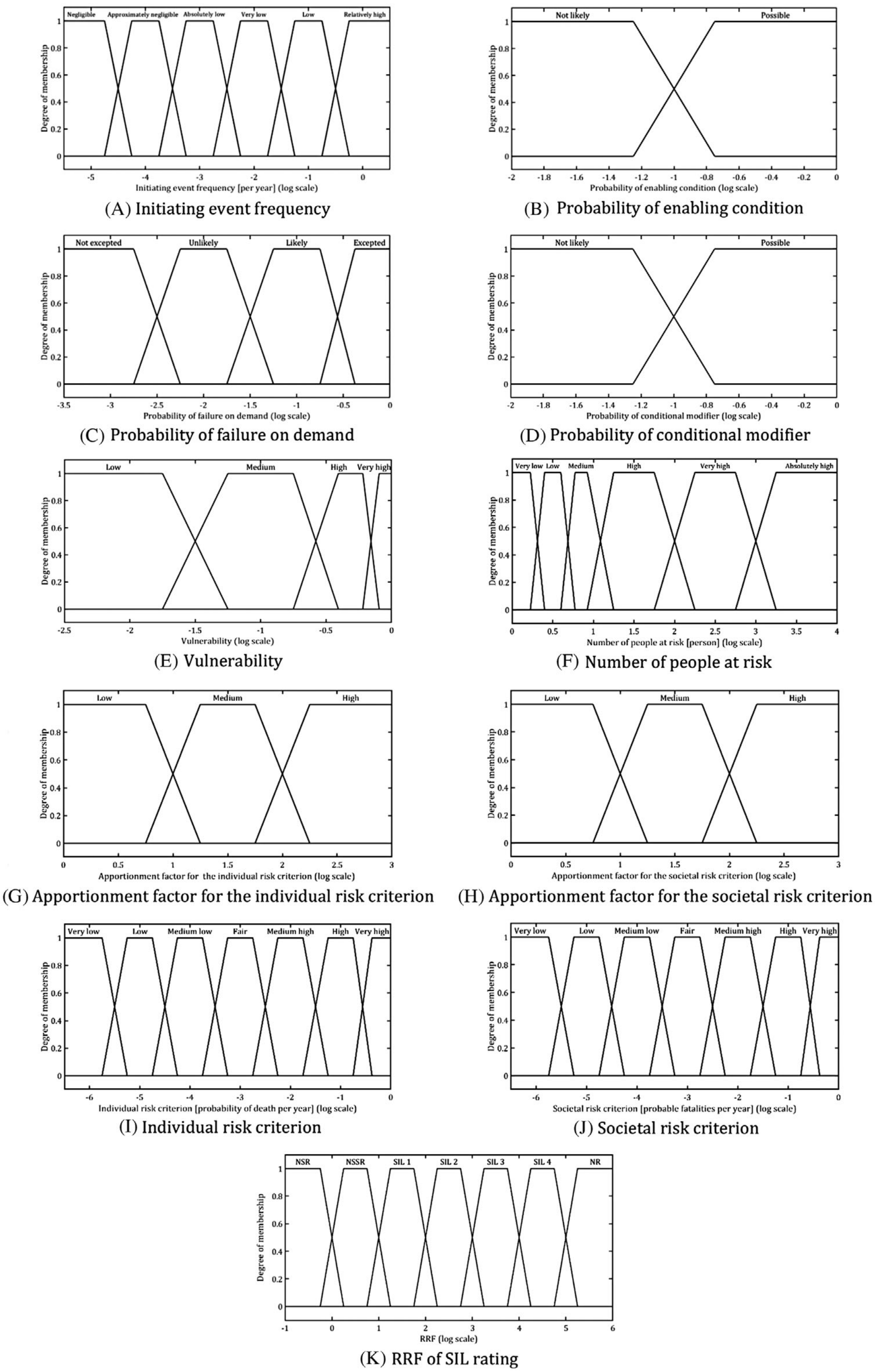


FIGURE 5 The generated fuzzy numbers for RRF of SIL rating and SIL parameters.

