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Neural-network-based adaptive tracking control for nonlinear pure-feedback systems subject to periodic disturbance

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ABSTRACT

This paper presents an adaptive neural control to solve the tracking problem of a class of pure-feedback systems with non-differentiable non-affine functions in the presence of unknown periodically time-varying disturbances. To handle with the design difficulty from non-affine structure of pure-feedback system, a continuous and positive control gain function is constructed to model the periodically disturbed non-affine function as a form that facilitates the control design. As a result, the non-affine function is not necessary to be differentiable with respect to control variables or input. In addition, the bounds of non-affine function are unknown functions, and some appropriate compact sets are introduced to investigate the bounds of non-affine function so as to cope with the difficulty from these unknown bounds. It is proven that the closed-loop control system is semi-globally uniformly ultimately bounded by choosing the appropriate design parameters. Finally, comparative simulations are provided to illustrate the effectiveness of the proposed control scheme.

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Pure-feedback system; non-affine function; periodic disturbance; robust compensator

1. Introduction

In recent decades, much attention has been focused on adaptive neural control of non-affine nonlinear systems, which have no affine appearance of state variables to be used in the control design and can be classified as strict-feedback and pure-feedback forms (Lv et al., 2021; Sakthivel et al., 2019). Specifically, pure-feedback systems have the more representative form that many actual systems can ultimately fall into this category, such as chemical process, aircraft flight control systems, Duffing oscillator, and mechanical systems, etc (Boulkroune et al., 2012; Kanellakopoulos et al., 1991; Kosmatopoulos & Ioannou, 2002; Li et al., 2015; W. Liu et al., 2017; Namadchian & Rouhani, 2018; Niu et al., 2018; Sun et al., 2013; Tong et al., 2012; Wu et al., 2019; Yang & Pei, 2020; Yoshimura, 2019; Zhang et al., 2017). In Yang and Pei (2020), a redesigned approximate dynamic inversion method is proposed for a class of pure-feedback nonlinear systems, where an intermediate subsystem is constructed to compensate the influence of input saturation. In Wu et al. (2019), a state observer-based adaptive fuzzy dynamic surface control is developed for uncertain discrete-time pure-feedback nonlinear systems with network-induced time-delay. In addition, the adaptive fuzzy tracking control problem is concerned in Tong et al. (2012) for a class of uncertain pure-feedback nonlinear systems with immeasurable states. In the above literatures, the mean value theorem and implicit function theorem are employed to transform the non-affine function into an affine form in order to solve the design difficulty for pure-feedback nonlinear systems. It is worth noting that the aforementioned theorems require the non-affine function must be differentiable with respect to the control variables or input. However, the differentiable condition is too restrictive since the dead-zone and hysteresis nonlinearity always present in real systems, which results in the non-differentiable for non-affine function. In recent years, many scholars try to use novel ways to relax the restrictive differential condition (Z. Liu et al., 2016, 2018). By using the piecewise functions to model the non-affine functions to an affine form, the differentiable assumption on the non-affine nonlinear function is removed as only a continuous condition for non-affine functions is given to guarantee the controllability of system in Z. Liu et al. (2016). Subsequently, this continuous condition is further relaxed in Z. Liu et al. (2018).

On the other hand, the control schemes for nonlinear systems with time-varying disturbances have received increasing attention since the time-varying disturbances exist in a wide range of mechanical systems and devices, such as industrial robots and numerical control machines (Chen, 2009; Chen & Jiao, 2010; Chen et al., 2010; Ding, 2007; Tian & Yu, 2003; Xu, 2004). As for nonlinear systems with unknown functions independent from unmeasured time-varying disturbances, one of the most common schemes is to employ the function approximators such as neural networks (NNs) or fuzzy logic systems (FLS) to approximate the unknown functions (Ding, 2007; Tian & Yu, 2003; Xu, 2004). Unfortunately, it is a challenging task to design the suitable function approximators to model the unknown functions affected by the unmeasured time-varying disturbances (Chen, 2009; Chen & Jiao, 2010; Chen et al., 2010).

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In Chen (2009), by introducing Fourier series expansion (FSE), a new function approximator is incorporated into the NNs-based adaptive control design framework for a class of nonlinear systems. However, the external disturbance of systems is not considered, and it still needs many important assumptions, namely, the known signs of control direction must be strictly positive or negative, and the bounded condition of gain functions with both upper and lower bounds, which restrict the applicability of control design. When there is no a priori knowledge about the signs of control gains and the bounds of the gain functions, the existing control schemes cannot be utilised directly.

Motivated by the aforementioned discussion, this paper investigates the tracking control problem of pure-feedback nonlinear systems possessing non-differentiable non-affine functions affected by unknown periodically time-varying disturbances. The main contributions are listed as follows.

(1) We investigate a more general case that all the control inputs and periodic disturbances appear implicitly in the system functions, which makes control design difficult and challenging. In addition, with the aid of a novel modelling method, the presented control strategy is free from the circular control construction problem, which is common but serious in the NNs-based control design.
(2) In contrast to the state of the art, a more relaxed assumption is constructed for the non-affine nonlinear function, removing the restrictive differential condition used widely in the existing literature. To be specific, the control gain function is modelled as positive and continue, facilitating the control design and engineering implementation.
(3) By utilising Lyapunov analysis, it is rigorously proved that all the signals of the closed-loop system are semi-globally uniformly ultimately bounded and the tracking error converges to a small neighbourhood of the origin by choosing the appropriate design parameters.

The rest of paper is organised as follows. The considered system and preliminary knowledge are given in Section 2. By incorporating the FSE and radial basis function NNs (RBFNNs), an adaptive tracking control scheme is designed in Section 3. Appropriate compact sets are introduced to investigate the bounds of non-affine function in Section 4. In Section 5, the system stability is rigorously proved via Lyapunov stability theorem. Two simulation examples are presented in Section 6 to show the effectiveness of the proposed theoretical results. Finally, the conclusion is obtained in Section 7.

2. Problem statement and preliminaries

2.1 Problem formulation

Consider a class of uncertain pure-feedback nonlinear systems as follows

$$\begin{cases} \dot{x}_i = f_i(\bar{x}_i, x_{i+1}, \omega_i(t)) + d_i(t), \quad i = 1, \ldots, n-1, \\ \dot{x}_n = f_n(x, u, \omega_n(t)) + d_n(t), \\ y = x_1, \end{cases} \tag{1}$$

where $\bar{x}_i = [x_1, \ldots, x_i]^\top \in R^i$ and $x = [x_1, \ldots, x_n]^\top \in R^n$ are system state variables, $u \in R$ is system control input, $y \in R$ is system output, $f_i(\cdot)$ are unknown non-affine functions, $d_i(t)$ are the unknown external disturbances, $\omega_i(t) : [0, +\infty) \to R^{m_i} (i = 1, \ldots, n)$ are unknown and continuously time-varying disturbances with known periods T_i, that is, $\omega_i(t + T_i) = \omega_i(t)$. For the sake of brevity, $\omega_i(t)$ is denoted by ω_i throughout this paper.

Assumption 2.1 (Z. Liu et al., 2016, 2018)**:** Define $F_i(\bar{x}_i, x_{i+1}, \omega_i) = f_i(\bar{x}_i, x_{i+1}, \omega_i) - f_i(\bar{x}_i, 0, \omega_i) (i = 1, \ldots, n)$, and denote $x_{n+1} = u$ and $\bar{x}_{n+1} = [x^\top, u]^\top$ for the sake of convenience. We assume that functions $F_i(\bar{x}_i, x_{i+1}, \omega_i)$ satisfy

$$\begin{cases} \underline{F}_i(\bar{x}_i, \omega_i)x_{i+1} + C_{i,1}(\bar{x}_i, \omega_i) \\ \quad \le F_i(\bar{x}_i, x_{i+1}, \omega_i) & x_{i+1} \ge 0, \\ \le \overline{F}_i(\bar{x}_i, \omega_i)x_{i+1} + C_{i,2}(\bar{x}_i, \omega_i), \\ \underline{F}'_i(\bar{x}_i, \omega_i)x_{i+1} + C_{i,3}(\bar{x}_i, \omega_i) \\ \quad \le F_i(\bar{x}_i, x_{i+1}, \omega_i) & x_{i+1} < 0, \\ \le \overline{F}'_i(\bar{x}_i, \omega_i)x_{i+1} + C_{i,4}(\bar{x}_i, \omega_i), \end{cases} \tag{2}$$

where $\underline{F}_i(\bar{x}_i, \omega_i)$, $\overline{F}_i(\bar{x}_i, \omega_i)$, $\underline{F}'_i(\bar{x}_i, \omega_i)$, and $\overline{F}'_i(\bar{x}_i, \omega_i)$ are unknown positive continuous functions, while $C_{i,1}(\bar{x}_i, \omega_i)$, $C_{i,2}(\bar{x}_i, \omega_i)$, $C_{i,3}(\bar{x}_i, \omega_i)$, and $C_{i,4}(\bar{x}_i, \omega_i)$ are unknown continuous functions.

Remark 2.1: It should be noted that the bounds of non-affine functions $f_i(\bar{x}_i, x_{i+1}, \omega_i)$ are some unknown positive functions $\underline{F}_i(\bar{x}_i, \omega_i)$, $\overline{F}_i(\bar{x}_i, \omega_i)$, $\underline{F}'_i(\bar{x}_i, \omega_i)$, and $\overline{F}'_i(\bar{x}_i, \omega_i)$, which makes the control design difficult and challenging. In the following, some appropriate compact sets will be introduced to investigate the bounds of these unknown functions so as to cope with this difficulty.