3.2.7 | Selecting the final required SIL

Since the SIF should satisfy both individual and societal risk criteria, the final required SIL is calculated as:

$$SIL_{Req}^{Final} = \max\left(SIL_{Req}^{Individual}, SIL_{Req}^{Societal}\right) \tag{41}$$

TABLE 4 Weighting score of different experts' attributes.[63,69]

Attribute	Classification	Weighting score
Professional position	Senior academic/manager	5
	Junior academic/manager	4
	Engineer	3
	Technician	2
	Worker	1
Education background	Doctor of Philosophy (Ph.D.)	5
	Master of Science (MSc)	4
	Bachelor of Science (BSc)	3
	Higher National Diploma (HND)	2
	School level	1
Age (year)	≥ 50	4
	40−49	3
	30−39	2
	< 30	1
Service time (year)	≥ 30	5
	20−29	4
	10−19	3
	6−9	2
	≤ 5	1

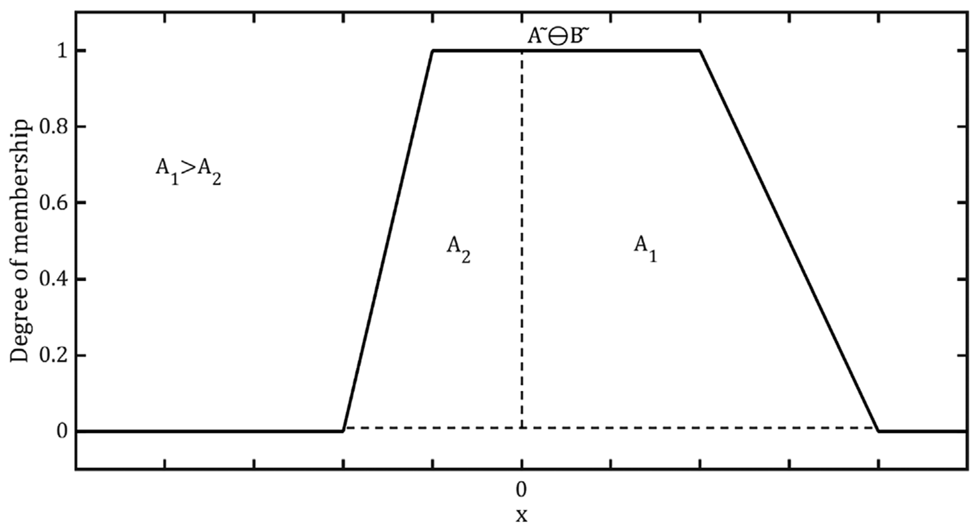


FIGURE 6 An example of comparing two membership functions $\tilde{A}$ and $\tilde{B}$ when $\tilde{A} > \tilde{B}$.

4 | APPLICATION OF THE PROPOSED METHODOLOGY TO A CASE STUDY

To illustrate the capability of the proposed model in chemical industry, consider the flammable liquid vessel in Figure 7.

The vessel liquid level is controlled with a Basic Process Control System (BPCS) that monitors the signal from LT-101, and controls the operation of LCV-101. If a BPCS failure occurs, which requires an immediate shutdown (high level trip), ESDV-101 can be shut by a signal from LSHH-101 to reduce the likelihood of vessel damage and potential fatalities due to a subsequent fire. An independent automatic fire detection system and a firewater deluge system are installed to quench incipient fire. In this case study, the failure of BPCS is considered as the initiating event without further exploring the root causes of such failure, which could otherwise be identified using a comprehensive hazard analysis (e.g., via HAZOP).