For $\forall a, b \in R$, if $a \le x \le b$, then $x = \theta a + (1-\theta)b$, where $\theta = \frac{b-x}{b-a}$. Thus there exist functions $\theta_{i,1}(\bar{x}_i, x_{i+1}, \omega_i)$ and $\theta_{i,2}(\bar{x}_i, x_{i+1}, \omega_i)$ taking values in the closed interval $[0, 1]$ and satisfying

$$F_i(\bar{x}_i, x_{i+1}, \omega_i) = \begin{cases} (1 - \theta_{i,1}(\bar{x}_i, x_{i+1}, \omega_i)) \\ (\underline{F}_i(\bar{x}_i, \omega_i)x_{i+1} + C_{1,i}(\bar{x}_i, \omega_i)) \\ \quad +\theta_{i,1}(\bar{x}_i, x_{i+1}, \omega_i) \\ \quad (\overline{F}_i(\bar{x}_i, \omega_i)x_{i+1} \\ \quad +C_{2,i}(\bar{x}_i, \omega_i)), \quad x_{i+1} \ge 0, \\ (1 - \theta_{i,2}(\bar{x}_i, x_{i+1}, \omega_i)) \\ (\underline{F}'_i(\bar{x}_i, \omega_i)x_{i+1} + C_{3,i}(\bar{x}_i, \omega_i)) \\ \quad +\theta_{i,2}(\bar{x}_i, x_{i+1}, \omega_i) \\ (\overline{F}'_i(\bar{x}_i, \omega_i)x_{i+1} + C_{4,i}(\bar{x}_i, \omega_i)), \quad x_{i+1} < 0. \end{cases} \tag{3}$$

Define functions $G_i(\bar{x}_{i+1}, \omega_i)$ and $\Delta_i(\bar{x}_{i+1}, \omega_i)$ as follows

$$G_i(\bar{x}_{i+1}, \omega_i) = \begin{cases} (1 - \theta_{i,1}(\bar{x}_i, x_{i+1}, \omega_i))\underline{F}_i(\bar{x}_i, \omega_i) \\ \quad +\theta_{i,1}(\bar{x}_i, x_{i+1}, \omega_i)\overline{F}_i(\bar{x}_i, \omega_i), \quad x_{i+1} > a, \\ g_i(\bar{x}_{i+1}, \omega_i), \quad -a \le x_{i+1} \le a, \\ (1 - \theta_{i,2}(\bar{x}_i, x_{i+1}, \omega_i))\underline{F}'_i(\bar{x}_i, \omega_i) \\ \quad +\theta_{i,2}(\bar{x}_i, x_{i+1}, \omega_i)\overline{F}'_i(\bar{x}_i, \omega_i), \quad x_{i+1} < -a, \end{cases} \tag{4}$$

$$\Delta_i(\bar{x}_{i+1},\omega_i) = \begin{cases} (1-\theta_{i,1}(\bar{x}_i,x_{i+1},\omega_i))C_{1,i}(\bar{x}_i,\omega_i) \\ \quad +\theta_{i,1}(\bar{x}_i,x_{i+1},\omega_i)C_{2,i}(\bar{x}_i,\omega_i), \quad x_{i+1} > a, \\ F_i(\bar{x}_i,x_{i+1},\omega_i) - g_i(\bar{x}_{i+1},\omega_i)x_{i+1}, \\ -a \le x_{i+1} \le a, \\ (1-\theta_{i,2}(\bar{x}_i,x_{i+1},\omega_i))C_{3,i}(\bar{x}_i,\omega_i) \\ \quad +\theta_{i,2}(\bar{x}_i,x_{i+1},\omega_i) \\ \quad C_{4,i}(\bar{x}_i,\omega_i), \quad x_{i+1} < -a, \end{cases} \tag{5}$$

where

$$g_i(\bar{x}_{i+1},\omega_i) = (y_{i,2}(\bar{x}_i,\omega_i) - y_{i,1}(\bar{x}_i,\omega_i))\frac{a-x_{i+1}}{2a} + y_{i,1}(\bar{x}_i,\omega_i), \tag{6}$$

$$y_{i,1}(\bar{x}_i,\omega_i) = (1-\theta_{i,1}(\bar{x}_i,a,\omega_i))\underline{F}_i(\bar{x}_i,\omega_i) + \theta_{i,1}(\bar{x}_i,a,\omega_i)\overline{F}_i(\bar{x}_i,\omega_i), \tag{7}$$

$$y_{i,2}(\bar{x}_i,\omega_i) = (1-\theta_{i,2}(\bar{x}_i,-a,\omega_i))\underline{F}'_i(\bar{x}_i,\omega_i) + \theta_{i,2}(\bar{x}_i,-a,\omega_i)\overline{F}'_i(\bar{x}_i,\omega_i), \tag{8}$$

with a being an arbitrary positive constant.

From (7) and (8) we can have

$$0 < \min\{\underline{F}_i(\bar{x}_i,\omega_i),\overline{F}_i(\bar{x}_i,\omega_i)\} \le y_{i,1}(\bar{x}_i,\omega_i) \le \max\{\underline{F}_i(\bar{x}_i,\omega_i),\overline{F}_i(\bar{x}_i,\omega_i)\}, \tag{9}$$

$$0 < \min\{\underline{F}'_i(\bar{x}_i,\omega_i),\overline{F}'_i(\bar{x}_i,\omega_i)\} \le y_{i,2}(\bar{x}_i,\omega_i) \le \max\{\underline{F}'_i(\bar{x}_i,\omega_i),\overline{F}'_i(\bar{x}_i,\omega_i)\}. \tag{10}$$

By noting the definition of $g_i(\bar{x}_{i+1},\omega_i)$, we have

$$0 < \min\{y_{i,1}(\bar{x}_i,\omega_i),y_{i,2}(\bar{x}_i,\omega_i)\} \le g_i(\bar{x}_{i+1},\omega_i) \le \max\{y_{i,1}(\bar{x}_i,\omega_i),y_{i,2}(\bar{x}_i,\omega_i)\}. \tag{11}$$

Then, substituting (9) and (10) into (11), we have

$$0 < \min\{\underline{F}_i(\bar{x}_i,\omega_i),\overline{F}_i(\bar{x}_i,\omega_i),\underline{F}'_i(\bar{x}_i,\omega_i),\overline{F}'_i(\bar{x}_i,\omega_i)\} \le g_i(\bar{x}_{i+1},\omega_i) \le \max\{\underline{F}_i(\bar{x}_i,\omega_i),\overline{F}_i(\bar{x}_i,\omega_i),\underline{F}'_i(\bar{x}_i,\omega_i),\overline{F}'_i(\bar{x}_i,\omega_i)\}. \tag{12}$$

From (4), (6), and (12), we know $G_i(\bar{x}_{i+1},\omega_i)$ is a continuous function and satisfy

$$0 < G_{i,0}(\bar{x}_i,\omega_i) \le G_i(\bar{x}_{i+1},\omega_i) \le G_{i,1}(\bar{x}_i,\omega_i), \tag{13}$$

where $G_{i,0}(\bar{x}_i,\omega_i) = \min\{\underline{F}_i(\bar{x}_i,\omega_i),\overline{F}_i(\bar{x}_i,\omega_i),\underline{F}'_i(\bar{x}_i,\omega_i),\overline{F}'_i(\bar{x}_i,\omega_i)\}$ and $G_{i,1}(\bar{x}_i,\omega_i) = \max\{\underline{F}_i(\bar{x}_i,\omega_i),\overline{F}_i(\bar{x}_i,\omega_i),\underline{F}'_i(\bar{x}_i,\omega_i),\overline{F}'_i(\bar{x}_i,\omega_i)\}$.

For $-a \le x_{i+1} \le a$, there exist unknown continuous functions $\kappa_{\Delta_i}(\bar{x}_i,\omega_i)$ satisfy $|\Delta_i(\bar{x}_{i+1},\omega_i)| = |F_i(\bar{x}_i,x_{i+1},\omega_i) - g_i(\bar{x}_{i+1},\omega_i)x_{i+1}| \le \kappa_{\Delta_i}(\bar{x}_i,\omega_i)$.

And from (5), it can be known that

$$0 \le |\Delta_i(\bar{x}_{i+1},\omega_i)| \le C_{i,M}(\bar{x}_i,\omega_i), \tag{14}$$

in which $C_{i,M}(\bar{x}_i,\omega_i) = \max\{|C_{i,1}(\bar{x}_i,\omega_i)|,|C_{i,2}(\bar{x}_i,\omega_i)|,|C_{i,3}(\bar{x}_i,\omega_i)|,|C_{i,4}(\bar{x}_i,\omega_i)|,\kappa_{\Delta_i}(\bar{x}_i,\omega_i)\}$. Therefore, from (4) and (5), we can rewrite (3) as

$$F_i(\bar{x}_i,x_{i+1},\omega_i) = G_i(\bar{x}_{i+1},\omega_i)x_{i+1} + \Delta_i(\bar{x}_{i+1},\omega_i). \tag{15}$$

Remark 2.2: Note that $G_i(\bar{x}_{i+1},\omega_i)$ in (15) can be seen as the control gain functions which are continuous and positive. Here we investigate the case that $f_i(\bar{x}_i,x_{i+1},\omega_i)$ are non-affine functions for ω_i and $\bar{x}_{i+1}$, which has not been considered in the available literature. Compared with the existing results, this case has a more general form and can represent many practical systems such as industrial robots, numerical control machines and autonomous underwater vehicles (Peng et al., 2019).

Assumption 2.2 (Peng et al., 2019; Wen & Ren, 2011): The reference trajectory y_d is sufficiently smooth function of t, and $y_d, \dot{y}_d$, and $\ddot{y}_d$ are bounded, that is, there exists a positive constant B_0 such that $\Pi_0 = \{(y_d,\dot{y}_d,\ddot{y}_d) \mid (y_d)^2 + (\dot{y}_d)^2 + (\ddot{y}_d)^2 \le B_0^2\}$.

Assumption 2.3 (Z. Liu et al., 2016, 2018): For $i = 1,\ldots,n$, there exist unknown positive constants d_i^* such that $|d_i(t)| \le d_i^*$.

2.2 FSE-RBFNNs-based approximator

The RBFNNs and FSE are combined to be used for the controller design in this paper. We first employ FSE to estimate ω_i, and then employ estimated values of ω_i as one of the RBFNNs inputs to approximate unknown functions $h_i(\chi_i,\omega_i)$.

Without loss of generality, we consider an unknown function $h_i(\chi_i,\omega_i)$, where $\chi_i \in \Pi_i \times \Pi_0$ is a measured signal with $\Pi_i \times \Pi_0$ a compact set, and $\omega_i = [\omega_{i,1},\ldots,\omega_{i,m}]^\top \in \Pi_\omega$ is an unknown continuous disturbance vector of known period T with Π_ω a compact set, $\Pi_\omega = \{(\omega_1,\omega_2,\ldots,\omega_n) | \sum_{j=1}^n \omega_j^\top \omega_j \le M_\omega^2\}$ with M_ω being a positive constant. On the one hand, the continuous and periodic disturbance vector ω_i can also be expressed by a linearly parameterised FSE as

$$\omega_i = S_i^\top \phi_i + \delta_{\omega_i}, \tag{16}$$

where $S_i = [S_{i,1},\ldots,S_{i,m}] \in R^{q\times m}$ is a constant matrix with $S_{i,j} \in R^q$ being a vector consisting of the first q coefficients of the FSE of $\omega_{i,j}$ (q is an odd integer), δ_{ω_i} is the truncation error with the minimum upper bound $\delta_{\omega_i}^* > 0$, which can be arbitrarily decreased by increasing q, and $\phi_i(t) = [\phi_{i,1}(t),\ldots,\phi_{i,q}(t)]^\top$ with $\phi_{i,1}(t) = 1, \phi_{i,2j}(t) = \sqrt{2}\sin(2\pi jt/T)$, and $\phi_{i,2j+1}(t) = \sqrt{2}\cos(2\pi jt/T)(j = 1,\ldots,(q-1)/2)$, whose derivatives up to n-order are smooth and bounded.

On the other hand, if ω_i is measured, the unknown function $h_i(\chi_i,\omega_i)$ can be approximated over the compact set $\Pi_i \times \Pi_0 \times \Pi_\omega$ by a RFBNNs as follows (Kurdila et al., 1995; Y. Liu et al., 2016; Park & Sandberg, 1991)

$$h_i(\chi_i,\omega_i) = W_i^\top \psi_i(\chi_i,\omega_i) + \delta_{h_i}(\chi_i,\omega_i), \tag{17}$$

where $\psi_i(\chi_i,\omega_i) = [\psi_{i,1}(\chi_i,\omega_i),\ldots,\psi_{i,p}(\chi_i,\omega_i)]^\top$ is a known smooth vector-valued function with the component $\psi_{i,j}(\chi_i,\omega_i) = \exp[-\|Z_i - \mu_{i,j}\|^2/\kappa^2](j = 1,\ldots,p)$, here $Z_i = [\chi_i^\top,\omega_i^\top]^\top$, $\mu_{i,j} \in \Pi_i \times \Pi_0 \times \Pi_\omega$ is a constant that is called the centre of $\psi_{i,j}(\chi_i,\omega_i)$, and $\kappa > 0$ is a real number that is called the width of $\psi_{i,j}(\chi_i,\omega_i)$. The optimal weight vector $W_i = [W_{i,1},\ldots,W_{i,p}]^\top$ is defined as $W_i := \arg\min_{\hat{W}_i\in R^p}\{\sup_{(\chi_i,\omega_i)\in\Pi_i\times\Pi_0\times\Pi_\omega}|h_i(\chi_i,\omega_i) - \hat{W}_i^\top \psi_i(\chi_i,\omega_i)|\}$, and $\delta_{h_i}(\chi_i,\omega_i)$ is the inherent NNs approximation error with the minimum upper bound $\delta_{h_i}^* > 0$, which can be decreased by increasing the NNs node number p (Park et al., 2009; Seshagiri & Khalil, 2000; Zuo et al., 2019).

By replacing ω_i in (17) with (16), we have

$$h_i(\chi_i, \omega_i) = W_i^\top \psi_i(\chi_i, S_i^\top \phi_i + \delta_{\omega_i}) + \delta_{h_i}, \\ = W_i^\top \psi_i(\chi_i, S_i^\top \phi_i) + \varepsilon_i(\chi_i), \tag{18}$$

where $\varepsilon_i(\chi_i) = \delta_{h_i} + W_i^\top \psi_i(\chi_i, S_i^\top \phi_i + \delta_{\omega_i}) - W_i^\top \psi_i(\chi_i, S_i^\top \phi_i)$.

Lemma 2.1 (Chen, 2009): *For $(\chi_i, \omega_i) \in \Pi_i \times \Pi_0 \times \Pi_\omega$, the approximation error $\varepsilon_i(\chi_i)$ in (18) satisfies*

$$|\varepsilon_i(\chi_i)| \le \varepsilon_i^*, \tag{19}$$

where ε_i^* denotes the minimum upper bound of $\varepsilon_i(\chi_i)$, which can be arbitrarily decreased by increasing p and q.

Lemma 2.2 (Chen, 2009): *For approximator (18), the estimation error can be expressed as*

$$W_i^\top \psi_i(\chi_i, S_i^\top \phi_i) - \hat{W}_i^\top \psi_i(\chi_i, \hat{S}_i^\top \phi_i) = \tilde{W}_i^\top (\hat{\psi}_i - \hat{\psi}_i' \hat{S}_i^\top \phi_i) \\ + \hat{W}_i^\top \hat{\psi}_i' \tilde{S}_i^\top \phi_i + z_i, \tag{20}$$

in which $\hat{\psi}_i = \psi_i(\chi_i, \hat{S}_i^\top \phi_i)$, $\hat{\psi}_i' = [\hat{\psi}_{i,1}', \hat{\psi}_{i,2}', \ldots, \hat{\psi}_{i,p}']^\top \in R^{p\times m}$ with $\hat{\psi}_{i,j}' = (\partial \psi_{i,j}(\chi_i, \omega_i))/\partial \omega_i|_{\omega_i = \hat{S}_i^T \phi_i} (j = 1, \ldots, p)$, and the residual term z_i is bounded by

$$|z_i| \le z_i^* = \|S_i\|_F \, \|\phi_i \hat{W}_i^\top \hat{\psi}_i'\|_F + \|W_i\| \, \|\hat{\psi}_i' \hat{S}_i^\top \phi_i\| + |W_i|_1. \tag{21}$$

In this paper, let $||\cdot||$ denotes the Euclidean norm of a vector, $||\cdot||_F$ denotes the Frobenius norm of a matrix, $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the largest and smallest eigenvalues of a square matrix A, respectively.

3. adaptive neural controller design

In this section, adaptive neural control is presented for the system (1). First of all, consider the following change of coordinates:

$$\begin{cases} e_1 = x_1 - y_d, \\ e_i = x_i - \alpha_{if}, \quad i = 2, \ldots, n, \end{cases} \tag{22}$$

where α_{if} is the output of the second-order filter with α_{i-1} as the input.

The recursive design procedure contains n steps. At step $i(i = 1, \ldots, n-1)$, the virtual control law α_i is designed to make the system toward stability position. Finally, the actual control law u is designed for stability analysis in step n.

Step i (i = 1,…,n-1): Noting $F_i(\bar{x}_i, x_{i+1}, \omega_i) = f_i(\bar{x}_i, x_{i+1}, \omega_i) - f_i(\bar{x}_i, 0, \omega_i)$ and denoting $\alpha_{1f} = y_d$, the time derivatives of e_i can be expressed as

$$\dot{e}_i = f_i(\bar{x}_i, 0, \omega_i) + G_i(\bar{x}_{i+1}, \omega_i) x_{i+1} + \Delta_i + d_i - \dot{\alpha}_{if}. \tag{23}$$

Construct an intermediate virtual control law α_i and the adaptation laws for $\hat{S}_i$ and $\hat{W}_i$ as follows

$$\alpha_i = -k_i e_i - \hat{W}_i^\top \psi_i(\chi_i, \hat{S}_i^\top \phi_i) \varpi_i, \tag{24}$$

$$\varpi_i = \tanh\left(\frac{e_i \hat{W}_i^\top \psi_i(\chi_i, \hat{S}_i^\top \phi_i)}{\upsilon_i} \right), \tag{25}$$

$$\dot{\hat{S}}_i = \Gamma_{S_i} [e_i \phi_i \hat{W}_i^\top \hat{\psi}_i' - \sigma_i \hat{S}_i], \tag{26}$$

$$\dot{\hat{W}}_i = \Gamma_{W_i} [e_i (\hat{\psi}_i - \hat{\psi}_i' \hat{S}_i^\top \phi_i) - \sigma_i \hat{W}_i], \tag{27}$$

where $\Gamma_{S_i} = \Gamma_{S_i}^\top > 0$ and $\Gamma_{W_i} = \Gamma_{W_i}^\top > 0$ are the adaptive gain matrices, and $k_i > 0, \sigma_i > 0, \upsilon_i > 0$ are the design parameters. Recalling the construction of $\dot{\hat{S}}_i$ and $\dot{\hat{W}}_i$ in (26) and (27), it is straightforward to deduce that for any given bounded initial condition $\hat{S}_i(0) \ge 0$ and $\hat{W}_i(0) \ge 0$, we have $\hat{S}_i(t) \ge 0$ and $\hat{W}_i(t) \ge 0$ for $\forall t \ge 0$, respectively.