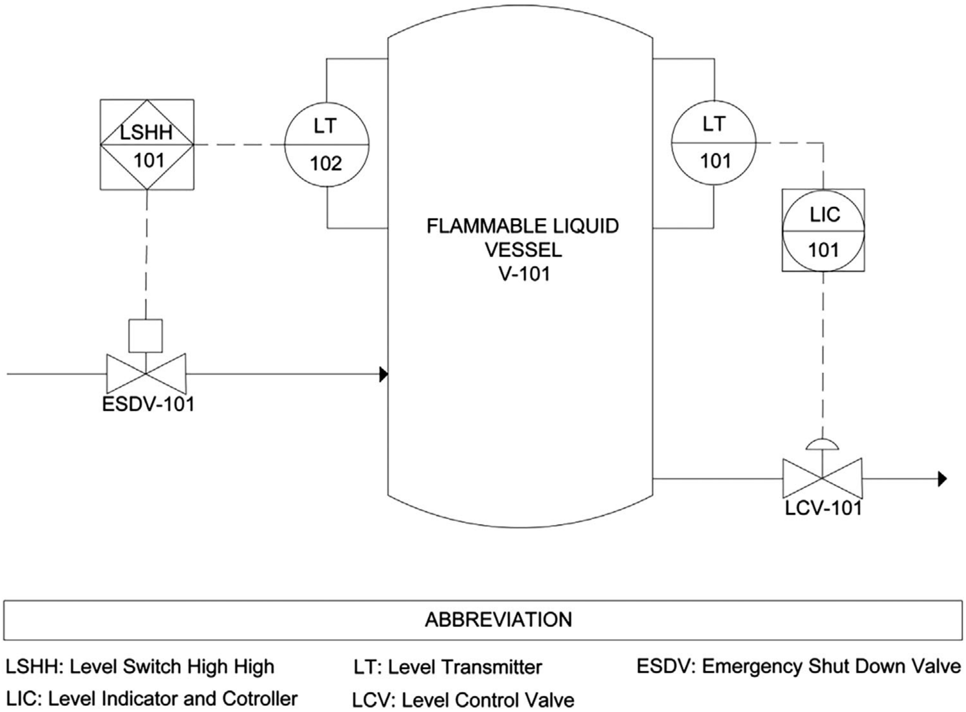


FIGURE 7 Flammable liquid vessel with level protection layers.

LOPA, risk graph, and the proposed model were utilized for the vessel. The results are presented and discussed in the following subsections.

4.1 | Results of the LOPA

Using LOPA, the SIL parameters for the vessel along with their values can be defined as:

Failure frequency of BPCS is considered to be 0.1/yr.[41]

- PFD of the IPL (automatic fire detection and firewater deluge system) is considered to be 0.1.[41]
- The probability of ignition depends on the characteristics of the release and ignition sources. The probability of ignition is 0.5 for releases in general process areas.[12] Consequently, the probability of ignition is considered to be 0.5.
- The probability of personnel presence at the time of accident is assumed to be 0.5.
- Since it is assumed that there is a high probability of catching fire, the vulnerability (probability of death per event) is considered to be 0.5 (see Table 2; V = 0.5)
- The individual risk tolerance criterion for any scenario is chosen to be 10^{-4} per year.[12]

Given the above assumptions, RRF_{Req} can be calculated as:

$$f_i^C = f_i^I \times \prod_{j=1}^{J} PFD_{ij} \times \prod_{k=1}^{K} PCM_k$$

$f^C = f^{BPCS} \times PFD^{PL} \times P^{Ignition} \times P^{Person\ present} \times V = 0.1\ event\ per\ year \times 0.1 \times 0.5 \times 0.5 \times 0.5\ probability\ of\ death\ per\ event = 1.25 \times 10^{-3}\ probability\ of\ death\ per\ year.$

$$RRF_{Req} = \frac{1.25 \times 10^{-3}\ probability\ of\ death\ per\ year}{10^{-4}\ probability\ of\ death\ per\ year} = 12.5$$

TABLE 5 Weighting scores and relative weight of experts.

Expert No.	Professional position	Education background	Age (year)	Service time (year)	Sum of scores	Weight
E1[a]	Senior academic/manager	Ph.D.	40–49	10–19	16	0.222
E2[b]	Senior academic /manager	BSc	≥50	20–29	16	0.222
E3[c]	Senior academic /manager	BSc	40–49	20–29	15	0.208
E4[d]	Junior academic/manager	BSc	40–49	20–29	14	0.195
E5[e]	Engineer	MSc	30–39	6–9	11	0.153
Sum					72	1

[a]Expert 1 is HAZOP and SIL facilitator, PhD in chemical engineering, 44 years old and has 15 years of experience.
[b]Expert 2 is the head of process department, BSc in chemical engineering, 53 years old and has 28 years of experience.
[c]Expert 3 is the head of process safety department, BSc in chemical engineering, 48 years old and has 26 years of experience.
[d]Expert 4 is the head of control and instrumentation department, BSc in control and instrumentation engineering, 44 years old and has 21 years of experience.
[e]Expert 5 is HAZOP and SIL scribe, MSc in chemical and process engineering, 35 years old and has 8 years of experience.

Since RRF_{Req} is 12.5, using Table 1, SIL1 can be determined for the SIF of the vessel.

4.2 | Results of the risk graph

Using the risk graph technique, the SIL parameters and their respective values (see Table 2) can be defined as:

- Twenty operators usually attend the vessel (NP = 20); the consequence parameter is considered as C_d ($C = NP \times V = 10$ probable fatalities per event) (see Table 2). But, if the impact of the IPL is considered in parameter C (the IPL can decrease the number of fatalities), the parameter becomes C_C ($C = NP \times V \times$ PFD of IPL $= 20 \times 0.5 \times 0.1 = 1$).
- The probability of personnel presence is considered to be 0.5, similar to the LOPA technique. Therefore, the occupancy parameter is chosen as F_B (see Table 2).
- Using Table 2, the unavoidability parameter should be considered as P_A, only if the three required conditions (see column 4 of Table 2) are true. Since it is assumed that these three conditions are not true, in this case study, this parameter is considered as P_B.
- If the calibration factor in the risk graph is considered as 0.33 (this value is proposed by UKOOA[37]), then, the demand rate parameter is obtained as W_2 (according to Table 2 and since the failure of BPCS is assumed to be 0.1 per year (i.e., $0.033 < 0.1 < 0.33$)).