The backstepping method suffers from the problem of 'explosion of complexity', which is caused by repeatedly differentiating α_i. The dynamic surface control (DSC) scheme is therefore used here to hand with this problem (Swaroop et al., 2000). Since the first-order filter may be sensitive to measurement noises, in this paper, we replace it with the second-order filter, which is different from the traditional DSC scheme. For $i = 1, \ldots, n-1$, define the state space implementation of the second-order filters as

$$\dot{z}_{i,1} = \omega_n z_{i,2}, \tag{28}$$

$$\dot{z}_{i,2} = -2\zeta \omega_n z_{i,2} - \omega_n (z_{i,1} - \alpha_i), \tag{29}$$

with $\alpha_{i+1f} = z_{i,1}$ and $\dot{\alpha}_{i+1f} = \omega_n z_{i,2}$ as the outputs of each filter. The filter initial conditions are $z_{i,1}(0) = \alpha_i(0)$ and $z_{i,2}(0) = 0$. The filter design parameters are $\omega_n > 0$ and $\zeta \in (0, 1]$. Each command filter is designed to compute α_{i+1f} and $\dot{\alpha}_{i+1f}$ without differentiation. The transfer functions corresponding to (28) and (29) are

$$\frac{\begin{bmatrix} \omega_n^2 \\ s\omega_n^2 \end{bmatrix}}{s^2 + 2\zeta \omega_n s + \omega_n^2}.$$

The natural frequency of the command filter is equal to the parameter ω_n; the filter has unit dc gain to the first output; and the first output is the integral of the second output.

By defining the output error of this filter as $y_{i+1} = \alpha_{i+1f} - \alpha_i$, it yields $\dot{\alpha}_{i+1f} = -(\ddot{\alpha}_{i+1f}/2\zeta\omega_n) - (\omega_n y_{i+1}/2\zeta)$ and

$$\dot{y}_{i+1} = -\frac{\omega_n y_{i+1}}{2\zeta} + \left(-\frac{\ddot{\alpha}_{i+1f}}{2\zeta\omega_n} - \frac{\partial \alpha_i}{\partial e_i}\dot{e}_i - \frac{\partial \alpha_i}{\partial \hat{S}_i}\dot{\hat{S}}_i - \frac{\partial \alpha_i}{\partial \hat{W}_i}\dot{\hat{W}}_i \right. \\ \left. - \frac{\partial \alpha_i}{\partial \bar{x}_i}\dot{\bar{x}}_i - \frac{\partial \alpha_i}{\partial y_i}\dot{y}_i \right), \\ \le -\frac{\omega_n y_{i+1}}{2\zeta} + B_{i+1}(\bar{e}_{i+1}, \bar{y}_{i+1}, \bar{\hat{S}}_i, \bar{\hat{W}}_i, y_d, \dot{y}_d, \ddot{y}_d), \tag{30}$$

where $\bar{e}_{i+1} = [e_1, \ldots, e_{i+1}]^\top, \bar{y}_{i+1} = [y_2, \ldots, y_{i+1}]^\top, \bar{\hat{S}}_i = [\hat{S}_1, \ldots, \hat{S}_i]^\top, \bar{\hat{W}}_i = [\hat{W}_1, \ldots, \hat{W}_i]^\top$, and $B_{i+1}(\cdot)$ is the introduced continuous function. From the strict proof in the later Section 4, we can obtain

$$0 < G_{i,m} \le G_i(\bar{x}_{i+1}, \omega_i) \le G_{i,M}, \tag{31}$$

$$0 \le |\Delta_i(\bar{x}_{i+1}, \omega_i)| \le C_i^*. \tag{32}$$

From (31), we can rewrite (23) as

$$\dot{e}_i = G_{i,m}(h_i(\chi_i,\omega_i) + x_{i+1} + G_{i,0}x_{i+1}) + \Delta_i + d_i, \quad (33)$$

where $G_{i,0} = G_i(\bar{x}_{i+1},\omega_i)/G_{i,m} - 1 > 0, h_i(\chi_i,\omega_i) = G_{i,m}^{-1}(f_i(\bar{x}_i, 0,\omega_i) - \dot{\alpha}_{if})$ with $\chi_i = [\bar{x}_i, \dot{\alpha}_{if}]^\top$.

Remark 3.1: Notice that traditional control method commonly design $h_i(\bar{x}_{i+1},\omega_i) = f_i(\bar{x}_i,0,\omega_i)G_i^{-1}(\bar{x}_{i+1},\omega_i)$. Unfortunately, $h_i(\bar{x}_{i+1},\omega_i)$ is a function with respect to x_{i+1}, and $h_i(\bar{x}_{i+1},\omega_i)$ is unknown and is approximated by NNs, thus the circular control construction problem will arise since x_{i+1} has to be chosen as an input of the NNs approximation, which is one part of the virtual control law α_i. To avoid this problem, we design $h_i(\chi_i,\omega_i) = G_{i,m}^{-1}(f_i(x_i,0,\omega_i) - \dot{\alpha}_{if})$ such that $h_i(\chi_i,\omega_i)$ can be independent of the state x_{i+1}. Furthermore, it will be proved that the coupling term $e_iG_{i,0}\alpha_i < 0$, which can be removed in the later controller design.

Construct the Lyapunov function candidate $V_{e_i} = \frac{1}{2}e_i^2$. Utilizing the FSE-RBFNNs-based approximator (18) to approximate the unknown function $h_i(\chi_i,\omega_i)$, it follows from (33) that the time derivatives of V_{e_i} is

$$\dot{V}_{e_i} \le e_iG_{i,m}(W_i^\top\psi_i(\chi_i,S_i^\top\phi_i) + x_{i+1} + G_{i,0}x_{i+1}) + |e_i|G_{i,m}\varepsilon_i^* + |e_i|C_i^* + |e_i|d_i^*. \quad (34)$$

Substituting (24), (25), and (31) into (34), and applying the inequality $|q| - q\tanh(\frac{q}{\upsilon}) \le 0.2785\upsilon$ for any $q \in R$ and $\forall\upsilon > 0$, we obtain

$$\begin{aligned}\dot{V}_{e_i} \le{}& e_iG_i(\bar{x}_{i+1},\omega_i)(e_{i+1}+y_{i+1}) - k_iG_{i,m}e_i^2 + e_iG_{i,m}G_{i,0}\alpha_i\\ &+ e_iG_{i,m}(W_i^\top\psi_i(\chi_i,S_i^\top\phi_i) - \hat{W}_i^\top\psi_i(\chi_i,\hat{S}_i^\top\phi_i))\\ &+ 0.2785G_{i,m}\upsilon_i + |e_i|d_i^* + |e_i|G_{i,m}\varepsilon_i^* + |e_i|C_i^*. \end{aligned} \quad (35)$$

From (24), (25), and (31), the following inequality holds

$$e_iG_{i,m}G_{i,0}\alpha_i = G_{i,m}G_{i,0}(-k_ie_i^2 - e_i\hat{W}_i^\top\psi_i(\chi_i,\hat{S}_i^\top\phi_i)\varpi_i) < 0. \quad (36)$$

Noting (30) and (37), we rewrite (35) as follows

$$\begin{aligned}\dot{V}_{e_i} \le{}& e_iG_i(\bar{x}_{i+1},\omega_i)(e_{i+1}+y_{i+1}) - k_iG_{i,m}e_i^2 + 0.2785G_{i,m}\upsilon_i\\ &+ e_iG_{i,m}(\tilde{W}_i^\top(\hat{\psi}_i - \hat{\psi}_i'\hat{S}_i^\top\phi_i) + \hat{W}_i^\top\hat{\psi}_i'\tilde{S}_i^\top\phi_i + z_i)\\ &+ |e_i|G_{i,m}\varepsilon_i^* + |e_i|C_i^* + |e_i|d_i^*, \end{aligned} \quad (37)$$

with z_i being bounded by

$$|z_i| \le z_i^* = \|S_i\|_F\,\|\phi_i\hat{W}_i^\top\hat{\psi}_i'\|_F + \|W_i\|\,\|\hat{\psi}_i'\hat{S}_i^\top\phi_i\| + |W_i|_1. \quad (38)$$

Consider the Lyapunov function as

$$V_i = V_{e_i} + \mathrm{tr}\left\{\frac{G_{i,m}\tilde{S}_i^\top\Gamma_{S_i}^{-1}\tilde{S}_i}{2}\right\} + \frac{G_{i,m}\tilde{W}_i^\top\Gamma_{W_i}^{-1}\tilde{W}_i}{2}. \quad (39)$$

From (37) and (38), the time derivative of V_i is

$$\begin{aligned}\dot{V}_i \le{}& -k_iG_{i,m}e_i^2\upsilon_i + e_iG_i(\bar{x}_{i+1},\omega_i)(e_{i+1}+y_{i+1}) + |e_i|G_{i,m}\theta_i^*\\ &+ \sigma_i\mathrm{tr}\{G_{i,m}\tilde{S}_i^\top\hat{S}_i\} + \sigma_iG_{i,m}\tilde{W}_i^\top\hat{W}_i + 0.2785G_{i,m}, \end{aligned} \quad (40)$$

where $\theta_i^* = z_i^* + \varepsilon_i^* + G_{i,m}^{-1}(C_i^* + d_i^*)$.

Step n: By using the analysis similar to the previous steps, the actual control law u and the adaptation laws for $\hat{S}_n$ and $\hat{W}_n$ are derived recursively as summarised in Table 1, where $k_n > 0, \sigma_n > 0, \upsilon_n > 0$ are design parameters, and $\Gamma_{S_n} = \Gamma_{S_n}^\top > 0, \Gamma_{W_n} = \Gamma_{W_n}^\top > 0$ are adaptive gain matrices.

Table 1. The actual control law and adaptation laws.

Actual Control Law
$u = -k_ne_n - \hat{W}_n^\top\psi_n(\chi_n,\hat{S}_n^\top\phi_n)\varpi_n, \varpi_n = \tanh\left(\frac{e_n\hat{W}_n^\top\psi_n(\chi_n,\hat{S}_n^\top\phi_n)}{\upsilon_n}\right)$, (41)
Adaptation Laws
$\dot{\hat{S}}_n = \Gamma_{S_n}[e_n\phi_n\hat{W}_n^\top\hat{\psi}_n' - \sigma_n\hat{S}_n], \dot{\hat{W}}_n = \Gamma_{W_n}[e_n(\hat{\psi}_n - \hat{\psi}_n'\hat{S}_n^\top\phi_n) - \sigma_n\hat{W}_n]$. (42)

4. bounds of compact sets

In this section, the bounds of the unknown functions $G_i(\bar{x}_{i+1},\omega_i)$ and $\Delta_i(\bar{x}_{i+1},\omega_i)$ are considered. It can be seen from Assumption 1 that $\underline{F}_i(\bar{x}_i,\omega_i), \overline{F}_i(\bar{x}_i,\omega_i), \underline{F}_i'(\bar{x}_i,\omega_i), \overline{F}_i'(\bar{x}_i,\omega_i), C_{i,1}(\bar{x}_i,\omega_i), C_{i,2}(\bar{x}_i,\omega_i), C_{i,3}(\bar{x}_i,\omega_i), C_{i,4}(\bar{x}_i,\omega_i)$, and $\kappa_{\Delta_i}(\bar{x}_i,\omega_i)$ are continuous functions. By using (24), (25), and (30), these functions can be expressed in new forms as follows

$$\underline{F}_i(\bar{x}_i,\omega_i) = \kappa_{\underline{F}_i}(\bar{e}_i,\bar{y}_i,\bar{\hat{S}}_{i-1},\bar{\hat{W}}_{i-1},y_d,\dot{y}_d,\omega_i), \quad (43)$$

$$\overline{F}_i(\bar{x}_i,\omega_i) = \kappa_{\overline{F}_i}(\bar{e}_i,\bar{y}_i,\bar{\hat{S}}_{i-1},\bar{\hat{W}}_{i-1},y_d,\dot{y}_d,\omega_i), \quad (44)$$

$$\underline{F}_i'(\bar{x}_i,\omega_i) = \kappa_{\underline{F}_i'}(\bar{e}_i,\bar{y}_i,\bar{\hat{S}}_{i-1},\bar{\hat{W}}_{i-1},y_d,\dot{y}_d,\omega_i), \quad (45)$$

$$\overline{F}_i'(\bar{x}_i,\omega_i) = \kappa_{\overline{F}_i'}(\bar{e}_i,\bar{y}_i,\bar{\hat{S}}_{i-1},\bar{\hat{W}}_{i-1},y_d,\dot{y}_d,\omega_i), \quad (46)$$

$$C_{i,1}(\bar{x}_i,\omega_i) = \kappa_{C_{i,1}}(\bar{e}_i,\bar{y}_i,\bar{\hat{S}}_{i-1},\bar{\hat{W}}_{i-1},y_d,\dot{y}_d,\omega_i), \quad (47)$$

$$C_{i,2}(\bar{x}_i,\omega_i) = \kappa_{C_{i,2}}(\bar{e}_i,\bar{y}_i,\bar{\hat{S}}_{i-1},\bar{\hat{W}}_{i-1},y_d,\dot{y}_d,\omega_i), \quad (48)$$

$$C_{i,3}(\bar{x}_i,\omega_i) = \kappa_{C_{i,3}}(\bar{e}_i,\bar{y}_i,\bar{\hat{S}}_{i-1},\bar{\hat{W}}_{i-1},y_d,\dot{y}_d,\omega_i), \quad (49)$$

$$C_{i,4}(\bar{x}_i,\omega_i) = \kappa_{C_{i,4}}(\bar{e}_i,\bar{y}_i,\bar{\hat{S}}_{i-1},\bar{\hat{W}}_{i-1},y_d,\dot{y}_d,\omega_i), \quad (50)$$

$$\kappa_{\Delta_i}(\bar{x}_i,\omega_i) = \kappa_{\Delta_i}(\bar{e}_i,\bar{y}_i,\bar{\hat{S}}_{i-1},\bar{\hat{W}}_{i-1},y_d,\dot{y}_d,\omega_i), \quad (51)$$

where $\kappa_{\underline{F}_i}(\cdot), \kappa_{\overline{F}_i}(\cdot), \kappa_{\underline{F}_i'}(\cdot), \kappa_{\overline{F}_i'}(\cdot), \kappa_{C_{i,1}}(\cdot), \kappa_{C_{i,2}}(\cdot), \kappa_{C_{i,3}}(\cdot), \kappa_{C_{i,4}}(\cdot)$, and $\kappa_{\Delta_i}(\cdot)$ are continuous functions. Define compact sets $\Pi_i (i = 1,\ldots,n)$ as follows

$$\Pi_i := \left\{(\bar{e}_i,\bar{y}_i,\bar{\hat{S}}_{i-1},\bar{\hat{W}}_{i-1}) \middle| \sum_{j=1}^{i} e_j^2 + \sum_{j=2}^{i} y_j^2 + \sum_{j=1}^{i-1}(G_{j,m}\tilde{W}_j^\top\Gamma_{W_j}^{-1}\tilde{W}_j + tr\{G_{j,m}\tilde{S}_j^\top\Gamma_{S_j}^{-1}\tilde{S}_j\}) \le 2\xi\right\}. \quad (52)$$

From (52), Assumption 2.2, and the definition of ω_i, it can be seen that all the variables of $\kappa_{\underline{F}_i}(\cdot), \kappa_{\overline{F}_i}(\cdot), \kappa_{\underline{F}_i'}(\cdot), \kappa_{\overline{F}_i'}(\cdot), \kappa_{C_{i,1}}(\cdot), \kappa_{C_{i,2}}(\cdot), \kappa_{C_{i,3}}(\cdot), \kappa_{C_{i,4}}(\cdot)$, and $\kappa_{\Delta_i}(\cdot)$ are included in the compact set $\Pi_i \times \Pi_0 \times \Pi_\omega$. Therefore, these functions have maximums and minimums on $\Pi_i \times \Pi_0 \times \Pi_\omega$, namely

$$\underline{F}_{i,m} \le \underline{F}_i(\bar{x}_i,\omega_i) \le \underline{F}_{i,M}, \quad (53)$$

$$\overline{F}_{i,m} \le \overline{F}_i(\bar{x}_i,\omega_i) \le \overline{F}_{i,M}, \tag{54}$$
$$\underline{F}'_{i,m} \le \underline{F}'_i(\bar{x}_i,\omega_i) \le \underline{F}'_{i,M}, \tag{55}$$
$$\overline{F}'_{i,m} \le \overline{F}'_i(\bar{x}_i,\omega_i) \le \overline{F}'_{i,M}, \tag{56}$$
$$|C_{i,1}(\bar{x}_i,\omega_i)| \le C_{i,1M}, \tag{57}$$
$$|C_{i,2}(\bar{x}_i,\omega_i)| \le C_{i,2M}, \tag{58}$$
$$|C_{i,3}(\bar{x}_i,\omega_i)| \le C_{i,3M}, \tag{59}$$
$$|C_{i,4}(\bar{x}_i,\omega_i)| \le C_{i,4M}, \tag{60}$$
$$|\kappa_{\Delta_i}(\bar{x}_i,\omega_i)| \le C_{\Delta_i}, \tag{61}$$

on $\Pi_i \times \Pi_0 \times \Pi_\omega$, where $\underline{F}_{i,m}, \underline{F}_{i,M}, \overline{F}_{i,m}, \overline{F}_{i,M}, \underline{F}'_{i,m}, \underline{F}'_{i,M}, \overline{F}'_{i,m}, \overline{F}'_{i,M}, C_{i,1M}, C_{i,2M}, C_{i,3M}, C_{i,4M}$, and C_{Δ_i} are unknown positive constants.

From (13) and (14) we can further obtain

$$0 < G_{i,m} \le G_i(\bar{x}_{i+1},\omega_i) \le G_{i,M}, \tag{62}$$
$$0 \le |\Delta_i(\bar{x}_{i+1},\omega_i)| \le C_i^*, \tag{63}$$

where $G_{i,m} = \min\{\underline{F}_{i,m}, \overline{F}_{i,m}, \underline{F}'_{i,m}, \overline{F}'_{i,m}\}$, $G_{i,M} = \max\{\underline{F}_{i,M}, \overline{F}_{i,M}, \underline{F}'_{i,M}, \overline{F}'_{i,M}\}$, and $C_i^* = \max\{C_{i,1M}, C_{i,2M}, C_{i,3M}, C_{i,4M}, C_{\Delta_i}\}$.

5. Stability analysis

We are now in a position to state our main result.

Theorem 5.1: *Consider the class of non-affine pure-feedback nonlinear system (1) under Assumptions 2.1–2.3. The intermediate virtual control law is constructed as (24), the actual control law is constructed as (41) with the adaptation laws given by (26), (27), and (42). Furthermore, for initial conditions satisfying $\hat{S}_i(0) \ge 0$, $\hat{W}_i(0) \ge 0$ and $V(0) \le \xi$ with ξ being any given positive constant, then, there exist $k_i, \sigma_i, \upsilon_i$, and τ_i such that:*

(1) $V(t) \le \xi$ for $\forall t > 0$, and all of the signals in the closed-loop system are semi-globally uniformly ultimately bounded;
(2) The tracking error $e_1 = x_1 - y_d$ will converge to an arbitrarily small neighbourhood by appropriately choosing design parameters.