Thus, using the risk graph in Figure 2, SIL4 is determined as the needed level. It is worth noting that if the impact of the IPL were considered in parameter C, this level would be reduced to SIL3.

4.3 | Results of the proposed model

A multi-disciplinary group of five experienced members was consulted, including a HAZOP and SIL facilitator, a head of process department, a head of process safety department, a head of control and instrumentation department, and a HAZOP and SIL scribe in a petrochemical facility. The experts' profiles and weight of each expert's opinion calculated using Table 4 and Equation (38) are shown in Table 5.

The experts were asked to express their opinions about the SIL parameters (which one is applicable) in Equation (9) and Equation (12) using the linguistic variables shown in Figure 5 and Table 3 (column 1). Moreover, the CEO was asked to define the parameters "individual risk criterion" and "societal risk criterion" through linguistic variables and based on the organization's policy. Both the CEO's decision and experts' opinions, related to SIL parameters, are shown in Table 6. Using the relative weights of the experts and Equation (37), experts' opinions were aggregated as presented in Table 6.

TABLE 6 Experts' opinions and CEO's decision for determination of SIL parameters in fuzzy scale.

Parameter	Linguistic variable symbol E1	E2	E3	E4	E5	CEO	Fuzzy number E1	E2	E3	E4	E5	CEO	Aggregated Fuzzy number
Failure frequency of BPCS	LO	LO	LO	RH	RH	–	(−1.75 −1.25 −0.75 −0.25)	(−1.75 −1.25 −0.75 −0.25)	(−1.75 −1.25 −0.75 −0.25)	(−0.75 −0.25 0.5 0.5)	(−0.75 −0.25 0.5 0.5)	–	(−1.402 −0.902 −0.315 −0.011)
PFD of the automatic fire detection and firewater deluge system	LI	LI	EX	EX	LI	–	(−1.75 −1.25 −0.75 −0.375)	(−1.75 −1.25 −0.75 −0.375)	(−0.75 −0.375 0 0)	(−0.75 −0.375 0 0)	(−1.75 −1.25 −0.75 −0.375)	–	(−1.347 −0.897 −0.448 −0.224)
Probability of ignition	PO	PO	PO	NL	PO	–	(−1.25 −0.75 0 0)	(−1.25 −0.75 0 0)	(−1.25 −0.75 0 0)	(−2 −2 −1.25 −0.75)	(−1.25 −0.75 0 0)	–	(−1.396 −0.994 −0.244 −0.146)
Probability of personnel presence	PO	PO	NL	NL	PO	–	(−1.25 −0.75 0 0)	(−1.25 −0.75 0 0)	(−2 −2 −1.25 −0.75)	(−2 −2 −1.25 −0.75)	(−1.25 −0.75 0 0)	–	(−1.552 −1.254 −0.504 −0.302)
Vulnerability	HI	ME	HI	VH	HI	–	(−0.75 −0.406 −0.219 −0.094)	(−1.75 −1.25 −0.75 −0.406)	(−0.75 −0.406 −0.219 −0.094)	(−0.219 −0.094 0 0)	(−0.75 −0.406 −0.219 −0.094)	–	(−0.870 −0.532 −0.294 −0.145)
Number of people at risk	HI	HI	HI	ME	ME	–	(0.925 1.25 1.75 2.25)	(0.925 1.25 1.75 2.25)	(0.925 1.25 1.75 2.25)	(0.6 0.775 0.925 1.25)	(0.6 0.775 0.925 1.25)	–	(0.812 1.085 1.463 1.902)
Apportionment factor for the individual risk criterion	ME	LO	ME	LO	ME	–	(0.75 1.25 1.75 2.25)	(0 0 0.75 1.25)	(0.75 1.25 1.75 2.25)	(0 0 0.75 1.25)	(0.75 1.25 1.75 2.25)	–	(0.437 0.729 1.333 1.833)
Apportionment factor for the societal risk criterion	HI	HI	HI	ME	ME	–	(1.75 2.25 3 3)	(1.75 2.25 3 3)	(1.75 2.25 3 3)	(0.75 1.25 1.75 2.25)	(0.75 1.25 1.75 2.25)	–	(1.402 1.902 2.565 2.739)
Individual risk criterion	–	–	–	–	–	FA	–	–	–	–	–	(−3.75 −3.25 −2.75 −2.25)	(−3.75 −3.25 −2.75 −2.25)
Societal risk criterion	–	–	–	–	–	MH	–	–	–	–	–	(−2.75 −2.25 −1.75 −1.25)	(−2.75 −2.25 −1.75 −1.25)

Using Equation (9), $log\ \widetilde{RRF}_{Req}^{Individual}$ is calculated as:

$$\begin{aligned} log\ \widetilde{RRF}_{Req}^{Individual} &= (-1.402\ -0.902\ -0.315\ -0.011) \oplus (-1.347\ -0.897\ -0.448\ -0.224) \\ &\oplus (-1.396\ -0.994\ -0.244\ -0.146) \oplus (-1.552\ -1.254\ -0.504\ -0.302) \\ &\oplus (-0.870\ -0.532\ -0.294\ -0.145) \ominus (-3.75\ -3.25\ -2.75\ -2.25) \oplus (0.437\ 0.729\ 1.333\ 1.833) \\ &= (-3.880\ -1.100\ 2.778\ 4.755) \end{aligned}$$

Using Equation (9), $log\ \widetilde{RRF}_{Req}^{Societal}$ is calculated as:

$$\begin{aligned} log\ \widetilde{RRF}_{Req}^{Societal} &= (-1.402\ -0.902\ -0.315\ -0.011) \oplus (-1.347\ -0.897\ -0.448\ -0.224) \\ &\oplus (-1.396\ -0.994\ -0.244\ -0.146) \oplus (-1.552\ -1.254\ -0.504\ -0.302) \\ &\oplus (-0.870\ -0.532\ -0.294\ -0.145) \oplus (0.812\ 1.085\ 1.463\ 1.902) \ominus (-2.75\ -2.25\ -1.75\ -1.25) \\ &\oplus (1.402\ 1.902\ 2.565\ 2.739) = (-3.103\ 0.158\ 4.473\ 6.563) \end{aligned}$$

The comparison of fuzzy RRF of SIL ratings and calculated fuzzy required RRF in both individual risk and societal risk approaches for this case study are illustrated in Figures 8 and 9, respectively. As can be seen, based on the individual risk, for SILs 1, 2, 3, and 4, $A_1 > A_2$. Thus, after implementing any of these levels, the risk reduction is higher than what is required. Therefore, for the case study of interest, the minimum of these levels (SIL1) can be selected as the required level based on the individual risk. Also, based on the societal risk, for SILs 2, 3, and 4, $A_1 > A_2$. Therefore, SIL2 can be selected as the required level based on the societal risk. Consequently, for meeting both individual and societal risk tolerance criteria in determining the required SIL, SIL2 can be selected as the final required SIL for this case study.

4.4 | The proposed model versus the conventional methods

Compared to the LOPA technique (resulting in SIL1), the result of the risk graph (resulting in SIL 3) is more conservative. In other words, the LOPA technique due to using additional SIL parameters (e.g., PFD of IPL and probability of ignition) results in more precise outcomes in comparison with the risk graph.[10,38,65] In this case, the PFD of the automatic fire detection and firewater deluge system are considered as 0.1, and the probability of ignition is considered as 0.5. So, the amount of current risk in the LOPA is less than the risk graph's. In addition, the difference between the results of these techniques can be due to different types of risk (individual and societal) considered in these techniques. In other words, the risk graph considers societal risk in the form of probable fatalities/year, whereas the LOPA considers the individual risk in the form of probability of death/year. Meanwhile, risk tolerance criteria in the risk graph has been set to 10^{-6} (probable fatalities/year) while in the LOPA it is 10^{-4} (probability of death/year). These unequal values for risk tolerance criteria can be another reason for differences in the results. In the case of the flammable liquid vessel of interest, although the result of the risk graph (SIL3), was more conservative than the result of the LOPA (SIL1), due to considering different types of risk, it cannot be concluded with certainty that the results of the risk graph are always more pessimistic than the results of the LOPA. In other words, without considering the type of risk and the value of risk tolerance criteria, any comparison between the two techniques can lead to an incorrect decision. This challenge makes decision-makers confused about determining the required SIL for their system. To deal with it, a more precise model, such as the proposed model, would be required for determining the required SIL.