Proof: Consider the Lyapunov function as follows

$$V = \sum_{i=1}^{n} V_i + \frac{1}{2}\sum_{i=1}^{n-1} y_{i+1}^2. \tag{64}$$

Using (40) and (64), the time derivative of V is

$$\begin{aligned}\dot{V} \le &\sum_{i=1}^{n}(-k_i G_{i,m} e_i^2 + 0.2785 G_{i,m}\upsilon_i + |e_i| G_{i,m}\theta_i^*)\\ &+ \sum_{i=1}^{n}(\sigma_i tr\{G_{i,m}\tilde{S}_i^\top \hat{S}_i\} + \sigma_i G_{i,m}\tilde{W}_i^\top \hat{W}_i)\\ &+ \sum_{i=1}^{n-1}(e_i G_i(\bar{x}_{i+1},\omega_i)(e_{i+1}+y_{i+1}))\\ &+ \sum_{i=1}^{n-1}(|y_{i+1}B_{i+1}(\cdot)| - \omega_n y_{i+1}^2/2\zeta).\end{aligned} \tag{65}$$

It can be seen from (30) that $B_{i+1}(\cdot)$ is a continuous function of variables $e_1 \ldots e_{i+1}, y_2 \ldots y_{i+1}, \hat{\bar{S}}_i, \hat{\bar{W}}_i, y_d, \dot{y}_d$, and $\ddot{y}_d$, thus, all the variables of $B_{i+1}(\cdot)$ are included in the compact set $\Pi_{i+1} \times \Pi_0 \times \Pi_\omega$. Consequently, there exists a maximum M_{i+1} such that

$$|B_{i+1}| \le M_{i+1}, \tag{66}$$

on $\Pi_{i+1} \times \Pi_0 \times \Pi_\omega$.

Invoking (66) and Young's inequality yields

$$\begin{aligned}\dot{V} \le &\sum_{i=1}^{n}\left(\frac{1}{2}G_{i,m}\sigma_i(\|W_i\|^2 - \|\tilde{W}_i\|^2 + \|S_i\|_F^2 - \|\tilde{S}_i\|_F^2)\right)\\ &+ \sum_{i=1}^{n}\left(-k_i G_{i,m} e_i^2 + 0.2785 G_{i,m}\upsilon_i + \frac{G_{i,m}^2 e_i^2}{2c_3} + \frac{c_3\theta_i^{*2}}{2}\right)\\ &+ \sum_{i=1}^{n-1}\left(\frac{c_2 G_{i,M}^2 y_{i+1}^2}{2} + \frac{e_i^2}{2c_2} + \frac{e_i^2}{2} + \frac{G_{i,M}^2 e_{i+1}^2}{2}\right)\\ &+ \sum_{i=1}^{n-1}\left(\frac{y_{i+1}^2 M_{i+1}^2}{2c_1} + \frac{c_1}{2} - \frac{\omega_n y_{i+1}^2}{2\zeta}\right),\end{aligned} \tag{67}$$

where c_1, c_2, and c_3 are positive constants.

Then, by defining $C_1 = \sum_{i=1}^{n}(\frac{1}{2}G_{i,m}\sigma_i(\|W_i\|^2 + \|S_i\|_F^2) + 0.2785 G_{i,m}\upsilon_i + \frac{c_3\theta_i^{*2}}{2}) + (n-1)\frac{c_1}{2}$, we can further have

$$\begin{aligned}\dot{V} \le &-\left(k_1 G_{1,m} - \frac{1}{2} - \frac{1}{2c_2} - \frac{G_{1,m}^2}{2c_3}\right)e_1^2\\ &- \sum_{i=2}^{n-1}\left(k_i G_{i,m} - \frac{1}{2} - \frac{1}{2c_2} - \frac{G_{i-1,M}^2}{2} - \frac{G_{i,m}^2}{2c_3}\right)e_i^2\\ &- \left(k_n G_{n,m} - \frac{G_{n-1,M}^2}{2} - \frac{G_{n,m}^2}{2c_3}\right)e_n^2\\ &- \sum_{i=1}^{n}\left(\frac{\sigma_i}{\lambda_{\max}(\Gamma_{S_i}^{-1})} tr\left\{\frac{G_{i,m}\tilde{S}_i^\top \Gamma_{S_i}^{-1}\tilde{S}_i}{2}\right\}\right)\\ &- \sum_{i=1}^{n-1}\left(\frac{\omega_n}{2\zeta} - \frac{M_{i+1}^2}{2c_1} - \frac{c_2 G_{i,M}^2}{2}\right)y_{i+1}^2\\ &- \sum_{i=1}^{n}\left(\frac{\sigma_i}{\lambda_{\max}(\Gamma_{W_i}^{-1})}\frac{G_{i,m}\tilde{W}_i^\top \Gamma_{W_i}^{-1}\tilde{W}_i}{2}\right) + C_1.\end{aligned} \tag{68}$$

Choose $k_i \ge G_{i,m}^{-1}(\frac{1}{2} + \frac{1}{2c_2} + \frac{G_{i-1,M}^2}{2} + \frac{G_{i,m}^2}{2c_3} + C_2)(i = 2, \ldots, n-1)$, $k_1 \ge G_{1,m}^{-1}(\frac{1}{2} + \frac{1}{2c_2} + \frac{G_{1,m}^2}{2c_3} + C_2)$, $k_n \ge G_{n,m}^{-1}(\frac{G_{n-1,M}^2}{2} + \frac{G_{n,m}^2}{2c_3} + C_2)$, and $\frac{\omega_n}{2\zeta} \ge \frac{M_{i+1}^2}{2c_1} + \frac{c_2 G_{i,M}^2}{2} + C_2 (i = 1, \ldots, n-1)$, where $C_2 = \min\{\frac{\sigma_1}{\lambda_{\max}(\Gamma_{S_1}^{-1})}, \ldots, \frac{\sigma_n}{\lambda_{\max}(\Gamma_{S_n}^{-1})}, \frac{\sigma_1}{\lambda_{\max}(\Gamma_{W_1}^{-1})}, \ldots,$

$\frac{\sigma_n}{\lambda_{\max}(\Gamma_{W_n}^{-1})}\}$. Thus, we have

$$\dot{V} \leq -C_2 V + C_1, \tag{69}$$

which implies

$$V(t) \leq (V(0) - C_3)e^{-C_2 t} + C_3 \leq V(0) + C_3, \tag{70}$$

where $C_3 = C_1/C_2$ can be made arbitrarily small by decreasing $\lambda_{\max}(\Gamma_{S_i}^{-1}), \lambda_{\max}(\Gamma_{W_i}^{-1}), \sigma_i$, and υ_i, meanwhile increasing k_i. Thus we can have $C_1/C_2 \leq \xi$ by choosing the appropriate design parameters.

Above stability analysis was given based on the condition that all the state variables must stay inside of the set $\Pi_n \times \Pi_0 \times \Pi_\omega$ since $\Pi_n \subset \Pi_{n-1} \times R^4 \subset \cdots \subset \Pi_2 \times R^{4(n-2)} \subset \Pi_1 \times R^{4(n-1)}$. From Theorem 1 we have initial condition $V(0) \leq \xi$, which means that the initial conditions of all the state variables are assumed to be in the set $\Pi_n \times \Pi_0 \times \Pi_\omega$. Then, According to $C_1/C_2 \leq \xi$ and (69), we have $\dot{V} \leq 0$ on $V = \xi$. Therefore, we have $V(t) \leq \xi$ for $\forall t > 0$, namely, $\Pi_n \times \Pi_0 \times \Pi_\omega$ is an invariant set. Hence all the variables will stay inside of the set $\Pi_n \times \Pi_0 \times \Pi_\omega$ and the property 1) of Theorem 1 is proved.

On the other hand, from (39) and (64), we have $\sum_{i=1}^{n} e_i^2/2 \leq V$. Using the first inequality of (70) and noting $\sum_{i=1}^{n} e_i^2/2 \leq V$, the tracking error e_1 satisfies

$$\overline{\lim_{t\to\infty}} |e_1| \leq \lim_{t\to\infty} \sqrt{2V(t)} \leq \sqrt{2C_3}. \tag{71}$$

Note that the size of C_3 can be adjusted to arbitrarily small by decreasing $\lambda_{\max}(\Gamma_{S_i}^{-1}), \lambda_{\max}(\Gamma_{W_i}^{-1}), \sigma_i$, and υ_i, meanwhile increasing k_i. Thus, by appropriately online-tuning the design parameters, the tracking error e_1 can be regulated to a neighbourhood of the origin as small as desired and property 2) of Theorem 1 is proved. This completes the proof. ■

Remark 5.1: In Assumption 2.1, the unknown continuous functions $\underline{F}_i(\cdot), \overline{F}_i(\cdot), \underline{F}_i'(\cdot), \overline{F}_i'(\cdot), C_{i,1}(\cdot), C_{i,2}(\cdot), C_{i,3}(\cdot)$, and $C_{i,4}(\cdot)$ are unbounded and cannot be applied in the control design directly, which makes the control design difficult or even impossible. To handle this difficulty, we skillfully introduce $\kappa_{\underline{F}_i}(\cdot), \kappa_{\overline{F}_i}(\cdot), \kappa_{\underline{F}_i'}(\cdot), \kappa_{\overline{F}_i'}(\cdot), \kappa_{C_{i,1}}(\cdot), \kappa_{C_{i,2}}(\cdot), \kappa_{C_{i,3}}(\cdot), \kappa_{C_{i,4}}(\cdot)$, and $\kappa_{\Delta_i}(\cdot)$ as shown in (43)–51. All of these introduced functions have bounds on $\Pi_i \times \Pi_0 \times \Pi_\omega$, and we utilise these bounds to design the controller and robust compensators. It should be pointed out that (43)–51 are only satisfied on $\Pi_i \times \Pi_0 \times \Pi_\omega$, however, Assumption 1 is for all the condition. Therefore, it is not necessary to assume the unknown continuous functions are bounded by known lower and upper bounds.