In the case study, the LOPA, risk graph and the proposed model determine the required SIL as SIL1, SIL3, and SIL2, respectively. Contrary to the outcomes of the conventional methods, which are crisp numbers or intervals, the result of the proposed model is a fuzzy membership function, which provides a broader range of possible values for the required RRF and SIL by accounting for the uncertainty of the input data. Figure 10 shows the results of the risk graph, LOPA technique, and the proposed model based on both individual and societal risks.

Since the available information and data for real situations are often not crisp and deterministic,[62,66,67] the results of the proposed model are expected to be more accurate than the results of the conventional methods. As can be predicted, the case study also demonstrates that the uncertainty of experts' opinions has considerable influence on the SIL

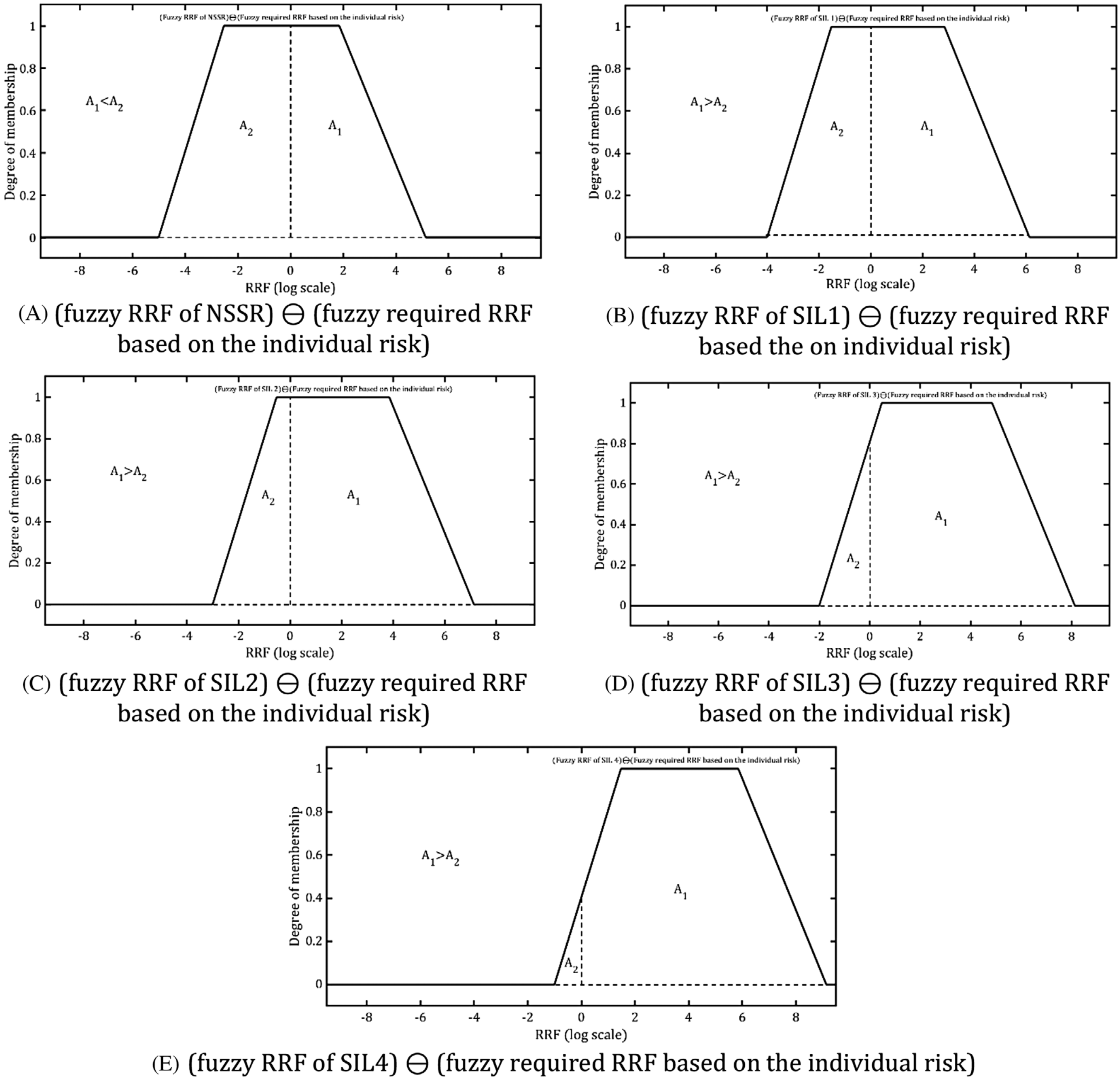
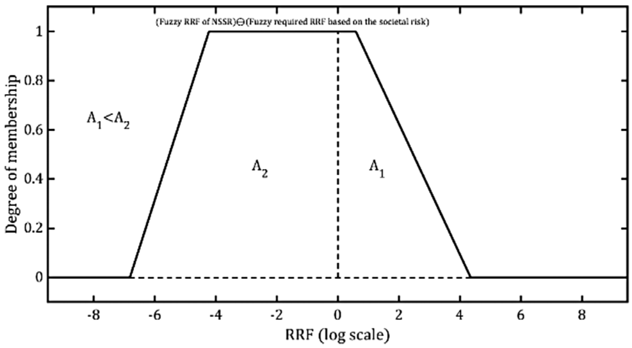


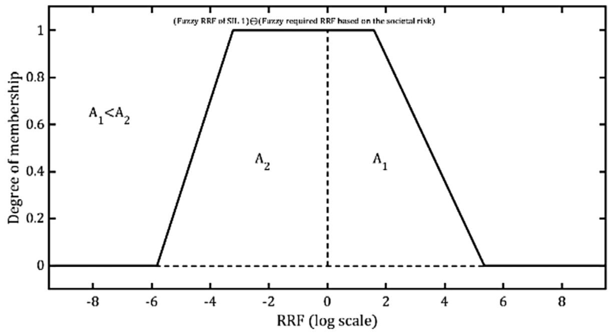
FIGURE 8 The comparison of fuzzy RRF of SIL ratings and the calculated fuzzy required RRF based on the individual risk.

determination. In addition, the difference of the outputs from individual risk and societal risk approaches indicates the important role of risk approaches, and the necessity of a policy for selecting the final SIL. Furthermore, explicitly considering the risk tolerance criteria as separate input parameters, ensures that these influential factors are not ignored. Moreover, assigning the apportionment factor guarantees that the applied individual and societal risks criteria are compliant with the system requirements. In the case study, the proposed model provides two outputs for the required level; SIL1 to meet the individual risk and SIL2 to fulfil the societal risk, meaning that the societal risk plays a pivotal role here. However, the LOPA and risk graph are not able to consider these two risks—individual and societal—together.

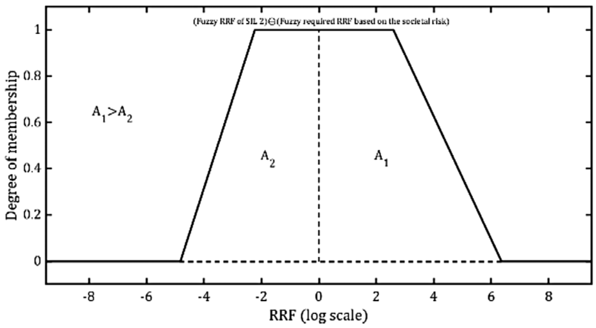
If the public risk (risk to people living near the facility) is concerned, the developed methodology can be expanded to determine the required SIL by considering the individual and societal risks for both on-site personnel and off-site



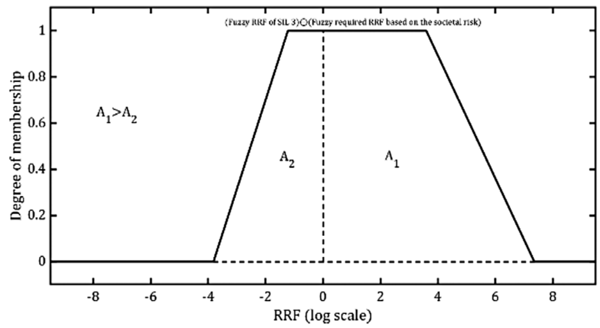
(A) (fuzzy RRF of NSSR) ⊖ (fuzzy required RRF based on the societal risk)



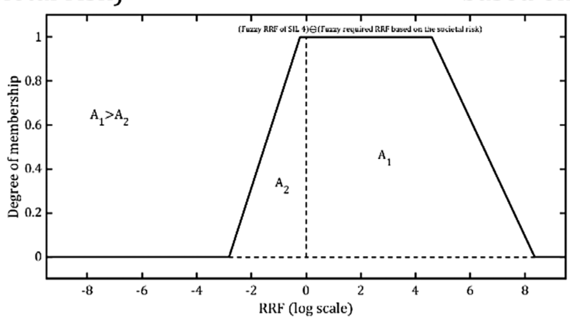
(B) (fuzzy RRF of SIL1) ⊖ (fuzzy required RRF based the on societal risk)



(C) (fuzzy RRF of SIL2) ⊖ (fuzzy required RRF based on the societal risk)



(D) (fuzzy RRF of SIL3) ⊖ (fuzzy required RRF based on the societal risk)



(E) (fuzzy RRF of SIL4) ⊖ (fuzzy required RRF based on the societal risk)

FIGURE 9 The comparison of fuzzy RRF of SIL ratings and the calculated fuzzy required RRF based on the societal risk.