Remark 5.2: In contrast to most previous studies investigating the periodic disturbances (Chen, 2009; Chen & Jiao, 2010; Chen et al., 2010; Ding, 2007; Tian & Yu, 2003; Xu, 2004), we have considered a more general case that all the control inputs and periodic disturbances appear implicitly in the system functions, which can accommodate more general classes of nonlinear systems. In addition, different from the previous works aiming at pure-feedback systems Z. Liu et al. (2016, 2018), we have developed a much more general model in which the gain functions can be positive and continue, facilitating the control design and engineering implementation. On this basis, the difficulty in dealing with the non-affine appearances of control variables or inputs is tackled and the restrictive differential conditions on the non-affine nonlinear functions are removed.

6. Simulation

In this section, comparative simulations are carried out between the proposed FSE-RBFNNs-based control scheme (PCS) and the conventional RBFNNs-based control scheme (CCS) (Z. Liu et al., 2016), whose specific expressions are in Table 2.

Example 6.1: Consider the following pure-feedback nonlinear system:

$$\begin{cases} \dot{x}_1 = f_1(x_1, x_2, \omega_1), \\ \dot{x}_2 = f_2(x, u, \omega_2), \\ y = x_1, \end{cases} \tag{72}$$

where the non-affine functions $f_1(x_1, x_2, \omega_1) = \frac{1-e^{-x_1}}{1+e^{-x_1}} + x_2^3 + x_2 e^{-1-x_1^2} + \frac{x_1^2\omega_1^2 + x_1\omega_1}{x_1^2\omega_1^2+1}$ and $f_2(x, u, \omega_2) = x_1^2 + 0.1(1 + x_2^2)u + (x_1^2 + x_2^2)u^3 + \sin(0.1u) + \sin(x_1 x_2 \omega_2)e^{-(x_1^2 x_2^2 \omega_2^2)}$, the unknown time-varying disturbances $\omega_1(t) = |\sin(0.5t)|$ and $\omega_2(t) = |\cos t|$ with known periods $T_1 = 2\pi$ and $T_2 = \pi$, respectively.

The reference model is taken as the following van der Pol oscillator

$$\begin{cases} \dot{x}_{d1} = x_{d2}, \\ \dot{x}_{d2} = -x_{d1} + \beta(1 - x_{d1}^2)x_{d2}, \\ y_d = x_{d1}, \end{cases} \tag{73}$$

which yields a limit cycle trajectory when $\beta > 0 (\beta = 0.2$ in this simulation) for initial values $[x_{d1}(0), x_{d2}(0)]^\top = [2.5, 2.5]^\top$. The control objective is to design an adaptive neural control scheme such that all the signals in the closed-loop control system are proven to be bounded, and the system output y follows the reference trajectory y_d.

In simulation, the design parameters are chosen as $\sigma_1 = \sigma_2 = 0.5, \Gamma_{W_1} = \text{diag}\{0.1\}, \Gamma_{W_2} = \Gamma_{S_1} = \Gamma_{S_2} = \text{diag}\{0.2\}, \upsilon_1 = \upsilon_2 = 0.75, \omega_n = 10, \zeta = 1$, and $k_1 = k_2 = 1$. We choose the numbers of FSE components as $q_1 = q_2 = 5$ and the numbers of NNs nodes as $p_1 = 11^3$ and $p_2 = 11^4$. The centres of radial basis functions (RBFs) evenly cover the compact sets $[-10, 10] \times [-10, 10]$ and $[-10, 10] \times [-10, 10] \times [-10, 10]$,

Table 2. The control structures of CCS.

Control Laws of CCS
$\alpha_1 = -k_1 e_1 - \frac{\hat{\theta}_1 e_1}{2a_1^2}\psi^\top(x_1)\psi(x_1) - \hat{\delta}_1 \tanh\left(\frac{e_1}{\upsilon_1}\right) - \zeta_1 \dot{y}_d \tanh\left(\frac{e_1 \dot{y}_d}{\upsilon_1}\right)$,
$u = -k_2 e_2 - \frac{\hat{\theta}_2 e_2}{2a_2^2}\psi^\top(x)\psi(x) - \hat{\delta}_2 \tanh\left(\frac{e_2}{\upsilon_2}\right) - \zeta_2 \dot{\alpha}_{2f} \tanh\left(\frac{e_2 \dot{\alpha}_{2f}}{\upsilon_2}\right)$,
where $k_i, a_i, \upsilon_i (i = 1, 2)$ are positive design parameters.
Adaptation Laws of CCS
$\dot{\hat{\delta}}_1 = \gamma_1 e_1 \tanh\left(\frac{e_1}{\upsilon_1}\right) - \sigma_1\gamma_1\hat{\delta}_1, \quad \dot{\hat{\theta}}_1 = \frac{\beta_1 e_1^2}{2a_1^2}\psi^\top(x_1)\psi(x_1) - \sigma_1\beta_1\hat{\theta}_1$,
$\dot{\hat{\delta}}_2 = \gamma_2 e_2 \tanh\left(\frac{e_2}{\upsilon_2}\right) - \sigma_2\gamma_2\hat{\delta}_2, \quad \dot{\hat{\theta}}_2 = \frac{\beta_2 e_2^2}{2a_2^2}\psi^\top(x)\psi(x) - \sigma_2\beta_2\hat{\theta}_2$,
where $\gamma_i, \upsilon_i, \sigma_i, \beta_i, a_i (i = 1, 2)$ are positive design parameters.

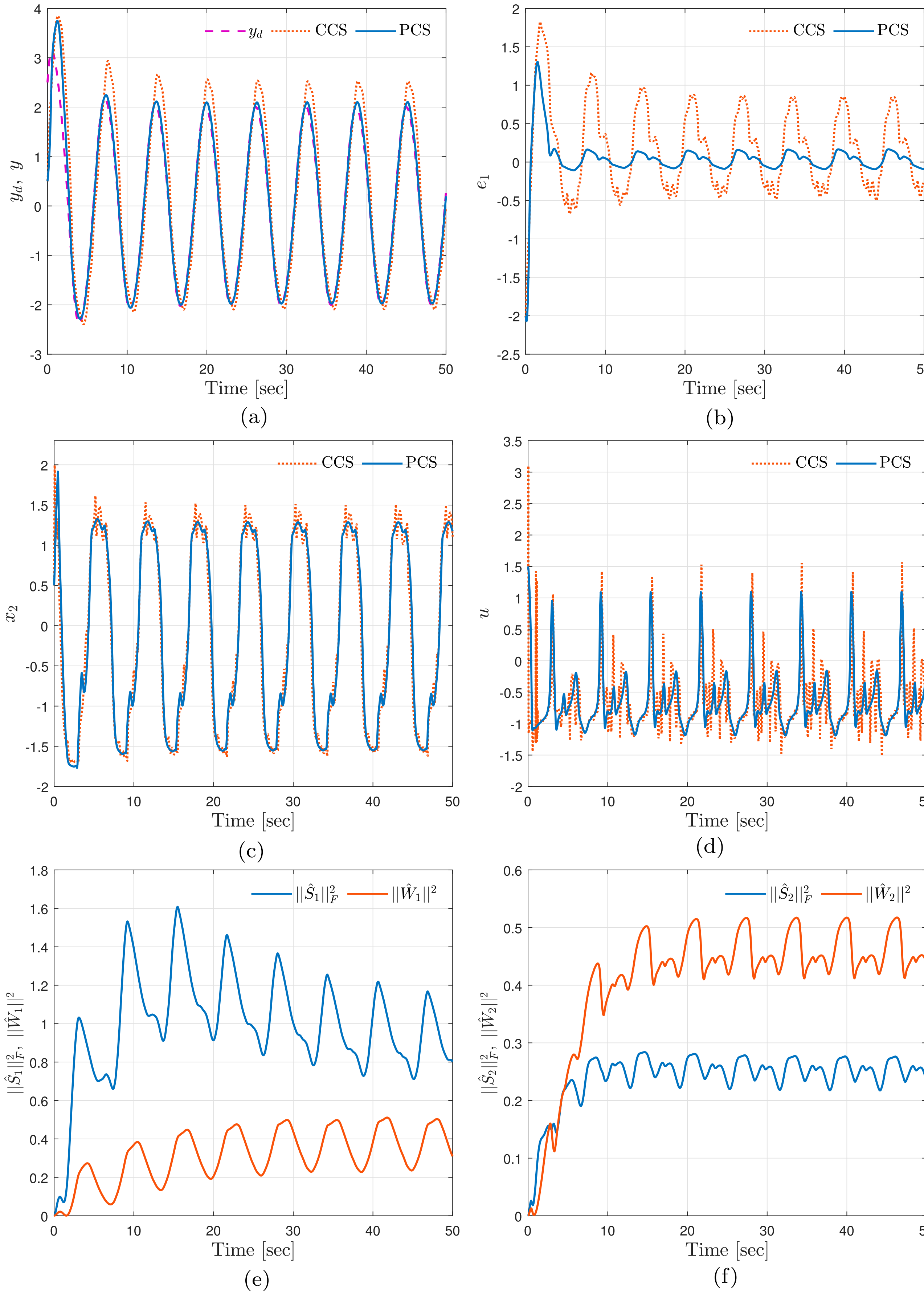

Figure 1. The simulation results of Example 1. (a) Reference trajectory y_d and system output y; (b) Output tracking error e_1; (c) System state x_2; (d) Actual control law u; (e) Adaptive parameters $\|\hat{S}_1\|_F^2$ and $\|\hat{W}_1\|^2$ and (f) Adaptive parameters $\|\hat{S}_2\|_F^2$ and $\|\hat{W}_2\|^2$.

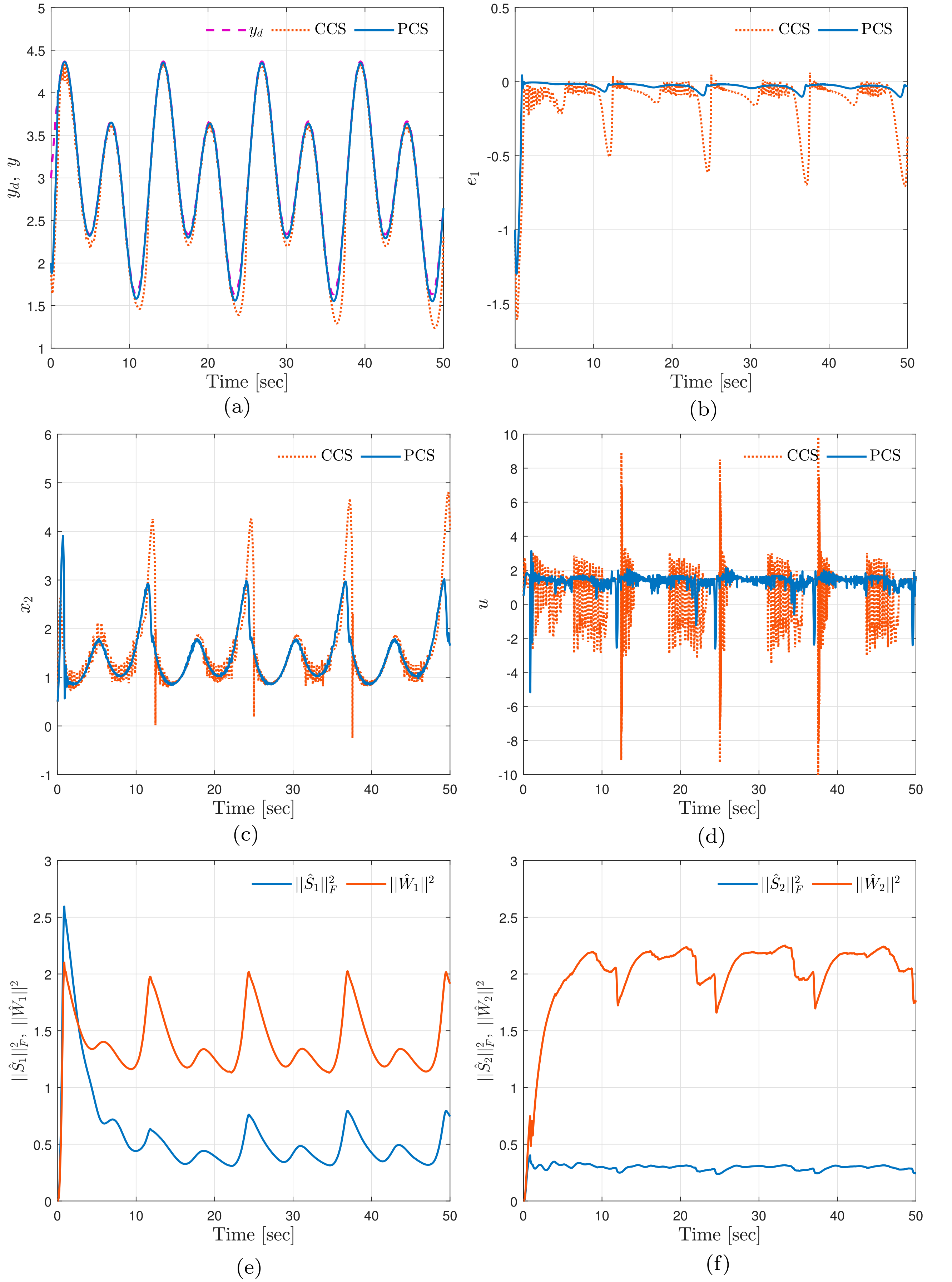

Figure 2. The simulation results of Example 2. (a) Reference trajectory y_d and system output y, (b) Output tracking error e_1, (c) System state x_2, (d) Actual control law u, (e) Adaptive parameters $\|\hat{S}_1\|_F^2$ and $\|\hat{W}_1\|^2$ and (f) Adaptive parameters $\|\hat{S}_2\|_F^2$ and $\|\hat{W}_2\|^2$.

and the widths of RBFs are set to be $\kappa_1 = 10$ and $\kappa_2 = 5$. Set the initial values as: $[x_1(0), x_2(0)]^\top = [0.5, 0.5]^\top, \hat{S}_1(0) = \hat{S}_2(0) = 0$ and $\hat{W}_1(0) = \hat{W}_2(0) = 0$. For fair comparison, the design parameters in CCS are chosen to be the same as those in PCS, that is, $k_i = 1, a_i = 1, \upsilon_i = 0.75, \gamma_i = 0.2, \sigma_i = 0.5, \beta_1 = 0.1, \beta_2 = 0.2 (i = 1, 2)$.

The simulation results are shown in Figure 2. Figure 2(a,b) reveal that, PCS provides better transient and steady tracking performance in comparison with CCS, and there is no high-frequency chattering in the system output y obtained using PCS. It can be observed from Figure 2(c,d) that if the pure-feedback nonlinear system (72) is subject to the unknown time-varying disturbances $\omega_1(t)$ and $\omega_2(t)$, the system state x_2 and actual control law u obtained using PCS are smoother than the ones of CCS. Figure 2(e,f) show the boundedness of the adaptive parameters $\|\hat{S}_1\|_F^2$, $\|\hat{W}_1\|^2$, $\|\hat{S}_2\|_F^2$, and $\|\hat{W}_2\|^2$.

Example 6.2: To further show the applicability of the proposed adaptive neural controller, consider the following Brusselator model in dimensionless form (Ge & Wang, 2002).

$$\begin{cases} \dot{x}_1 = C - (D+1)x_1 + x_1^2 x_2 + d_1(x_1, x_2, \omega_1), \\ \dot{x}_2 = Dx_1 - x_1^2 x_2 + (2 + \cos(x_1))u + d_2(x, u, \omega_2), \\ y = x_1, \end{cases} \tag{74}$$

where x_1 and x_2 denote the concentrations of the reaction intermediates, C, $D > 0$ are parameters which describe the supply of reservoir chemicals. $d_1(x_1, x_2, \omega_1)$ and $d_2(x, u, \omega_2)$ are the external disturbance terms. It is assumed that $x_1 \neq 0$ as in Ge and Wang (2002). In this simulation, choose $d_1(x_1, x_2, \omega_1) = 0.1\cos(x_1\omega_1)x_2 + 0.2\sin t, \omega_1(t) = |\cos(0.5t)|, \omega_2(t) = |\cos(0.25t)|$, and $d_2(x, u, \omega_2)$ is chosen as follows

$$d_2(x, u, \omega_2) = \begin{cases} 0.1\omega_2^2 \sin^2(x_1x_2) + u + \frac{u^3}{7}, & u \geq 1.5, \\ 0.1\omega_2^2 \sin^2(x_1x_2), & -2.5 < u < 1.5, \\ 0.1\omega_2^2 \sin^2(x_1x_2) + u + \frac{u^3}{7}, & u \leq -2.5. \end{cases} \tag{75}$$

It can be seen that function $d_2(x, u, \omega_2)$ is non-differentiable with respect to u as shown in (75).

The design parameters are chosen as $\sigma_1 = \sigma_2 = 0.3, \upsilon_1 = \upsilon_2 = 0.25, k_1 = k_2 = 1, \Gamma_{W_1} = \Gamma_{W_2} = \Gamma_{S_1} = \Gamma_{S_2} = \text{diag}\{0.5\}$, $\omega_n = 10, \zeta = 1, C = 1$, and $D = 3$. We choose the numbers of FSE components as $q_1 = q_2 = 5$ and the numbers of NNs nodes as $p_1 = 11^3$ and $p_2 = 11^4$. The centres of radial basis functions (RBFs) evenly cover the compact sets $[-10, 10] \times [-10, 10]$ and $[-10, 10] \times [-10, 10] \times [-10, 10]$, and the widths of RBFs are set to be $\kappa_1 = 10$ and $\kappa_2 = 20$. Set the initial values as: $[x_1(0), x_2(0)]^\top = [2, 0.5]^\top, \hat{S}_1(0) = \hat{S}_2(0) = 0$, and $\hat{W}_1(0) = \hat{W}_2(0) = 0$. The desired reference trajectory $y_d = 3 + \sin t + 0.5\sin(0.5t)$. In addition, the design parameters in CCS are chosen as $k_i = 1, a_i = 1, \upsilon_i = 0.25, \gamma_i = 0.5, \sigma_i = 0.3, \beta_i = 0.5 (i = 1, 2)$. The simulation results are shown in Figure 2. It can be seen that PCS provides better transient and steady state performances in contrast to CCS, and PCS is effective in fast suppressing the unknown time-varying disturbances due to the introduction of FSE-RBFNNs-based approximators.

7. Conclusion

A novel and effective control approach has been presented for non-affine pure-feedback system with non-differentiable non-affine functions affected by the periodic disturbances. All the bounds of non-affine functions are unknown functions, therefore, some important assumptions, such as known signs of control direction and bounded gain functions are cancelled. Moreover, the DSC technique has been utilised for handling with the problem of 'explosion of complexity'. Finally, it is proven that all the variables will stay in these introduced compact sets by choosing the appropriate design parameters, and the system stability is therefore achieved. Future research will be concentrated on the sampled-data control for multi-agent systems in pure-feedback from (Luo et al., 2020; Lv et al., 2020; Shi et al., 2020; Xie et al., 2020).

Disclosure statement

No potential conflict of interest was reported by the author(s).

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