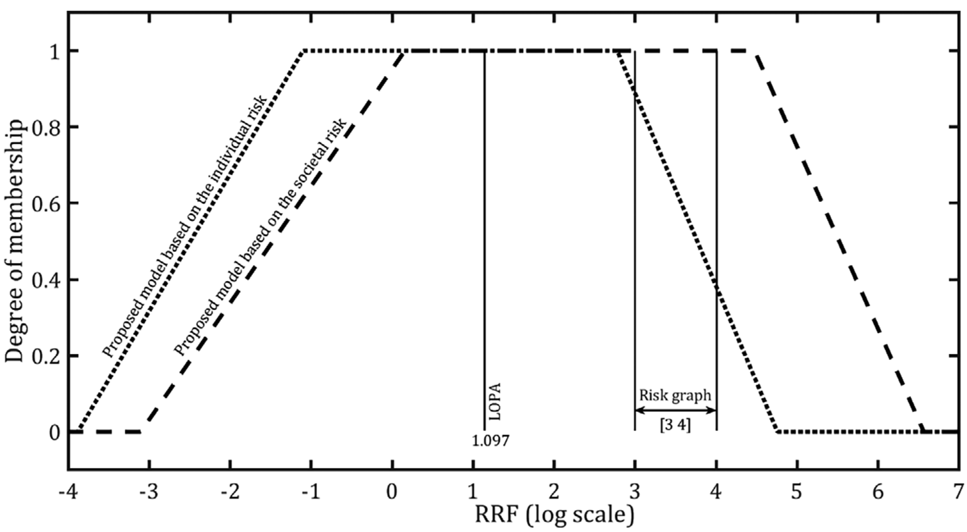


FIGURE 10 Results of the risk graph, LOPA and the proposed model.

people. However, it should be noted that due to differences between off-site and on-site risk tolerance criteria (the former is usually stricter), the public risk tolerance criteria should be set. The developed methodology can further be modified to consider—on top of individual and societal risk criteria—other risk tolerance criteria that organizations may assign to their assets (e.g., environmental impacts and property damage). Due to an ever-increasing physical and cyber attacks to process facilities and due to many similarities between safety risk and security risk,[68] another interesting amendment to the developed methodology could be the inclusion of security risks, in addition to safety risks, in determining a S^2IL (Safety and Security Integrity Level) instead of a SIL. It is also noteworthy that type-1 fuzzy sets are two dimensional, allowing more convenient reference function processing and simple arithmetic operations. In order to improve the consistency of the findings, application of type-2 fuzzy sets, which are three dimensional, can be considered in the future studies.

5 | CONCLUSION

In the present study, an innovative comprehensive fuzzy arithmetic model was developed to determine the required SILs in a process industry as an alternative methodology to LOPA and risk graph techniques. The required RRF was calculated in a fuzzy environment for both individual and societal risks. An apportionment factor was employed to ensure that the effective risk criteria are used based on the level of study. Finally, the required SIL was determined based on a comparison between the fuzzy RRF of SIL ratings and the fuzzy required RRF. Applying the risk graph, LOPA, and the proposed model to determining the required SIL for a storage vessel, it was demonstrated that the proposed model not only benefits from the advantages of risk graph and LOPA techniques but alleviates their limitations. These limitations include: ignoring uncertainty of the input parameters, not evaluating the individual risk and societal risk simultaneously, and the possibility of error in using risk tolerance criteria. Compared with risk graph and LOPA, which both result in crisp values for the required SIL, the proposed model results in a fuzzy membership function, which is more accurate as it can account for the uncertainty of the input data. The proposed model maintains the advantages of conventional techniques, such as their simplicity. However, it requires experienced experts with knowledge of fuzzy logic. It is also more time-consuming in terms of setup and analysis.

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DATA AVAILABILITY STATEMENT

The data underlying this article are available in the article.

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